

Osnove elektrotehnike

$\vec{w}$ - vektor hitrosti delca ki nosi naboj Q

1) Lorentzova sila

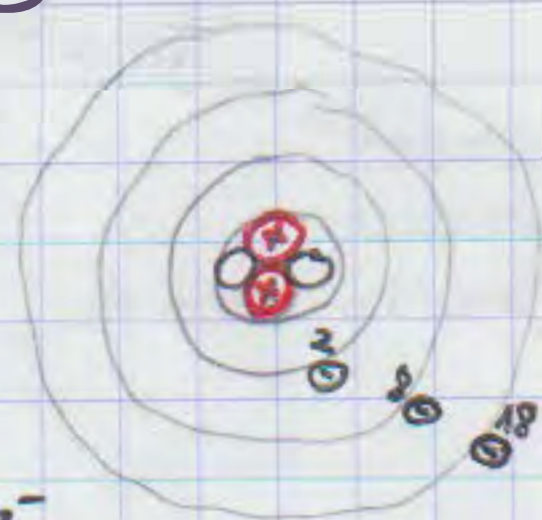
malá delta

$$\oint \vec{F}_L = \oint Q (\vec{E} + \vec{w} \times \vec{B})$$

hitrost delca Q

Q ... električni naboj → elektrina

- nosilci naboja:
 - ↳ elektroni $\ominus$
 - ↳ protoni $\oplus$



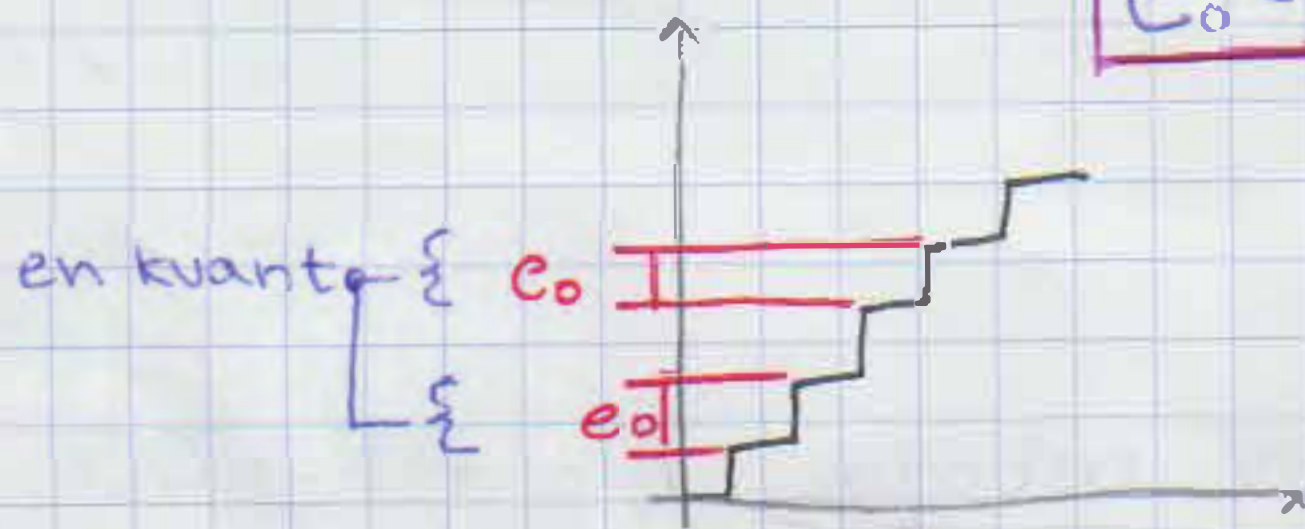
prosti nosilci naboja ... e^-

- naboj merimo:

$[As] [C]$

- naboj je kvantiziran:

$$e_0 = 1,602 \cdot 10^{-19} C$$



$\vec{E}$... vektor električne poljske jakosti $[V/m]$

izpeljava zgornje enačbe ①

$$\oint Q \vec{E} + \oint Q (\vec{w} \times \vec{B})$$

$\oint \vec{F}_e = \vec{F}_c$
Coulombova sila
elektrostatična sila

$\oint \vec{F}_m = \oint \vec{F}_a$
amperova sila

$\vec{w}$ = hitrost gibanja nabojev
hitrost $[m/s]$

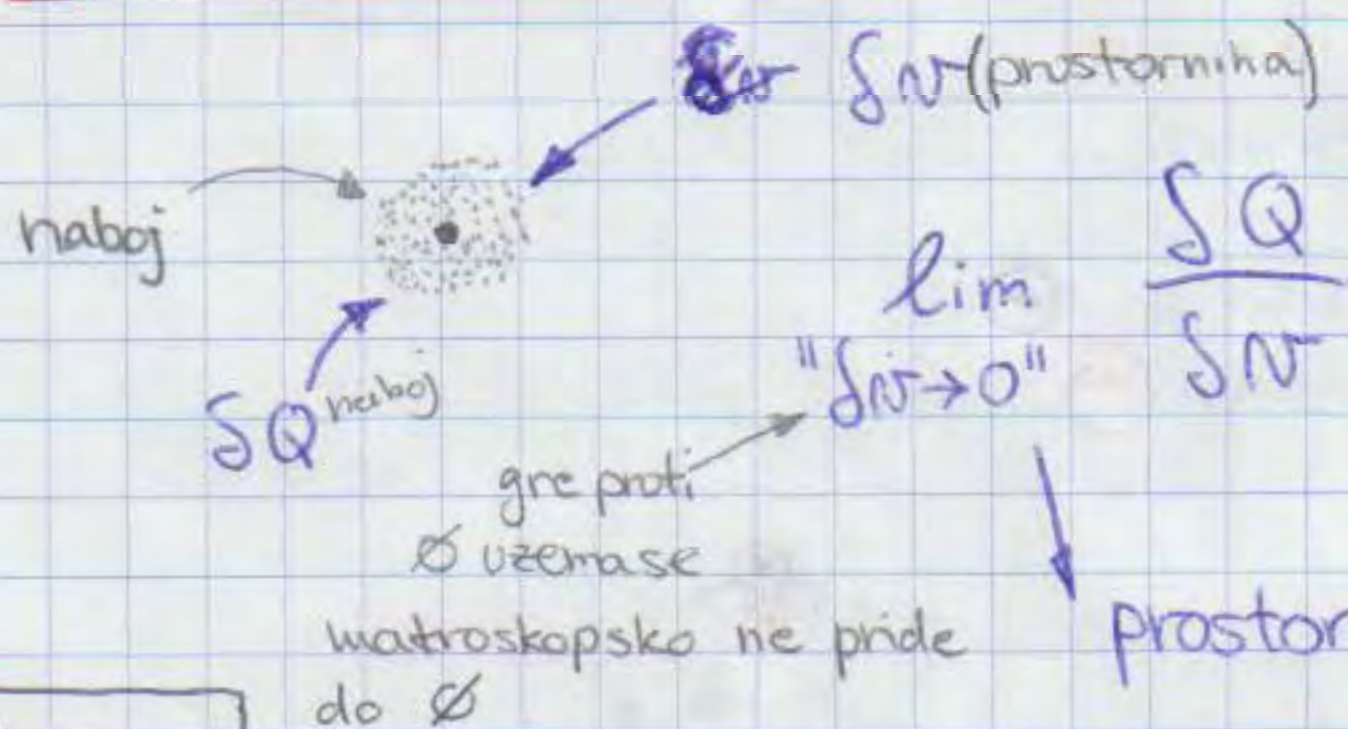
$\vec{B}$ = vektor mag. pdja $[T]$
gostota mag. pretoka

$$\vec{F}_L = \int \underbrace{Q \vec{E}}_{\text{OE}} + \underbrace{\left(\int \underbrace{d\vec{\ell}}_{\vec{r}} \times \vec{B} \right)}_{\text{magnetni moment}}$$

osnovni gradnik $\underbrace{\quad}_{\text{OE}}$

2. Porazdelitev nabojev

1) Prostorska porazdelitev naboja ρ



$$\rho = \lim_{\substack{\text{delta} \\ \text{"delta V to 0"}}} \frac{\delta Q}{\delta V} \left[\frac{\text{C}}{\text{m}^3} \right]$$

naboj je ob času t
volumen v prostoru
Volumen gre proti 0 (volumen ki je fizično majhen)

prostorska gostota naboja $[\text{C/m}^3]$

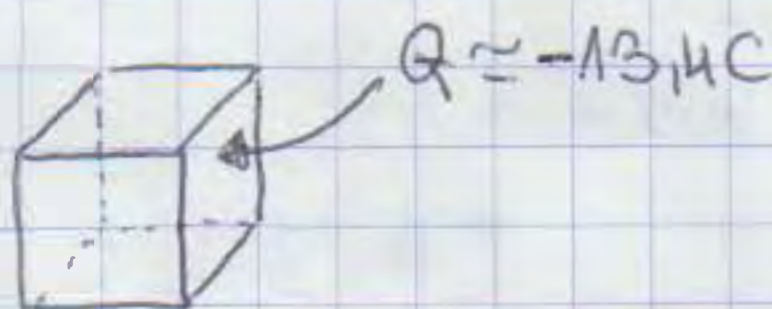
Zgled:
 Cu baker
št. atomov

$$\eta = 8,4 \cdot 10^{28} / \text{m}^3$$

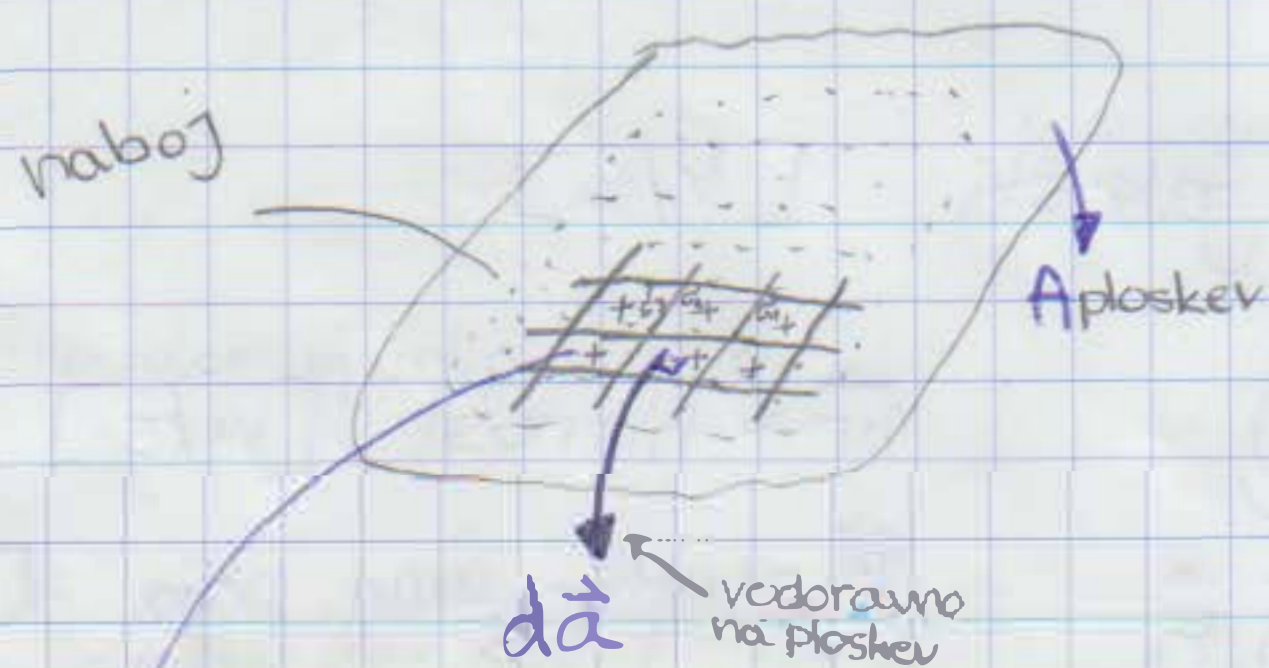
št. prostih elektronov v bakru

$$\rho = -e \cdot \eta = -1,34 \cdot 10^{10} \text{ C/m}^3$$

$$\text{za } 1 \text{ mm}^3 \rightarrow -13,4 \text{ C}$$



2) Ploskovna porazdelitev naboja v razredih $\mu\text{C/m}^2$



$$\sigma = \lim_{da \rightarrow 0} \frac{dQ}{da} \rightarrow Q = \int \sigma \cdot dA$$

prostorska gostota
ploskovna gostota naboja $[\text{C/m}^2]$
ploskev

vsaka ploskev ima svojo σ (sigma), ploskve se seštevajo

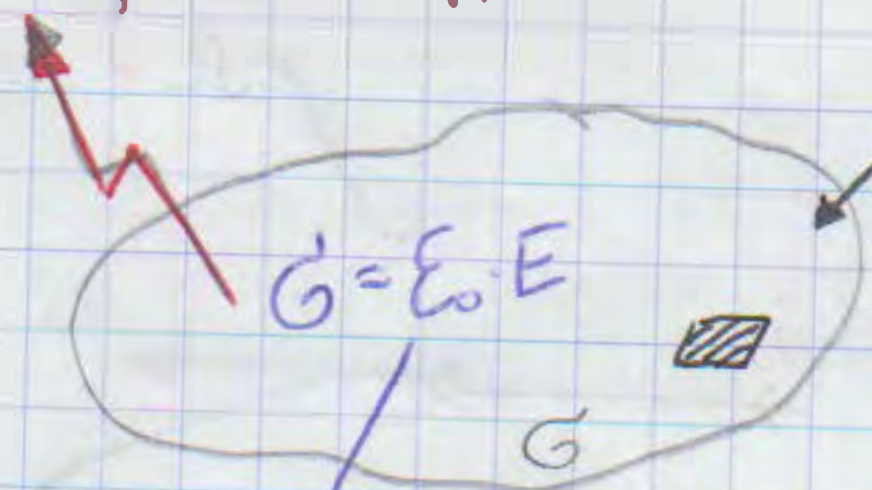
$$Q = \sigma_1 \cdot da_1 + \sigma_2 \cdot da_2 + \dots$$

$$Q = \sum_{i=1}^k \sigma_i \cdot da_i$$

$$Q = \int_A dQ = \int_A \sigma \cdot da$$

$$E_p = 3 \cdot 10^6 \frac{V}{m} \text{ (preboj trdnost bakra)}$$

preboj



dQ

davajamo Q pride do preboja E_p

$$\epsilon_0 = 8,85 \cdot 10^{-12} \left[\frac{As}{Vm} \right]$$

$$G = 26,5 \mu C/m^2$$

$$\text{vs} \rightarrow \eta_p = 25 \cdot 10^{18} / m^2$$

$$\eta_{ff} = 166 \cdot 10^{12} / m^2 \rightarrow \text{preseček}$$

$e_0 = G \text{ preseček}$



fun fact

vsak 150.000 atom ima presežen elektron. !

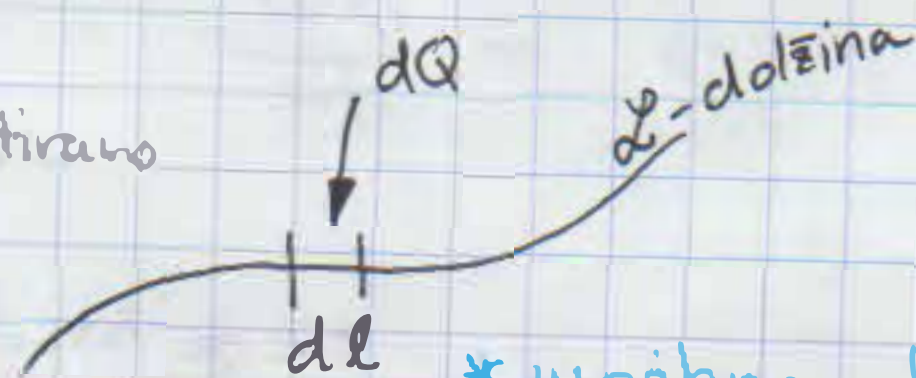
3) Linijska porazdelitev naboja

v razredih $\mu C/m$

↑ ker je žica

$$\lambda = \lim_{dl \rightarrow 0} \frac{dQ}{dl}$$

razdaljo
zmanjšamo - limitiramo

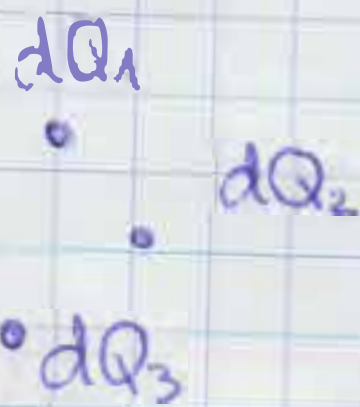


naboj
na celotni
dolžini L

$$Q = \int_L \lambda \cdot dl$$

* majhna količina Q na
majhni dolžini L zato
je dQ, dl *

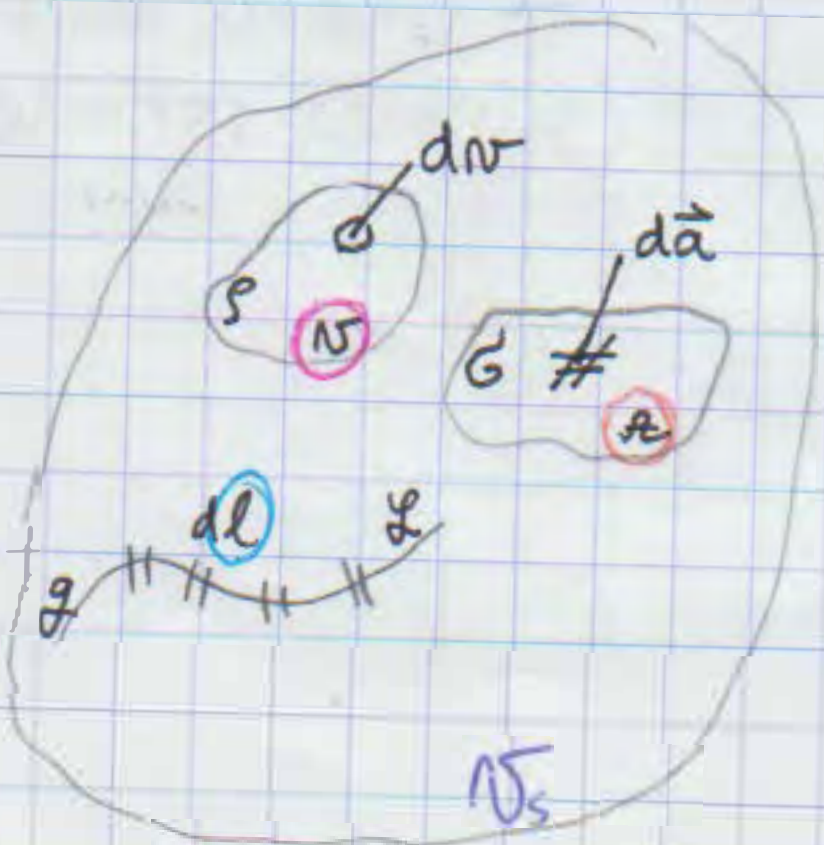
4) Točkasti naboj



$$Q = \sum_{i=1}^n dQ_i$$

seštevek različnih vrst naboja:

$$Q_{\text{vs}} = \int_V \rho dV + \int_A \sigma dA + \int_L \lambda dl + \sum_{i=1}^n dQ_i$$

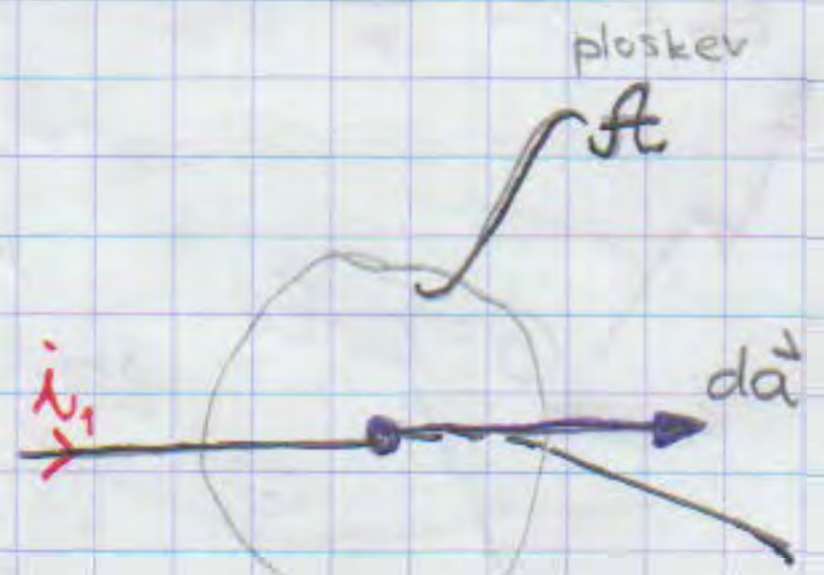


Alta

Električni tok [A]

velike črke časovno so nespremenjive (I, U, ...)

I - i



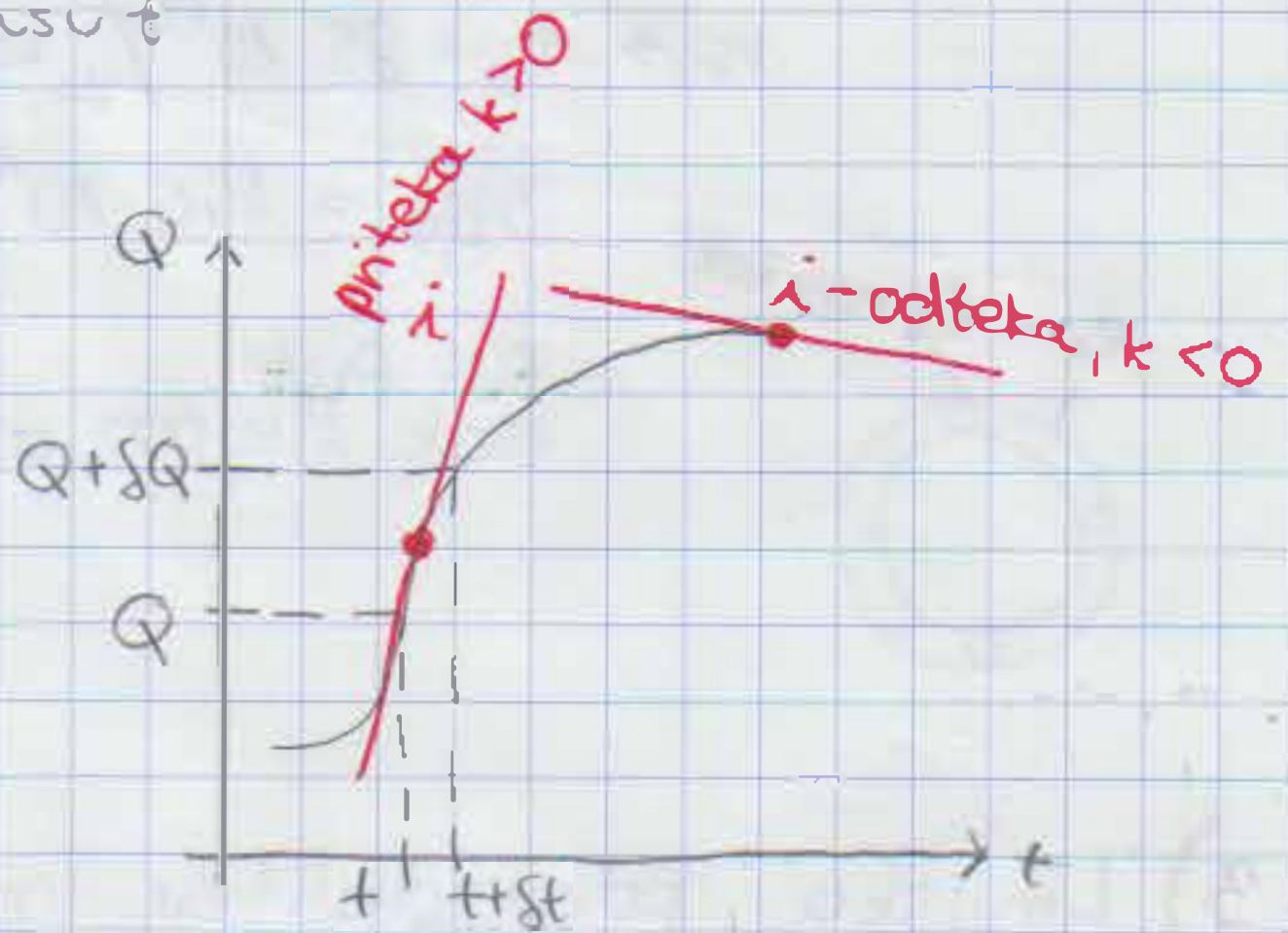
definicija pretakanja teka

$$i = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t}$$

$$\frac{\int Q_{\text{skoz}} \text{ v } \Delta t}{\Delta t}$$

mnostina naboja v nekem casu t

$$i = \frac{dQ}{dt}$$



$$i = \lim_{\Delta t \rightarrow 0} \frac{Q + \Delta Q - Q}{t + \Delta t - t} = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

zglede tok v žici



tok je čas. funkcija

$$i(t) = i = I_{\text{max}} \cdot \sin \omega t$$

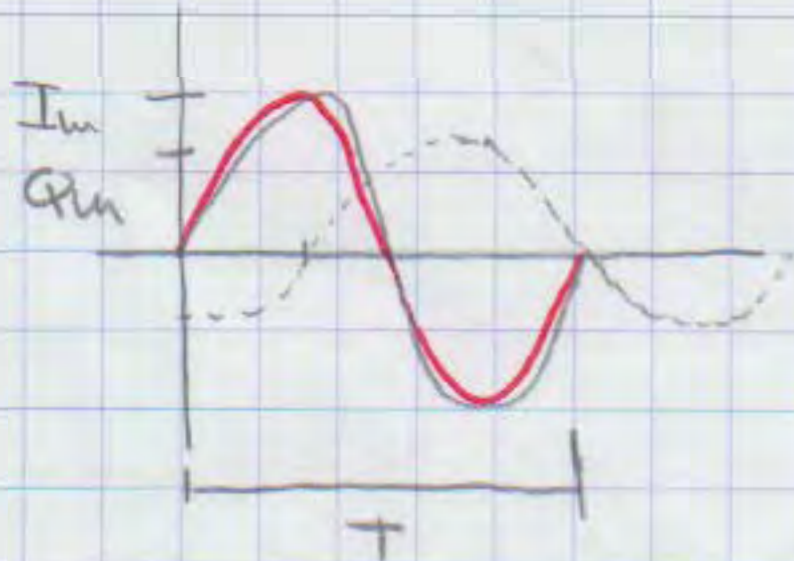
amplituda

I - časovno konstanten tok

časovno spremenljiv tok i

kržna frekvenca / kotna hitrost

$$\omega = 2\pi \cdot f$$



$$T = 1/f$$

$$i = Q' \rightarrow dQ = i dt$$

$$Q = \int I_{\text{max}} \cdot \cos \omega t \cdot \omega^{-1} (-1) + Q_0 = 0$$

$$Q_t = -\frac{I_{\text{max}}}{\omega} \cos \omega t = \frac{I_{\text{max}}}{\omega} \cos(\omega t + \pi)$$

$$Q_{\text{m}} = \frac{I_{\text{max}}}{\omega}$$

π se doda da ni več v minusu periode

vaja:

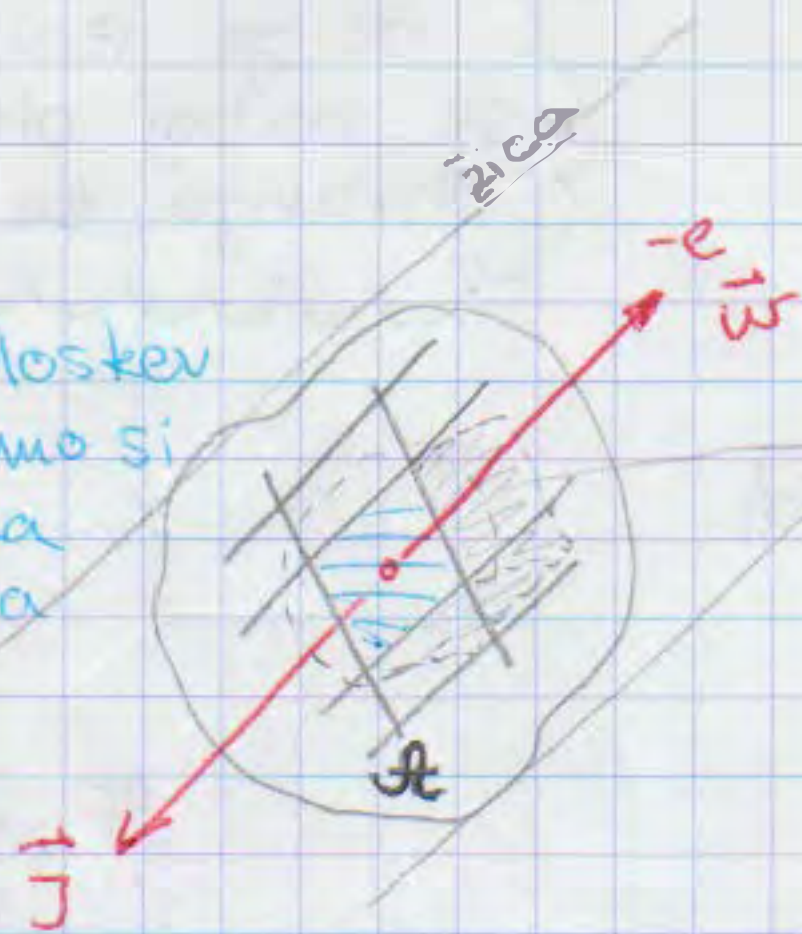
$$f = 50 \text{ Hz}$$

$$I_{\text{m}} = 1 \text{ A} \Rightarrow Q_{\text{m}} = \frac{1 \text{ A}}{2\pi \cdot 50 \text{ s}^{-1}} = 3,18 \text{ mC}$$

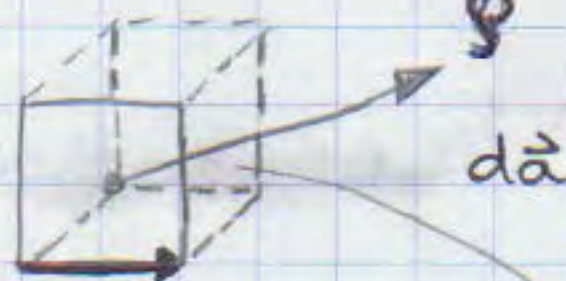
$$n = \frac{3 \cdot 10^{-3} \text{ C}}{1,6 \cdot 10^{-19} \text{ C}} = 2 \cdot 10^{16} \text{ elektronov}$$

J - Gostota električnega toka - ploskovna gostota

Vzame ploščo predstavljamo si da je kocka ker je izrezeta iz žice



prostorsko porazdeljen naboj



$$\vec{dl} = \vec{w} \cdot \vec{t}$$

$$\delta Q = \rho \cdot dV = \rho$$

tok iz majhnega izseka

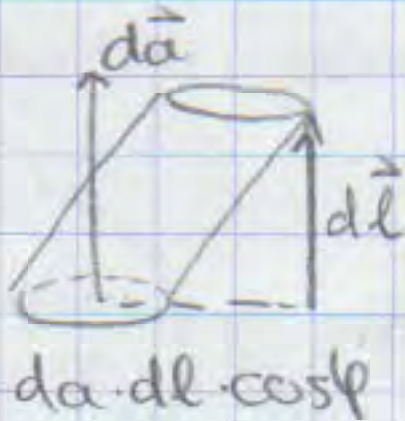
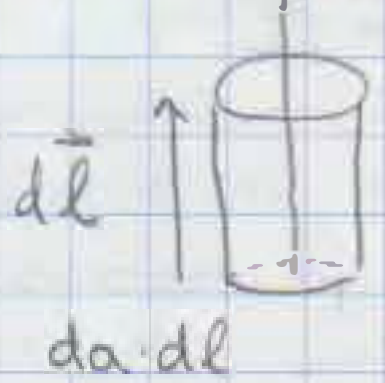
$$di = \frac{\delta Q}{\delta t} = \rho \frac{d\vec{a} \cdot d\vec{l}}{dt} = \rho \cdot \vec{w} \cdot d\vec{a}$$

$$\vec{J} = \rho \cdot \vec{w}$$

$$di = \vec{J} \cdot d\vec{a} \Rightarrow J = \frac{di}{da} \left[\frac{A}{m^2} \right]$$

$$\delta i = \vec{J} \cdot \vec{sa}$$

žica



zgled:

Cu

$$J = 4 A/m^2 = 4 \cdot 10^6 A/m^2$$

prostih nabojev $|\rho_{Cu}| = 1,34 \cdot 10^{10} C/m^3$

hitrost elektronov na meter $w = \frac{J}{\rho} = 300 \mu m/s$

fun fact

hitrost med elektroni $10^5 m/s$



pot elektrona

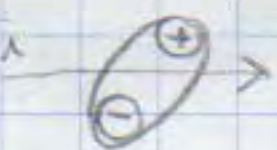
↳ Konduktivni tok

↳ Konvektivni tok

merita se

makroskopska

↳ Polarizacijski tok



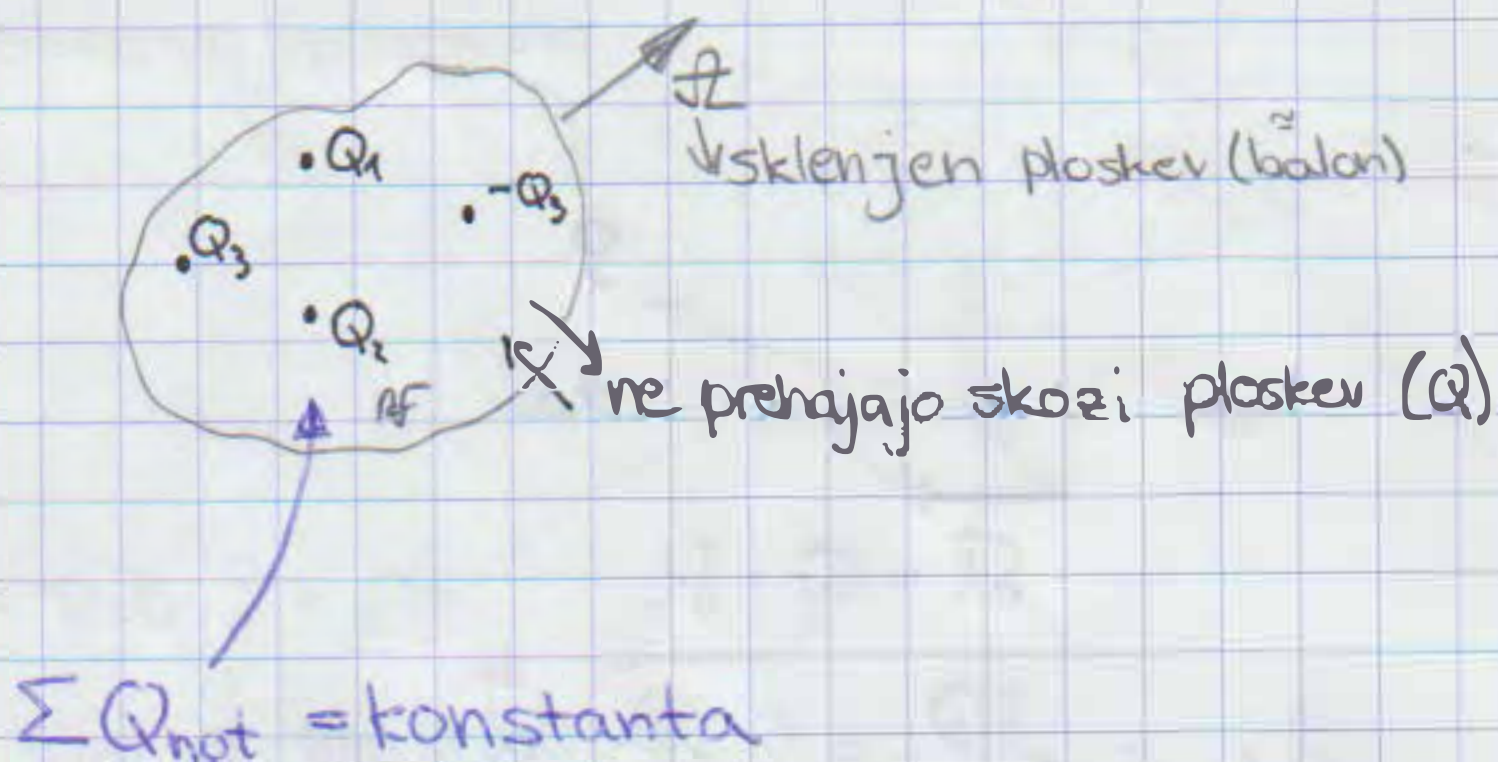
poli se obračajo v smeri toka

↳ Magnetizacijski tok

v notranjosti snovi

Zakon o ohranitvi naboja

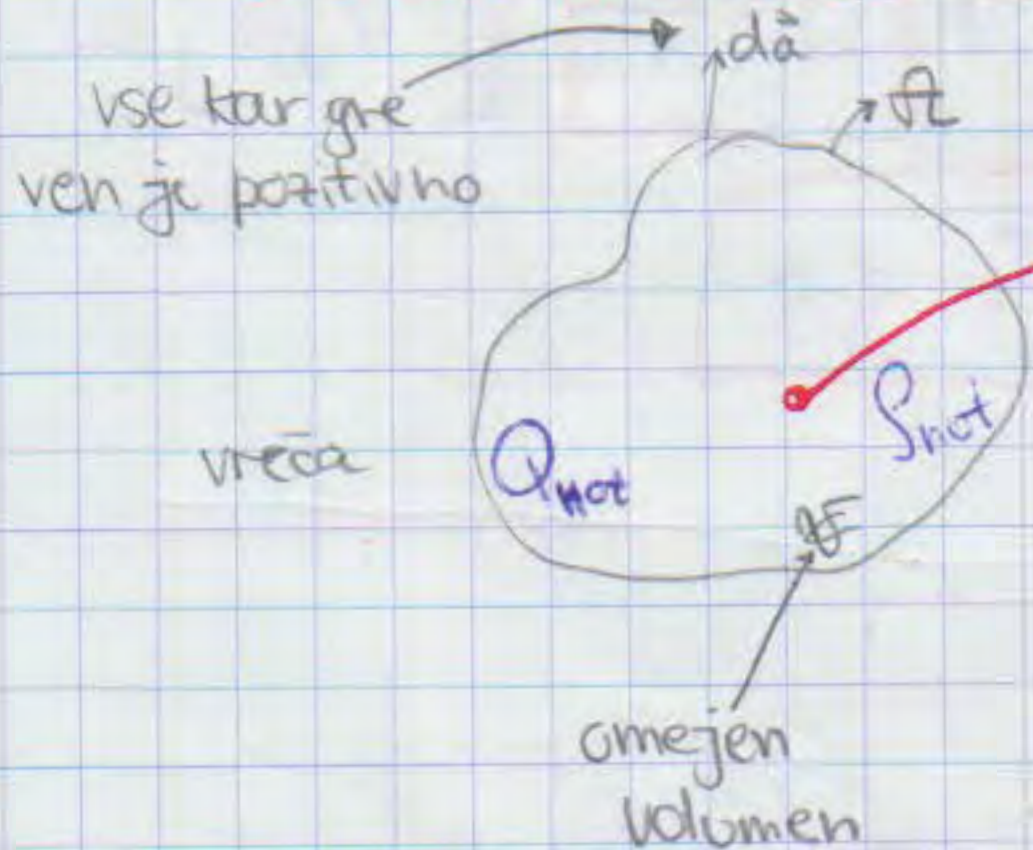
če je izoliran sistem se naboj ohranja... ohranja se množina



if...volumen sklenjene ploskve

Kontinuitetna enačba

pove kaj se dogaja če naboji prehajajo



$Q_{not} + Q_{zun} = K$

konstan

$\frac{d}{dt}$

$\frac{dQ_{not}}{dt} + \frac{dQ_{zun}}{dt} = 0$

če se pojavi migracija naboja govorimo o toku

$\frac{dQ_{not}}{dt} = - \frac{dQ_{zun}}{dt}$

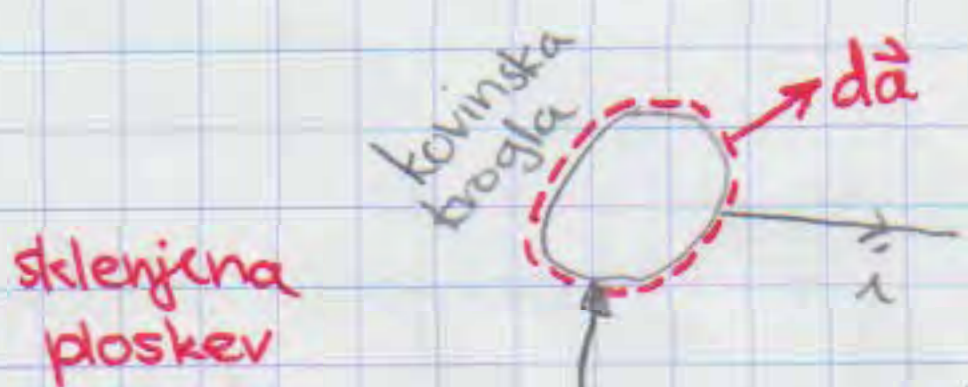
odvod

večanje not. naboja

zmanjšanje not. naboja

$i = \frac{Q_{zun}(t+dt) - Q_{zun}(t)}{t+dt - t} = \frac{dQ_{zun}}{dt} = - \frac{dQ_{not}}{dt}$

kok jih gre in za tok se zuni zmanjša naboj



$i = - \frac{dQ}{dt}$

notranji naboj

naboj ki gre v časovni funkciji t ven iz ploskve

KONTINUITETNA ENAČBA

$i = di = \oint \vec{j} \cdot d\vec{a}$

krog označuje sklenjeno ploskev

$Q = \int \rho \cdot dV$

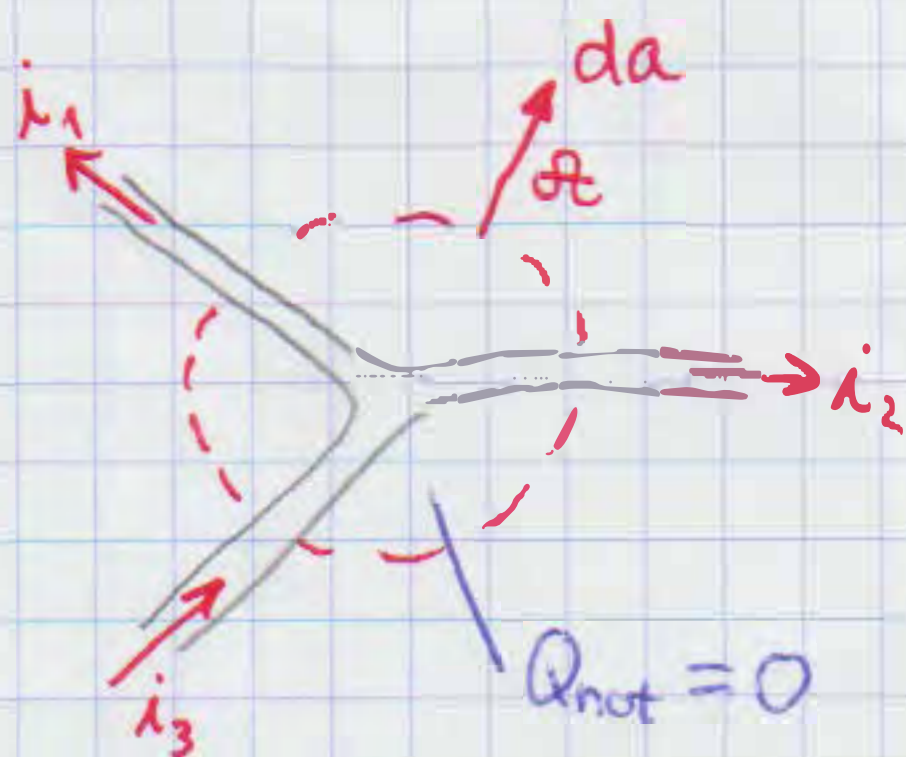
$\oint \vec{j} \cdot d\vec{a} = - \frac{d}{dt} \left(\int \rho \cdot dV \right)$

$\sum_{k=1}^n i_k$

pove, da gre za zmanjšanje

I. Kirchhoffov zakon

po definiciji:
 $i_1 + i_2 = i_3$



definicija:
 Vsota pritekajočih tokov je enaka vsoti odtekajočih tokov

$$\sum_{k=1}^n i_k = 0$$

$$\sum i_{\text{odtekajoči}} - \sum i_{\text{pritekajoči}} = 0$$

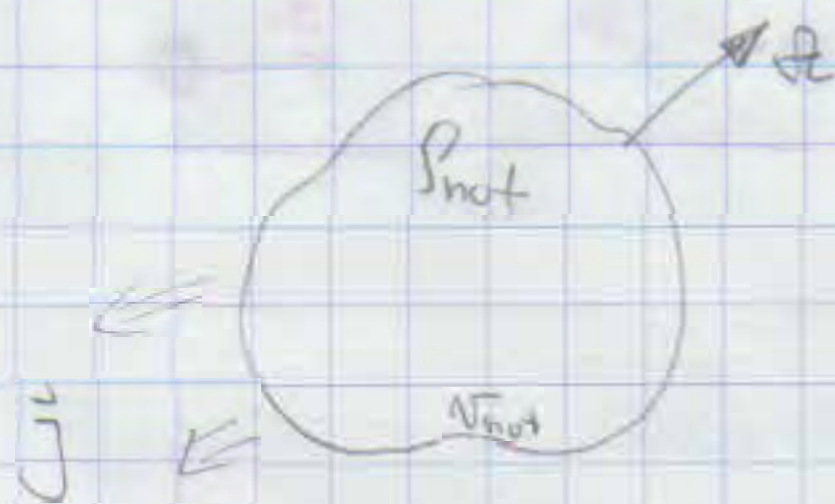
$$\boxed{\sum i_{\text{odt.}} = \sum i_{\text{pri.}}}$$

Razrede polj

$$\oint \vec{j} \cdot d\vec{a} = - \frac{d}{dt} \int_{S_{\text{not}}} \rho_{\text{not}} \cdot dV$$

prostorska porazdelitev naboja po ploskvi S

1) Elektrostatika: $j=0$
 naboji se ne gibljejo



$$0 = - \frac{d}{dt} \int_{V_{\text{not}}} \rho_{\text{not}} \cdot dV \rightarrow \rho_{\text{not}} = \text{konstanta (časovno)}$$

↳ prostorska funkcija

Kondenzatorska vezja C

2) Tokovna polja

$j = \text{konstanta}$ (Enosmerni toki)

$$\oint_{S \parallel j} \vec{j} \cdot d\vec{a} = k = - \frac{d}{dt} \int \rho_{\text{not}} \cdot dV$$

$j = k$

$$k_1 t + k_2 = \int \rho_{\text{not}} \cdot dV$$

⇓

$$\boxed{k_1 = 0}$$

naboj s časom narašča

Alta

3) Magnetostatika

$I = \text{konstanta}$

4) Dinamika

↳ indukcija

↳ vezje časovno spr. količin

* Prehodni pojavi

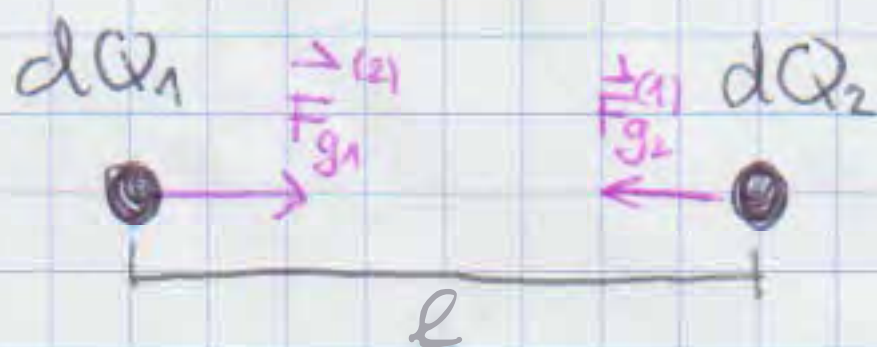
* harmonična vezja

ELEKTROSTATIKA

Coulombova sila 1785 ponra o silah, ki delujejo ena na drugo

$$y \propto x$$

$$y = k \cdot x$$



$$F_e \propto \frac{dQ_1 \cdot dQ_2}{R^2}$$

sorazmerno

$$\vec{F}_e = k \cdot \frac{dQ_1 dQ_2}{R^2} \cdot \vec{e}_R$$

$$\vec{F}_g = -G \frac{dm_1 \cdot dm_2}{R^2} \cdot \vec{e}_{R_{gij}}$$

dielektričnost
vakuma / influenčna
konstanta

$$\frac{1}{4\pi\epsilon_0} = k$$

↳ sili sta enaki

↳ po smeri sta nasprotni

$\oplus \oplus$ $\ominus \ominus$ odbojne sile
 $\oplus \ominus$ privlačne

↳ sila sorazmerna $dQ_1 \cdot dQ_2$

↳ sila sorazmerna $\frac{1}{R^2}$

$$k = \frac{1}{4\pi\epsilon_0}$$

4π polni prostorski kot $4\pi R^2 \rightarrow R^2$ površina krogle
ker je volumen 3D je 4π

2π navaden kot ima $d\pi$ ker je v ravnini

ϵ_0 dielektričnost praznega prostora

⑧ μ_0 permeabilnost praznega prostora

$$c^2 = \frac{1}{\mu_0 \cdot \epsilon_0}$$

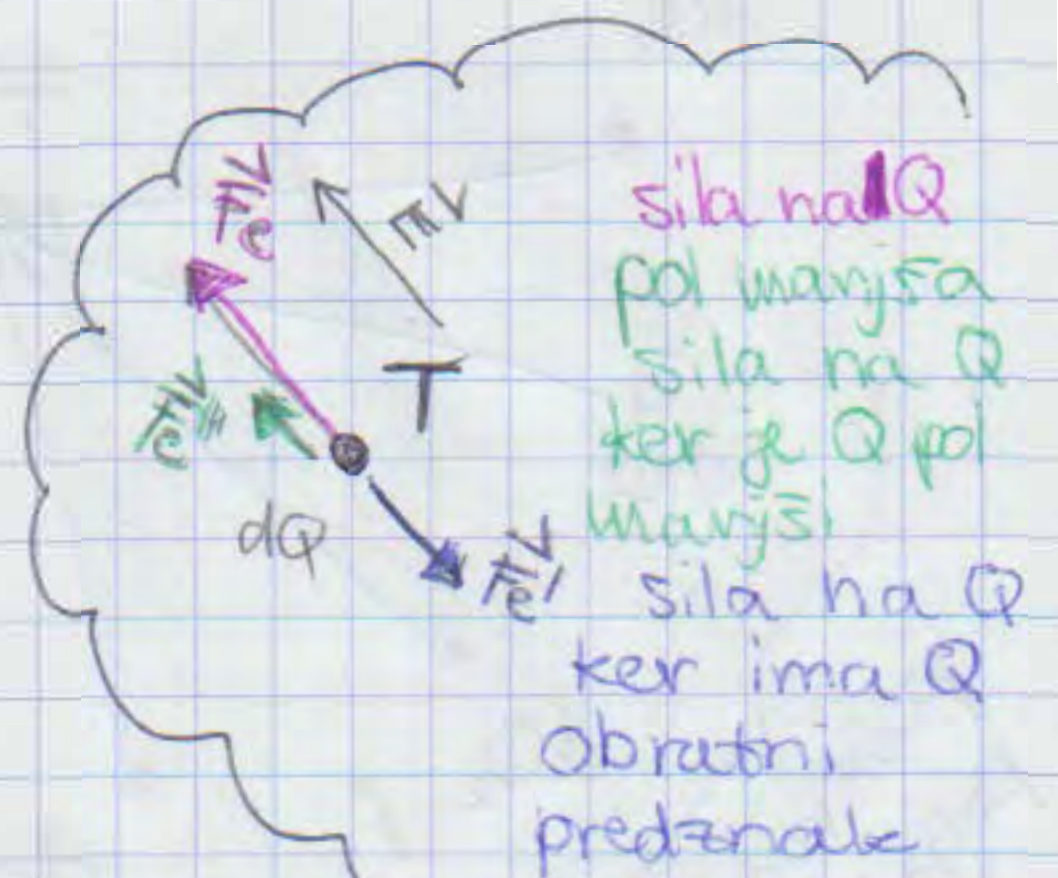
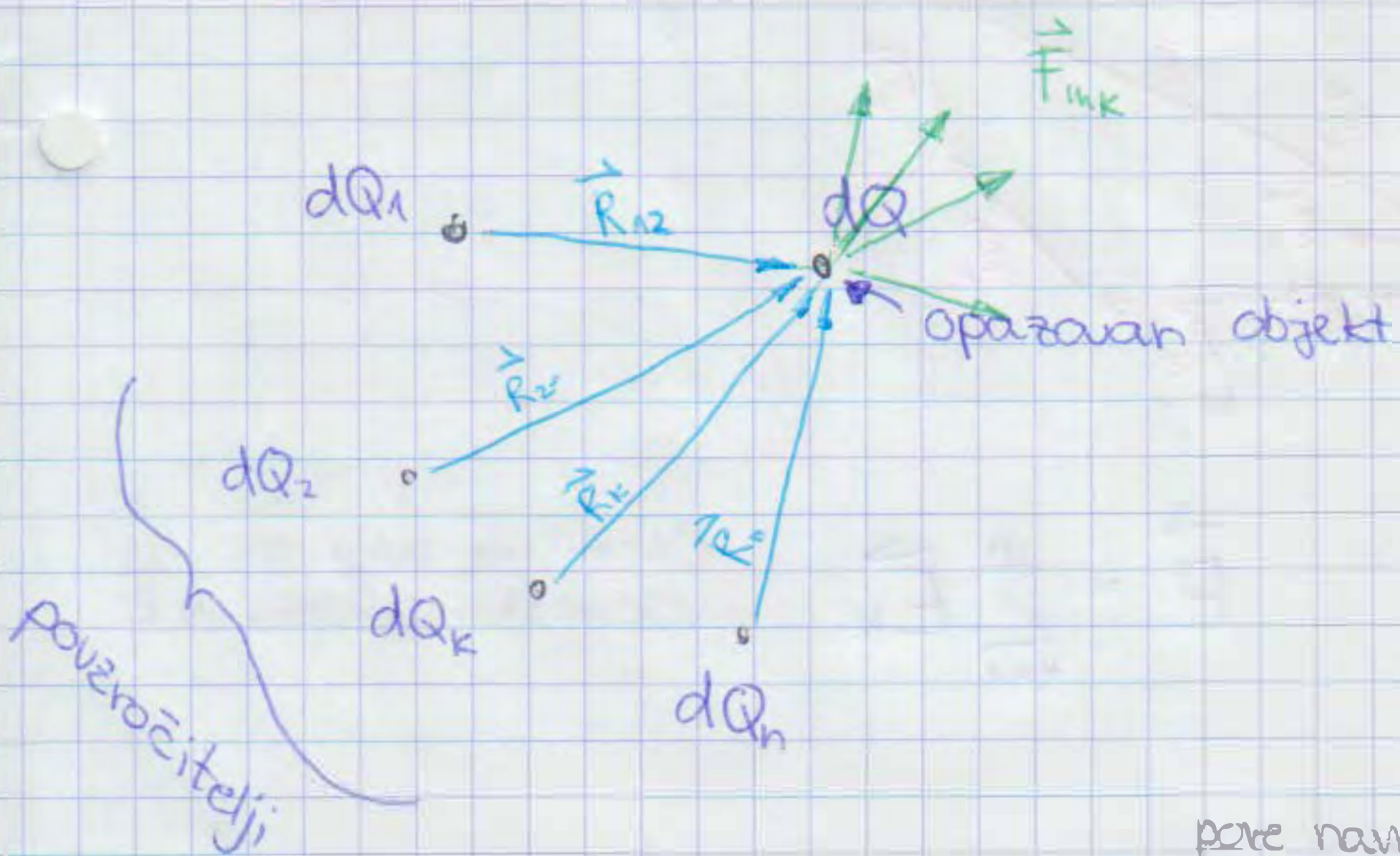
$$1 = \epsilon_0^2 \mu_0 \epsilon_0$$

$$\frac{10^{-3}}{9 \cdot 4\pi} \frac{As}{Vm}$$

$$8,85 \cdot 10^{-12} \frac{As}{Vm}$$

$$3 \cdot 10^8 \frac{m}{s}$$

Sila in sosesčini drugih nabojev



$$\vec{dF}_{12} = \frac{dQ_1 dQ}{4\pi\epsilon_0 R_{12}^2} \cdot \frac{\vec{R}_{12}}{R_{12}} \quad \left\{ \begin{array}{l} \text{pore navn smer} \\ \text{enotski vektor} \end{array} \right.$$

$$\vec{dF}_{ek} = \frac{dQ_k \cdot dQ}{4\pi\epsilon_0 R_k^2} \cdot \frac{\vec{R}_k}{R_k}$$

$$\vec{dF}_{ek} = \frac{dQ_k dQ}{4\pi\epsilon_0 R_k^3} \cdot \vec{R}_k$$

seštevek sil ki delujejo na naboj dQ

$$\vec{dF}_e = \sum \vec{dF}_k = \frac{dQ}{4\pi\epsilon_0} \sum_{k=1}^n \frac{dQ_k \vec{R}_k}{R_k^3}$$

Vektor električne poljske jakosti $\vec{E}$

$\vec{E}$ upada s kvadratom oddaljenosti

$$\vec{E} = \frac{\vec{dF}_e}{dQ}$$

normiramo silo z nabojem

vektor električne poljske jakosti

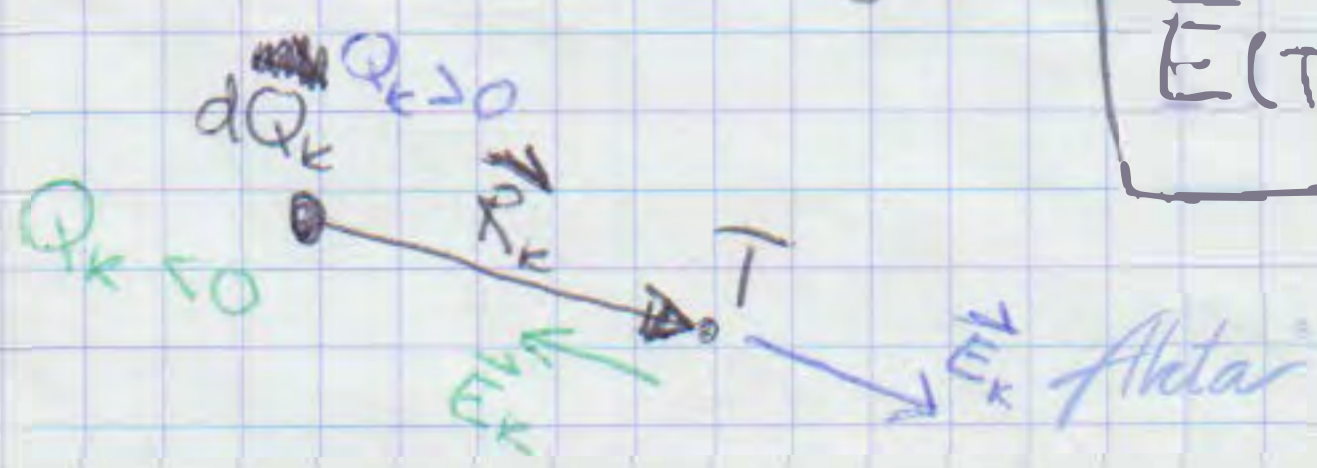
$$\vec{E} = \frac{\vec{dF}_e}{dQ} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{dQ_k \vec{R}_k}{R_k^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum \frac{Q_k \vec{R}_k}{R_k^3}$$

v smeri vektorja $\vec{R}$

Coulonovo ELE. POLJE

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \int \frac{dQ(r) \vec{R}}{R^3}$$

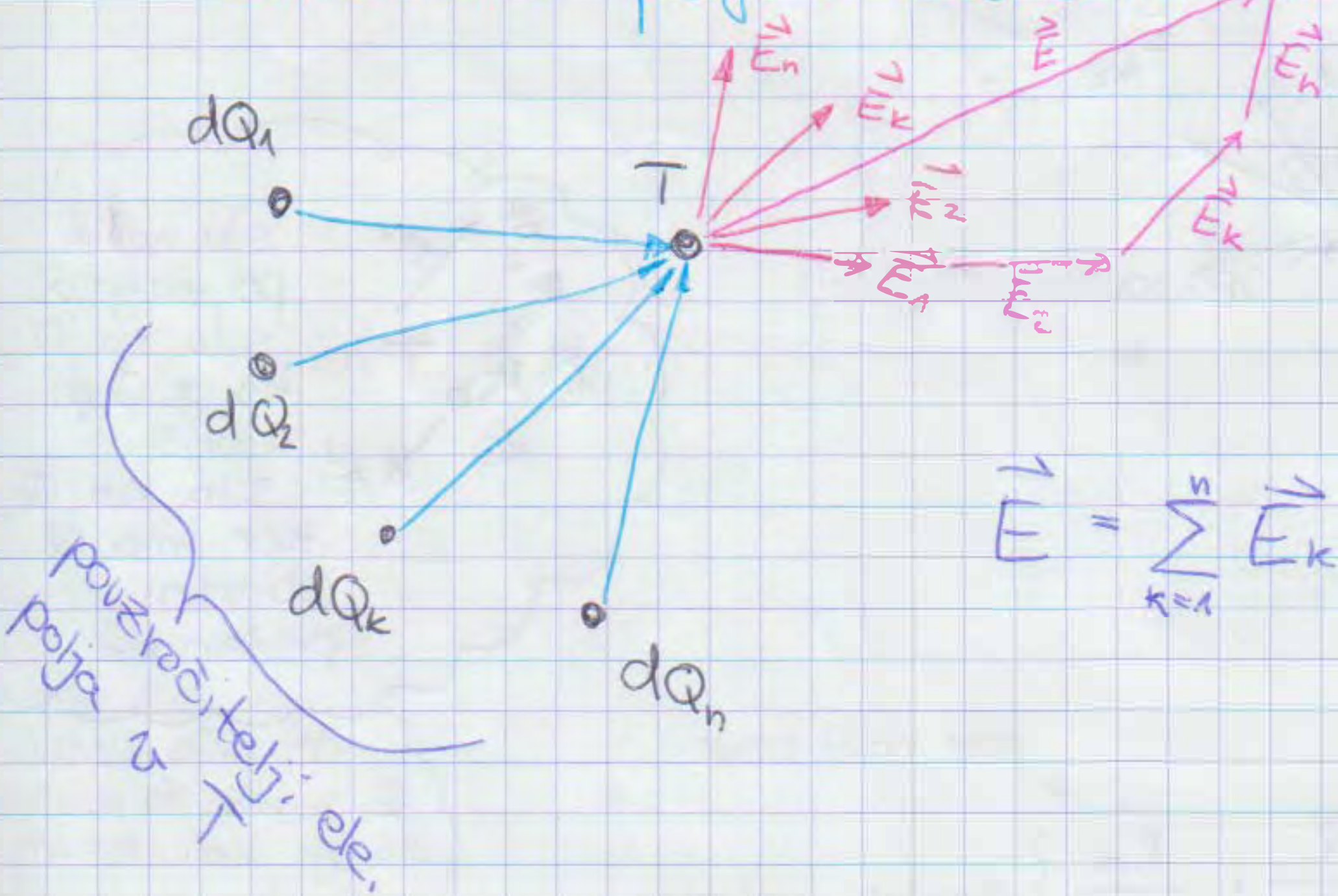


$$F_g = G \frac{m_1 m_2}{R^2}$$

$$F_g = g \cdot h$$

FIZIKA

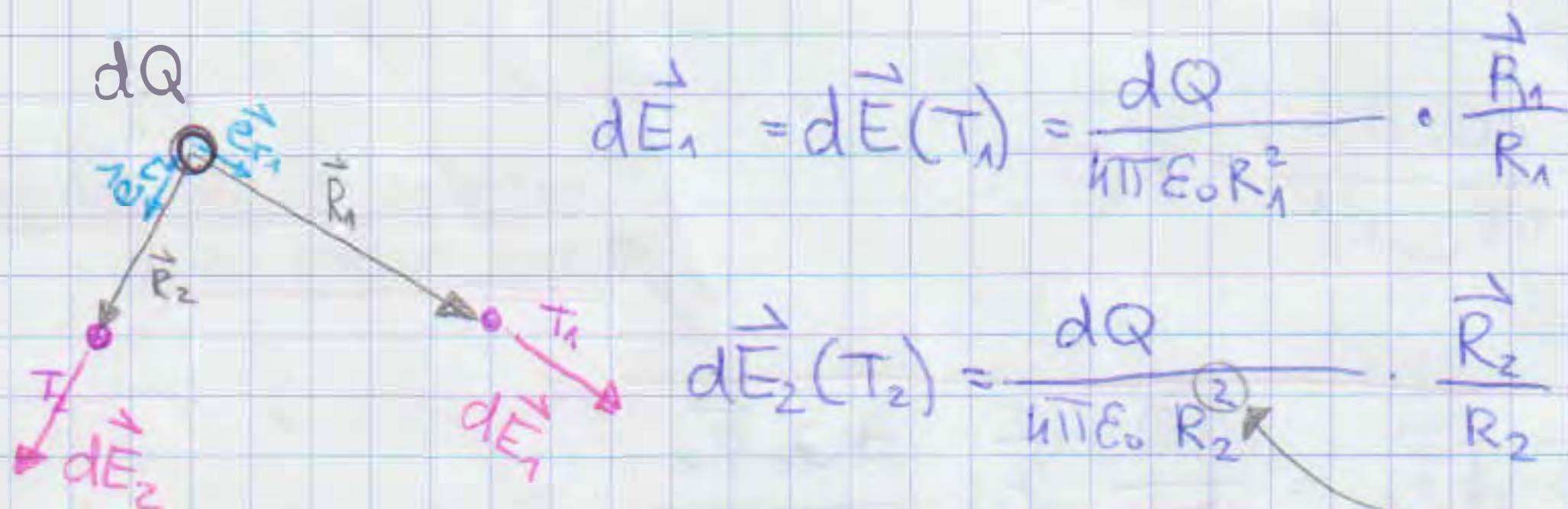
Električno polje v sosesčini n nabojev



$$\vec{E} = \sum_{k=1}^n \vec{E}_k$$

električno polje se vektorsko seštevaja v $\vec{E}$

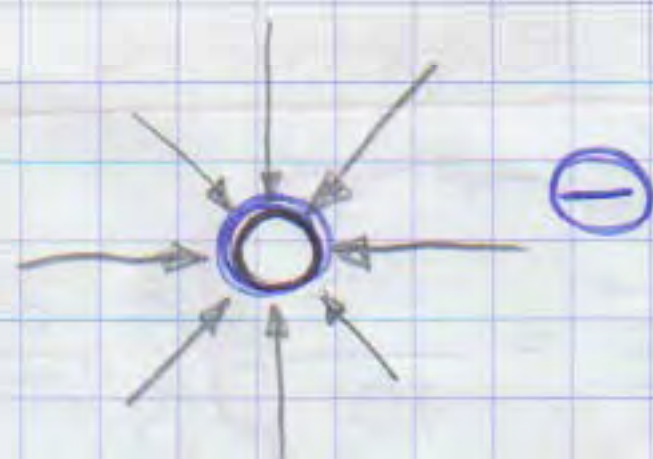
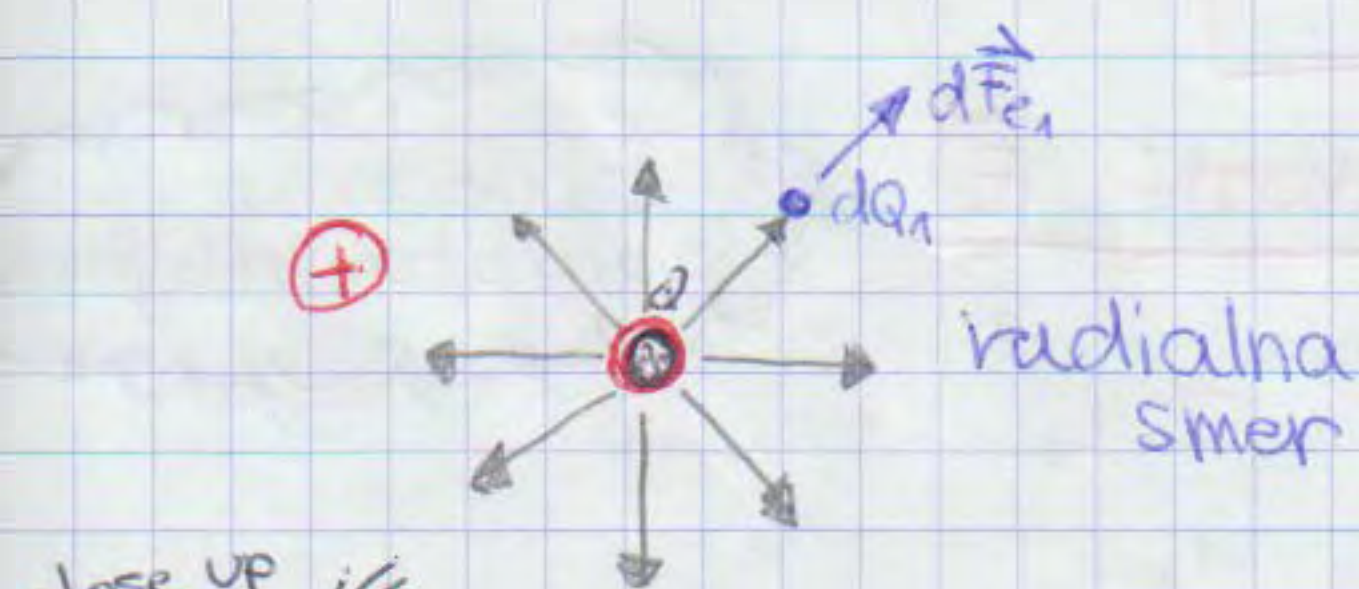
Električno polje točkastega naboja



$$d\vec{E}_1 = d\vec{E}(T_1) = \frac{dQ}{4\pi\epsilon_0 R_1^2} \cdot \frac{\vec{R}_1}{R_1}$$

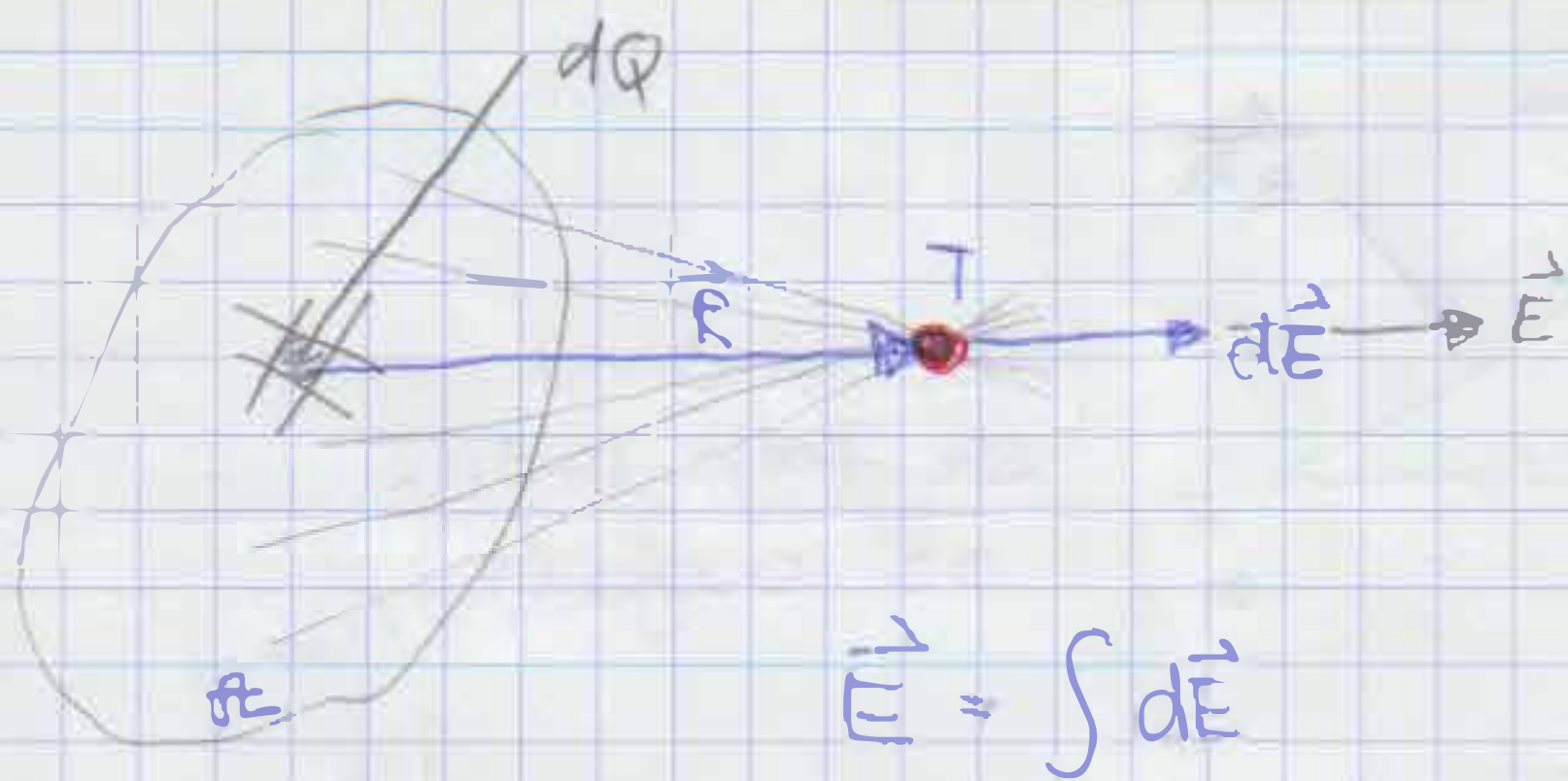
$$d\vec{E}_2(T_2) = \frac{dQ}{4\pi\epsilon_0 R_2^2} \cdot \frac{\vec{R}_2}{R_2}$$

električno polje pada s 2. potenco razdalje



Ele. Polje je stanje prostora na katerega deluja sila.

na električen objekt



$\vec{E}$ je vsi naelektronega območja

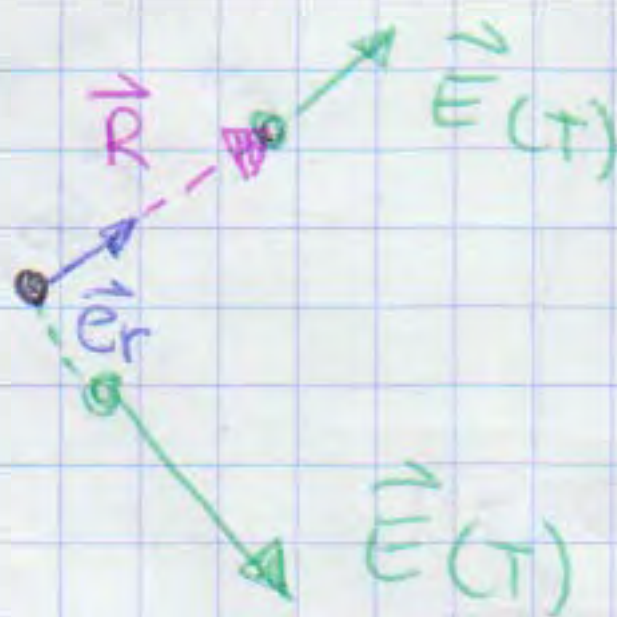
$$d\vec{E} = \frac{d\vec{F}}{dQ}$$



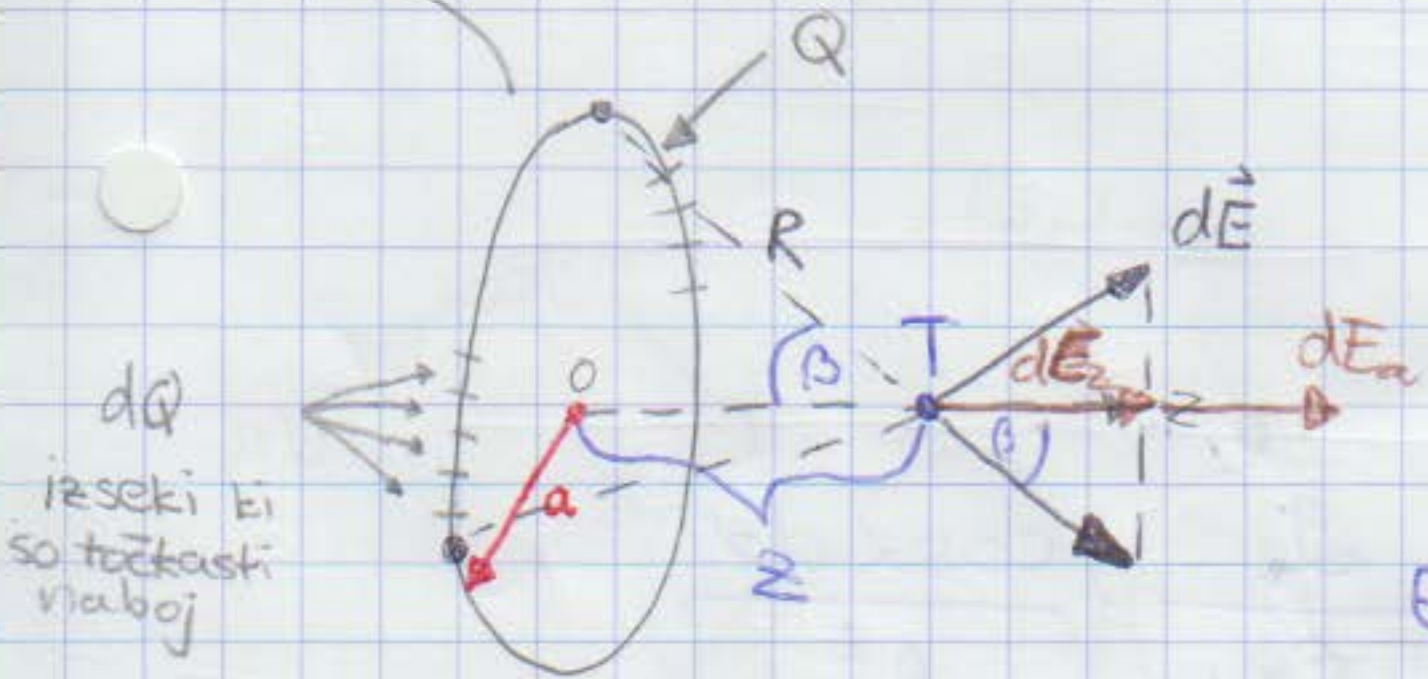
paraznčitelj

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \cdot \vec{e}_R$$

načrta naboj



območje



$$dE_z = \frac{dQ}{4\pi\epsilon_0 R^2} \cdot \cos\beta = \frac{z \cdot dQ}{4\pi\epsilon_0 R^3}$$

$\cos\beta = \frac{z}{R}$

$$E_z(T) = \int dE_z = \int \frac{z \cdot dQ}{4\pi\epsilon_0 R^3} = \frac{z}{R^3 4\pi\epsilon_0} \int dQ =$$

$$E_z(T) = \frac{z \cdot Q}{4\pi\epsilon_0 R^3} = \frac{Qz}{4\pi\epsilon_0 (\sqrt{a^2 + z^2})^{3/2}}$$

$R = \sqrt{a^2 + z^2}$

$$\vec{E}(T) = \frac{Q \cdot z \cdot \vec{e}_z}{4\pi\epsilon_0 (\sqrt{a^2 + z^2})^{3/2}}$$

Alstar

seštevek delnih prispevkov

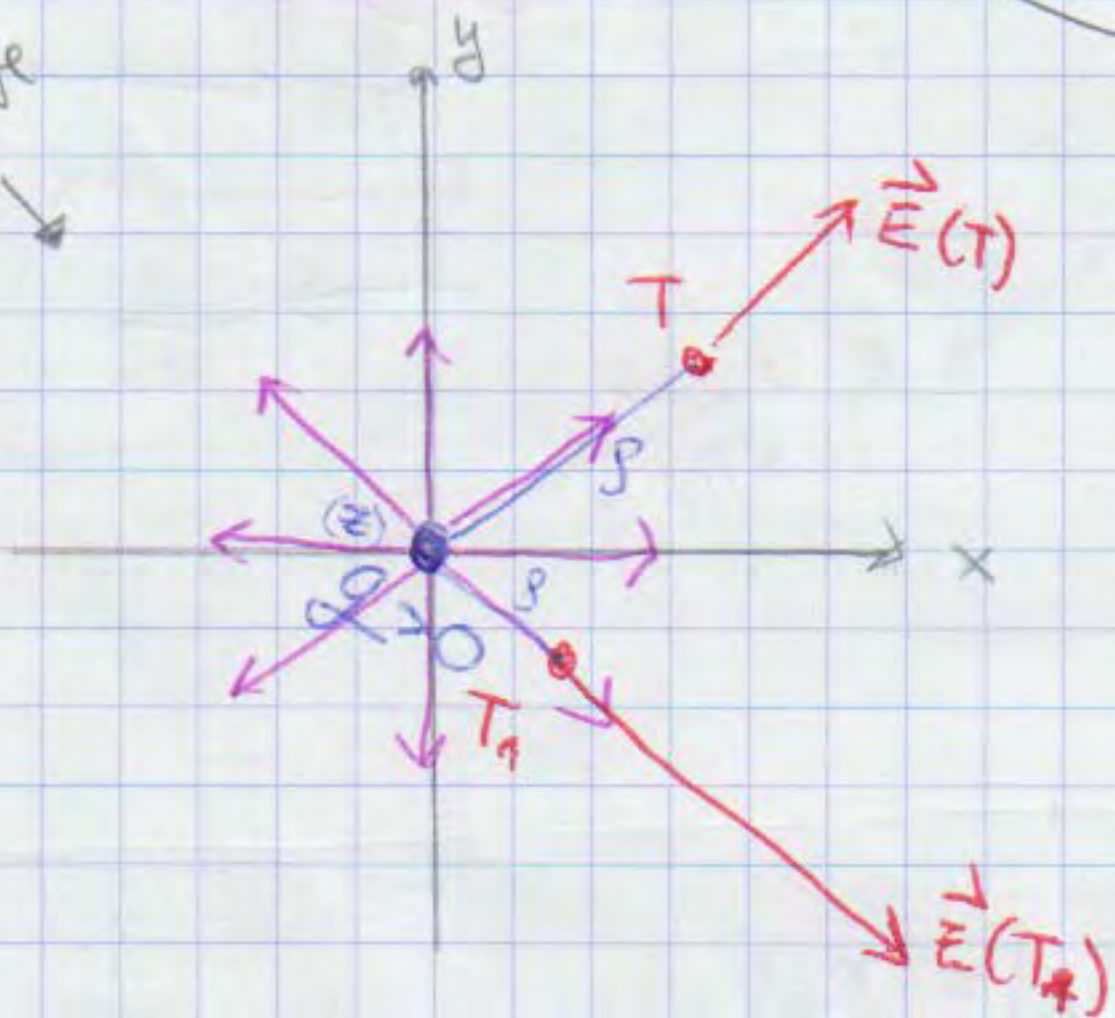
$$E_s(T) = \int dE_s(T) = \int_0^{\frac{\pi}{2}} \frac{Q}{2\pi\epsilon_0 r} \cdot \cos\alpha \cdot d\alpha =$$

$$E_s(T) = \frac{Q}{2\pi\epsilon_0 r}$$

meja je od 0° do $\frac{\pi}{2}$ ker jemljemo pare

pada s prvo potenco razdalje r

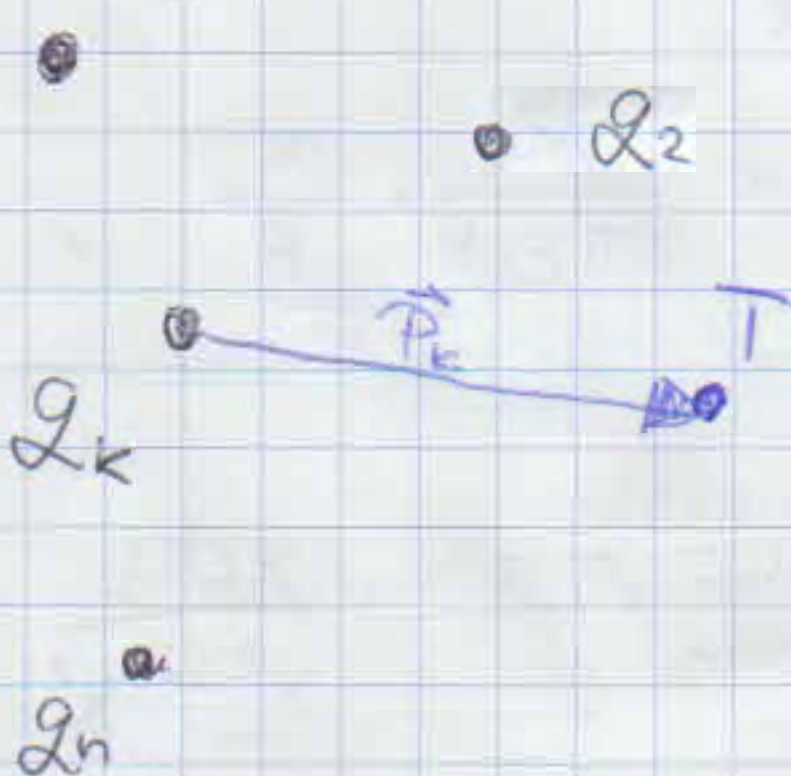
če je $Q > 0$



če je $Q < 0$ so silnice obrnjen proti središču koordinatnega sistema

Snop vzp. premih nabojev

Q_1

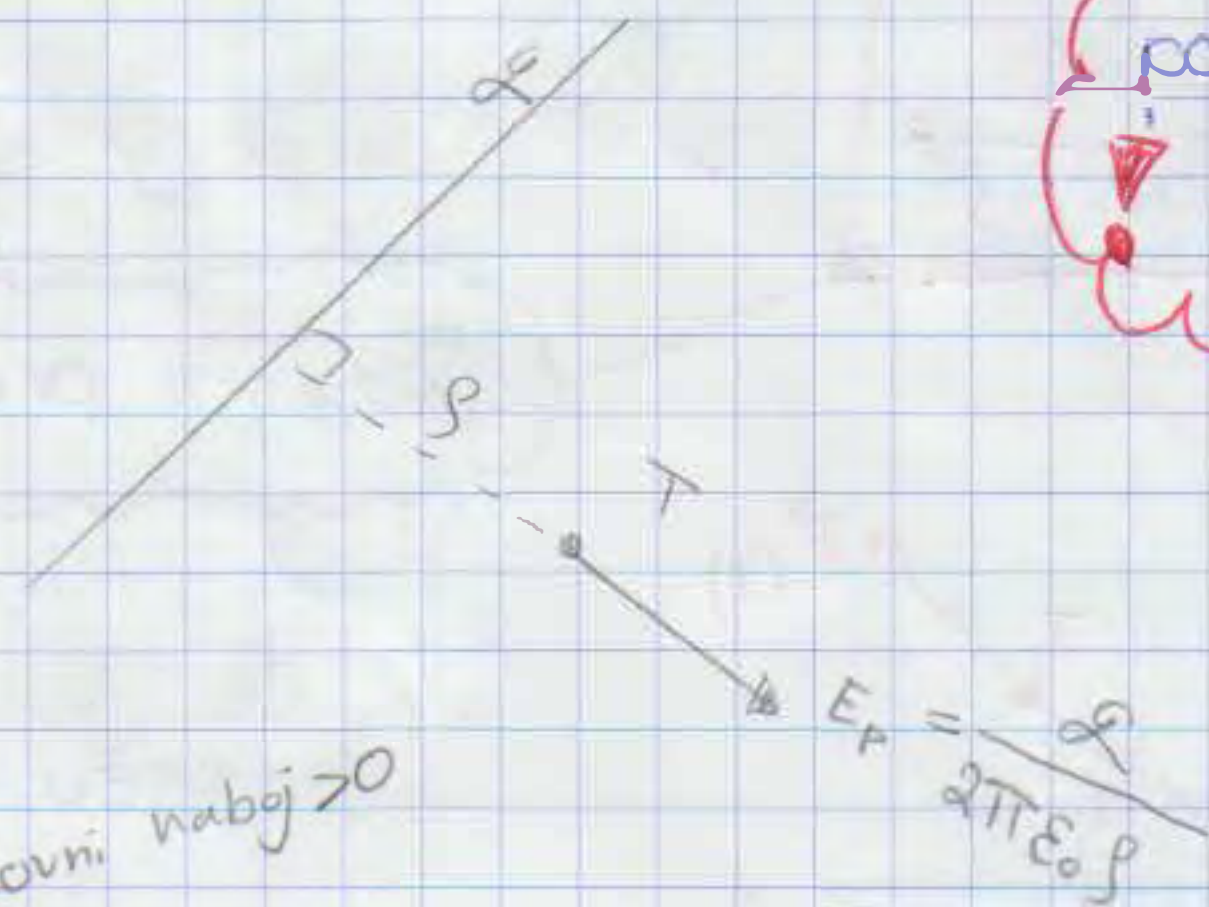
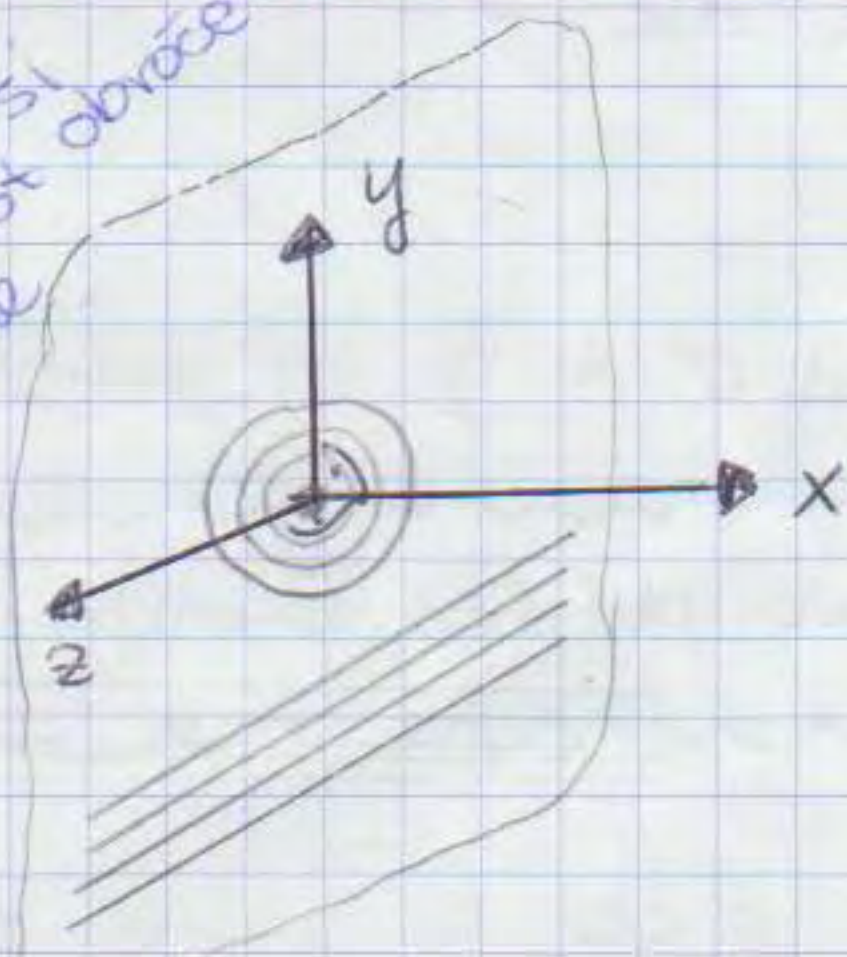


$$\vec{E}(T) = \sum_{k=1}^n \frac{Q_k \vec{r}_k}{2\pi\epsilon_0 r_k^2}$$

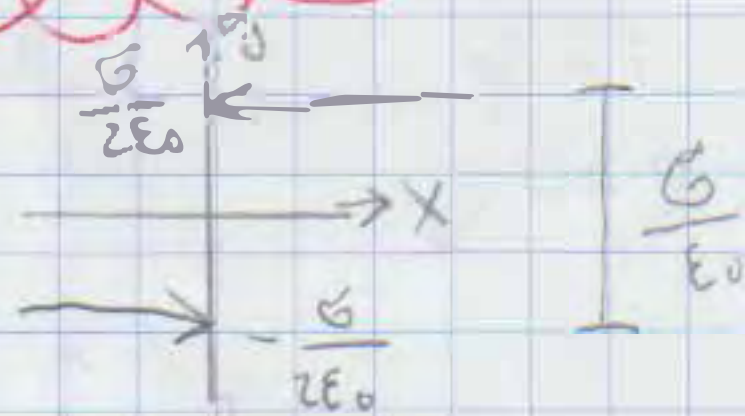
Ele. polje naelektrene "ravnine"

ravnina je neskončna
vzamemo z rezervo besedo

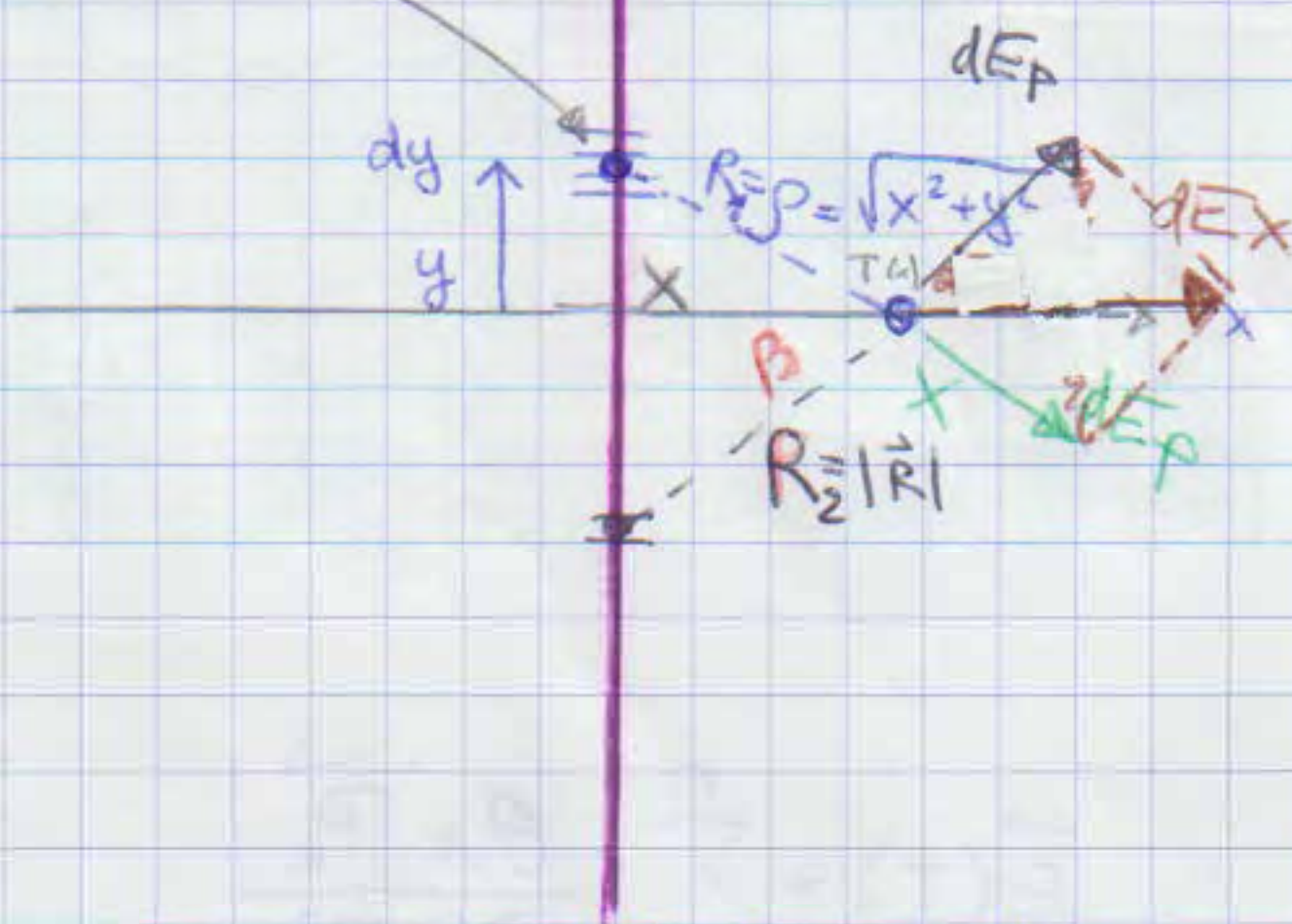
ravnino si
predstavljamo kot obroč



polje v ravnini
je 0.



