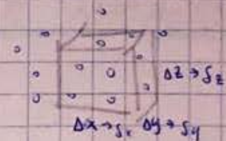
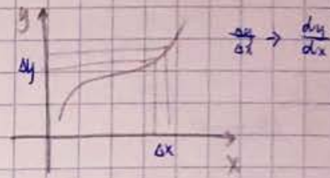
LORENTZOVA SILA
(1895)

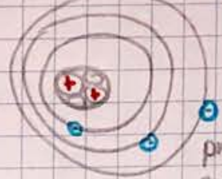


$$\boxed{\delta \vec{F}_L = \delta Q \left(\vec{E} + \vec{w} \times \vec{B} \right)}$$



Q ..... ele. naboj = elektrina

nosilci naboja → elektroni ⊖, → protoni ⊕

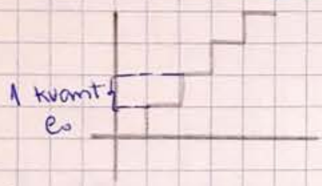


prosti nosilci naboja

presežni nosilci naboja → + ali + → -

oznaka [As] [C]

naboj je **KVANTIZIRAN** : $e_0 = 1,602 \cdot 10^{-19}$ C



$\vec{E}$ ...... vektor ele. polja oz. ele. poljske jakosti

oznaka [V/m]

$$\underbrace{\delta Q \cdot \vec{E}}_{\delta \vec{F}_e = \delta \vec{F}_c} + \underbrace{\delta Q \left(\vec{w} \times \vec{B} \right)}_{\delta \vec{F}_m = \delta \vec{F}_A}$$

električna = Coulonova magnetna = Amperova

$\vec{w}$ .... hitrost

oznaka [m/s]

V - potencial

v - volumen, dv

w - hitrost

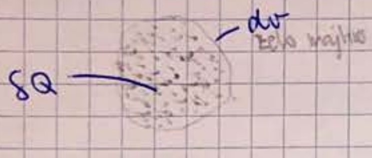
$\vec{B}$ ...... vektor magnetnega polja oz. gostote magnetnega pretoka

oznaka [T]

$$\boxed{\delta \vec{F}_L = \delta Q \vec{E} + I \, d\vec{l} \times \vec{B}}$$

Porazdelitve nabojev

1) PROSTORSKA PORAZDELITEV NABOJA



$$\boxed{\rho = \lim_{\text{"}\delta v \to 0\text{"}} \frac{\delta Q}{\delta v}}$$

prostorska gostota naboja $\left[\frac{C}{m}\right]$

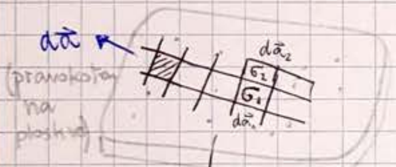
Cu
št. atomov

$n = 8{,}4 \cdot 10^{23} / m^3$
↳ št. prostih elektronov

$\rho = -e_0 \cdot n = -1{,}34 \cdot 10^{10}\ C/m^3$

za $1\,mm^3 \to -13{,}4\ C$

2) PLOSKOVNA PORAZDELITEV NABOJA



vse ploskve je potrebno sešteti

$$\boxed{\sigma = \lim_{da \to 0} \frac{dQ}{da}}$$

ploskovna gostota naboja $\left[\frac{C}{m^2}\right]$

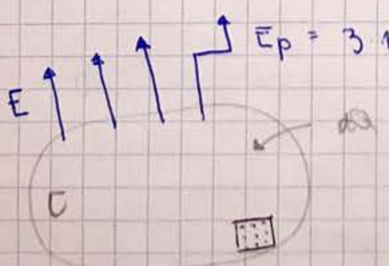
$Q = \sigma_1 da_1 + \sigma_2 a_2 + \sigma_3 a_3 + \ldots$

$Q = \sum^{k} \sigma_1 da_1$

$Q = \int dQ = \int^{A} \sigma \cdot da$

• ploskve toliko zmanjšamo, da dobimo homogeno polje

PREBOJ:



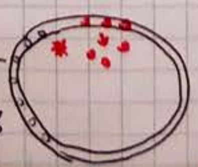
$\sigma = \varepsilon_0 \cdot E = 26{,}5\ \mu C/m^2$
↳ $8{,}85 \cdot 10^{-12}$

* da bi moral naboj iz površine v notranjost

$n_{pp} = 166 \cdot 10^{12} / m^2$

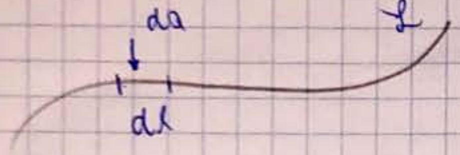
$n_p = 25 \cdot 10^{18} / m^2$

$$\boxed{\text{vsak 150000 atom ima presežen elektron}}$$



presežni prosti naboji so VEDNO NA POVRŠINI

3) LINIJSKA PORAZDELITEV NABOJA

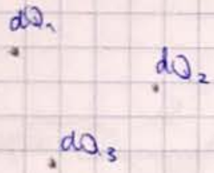


$$\boxed{q = \lim_{dl \to 0} \frac{dQ}{dl}}$$

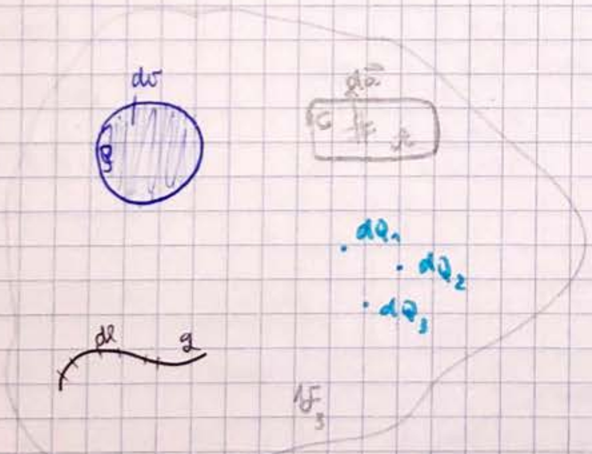
linijska gostota naboja $\left[\frac{C}{m}\right]$

$$Q = \int_L q \cdot dl$$

4) TOČKASTI NABOJ



$$Q = \sum_{i=1}^{n} dQ_i$$



Vse moramo sešteti:

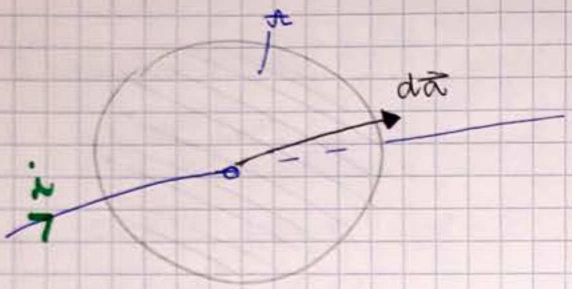
$$Q_{V_3} = \int \rho \, dv + \int_A \sigma \, da + \int_L q \, dl + \sum_{i=1}^{n} dQ_i$$

ELEKTRIČNI TOK

Tok je usmerjeno (urejeno) gibanje naboja.

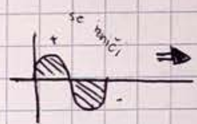
* I - časovno nespremenljiv
* i - časovno spremenljiv

* določiti moramo smer ($d\vec{a}$)
* pozitivna smer: tok gre ven



↳ Govorili bomo o nekem toku skozi ploskev $\mathcal{A}$:

↳ Vprašali se bomo koliko naboja preteče v 1s?
vendar skozi $\mathcal{A}$ v δt

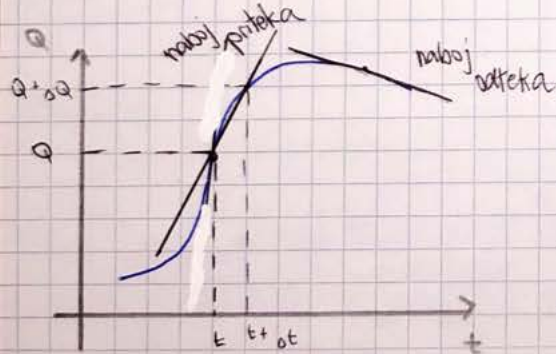


⇒ odgovor 0

$$i\Big|_{\text{skozi } \mathcal{A} \text{ v } \delta t} = \lim_{\delta t \to 0} \frac{\delta Q \text{ skozi } \mathcal{A} \text{ v } \delta t}{\delta t}$$

↓

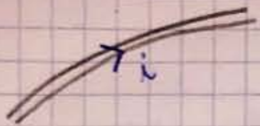
$$\boxed{i = \frac{dQ}{dt}}$$



$$i\Big|_{\Delta t}^{i} = \lim_{\Delta t \to 0} \frac{\Delta Q \text{ skozi } A \text{ v } \Delta t}{\Delta t}$$

Pretečeni tok je časovni tok

Zgled: Tok v žici



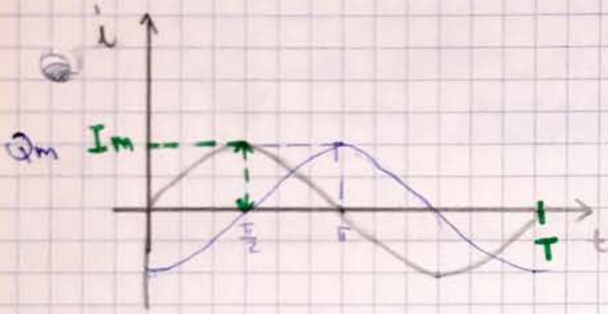
krožna frekvenca

$$\boxed{i(t) = i = I_m \sin \omega t}$$

HARMONIČNI TOK

AMPLITUDA

$\omega = 2\pi f$ $T = \frac{1}{f}$



$i = Q'$

$\Downarrow$

$dQ = i\,dt$

$\Downarrow$ integriraš!

$Q = I_m \cos \omega t \cdot \omega^{-1}(-1) + \overset{0}{Q_0}$

$Q(t) = -\frac{I_m}{\omega} \cos \omega t =$

$= \frac{I_m}{\omega} \cos(\omega t + \pi)$

$$\boxed{Q_m = \frac{I_m}{\omega}}$$

št. primer:

$f = 50\,Hz$

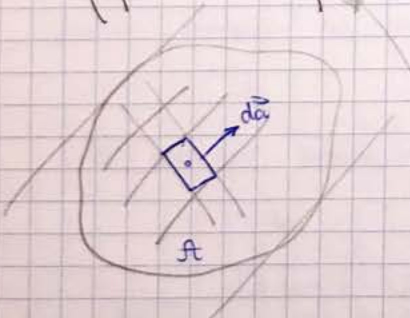
$I = 1\,A$

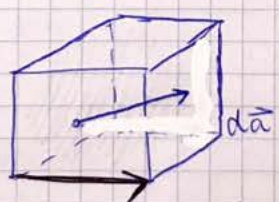
$Q_m = \frac{1\,A}{2\pi \cdot 50\,Hz} \simeq \underline{\underline{3{,}2\,mC}}$

$n = \frac{3{,}18 \cdot 10^{-3}}{1{,}6 \cdot 10^{-19}}$

GOSTOTA TOKA

(ploskovna)





$\delta \vec{l} = \underset{\text{hitrost}}{\vec{w}} \cdot \delta t$

$di = \frac{\delta Q}{\delta t} = \rho \cdot d\vec{a} \cdot \frac{d\vec{l}}{dt} =$

$= \rho \cdot \vec{w} \cdot d\vec{a}$

$$\boxed{\vec{J} = \rho \vec{w}}$$

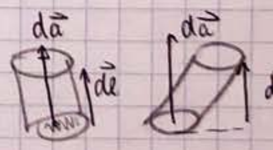
$$\boxed{di = \vec{J} \cdot d\vec{a}}$$

$\vec{J} = \frac{di}{d\vec{a}}$

Naboj znamo izračunati:

$\delta Q = \rho \cdot dv$

$\Downarrow$ izračunamo volumen



$d\vec{a} \cdot d\vec{l}$

$d\vec{a} \cdot dl \cdot \cos\varphi$

(projekcija)

$\Downarrow$

$$\boxed{\delta Q = \rho \cdot d\vec{a} \cdot d\vec{l}}$$

Zgled: Cu

$J = 4\,A/mm^2 = 4 \cdot 10^6\,A/m^2$

$\rho = 1{,}34 \cdot 10^{10}\,C/m^3$

$w_e = \frac{J}{\rho} = \underline{\underline{300\,\mu m/s}}$

Poznamo več vrst toka:

↳ KONDUKTIVNI TOK

↳ KONVEKTIVNI TOK

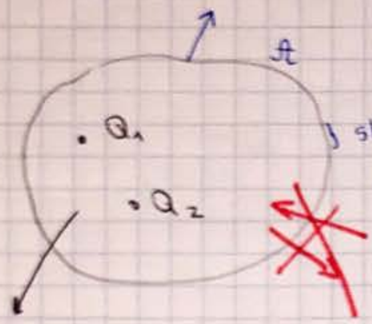
↳ POLARIZACIJSKI TOK nanaša se na pole, ki se obračajo

zaradi polov, se delec (molekula, naboj) **suka** sem in tja

↳ MAGNETIZACIJSKI TOK

težko se jih meri

ZAKON O OHRANITVI NABOJA



$\sum Q_{not.} = konst.$

* naboji ne prehajajo med ploskvami
* notranje št. nabojev je <u>KONSTANTNO</u>!

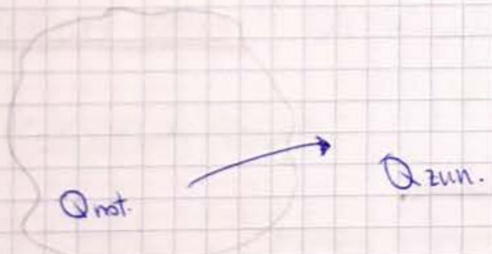
$\Downarrow$ če pustimo, da naboji <u>PREHAJAJO</u>

KONTINUITETNA ENAČBA

$$Q_{not.} + Q_{zun.} = K \quad \Big| \frac{d}{dt}$$ odvajamo

$$\frac{dQ_{not}}{dt} + \frac{dQ_{zun.}}{dt} = 0 \qquad \boxed{K' = 0}$$

$$\boxed{\frac{dQ_{not}}{dt} = -\frac{dQ_{zun}}{dt}}$$



Naboj prehaja od znotraj ven

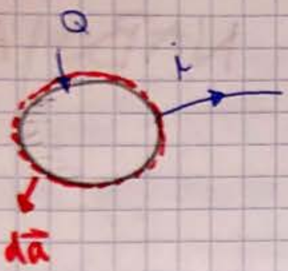
↓

št. nabojev zunaj se POVEČUJE

↓

$$i = \frac{Q_{zun}(t+dt) - Q_{zun}(t)}{t+dt-t}$$

$$i = \frac{dQ_{zun}}{dt} = -\frac{dQ_{not.}}{dt}$$



! Tok, ki odteka je enak ZMANJŠANJU NABOJA

$$\boxed{i = -\frac{dQ}{dt}}$$

KONTINUITETNA ENAČBA

i ..... tok, ki odteka

dQ ..... notranji naboji

Zdaj smo definirali, kaj ta tok pomeni.

$$i = -\frac{dQ}{dt} \quad \cancel{\Leftrightarrow} \quad i = \frac{dQ}{dt}$$

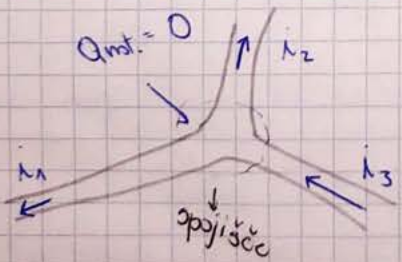
Vse dele, skozi katere naboj odtega, je potrebno SEŠTETI.

Ko govorimo o kontinuitetni enačbi, moramo imeti ZAPRT PROSTOR, torej SKLENJENA PLOSKEV.

Tedaj govorimo tudi o ZAPRTEM VOLUMNU

$$\boxed{\oint \vec{J} \cdot d\vec{a} = -\frac{d}{dt} \int_V \rho \cdot dV}$$

I. KIRCHHOFFOV ZAKON



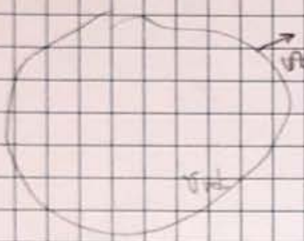
$$\sum_{k=1}^{n} i_k = 0$$

$$\sum i_{odtekajoči} - \sum i_{pritekajoči} = 0$$

$$\boxed{\sum i_{odt.} = \sum i_{prit.}}$$

RAZREDI POLJ

$$\oint \vec{J} \cdot d\vec{a} = -\frac{d}{dt} \int_{\text{Q}_{not}} \rho \, dv$$ } naboj, ki odteka ven



1) Elektro statika $J = 0$ levi del enačbe

$$0 = -\frac{d}{dt} \int_{\text{Q}_{not}} \rho_{not} \cdot dv$$

$\Downarrow$ kdaj je $f' = 0$?

$$\boxed{\rho_{not} = konst.}$$

Notranji naboj se časovno NE spreminja, je PROSTORSKA FUNKCIJA.

KONDENZATORSKA VEZ

2) TOKOVNA POLJA

$J = konst.$ (Imeli bomo enosmerni tok)

$$\oint \vec{J} \, d\vec{a} = k = -\frac{d}{dt} \int \rho_{not} \, dv$$

$\Downarrow$ odvajamo

$$k_1 t + k_2 = \int \rho_{not} \cdot dv$$

$\downarrow$

v praksi uporabljamo

$\boxed{k_1 = 0} \rightarrow \boxed{k_2 = \int \rho_{not} \, dv}$

da ne more iti v neskončnost

UPOROVNA VEZJA

3) MAGNETOSTATIKA

I = konst.

Prišli bomo do statičnega mag. polja

4) DINAMIKA

↳ Dovolili bomo, da tok steče

$\vec{J}(T, t)$

↳ Pridemo do indukcije

↳ Spoznavali bomo vezja čas. spremenljivih količin

PREHODNI POJAVI   HARMONIČNA VEZJA

ELEKTROSTATIKA

Coulombova sila (1785)

↳ Ukvarjal se je s silami med dvema točkastema naboja.

dQ_1 dQ_2

Ugotovitve

↳ Sili sta medseboj enaki po velikosti

↳ Po smeri sta si NASPROTNI !

↳ ⊕⊕ , ⊖⊖ ODBOJNE SILE

⊕⊖ , ⊖⊕ PRIVLAČNE SILE

↳ Sila je sorazmerna $dQ_1 \cdot dQ_2$

↳ Sila je obratnosorazmerna $\frac{1}{R^2}$

$$\Rightarrow \quad F_e \propto \frac{dQ_1 \cdot dQ_2}{R^2}$$

$y \propto x$

$y = kx$ konstanta $\frac{1}{4\pi\varepsilon_0}$

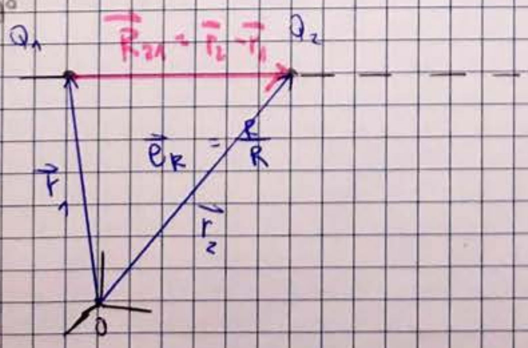
$\left(\frac{\vec{R}}{R}\right)$ enotski vektor

$$\boxed{\vec{F_e} = k \frac{dQ_1 \, dQ_2}{R^2} \vec{e_R}}$$

$$F_g = -G \frac{dm_1 \, dm_2}{R^2} \vec{e_R}$$

* Nekdo je povzročitelj, nekdo pa opazovani objekt !

ker premici skozi Q_1 in Q_2 ležijo sile, zato bo za nas pomemben vektor $\vec{R}_{21}$



1. Dobiti moramo, kje sta naboja ! $\vec{r}_1, \vec{r}_2$ (= vektorji)

2. Dobimo koordinatno izhodišče

$$\vec{F}_{e1} = k \frac{Q_1 Q_2}{R_{12}^2} \frac{\vec{R}_{12}}{R_{12}}$$

$$\vec{F}_{e2} = k \frac{Q_1 Q_2}{R_{21}^2} \frac{\vec{R}_{21}}{R_{21}}$$

$$\boxed{\vec{F}_{e1}^{(2)} + \vec{F}_{e2}^{(1)} = \vec{0}}$$

* sili sta vzajemni

* Navadno bomo imeli več nabojev v prostoru, zato bomo računali silo na Q kot vsoto DELOVANJA VEČIH SIL → superpozicija

Aktar

$\boxed{k = \frac{1}{4\pi\varepsilon_0}}$ v „SI"

Polni kot ... 4π ... **PROSTORSKI** ... $4\pi R^2$

Polni kot ... 2π ... PLOSKOVNI ... $2\pi R$

Govorimo o naboju, ki je v notranjosti, in ta naboj se pretaka skozi površino:

$\boxed{\varepsilon_0}$ - influenčna konst.

INFLUENCA:

presežek + na eni strani in presežek - na drugi strani

ε_0 ... dielektričnost

μ_0 ... permeabilnost

Obe sta povezani preko svetlobne hitrosti: $c_0 = 299792458$ m/s ($\simeq 3\cdot 10^8$ m/s)

$$\boxed{\varepsilon_0 \mu_0 c_0^2 = 1}$$

$\boxed{\mu_0 = 4\pi \cdot 10^{-7}\ Vs/Am}$ → $\varepsilon_0 = \frac{c_0^2}{\mu_0} =$ $\boxed{\varepsilon_0 = 8{,}8502 \cdot 10^{-12}\ As/Vm = \frac{10^{-9}}{9\cdot 4\pi}\frac{As}{Vm}}$

V času Coulumba je bila $k = 1$, zato se je bila to podlaga za

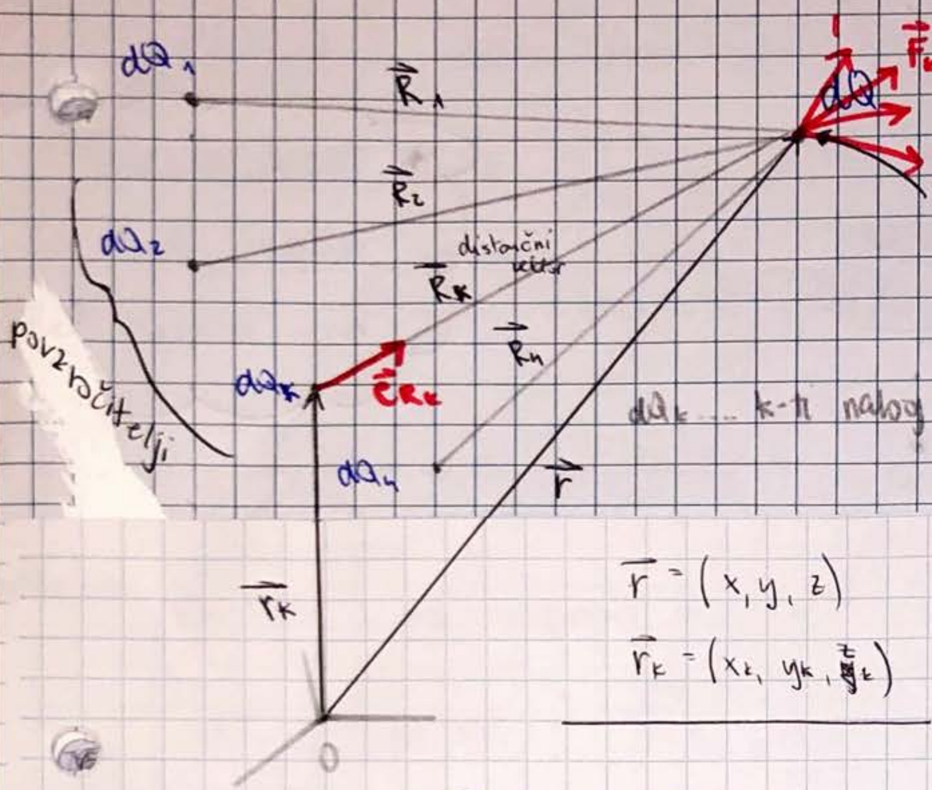
STATIČNI COULOMB

μ_0 in ε_0 sta povezani preko svetlobe, ker je sama ELEKTROMAGNETNI POJAV. (valovanje)

↳ znajde se v elektr. polju

↳ znajde se v mag. polje

SILA V SOSEŠČINI DRUGIH N NABOJEV



$$d\vec{F}_{m1} = \frac{dQ_1\, dQ}{4\pi\varepsilon_0 R_1^2}\cdot\underbrace{\frac{\vec{R}_1}{R_1}}_{\vec{e}_{R1}}$$

$$d\vec{F}_{mk} = \frac{dQ_k\, dQ}{4\pi\varepsilon_0 R_k^2}\cdot\frac{\vec{R}_k}{R_k}$$

$$d\vec{F}_{mk} = \frac{dQ_k\, dQ}{4\pi\varepsilon_0 R_k^3}\cdot\vec{R}_k \qquad \frac{\vec{R}_k}{R_k^3}$$

↓

sila na naboj Q zaradi Q_k

$$\vec{r} = (x, y, z)$$

$$\vec{r}_k = (x_k, y_k, z_k)$$

$$\vec{R}_k = \vec{r} - \vec{r}_k = (x - x_k,\ y - y_k,\ z - z_k)$$

$$|\vec{R}_k| = R_k = \sqrt{(x-x_k)^2 + (y-y_k)^2 + (z-z_k)^2}$$

Vse sile seštejemo v celotno silo:

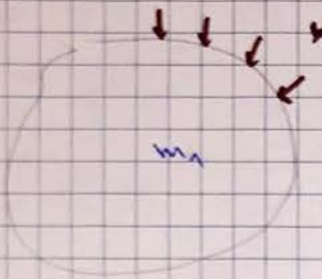
$$d\vec{F} = \sum_{k=1}^{n} \delta\vec{F}_k = \sum_{k=1}^{n} \frac{dQ\, dQ_k}{4\pi\varepsilon_0}\cdot\frac{\vec{R}_k}{R_k^3}$$

bodi pozoren →, gre za vektorsko količino ! (ne skalarno)

$$d\vec{F} = \frac{dQ}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{dQ_k}{R_k^3}\cdot\vec{R}_k$$

VEKTOR ELE. POLJSKE JAKOSTI $\vec{E}$

Razmišljanje:



zemlja

$$F_g = G\frac{m_1 m_2}{R^2}$$

dali smo v neke konstante

* Povsod na telo delujejo sile → kar bomo imenovali **POLJE**

$$F_g = g \cdot m$$

(če normiramo gravitacijsko silo, torej jo razkomenimo mase, dobimo pospešek. Enako bomo storili z $\vec{F_e}$)

Električno silo bi radi normirali (jo delimo) z nabojem:

$$\boxed{\vec{E} = \frac{d\vec{F_e}}{dQ}}$$

$\vec{E}$ ... vektor ele. polj. jakosti

$\vec{E}(T)$ → naboj q v točki T

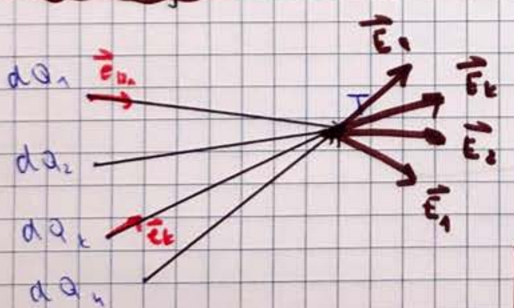
Definicija:

Vektor ele. poljske jakosti je vektor sile, normiran z množino ele. naboja. torej je razbremenjena naboja!

$$\vec{E} = \frac{d\vec{F_e}}{dQ} = \frac{dQ}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{dQ_k\vec{R_k}}{R_k^3} = \sum_{k=1}^{n}\delta\vec{E}_k(T)$$

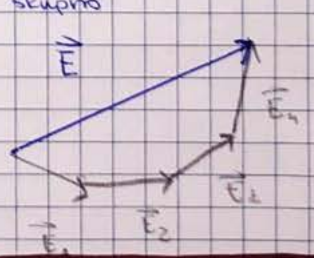
$$\boxed{\delta\vec{E}_k(T) = \frac{dQ_k\vec{R_k}}{4\pi\varepsilon_0 R_k^3}} = \frac{d\vec{F_k}}{dQ}$$

* ELE. POLJE V SOSEŠČINI N NABOJEV



→ vektorsko → seštejemo

skupno



$$\boxed{E(T) = \int d\vec{E}(T)}$$

$$\boxed{\vec{E}(T) = \frac{1}{4\pi\varepsilon_0}\int\frac{dQ(T')\vec{R}}{R^3}}$$

Coulombovo polje

Če seštevamo silo, lahko tudi vektorje ele. polj. jakosti

Električna poljska jakost (normirana sila) je integral delnih prispevkov.

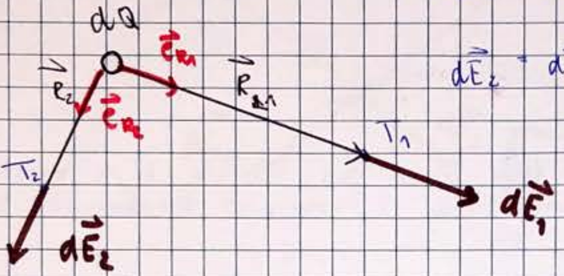
* ELE. POLJE TOČKASTEGA NABOJA

(notranjost in površino naboja pustimo pri miru)

Pomembno je, da se v prostoru okoli naboja dogovorimo za točke, v katerih bomo gledali ele. polja:

$$d\vec{E}_1 = d\vec{E}(T) = \frac{dQ}{4\pi\varepsilon_0 R^2}\cdot\frac{\vec{R}_1}{R_1}$$

$$d\vec{E}_2 = d\vec{E}(T_2) = \frac{dQ}{4\pi\varepsilon_0 R^2}\cdot\frac{\vec{R}_2}{R_2} \qquad \vec{e}_R$$



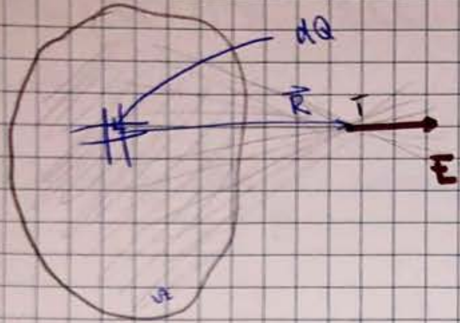
radialna smer

Puščice vizualizirajo **POLJE** (vektorji)

Vektorji niso spremenljivi, vendar lahko pa povemo, da če postavimo tja nek naboj, bo nanj začela delovati ele sila. → ustvari se polje.

ELEKTRIČNO POLJE je stanje prostora, ko na nek delec v tem prostoru deluje električna sila.

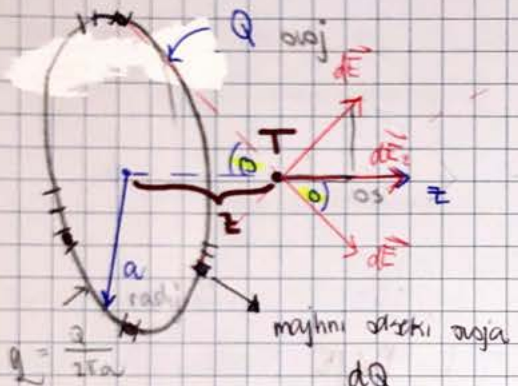
Kaj če imamo naelektreno ploskev?



Ploskev A razdelimo na več majhnih # in prispevke vsake te ploskvice seštejemo v SKUPNI VEKTOR ELE. POLJ. JAKOSTI

$$\vec{E} = \int d\vec{E}$$

$\vec{E}$ v osi naelektrenega obroča pistan, obroj



* Vzeli bomo par nabojev, ki se med seboj seštejeta v vodoravno komponento $\vec{E}$ v smer z.

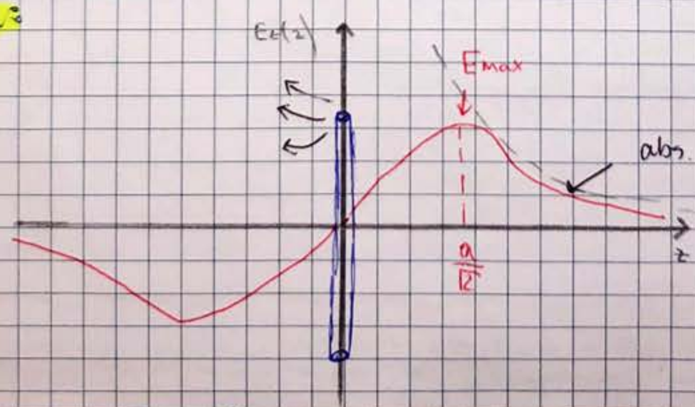
$$d\vec{E}_z = \underbrace{\frac{dQ}{4\pi\varepsilon_0 R^2}}_{dE} \cdot \underset{\frac{z}{R}}{\cos\beta} = \frac{z \cdot dQ}{4\pi\varepsilon_0 R^2}$$

$$E_z(T) = \int dE_z = \int_Q \frac{z \cdot dQ}{4\pi\varepsilon_0 R^2} = \frac{z}{R^3 \cdot 4\pi\varepsilon_0} \int_Q dQ$$

$$E_z(T) = \frac{z \cdot Q}{4\pi\varepsilon_0 R^3} = \frac{Qz}{4\pi\varepsilon\sqrt{a^2+z^2}} \to \to \to \boxed{\vec{E}(T) = \frac{Qz \cdot \vec{e}_z}{4\pi\varepsilon_0(\sqrt{a^2+z^2})^3}}$$

$$R = \sqrt{a^2+z^2}$$

Potek polja:



Naelektrena „premica"

V mislih imamo neko žico, ki nam deluje kot premica.

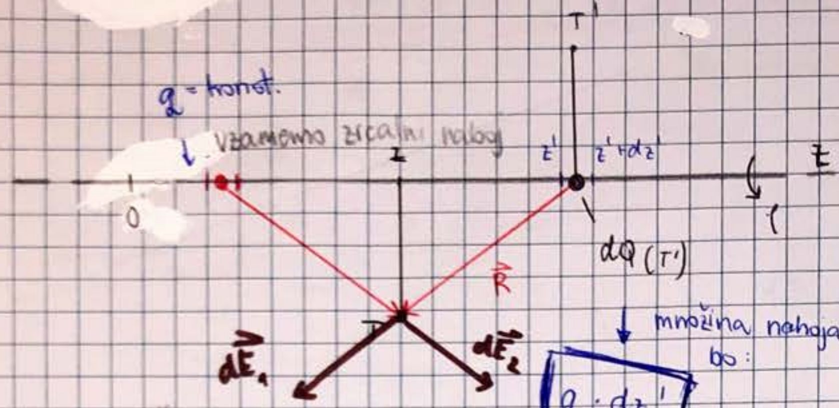
* Gostota naboja je največkrat konstantna

Tako bomo sami vzeli

* os izberemo vzdolž žice

ϱ = konst.

↓ vzamemo zrcalni naboj



↓ množina naboja bo:

$$\boxed{\varrho \cdot dz'}$$

gostota · dolžina

če je q pozitiven, bo $\vec{E}$ stran od njega

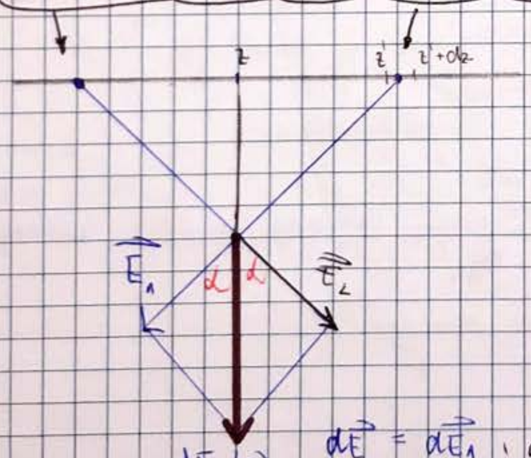
!!! če želimo priti do kon. rezultata moramo zajeti VSE NABOJE

$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \int\limits_{(Q\,cel.)} \frac{\vec{R}\, dQ(r')}{R^3}$$

Vse vektorje je treba sešteti:

! * seštevali bomo dva simanda, katerih vsota kaže stran od žice:

da ne rabimo seštevati celotne pahljače vektorjev



$$d\vec{E} = d\vec{E}_1 + d\vec{E}_2$$

$$\vec{E}_1 + \vec{E}_2 = dE_\varrho\, \vec{e}_\varrho$$ → radialna komp. · smernik

$$dE_\varrho(r) = \frac{\varrho\, dz'}{4\pi\varepsilon_0 R^2} \cdot 2 \cdot \cos\alpha =$$

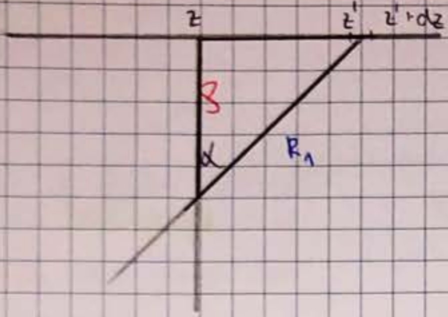
kot med njima

$$= \frac{\varrho}{2\pi\varepsilon_0} \cdot \frac{\cos\alpha\, dz}{R^2} =$$

izrazili s kotom

* povezane količine → določiti se moramo kateri bomo

spremenljivko, ki jo bomo spreminjali, drugi, da



$$z' - z = \rho \cdot \tan\alpha \qquad R_1 = \frac{\rho}{\cos\alpha}$$

odvajamo po α

$$dz' = \frac{\rho}{\cos^2\alpha}\, d\alpha$$

vstavimo v enačbo na prejšnji strani

(Mi smo se odločili, da razdaljo $z'-z$ in R_1 izrazimo s kotom α)

$$= \frac{q}{2\pi\varepsilon_0} \cdot \frac{\cos\alpha \cdot dz}{R^2} =$$

$$= \frac{q}{2\pi\varepsilon_0} \cdot \frac{\cos\alpha \cdot \frac{\rho}{\cos^2\alpha}\, d\alpha}{\frac{\rho^2}{\cos^2\alpha}}$$

$$= \frac{q}{2\pi\varepsilon_0\,\rho} \cdot \cos\alpha\, d\alpha$$

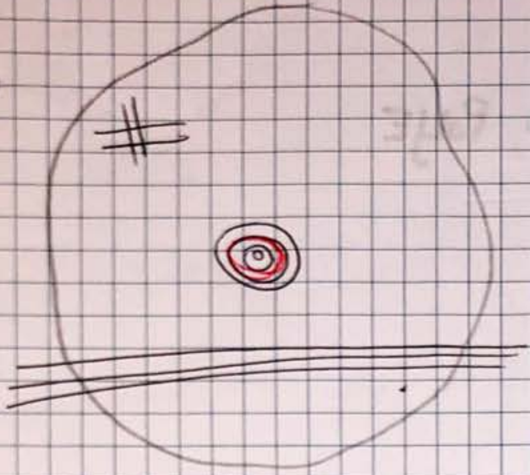
$$E_\rho(r) = \int dE_s(r) = \int_0^{\frac{\pi}{2}} \underbrace{\frac{q}{2\pi\varepsilon_0\,\rho}}_{\text{konst.}} \cos\alpha\, d\alpha = \frac{q}{2\pi\varepsilon_0\,\rho}$$

$$\boxed{E_\rho(r) = \frac{q}{2\pi\varepsilon_0\,\rho}}$$

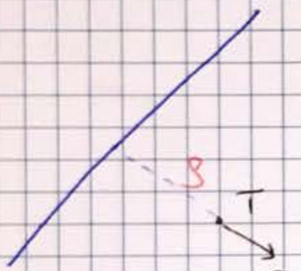
$\int$ → sprehodimo se po vseh nabojih

Ko ima naelektreno žico vidimo, da jakost el. polja upada ~~[illegible]~~ s prvo potenco.

