MATEMATIKA III 2 UNI

Formule

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Skeniranje	

UREJANJE DOKUMENTA	
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OPOMBE

Formule za Matematiko III *

Diferencialna geometrija	
Parametrična enačba krivulje	$\vec{r} = (x(t), y(t), z(t))$
smer tangente	$\dot{\vec{r}} = (\dot{x}(t),\ \dot{y}(t),\ \dot{z}(t))$
dolžina loka	$s = \int_{t_0}^{t} \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, dt$
	$\mathrm{d}s = \sqrt{\dot{\vec{r}}(t) \cdot \dot{\vec{r}}(t)}$
Parametrična enačba ploskve	$\vec{r} = (x(u,v),\ y(u,v),\ z(u,v))$
smer normale	$\vec{\nu} = \vec{r}_u \times \vec{r}_v$
Eksplicitna enačba ploskve	$z = f(x,y)$
smer normale	$\vec{\nu} = (\frac{\partial z}{\partial x},\ \frac{\partial z}{\partial y},\ -1)$
Implicitna enačba ploskve	$F(x,y,z) = 0$
smer normale	$\vec{\nu} = (\frac{\partial F}{\partial x},\ \frac{\partial F}{\partial y},\ \frac{\partial F}{\partial z})$
Enačba tangentne premice	$\frac{x-T_x}{\dot{r}_x} = \frac{y-T_y}{\dot{r}_y} = \frac{z-T_z}{\dot{r}_z}$
Premica skozi 2 točki	$\vec{r} = (A_x, A_y, A_z) + t(B_x - A_x, B_y - A_y, B_z - A_z)$
Enačba tangentne ravnine	$\vec{n} \cdot \vec{r}_t = d$

Integrali s parametrom
$F(x) = \int_{u(x)}^{v(x)} f(x,y)\mathrm{d}y$
$F'(x) = \int_{u(x)}^{v(x)} \frac{\partial f}{\partial x}\mathrm{d}y + f(x, v(x))v'(x) - f(x, u(x))u'(x)$

Dvojni integral		
polarne koordinate	$x = r\cos\varphi$	$0 \le \varphi \le 2\pi$
	$y = r\sin\varphi$	$r \ge 0$
	Jacobijeva det. $J(r,\varphi) = r$	
ploščina lika D	$pl = \iint_D dxdy$	
volumen telesa s streho $z = f(x,y)$ projekcijo D	$V = \iint_D f(x,y)\mathrm{d}x\mathrm{d}y$	
površina ploskve $z = f(x,y)$ s projekcijo D	$P = \iint_D \sqrt{1 + (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}\mathrm{d}x\mathrm{d}y$	

Trojni integral		
Cilindrične koordinate	$x = r\cos\varphi$	$0 \le \varphi \le 2\pi$
	$y = r\sin\varphi$	$r \ge 0$
Jacobijeva det. $= r$	$z = z$	$-\infty \le z \le \infty$
Sferične koordinate	$x = r\cos\varphi\cos\vartheta$	$0 \le \varphi \le 2\pi$
	$y = r\sin\varphi\cos\vartheta$	$-\frac{\pi}{2} \le \vartheta \le \frac{\pi}{2}$
Jacobijeva det.$= r^2\cos\vartheta$	$z = r\sin\vartheta$	$r \ge 0$
Prostornina območja V	$\iiint_V \mathrm{d}x\mathrm{d}y\mathrm{d}z$	
Težišče	$x_t = \frac{\iiint x\rho\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\iiint \rho\mathrm{d}x\mathrm{d}y\mathrm{d}z}$	
	$y_t = \frac{\iiint y\rho\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\iiint \rho\mathrm{d}x\mathrm{d}y\mathrm{d}z}$	
	$z_t = \frac{\iiint z\rho\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\iiint \rho\mathrm{d}x\mathrm{d}y\mathrm{d}z}$	

*To delo je oblikovano s programskim paketom LaTeXv operacijskem sistemu Linux. Verzija 0.3 popravljena izdaja.