$dq = \sigma \cdot dy$
 σ - ploskovni naboj > 0
ravnina



$$dE = \frac{2 \cdot dE_x}{\pi \epsilon_0} = 2 \cdot dE_P \cdot \cos \beta = \frac{\sigma}{2 \pi \epsilon_0 R} \cdot \frac{x}{R}$$

$$dE = \frac{\sigma}{\pi \epsilon_0} \cdot \frac{x}{(x^2 + y^2)}$$

$$E = \int dE$$

$$E = \int dE = \frac{\sigma}{\pi \epsilon_0} \int_0^{\infty} \frac{x dy}{x^2 + y^2}$$

$$E = \frac{\sigma}{\pi \epsilon_0} \arctg \frac{y}{x} \Big|_0^{\infty}$$

$$E = \frac{\sigma}{\pi \epsilon_0} \left(\frac{\pi}{2} - 0 \right)$$

$$E_n = \frac{\sigma}{\pi \epsilon_0} \cdot \frac{\pi}{2} = \frac{\sigma}{2 \epsilon_0}$$

normale

ker so prispevki
diferencialni jih seštevamo
z integracijo

$$\frac{x dy}{x^2 + y^2} = \frac{\frac{dx}{x}}{1 + \left(\frac{y}{x}\right)^2}$$

$$s = \frac{y}{x} \Rightarrow \frac{dy}{x} = ds$$

$$\frac{ds}{1 + s^2} \int \frac{ds}{1 + s^2} = \arctg$$

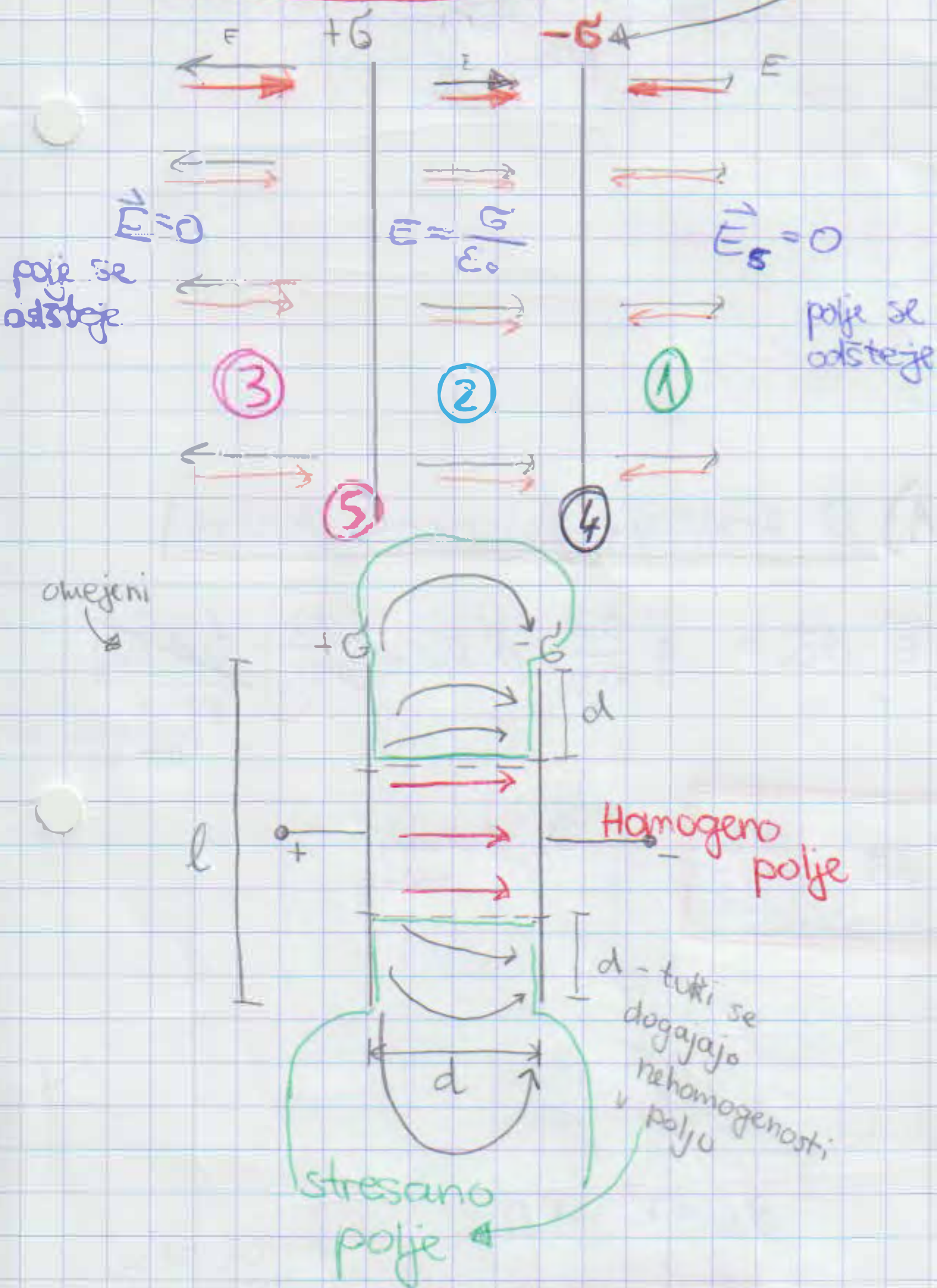
$$\vec{E} = \frac{\sigma}{2 \epsilon_0} \cdot \vec{e}_n$$

polje v prostoru
je konstanta
Homogeno polje



Dve naelektrani ravnini:

lastno polje v ravnini je 0 deluje samo polje od +G



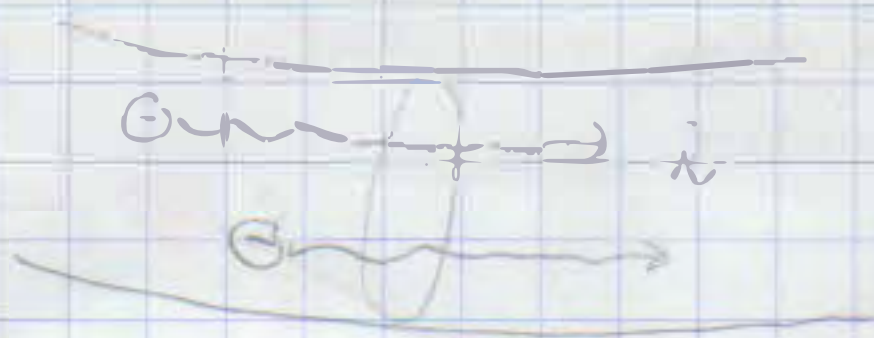
E je konstanten v kondenzatorju

$$E_x = \begin{cases} 0 & , x > d \quad (1) \\ \frac{G}{2\epsilon_0} & , x = d \quad (4) \\ \frac{G}{2\epsilon_0} & , 0 < x < d \quad (2) \\ \frac{G}{2\epsilon_0} & , x = 0 \quad (5) \\ 0 & , x < 0 \quad (3) \end{cases}$$

Električni pretok vektorja $\vec{E}$

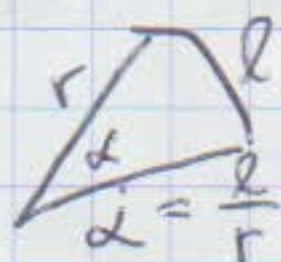
$$i = \int_S \vec{j} d\vec{a}$$

električni tok

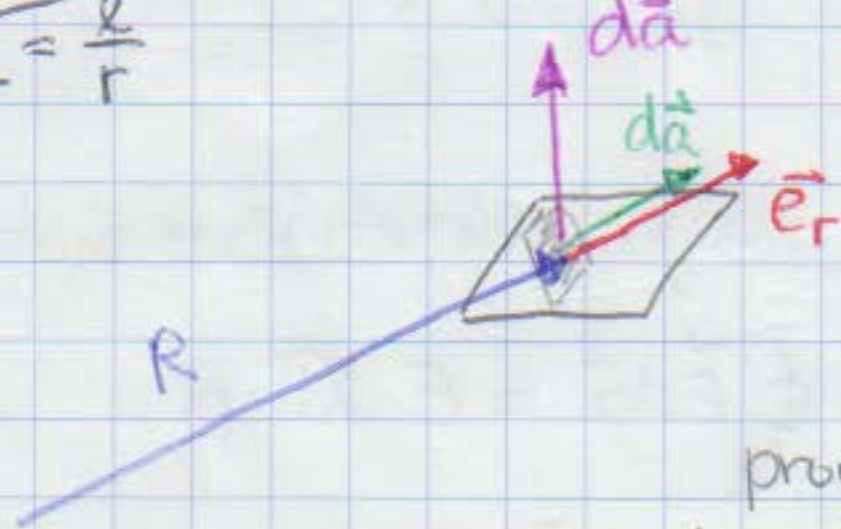


Fluks $\Phi_e = \int_S \vec{E} \cdot d\vec{a}$

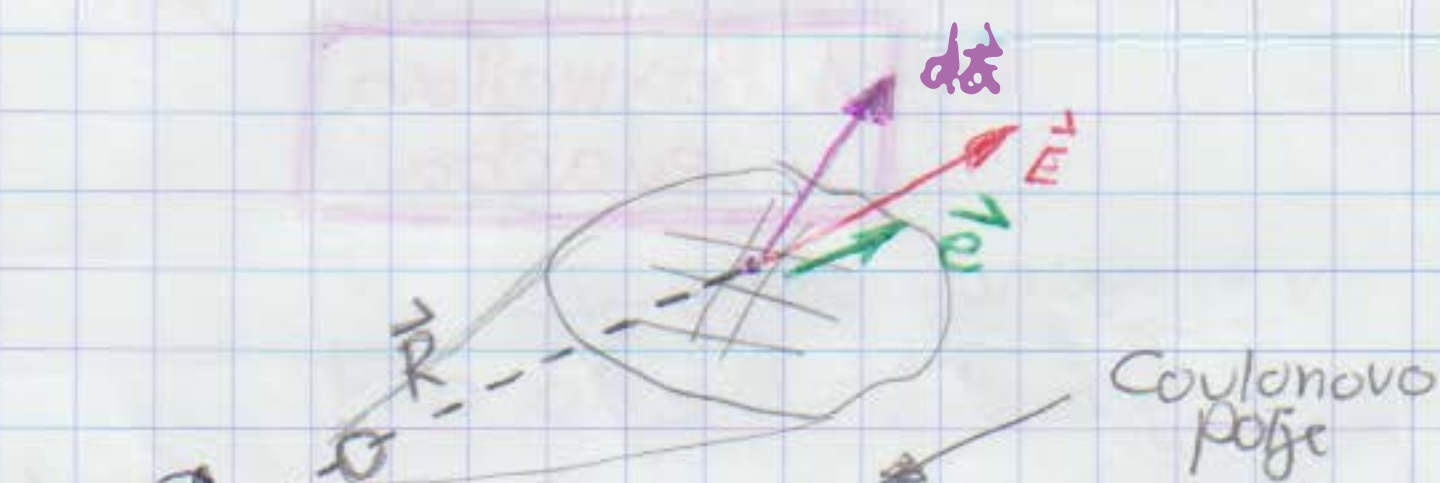
prostorski kot



$$\Omega = \frac{a}{R^2}$$



projiciramo vektor na drug vektor



$$d\Phi_e = \vec{E} \cdot d\vec{a}$$

$$d\Phi_e = \frac{dQ \cdot \vec{R}}{4\pi\epsilon_0 R^3} \cdot d\vec{a}$$

$$d\Phi_e = \frac{dQ}{4\pi\epsilon_0} \cdot \frac{(\frac{R}{R}) \cdot d\vec{a}}{R^2}$$

$$d\Omega = \frac{da'}{R^2}$$



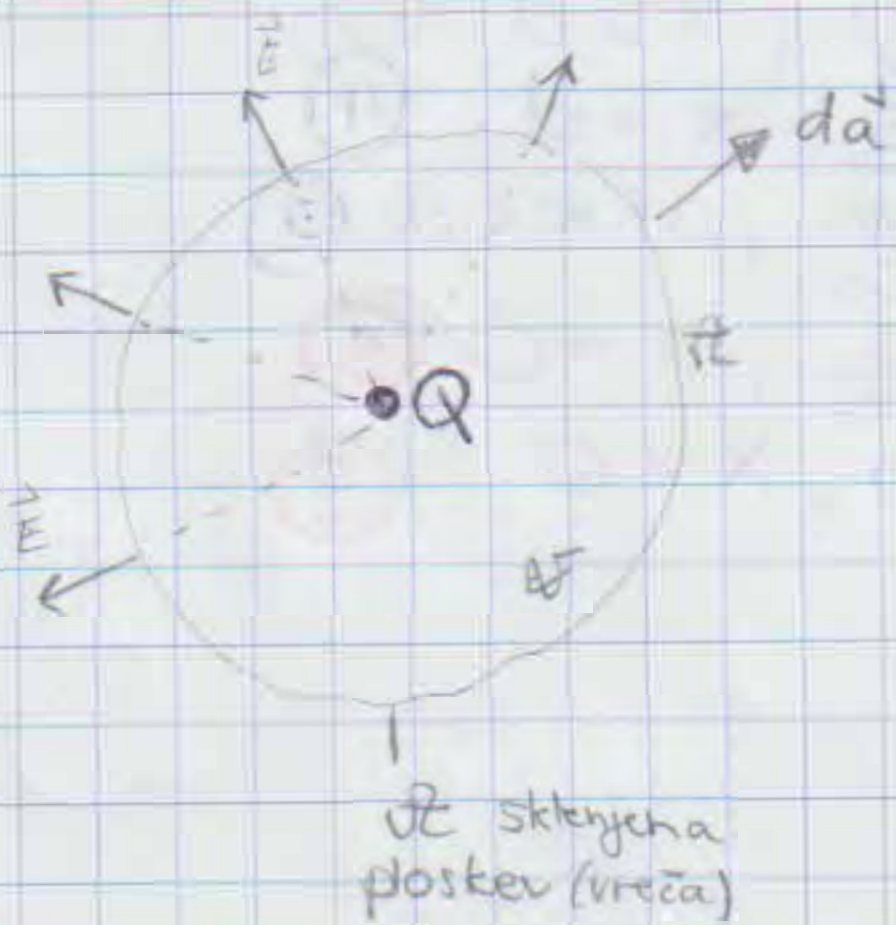
$$d\Phi_e = \frac{dQ}{4\pi\epsilon_0} \cdot d\Omega$$

prostorski kot

elektrni protok

Gaussov zakon

$\vec{E}$



je sklenjena ploštev (vreča)

1) Q znotraj sklenjene ploštre

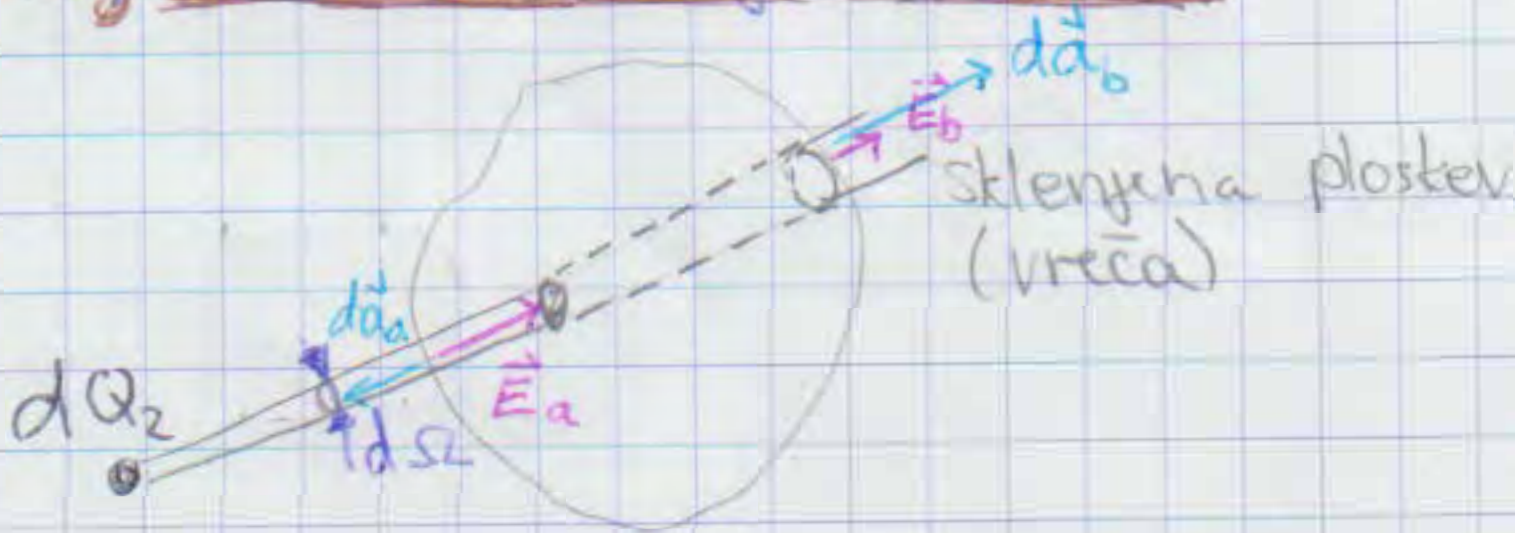
$$\oint_{\mathcal{A}} \vec{E} \cdot d\vec{a} = \oint \frac{dQ}{4\pi\epsilon_0} \cdot d\Omega = \frac{dQ}{4\pi\epsilon_0} \cdot \oint d\Omega$$

poni prostorski kot 4π

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{int}}{\epsilon_0}$$

Q_{int} - notranji naboj

2) Q izven sklenjene ploštre



$\vec{E}$ različne dolžine

E se s kvadratom povečuje

$d\vec{a}$ se pa s kvadratom zmanjšuje

$$da = R^2 \cdot d\Omega$$

$$\oint \vec{E} \cdot d\vec{a} = 0$$

pretok skozi 2 ploskvi je zmeraj enak 0

$$\oint_{\mathcal{A}} \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\epsilon_0}$$

Gaussov stavek

$$\oint_{\mathcal{A}} \vec{x} \cdot d\vec{a}$$

po tej kontinuitetno enačbo

$$\oint \vec{j} \cdot d\vec{a}$$

spremenljiv in konstanta stavka

1. Maxwellova enačba

v kroglnem sistemu

$$\oint_{\mathcal{A}} \vec{E} \cdot d\vec{a} = \oint_{\mathcal{A}} E da \cos\varphi$$

$$\cos\varphi = 1$$

zato ker je kot med vektorjema enak 1

kroglno simetričen sistem

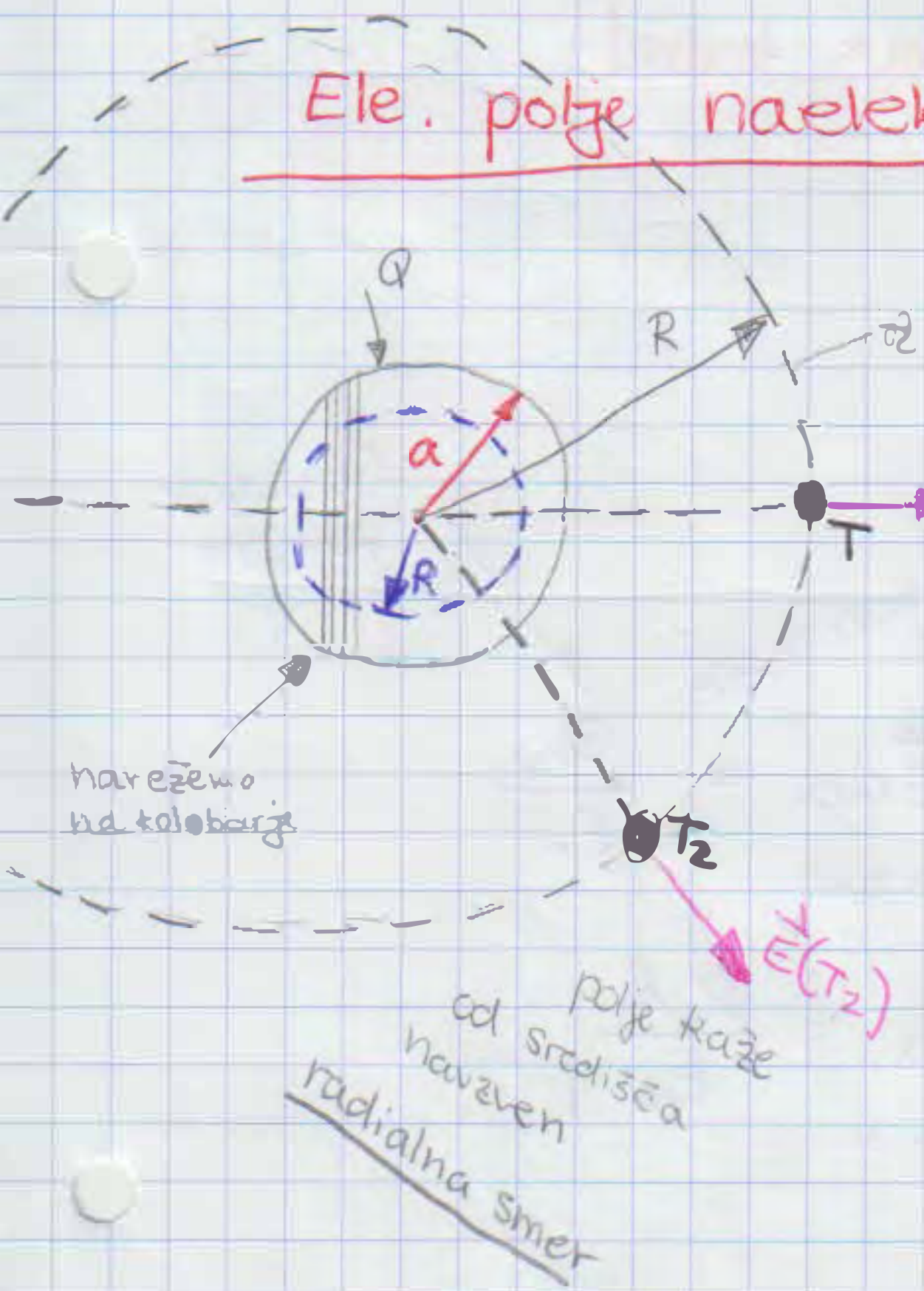
$$\oint_{\mathcal{A}} \vec{E} \cdot d\vec{a} = E \cdot 4\pi r^2$$

valjno simetrični sistem

$$\oint_{\mathcal{A}} \vec{E} \cdot d\vec{a} = E \cdot 2\pi \cdot pl$$

ploščina

Ele. polje naelektrene krogle - sfere



$$\vec{E}(T) = ?$$

$$\oint_{d_{\vec{E}}} \vec{E} d\vec{a} = \frac{Q_{not}}{\epsilon_0}$$

$$\oint_{d_{\vec{E}}} E \cdot d\vec{a} = \frac{Q_{not}}{\epsilon_0}$$

konstanten po dolžini

$$E \int da = \frac{Q}{\epsilon_0}$$

A - površina krogle $4\pi R^2$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \cdot \vec{e}_r$$

velja za zunanost krogle

! $\vec{E} = 0$ notranjost krogle

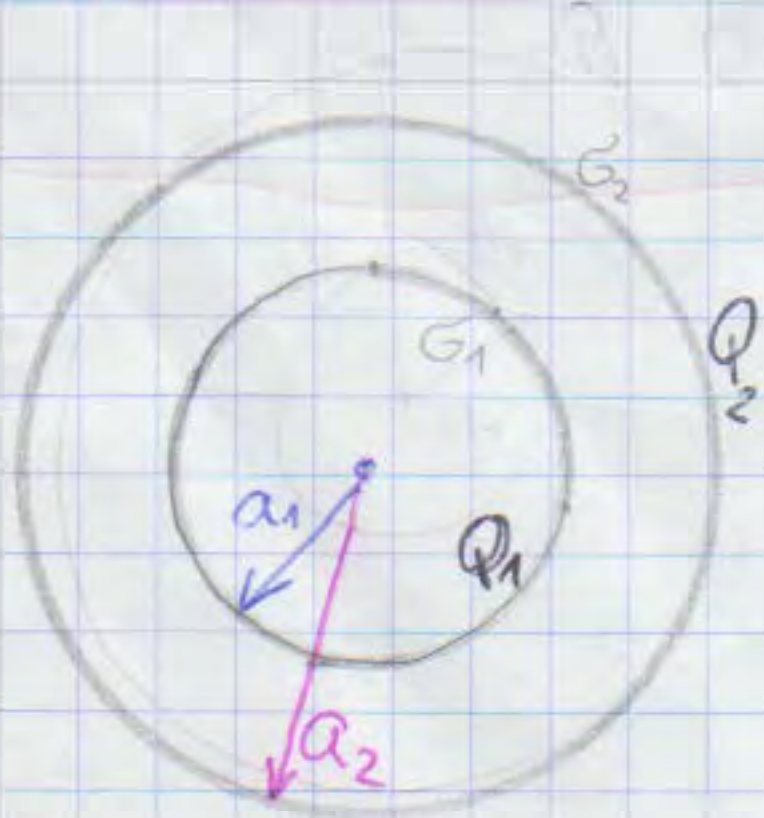
E krogle

$$\vec{E} = \begin{cases} 0 & R < a \\ \vec{e}_r \frac{Q}{4\pi\epsilon_0 R^2} & R > a \end{cases}$$

— naboj znotraj krogle

— naboj zunaj krogle

2 koncentrični sfere

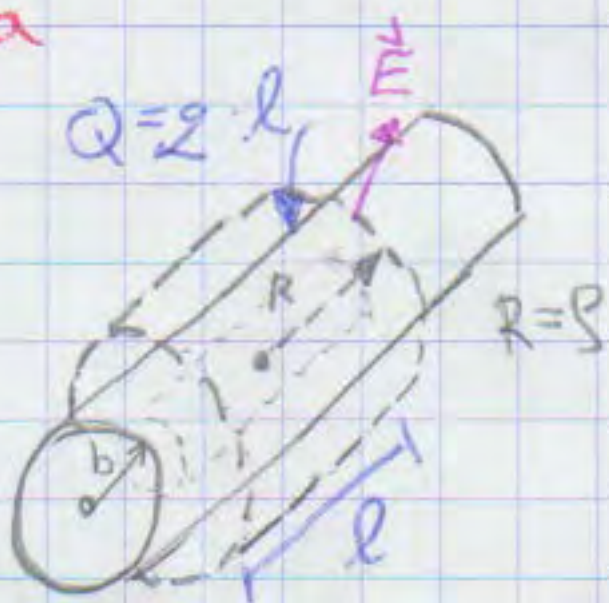


$$\vec{E} = \begin{cases} 0 & R < a_1 \text{ notranjost} \\ \frac{Q_1}{4\pi\epsilon_0 R^2} & a_1 < R < a_2 \text{ med sferama} \\ \frac{Q_1 + Q_2}{4\pi\epsilon_0 R^2} & R > a_2 \text{ izven} \end{cases}$$

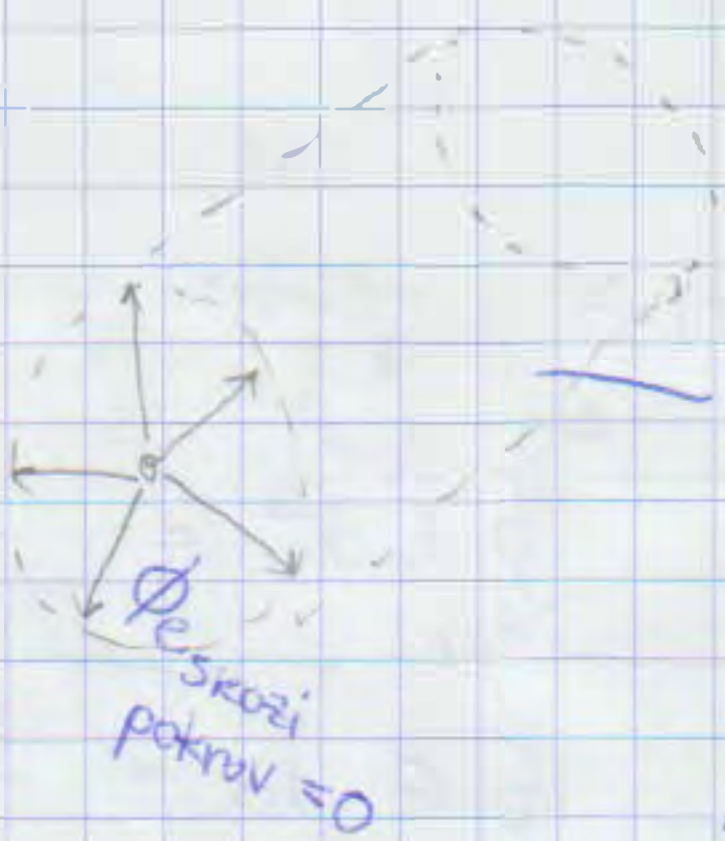
Q_2 je enaka

$\vec{E}$ na elektrenega valja (žica, coax kabl)

1) žica



smer polja je radialna



Gaussov zakon

$$\oint_{\text{skozi PLAFEC}} \vec{E} da = \frac{Q_{\text{noto}}}{\epsilon_0}$$

$$E \cdot \int_{\text{skozi PLAFEC}} da = \frac{q \cdot l}{\epsilon_0}$$

$$E \cdot 2\pi R l = \frac{q \cdot l}{\epsilon_0}$$

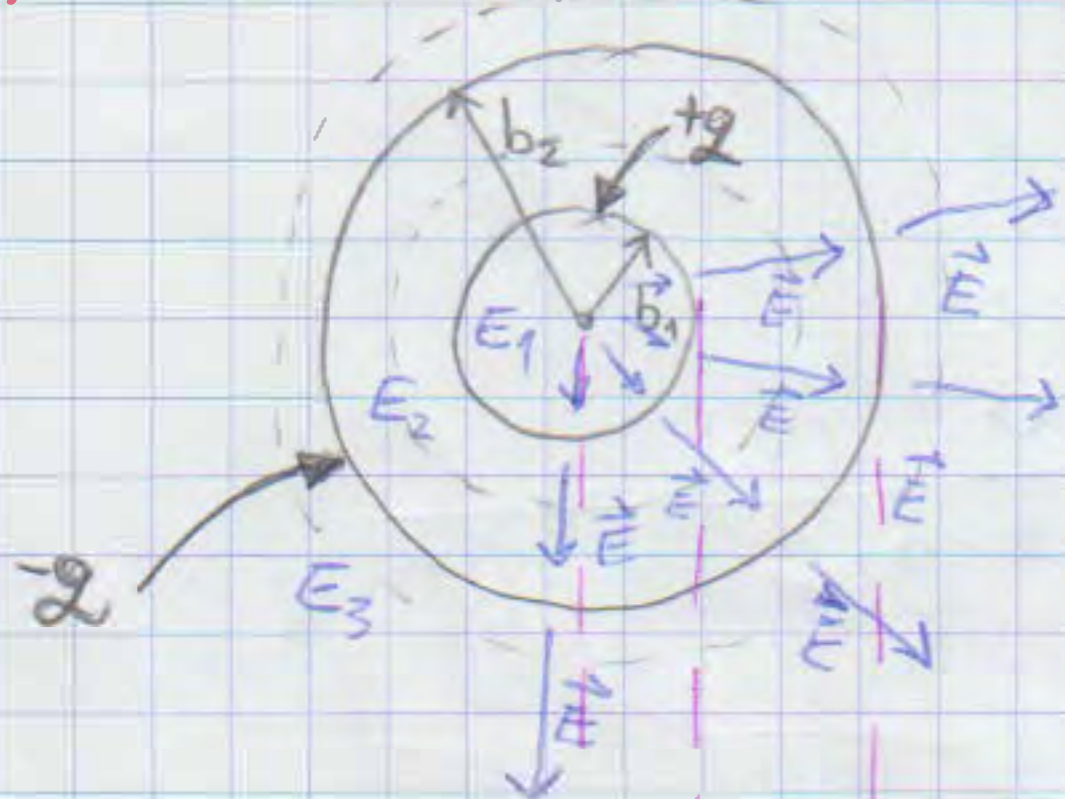
$$\vec{E} = \frac{q}{2\pi \epsilon_0 R} \vec{e}_R$$

$$\vec{E} = \vec{e} \cdot \frac{q}{2\pi \epsilon_0 r} \quad \text{zunaj valja}$$

$$\vec{E} = 0 \quad \text{znotraj}$$

$$\vec{E} = \begin{cases} 0 & r < b \\ \frac{q}{2\pi \epsilon_0 r} \vec{e}_r & r > b \end{cases}$$

2) koaksialni kabl



$$\vec{E} = \begin{cases} 0 & E_1 \quad r < b_1 \\ \frac{q}{2\pi \epsilon_0 r} & E_2 \quad b_1 < r < b_2 \\ \frac{-q+q}{2\pi \epsilon_0 r} = 0 & E_3 \quad b_2 < r \end{cases}$$

polje je samo med plastmi

$$E_1 = \frac{q}{2\pi \epsilon_0 b_1}$$

$$E_2 = \frac{q}{2\pi \epsilon_0 r}$$

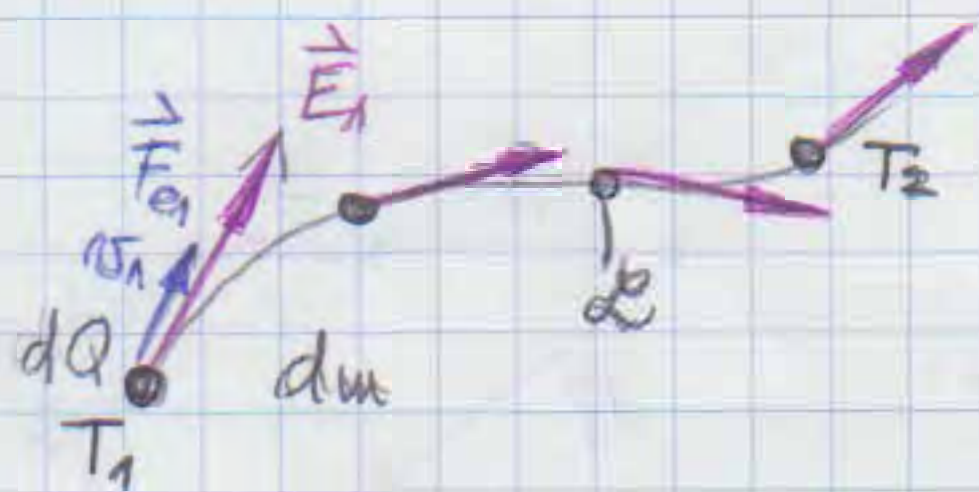
$$\frac{q}{2\pi \epsilon_0 b_2} = E_3$$

graf $\vec{E}$ na koaksialnem kabl

fact
3MV je že prebojna jakost

Delo električnega polja - Coulonove sile

→ za delo ni pomembna pot & samo začetna in končna vrednost (T_1, T_2)



sila & fiziki $d\vec{F} = dm\vec{a} = dm \cdot \vec{v} = dm \cdot \frac{d\vec{v}}{dt}$

električna sila $d\vec{F}_e = dQ\vec{E}$

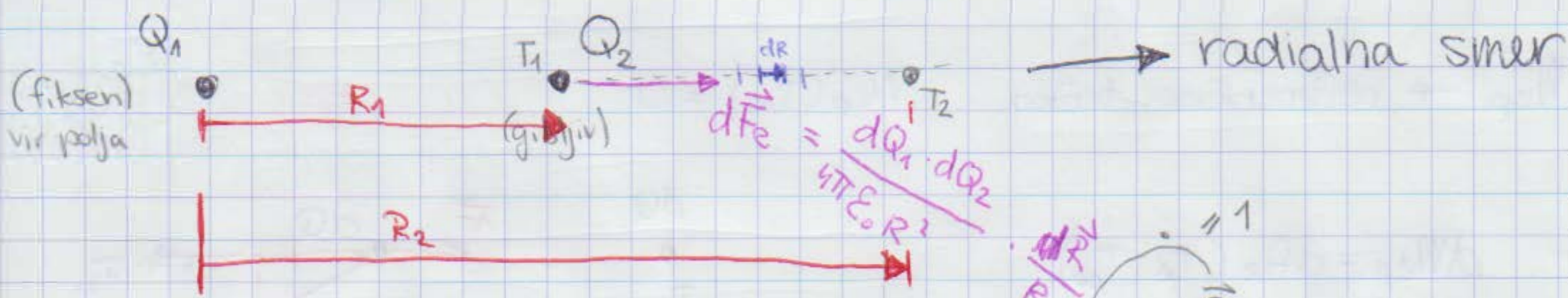
električno delo $dA_e(T_1, \gamma, T_2) = \int_{T_1, \gamma, T_2} d\vec{F}_e \cdot d\vec{l} = \int_{T_1, \gamma, T_2} dQ\vec{E} \cdot d\vec{l}$

$$= \int_{T_1}^{T_2} dm \frac{d\vec{v}}{dt} \cdot \vec{v} dt = dm \cdot \vec{v} \cdot d\vec{v} = \int dm d\left(\frac{v^2}{2}\right) =$$

$$= dm \int \frac{v^2}{2} \Big|_{T_1}^{T_2} = \frac{dm v_2^2}{2} - \frac{dm v_1^2}{2}$$

$$(v^2)' = 2 \cdot \vec{v} \cdot d\vec{v}$$

1) opazujemo 2 točkasta naboja - gibanje v radialni smeri



$$A_e(T_1, T_2) = \int_{R_1}^{R_2} d\vec{F}_e \cdot d\vec{l} = \frac{dQ_1 dQ_2}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{\vec{R}}{R^2} \cdot \frac{\vec{R}}{R} dR$$

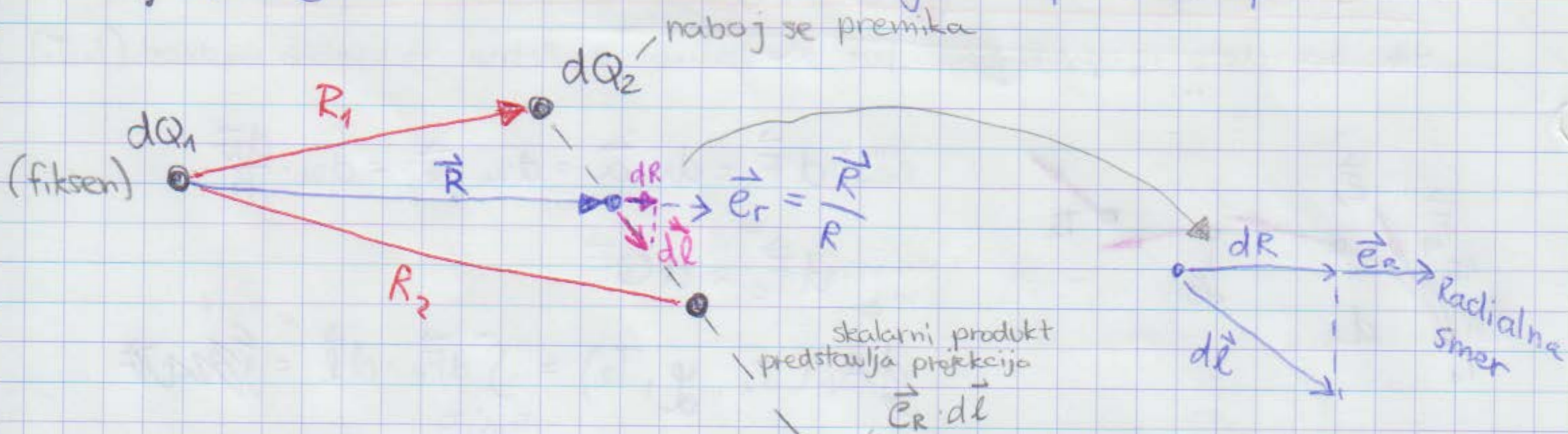
$$= \frac{dQ_1 dQ_2}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{dR}{R^2} =$$

integriramo od R_1 do R_2
od manjše do večje

$$\int \frac{dx}{x^2} = -\frac{1}{x} + C$$

$$A_e(T_1, T_2) = \frac{dQ_1 dQ_2}{4\pi\epsilon_0} \left(-\frac{1}{R_2} + \frac{1}{R_1} \right)$$

2) Opazujemo dva točkasta naboja - splošna pot



$$A_e(T_1, T_2) = \int_{R_1}^{R_2} \vec{F}_e \cdot d\vec{l} = \frac{dQ_1 dQ_2}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{\vec{R}}{R} \cdot \frac{1}{R^2} d\vec{l} = \frac{dQ_1 dQ_2}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{dR}{R^2}$$

$$A_e(T_1, T_2) = \frac{dQ_1 dQ_2}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

enaka enačba kot pod točko 1

pot gibanja ni pomembna
vendar sta pomembni začetna in
končna lega

T_1 T_2
 R_1 R_2

Elektronska potencialna energija W_{ep}

$W_{ep} \rightarrow$ referenčna točka $W_{ep}(T_{\infty}) = 0$

$$dW_{ep} = dA_e(T, T_{\infty})$$

$$= \int_T^{T_{\infty}} d\vec{F}_e \cdot d\vec{l} = dQ \cdot \int_T^{T_{\infty}} \vec{E} \cdot d\vec{l} =$$

$$dW_{ep} = \frac{Q_2}{4\pi\epsilon_0} \int_{Q_1} \frac{dQ}{R}$$

vir polja

Definicija ele. poljske jakosti

$$\vec{E} = \frac{d\vec{F}_e}{dQ}$$

normirano z
nabojem

$$V = \frac{dW_{ep}}{dQ}$$

izločimo objekt

$$dW_{ep} = A_e(T, T_{\infty}) = \int_T^{T_{\infty}} \frac{dQ_1 dQ_2}{4\pi\epsilon_0 R^2} dR$$

$$V = \int_T^{T_{\infty}} \frac{dQ_1}{4\pi\epsilon_0 R^2} dR = \int_T^{T_{\infty}} \vec{E} \cdot d\vec{l}$$

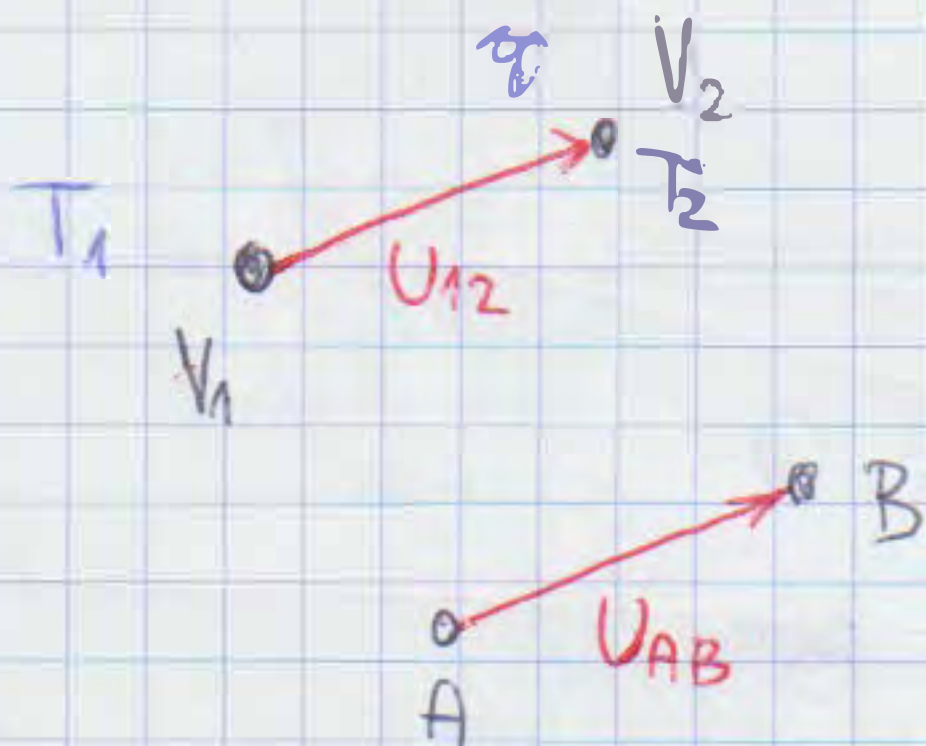
Električna napetost U

$$\left[\frac{J}{C} = \frac{V \cdot AS}{AS} = V \right]$$

$$U_{12} = V_1 - V_2 = \frac{\Delta W_{ep,12}}{dQ} = \frac{dA_e(T_1, T_2)}{dQ}$$

Ele napetost je razlika potencialov

Označevanje napetosti:



$$U = \int_{T_1}^{T_2} \vec{E} \cdot d\vec{l}$$

odvod polja po dolžini med točkama T_1 in T_2

$$\vec{E} = \frac{d\vec{F}_e}{dQ}$$

$$V = \frac{dW_e}{dQ}$$

$$U = \frac{dA_e}{dQ}$$

(Ne) Vrtinčnost $\vec{E}$ velja za elektrostatično polje Stokesov stavek

polje se ne vrti ni vrtinčno

$$\oint_L \vec{E} d\vec{l} = ?$$



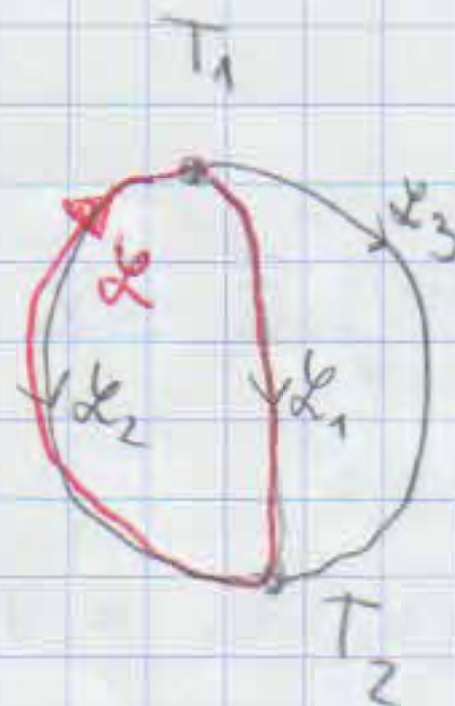
$$\int_{T_1, L, T_2} \vec{E} d\vec{l} = \int_{T_1, L, T_2} \vec{E} d\vec{l} = \int_{T_1, L, T_2} \vec{E} d\vec{l}$$

T_1, L, T_2 pot je nepomembna

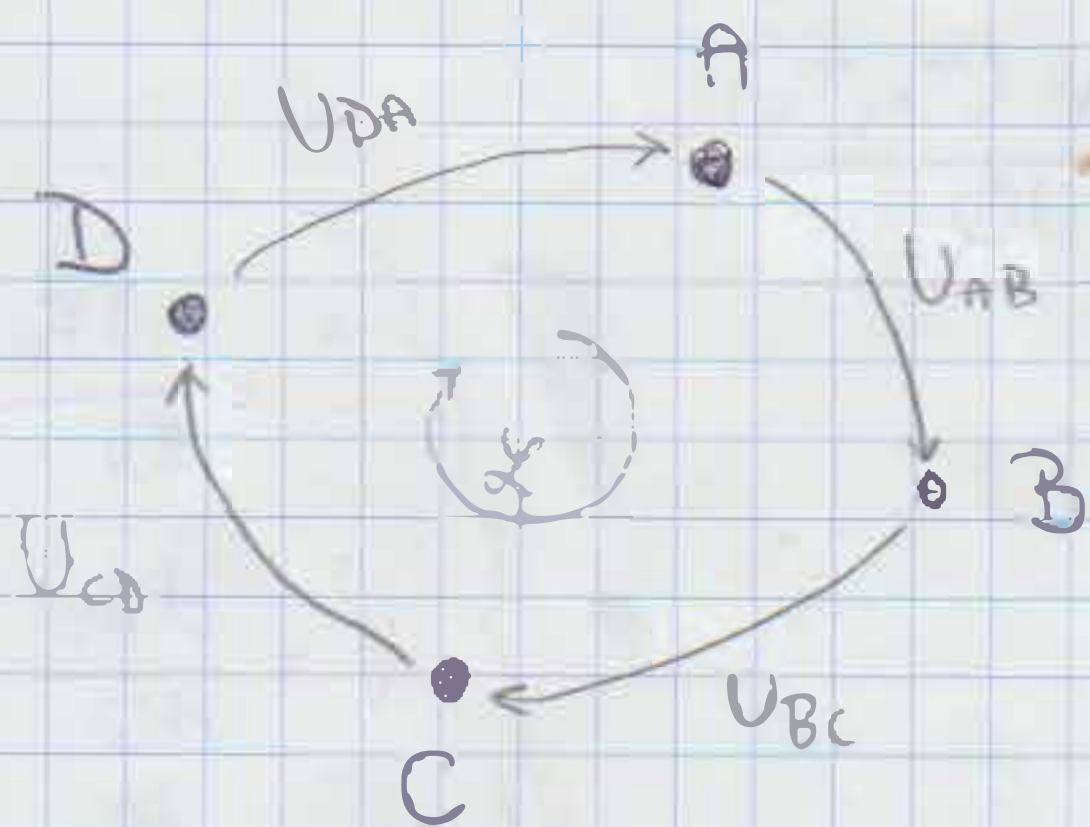
$$\int_{T_1, L, T_2} \vec{E} d\vec{l} = - \int_{T_2, L, T_1} \vec{E} d\vec{l}$$

|| vemo da je predznak -

$$\oint_L \vec{E} d\vec{l} = \int_{T_1, L, T_2} \vec{E} d\vec{l} + \int_{T_2, L, T_1} \vec{E} d\vec{l} = 0$$



II Kirchhoffov zakon



$$\sum_{j=1}^n U_j = 0$$

$$\oint \vec{E} d\vec{l} = \int_A^B \vec{E} d\vec{l} + \int_B^C \vec{E} d\vec{l} + \int_C^D \vec{E} d\vec{l} + \int_D^A \vec{E} d\vec{l} = 0$$

$$U_{AB} + U_{BC} + U_{CD} + U_{DA} = 0$$

Vsota vseh napetosti v sklenjeni zanki je 0.

To je posledica neizvirnosti elek. polja.

Ponovitev: koncepti debčauja potenciala in napetosti:

$$1) V(r) = \frac{1}{4\pi\epsilon_0} \int_{Q_{ed}} \frac{dQ}{R}$$



$$V(r) = ?$$

ne potrebujemo
vektorsko razdaljo
samo absolutno

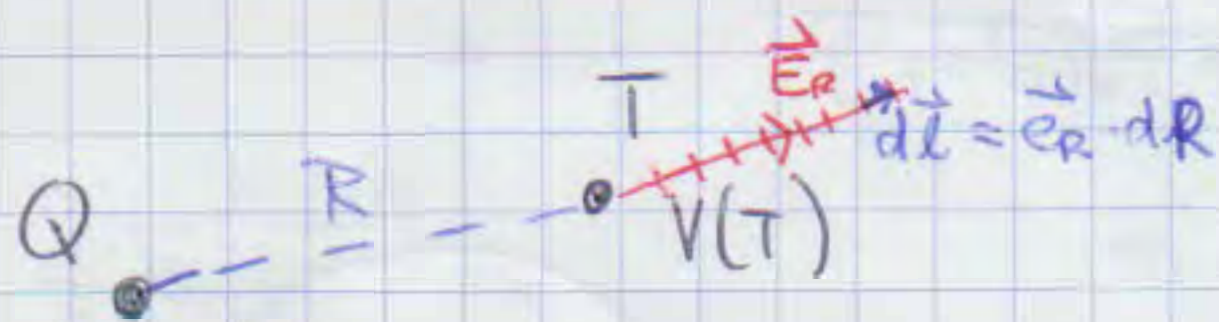
$$2) V(r) = \int_T^{T_2} \vec{E} \cdot d\vec{l} = - \int_{T_2}^T \vec{E} d\vec{l}$$

$$U_{12} = V_1 - V_2 = V(T_1) - V(T_2) = \int_{T_1}^{T_2} \vec{E} d\vec{l}$$



zgleđ

1) potencial točkastega naboja Q



I. način

$$V(T) = \frac{1}{4\pi\epsilon_0} \int_Q \frac{dQ}{R} = \frac{1}{4\pi\epsilon_0 R} \int_Q dQ = \boxed{\frac{Q}{4\pi\epsilon_0 R}} \quad \checkmark$$

konstanta

Upada s 1. potencko

II. način

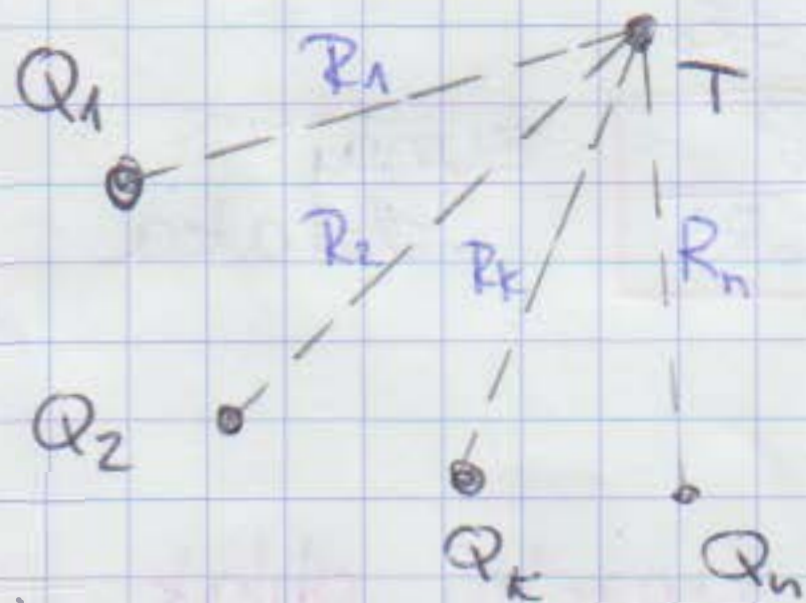
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \cdot \vec{e}_R$$

$$V(T) = \int_T^{\infty} \vec{E} \cdot d\vec{l} = \int_R^{\infty} \frac{Q}{4\pi\epsilon_0 R^2} \vec{e}_R \cdot \vec{e}_R \cdot dR = \frac{Q}{4\pi\epsilon_0} \int_R^{\infty} \frac{dR}{R^2} =$$

$$= \boxed{\frac{Q}{4\pi\epsilon_0 R}} \quad \checkmark$$

$-\frac{1}{R} \Big|_R^{\infty} = -\frac{1}{\infty} + \frac{1}{R} = \frac{1}{R}$

2) V gručo točkastih nabojev



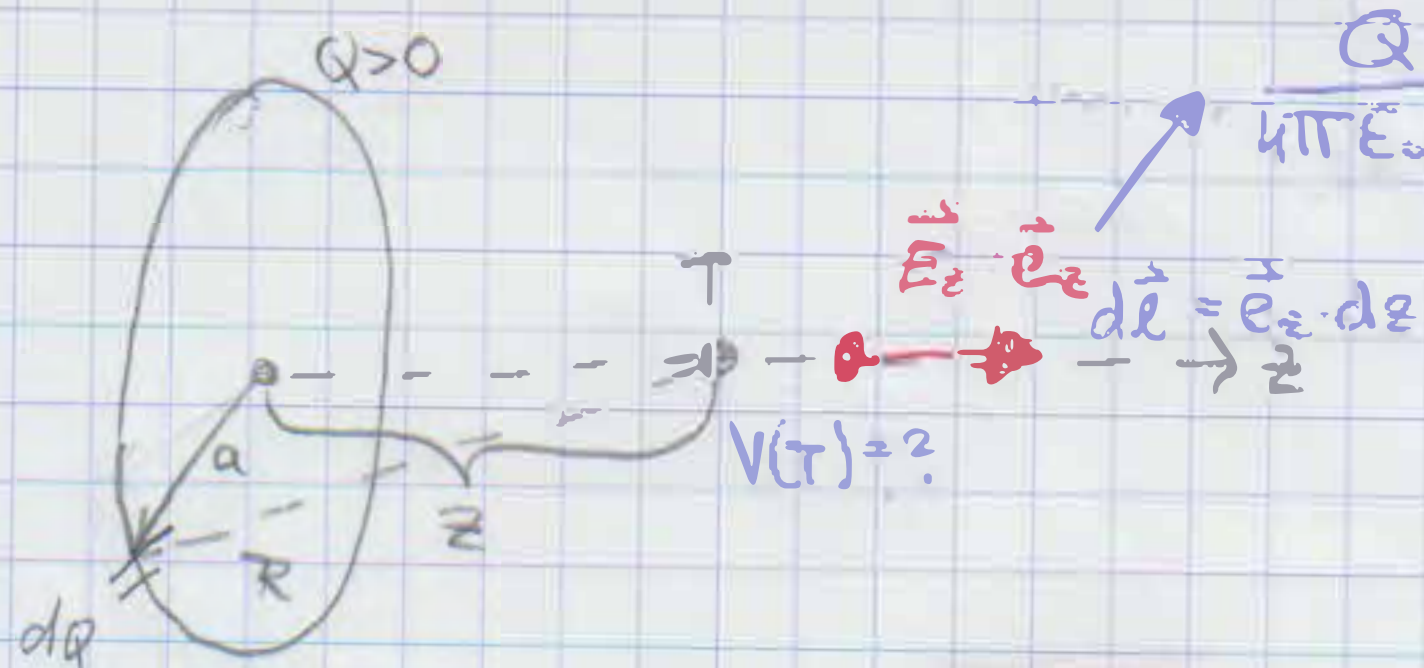
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_k + \vec{E}_n = \sum_{k=1}^n \vec{E}_k$$

$$V = \int_T^{\infty} \vec{E} \cdot d\vec{l} = V_1 + V_2 + \dots + V_k + V_n = \sum_{k=1}^n V_k$$

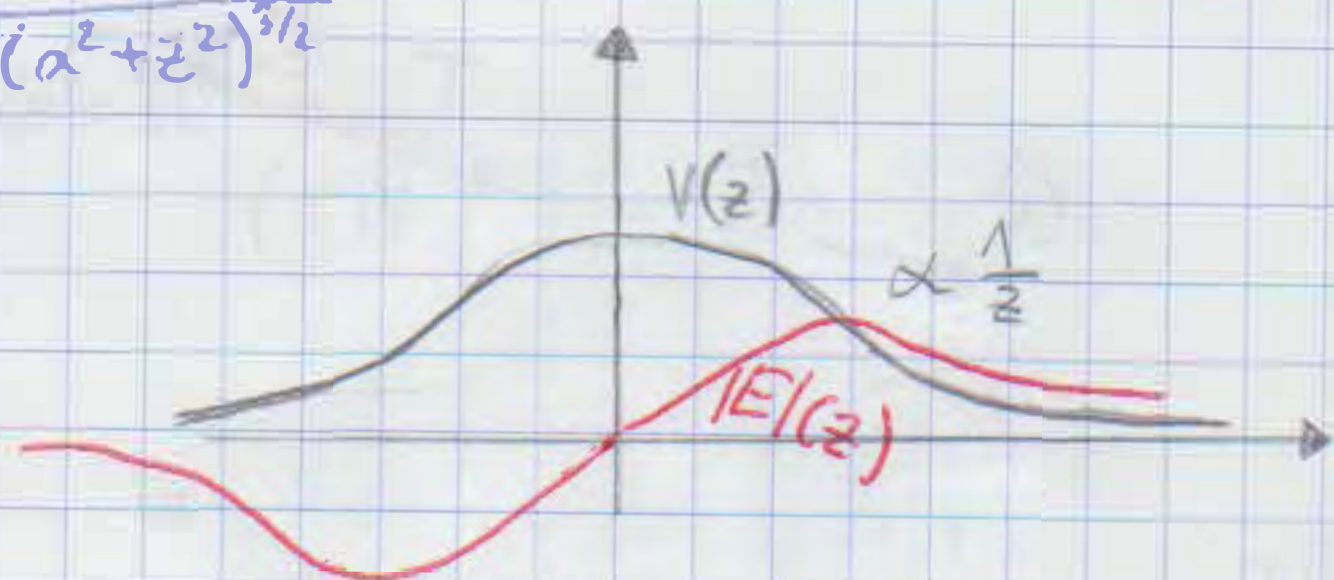
$$\boxed{V = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{Q_k}{R_k}} = \frac{1}{4\pi\epsilon_0} \sum_{k=1}^n \frac{Q_k}{R_k}$$

Če imamo več Q, potencial napišemo kot seštevek vseh točkastih nabojev

3) Potencial V n. elektnenega obroča



$$\frac{Q \cdot z}{4\pi\epsilon_0 (a^2 + z^2)^{3/2}}$$



I. koncept

$$V = \frac{1}{4\pi\epsilon_0} \int_Q \frac{dQ}{R} = \frac{1}{4\pi\epsilon_0 R} \cdot Q = \frac{Q}{4\pi\epsilon_0 \sqrt{a^2 + z^2}}$$

ves čas integracije je konstanten

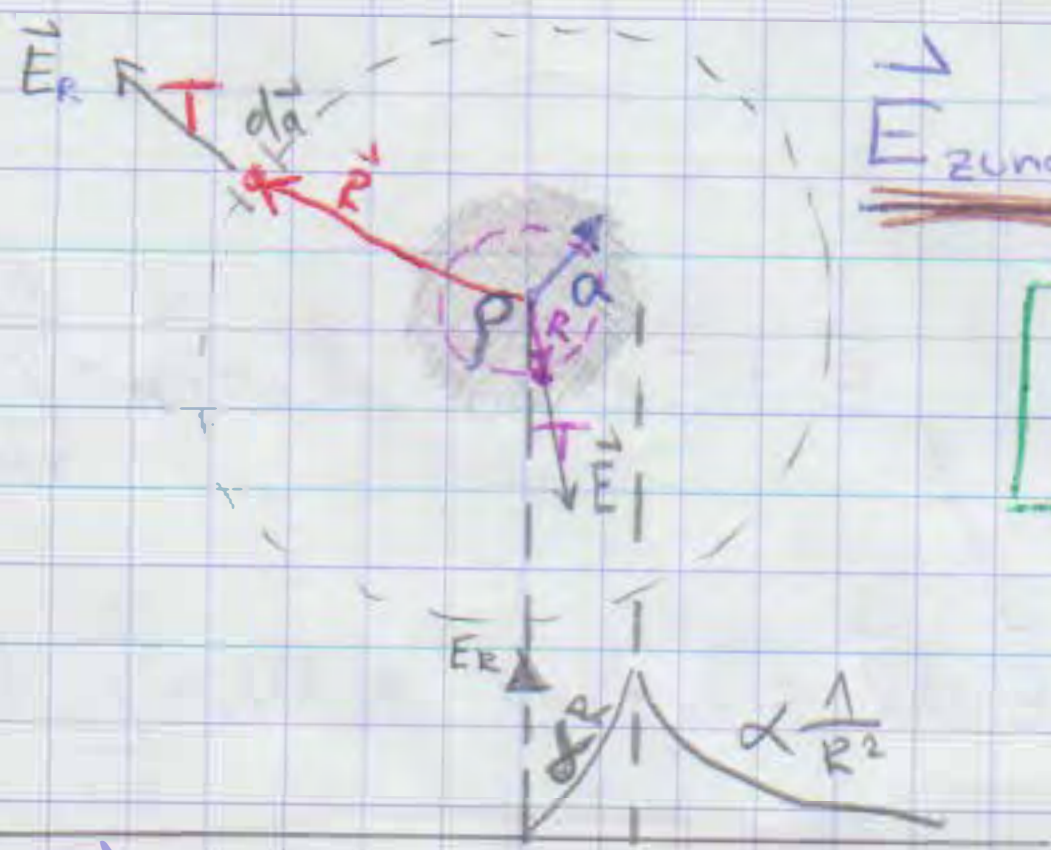
II. koncept

$$V = \int_T^{T_0} \vec{E} d\vec{l} = \int_T^{T_0} \frac{Q \cdot z \cdot dz}{4\pi\epsilon_0 (a^2 + z^2)^{3/2}} = \text{težji način}$$

$$= \frac{Q}{4\pi\epsilon_0} \int_z^\infty \frac{dz \cdot z}{(a^2 + z^2)^{3/2}} = \frac{Q}{4\pi\epsilon_0} \left(-\frac{1}{\sqrt{a^2 + z^2}} \right) \Big|_z^\infty =$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{1}{(a^2 + z^2)^{1/2}}$$

4) V in ob krogelnem oblaku



E zunaj oblaka

$$\oint \vec{E} d\vec{a} = \frac{Q_{\text{not}}}{\epsilon_0}$$

$$E_R \cdot 4\pi R^2 = \rho \cdot \frac{4\pi a^3}{3\epsilon_0}$$

$$E_R = \rho \frac{a^3}{3\epsilon_0 R^2}$$

Gaussov stavek

zunaj oblaka

E znotraj oblaka

$$\oint \vec{E} d\vec{a} = \frac{Q_{\text{not}}}{\epsilon_0}$$

$$E_R \cdot 4\pi R^2 = \frac{\rho \cdot 4\pi R^3}{3\epsilon_0}$$

$$E_R = \frac{\rho R}{3\epsilon_0}$$

znotraj

Gaussov stavek

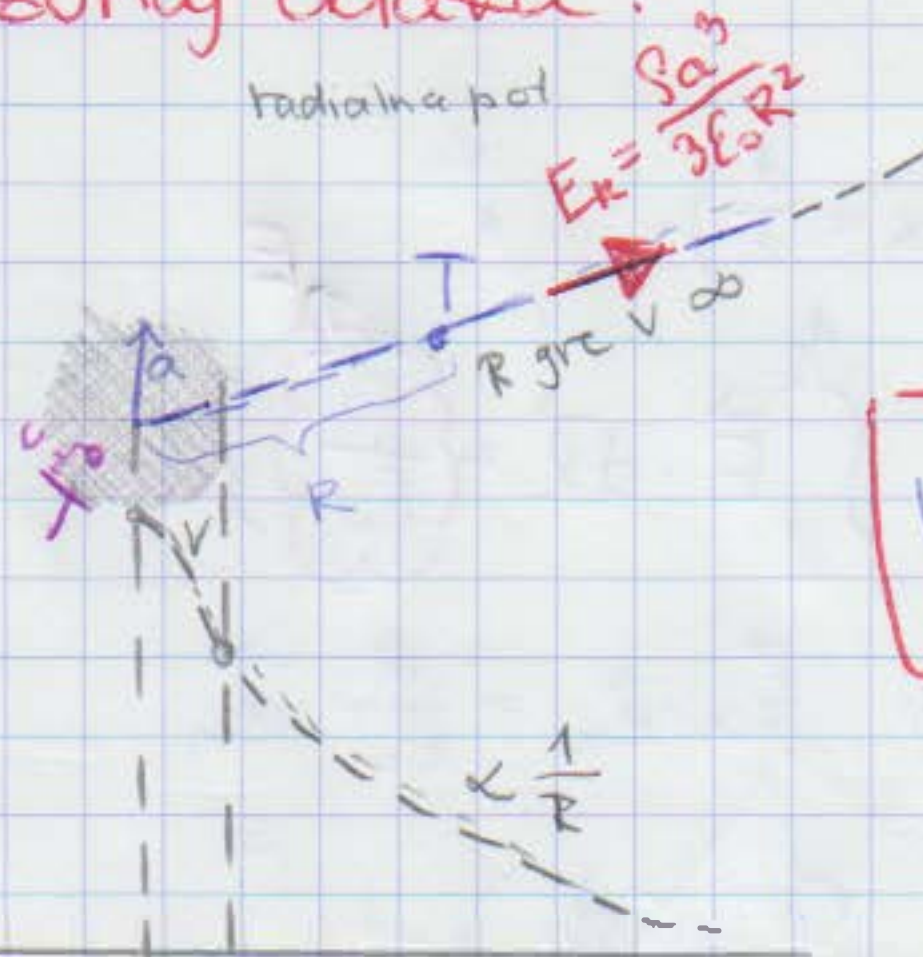
krogelni oblak

$$E_R = \begin{cases} \frac{\rho R}{3\epsilon_0}, & \text{znotraj oblaka} \\ \frac{\rho a^3}{3\epsilon_0 R^2}, & \text{zunaj oblaka} \end{cases}$$

V zunaj oblake:

1) zunaj

$$\int \frac{dR}{R^2} = -\frac{1}{R}$$



$$V = \int_T^{T_0} \vec{E} d\vec{l} = \frac{\rho a^3}{3\epsilon_0} \int_R^\infty \frac{1}{R} \vec{e}_r \cdot \vec{e}_r dR$$

konstanta

$$V = \frac{\rho a^3}{3\epsilon_0} \cdot \frac{1}{R} \quad V(R > a)$$

2) V na meji $R=a$

$$V(R=a) = \frac{\rho a^3}{3\epsilon_0 a} = \frac{\rho a^2}{3\epsilon_0}$$

3) V znotraj $R < a$

$$\int R dR = \frac{R^2}{2}$$

$$V(R) - V(a) = \int_R^a \frac{\rho R}{3\epsilon_0} dR = \frac{\rho}{3\epsilon_0} \int_R^a R dR = \frac{\rho}{3\epsilon_0} \left(\frac{a^2}{2} - \frac{R^2}{2} \right)$$

$$V(R) = \frac{\rho a^2}{3\epsilon_0} + \frac{\rho a^2}{6\epsilon_0} - \frac{\rho R^2}{6\epsilon_0} = \frac{\rho}{2\epsilon_0} \left(a^2 - \frac{R^2}{3} \right)$$

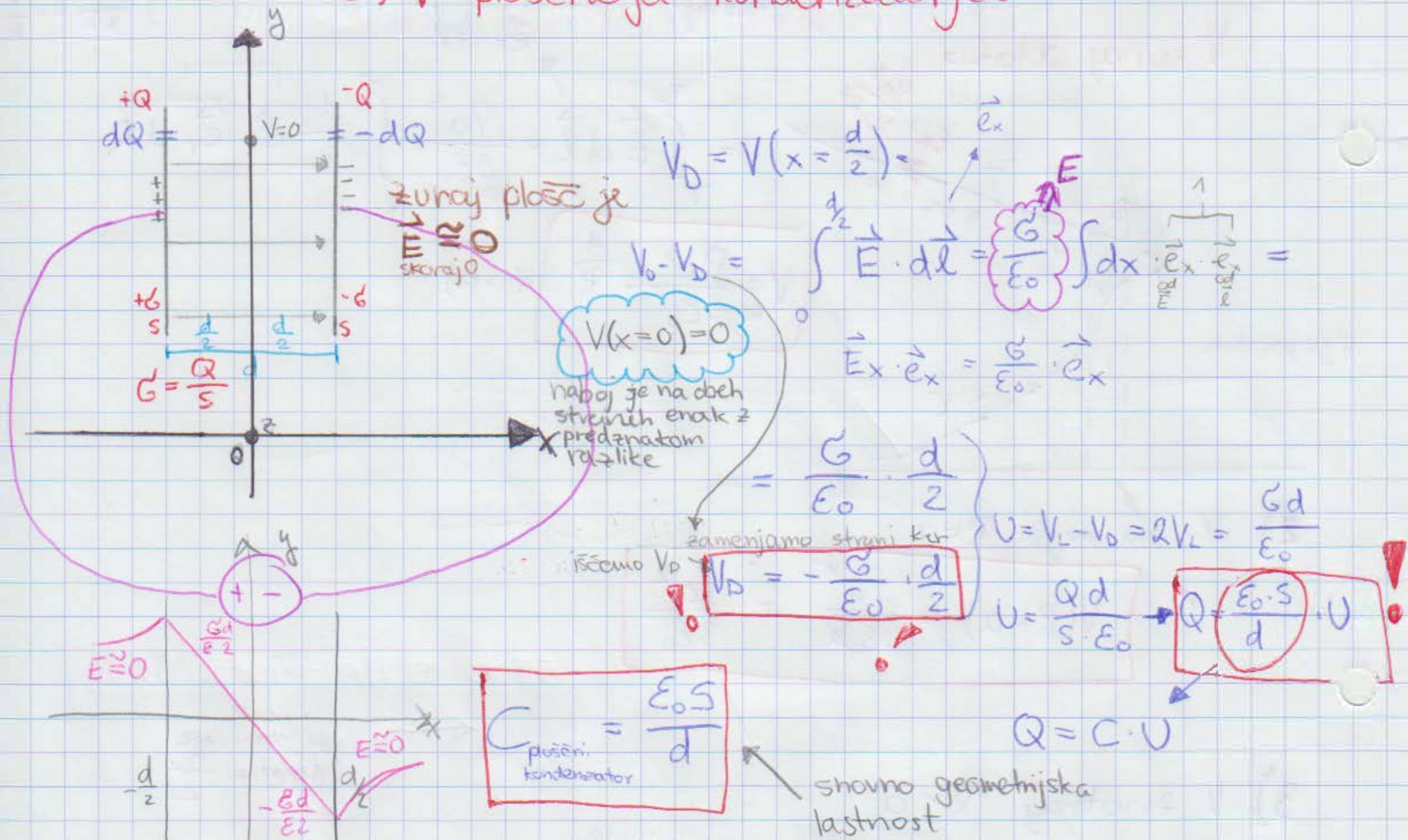
$$V(R) = \frac{\rho}{2\epsilon_0} \left(a^2 - \frac{R^2}{3} \right)$$



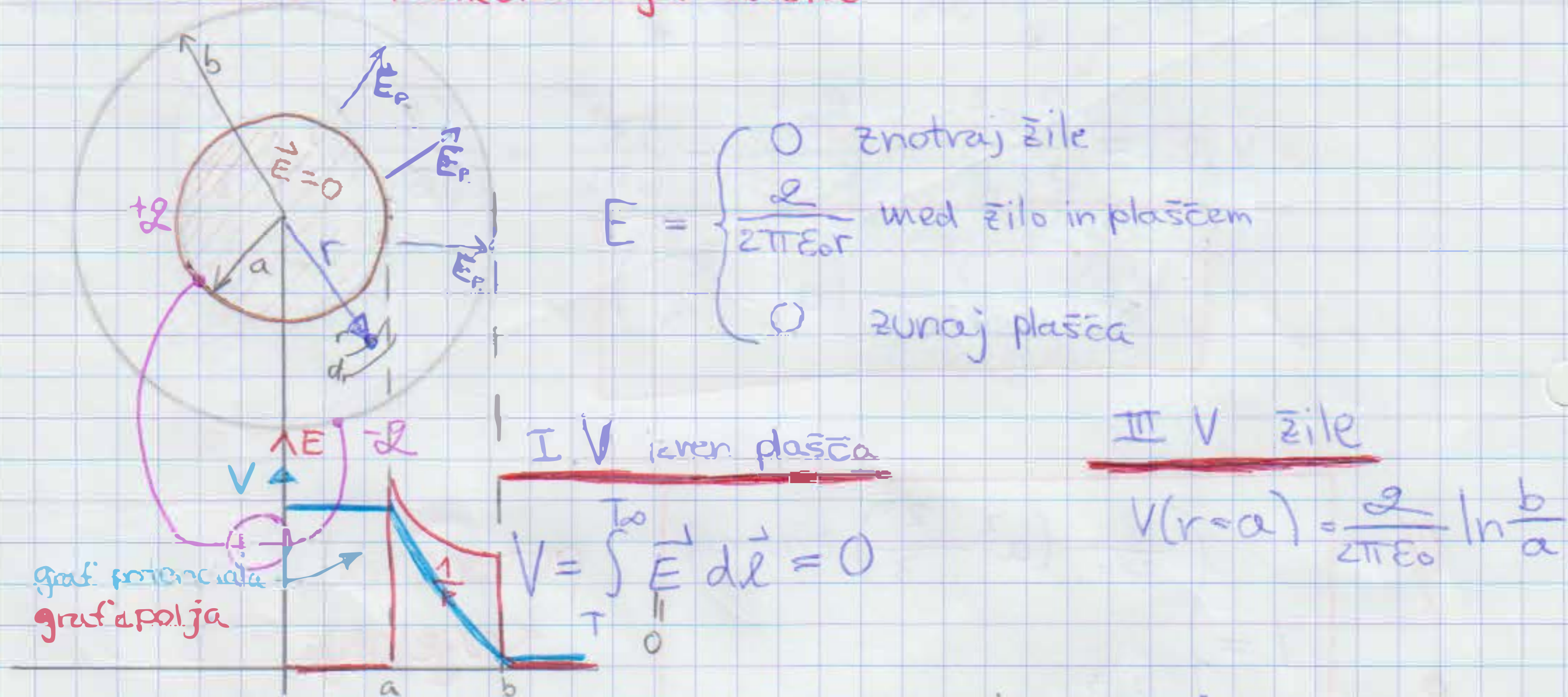
$$V = \begin{cases} \frac{\rho}{2\epsilon_0} \left(a^2 - \frac{R^2}{3} \right) & \text{znotraj} \\ \frac{\rho a^3}{3\epsilon_0 R} & \text{zunaj} \end{cases}$$

potencial
krogelnega
oblaka

5) V ploščnega kondenzatorja



6) Potencial koaksialnega kabla



II V med žilo in plaščem

$$V(r) - V(b) = \int_r^b \frac{Q}{2\pi\epsilon_0 r} \cdot dr = \frac{Q}{2\pi\epsilon_0} \ln \frac{b}{r} \quad \text{med žilo in plaščem}$$

IV. V znotraj žile: konstanten $\vec{E}$ polje nam pove koliko se potencial spremeni, ker ga ni je

$$V = \frac{Q}{2\pi\epsilon_0} \ln \frac{b}{a} \neq 0 \Rightarrow V(a) - V(b) = V$$

$$U = \frac{Q}{2\pi\epsilon_0} \ln \frac{b}{a}$$

med žilo in
plaščem

$$Q = l \cdot q = \frac{2\pi\epsilon_0 U}{\ln \frac{b}{a}} \cdot l = C$$

$$C = \frac{2\pi\epsilon_0}{\ln \frac{b}{a}} \cdot l$$

kapacitivnost
celotnega kabla ker vstavimo
celotno dolžino kabla

$$C = \frac{C}{l} = \frac{2\pi\epsilon_0}{\ln \frac{b}{a}}$$

kapacitivnost na
dolžinsko enoto kabla.