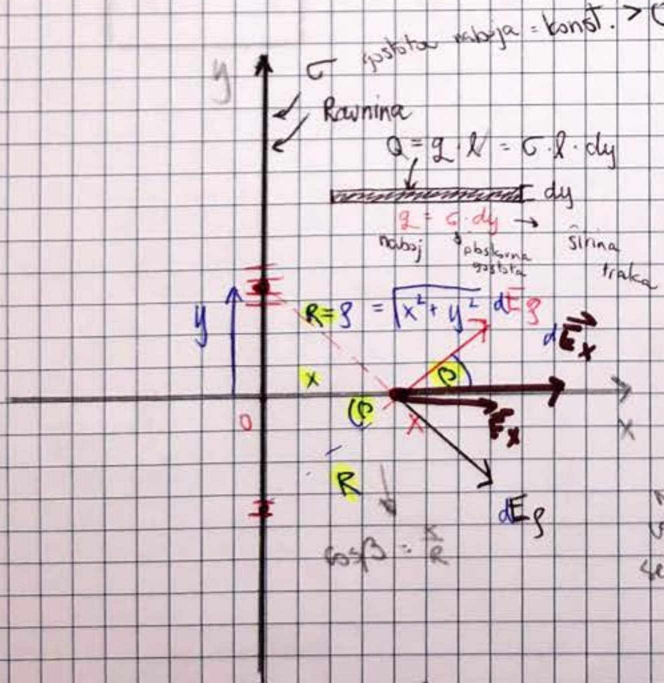
Naelektrena ravnina - E polje



Ravnino razdelimo na:
1. Na majhne delčke
2. Na majhne kolobarje
3. Na majhne trakove



$$E_\rho = \frac{q}{2\pi \varepsilon_0 \rho}$$



$$\underline{d\vec{E} = (dE_x, 0, 0)}$$

$$dE_x = 2 \cdot dE_\rho = 2 \cdot d\vec{E}_\rho \cdot \cos\beta$$

$$\cos\beta = \frac{x}{R}$$

$$dE = \cancel{2} \underbrace{\frac{q}{\cancel{2}\pi\varepsilon_0 R} \cdot \frac{x}{R}}$$

$$R^2 = x^2 + y^2$$

$$dE = \frac{q}{\pi\varepsilon_0} \frac{x}{x^2+y^2}$$

spet moramo vse prispevke sešteti → ∫

$$E_x = \int dE_x = \frac{\sigma}{\pi\varepsilon_0} \int_0^\infty \frac{x\, dy}{x^2+y^2} =$$

$$= \frac{\sigma}{\pi\varepsilon_0} \arctg \frac{y}{x} \Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{\sigma}{\pi\varepsilon_0}\left(\frac{\pi}{2} - 0\right) = \frac{\sigma}{\cancel{\pi}\varepsilon_0} \frac{\cancel{\pi}}{2}$$

$$\frac{x\, dy}{x^2+y^2} = \frac{\frac{dy}{x}}{1 + \left(\frac{y}{x}\right)^2} \quad :x^2 = \frac{ds}{1+s^2}$$

$$\frac{y}{x} = s$$

integriramo

$$\int \frac{ds}{1+s^2} = \cancel{\text{arctan}} \;\; \text{arctang}$$

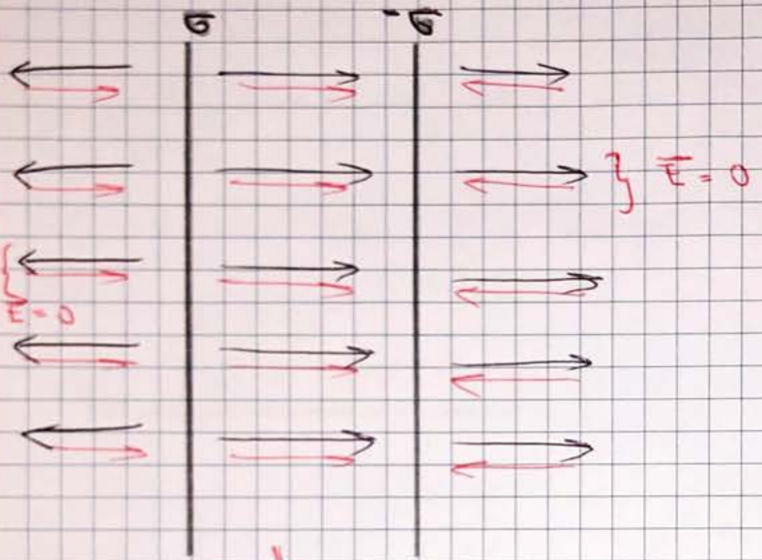
$$\boxed{E = \frac{\sigma}{\varepsilon_0 \cdot 2}}$$

!

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\,\vec{e}}$$

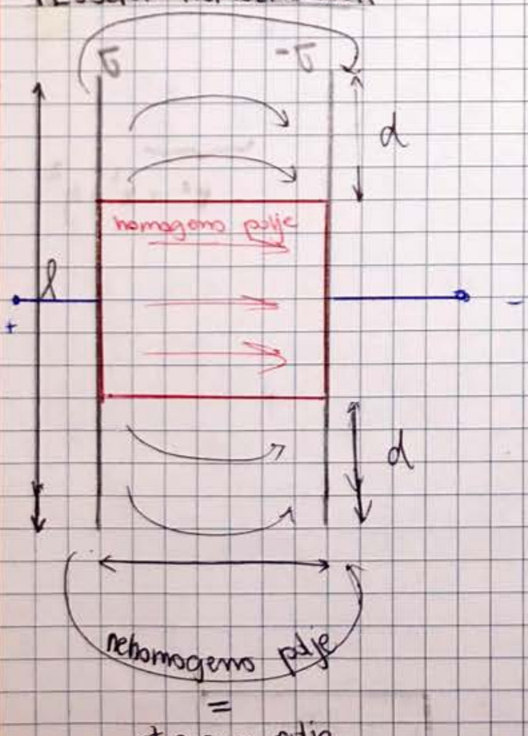
↳ polje polprostra je konstantno ⇒ HOMOGENO POLJE

Dve naelektreni ravnini



↓ polje 2x večje → $\vec{E} = \frac{\sigma}{2\cdot\varepsilon_0}\cdot 2$ → $\boxed{E = \frac{\sigma}{\varepsilon_0}}$

PLOŠČNI KONDENZATOR



$Q(t) = Q_0 e^{-\lambda t}$

$t = 0 \rightarrow$

$Q_0 = 4\cdot10^{-6}\ C$

$\lambda = 4\ s^{-1}$

$t_1 = 2ms$

$Q(t) = 4\cdot10^{-6} e^{-4\cdot t_0}$

$Q(t_1) =$

$Q_1 = 0{,}12\ \mu C$

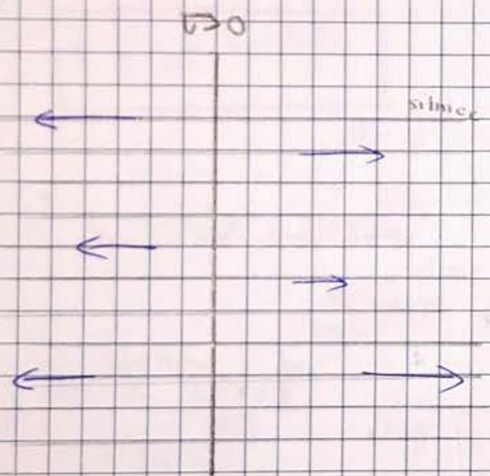
$Q_2 = -150\ \mu C$

0,21 $\rightarrow$ $\leftarrow$ 150

Q_1 Q_2

120 nC

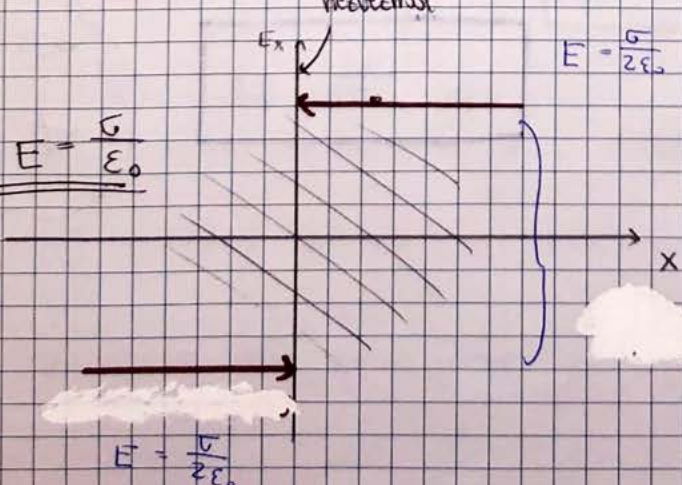
−150 nC



$$E_x(x) = \begin{cases} \frac{\sigma}{2\varepsilon_0}, & x > 0 \\ 0, & x = 0 \\ -\frac{\sigma}{2\varepsilon_0}, & x < 0 \end{cases}$$

Grafično:

$E = \frac{\sigma}{\varepsilon_0}$



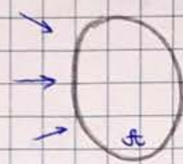
ELEKTRIČNI PRETOK

$$I = \int_A \vec{J}\, d\vec{a}$$

električni tok

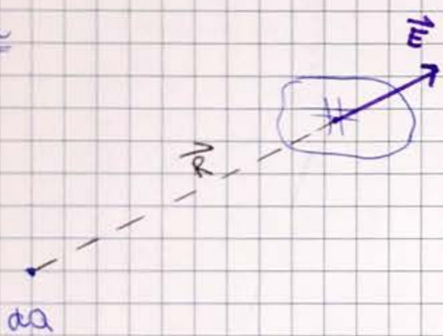
Zdaj se bomo pogovarjali o pretoku $\vec{E}$ skozi neko ploskev:

$$\boxed{\phi = \vec{E} \cdot d\vec{a}}$$



Ni nujno, da se nekaj premika za pretok, zadostuje že polje

Zgled



$$d\phi_e = \vec{E} \cdot d\vec{a}$$

$$d\phi_e = \frac{dQ\ \vec{R}}{4\pi\varepsilon_0 R^3} \cdot d\vec{a}$$

⇓ izpostavimo konst.

$$d\phi_e = \frac{dQ}{4\pi\varepsilon_0} \frac{\left(\frac{\vec{R}}{R}\right) \cdot d\vec{a}}{R^2}$$

projekcija da'

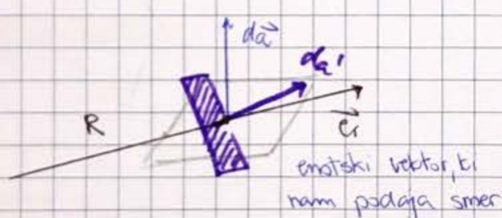
$$d\Omega = \frac{da'}{R^2}$$

PROSTORSKI KOT

$$\alpha = \frac{l}{r}$$

$$d\Omega = \frac{da}{R^2}$$

$$\boxed{d\phi_e = \frac{dQ}{4\pi\varepsilon_0} \cdot d\Omega}$$



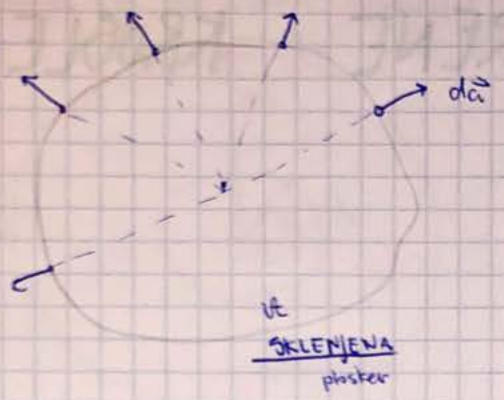
da' - projekcija da

GAUSSOV ZAKON $\vec{E}$

* Njegovo poznavanje je priporočljivo, da bomo lahko rešili in razumeli ZAHTEVNEJŠE PRIMERE

↓

Mi se bomo spraševali, kako je pretok v SKLENJENI PLOSKVI !!



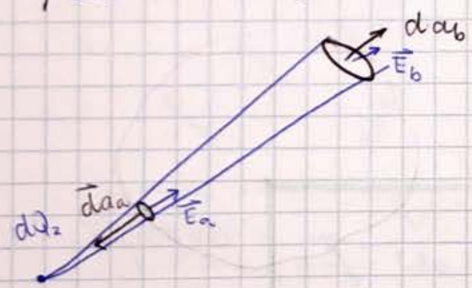
1) Q znotraj skl. ploskve

$$\oint \vec{E} \cdot d\vec{a} = \oint \frac{dQ}{4\pi\varepsilon_0} \, d\Omega = \Longrightarrow \frac{dQ_1}{4\pi\varepsilon_0} \int d\Omega$$

integriramo, saj nas zanima po CELOTNI PLOSKVI

$$\boxed{\oint \vec{E} \cdot d\vec{a} = \frac{dQ_1}{\varepsilon_0}}$$

2) Q izven skl. ploskve



$$\oint_{vt} \vec{E} \cdot d\vec{a} = 0$$

$\Downarrow$ ugotovitev

$$\boxed{\oint \vec{E} \cdot d\vec{a} = \frac{Q_{not.}}{\varepsilon_0}}$$

I. Maxwellova enačba

Gaussov zakon uporabljamo za IZVORNOST TOKA !

Samo NOTRANJI NABOJI so vzrok za to:

Q_4 Q_1 Q_2 Q_5 Q_3 ...

$$\frac{Q_1 + Q_2 + Q_3}{\varepsilon_0} \; ...$$

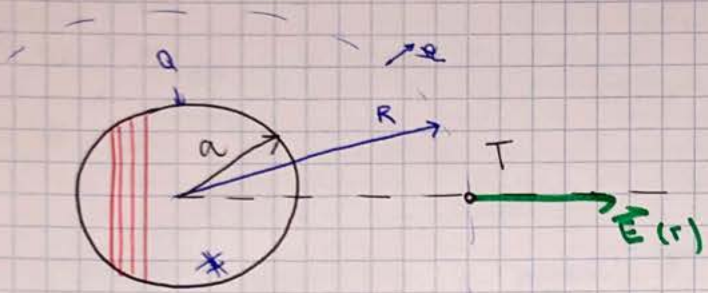
Gaussov stavek:

$$\oint \vec{J} \cdot d\vec{a} = -\frac{Q_{not}}{dt}$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\varepsilon_0}$$

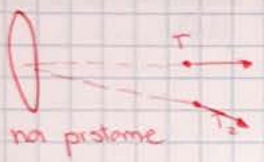
$$\boxed{\oint \vec{X} \cdot d\vec{a}} = \; ...$$

EL. POLJE NAELEKTRENE KROGLE



Kaj lahko povemo v točki T_1 o $\vec{E}$?

↳ Kroglo razrežemo:



ZAKLJUČEK: $\vec{E}$ kaže v RADIALNI SMERI

$$\oint_{r=R} \vec{E} \, d\vec{a} = \frac{Q_{not}}{\varepsilon_0}$$

oba kažeta v rad. smeri

$$\oint \vec{E} \, da = \frac{Q_{not}}{\varepsilon_0} \Rightarrow E \cdot \int da = \frac{Q}{\varepsilon_0}$$

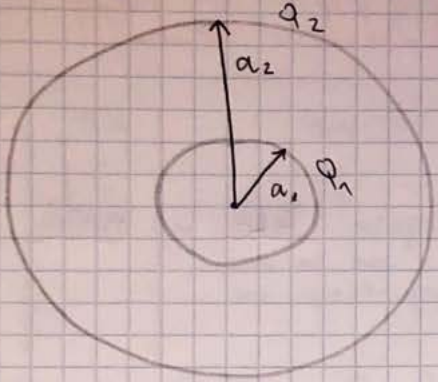
konstanten po dolžini

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \vec{e}_R}$$ ZUNANJOST

V notranjosti polja NI.

$$\boxed{\vec{E} = \begin{cases} 0 & R < a \\ \vec{e}_R \dfrac{Q}{4\pi\varepsilon_0 R^2} & R > a \end{cases}}$$

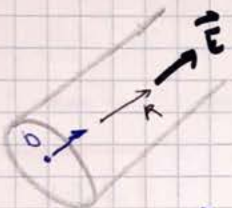
Kako je pri dveh kroglah?



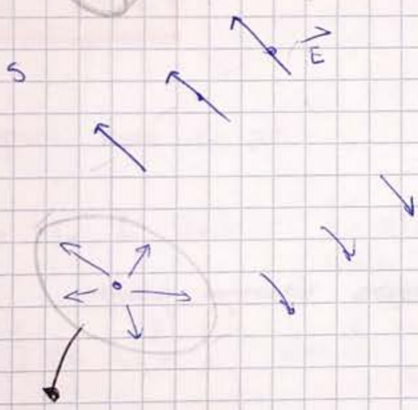
$$\vec{E} = \begin{cases} 0 & R < a_1 \\ \dfrac{Q_1}{4\pi\varepsilon_0 R^2}\,\vec{e}_R & a_1 < R < a_2 \\ \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 R^2}\,\vec{e}_R & R > a_2 \end{cases}$$

$\vec{E}$ NAELEKTRENEGA VALJA

(žice, koax kabla)



Smer $\vec{E}$ je RADIALNA



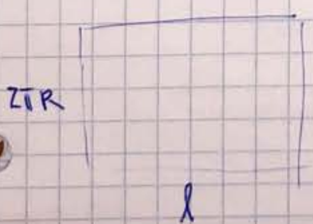
Φ_e skozi dno = 0

Φ_e skozi plašč

$$\oint_{\text{čez skozi plašč}} \vec{E}\, da = \frac{Q_{notr.}}{\varepsilon_0}$$

Φ skozi pokrov je 0

⇓ če raztegnemo plašč



$$E \int da = \frac{q \cdot l}{\varepsilon_0}$$

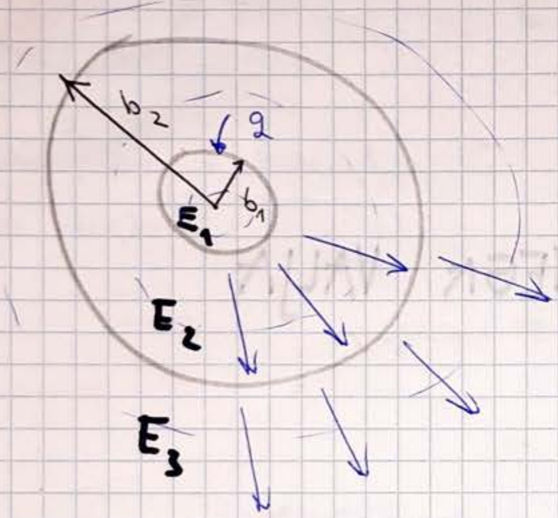
$$E\, 2\pi R \cancel{l} = \frac{q \cancel{l}}{\varepsilon_0}$$

$$\vec{E} = \vec{e}_R \frac{q}{2\pi\varepsilon_0 R}$$

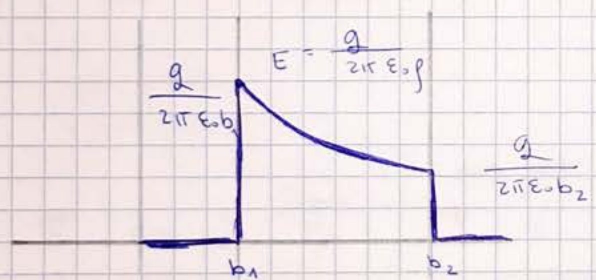
$$\boxed{\vec{E} = \vec{e}_\rho \frac{q}{2\pi\varepsilon_0 \rho}}$$ zunanjost

$$\vec{E} = \begin{cases} 0 & , \ \rho < \\ \dfrac{q}{2\pi\varepsilon_0\rho}\vec{e}_\rho & , \ \rho > \end{cases}$$

KOEX. KABEL: to je vodnik, ki ga sestavljata ŽILA in PLAŠČ

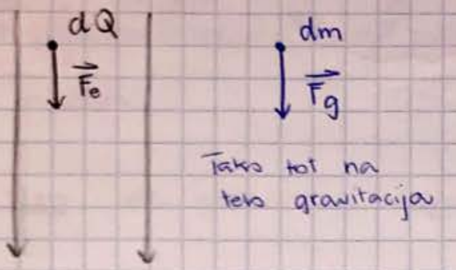


$$\vec{E} \begin{cases} 0 & E_1 & \rho < b_1 \\ \dfrac{q}{2\pi\varepsilon_0\rho} & E_2 & b_1 < \rho < b_2 \\ \dfrac{-q+q}{2\pi\varepsilon_0\rho} = 0 & E_3 & b_2 < \rho \end{cases}$$



Koex kabel odzunaj NE VPLIVA s svojim ~~[illegible]~~ notranjim nabojem

DELO ELEKTRIČNEGA POLJA



Na naboj deluje
sila v ele. polju

Delec se giba po traektoriji $(\mathcal{L})$:

$$d\vec{F} = dm\,\vec{a} = dm\,\dot{\vec{v}} = dm\,\frac{d\vec{v}}{dt}$$

$$\boxed{d\vec{F_e} = dQ\,\vec{E}}$$

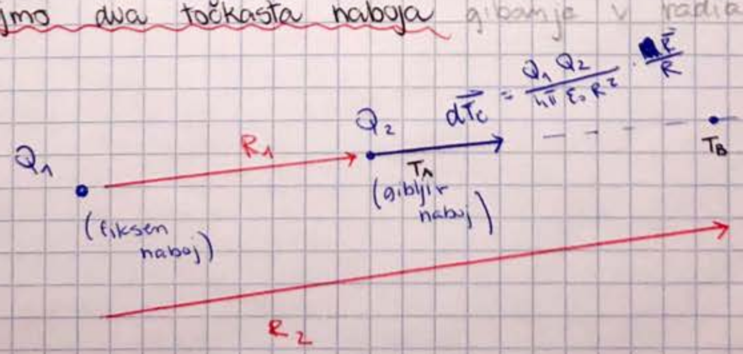
$$(\vec{v}^2)' = 2 \cdot v\, d\vec{v}$$

Integriral bomo po poti:

$$A_e(T_1, \mathcal{L}, T_2) = \int_{T_1, \mathcal{L}, T_2} d\vec{F_e} \cdot d\vec{\mathcal{L}} = \int dm \frac{d\vec{v}}{\cancel{dt}} \cdot \vec{v}\,\cancel{dt} = \int dm\, \vec{v} \cdot d\vec{v} =$$

$$\int dm\, d\left(\frac{v^2}{2}\right) = dm \frac{v^2}{2}\Big|_{T_1}^{T_2} = \frac{dm\, v_2^2}{2} - \frac{dm\, v_1^2}{2} \quad \boxed{\frac{dm}{2}\left(v_2^2 - v_1^2\right)}$$

1) Opazujmo dva točkasta naboja gibanje v radialni smeri



T_A... začetna točka

T_B... končna točka

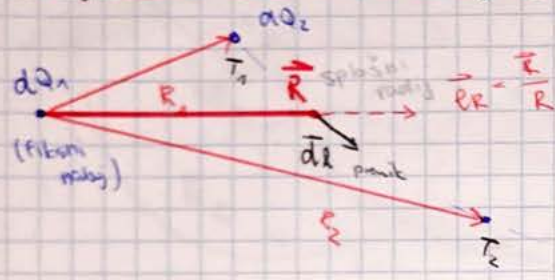
* Q_2 bo odletel daleč stran

Zanimalo nas bo koliko dela se sprosti pri premiku $T_A \to T_B$: $\int \frac{dx}{x^2} = -\frac{1}{x}$

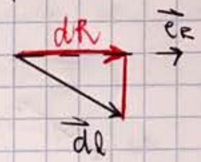
$$A_e(T_1,T_2) = \int_{R_1}^{R_2} d\vec{F}_e \, d\vec{l} = \frac{dQ_1 Q_2}{4\pi\varepsilon_0} \int \underbrace{\frac{\vec{R}}{R}\cdot\frac{\vec{R}}{R}} \, dR = \frac{dQ_1 Q_2}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dR}{R^2} =$$

$$\boxed{A_e(T_1,T_2) = \frac{dQ_1 Q_2}{4\pi\varepsilon_0}\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

2) Opazujemo dva točkasta naboja splošna pot



* pogovarjamo se o skalarnem produktu:



skalarni produkt = projekcija

$\vec{e}_R \cdot d\vec{l}$

diferencial poti se nam preslika v radialni smeri

$$A_e(T_1,T_2) = \int_{R_1}^{R_2} \vec{F}_e \, d\vec{l} = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\int \frac{\vec{R}}{R}\cdot\frac{1}{R^2}\cdot d\vec{l} =$$

$$= \frac{Q_1 Q_2}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dR}{R^2}$$

$$\boxed{A_e(T_1,T_2) = \frac{Q_1 Q_2}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

Dobili smo enak rezultat

UGOTOVITEV in ZAKLJUČEK:

* Trajektorija (pot, krivulja) ni pomembna

* Pomembni sta samo T_1 in T_2

ELEKTRIČNA POTENCIALNA ENERGIJA

* Električna dodamo samo zato, da ne bi zamenjali to energijo s potencialno ener
pri fiziki.

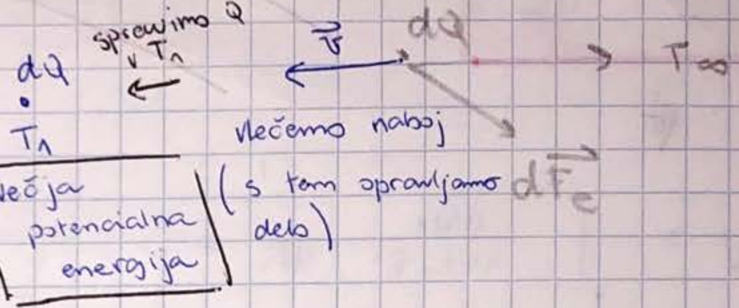
Wep ... ele. potencialna energija → referenčna točka $W_{ep}(T_\infty) = 0$

v neskončni okolici je enaka 0

$$dW_{ep} = dA_e(T, T_\infty) =$$

$$= \int_T^{T_\infty} d\vec{F}_e \cdot d\vec{l} =$$

$$= dQ \cdot \int_T^{T_\infty} \vec{E} \cdot d\vec{l} =$$



$$\boxed{= \frac{Q_2}{4\pi\varepsilon_0} \int_{Q_1} \frac{dQ}{R}}$$

viri polja

pomeni, da seštevamo po celotnem naboju →

} torej po celotnem Q_1 seštejemo vse dQ

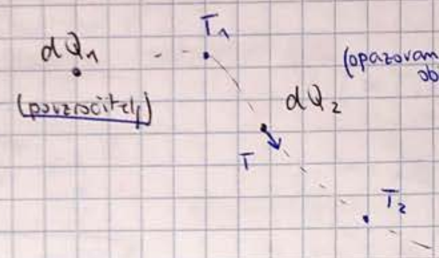
ELEKTRIČNI POTENCIAL

Definicija ele. poljske jakosti:

$$E = \frac{d\vec{F_e}}{dQ}$$ } ko ele. silo NORMIRAMO z nabojem dobimo E

Vpeljali bomo potencial:

$$\boxed{V = \frac{dW_{ep}}{dQ}}$$



Za Wep smo rekli:

$$W_{ep} = \cancel{\quad} A_e (T_1, T_\infty)$$

$$\int_{T_1}^{T_\infty} \frac{dQ_1\, Q_2}{4\pi \varepsilon_0 R^2} \, dR$$

to želimo izločiti

$$\Downarrow$$

$$V = \int_{T}^{T_\infty} \frac{dQ_1}{4\pi \varepsilon_0 R^2} \cdot dR =$$

$$\boxed{V = \int_{T}^{\infty} \vec{E} \cdot d\vec{l}}$$

Vpeljali bomo še električno napetost, potem pa se vrnemo na potencial

ELEKTRIČNA NAPETOST U [V]

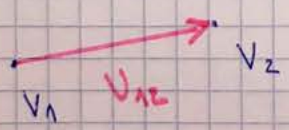
↳ definirana je kot razlika med potenciali.

$$\boxed{U_{12} = V_1 - V_2}$$

Če se sklicujemo na Wep je to enako spremembi Wep, ~~...~~ normirana z Q:

$$U = \frac{\Delta W_{ep_{1,2}}}{dQ} \Rightarrow \boxed{U = \frac{dA_e(T_1, T_2)}{dQ}}$$

Označevanje el. nap.:



Zapis napetosti kot integral poti:

$$\boxed{U = \int_{T_1}^{T_2} \vec{E}\cdot\vec{dl}}$$

I. lastnost polja je bil **GAUSSOV STAVEK**: izvornost E

↓ spoznali pa bomo še

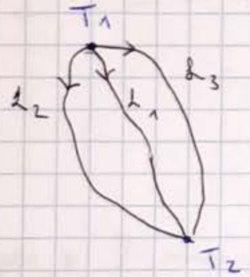
II. lastnost polja: **STOKESOV STAVEK**: vrtinčnost E

$$\boxed{\oint_L \vec{E}\cdot\vec{dl} = 0}$$

to velja za elektrostatično polje!

Razmislek:

Imeli bomo dve točki povezani s tremi L_1, L_2, L_3 (ki niso skupaj).

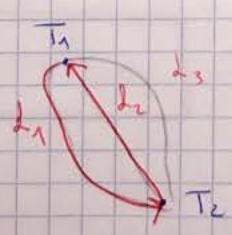


$$\int_{T_1, L_1, T_2} \vec{E}\cdot\vec{dl} = \int_{T_1, L_2, T_2} \vec{E}\cdot\vec{dl} = \int_{T_1, L_3, T_2} \vec{E}\cdot\vec{dl}$$

Razmišljanje: $\int_{T_1 L_1 T_2} \vec{E}\cdot\vec{dl} = -\int_{T_2, L_2, T_1} \vec{E}\cdot\vec{dl}$

Ugotovitev: Če gremo po eni poti iz T_1 v T_2 in po drugi poti nazaj, dobimo, da je to $\underline{0}$

$$\oint_L \vec{E}\cdot\vec{dl} = \int_{T_1, L_1, T_2} \vec{E}\, d\vec{l} + \left(-\int_{T_2 L_2, T_1} \vec{E}\, d\vec{l}\right) = 0$$



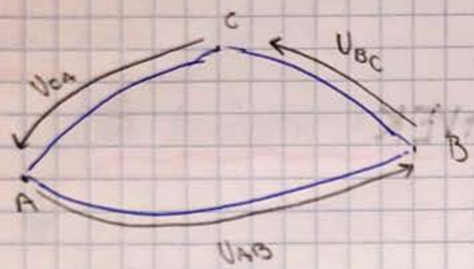
* Dobili smo sklenjeno zanko

Na podlagi tega bomo zapisali:

II. KIRCHHOFFOV ZAKON

$$\oint_L \vec{E}\cdot d\vec{l} = 0$$

$$\int_A^B \vec{E}\cdot d\vec{l} + \int_B^C \vec{E}\cdot d\vec{l} + \int_C^A \vec{E}\cdot d\vec{l} = 0$$

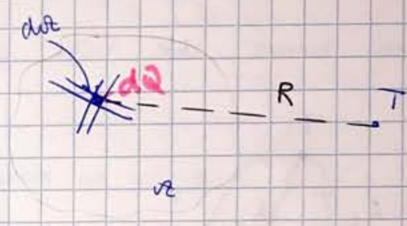


$$\boxed{U_{AB} + U_{BC} + U_{CA} = 0}$$

Ponovitev: Koncepti določanja potencialov in napetosti

I. Po celotnem naboju seštevamo prispevke:

$$\boxed{V(T) = \frac{1}{4\pi\varepsilon_0}\int_{Q_{cel.}} \frac{dQ}{R}}$$



II. Preko električnega polja:

$$\boxed{V(T) = \int_T^{T_\infty} \vec{E}\cdot d\vec{l} = -\int_{T_\infty}^{T} \vec{E}\cdot d\vec{l}}$$

* Integriramo $\vec{E}$ iz neke REFERENČNE TOČKE

$$U_{12} = V_1 - V_2 = V(T_1) - V(T_2) = \int_{T_1}^{T_2} \vec{E}\cdot d\vec{l}$$

* Pot ni pomembna, temveč je samo PROJEKCIJA $\vec{E}$ v smeri POTI.

POTENCIAL TOČKASTEGA NABOJA

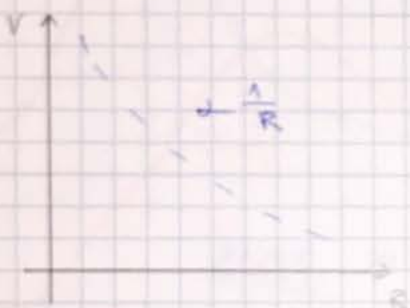
I. koncept

$$V = \frac{1}{4\pi\varepsilon_0} \int_Q \frac{dQ}{R} = \frac{1}{4\pi\varepsilon_0 R} \int_Q dQ$$

konstanta

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 R}}$$

parla s 1. polovico



II. koncept (integracija $\vec{E}$ po poti od referenčne točke $T_\infty \to T$)

V =

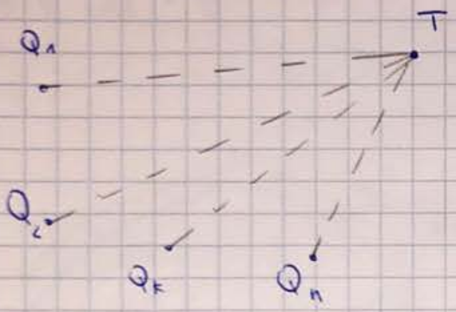
$$V = \int_T^{T_\infty} \vec{E} \cdot \vec{dl} = \int_R^\infty \frac{Q}{4\pi\varepsilon R^2} \cdot \underbrace{\vec{e_R} \cdot \vec{e_R}}^{1} \cdot dR = \frac{Q}{4\pi\varepsilon_0} \int_R^\infty \frac{dR}{R^2}$$

$$-\frac{1}{R} \Big|_R^\infty = -\frac{1}{\infty} + \frac{1}{R} = \frac{1}{R}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 R}}$$

Kaj dobimo, če vzamemo več točkastih nabojev?

II. SRUČE TOČKASTIH NABOJEV



$$V = \int_T^{\infty} \vec{E} \cdot d\vec{l}$$

Še vedno velja, da lahko $\vec{E}$ vektorsko seštevamo:

$$E = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_k + \ldots + \vec{E}_n = \sum_{k=1}^{n} \vec{E}_k$$

Torej lahko tudi potencial

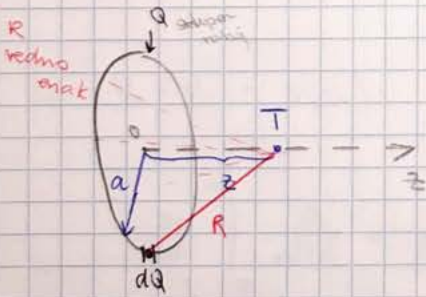
$$V = V_1 + V_2 + \ldots + V_k + \ldots V_n = \sum_{k=1}^{n} V_k$$

$$\boxed{V = \sum_{k=1}^{n} \frac{Q_k}{4\pi\varepsilon_0 R_k} = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{Q_k}{R_k}}$$

Edina razlika z $\vec{E}$ je, da potencial seštevamo kot SKALAR.

