Trojni integral

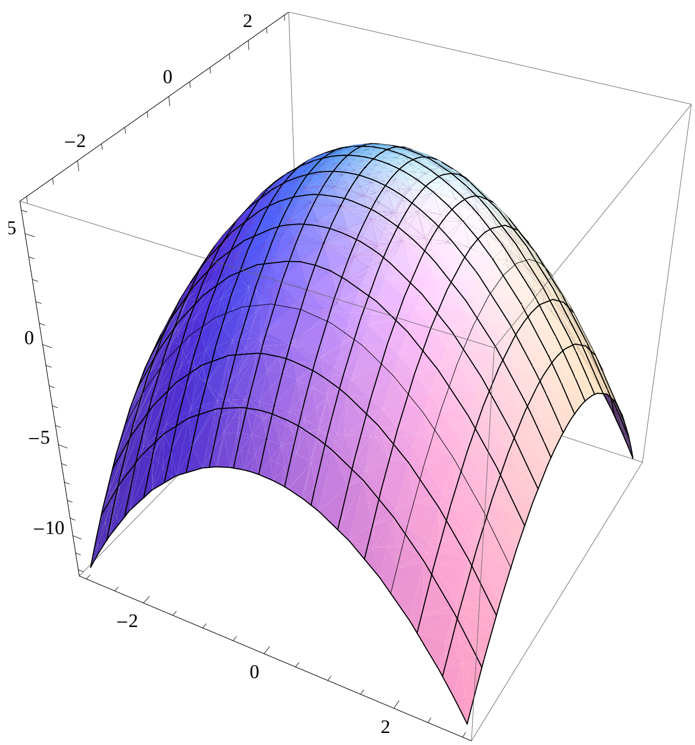
1. Izra■unaj ***prostornino*** telesa, omejenega z danima ploskvama:

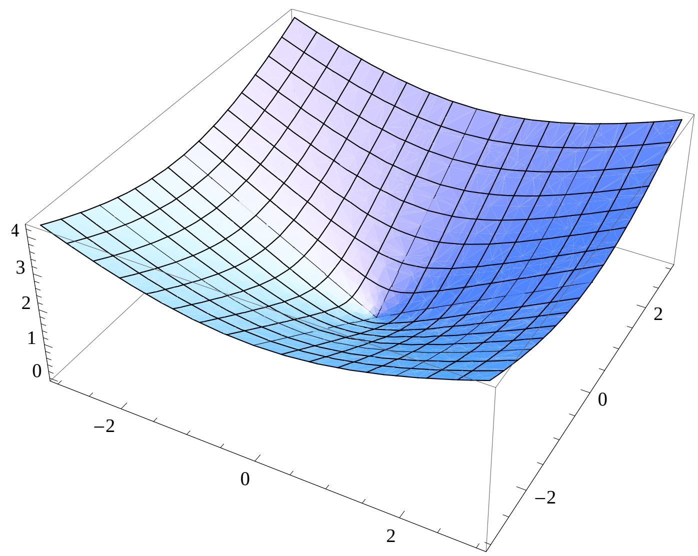
   $z = 6 - x^2 - y^2$ in $z = \sqrt{x^2 + y^2}$

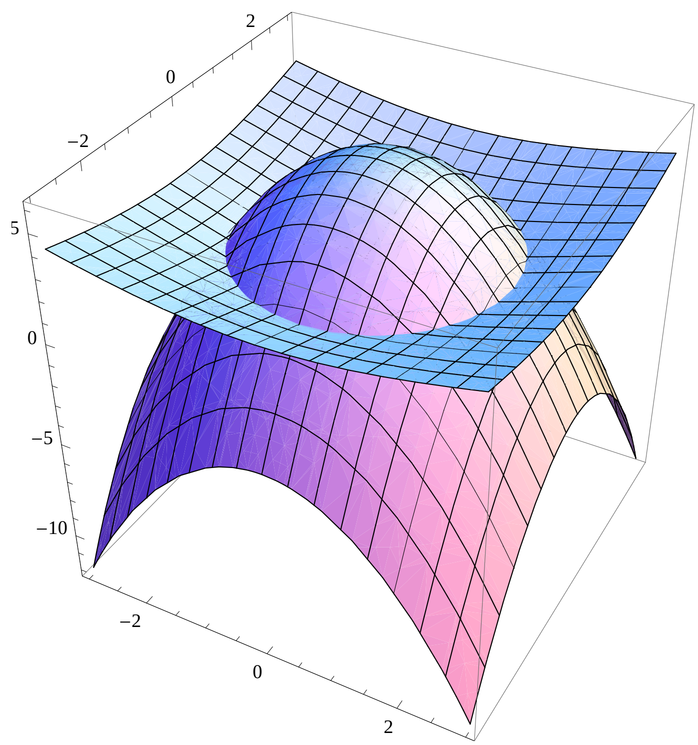
   Navodilo: vpelji cilindri■ne kooedinate!