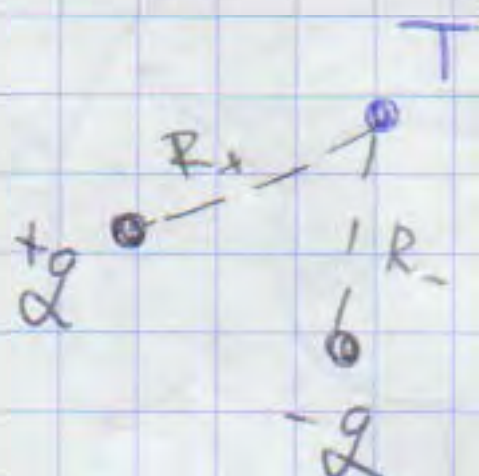
7) V električnih prenic

• $V(T)$

• Q

$$V = \int \vec{E} d\vec{l}$$

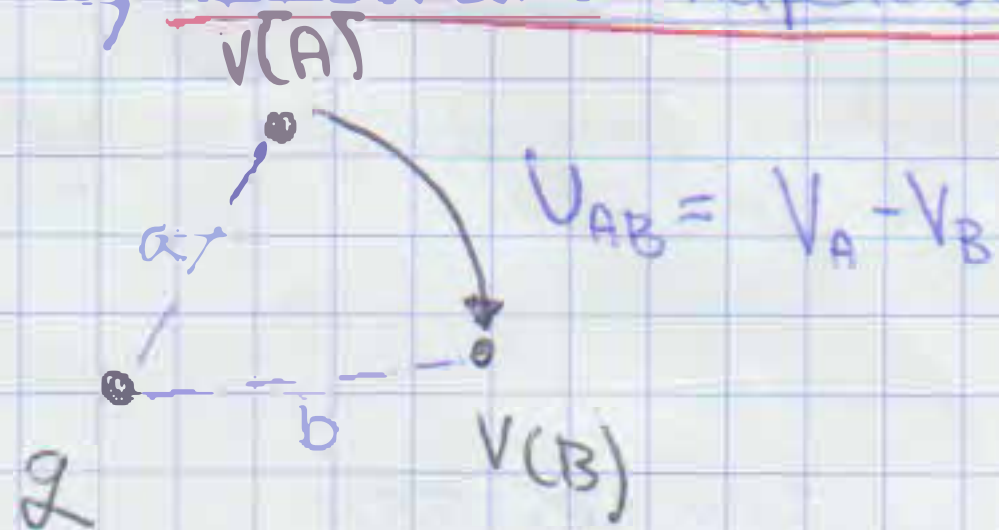
1) Računamo potencial v okolici dveh nabojev $\pm q$



$$V(T) = \frac{q}{2\pi\epsilon_0} \ln \frac{R_-}{R_+}$$

enaka
enačba

2) Računamo napetost v okolici premega naboja



$$U_{AB} = \int_A^B \vec{E} d\vec{l} = \frac{q}{2\pi\epsilon_0} \ln \frac{b}{a}$$

$$E \rightarrow V$$

$$V \rightarrow \vec{E} \quad ?$$

$$V(T) = \int_T^{T_0} \vec{E} d\vec{l}$$

$$V(T) = - \int_{T_0}^T \vec{E} d\vec{l}$$

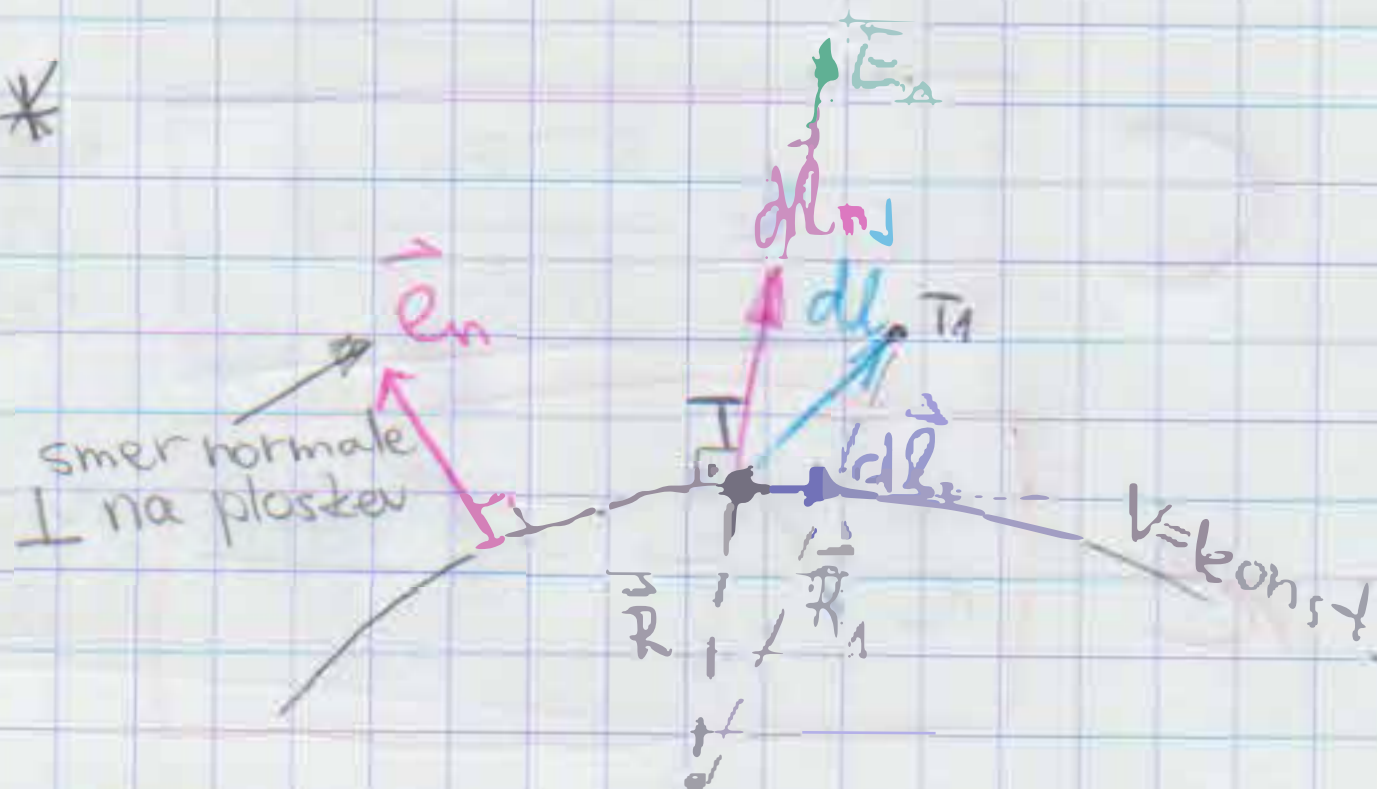
INTEGRAL

TOČKA 1) 2) na STR. 29

$$V \rightarrow \vec{E} = ?$$

ODVOD

*



$$dV = (V(T_1) - V(T))$$

$$V(T) - V(T_1) = \int_T^{T_1} \vec{E} d\vec{l} = \vec{E} \cdot d\vec{l} = -dV$$

$$dV = -\vec{E} \cdot d\vec{l}$$

$$\vec{E} = - \frac{dV}{d\vec{l}}$$

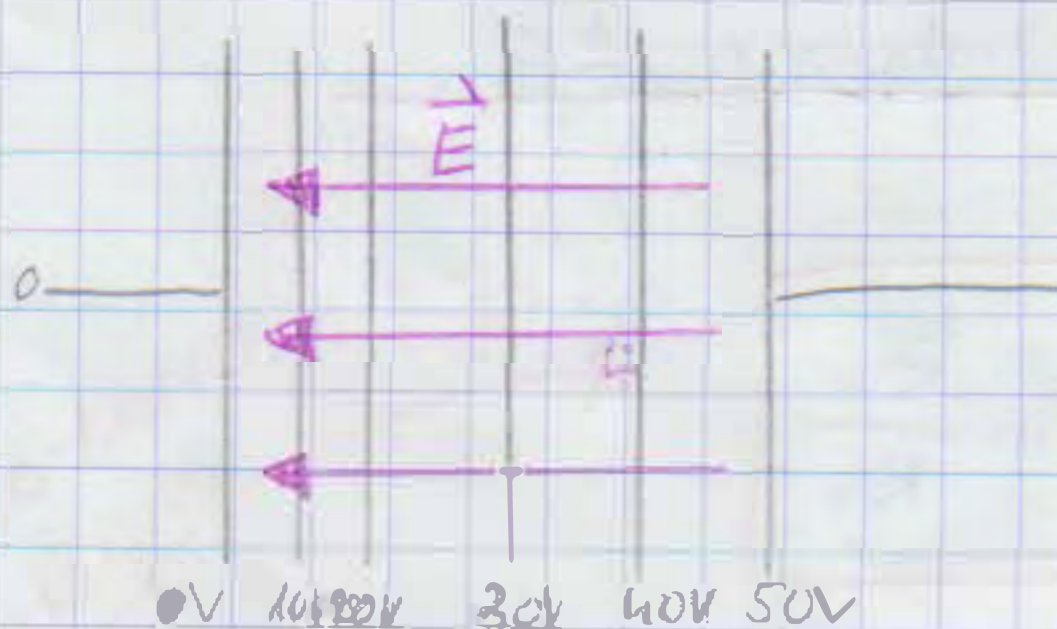
ne moreš deliti z vektorjem

Ekvipotencialne plošče

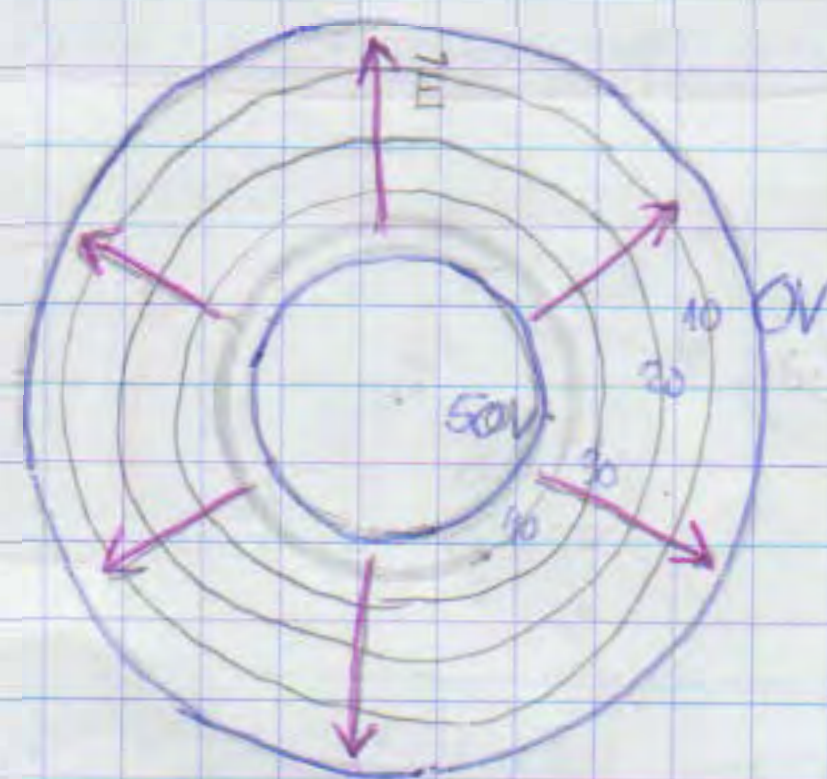
ekvi

so = enak

1) ploščni kondenzator



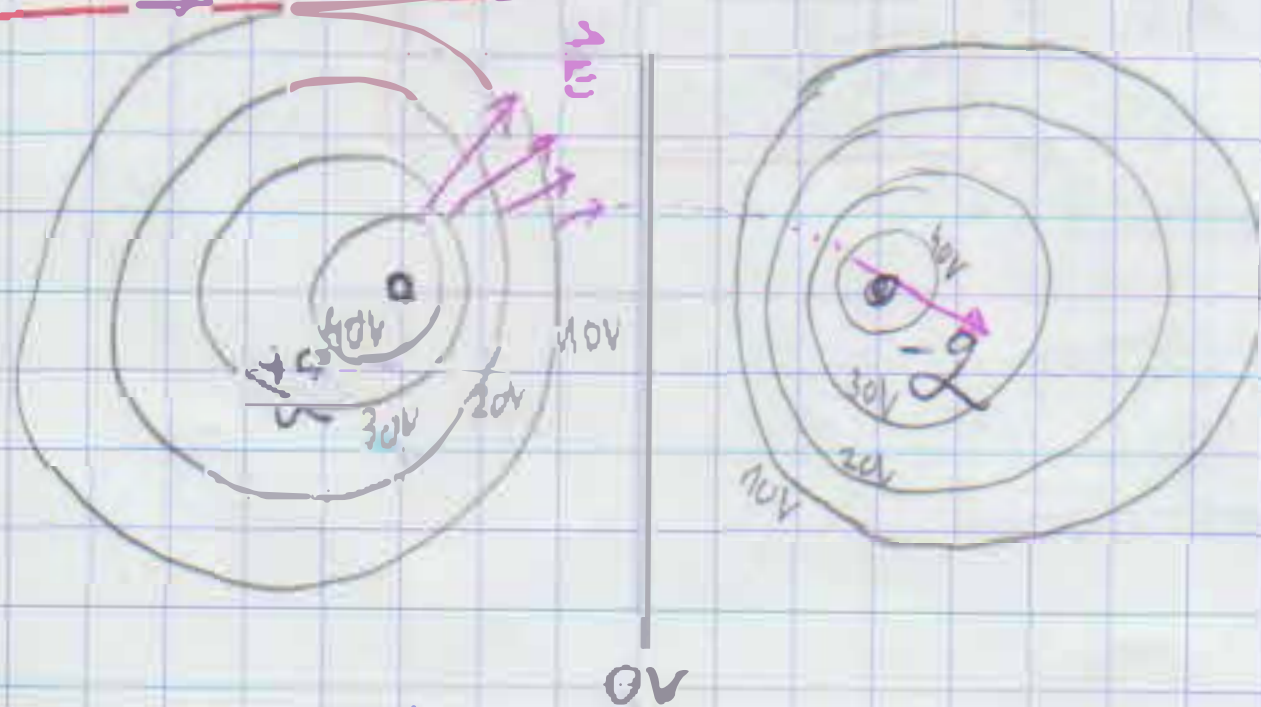
2) Valjni K.S. koax kabel



ekvipotencialne plošče so gostejše od 0V do 50V

silnice so pravokotne

3) Dvovod



polje je manjše kjer so plošče bolj razčlen

1) $d\vec{l}$ ^{tangentni odmik} tangento smer dl

$$dV = -\vec{E} \cdot d\vec{l} = 0$$

$$? \vec{E} = d\vec{l}_n \cdot \vec{e}_n ?$$

2) dl ^{premik v smeri normale} v smeri normale

$$dV = \vec{E} \cdot \vec{e}_n \cdot dl_n = -E \cdot dl_n$$

$$E_n = -\frac{dV}{dl_n}$$

minus je zato ker greš stran od izhodišča in ekvipotencialne prosteke padajo z razdaljo od središča

računaš v smeri normale ki je $\perp$ na ekvipotencialko

* skica na prejšnji strani desno zgoraj

Zgled:

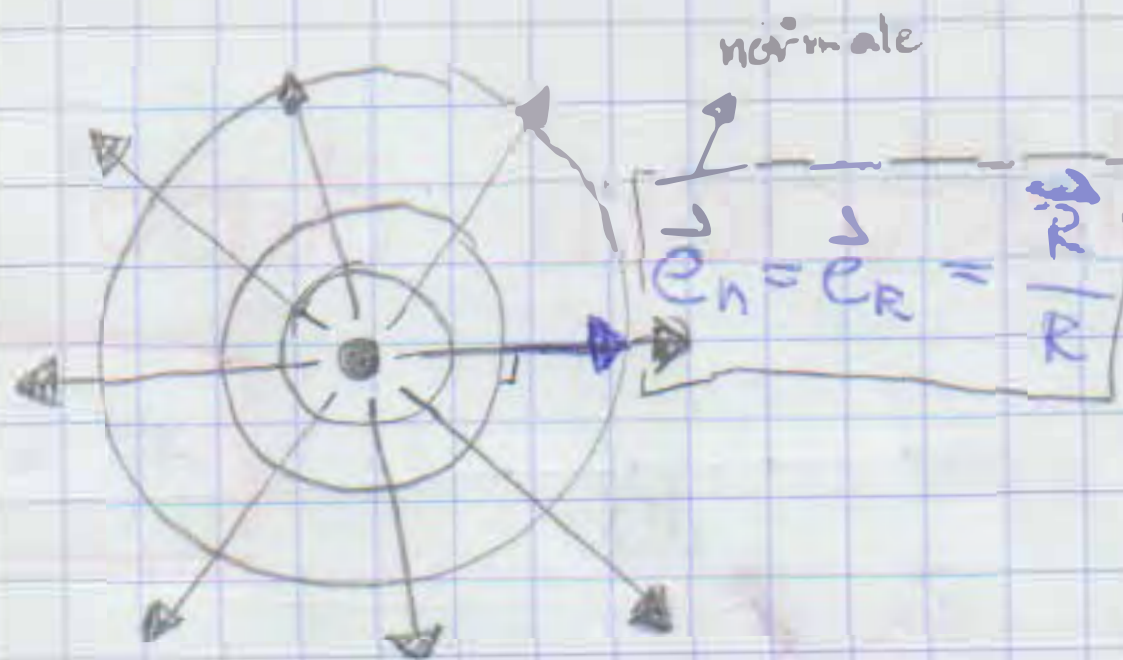
1) Točkast naboj

$$\vec{E} = \frac{Q}{4\pi\epsilon_0} \frac{\vec{R}}{R^3}$$

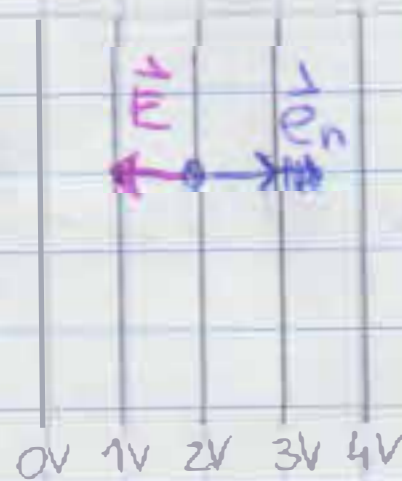
$$V = \frac{Q}{4\pi\epsilon_0 R}$$

$$E_n = -\frac{dV}{dR} = \frac{Q}{4\pi\epsilon_0} \frac{1}{R^2}$$

$$\vec{E} = \vec{e}_R \cdot E_n = \frac{Q\vec{R}}{4\pi\epsilon_0 R^3}$$

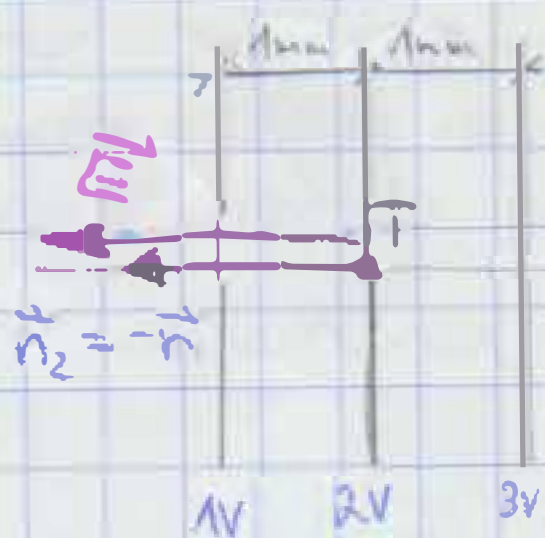


2) Homogeno polje



$$E_n = \frac{dV}{dn} = -\frac{(3-1)V}{1mm} = -\frac{2V}{1mm}$$

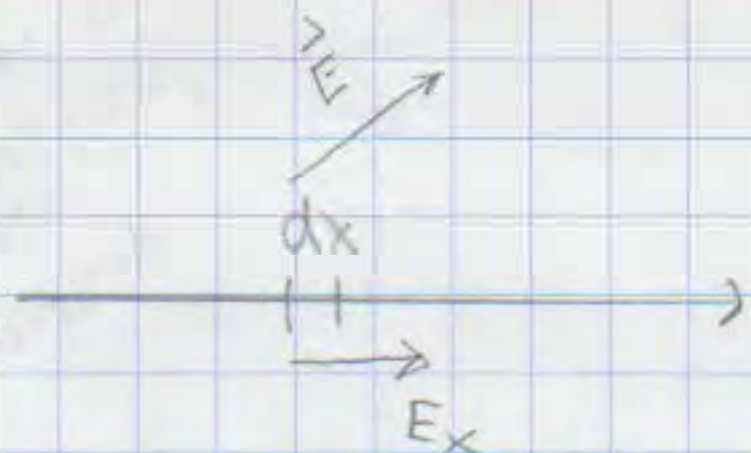
pove da greš stran od potenciala



$$\frac{dV}{dn} = \frac{1V-3V}{2mm} = -1kV/m$$

$$\vec{E} = -\vec{n}_2 (-1kV/mm) = \vec{n}_2 10^3 V/m$$

$\vec{E}$ kot trojica komponent K.S.



$$\vec{E} = (\vec{E}_x, \vec{E}_y, \vec{E}_z) \begin{cases} E_x = -\frac{dV}{dx} \\ E_y = -\frac{dV}{dy} \\ E_z = -\frac{dV}{dz} \end{cases}$$

kartezijev K.S. $\vec{E} = -\left(\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right) = \text{funkcija gradient}$

$-\text{grad}(V)$

velikost
strmine

$\vec{E} =$

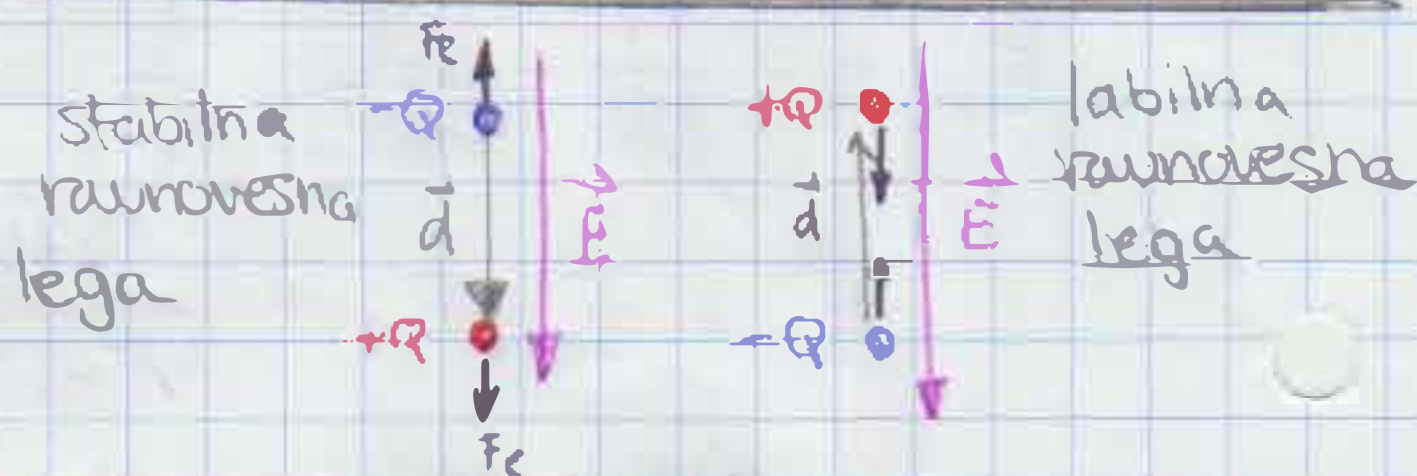
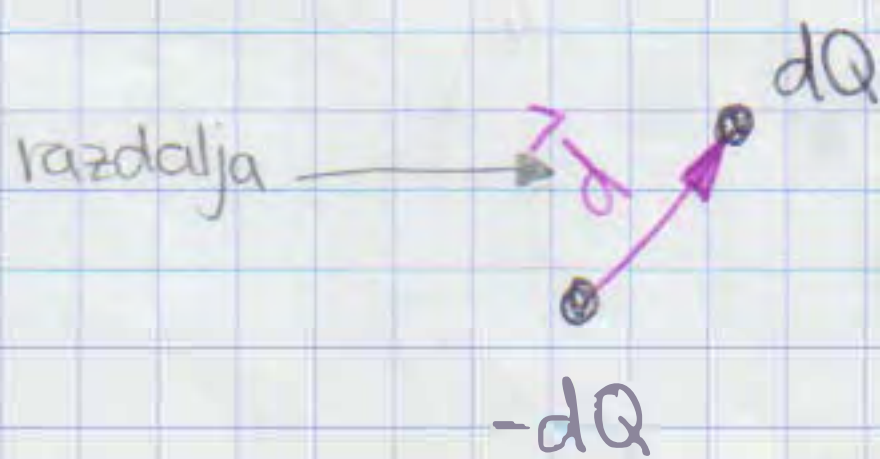
Električni dipol

$$\vec{p} = dQ \cdot \vec{d}$$

velikost dipola

DIPOLSKI MOMENT

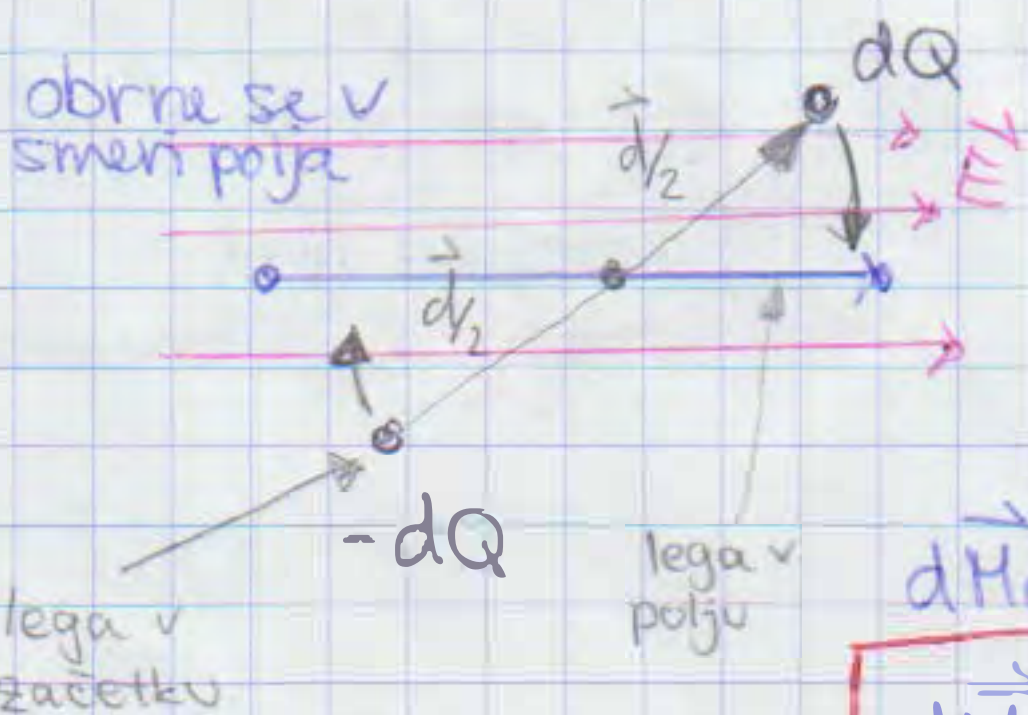
→ bolj kot sta razdaljena močnejši je dipol



Navor na dipol

$d\vec{M} = d\vec{r} \times \vec{F}$ ročica kvant sila ne sme se obratovati besed

$$d\vec{M}_e = dQ \cdot \frac{d}{2} = \frac{d}{2} \times dQ\vec{E} - \frac{d}{2} \times (-dQ)$$



$$d\vec{F}_e = dQ\vec{E}$$

$$d\vec{M}_e = dQ\vec{d} \times \vec{E}$$

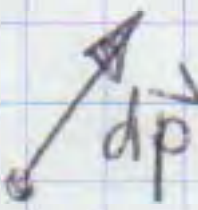
$$d\vec{M}_e = d\vec{p} \times \vec{E}$$

$|dM_e| = Q \rightarrow$ to je to

navor M_e so pojavi samo ko se naboj obrača v smeri polja

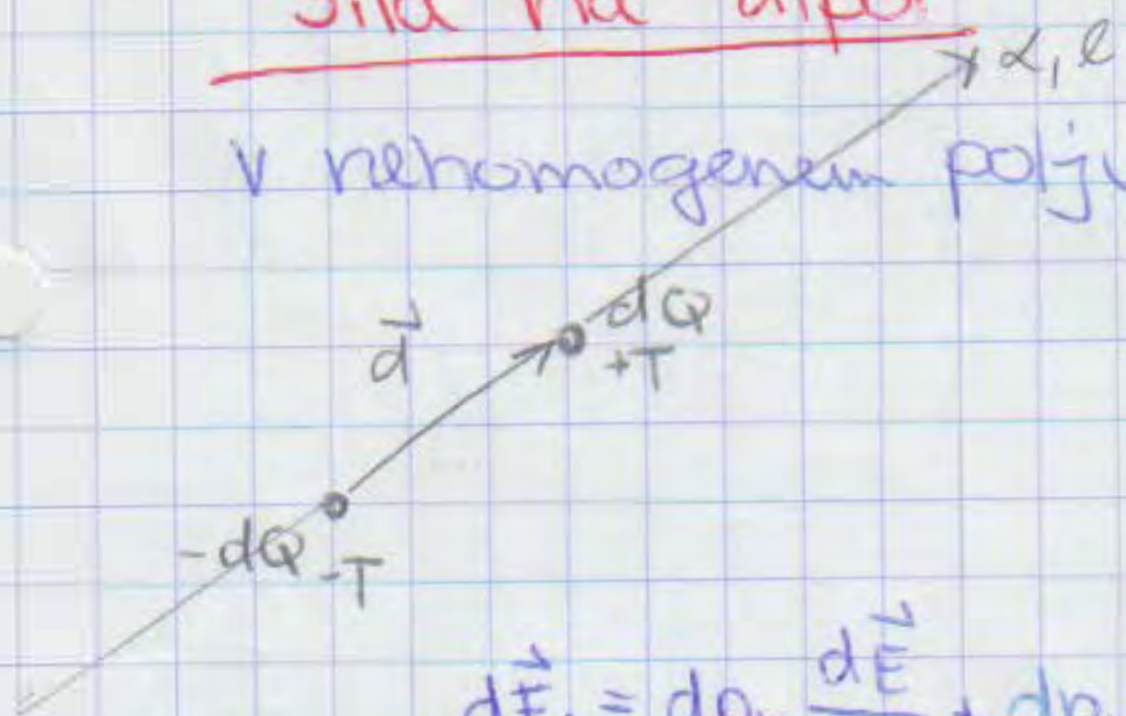
potencialna energija

$$dW_{ep} = -d\vec{p} \cdot \vec{E}$$



Sila na dipol

v nehomogenem polju



$$\vec{dF}_e = dQ \cdot \vec{E}(T_+) + (-dQ) \cdot \vec{E}(T_-) =$$

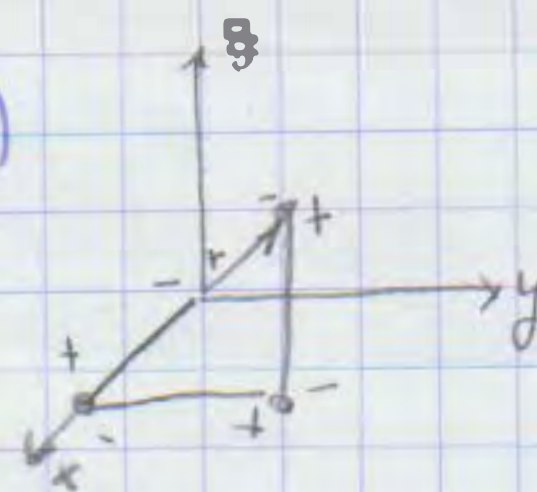
$$= dQ [\vec{E}(T_+) - \vec{E}(T_-)]$$

$$d\vec{p} = (dp_x, dp_y, dp_z)$$

$$d\vec{F}_e = dp_x \frac{d\vec{E}}{dx} + dp_y \frac{d\vec{E}}{dy} + dp_z \frac{d\vec{E}}{dz}$$

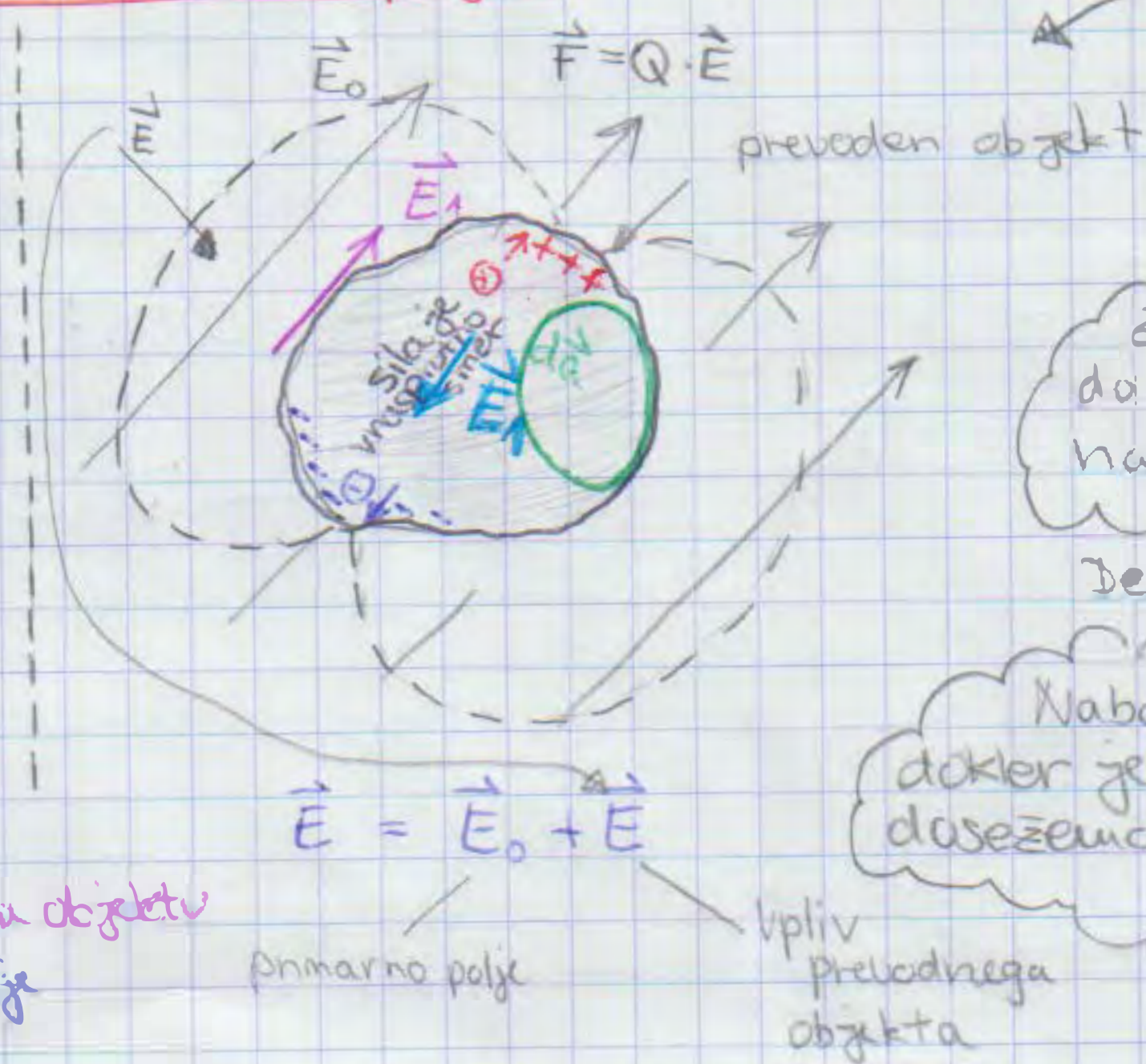
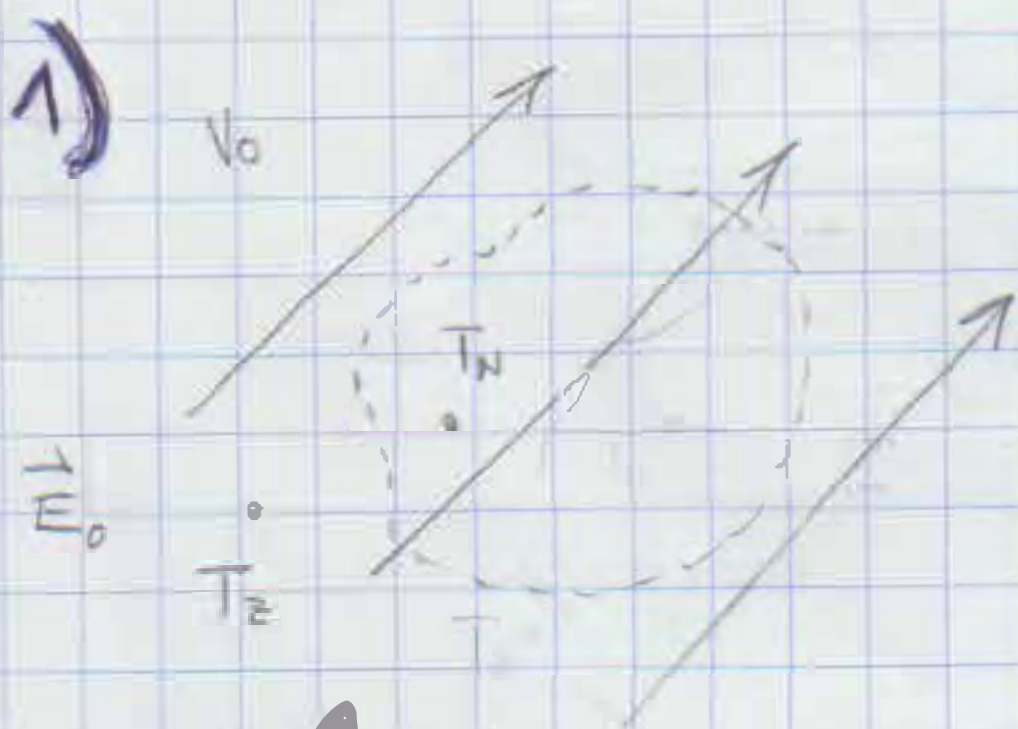
dipol vleče proti večjemu/močnejšemu polju

homogeno polje



Prevodnik v električnem polju - INFLUENCA

zunanje polje se spremeni



zaradi E pride do preazporoditve naboja - **INFLUENCA**

Delitev naboja

Naboj se preazporodja dokler je $\vec{E}_1 \neq 0$, ravnoesje dosežemo dokler ni postal $\vec{E}_{int} = 0$ zaradi preazporoditve

$E_{not} = 0$

$\vec{E}_1$ polje na objektu
 $\vec{E}_0$ vse polje

primarno polje

vpliv prevodnega objekta

2) kje se nahajajo naboji?

$$\oint_a \vec{E} d\vec{a} = \frac{Q_{not}}{\epsilon_0} = 0$$

naboj se nahaja na površini objekta

$$Q_{not} = \epsilon_0 \oint_{f_{izum}} \vec{E} d\vec{a}$$

če ima telo presežek naboja se ta nahaja na površini

Celoten objekt je ekvipotencialka

3) Potencial prevodnika

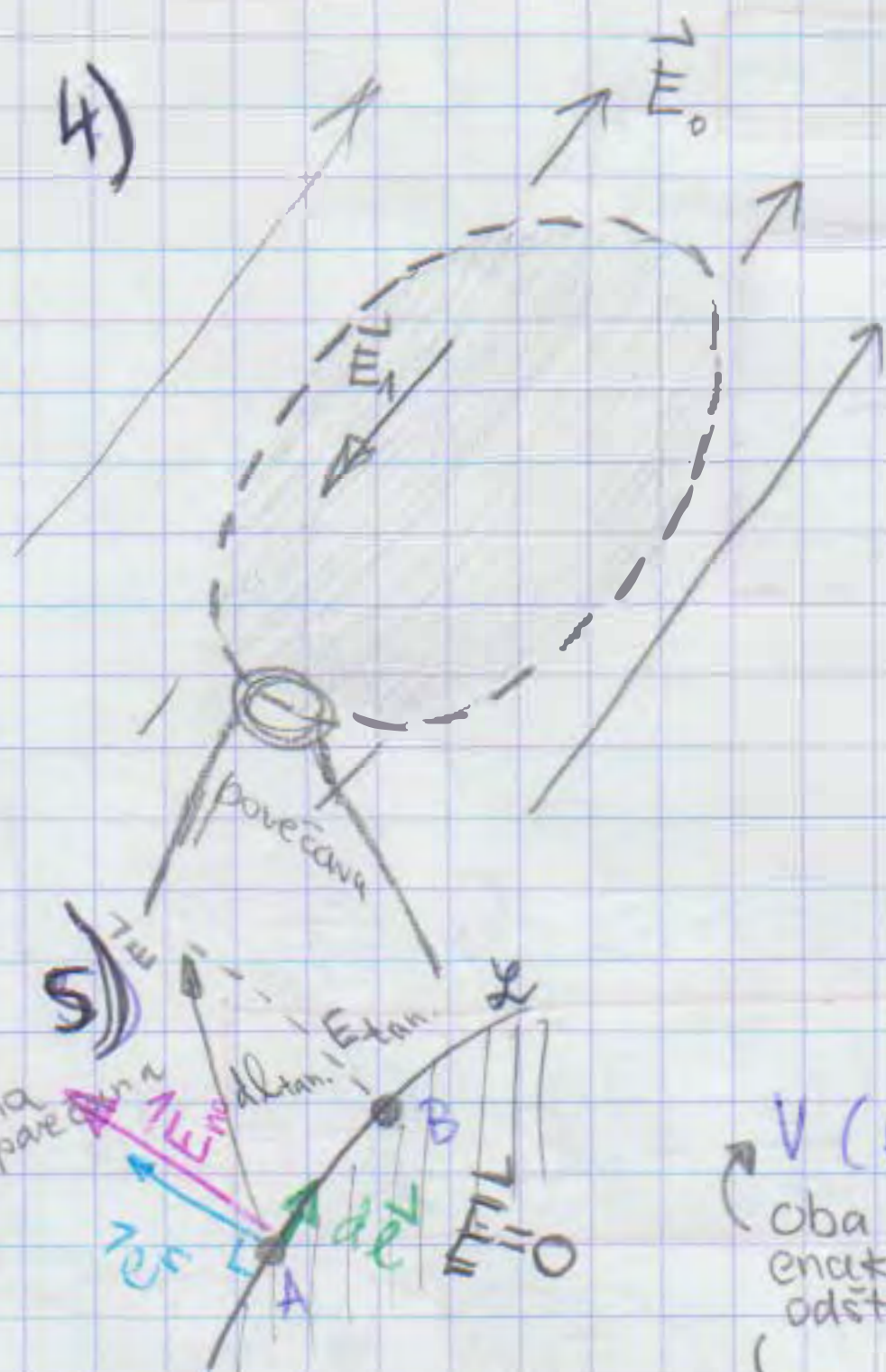


$$V(A) - V(B) = \int_A^B \vec{E} \cdot d\vec{l}$$

$\vec{E}_{\text{not}} = 0$

$$V(A) = V(B) \quad \text{ker bi vpliva } \vec{E} \text{ na potencial}$$

Celoten prevodnik je na istem potencialu.
(predstavlja ekvipotencialno ploščev)
ker je potencial povsod enak



$$\vec{E} = \vec{E}_0 + \vec{E}_1 = \vec{E}_{\text{not}} = 0$$

$$\vec{E}_1 = -\vec{E}_0$$

naboji se po površini razporedijo, da v vsaki točki nasprotujejo zunanjemu polju

$$V(A) - V(B) = \int_A^B \vec{E} \cdot d\vec{l}$$

Oba potenciala sta enaka in se odštejeta

$$0 = \int_{A, B} E_{\text{tange}} \cdot dl_{\text{tangenci.}}$$

$$\vec{E} \rightarrow E_{\text{tan}} = 0$$

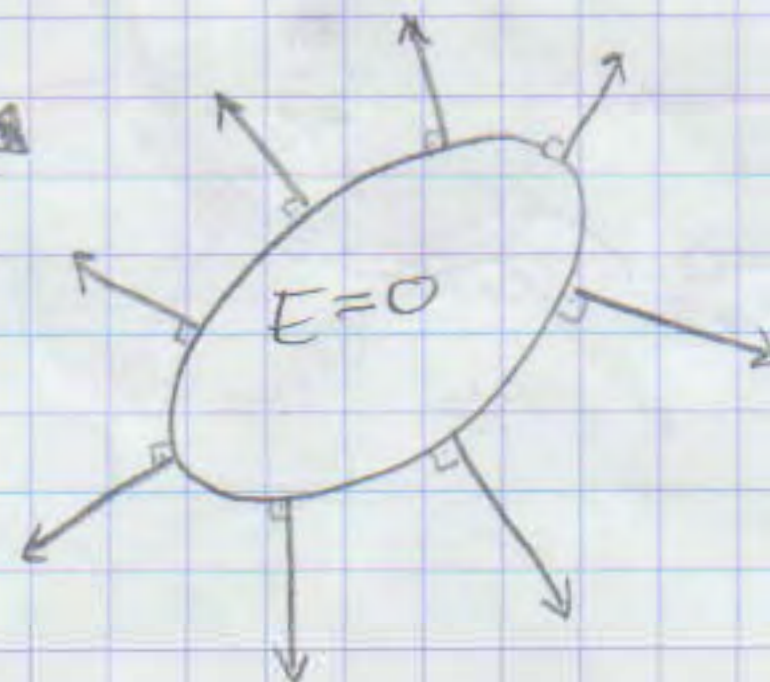
$$E_{\text{normale}} \neq 0$$

$$\vec{E} = \vec{E}_n \vec{n} = -\vec{n} \frac{dV}{dn}$$

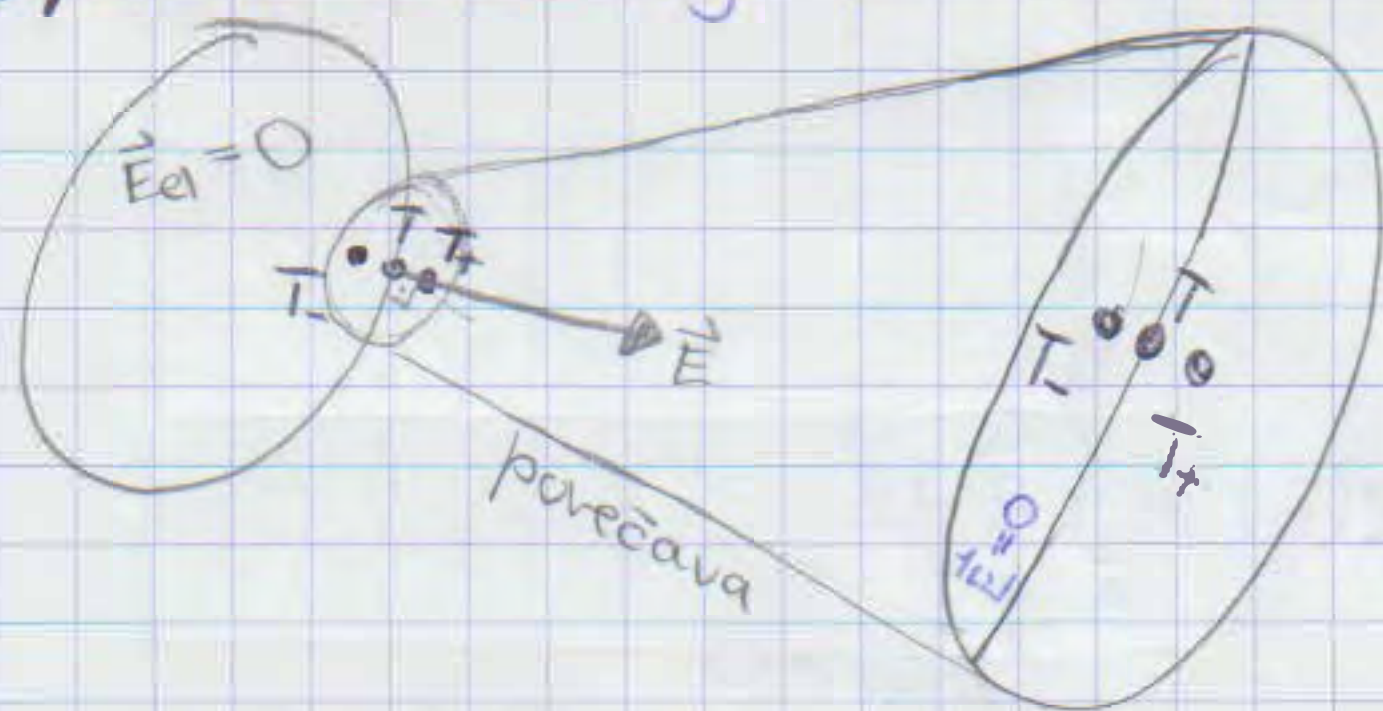
elektrostatično polje je pravokotno na prevodnik

$$E_{\text{tan.}} = 0$$

električno polje je $\perp$ na ploščev v smeri normale



6) polje na meji prevodnega objekta



$$\vec{E} = \vec{E}' + \vec{E}''$$

polje "diske" ravnine

polje nabojev izven diska

$$\vec{E}(T_+) = \vec{E}'(T_+) + \vec{E}''(T_+) = \frac{\vec{E}_n}{\epsilon_0} \frac{G}{2\epsilon_0} \quad \text{do površini}$$

$$\vec{E}(T) = \vec{E}'(T) + \vec{E}''(T) = \frac{\vec{E}_n}{2\epsilon_0} \quad \text{na površini}$$

$$\vec{E}_{\text{not}} = \vec{E}(T_-) = \vec{E}'(T_-) + \vec{E}''(T_-) = 0 \quad \text{v površini}$$

$$\vec{E}_n = \frac{\vec{E}_n}{2\epsilon_0}$$

$$E_n = \begin{cases} \frac{G}{\epsilon_0} & \text{v } T_+ \\ \frac{G}{2\epsilon_0} & \text{v } T \\ 0 & \text{v } T_- \end{cases}$$

$$\vec{E} = 0$$

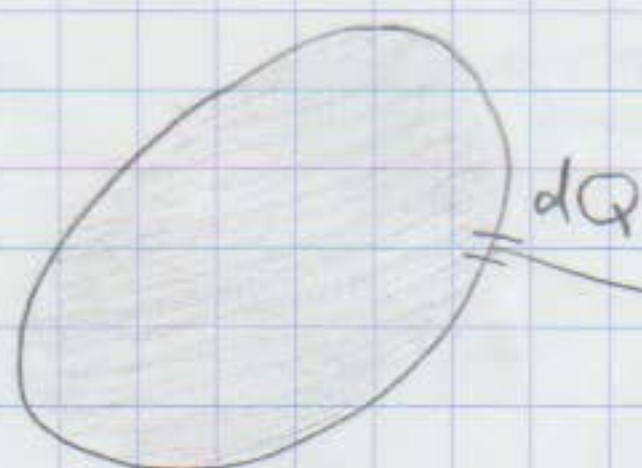
znotraj

izven

$$\vec{E} = \vec{E}_n \frac{G}{\epsilon_0}$$

$$\vec{E} = \vec{E}_n \frac{G}{2\epsilon_0}$$

7) Elektrostatični tlak



$$d\vec{F} = dQ \cdot \vec{E} = \vec{E}_n \cdot G \frac{G}{2\epsilon_0} da \quad | : da$$

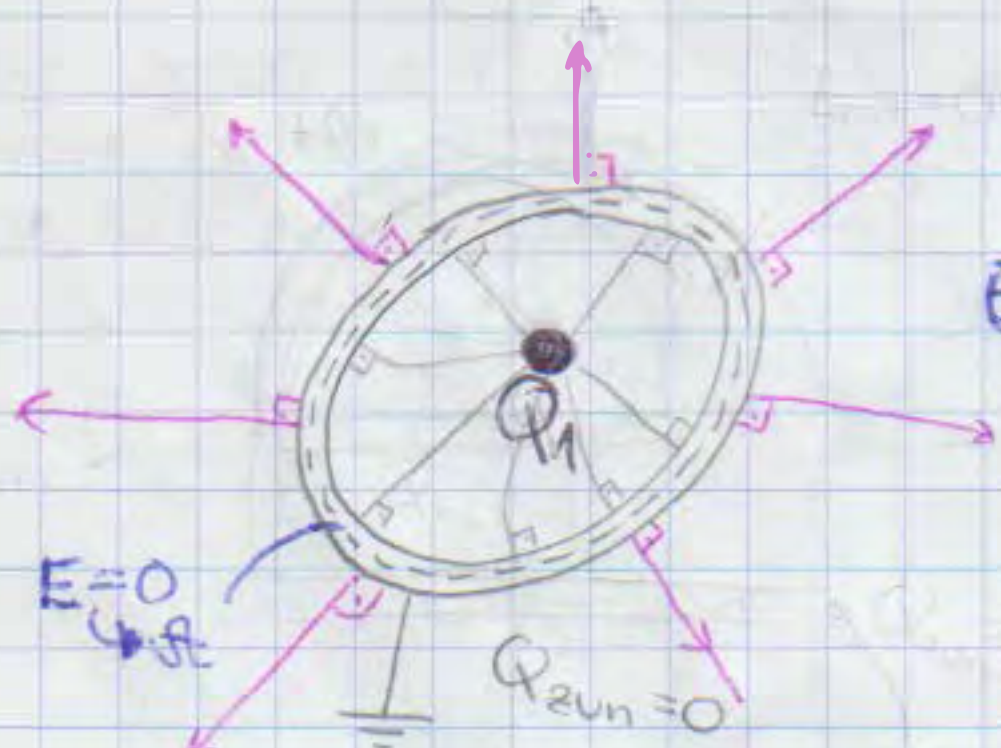
$$dpe = \frac{dF}{da} = \vec{E}_n \frac{G}{2\epsilon_0} = \epsilon_0 \frac{E^2}{2} = W_{pe} \quad \text{!}$$

$$p_e = \epsilon_0 \frac{G^2}{2\epsilon_0}$$

zmeraj je rezultat

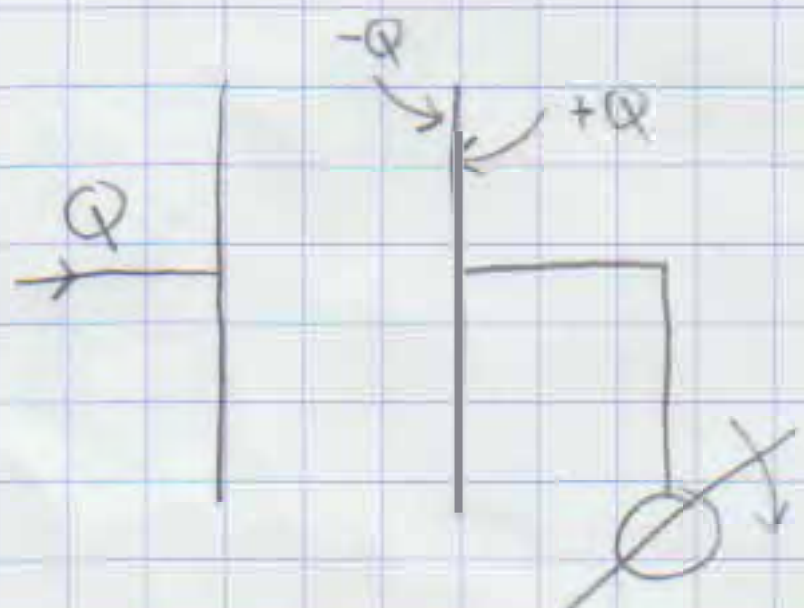
TLAK JE ENAK ELEKTROSTATIČNI ENERGIJI

Električno zaključčen sistem, avtonomnost



$$\oint \vec{E} d\vec{a} = 0 = \frac{Q_{\text{not}}}{\epsilon_0} \quad \text{1. sistem}$$

$$E_{\text{zun}} = 0 \quad \text{Faradayeva kletka} \quad \text{2. sistem}$$

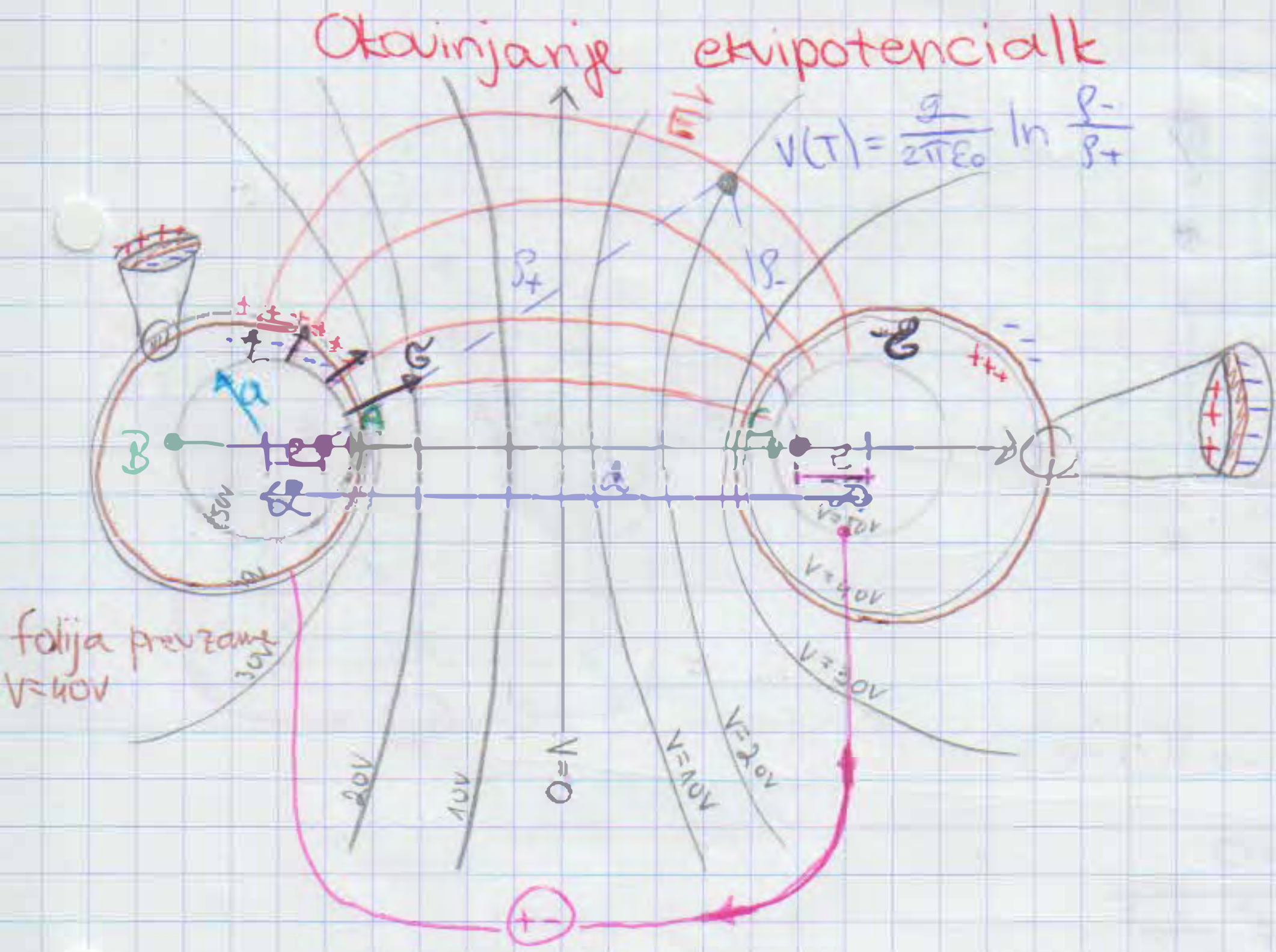


menjenje naboja

Oskvinjanje ekvipotencialk

$$V(r) = \frac{q}{2\pi\epsilon_0} \ln \frac{r_-}{r_+}$$

G pada z dolžino silnic E

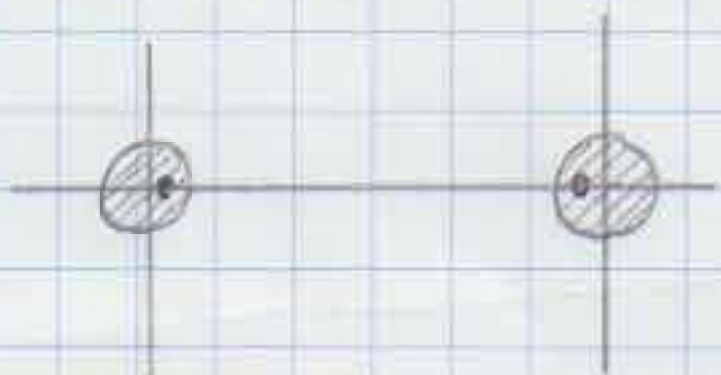


folija prevzame
V=40V

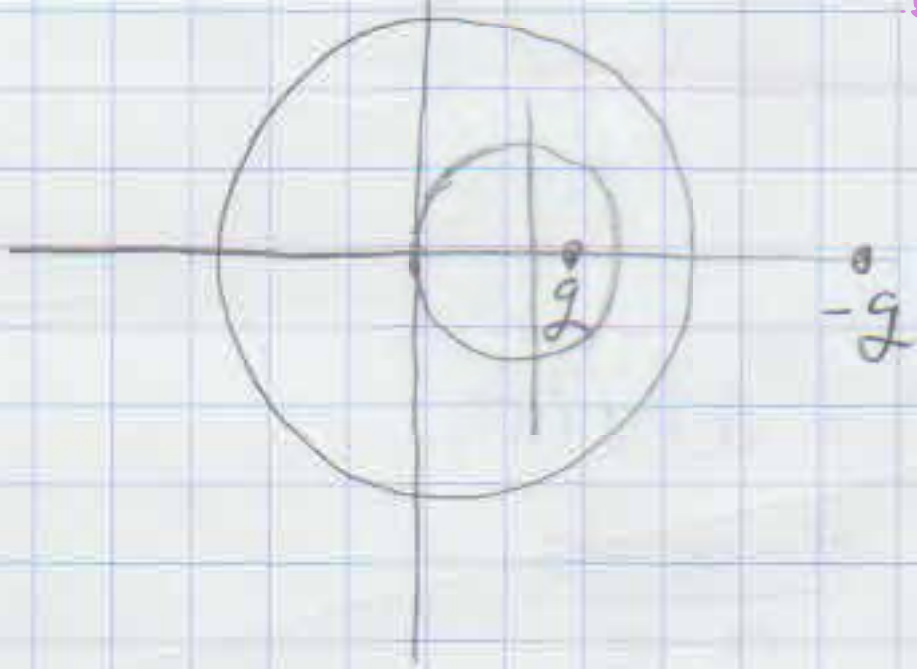
• Oskvinimo ekvipotencialke in dobimo žico. S pomočjo pomožnih nabojev q računamo vse ekscentričnost

• Oskvinimo ekvipotencialko dobimo žico iz naboja

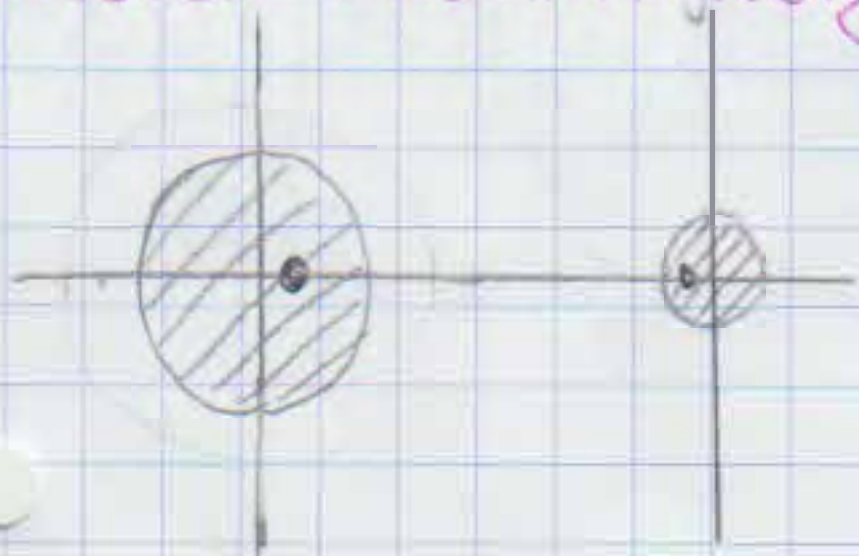
Zgled: dvovod



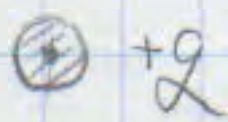
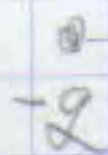
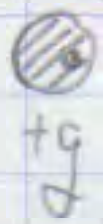
ekscentričen koax



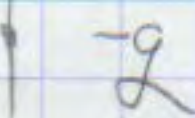
dvovod različnih radijev



vodnik pred
pre vodno
steno



vodnik nad
zemljo



Alta

simetričen dvovod

$$V(A) = \frac{q}{2\pi\epsilon_0} \ln \frac{d-a-e}{a-e} \quad !$$

$$V(B) = \frac{q}{2\pi\epsilon_0} \ln \frac{d+a-e}{a+e}$$

da sta zgornja potenciala enaka:

$$\frac{d-a-e}{a-e} \neq \frac{d+a-e}{a+e}$$

$$(d-a-e)(a+e) = (d+a-e)(a-e)$$

$$e^2 - de + a^2 = 0$$

$$e_{1,2} = \frac{d \pm \sqrt{d^2 - 4a^2}}{2}$$

$$e = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - a^2}$$

pove nam razdaljo od središča

$$V(C) = -V(A)$$

$$U = V(A) - V(C) = 2 \cdot V(A)$$

$$U = \frac{q}{\pi\epsilon_0} \ln \frac{d-a-e}{a-e}$$

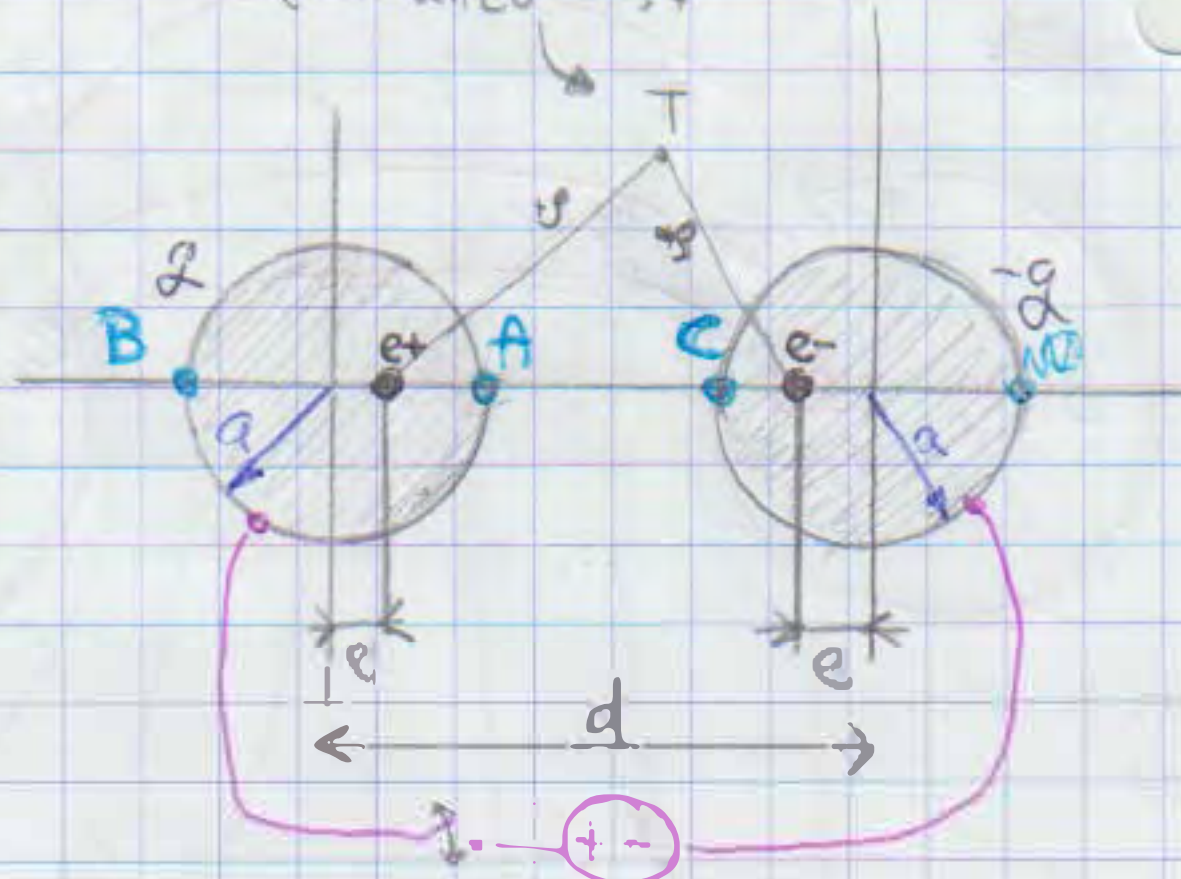
$$q = \frac{\pi\epsilon_0 U}{\ln \frac{d-a-e}{a-e}}$$

kapacitivnost na dolžinsko enoto

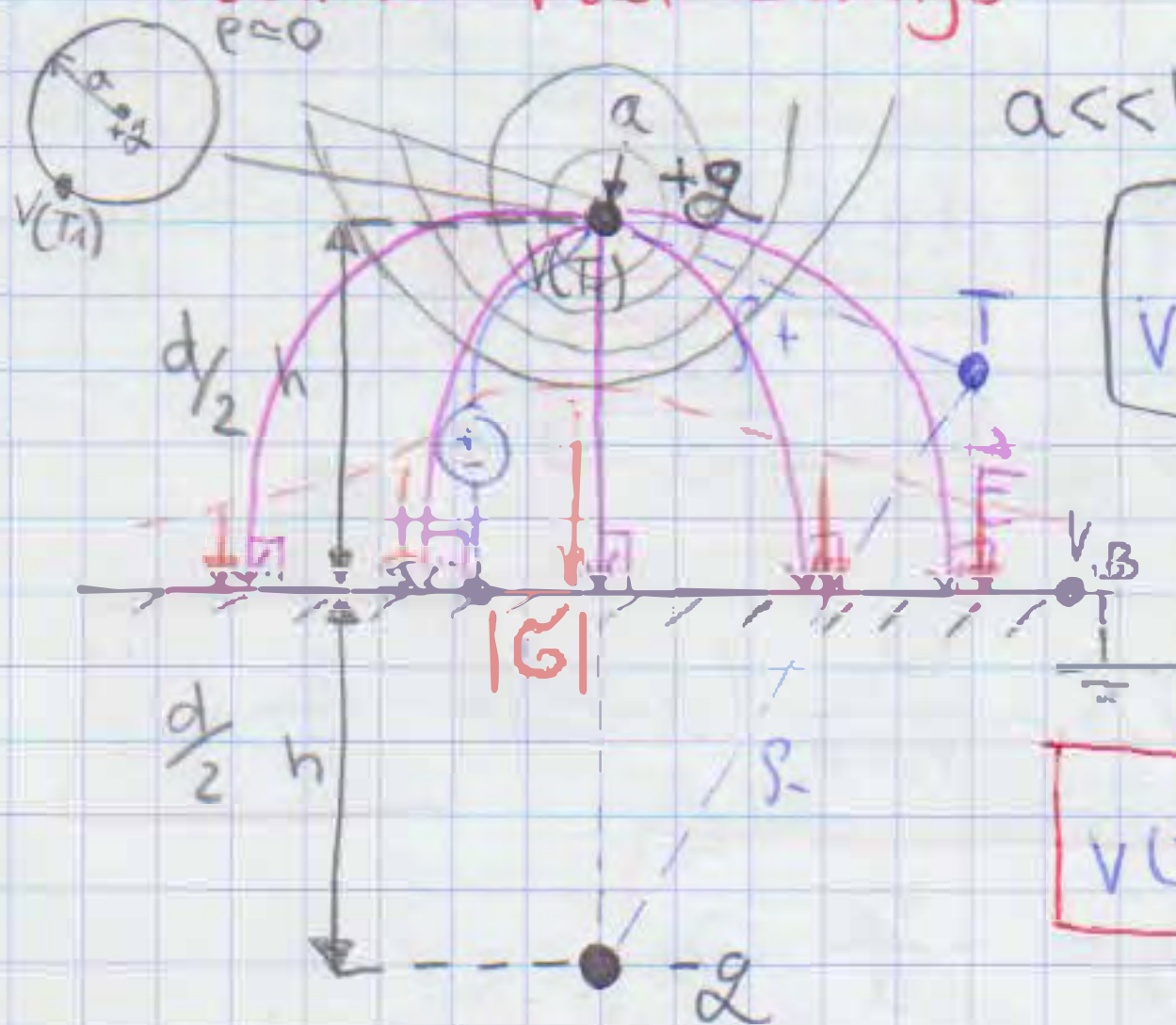
Q dobimo da q pomnožimo z l

$$Q = q \cdot l$$

$$V(T) = \frac{q}{2\pi\epsilon_0} \ln \frac{r_+}{r_-}$$



Vodnik nad zemljo



$$a \ll h \quad e \approx 0$$

$$V(T) = \frac{Q}{2\pi\epsilon_0 l} \ln \frac{r_-}{r_+}$$

splošna enačba

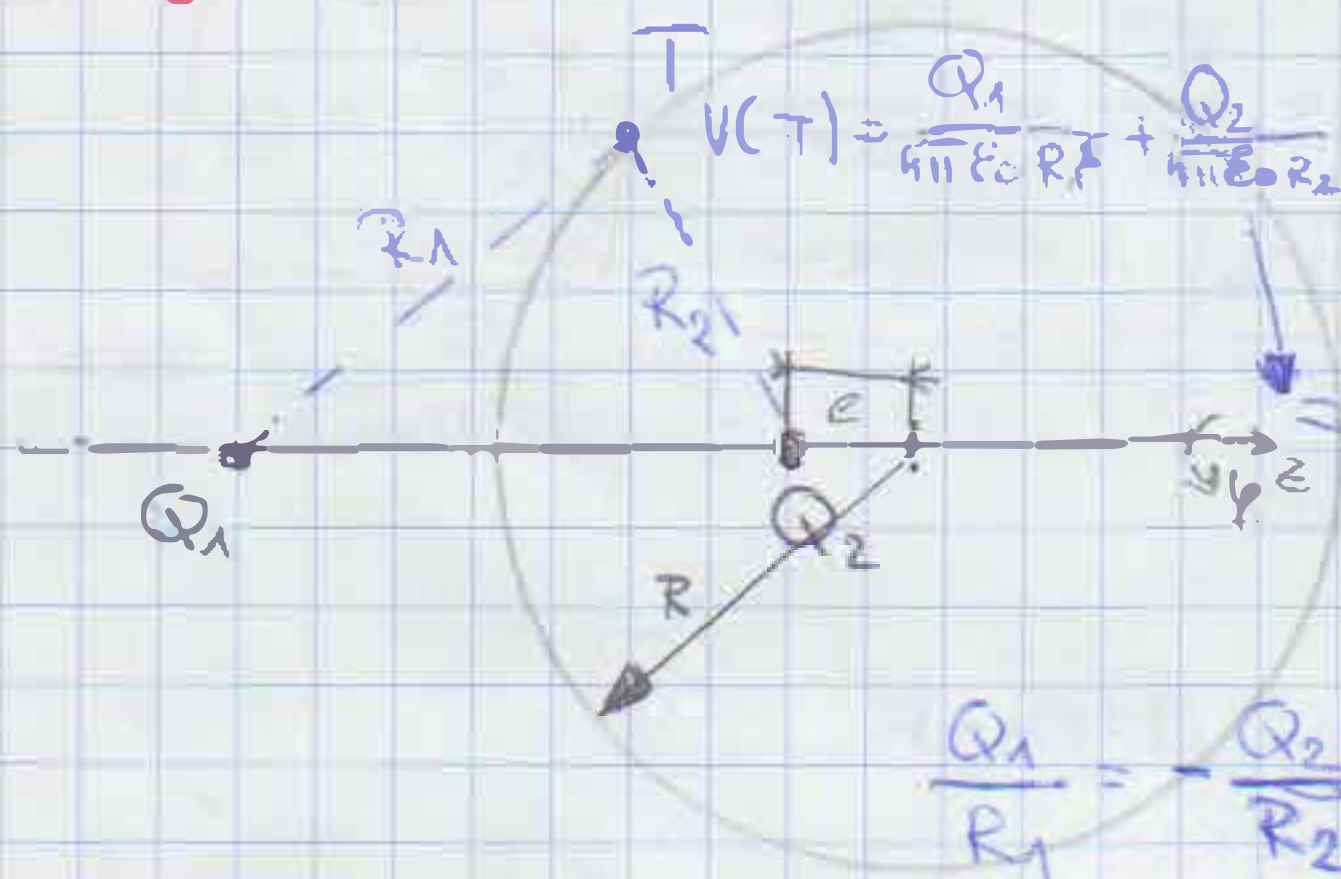
$$V(B) = 0 \text{ ker je na zemlji}$$

$$V(T) \approx \frac{Q}{2\pi\epsilon_0 l} \ln \frac{2h}{a}$$

Ku je polmer žice zanemarljiv glede na medosno razdaljo

$$Q = \frac{2\pi\epsilon_0 l \cdot U}{\ln \frac{2h}{a}} = C \cdot U$$

Polje v okolici dveh točkastih nabojev



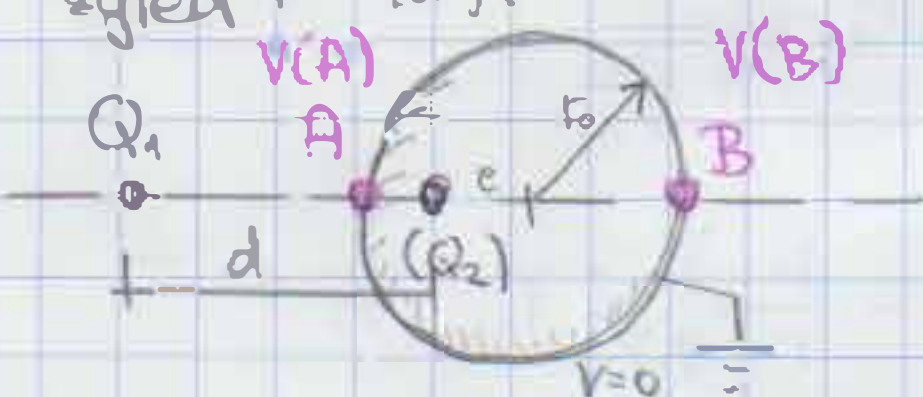
$$Q_1 \cdot Q_2 < 0 \quad |Q_1| \geq |Q_2|$$

$$\frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2} \right)$$

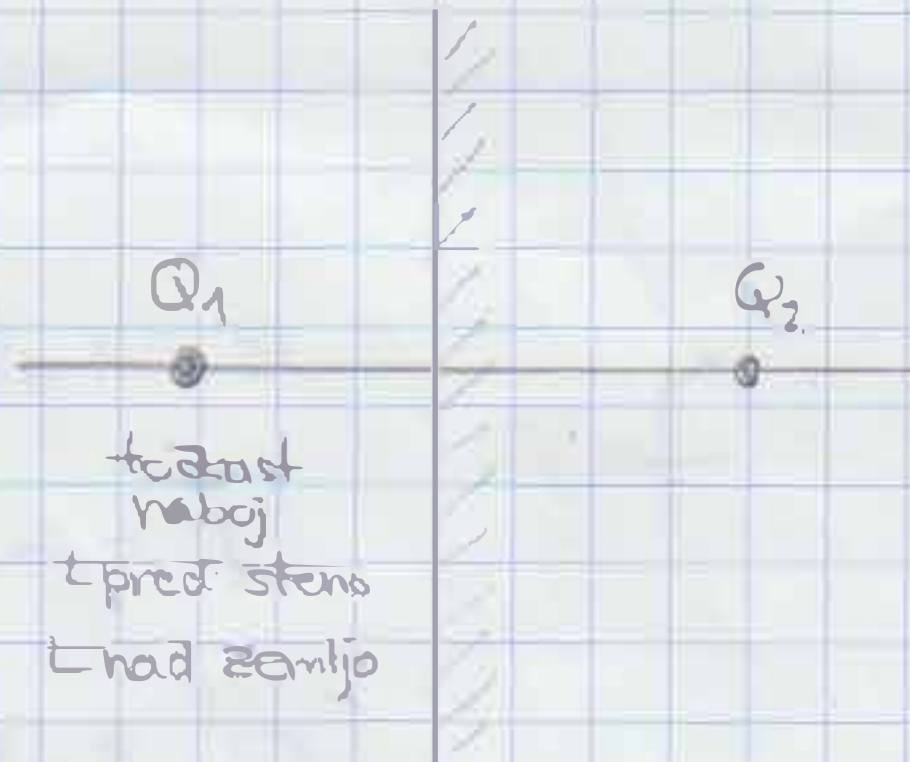
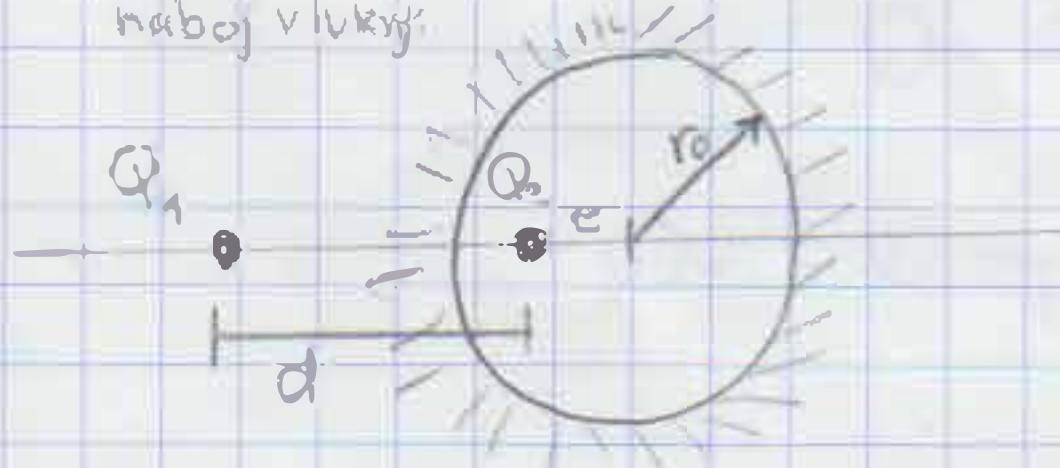
$$\frac{Q_1}{R_1} = -\frac{Q_2}{R_2}$$

ekvipotencialka je sfera
 $V = \text{const}$

zgled 1 zemlja



naboj v luknji



točkast naboj
pred steno
nad zemljo

Zgled 1 zemlja

ko izenačimo dobimo

$$V(A) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{d-r_0} + \frac{Q_2}{r_0-e} \right)$$

$$V(B) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{d+r_0} + \frac{Q_2}{r_0+e} \right)$$

$$de = r_0^2$$

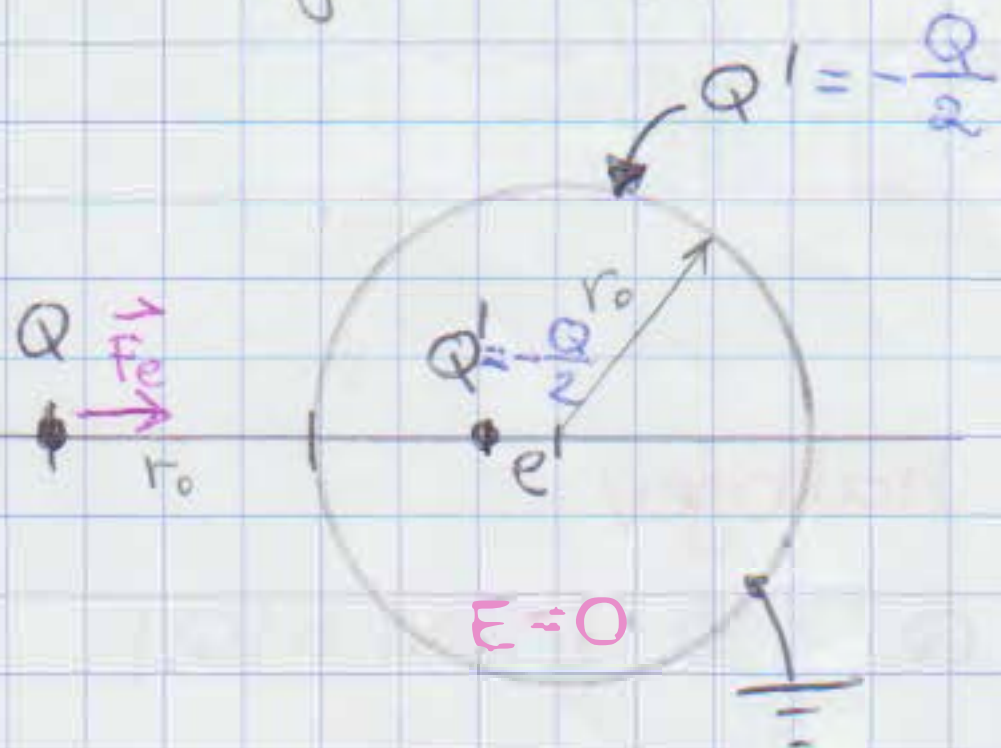
$$e = \frac{r_0^2}{d}$$

ekscentričnost

$$Q_1 = -\frac{Q_2}{r_0-e} (d-r_0) = -\frac{d}{r_0} Q_2$$

$$Q_2 = -\frac{r_0}{d} Q_1$$

Zgled: naboj pred ozemljeno kroglo



$$Q' = -\frac{r_0}{d} Q = -\frac{r_0}{2r_0} Q = -\frac{Q}{2} = Q'$$

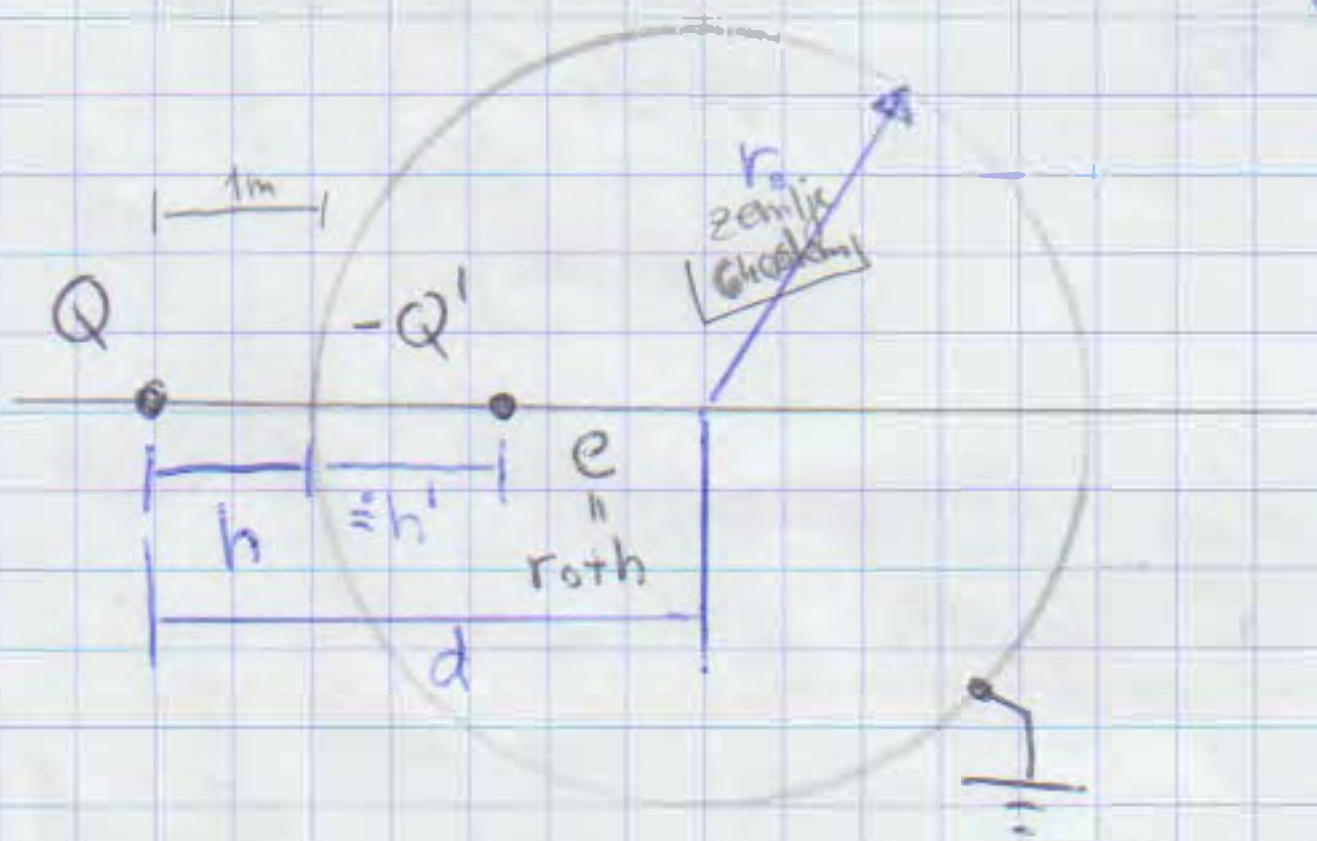
$$e = \frac{r_0^2}{d} = \frac{r_0^2}{2r_0} = \frac{r_0}{2} = e$$

$$|\vec{F}_e| = \frac{|QQ'|}{4\pi\epsilon_0 (\frac{3r_0}{2})^2} = \frac{Q^2}{28\pi\epsilon_0 3r_0^2} = \frac{Q^2}{18\pi\epsilon_0 r_0^2}$$

delci se lepijo

Vloga zemlje

$$d = r_0 + h$$

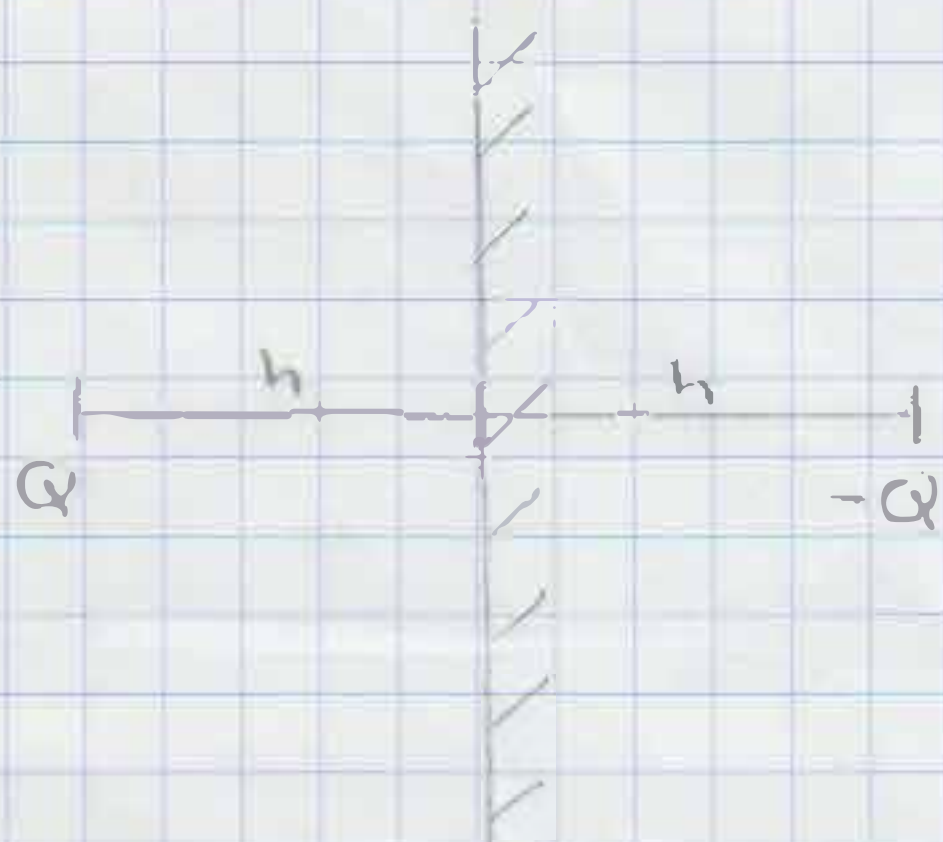


$$e = \frac{r_0^2}{d} = \frac{r_0^2}{r_0 + h} = r_0 \frac{r_0}{r_0 + h} = r_0 \frac{1}{1 + \frac{h}{r_0}}$$

$$e = r_0 \left(1 - \frac{h}{r_0} \right) = r_0 - h$$

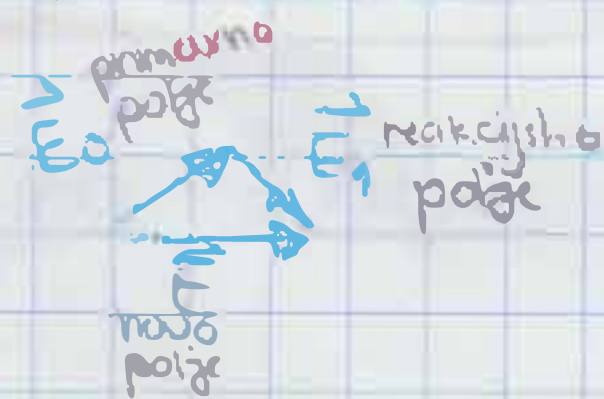
$$Q' = -\frac{r_0}{d} Q$$

$$Q' = -\frac{r_0}{r_0 + h} Q = -Q$$



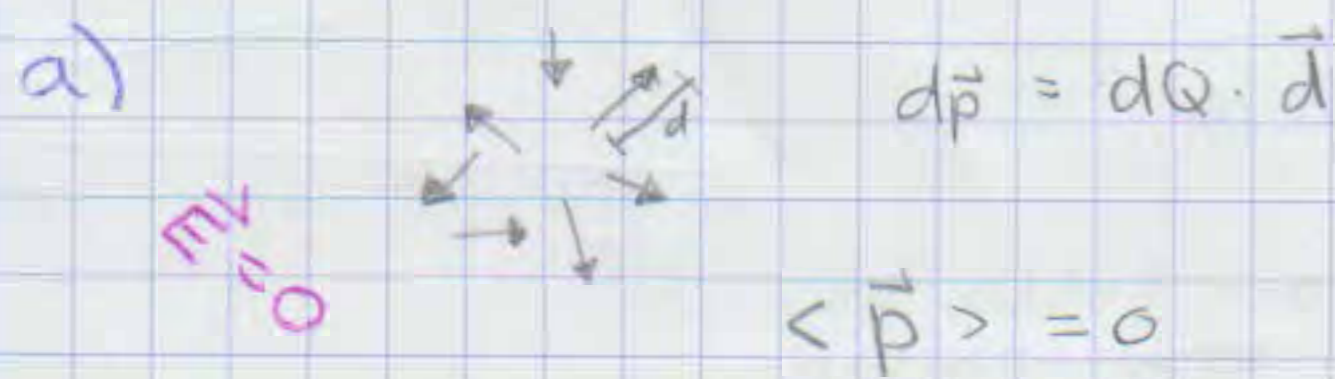
Izolant / dielektrik v električnem polju - POLARIZACIJA