III. NAELEKTREN OBROČ

Ele. polj jakost smo zaključili, da analitično izračunamo samo vzdolž osi

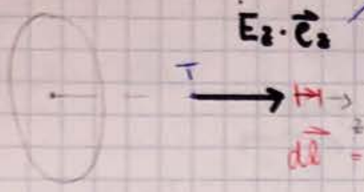


I. Koncept (seštejemo vse prispevke)

$$V = \frac{1}{4\pi\varepsilon_0} \int_Q \frac{dQ}{R} = \frac{1}{4\pi\varepsilon_0 R} \cdot Q \Rightarrow \boxed{V = \frac{Q}{4\pi\varepsilon_0 \sqrt{z^2 + a^2}}}$$

konstanta

II. koncept



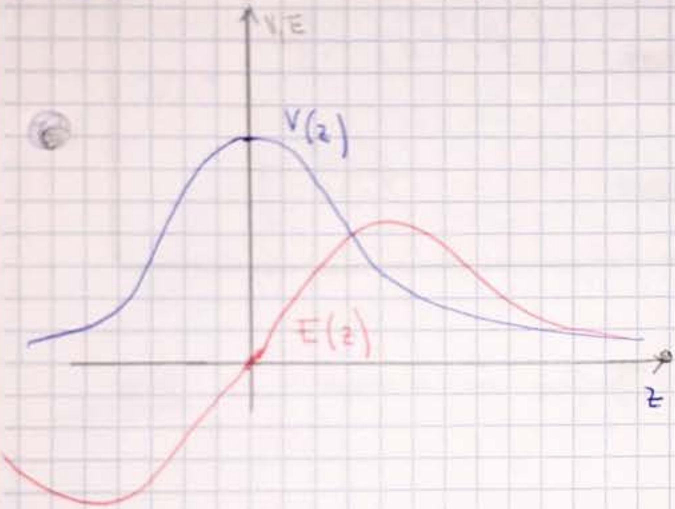
$$E_z \cdot \vec{e}_z \nearrow \frac{Q \cdot z}{4\pi\varepsilon_0 (a^2+z^2)^{3/2}}$$

$$\vec{dl} \stackrel{!}{=} \vec{e}_z \cdot dz$$

$$V = \int_T^{T\infty} \vec{E} \, \vec{dl} = \int_z^{\infty} \frac{Q}{4\pi\varepsilon_0} \frac{z \cdot dz}{(a^2+z^2)^{3/2}} \; \ldots$$

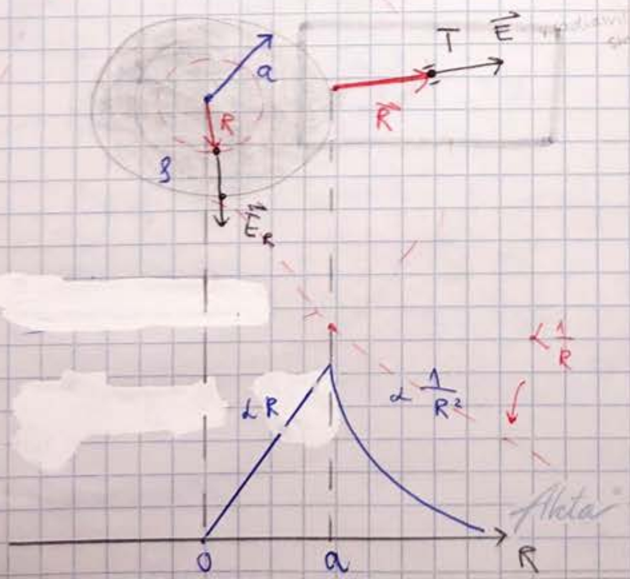
→ pripelje do enakega rezultata

Graf:



IV. KROGELNI OBLAK

Predstavljamo si krogelni oblak z enakomerno razporejenim nabojem:



Zunaj oblaka

$$\oint \vec{E} \cdot \vec{da} = \frac{Q_{not}}{\varepsilon_0}$$

el. polj jakost in naša pot sta vzporedni zato jih ne rabimo vektorsko sešteti

Vprašamo se koliko je zaobjetega naboja v volumnu

$$E \cdot 4\pi R^2 = \rho \cdot \frac{4\pi a^3}{3\varepsilon_0}$$

$$\boxed{E_R = \frac{\rho a^3}{3\varepsilon_0 R^2}}$$

Znotraj oblaka

$$\oint \vec{E}\, d\vec{a} = \frac{Q_{not.}}{\varepsilon_0}$$

$$E_R \cdot \cancel{4\pi} R^2 = \frac{\rho \cdot \cancel{4\pi} R^{\cancel{3}}}{3\varepsilon_0}$$

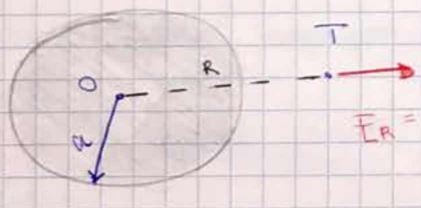
$$\boxed{E_R = \frac{\rho R}{3\varepsilon_0}}$$

$$\boxed{E_R \begin{cases} \dfrac{\rho R}{3\varepsilon_0} & \text{znotraj} \\[2ex] \dfrac{\rho a^3}{3\varepsilon_0 R^2} & \text{zunaj} \end{cases}}$$

→ **POTENCIAL**

ZUNAJ OBLAKA

Integrirali bomo $\vec{E}$ v okolici oblaka:



$$E_R = \frac{\rho a^3}{3\varepsilon_0 R^2}$$

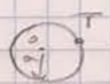
$$V = \int_T^{\infty} \vec{E} \cdot d\vec{l} = \frac{\rho a^3}{3\varepsilon_0} \int_R^{\infty} \frac{1}{R^2} \overbrace{\vec{e}_R \cdot \vec{e}_R}^{1}\, dR$$

$\vec{e}_R\, dR \rightarrow$ (radialna smer)

$$\int_R^{\infty} \frac{dR}{R^2} = -\frac{1}{R}\Big|_R^{\infty}$$

$$\boxed{V = \frac{\rho \cdot a^3}{3\varepsilon_0 R}}$$

Na meji: $\underline{\underline{R = a}}$



$$V(R=a) = \frac{\rho \cdot a^{\cancel{3}\,2}}{3\varepsilon_0 \cancel{a}} =$$

$$\boxed{V = \frac{\rho \cdot a^2}{3\varepsilon_0}}$$

ZNOTRAJ OBLAKA

$$\int R\,dR = \frac{R^2}{2}$$

$$V(R) - V(a) = \int_R^a \frac{\rho \cdot R}{3\varepsilon_0}\cdot dR = \frac{\rho}{3\varepsilon_0}\int_R^a R\,dR = \frac{\rho}{3\varepsilon_0 2}\left(a^2 - R^2\right)$$

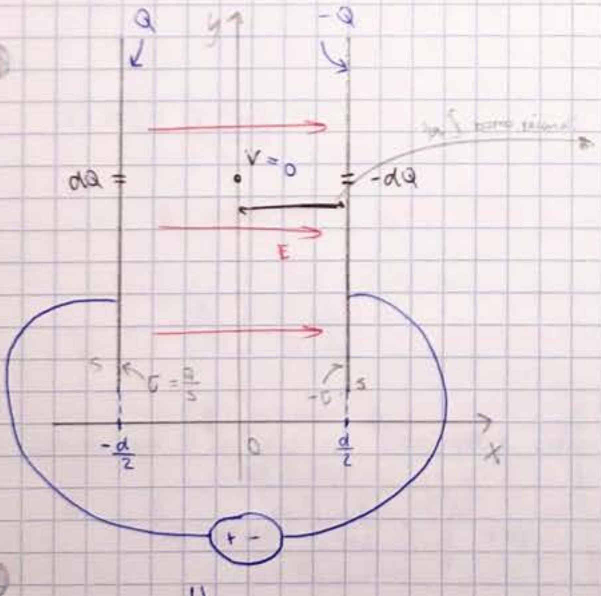
$$V(R) = \overbrace{\frac{\rho\, a^2}{3\varepsilon_0}}^{V(a)} + \frac{\rho\, a^2}{6\varepsilon_0} - \frac{\rho R^2}{6\varepsilon_0}$$

$$\boxed{V(R) = \frac{\rho}{2\varepsilon_0}\left(a^2 - \frac{R^2}{3}\right)}$$

$$\boxed{V = \begin{cases} \frac{\rho}{2\varepsilon_0}\left(a^2 - \frac{R^2}{3}\right), & \text{znotraj} \\ \frac{\rho a^3}{3\varepsilon_0 R}, & \text{zunaj} \end{cases}}$$

Graf V je narisan na prejšnji strani z: - - - -

V. PLOŠČNI KONDENZATOR



$$V_D = V\left(x = \frac{d}{2}\right)$$

kaže v smer x: $-\vec{E_x}\cdot\vec{e_x} = \frac{\sigma}{\varepsilon_0}\cdot\vec{e_x}$

$$\underset{\underset{0}{\parallel}}{V_0} - V_d = \int_0^{\frac{d}{2}} \vec{E}\cdot\underbrace{\vec{dl}}_{\vec{e_x}\,dx} =$$

$$= \int_0^{\frac{d}{2}} \underbrace{\frac{\sigma}{\varepsilon_0}}_{\text{konst.}} \underbrace{\vec{e_x}\cdot\vec{e_x}}_{1}\,dx =$$

$$- \frac{\sigma}{\varepsilon_0}\int_0^{\frac{d}{2}} dx =$$

$$\boxed{V_D = -\frac{\sigma}{\varepsilon_0}\cdot\frac{d}{2}}$$

$$\boxed{V_L = \frac{\sigma}{\varepsilon_0}\cdot\frac{d}{2}}$$

$$U = V_L - V_D = 2V_L = \frac{\sigma\cdot d}{\varepsilon_0}$$

$$\boxed{U = \frac{Q\,d}{S\,\varepsilon_0}}$$

Iz prejšnje enačbe izrazimo Q:

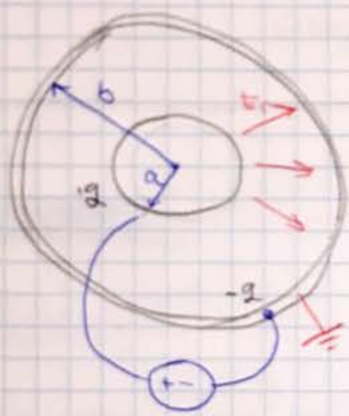
$$Q = \frac{\varepsilon_0 S}{d} \cdot U$$

$$Q = C \cdot U$$

$$\boxed{C = \frac{\varepsilon_0 S}{d}}$$

KAPACITIVNOST
pl. kondenzatorja

VI KOEXIALNI KABEL



$$E_r = \begin{cases} 0 & , \text{ znotraj} \\ \frac{q}{2\pi\varepsilon_0 r} & , \text{ med žilo in plaščem} \\ 0 & , \text{ zunaj} \end{cases}$$

V IZVEN PLAŠČA

$$\boxed{V = \int \underbrace{\vec{E}}_{0} \cdot dx = 0}$$

V MED ŽILO IN PLAŠČEM

Izberemo si točko, za katero napišemo razliko potencialov v točki in v koor. izhodišču, kjer je 0

$$V(r) - \underbrace{V(b)}_{0} = \int_r^b \frac{q}{2\pi\varepsilon_0 r}\, dr = \frac{q}{2\pi\varepsilon_0} \cdot \ln r \Big|_r^b = \frac{q}{2\pi\varepsilon_0} \cdot \ln b - \ln r$$

$$\boxed{V(r) = \frac{q}{2\pi\varepsilon_0} \cdot \ln \frac{b}{r}}$$

V ŽILE

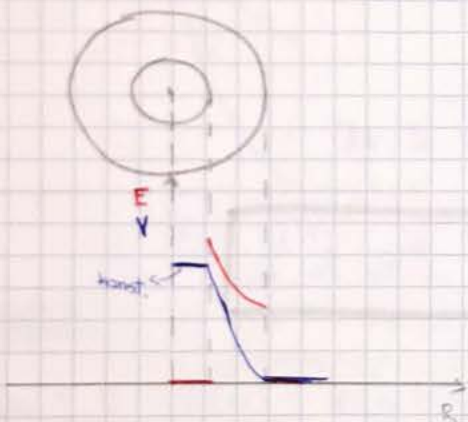
$$\boxed{V(r=a) = \frac{q}{2\pi\varepsilon_0}\ln\frac{b}{a}}$$

V ZNOTRAJ ŽILE:

Potencial znotraj žile je **konstanten!**

$$\boxed{V = \frac{q}{2\pi\varepsilon_0}\cdot\ln\frac{b}{a}} \neq 0$$

GRAF



$$V(a) - \underbrace{V(b)}_{0} = 0$$

↓ vstavimo v zgornjo enačbo

$$U = \frac{q}{2\pi\varepsilon_0}\ln\frac{b}{a}$$

↓ izrazimo q

$$\text{I.}\quad q = \frac{2\pi\varepsilon_0 \cdot U}{\ln\frac{b}{a}} \quad \Big| \cdot l$$

$$Q = q \cdot l = \frac{2\pi\varepsilon_0 U}{\ln\frac{b}{a}} \cdot l$$

↓

$$\boxed{C = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}} \cdot l}$$

$$c = \frac{C}{l}$$

$$\boxed{c = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}}}$$

kapacitivnost na dolžinsko enoto kabla

VI NAELEKTRENE PREMICE

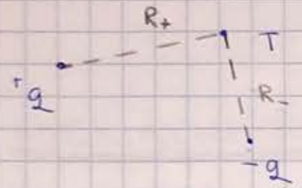
* Potencial v neki točki z eno samo premo elektrino NE MOREMO izračunati

Lahko ga dobimo le do KONST. NATANČNO

$$V = \int \vec{E}\, d\vec{l}$$

Zato ga lahko računamo le v dveh primerih:

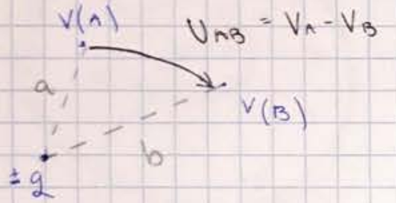
① Potencial v okolici dveh nabojev $\pm q$



$$\boxed{V(T) = \frac{q}{2\pi\varepsilon_0} \ln \frac{R_-}{R_+}}$$

(izpeljava poglej na posnetkih!)

② Napetost v okolici premega naboja



$$U_{AB} = V_A - V_B$$

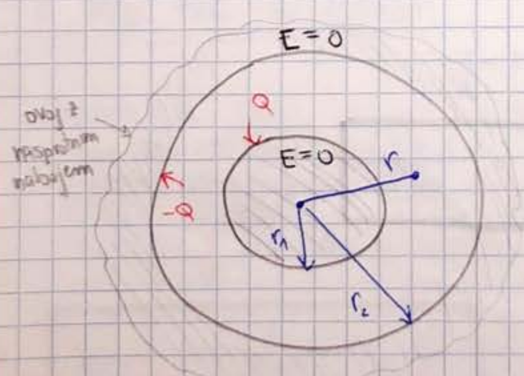
$$U_{AB} = \int_A^B \vec{E}\, d\vec{l} =$$

$$\boxed{U_{AB} = \frac{q}{2\pi\varepsilon_0} \ln \frac{b}{a}}$$

(izpeljava v knjigi / posnetki)

VII. SFERIČNI KONDENZATOR kroglni

↳ (ena naelektrena sfera, druga v njej kocentrična z nasprotnima nabojema, ugotavljamo dve področji brez polja...)



$$E_r (r_1 < r < r_2) = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Zunanjost:

$$\boxed{V(r \geq r_2) = 0}$$

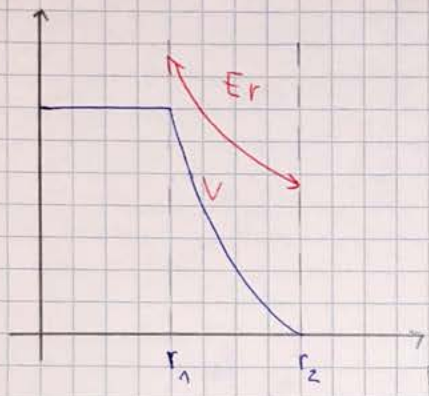
- Notranjost:

$$V(r) - \underbrace{V(r_2)}_{=0} = \int_r^{r_2} \frac{Q}{4\pi\varepsilon_0 s^2}\, ds =$$

$$\boxed{V(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_2}\right)}$$

- $r < r_1$

$$V(r < r_1) = \boxed{V(r_1) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$



Napetost:

$$U_{12} = U = V(r_1) - V(r_2) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r_2 - r_1}{r_1 r_2}$$

$$\boxed{C = \frac{Q}{U} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_2 - r_1}}$$

↳ „Osamljena krogla" $r_2 \to \infty$

$$C = 4\pi\varepsilon_0 r_1$$

↳ C zemlje brez ionosfere"

$$C = 4\pi \frac{10^{-9}}{36\pi} 6{,}4 \cdot 10^{6} \approx \underline{0{,}7\,mF}$$

↳ C zemlja – ionosfera

$$C = \frac{4\pi\varepsilon r_1^2}{h} = \frac{4\pi 10^{-9} \cdot 6{,}4^2 \cdot 10^{12}}{36\pi \cdot 2 \cdot 10^5} = \underline{2{,}25 \cdot 10^{-6}}\,F$$

Primer:

Naj sta dana napetost U in polmer r_2, iščemo pa najugodnejši polmer r_1, da bo največja jakost polja minimalna.

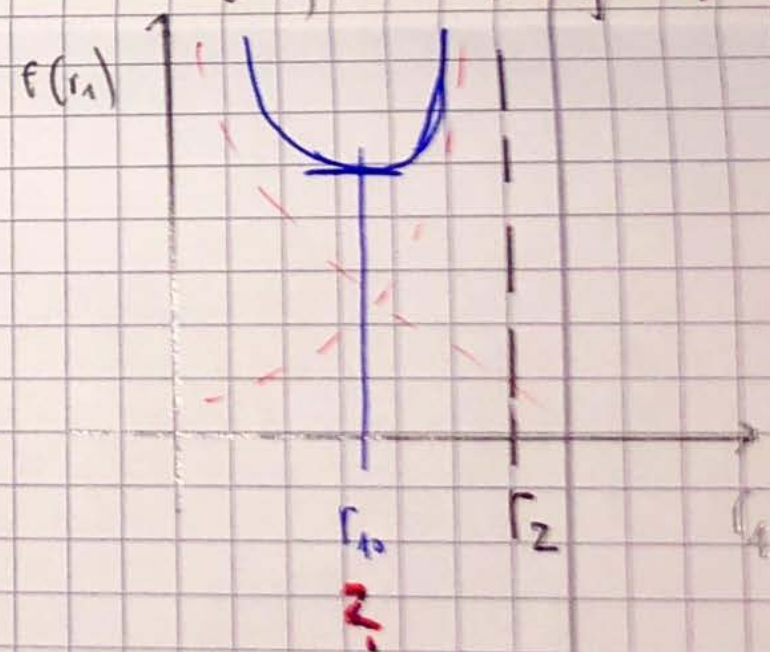
* Torej iščemo najboljšo geometrijo

* Max $\vec{E}$ je na notranji krogli

$$E_r(r_1) = \frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{r_1^2} = \frac{U r_1 r_2}{r_2 - r_1}\cdot\frac{1}{r_1^2} = \frac{U r_2}{\underbrace{r_1(r_2 - r_1)}_{g(r_1)}} = f(r_1)$$

funkcija r_1

$$\frac{Q}{4\pi\varepsilon_0} = \frac{U r_1 r_2}{r_2 - r_1}$$



$$g(r_1) = r_1 r_2 - r_1^2$$

$$g' = r_2 - 2r_1$$

$$r_2 - 2r_{10} = 0$$

$$r_{10} = \frac{r_2}{2}$$

$V \rightarrow \vec{E}$?

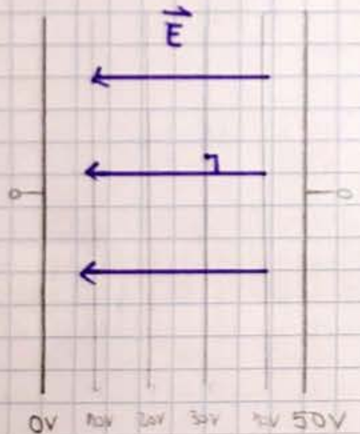
$\vec{E}$ integriramo, da dobimo V, obratna operacija pa je **ODVOD**, vendar potrebujemo nekaj več.

Najprej bomo <u>VIZUALIZIRALI POLJE</u>

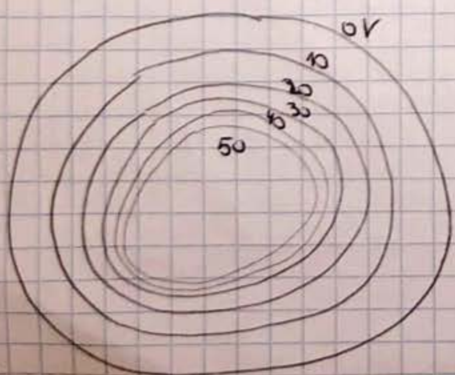
① EKVIPOTENCIALKE

↳ ekvi = iso = enak

② PLOŠČNI KONDENZATOR



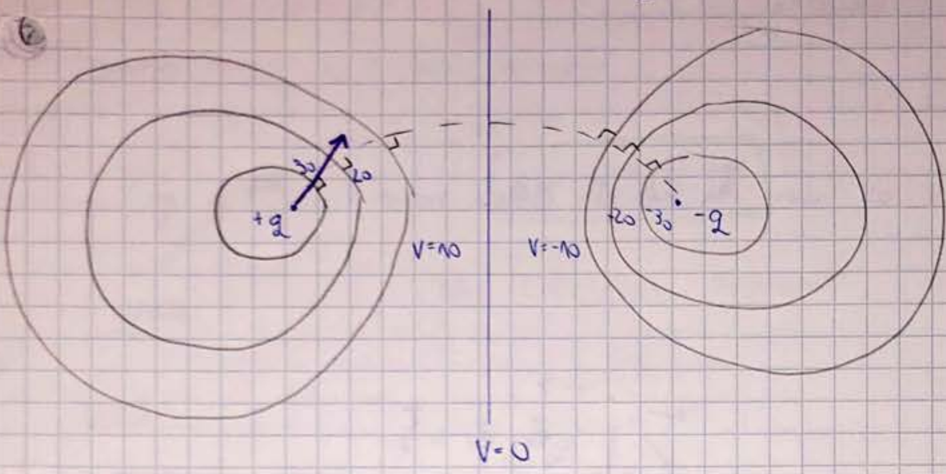
③ VALJNI KS KOEX KABEL

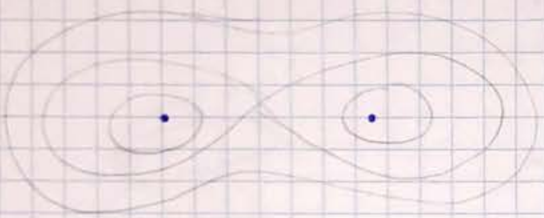


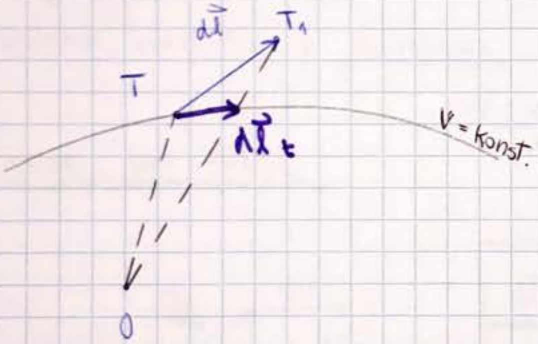
* Ekvipotencialke se proti središču zgostijejo.

④ DVOVOD

Ekvipotencialke so ekscentrični krogi → imeli bomo simetričnost







$$dV = \left(V(T_1) - V(T) \right)$$

$$V(T) - V(T_1) = \int_T^{T_1} \vec{E} \cdot \vec{dl} = \underbrace{\vec{E} \cdot \vec{dl}}_{-dV}$$

$$dV = -\vec{E} \cdot \vec{dl}$$

$$\cancel{\vec{E} = -\frac{dV}{dl}}$$ tega ne moremo!

1) $\vec{dl}$ v tangentni smeri $\vec{dl}_t$

$$dV = \vec{E} \cdot \underbrace{\vec{dl}}_{\text{tang. smer}} = 0$$

$$\underline{\underline{E_t = 0}}$$

Če je $E_t = 0$, bomo gledali še drug premik → NORMALA

2) dl v smeri NORMALE

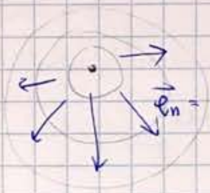
$$dV = \vec{E} \cdot \vec{e}_n \cdot dl_n = E_n \, dl_n$$

$$\boxed{E_n = -\frac{dV}{dl_n}}$$

Z zgledi bomo preverili, ali nam ta enačba sploh koristi:

I. TOČKAST NABOJ

$$\vec{E} = \frac{Q \, \vec{R}}{4\pi\varepsilon_0 R^3}$$



$$\vec{e}_n = \vec{e}_R = \frac{\vec{R}}{R}$$

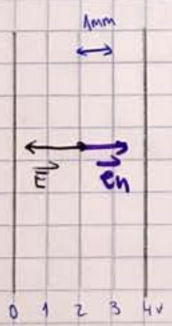
$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

$$E_n = -\frac{dV}{dR} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{R^2}$$

$$\vec{E} = \vec{e}_R \cdot E_n = \frac{Q \, \vec{R}}{4\pi\varepsilon_0 R^3}$$

enako smo dobili

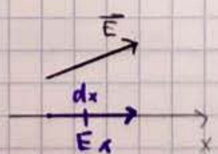
II. PLOŠČNI KONDENZATOR



$$E_n = -\frac{dV}{d_n} = -\frac{(3-2)V}{1mm}$$

$$E_n = -1 \text{ V/mm}$$

$\vec{E}$ kot trojica komponent KS

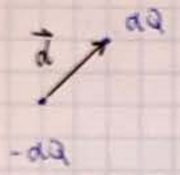


$$\vec{E} = (\vec{E}_x, \vec{E}_y, \vec{E}_z) \quad \begin{cases} E_x = -\frac{dV}{dx} \\ E_y = -\frac{dV}{dy} \\ E_z = -\frac{dV}{dz} \end{cases} \Rightarrow \boxed{\vec{E} = -\left(\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right)} = -\mathrm{grad}\,(V)$$

(funkcija)

ELEKTRIČNI $\underbrace{\text{DI}}_{\text{dva}}$POL

Opravka imamo z 2 točk nabojema



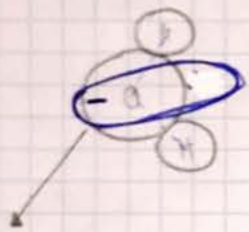
matematični zapis

$$\boxed{d\vec{p} = dQ \cdot \vec{d}}$$

EL. DIPOLSKI MOMENT

Zgled

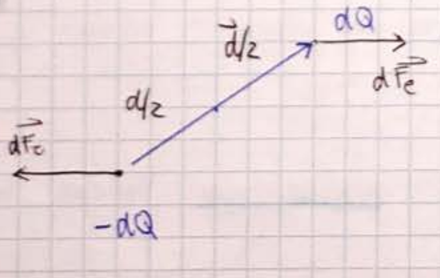
* Molekula vode



* vodik prispeva svoj naboj kisiku, zato je presežek -
* Na drugi strani pa je presežek +

Dipolska molekula je obvladljiva (sile...)

NAVOR NA DIPOL



$$\boxed{d\vec{M} = d\vec{r} \times \vec{F}}$$

ročica x sila !
(ravno tak vrstni red)

$$\boxed{d\vec{M_e} = \frac{\vec{d}}{2} \times dQ\vec{E}} = -\frac{\vec{d}}{2} \times \left(-dQ\right)$$

Gledamo kot, da imamo HOMOGENO POLJE.

$$d\vec{F_e} = dQ\vec{E}$$

$$d\vec{M_e} = dQ\ \vec{d} \times \vec{E}$$

* Navor bo deloval toliko časa, da bo dipol usmerjen v smeri $\vec{E}$

$$\boxed{d\vec{M_e} = d\vec{p} \times \vec{E}}$$

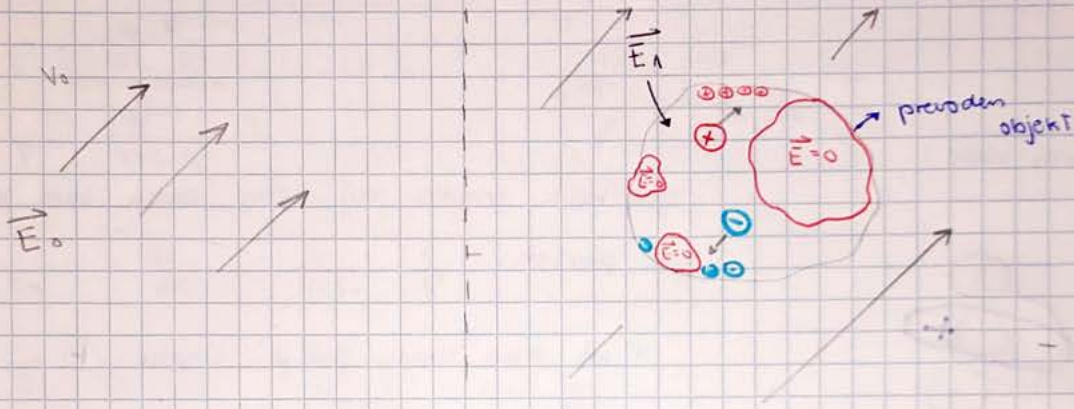
$|dM_e| = 0 \rightarrow$ ko je kot 0

PREVODNIK V ELEKTRIČNEM POLJU

PREVODNIK je snov, ki ima veliko prostih elektronov, ki lahko nosijo naboj.

I. PRIMARNO EL. POLJE

V ta prostor bomo postavili neko prevodno snov / objekt, primarno polje, kot rezultat pa bomo dobili nek vpliv prevodne snovi → SEKUNDARNO POLJE



Skupna $\vec{E}$: $\boxed{\vec{E} = \vec{E}_0 + \vec{E}_1}$

primarno el. polje ($\vec{E}_0$)

vpliv prevodnega telesa ($\vec{E}_1$)

Pride do PRERAZPOREDITVE NABOJA → **EL. INFLUENCA**

⊕ se nabirajo na eni strani, ⊖ pa na drugi → ustvari se **DIPOL**

Naboj se bo prerazporejal dokler ne pride do RAVNOVESJA. Takrat bo

$\boxed{E_{notr.} = 0}$ $\quad \vec{E} = \vec{E}_0 + \vec{E}_1 = 0$

Ko je naboj v ravnovesju, nanj ne deluje nobena sila.

SLIKA DIPOLA : učb.

Sledijo zaključki

↓

1) $\vec{E} = 0$

2) Kje se nahajajo naboji?

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\varepsilon_0} = 0$$

* V notranjosti snovi bomo narisali zaključeno ploskev

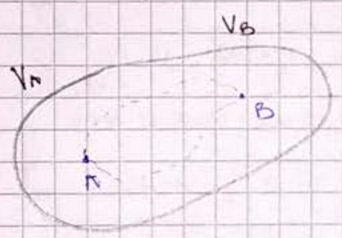
Kamor koli postavimo sklenjeno ploskev je $\vec{E} = 0$.

ZAKLJUČEK: Naboji se nahaja samo NA ZUNANJI POVRŠINI SNOVI

Torej ga bomo tam sešteli:

$$\boxed{Q_{cel} = \varepsilon_0 \oint_{po\ zun} \vec{E}\, d\vec{a}}$$

3) Potencial prevodnika

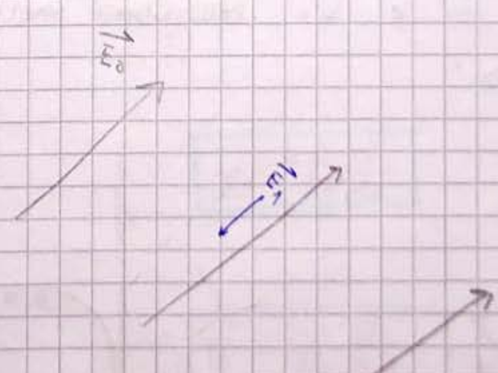


$$V_A - V_B = \int_A^B \underbrace{\vec{E}}_{0}\, \vec{dl}$$

$$\boxed{V_A = V_B}$$

Celoten prevodnik je na ISTEM POTENCIALU predstavlja ekvipotencialno ploskev

4)

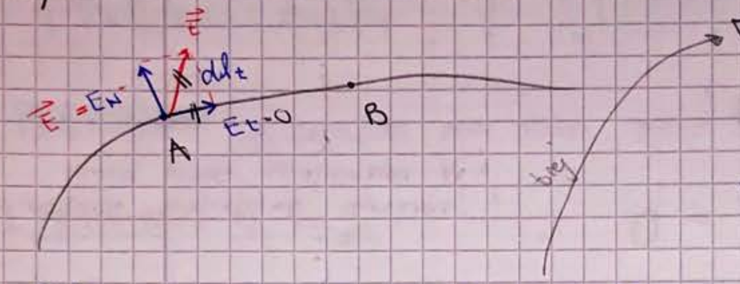


* Naboji se razporedijo ravno tako, da je $\vec{E_1}$ v notranjosti RAVNO NASPROTEN:

$$\boxed{\vec{E_1} = -\vec{E_0}}$$

$$\vec{E} = \vec{E_0} + \vec{E_1} = \vec{E}_{not} = 0$$

5)



$$\overbrace{V_A - V_B}^{0} = \int_{A,l} \vec{E} \cdot \vec{dl} = \int_{A,l,B} E_t \cdot dl_t$$

v TANGENCIALNI SMERI !

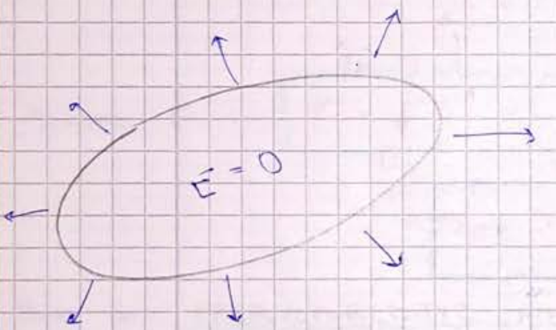
Točki A in B sta na EKVIPOTENCIALKI

$$\boxed{E_t = 0}$$

Torej pridemo do zaključka, da je $\vec{E}$ v SMERI NORMALE

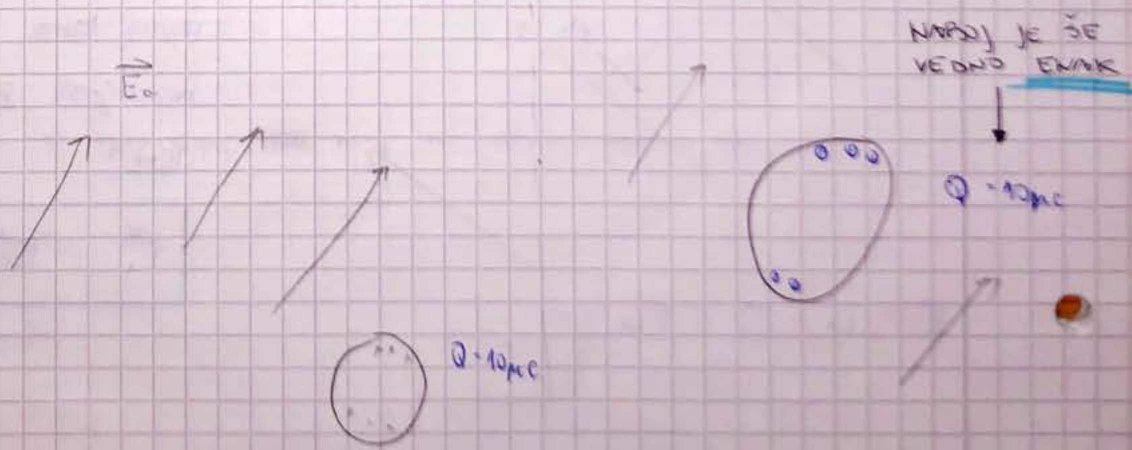
$$\boxed{\vec{E} = \vec{e}_N E_N} \qquad \vec{E}_N \neq 0$$

↳ $\vec{E}_N$ je pravokotna na točko, torej je tudi celotno ELEKTROSTATIČNO POLJE PRAVOKOTEN NA PREVODNIK



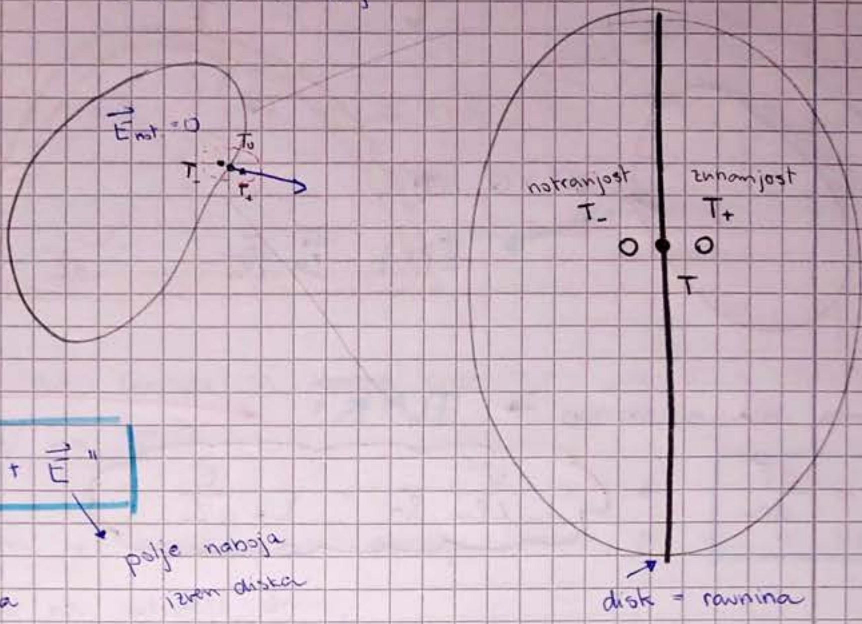
Če hočemo narisati el polje na neko telo, si predstavljamo ekvipotencialne ploskve

Če damo v primarno polje telo, ki je bilo PREDHODNO NAELEKTRENO se naboj ZGOLJ PRERAZPOREDI:



6) EL. POLJE NA MEJI

Govorili bomo o 3 točkah v meji:



$$\boxed{\vec{E} = \vec{E}' + \vec{E}''}$$

$\vec{E}'$ → polje diska

$\vec{E}''$ → polje naboja izven diska

Govorili bomo o $\vec{E}$ v točkah T_-, T, T_+:

↳ T_+

$$\vec{E}(T_+) = \underbrace{\vec{E}'(T_+)}_{\vec{e}_n \frac{\sigma}{2\varepsilon_0}} + \vec{E}''(T_+) = \vec{e}_n \frac{\sigma}{\varepsilon_0}$$

|| te tri so med sabo enake

↳ T

$$\vec{E}(T) = \underset{0}{\vec{E}'(T)} + \vec{E}''(T) = \vec{e}_n \frac{\sigma}{2\varepsilon_0}$$

||

↳ T_-

$$\vec{E}(T_-) = \underbrace{\vec{E}'(T_-)}_{-\vec{e}_n \frac{\sigma}{2\varepsilon_0}} + \vec{E}''(T_-) = 0$$

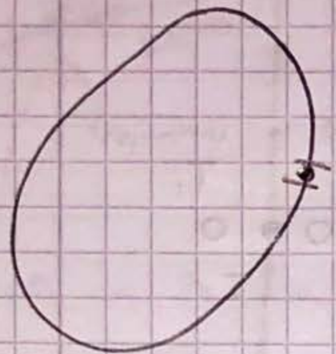
$$\boxed{\vec{E} = \vec{e}_n \frac{\sigma}{\varepsilon_0}}$$

$$\boxed{\vec{E} = \vec{e}_n \frac{\sigma}{2\varepsilon_0}}$$ NA POVRŠINI

$$\boxed{E = 0}$$

ZNOTRAJ | ZUNAJ

7) Elektrostatični tlak



$dQ = \sigma \cdot da$

$\vec{E}(T) = \vec{e}_n \frac{\sigma}{2\varepsilon_0}$; $d\vec{F} = dQ\,\vec{E} = \vec{e}_n\, \sigma \frac{\sigma}{2\varepsilon_0}\, da$

$: da$

delimo

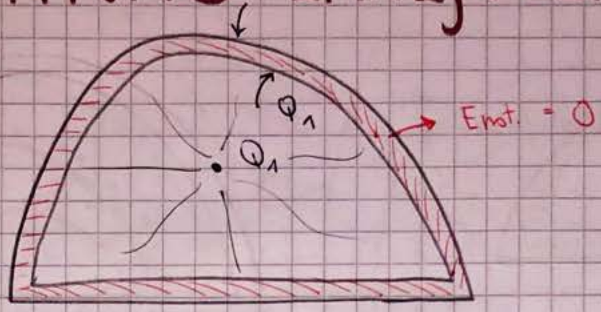
Normiramo silo s ploskvijo → TLAK:

$$\boxed{dp = \frac{d\vec{F}}{da}} \rightarrow = \frac{\sigma^2}{2\varepsilon_0}\vec{e}_n = \varepsilon_0 \frac{E^2}{2}$$

Sila je usmerjena v smeri normale

ELEKTRIČNO ZAKLJUČEN SISTEM

autonomnost



$$\oint \vec{E}_{not} \, da = 0 = \frac{Q_{not}}{\varepsilon_0}$$

Imeli bomo dve posodi in ~~[illegible]~~ razmišljali bomo o VPLIVU TEH DVEH POSOD NA $\vec{E}$.

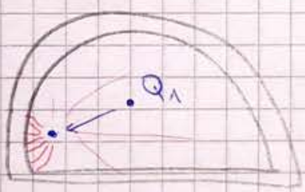
Če postavimo v posodo nek naboj, pomeni, da se bo naboj prestavil tudi na notranjo steno.