   Rezultat: $\frac{32\pi}{3}$

```
Clear[x, y, z, S1, S2]
S1 = Plot3D[6 - x^2 - y^2, {x, -3, 3}, {y, -3, 3}, BoxRatios → {1, 1, 1}]
S2 = Plot3D[Sqrt[x^2 + y^2], {x, -3, 3}, {y, -3, 3}]
Show[S1, S2]
```







2. Dolo■i ***tezis■e*** 1/8 krogle $x^2 + y^2 + z^2 \leq 1$ v prvem oktantu,
■e je gostota enaka $\rho = \frac{1}{\sqrt{1-(x^2+y^2+z^2)}}$.

Rezultat: $\left\{\frac{4}{3\pi}, \frac{4}{3\pi}, \frac{4}{3\pi}\right\}$

Formula za posamezno koordinato tezis■a se glasi

$$x_T = \frac{\iiint_V \rho x \, dx \, dx \, dz}{\iiint_V \rho \, dx \, dx \, dz}$$

Uvedi sferi■ne koordinate!

```
ParametricPlot3D[{Cos[φ] Cos[θ], Sin[φ] Cos[θ], Sin[θ]},
 {φ, 0, Pi / 2}, {θ, 0, Pi / 2}, ViewPoint -> {5, 3, 4}]
```

Teorija polja

3. Pois■i ***nivojske ploskve*** skalarnega polja u=arcsin$\left[\frac{z}{\sqrt{x^2+y^2}}\right]$ in skiciraj nivojske ploskve za u=0, u=-π/2, u=π/6!

u = ArcSin$\left[\frac{z}{\sqrt{x^2+y^2}}\right]$

ArcSin$\left[\frac{z}{\sqrt{x^2+y^2}}\right]$

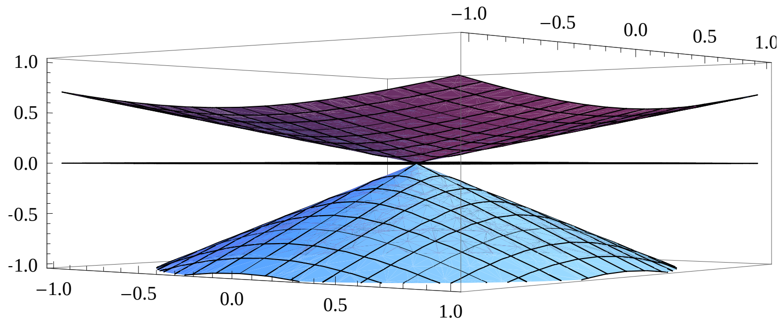
```
Solve[u == 0, z]
Solve[u == -Pi / 2, z]
Solve[u == Pi / 6, z]
```

$\{\{z \to 0\}\}$

$\left\{\left\{z \to -\sqrt{x^2+y^2}\right\}\right\}$

$\left\{\left\{z \to \frac{1}{2}\sqrt{x^2+y^2}\right\}\right\}$

```
p1 = Plot3D[z = 0, {x, -1, 1}, {y, -1, 1}];
p2 = Plot3D[z = -Sqrt[x^2 + y^2], {x, -1, 1}, {y, -1, 1}];
p3 = Plot3D[z = 1/2 Sqrt[x^2 + y^2], {x, -1, 1}, {y, -1, 1}];
Show[p1, p2, p3]
```




```
Clear[p1, p2, p3]
p1 = ParametricPlot3D[z = 0, {r, -1, 1}, {ϕ, -1, 1}];
p2 = ParametricPlot3D[-r, {r, -1, 1}, {ϕ, -1, 1}];
p3 = ParametricPlot3D[1/2 r, {r, -1, 1}, {ϕ, -1, 1}];
Show[p1, p2, p3] (* NAREDI DO KONCA!!! *)
```

```
<< VectorAnalysis`
```

4. Izra■unaj ***divergenc***o vektorskega polja V= $\frac{1}{\sqrt{x^2+y^2}}$ arctg $\left(z^2 - \sin(x) + \cos(xyz)\right)$(-x,y,z) v to■ki T(0,1,2)!