Tri vrste polarizacije

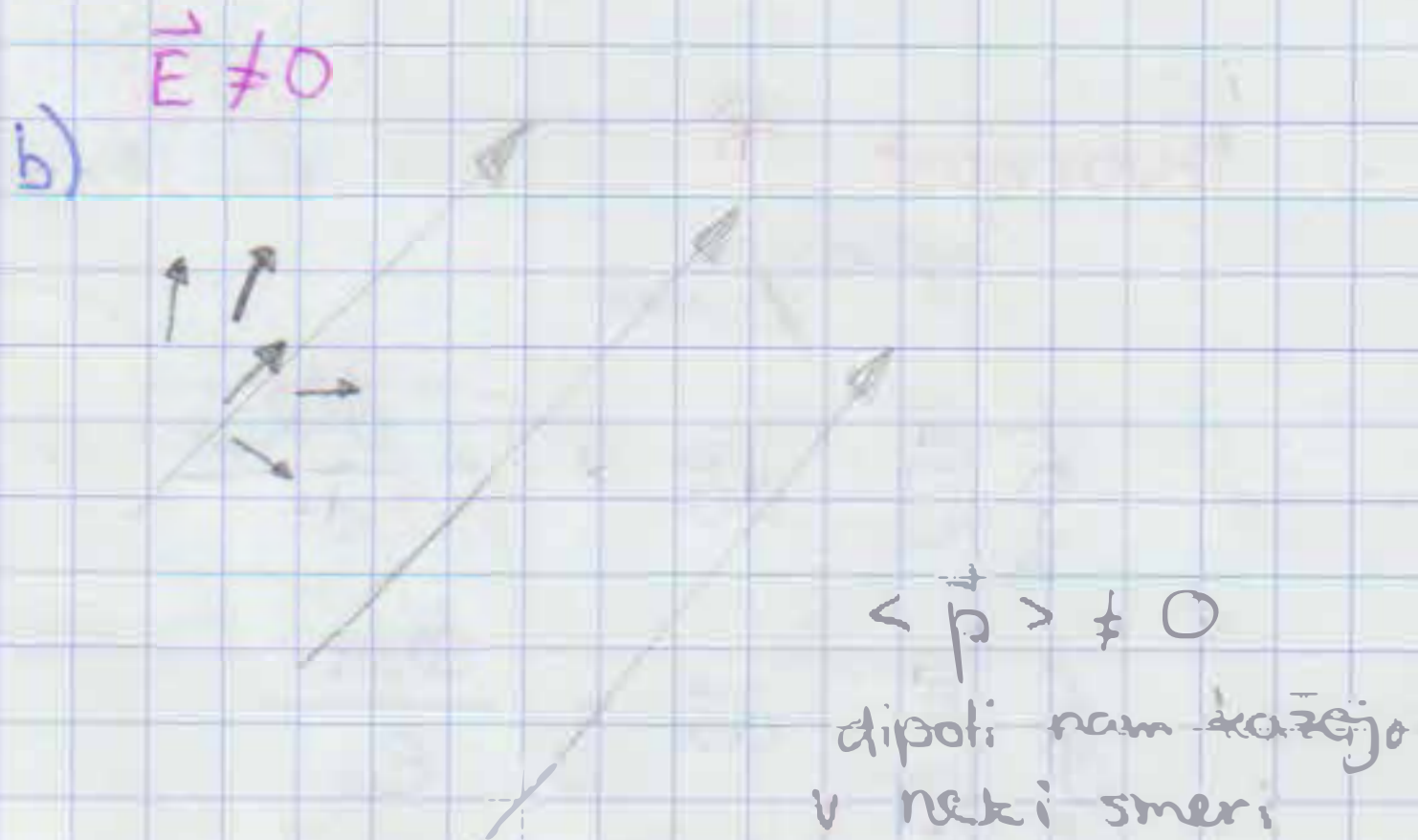


1) Dipolska polarizacija - prisotna elektronska polarizacija

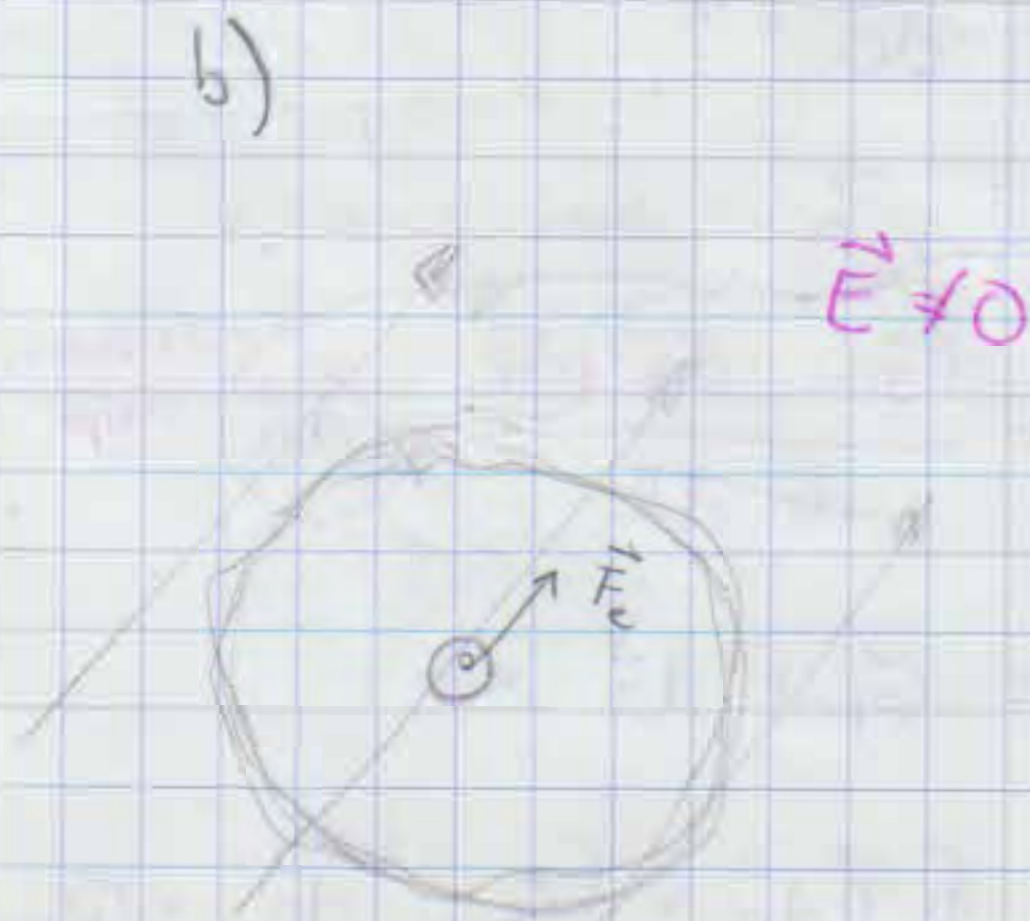
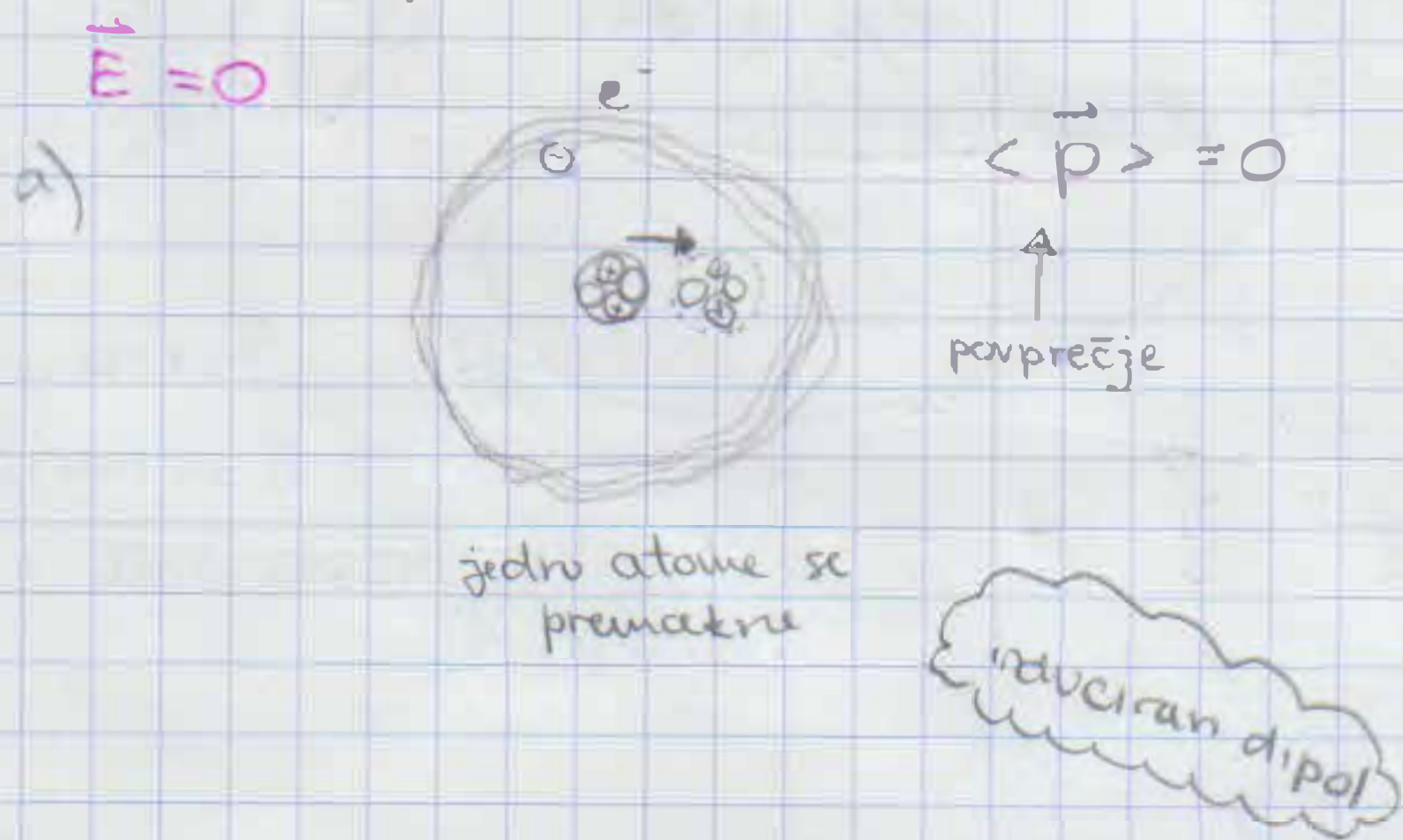
↳ snov s permanentno izraženimi dipoli



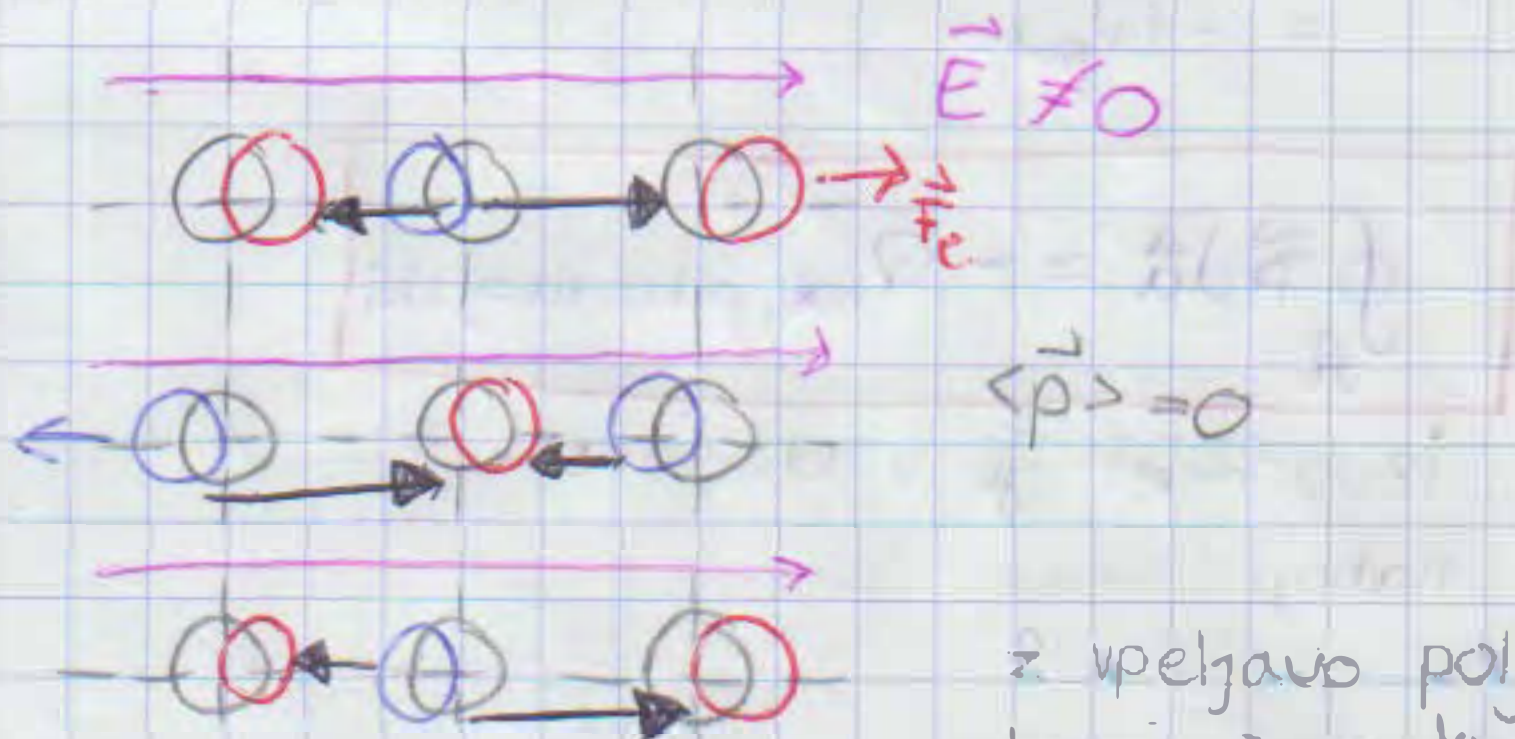
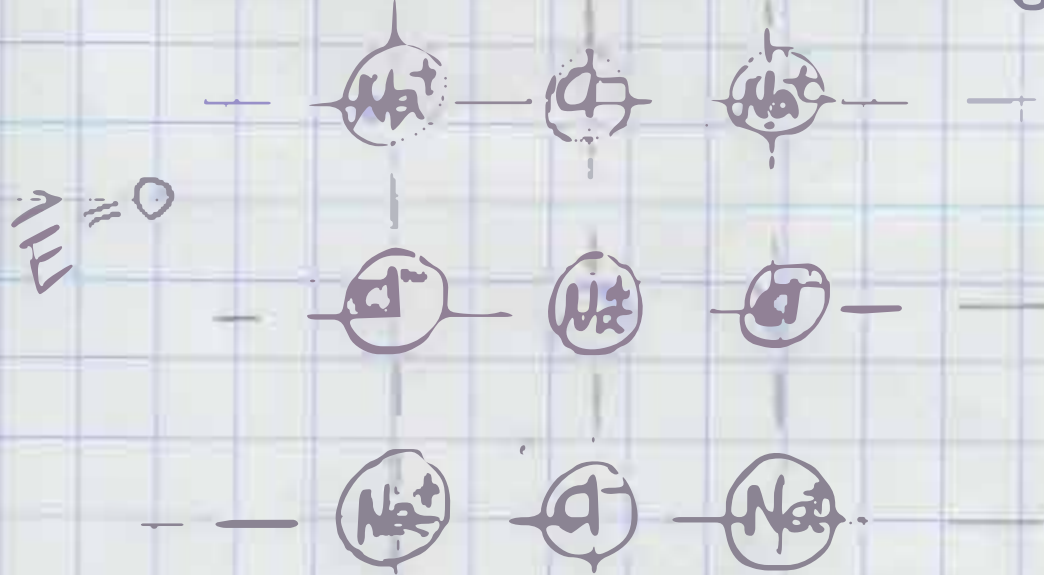
($\langle \rangle$) - krajevno povprečje



2) Elektronska polarizacija prisotna elektronska polarizacija



3) Ionska polarizacija



v povprečju je
vsota dipolskih momentov
enaka nič

z vpljavo polja se
atomii zamaknejo v levo
in desno odvisno od $\pm / -$

* premiki so v
razredu 10^{-14} m

Anizotropna snov
polarizacijski vektor ni
v isti smeri kot elek. polje

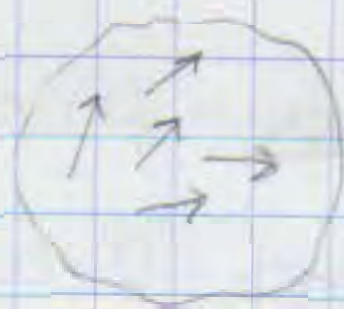
Ele. polarizacija, vektor $\vec{P}$

UCB. 14-4

Učinek polja na dielektrik

$$\vec{P} = \lim_{dV \rightarrow 0} \frac{\sum d\vec{p} [Cm]}{dV [m^3]} \left[\frac{C}{m} \right]$$

↑
volumen



Izurnost $\vec{P}$

↓
gaussov stavek

UCB. 14-4

pove nam koliko toka gre ven

$$\oint \vec{J} \cdot d\vec{a} = - \frac{dQ_{not}}{dt}$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\epsilon_0}$$

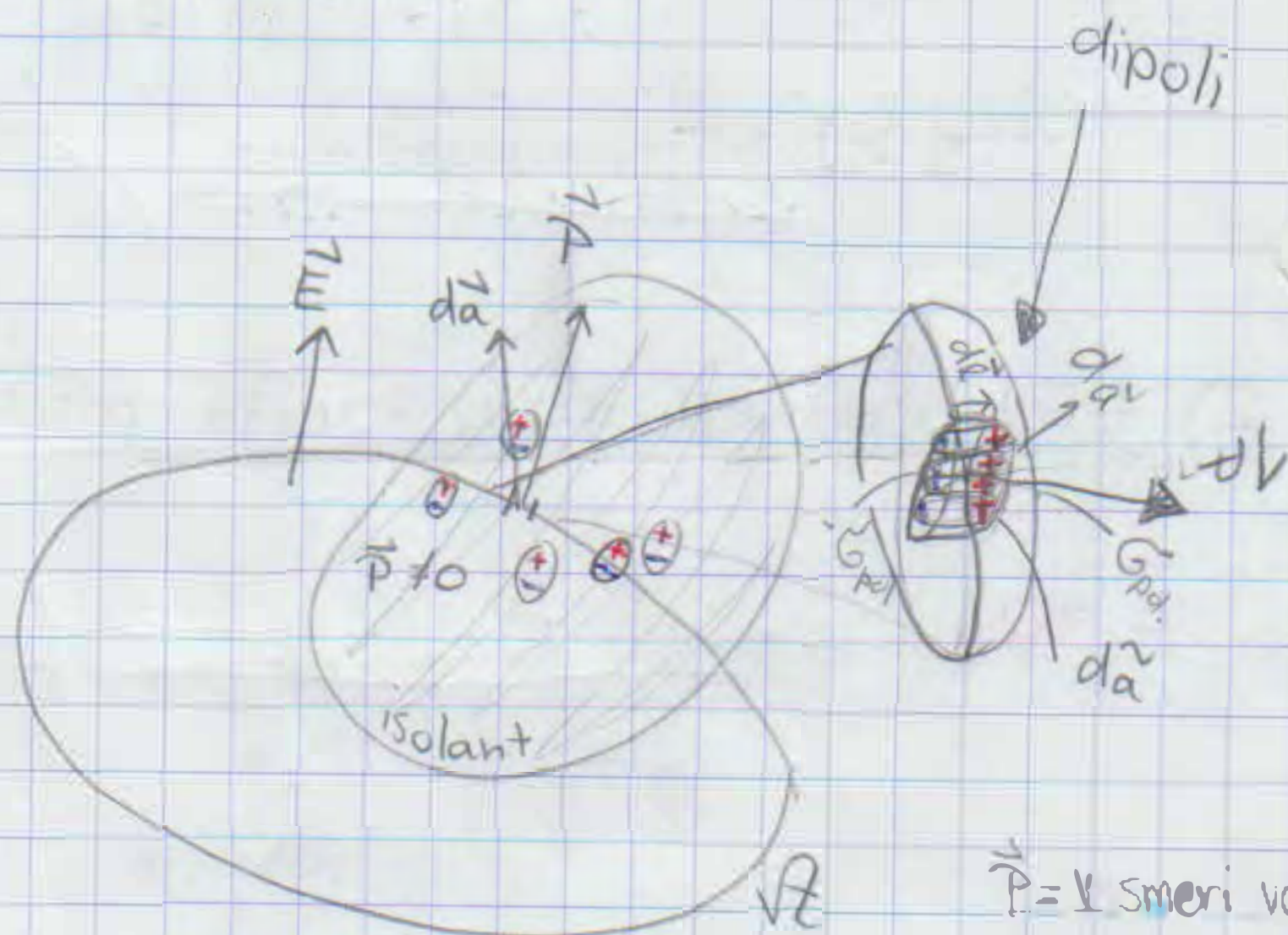
$$\oint \vec{P} \cdot d\vec{a} = ?$$

naboj

seštevek dipols. momentov

$$\vec{P} \cdot d\vec{a} = \frac{(\vec{G}_{pol} \cdot d\vec{a}) d\vec{a}}{d\vec{a} d\vec{a}}$$

(volumen valja)



$\vec{P} = \vec{p}$ smeri valja

$\vec{a} = \vec{v}$ smeri ploščke

$G = na$ ploški +

mnostina
naboja preseka
na + ploški

$$\vec{G}_{pol} \cdot d\vec{a} =$$

$$\oint \vec{P} \cdot d\vec{a} = \int \vec{G}_{pol} \cdot d\vec{a} = Q_{pol, zunaj}$$

$$= -Q_{pol, notranji}$$

$$\oint \vec{P} \cdot d\vec{a} = -Q_{not, polarizacijski}$$

izurnost je v odvisnosti z
naboji (notranji, izunanji)

polarizacijski tok

UČ. 14-4

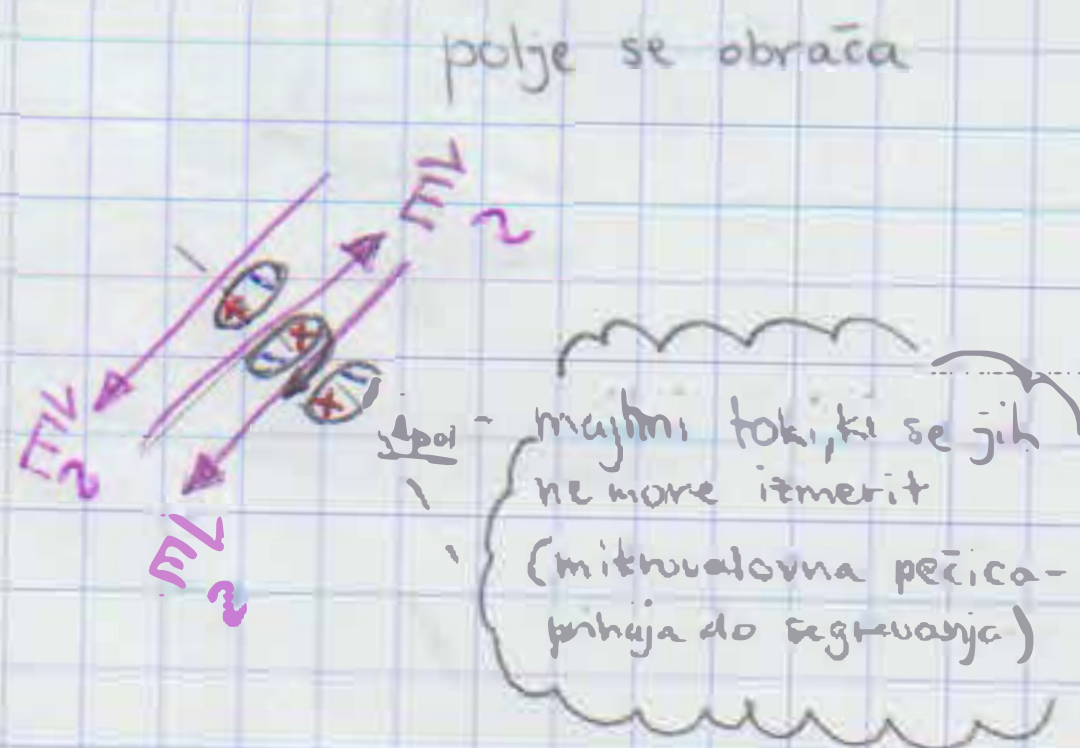
$$\oint \frac{d\vec{P}}{dt} \cdot d\vec{a} = - \frac{dQ_{pol.net}}{dt} = \dot{I}_{pol.} \quad \rightarrow \quad I = \oint \vec{J} d\vec{a}$$

$$\frac{d}{dt} \oint \vec{P} \cdot d\vec{a} = \oint \vec{J}_{pol.} \cdot d\vec{a}$$

gostota polarizacijskih tokov

$$\frac{d\vec{P}}{dt} = \vec{J}_{pol.}$$

gostota vezanih nabojev



polarizacijski naboji so tisti ki so vezani v strukturo

vektor gostote električnega pretoka $\vec{D}$

UČ. 14-5

$$\oint (\epsilon_0 \vec{E} + \vec{P}) \cdot d\vec{a} = Q_{net} - Q_{napolar.} = Q_{net.prosti}$$

$$\oint \vec{D} \cdot d\vec{a} = Q_{net.prosti} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E} = (1 + \chi_e) \cdot \epsilon_0 \vec{E}$$

seštejemo
gaussova
stavka
dobimo $\rightarrow$

$$\vec{D} = [C/m^2]$$

gostota el. pretoka $[C/m^2]$

ϵ_r - relativna dielektričnost snovi

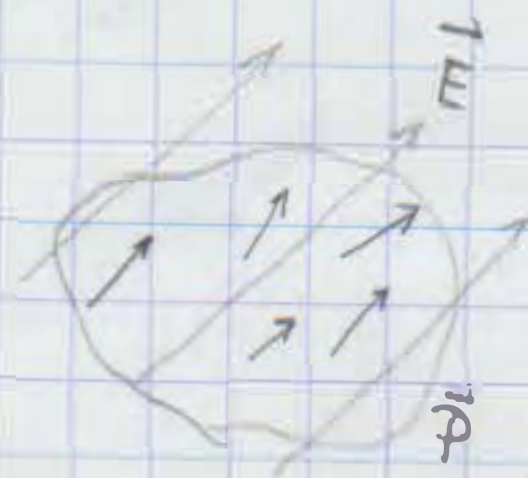
$$Def.: \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\lim \frac{\sum \vec{P}}{dV}$$

$$\vec{D} = \epsilon_0 \cdot \epsilon_r \cdot \vec{E}$$

Aleza med
ele. poljem in
gostote el. pretoka

obnašanje snovi



$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

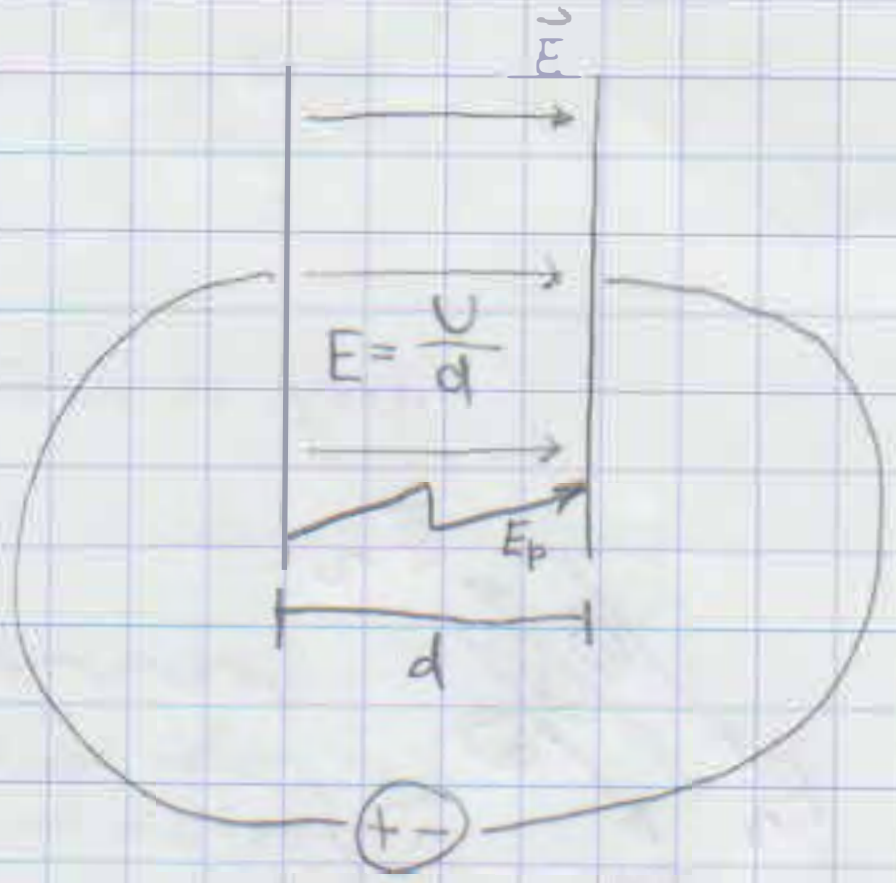
elek. susceptibilnost snovi

snov	relativno die. snovi	prebojna trdnost
prazen prostor	1	
zrak	1,0006	3 MV/m
steklo	5-10	10-30 MV/m
olje	2-15	
specialne snovi	10^6	

SUSCEPTIBILNOST
lastnost / dovzetnost snovi
za polariziranje / usmerjanje

Aktaar

Električni preboj

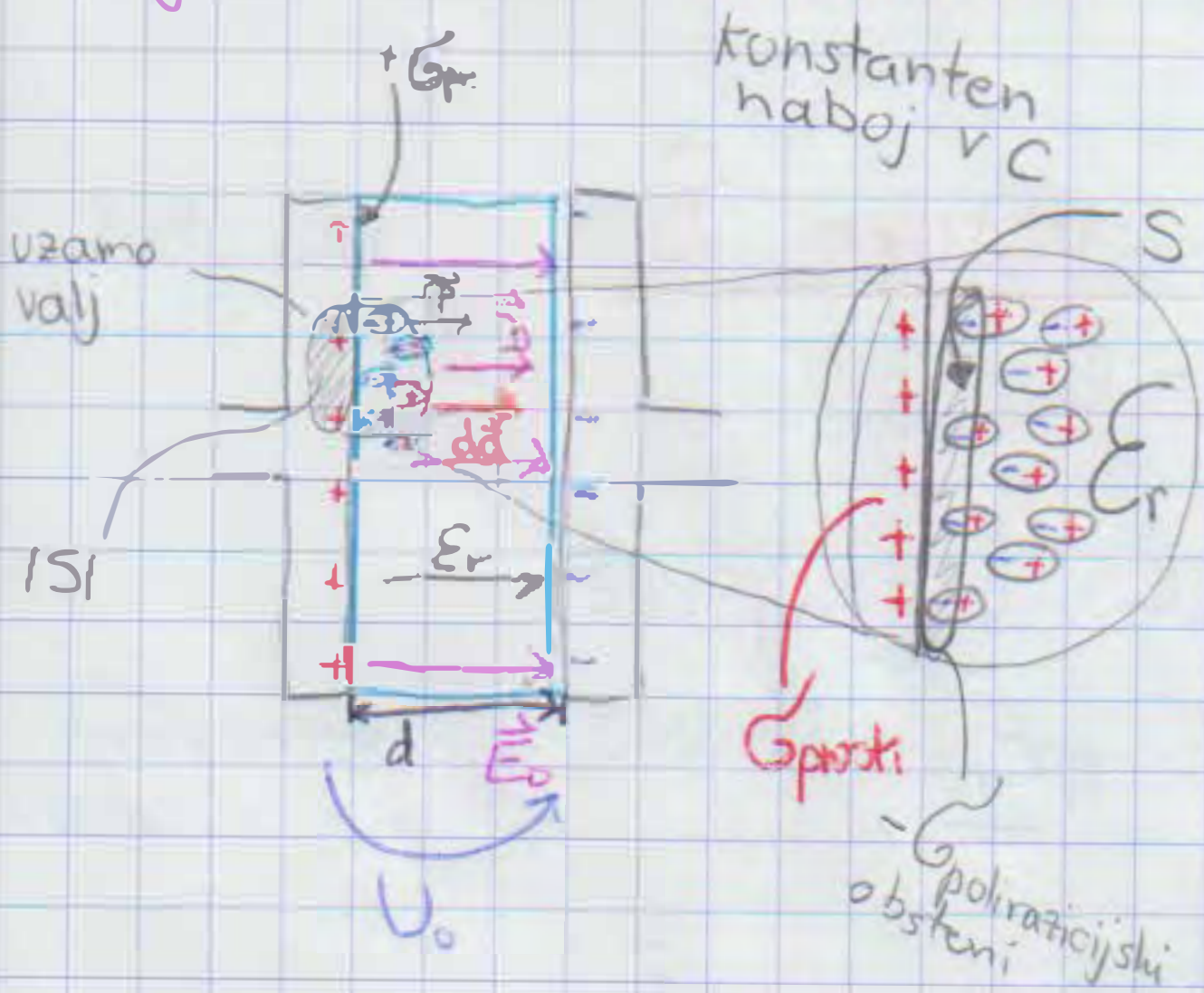


bolj kot se plošče približujejo večji je $\vec{E}$ in natančno pride do preboja $\vec{N}$

$$\vec{E} > E_p$$

prebojna trdnost snovi

žgled: Ploščni kondenzator z in brez dielektrika



1) brez dielektrika

$$E_0 = \frac{G_{\text{prosti naboj}}}{\epsilon_0}$$

$$U_0 = E_0 \cdot d$$

2) z dielektrikom - dielektrik debeline d

$$\vec{E} = \vec{E}_0 + \vec{E}_1$$

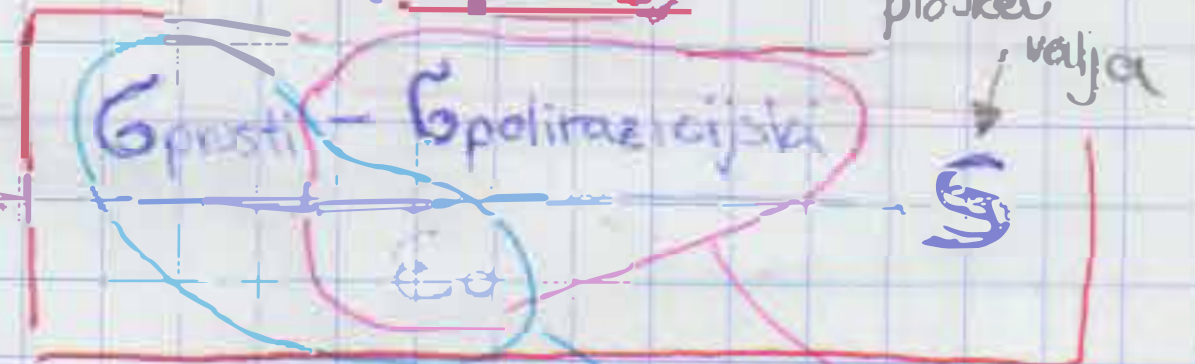
$$\oint_{\vec{A}} \vec{D} d\vec{a} = D_x \cdot S = G_{\text{prosti}} \cdot S$$

$$D_x = G_{\text{prosti naboj v vrsto}}$$

vektor polarizacije skozi snov

$$\oint \vec{P} d\vec{a} = P_x \cdot S = (E_{\text{pol}} \cdot S)$$

$$\oint \vec{E} d\vec{a} =$$



polje vezanih nabojev
prvotno polje

$$D_x = \epsilon_0 \cdot E_r \cdot E_0$$

$$G_{pr} = \epsilon_0 \epsilon_r \cdot \frac{G_{prosti} - G_{poliracijski}}{\epsilon_0}$$

$$\epsilon_r G_{poli.} = G_{prosti} (\epsilon_r - 1)$$

$$G_{poli.} = G_{prosti} \frac{(\epsilon_r - 1)}{\epsilon_r}$$

npr. $\epsilon_r = 3$

$$\frac{G_{poli.}}{G_{prosti}} = \frac{2}{3}$$

$$E_x = \frac{G_{pro.}}{\epsilon_0 \cdot \epsilon_r}$$

polje z dielektrikom

$$U = E d = \frac{U_0}{d}$$

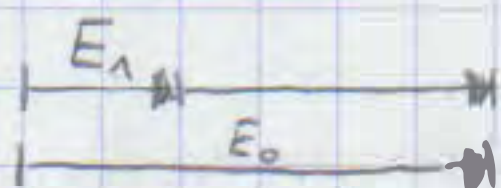


$$E_x = \frac{\epsilon_{\text{prosti}} - \epsilon_{\text{poli.}}}{\epsilon_0} = \frac{\epsilon_{\text{prosti}} - \epsilon_{\text{prosti}} \frac{\epsilon_r - 1}{\epsilon}}{\epsilon_0} = \frac{\epsilon_{\text{prosti}}}{\epsilon_0} \cdot \frac{\epsilon_r - \epsilon_r + 1}{\epsilon_r} = \frac{\epsilon_{\text{prosti}}}{\epsilon_0} \cdot \frac{1}{\epsilon_r} = \frac{E_0}{\epsilon_r}$$

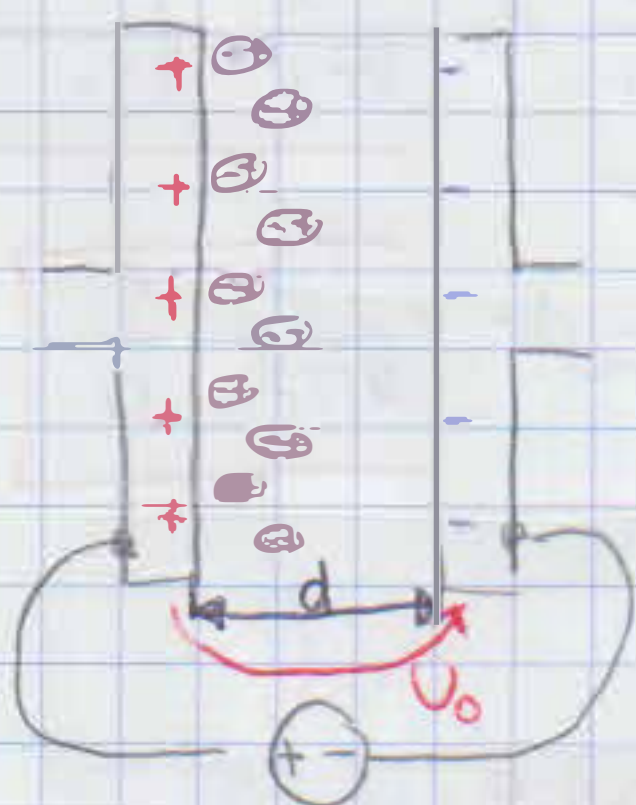
polje brez dielektrika

$$E = \frac{E_0}{\epsilon_r}$$

novi polje



Vključen vir U_0



$$1) E_0 = \frac{U_0}{d}$$

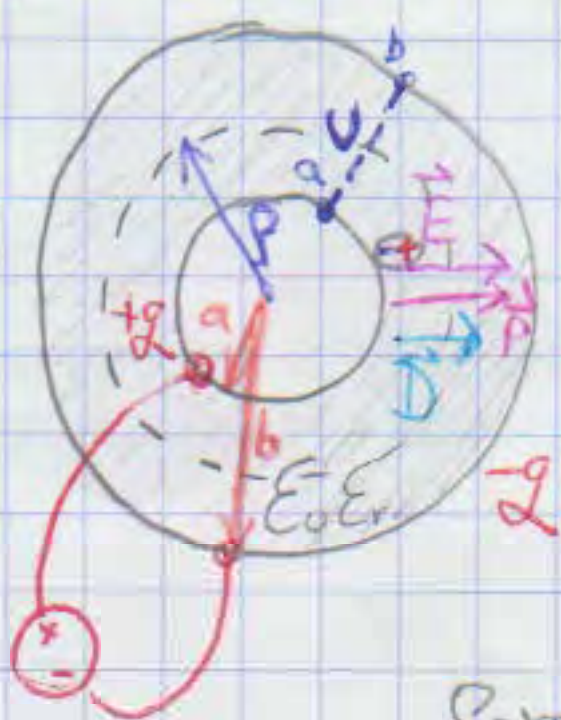
$$\epsilon_{\text{pr.}} = \epsilon_0 \cdot \epsilon_r$$

$$2) E = \frac{U_0}{d} = E_0$$



$\vec{E}_0$ = polje brez izolanta
 $\vec{E}$ = končni E
 $\vec{E}_{\text{onovo}}$ = ko vstavimo ϵ_r
 $\vec{E}_r$ = reakcija dipolov

zgled KOAX



ρ -radialna smer

$$\oint \vec{D} \cdot d\vec{a} = Q \cdot l$$

$$D \cdot 2\pi \cdot \rho \cdot l = Q \cdot l$$

plašč valja

$$\rho = \frac{Q}{2\pi l \rho}$$

$$E_\rho = \frac{D_\rho}{\epsilon_0 \epsilon_r} = \frac{Q}{2\pi \epsilon_0 \epsilon_r \rho}$$

$$U = \int_a^b E_\rho \cdot dl$$

$$U = \frac{Q}{2\pi \epsilon_0 \epsilon_r} \ln \frac{b}{a}$$

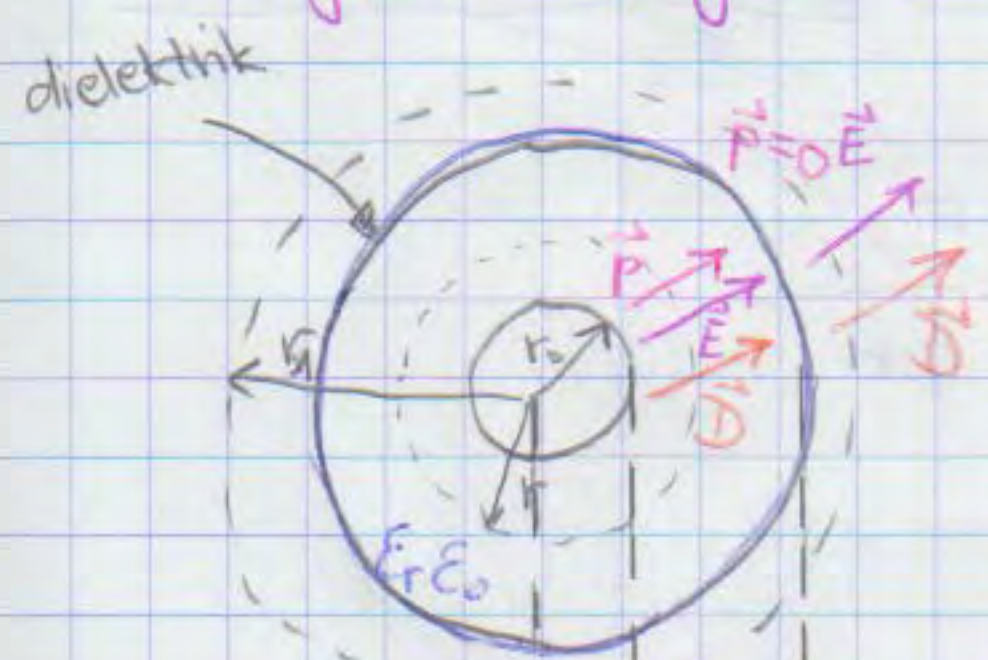
$$\epsilon = \epsilon_0 \epsilon_r$$

$$C = \frac{2\pi \epsilon_0 \epsilon_r}{\ln b/a}$$

Alta

42

zgleđ kroga v dielektričnem omotu UC. 14-9



$$\oint \vec{D} d\vec{a} = Q_{\text{prosti}} = Q$$

$$D_r = \frac{Q}{\underbrace{4\pi r^2}_{\text{obseg kroga}}}$$

* kroglo ovijemo
z dielektrikom *

$$E_r = \begin{cases} 0, & r < r_0 \\ \frac{Q}{4\pi\epsilon_r r^2}, & r_0 < r < r_1 \\ \frac{Q}{4\pi\epsilon_0 r^2}, & r_1 < r \end{cases}$$

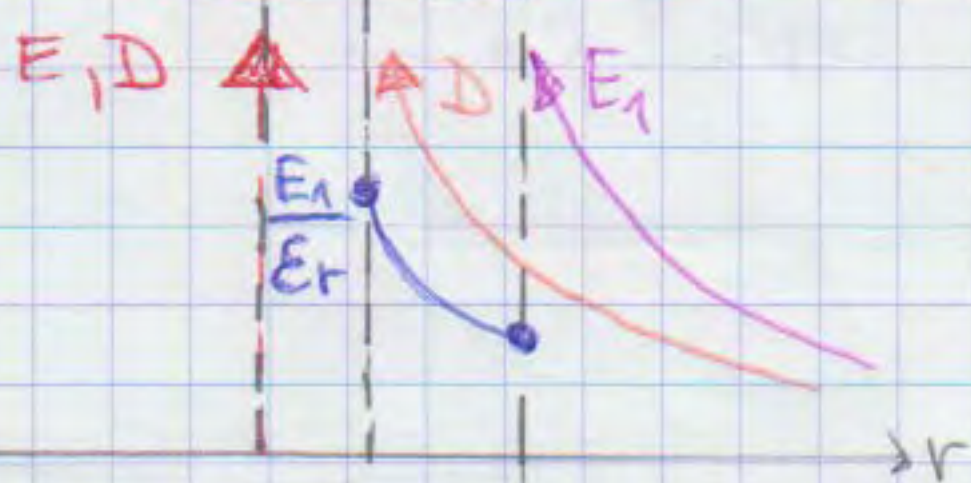
znotraj
krogla

med omotom in kroglo

izven omota

$$\vec{D} = \epsilon \vec{E} \quad \text{v izdanku}$$

$$\vec{D} = \epsilon_0 \vec{E} \quad \text{v zraku}$$



$$V_{01} = \int_{r_0}^{r_1} \frac{Q}{4\pi\epsilon_r r^2} dr = \frac{Q}{4\pi\epsilon} \left(\frac{1}{r_0} - \frac{1}{r_1} \right)$$

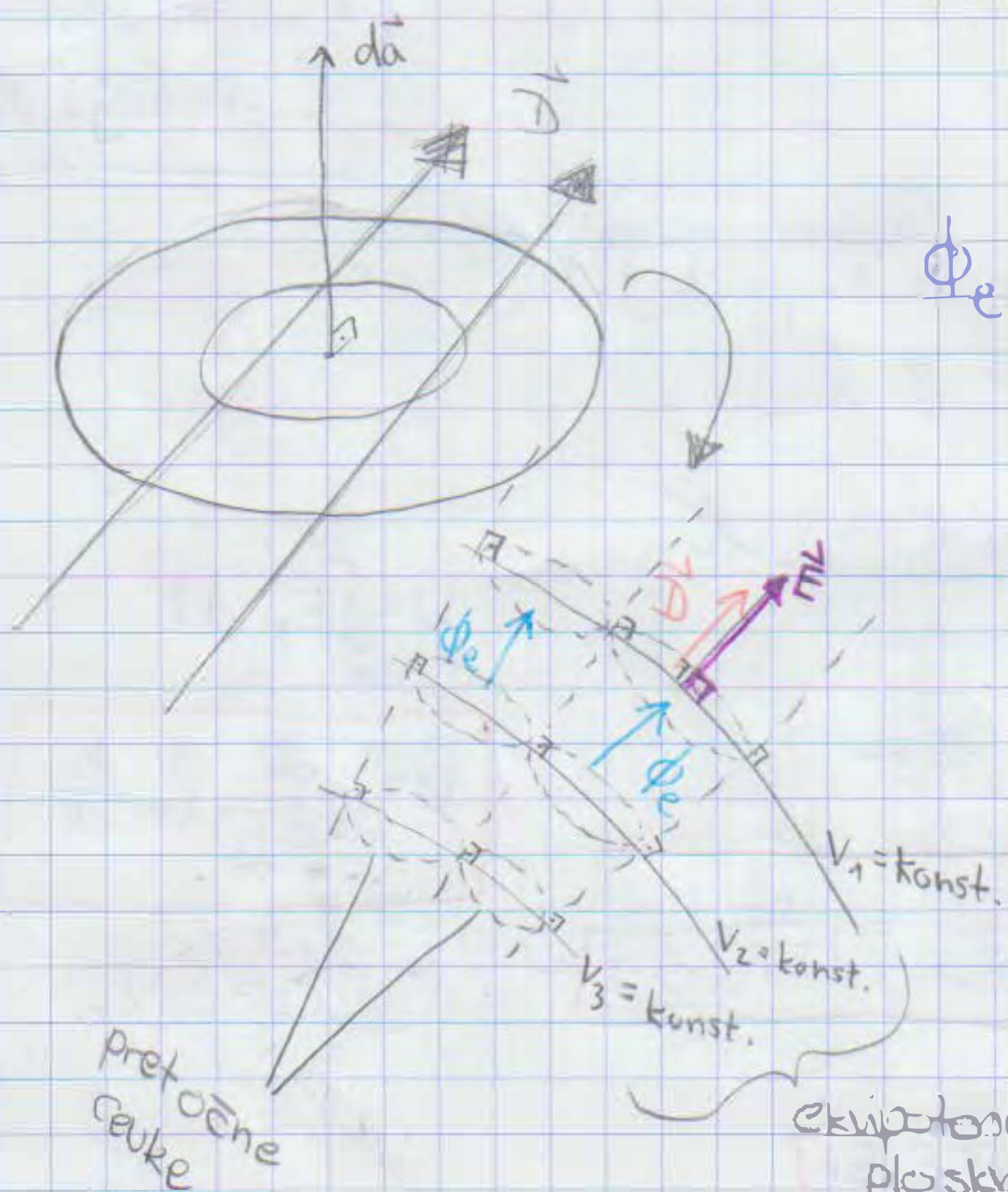
$$V_{1\infty} = \int_{r_1}^{\infty} \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0} \frac{1}{r_1}$$

Električni pretok Φ_e , pretočne cevke

* ploščina gostota ele. pretoka

$$\vec{D}$$

$$\Phi_e = \int \vec{D} d\vec{a} \quad [C/m^2]$$

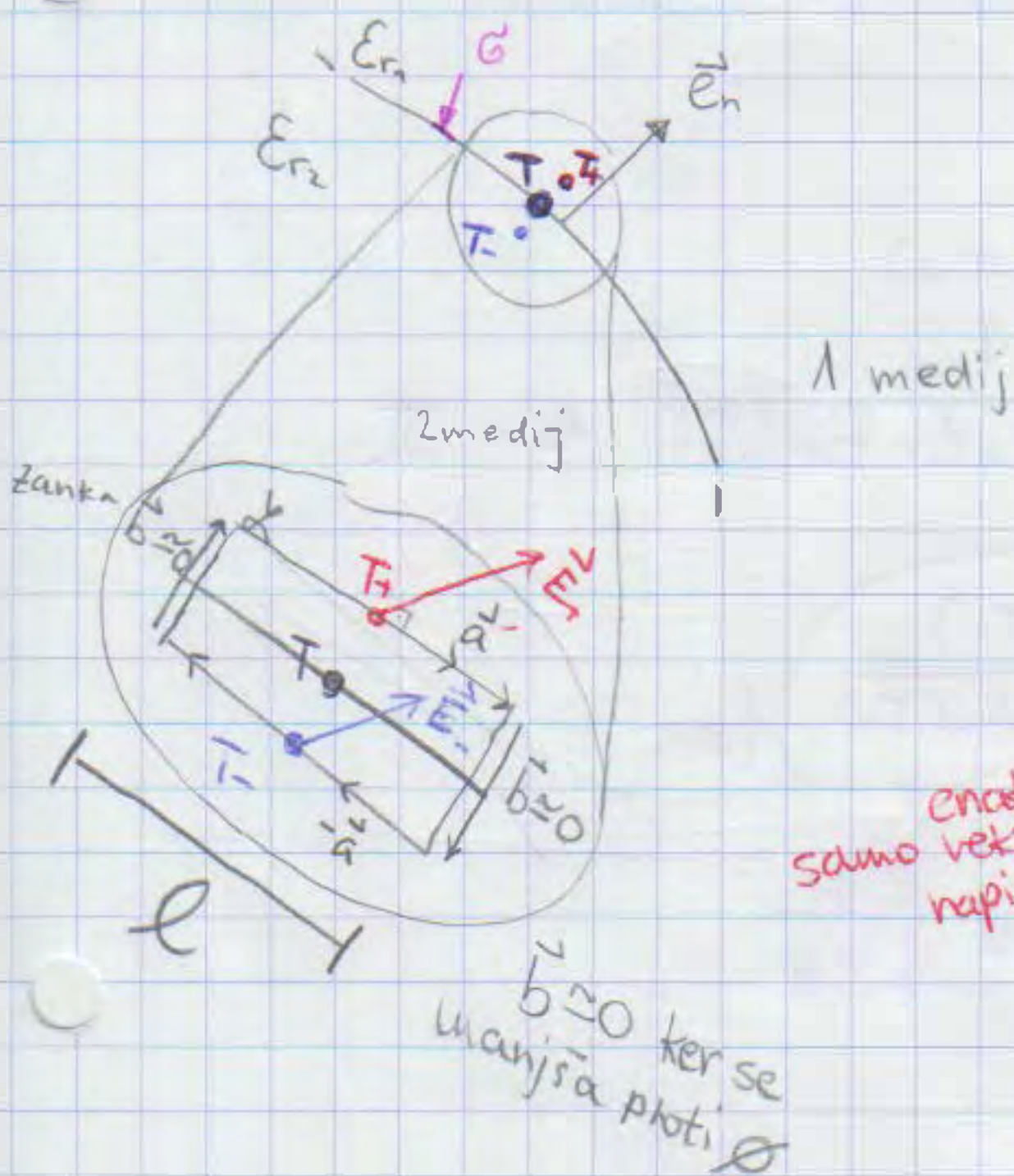


ekvipotencialne
ploskve
 V_1, V_2, V_3

pretočne
cevke

Mejni pogoji VČ. 15-2

1) Mejni pogoj $\vec{E}$



$$\oint_L \vec{E} \cdot d\vec{l} = 0$$

$$\underbrace{\vec{E}(T_+) \cdot \vec{a}}_{\vec{a}} + \underbrace{0}_{\vec{b}} + \underbrace{\vec{E}(T_-) \cdot (-\vec{a})}_{-\vec{a}} + \underbrace{0}_{\vec{b}} = 0$$

projiciramo na $\vec{a}$

$$E_t(T_+) \cdot \vec{a} - E_t(T_-) \cdot \vec{a} = 0$$

$$E_t(T_+) = E_t(T_-)$$

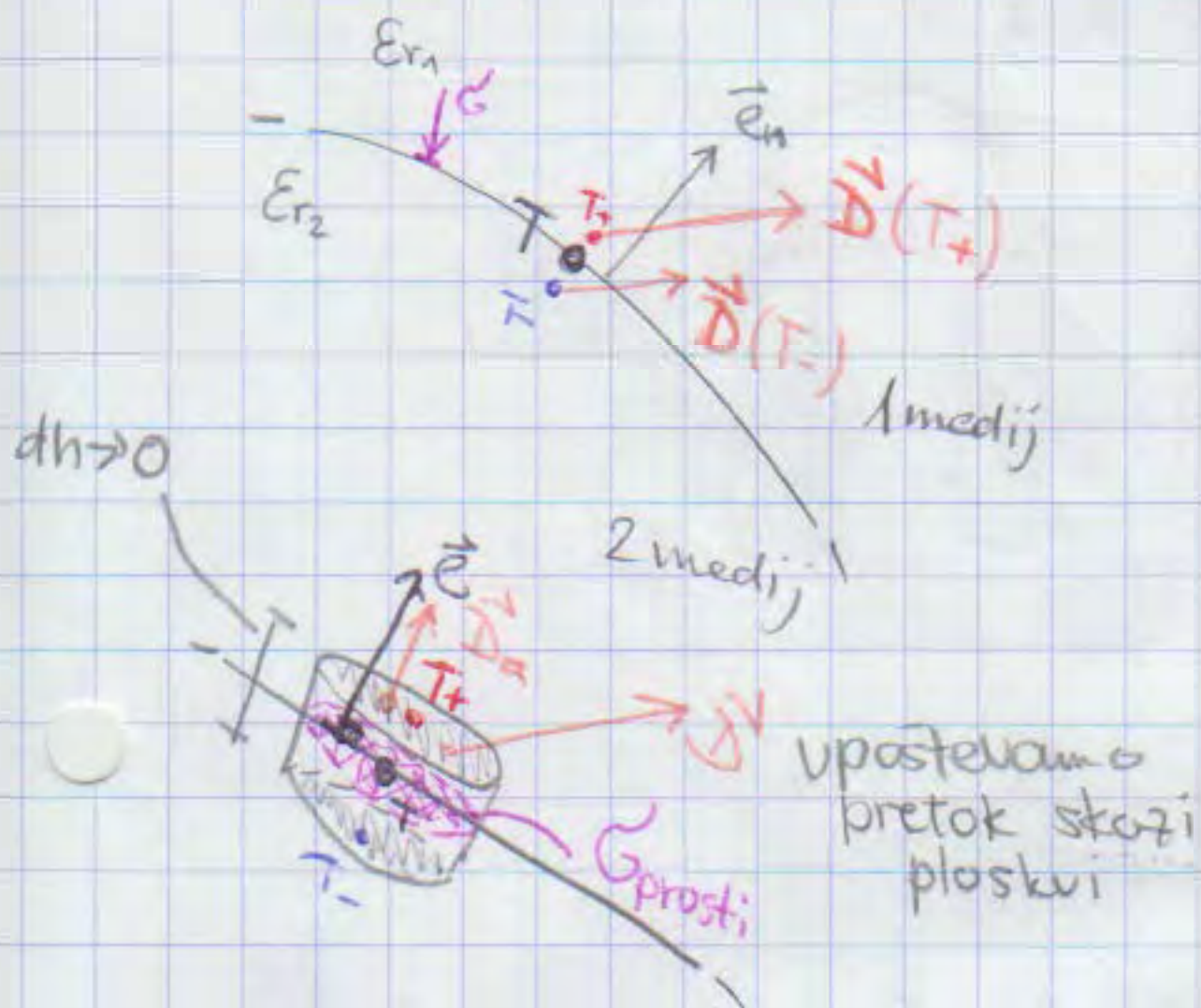
tangencialna komponenta se ohrani

1. pogoj

Enako samo vektorsko napisano

$$\vec{e}_n \times (\vec{E}(T_+) - \vec{E}(T_-)) = 0$$

2) Mejni pogoj za $\vec{D}$



$$\oint_S \vec{D} \cdot d\vec{a} = Q_{\text{prostih-notrajnih}}$$

produkt 2 skalarjev

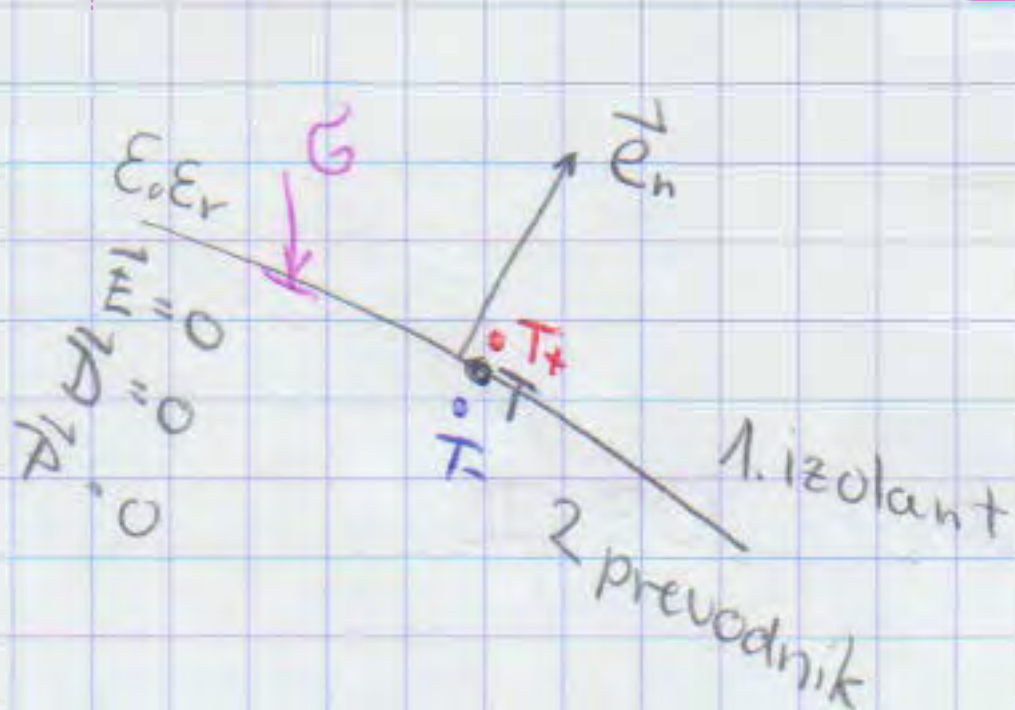
$$0 + 0 + \underbrace{D(T_+) \cdot \vec{a}}_{\vec{a}} + \underbrace{D(T_-) \cdot (-\vec{a})}_{-\vec{a}} = \sigma \cdot \vec{a}$$

normalne komponente

$$D_n(T_+) - D_n(T_-) = \sigma$$

$$\vec{e}_n \cdot (\vec{D}(T_+) - \vec{D}(T_-)) = \sigma$$

Mejni prevodnik / izolant VČ. 15-3



$$E_t(T_+) = E_t(T_-)$$

$$D_n(T_+) - D_n(T_-) = 0$$

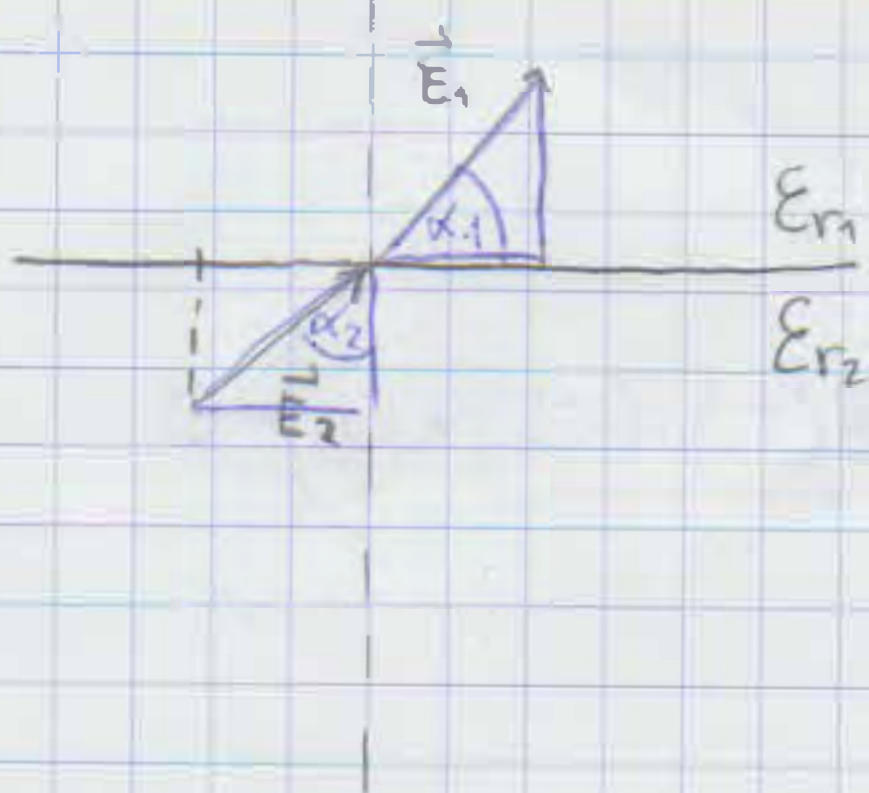
$$\epsilon_0 \epsilon_r E(T_+) = \sigma$$

tangencialne komponente so 0
v izolantu so $\vec{E} \perp$ na mejo

$$E_n(T_+) = \frac{\sigma}{\epsilon_0 \epsilon_r}$$

Alstar

Meje izolant / izolant brez naboja na meji UC. 15-4



$$\vec{E}(T_+) = \vec{E}_+(T_-) \quad \leftarrow \text{I.}$$

Pri $G=0$!

$$D_n(T_+) = D_n(T_-)$$

$$\epsilon_r \epsilon_n(T_+) = \epsilon_r \epsilon_n(T_-) \quad \leftarrow \text{II}$$

"lomni zakon" pri prestopu

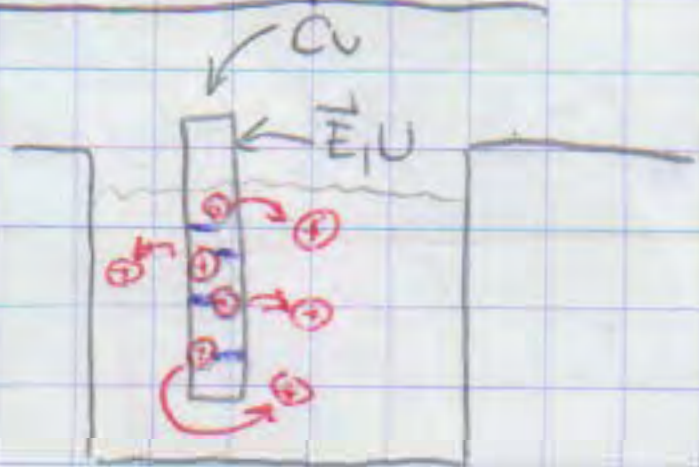
$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

$$\frac{\frac{E_+(T_+)}{\epsilon_r \epsilon_n(T_+)}}{\tan \alpha_1} = \frac{\frac{E_+(T_-)}{\epsilon_r \epsilon_n(T_-)}}{\tan \alpha_2}$$

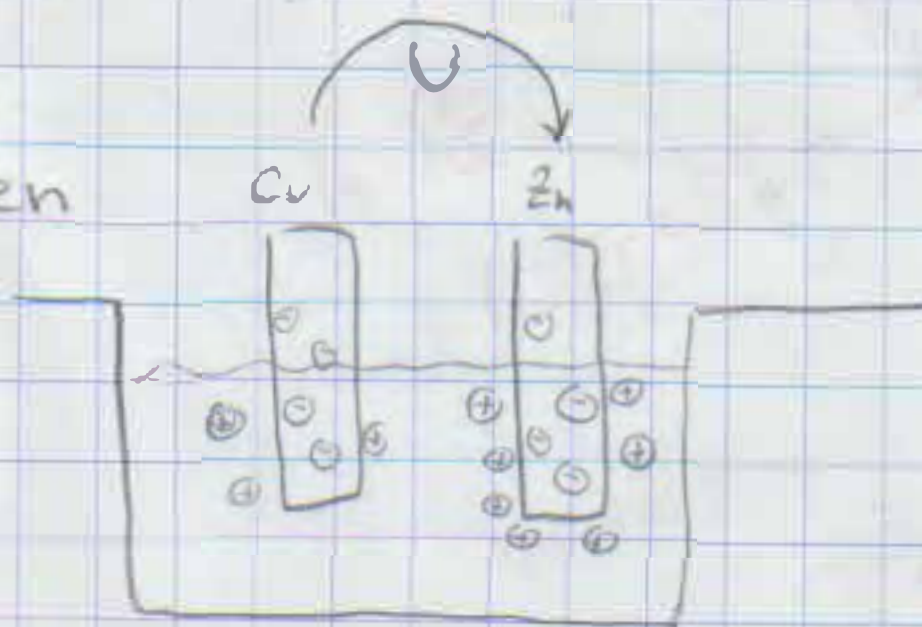
Idealen napetostni vir

UC. 16-3

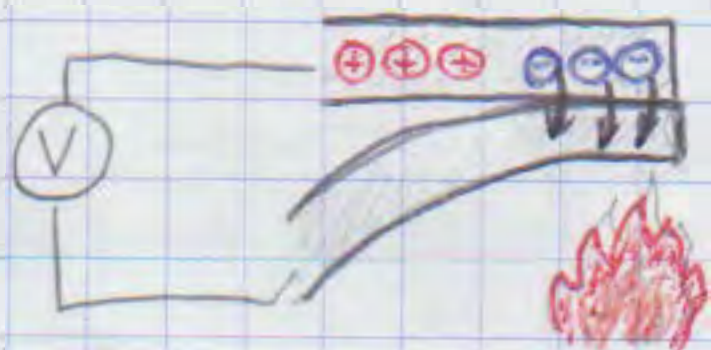
1) Kemični proces



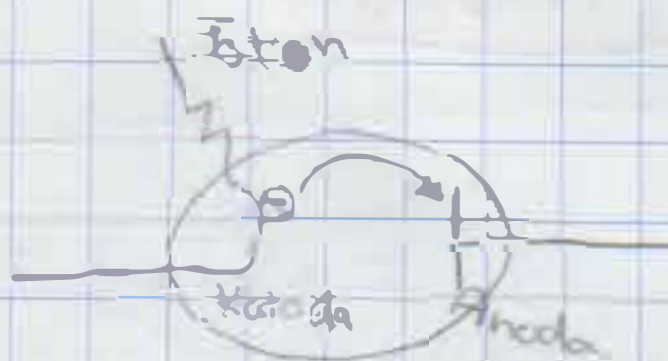
Voltov člen



2) Termo člen



3) Fotoefekt (sončne celice)



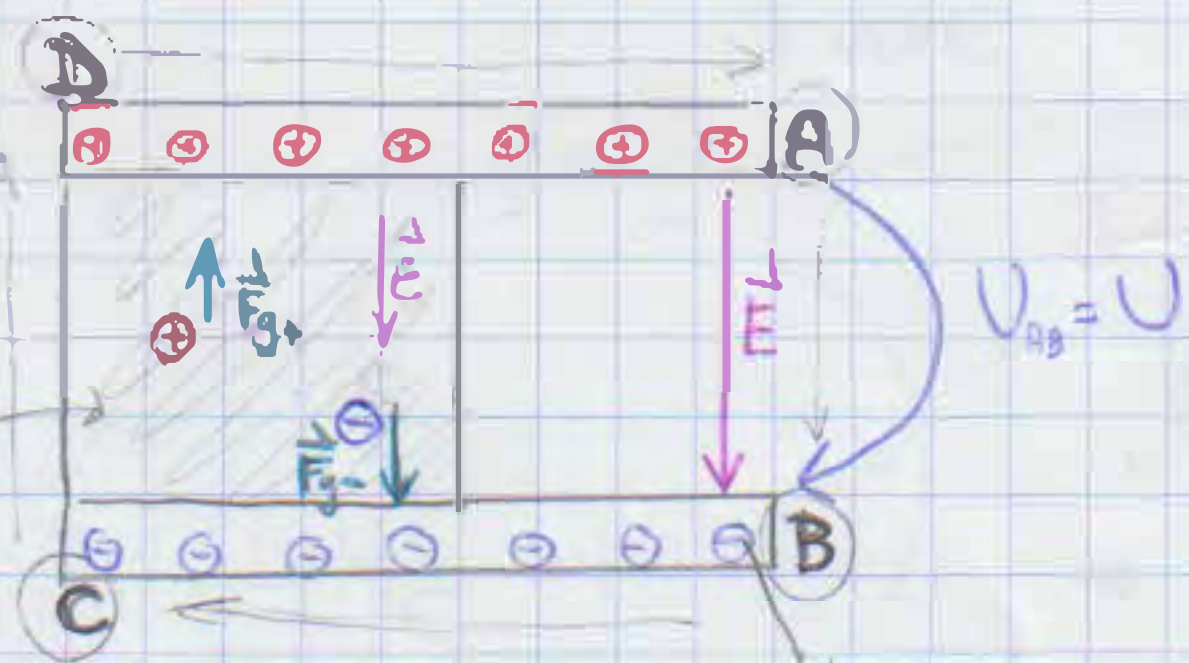
ni tako pogost
slabi izkoristki

4) Influenca OE I.