↓

Torej naboj se prerazporedi → pojavi se INFLUENCA

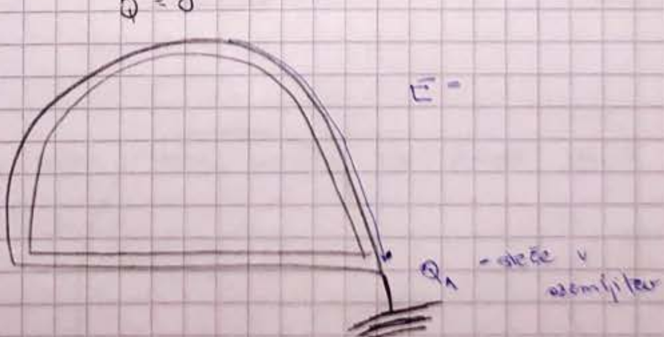
PREMIK Q_{not}:

- če bomo Q premaknili proti levi, se bojo tam gostotnice ZGOSTILE, na desni pa razredčile

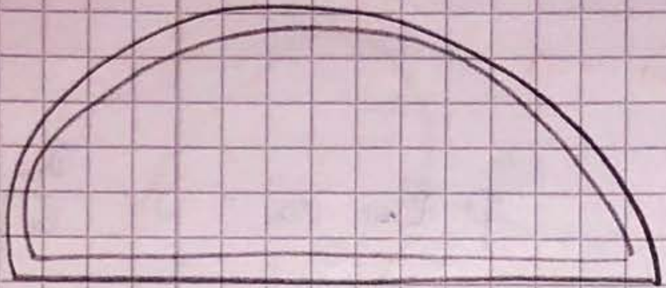


! Vendar Q_1 na zunanji naboj NE BO VPLIVAL

FARADEYEVA KLETKA

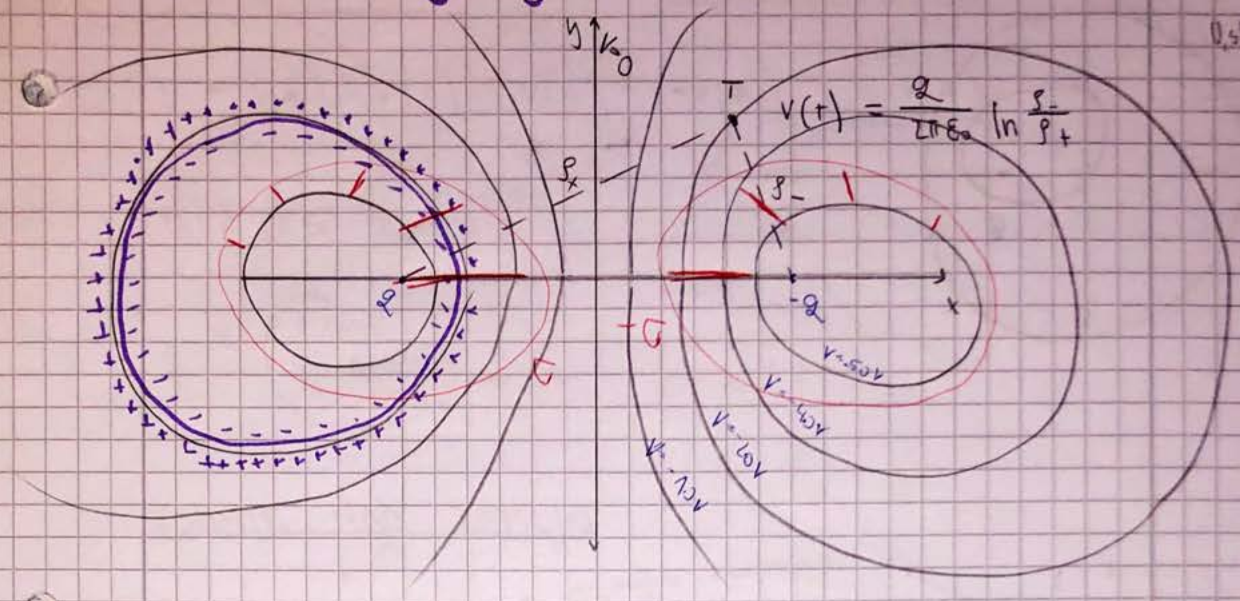


$\boxed{Q_1 = 0}$



ZAKLJUČEK: zaradi zunanjega naboja v notranjosti ni el. polja → $\underline{\underline{E = 0}}$

OKOVINJANJE EKVIPOTENCIALK



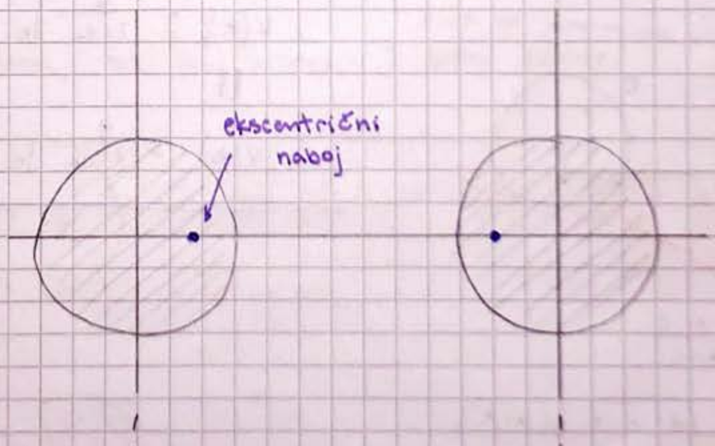
Predstavljamo si tanko folijo, ki jo zvijemo in jo damo med potencialke:

- ↳ **+ naboj** zunanji
- ↳ **− naboj** bi bil na notranji strani

} folija bi bila še vedno NEVTRALNA

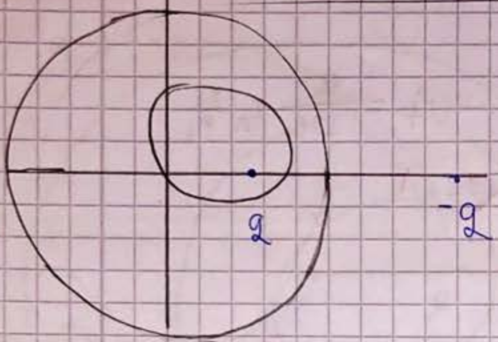
Zgledi

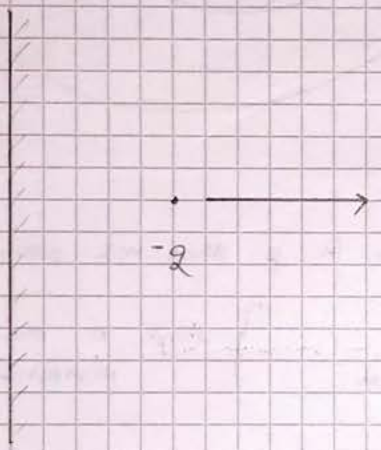
Rišemo dvovod – kot da bi okovinili notranji ekvipotencialki iz zgornje slike



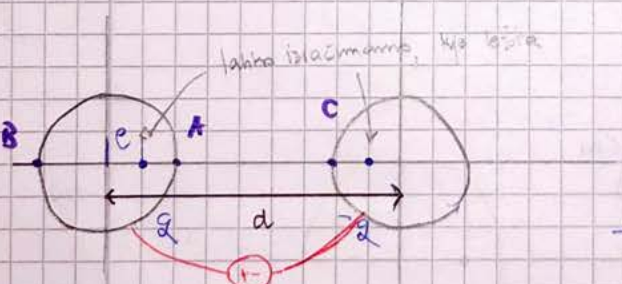
Velik sistem bomo prenesli na dvovod premih elektrod

Ekscentričen koax





SIMETRIČEN DVOVOD



$$V(A) = \frac{q}{2\pi\varepsilon_0}\ln\frac{d-a-e}{a-e}$$

$$\parallel$$

$$V(B) = \frac{q}{2\pi\varepsilon_0}\ln\frac{d+a-e}{a+e}$$

$$\frac{d-a-e}{a+e} = \frac{d+a-e}{a+c}$$

$$(d-a-e)(a+e) = (d+a-e)(a-e)$$

$$\cancel{da} + de - a^2 - \cancel{ae} - \cancel{ea} - e^2 = \cancel{da} - de + a^2 - \cancel{ae} - \cancel{ea} + e^2$$

$$+de + de - a^2 - e^2 - a^2 - e^2 = 0$$

$$-2a^2 + 2de - 2e^2 = 0$$

$$e^2 - de + a^2 = 0$$

$$e_{12} = \frac{d \pm \sqrt{a^2 - 4a^2}}{2}$$

$$\boxed{e = \frac{d}{2} - \sqrt{\left(\frac{d}{2}\right)^2 - a^2}}$$

$V(A) = -V(C)$

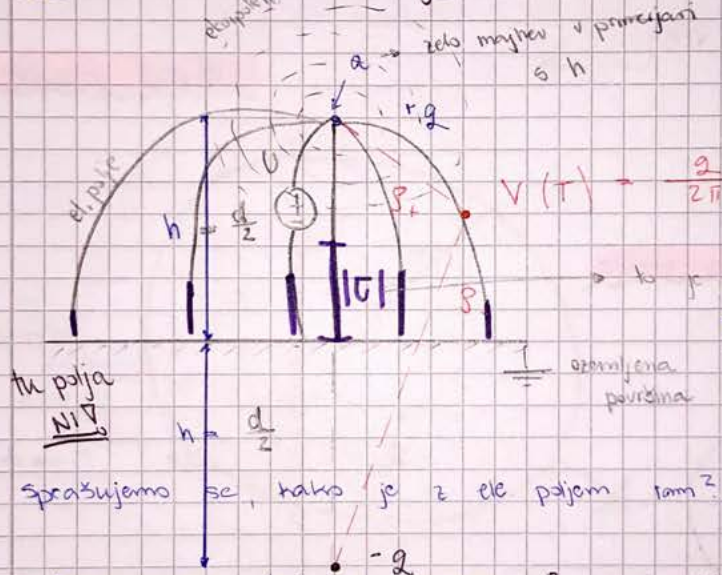
* Napetost med dvema vodnikoma

$U = V(A) - V(C) = V(A) + (-V(A)) = \underline{2V(A)}$

$U = 2\frac{q}{2\pi\varepsilon_0} \ln \frac{d-a-e}{a-e}$

$q = \frac{\pi \varepsilon_0 U}{\ln \frac{d-a-e}{a-e}} \Rightarrow C \qquad \boxed{C = c \cdot l}$

VODNIK NAD ZEMLJO



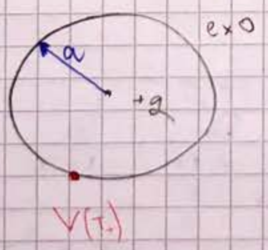
$V(T) = \frac{q}{2\pi\varepsilon_0} \ln \frac{\rho_-}{\rho_+}$

→ to je absolutna vrednost, sicer pa je

− PREDZNAK

sprašujemo se, kako je z ele poljem tam?

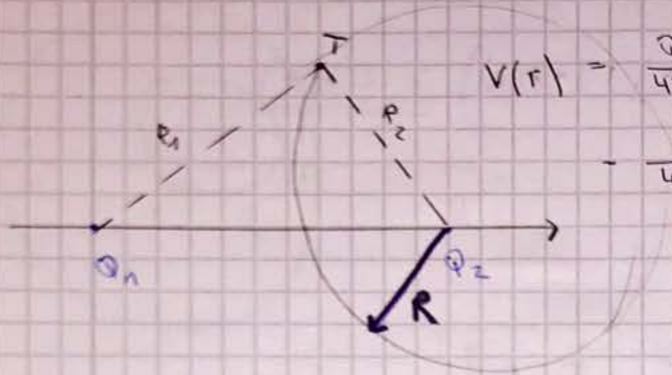
* Obravnavamo, kot da imamo 2 premi elektroni



$U = V(T_1) = \frac{q}{2\pi\varepsilon_0} \ln \frac{2h}{a}$ ← koliko je do $-q$? ← koliko je do $+q$?

$q = \frac{2\pi\varepsilon_0 U}{\ln \frac{2h}{a}} - C$

POLJE V OKOLICI DVEH TOČKASTIH NABOJEV

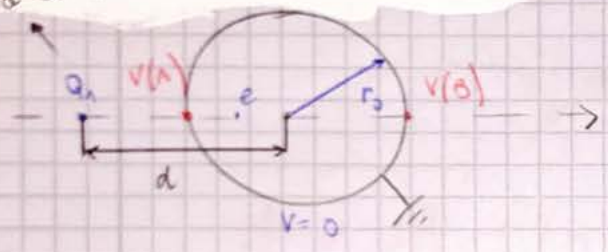


$$V(r) = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} = \frac{1}{4\pi\varepsilon_0}\underbrace{\left(\frac{Q_1}{R_1} + \frac{Q_2}{R_2}\right)}_{0}$$

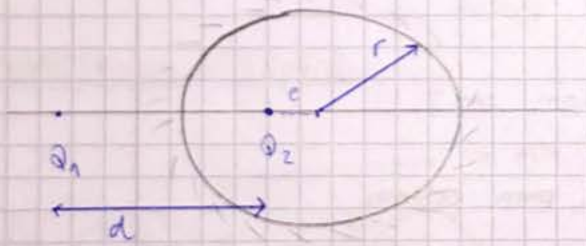
ekvipotencialka je SFERA
$$\boxed{\frac{Q_1}{R_1} = -\frac{Q_2}{R_2}}$$

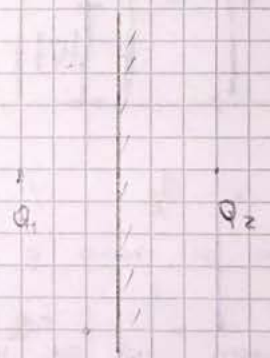
Zgledi

pred ozemljeno kroglo



Točkast naboj pred ozemljeno kroglo





točkast naboj
- pred prevodno steno
- med zemljo

$$V(A) = \frac{1}{4\pi\varepsilon_0}\overbrace{\left(\frac{Q_1}{d-r_0} + \frac{Q_2}{r_0-e}\right)}^{\emptyset}$$

$$V(B) = \frac{1}{4\pi\varepsilon_0}\underbrace{\left(\frac{Q_1}{d+r_0} + \frac{Q_2}{r_0+e}\right)}_{\emptyset}$$

izenačimo $\Longrightarrow$

$$\frac{Q_1}{d-r_0} + \frac{Q_2}{r_0-e} = \frac{Q_1}{d+r_0} + \frac{Q_2}{r_0+e}$$

$$d \cdot e = r_0^2$$

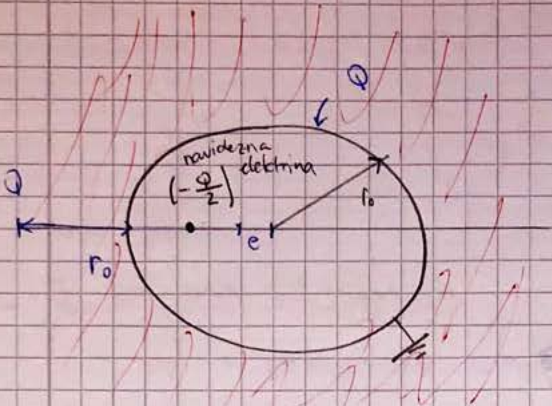
$$\boxed{e = \frac{r_0^2}{d}}$$ ekscentričnost

$$\frac{Q_1}{d-r_0} = -\frac{Q_2}{r_0-e}$$

$$Q_1 = -\frac{Q_2}{r_0-e}(d-r_0) = -\frac{Q_2}{r_0-\underbrace{\frac{r_0^2}{d}}_{e}}(d-r_0) = -\frac{d}{r_0}Q_2$$

$$\boxed{Q_2 = -\frac{r_0}{d}Q_1}$$

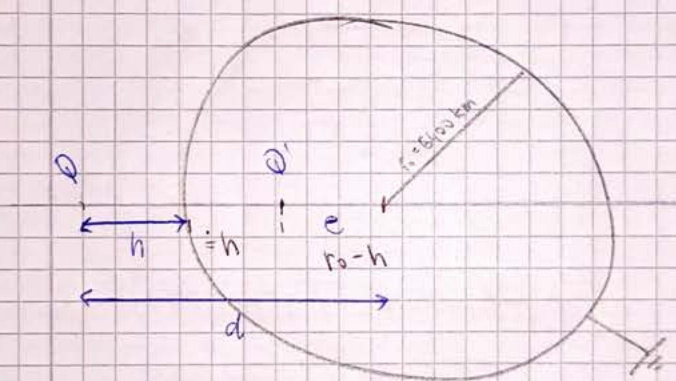
* samo v zmanjosti



$$Q' = -\frac{r_0}{d} Q = -\frac{r_0}{2r_0} Q = -\frac{Q}{2}$$

$$e = \frac{r_0^2}{d} = \frac{r_0^2}{2r_0} = \frac{r_0}{2}$$

Vloga zemlje



Q' - navidezen naboj, s katerim moduliramo

e = ?

d = ?

$$e = \frac{r_0^2}{d} = \frac{r_0^2}{r_0+h} = r_0 \frac{r_0}{r_0+h} = r_0 \frac{1}{1+\frac{h}{r_0}}$$ zelo malo

$$\frac{1}{1+0{,}01} \doteq 0{,}99 = 1 - 0{,}01$$

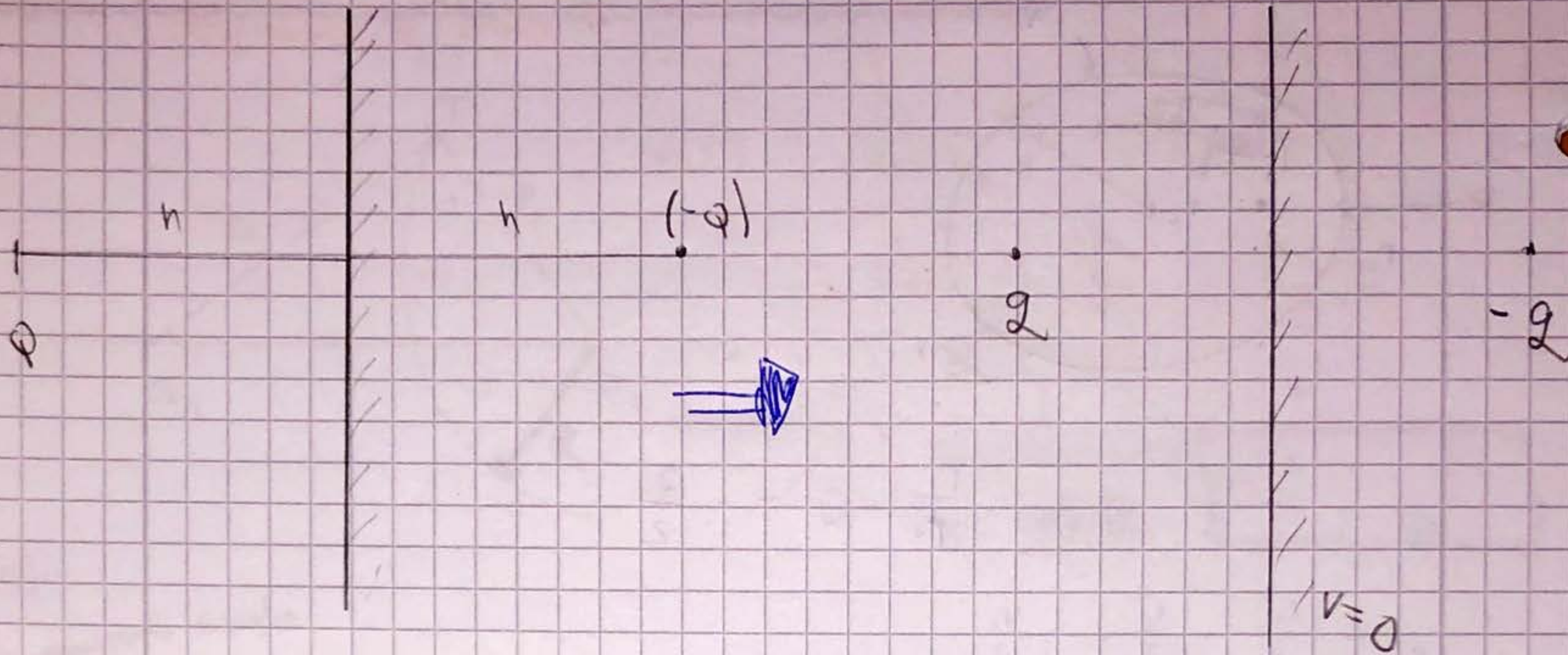
$$Q' = -\frac{r_0}{d} Q$$

$$Q' = -\underbrace{\frac{r_0}{r_0+h}}_{\text{zanemerljivo}} Q \doteq -Q$$

$$\frac{1}{1+\frac{h}{r_0}} \overset{*}{=} 1 - \frac{h}{r_0}$$

$$e = r_0 \cdot \left(1 - \frac{h}{r_0}\right)$$

$$\boxed{e = r_0 - h}$$


Q
h
h
(-q)
q
-q
V=0

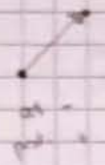
IZOLANT / DIELEKTRIK V ELEKTRIČNEM POLJU → polarizacija

Vrste polarizacije

I. DIPOLSKA POLARIZACIJA

↳ snov s permanentno izraženimi dipoli (voda)

a)



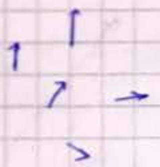
$\boxed{d\vec{p} = dQ \cdot \vec{d}}$ dipolski moment

$\langle \vec{p} \rangle = 0$ voda ni polarizirana

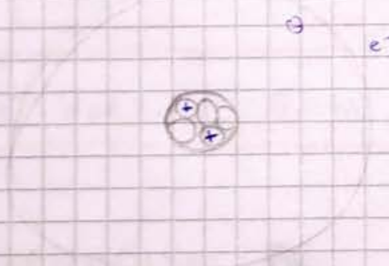
povprečni moment

} $E = 0$

b) $\vec{E} \neq 0$. mamo neko el. polje



I. ELEKTRONSKA POLARIZACIJA

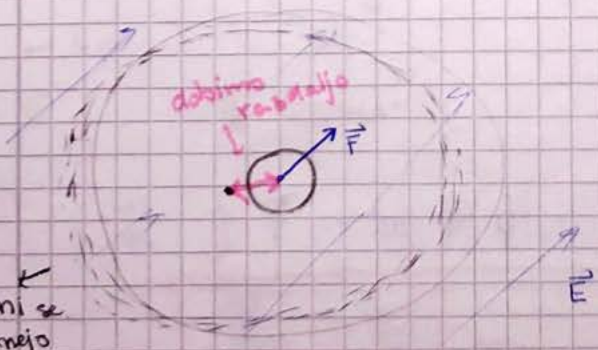


$\vec{E} = 0$

$\langle \vec{p} \rangle = 0$

↓ postavimo v polje

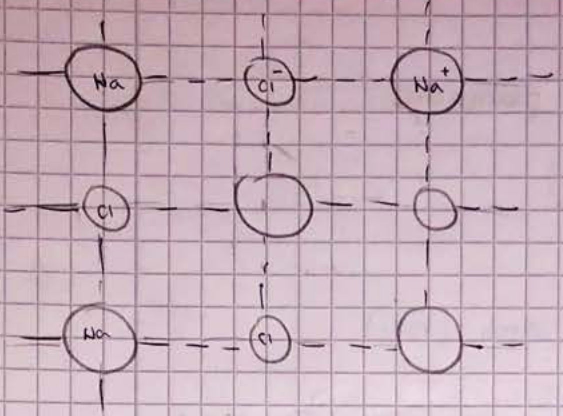
→ ko postavimo molekulo v $\vec{E}$ se ustvari dipol



elektroni se izmaknejo

$\vec{E} \neq 0$

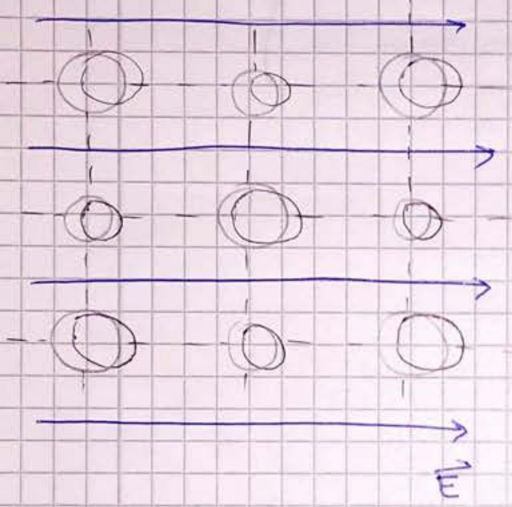
III. IONSKA POLARIZACIJA



$\vec{E} = 0$

$\langle \vec{p} \rangle = 0$

$\Downarrow$ damo v $\vec{E}$



$\vec{E} \neq 0$

$\langle \vec{p} \rangle \neq 0$

ELEKTRIČNA POLARIZACIJA: $\vec{P}$

↳ učinek el. polja na dielektrik

$$\boxed{\vec{P} = \lim_{\to 0} \frac{\sum d\vec{p}}{dv}} \quad [C/m^2]$$

def: $\vec{E}$, izvornost $\vec{E}$, vrtinčnost $\vec{E}$

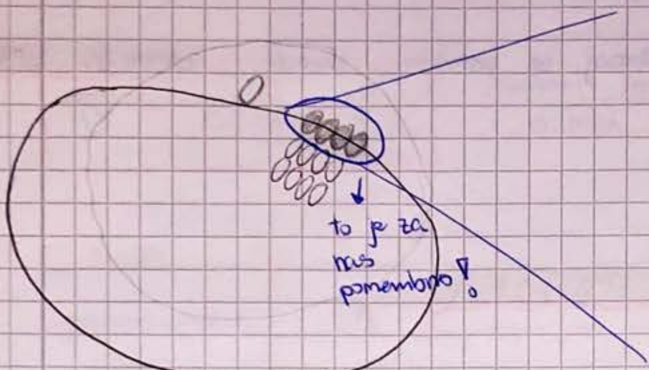
IZVORNOST $\vec{P}$

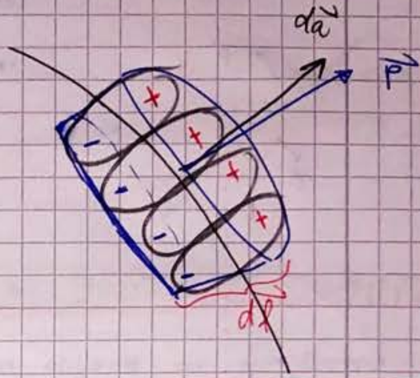
$$\oint_{V} \vec{J}\, d\vec{a} = -\frac{dQ_{not}}{dt}$$

$$\oint_{V} \vec{E}\, d\vec{a} = \frac{Q_{not}}{\varepsilon_0}$$

$$\oint \vec{P}\, d\vec{a} = ?$$

Smo postavimo v $\vec{E}$:





sklenjena ploskev → z njo smo nekatere DIPOLE PREREZALI

Zanima nas produkt $d\vec{a}\,\vec{P}$?

Vse zaobjete dipole bi radi SEŠTELI:

↳ POLARIZIRANI SO

$$d\vec{P}_{pol} = -dQ_{pol}\,\vec{d}$$

$$Q_{pol} = \sigma_{pol} \cdot d\vec{a}$$

$$\vec{P}\cdot d\vec{a} = \frac{Q_{pol}\ dl}{dV} = \frac{\sigma_{pol}\cdot \overbrace{d\vec{a}\ dl}^{\cancel{dV}}}{\cancel{dV}} = \sigma_{pol\,zun}\cdot d\vec{a}$$

$$\boxed{\vec{P}\cdot d\vec{a} = \sigma_{pol\,zun}\cdot d\vec{a}}$$

$$\boxed{\vec{P}\,d\vec{a} = Q_{pol\,zun} = -Q_{pol\,not}}$$

$$\boxed{\oint \vec{P}\,d\vec{a} = Q_{pol\,zun} = -Q_{pol\,not}}$$

POLARIZACIJSKI TOK

$$\leftarrow i_{pol} = -\frac{dQ_{pol\,not}}{dt}$$

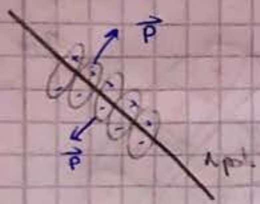
$i = \oint \vec{J}\cdot d\vec{a}$

⇓ ODVAJAMO

$$= \frac{d}{dt}\oint \vec{P}\cdot d\vec{a} = \cancel{\oint} \vec{J}_{pol}\cdot \cancel{d\vec{a}}$$

$$\boxed{\frac{d\vec{P}}{dt} = \vec{J}_{pol}}$$

POMEMBNO:



↳ Naboj se premika, zaradi ZASUK OV DIPOLOV

VEKTOR GOSTOTE EL PRETOKA

Naše razmišljanje bo temeljilo na:

I $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\varepsilon_0} \quad \Big| \cdot \varepsilon_0$

II $\oint \vec{P} \, d\vec{a} = -Q_{not.\ pol.}$

SKUPNO:
- Notranji naboj Q not.
- sklenjena ploskev

Najprej enačbi SEŠTEJEMO

$$\oint_{\text{sk}} \underbrace{\left(\varepsilon_0 \vec{E} + \vec{P}\right)}\, d\vec{a} = Q_{not.} - Q_{not\ pol.} = Q_{not.\ prosti}$$

$Q_{not\ pol.}$ ↓ ne znamo jih izmeriti

$Q_{not.\ prosti}$ → znamo jih inženirsko izmeriti

⇓

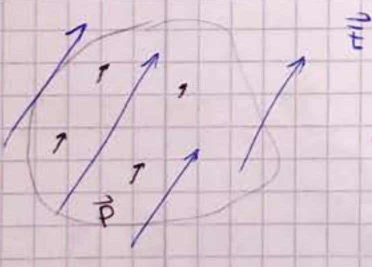
$$\boxed{\oint_{\text{sk}} \vec{D} \cdot d\vec{a} = Q_{not\ prosti}}$$

GOSTOTA EL. PRETOKA

Gauss ov stavek za $\vec{D}$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$ definicija

$\lim \frac{\Sigma d\vec{p}}{dv}$ – še vedno ne znamo izmeriti



sorazmernost α

$\vec{P} \propto \vec{E}$

$\vec{P} = k\vec{E}$

da bo enačba enake oblike

$\vec{P} = \chi_e \, \varepsilon_0 \vec{E}$

znamo empirično izmeriti

↓

ELEKTRIČNA SUSCEPTIBILNOST SNOVI

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\vec{E} + \chi_e\varepsilon_0\vec{E} = \underbrace{(1+\chi_e)}\varepsilon_0\vec{E}$$

ε_r - relativna dielektrična snovi

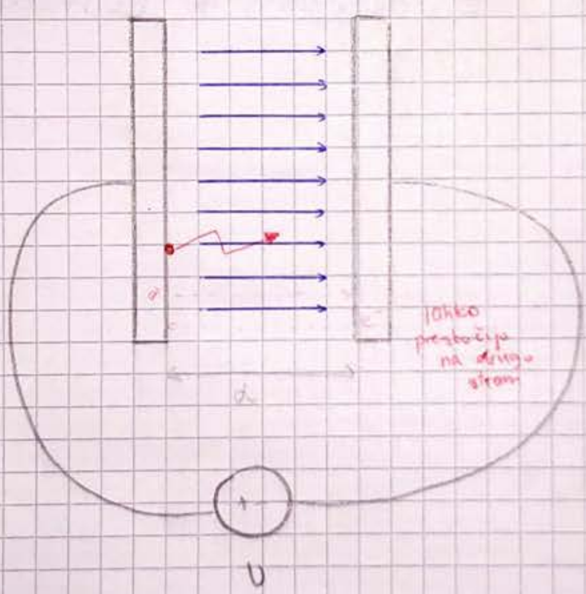
$$\boxed{\vec{D} = \varepsilon_0 \cdot \varepsilon_r \cdot \vec{E}}$$

obnašanje snovi v el. polju

SNOV	ε_r
prazen prostor	1
zrak	1,0006
steklo	5 - 10
olje	2 - 15
voda	81
spec. snovi	10^6

ELEKTRIČNI PREBOJ

↳ Preskok elektronov na drugo ploščo



To se zgodi:

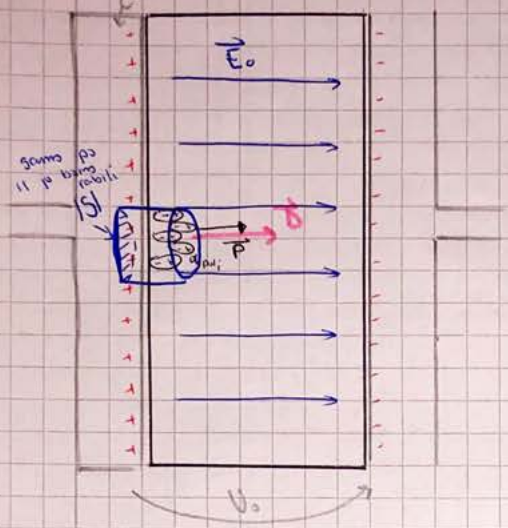
$$\boxed{E > E_p}$$

Vsaka snov ima koeficient prebojne trdnosti in če ga z $\vec{E}$ PRESEŽEMO

↓

EL. PREBOJ

Zgled: Ploščni kondenzator z in brez dielektrika



I. Brez dielektrika

$$E_0 = \frac{\sigma_{pr.}}{\varepsilon} \qquad U_0 = E \cdot d$$

- konstanten naboj
- priključeno na neko napetost

II. Z dielektrikom dielektrik debeline d

$$\vec{E} = \vec{E}_0 + \vec{E}_1 \leftarrow \text{[illegible]}$$

$$\oint_{vt} \vec{D} \cdot d\vec{a} = D_x \cdot S = \boxed{\sigma_{pr.} \cdot S}$$

$\hookrightarrow D_x = \sigma_{pr.}$

valj
- 2 osnovnici
- 1 plašč

* Dobili smo gostoto pretoka, D_x, ki kaže samo v smeri x in je enaka gostoti prostega naboja $\sigma_{pr.}$

$$\oint_{vt} \vec{P} \cdot d\vec{a} = P_x \cdot S = \boxed{-\sigma_{pol.} \cdot S}$$

$$\oint_{vt} \vec{E} \cdot d\vec{a} = \frac{\sigma_{pr.} - \sigma_{pol}}{\varepsilon_0} \cdot S \qquad E_x = \frac{\sigma_{pr.} - \sigma_{pol.}}{\varepsilon_0}$$

$$D_x = \varepsilon_0 \varepsilon_r E_x$$

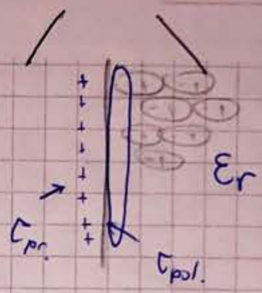
$$\sigma_{pr} = \varepsilon_0 \varepsilon_r \cdot \frac{\sigma_{pr} - \sigma_{pol}}{\varepsilon_0}$$

$$\varepsilon_r \sigma_{pol} = \sigma_{pr} (\varepsilon_r - 1)$$

$$\boxed{\sigma_{pol} = \sigma_{pr.} \frac{(\varepsilon_r - 1)}{\varepsilon_r}}$$

Primer: $\varepsilon_r = 3$

$$\frac{\sigma_{pol.}}{\sigma_{pr.}} = \frac{2}{3}$$



$$E_x = \frac{\sigma_{pr} - \sigma_{pol.}}{\varepsilon_0} = \frac{\sigma_{pr.} - \sigma_{pr.}\frac{(\varepsilon_r - 1)}{\varepsilon_r}}{\varepsilon_0} = \frac{\sigma_{pr}}{\varepsilon_0} \cdot \frac{\varepsilon_r - \varepsilon_r + 1}{\varepsilon_r} =$$

$$= \frac{\overbrace{\sigma_{pr}/\varepsilon_0}^{E_0}}{\varepsilon_r} = \frac{E_0}{\varepsilon_r}$$

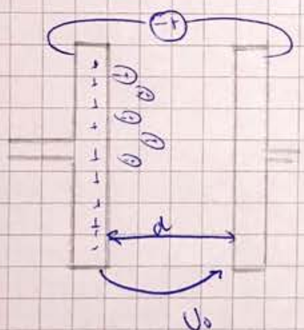
$$\boxed{E = \frac{E_0}{\varepsilon_r}}$$

* ε_r zmanjša el. polje

b) Vključen vir U_0

Napetostni vir nam vzdržuje:

* konstantno napetost

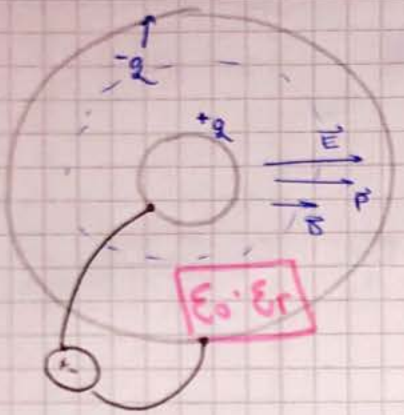


1) $E = \frac{U_0}{d}$

2) $E = \frac{U_0}{d} = E_0$

KAJ SE SPREMENI?
Napetostni vir bo DODAL NABOJE, ki bojo vzdržali el. polje, da bo enako kot prej.

Zgled: koaxialni kabel



prosti zaobjeti naboji

$$\oint_{\mathcal{A}} \vec{D} \cdot d\vec{a} = \underbrace{q \cdot l}$$

$$D \cdot 2\pi \cdot \rho \cdot l = q \cdot l$$

$$D_\rho = \frac{q}{2\pi \rho}$$ ← radij

$$\boxed{E_\rho = \frac{D_\rho}{\varepsilon_0 \cdot \varepsilon_r} = \frac{q}{2\pi \varepsilon_0 \boxed{\varepsilon_r} \rho}}$$

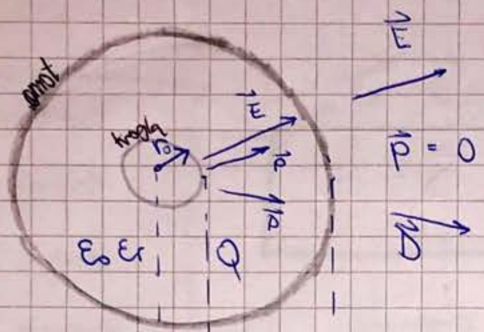
→ enaka enačba samo dodana je relativna del

$$U = \int_a^b E_\rho \, d\rho =$$

$$\boxed{U = \frac{q}{2\pi \varepsilon_0 \varepsilon_r} \ln \frac{b}{a}}$$

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

Zgled: Krogla v dielektričnem omotu

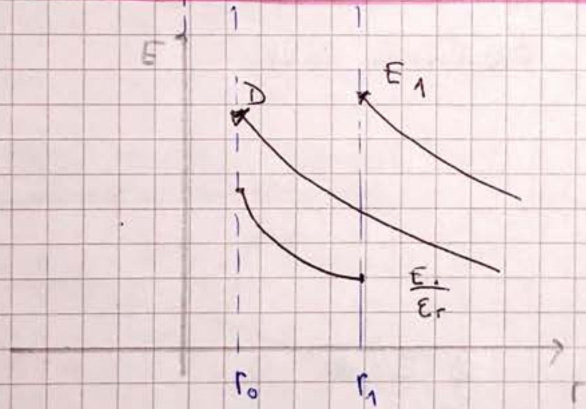


$$\oint_{\partial V} \vec D \cdot d\vec a = Q_{prosti} = Q$$

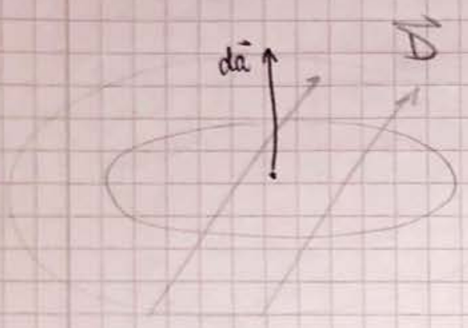
$$\boxed{D_r = \frac{Q}{4\pi r^2}}$$

Pri $\vec E$ ne velja za oba r:

$$\boxed{E_r = \begin{cases} 0 & , \quad r < r_0 \\ \dfrac{Q}{4\pi \varepsilon r^2} & , \quad r_0 < r < r_1 \\ \dfrac{Q}{4\pi \varepsilon_0 r^2} & , \quad r_1 < r \end{cases}}$$



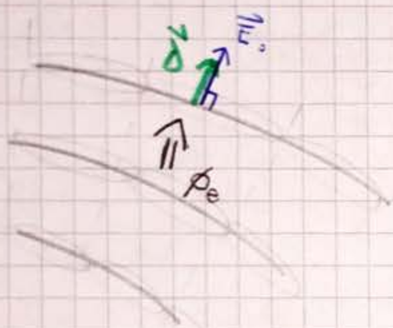
ELEKTRIČNI PRETOK - ϕ_e, pretočne cevke



$$\boxed{\phi_e = \int_{vt} \vec{D} \cdot d\vec{a}}$$

↓
ploskovna gostota el pretoka

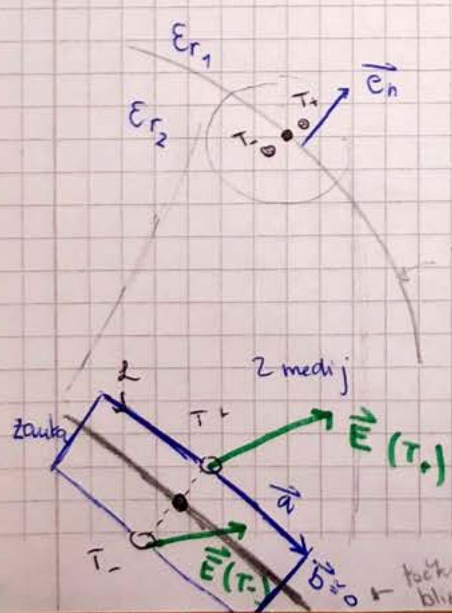
Podobno kot Gaussov stavek, vendar mi nimamo sklenjene ploskve in ga ne računamo po celi ploskvi.



* ϕ_e ogradimo v neke „ograje" oz. „cevi" ki so VZPOREDNE z $\vec{E}$ in $\vec{D}$, ter v njih je ϕ_e ENAK. Imenujemo jih PRETOČNE CEVI.

