DELNI ZAPISKI

ELEKTROENERGETSKI ELEMENTI

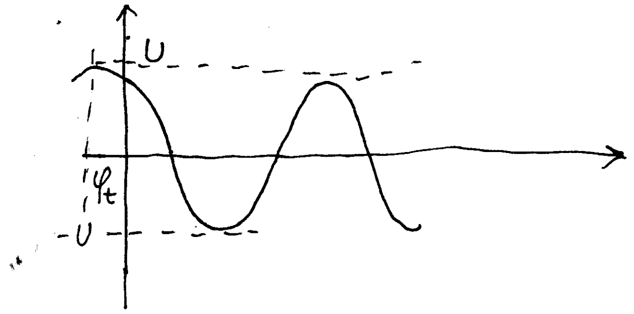
zapiski z avditornih vaj

Šolsko leto	2009 / 2010
Izvajalec	
Avtor	
Ver.:	1

OPOMBE

Obseg snovi: do vključno 18. 11. 2009

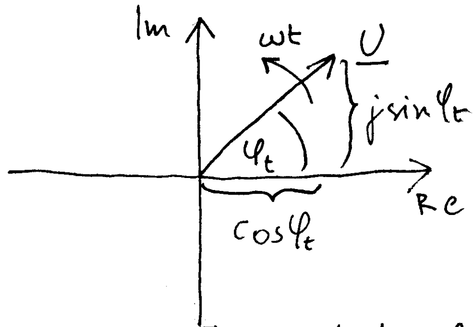
26.10.09



$$u(t) = U\cdot\cos(\omega t+\varphi_t)$$

$$\omega = 2\pi f$$

$$e^{j\alpha} = \cos\alpha + j\cdot\sin\alpha$$



$$e^{j(\omega t+\varphi_t)} = e^{j\omega t} + e^{j\varphi_t} =$$

$$= \cos(\omega t+\varphi_t) + j\sin(\omega t+\varphi_t)$$

$$u(t) = Re\left[U e^{j(\omega t+\varphi_t)}\right] = Re\left[U e^{j\omega t} e^{j\varphi_t}\right]$$

$u(t) = Re\left[\underline{U} e^{j\omega t}\right]$, kjer $\underline{U} = U e^{j\varphi_t} = U(\cos(\varphi_t) + j\sin(\varphi_t))$

Povezava med tokom in napetostjo

- $u = R\cdot i$

$i = I\cos(\omega t+\varphi_i)$

$u = R\cdot I\cdot\cos(\omega t+\varphi_i) \rightarrow *$

$i = I\cos(\omega t+\varphi_i) \rightarrow \underline{I} = I\cdot e^{j\varphi_i}$

$u = U\cdot\cos(\omega t+\varphi_u) \rightarrow \underline{U} = U\cdot e^{j\varphi_u}$

$* \rightarrow \underline{U} = R\cdot I\cdot e^{j\varphi_i}$, $\boxed{\underline{U} = R\cdot\underline{I}}$

- $i = C\,du/dt$

$u = U\cos(\omega t+\varphi_u)$

$i = -\omega CU\sin(\omega t+\varphi_u)$ $\quad i = \omega CU\cos(\omega t+\varphi_u+\pi/2)$

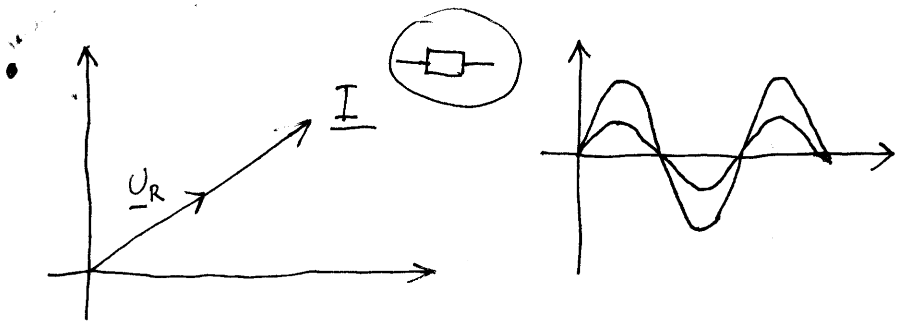
$\underline{I} = \omega CUe^{j(\varphi_u+\pi/2)}$

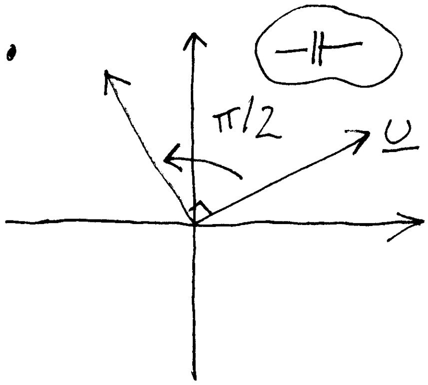
$e^{j(\varphi_u+\pi/2)} = e^{j\varphi_u}\cdot e^{j(\pi/2)} = je^{j\varphi_u}$

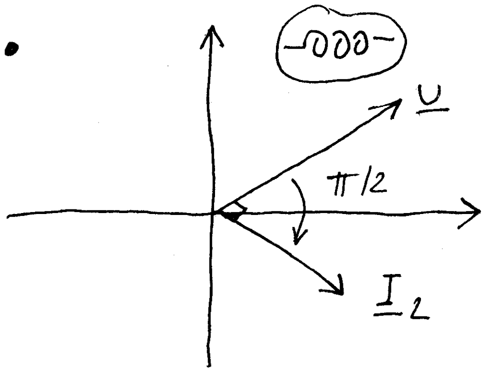
$$\boxed{\underline{I} = \omega CU\,je^{j\varphi_u} = j\omega C\underline{U}}$$