5) Indukcija OE II

Model napetostnega vira (idealnega)

F_g - generatorjska sila



$$d\vec{F}_e + d\vec{F}_g = 0$$

prostori kjer
se naboj
ločuje

Fizikalna sila razdvaja naboj

normirana
električna
sila

elek. polj.
jakost

$$\vec{E}_g = \frac{\vec{F}_g}{Q}$$

$$(Q \cdot \vec{E} + \vec{F}_g) = 0$$

$$Q(\vec{E} + \vec{E}_g) = 0$$

$$\vec{E} = -\vec{E}_g$$

$$\vec{E}_g = -\vec{E}$$

stanje
ravnovesja

najpomembnejši

$$\oint \vec{E} d\vec{l} = 0 = \int_A^B \vec{E} d\vec{l} + \int_B^C \vec{E} d\vec{l} + \int_C^D \vec{E} d\vec{l} + \int_D^A \vec{E} d\vec{l} = 0$$

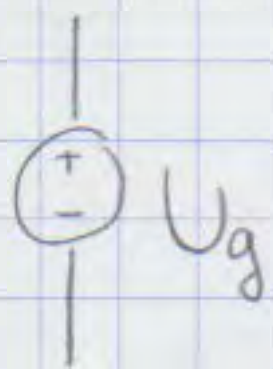
$-\int_C^D \vec{E}_g d\vec{l} = U_g$ notranja napetost

$$A_g = \int \vec{F}_g \cdot d\vec{l} = Q \int \vec{E}_g \cdot d\vec{l} = Q \cdot U_g$$

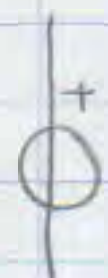
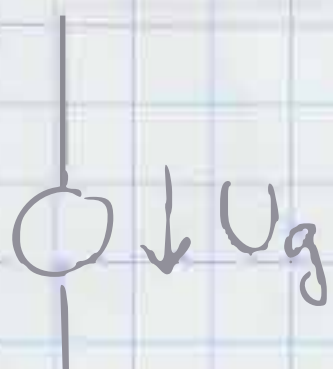
$$dA = dQ \cdot U_g$$

$$U - U_g = 0 \Rightarrow U = U_g = \int_C^D \vec{E}_g d\vec{l}$$

Simboli za označevanje nap. vira

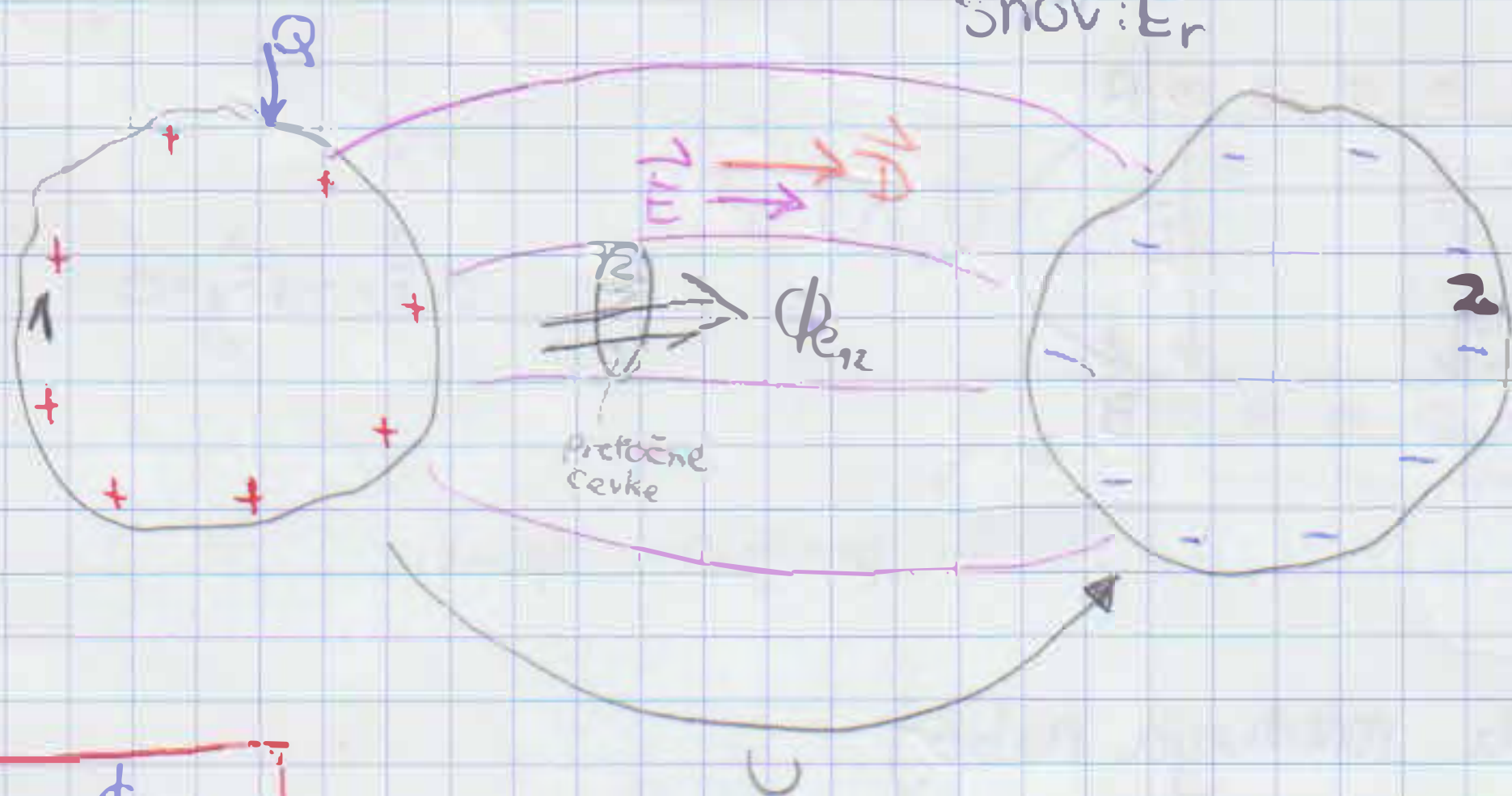


$$U_g = 3V \cdot \sin(2\pi 50Hz + \frac{\pi}{2})$$



Kapacitivnost in kondenzator (kot strujni element) ULC-5

Snov: ϵ_r



$$\Rightarrow \frac{U}{Q} = \frac{1}{C}$$

$$C = \frac{\Phi_{E12}}{U_{12}}$$

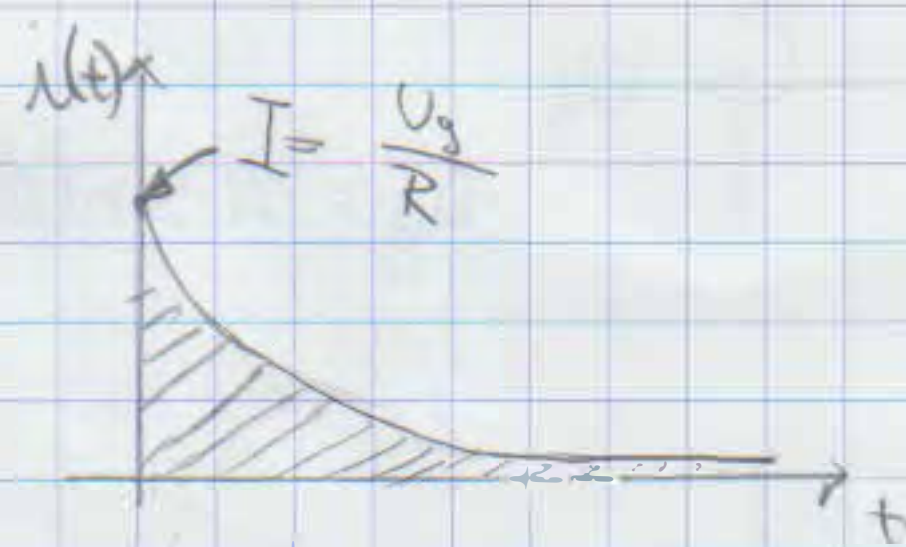
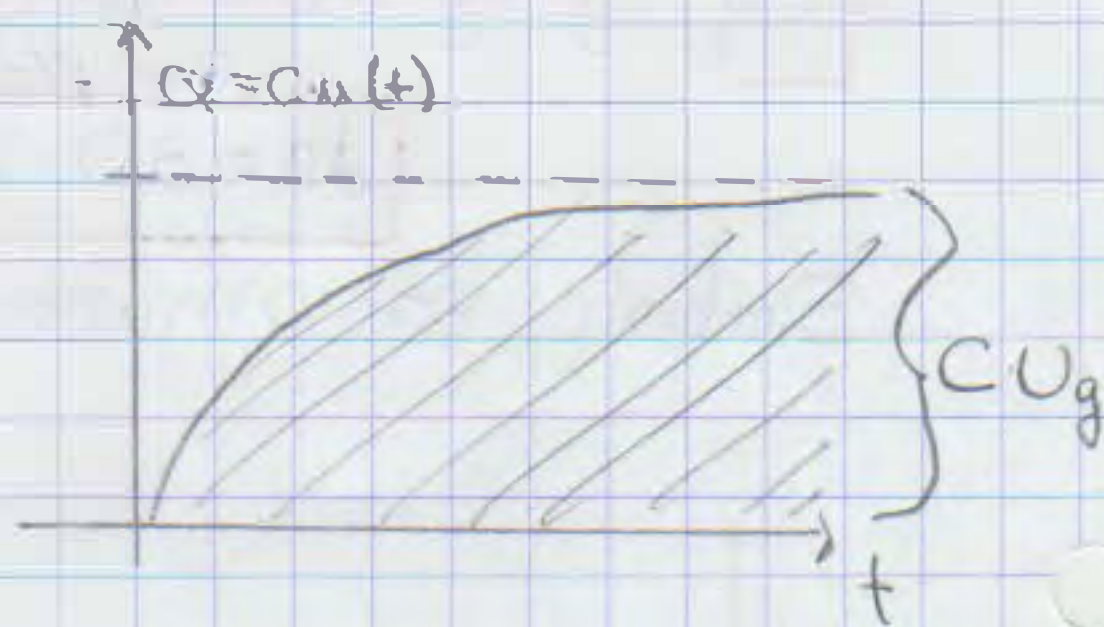
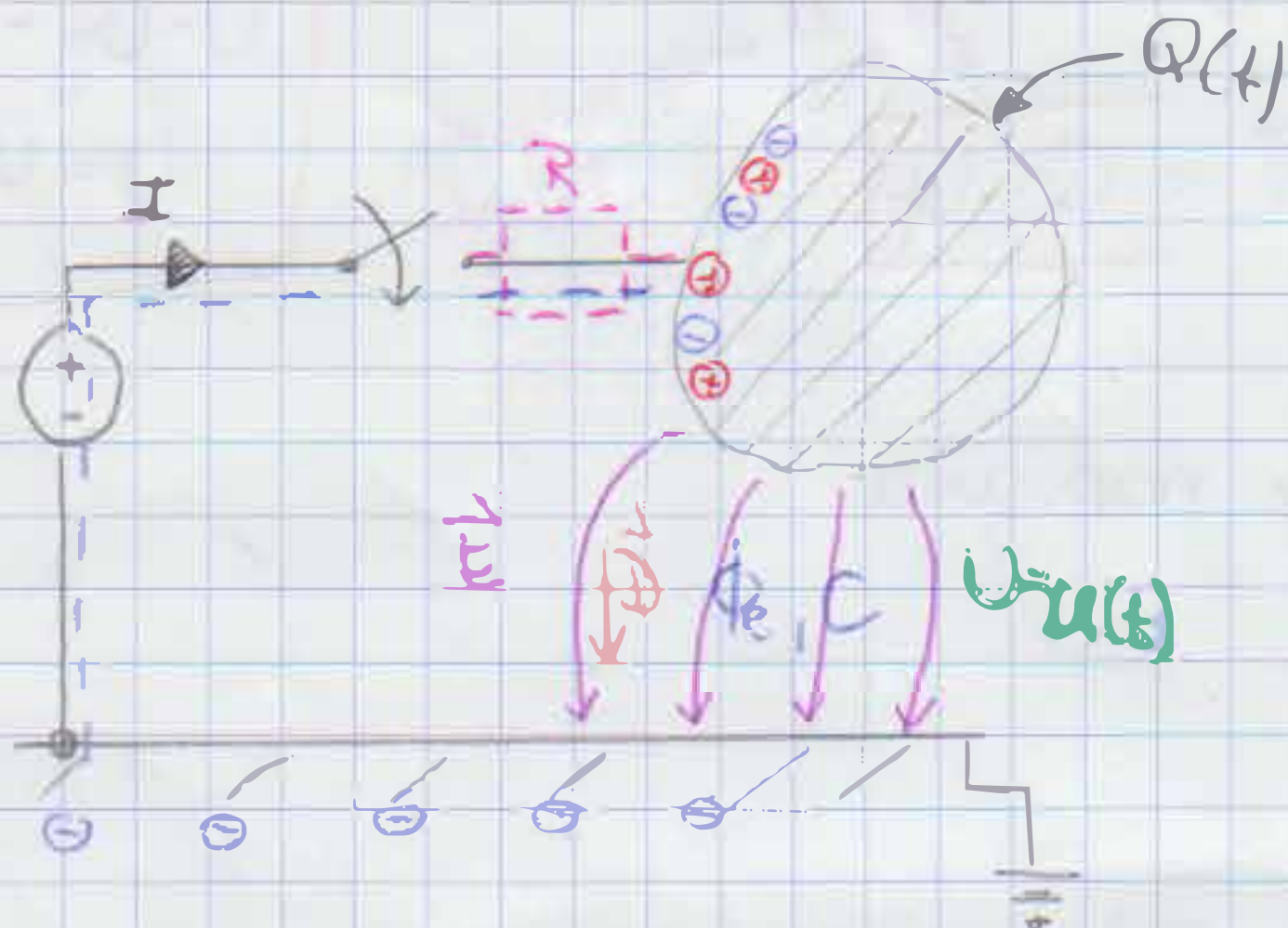
$$C = \frac{Q_{12}}{U_{12}}$$

→ že znani primeri: $\epsilon_r \epsilon_0$

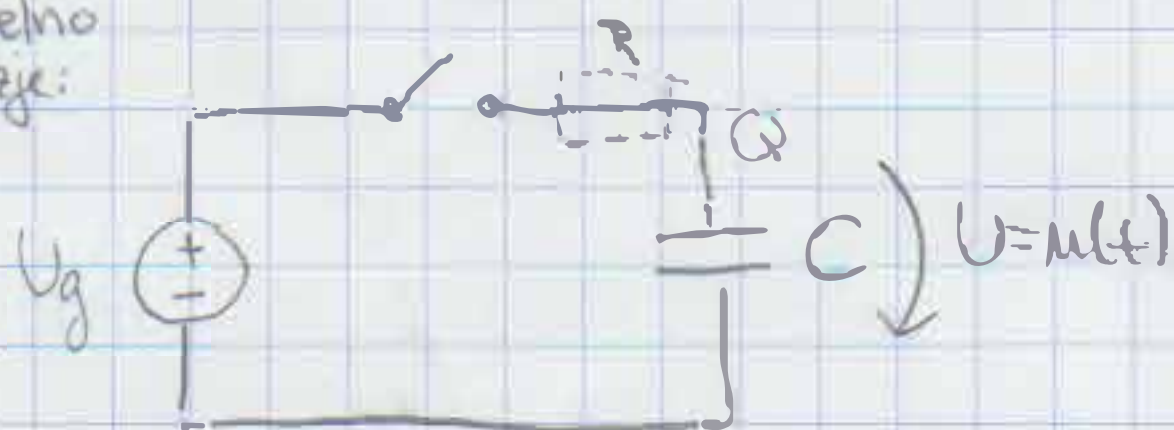
koax: $C = \frac{2\pi\epsilon\ell}{\ln \frac{b}{a}}$, $C = 4\pi\epsilon_0 r_0$, $C = \frac{4\pi\epsilon r_1 r_2}{r_2 - r_1}$

$$C = \frac{2\pi\epsilon\ell}{\ln 2h/a}$$

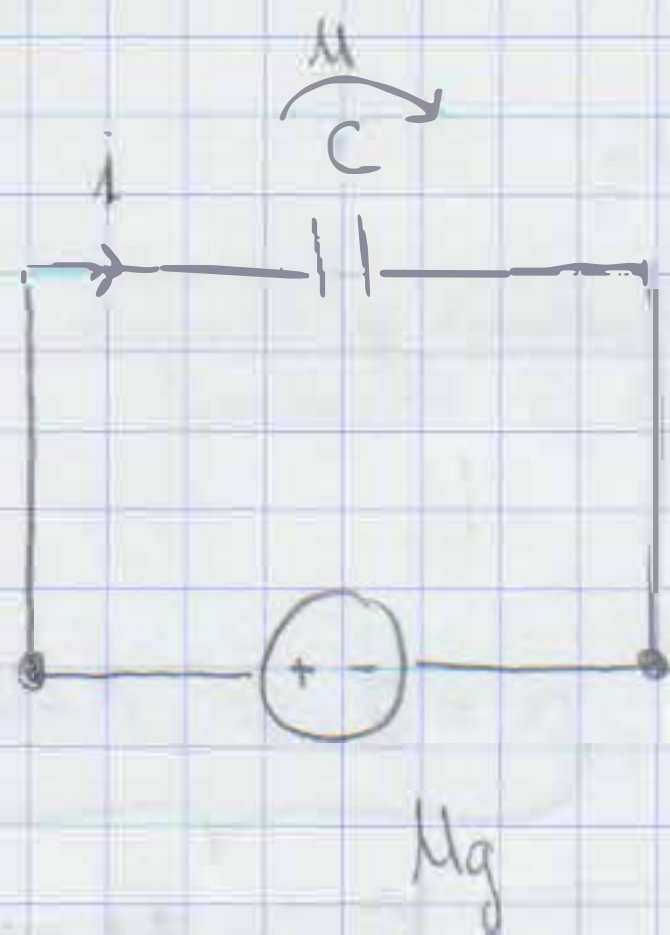
Naelektirani kondenzatorja



Modelno vezje:



U16-6 Kondenzator v živih strukturah (časovno spremenljive strukture)



$$U - U_g = 0$$

$$U = U_g$$

Kontinuitetna enačba

$$i = + \frac{dQ}{dt} = \frac{d(C \cdot u(t))}{dt} = C \cdot \frac{du}{dt} = C u'$$

$$i = C \cdot \frac{du}{dt}$$

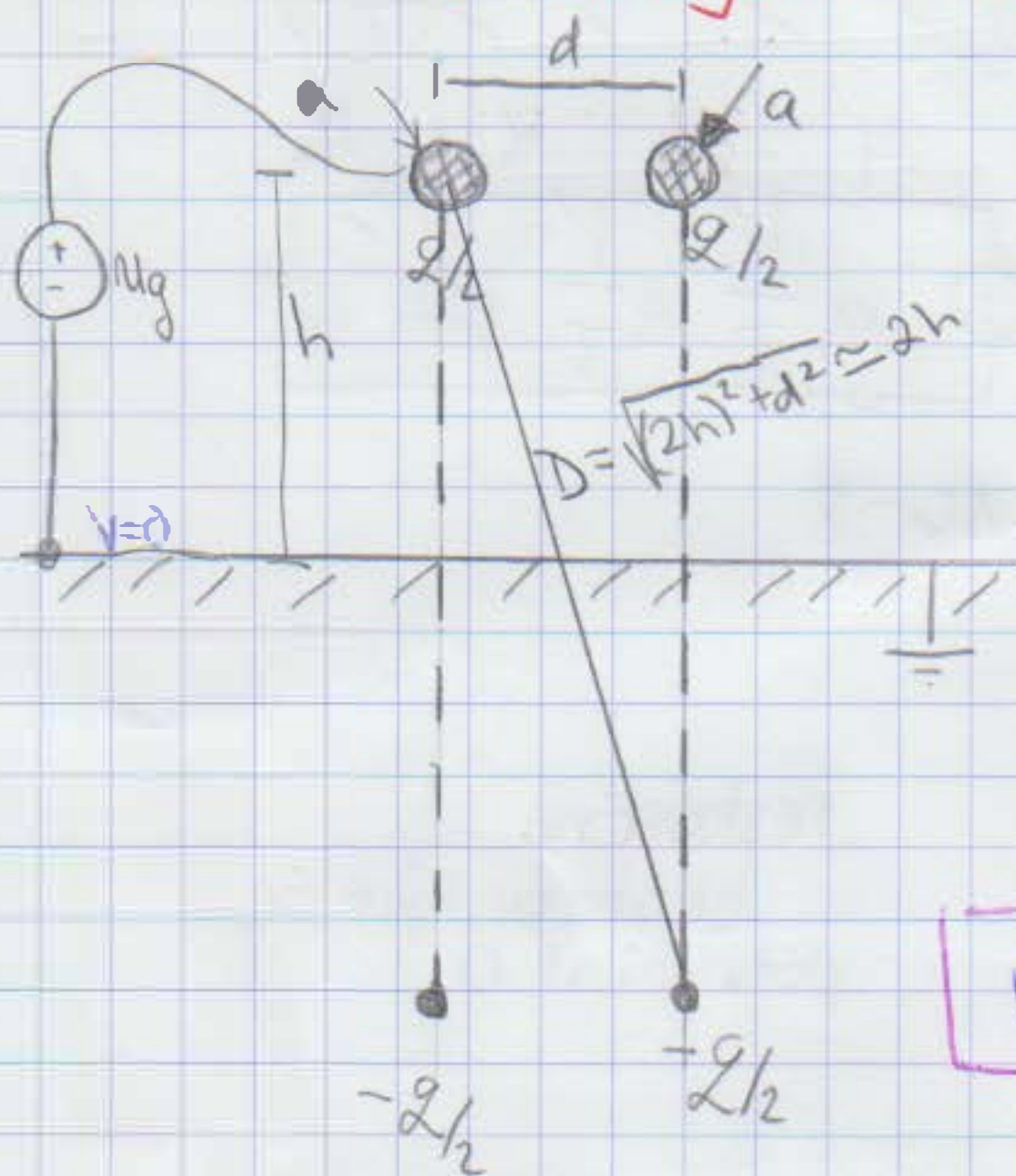
$$i = C u'$$

$$\int_{t_1}^{t_2} du = \int_{t_1}^{t_2} \frac{1}{C} i(t) dt$$

$$u(t_2) - u(t_1) = \frac{1}{C} \int_{t_1}^{t_2} i dt$$

prirastek napetosti pritečen naboj

Harmonično vzbujevanje "dvojčka" nad zemljo



$$V_{\text{dvojčka}} = \frac{(Q/2)}{2\pi\epsilon_0} \ln \frac{2h}{a} + \frac{(Q/2)}{2\pi\epsilon_0} \ln \frac{D}{d}$$

$$= \frac{Q}{2\pi\epsilon_0} \ln \sqrt{\frac{(2h)(2h)}{ad}} = \frac{Q}{2\pi\epsilon_0} \ln \frac{2h}{\sqrt{ad}}$$

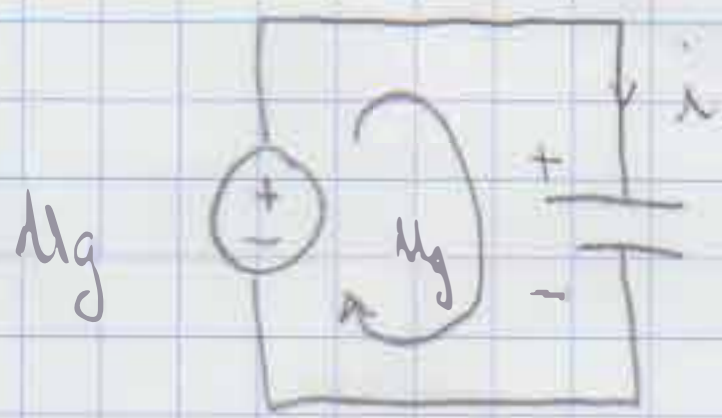
$$C = \frac{Q}{V_{\text{dvojčka}}} = \frac{2\pi\epsilon_0}{\ln \frac{2h}{\sqrt{ad}}} = \frac{2\pi\epsilon_0}{\ln \frac{2h}{r_{\text{ekvivalentni}}}}$$

$$r_{\text{ekvivalentni}} = \sqrt{ad}$$

$$r_{\text{ekv.}} = \sqrt[3]{ad^2}$$

$$r_{\text{ekv.}} = \sqrt[4]{ad^3\sqrt{2}}$$

$$u_g = \sin \omega t \cdot U_{gmax}$$



$$-u_g + u = 0$$

$$u = u_g$$

$$i = C u' = C u_g'$$

napetost je sinusna
tok je kosinusni

žgled:

$$h = 10m$$

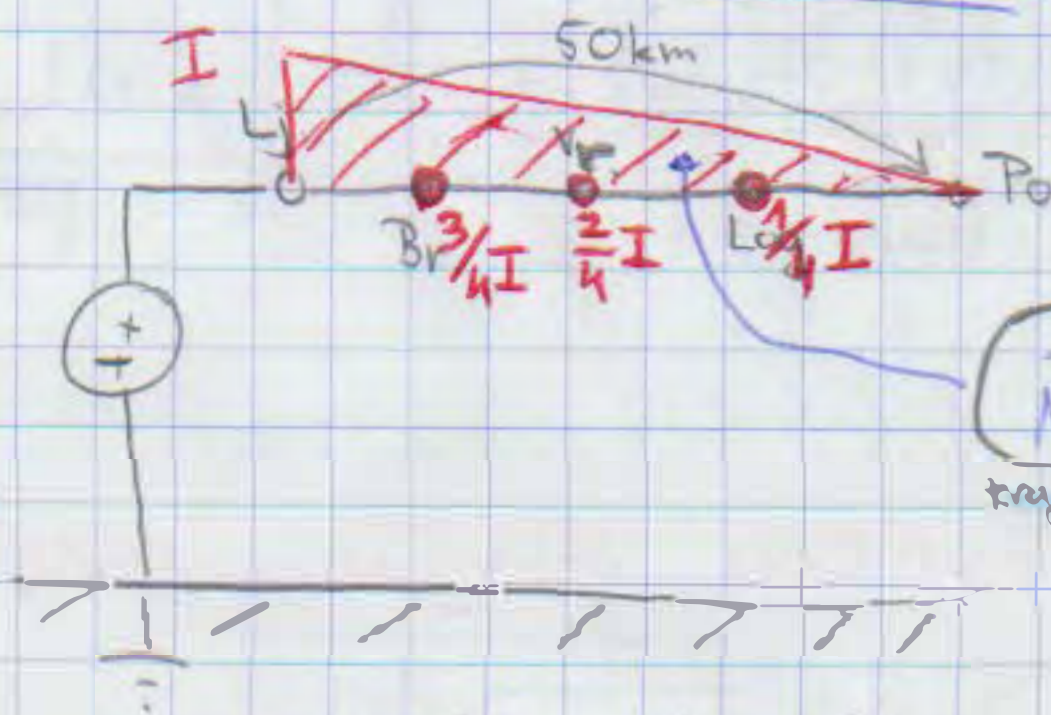
$$a = 3cm, d = 40cm$$

$$\omega = 2\pi f = 314s^{-1}$$

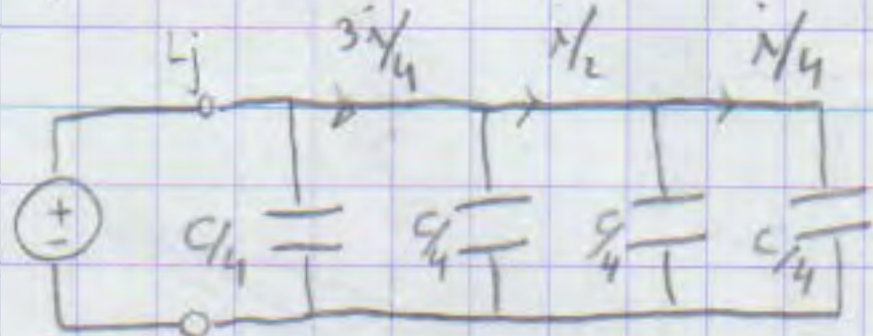
$$r_{ekv} = 11cm \sqrt{\epsilon}$$

$$l = 50km$$

$$C = \frac{2\pi \cdot 10^{-6} \cdot 5 \cdot 10^4}{\ln \frac{20m}{0.11m}} \approx 0.053 \cdot 10^{-5} = 0.53 \mu F$$

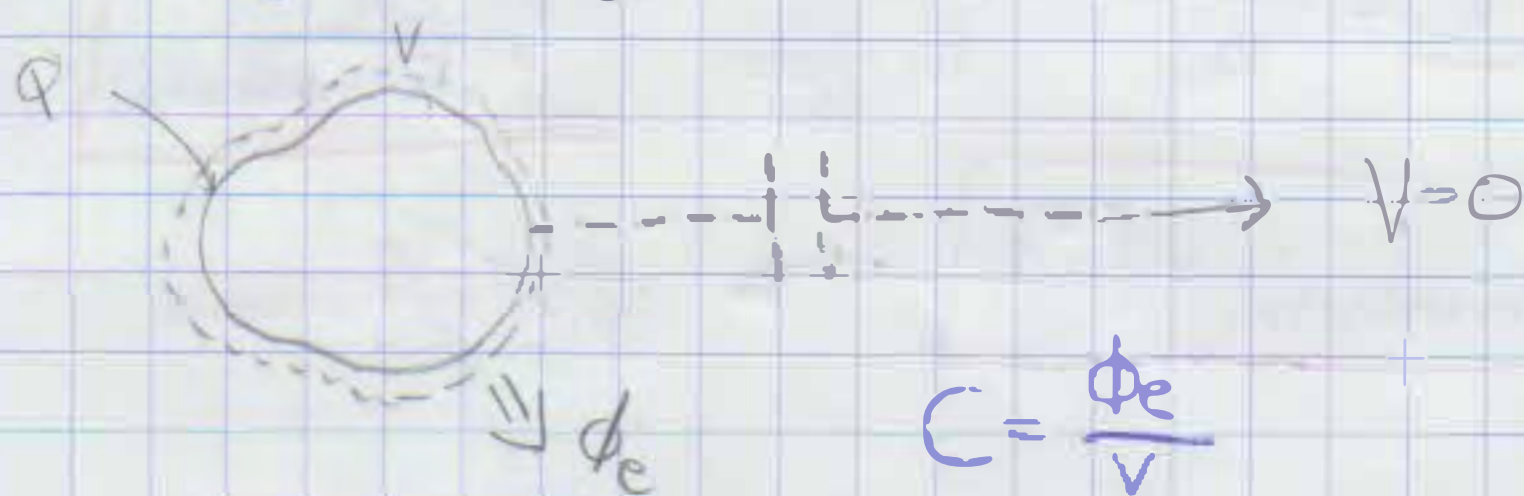


valovna funkcija
 $i(x, t)$
prostor, časovna funkcija



Delne kapacitivnosti $\epsilon \approx 10^{-8}$

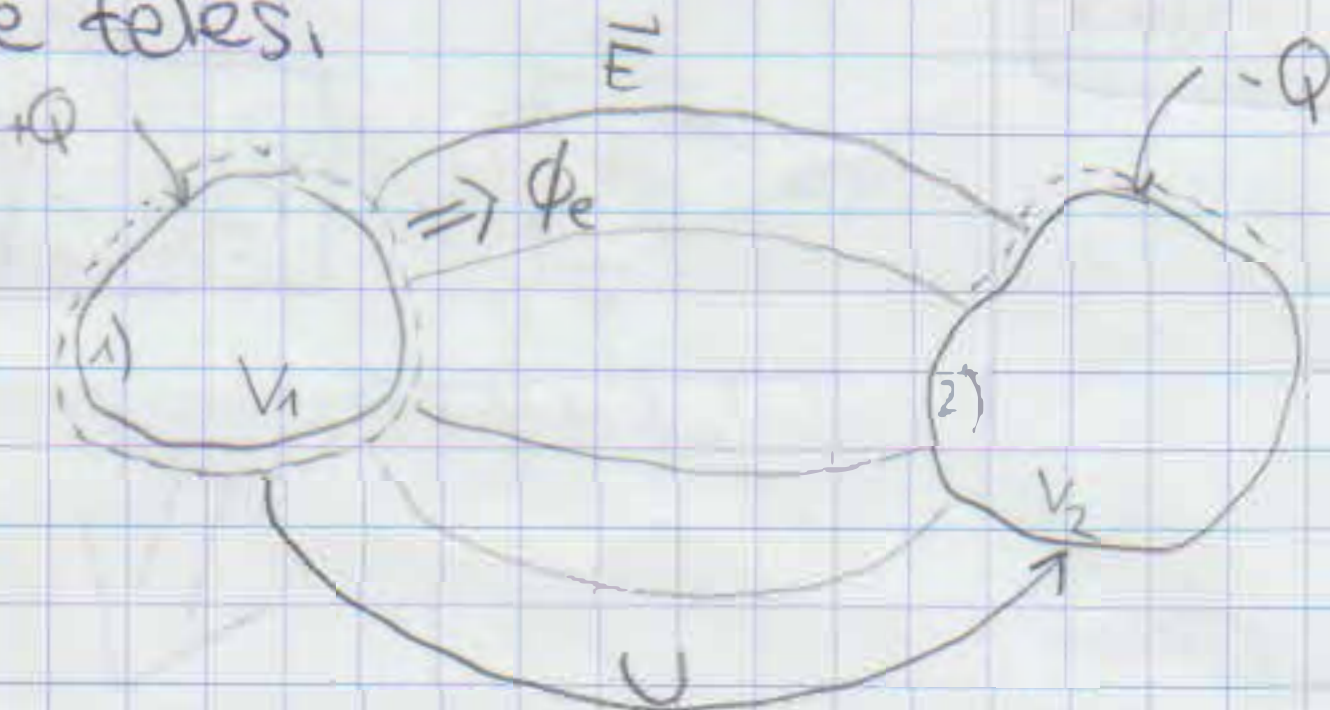
1) Osamljen objekt



neskončna
skolica, kjer je
potencial 0

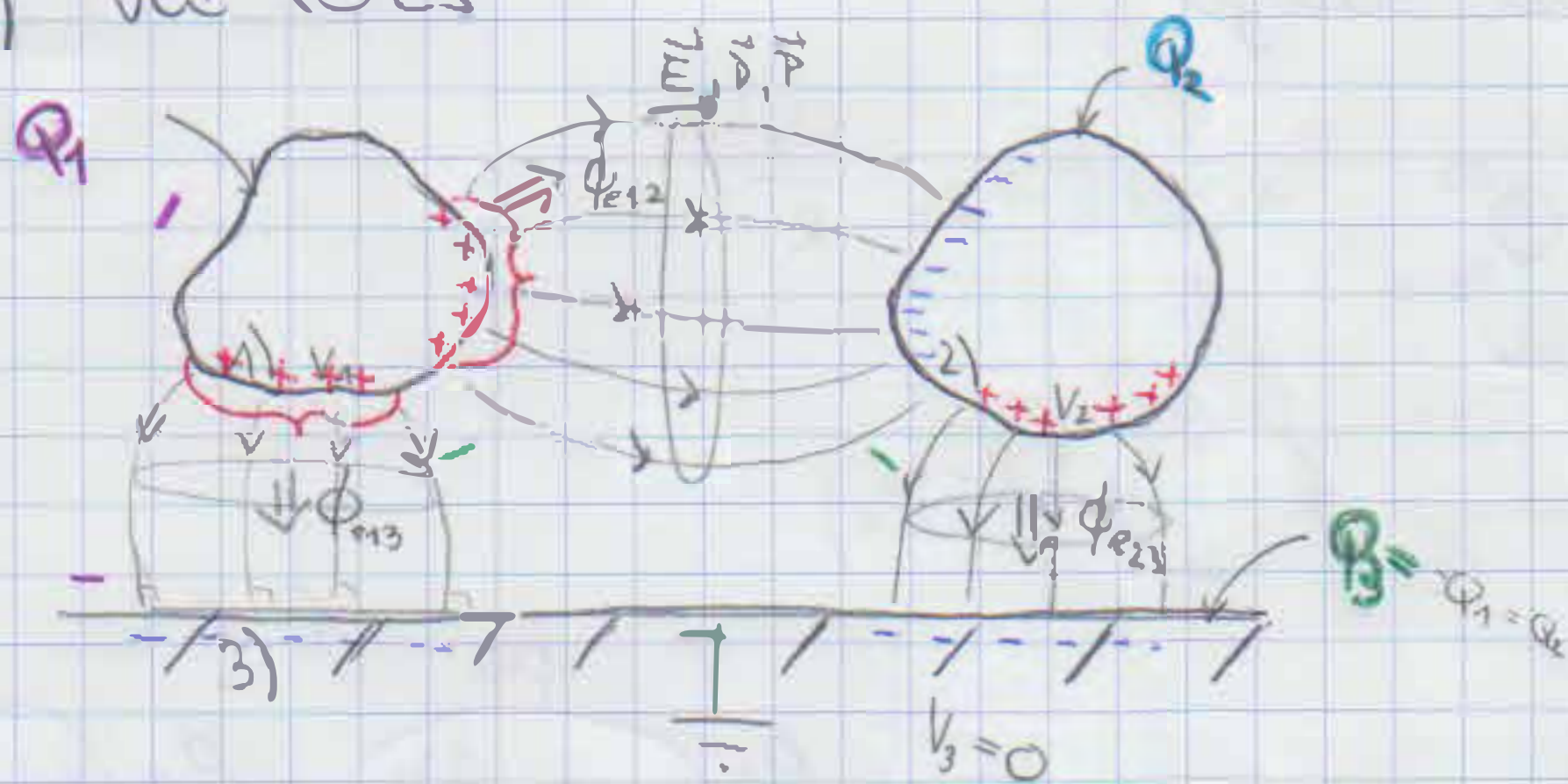
$$C = \frac{Q}{V}$$

2) dve telesi



$$C = \frac{Q}{V}$$

3) Več teles



$$C = \frac{Q}{U} \Rightarrow C_{jk} = \frac{\Phi_{ejk}}{U_{jk}}$$

$$C_{jk} = \frac{\Phi_{ejk}}{V_j - V_k} = \frac{+\Phi_{ejk}}{+(V_k - V_j)} = C_{kj}$$

zauzemanje desnih kapacitivnosti κ
 $C_{12} = C_{21}$ smer neodvisna

$$Q_1 = \underbrace{\Phi_{e12}}_{Q_{12}} + \underbrace{\Phi_{e13}}_{Q_{13}} = C_{12} \cdot U_{12} + C_{13} \cdot U_{13} = C_{12}(V_1 - V_2) + C_{13}(V_1 - V_3)$$

$$Q_2 = \Phi_{e21} + \Phi_{e23} = C_{12} \cdot U_{21} + C_{23} \cdot U_{23} = C_{21}(V_2 - V_1) + C_{23}(V_2 - V_3)$$

$$Q_1 = (C_{12} + C_{13})V_1 - C_{12}V_2$$

$$Q_2 = -C_{12}V_1 + (C_{12} + C_{23})V_2$$

sistem dveh enačb

$$V_1 = p_{11}Q_1 + p_{12}Q_2$$

$$V_2 = p_{21}Q_1 + p_{22}Q_2$$

$$Q_1 = \frac{p_{22}V_1 - p_{12}V_2}{p_{11}p_{22} - p_{12}p_{21}}$$

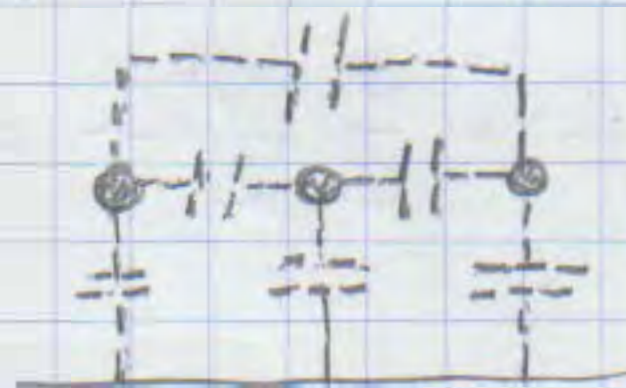
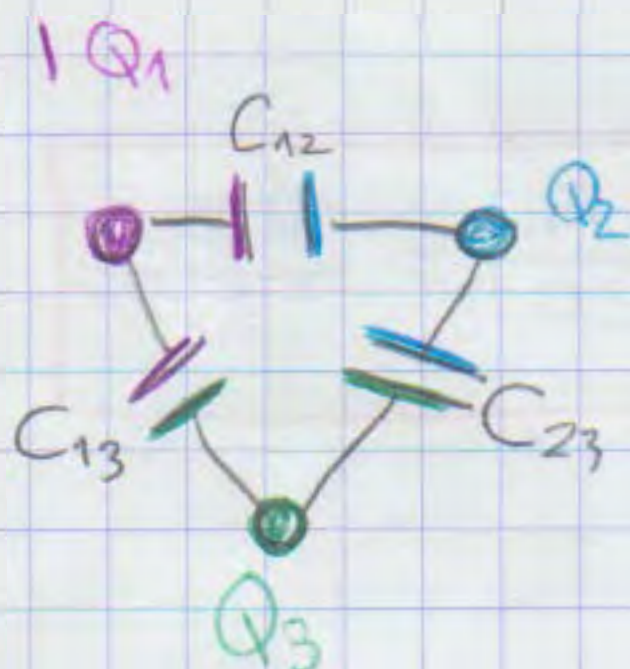
$$Q_2 = \frac{-p_{21}V_1 + p_{11}V_2}{p_{11}p_{22} - p_{12}p_{21}}$$

$$C_{12} = \frac{p_{12}}{[]}$$

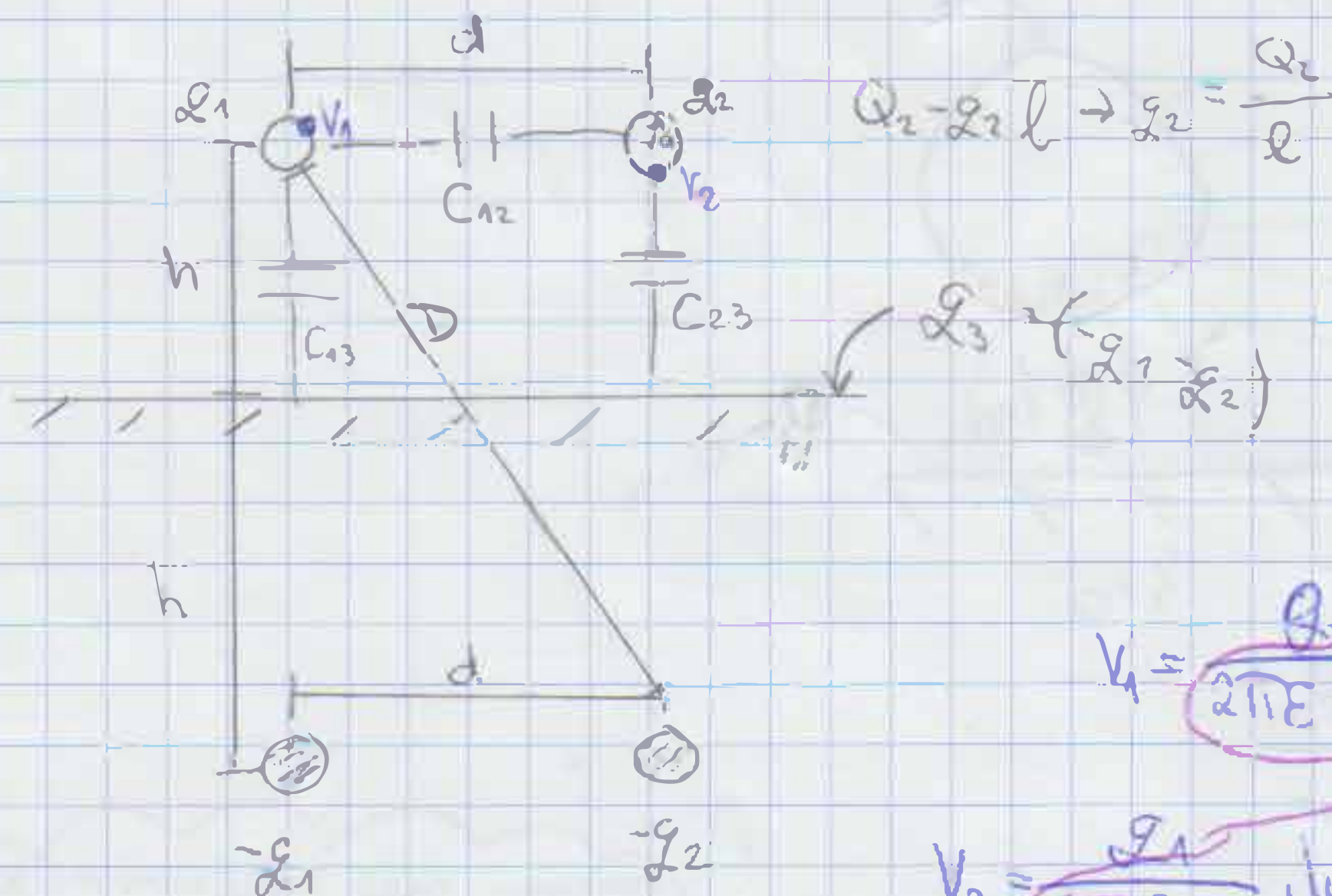
$$C_{13} = \frac{p_{22} - p_{12}}{[]}$$

$$C_{23} = \frac{p_{11} - p_{21}}{[]}$$

zgornje vezje modeliramo z kondenzatorji



žgled duovod nad zemljo



$$Q_2 - Q_1 \ell \rightarrow Q_2 = \frac{Q_1}{\ell}$$

$$Q_3 = -(Q_1 + Q_2)$$

$$V_1 = \frac{Q_1}{2\pi\epsilon} \ln \frac{2h}{a} + \frac{Q_2}{2\pi\epsilon} \ln \frac{D}{d}$$

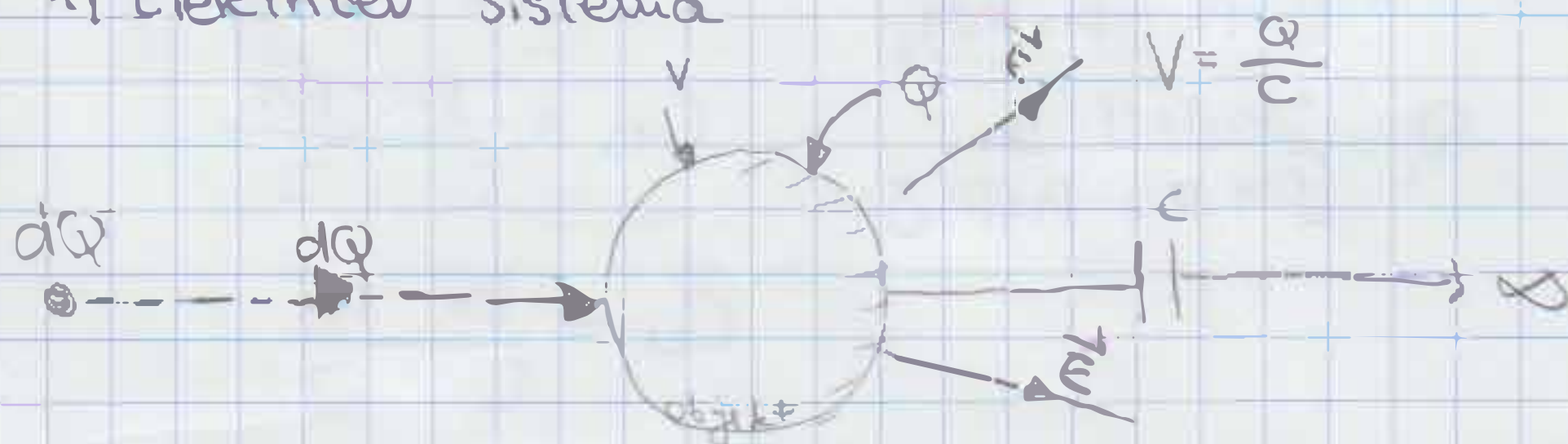
$$V_2 = \frac{Q_1}{2\pi\epsilon} \ln \frac{D}{d} + \frac{Q_2}{2\pi\epsilon} \ln \frac{2h}{a}$$

Električna energija

UČ. 18-1

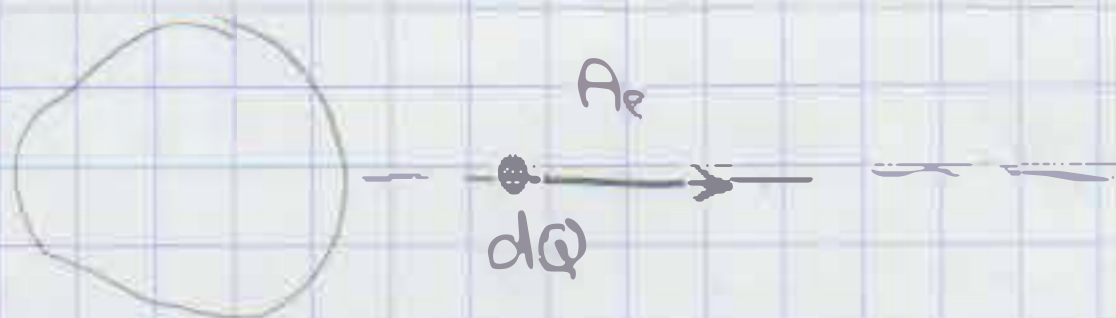


1) Elektnitev sistema



$$A_z = \int_0^{Q_0} V dQ = \int_0^{Q_0} \frac{Q}{C} dQ = \frac{1}{C} \frac{Q_0^2}{2} = A_e = \frac{Q_0^2}{2C} = \frac{Q_0 V_0}{2}$$

zunanji



$$A_e = \int dA_e = \int \frac{Q_0 - Q}{C} = \frac{Q_0^2}{C} - \frac{Q^2}{2C} \Big|_0^{Q_0} = \frac{Q_0^2}{C} - \frac{Q_0^2}{2C} =$$

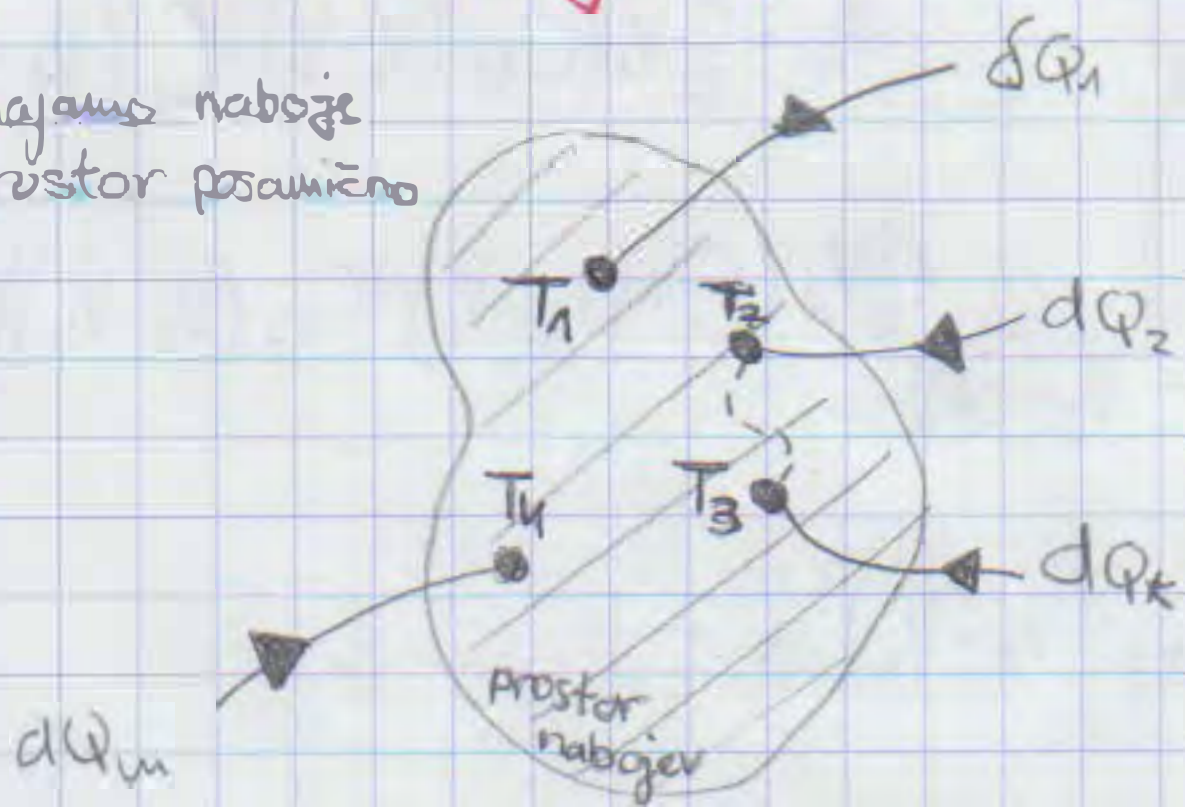
$$A_e = \frac{Q_0^2}{2C}$$

toliko kot smo potrebovali energije da smo vzpostavili sistem toliko ga dobimo

Elektenje prostora

UC. 18-4

dodajamo naboje v prostor posamično



$$dQ \rightarrow dQ$$

$$dA_{z1} = 0$$

$$dA_{z2} = dQ_2 dV_1(T_2)$$

$$dA_{z3} = dQ_3 [dV_1(T_3) + dV_2(T_3)]$$

$$dA_{zk} = dQ_k [dV_1(T_k) + \dots + dV_{k-1}(T_k)]$$

$$dA_{zm} = dQ_m [dV_1(T_m) + \dots + dV_{m-1}(T_m)]$$

$$dQ_k dV_j(T_k) =$$

$$= \frac{dQ_k dQ_j}{4\pi\epsilon_0 \cdot r_{jk}} = dQ_j dV_k(T_j)$$

$$dV_k = \frac{P(T_k) \cdot dr_k^2}{2\epsilon_0}$$

$$A_z = \sum_{k=1}^n \sum_{j=k}^{k-1} dQ_k \cdot dV_j(T_k)$$

$$A_z = \sum_{k=1}^n \sum_{j=k}^{k-1} dQ_j \cdot dV_k(T_j)$$

$$2A_z = \sum_{k=1}^n \sum_{j=1, j \neq k}^n dQ_k dV_j(T_k) \quad j \neq k$$

$$A_z = \frac{1}{2} \lim_{n \rightarrow \infty} \left[\sum_{k=1}^n dQ_k \cdot \lim_{n \rightarrow \infty} \sum_{j=1, j \neq k}^n dV_j(T_k) \right]$$

$$\lim_{n \rightarrow \infty} \sum_{j=1}^n dV_j(T_k) = V(T_k) \quad (52)$$

Alta

$$A_z = \frac{1}{2} \lim_{n \rightarrow \infty} \sum_{k=1}^n dQ_k V(T_k)$$

$$A_z = \frac{1}{2} \int V dQ = A_e = W_e$$

$$W_e = \frac{1}{2} \int V dQ \quad (Q_{\text{cel}})$$

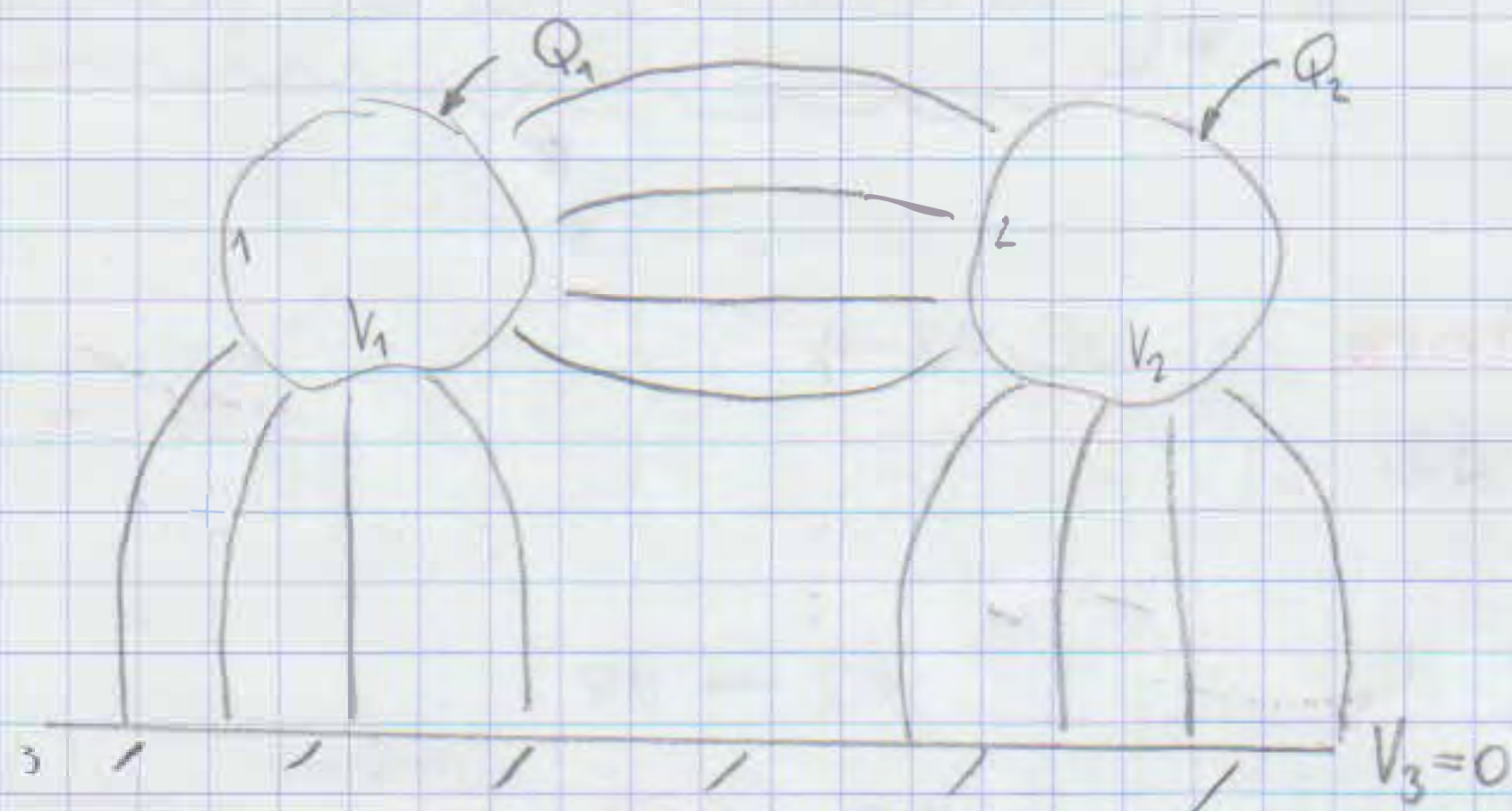
evako smo dokazali kot prej

Energija pri elektivni večih teles

$$W_e = \frac{1}{2} (V_1 Q_1 + V_2 Q_2 + \dots)$$

$$W_e = \frac{1}{2} \sum_{j=1}^n V_j Q_j$$

$n=3$



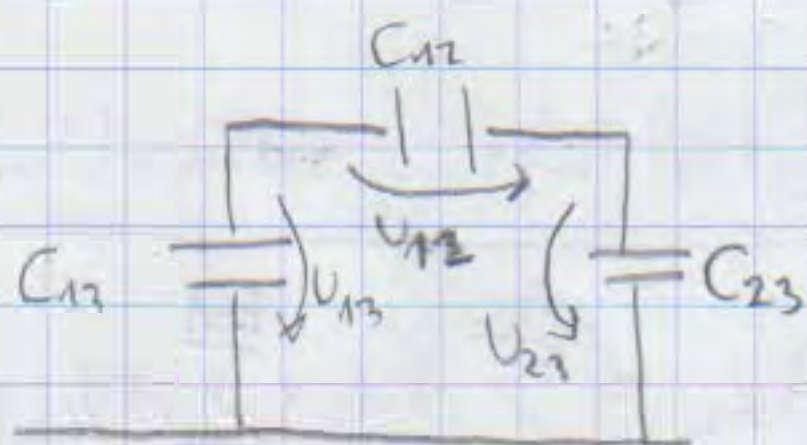
$$W_e = \frac{1}{2} V_1 Q_1 + \frac{1}{2} V_2 Q_2 + \frac{1}{2} V_3 Q_3$$

Annotations: $\phi_{12} + \phi_{13}$ (pink arrow to Q_1), $\phi_{21} + \phi_{23}$ (blue arrow to Q_2), $V_3=0$ (black arrow to Q_3)

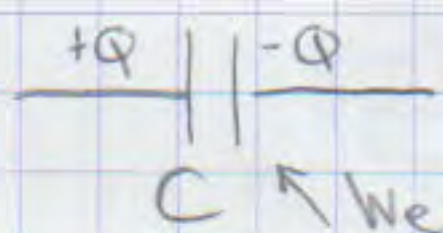
$$W_e = \frac{1}{2} V_1 (C_{12} (V_1 - V_2) + C_{13} (V_1 - V_3)) + \frac{1}{2} V_2 (C_{12} (V_2 - V_1) + C_{23} (V_2 - V_3))$$

$$W_e = \frac{1}{2} C_{12} (V_1 - V_2)^2 + \frac{1}{2} C_{13} (V_1 - V_3)^2 + \frac{1}{2} C_{23} (V_2 - V_3)^2 =$$

$$W_e = \frac{1}{2} C_{12} U_{12}^2 + \frac{1}{2} C_{13} U_{13}^2 + \frac{1}{2} C_{23} U_{23}^2$$



Modelno vezje



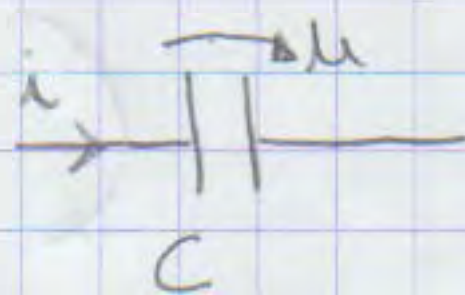
$$Q = CU$$

$$W_e = \frac{CU^2}{2} = \frac{QU}{2}$$

moč potrebna za elektrenje

$$p = W_e' = \frac{C}{2} \frac{dU}{dt} \cdot U' = U \cdot C \cdot U' = U \cdot i$$

$$p = U \cdot i$$



$$i = C \cdot U'$$

U' vsebuje časovni odvod

$$W_e = \frac{CU^2}{2}$$

Gostota električne energije

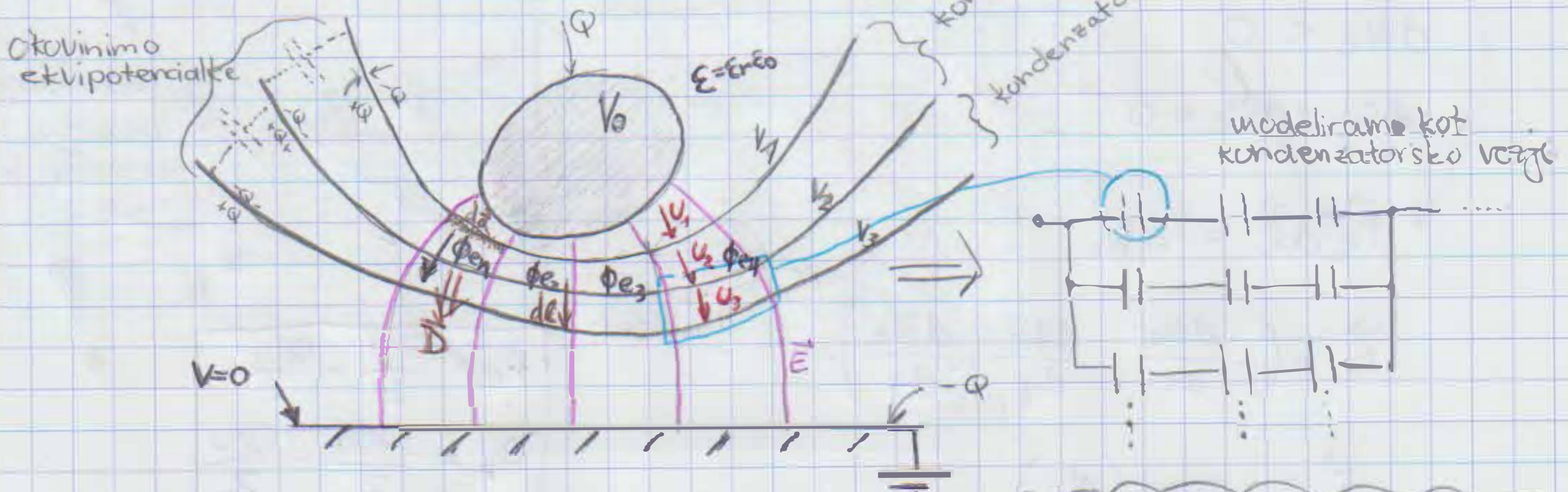
ek. energija

$$W_e = \frac{QV}{2}$$

$$\frac{Q^2}{2C}$$

$$W_e = \frac{CU^2}{2}$$

energija



$$W_e = \frac{Q}{2} \sum_{k=1}^n U_k$$

imamo ločene sisteme zaradi okvirjenja
 $W_e = \frac{1}{2} Q(V_0 - V_1) + \frac{1}{2} Q(V_1 - V_2) + \frac{1}{2} Q(V_2 - V_3) + \dots$
 modeliranje sk. e.
 pomemben je prostor kjer se nahaja sistem ne naboj.

vsota vseh
napetosti
in nabojev

$$W_e = \frac{1}{2} \sum_{j=1}^m \phi_{ej} \cdot \sum_{k=1}^n U_k$$

$$W_e = \frac{1}{2} \sum_{j=1}^m \sum_{k=1}^n \phi_{ej} \cdot U_k = \frac{1}{2} \sum_{k=1}^n \sum_{j=1}^m \vec{D}_j \cdot \vec{E}_k \underbrace{d\vec{a}_j d\vec{l}_k}_{d\tau \text{ volumen celic}} = \sum_{j,k} \left(\frac{1}{2} \vec{E} \cdot \vec{D} \right) d\tau$$

suma vseh celic
suma je integral

lim j in k $\rightarrow \infty$

$$W_e = \int \frac{\vec{D} \cdot \vec{E}}{2} d\tau$$

celota v neki točki prostora

volumska/prostorska
gostota energije

energija se shrani v prostor, kjer je polje v okolici objekta

[J/m³]

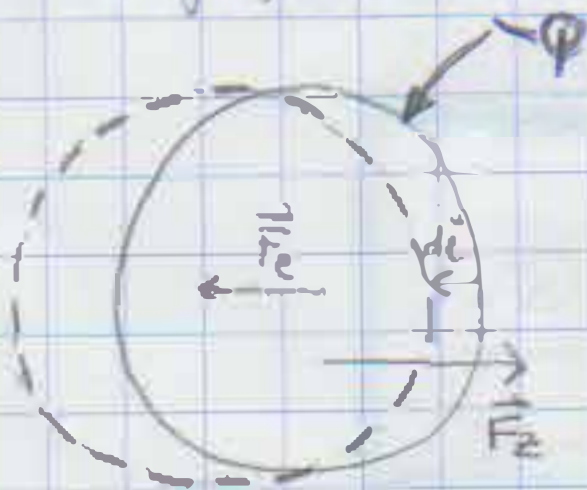
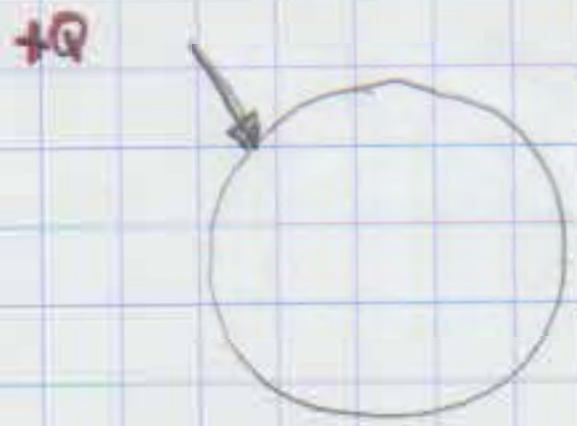
$$W_e = \frac{\vec{D} \cdot \vec{E}}{2} = \frac{\epsilon E^2}{2} = \frac{D^2}{2\epsilon}$$

gostota električne energije

$$W_e = \int_{\text{cel}} W_e \cdot d\tau$$

energija je v izolantu polje je nosilec energije

Gibalni procesi sistema brez virov



* kapacitivnost se povečuje
energija se manjša *

brez napajanja

$$dA_e = \vec{F}_e d\vec{l}$$

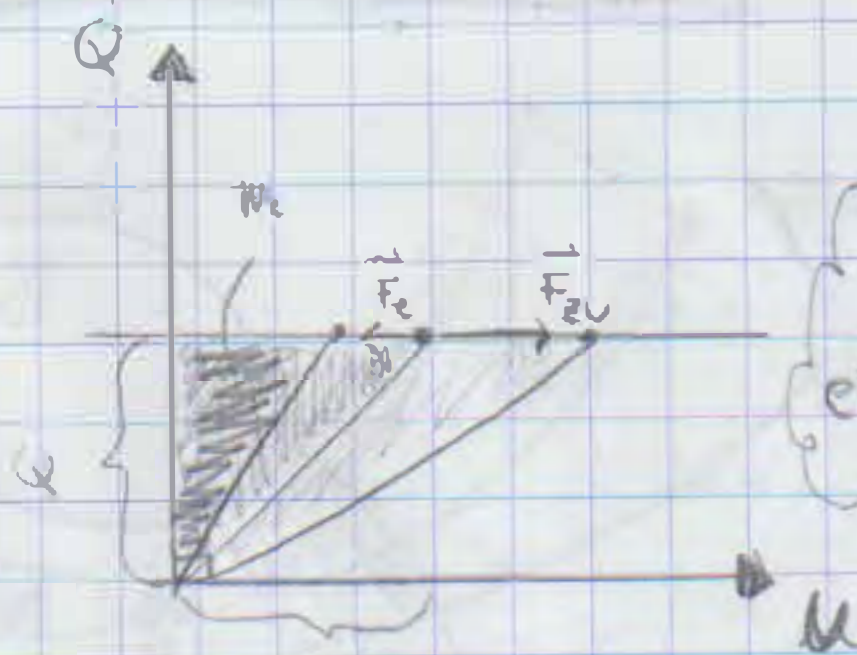
$$dW_e < 0$$

$$dA_e + dW_e = 0$$

$$\vec{F}_e d\vec{l} = -dW_e$$

$$\vec{F}_e = \left(-\frac{dW_e}{dx}, \frac{dW_e}{dy}, \frac{dW_e}{dz} \right)$$

$$\vec{F}_e = -\text{grad}(W_e)$$



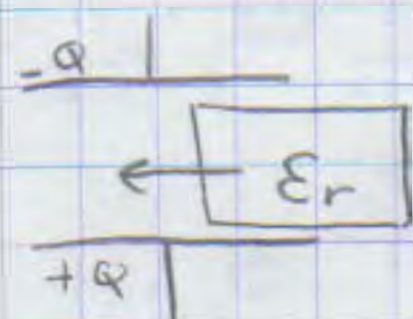
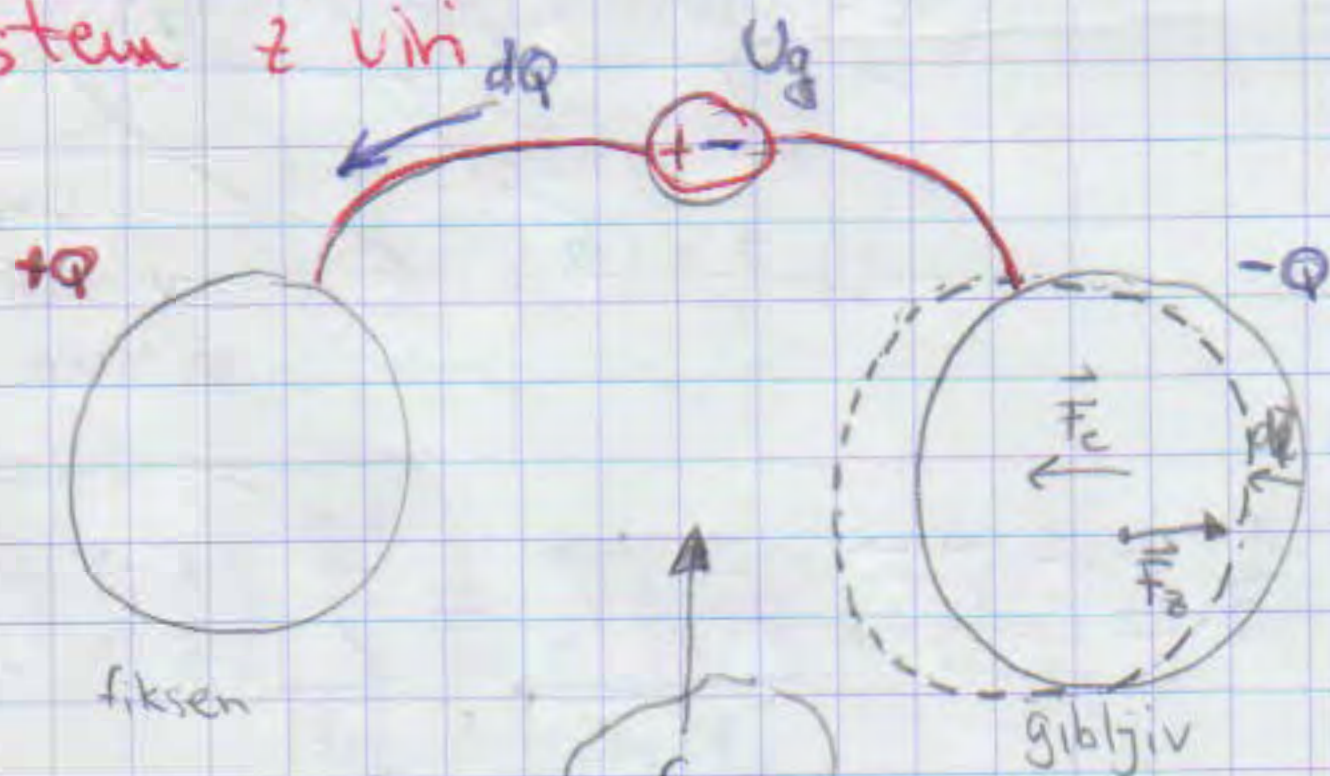
premikanje desnega
naboja z delovanjem
električne in zunanje
sile

-gradient

$$A_e = \frac{QU}{2} - \frac{QU_1}{2}$$

$$A_z = \frac{QU_2}{2} - \frac{QU}{2}$$

b) sistem z viri



$$dA_e = \vec{F}_e d\vec{l}$$

$$dA_g = U_g \cdot dQ$$

$$dW_e \geq 0 = \frac{1}{2} U_g (Q + dQ) - \frac{1}{2} U_g \cdot Q$$

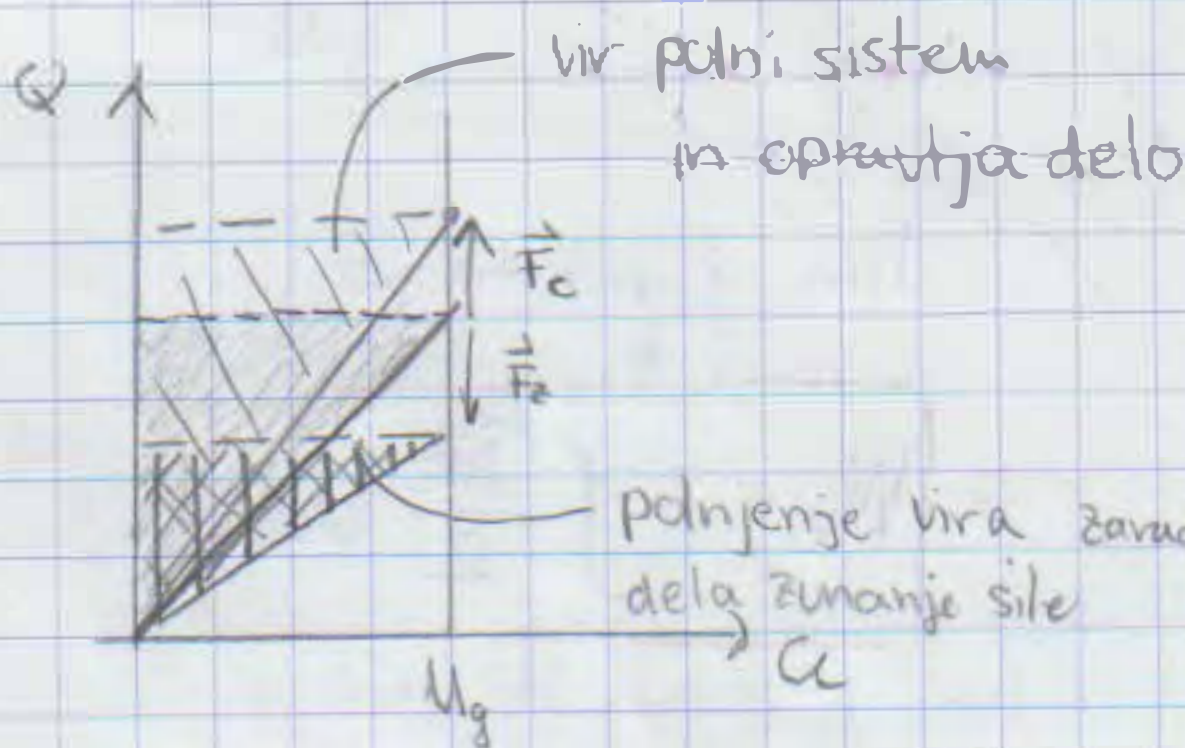
$$dA_e + dW_e = dA_g \text{ delo vira}$$

$$\vec{F}_e d\vec{l} + \frac{1}{2} U_g dQ = U_g dQ \text{ na začetku}$$

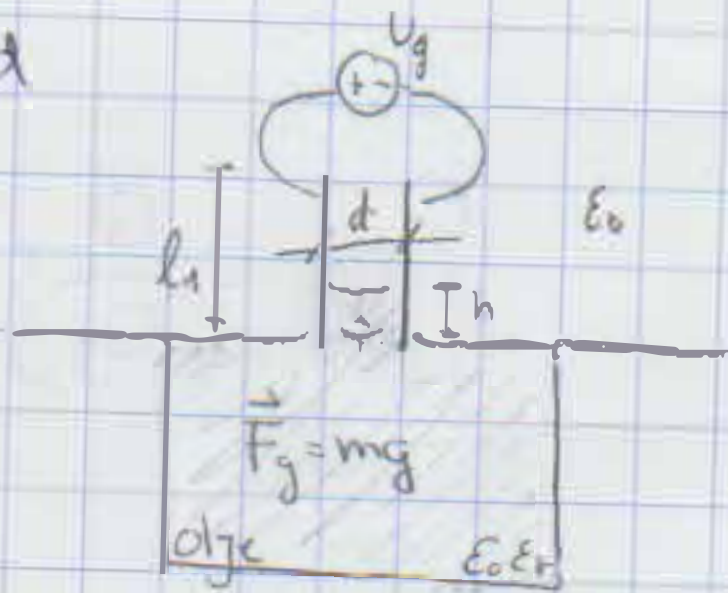
$$\vec{F}_e d\vec{l} = \frac{1}{2} U_g dQ = +dW_e$$

$$\vec{F}_e d\vec{l} = +dW_e \text{ delo ele. energije = pozitivni spremembi energije}$$

$$\vec{F}_e = \left(\frac{dW_e}{dx}, \frac{dW_e}{dy}, \frac{dW_e}{dz} \right) = \text{grad}(W_e)$$

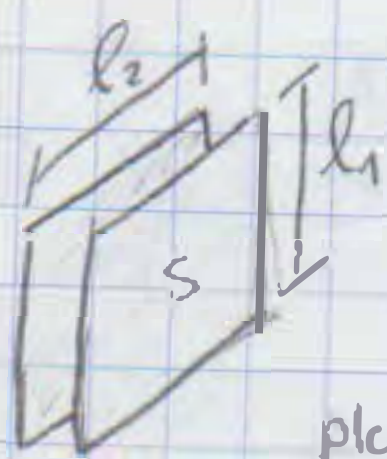
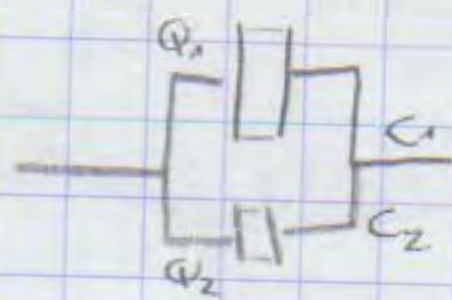
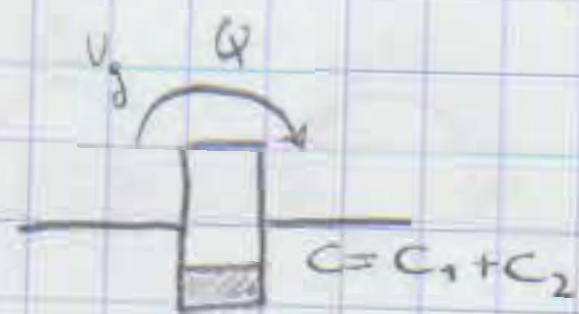


zgled



$$C = \epsilon \frac{S}{d}$$

$$W_e = \frac{1}{2} C U_g^2$$



plošči

$$C = C_1 + C_2$$

$$C = \epsilon_0 \frac{(l_1 - h) l_2}{d} + \frac{\epsilon_0 \epsilon_r h l_2}{d}$$

$$F_e = \frac{dW_e}{dh}$$

ker se obje premika le v z komponenti

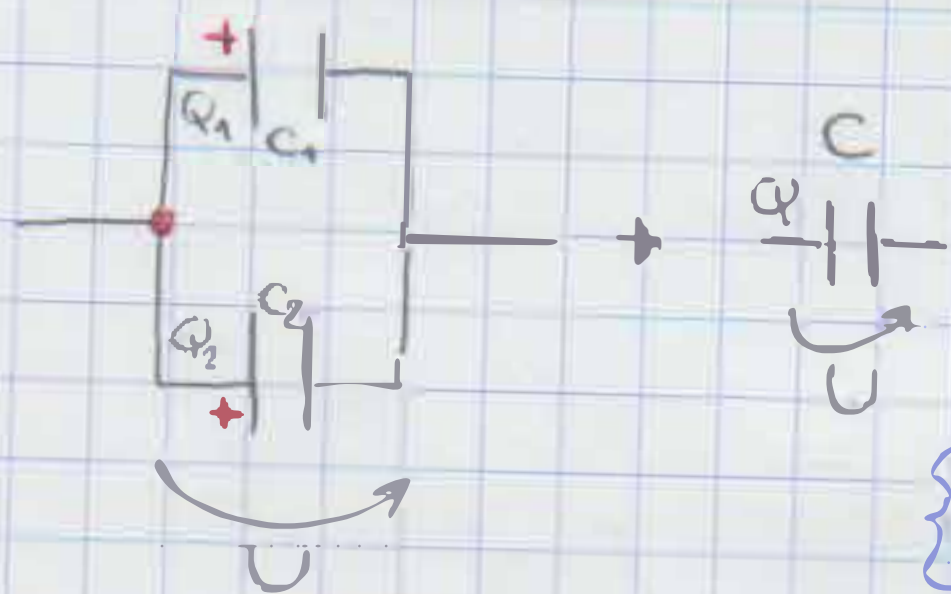
$$F_e = \frac{dW_e}{dC} \cdot \frac{dC}{dh} = \frac{1}{2} U_g^2 \left(\frac{\epsilon_0 l_2}{d} + \epsilon_0 \epsilon_r \frac{l_2}{d} \right)$$

$$\rho l_2 h d g = \rho V g = m g = F_g = F_e = \frac{\epsilon_0 l_2 U_g^2}{2d} (\epsilon_r - 1)$$

$$h = \frac{\epsilon_0 U_g^2 (\epsilon_r - 1)}{2 \rho \cdot d^2 \cdot g}$$

Kondenzatorska vezja

1) Vzporedna vezava kondenzatorjev



$$\{Q_i = C_i \cdot U\}$$

$$Q = Q_1 + Q_2$$

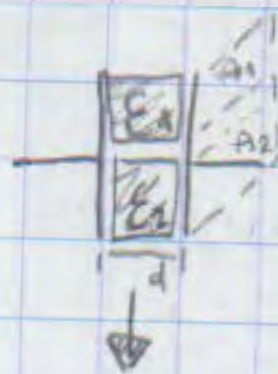
$$C \cdot U = C_1 \cdot U + C_2 \cdot U$$

$$C = C_1 + C_2$$

$$C = \sum_{i=1}^n C_i$$

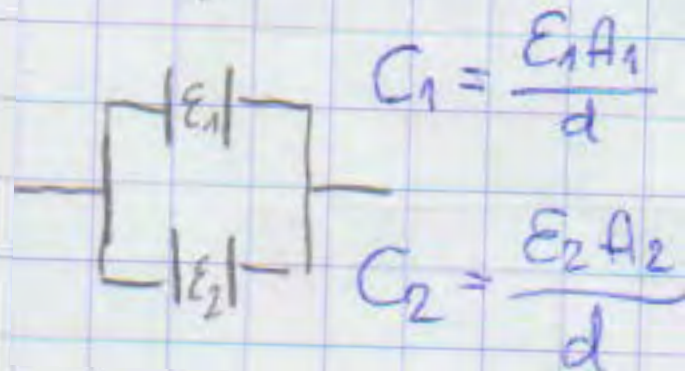
+ sponke skupaj zvezane

zgled:

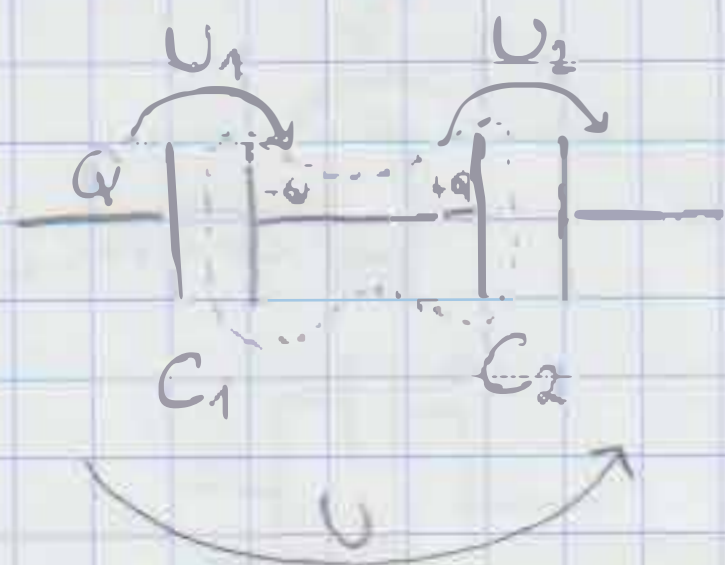


$$C = \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{d}$$

$$Q = C \cdot U$$



2) Záporedná vežawa. kondenzatorjev



$$U = U_1 + U_2$$

$$\left\{ U_i = \frac{Q}{C_i} \right\}$$

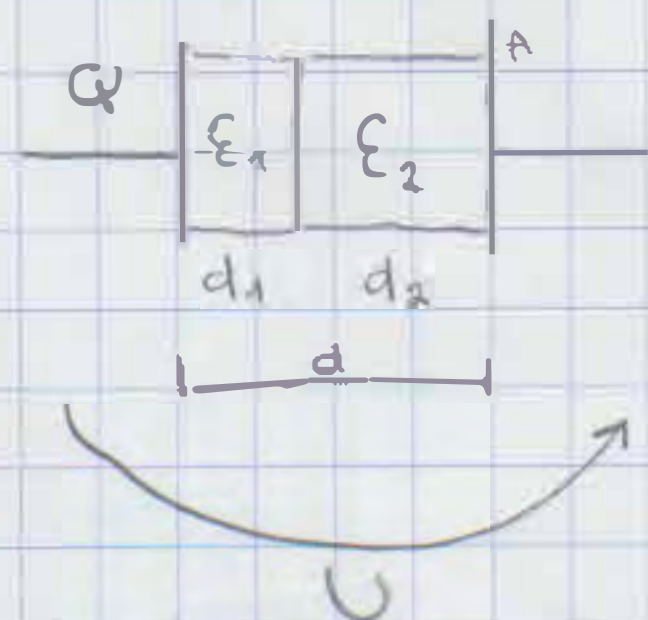
$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \quad | : Q$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \sum_{j=1}^n \frac{1}{C_j}$$

$$C = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

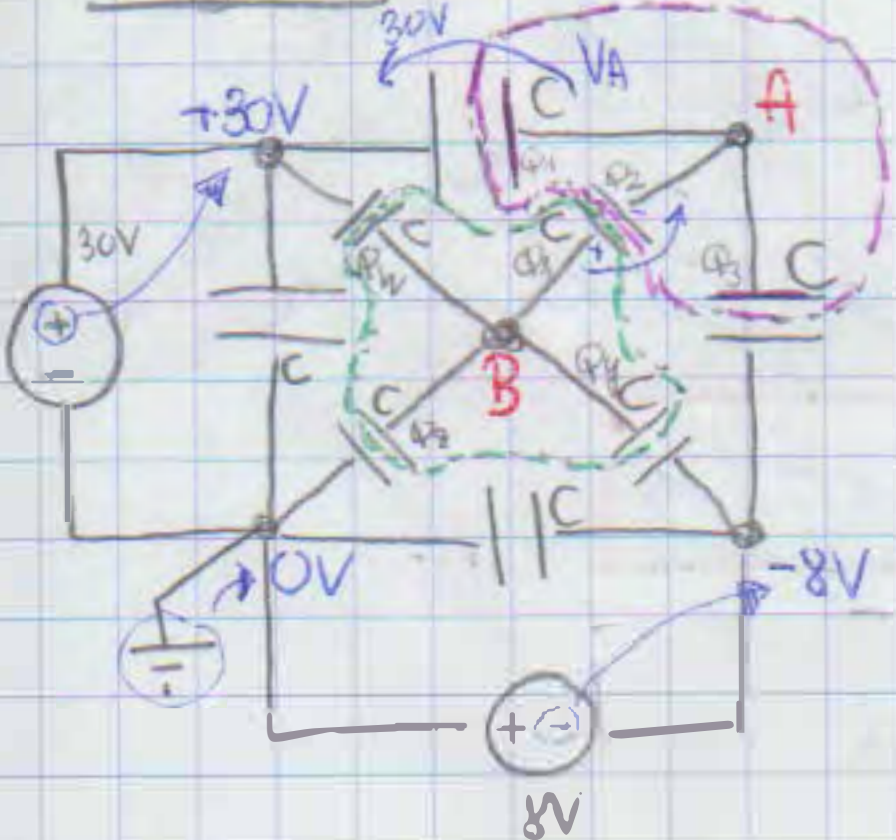
2gled



$$C_1 = \frac{\epsilon_1 A}{d_1} \quad C_2 = \frac{\epsilon_2 A}{d_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} \rightarrow Q = C \cdot U$$

2gled



$$C = 10 \text{ nF}$$

$$V_A, V_B = ?$$

$$Q_1 + Q_2 + Q_3 = 0$$

$$Q_x + Q_y + Q_z + Q_w = 0$$

$$B: \varphi(V_A - 30V) + \varphi(V_A - V_B) + \varphi(V_A - (-8V)) = 0$$

|: C sistem
linearnih
enačb

$$A: \varphi(V_B - V_A) + \varphi(V_B - (-8V)) + \varphi(V_B - 0) + C(V_B - 30) = 0$$

$$3V_A - V_B = 22V$$

$$-V_A + 4V_B = 22V \quad | \cdot 3$$

$$\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 22 \\ 22 \end{bmatrix}$$

$$[C] [V] = [Q]$$

$$[V] = [C]^{-1} [Q]$$

$$V_A = 10V$$

$$V_B = 8V$$

$$Q_z = C(V_B - 0) = 80 \text{ nC}$$

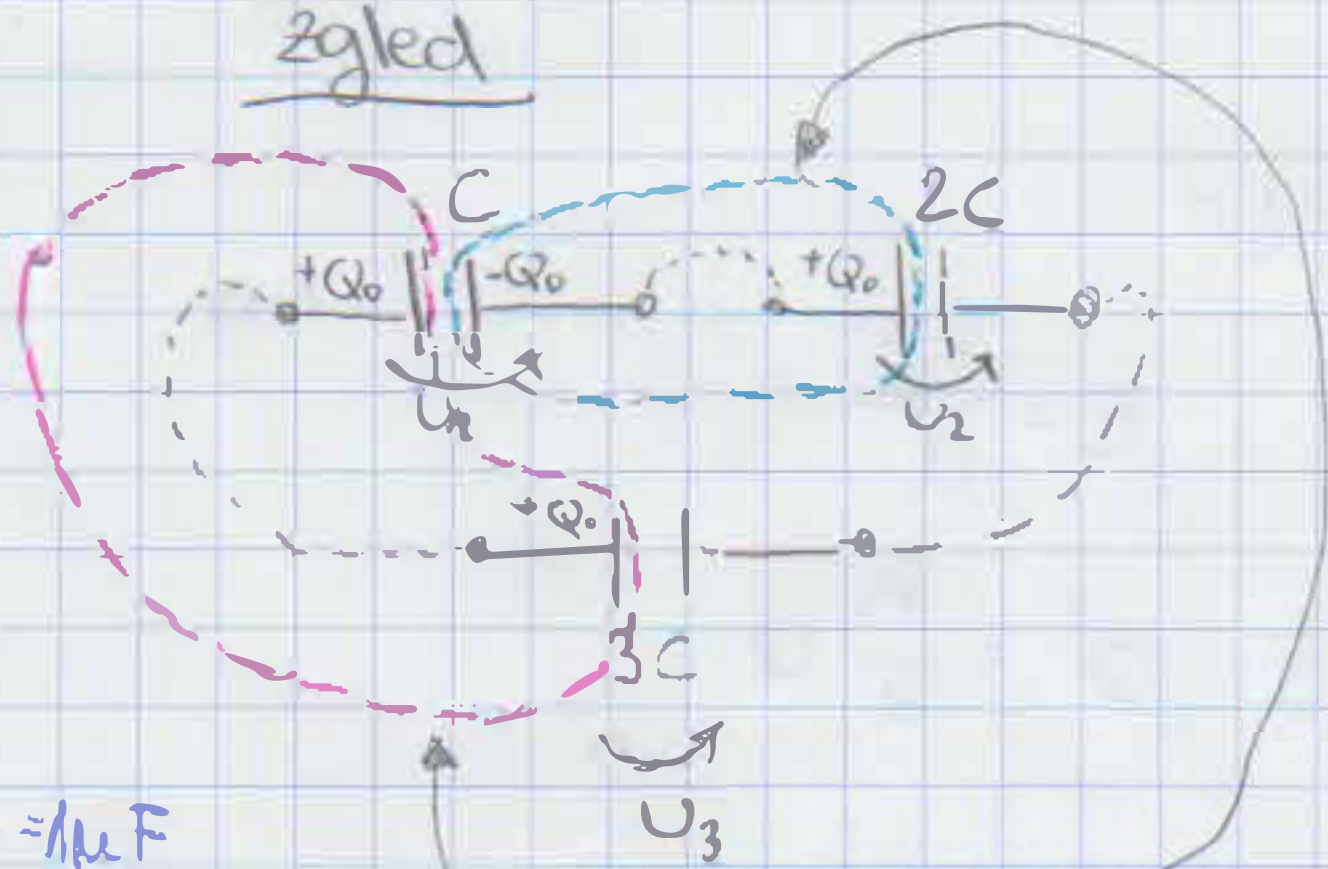
$$W_y = \frac{CU^2}{2} \quad U = (V_B + 8)$$

$$W_y = 1,28 \mu J$$

Zgled

$$C_1 = 1 \mu F$$

$$Q_0 = 10 \mu C$$



stare napetosti

$$U_1' = \frac{Q_0}{C}$$

$$U_2' = \frac{Q_0}{2C}$$

$$U_3' = \frac{Q_0}{3C}$$

nove napetosti U_1, U_2, U_3

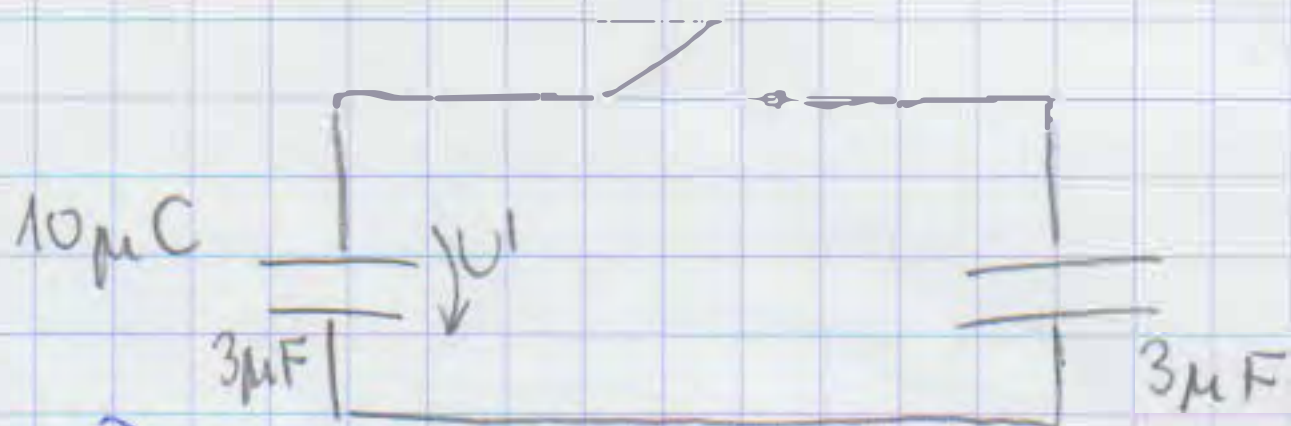
novi naboji $CU_1, 2CU_2, 3CU_3$

Kirchoffov zakon 1) $U_1 + U_2 - U_3 = 0$

$$2) \quad CU_1 + 3CU_3 = 2Q_0(Q_0 + Q_0) \quad \left. \begin{array}{l} U_1 = 3,63V \\ U_2 = 1,81V \\ U_3 = 5,45V \end{array} \right\} \text{zračunano z mehaniko}$$

$$3) \quad -CU_1 - 2CU_2 = 0(-Q_0 + Q_0)$$

Energijski defekt

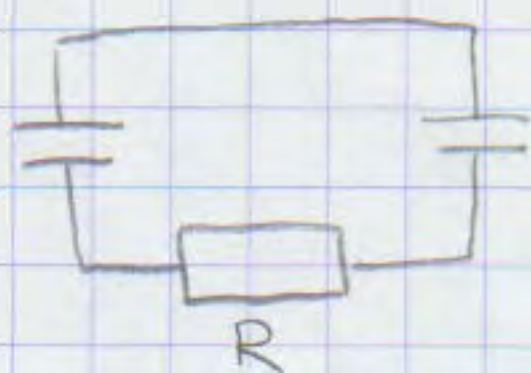


1) pred preklpom

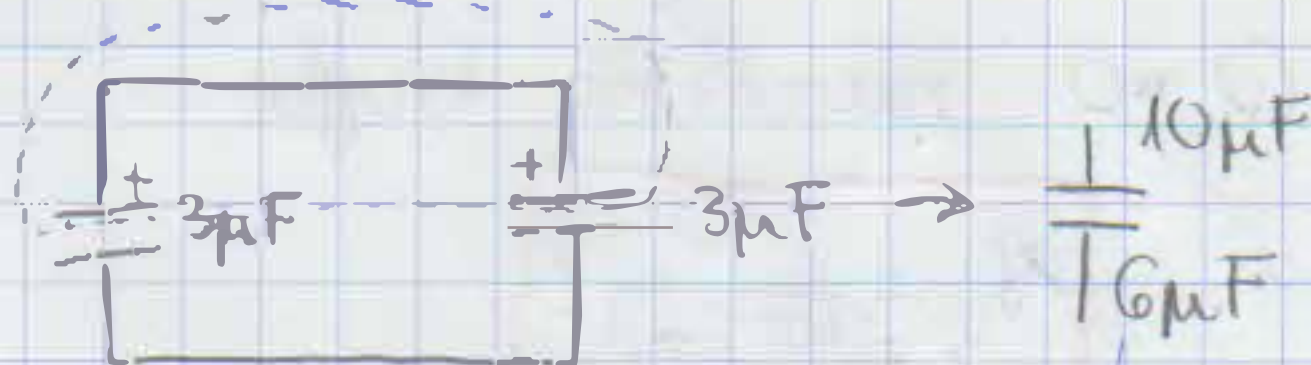
$$U_1' = \frac{10 \mu C}{3 \mu F} = 3,33V$$

$$W_{e1}' = \frac{Q'^2}{2C} = \frac{10^2 \mu C^2}{2 \cdot 3 \mu F} = \frac{100}{6} \cdot 10^{-6} J$$

$$W_{e2}' = 0$$



2) po preklpu



$$U = \frac{10 \mu C}{6 \mu F} = 1,66 = \frac{U}{2}$$

$$Q = Q'$$

$$W_{e1+2} = \frac{C \cdot U^2}{2} = \frac{6 \cdot 10^{-6}}{2} \cdot \left(\frac{10}{6}\right)^2 = 80 \cdot \frac{100 \cdot 10^{-6}}{12} J$$

$$W_e = W_{e1} + W_{e2} = \frac{100 \cdot 10^{-6}}{12} J$$

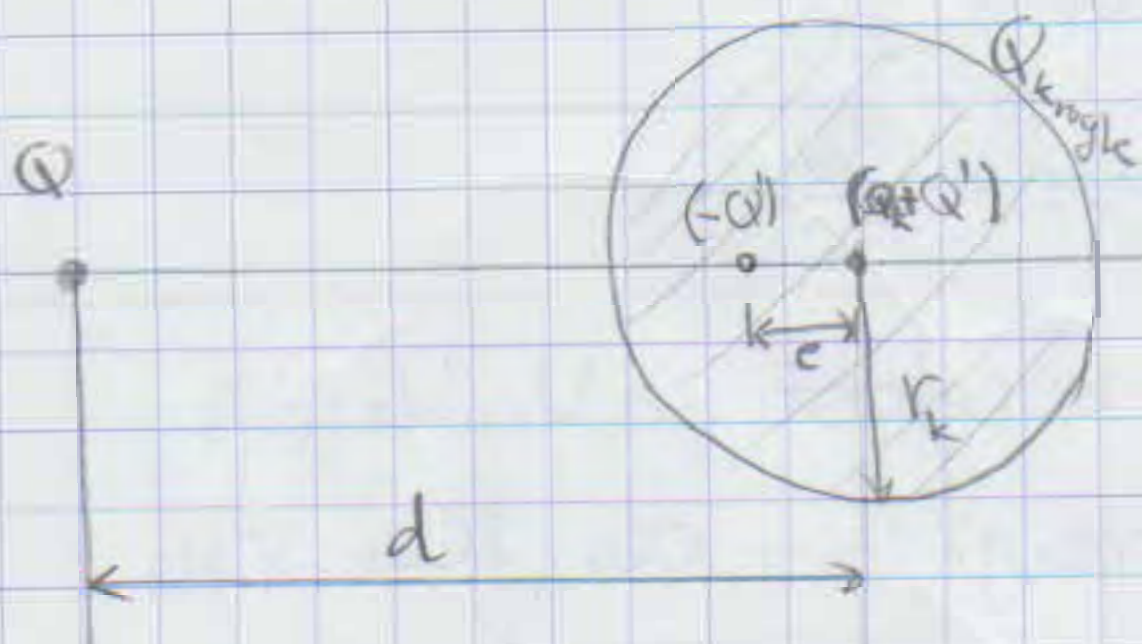
$$\Delta W_e = W_e' - W_e = \frac{W_e}{2} \text{ energijski defekt}$$