MEJNA POGOJA:

1) Mejni pogoj $\vec{E}$



← točki sta tako blizu, da je praktično 0

#Skulvajb

$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0$$

projekcija $\vec{E}$ na $\vec{a}$

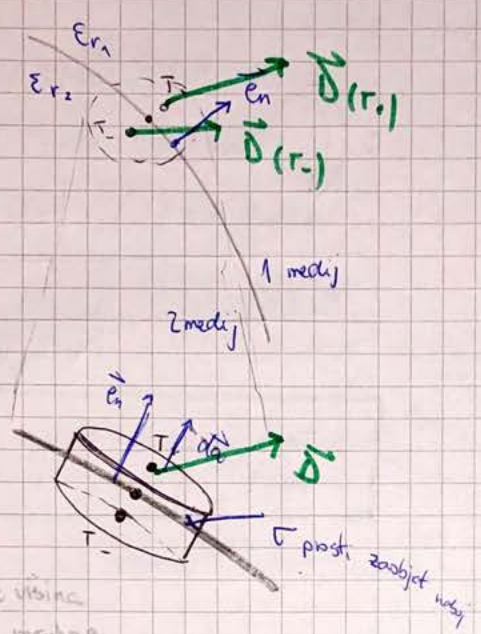
$$\vec{E}(T_+) \cdot \vec{a} + \vec{E}(T_-)(-\vec{a}) = 0$$

$$E_t(T_+) \cdot a - E_t(T_-) \cdot a = 0$$

tangencialno $\vec{E}$ se OHRANJA ← $\underline{E_t(T_+) = E_t(T_-)}$

$$\underline{\vec{e}_n \times (\vec{E}(T_+) - \vec{E}(T_-)) = 0}$$

2) Mejni pogoj $\vec{D}$



$$\oint_{u} \vec{D}\, d\vec{a} = Q_{\text{prosti zaobjeti naboji}}$$

normala na ploskev je v drugo smer

$$D_n(T_+) \cdot da - D_n(T_-) \cdot da = \sigma_{\text{prosti zaobj.}}\, da$$

$$\boxed{D_n(T_+) - D_n(T_-) = \sigma}$$

$$\boxed{\vec{e}_n \cdot (\vec{D}(T_+) - \vec{D}(T_-)) = \sigma}$$

* ker je višina valja majhna bomo računali pretok le za površini T_- in T_+

Meja prevodnih | izolant

$\vec{E} = 0$

$\vec{\varphi} = 0$

$\vec{D} = 0$

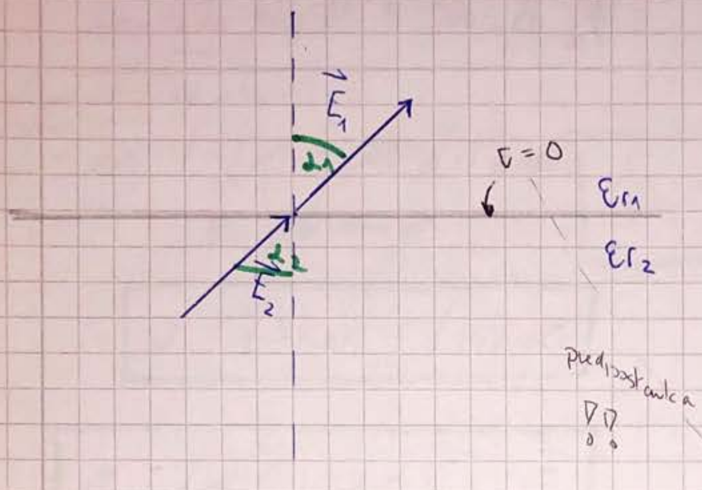
$E_t(T_+) = E_t(T_-) \rightarrow$ v izolantu $\vec{E} \perp$ na mejo

$$D_n(T_+) - D_n(T_-) = \sigma$$

$$\varepsilon_0 \varepsilon_r E_n(T_+) = \sigma$$

$$\boxed{E_n(T_+) = \frac{\sigma}{\varepsilon_0 \varepsilon_r}}$$

Meja izolant / izolant brez naboja na meji



$$E_t(T_1) = E_t(T_2)$$
$$D_n(T_1) = D_n(T_2)$$
$$\varepsilon_0 \varepsilon_r E_n(T_1) = \varepsilon_0 \varepsilon_r E_n(T_2)$$

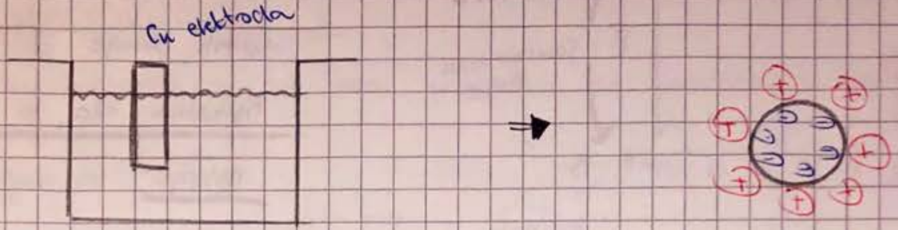
$$\frac{E_t(T_1)}{\varepsilon_0 \varepsilon_{r1} E_n(T_1)} = \frac{E_t(T_2)}{\varepsilon_0 \varepsilon_{r2} E_n(T_2)}$$

$$\boxed{\frac{\mathrm{tg}\,\alpha_1}{\mathrm{tg}\,\alpha_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}}$$

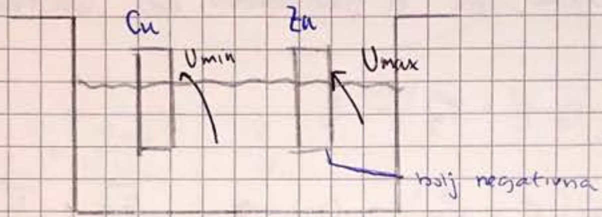
„lomni zakon"

IDEALNI NAPETOSTNI VIR

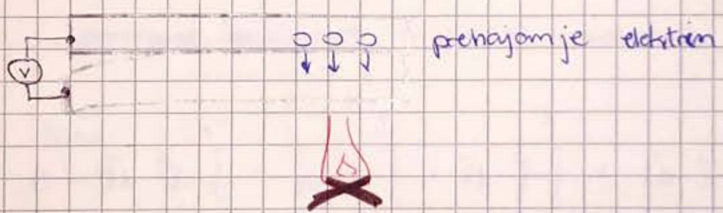
1) KEMIČNI PROCES



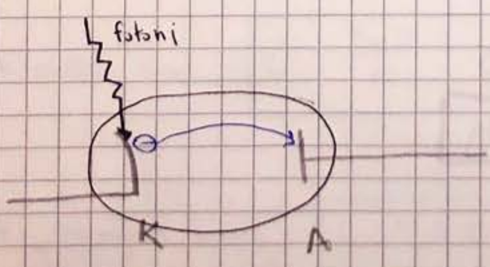
Voltov člen



2) TERMIČNI PROCES



3) FOTOEFEKT

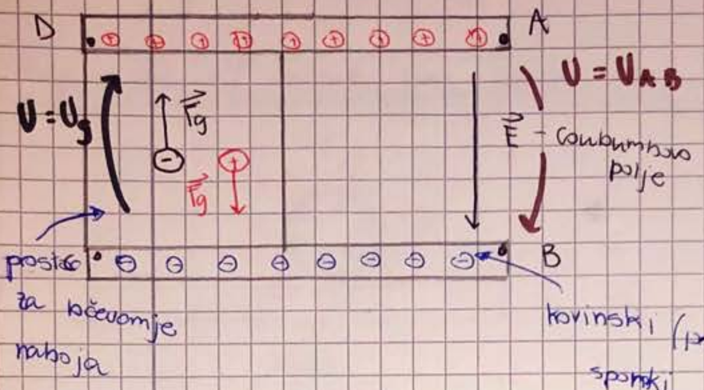


- fotoni izbijejo elektrone iz anode in odletijo do katode.
- dajo jim začetno energijo, s katero lahko grejo na katodo.

4) INFLUENCA OE I

5) INDUKCIJA OE II

Model napetostnega vira



$\vec{F}_g$ .... sila generatorja (ki meče ene naboje na eno stran sponk, druge pa na drugo)

Fizikalna sila, ki razdvaja naboje ni nujno električna

Neelektrično silo bomo interpretirali kot el. polje, vendar gre ubistvu za:

$$\vec{E}_g = \frac{\vec{F}_g}{Q} \qquad \left(Q\vec{E} + \vec{F}_g \right) = 0$$

normirana sila z nabojem

$$Q\left(\vec{E} + \vec{E}_g\right) = 0 \rightarrow \vec{E}_g = -\vec{E}$$

Izbrali si bomo 4 točke, da dobimo SKLENJENO PLOSKEV in zapišemo

$$\oint_L \vec{E}\cdot d\vec{l} = 0 = \underbrace{\int_A^B \vec{E}\, d\vec{l}}_{U = U_{AB}} + \int_B^C \vec{E}\cdot d\vec{l} + \underbrace{\int_C^D \vec{E}\cdot d\vec{l}}_{-\int_C^D \vec{E}_g\cdot d\vec{l} = U_g} + \int_D^A \vec{E}\, d\vec{l} = 0$$

$$U - U_g = 0$$

$$\boxed{U = U_g}$$

* Delo generatorja:

$$A_g = \int \vec{F}_g\, d\vec{l} = Q\int \vec{E}_g\, d\vec{l} = Q\cdot U_g \rightarrow \boxed{A_g = Q\,U_g}$$

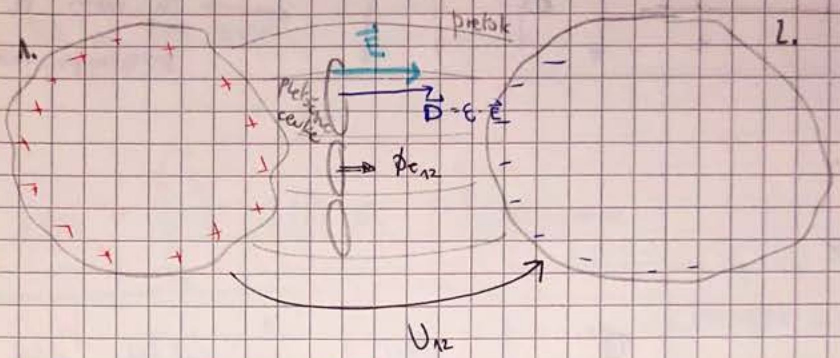
KAPACITIVNOST IN KONDENZATOR

(kot strjen element)

KAPACITIVNOST

Predstavljajmo si dve telesi, priključeni na napetost, z nekima nabojema

vmes je snov : ε_r



$$\boxed{C = \frac{\Phi_{e_{12}}}{U_{12}} = \frac{Q}{U_{12}}}$$

Kapacitivnost je lastnost strukture, da pod vplivom el. napetosti shrani el. naboj oz. energijo.

Že znani primeri za C :

* koax : $\boxed{C = \frac{2\pi\varepsilon l}{\ln\frac{b}{a}}}$

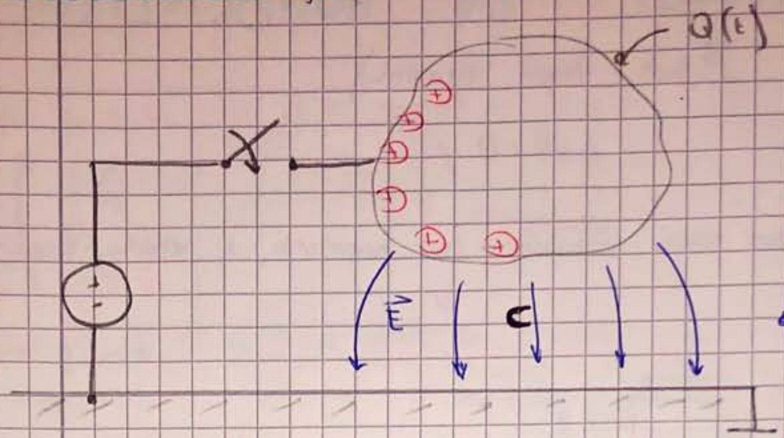
* sferični kond. $\boxed{C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_2 - r_1}}$

* ploščni kond. : $\boxed{C = \frac{\varepsilon_0 \cdot S}{d}}$

* „Osamljena krogla" $\boxed{C = 4\pi\varepsilon_0 r_1}$
  zemlja brez ionosfere

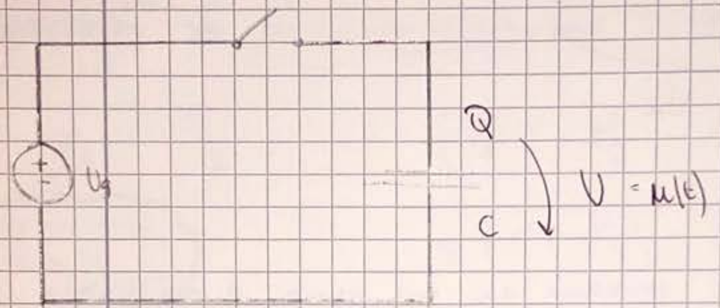
* Zemlja_ ionosfera $\boxed{C = \frac{4\pi\varepsilon_0 r_1^2}{h}}$

Naelektritev kondenzatorja

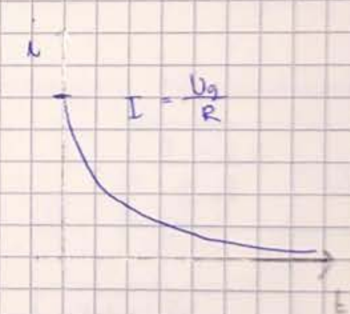


Ta norm el. polje predstavlja neko C

- Modelno vezje



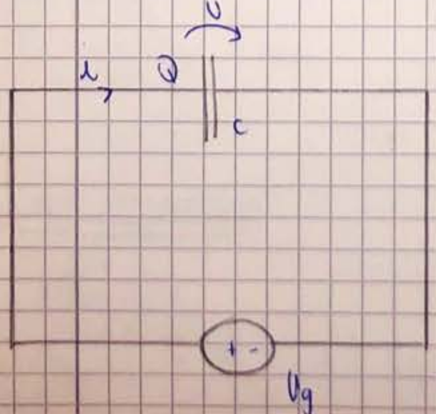
$Q(t) = C\,u(t)$



$$i = \frac{dQ}{dt}$$

Ta limitni proces je za nas še NEZNANKA

Kondenzator v živih strukturah



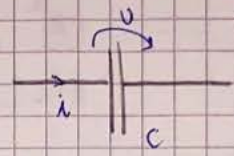
$u - u_g = 0$

$u = u_g$

Kontinuitetna enačba:

$$i = \frac{dQ(t)}{dt} = \frac{d\left(C\cdot u(t)\right)}{dt} = C\cdot\frac{du}{dt} = Cu'$$

$$\boxed{i = C\cdot\frac{du}{dt}}$$



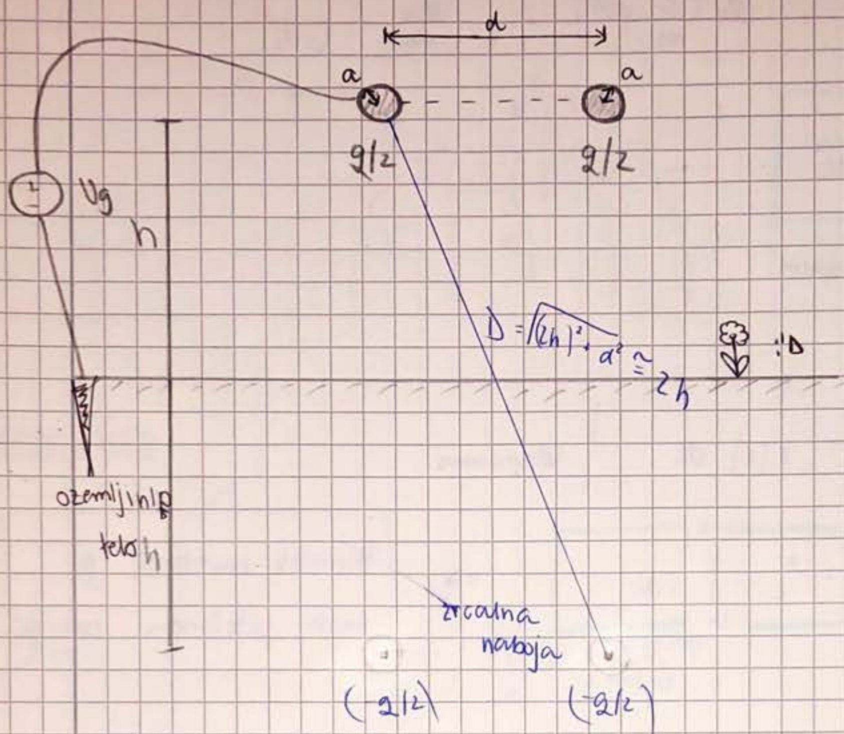
tok pri čas. sprem. strukturah

$$\int_{t_1}^{t_2} du = \int_{t_1}^{t_2} \frac{1}{C}\, i(t)\, dt \quad \Big|\ \text{integriramo}$$

$$\boxed{\underbrace{u(t_2) - u(t_1)}_{\text{prirastek napetosti}} = \frac{1}{C}\underbrace{\int_{t_1}^{t_2} i\,dt}_{\text{pritečen naboj}}}$$

$\Rightarrow$ Prirastek napetosti je enak pritečenem naboju

HARMONIČNO VZBUJEN „DVOJČEK" NAD ZEMLJO



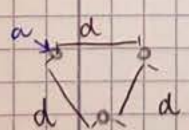
$$V_{dvojčka} = \frac{q/2}{2\pi\varepsilon_0}\ln\frac{2h}{a} + \frac{q/2}{2\pi\varepsilon_0}\ln\underbrace{\frac{D}{d}}_{\simeq 2h/d} =$$

$$= \frac{q}{2\pi\varepsilon_0}\ln\sqrt{\frac{(2h)(2h)}{ad}} = \frac{q}{2\pi\varepsilon_0}\ln\frac{2h}{\sqrt{ad}}$$

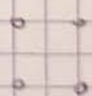
$$C = \frac{q}{V_{dvojček}} = \frac{2\pi\varepsilon_0}{\ln\frac{2h}{\sqrt{ad}}} = \frac{2\pi\varepsilon_0}{\ln\frac{2h}{r_{ekvivalentni}}}$$

$$\boxed{r_{ek} = \sqrt{ad}}$$

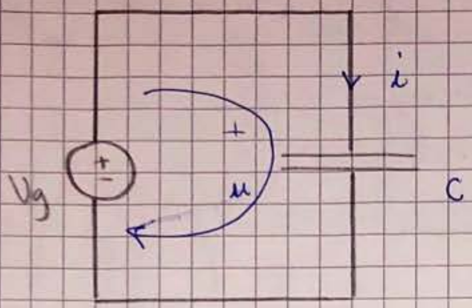
Primer trojčka



$$\boxed{r_{ekv} = \sqrt[3]{ad^2}}$$



$U_g = U_{gm} \sin \omega t$, $U_{gm} = \frac{400\ kV}{\sqrt{3}} \sqrt{2} \cong 326 \cdot 10^3\ V$, $\omega = 2\pi f = 100\pi \cong 314\ s^{-1}$



$-u_g + u = 0$

$u = u_g$

$i = C u' = C u_g'$

$h = 10m$

$a = 3cm$

$d = 40cm$

$r_{ekv} = \sqrt{ad} = \sqrt{3 \cdot 40} \cong 11\ cm$

Vzeli bomo eno dolžino l cca:

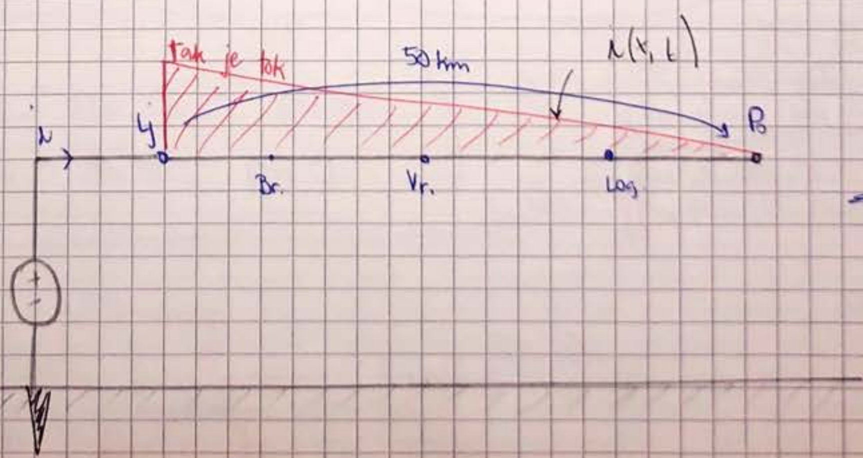
$l = 50\ km$

$$C = \frac{2\pi \varepsilon \cdot l}{\ln \frac{2h}{r_{ekv}}} = \frac{2\pi \cdot 10^{-9} \cdot 5 \cdot 10^4}{36\pi \ln \frac{20m}{0{,}11m}} \cong 0{,}053 \cdot 10^{-5} = 0{,}53\ \mu F$$

$$i = C u_g' = \underbrace{C \omega U_{gm}}_{I_m} \cos \omega t = I_m \cos \omega t$$

$$I_m = \omega \cdot C \cdot U_{gm} = 314 \cdot 0{,}53 \cdot 10^{-6} \cdot 326 \cdot 10^3 = 55\ A$$

Če skiciramo



* imamo odprt krog, pa tok še vedno teče

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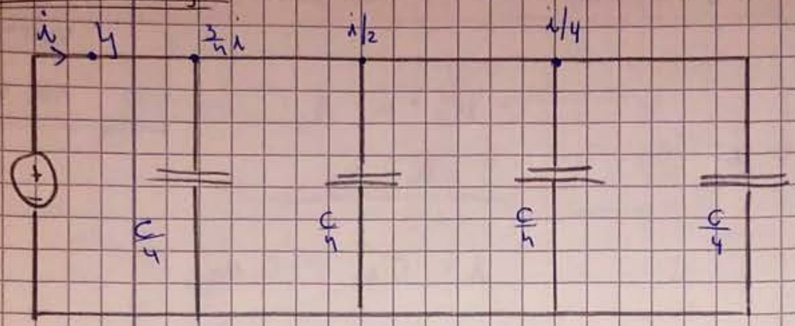
Modelno vezje:

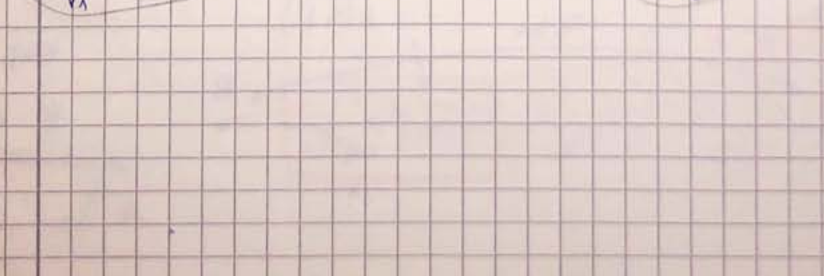


DELNE KAPACITIVNOSTI

1) OSAMLJENI OBJEKT

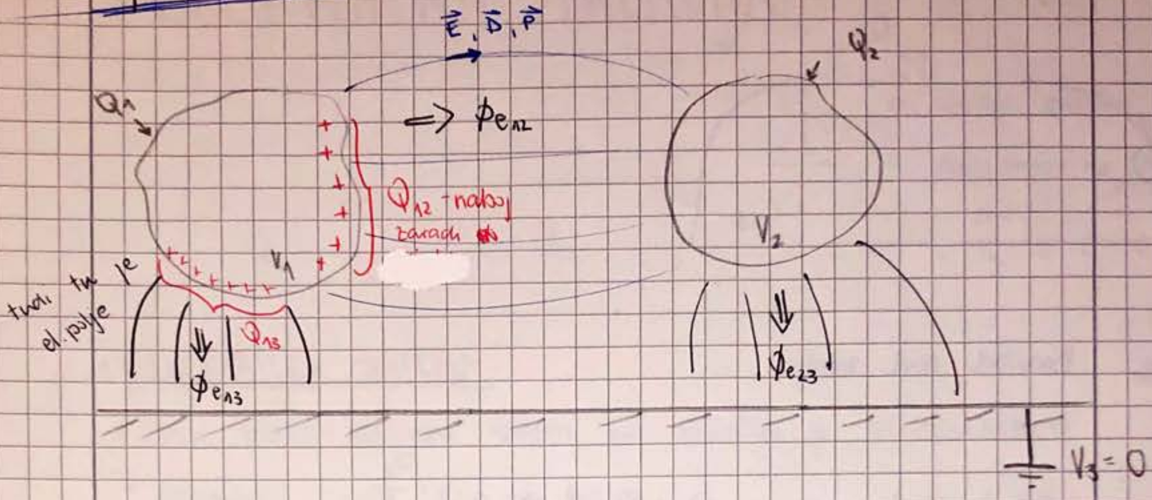
$$C = \frac{\Phi_e}{V}$$

2) DVE TELESI



$$C = \frac{\Phi_e}{U}$$

3) VEČ TELES



$$C = \frac{\phi_e}{U} \Rightarrow C_{jk} = \frac{\phi_{ejk}}{U_{jk}} \qquad C_{12} = C_{21}$$

$$\underbrace{C_{jk} = \frac{\phi_{ejk}}{V_j - V_k} = \frac{-\phi_{ejk}}{-(V_k - V_j)} = \frac{\phi_{ejk}}{V_k - V_j}}$$

$$Q_1 = Q_{12} + Q_{13} = \phi_{e12} + \phi_{e13} = C_{12} \cdot U_{12} + C_{13} \cdot U_{13}$$

$$Q_2 = \phi_{e21} + \phi_{e23} = C_{12} \cdot U_{21} + C_{23} \cdot U_{23}$$

$$Q_1 = C_{12}(V_1 - V_2) + C_{13}(V_1 - \overset{0}{V_3})$$

$$Q_2 = C_{12}(V_2 - V_1) + C_{23}(V_2 - \overset{0}{V_3})$$

$$Q_1 = (C_{12} + C_{13})V_1 - C_{12} V_2$$

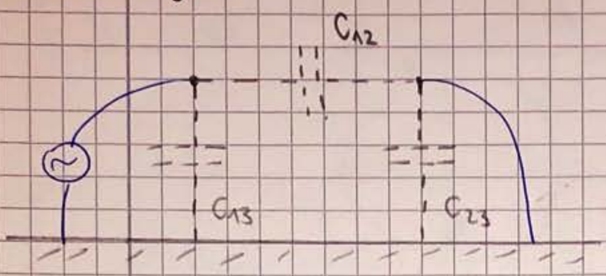
$$Q_2 = (C_{12} + C_{23})V_2 - C_{12} V_1$$

$$\left.\begin{aligned} V_1 &= p_{11} Q_1 + p_{12} Q_2 \\ V_2 &= p_{21} Q_1 + p_{22} Q_2 \end{aligned}\right\} \qquad Q_1 = \frac{p_{22} V_1 - p_{12} V_2}{[p_{11} p_{22} - p_{12} p_{21}]} \qquad Q_2 = \frac{-p_{21} V_1 + p_{11} V_2}{[p_{11} p_{22} - p_{12} p_{21}]}$$

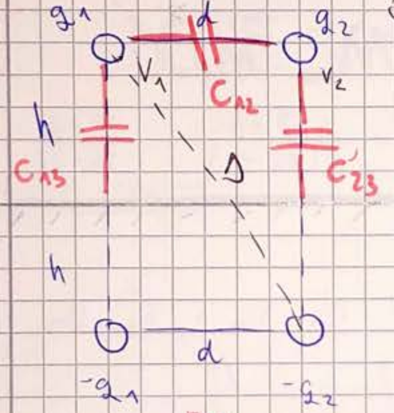
$$C_{12} = \frac{p_{12}}{[\quad]} \qquad C_{13} = \frac{p_{22} - p_{12}}{[\quad]} \qquad C_{23} = \frac{p_{11} - p_{21}}{[\quad]}$$

p_{11} - potencialni koeficienti

Modelna vezje



Zgled: Dvovod nad zemljo



$$Q_2 = q_2 \cdot l \rightarrow q_2 = \frac{Q_2}{l}$$

$$q_3 = (-q_1 - q_2)$$

$$V_1 = \underbrace{\frac{q_1}{2\pi\varepsilon} \ln \frac{2h}{a}}_{p_{11}} + \underbrace{\frac{q_2}{2\pi\varepsilon} \ln \frac{D}{d}}_{p_{12}}$$

$$V_2 = \underbrace{\frac{Q_1}{2\pi\varepsilon l} \ln \frac{D}{a}}_{p_{21}} + \underbrace{\frac{Q_2}{2\pi\varepsilon l} \ln \frac{2h}{a}}_{p_{22}}$$

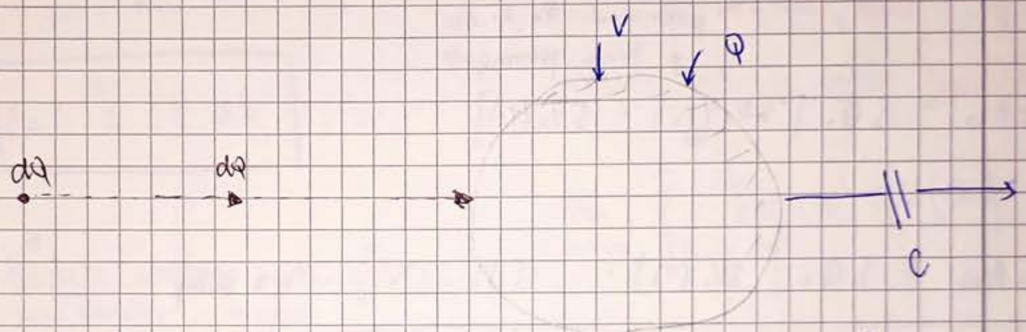
ELEKTRIČNA ENERGIJA

* Znanje energija, ki jo dovajamo se SHRANJUJE

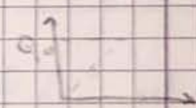
1) ELEKTRITEV SISTEMA

- Imeli bomo kovinski sistem, na začetku je NENAELEKTREN
- Dovajali bomo dQ na sistem

↳ prvi naboj bo šel najlažje gor, drugi težje, (ker je bil že 1 notri) tretji še težje...



Če nam narašča naboj, nam tudi potencial



$$V_0 = \frac{Q_0}{C}$$

$$A_z = \int_0^{Q_0} V\,dQ = \int_0^{Q_0} \frac{Q\,dQ}{C} = \frac{1}{C} \cdot \frac{Q_0^2}{2}$$

$$\boxed{A_z = \frac{Q_0^2}{2C} = \frac{Q_0 V_0}{2}}$$

2) RAZELEKTRITEV SISTEMA

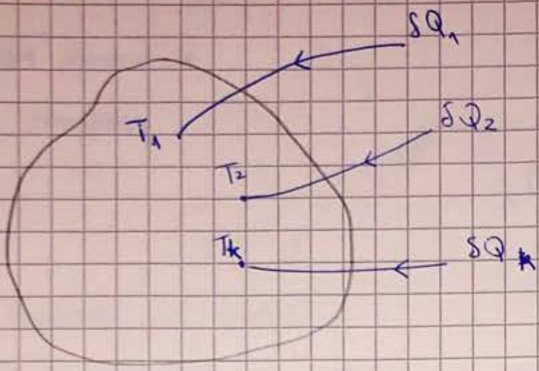
- Zaradi nekega razloga nam naboji ZAPUŠČAJO SISTEM

$$A_e = \int dA_e = \int_0^{Q_0} \frac{Q_0 - Q}{C}\,dQ = \frac{Q_0 Q}{C} - \frac{Q^2}{2C} \Big|_0^{Q_0} = \frac{Q_0^2}{C} - \frac{Q_0^2}{2C} =$$

$$\boxed{A_e = \frac{Q_0^2}{2C}}$$

* Toliko, kolikor smo vložili v sistem, da smo ga naelektrili, toliko dobimo NAZAJ, ko GA RAZELEKTR

3) ELEKTRENJE PROSTORA



* Kakšno delo bi mogli oprant, da prenesemo naboje na telo?

$\delta A_{z1} = 0$ ← naboj, ki ga vnesemo

$\delta A_{z2} = \delta Q_2 \, \delta V_1 (T_2)$

potencial Q_1, ki ga je treba premagat

$\delta A_{z3} = \delta Q_3 \left[\delta V_1 (T_3) + \delta V_2 (T_3) \right]$

$\vdots$

$\delta A_{zk} = \delta Q_k \left[\delta V_1 (T_k) + \dots + \delta V_{k-1} (T_k) \right]$ k-ti naboj

$\delta A_{zn} = \delta Q_n \left[\delta V_1 (T_n) + \dots + \delta V_{n-1} (T_n) \right]$ zadnji naboj

$$A_z = \sum_{k=1}^{n} \delta A_{zk} = \sum_{k=1}^{n} \delta Q_k \sum_{j=1}^{k-1} \delta V_j (T_k)$$

$$A_z = \sum_{k=1}^{n} \sum_{j=1}^{k-1} \underbrace{\delta Q_k \, \delta V_j (T_k)}$$

→ pomnožit z V nižjega indeksa

$$\delta Q_k \cdot \delta V_j (T_k) = \frac{\delta Q_k \, \delta Q_j}{4\pi\varepsilon_0 \, r_{T_j T_k}} = \delta Q_j \, \delta V_k (T_j)$$

$$A_z = \sum_{k=1}^{n} \sum_{j=1}^{k-1} \delta Q_j \, \delta k (T_j)$$

seštejem

$$2A_z = \sum_{k=1}^{n} \sum_{\substack{j=1 \\ j \neq k}}^{n} \delta Q_k \, \delta V_j(T_k);$$

Zapisali bomo limito, ko gre $n \to \infty$:

$$A_z = \frac{1}{2} \lim_{n\to\infty} \Big[\sum_{k=1}^{n} \delta Q_k \cdot \underbrace{\lim_{n\to\infty} \sum_{\substack{j=1 \\ j \neq k}}^{n} \delta V_j(T_k)}_{} \Big]$$

v tej vsoti manjka lastni prispevek, ki je $\to$ $\cancel{\delta V_k(T_k) = \frac{\delta(T_k)\, \delta r_k^2}{2\varepsilon_0}}$

vendar je majhen, kolikor hočem, zato ga lahko izpustim

$\Downarrow$

$$\lim_{n\to\infty} \sum_{j=1}^{n} \delta V_j(T_k) = V(T_k)$$

$$A_z = \frac{1}{2} \lim_{n\to\infty} \sum_{k=1}^{n} \delta Q_k V(T_k)$$

$$\boxed{A_z = \frac{1}{2} \int V \, dQ}$$

$\downarrow$

Šešteti moramo potenciale na mestu nabojev

$$\boxed{A_z = A_e = W_e}$$

~~ENERGIJA~~

ENERGIJA PRI ELEKTRITVI VEČIH TELES

$$W_e = \frac{1}{2}\left(V_1 Q_1 + V_2 Q_2 + \dots\right)$$

$$W_e = \frac{1}{2}\sum_{j=1}^{n} V_j Q_j$$

$n = 3$

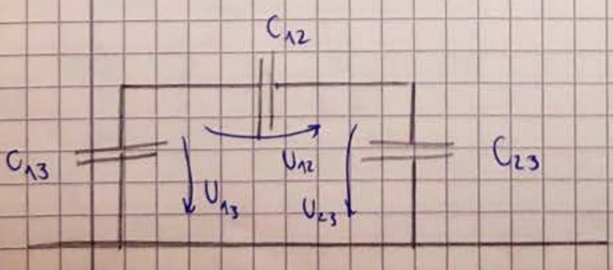
$$W_e = \frac{1}{2} V_1 \underbrace{Q_1}_{\Phi_{13} + \Phi_{12}} + \frac{1}{2} V_2 \underbrace{Q_2}_{\Phi_{21} + \Phi_{23}} + \cancel{\frac{1}{2} \overset{0}{V_3} Q_3}$$

$$W_e = \frac{1}{2} V_1 \left(C_{12}(V_1 - V_2) + C_{13}(V_1 - \underset{0}{V_3}) \right) + \frac{1}{2} V_2 \left(C_{12}(V_2 - V_1) + C_{23}(V_2 - \underset{0}{V_3}) \right) =$$

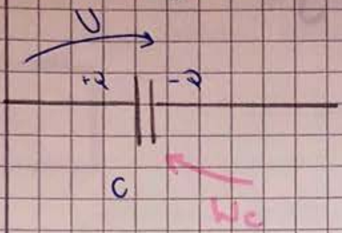
$$W_e = \frac{1}{2} V_1 \left((C_{12} + C_{13}) V_1 - C_{12} V_2 \right) + \frac{1}{2} V_2 \left((C_{12} + C_{23}) V_2 - C_{12} V_1 \right) =$$

$$= \frac{1}{2} C_{12} (V_1 - V_2)^2 + \frac{1}{2} C_{13} (V_1 - V_3)^2 + \frac{1}{2} C_{23} (V_2 - V_3)^2 =$$

$$W_e = \frac{1}{2} C_{12} U_{12}^2 + \frac{1}{2} C_{13} U_{13}^2 + \frac{1}{2} C_{23} U_{23}^2$$



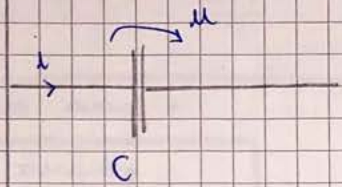
Modelno rezje



$Q = C \cdot U$

$$\boxed{W_e = \frac{C \cdot U^2}{2} = \frac{QU}{2}}$$

V „živih" strukturah



$$\boxed{i = C \cdot u'}$$

$$W_e = \frac{C \cdot u^2}{2}$$

* Potrebujemo moč, za elektrenje: V zgornjem primeru, ko imamo statiko, ~~nal~~ predpostavimo, da imamo neskončno moči

$$p = W_e' = \frac{C}{2} \cdot 2 \cdot u \cdot u' = u \cdot C \cdot u' = u \cdot i$$

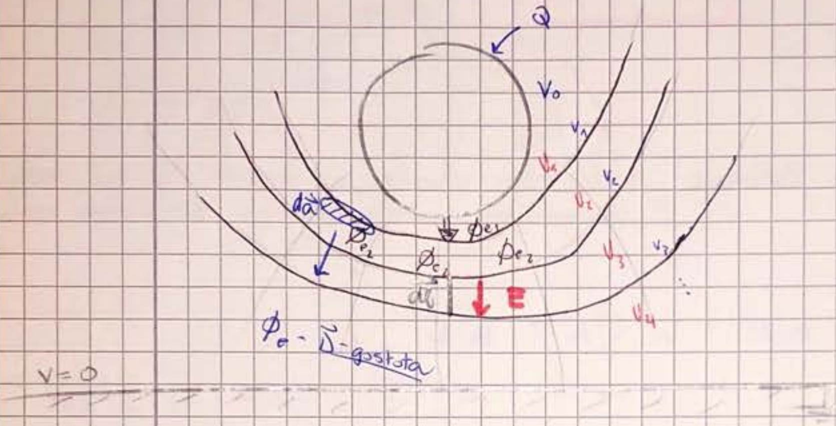
GOSTOTA EL. ENERGIJE

Električna energija:

$$W_e = \frac{QV}{2}$$

$$W_e = \frac{CU^2}{2}$$

PREMISLEK: Kje je energija shranjena?



→ zaradi različnih potencialov imamo el. polje, ki ga predstavimo z ekvipoten.

$$W_e = \frac{Q\,V_0}{2} \Rightarrow W_e = \frac{Q}{2}\sum_{k=1}^{n} U_k$$ ← vsota vseh napetosti

Tudi ϕ_e lahko seštejemo:

$$W_e = \frac{1}{2}\sum_{j=1}^{m} \phi_{ej} \sum_{k=1}^{n} U_k$$

⇓ ϕ_{ej} je pri vsaki U konstanten, zato ga lahko damo k U

$$W_e = \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{n} \phi_{ej} \cdot U_k \Rightarrow \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{n} \vec{D}_j \vec{E}_k \underbrace{d\vec{a}_j\, d\vec{l}_k}_{dv}$$

U_k ↳ $d\vec{l} \cdot \vec{E}$

ϕ_{ej} ↳ $d\vec{a} \cdot \vec{D}$

lim j in k → ∞

$$\boxed{W_e = \int \frac{\vec{D}\,\vec{E}}{2}\, dv}$$ pretok v prostorni ploščice

Energija se shranjuje v okolici → v EL. POLJU

$$\boxed{\frac{\vec{D}\vec{E}}{2}}$$

↓ prostorska gostota energije
volumska - - - - - -

$$\boxed{w_e = \frac{\vec{D}\vec{E}}{2} = \frac{\varepsilon E^2}{2} = \frac{D^2}{2\varepsilon}}$$

$$\boxed{W_e = \int_{vol.} w_e \cdot dV}$$

PREMISLEK: Kakšne so posledice te energije v prostoru?