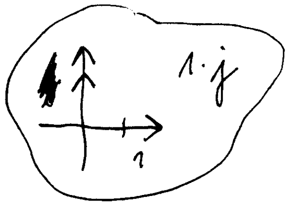
• $u = L\,di/dt$ podobno

$$\boxed{\underline{U} = j\omega L\,\underline{I}}$$

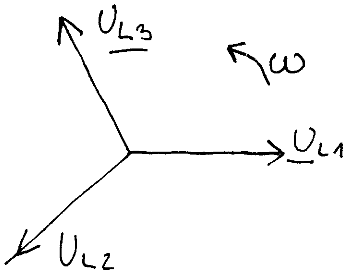
• 

• 

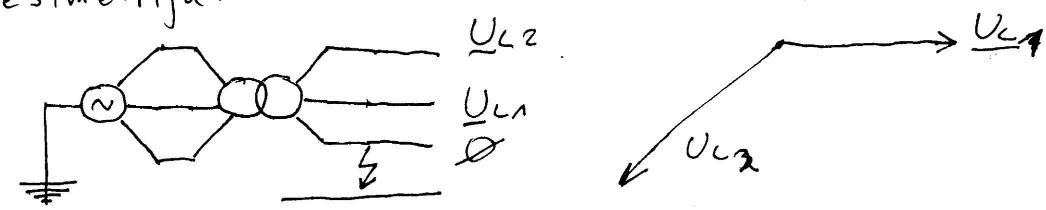
• 



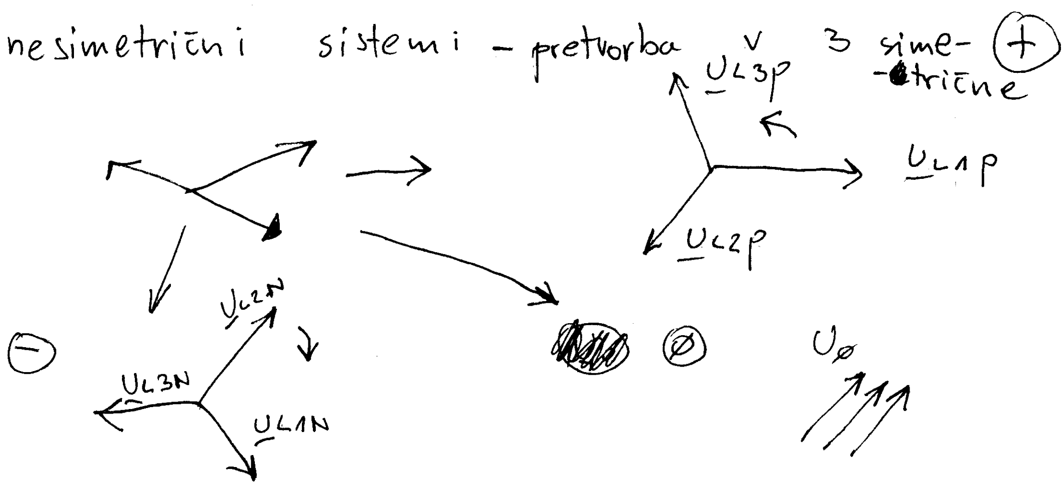
3F SISTEMI

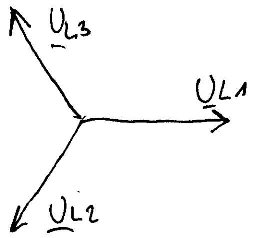


nesimetrija:



• nesimetrični sistemi – pretvorba v 3 sime-trične



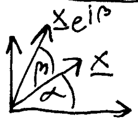
- 

$$u_{L1} = U \cdot \cos(\omega t)$$
$$u_{L2} = U \cdot \cos(\omega t - 120°)$$
$$u_{L3} = U \cdot \cos(\omega t + 120°)$$

$\underline{U}_{L1}$

$$\underline{U}_{L2} = \underline{U}_{L1}\, e^{j(-120)} = \underline{U}_{L1}\, e^{j(240)}$$

$$\underline{U}_{L3} = \underline{U}_{L1}\, e^{j(120)}$$



$$\boxed{\underline{x} = X e^{j\alpha} \qquad \underline{X} \cdot e^{j\beta} = X e^{j\alpha} e^{j\beta} = X e^{j(\alpha+\beta)}}$$

$e^{j120} = \underline{a}$	$e^{j240} = \underline{a}^2$

$$e^{j120} = \cos 120° + j \sin 120°$$
$$= -1/2 + j\ \sqrt{3}/2$$

$$\underline{U}_{L2} = \underline{a}^2 \underline{U}_{L1} \qquad \underline{U}_{L3} = \underline{a}\,\underline{U}_{L1}$$

- $$\begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix}$$

$\underline{U}_1 \to$ pozitivni

$\underline{U}_2 \to$ negativni

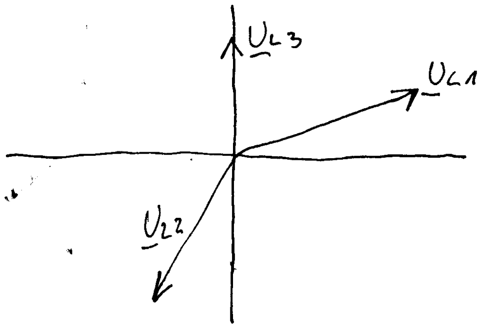
$$\underline{U}_0 = \frac{1}{3}\left(\underline{U}_{L1} + \underline{U}_{L2} + \underline{U}_{L3}\right)$$

$$\underline{U}_1 = \frac{1}{3}\left(\underline{U}_{L1} + \underline{a}\,\underline{U}_{L2} + \underline{a}^2 \underline{U}_{L3}\right) \qquad \underline{U}_2 = \frac{1}{3}\left(\underline{U}_{L1} + \underline{a}^2 \underline{U}_{L2} + \underline{a}\,\underline{U}_{L3}\right)$$

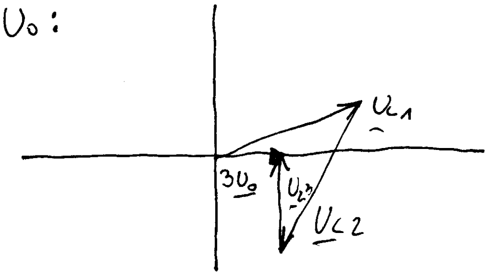
- Inverzna operacija

$$\begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix} \begin{bmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

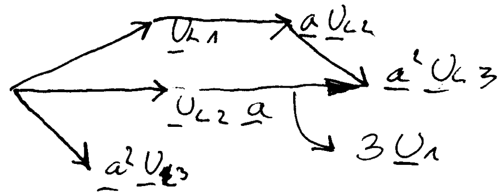
- grafično



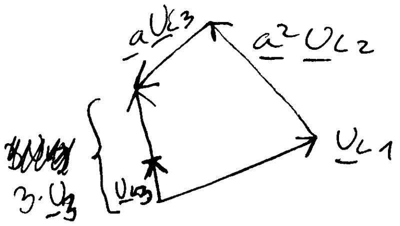
$*$ U_0:



$*$ U_1:



$*$ U_2:

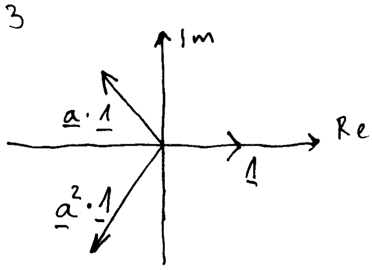


- Primer:

$$\underline{U}_{L1}$$
$$\underline{U}_{L2} = \underline{a}^2\underline{U}_{L1}$$
$$\underline{U}_{L3} = \underline{a}\cdot\underline{U}_{L1}$$

$$U_0 = \frac{1}{3}\left(\underline{U}_{L1} + \underline{U}_{L2} + \underline{U}_{L3}\right) =$$

$$= \frac{1}{3}\left(\underline{U}_{L1} + \underline{a}^2\underline{U}_{L1} + \underline{a}\,\underline{U}_{L1}\right) = \frac{1}{3}\underline{U}_{L1}\left(1 + a^2 + a\right) =$$



$$= \frac{1}{3}\underline{U}_{L1}\cdot\emptyset = \emptyset$$

$\underline{U}_1 = \frac{1}{3}(\underline{U}_{L1} + a\underline{U}_{L2} + a^2\underline{U}_{L3}) = \frac{1}{3}(\underline{U}_{L1} + a^3\underline{U}_{L1} + \underline{a}^3 U_{L1})$

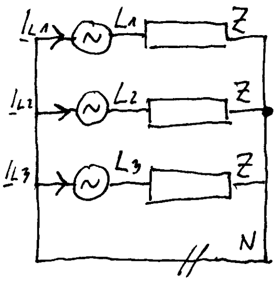
$= \frac{1}{3}\underline{U}_{L1}(1 + \underline{a}^3 + \underline{a}^3) = \frac{1}{3}\underline{U}_{L1}(1+1+1) = \underline{U}_{L1}$

$\underline{U}_2 = \frac{1}{3}(U_{L1} + \underline{a}^2 U_{L2} + \underline{a}\,\underline{U}_{L3}) = \frac{1}{3}(U_{L1} + \underline{a}^4 \underline{U}_{L1} + \underline{a}^2 \underline{U}_{L1}) =$

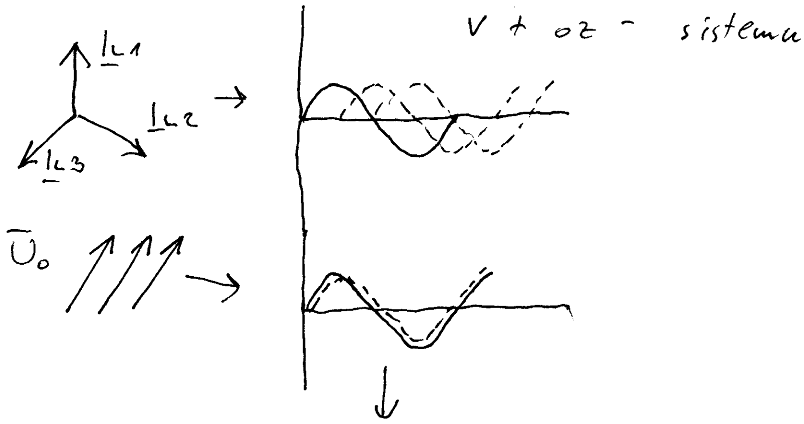
$= \frac{1}{3}\underline{U}_{L1}(1 + \underline{a}^4 + \underline{a}^2)$

$a^4 = a$

$\underline{U}_2 = \emptyset$

- 

$\Sigma I = \emptyset$



V ničnem sistemu $\Sigma \underline{I}_0 = \underline{3} I_0$

- $\underline{U}_{L1} =$ eno fazni sistem
  $\underline{U}_{L2} = \underline{U}_{L3} = \emptyset$

  $\underline{U}_0 = \frac{1}{3}\underline{U}_{L1}$ $\quad \underline{U}_1 = \frac{1}{3}\underline{U}_{L1} = \underline{U}_2$

- enopolni kratek stik na fazi 3

  $\underline{U}_{L1}$
  $\underline{U}_{L2} = \underline{a}^2 \underline{U}_{L1}$
  $\underline{U}_{L3} = \emptyset$

  $\underline{U}_0 = \frac{1}{3}(\underline{U}_{L1} + \underline{a}^2 \underline{U}_{L1}) = \frac{1}{3}\underline{U}_{L1}(1 + a^2) = -\frac{1}{3}\cdot\underline{a}\cdot\underline{U}_{L1}$

  $\underline{U}_1 = \frac{1}{3}(\underline{U}_{L1} + a^3 U_{L1}) = \frac{1}{3}\underline{U}_{L1}(1 + a^3) = \frac{1}{3}\underline{U}_{L1}(1+1) =$

  $= \frac{2}{3}\underline{U}_{L1}$

  $\underline{U}_2 = \frac{1}{3}(\underline{U}_{L1} + \underline{a}^4 \underline{U}_{L1}) = \frac{1}{3}\underline{U}_{L1}(1 + a^4) = -\frac{1}{3}a^2 \underline{U}_{L1}$

• dvofazni sistem, tanik $180°$

$\underline{U}_{L1}$

$\underline{U}_{L2} = -\underline{U}_{L1}$ $\qquad \underline{U}_{L2} \longleftarrow \bullet \longrightarrow \underline{U}_{L1}$

$\underline{U}_{L3} = \varnothing$

$$\underline{U}_0 = \frac{1}{3}\left(\underline{U}_{L1} - \underline{U}_{L1}\right) = \varnothing$$

$$\underline{U}_1 = \frac{1}{3}\left(\underline{U}_{L1} - \underline{a}\,\underline{U}_{L1}\right) = \frac{1}{3}\underline{U}_{L1}(1-\underline{a}) = *$$

$$(1-a) = 1-(-1/2 + j\sqrt{3}/2) = \frac{3}{2} - j\frac{\sqrt{3}}{2}$$

$$* = \left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right)\frac{1}{3}\underline{U}_{L1} = \frac{1}{\sqrt{3}}\underline{U}_{L1}\left(-\frac{\sqrt{3}}{2}j - \frac{1}{2}\right) =$$