Metoda zrcaljenja

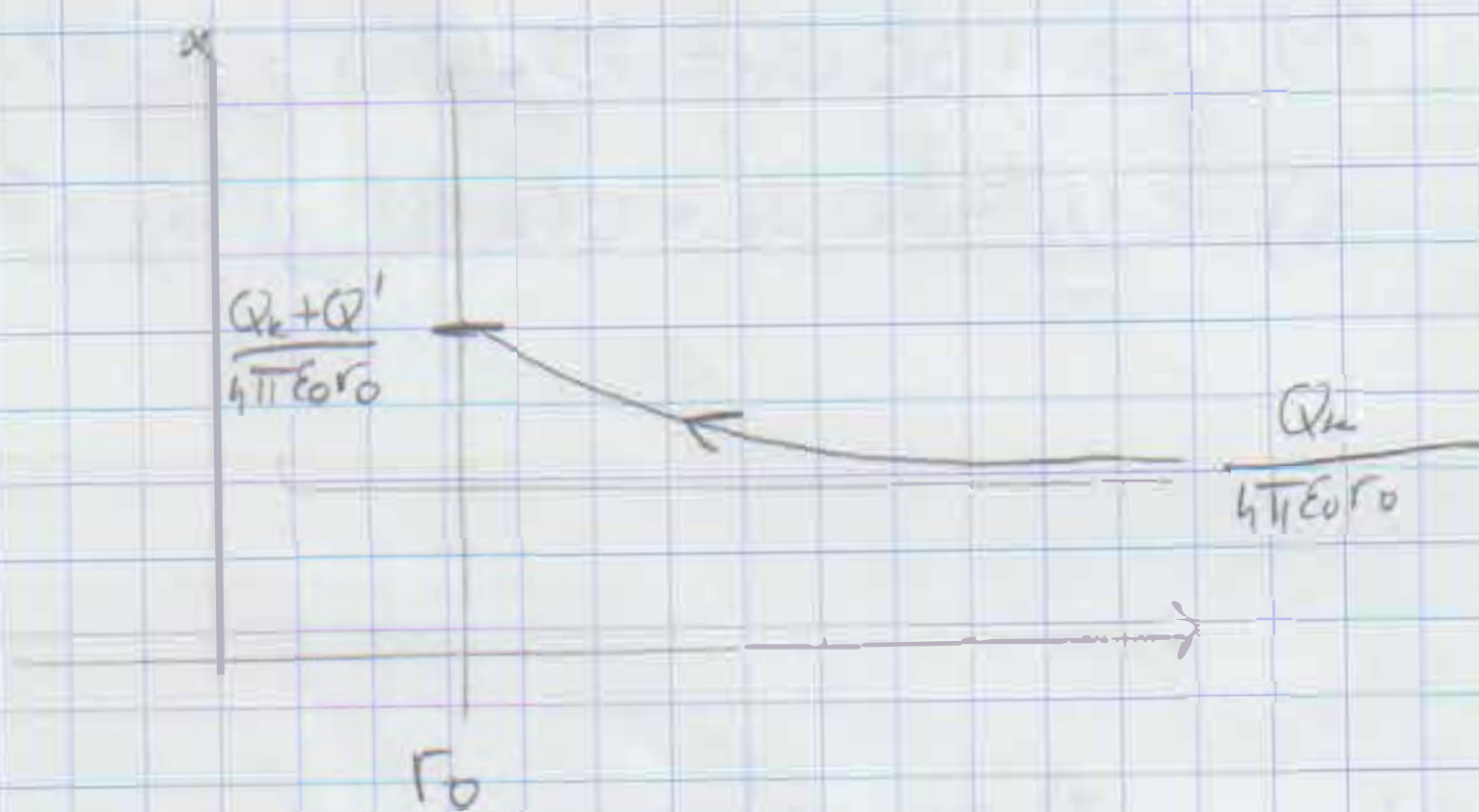
A) Izgled naboja pred naelektrenu kruglo

$$e = \frac{r_0^2}{d}$$

$$Q' = \frac{r_0}{d} Q$$

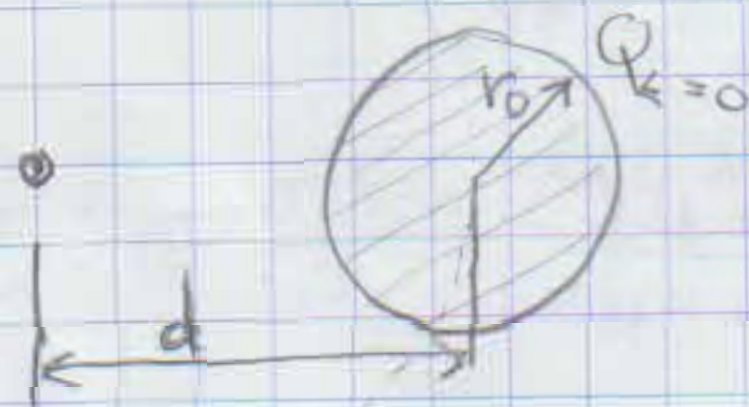


$$V_{\text{angle}} = \underbrace{V(Q - Q')}_{\ominus} + V(Q_k + Q') = \frac{Q_k + Q'}{4\pi\epsilon_0 r_0} = \frac{Q_k + \frac{r_0}{d}Q}{4\pi\epsilon_0 r_0}$$

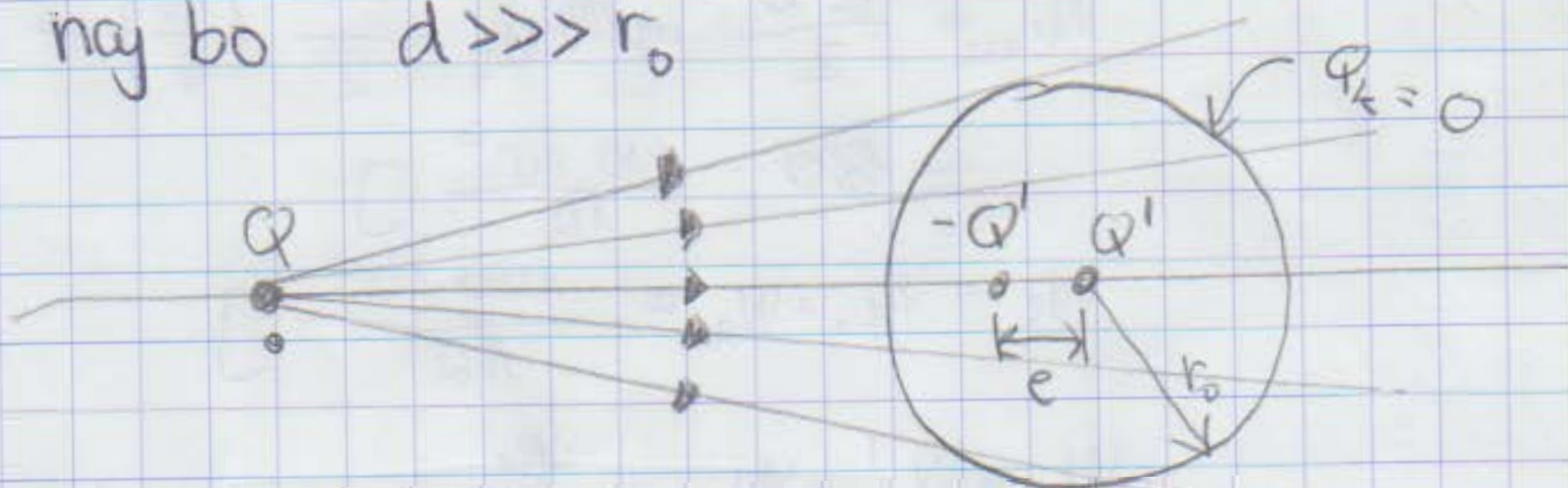


$Q_k = 0$ neutralna krogla

$$V_k = \frac{Q}{4\pi\epsilon_0 d}$$



may be $d \gg r_0$



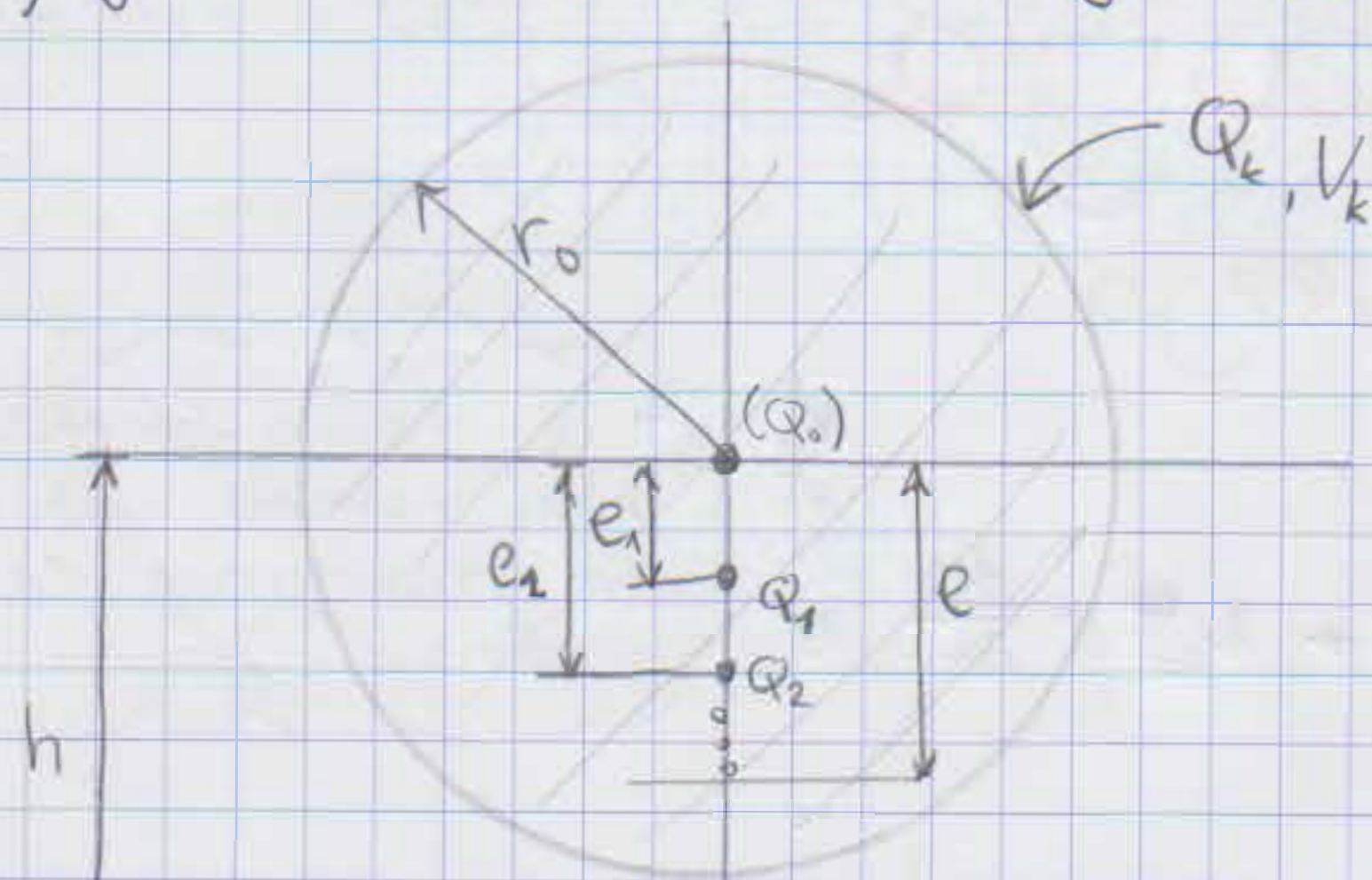
$$p = Q' \cdot e = \frac{r_0 Q}{d} \cdot \frac{r_0^2}{d} = \frac{4\pi \epsilon_0}{4\pi \epsilon_0} =$$



$$\epsilon = \frac{Q}{4\pi\epsilon_0 d^2} = 4\pi\epsilon_0 r_0^3 = 4\pi\epsilon_0 r_0^2 E_0$$

$$E = E_0 + E_p$$

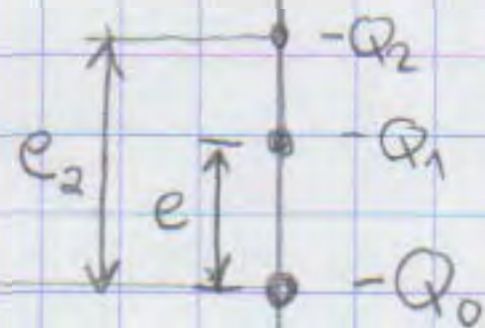
2) zgled kapacitivnost kropla - zemlja



Q_k, V_k

$V=0$

h



$$Q_0 = 4\pi\epsilon_0 r_0 V_k = C_0 V_k$$

$$Q_1 = \frac{r_0}{2h} Q_0, \quad e_1 = \frac{r_0^2}{2h}$$

$$Q_2 = \frac{r_0}{2h-e_1} Q_1, \quad e_2 = \frac{r_0^2}{2h-e_1}$$

$$Q_{n+1} = \frac{r_0}{2h-e_n} Q_n$$

$$e_{n+1} = \frac{r_0^2}{2h-e_n}$$

$$\lim_{n \rightarrow \infty} e_n = e$$

$$e = \frac{r_0^2}{2h-e}$$

$$2he - e^2 + r_0^2 = 0$$

$$e^2 - 2he + r_0^2 = 0$$

$$e = \frac{2h \pm \sqrt{(2h)^2 - 4r_0^2}}{2}$$

$$e = h - \sqrt{h^2 - r_0^2}$$

$$Q_{n+1} = \frac{r_0}{2h-e_n} Q_n$$

$$\frac{r_0}{2h-e} < 1$$

vsak naslednji nabij je manj za faktor

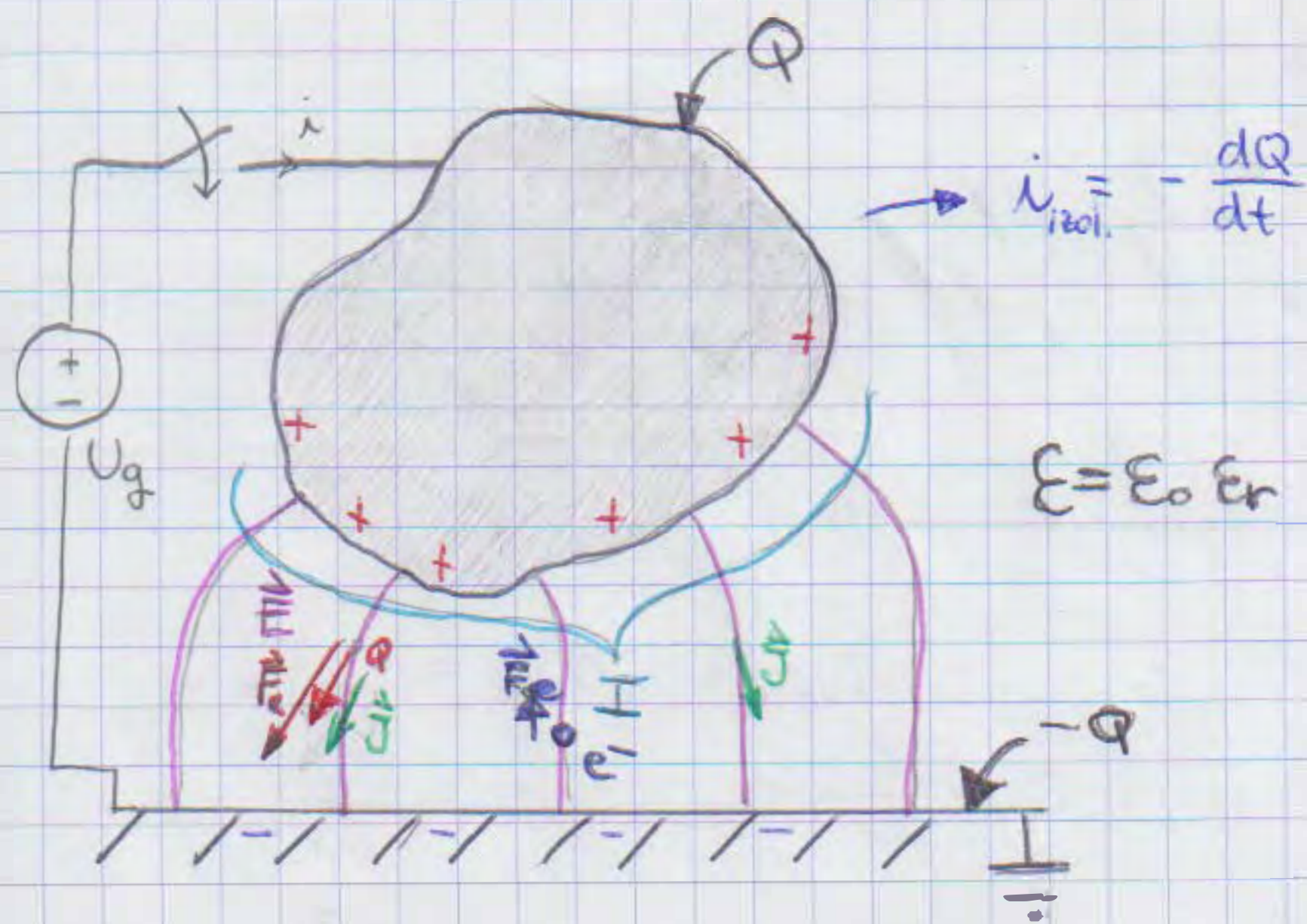
$$Q_k = Q_0 + Q_1 + Q_2 + \dots$$

$$Q_k = C_0 \left[1 + \frac{r_0}{2h} + \frac{r_0^2}{2h(2h-e_1)} + \frac{r_0^3}{2h(2h-e_1)(2h-e_2)} + \dots \right] V_k$$

C

Tokovna polja

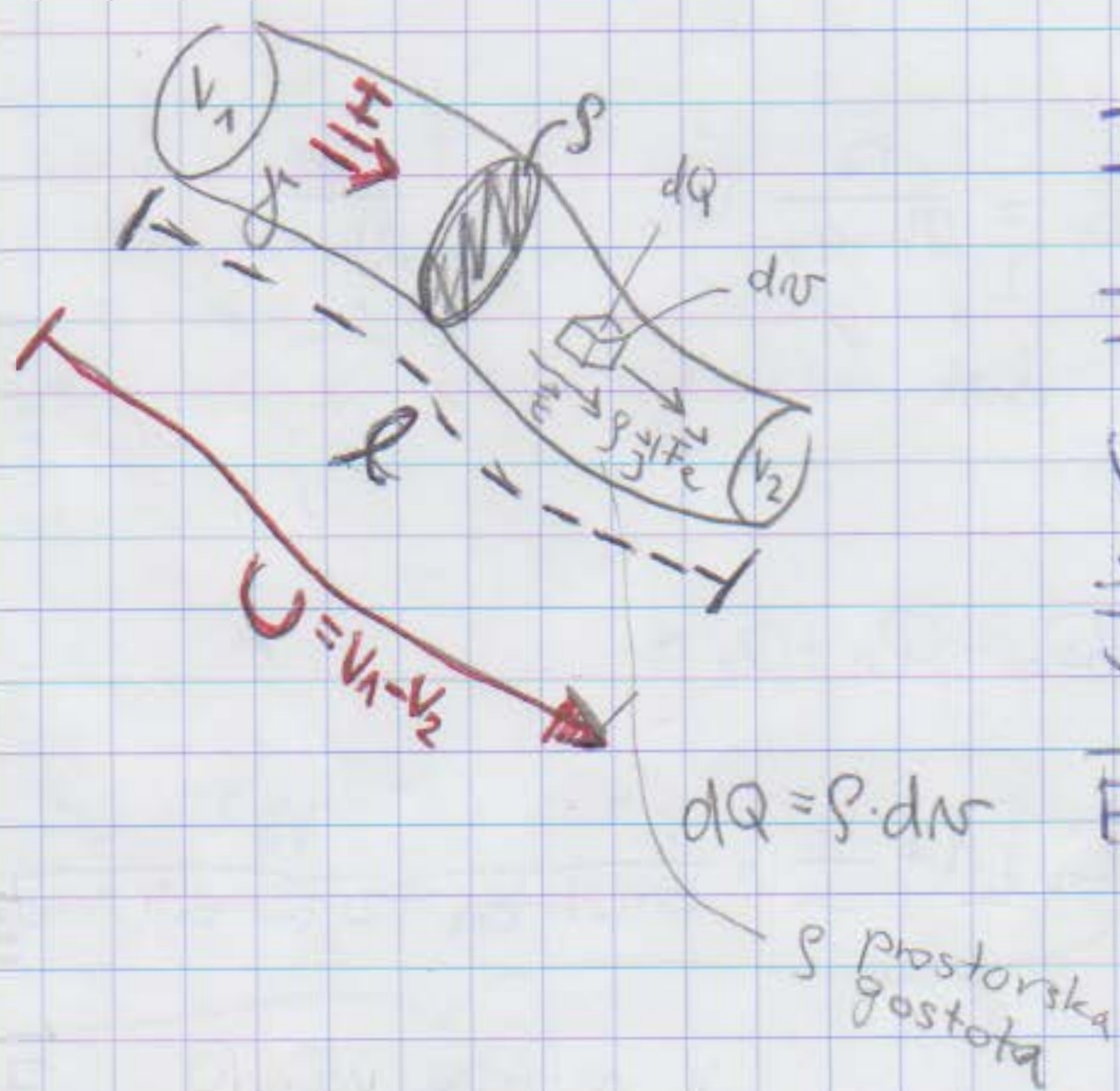
uporovna vezja



če želimo vzdrževati el. polje moramo imeti prisoten vir napajanja, če ne polje poihne

Pravodne lastnosti snovi

Ohmov zakon || v omejenih pogojih ||



SORAZMERNOST

$I \propto U$ za toliko kot se napetost dvigne za toliko se tok

$I \propto j$ tok sorazmeren z gostoto toka

$U \propto E$

$j = \sigma \cdot E$

$E = \frac{dV}{dQ}$

$j \propto E$

$j = \sigma E$

OHMOV ZAKON V
DERIVACIJSKI OBLIKI

specifična
prevodnost snovi $\left[\frac{A}{Vm} = \frac{S}{m} \right]$

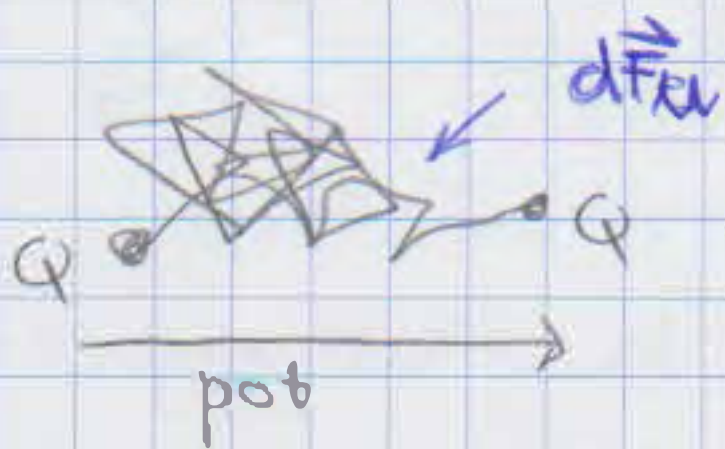
$$\vec{J} \propto \vec{E}$$

$$S \cdot \vec{w} \propto \frac{d\vec{F}_e}{dQ} \leftarrow S \cdot d \cdot v$$

$\vec{w}$
L hitrost
pretakanja
nabojev

$$d\vec{F}_e$$

L ele. sila na naboje



$$d\vec{F}_e + d\vec{F}_u = 0$$

$$\vec{w} \propto d\vec{F}_u$$

$$\vec{J} = \left[\frac{ne^2 T_e}{m} \cdot \vec{E} \right] \left[\Omega^{-1} = \frac{A}{Vm} \right]$$

$e \dots$ osnovni naboj

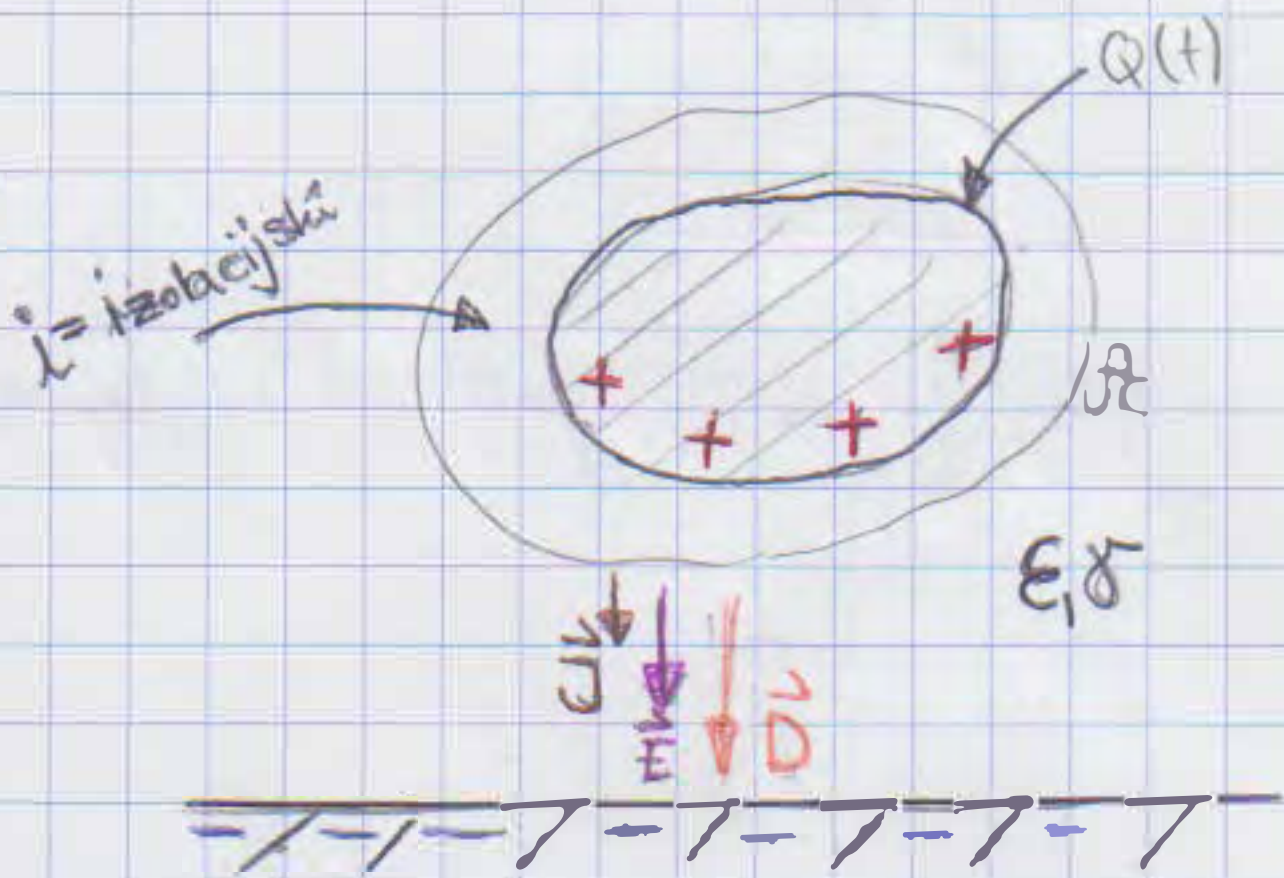
$n \dots$ št. prostih nosilcev

$T_e \dots$ povprečen čas med zaporednimi trki ($\sim 10^{-15}s$)

$m \dots$ masa prostih nosilcev naboja

$T^{-1} \dots$ frekvenca trkov ($10^{15}s$)

Praznjenje kondenzatorja



$$\oint \vec{D} \cdot d\vec{a} = Q = \oint \frac{\epsilon}{\gamma} \vec{J} \cdot d\vec{a} = \frac{\epsilon}{\gamma} \cdot \lambda \cdot da =$$

$$= \frac{\epsilon}{\gamma} \left(- \frac{dQ}{dt} \right)$$

$$\frac{dQ}{dt} + \left(\frac{\gamma}{\epsilon} \right) Q = 0$$

$$Q = A \cdot e^{\lambda t}$$

$$Q' = \lambda A \cdot e^{\lambda t}$$

$$\left(\lambda + \frac{\gamma}{\epsilon} \right) A e^{\lambda t} = 0$$

$$\lambda = - \left(\frac{\epsilon}{\gamma} \right)^{-1} = - \frac{1}{\frac{\epsilon}{\gamma}} \rightarrow - \frac{1}{T}$$

naj je Q ob $t=0$ enak Q_0

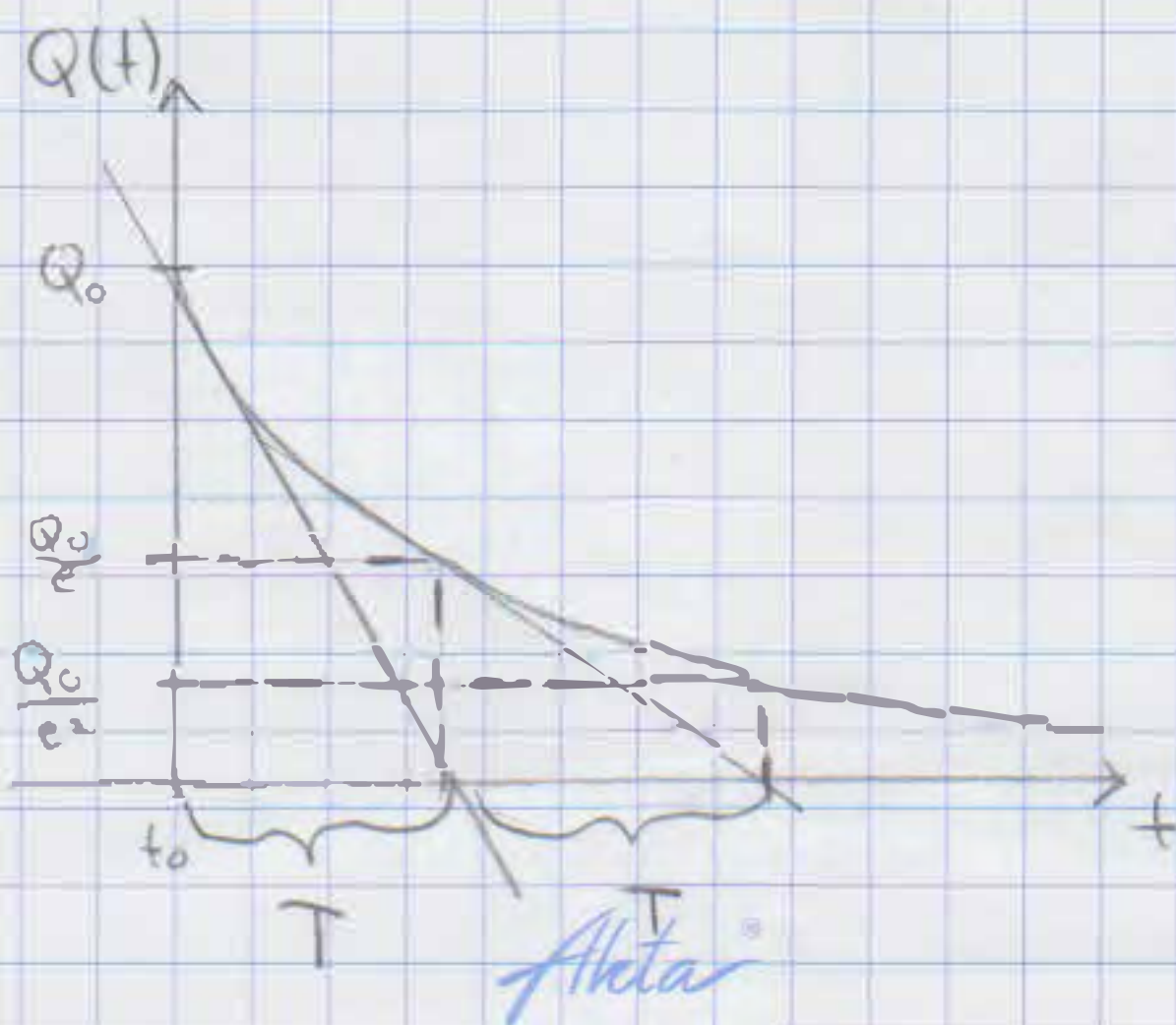
$$Q_0(0) = Q_0 = A e^{\lambda \cdot 0} = A$$

$$Q_0 = A$$

$$Q(t) = Q_0 e^{-\frac{\gamma}{\epsilon} t} =$$

$$= Q_0 e^{-t/T}$$

$$T = \frac{\epsilon}{\gamma}$$



$$Q_0(t=T) = \frac{Q_0}{e}$$

$$Q_0(t=2T) = \frac{Q_0}{e^2}$$

primer

$$\epsilon = 10^{-10}$$

$$\gamma = 10^{-13}$$

$$\epsilon/\gamma = T = 10^3 \text{ s}$$

$$u(t) = \frac{Q_0}{C} e^{-t/T}$$

$$i = + \frac{Q_0}{T} e^{-t/T}$$

$$\vec{j}(T, t) = \vec{j}(T, 0) e^{-t/T}$$

$$w_e(T) = \int_0^\infty p_t dt = \int_0^\infty \frac{j^2}{\gamma} dt = \frac{j^2(T, 0)}{\gamma} \int_0^\infty e^{-2t/T} dt =$$

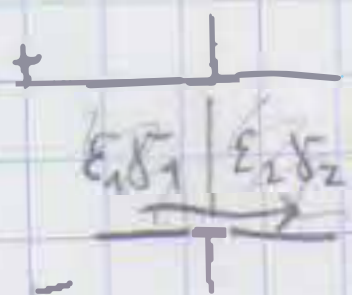
$$= \frac{\epsilon}{2\gamma} \frac{j^2(T, 0)}{\gamma}$$

$$= \frac{\epsilon \cdot E^2(T, 0)}{2} = w_e(T)$$

$$\left[-\frac{T}{2} e^{-2t/T} \right]_0^\infty$$

$T/2$

energija je ostala ele. energija $\rightarrow$ toplotno



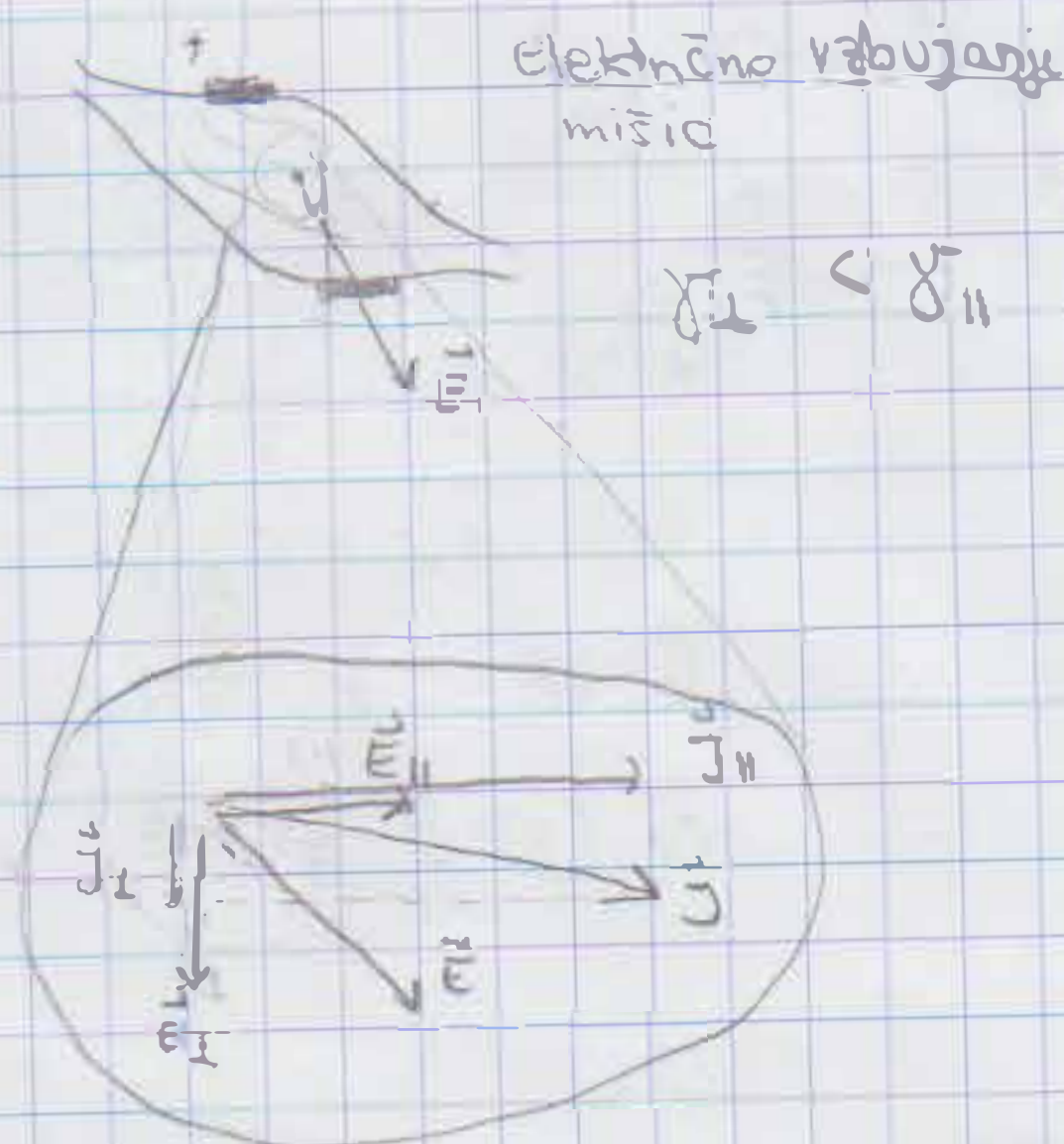
$$\vec{j} = \gamma \vec{E}$$

$$\gamma \approx \frac{ne^2 T}{m}, \quad \rho = \frac{1}{\gamma} \rightarrow \rho(T) = \rho(20^\circ\text{C}) [1 + \alpha(T - 20^\circ\text{C})]$$

anizotropnost

$$\left\{ \vec{j} \times \vec{E}, \vec{j} \parallel \vec{E} \right\}$$

$$\vec{j} \neq \vec{E}$$



|| vzdolž je večja prevodnost kot čez +

Temperaturna odvisnost specifične upornosti

γ ... specifična prevodnost snovi [S/m]

ρ ... specifična upornost snovi [Ωm]

$$\rho = \frac{1}{\gamma}$$



$$\rho(T) = \rho(20^\circ) (1 + \alpha (T - 20^\circ))$$

$\underbrace{293K}_{\Delta T}$

temperaturni koeficient snovi

$$Cu \Rightarrow \alpha_{Cu} = 0,0039/K$$

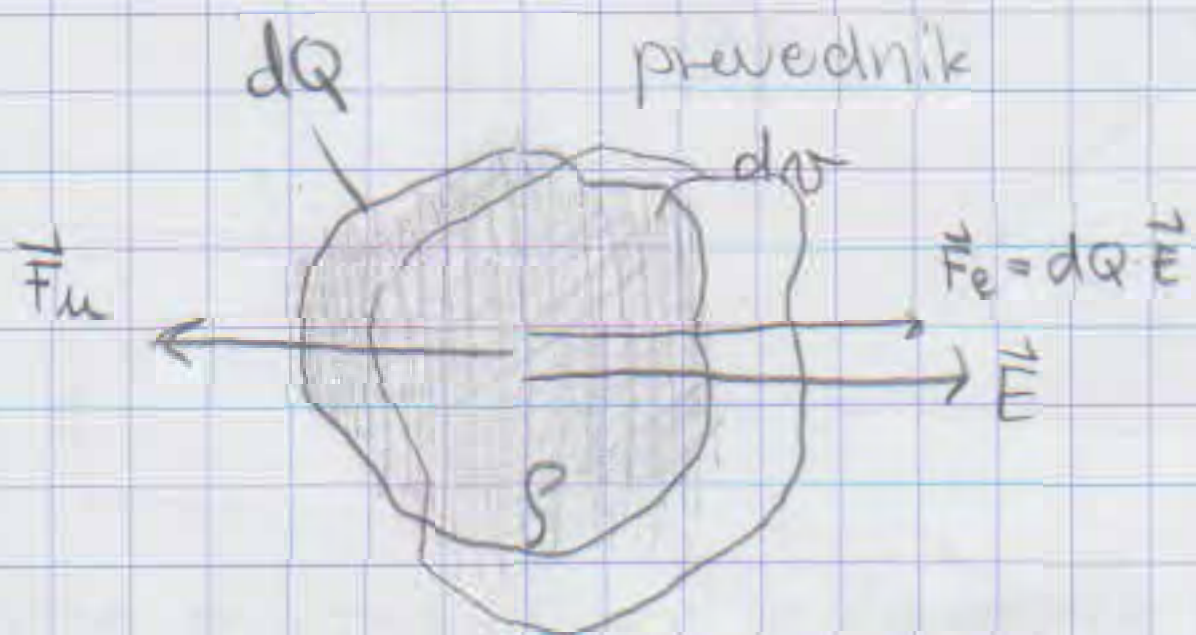
zglede

$$\alpha_{Cu} = 0,004/K$$

$$\rho_{Cu}(80^\circ) = \rho_{Cu}(20^\circ) (1 + 0,004 \cdot (80 - 20)) = \rho_{Cu}(20^\circ) \cdot 1,24$$

$$= \rho_{Cu}(20^\circ) \cdot 124\%$$

Joulov zakon



$\vec{dl}$
premik objekta

$$\vec{w} = \frac{d\vec{l}}{dt} \Rightarrow d\vec{l} = \vec{w} \cdot dt$$

$$dA_e = \vec{F}_e \cdot d\vec{l} \rightarrow \rho \cdot dV$$

$$dA_e = \vec{E} \cdot dQ \cdot d\vec{l} = \rho \cdot \vec{E} \cdot dV \cdot \vec{w} \cdot dt$$

$\vec{j} = \rho \vec{w}$

$$dA_e = \vec{j} \cdot \vec{E} \cdot dV \cdot dt$$

$$dP_e = \frac{dA_e}{dt} = \vec{j} \cdot \vec{E} \cdot dV = p_e \cdot dV$$

Joulov zakon
v diferencialni obliki

$$p_e = \vec{j} \cdot \vec{E} \quad \text{— prostorska gostota moči} \quad [W/m^3]$$

$$p_e = \vec{j} \cdot \vec{E} = \gamma E^2 = \frac{j^2}{\gamma}$$

Alta

64

Zgled

$$J = 4 \text{ MA/mm}^2$$

$$\gamma = 56 \cdot 10^6 \text{ S/m}$$

$$p_e = \frac{J^2}{\gamma} = \frac{16 \cdot 10^{12}}{56 \cdot 10^6} = 280 \text{ k W/m}^3$$

Razširjen Ohmov zakon

$$\vec{J} = \gamma (\vec{E} + \frac{\vec{J}}{c} \times \vec{B} + \vec{E}_g)$$

$$\vec{F}_L = Q(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{\vec{J}}{\gamma} - \frac{\vec{J}}{c} \times \vec{B} = \vec{E} + \vec{E}_g$$

$\downarrow \quad \downarrow$
 $|J| \gg |J|$

$\Rightarrow$

$$\vec{J} = \gamma (\vec{E} + \vec{E}_g)$$

Upoštevamo še generator (E_g)

Enačbe časovno stalnega tokovnega polja $\vec{E}, \vec{D}, \vec{J}$

neprimenljivo: $\oint \vec{E} d\vec{l} = 0$

$$\vec{D} = \epsilon \cdot \vec{E}$$

$$\oint \vec{D} d\vec{a} = + Q_{\text{not}}$$

$$\vec{J} = \gamma \vec{E}$$

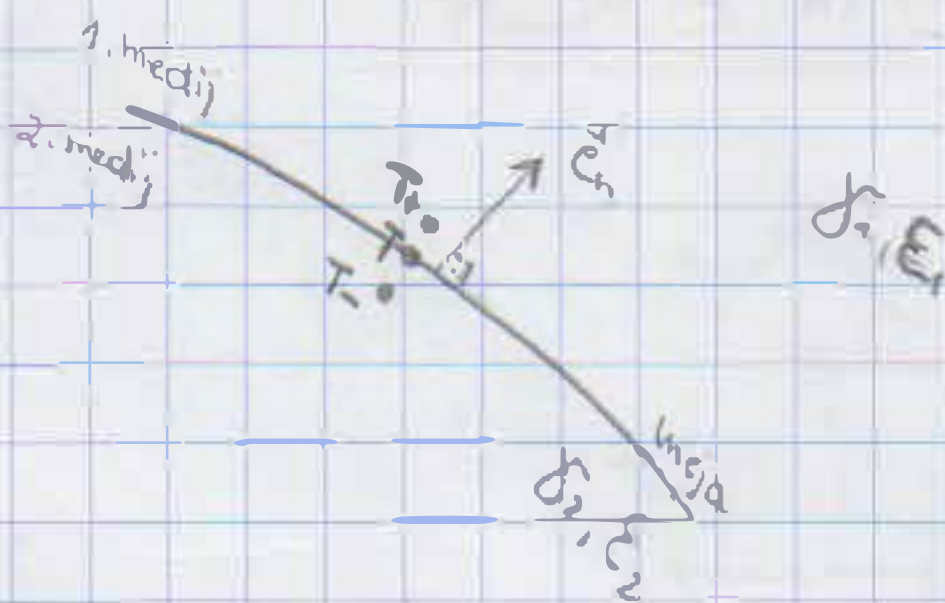
$$\oint \vec{J} d\vec{a} = - \frac{dQ_{\text{not}}}{dt}$$

$$= 0$$

$$E_t(T_+) = E_t(T_-)$$

$$D_n(T_+) - D_n(T_-) = \sigma_{\text{povr}}(T)$$

$$J_n(T_+) - J_n(T_-) = 0 \Rightarrow J_n(T_+) = J_n(T_-) \text{ (normalna komponenta je zara)}.$$

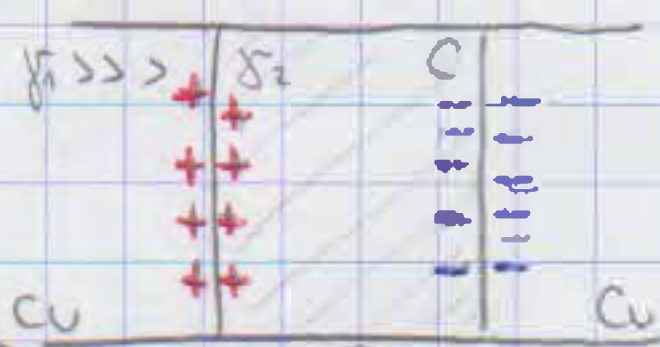


zglede 1: Prehajanje polja med prevodniki

$$\frac{\epsilon_1}{\gamma_1} J_n(T_+) - \frac{\epsilon_2}{\gamma_2} J_n(T_-) = G_{\text{prosti}}(T) \leftarrow (D_n(T_+) - D_n(T_-) = G_{\text{prosti}}(T))$$

$$\left(\frac{\epsilon_1}{\gamma_1} - \frac{\epsilon_2}{\gamma_2} \right) J_n(T) = G_{\text{prosti}}(T)$$

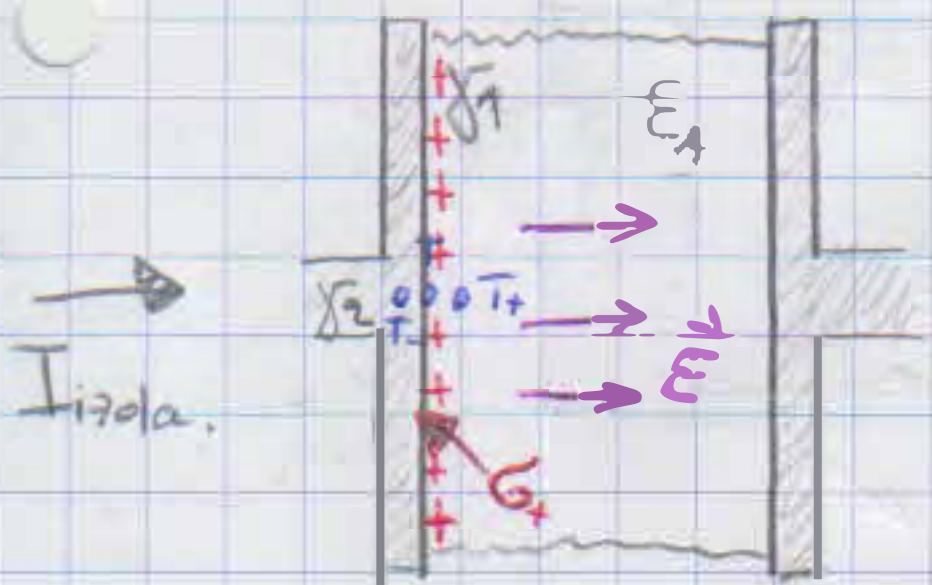
zglede 1:



večja kot je tokovna gostota (J_n)
večji je presežek (G_{prosti})

zglede 2: Realni dielektrik v kondenzatorju

zanemarljiv ≈ 0



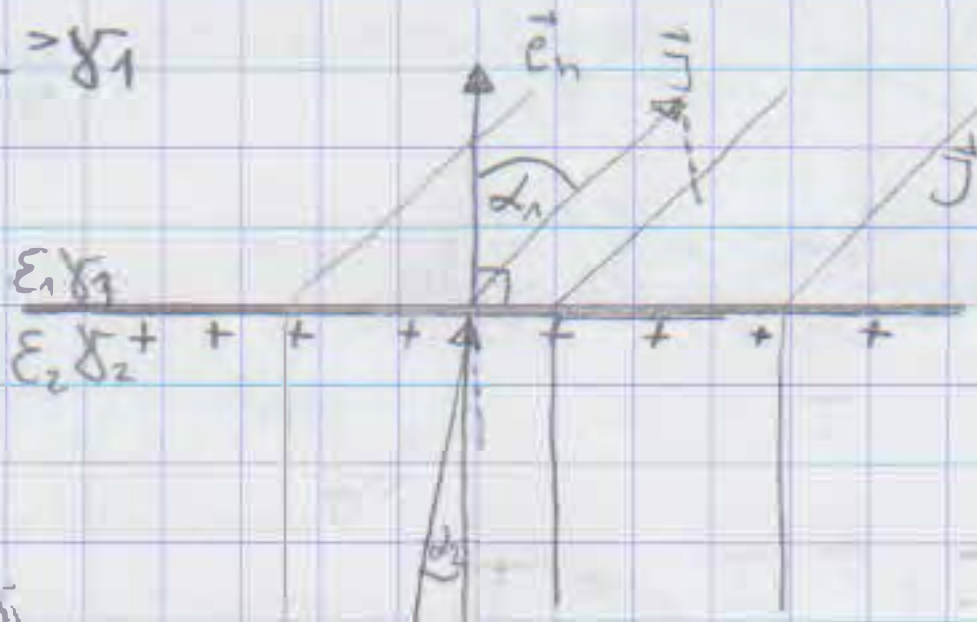
$$\left(\frac{\epsilon_1}{\gamma_1} - \frac{\epsilon_2}{\gamma_2} \right) J_n = G_{\text{prosti}}$$

$$\epsilon_1 \cdot E_n = G_{\text{prosti}}$$

$$E_n(T_+) = \frac{G_{\text{prosti}}}{\epsilon_1}$$

Lomni zakon

$\gamma_2 > \gamma_1$



normalni
komponenti sta
enaki

$$E_t(T_+) = E_t(T_-)$$

$$J_n(T_+) = J_n(T_-)$$

$$\gamma_1 E_n(T_+) = \gamma_2 E_n(T_-)$$

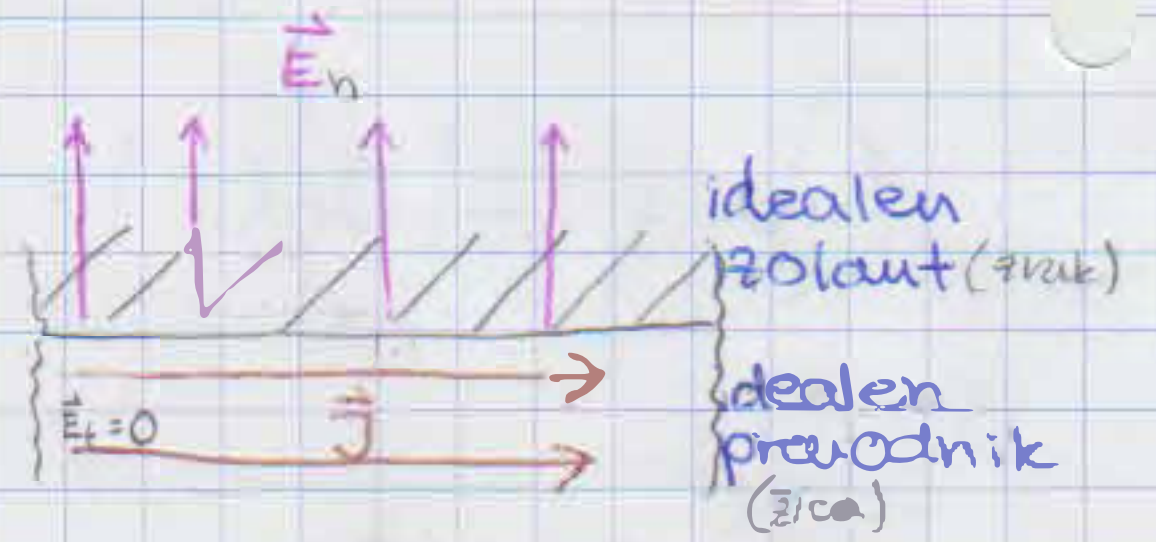
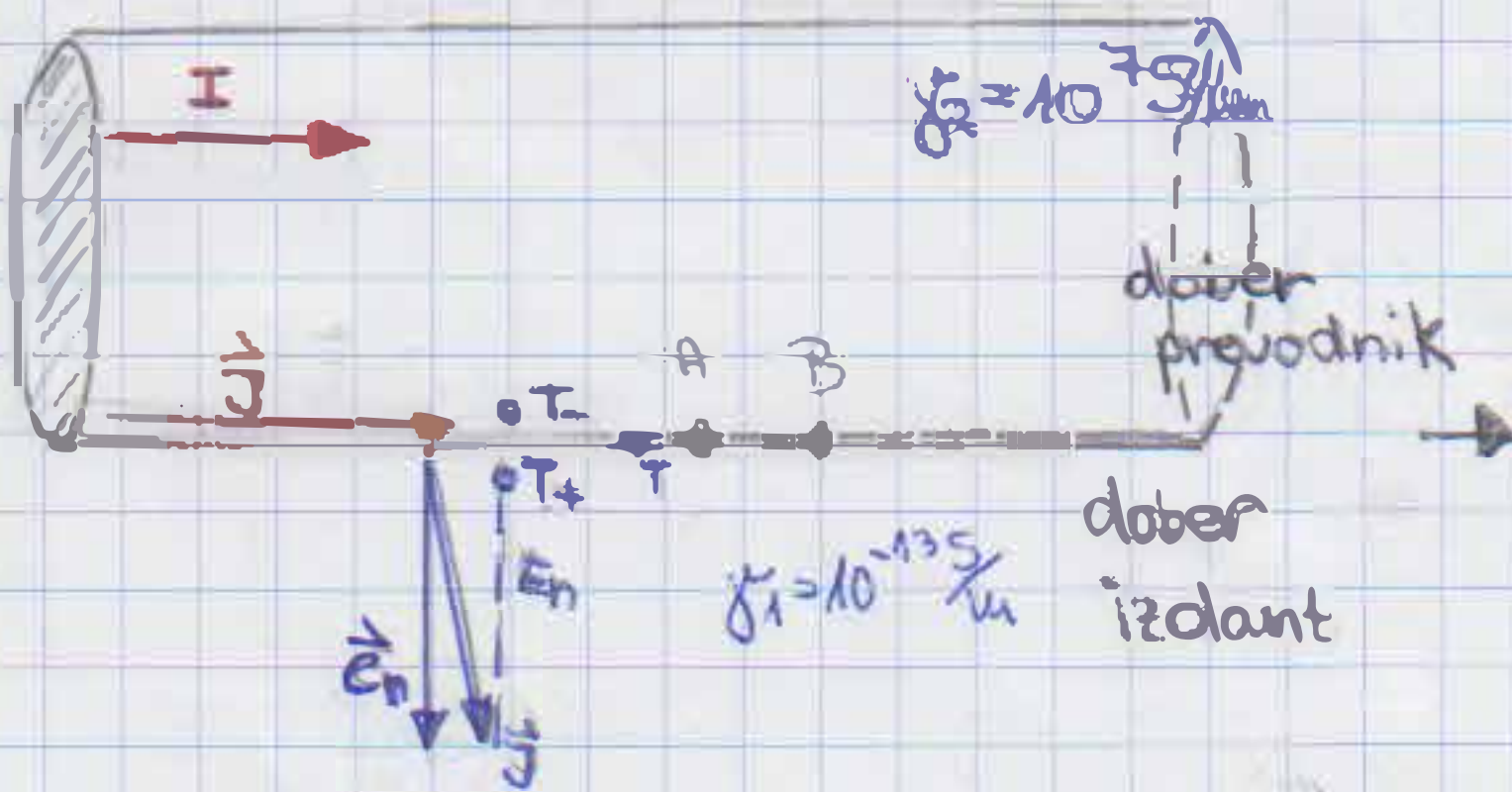
$$\frac{E_t(T_+)}{\gamma_1 E_n(T_+)} = \frac{E_t(T_-)}{\gamma_2 E_n(T_-)}$$

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}$$

gostotnice toka se
prelomijo pri prehodu v
manj prevodno snov

Meja prevodnik in izolant

žica / zrbk



meje pogoje upoštevaj

$$E_n(T+) = 10^6 \text{ V/m}$$

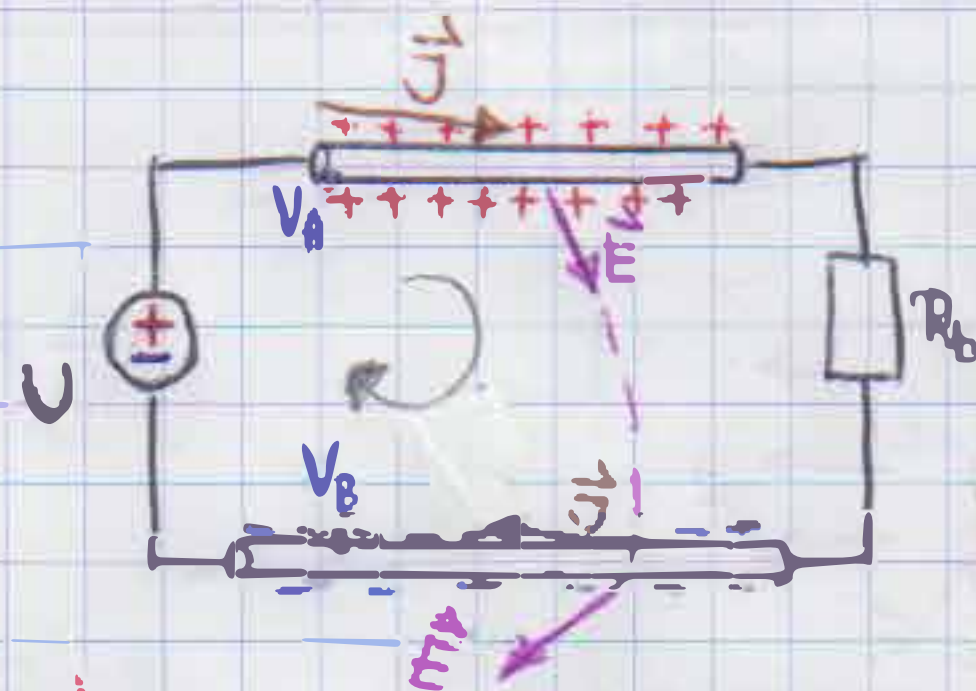
$$J_t(T-) = \frac{10^6 \text{ A/m}}{\gamma_2} \rightarrow J_n = 10^{-6} \text{ A/m}^2$$

$$E_t(T-) = 0,1 \text{ V/m}$$

! imamo padec napetosti ker ni več superprevodnik

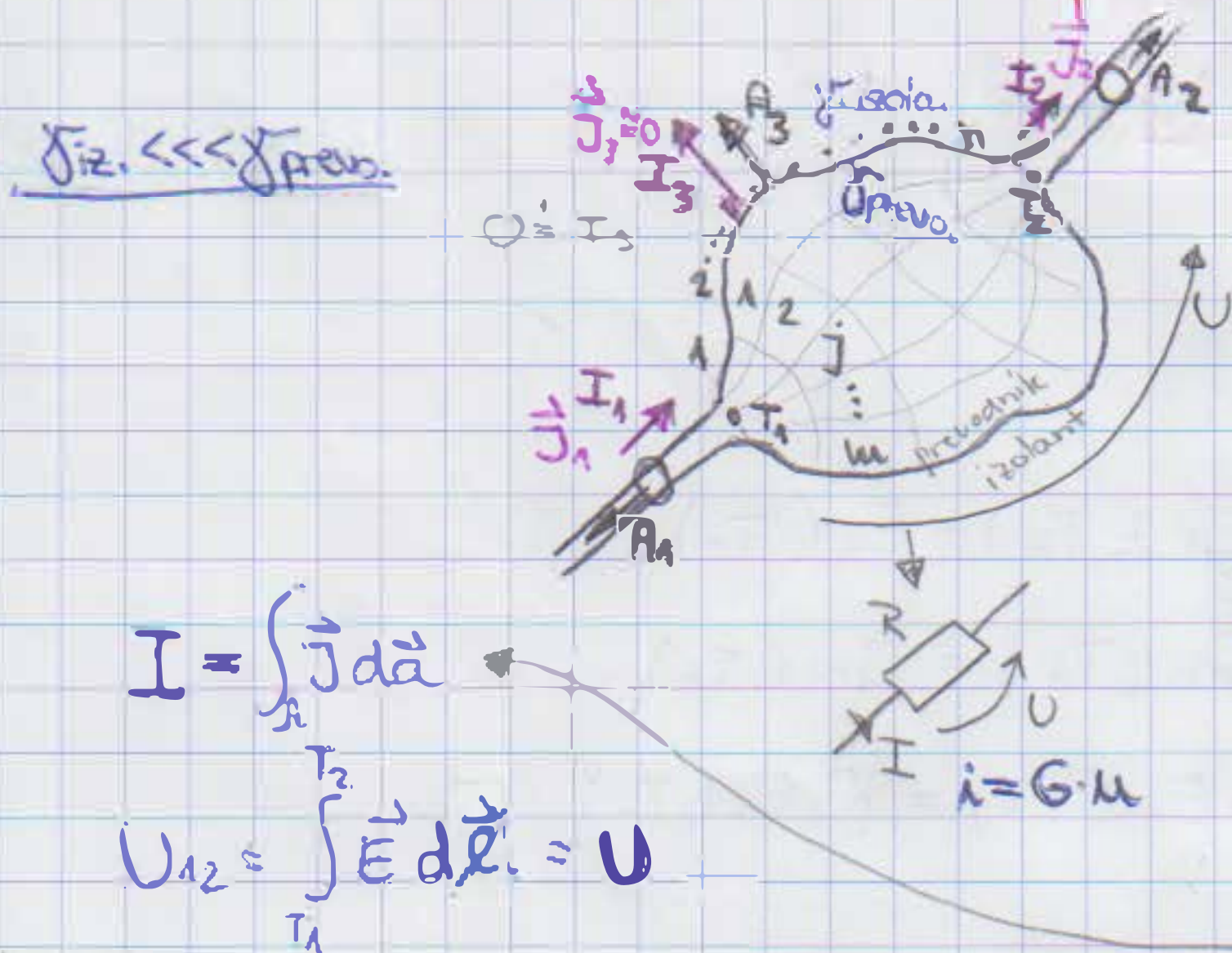
$$U_{AB} = V_A - V_B = \int_A^B \vec{E}_t d\vec{l} \approx 0,1 \frac{\text{V}}{\text{m}} \cdot \overline{AB}$$

$V_A > V_B$



Silnice prevodnika so rahlo nagibene na smer toka v žici

Električna Upornost / prevodnost, upor



$A = A_1 \cup A_2 \cup A_3$ sklenjena ploskev

$$\oint_A \vec{j} d\vec{a} = 0$$

skalarni produkt γ negativen zato $-I_1$

$$\int_{A_1} \vec{j} d\vec{a} + \int_{A_2} \vec{j} d\vec{a} + \int_{A_3} \vec{j} d\vec{a} = 0$$

$$-I_1 + I_2 + I_3 = 0$$

$I_1 \approx I_2 = I$

Če imamo na vnosu 1A γ na izhodno manj odvisno od pogojev

ker velja:

$$\vec{j} = \gamma \vec{E}$$

odvisnost

ker γ

$$I \propto U \rightarrow I = k \cdot U$$

$$G = \frac{1}{R}$$

$$U = R \cdot I \quad [\Omega]$$

Ohmov zakon v integralni obliki

upornost strukture

Ohmov zakon v celotni strukturi

Moč v konduktivnem kanalu

$$p = \vec{j} \cdot \vec{E}$$

$$P = \sum_{j=1}^m \sum_{k=1}^n \vec{j} \cdot \vec{E} d\vec{r}_{jk}$$

$$\underbrace{d \cdot U_j}_{\vec{E} d\vec{l}_j} \underbrace{dI_k}_{(\vec{j} \cdot d\vec{a}_k)} \Rightarrow (\vec{E} d\vec{l}_j)(\vec{j} \cdot d\vec{a}_k)$$

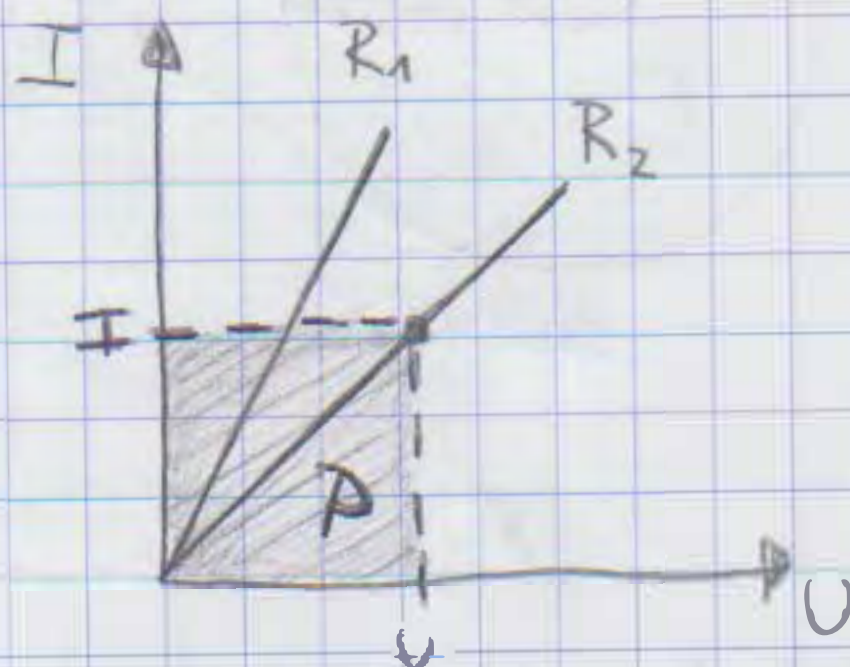
$$P = \sum_{j=1}^m \sum_{k=1}^n (\vec{j} \cdot \vec{E})(d\vec{l}_j \cdot d\vec{a}_k)$$

$$P = \sum_{j=1}^m dU \cdot \sum_{k=1}^n dI = U \cdot I$$

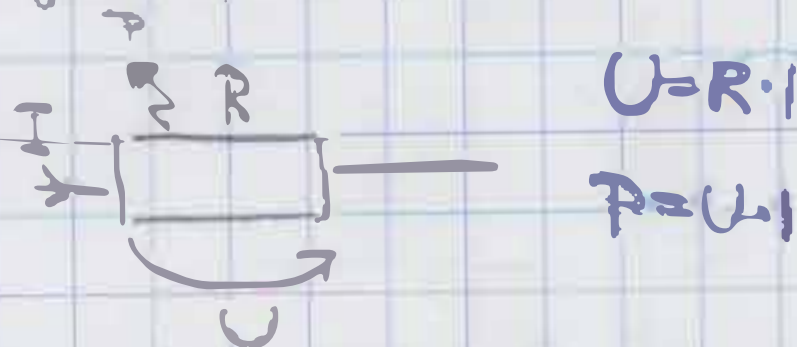
$$P = U \cdot I$$

Jeulov zakon
v integralni obliki

$$P = G I^2 = \frac{U^2}{R}$$



Strujen element upor.



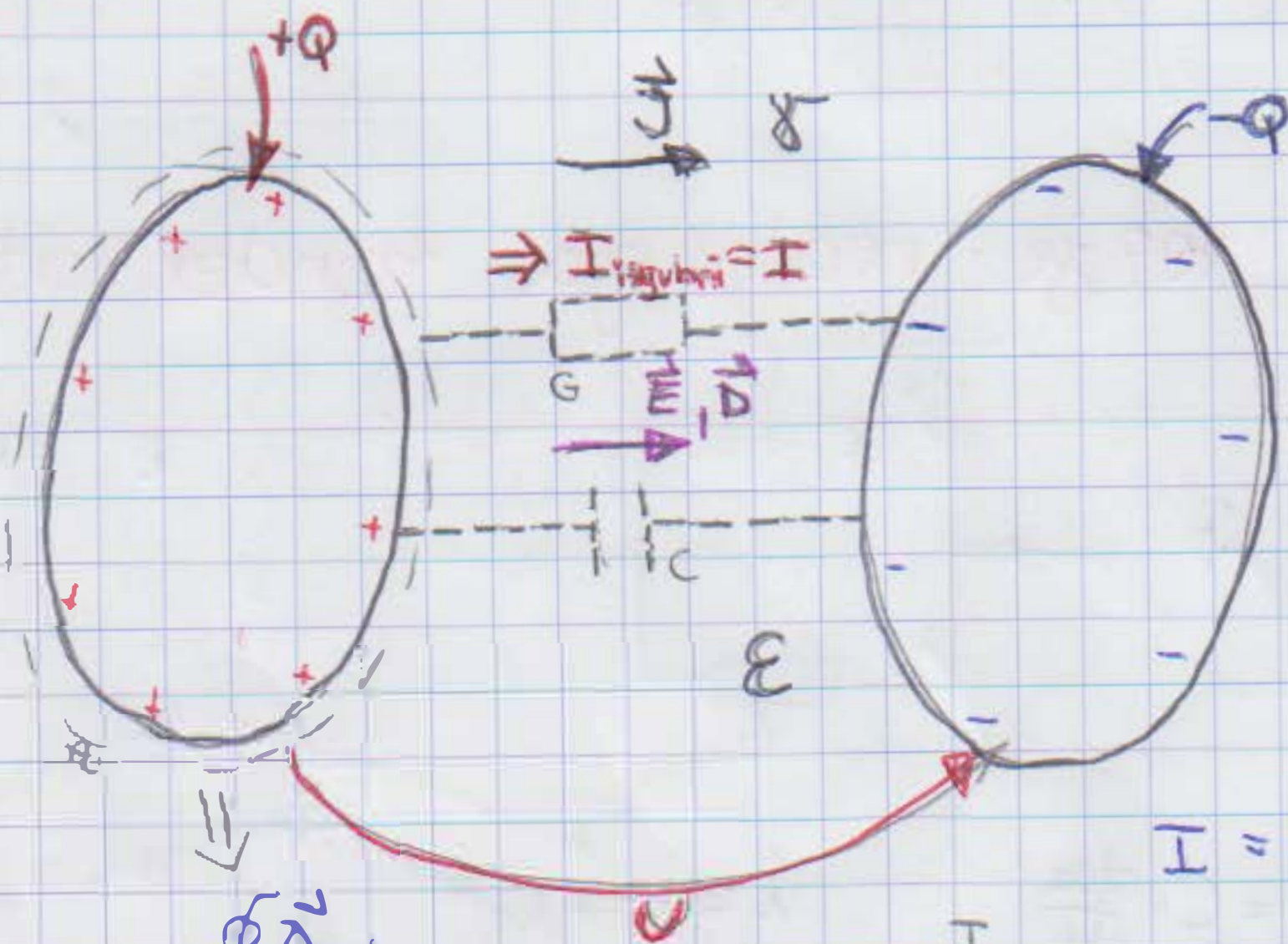
v časovno spremenljivih razmerah

$$U = R \cdot i$$

$$P(t) = u \cdot i = G u^2 = R i^2$$

Dvojnost (dualnost) električnega in tokovnega polja

in ploščev
in sklenjenca



$$\vec{D} = \epsilon \cdot \vec{E}$$

$$\vec{B} = \mu \cdot \vec{H}$$

$$Q = C \cdot U$$

$$I = G \cdot U$$

$$\oint_{\partial} \vec{D} \cdot d\vec{a} = +Q$$

$$\oint_{\partial} \vec{j} \cdot d\vec{a} = I$$

$$I = \oint_{\partial} \vec{j} \cdot d\vec{a} = \frac{\sigma}{\epsilon} \oint_{\partial} \vec{D} \cdot d\vec{a} =$$

$$\frac{I}{G U} = \frac{\sigma}{\epsilon} Q$$

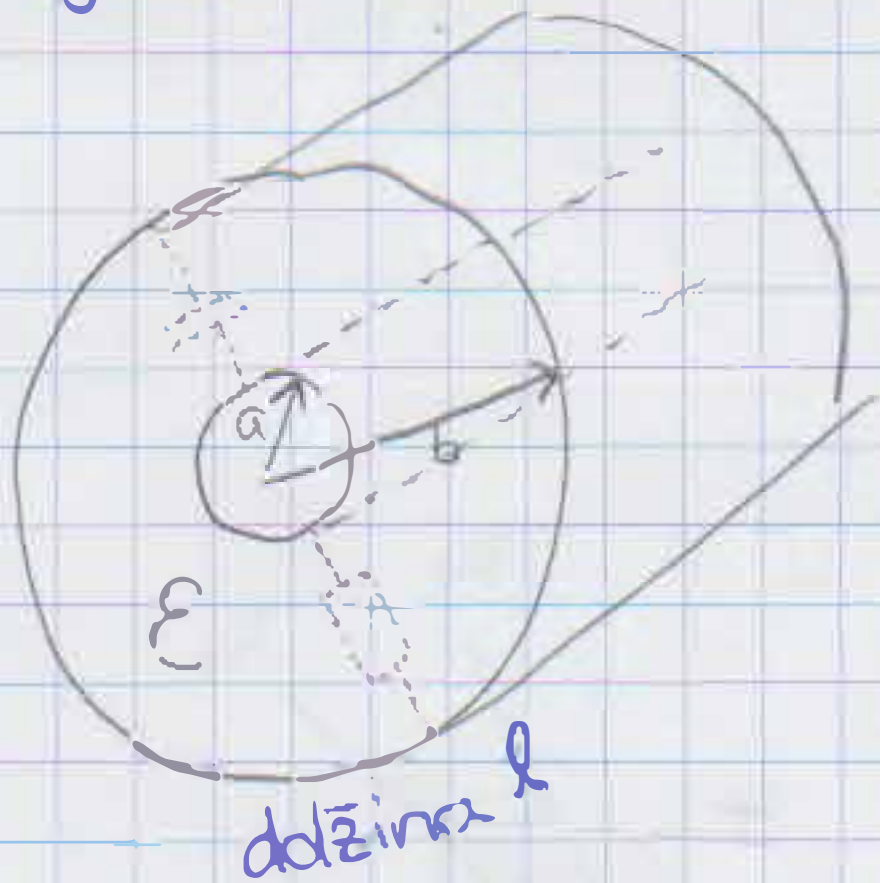
$$G \cdot U = \frac{\sigma}{\epsilon} \cdot C \cdot U$$

habeto je korelaciji z napetostjo preko C

$$\frac{G}{C} = \frac{\sigma}{\epsilon}$$

Dualnost

Zgled: koax kabel

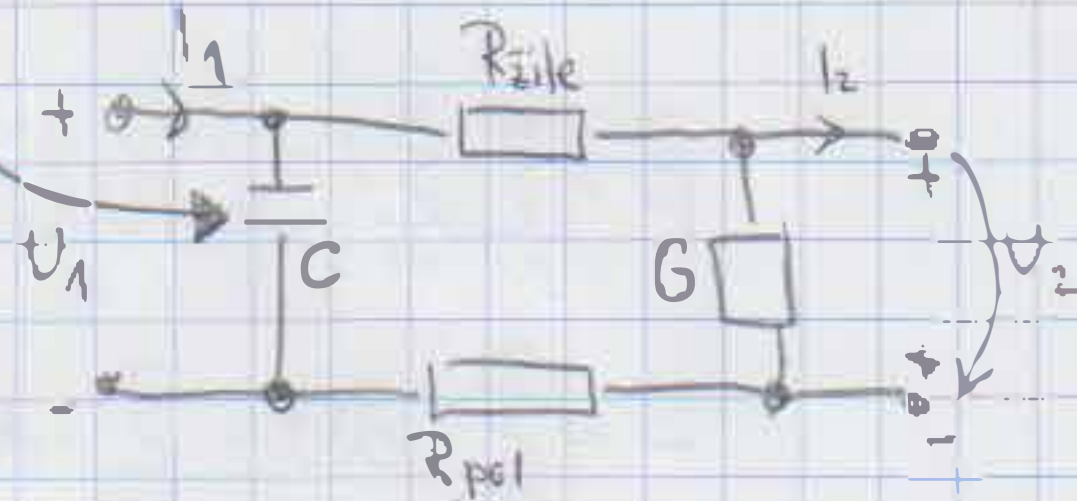


$$C = \frac{2\pi\epsilon l}{\ln \frac{b}{a}}$$

$$G_{\text{izolacija}} = \frac{2\pi\gamma l}{\ln \frac{b}{a}}$$

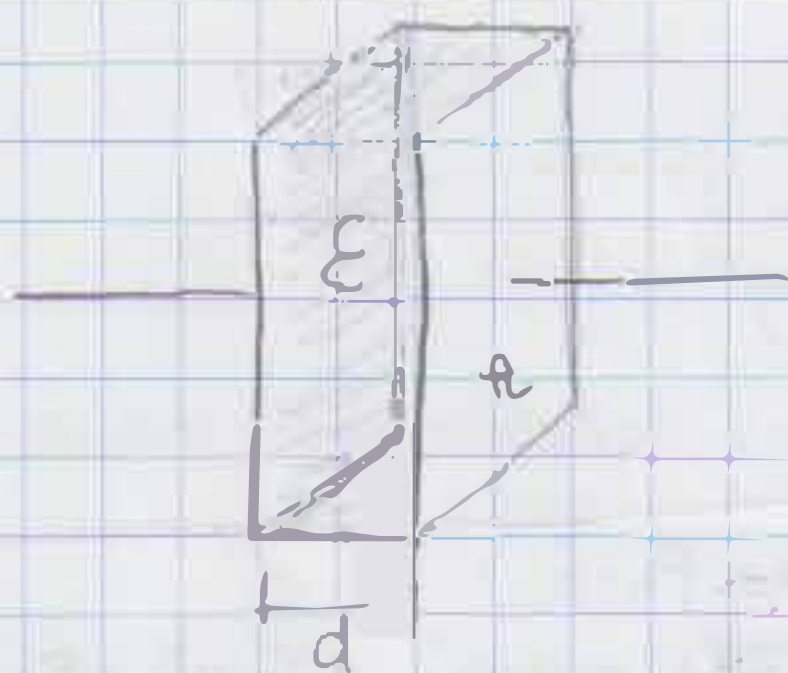
$$R_{\text{izolacija}} = \frac{1}{G_{\text{izolacija}}}$$

splosnost sistema
sprejeti energijo (C)



$$\frac{G}{C} \propto \frac{\gamma}{\epsilon}$$

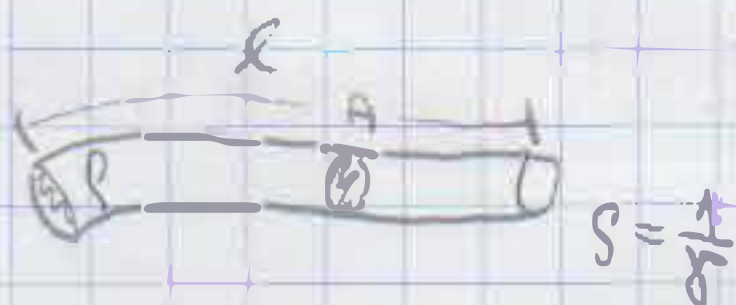
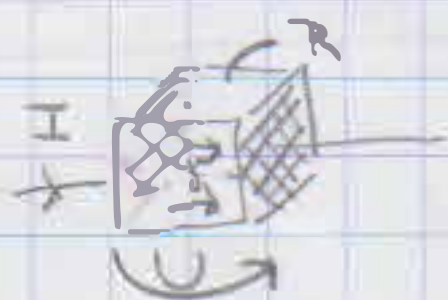
Zgled: ploščni kondenzator



$$C = \frac{\epsilon \cdot A}{d}$$

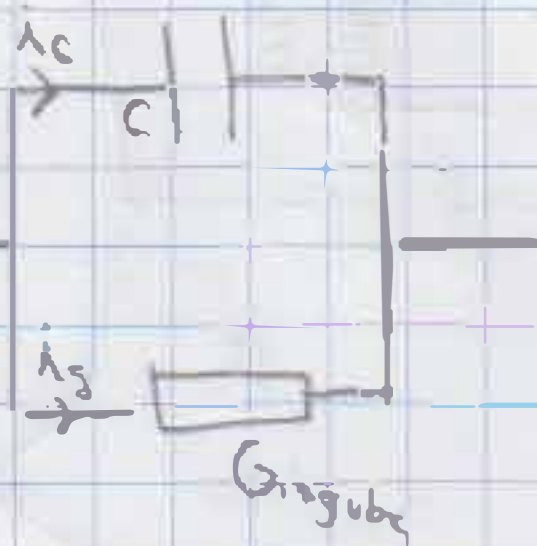
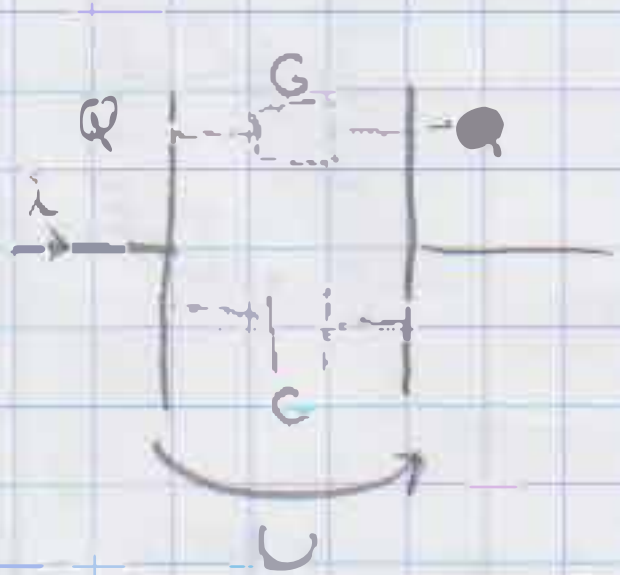
$$G = \frac{\gamma A}{d}$$

$$R = \frac{d}{\gamma A}$$



$$R = \frac{\rho \cdot l}{A}$$

Nadomestno vezje realnega kondenzatorja



$$i = \frac{dQ}{dt} = C \frac{du}{dt}$$

$$i_g = G \cdot u$$

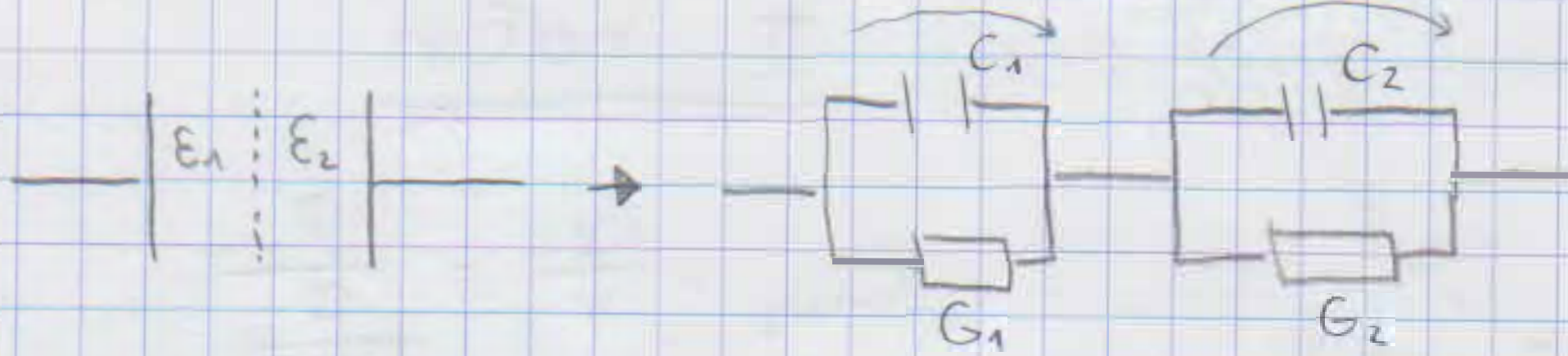
$$i = i_c + i_g$$

$$i = C \cdot u' + G \cdot u$$

i_c

parazitna lastnost

tolerancijski polni vezje
izolacijski teh. izgubi

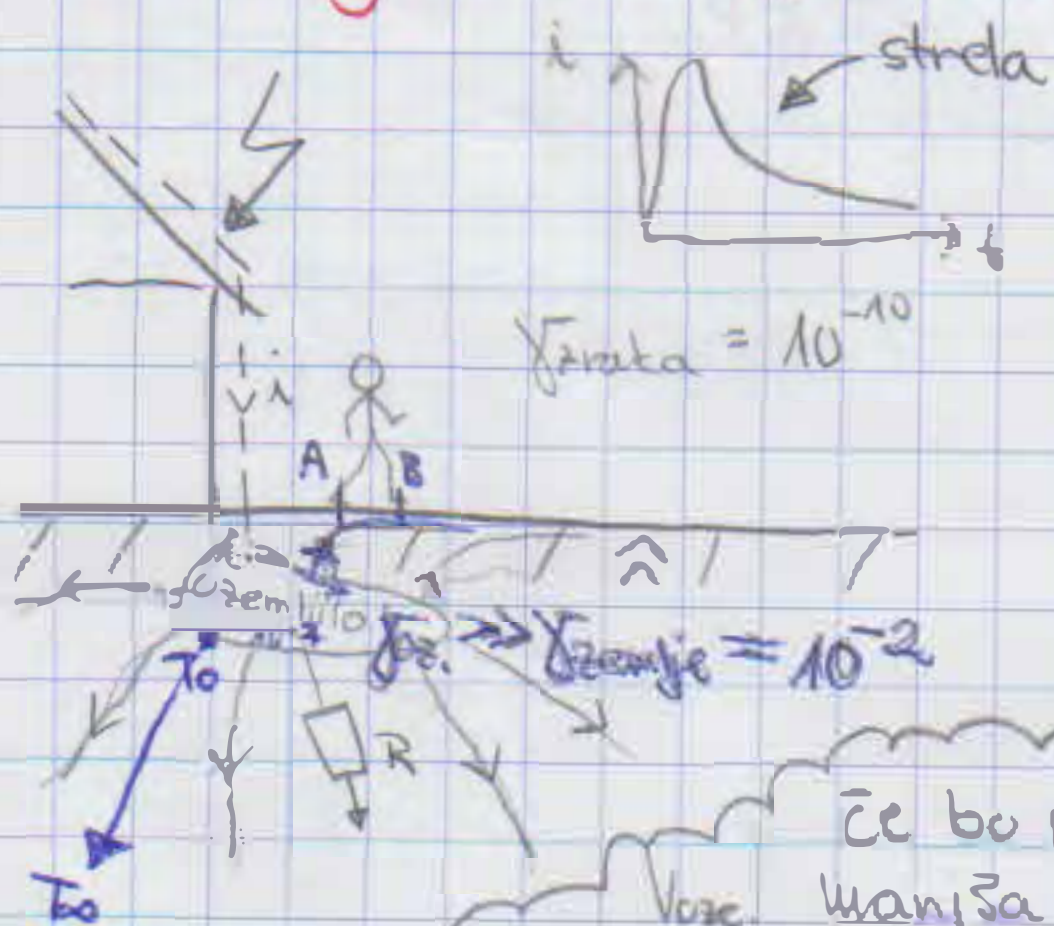


$$\frac{U_1}{U_2} = \frac{C_2}{C_1}$$

→

$$\frac{U_1}{U_2} = \frac{R_1}{R_2}$$

Ozemljitvena (ponikalna) upornost



γ_{zrak} = 10^{-10}

γ_{zemlje} = 10^{-2}

Če bi peak vrednost
manjša bi tudi nap.
koraka manjša. To dosežemo z manjšanjem
upornosti