Rezultat: 4/13
Pomo■:
divV/.{x→0, y→1, z→2}   vstavi to■ko T(0,1,2) v splosen izraz za divV

```
? Div
```

Div[f] gives the divergence, ∇ · f, of the vector field f in the default coordinate system.
Div[f, coordsys] gives the divergence of f in the coordinate system coordsys. ≫

```
Divergenca[v_] := D[v[[1]], x] + D[v[[2]], y] + D[v[[3]], z]
```

```
SetCoordinates[Cartesian[x, y, z]]
```

Cartesian[x, y, z]

```
Clear[V, x, y, z]
```

$$V := \frac{1}{\sqrt{x^2 + y^2}} * \text{ArcTan}\left[z^2 - \text{Sin}[x] + \text{Cos}[x\,y\,z]\right] * \{-x, y, z\}$$

```
Divergenca[V] /. {x → 0, y → 1, z → 2}
Div[V] /. {x → 0, y → 1, z → 2}
```

$$\frac{4}{13}$$

$$\frac{4}{13}$$

5. Izra■unaj ***rotor*** vektorskega polja $V=\left(\frac{\arcsin\left(\frac{z}{y}\right) y}{\sqrt{z+1}},\ -\frac{e^{xz+\sin\left(\frac{z}{y}\right)} x}{\sqrt{z+1}},\ \sqrt{xy}\,\log\left(\frac{y}{x}\right)\right)$ v to■ki T(2,2,0)!

Rezultat: {5,2,-1}
Pomo■: glej nalogo 4

```
Clear[V, x, y, z]
```

$$V = \left\{\frac{\text{ArcSin}\left[\frac{z}{y}\right] y}{\sqrt{z+1}},\ -\frac{E\,\hat{}\left(x\,z + \text{Sin}\left[\frac{z}{y}\right]\right) x}{\sqrt{z+1}},\ \sqrt{x\,y}\ \text{Log}\left[\frac{y}{x}\right]\right\}$$

```
P = V[[1]]
Q = V[[2]]
R = V[[3]]
```

$$\left\{\frac{y\,\text{ArcSin}\left[\frac{z}{y}\right]}{\sqrt{1+z}},\ -\frac{e^{x\,z+\text{Sin}\left[\frac{z}{y}\right]}\,x}{\sqrt{1+z}},\ \sqrt{x\,y}\ \text{Log}\left[\frac{y}{x}\right]\right\}$$

$$\frac{y\,\text{ArcSin}\left[\frac{z}{y}\right]}{\sqrt{1+z}}$$

$$-\frac{e^{x\,z+\text{Sin}\left[\frac{z}{y}\right]}\,x}{\sqrt{1+z}}$$

$$\sqrt{x\,y}\ \text{Log}\left[\frac{y}{x}\right]$$

```
rotor = {D[R, y] - D[Q, z], D[P, z] - D[R, x], D[Q, x] - D[P, y]} /. {x → 2, y → 2, z → 0}
```

{5, 2, -1}

```
Curl[V] /. {x → 2, y → 2, z → 0}
```

{5, 2, -1}

6. Izra■unaj Δu v to■ki T(1,0), kjer je u podan v polarnih koordinatah
u=$e^{r \sin(r\varphi + \ln(\varphi+1))} \operatorname{arctg}(\varphi\sqrt{r})$!

Rezultat: 4
Pomo■: ■e v polarnih koordinatah zapisemo u=u(r,φ), velja
$\triangle u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}$

```
SetCoordinates[Cylindrical[r, fi, z]]
```

```
Cylindrical[r, fi, z]
```

```
Clear[V, x, y, z, u, r, fi]
u = E^(r * Sin[r * fi + Log[fi + 1]]) * ArcTan[fi * Sqrt[r]]
```

$e^{r \operatorname{Sin}[fi\, r+\operatorname{Log}[1+fi]]} \operatorname{ArcTan}\left[fi \sqrt{r}\right]$

```
Lapl = D[u, {r, 2}] + D[u, r] / r + D[u, {fi, 2}] / r^2 /. {r → 1, fi → 0}
```

4

```
Laplacian[u] /. {r → 1, fi → 0}
```

4

```
? D
```

D[f, x] gives the partial derivative $\partial f / \partial x$.
D[f, {x, n}] gives the multiple derivative $\partial^n f / \partial x^n$.
D[f, x, y, …] differentiates f successively with respect to x, y, ….
D[f, {{x_1, x_2, …}}] for a scalar f gives the vector derivative $(\partial f / \partial x_1, \partial f / \partial x_2, \ldots)$.
D[f, {*array*}] gives a tensor derivative. ≫