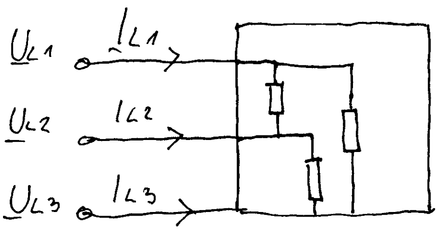
$$= \frac{1}{\sqrt{3}}j\,\underline{U}_{L1}\,a^2$$

$$U_2 = \frac{1}{3}\left(U_{L1} - a^2 U_{L1}\right) = \frac{1}{3}U_{L1}(1-a^2) = *$$

$$(1-a^2) = \frac{3}{2} + j\frac{\sqrt{3}}{2} = 1-\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

$$* = \frac{1}{3}\underline{U}_{L1}\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right)$$

IMPEDANCA



$$\underline{U}_f = \begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix} \qquad \underline{I}_f = \begin{bmatrix} \underline{I}_{L1} \\ \underline{I}_{L2} \\ \underline{I}_{L3} \end{bmatrix}$$

$$\underline{U}_f = \underline{Z}_f\,\underline{I}_f$$

$$\underline{Z}_f = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{33} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

lastne impedance
ostale so medsebojne/medfazne impedance

$$\underline{Z}_f = \begin{bmatrix} Z_p & Z_m & Z_m \\ Z_m & Z_p & Z_m \\ Z_m & Z_m & Z_p \end{bmatrix}$$ → velja vkolikor sistem simetričen

$\underline{U}_s = \underline{Z}_s \underline{I}_s$ za simetričen sistem 9. 11. 2009

zanima nas $\underline{Z}_s$

$\underline{U}_F = T \cdot \underline{U}_s$ in $\underline{U}_s = S \cdot \underline{U}_F$

$\underline{I}_F = T \cdot \underline{I}_s$ in $\underline{I}_s = \underline{S}\, \underline{I}_s$

} kjer T in S transformacijski matriki, $T^{-1} = S$

$T \cdot \underline{U}_s = \underline{Z}_F \cdot T \cdot \underline{I}_s \quad / \cdot T^{-1}$

$\underline{U}_s = T^{-1} \underline{Z}_F \cdot T \cdot \underline{I}_s$

$\underline{U}_s = S \cdot \underline{Z}_F \cdot T \cdot \underline{I}_s$

$$\boxed{\underline{Z}_s = S \cdot \underline{Z}_F \cdot T}$$

$T = S^{-1}$

$$S = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$\underline{Z}_s = \begin{bmatrix} Z_p + 2Z_m & \emptyset & \emptyset \\ \emptyset & Z_p - Z_m & \emptyset \\ \emptyset & \emptyset & Z_p - Z_m \end{bmatrix}$$

za simetričen sistem

- Signali: $\rho = 60^\circ$

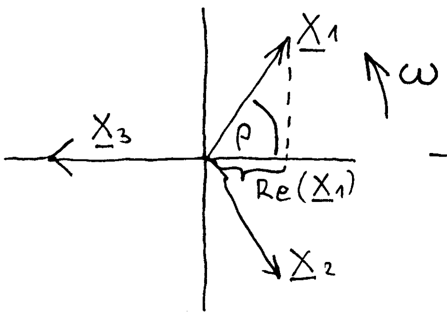
$X_{L1} = X \cos(\omega t + \rho)$

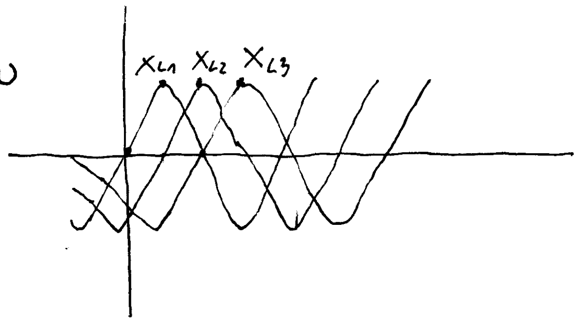
$X_{L2} = X \cos(\omega t - 120^\circ + \rho)$

$X_{L3} = X \cos(\omega t + 120^\circ + \rho)$

$X_{L1}(t) = \mathrm{Re}[X e^{j(\omega t + \rho)}] = \mathrm{Re}[X e^{j\rho} e^{j\omega t}]$

$\underline{X} = X e^{j\rho}$



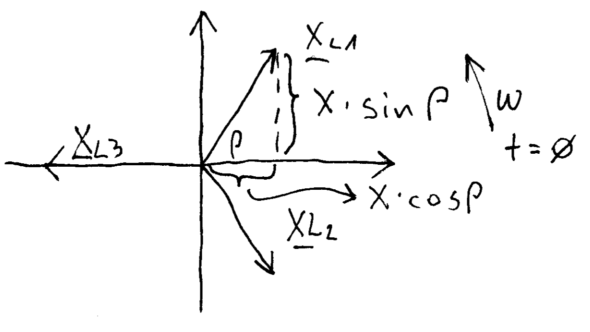


trenutne vrednosti: $\mathrm{Re}(\underline{X}_1)$

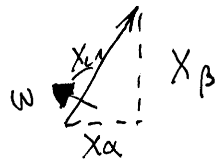
Pretvorba v α, β

$X_\alpha = X \cos(\omega t + \rho)$

$X_\beta = X \sin(\omega t + \rho)$



Dovolj je, da sistem ponazorimo z enim vektorjem.



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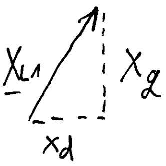
Pretvorba v X_d, X_q : $X_d = X \cdot \cos\varphi$

$X_q = X \cdot \sin\varphi$



Če pri Parkovi transformaciji postavimo $\omega t = 0$, dobimo Clarkovo.