žgled: polkrožno ozemljilo

$$V_{ozemlj.} = \int_{T_0}^{T_{\infty}} \vec{E} \cdot d\vec{l}$$

žemlja v neposredni
bližini ozemljila
ni ϕ v poslednji
bližini pa je

napetost koraka

$$U_2 = \int_A^B \vec{E} \cdot d\vec{l}$$

$$R_{oz} = \frac{V_{oz}}{I}$$

I. način

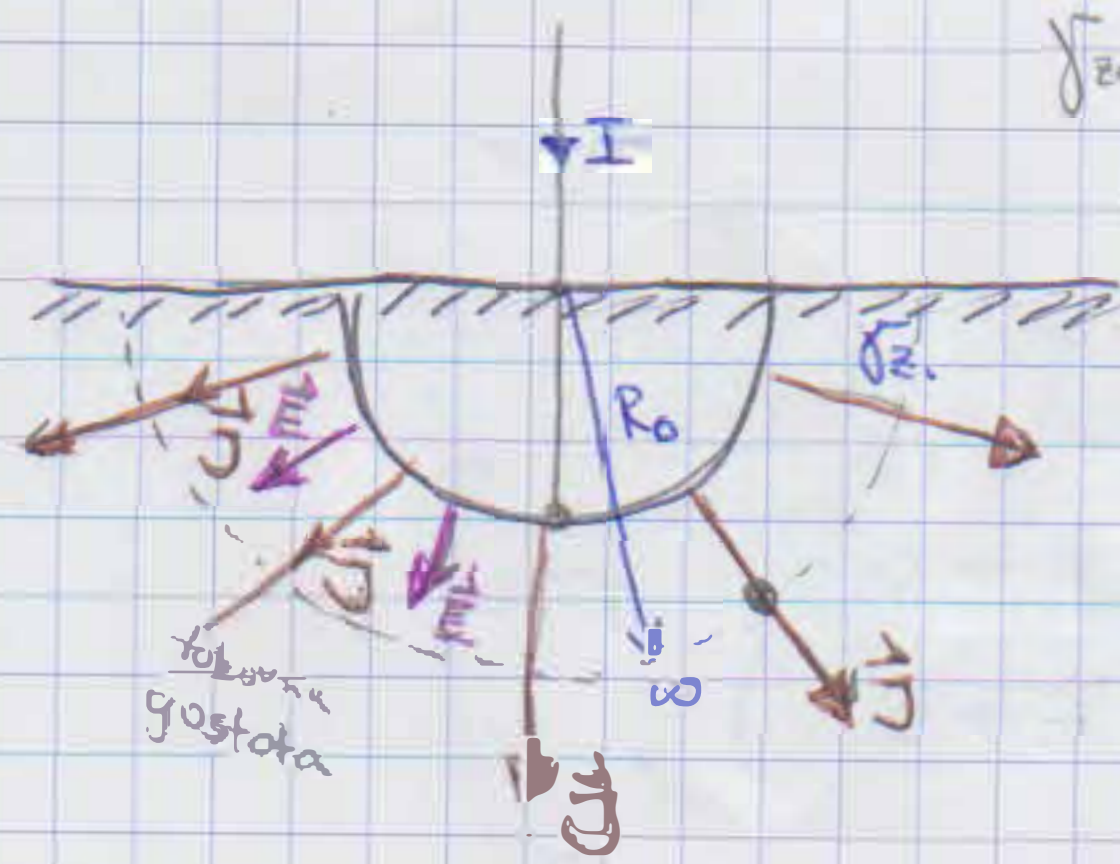
$$J = \frac{I}{2\pi r^2}$$

$$E = \frac{J}{\gamma_z} = \frac{I}{2\pi \gamma_z r^2}$$

$$U = \int_{R_0}^{\infty} E dr = \frac{I}{2\pi \gamma_z} \int_{R_0}^{\infty} \frac{dr}{r^2} =$$

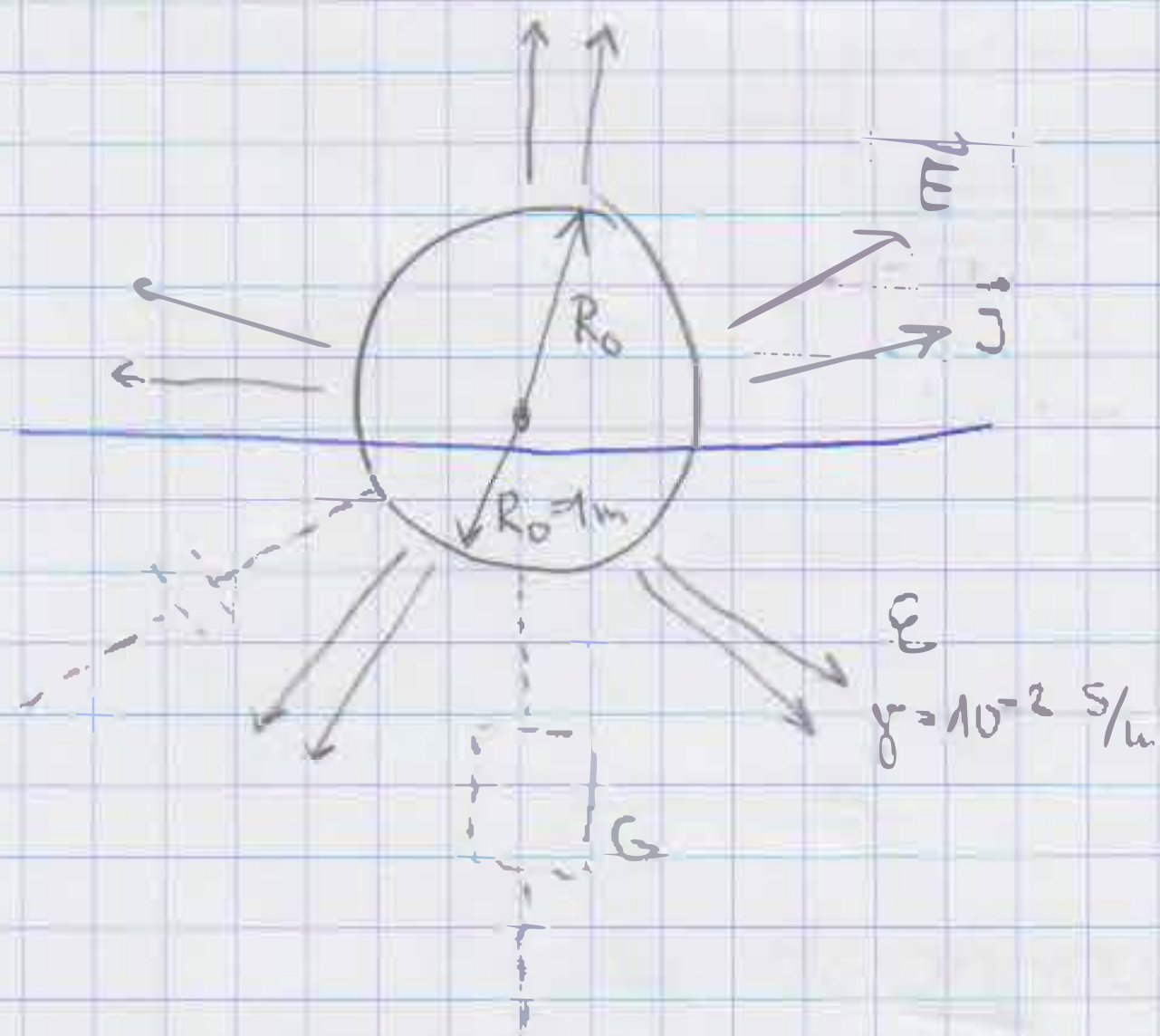
$$V_{oz} = U = \frac{I}{2\pi \gamma_z R_0}$$

$$R_{oz} = \frac{U}{I} = \frac{1}{2\pi \gamma_z R_0}$$



γ_{zemlje} = 10^{-2} S/m

tokovna
gostota



II. način

$$C_{\text{krog.}} = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\epsilon_0 R_0}}$$

pravilo dualnosti

$$C_{\text{krog}} = 4\pi\epsilon R_0 = 4\pi\gamma R_0$$

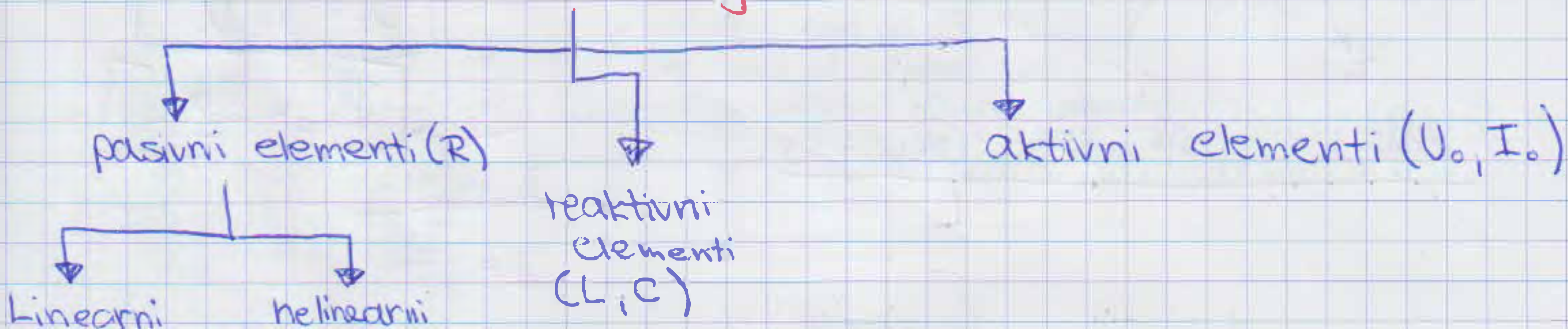
$$C_{\text{pokr.}} = 2\pi\gamma R_0$$

$$R = \frac{1}{2\pi\gamma R_0}$$

$$R = \frac{10^2}{2\pi \cdot 1\text{m}} = 16 \Omega$$

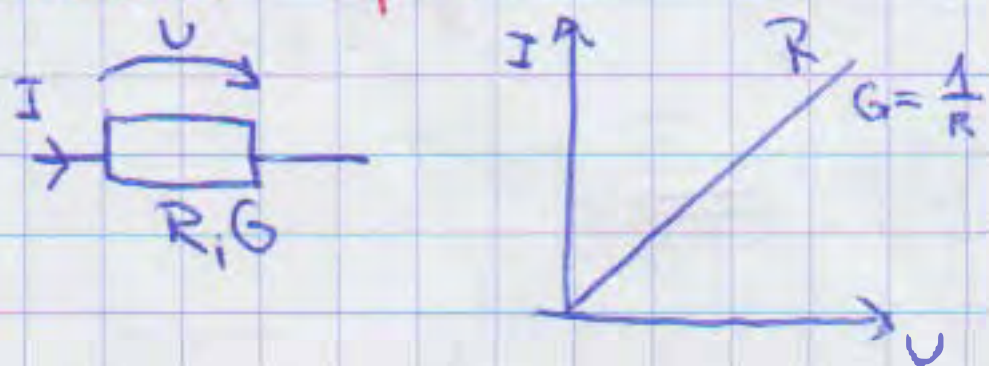
R-vezja

Elementi električnih vezij



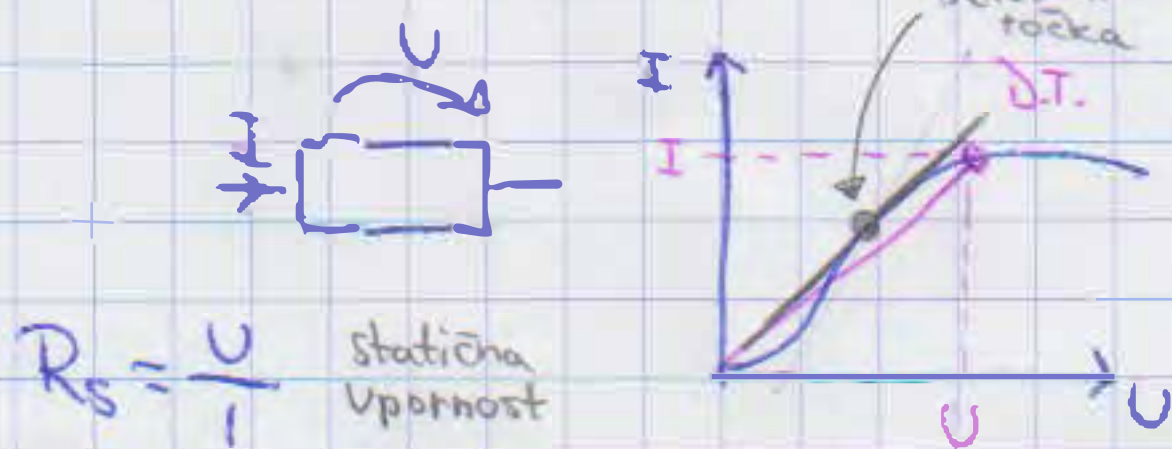
Pasivni elementi

Linearni: upor



$$\left. \begin{array}{l} G \rightarrow \infty ; R \rightarrow 0 \\ R \rightarrow \infty ; G \rightarrow 0 \end{array} \right\} G^{-1} = R$$

Nelinearni elementi



$$R_s = \frac{U}{I} \text{ statična upornost}$$

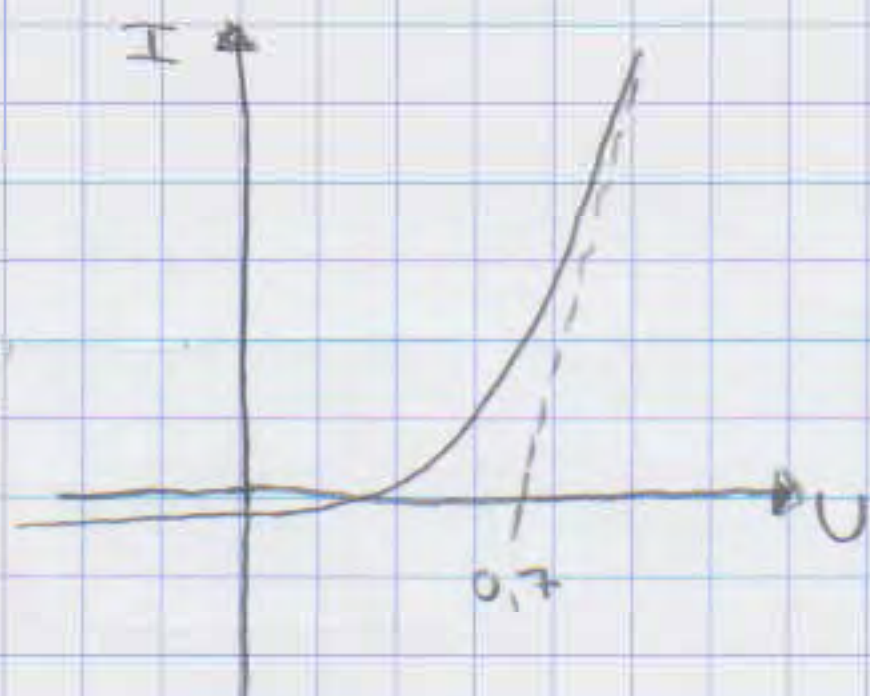
$$I = f(U)$$

$$\text{D.T.} \quad \frac{I'}{U'}$$

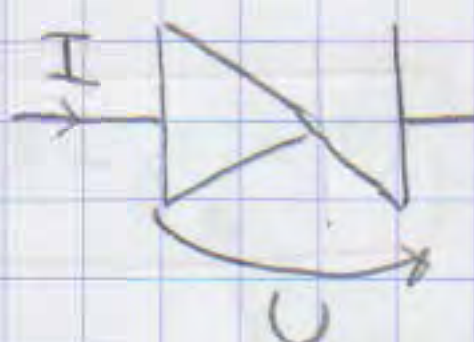
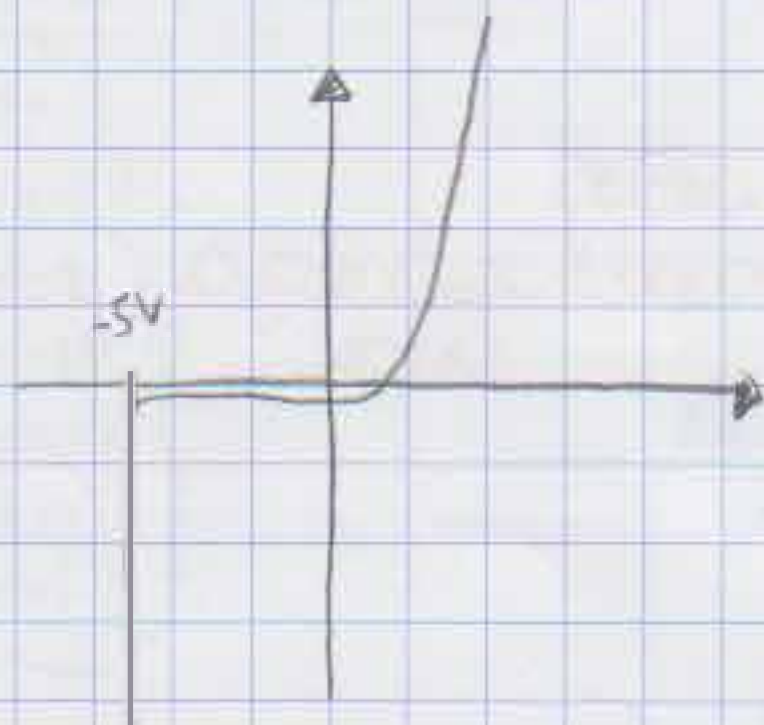
$$R_D = \frac{dU}{dI}$$

* fluorescentna luč *

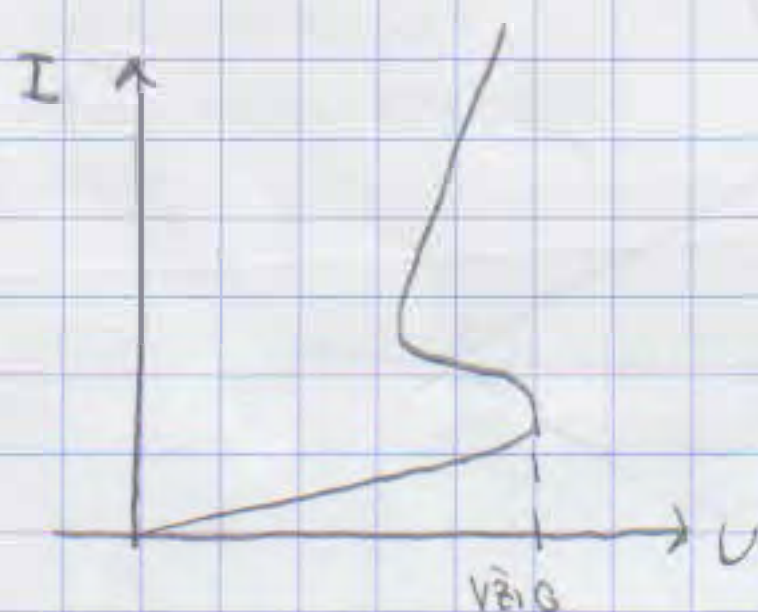
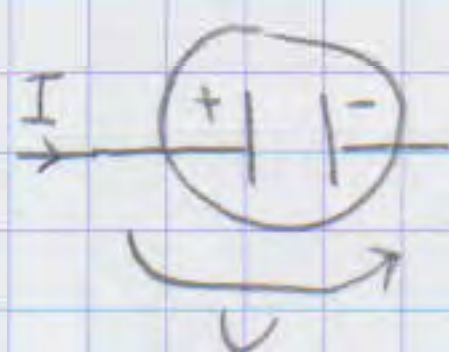
↳ dioda



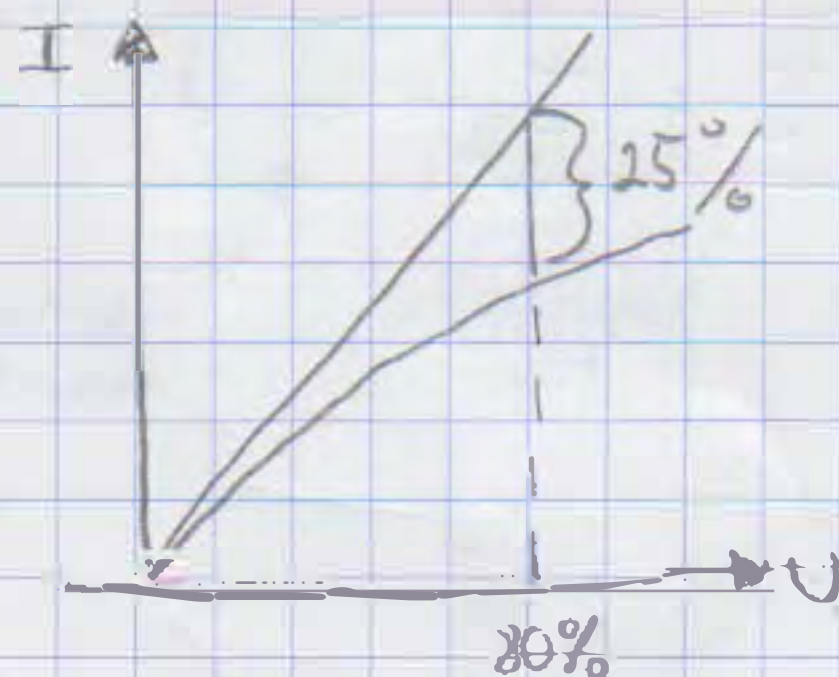
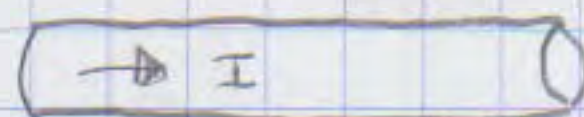
↳ zener dioda



↳ tljivke

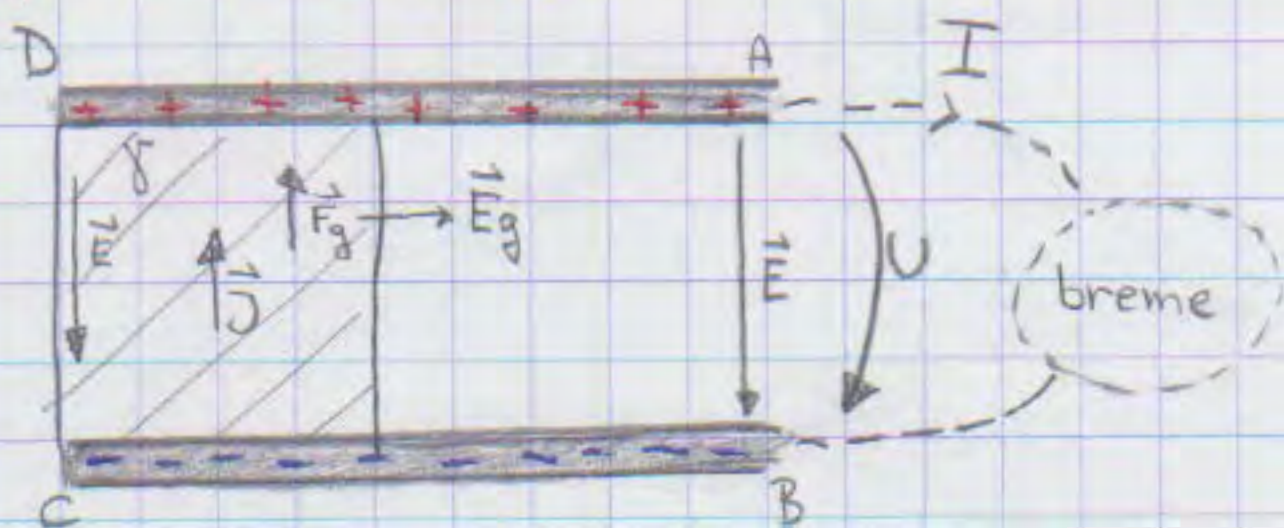


↳ žica



Aktivni elementi

Napetostni vir



$$\oint \vec{E} d\vec{l} = 0$$

notranja upornost vira

$$U + R_g \cdot I$$

upornost medija in priključkov

$$U = U_g - R_g \cdot I$$

enaka napetostnega vira

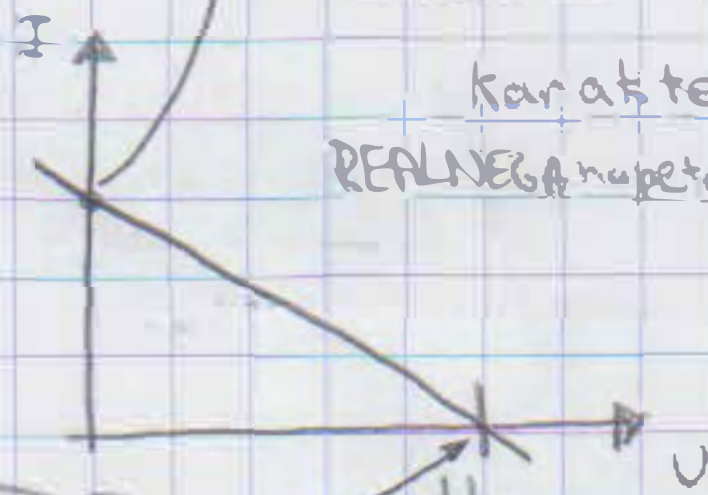
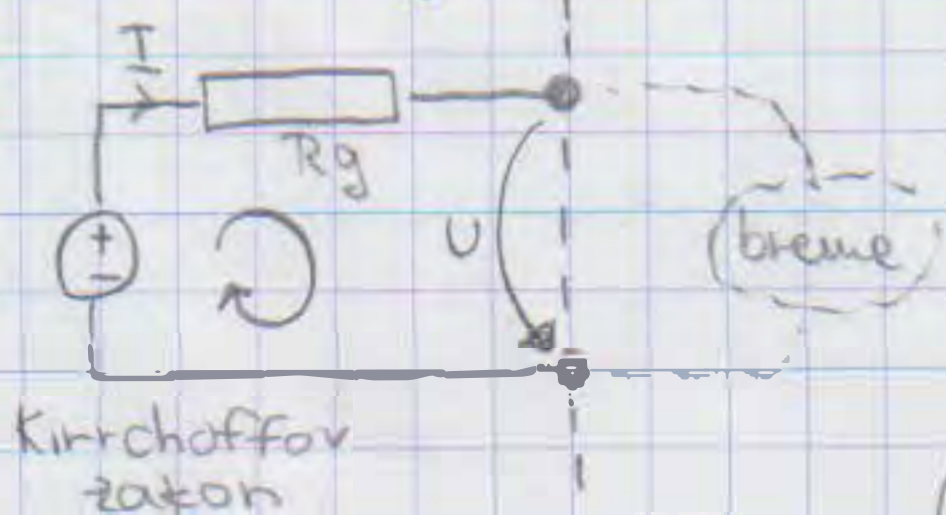
napetost odprtih sponk

upornost vira

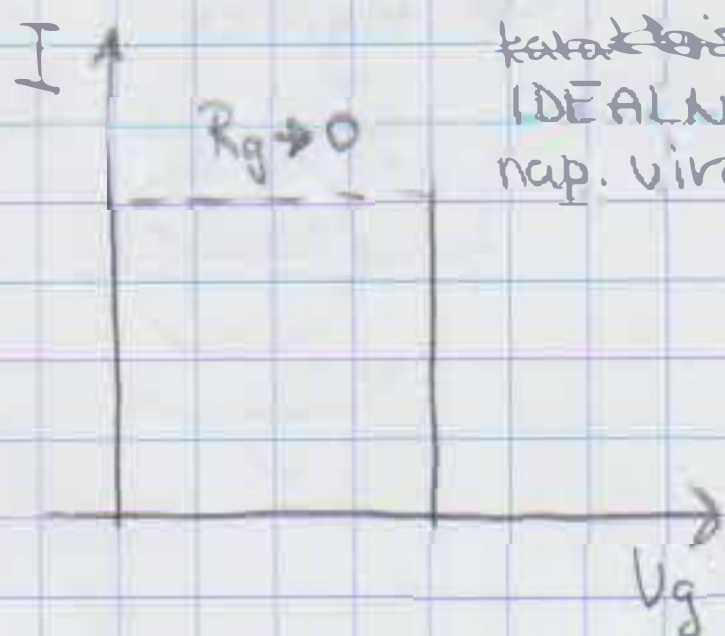
realna napetost nam pove koliko dobimo U na sponkah brez napetosti, ki jo izgubimo v notranjosti vira

tok kratkega stika

Modelno vezje N.V.



Karakteristika REALNEGA napetostnega vira



Karakteristika IDEALNEGA nap. vira

točka odprtih sponk

Realni Tokovni vir

$$U = U_g - R_p \cdot I$$

$$I = \frac{U_g}{R_p} - \frac{U}{R_g}$$

I_g

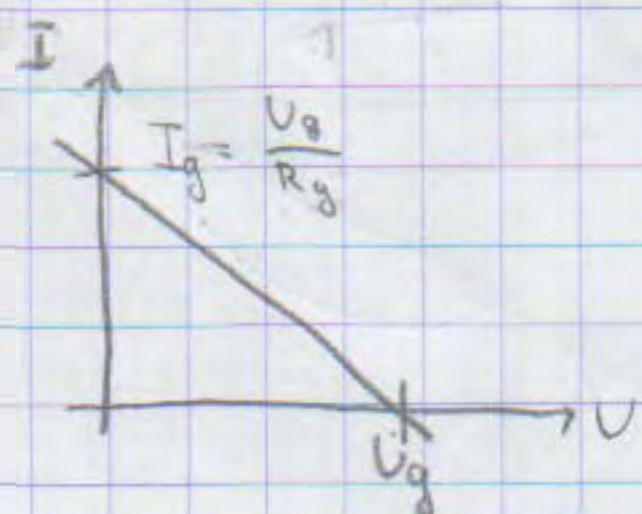
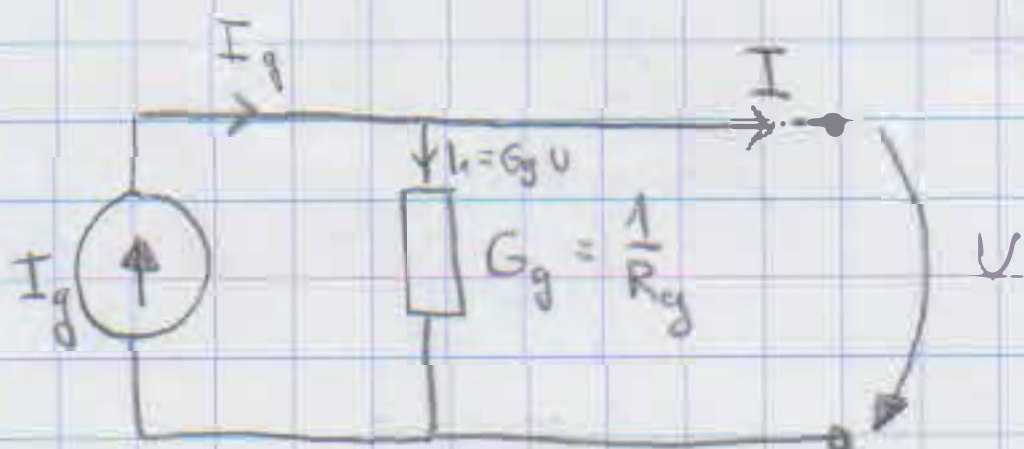
$$I = I_g - G_p \cdot U$$

enaka tokovnega vira

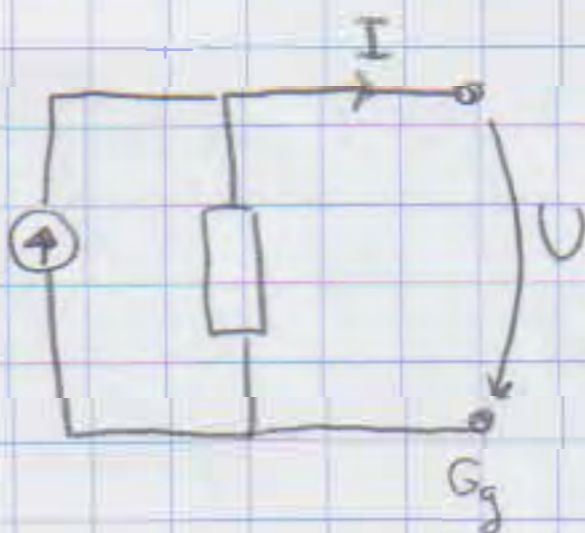
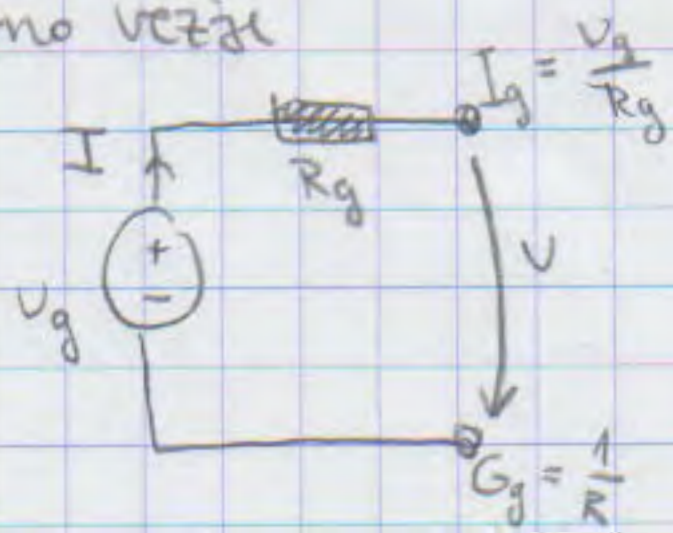
kratkostični tok

notranja prevodnost vira

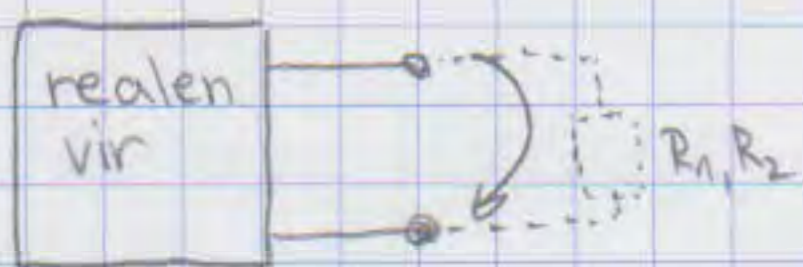
tokovni vir



Ekvivalentno vezje



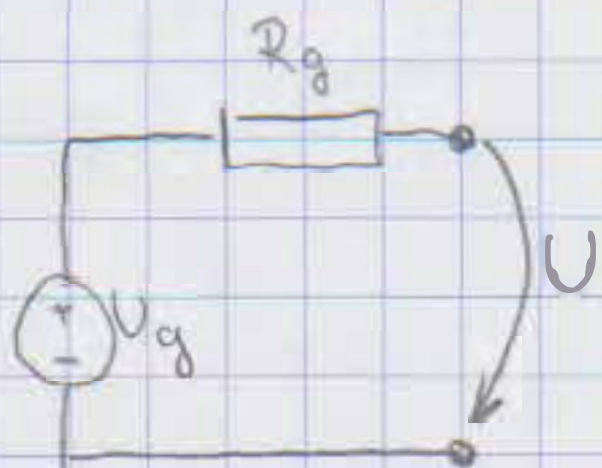
Določanje parametrov vira



- izmerimo napetost odprtih sponk

- ne merimo $I_g = I_o = I_k$

- izmerimo dve delovni točki vira



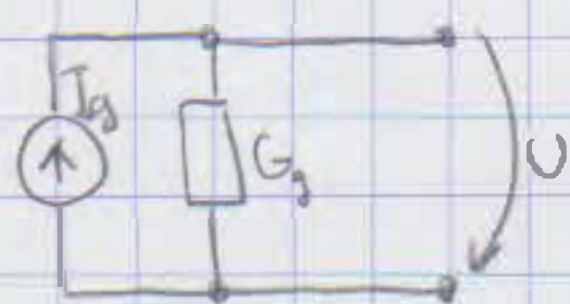
$$U_g = I_g R_g$$

$$U = U_g - I R_g$$

$$\left. \begin{aligned} U_1 &= U_g - I_1 R_g \\ U_2 &= U_g - I_2 R_g \end{aligned} \right\} R_g = \frac{U_2 - U_1}{I_1 - I_2}$$

$$U_g = U_1 + \frac{U_2 - U_1}{I_1 - I_2} \cdot I_1$$

$$I_g = \frac{U_g}{R_g}$$



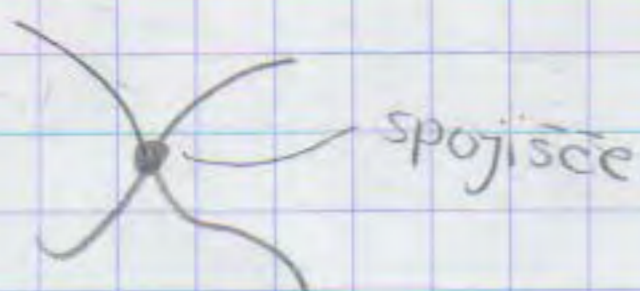
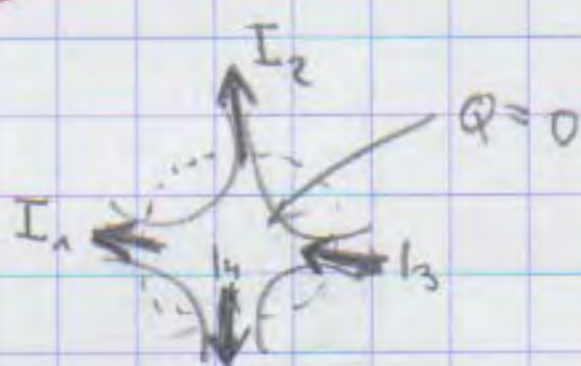
Zakoni ele. krogov

I. Kirchhoffov zakon

$$\oint_{\mathcal{L}} \vec{J} d\vec{a} = 0$$

$$\sum_{k=1}^n I_k = 0$$

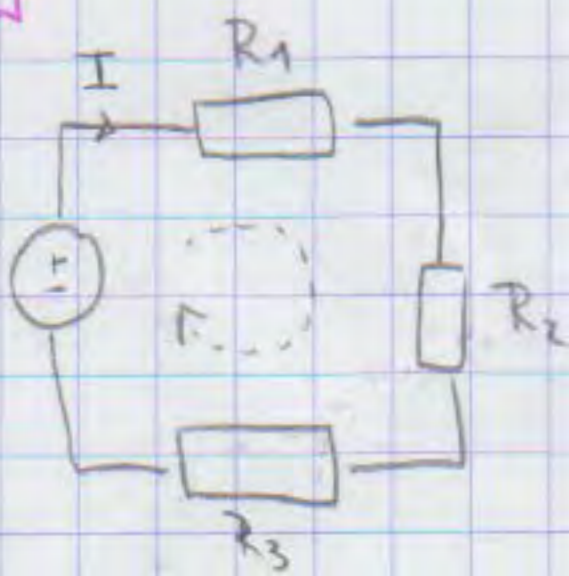
$$I_1 + I_2 - I_3 + I_4 = 0$$



II. Kirchhoffov zakon

$$\oint_{\mathcal{L}} \vec{E} d\vec{l} = 0$$

$$\sum_{j=1}^m U_j = 0$$



$$U_1 = 1 \text{ MV}$$

$$R_1 = 30 \Omega = R_3$$

$$R_2 = 50 \Omega$$

$$I \cdot R_1 + I \cdot R_2 + I \cdot R_3 = 0$$

$$I(110 \Omega) = 1 \text{ MV}$$

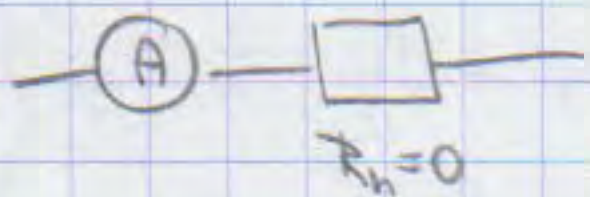
$$I = 1 \text{ A}$$

Merjenje električnih količin

1) merjenje toka : A-meter



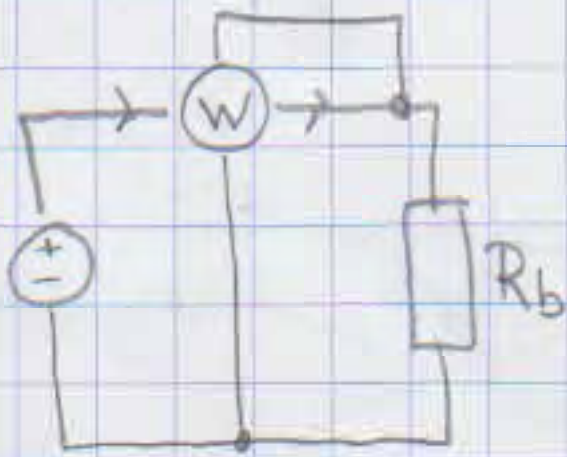
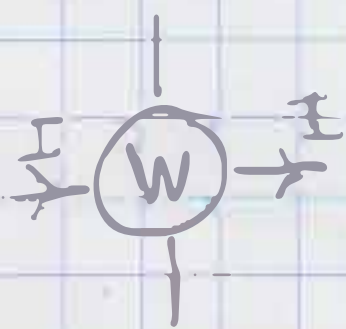
želimo, da je
notranja upornost
A-metra 0 Ω



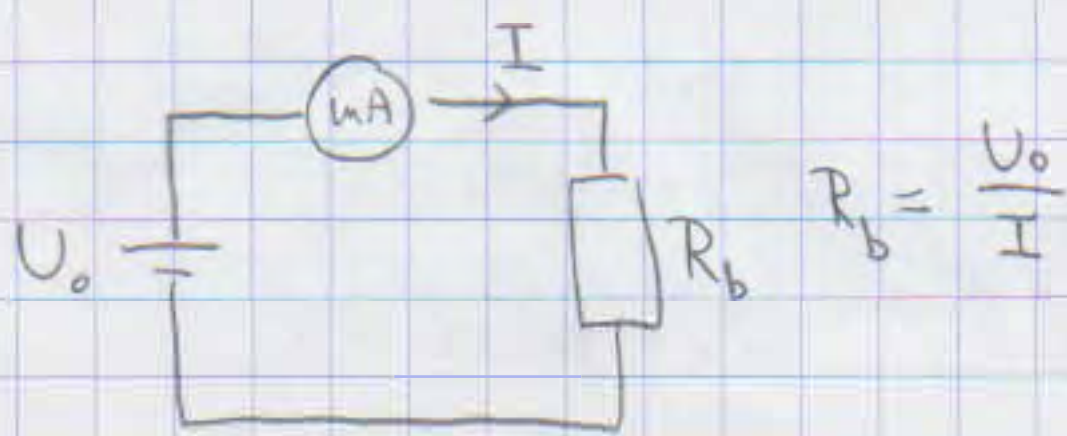
2) merjenje napetosti : V-meter



3) merjenje moči

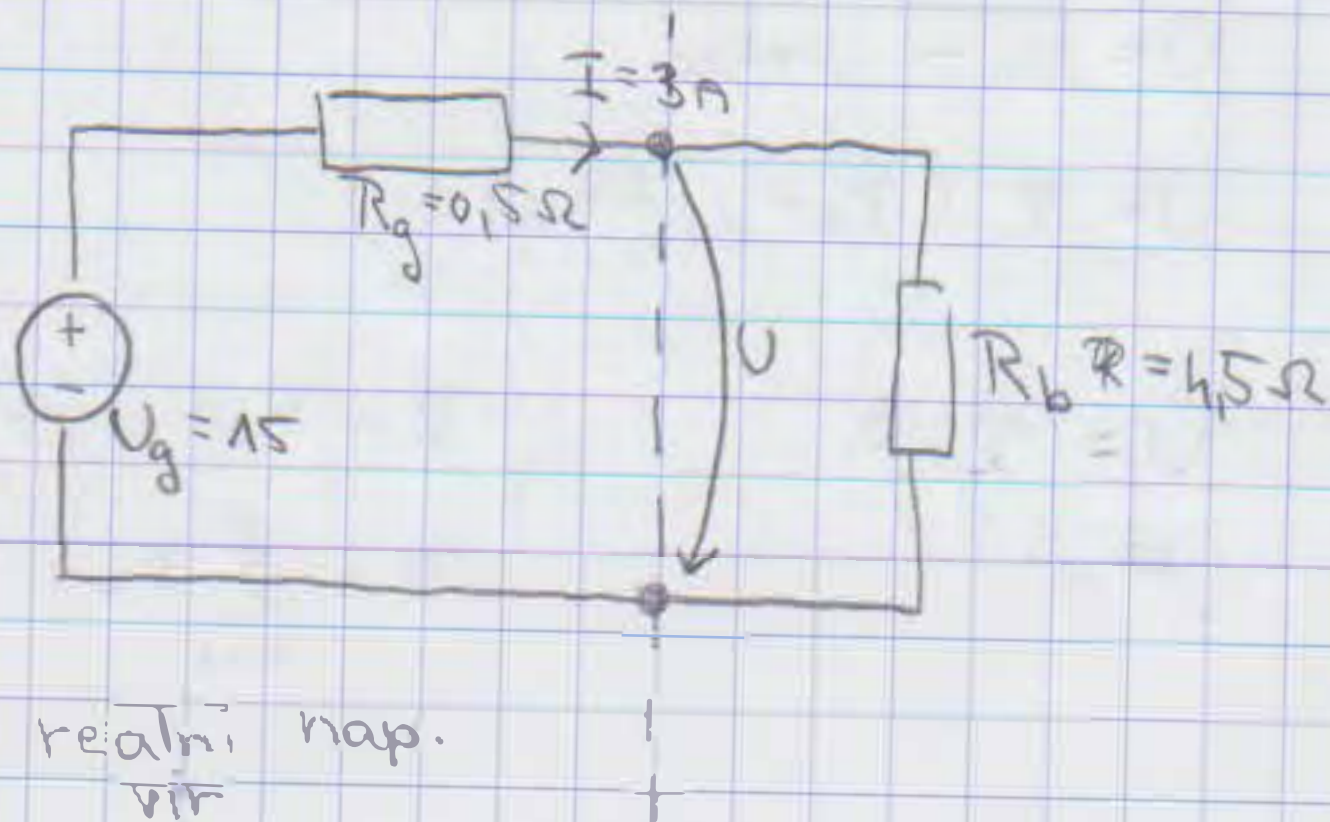


4) merjenje upornosti



Enostavna enosmerna električna vezja

1) Linearen krog



analitično reševanje

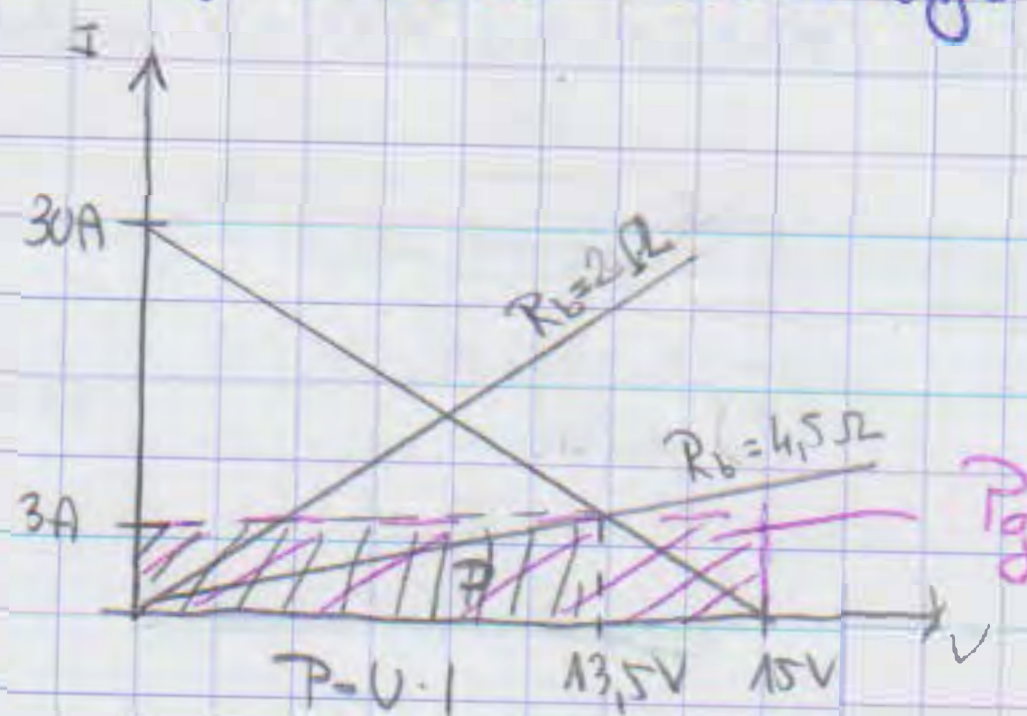
$$U = U_g - R_g \cdot I$$

$$U = R_b \cdot I = 13,5$$

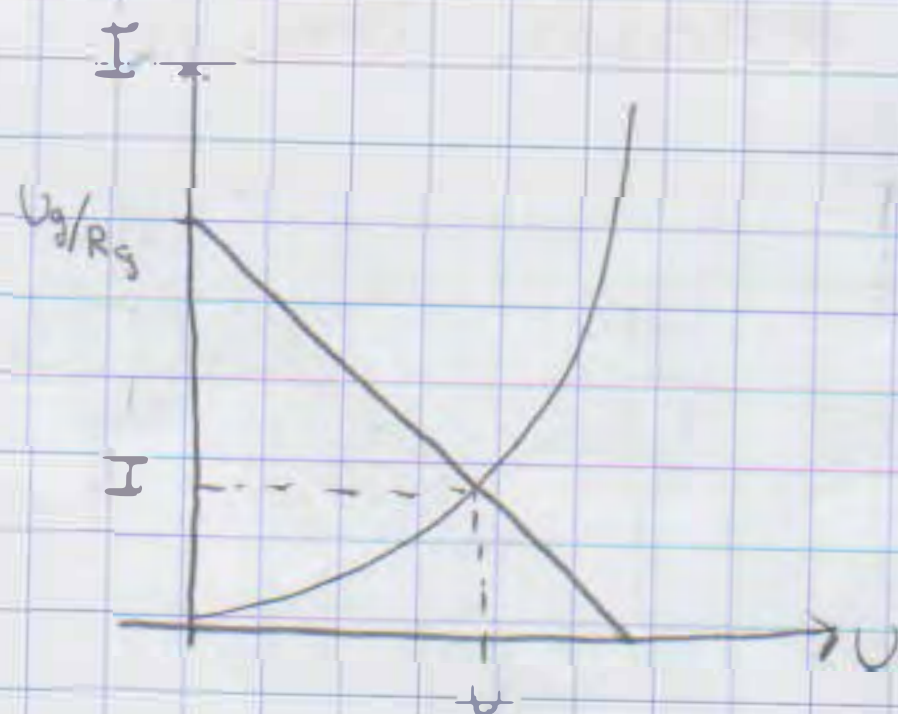
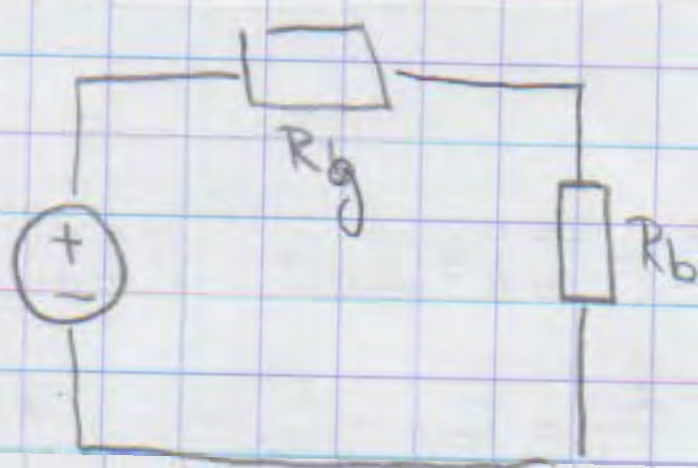
$$R_b \cdot I = U_g - R_g \cdot I$$

$$R \cdot I = \frac{U_g}{R_b + R_g} = 3A$$

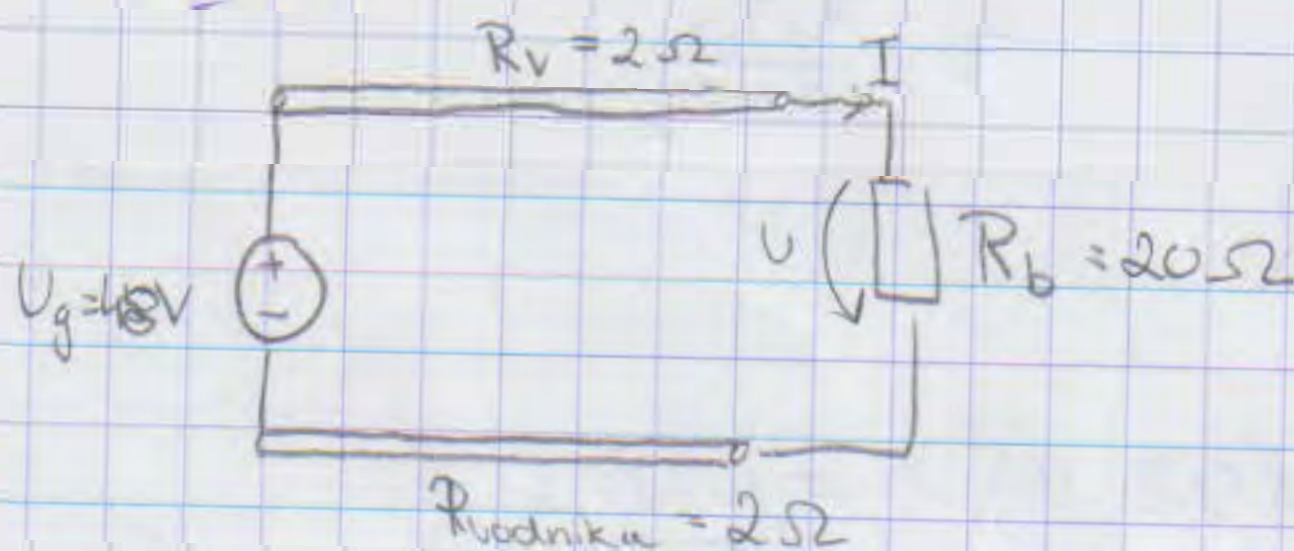
grafično reševanje



2) Nelinearen krog



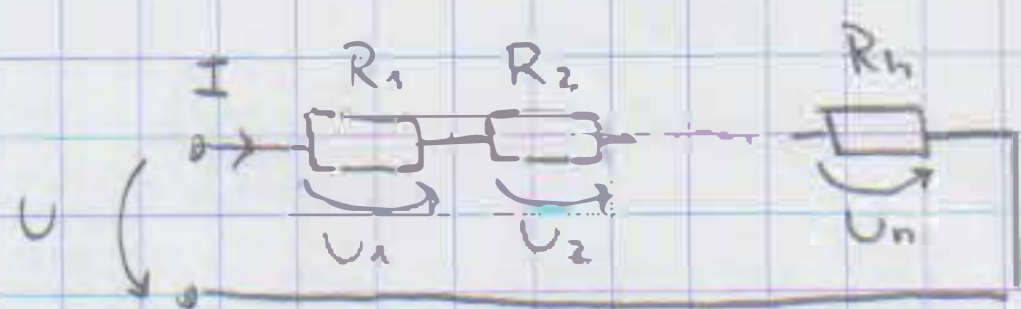
Zgled:



$$\eta = \frac{P_b}{P_g} = \frac{I U_b}{I U_g} = \frac{I \cdot 20\Omega}{48} = \frac{40}{48}$$

$$I = \frac{48}{20+2+2} = 2A$$

Zaporedna vezava bremen



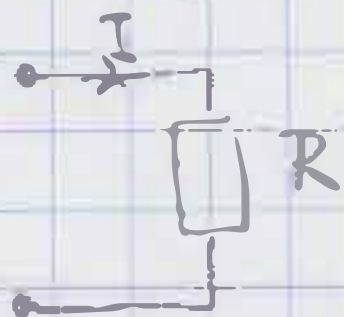
$$U = U_1 + \dots + U_n$$

$$U = I (R_1 + \dots + R_n)$$

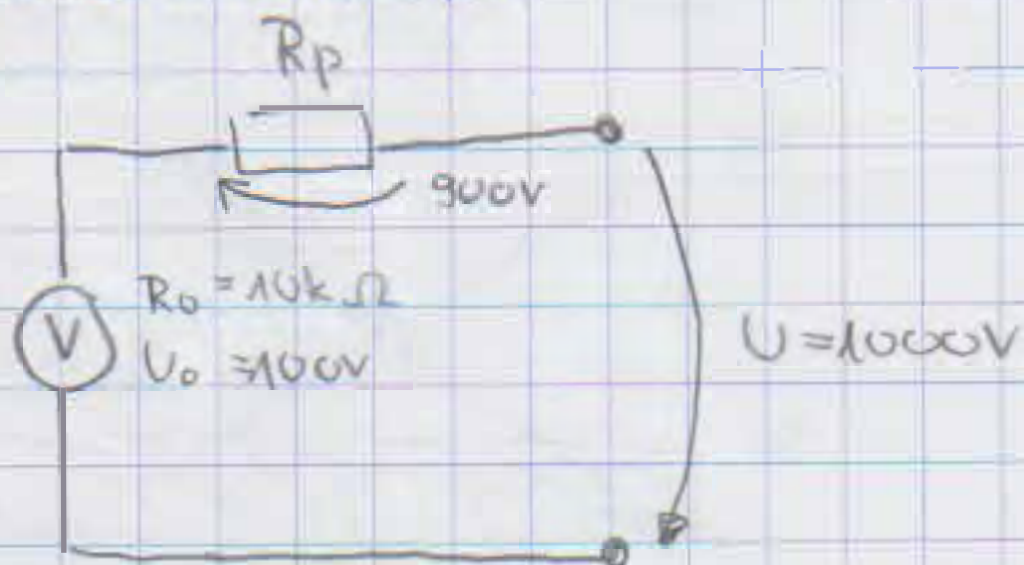
$$U = I \cdot R$$

$$R = R_1 + \dots + R_n$$

$$R = \sum_{k=1}^n R_k$$



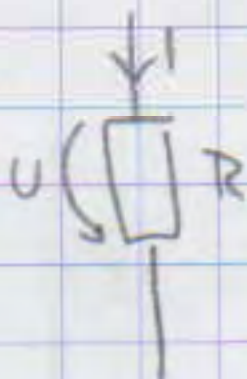
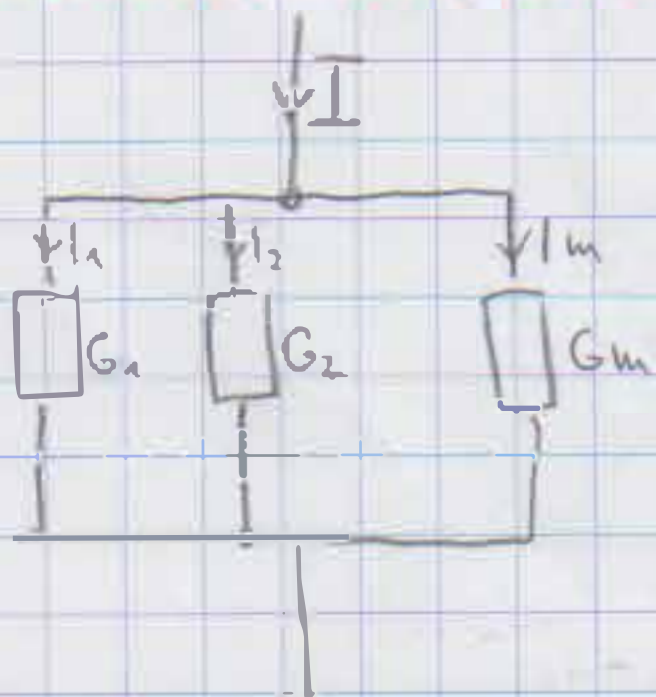
zgled: razširitev M.O. v-metra



$$\frac{R_p}{R_0} = \frac{900V}{100V} \Rightarrow R_p = 9 \cdot R_0$$

$$R_p = 90k\Omega$$

Vzporedna vezava bremen



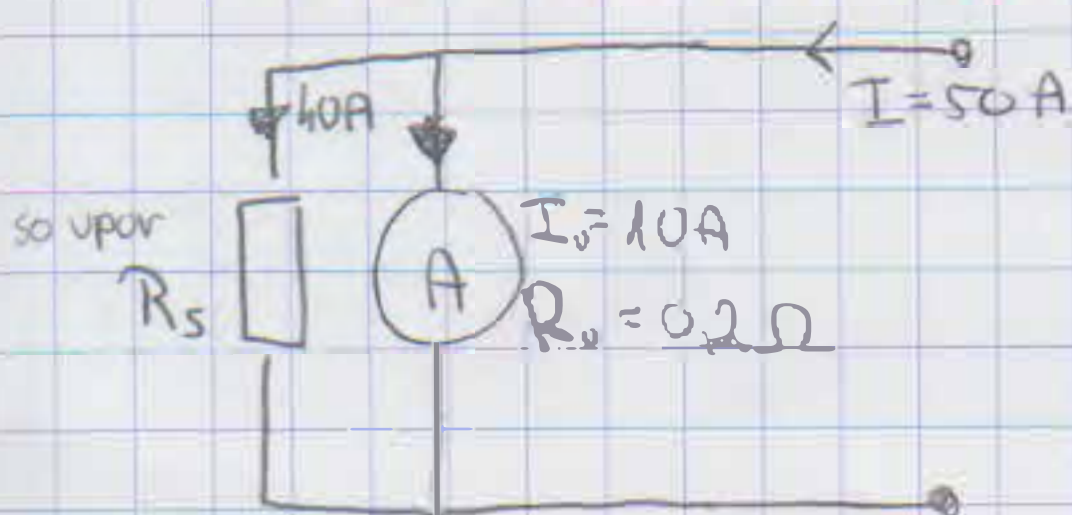
$$I = I_1 + I_2 + \dots + I_m \quad | : U$$

$$G = \frac{I}{U} = G_1 + G_2 + \dots + G_m$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_m}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

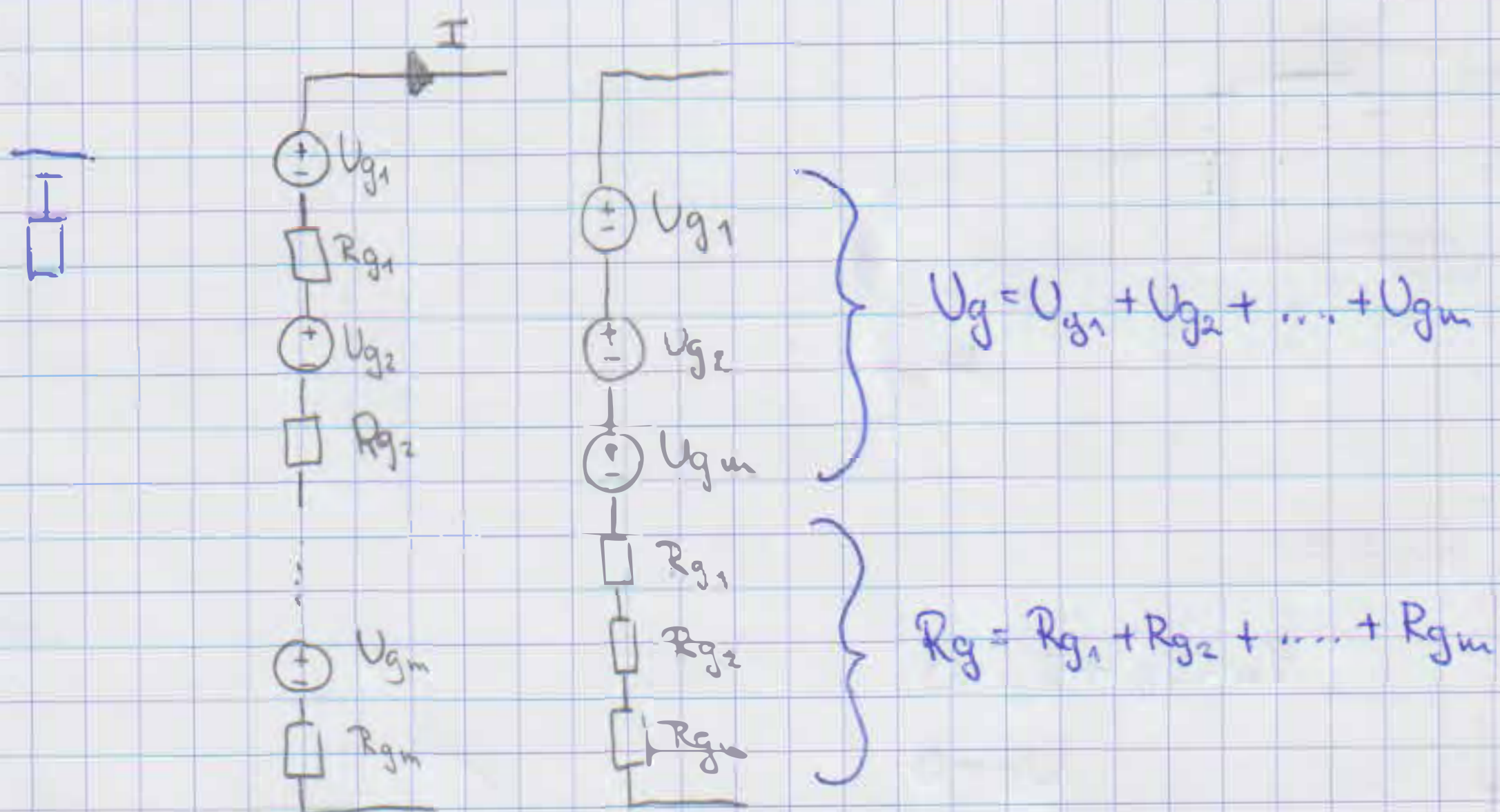
Razširitev merilnega območja A-metra



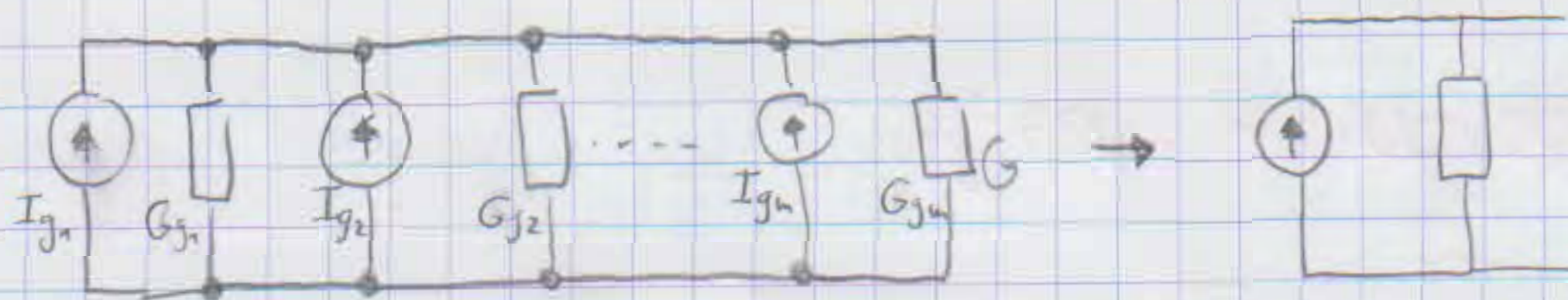
$$R_s = \frac{I_0 R_0}{40A} = \frac{(0.2 \cdot 4)}{40} = \underline{\underline{0.05 \Omega}}$$

$$I_A = 4A \Rightarrow I = \frac{4A}{40A} \cdot 50A = \underline{\underline{20A}}$$

Łaporedna vezawa virov

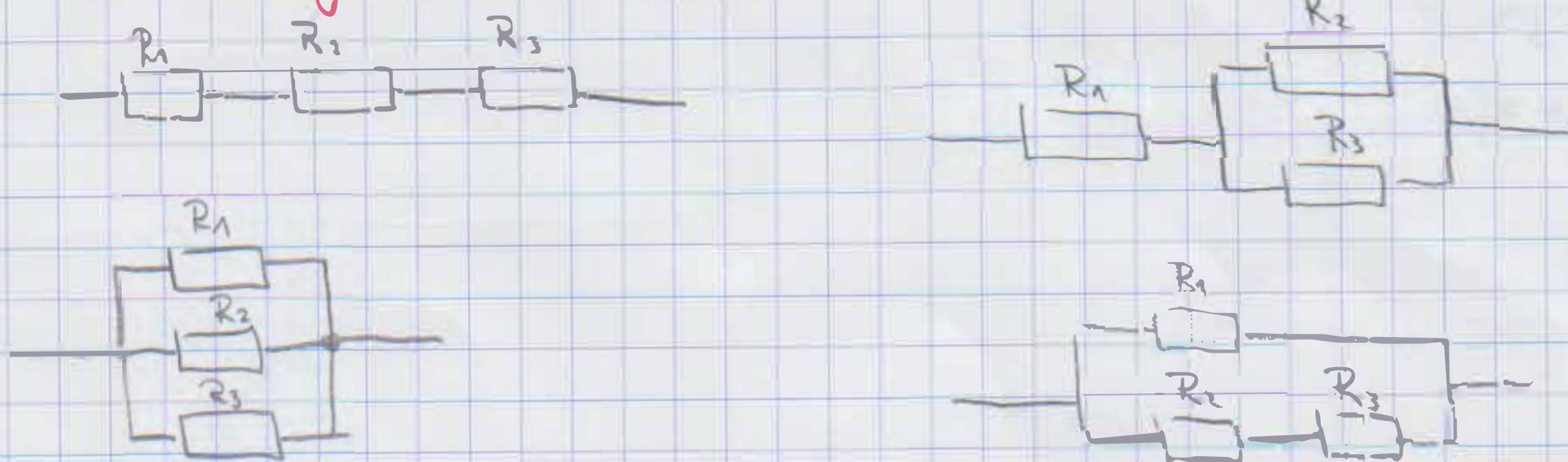


Łaporedna vezawa virov

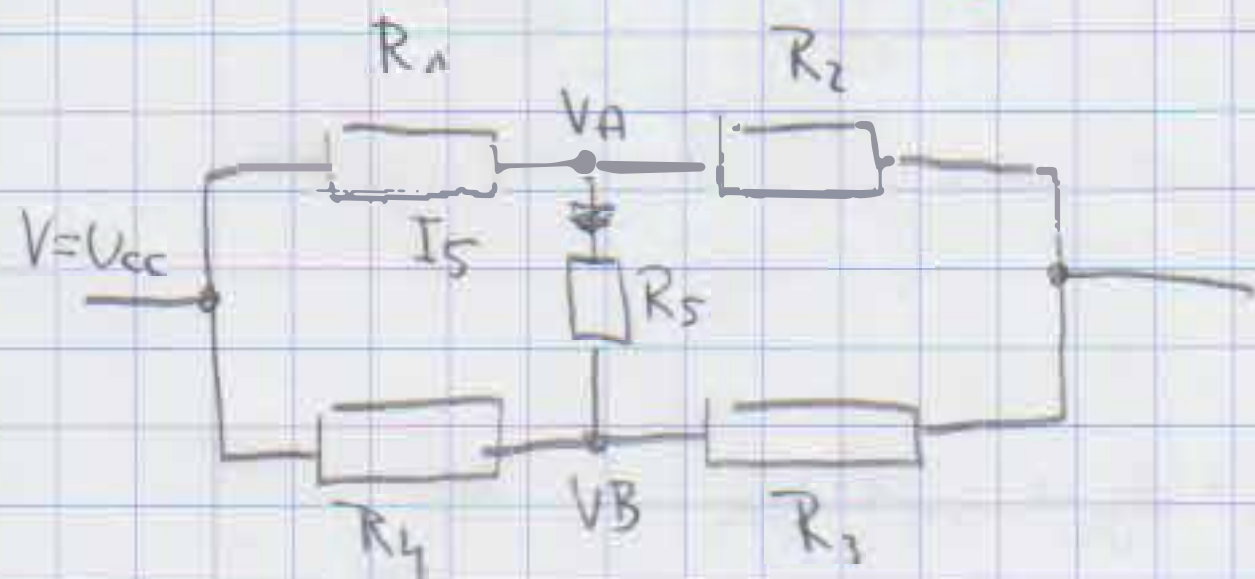


$$I_p = \sum_{k=1}^m I_{gk} \quad G_g = \sum_{k=1}^m G_{gk}$$

Sestawijencja vezawa



Mostično vezje



Uravnotežen mostič

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

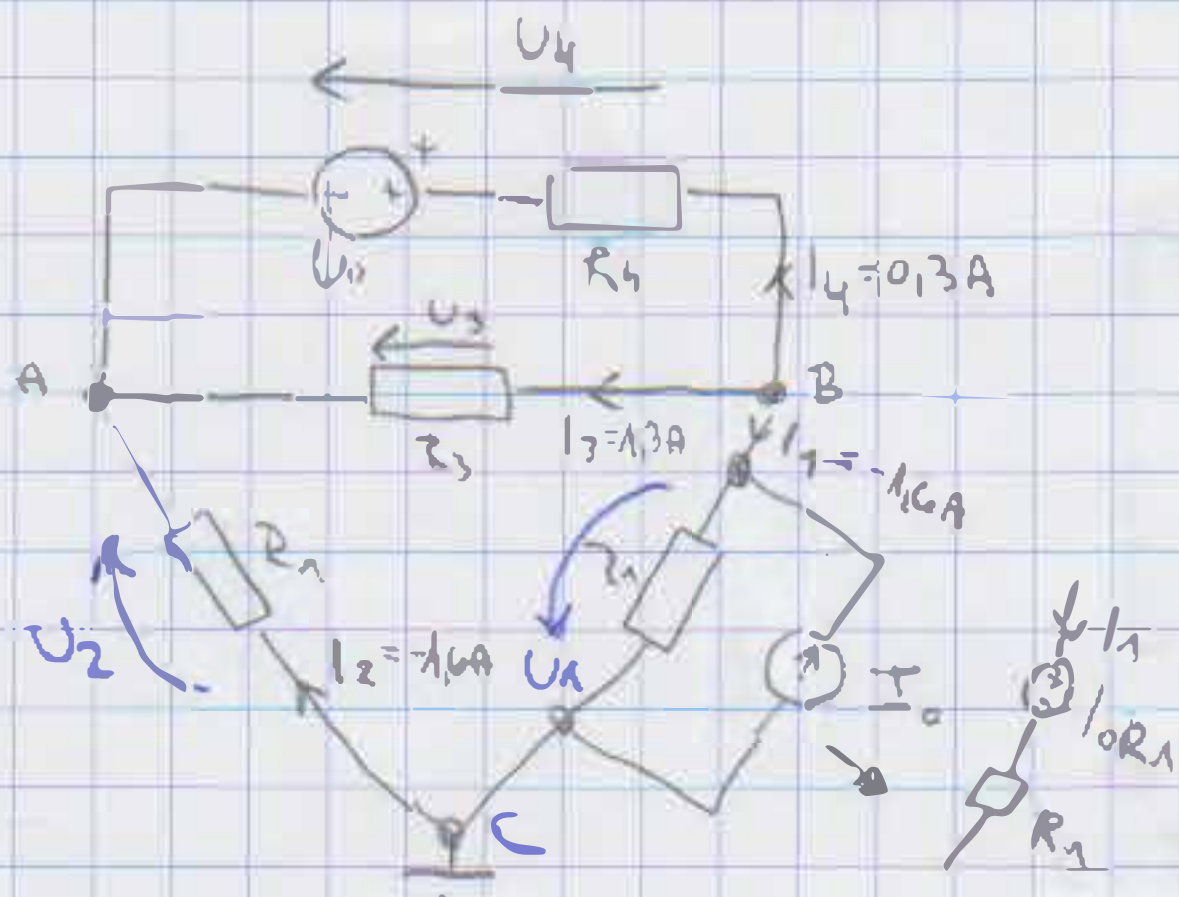
$$V_A = V_B \rightarrow I_5 = 0$$

$$U_5 = 0$$

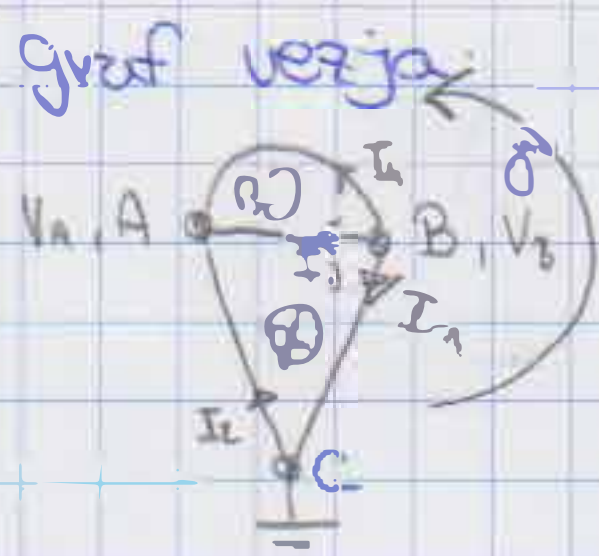
$$\frac{1}{R} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$$

Analiza uporovnih vezji

- ↳ vezje
- ↳ veja
- ↳ vozlišče
- ↳ zanka
- ↳ graf vezja
- ↳ drevo vezja



$$\begin{aligned} R_1 &= 10 \Omega \\ R_2 &= 20 \Omega \\ R_3 &= 40 \Omega \\ R_4 &= 40 \Omega \\ I_0 &= 10 \text{ A} \\ U_0 &= 40 \text{ V} \end{aligned}$$



drevo vezja



ali



Metode za reševanje vezij

- ① L neposredna Kirchhoffova metoda
- ② L metoda zanknih tokov
- ③ L metoda spojiščnih potencialov

$$P_T = I_0 U_A = 840 \text{ W}$$

$$P_N = -U_0 I_k = -12 \text{ W}$$

Sistem enačb za vezj:

$$U_1 = R_1 \cdot I_1 + R_1 I_0 = 84 \text{ V}$$

$$U_2 = R_2 \cdot I_2 = -32 \text{ V}$$

$$U_3 = R_3 \cdot I_3 = 52 \text{ V}$$

$$U_4 = R_4 \cdot I_4 + U_0 = 52 \text{ V}$$

$$I_1 = G_1 U_1 - I_0$$

$$I_2 = G_2 \cdot U_2$$

$$I_3 = G_3 \cdot I_3$$

$$I_4 = G_4 \cdot U_4 - G_4 \cdot U_0$$

① I. K. Z.

$$A: -I_2 - I_3 - I_4 = 0 \text{ v. pntekajo}$$

$$B: +I_1 + I_2 + I_4 = 0$$

$$C: -I_1 + I_2 = 0 \quad \text{izbčino omejeno spojišče}$$

$$0 = 0$$

$$\alpha: R_1 I_1 + R_2 I_2 - R_3 I_3 = -R_4 I_0$$

$$\beta: R_3 I_3 - R_4 I_4 = U_0$$

II. K. Z.

$$\alpha: U_1 + U_2 - U_3 = 0$$

$$\beta: U_3 - U_4 = 0$$

$$\gamma: -U_1 - U_2 + U_4 = 0$$

$$0 = 0$$

znanja
zanka



$$\begin{array}{l}
 A: \\
 B: \\
 C: \\
 D:
 \end{array}
 \begin{bmatrix}
 0 & -1 & -1 & -1 \\
 1 & 0 & 1 & 1 \\
 10 & 20 & -40 & 0 \\
 0 & 0 & 40 & -40
 \end{bmatrix}
 \begin{bmatrix}
 I_1 \\
 I_2 \\
 I_3 \\
 I_4
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 -100 \\
 40
 \end{bmatrix}$$

$[R] \quad [I] \quad [U]$
 $[I] = [R]^{-1} [U]$

$$[I] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -1,6 \\ -1,6 \\ 1,3 \\ 0,3 \end{bmatrix}$$

③ Metoda spojinskih potencijalov

$$V_A, V_B = ?$$

$$\begin{aligned}
 V_A &= 32V \\
 V_B &= 48V
 \end{aligned}$$

zberes si eno referenčno točko in se orientiraš na njo s celotnim vezjem (jo opredeliš).

$$U_1 = V_B - 0 = V_B = 48V$$

$$U_2 = 0 - V_A = -V_A = -32V$$

$$U_3 = V_B - V_A = 52V$$

$$U_4 = V_B - V_A = 52V$$

$$A: V_B - V_A - V_B + V_A = 0$$

$$B: V_B - V_A - (V_B - V_A) = 0$$

$$A: -(-V_A) \cdot G_2 - (V_B - V_A) \cdot G_3 - (V_B - V_A) \cdot G_4 + U_0 G_4 = 0$$

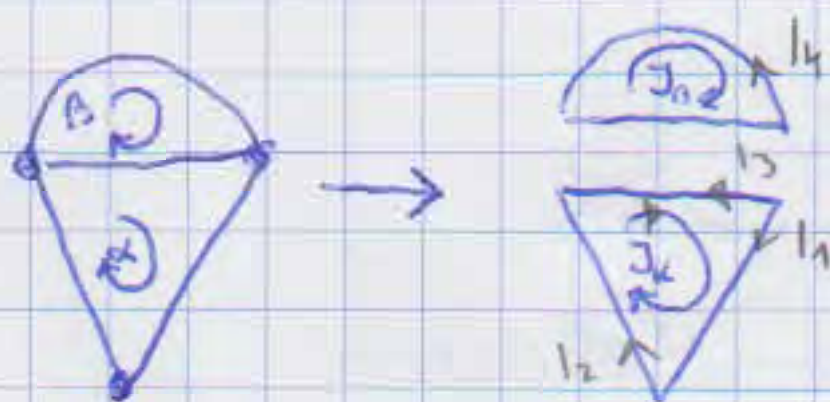
$$B: V_B \cdot G_1 + (V_B - V_A) \cdot G_3 + (V_B - V_A) \cdot G_4 = I_0 + U_0 G_4$$

$$\begin{array}{l}
 A \\
 B
 \end{array}
 \begin{bmatrix}
 G_2 + G_3 + G_4 & -G_3 - G_4 \\
 -G_3 - G_4 & G_1 + G_3 + G_4
 \end{bmatrix}
 \begin{bmatrix}
 V_A \\
 V_B
 \end{bmatrix}
 =
 \begin{bmatrix}
 -U_0 G_4 \\
 I_0 + U_0 G_4
 \end{bmatrix}$$