Teorija polja	
$\vec{r} = (x, y, z)$ $r = \sqrt{x^2 + y^2 + z^2}$ $\mathrm{grad}\, u = (\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z})$ smerni odvod skal. polja, $F(T)$ nivojska ploskev $\mathrm{grad}\, f(r) = \frac{f'}{r}\vec{r}$ $\frac{\partial u}{\partial l} = \mathrm{grad}\, u \frac{\vec{l}}{\lvert\vec{l}\rvert}$	$\mathrm{div}(P, Q, R) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$ $\mathrm{rot}\, \vec{V} = det = (R_y - Q_z, P_z - R_x, Q_x - P_y)$ Če $\mathrm{rot}\, \vec{V} = \vec{0}$ potem je $\int_C$ neodvisen od poti $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$
Krivuljni in ploskovni integral	
$\int_C f(x, y, z)\, \mathrm{d}s = \int_{t_A}^{t_B} f(x(t), y(t), z(t))\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}\, \mathrm{d}t$ $\int_C P\, \mathrm{d}x + Q\, \mathrm{d}y + R\, \mathrm{d}z = \pm \int_{t_A}^{t_B} (P\dot{x} + Q\dot{y} + R\dot{z})\, \mathrm{d}t$ $C : \vec{r} = (x(t), y(t), z(t))\ ,\ t_A \le t \le t_B$	
$\iint_S f(x, y, z)\, \mathrm{d}S = \iint_D f(x(u, v), y(u, v), z(u, v))\sqrt{EG - F^2}\, \mathrm{d}u\mathrm{d}v$ $\iint_S P\, \mathrm{d}y\mathrm{d}z + Q\, \mathrm{d}z\mathrm{d}x + R\, \mathrm{d}x\mathrm{d}y = \iint_S \vec{V} \cdot \vec{\nu}\, \mathrm{d}S = \pm \iint_D (\vec{V}, \vec{r}_u, \vec{r}_v)\, \mathrm{d}u\mathrm{d}v$ $S : \vec{r} = (x(u, v), y(u, v), z(u, v))\ ,\ (u, v) \in D$ $E = \vec{r}_u \cdot \vec{r}_u \quad F = \vec{r}_u \cdot \vec{r}_v \quad G = \vec{r}_v \cdot \vec{r}_v$	
Stokesova formula Gaussova formula(`\oint`) pretok vektorskega polja, ploskovni 2. vrste Greenova formula	$\oint_{\partial S} \vec{V} \cdot \mathrm{d}\vec{r} = \iint_S \mathrm{rot}\vec{V} \cdot \vec{\nu}\, \mathrm{d}S$ $\iint_{\partial V} \vec{V} \cdot \vec{\nu}\, \mathrm{d}S = \iiint_V \mathrm{div}\, \vec{V}\, \mathrm{d}x\mathrm{d}y\mathrm{d}z$ $\oint_{\partial D} P\, \mathrm{d}x + Q\, \mathrm{d}y = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\, \mathrm{d}x\mathrm{d}y$
Funkcije kompleskne spremenljivke	
$z = x + iy \quad \bar{z} = x - iy$ $e^z = e^x(\cos y + i \sin y)$ $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$ $\mathrm{sh}\, z = \frac{e^z - e^{-z}}{2}$	$w = f(z) = u(x, y) + iv(x, y)$ $\ln z = \ln \lvert z\rvert + i(\varphi + 2k\pi)$ $\cos z = \frac{e^{iz} + e^{-iz}}{2}$ $\mathrm{ch}\, z = \frac{e^z + e^{-z}}{2}$
$\int_C f(z)\, \mathrm{d}z = \int_C (u(x, y) + iv(x, y))(\mathrm{d}x + i\mathrm{d}y) =$ $= \int_C u(x, y)\, \mathrm{d}x - v(x, y)\, \mathrm{d}y + i \int_C v(x, y)\, \mathrm{d}x + u(x, y)\, \mathrm{d}y$ Cauchy-Riemannovi enačbi: Cauchyjeva formula:	 $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad , \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ $f(z_0) = \frac{1}{2\pi i} \int_{C+} \frac{f(z)}{z - z_0}\, \mathrm{d}z$ $\oint_C \frac{f(z)}{(z - z_0)^{n+1}} = \frac{2\pi i}{n!} f^{(n)}(z_0)$
Residuumi: $\mathrm{Res}_{z=z_0} f(z) = \frac{1}{(n-1)!} \lim_{z \to z_0} \frac{d^{(n-1)}}{dz^{(n-1)}} f(z)(z - z_0)^n$	