<u>Diagonalne komponente</u>

$$\begin{bmatrix} \underline{U}_0 \\ \underline{U}_\alpha \\ \underline{U}_\beta \end{bmatrix} = \underbrace{\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 0 & \sqrt{3} & \sqrt{3} \end{bmatrix}}_{\to K} \begin{bmatrix} \underline{U}_{L1} \\ \underline{U}_{L2} \\ \underline{U}_{L3} \end{bmatrix}$$

- $\underline{U}_{L1}$

$\underline{U}_{L2} = \underline{a}^2 \underline{U}_{L1}$ $\quad \underline{U}_{L3} = \underline{a}\, \underline{U}_{L1}$

$$\underline{U}_0 = \frac{1}{3}\left(\underline{U}_{L1} + \underline{a}^2 \underline{U}_{L1} + \underline{a}\,\underline{U}_{L1}\right) =$$

$$= \frac{1}{3} \underline{U}_{L1} \left(1 + \underline{a} + \underline{a}^2\right) = 0$$

$$\underline{U}_\alpha = \frac{1}{3}\left(2\,\underline{U}_{L1} - \underline{a}^2 \underline{U}_{L1} - \underline{a}\,\underline{U}_{L1}\right) =$$

$$= \frac{1}{3} \underline{U}_{L1} \left(2 - \underline{a}^2 - \underline{a}\right) =$$

$1 + a + a^2 = 0$

$a + a^2 = -1$

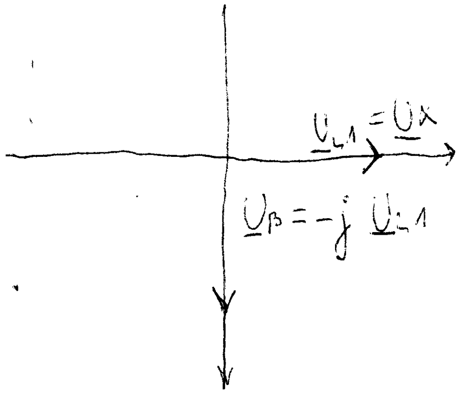
$$= \underline{U}_{L1}$$

$$\underline{U}_\beta = \frac{1}{3}\left(\sqrt{3}\,\underline{a}^2 \underline{U}_{L1} - \sqrt{3}\,\underline{a}\,\underline{U}_{L1}\right) =$$

$$= \frac{1}{\sqrt{3}} \underline{U}_{L1} \left(\underline{a}^2 - \underline{a}\right)$$

$$a^2 - a = \cancel{-\frac{1}{2}} - j\frac{\sqrt{3}}{2} \cancel{+\frac{1}{2}} - j\frac{\sqrt{3}}{2} = -\sqrt{3}\,j$$

$$= -j\,\underline{U}_{L1}$$



- $\underline{U}_{L1} \qquad \underline{U}_{L2} = \underline{U}_{L3} = \varnothing$

$$\underline{U}_0 = \frac{1}{3}\,\underline{U}_{L1}$$

$$\underline{U}_\alpha = \frac{1}{3}\cdot 2 \cdot \underline{U}_{L1} = \frac{2}{3}\,\underline{U}_{L1}$$

$$\underline{U}_\beta = \varnothing$$

- $\underline{U}_{L1} \qquad \underline{U}_{L2} = \underline{a}^2\,\underline{U}_{L1} \qquad \underline{U}_3 = \varnothing$

$$\underline{U}_0 = \frac{1}{3}\,\underline{U}_{L1}\,(1 + \underline{a}^2) = -\frac{\underline{a}}{3}\cdot\underline{U}_{L1} = -\frac{1}{3}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\underline{U}_{L1}$$

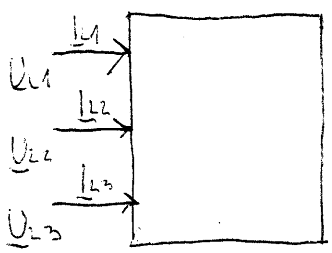
$$\underline{U}_\alpha = \frac{1}{3}\,(2\,\underline{U}_{L1} - \underline{a}^2\underline{U}_{L1}) = \frac{1}{3}\,\underline{U}_{L1}\,(2 - a^2) = *$$

$$1 + a + a^2 = \varnothing$$
$$-a^2 = 1 + a$$

$$* = \frac{1}{3}\,\underline{U}_{L1}\,(2 + 1 + a) = \frac{1}{3}\,(3 + a)\,\underline{U}_{L1} = \left(1 + \frac{a}{3}\right)\underline{U}_{L1}$$

$$\underline{U}_\beta = \frac{1}{3}\sqrt{3}\,\underline{U}_{L2} = \frac{1}{3}\sqrt{3}\,\underline{a}^2\,\underline{U}_{L1} = \frac{1}{\sqrt{3}}\,\underline{U}_{L2}$$

- Imamo 3F porabnik



Fazna impedančna matrika

$$\underline{Z}_f = \begin{bmatrix} \underline{Z}_p & \underline{Z}_m & \underline{Z}_m \\ \underline{Z}_m & \underline{Z}_p & \underline{Z}_m \\ \underline{Z}_m & \underline{Z}_m & \underline{Z}_p \end{bmatrix}$$

$$\underline{U}_F = \underline{Z}_F \cdot \underline{I}_F \qquad \underline{Z}_p = (10 + j30)\,\Omega \qquad \underline{Z}_m = (5 + j20)\,\Omega$$

$$\underline{U}_F = \begin{bmatrix} 277\angle 0^\circ \\ 260\angle -120^\circ \\ 295\angle 115^\circ \end{bmatrix} V = *$$

*določimo matriko faznih tokov $\underline{I}_F$, nato $\underline{Z}_S$ (simetrične komponente impedančne matrike), $\underline{I}_S$ in $\underline{U}_S$ ter $\underline{I}_D$ in U_D

$$* = \begin{bmatrix} 277(\cos 0^\circ + j\sin 0^\circ) \\ 260(\cos(-120^\circ) + j\sin(-120^\circ)) \\ 295(\cos(115^\circ) + j\sin(115^\circ)) \end{bmatrix} V =$$

$$= \begin{bmatrix} 277 \\ -130 - j225{,}17 \\ -124{,}7 + j267{,}36 \end{bmatrix} V$$

$$[\underline{I}_F] = \underline{Z}_F^{-1}\,\underline{U}_F$$

$$= \begin{bmatrix} 9{,}87 - j22{,}17 \\ -24{,}42 + j1{,}38 \\ 15{,}19 + j20{,}66 \end{bmatrix} = \begin{bmatrix} 24{,}27\angle -66^\circ \\ 24{,}46\angle 176{,}8^\circ \\ 25{,}64\angle 53{,}7^\circ \end{bmatrix}$$

$$\underline{Z}_S = \begin{bmatrix} 20+j70 & 0 & 0 \\ 0 & 5+j10 & 0 \\ 0 & 0 & 5+j10 \end{bmatrix}$$

$$\underline{U}_S = \underline{S}\cdot\underline{U}_F = \begin{bmatrix} 7{,}44 + j14{,}06 \\ 276{,}96 - j8{,}57 \\ -7{,}4 - j5{,}49 \end{bmatrix} = \begin{bmatrix} 15{,}91\angle 62{,}11^\circ \\ 277{,}09\angle -1{,}77^\circ \\ 9{,}22\angle -143{,}41 \end{bmatrix}$$