* Pustili bomo premik oz gibanje:

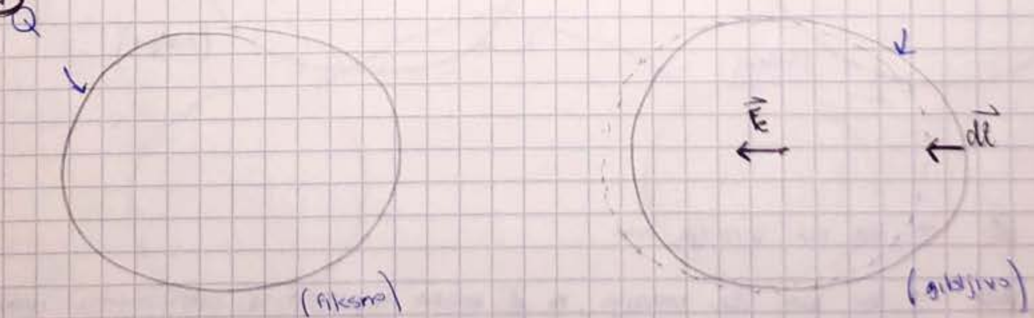
GIBALNI PROCESI

Imeli bomo dva sistema

I brez vira

II. z virom

I



* do premika pride samo pri desnem VODNIKU

$dA_e = \vec{F_e}\,\vec{dl}$

$dW_e < 0$

$$\boxed{dA_e + dW_e = 0}$$

$\vec{F_e}\, d\vec{l} = -dW_e$

$\downarrow$

$$\boxed{\begin{aligned}\vec{F_e} &= -\left(\frac{dW_e}{dx}, \frac{dW_e}{dy}, \frac{dW_e}{dz}\right)\\ \vec{F_e} &= -\operatorname{grad}(V)\end{aligned}}$$

* matematično bi lahko z $d\vec{l}$ delili obe strani, vendar

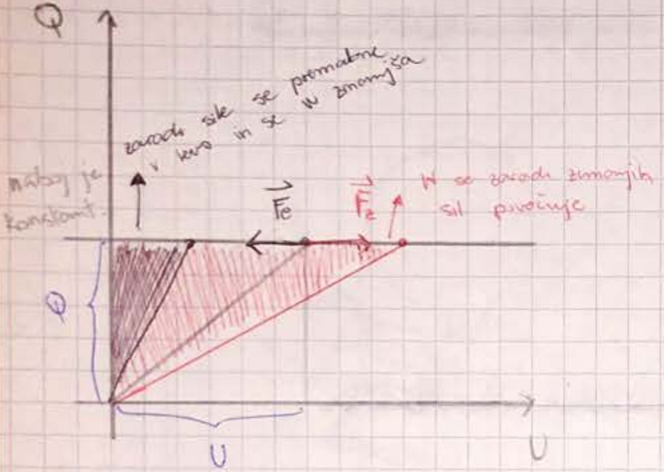
VEKTORJA NE MOREMO TAKO DELITI

$\vec{E}\, d\vec{l} = -dV$

$\downarrow$

$$\vec{E} = -\left(\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right)$$

$$\vec{E} = -\operatorname{grad}(V)$$



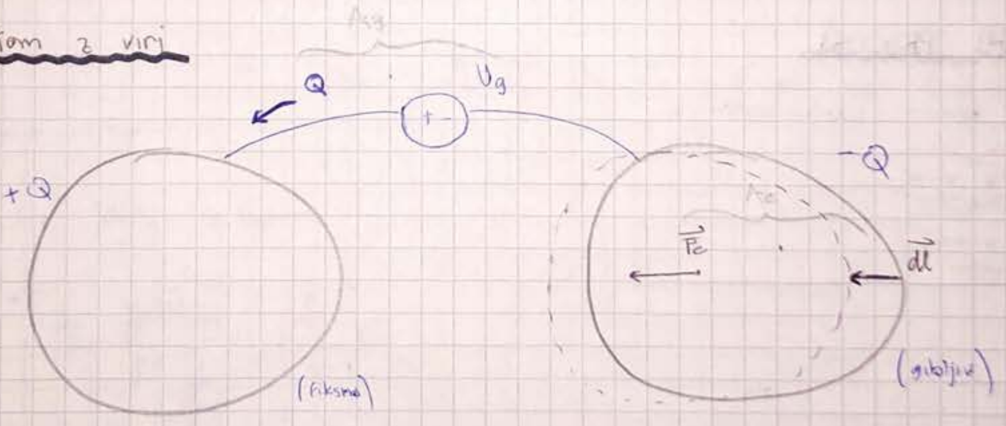
I. Objekt spustimo

- We se zmanjšuje
- C se povečuje

II. Objekt vlečemo stran

- We se povečuje
- C se zmanjšuje

II Sistem z viri



1. $dA_e = \vec{F_e} \cdot \vec{dl}$ → Ele sila opravlja delo

2. $dA_g = U_g\, dQ$ → Vir nam da energijo, ki je enaka napetosti in pretečenemu naboju

3. $dW_e > 0$

$\downarrow$

$$dW_e = \frac{1}{2} U_g (Q + dQ) - \frac{1}{2} U_g Q > 0$$

$dA_e + dW_e = dA_g$

$\vec{F_e}\, d\vec{l} + \frac{1}{2} U_g (Q + dQ) - \frac{1}{2} U_g\, Q = U_g \cdot dQ$

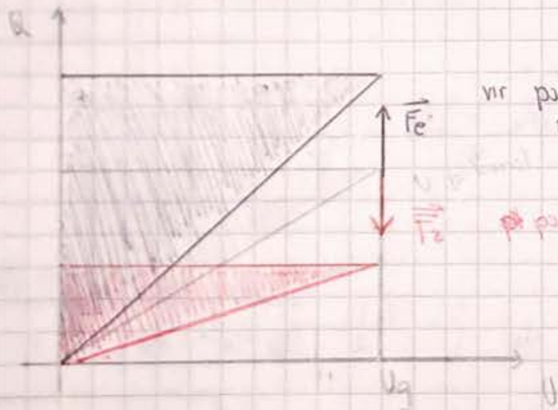
$\vec{F_e} \cdot d\vec{l} + \frac{1}{2} U_g\, dQ = U_g \cdot dQ$

$\vec{F_e}\, d\vec{l} = \frac{1}{2} U_g\, dQ = + dW_e$ pozitivna energija prej je bila –

$\vec{F_e}\, d\vec{l} = dW_e$

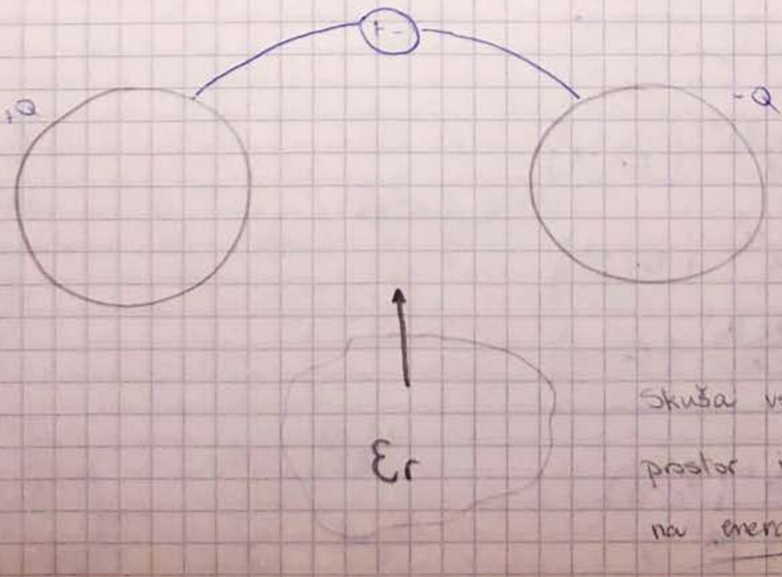
$\Downarrow$

$$\boxed{\begin{gathered} \vec{F_e} = \left(\frac{dW_e}{dx}, \frac{dW_e}{dy}, \frac{dW_e}{dz} \right) \\ \vec{F_e} = \text{grad}\,(W_e) \end{gathered}}$$



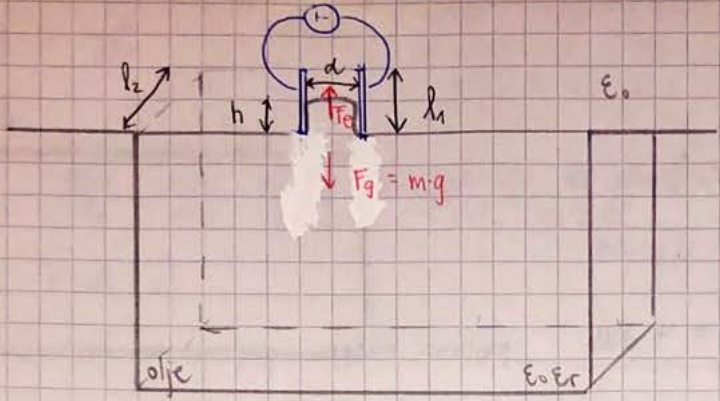
vir polni sistem in opravlja dela

polnjenje vira zaradi dela zunanje sile

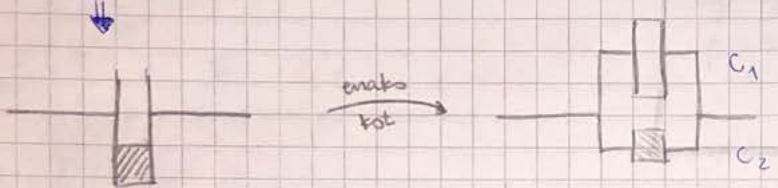


Skuša vstopiti v prostor in vplivati na energijo sistema

Izgled



$$W_e = \frac{1}{2} C U_g^2$$



$$C = C_1 + C_2$$

$$\left.\begin{aligned} C &= C_1 + C_2 \\ C &= \varepsilon \cdot \frac{S}{d} \end{aligned}\right\} \quad C = \varepsilon_0 \frac{(h_1 - h) l_2}{d} + \varepsilon_0 \varepsilon_r \frac{h l_2}{d}$$

$$F_e = \frac{dW_e}{dh} = \frac{dW_e}{dC} \frac{dC}{dh} = \frac{1}{2} U_g^2 \left(- \frac{\varepsilon_0 l_2}{d} + \varepsilon_0 \varepsilon_r \frac{l_2}{d} \right)$$

$$F_e = \frac{\varepsilon_0 l_2 U_g^2}{2 d} \left(\varepsilon_r - 1 \right)$$

$$F_g = F_e$$

$$m g = \frac{\varepsilon_0 l_2 U_g^2}{2 d} \left(\varepsilon_r - 1 \right)$$

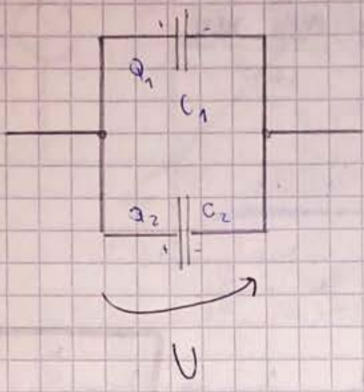
$$\rho \cdot V \cdot g = \frac{\varepsilon_0 l_2 U_g^2}{2 d} \left(\varepsilon_r - 1 \right)$$

$$d \cdot \rho \, h \cdot \cancel{l_2} \cdot g = \frac{\varepsilon_0 \cancel{l_2} U_g^2}{2 d} \left(\varepsilon_r - 1 \right)$$

$$h = \frac{\varepsilon_0 U_g^2 \left(\varepsilon_r - 1 \right)}{2 \rho d^2 g}$$

KONDENZATORSKA VEZJA

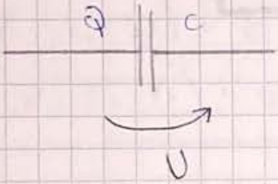
Vzporedna vezava kondenzatorjev



* Pozitivni plošči sta med seboj GALVANSKO POVEZANI

* Negativni plošči sta tudi med seboj povezani.

⇓ ZANIMA NAS NADOMESTNI KONDEN. → ki bo imel enake lastnosti pod napetostjo U, kot zgornja dva

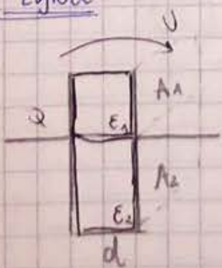


$$Q = Q_1 + Q_2$$

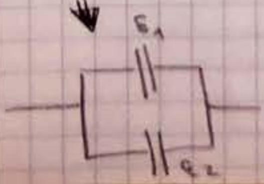
$$C \cdot \cancel{U} = C_1 \cancel{U_1} + C_2 \cancel{U_2}$$

$$\boxed{C = C_1 + C_2}$$

Zgled



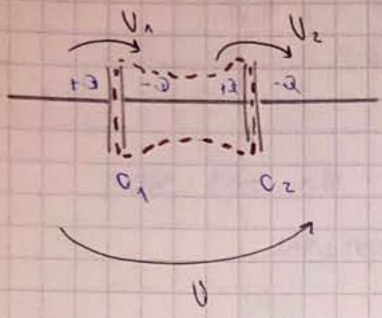
$$C = \frac{\varepsilon_1 A_1 + \varepsilon_2 A_2}{d}$$



$$C_1 = \frac{\varepsilon_1 A_1}{d}$$

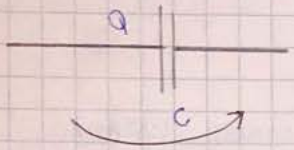
$$C_2 = \frac{\varepsilon_2 A_2}{d}$$

Zaporedna vezava kondenzatorja



GALVANSKO SKLENJENO - če je bila vsota Q pred priključitvijo 0 je tudi zdaj.

NAD. KON.



$$\{U_j = \frac{Q}{C_j}\}$$

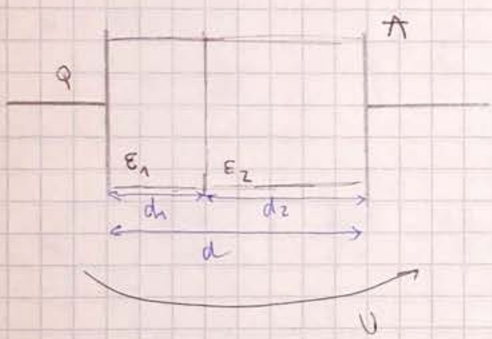
$$U = U_1 + U_2$$

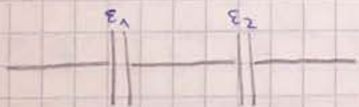
$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \rightarrow \boxed{\frac{1}{C} = \sum_{j=1}^{m} \frac{1}{C_j}}$$

$$\boxed{C = \frac{C_1 C_2}{C_1 + C_2}}$$

Zgled

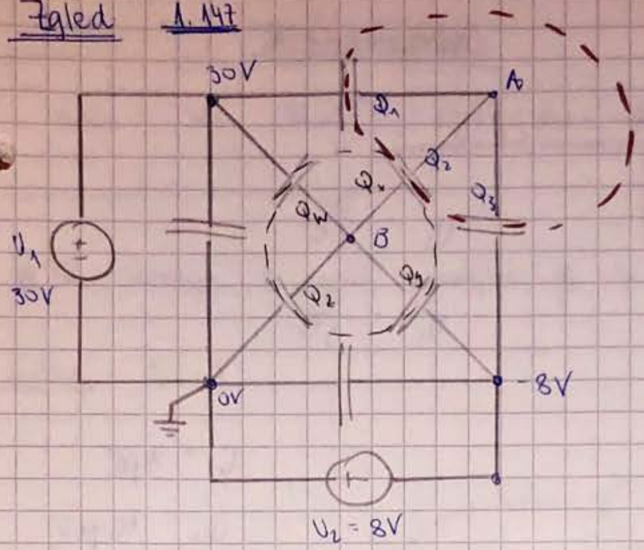




$$C_1 = \frac{\varepsilon_1 A}{d_1} \qquad C_2 = \frac{\varepsilon_2 A}{d_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow Q = CU$$

Zgled 1.147



$C = 10\,nF$

$U_1 = 30V$

$U_2 = 8V$

$V_A = ?$ $V_B = ?$

* Pred vklopom je ... nevtralno,
prav tako po vklopu
Torej $\Sigma Q = 0$

$Q_1 + Q_2 + Q_3 = 0$

$Q_x + Q_y + Q_z + Q_w = 0$

$\cancel{C}\cdot(V_A - 30) + \cancel{C}(V_A - V_B) + \cancel{C}(V_A - (-8)) = 0$

$\cancel{C}(V_B - V_A) + \cancel{C}(V_B - (-8)) + \cancel{C}(V_B - 0) + \cancel{C}(V_B - 30) = 0$

$3V_A - V_B = 22$

$-V_A + 4V_B = 22$

$V_A = 10V$

$V_B = 8V$

$Q_z = C(V_B - 0)$

$W_y = \frac{CU^2}{2}$ $U = (V_B + 8)$

(Matrika):

$$\begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 22 \\ 22 \end{bmatrix}$$

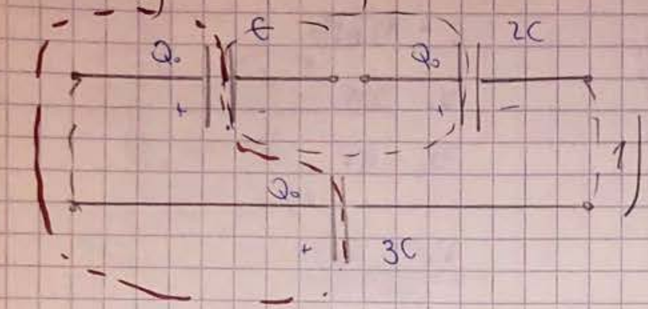
$[C] \cdot [V] = [Q]$

$[V] = [C]^{-1} [Q]$

inverzna matrika

matrično množenje

Zgled



stare napetosti

$$U_1' = \frac{Q_0}{C}$$

$$U_2' = \frac{Q_0}{2C}$$

$$U_3' = \frac{Q_0}{3C}$$

nove napetosti: U_1, U_2, U_3

novi naboji: $CU_1, 2CU_2, 3CU_3$

$C = 1\,\mu F$

$Q_0 = 10\,\mu C$

1) $U_1 + U_2 - U_3 = 0$

2) $CU_1 + 3CU_3 = 2Q_0$

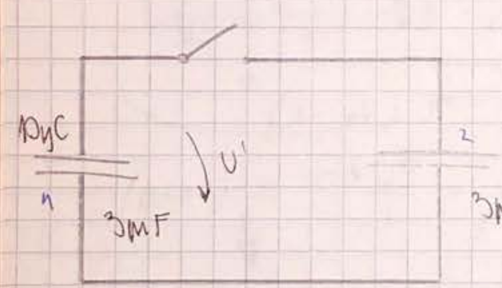
3) $-CU_1 + 2CU_2 = 0$

$U_1 = 3{,}63\,V$

$U_2 = 1{,}81\,V$

$U_3 = 5{,}45\,V$

ENERGIJSKI DEFEKT



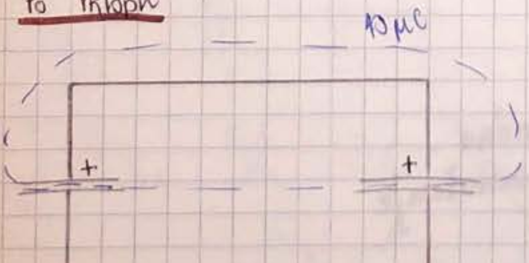
Pred vklopom

$$U' = \frac{Q}{C} = \frac{10\,\mu C}{3\,\mu F} = 3{,}33\,V$$

$$W_{e1}' = \frac{Q'^2}{2C} = \frac{(10\cdot 10^{-6})^2}{2\cdot 3\cdot 10^{-6}} = \frac{100}{6}\cdot 10^{-6}\,J$$

$$W_{e2}' = 0$$

Po vklopu



$$U = \frac{Q}{C_1 + C_2} = \frac{10\,\mu C}{6\,\mu F} = 1{,}66 = \frac{U'}{2}$$

$$Q = Q'$$

$$W_{e1} + W_{e2} = W_{esk} = \frac{\overbrace{6\cdot 10^{-6}}^{C_1 + C_2}\cdot\left(\frac{10}{6}\right)^2}{2}$$

$$= \frac{3\cdot 10^{-6}\cdot 100}{36} = \frac{1\cdot 10^{-4}}{12}\,J$$

+ + sponki sta povezani → vzporedna vezava

Dobili smo <u>manjšo energijo</u>

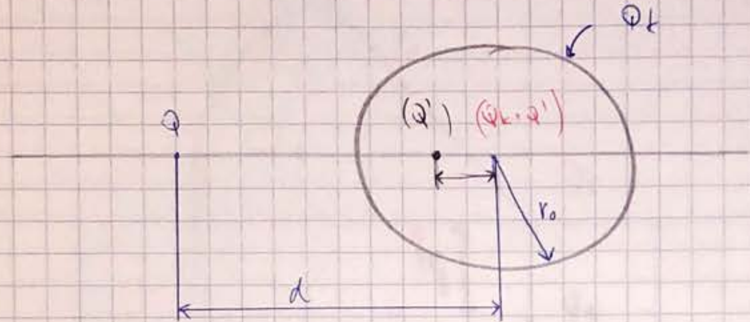
$\Delta W_e = W_e' - W$ = <u>energijski defekt</u>

Ta energija, ki nam zmanjka se nekje uporabi → <u>upor</u> žice npr.

<u>Izgled</u> 20-1

$$\boxed{e \cdot \frac{r_0^2}{d}}$$

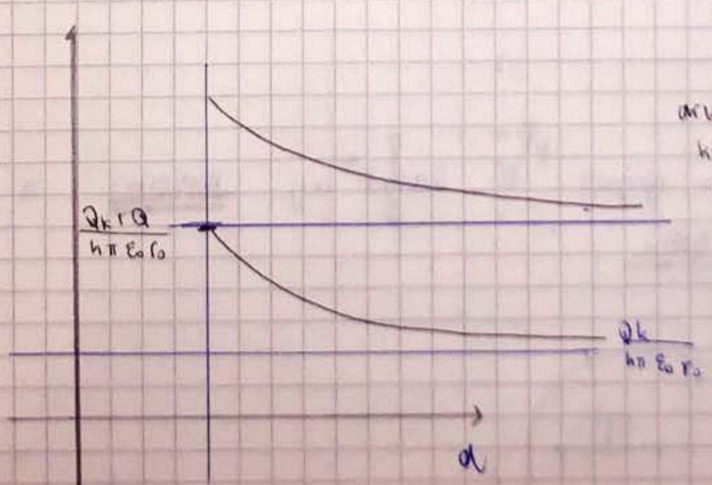
$$\boxed{Q' = \frac{r_0}{d} \cdot Q}$$



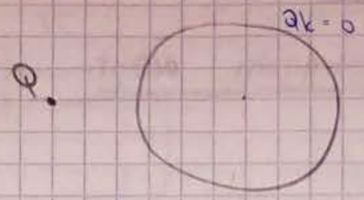
Q, Q' in Q_k so

$$V_k = \underbrace{V(Q, -Q')}_{0} + V(Q_k + Q') = \frac{Q_k + Q}{4\pi\varepsilon r_0} =$$

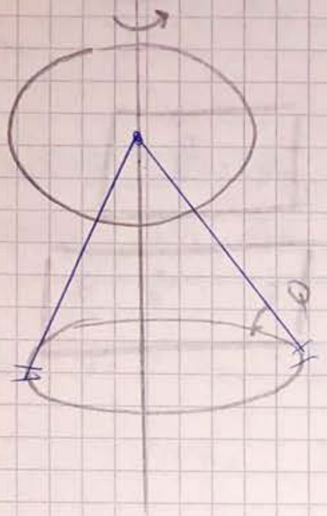
$$\boxed{V_k = \frac{Q_k + \frac{r_0}{d} Q}{4\pi\varepsilon r_0}} = V_k(d)$$



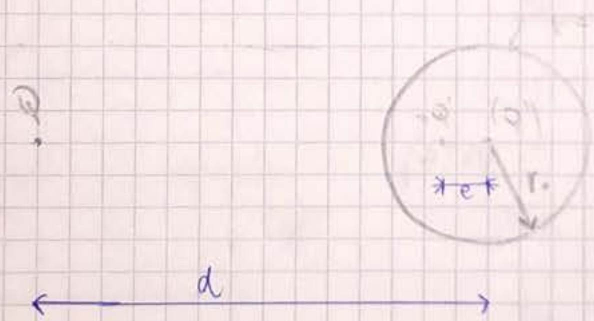
$Q_k = 0$ (nevtralna krogla)



$$V_k = \frac{Q}{4\pi\varepsilon_0 d}$$

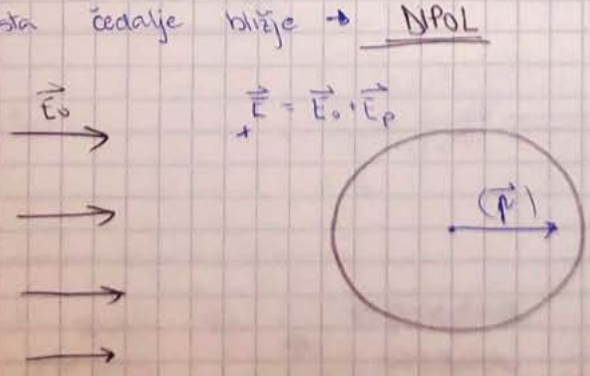


Nevtralna krogla → naj bo $d >>> r_0$



Ko bomo Q vlekli stran, bo njegov $\vec{E}$ čedalje bolj HOMOGENO in $-Q$, Q bosta čedalje bližje → DIPOL

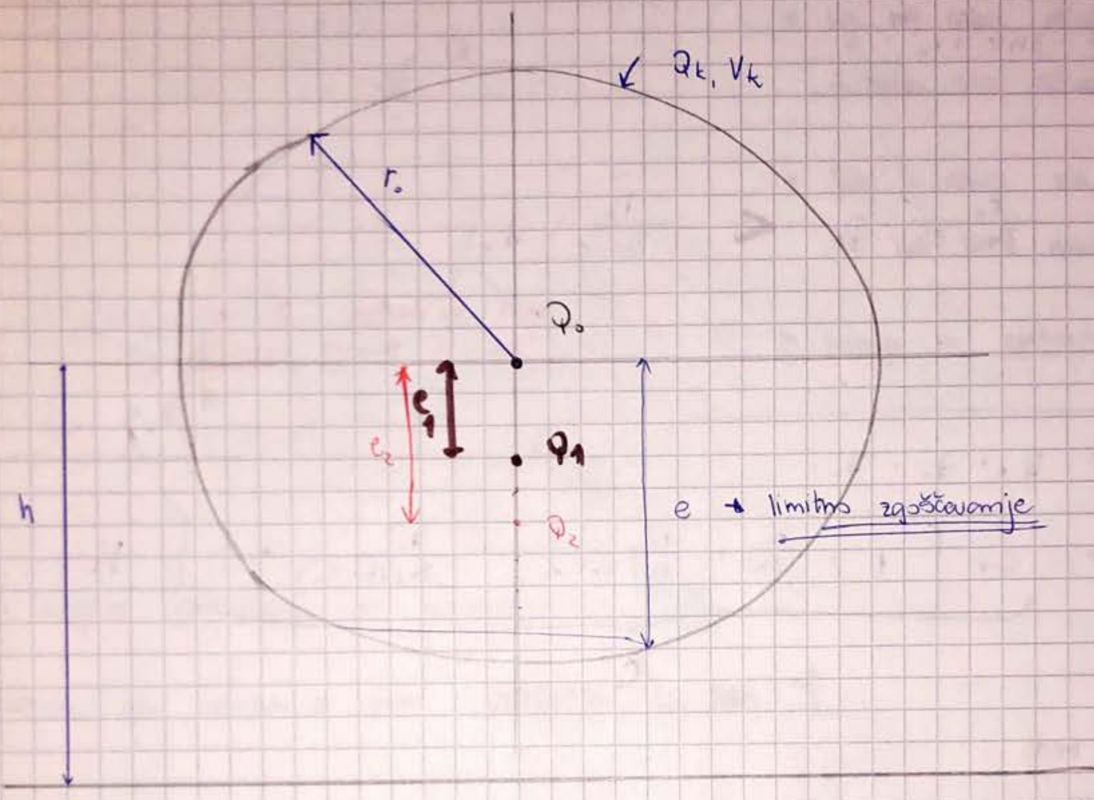
$$\vec{E} = \vec{E}_0 + \vec{E}_p$$



Dobimo primer dipola v homogenem polju

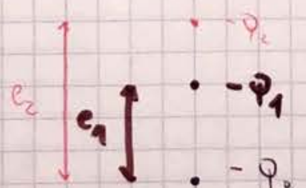
$$p = Q' e = \frac{r_0}{d} \cdot Q \, \frac{r_0^2}{d} \, \frac{4\pi\varepsilon_0}{4\pi\varepsilon_0} = \underbrace{\frac{Q}{4\pi\varepsilon d^2}}_{E} \cdot 4\pi\varepsilon r_0^3 = 4\pi\varepsilon_0 r_0^3 E_0$$

Zgled: Kapacitivnost sistema krogla - zemlja



$$Q_0 = \overset{C_0}{4\pi\varepsilon_0 r_0} V_k = C_0 V_k$$

$$Q_1 = -\frac{r_0}{2h} Q_0 \,, \quad e_1 = \frac{r_0^2}{2h}$$



$$Q_2 = \frac{r_0}{2h - e_1} Q_1 \,, \quad e_2 = \frac{r_0^2}{2h - e_1}$$

$$\vdots$$

$$Q_{n+1} = \frac{r_0}{2h - e_n} Q_n \,, \quad e_{n+1} = \frac{r_0^2}{2h - e_n}$$

$$\vdots$$

$$\lim_{n\to\infty} e_n = e$$

$$e_{12} = \frac{2h \pm \sqrt{4h^2 - 4 r_0^2}}{2}$$

$$e = \frac{r_0^2}{2h - e}$$

$$\boxed{e = h \pm \sqrt{h^2 - r_0^2}}$$

$$-2he + e^2 + r_0^2 = 0$$

$$e^2 - 2he + r_0^2 = 0$$

$$Q_{n+1} = \frac{r_0}{2h - e_n} Q_n < \frac{r_0}{2h - e} Q_n$$

vsak naslednji Q je za ta faktor manjši

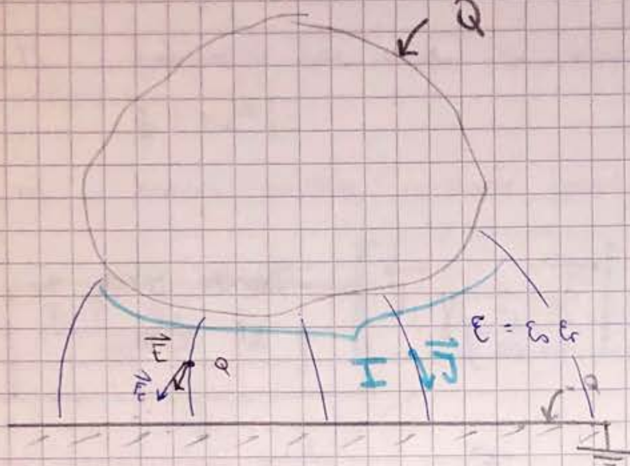
$$Q_k = Q_0 + Q_1 + Q_2 + \dots$$

$$Q_k = \underbrace{C_0 \left[1 + \frac{r_0}{2h} + \frac{r_0^2}{2h(2h - e_1)} + \frac{r_0^3}{2h(2h - e_1)} + \dots \right]}_{C} V_k$$

odnos med nabojem in potencialom

TOKOVNA POLJA

R - vezja



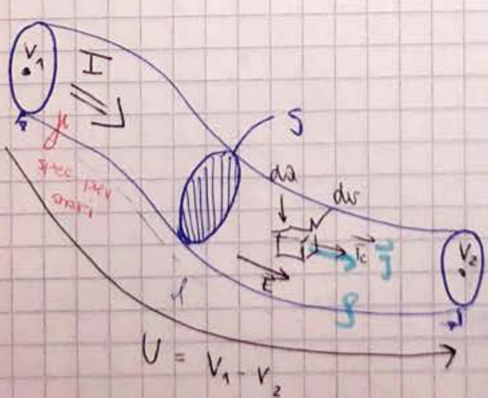
* Na nek naboj deluje sila, s katero smo definirali $\vec{E}$
* Torej če deluje sila, se začne ta naš prost naboj PREMIKAT
* Naboje se pretaka

To urejeno pretakanje pa imenujemo TOK

Znamo ga izračunat preko GOSTOTE EL. TOKA $\vec{J}$

PREVODNE LASTNOST SNOVI:

$\boxed{\text{OHMOV „ZAKON"}}$



* Napetost je posledica el polja oz razlike potencialov

sorazmernost

$\boxed{I \propto U}$

$\boxed{I \propto \vec{J}}$ $I = \oint \vec{J}\, d\vec{A}$

$\boxed{U \propto \vec{E}}$ $U = \oint \vec{E} \cdot d\vec{l}$

* Če na ▢ deluje $\vec{F_e}$, se začne premikat, torej ima

Povezali smo $\vec{J}$ in hitrost:

$$\vec{J} = \rho \cdot \vec{w}$$

$$\vec{E} = \frac{d\vec{F_e}}{dQ} \quad , \quad dQ = \rho \cdot dV$$

$$I \propto U$$

$$I \propto \vec{J} \qquad U \propto \vec{E}$$

$$\boxed{\vec{J} \propto \vec{E}} \Longrightarrow \boxed{\vec{J} = \gamma \vec{E}}$$

Ohmov zakon v dif. obliki

γ → specifična prevodnost snovi !

Dobili smo linearno zvezo med $\vec{E}$ in $\vec{J}$

$\vec{E}$ → predstavlja U

$\vec{J}$ → predstavlja I

$$\rho \cdot \vec{w} \propto \frac{d\vec{F_e}}{dQ} \leftarrow \rho \cdot dV$$

$$\vec{w} \propto d\vec{F_e} \propto d\vec{F_u}$$

$\vec{w}$ → hitrost pretakanja nabojev

$d\vec{F_e}$ → elektr. sila na naboj

$$\vec{F_e} + \vec{F_u} = 0$$

→ Kolikor el. sila pospešuje, mora sila upora zavirati

EMPIRIKA

$$\vec{j} = \frac{n \cdot e^2 \cdot \tau_t}{m} \cdot \vec{E}$$

e ..... osnovni naboj

n ..... št. prostih nosilcev naboja

τ_t ..... povprečni čas med zaporednimi trki 10^{-15} s

$f = 10^{15} / s$

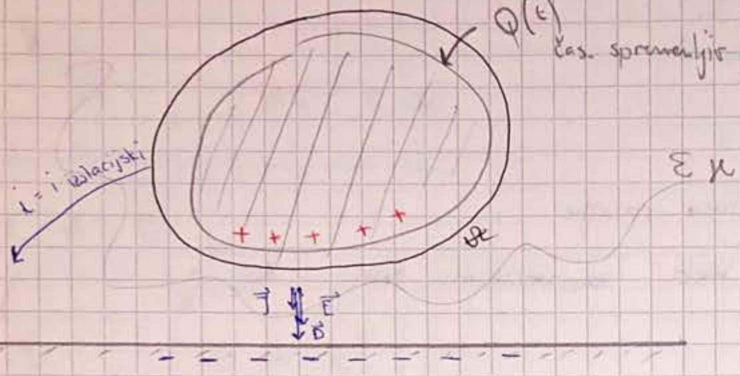
m ..... masa prostih nosilcev naboja

$$\boxed{\frac{n \cdot e^2 \cdot \tau_t}{m} = \gamma}$$

$\left[\frac{S}{m}\right]$ Simens / meter

čas med sosednjima trkoma

PRAZNJENJE KONDENZATORJA



$$\oint \vec{D} \cdot d\vec{a} = Q = \oint \frac{\varepsilon}{\gamma} \vec{J} \cdot d\vec{a} = \frac{\varepsilon}{\gamma} \, i_{izol.} = \frac{\varepsilon}{\gamma} \left(- \frac{dQ}{dt} \right)$$

$$\boxed{\frac{dQ}{dt} + \frac{\gamma}{\varepsilon} Q = 0}$$

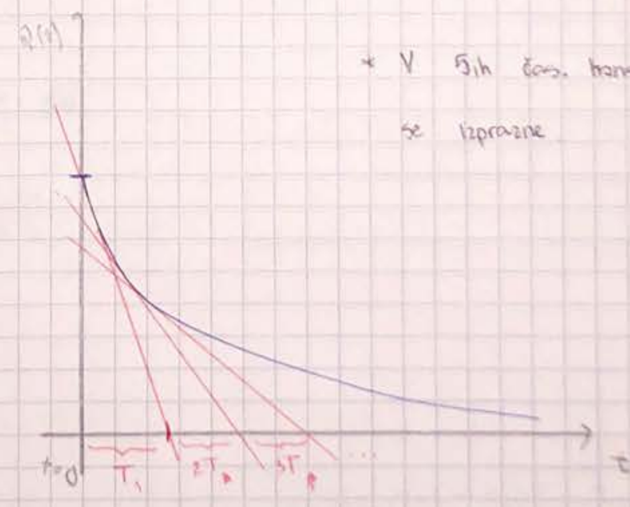
* Časovna dinamika spreminjanja je sorazmerna z nabojem

$$Q(t) = A \cdot e^{\lambda t}$$

$$Q'(t) = \lambda A e^{\lambda t}$$

$$\left(\lambda + \frac{\gamma}{\varepsilon} \right) \cdot A e^{\lambda t} = 0$$

$$\lambda = - \left(\frac{\varepsilon}{\gamma} \right)^{-1} = - \frac{1}{\varepsilon / \gamma}$$



* V 5ih čas. konstantah se izpraznejo

Naj bo Q ob $t = 0$ enak Q_0

$$Q(0) = Q_0 = A \cdot e^{\lambda 0} = A$$

$$Q_0 = A$$

$$Q(t) = Q_0 \, e^{-\frac{\gamma}{\varepsilon} t} = Q_0 \, e^{-t/T}$$

$$\boxed{T = \frac{\varepsilon}{\gamma}}$$

* primer

$\varepsilon = 10^{-10}$

$\gamma = 10^{-13}$

$\frac{\varepsilon}{\gamma} = T = 10^{3}\,s$

$Q(t+T) = \frac{Q_0}{e}$

$Q(t+2T) = \frac{Q_0}{e^2}$

$\vdots$

Če se manjša naboja, se tudi **NAPETOST** in **TOK**

$$u(t) = \frac{Q_0}{C} e^{-t/T}$$

$$i(t) = \frac{Q_0}{T} e^{-t/T}$$

$$\vec{J}(T,t) = \vec{J}(T,0)\, e^{-t/T}$$

Sproščena toplota:

$$\underline{\underline{W_t(T)}} = \int_0^\infty p_t \, dt = \int_0^\infty \frac{J^2}{\gamma} \, dt = \frac{J^2(T,0)}{\gamma} \underbrace{\int_0^\infty e^{-2t/T} dt}_{\underbrace{\left[-\frac{T}{2} e^{-2t/T}\right]_0^\infty}_{\frac{T}{2}}} =$$

$$= \frac{J^2(T,0)}{\gamma} \cdot \frac{T}{2} = \frac{\varepsilon}{2\gamma} \cdot \frac{J^2(T,0)}{\gamma} =$$

$$= \frac{\varepsilon E^2(T,0)}{2} = \underline{\underline{W_e(T)}}$$

energijo spremeni iz el. v toplotno

$$\vec{J} = \gamma \vec{E}$$

$$\gamma = \frac{n e^2 T}{m}$$

Toplotna odvisnost

$$\rho = \frac{1}{\gamma} \Rightarrow \boxed{\rho(\vartheta) = \rho(20^\circ)\left[1 + \alpha(\vartheta - 20^\circ C)\right]}$$