Spojnica

2

Metoda zvančnih tokov



$$J_\alpha, J_\beta = ?$$

$$J_\alpha = -1,6 \text{ A}$$

$$J_\beta = -0,3 \text{ A}$$

$$I_1 = J_\alpha = -1,6 \text{ A}$$

$$I_2 = J_\alpha = -1,6 \text{ A}$$

$$I_3 = -J_\alpha + J_\beta = 1,3 \text{ A}$$

$$I_4 = -J_\beta = 0,3 \text{ A}$$

$$A: -J_\alpha - (-J_\alpha + J_\beta) - (-J_\beta) = 0$$

$$B: J_\alpha - J_\alpha + J_\beta - J_\beta = 0$$

$$R_1 \cdot J_\alpha + R_2 \cdot J_\alpha + R_3 (J_\alpha - J_\beta) = -U_0 \cdot R_1$$

- spinka

+ spinka

2 neznanke J_α, J_β

$$R_3 (J_\beta - J_\alpha) + R_4 \cdot J_\beta = +U_0$$

$$\begin{bmatrix} (R_1 + R_2 + R_3) & -R_3 \\ -R_3 & (R_3 + R_4) \end{bmatrix} \begin{bmatrix} J_\alpha \\ J_\beta \end{bmatrix} = \begin{bmatrix} -U_0 R_1 \\ U_0 \end{bmatrix}$$

$\Downarrow$

$$\begin{bmatrix} 70 & -40 \\ -40 & 80 \end{bmatrix} \begin{bmatrix} J_\alpha \\ J_\beta \end{bmatrix} = \begin{bmatrix} -100 \\ 40 \end{bmatrix}$$

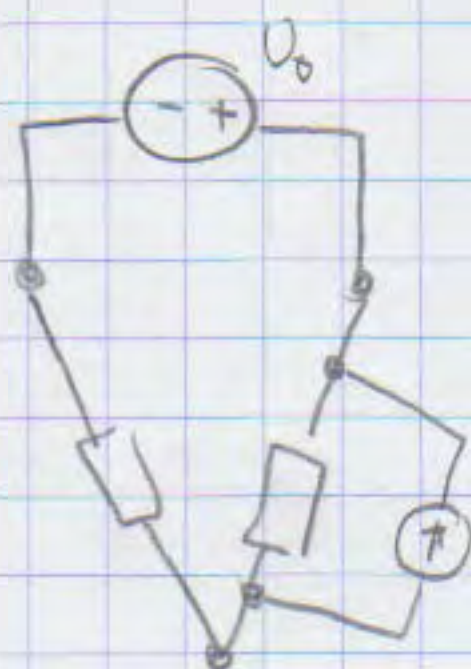
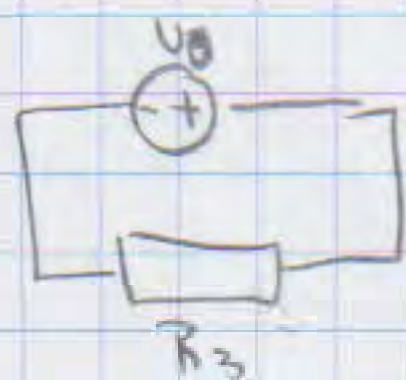
zamrjeno tokovi
vr z napetostnim

Vežje z idealnimi elementi

L idealni napetostni vir

- metoda zanknih tokov Ok
- metoda spoji. potencialov težavno

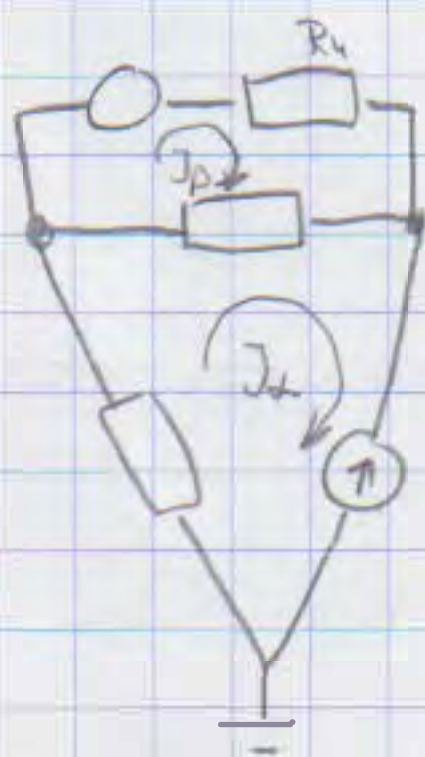
vežje razdelimo



L idealni tokovni vir

- metoda spojišnih potencialov Ok
- metoda zanknih tokov

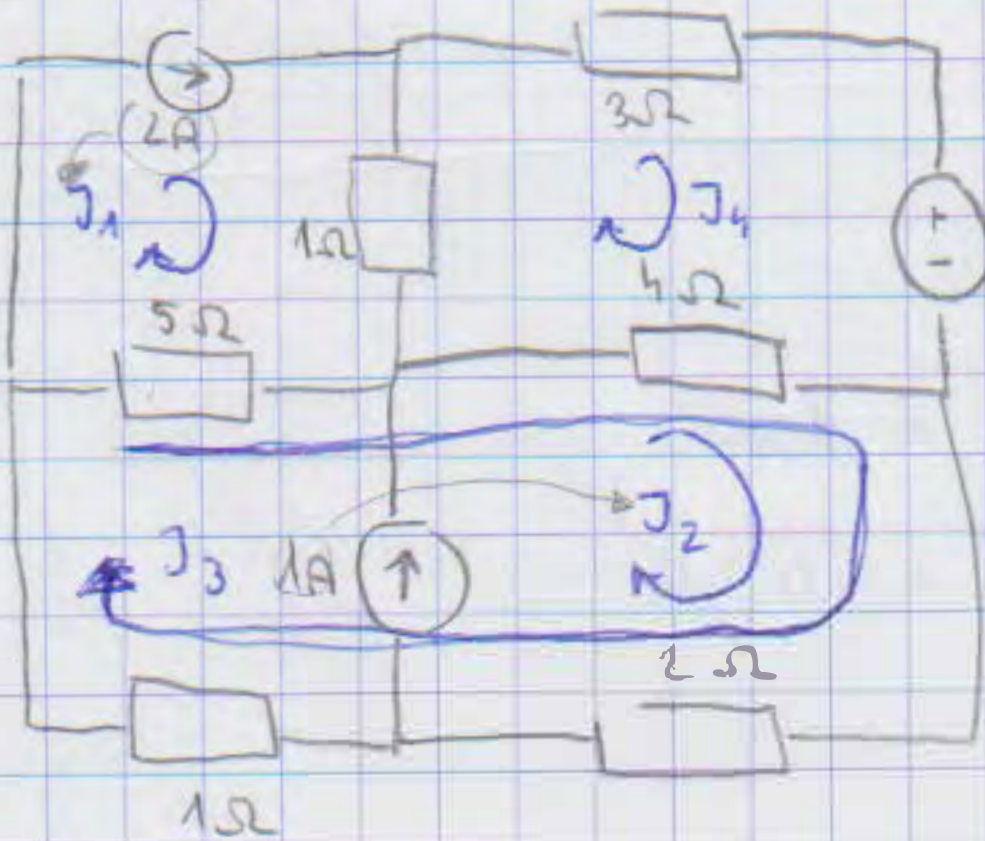
vsak tokovni vir "zaobjamemo" z eno zanko



$$I_\alpha = -I_0$$

$$I_\beta =$$

Zgled 3.37



$$P_{10V} = -J_4 \cdot 10V$$

graf vezja



če imamo tokovi
generator gremo čez
njega samo 1x

lažji postopek: zunanji tokovi

$$J_1 = 2A$$

$$J_2 = 1A$$

$$\left. \begin{matrix} J_3 \\ J_4 \end{matrix} \right\} ?$$

gremo čez vse elemente

$$J_3 \quad A \quad 5\Omega (J_3 - 2A) + 4\Omega (1A + J_3 - J_4) + 2\Omega (1A + J_3) + 1\Omega (J_3) = 0$$

$$J_4 \quad B \quad 1\Omega (J_4 - 2A) + 3\Omega J_4 + 10V + 4\Omega (J_4 - J_3 - 1A) = 0$$

$$\begin{matrix} A & 12\Omega J_3 - 4\Omega J_4 = 4V \\ B & -4\Omega J_3 + 8\Omega J_4 = -4V \end{matrix}$$

delimo s 4Ω

$$\begin{matrix} \rightarrow 3J_3 - J_4 = 1A \\ -J_3 + 2J_4 = -1A \end{matrix}$$

$$\begin{matrix} J_3 = 0,2A \\ J_4 = -0,4A \end{matrix}$$

$$P_{10V} = 4W$$

Štawki (Teoremi) o enosmernih ele. vezjih

1) Superpozicija (linearna vezja) (velja v linearnih vezjih)

! Deaktivacija vira !

napetostni vir		sledi	
tokovni vir		sledi	

nadaljeujemo s krcnim stikom

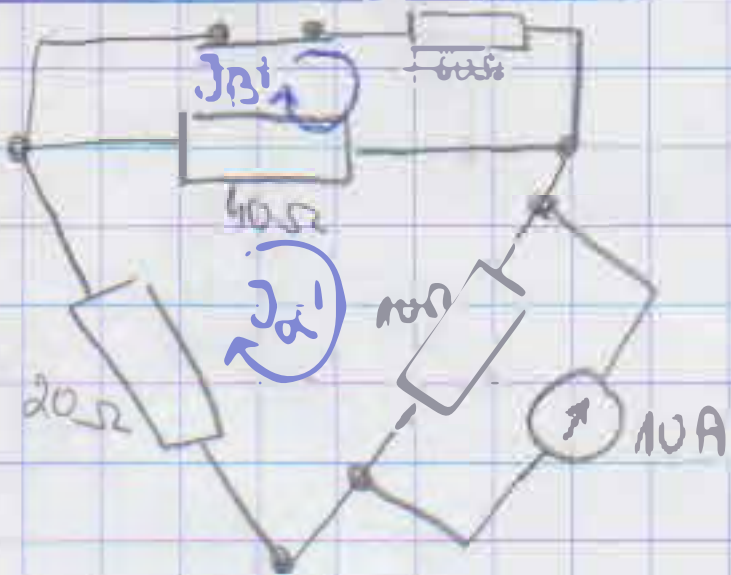
$$[R] \cdot [J] = [U]$$

$$[J] = [R^{-1}] \cdot [U_g]$$

$$[J] = \overbrace{[R^{-1}] \cdot [U_{g1}]}^{J'} + \overbrace{[R^{-1}] \cdot [U_{g2}]}^{J''} + \underbrace{[R^{-1}] \cdot [U_{g3}]}_{J'''} = \begin{bmatrix} U_{g1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ U_{g2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ U_{g3} \end{bmatrix}$$

$$[J] = [J'] + [J''] + [J''']$$

a) aktiven tokovni vir



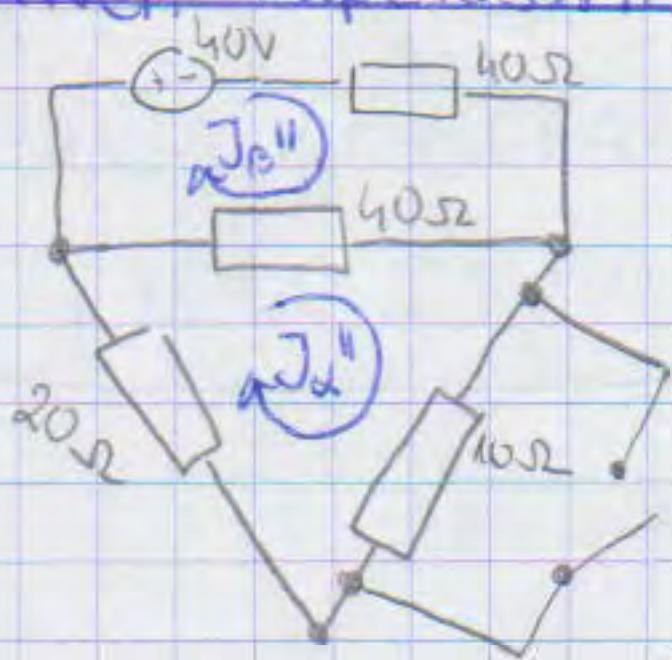
$$J_A' = -2A$$

$$J_B' = -1A$$

$$J_A'' = J_A' + J_A'' = -1,6A$$

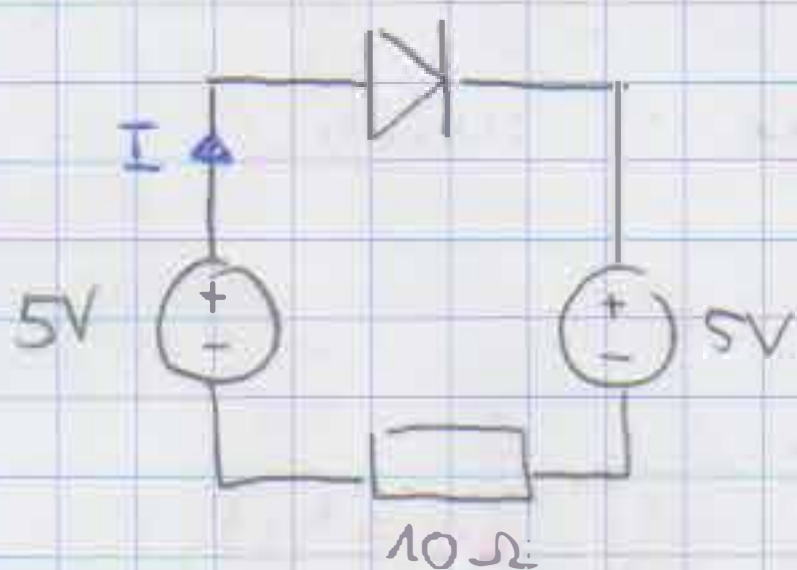
$$J_B = J_B' + J_B'' = -0,3A$$

b) aktiven napetostni vir



$$J_A'' = 0,4A$$

$$J_B'' = 0,7A$$

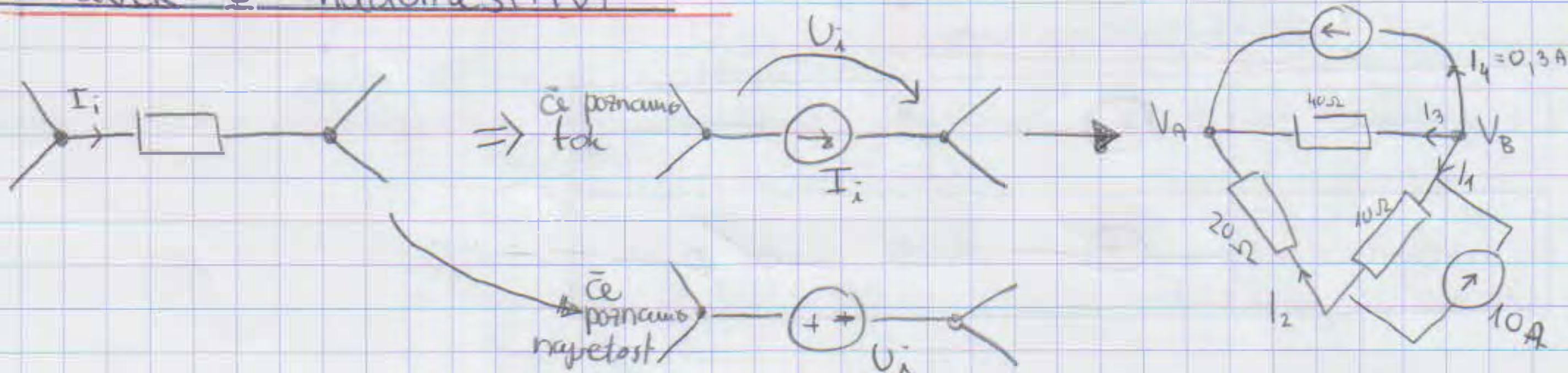


$$I_e = 0,5A$$

$$\frac{I_d = 0}{I = 0,5A}$$

Če imamo nelinearen element v vezju nam superpozicija odpade

2) Stavak 6 nadomestitvi



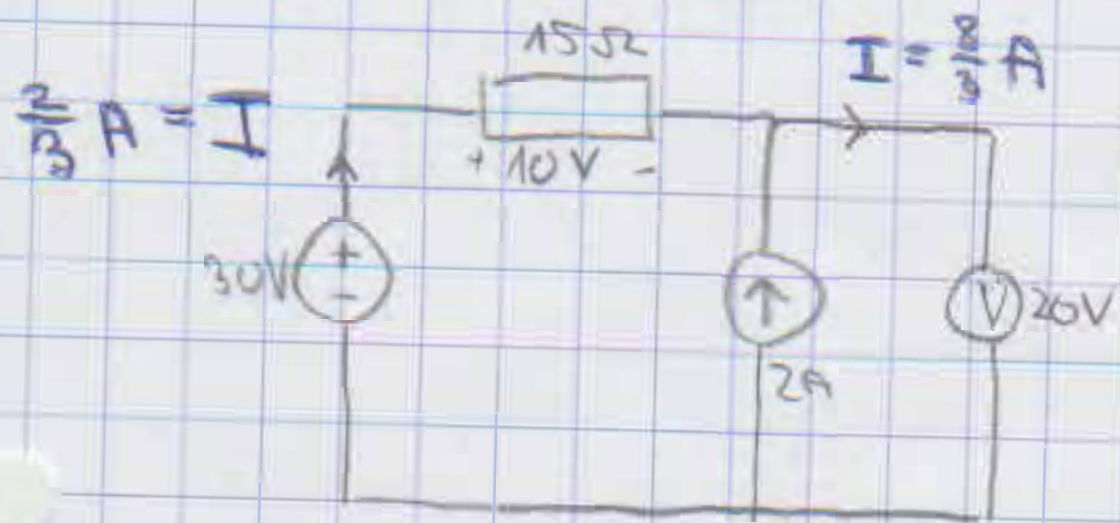
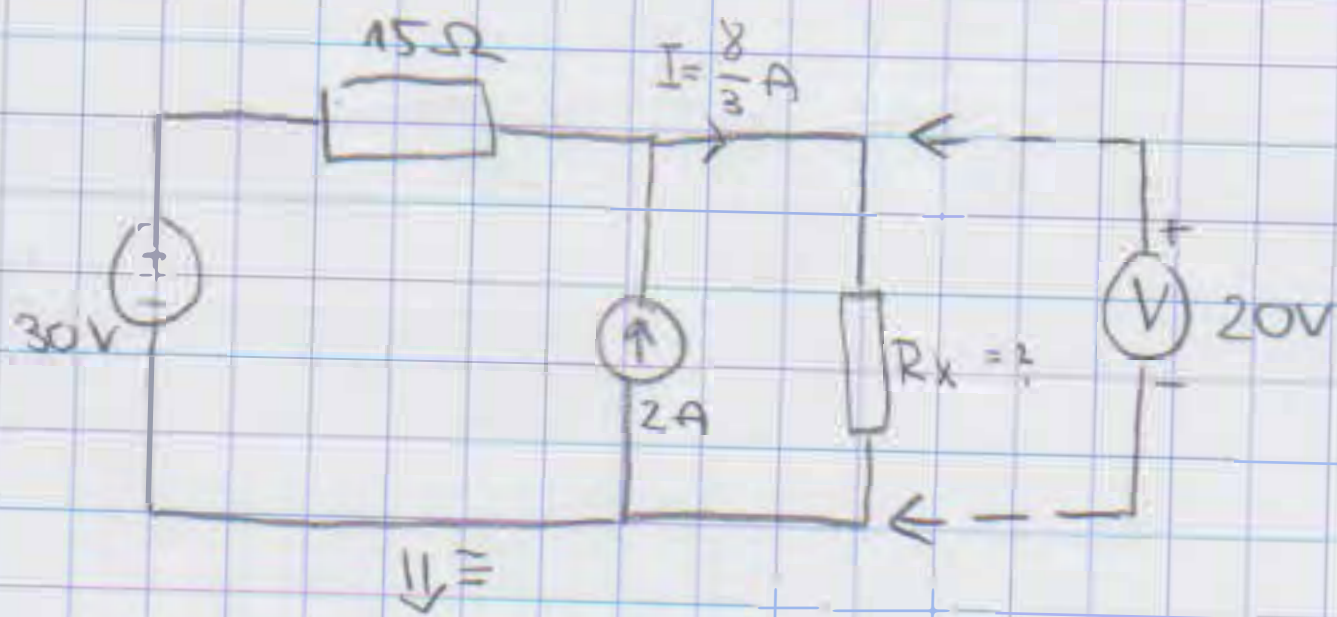
$$V_A = 32V$$

$$V_B = 84V$$

$$\begin{bmatrix} \frac{1}{20} + \frac{1}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{1}{40} + \frac{1}{10} \end{bmatrix} \cdot \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0,3 \\ 10 - 0,3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 12 \\ 388 \end{bmatrix}$$

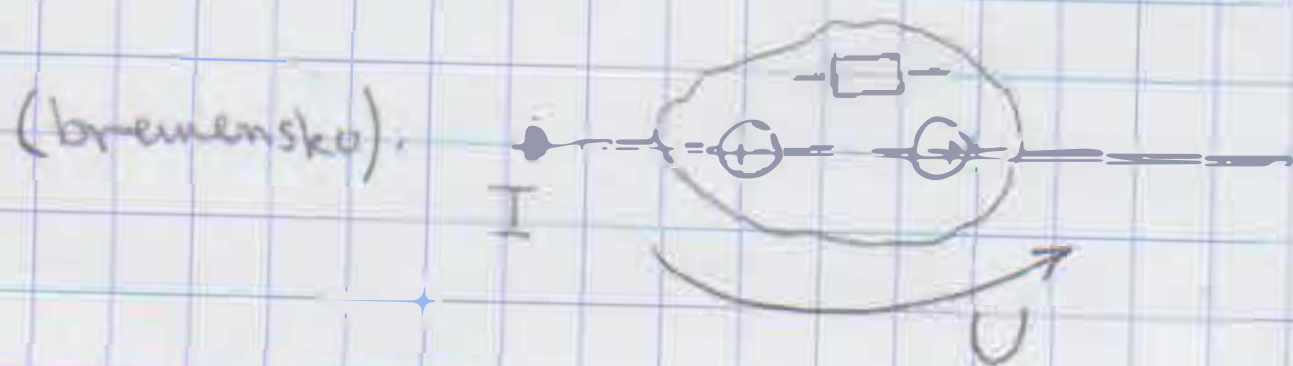
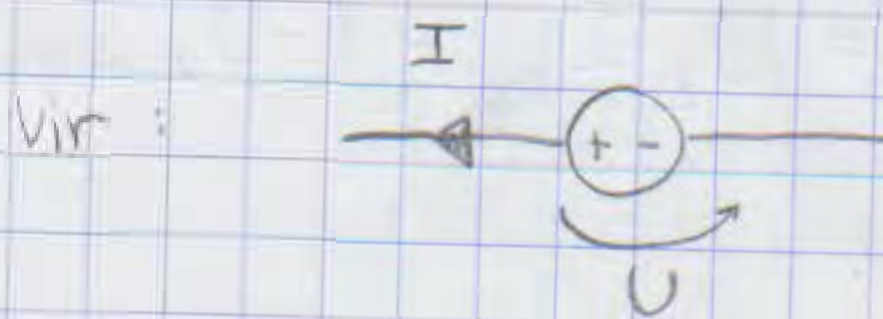
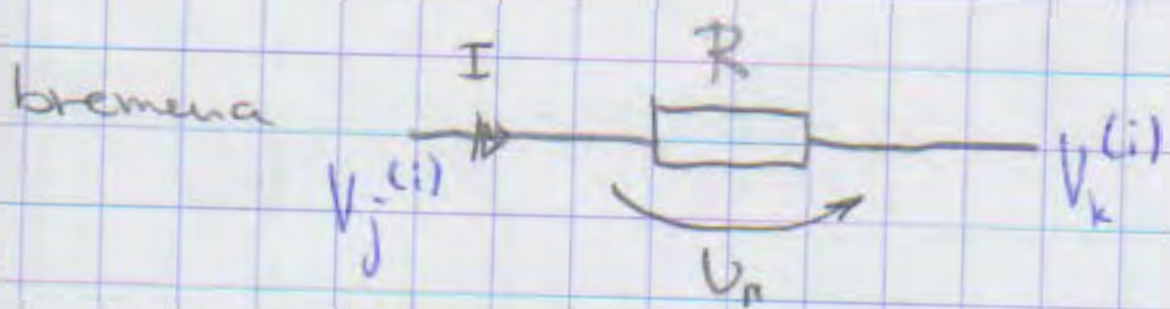
3.42



$$R_x = \frac{20V}{\frac{8}{3}A} = 7,5\Omega$$

3) Tellegenov stavek

istosmiselno označevanje



istosmiselno označevanje

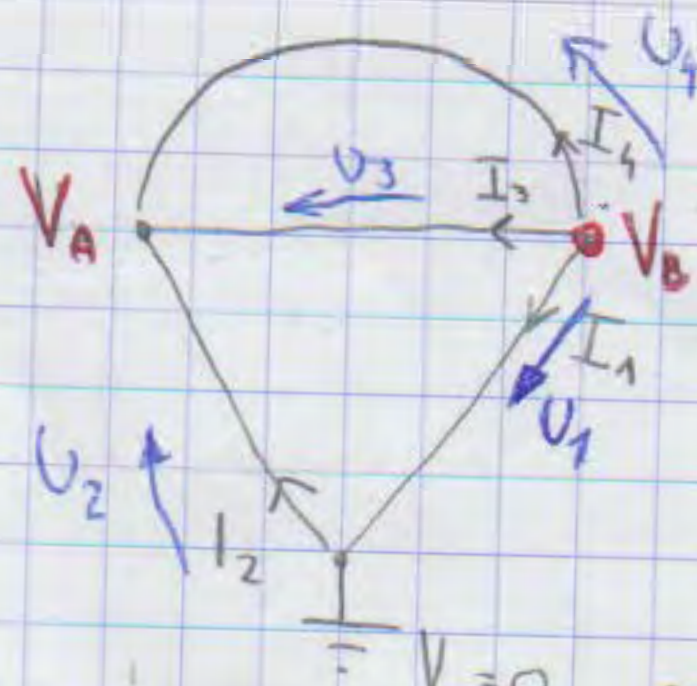
$$\sum_{i=1}^n u_i i_i = 0$$

$$\sum_{k=1}^n P_{gk} = \sum_{k=1}^n P_{bk}$$

$$\sum_{i=1}^n U_i I_i = 0$$

$$\sum_{i=1}^n I_i (V_j^{(i)} - V_k^{(i)})$$

$$+ V_j (\pm I \dots) + V_k (\pm I \pm \dots)$$



$$I_1 \cdot \underbrace{(V_B)}_{U_1} + I_2 \cdot (V_C - V_A) + I_3 (V_B - V_A) + I_4 \underbrace{(V_B - V_A)}_{U_4} =$$

$$= V_A (-I_2 - I_3 - I_4) + V_B (I_1 + I_3 + I_4) =$$

$$= \underbrace{I \cdot K \cdot Z.}_{\emptyset} + \underbrace{I \cdot K \cdot Z.}_{\emptyset}$$

$$U_k I_k = -P_{gk} + P_{bk}$$

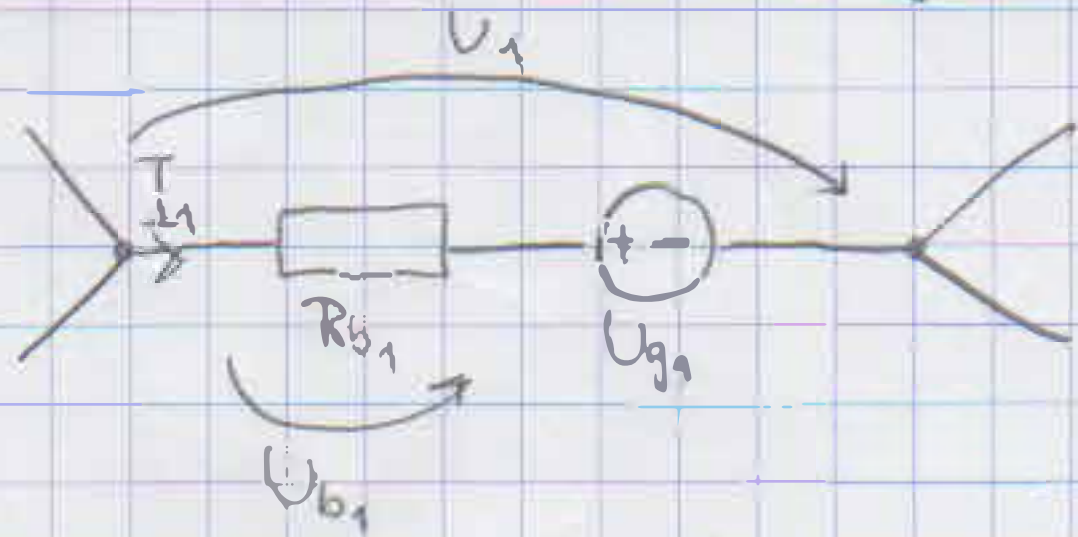
moč ki jo
dobimo iz gener.

toplota ki se
sproščana na upor
v tej vezi

Alkatar

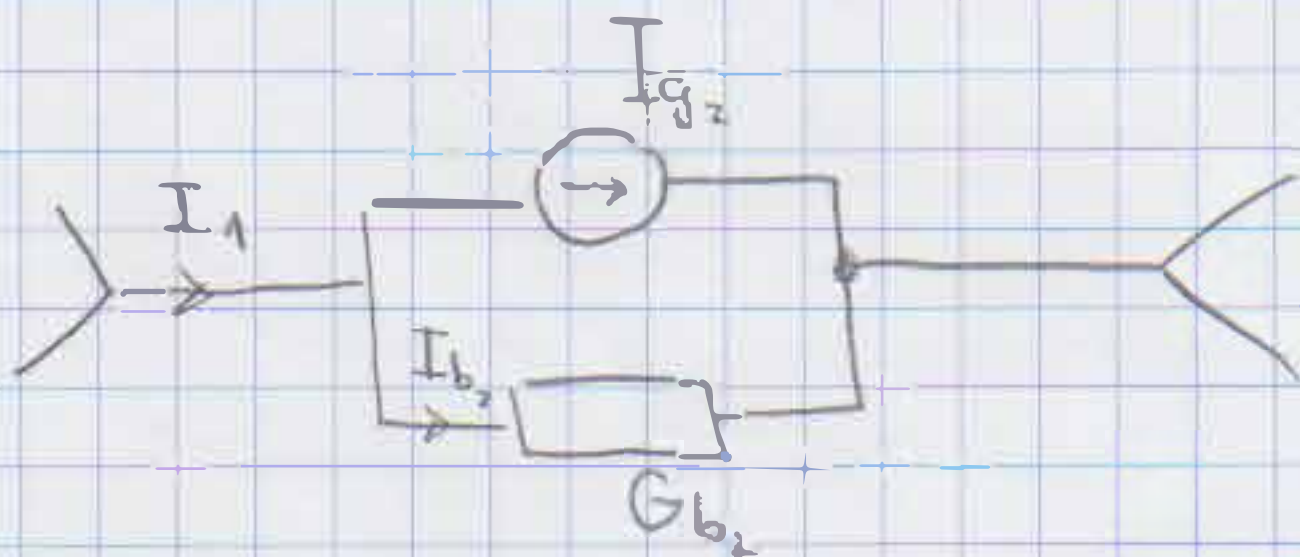
(82)

Vsebinska osvetlitev Tellegenovega stavka

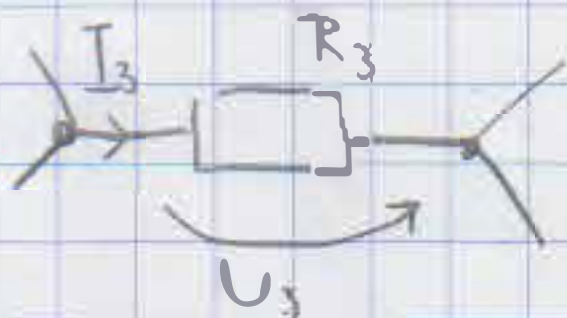


$$U_1 I_1 = U_{b1} I_1 + U_{g1} I_1$$

$$\underbrace{U_1 I_1}_{P_{v1}} = \underbrace{U_{b1} I_1}_{P_{b1}} - \underbrace{(-U_{g1} I_1)}_{P_{g1}}$$



$$U_2 \cdot I_2 = U_2 I_{b2} - (-U_2 I_{g2})$$



$$U_3 \cdot I_3$$

$$U_1 = 84V$$

$$U_2 = -32V$$

$$U_3 = 52V$$

$$U_4 = 52V$$

$$I_1 = -1,6A$$

$$I_2 = -1,6A$$

$$I_3 = 1,3A$$

$$I_4 = 0,3A$$

$$P_{V1} = -134,4W$$

$$P_{V2} = 51,2W$$

$$P_{V3} = 67,6W$$

$$P_{V4} = 15,6W$$

$$P_s = 0W$$

MOCI
VEJ

$$G_1 U_1^2 = 705,6W$$

$$R_2 \cdot I_2^2 = 51,2W$$

$$R_3 \cdot I_3^2 = 67,6W$$

$$R_4 \cdot I_4^2 = 3,6W$$

ker je samo en
element v reži

$$P_s > 0$$

$$P_s < 0$$

$$P_{g1} = -840W$$

daje moč

$$P_{g4} = 12W$$

sprejema moč

4) Stavci Thevenina in Nortona

napetostni
vir

tokovni
vir

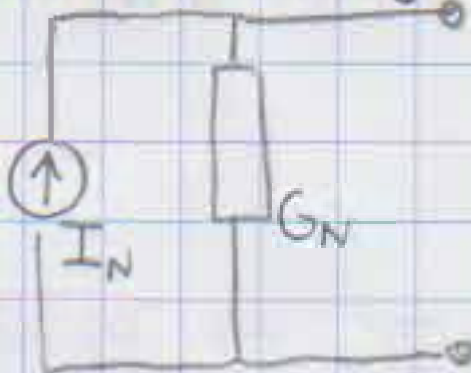
deaktiviramo



Aktivno
dvopolno
linearno
vezje

ADLV

Nortonovo vezje:



Norton

I_N = tok kratkega stika ALDV

G_N = nadomestna prevodnost LDV
vire deaktiviramo

$$G_N = \frac{1}{R_T} = R_T^{-1}$$

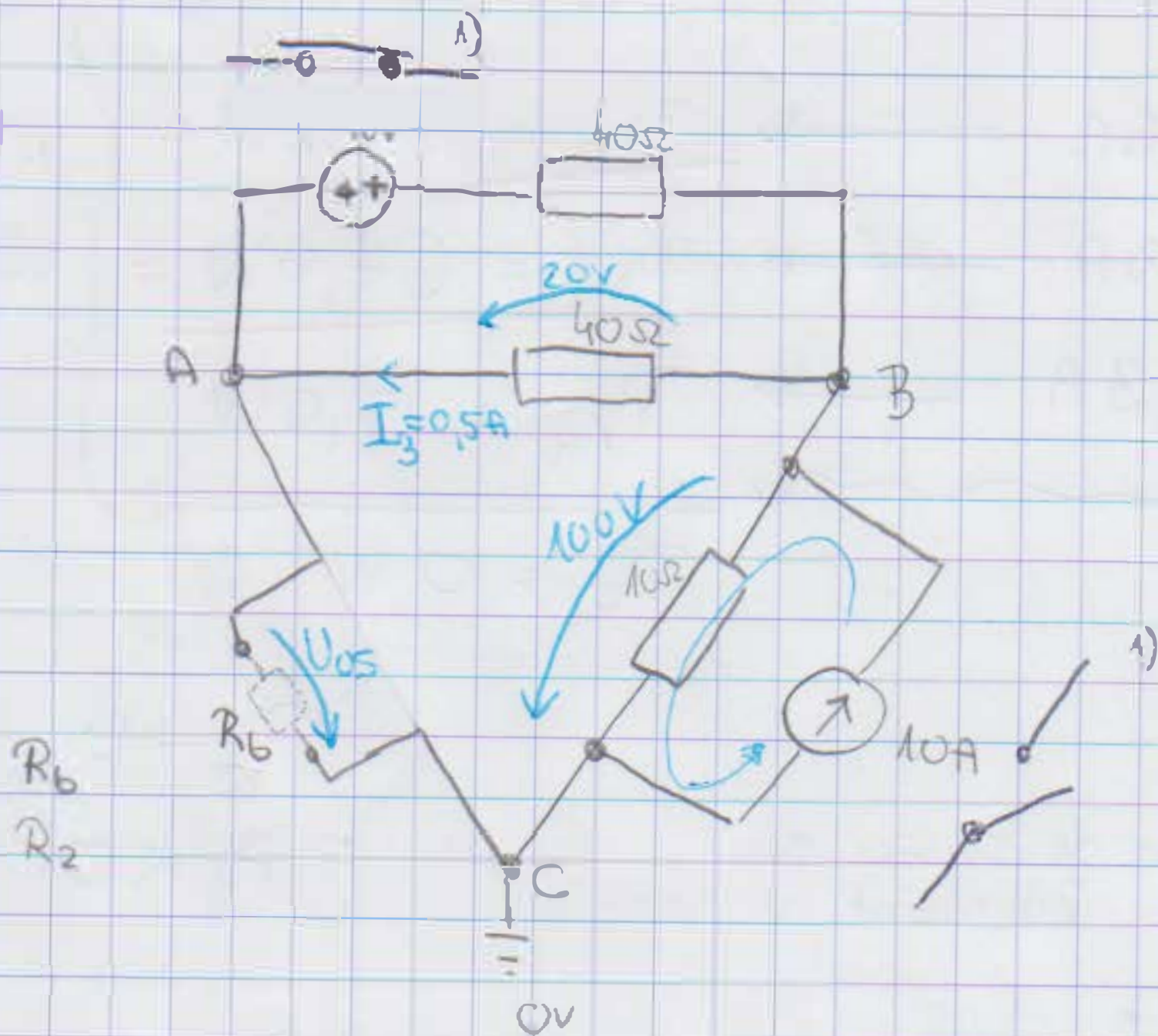
Theveninovo vezje

U_T = napr. odprtih spojk ALDV

R_T = nadom. not. upor. LDV (vire deaktiviramo)

Alketa

egled:



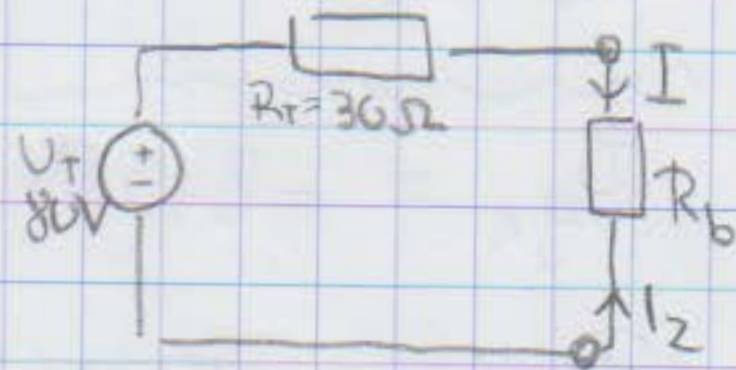
Theveninov nadomestni vir:

1) Vir deaktiviramo in določimo $R_{Theveninov}$

$$R_T = R_{NAD.} = 40\Omega \parallel 40\Omega + 10\Omega = 30\Omega$$

2) napetost odprtih spojk

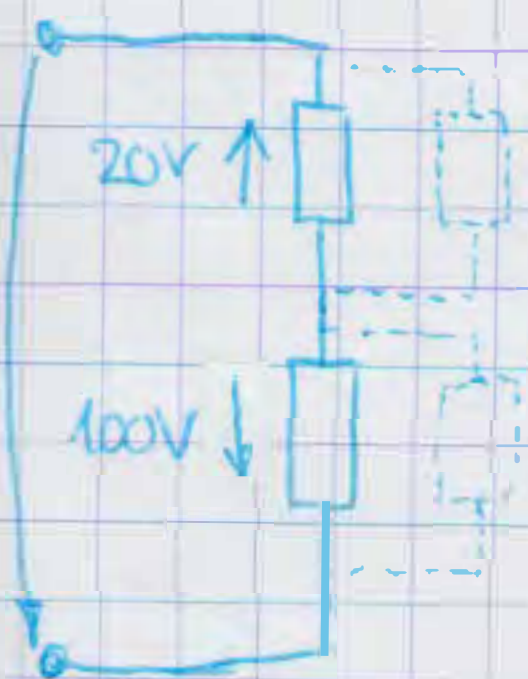
$$U_T = U_{os} = 100 - 20 = 80V$$



$$R_b = R_T = 30\Omega$$

$$(-I_2) = I = \frac{80}{60} = 1,33A$$

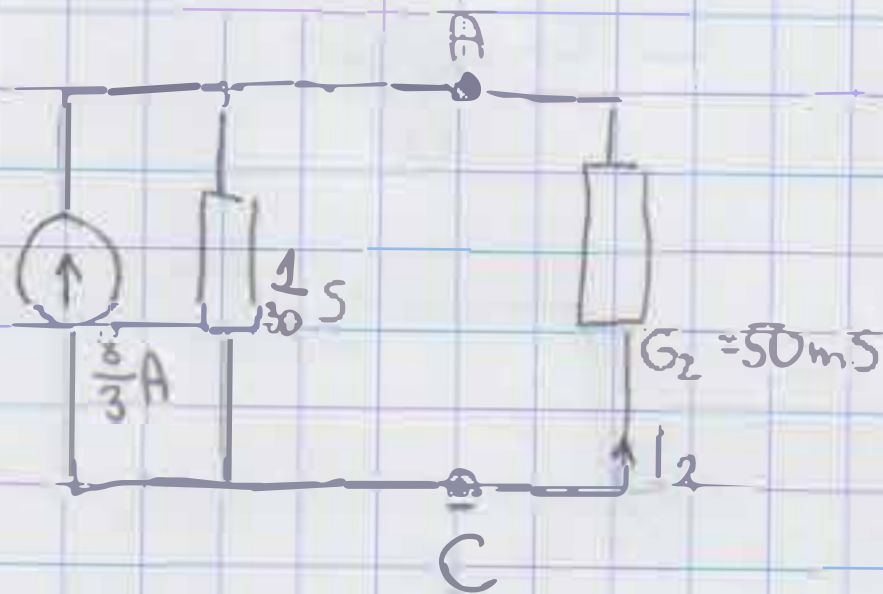
$$I_2 = -1,33A$$



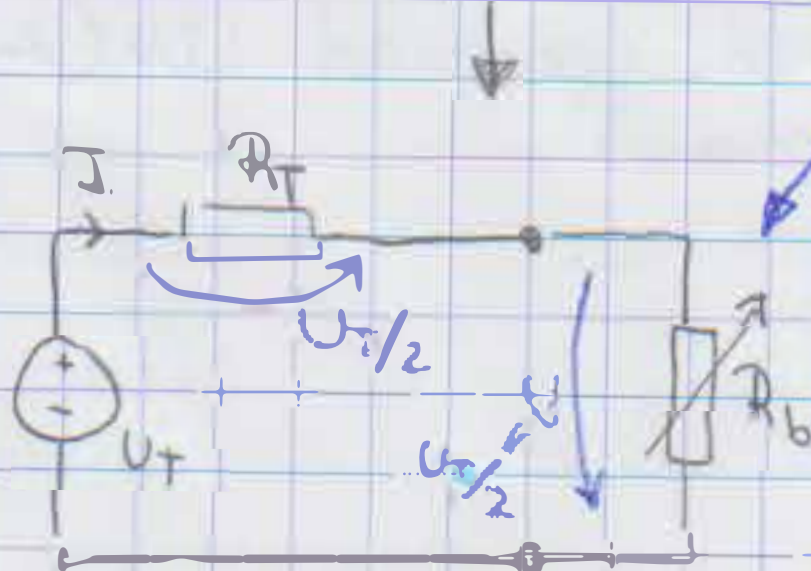
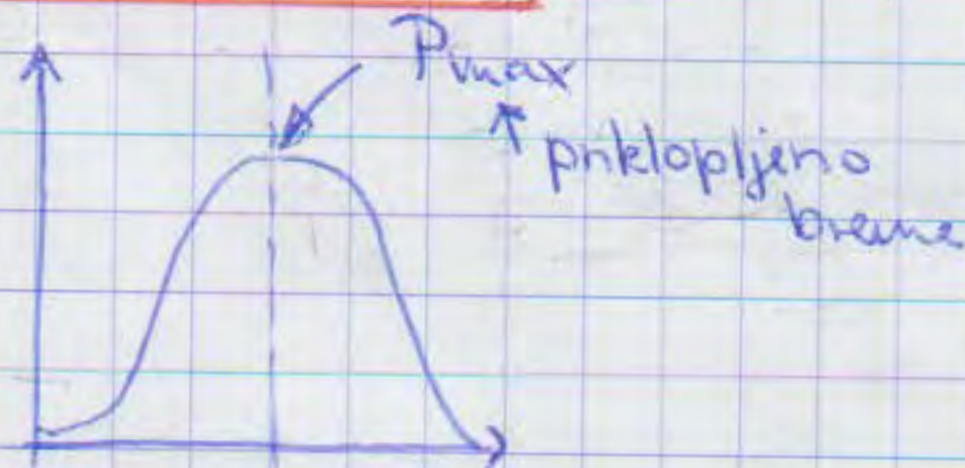
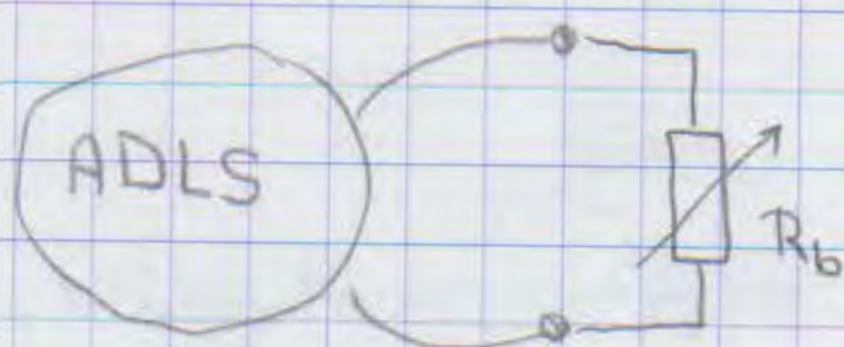
3) Nortonov nadomestni vir

$$G_N = \frac{1}{R_T} = \frac{1}{30} S$$

$$I_N = \frac{U_T}{R_T} = \frac{80}{30} A$$



5) Stavak maksimalne moči / prilagoditev



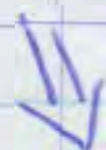
$$P_b = U \cdot I$$

$$1) R_b \rightarrow 0 \quad U=0 \quad I=I_k \Rightarrow P_b=0$$

$$I = \frac{U_T}{R_T + R_b}$$

$$2) R \rightarrow \infty \quad U=U_{os} \quad I=0 \quad P_b=0$$

$$U = U_T - R_T \cdot I$$



↓ vstavimo

$$P_b = (U_T - R_T I) \cdot I \xrightarrow{\text{odvajamo}}$$

P_{bmax} :

$$\frac{dP_b}{dI} = 0 = -R_T I + (U_T - R_T I) \cdot 1$$

$$2R_T I = U_T$$

$$I = \frac{U_T}{2R_T}$$



$$I = \frac{U_T}{R_T + R_b}$$

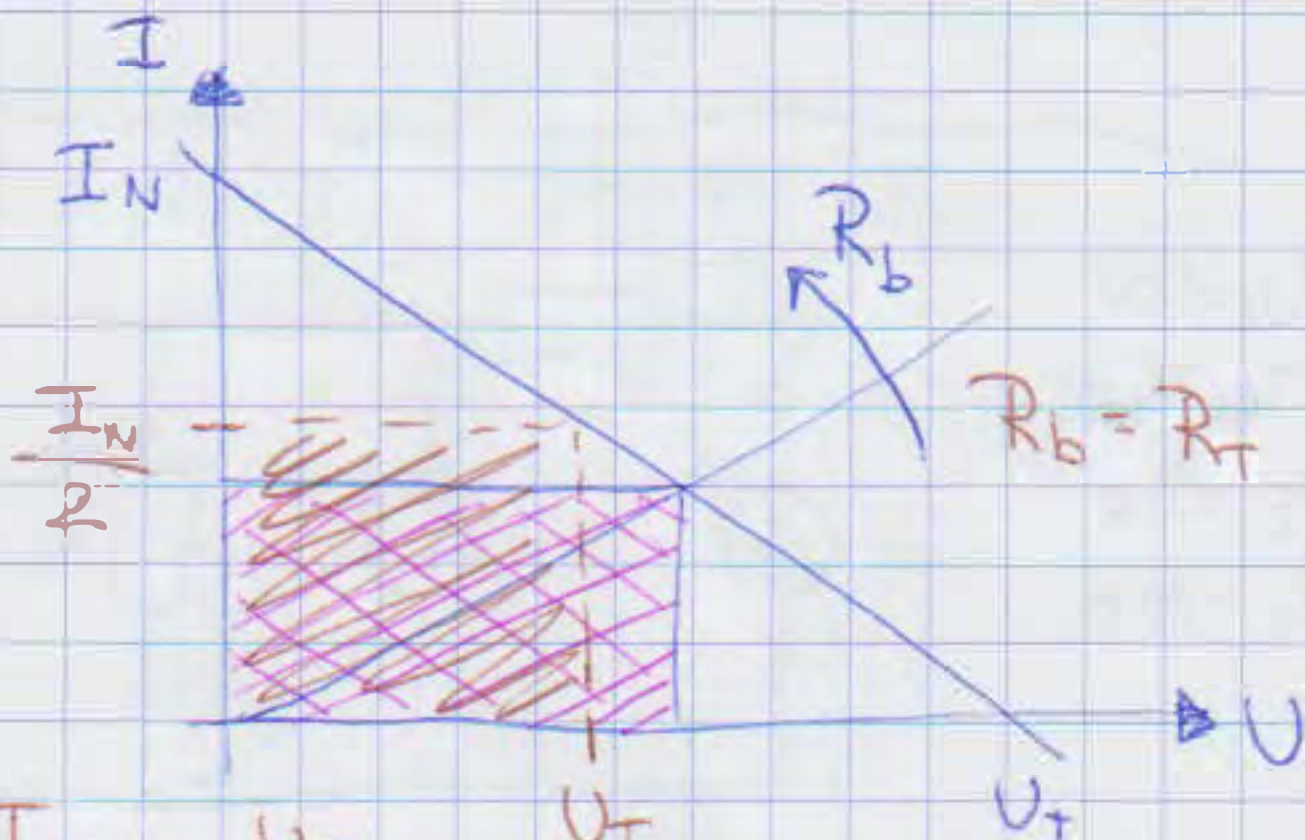
če je $R_T = R_b \Rightarrow 2R_T$

P_{bmax} : $R_b = R_T$ - prilagojeno frekvenca

$$P_b = \frac{U_T^2}{R_b} = \frac{(U_T/2)^2}{R_T} = \frac{U_T^2}{4R_b} = P_{bmax}$$

$$P_{bmax} = R_b = R_T = 30 \Omega$$

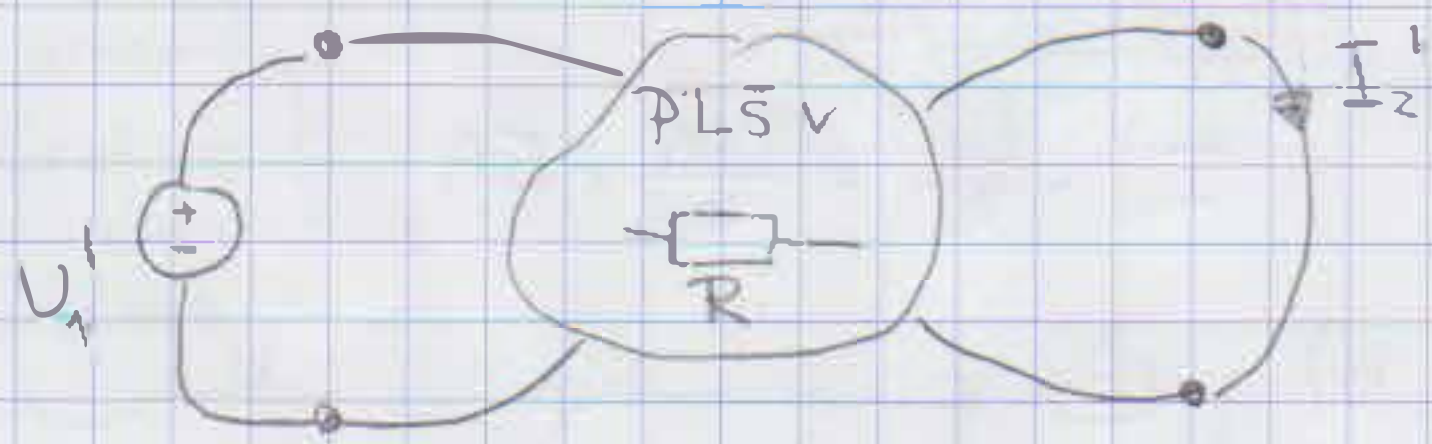
$$P_{bmax} = \frac{80^2}{4 \cdot 30} = 53,3 W$$



$$P_{bmax} = \frac{I_N}{2} \cdot \frac{U_T}{2}$$

6) Reciprocnost

Pasivno, linearno, štiripolno (dvovhodno) vezje

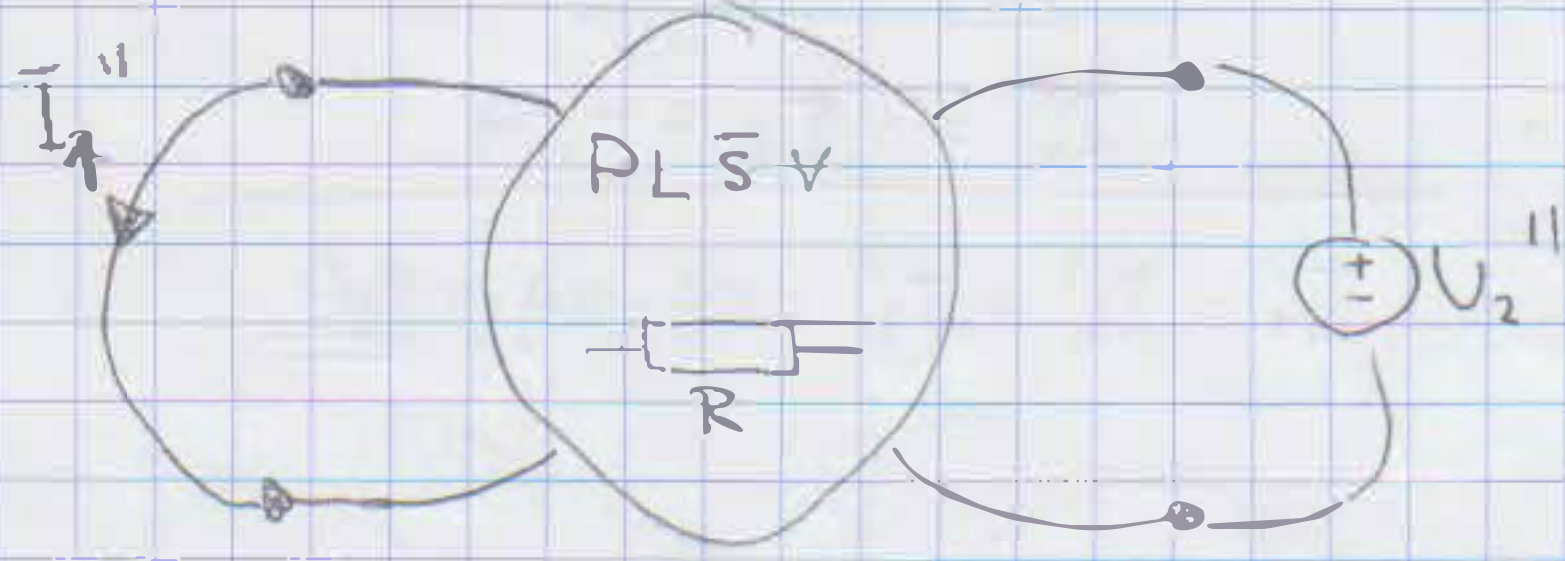


velja za napetostne generatorje

$$\frac{U_1'}{I_2'} = \frac{U_2''}{I_1''}$$

za tokovni generator

$$\frac{U_2'}{I_1'} = \frac{U_1''}{I_2''}$$



↳ Superpozicija

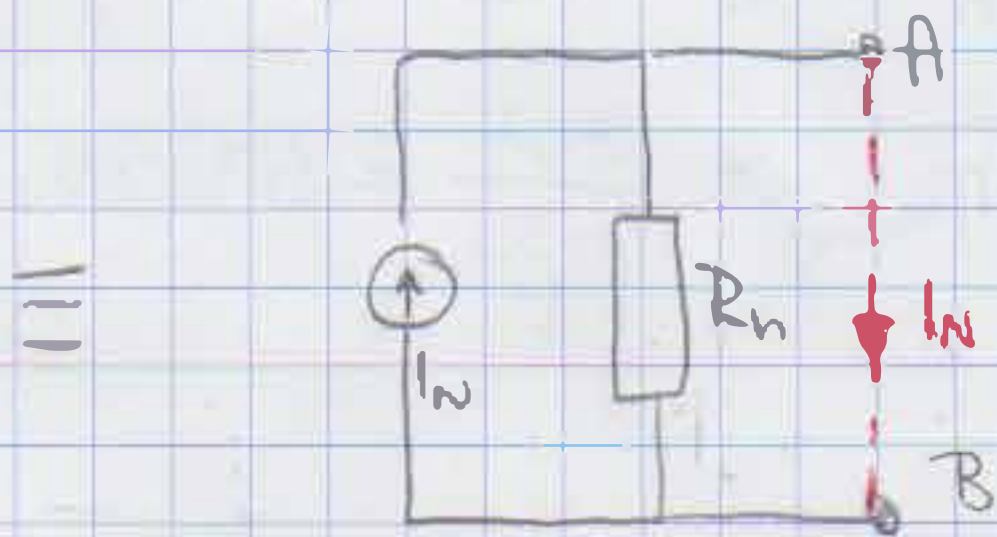
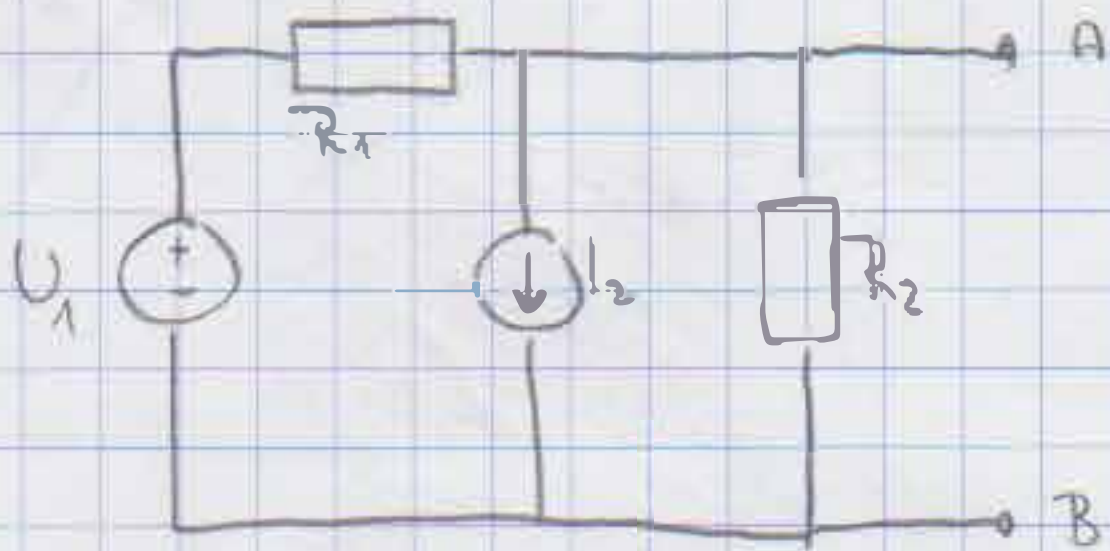
- izklopimo vse vire (razen enega)
če sta zap. (2) oba pustimo

↳ Nadomestitev

↳ Tellegenov teorem

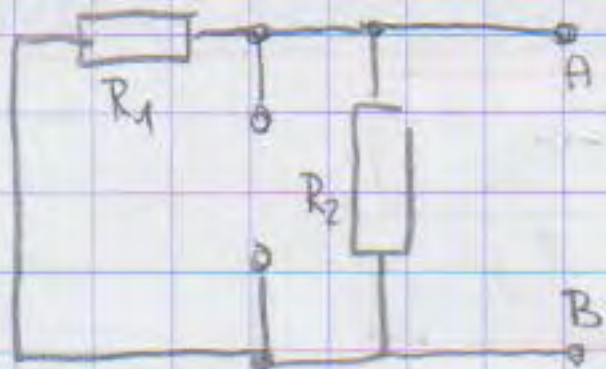
$$\sum P_{\text{virov}} = \sum P_{\text{uporov}}$$

$$\begin{aligned} U_1 &= 20V \\ R_1 &= 4\Omega \\ I_2 &= 2A \\ R_2 &= 12\Omega \end{aligned}$$



① Določimo R_n

- izklopimo vse vire

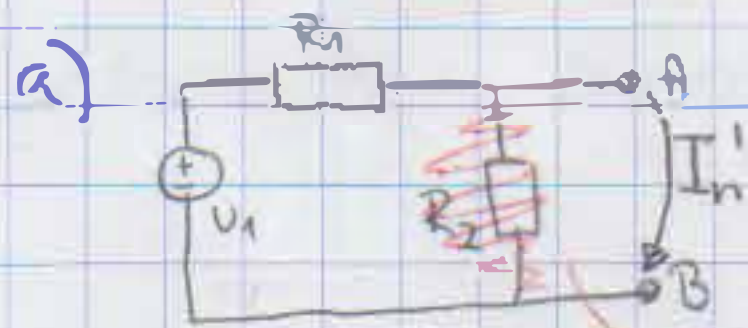


$$R_n = R_1 \parallel R_2$$

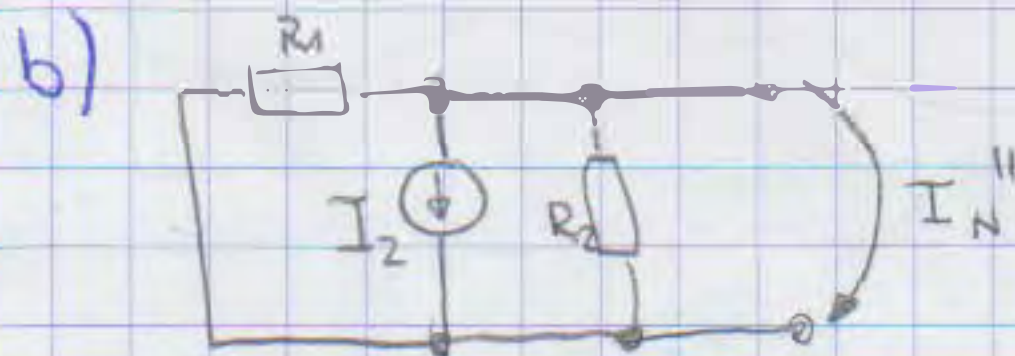
$$R_n = \frac{R_1 \cdot R_2}{R_1 + R_2} = 3\Omega$$

● Določitev I_n :

- uporabimo superpozicijo



$$I_n' = \frac{U_1}{R_1} = \frac{20V}{4\Omega} = 5A$$

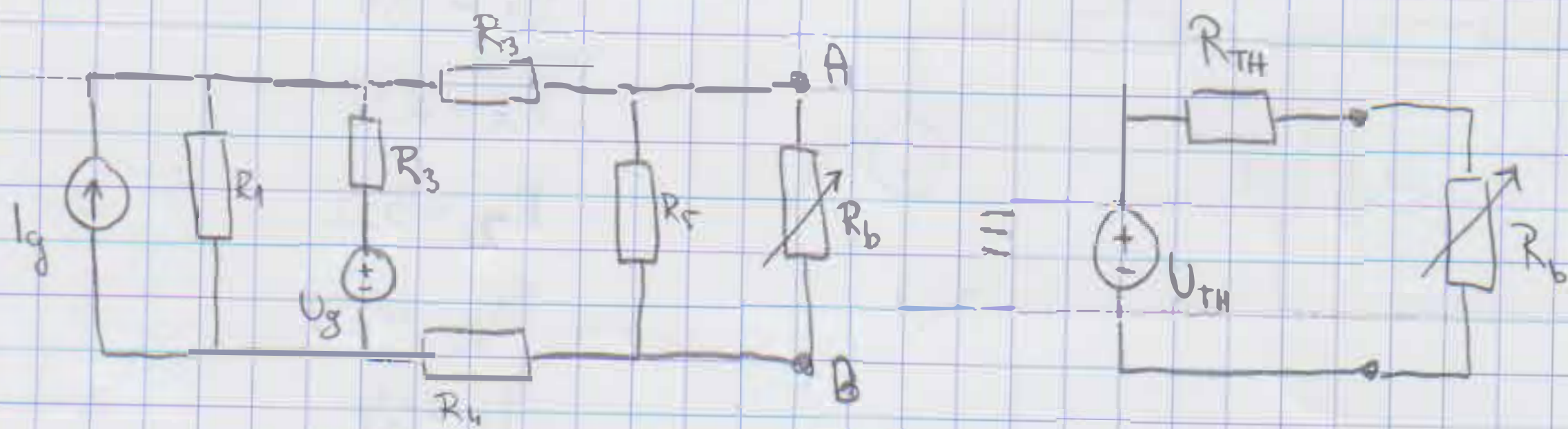


$$I_n'' = -I_2 = -2A$$

$$I_n = I_n' + I_n'' = 5A - 2A = \underline{\underline{3A}}$$

k da ga ni

zmanj ko
je kratak
stik odpade
vneshi upor



$$R_1 = R_2 = R_4 = 2\Omega$$

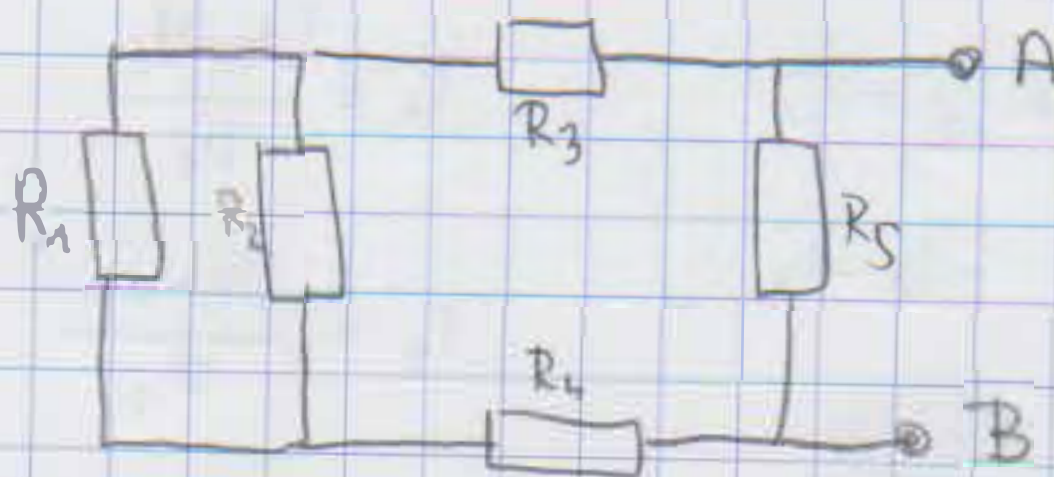
$$R_3 = 1\Omega$$

$$R_5 = 4\Omega$$

$$I_g = 1A$$

$$U_g = 12V$$

1) Določimo R_{TH}



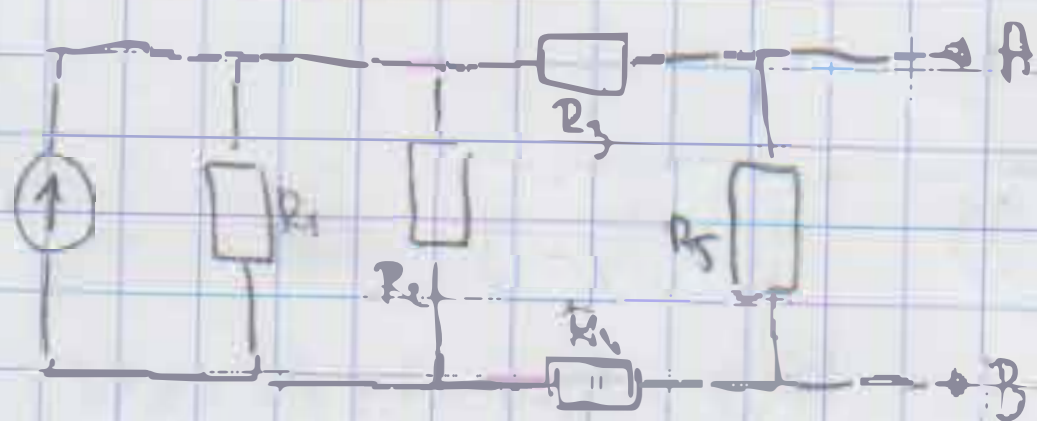
$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 1\Omega$$

$$R_{1234} = R_{12} + R_3 + R_4 = 4\Omega$$

$$R_{TH} = R_{1234} \parallel R_5 = 2\Omega$$

2) Določimo U_{TH} :

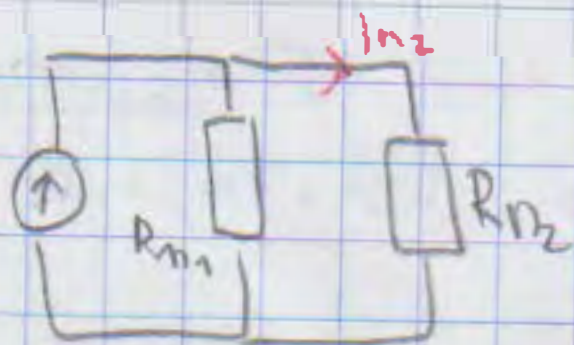
a) uporabimo superpozicijo



(+) kratek stik

$$R_{n1} R_{12} = R_1 \parallel R_2 = 1\Omega$$

$$R_{n2} R_{345} = R_3 + R_4 + R_5 = 7\Omega$$



$$I_{n2} = I_g \cdot \frac{R_{n1}}{R_{n1} + R_{n2}}$$

$$I = \frac{U_g}{R_2} = 6A$$

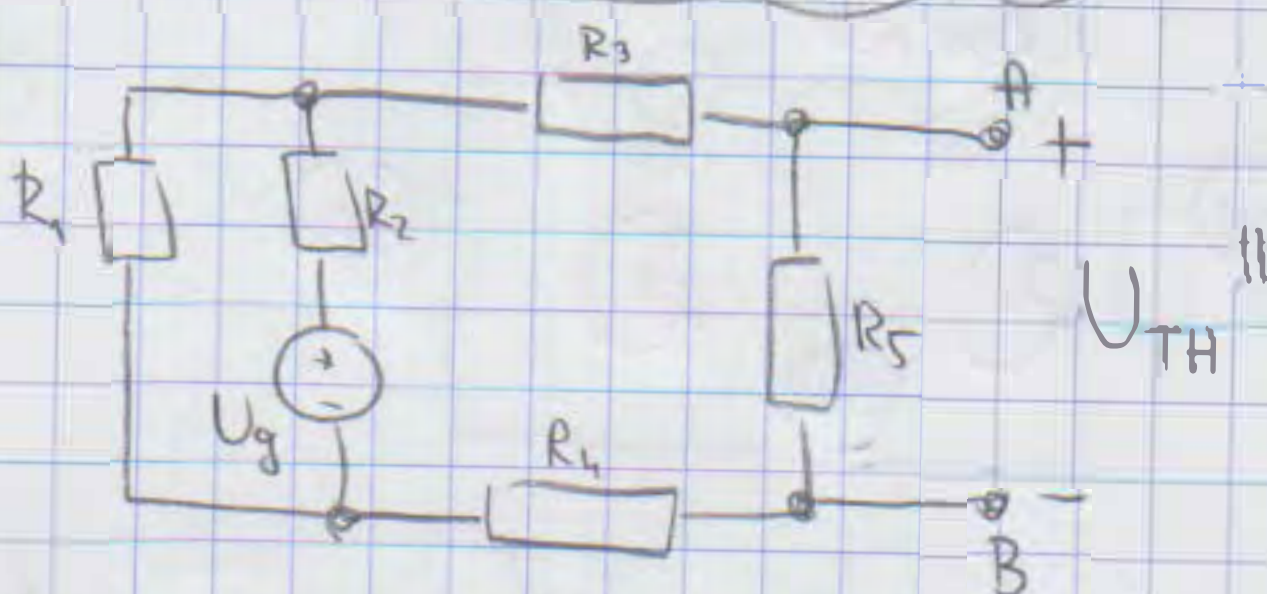
$$I_{n2} = I \cdot \frac{R_{n1}}{R_{n1} + R_{n2}} = 6A \cdot \frac{1}{8} = 0,75A$$

$$U_{TH}'' = U_{R5} = R_5 \cdot I_{n2} = 3V$$

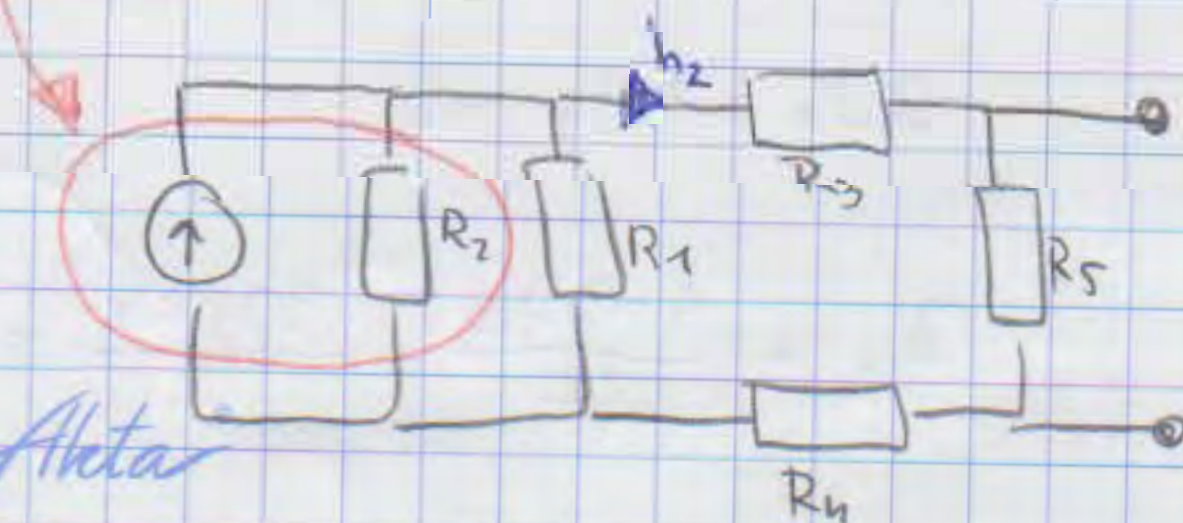
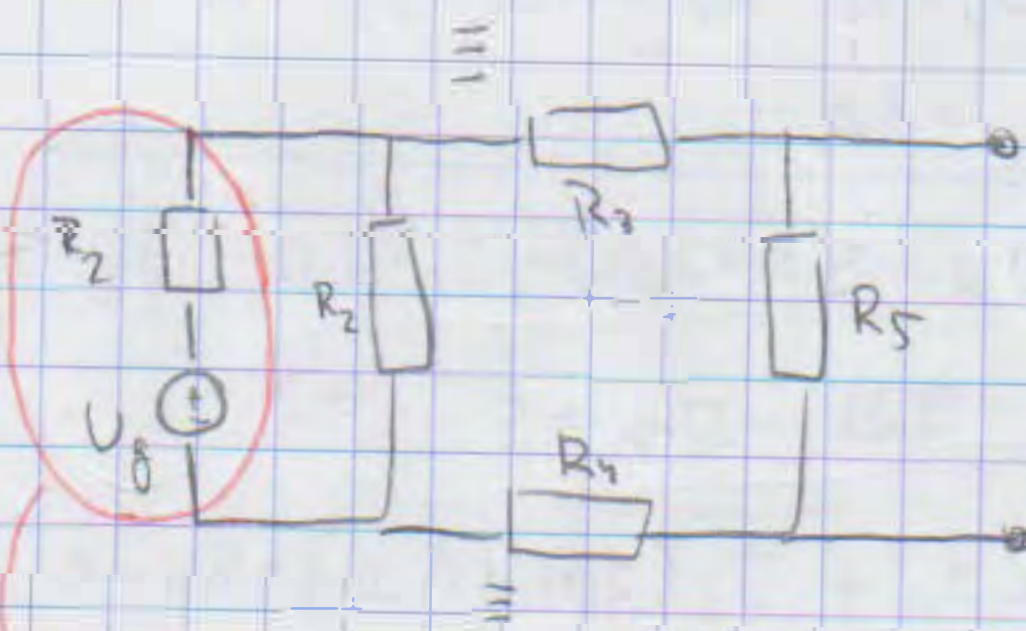
$$U_{TH} = U_{TH}' + U_{TH}'' = 3,5V$$

$$U_{R5} = I_{n2} \cdot R_5 = 0,5V$$

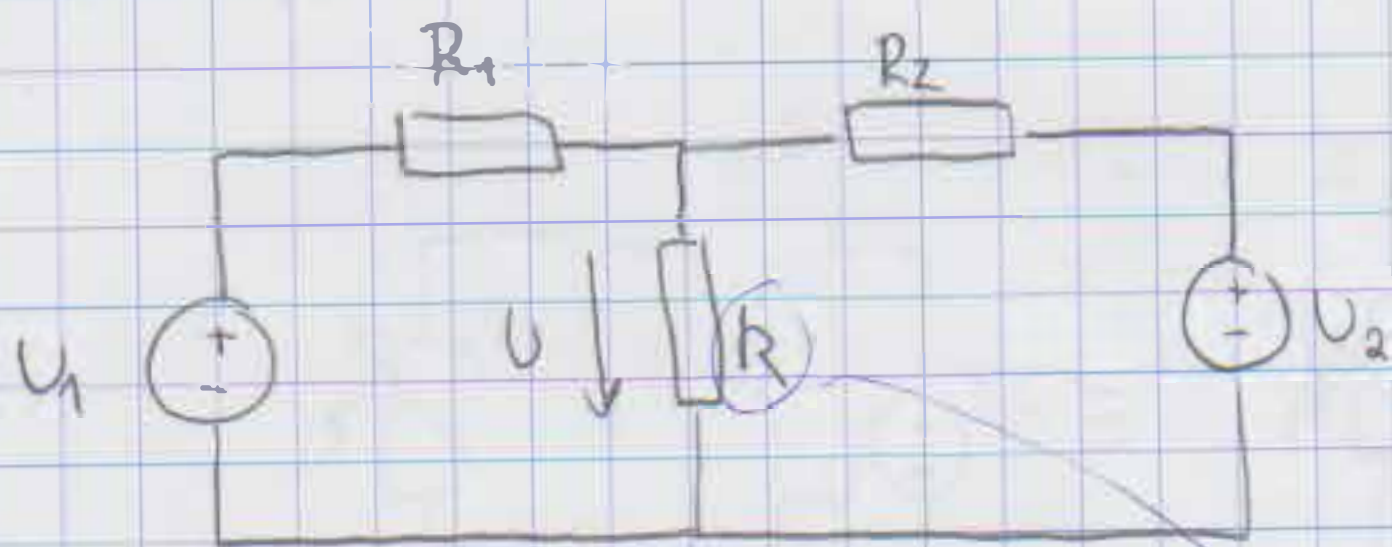
b)



$$U_{TH}'' = U_{R5}$$



Alta



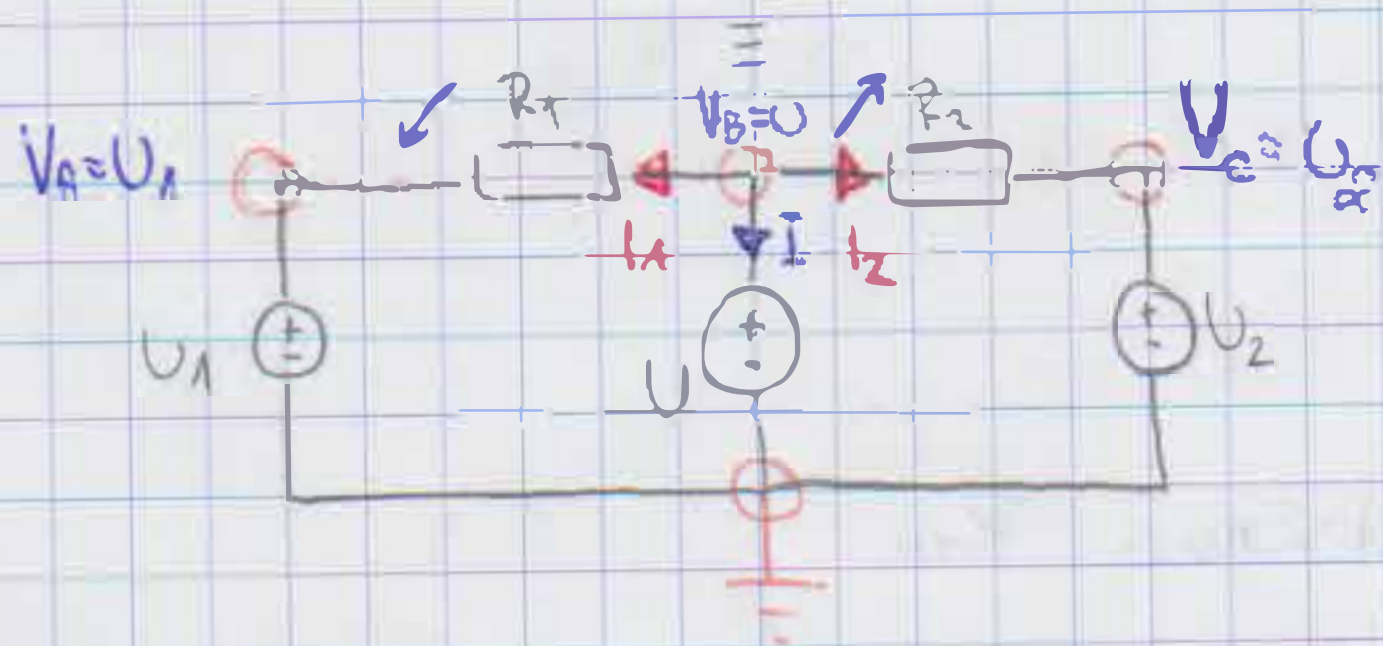
$$U_1 = 8V$$

$$R_1 = 2\Omega$$

$$R_2 = 3\Omega$$

$$U_2 = 12V$$

$$U = 6V$$



$$I + I_1 + I_2 = 0$$

$$I = -I_1 - I_2$$

$$I = -\frac{U_B - U_1}{R_1} - \frac{U_B - U_2}{R_2}$$

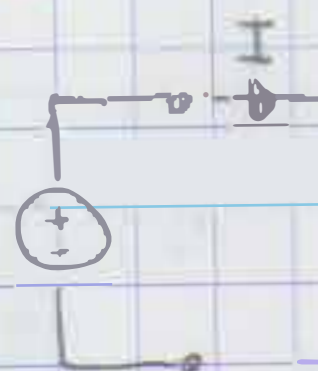
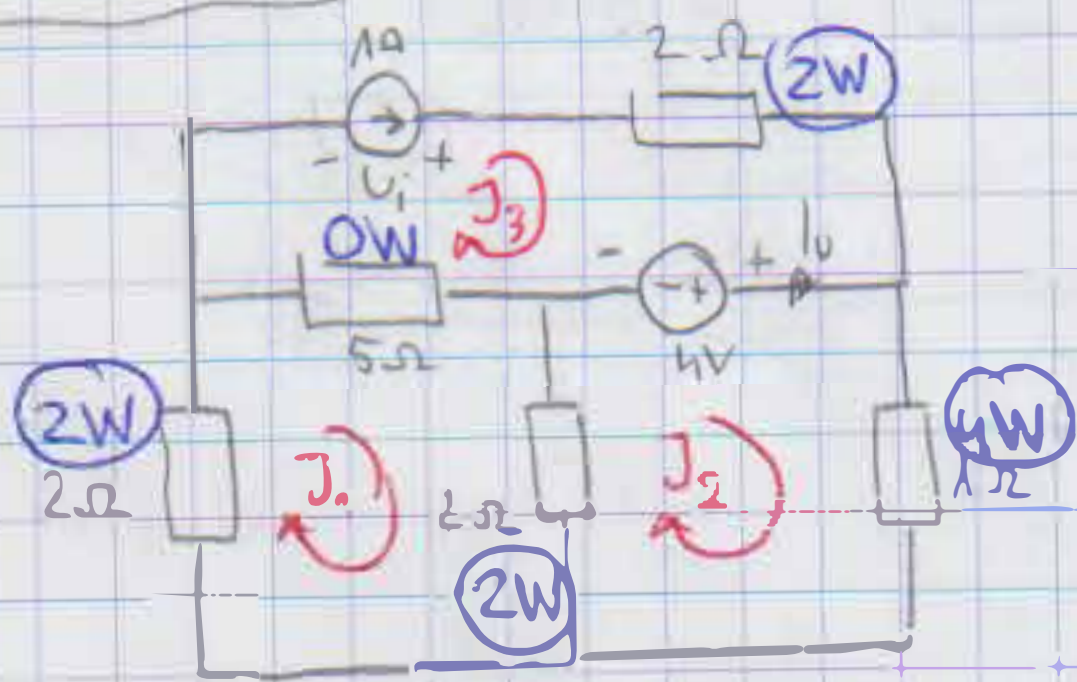
$$I = -\frac{(6-8)V}{2\Omega} - \frac{(6-12)V}{3\Omega}$$

$$I = \underline{\underline{3A}}$$

$$I_1 = \frac{U_B - U_1}{R_1}$$

$$I_2 = \frac{U_B - U_2}{R_2}$$

$$R = \frac{U}{I} = \frac{6}{3} = 2\Omega$$



$$I_0 = I_2 - I_3 = 1A$$

$$|P_0| = U \cdot I = 4V \cdot 1A = 4W$$

$$-U_1 + 2\Omega \cdot I_3 + 4V + 5\Omega (I_3 - I_1) = 0$$

$$U_I = 2\Omega \cdot 1A + 4V + 5\Omega (1 - 1) = 6V$$

$$|P_I| = U_I \cdot I = 6V \cdot 1A = \underline{\underline{6W}}$$

$$J_1: 2\Omega \cdot J_1 + 5\Omega (J_1 - J_3) + 2\Omega (J_1 - J_2) = 0$$

$$J_2: 2\Omega (J_2 - J_1) - 4V + 1\Omega \cdot J_2 = 0$$

$$J_3: J_3 = 1A$$

$$J_1: J_1 (2\Omega + 5\Omega + 2\Omega) - J_2 \cdot 2\Omega - J_3 \cdot 5\Omega = 0$$

$$9J_1 - 2J_2 = 5 \Rightarrow$$

$$J_2: -2\Omega J_1 + J_2 (2\Omega + 1\Omega) - 4V = 0$$

$$-2J_1 + 3J_2 = 4$$

$$J_1 = \frac{4 - 3J_2}{-2} = \frac{3J_2 - 4}{2} = \frac{3J_2}{2} - 2 = \underline{\underline{1A}}$$

$$N_1 \text{ uponih se troši } \sum P_{\text{vrat}} + \sum P_{\text{pot}} = 0$$

$$P_R = P_2 + P_0 = 10W \quad \text{Test: } P = 12 \cdot R$$

$$9\left(\frac{3J_2}{2} - 2\right) - 2J_2 = 5$$

$$\frac{27J_2}{2} - 18 - 2J_2 = 5$$

$$23\frac{J_2}{2} = 5 + 18$$

$$J_2 = \frac{23 \cdot 2}{23} = \underline{\underline{2A}}$$