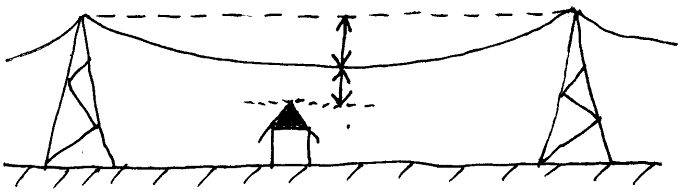
$$\underline{I}_S = \underline{S}\,\underline{I}_F = \begin{bmatrix} 0{,}21 - j0{,}05 \\ 10{,}39 - j22{,}5 \\ -0{,}74 + j0{,}37 \end{bmatrix} = \begin{bmatrix} 0{,}22 \ \underline{\llcorner -11{,}94^\circ} \\ 24{,}78 \ \underline{\llcorner -1{,}77^\circ} \\ 0{,}82 \ \underline{\llcorner 153{,}15^\circ} \end{bmatrix}$$

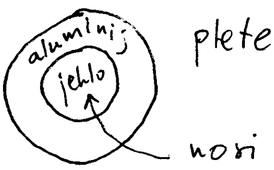
- Diagonalne komponente:

$$\underline{U}_d = K \cdot \underline{U}_F = \begin{bmatrix} 15{,}91^\circ \ \underline{\llcorner 62{,}11^\circ} \\ 269{,}92 \ \underline{\llcorner -3^\circ} \\ 284{,}38 \ \underline{\llcorner -90{,}62^\circ} \end{bmatrix} = \begin{bmatrix} \underline{U}_0 \\ \underline{U}_\alpha \\ U_\beta \end{bmatrix}$$

$$I_d = \underline{K} \cdot \underline{I}_F = \begin{bmatrix} 0{,}22 \ \underline{\llcorner -11{,}94^\circ} \\ 24{,}14 \ \underline{\llcorner -66{,}92^\circ} \\ 25{,}44 \ \underline{\llcorner -154{,}5^\circ} \end{bmatrix}$$

Mehanski parametri vodov





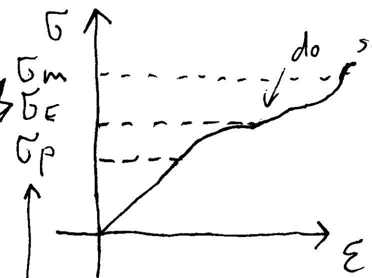
pleteno $\quad$ Al/Je : 490/65 mm² $\quad$ 54 / 7 žic

- specifična teža $\gamma \left(\frac{N}{m^3} , \frac{N}{m \cdot mm^2} \right)$
- specifična masa ρ (kg/m^3)
- temperaturni razteznostni koeficient α $(1/K)$
- modul elastičnosti E (N/mm^2)
- natezna napetost σ (N/mm^2)
- relativni raztezek

$$\varepsilon = \Delta l / l_0$$

$$\varepsilon = \left. \frac{\Delta l}{l_0} \right|_{\sigma} = \frac{\sigma}{E}$$ zaradi natezne napetosti

$$\varepsilon = \left. \frac{\Delta l}{l_0} \right|_{\vartheta} = \alpha \cdot \Delta \vartheta$$ zaradi spremembe temperature



do sem je še elastična

območje proporcionalnosti

območje elastičnosti

mejni σ, se pretrga

- specifična teža

$$\gamma = \frac{\gamma_{Fe} + \gamma_{Al} \cdot \eta}{1+\eta} \left[\frac{N}{m\ mm^2}\right] \quad \eta = \frac{A_{Al}}{A_{Fe}}$$ presečno razmerje

- modul elastičnosti

$$E = \frac{E_{Fe} + E_{Al} \cdot \eta}{1+\eta} \left[\frac{N}{mm^2}\right]$$

- temperaturni raztezenostni koeficient

$$\alpha = \frac{\alpha_{Fe} E_{Fe} + \alpha_{Al} E_{Al} \eta}{E(1+\eta)} \left[\frac{1}{K}\right]$$

- izpeljava

$$\gamma A = \gamma_{Al} A_{Al} + \gamma_{Fe} A_{Fe}$$

$$\gamma = \frac{\gamma_{Al} A_{Al} + \gamma_{Fe} A_{Fe}}{A_{Al} + A_{Fe}}$$

$$\boxed{\eta = \frac{A_{Al}}{A_{Fe}}}$$

po vstavitvi dobimo zgornjo enačbo

podobno za ostale

- α za celotno vrv

$$\alpha(\vartheta - \vartheta_{15}) + \frac{\sigma}{E}$$ izhodiščna temperatura (pri 15 °C)

$$= \alpha_{Al}(\vartheta - \vartheta_{15}) + \frac{\sigma_{Al}}{E_{Al}}$$ predpostavitev, da se snovi enako raztezata

$$\sigma = (\alpha_{Al} - \alpha)(\vartheta - \vartheta_{15}) E + \frac{E}{E_{Al}} \sigma_{Al}$$

$\sigma_{DOPUSTNI\_AL}$ podajamo ~~za~~ $\sigma_{DOP-AL} = 60\ N/mm^2$

Izraz za pojav žledi (aproksimacija)

$$\Delta\gamma \doteq \frac{2}{A^{3/4}} \left[\frac{N}{m\ mm^2}\right] \quad A = A_{Al} + A_{Fe}$$

povečanje specifične teže

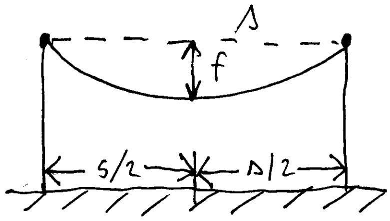
ni pravilno po enotah

$$\boxed{\sigma_{DOPUSTNI} = (\alpha_{Al} - \alpha)(\vartheta - \vartheta_{11}) E + \frac{E}{E_{Al}} \sigma_{DOP\_AL}}$$

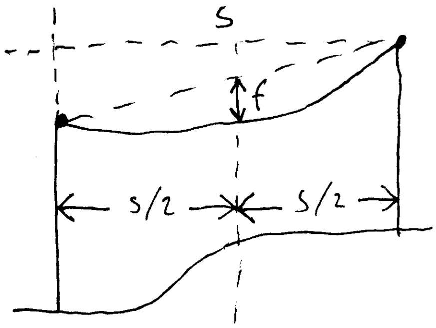
za celotno vrv

$\sigma_{DOP\_AL} = 60\ N/mm^2$

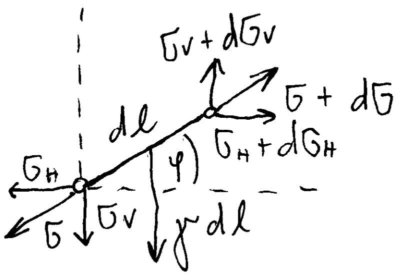
$\sigma_{dopustni}$ - gre za horizontalno komponento



f .... poves
s .... razpetina



f nastopi na s/2 !