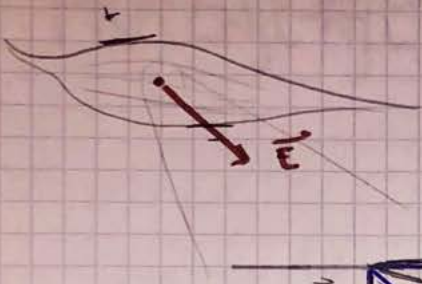
ANIZOTROPNOST

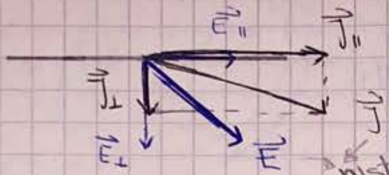
↳ smer $\vec{E}$ je pomembna glede na orientacijo objekta

$$\{\vec{P} \nparallel \vec{E},\ \vec{D} \nparallel \vec{E}\}$$

$$\underline{\vec{J} \nparallel \vec{E}}$$

Izkaže se: $\mu_\perp < \mu_\parallel$





nista vzporedna

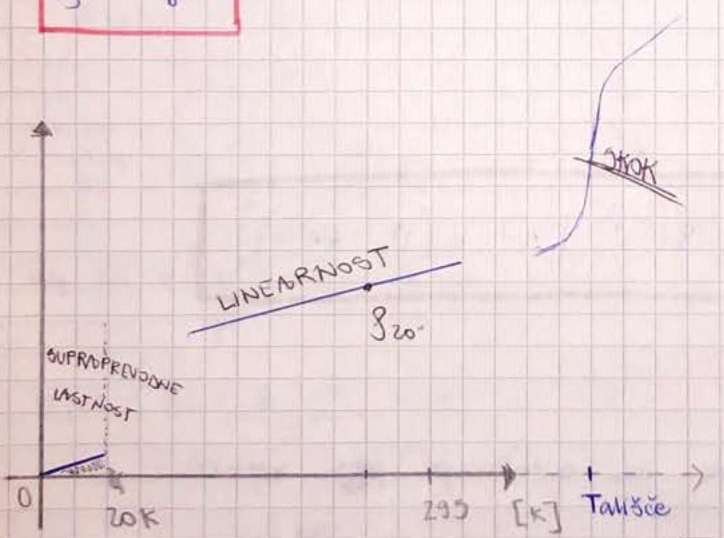
PREVODNOST SNOVI

TEMPERATURNA ODVISNOST (SPECIFIČNE) UPORNOSTI

μ ..... specifična prevodnost snovi [S/m]

ρ ..... specifična upornost snovi [Ωm]

$$\boxed{\rho = \frac{1}{\mu}}$$



* Pri nizkih temperaturah, se gibanje snovi zelo umiri in takrat se pogovarjamo o SUPRAPREVODNIH LASTNOSTIH

$\rho_{20°}$ - temperatura 20°C je REFERENČNA TOČKA

α .... temperaturni koeficient

$$\boxed{\rho(T) = \rho(20°C) \cdot \left(1 + \alpha\left(\overbrace{T - 20°C}^{\Delta T}\right)\right)}$$

Izgled

$\alpha_{cu} = 0{,}004 \,/K$

$\rho(80°C) = ?$

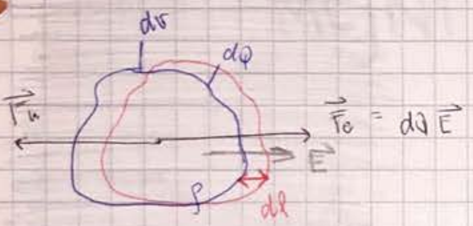
$$\rho(80°C) = \rho_{cu}(20°C)\left(1 + \underset{0{,}004}{\alpha}\left(\overbrace{80-20}^{60}\right)\right) =$$

$$\rho(80°C) = \rho_{cu}(20°C) \cdot 1{,}24$$

$$\rho(80°C) = \rho_{cu}(20°C) \cdot \underline{\underline{124\,\%}}$$

JOULOV ZAKON

Prevodnik



dt – v času se zgodi premik $d\ell$ $\Rightarrow$ $\vec{w} = \dfrac{\vec{d\ell}}{dt}$

* Diferencialno majhen naboj dQ se giblje zaradi $\vec{E}$, ~~...~~ ~~...~~ nanj deluje $\vec{F}_e$

$$dA_e = \vec{F}_e \cdot \vec{d\ell}$$

Silo bomo zapisali kot:

$$dA_e = \vec{E} \cdot \underset{\rho \cdot \vec{dV}}{\vec{dQ}}\; \vec{d\ell} = \rho\,\vec{E} \cdot \vec{dV} \cdot \vec{w}\, dt = \vec{J} \cdot \vec{E} \cdot \vec{dV} \cdot dt \quad \Big| : dt$$

$d\ell = \vec{w}\, dt$ $\qquad$ $\vec{J} = \rho\,\vec{w}$

gostota toka

$$\frac{dA_e}{dt} = \vec{J}\,\vec{E} \cdot \vec{dV}$$

$$dP_e = \vec{J}\,\vec{E}\,\vec{dV} = p_e \cdot \vec{dV}$$

$$\boxed{p_e = \vec{J}\,\vec{E}}$$

Prostorska gostota moči $[W/m^3]$ $\Rightarrow$ **Joulov zakon** v diferencialni obliki

Dodali bomo Ohmov zakon:

$$\vec{J} = \gamma \vec{E}$$

$$\boxed{p_e = \vec{J}\vec{E} = \gamma \vec{E}^2 = \frac{J^2}{\gamma}}$$

Zgled

$J = 4\ A/mm^2$

$\gamma = 56 \cdot 10^6\ S/m$

$$p_e = \frac{J^2}{\gamma} = \frac{16 \cdot 10^{12}}{56 \cdot 10^6} = \frac{2}{7} \cdot 10^6 = \underline{280\ kW/m^3}$$

Razširitev Ohmovega zakona: (in Joulovega)

$$\boxed{\vec{J} = \gamma \Big(\vec{E} + \underbrace{\frac{\vec{J}}{\rho_w} \times \vec{B}}_{\text{uvoz magnetne sile}} + \underbrace{\vec{E}_g}_{\text{normirana sila generatorja}} \Big)}$$

$$\left\{ \vec{F}_L = Q(\vec{E} + \vec{w} \times \vec{B}) \right.$$

$$\underbrace{\frac{\vec{J}}{\gamma}}_{\text{mnogo MOČNEJŠI ČLEN}} - \underbrace{\frac{\vec{J}}{\rho_w} \times \vec{B}}_{\text{izkaže se, da je ta člen ZANEMARLJIV}} = \vec{E} + \vec{E}_g \implies \boxed{\vec{J} = \gamma(\vec{E} + \vec{E}_g)}$$

$$| \ | \gg | \quad |$$

Joulov zakon: v knjigi

$J \cdot E$ = razlika moči

$J E = p_e - p$

[illegible] moč neelektričnih dejavnikov

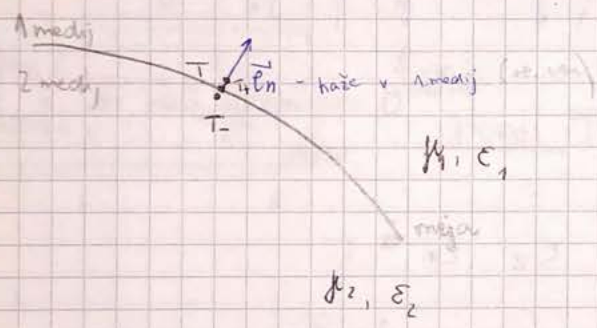
Enačbe časovno stalnega tokovnega polja

$$\oint_c \vec{E}\cdot d\vec{l} = 0$$

$$\oint_A \vec{D}\cdot d\vec{a} = Q_{not\ prosti} \qquad \vec{D} = \varepsilon\cdot\vec{E}$$

$$\oint_A \vec{J}\cdot d\vec{a} = -\frac{dQ_{not}}{dt} \text{ ali } 0 \qquad \vec{J} = \gamma\vec{E}$$

MEJNI POGOJ $\vec{J}$



Tangencialna komponenta $\vec{E}$ se OHRANJA:

* $\boxed{E_t(T_+) = E_t(T_-)}$

Normalna komponenta $\vec{D}$:

* $\boxed{D_n(T_+) - D_n(T_-) = \sigma_{prosti}(r)}$

Vzeli bomo $\oint \vec{D}\, d\vec{a} = Q$ in $\oint \vec{J}\, d\vec{a} = 0 \Rightarrow J_n(T_+) - J_n(T_-) = 0$

Normalna komponenta $\vec{J}$ se OHRANJA:

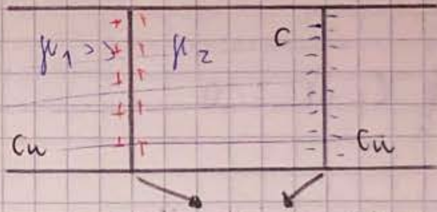
* $\boxed{J_n(T_+) = J_n(T_-)}$ mejni pogoj

Izviramo iz enačbe $D_n - D_n = \sigma$:

$$\frac{\varepsilon_1}{\gamma_1} J_n(T_+) - \frac{\varepsilon_2}{\gamma_2} J_n(T_-) = \sigma_{prosti}$$

$\Downarrow$ dodamo mejni pogoj za $\vec{J}$

$$\vec{J}_n(T)\left(\frac{\varepsilon_1}{\gamma_1} - \frac{\varepsilon_2}{\gamma_2}\right) = \sigma_{prosti}(T)$$



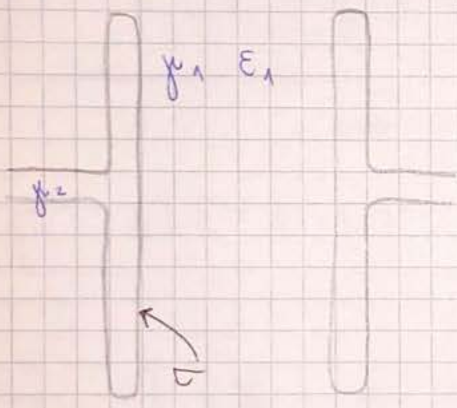
prosti naboj se nabere ravno na mejah

* iz bolj prevodnega v manj: +
* iz manj prevodnega v bolj: −

Zgled:

Realen dielektrik v kondenzatorju

↳ izguben izolant



$\gamma_2 >>> \gamma_1$

$$\left(\frac{\varepsilon_1}{\gamma_1} - \frac{\varepsilon_2}{\gamma_2}\right) J_n = \sigma$$

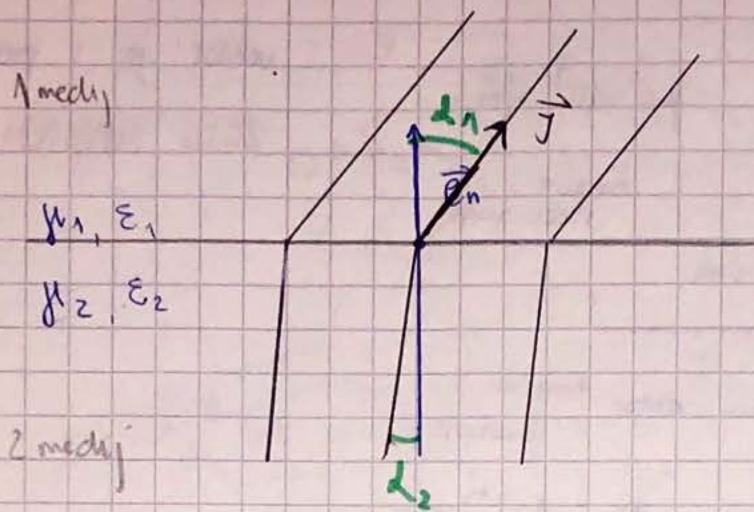
$\frac{\varepsilon_2}{\gamma_2} = 0$ zanemarljiv člen, ker je γ_2 tako velik

$$\varepsilon_1 \cdot E_n \doteq \sigma$$

$$E_n \doteq \frac{\sigma}{\varepsilon_1}$$

↳ približno enako

LOMNI ZAKON $\vec{J}$



Uporabili bom:

1. $* \; E_t(T_+) = E_t(T_-)$

$* \; J_n(T_+) = J_n(T_-)$

Zdaj lahko uporabimo Ohmov zakon:

2 $* \; \gamma_1 E_1(T_+) = \gamma_2 E_2(T_-)$

Podelimo 1. in 2.

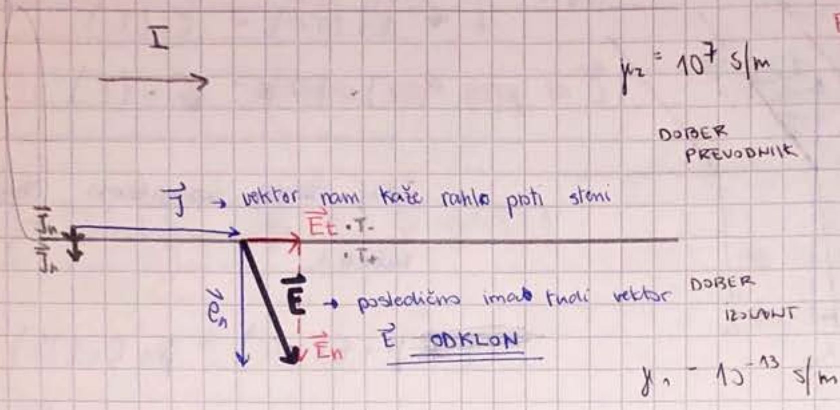
$$\frac{E_t(T_+)}{\gamma_1 E_1(T_+)} = \frac{E_t(T_-)}{\gamma_2 E_2(T_-)}$$

$$\boxed{\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\gamma_1}{\gamma_2}}$$

LOMNI ZAKON $\vec{J}$

* velja, ko tok teče in prejšnji lomni zakon, ki smo ga izpeljali, NE VELJA VEČ

MEJA PREVODNIK IN IZOLANT



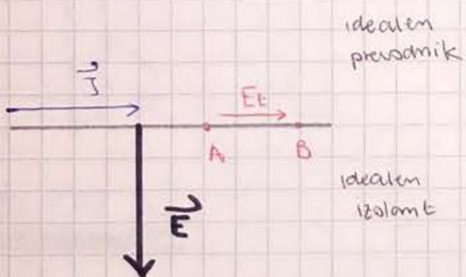
E_t - vektor je v prevodniku!
ZELO MAJHEN!

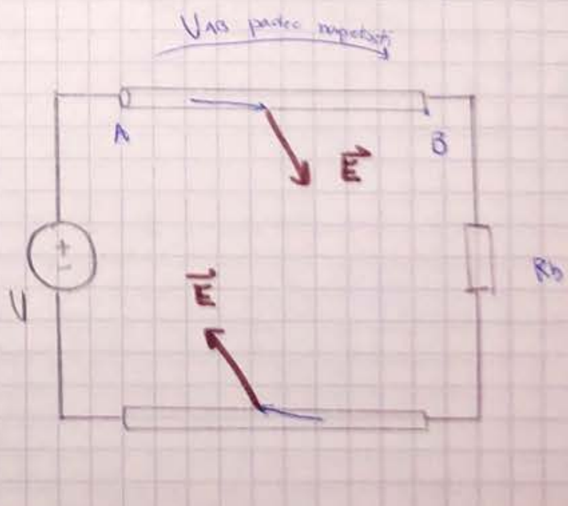
$E_n(T_+) = 10^6$ V/m

$J_t(T_-) = 10^6$ A/m

$E_t(T_-) = \frac{J_t}{\gamma_2} = \frac{10^6}{10^7} = \underline{0{,}1 \text{ V/m}}$

$J_n(T_+) = E_n \cdot \gamma_1 = 10^6 \cdot 10^{-13} = \underline{10^{-7} \text{ A/m}}$



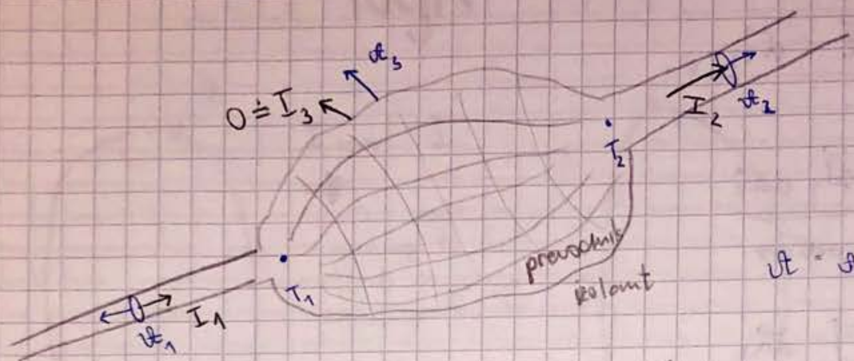


Če izberemo dve točki A in B:

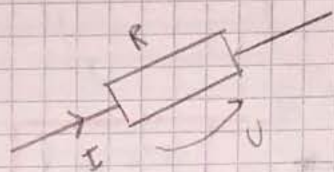
$$U_{AB} = V_A - V_B = \int_A^B \vec{E}\, \vec{dl}$$

* Ni dejansko ekvipotencialka

ELEKTRIČNA UPORNOST / PREVODNOST



$A = A_1 \cup A_2 \cup A_3$ - sklenjena ploskev



$$\oint_{A} \vec{J}\, d\vec{a} = 0$$

$$\int_{A_1} \underbrace{\vec{J}\, d\vec{a}}_{-I_1} + \int_{A_3} \underbrace{\vec{J} \cdot d\vec{a}}_{I_3 = 0} + \int_{A_2} \underbrace{\vec{J}\, d\vec{a}}_{I_2} = 0$$

$$-I_1 + I_3 + I_2 = 0$$

$$\boxed{I_1 = I_2 = I}$$

$$U_{12} = \int_{T_1}^{T_2} \vec{E} \cdot d\vec{l} = U$$

G - (integralna) prevodnost

Ker velja: $\vec{J} = \underbrace{\gamma \vec{E}}_{\text{linearna odvisnost}} \Rightarrow I = k \cdot U$

$$I = G \cdot U \rightarrow I = \frac{1}{R} U$$

$$U = R \cdot I$$

$$\boxed{U = R \cdot I}$$

R - (integralna) upornost $\underline{\underline{\Omega}}$

Moč

$P = \vec{J}\,\vec{E}$ moramo sešteti

$$P = \sum_{j=1}^{m}\sum_{k=1}^{n} \vec{J}\cdot\vec{E}\cdot d v_{jk}$$

$$P = \sum\sum \underbrace{\vec{J}\,\vec{E}\cdot d\vec{l}_j\, d\vec{a}_k}_{\text{prerazporedimo}}$$

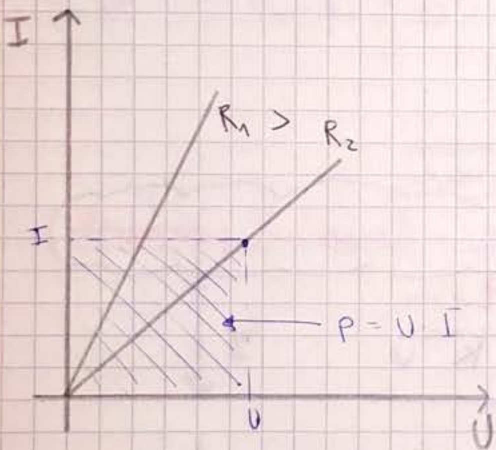
$$\underbrace{\vec{E}\cdot d\vec{l}_j}_{dU_j}\cdot\underbrace{\vec{J}\cdot d\vec{a}_k}_{dI_k}$$

$$P = \sum dU \sum dI \Longrightarrow \boxed{P = U\cdot I}$$

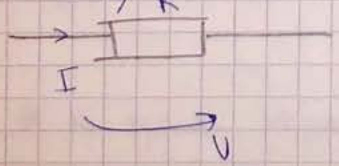
Joulov zakon

v integralni obliki

$$\boxed{P = G\cdot I^2 = \frac{U^2}{R}}$$



Strnjen el. upor $\rightarrow$ v časovno spr. razmerah



$u = R\cdot i$

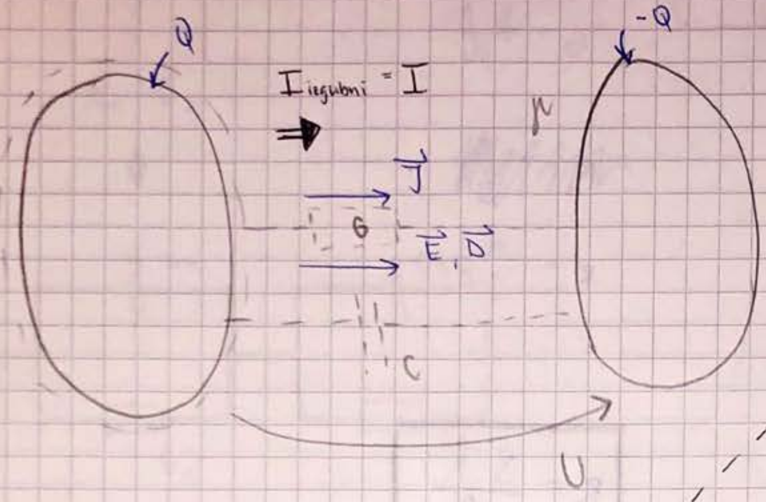
še vse velja

$P(t) = u\cdot i$

$U = R\cdot I$

$P = U\cdot I$

DUALNOST (DVOJNOST) EL. IN TOKOVNEGA POLJA



$$Q = C \cdot U$$
$$I = G \cdot U$$

$$\vec{D} = \varepsilon \cdot \vec{E}$$
$$\vec{J} = \mu \cdot \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon}$$

$$\oint_{\text{vt}} \vec{D}\, d\vec{a} = Q$$

$$\oint_{\text{vt}} \vec{J} \cdot d\vec{a} = I$$

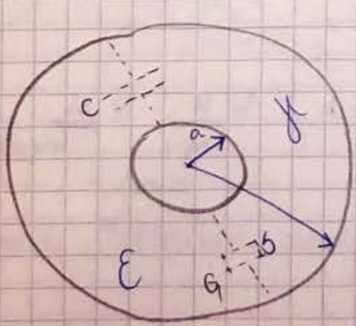
$$I = \oint_{\text{vt}} \vec{J} \cdot d\vec{a} = \oint_{\text{vt}} \mu \cdot E \cdot d\vec{a} =$$
$$= \frac{\mu}{\varepsilon} \oint \vec{D} \cdot d\vec{a} = \frac{\mu}{\varepsilon}\, Q$$

$$G \cdot U = \frac{\mu}{\varepsilon}\, Q$$
$$G \cdot \cancel{U} = \frac{\mu}{\varepsilon}\, C \cdot \cancel{U}$$

$$\boxed{\frac{G}{C} = \frac{\mu}{\varepsilon}}$$

DUALNOST

Zgled koax kabel

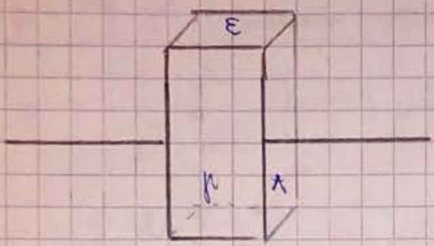


$$C = \frac{2\pi\, \varepsilon\, l}{\ln \frac{b}{a}}$$

$$G = \frac{2\pi\, \mu\, l}{\ln \frac{b}{a}}$$

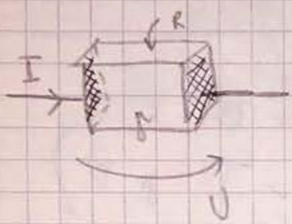
$$R_{izol} = \frac{1}{G_{izolacija}}$$

Zgled : Ploščni kondenzator



$$C = \frac{\varepsilon A}{d}$$

$$G = \frac{\mu A}{d}$$

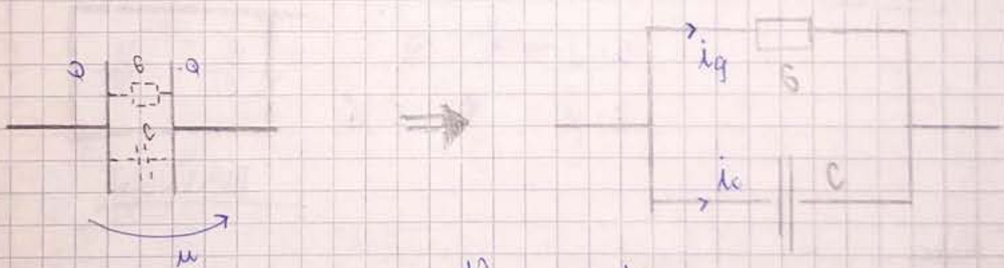


$$R = \frac{d}{\mu A}$$

$$\boxed{R = \frac{\rho \cdot l}{A}}$$

$$\rho = \frac{1}{\mu}$$

Zgled : NADOMESTNO VEZJE REALNEGA KONDENZATORJA



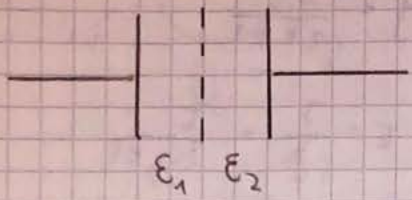
$$i_c = \frac{dQ}{dt} = C \cdot \frac{du}{dt}$$

$$i_g = G \cdot u$$

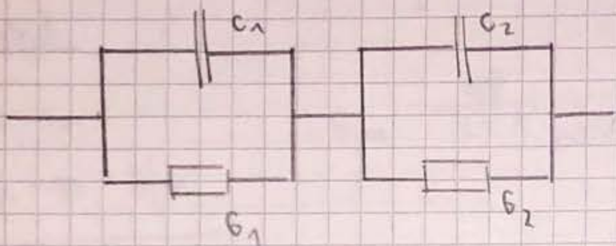
$$i = i_c + i_g$$

$$i = C \cdot \underbrace{u'}_{\text{njen odvod}} + G\,\underbrace{u}_{\text{napetost}}$$

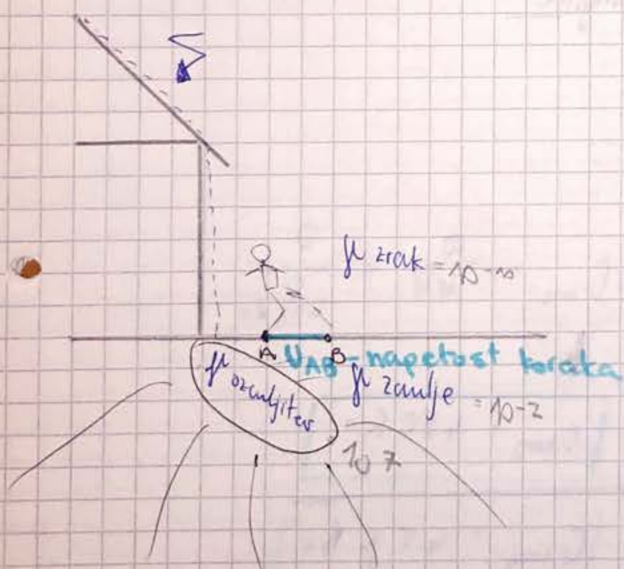
* DVEH KONDENZATORJEV



$$\frac{U_1}{U_2} = \frac{C_2}{C_1} \Rightarrow \frac{U_1}{U_2} = \frac{R_1}{R_2}$$

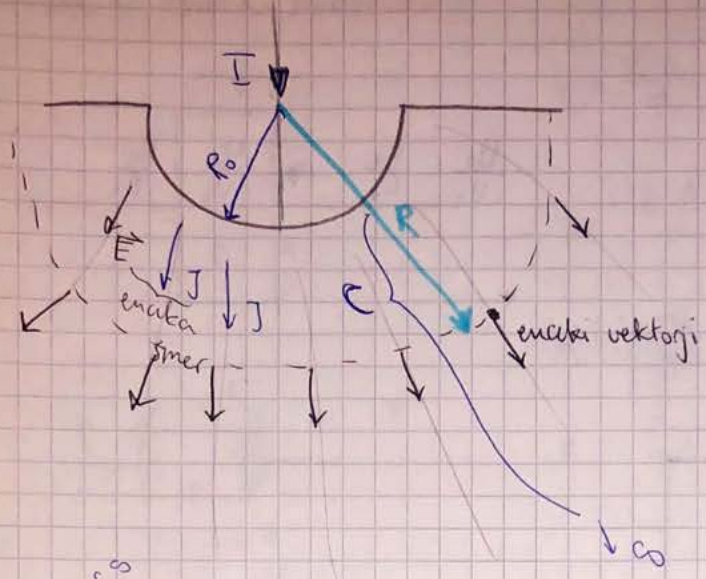


OZEMLJITVENA (PONIKALNA) UPORNOST



Zgled Polkrožno ozemljilo

druga stran



$$J = \frac{I}{2\pi R^2}$$

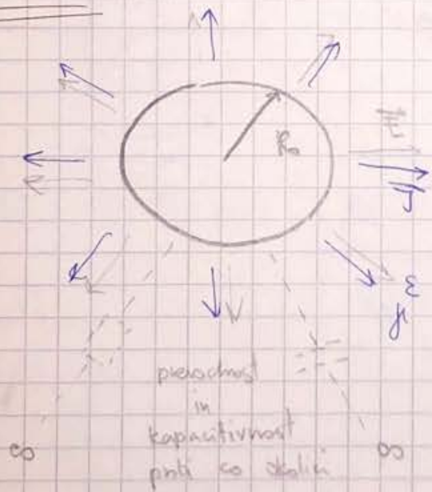
$$E = \frac{J}{\gamma} = \frac{I}{2\pi \gamma R^2}$$

$$V = \int_{R_0}^{\infty} \vec{E}\, dR =$$

$$= \frac{1}{2\pi\gamma} \int_{R_0}^{\infty} \frac{dR}{R^2} = \frac{I}{2\pi \gamma R_0}$$

$$\boxed{R_{oz} = \frac{U}{I} = \frac{1}{2\pi\gamma R_0}}$$ Upornost ozemljila

II. način



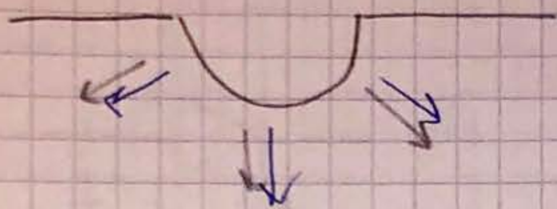
$$C_{krog} = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon R_0}}$$

$$\boxed{C_{krog} = 4\pi\varepsilon R_0}$$

$$\boxed{G_{krog} = 4\pi\gamma R_0}$$

če hočemo, da

je samo pol krogla



$$G_{polkrog} = \frac{G_{krog}}{2}$$: delimo z 2

$$\boxed{G_{polkr.} = 2\pi \gamma R_0}$$

$$\boxed{R = \frac{1}{2\pi \gamma R_0}}$$

Zgled

$R_0 = 1m$

$\gamma = 10^{-2} S/m$

$$R_{oz} = \frac{10^2}{2\pi 1} = \underline{\underline{16\ \Omega}}$$

R - VEZJA

ELEMENTI EL. VEZIJ

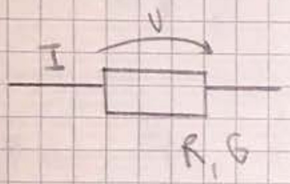
pasivni elementi (R)
* linearni
* nelinearni

re-aktivni elementi (C, L)

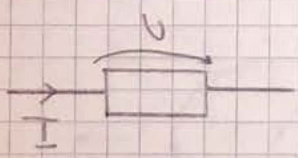
aktivni elementi (U_0, I_0)

* PASIVNI ELEMENTI

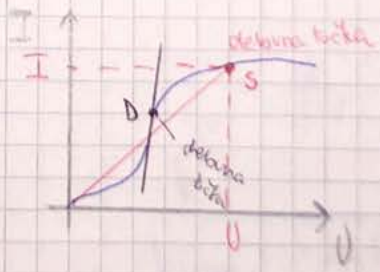
Linearni upor



Nelinearni elem.



ne moremo reči
da ima upornost

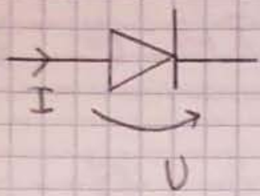


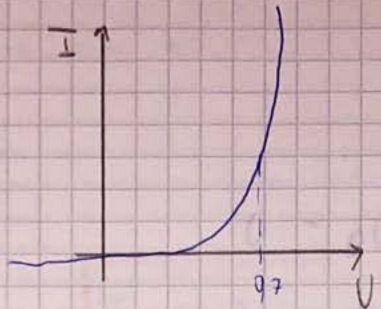
* Apoksimacije:

$R_S = \frac{U}{I}$ statična upornost

$R_D = \frac{du}{di}$

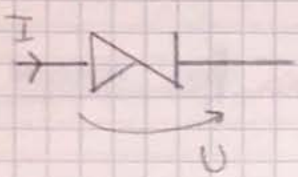
* diode

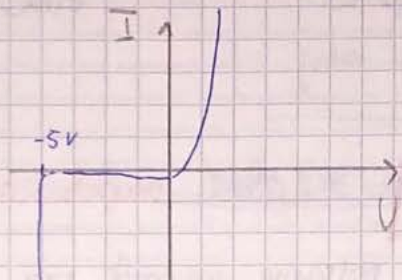




* V eno smer prepusti tok, v drugo pa ne

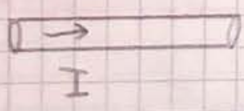
* Zenner dioda

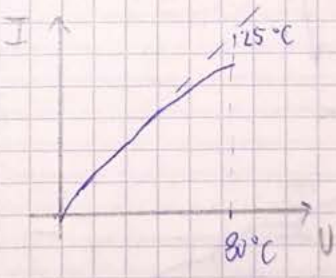




* Tlivka

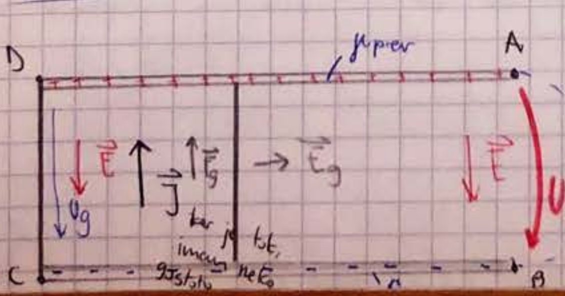
* Žica





* AKTIVNI ELEMENTI

↳ Napetostni vir (realni) ima notranjo R



↳ I - zdaj nam teče tok

breme

(prej smo imeli odprte sponke)

$$\oint_L \vec{E}\cdot d\vec{l} = 0$$

$$\underbrace{U + R_g \cdot I}_{\text{upornost medija in priključkov}} - U_g = 0$$

medij, kjer se razdvaja naboj = notranja upornost vira

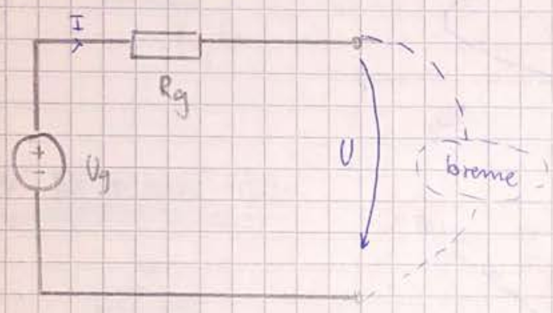
$$\boxed{U = U_g - R_g \cdot I}$$

Enačba napetostnega vira !