(1) $\sigma_H - (\sigma_H + d\sigma_H) = \emptyset$

$\sigma_H - \sigma_H - d\sigma_H = \emptyset$

$d\sigma_H = \emptyset$

(2) $\sigma_V + \gamma\, dl - (\sigma_V + d\sigma_V) = \emptyset$

$\sigma_V + \gamma\, dl - \sigma_V - d\sigma_V = \emptyset$

$d\sigma_V = \gamma \cdot dl$

$$dl = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \frac{dy^2}{dx^2}}$$

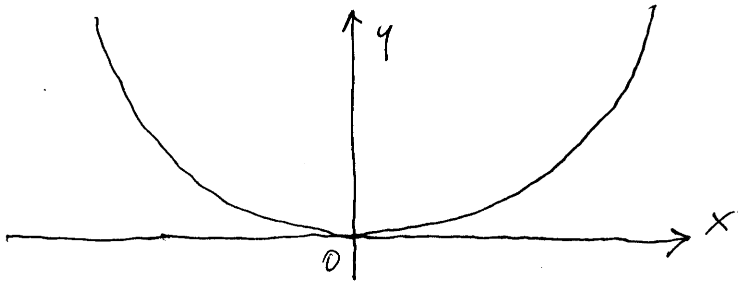
$$d\sigma_V = \gamma\, dl = \gamma\, dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\frac{d\sigma_V}{dx} = \gamma \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\mathrm{tg}\,\varphi = \frac{\sigma_V}{\sigma_H} = \frac{dy}{dx} \rightarrow \sigma_V = \sigma_H \cdot \frac{dy}{dx}$$

16.11.09

$$\sigma_H \frac{d^2y}{dx^2} = \gamma \sqrt{1+\left(\frac{dy}{dx}\right)^2}$$



kjer x majhen, zadeva podobna cosinusu.

$$y = a\,ch\frac{x}{a} + K \qquad \boxed{a = \frac{\sigma_H}{\gamma}}$$

$$y(x=0)=0$$

$$\boxed{y = a\cdot ch\frac{x}{a} - a}$$

kolokvij 30.11.09 do strani 92.

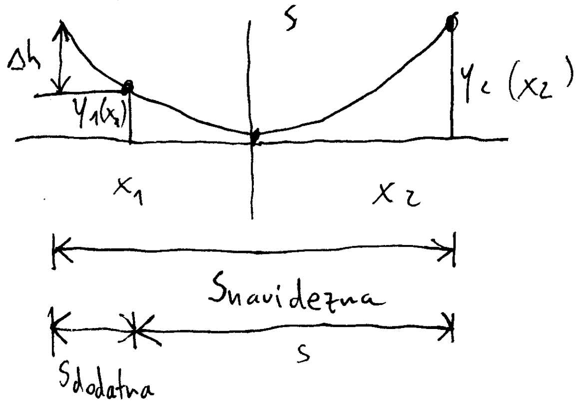
Dalje:

$$\sigma = \frac{\eta + \frac{E_{Fe}}{E_{Al}}}{\eta+1}\sigma_{Al}$$

$$E = \sigma\frac{E_{Fe}}{\sigma_{Fe}} = \sigma\frac{E_{Al}}{\sigma_{Al}} = \frac{\eta + \frac{E_{Al}}{E_{Fe}}}{\eta+1}\cancel{\sigma_{Al}}\,\frac{E_{Al}}{\cancel{\sigma_{Al}}}$$

$$E = \boxed{\frac{E_{Fe} + \eta E_{Al}}{1+\eta}}$$

skupni modul elastičnosti

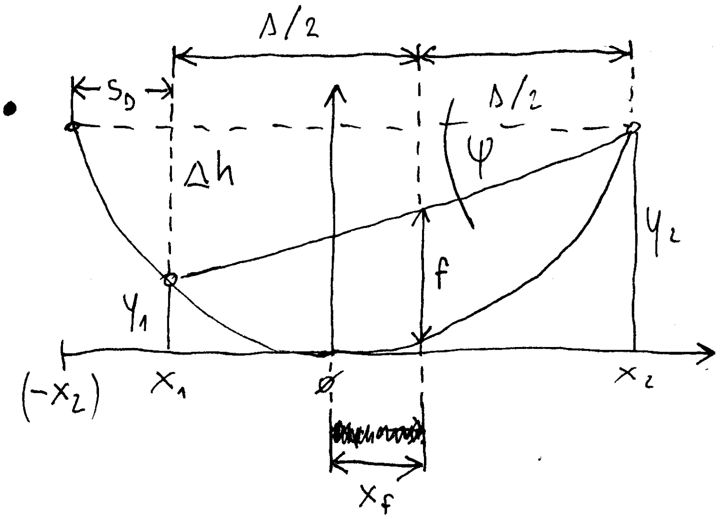


$$\Delta h = y_2 - y_1 = a \cdot ch\frac{x_2}{a} - a\, ch\frac{x_1}{a}$$

$$\Delta h = 2a \cdot sh\frac{s_d}{2a} \cdot sh\frac{s}{2a}$$

$$s_d = x_2 - x_1$$

$$s = x_2 + x_1$$



$$S_{celotni} = S + \underbrace{S_{DODATNI}}_{S_D}$$

Izračunamo lahko:

- $$\Delta h = y_2 - y_1 = a \cdot ch\left(\frac{x_2}{a}\right) - a \cdot ch\left(\frac{x_1}{a}\right)$$

$$= 2a \cdot sh\left(\frac{x_1 + x_2}{2a}\right) \cdot sh\left(\frac{x_2 - x_1}{2a}\right)$$

$$= 2a \cdot sh\left(\frac{s}{2a}\right) \cdot sh\left(\frac{s_D}{2a}\right)$$

- dolžina verižnice

$$dl = \sqrt{dx^2 + dy^2} = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\frac{l}{2} = \int_0^x dl = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = a \cdot sh\left(\frac{x}{a}\right)$$

dolžina od koordinatnega izhodišča, $l/2$ v kolikor upoštevamo od koordinatnega izhodišča

$$l = 2 \cdot a \cdot sh\left(\frac{x}{a}\right)$$

- Aproksimacija

$$y = a\,\mathrm{ch}\left(\frac{x}{a}\right) - a$$

$$\mathrm{ch}\,x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\mathrm{sh}\,x = x + \frac{x^3}{3!} + \dots$$

Upoštevamo prva dva člena, v kolikor vod ni predolg.

$$y = \frac{x^2}{2a}$$ izpeljava

$$y = a\left(\cancel{1} + \frac{x^2}{2a^2} - \cancel{1}\right) = \frac{x^2}{2a}$$

- poves f

$$f = \frac{\gamma s^2}{8\sigma_H}$$ dobimo, ko upoštevamo $y = \frac{x^2}{2a}$, $x = s/2$, $a = \sigma_H/\gamma$

V kolikor višine niso enake

$$f = \frac{1}{\cos\varphi}\frac{\gamma s^2}{8\sigma_H}$$

$$f = \frac{\gamma s^2}{8\sigma_H} + \frac{\gamma^3 s^4}{384\,\sigma_H^3}$$ v kolikor vod daljši izpeljava v knjigi (če prej upoštevamo dodaten člen)

- Δh z vrstami

$$\Delta h = \frac{s_D \cdot s}{2a}$$

- $$s_D = \frac{2a\,\Delta h}{s} = 2\frac{\sigma_H}{\gamma}\frac{\Delta h}{s}$$

• dolžina verižnice z vrstami

$$L_v = 2 \cdot a \cdot sh\left(\frac{x}{a}\right) \doteq 2a\left(\frac{s}{2a} + \frac{s^3}{48a^3}\right) = s + \frac{s^3}{24a^2}$$

$$= s + \frac{\gamma^2 s^3}{24 \sigma_H^2} = s + \frac{8f^2}{3s}$$

Klasična položajna enačba

σ, f = funkcija, odvisna od σ

$$L_{v\,geometrijska} = s\left(1 + \frac{\gamma^2 s^2}{24 \sigma_H^2}\right)$$

zaradi mase

$$L_{v\,fizikalna} = s\left(1 + \xi\right)\left(1 + \alpha \vartheta\right)\left(1 + \frac{\sigma_H}{E}\right)$$

ξ konstrukcijski raztezek

ϑ odvisnost od temp

$L_{vg} = L_{vf}$!

sledi

$$\frac{\gamma^2 s^2}{24 \sigma_H^2} = \xi + \alpha \vartheta + \frac{\sigma_H}{E}$$

$$\frac{\gamma_\varnothing^2 s^2}{24 \sigma_{\varnothing H}^2} = \xi + \alpha \vartheta_\varnothing + \frac{\sigma_{\varnothing H}}{E}$$

$$\frac{\gamma^2 s^2}{24 \sigma_H^2} - \frac{\gamma_\varnothing^2 s^2}{24 \sigma_{H\varnothing}^2} = \alpha\left(\vartheta - \vartheta_\varnothing\right) + \frac{\sigma_H - \sigma_{\varnothing H}}{E}$$

za različne višine obesišč:

$$\frac{\gamma^2 s^2}{24 \sigma_H^2} - \frac{\gamma_\varnothing^2 s^2}{24 \sigma_{H\varnothing}^2} = \alpha\left(\vartheta - \vartheta_\varnothing\right) + \frac{\sigma_H - \sigma_{\varnothing H}}{E \cos\psi}$$

$$\sigma_H^3 + m\,\sigma_H^2 = n^2 \qquad m = \frac{\gamma_\varnothing^2 s^2}{24 \sigma_{H\varnothing}^2} E\cos\psi + \alpha\left(\vartheta - \vartheta_\varnothing\right) E\cos\psi - \sigma_{\varnothing H}$$

$$n = \gamma s \sqrt{\frac{E\cos\psi}{24}}$$

$\sigma = funk(\vartheta)$

$f = funk(\vartheta) = \frac{s^2}{8}\frac{\gamma}{\sigma_H}$

Montažna tabela

	$-20°$ ...	$0°$ ...	$30°$ ...	$40°$
σ	~	~	~	~
f	~	~	~	~

Kritična razpetina

$-20°$
$-5°$ + dodatno breme } odločimo se, kaj je slabše v našem primeru

$$s_k = \sigma_{H\,DOPUSTNI}\sqrt{\frac{360\,\alpha}{(\gamma+\Delta\gamma)^2-\gamma^2}}$$

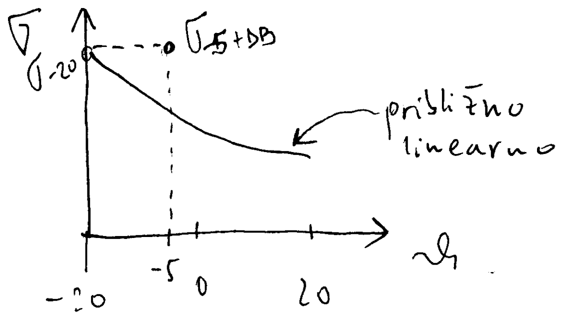
← zaradi dodatnega bremena

pove razpetini pri kateri je $\sigma_{-20} = \sigma_{-5+BREME}$!

najslabše pri:

$s > s_k$ .... σ_{-5+DB}

$s < s_k$ .... σ_{-20}



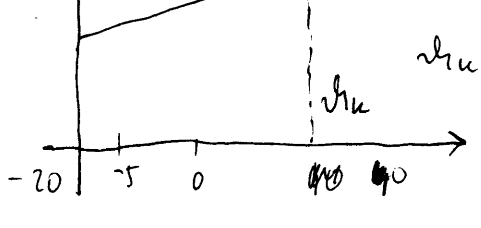
Najvišja σ_{dop}

$$\sigma_{dop} = (\alpha_{Al}-\alpha)(\vartheta-\vartheta_{15})E + \frac{E}{E_{Al}}\sigma_{dopAl}$$

KRITIČNA TEMPERATURA

Opciji: $+40°$
$-5° + D.B.$

$$\vartheta_k = \frac{\sigma_{-5}\,\Delta\gamma}{\alpha E(\gamma+\Delta\gamma)} - 5°$$



$\vartheta_k < 40° \rightarrow f_{max} = f(40°)$

$\vartheta_k > 40° \rightarrow f_{max} = f(-5° + DB)$