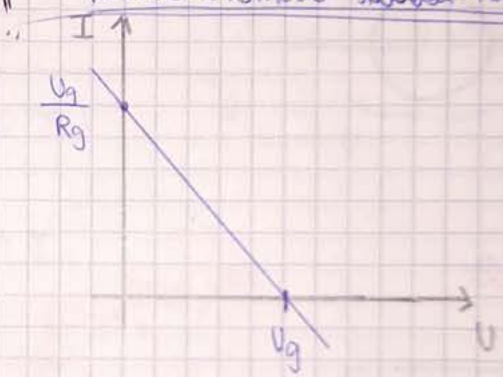
U_g – napetost odprtih sponk

R_g – notranja upornost vira

Modelno vezje:



Karakteristika model. nap. vira:



V mislih imejmo:

I. ENAČBO NAP. VIRA

II. MODELNO VEZJ

III. KARAKTERISTIKO NAP. VIRA

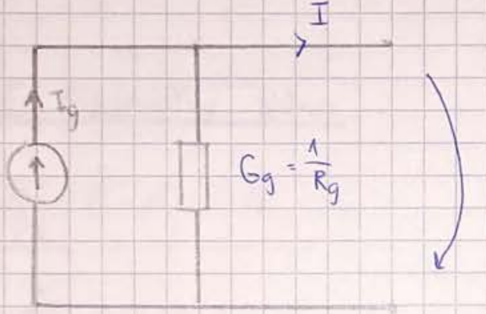
Tokovni vir (realni) ima notranjo R

$$U = U_g - R_g I \quad \Big| : R_g$$

$$I = \underbrace{\frac{U_g}{R_g}}_{I_g} - \underbrace{\frac{U}{R_g}}_{G_g} \Longrightarrow \boxed{I = I_g - G_g \cdot U}$$

Enačba tokovnega vira

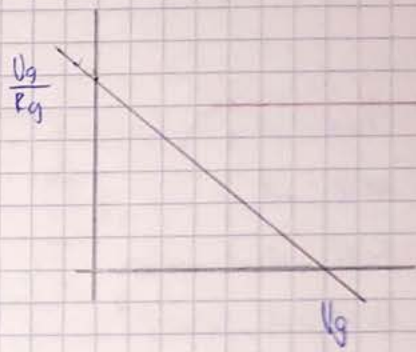
Modelno vezje:



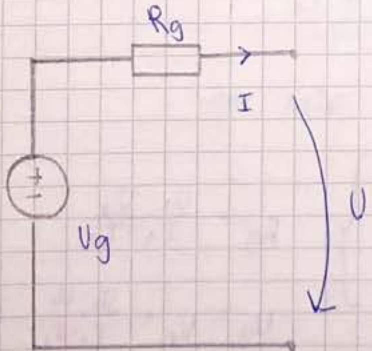
* Ta U bo 0, ko bomo imeli vmes KRATEK STIK

I_g ... kratkostični tok

Karakteristika

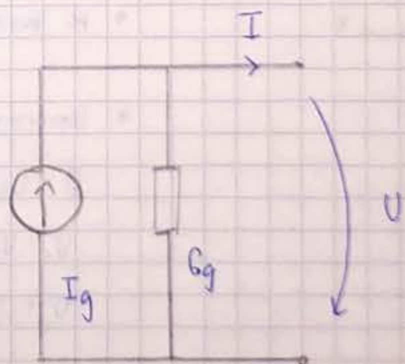


* Ekvivalenca virov



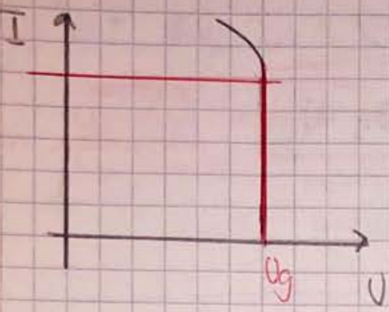
$I_g = \frac{U_g}{R_g}$

$G_g = \frac{1}{R}$

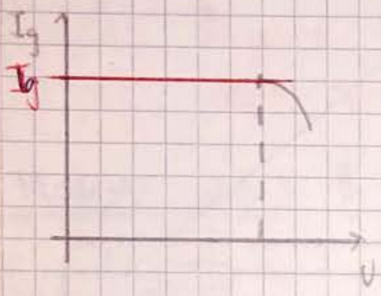


Idealni vir

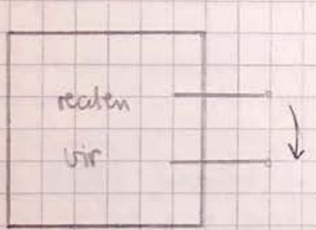
* Napetostni : $R_g = 0$



* Tokovni : $R_g = \infty$



DOLOČANJE PARAMETROV VIRA



* Izmerimo : napetost odprtih sponk : $U_g = U_0$

* ne merimo $I_g = I_0 = I_K$

* Izmerimo 2 DELOVNI TOČKI vira

$$\left.\begin{aligned} U &= U_g - I R_g \\ U_1 &= U_g - I_1 R_g \\ U_2 &= U_g - I_2 R_g \end{aligned}\right\} \quad R_g = \frac{U_2 - U_1}{I_1 - I_2}$$

$$U_g = U_1 + \frac{U_2 - U_1}{I_1 - I_2} I_1$$

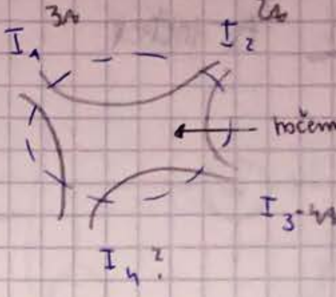
$$\underline{\underline{I_g = \frac{U_g}{R_g}}}$$

ZAKONI EL. KROGOV

I. Kirchhoffovi zakoni

$$\oint_{A} \vec{J}\, d\vec{a} = 0$$

$$\sum_{k=1}^{n} I_k = 0$$



hočemo, da je $Q = 0$

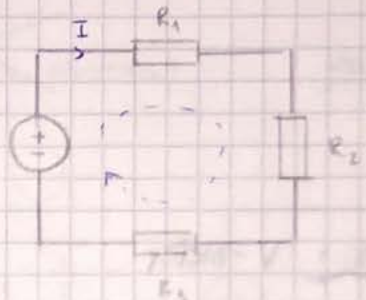
$$I_1 + I_2 + I_3 + I_4 = 0$$

$$3A + 2A - 4A + I_4 = 0 \rightarrow I_4 = -1A$$

II. Kirchhoffov zakon

$$\oint_{R} \vec{E} \cdot d\vec{l} = 0$$

$$\sum_{j=1}^{m} U_j = 0$$



$U_1 = 110V$

$R_1 = 30\Omega$

$R_2 = 50\Omega$

$R_3 = 30\Omega$

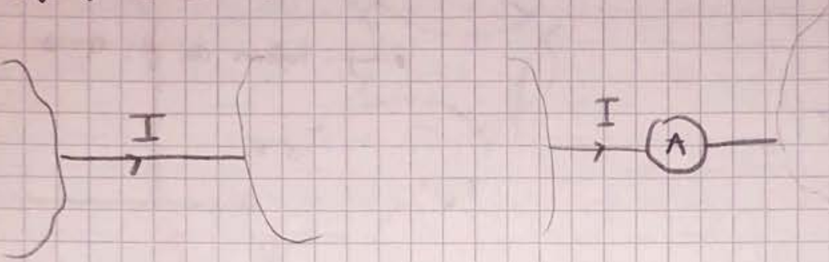
$$I R_1 + I R_2 + I R_3 - U_1 = 0$$

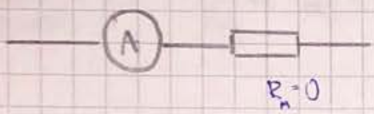
$$I (110\Omega) = 110V$$

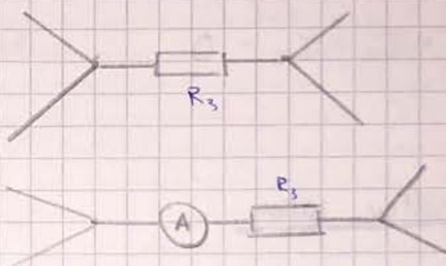
$$I = 1A$$

MERJENJE EL. KOLIČIN

1) Merjenje toka: A-meter

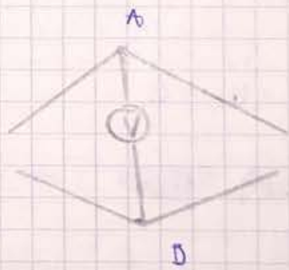


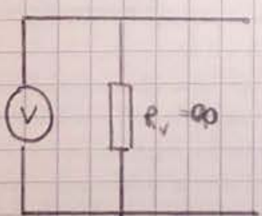


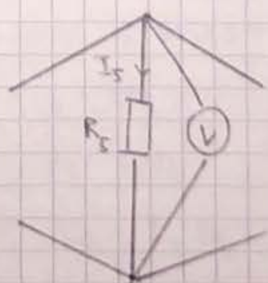


* Vežemo ZAPOREDNO

2) Merjenje napetosti: V-meter



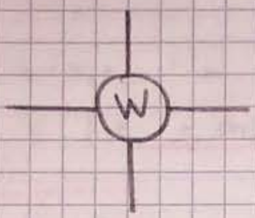


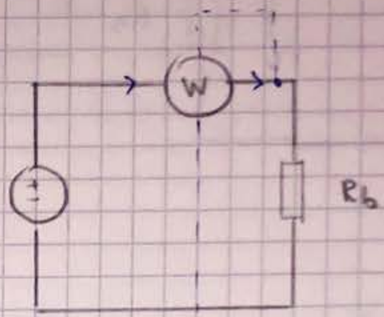


* Vežemo VZPOREDNO

3) Merjenje moči

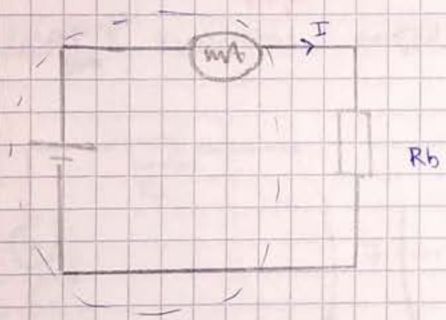
* Želi meriti PRODUKT napetosti in toka
* Ima 4 sponke
    - 2 za tokovni vir
    - 2 za napetostni vir



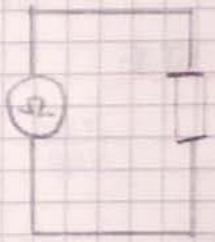


4) Merjenje upornosti

* natančno meri tok (vgrajen mA-meter)
* Ima stabilno napetost

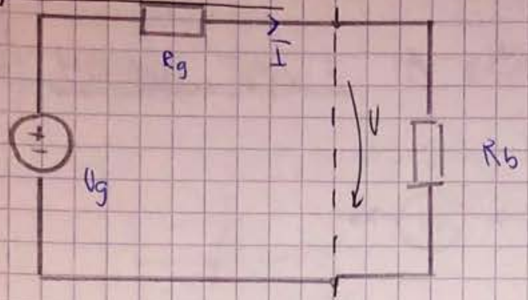


$$R_b = \frac{U_0}{I}$$



ENOSTAVNA ENOSMERNA EL. VEZJA

1) LINEAREN KROG



$R_g = 0{,}5\,\Omega$

$R_b = 4{,}5\,\Omega$

$U_g = 15\,V$

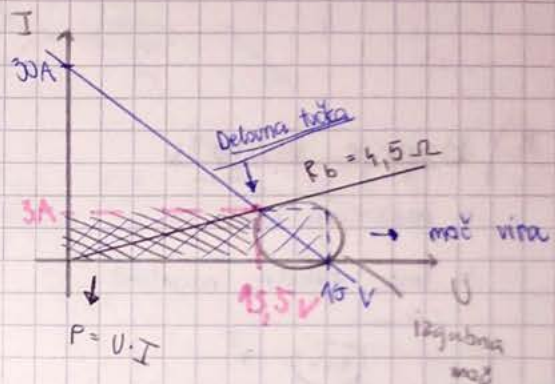
* Analitično

$U = U_g - R_g \cdot I$

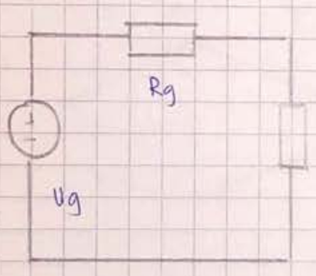
$R_b I = U_g - R_g I$

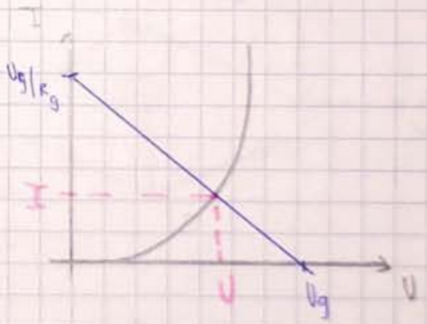
$I = \dfrac{U_g}{R_b + R_g} = \dfrac{15}{5} = 3A$

* Grafično

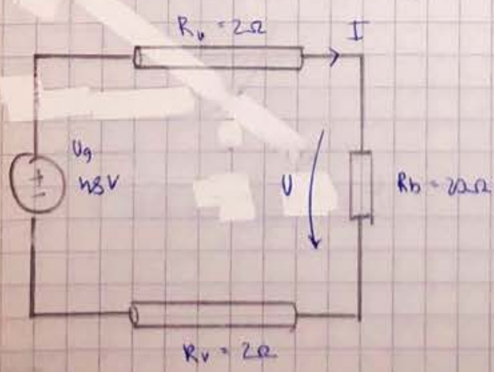


2) NELINEAREN KROG





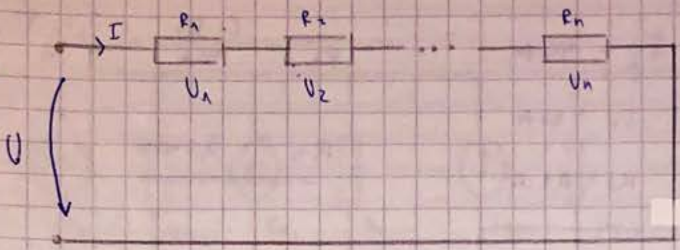
Zgled



$\eta = \dfrac{P_b}{P_g} = \dfrac{I \cdot U_b}{I \cdot U_g} = \dfrac{I \cdot 20\,\Omega}{48\,V} = \dfrac{40}{48}$

$I = \dfrac{48}{20 + 2 + 2} = 2A$

ZAPOREDNA VEZAVA BREMEN



$$U = U_1 + U_2 + \dots + U_n$$

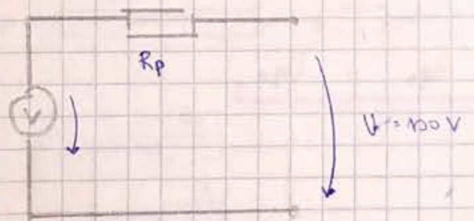
$$U = I \,(R_1 + R_2 + \dots + R_n)$$

$$\boxed{U = I \cdot R}$$

$$R = R_1 + R_2 + \dots + R_n$$

$$\boxed{R = \sum_{k=1}^{n} R_k}$$

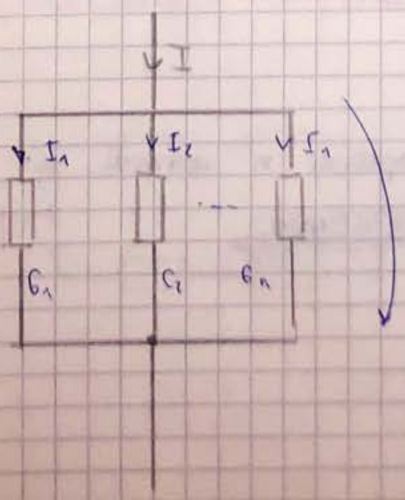
Zgled: Razširitev merilnega območja V-metra



$$\frac{R_p}{R_0} = \frac{900\,V}{100\,V} \rightarrow R_p = 9 R_0$$

$$R_p = 90\,k\Omega$$

VZPOREDNA VEZAVA BREMEN



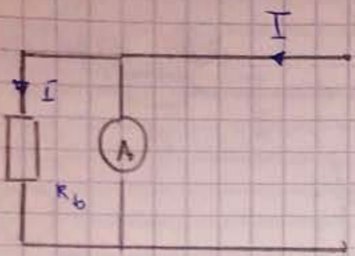
$$I = I_1 + I_2 + \dots + I_n \quad | \cdot U$$

$$G = \frac{I}{U} = G_1 + G_2 + \dots + G_n$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$\boxed{R = \frac{R_1 R_2}{R_1 + R_2}}$$

Zgled: Razširitev merilnega območja A-metra



$I = 50\,A$

$I_0 = 10\,A$

$R_0 = 0{,}2\,\Omega$

$R_s = \frac{I_0 R_0}{40\,A} = \frac{0{,}2 \cdot 10}{40}$

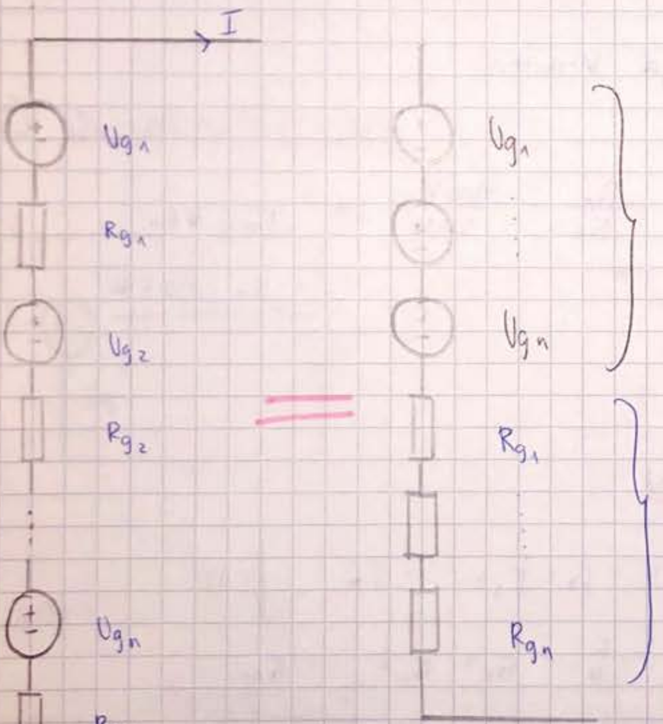
$R_s = 0{,}05\,\Omega$

$I_A = 4\,A \rightarrow I = \frac{4\,A}{10\,A} \cdot 50\,A$

$I = 20\,A$

* VIRI

ZAPOREDNA VEZAVA VIROV

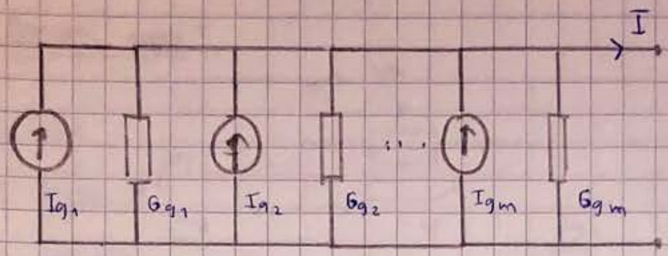


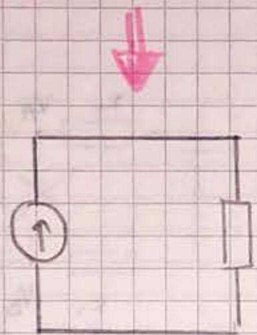
$U_g = U_{g1} + \dots U_{gn}$

$R_g = R_{g1} + \dots + R_{gn}$

* Napetosti in upornosti se SEŠTEVAJO.

VZPOREDNA VEZAVA VIROV

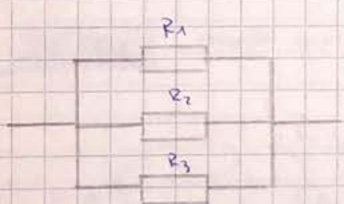




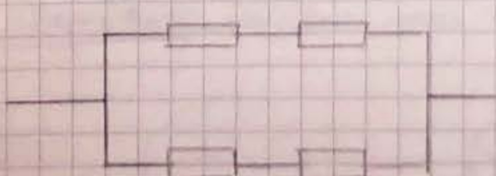
$$\underbrace{I_g = \sum_{k=1}^{m} I_{gk}}$$

$$\underbrace{G_g = \sum_{k=1}^{m} G_{gk}}$$

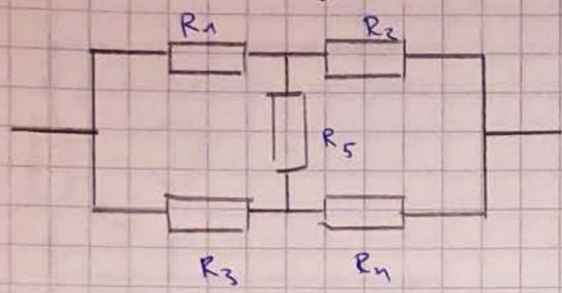
SESTAVLJENA BREMENA



50Ω | 100 W   50 | 25 W



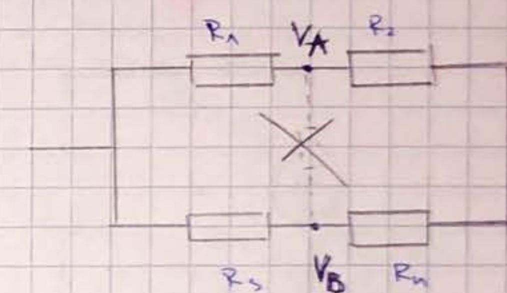
MOSTIČNO VEZJE



* Uravnotežen mostič

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$

$V_A = V_B \rightarrow \boxed{\begin{array}{l} I_5 = 0 \\ U_5 = 0 \end{array}}$ $\Rightarrow$

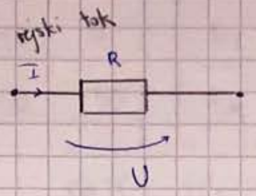


$\frac{1}{R} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4}$

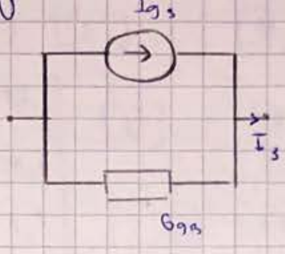
Analiza R vezij

Osnovni pojmi:

* vezje
* veja
* vozlišče
* zamke
* graf vezja
* drevo vezja

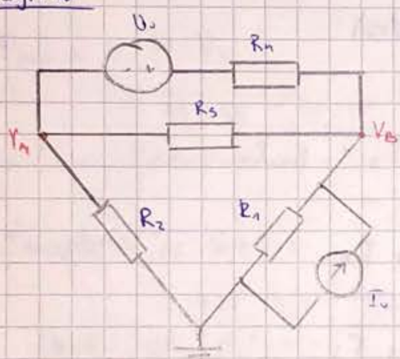


$U_g = R_g \cdot I_2 - U_{g2}$



$I_e = I_{g3} + U_3 \cdot G_{g3}$

Zgled



$R_1 = 10\Omega$

$R_2 = 20\Omega$

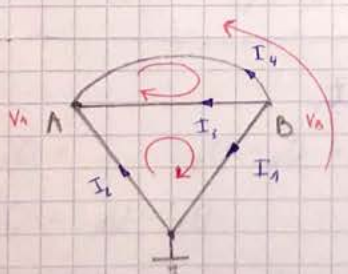
$R_3 = 40\Omega$

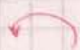
$R_4 = 40\Omega$

$I_0 = 10A$

$U_0 = 40V$

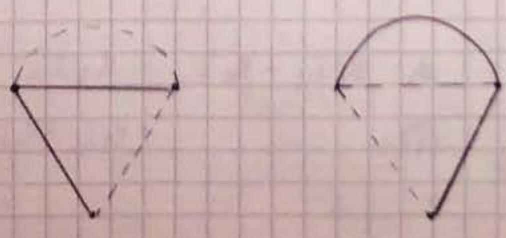
Graf vezja: prikaže nam poenostavljeno vezje, z vsemi vejami in spojišči





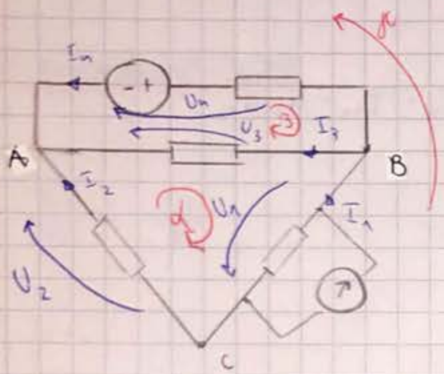
zamke

Drevo vezja: podmnožica grafa vezja, z vsemi spojišči in vsebeznimi zankami



Metode za reševanje vezij

* Neposredna Kirchhoffova metoda
* Metoda zančnih tokov
* Metode spojiščnih potencialov



imamo 4 veje in bomo ločeno pisali za toke in napetosti:

SISTEM ENAČB 5Ø

$I_1 = G_1 U_1 - I_0$ | $U_1 = R_1 I_1 + R_1 I_0 = 84\,V$

$I_2 = G_2 U_2$ | $U_2 = R_2 I_2 = -1{,}6 \cdot 20 = -32\,V$

$I_3 = G_3 U_3$ | $U_3 = R_3 I_3 = 1{,}3 \cdot 40 = 52\,V$

$I_4 = G_4 U_4 + - G_4 U_0$ | $U_4 = U_0 + R_4 I_4 = 52\,V$

I. KIR. Z

A: $-I_2 - I_3 - I_4 = 0$

B: $I_1 + I_3 + I_4 = 0$

~~C: $-I_1 + I_2 = 0$~~

$0 = 0$

II. KIR. Z

α: $U_1 + U_2 - U_3 = 0$

β: $U_3 - U_4 = 0$

~~γ: $-U_1 - U_2 + U_4 = 0$~~ zunanja zanka

$0 = 0$

α: $R_1 I_1 + R_2 I_2 - R_3 I_3 = -R_1 I_0$

β: $+ R_3 I_3 - R_4 I_4 = U_0$

$$\begin{bmatrix} 0 & -1 & -1 & -1 \\ 1 & 0 & 1 & 1 \\ 10 & 20 & -40 & 0 \\ 0 & 0 & 40 & -40 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -100 \\ 4 \end{bmatrix}$$

$$[R] \cdot [I] = [U]$$

$$[I] = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \begin{bmatrix} -1{,}6 \\ -1{,}6 \\ 1{,}3 \\ 0{,}3 \end{bmatrix} \qquad [I] = [R]^{-1} [U]$$

$I_1 = -1{,}6\,A$ $\quad$ $I_3 = 1{,}3\,A$

$I_2 = -1{,}6\,A$ $\quad$ $I_4 = 0{,}3\,A$

$$P_{\text{tokovni vir}} = I_0 \cdot U_1$$

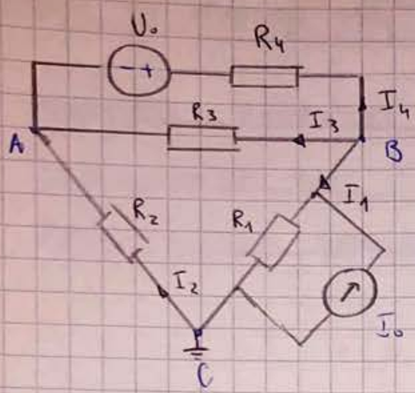
$$= 10 \cdot 84 = \underline{\underline{840\,W}}$$

– ta vir mora pokrit, kar baterija rabi, da se polne

$$P_{\text{napetostni vir}} = -U_0\, I_4$$

$$= -40\,V \cdot 0{,}3 = \underline{\underline{-12\,W}}$$

vir se polni
s to močjo



$R_1 = 10\Omega$

$R_2 = 20\Omega$

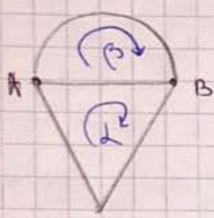
$R_3 = 40\Omega$

$R_4 = 40\Omega$

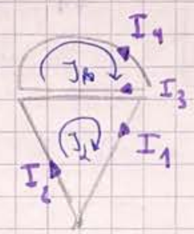
$I_0 = 10A$

$U_0 = 40V$

METODA ZANČNIH TOKOV



$\Longrightarrow$



$J_\alpha, J_\beta = ?$

$I_1 = J_\alpha$

$I_2 = J_\alpha$

$I_3 = -J_\alpha + J_\beta$

$I_4 = -J_\beta$

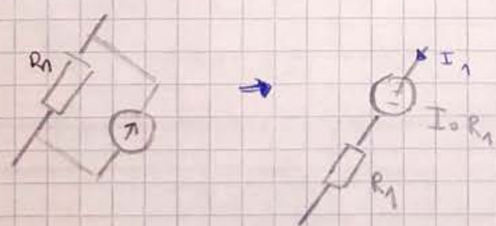
A: $\cancel{-J_\alpha - (-J_\alpha + J_\beta) - (-J_\beta) = 0}$

B: $\cancel{J_\alpha - J_\alpha + J_\beta - J_\beta = 0}$

enačbi nam data odgovor $0=0$

~~s te ne moremo pomagati~~

* Kirchhoff za vsako zanko:



$\alpha: \; R_1 J_\alpha + R_2 J_\alpha + R_3 (J_\alpha - J_\beta) = -I_0 R_1$

$\beta: \; R_3 (J_\beta - J_\alpha) + R_4 J_\beta = U_0$

samo dve enačbi, z dvema neznankama

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 \end{bmatrix} \begin{bmatrix} J_\alpha \\ J_\beta \end{bmatrix} = \begin{bmatrix} -I_0 R_1 \\ U_0 \end{bmatrix}$$

Do te matrike lahko pridemo iz samega vezja:

* $R_1 + R_2 + R_3 \rightarrow$ upori skozi α zanko
* $-R_3 \rightarrow$ mejni upor (skozi α in β)
* $R_3 + R_4 \rightarrow$ upori skozi β zanko

SIMETRIČNA MATRIKA

$$\begin{bmatrix} 70 & -40 \\ -40 & 80 \end{bmatrix} \begin{bmatrix} J_\alpha \\ J_\beta \end{bmatrix} = \begin{bmatrix} -100 \\ 40 \end{bmatrix}$$

$J_\alpha = -1{,}6\ A$

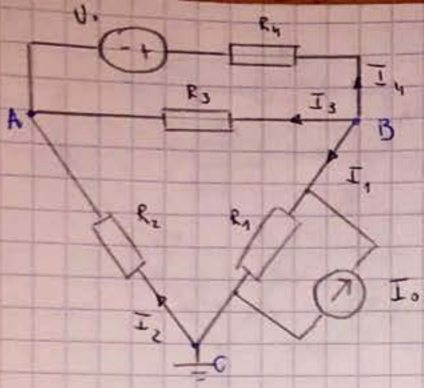
$J_\beta = -0{,}3\ A$

$I_1 = I_\alpha = -1{,}6 A$

$I_2 = I_\alpha = -1{,}6 A$

$I_3 = -I_\alpha + J_\beta = 1{,}6 - 0{,}3 = 1{,}3 A$

$I_4 = -J_\beta = 0{,}3 A$



$R_1 = 10\,\Omega$

$R_2 = 20\,\Omega$

$R_3 = 40\,\Omega$

$R_4 = 40\,\Omega$

$U_0 = 40\,V$

$I_0 = 10\,A$

METODA SPOJIŠČNIH POTENCIALOV

$V_A,\ V_B = ?$

$U_1 = V_B - 0 = V_B$

$U_2 = 0 - V_A = -V_A$

$U_3 = V_B - V_A$

$U_4 = V_B - V_A$

$\mathcal{L}: \quad V_B - V_A - V_B + V_A = 0$

$\mathcal{D}: \quad V_B - V_A - (V_B - V_A) = 0$

si ne moremo pomagati

$0 = 0$, vrtinčnost že všteta v enačbo

II. Kirchhoffov zakon

$$A: \quad -(-V_A)\,G_2 - (V_B - V_A)\,G_3 - (V_B - V_A)\,G_4 + U_0 G_4 = 0$$

$$B: \quad V_B G_1 - (V_B - V_A)\,G_3 + (V_B - V_A)\,G_4 = I_0 + U_0 G_4$$

$$\begin{bmatrix} G_2 + G_3 + G_4 & -G_3 - G_4 \\ -G_3 - G_4 & G_1 + G_3 + G_4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -U_0 G_4 \\ I_0 + U_0 G_4 \end{bmatrix}$$

* $G_2 + G_3 + G_4$ → se iztekajo v spojišče A
* $-G_3 - G_4$ → sta med obemi spojišči zato - predznak !
* $G_1 + G_3 + G_4$ → se iztekajo v spojišče B

$V_A = 32\,V$

$V_B = 84\,V$

$U_1 = V_B - 0 = V_B = 84\,V$

$U_2 = 0 - V_A = -V_A = -32\,V$

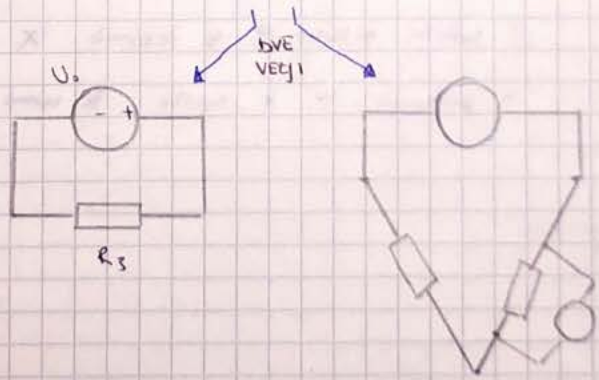
$U_3 = V_B - V_A = 52\,V$

$U_4 = V_B - V_A = 52\,V$

Primer: Vezje z idealnimi elementi

1) Idealni napetostni vir

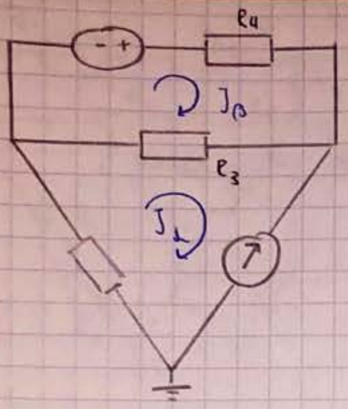
* metoda zančnih tokov OK
* metoda spojiščnih potencialov - težave



2) Idealni tokovni vir

* metoda spojiščnih potencialov OK
* metoda zančnih tokov

[vsak tokovni vir zaobjamemo z ENO ZANKO] !



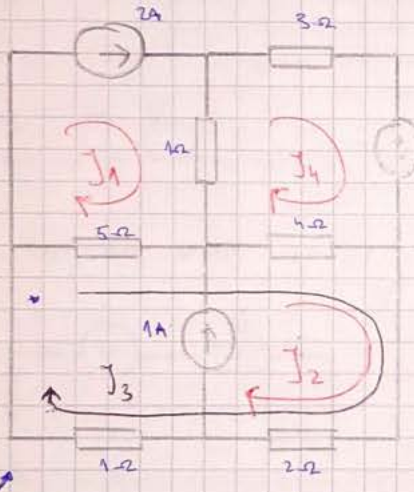
$J_L = -I_0$

vrtinčnost že vključena

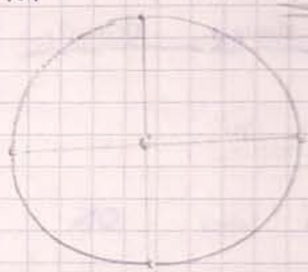
Spojiščni potenciali → o potencialu smo lahko govorili šele, ko $\oint \vec{E} \cdot dl = 0$

Zančni tokovi → tudi tu nismo podali tokovnih enačb, ker vsak tok že SAM PO SEBI ZADOŠČA KIRCHHOFFU

Zgled 3.37



$P_{10V} = -J_4 \cdot 10V = 0,4 \cdot 10 = 4W$



* osnovna metoda → 8 neznank ✗ preveč
* potenciali → 4 enačbe še vedno preveč

samo to okno ni primerno za izbiro, kajti potem bo tokovni vir seštevek J_2 in J_3 dodatna neznanka

! zanke določujemo tako, da gremo samo 1x čez tokovni vir

$J_1 = 2A$

$J_2 = 1A$

$J_3 = ?$

$J_4 = ?$

J_3: $5\,\Omega\,(J_3 - 2A) + 4\,\Omega\,(1A + J_3 - J_4) + 2\,\Omega\,(1A + J_3) + 1\,\Omega \cdot J_3 = 0$

J_4: $1\,\Omega\,(J_4 - 2A) + 3\,\Omega\,J_4 + 10\,V + 4\,\Omega\,(J_4 - J_3 - 1A) = 0$

$$\begin{aligned} 12\,\Omega\,J_3 - 4\,\Omega\,J_4 &= 4V \\ -4\,\Omega\,J_3 + 8\,\Omega\,J_4 &= -4V \end{aligned} \quad \Big| :4$$

$$\begin{aligned} 3J_3 - J_4 &= 1V \quad \Big| \cdot 2 \\ -J_3 + 2J_4 &= -1V \end{aligned}$$

$$\begin{aligned} 6J_3 - \cancel{2J_4} &= 2 \\ -J_3 + 2J_4 &= -1 \end{aligned}$$

$5J_3 = 1$

$\underline{\underline{J_3 = 0{,}2A}}$

$\underline{\underline{J_4 = -0{,}4\,A}}$

STAVKI (TEOREMI) O ENOSMERNIH EL. VEZJIH

1.) STAVEK SUPERPOZICIJE

V enosmernem lin. vezju lahko izračunamo vejske in zančne tokove po načinu superpozicije in seštejemo prispevke virov, tako da enega DEAKTIVIRAMO

Deaktivacija:

J = zančni tokovi

I = vejski tokovi

$$[R] \cdot [J] = [U_g]$$

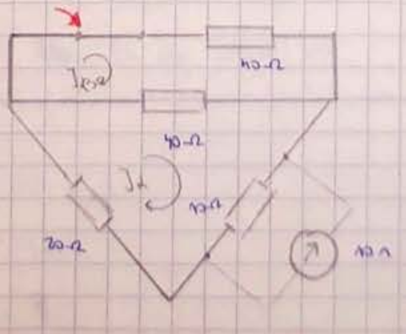
$$[J] = [R^{-1}]\,[U_g]$$

$$\begin{bmatrix} U_{g1} \\ U_{g2} \\ U_{g3} \end{bmatrix} = \begin{bmatrix} U_{g1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ U_{g2} \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ U_{g3} \end{bmatrix}$$

$$[J] = [R^{-1}]\,[U_{g1}] + [R^{-1}]\,[U_{g2}] + [R^{-1}]\,[U_{g3}]$$

$$[J] = [J'] + [J''] + [J''']$$

I.) Aktiven je TOKOVNI VIR

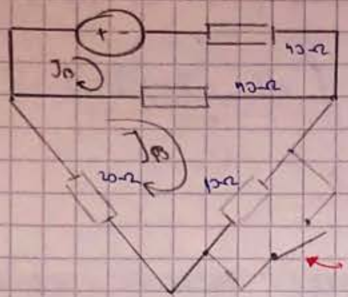


- Napetostni vir nadomestili s ~~[illegible]~~ slikalom (kratek stik)

$J_2' = -2A \rightarrow J_2 = J_2' + J_2'' = -1.6A$

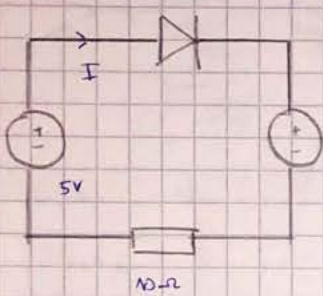
$J_a' = -1A \rightarrow J_a = J_a' + J_a'' = 3.3A$

II) Aktiven je NAPETOSTNI VIR



* Tokovni vir nadomestimo z odprtimi sponkami.

Primer:



$I_L = \frac{U}{R} = \frac{5}{10} = 0,5A$

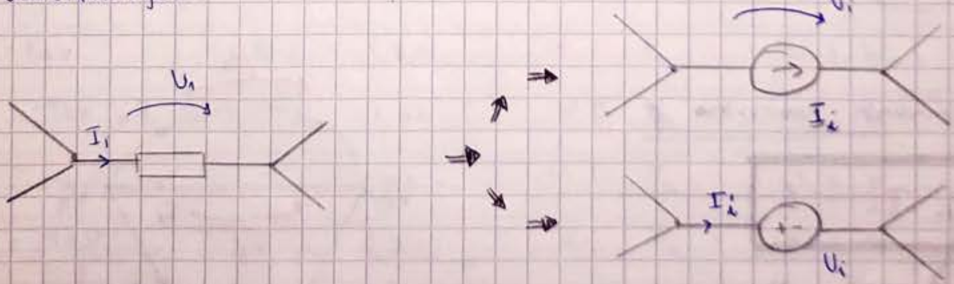
$I_d = 0A$

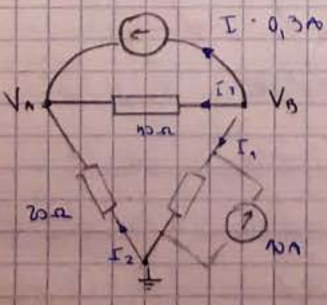
~~$I = 0,5A$~~ → Nimamo linearnega vezja

2.) STAVEK O NADOMESTITVI

Če poznamo tok ene veje, lahko tok nadomestimo z VEJNIM GENERATORJEM, pa se ostale

Če poznamo vejsko napetost, lahko element te veje nadomestimo z NAPETOSTNIM GENERATORJEM.



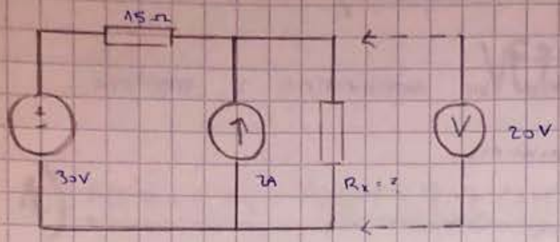


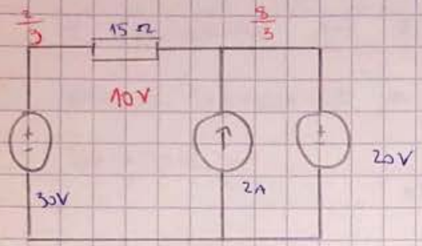
$$\begin{bmatrix} \frac{1}{20} + \frac{1}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{1}{40} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0,3 \\ 10 - 0,3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 12 \\ 388 \end{bmatrix}$$

$\left.\right\} V_A = 32V$, $V_B = 84V$

Zgled





$$R_x = \frac{20V}{\frac{8}{3}} = \underline{7{,}5\ \Omega}$$

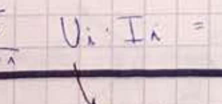
3.) TELLEGENOV STAVEK

Istosmiselno označevanje:

* bremena:

vrišemo → en način

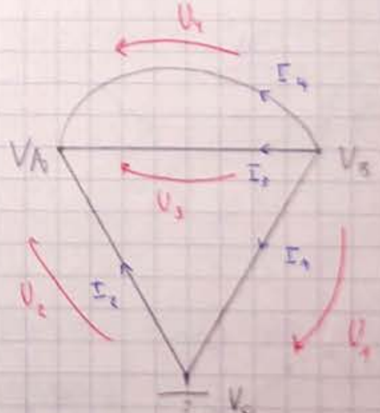
* viri



Vsota istosmiselnih produktov je 0:

$$\boxed{\sum_{i=1}^{n} U_i \cdot I_i = 0}$$

$\rightarrow V_j^{(i)} - V_k^{(i)}$



$$\sum_{j=1}^{n} I_i \left(V_j^{(i)} - V_k^{(i)}\right) = ?$$

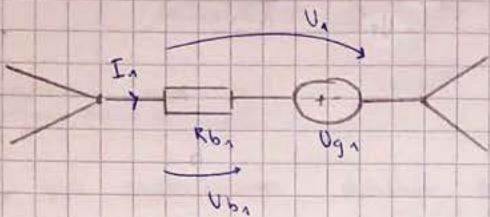
$$\ldots + V_j(+I) + V_k(-I) + \ldots$$

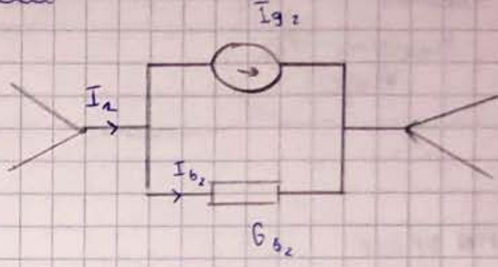
$$I_1 \cdot (V_B) + I_2(-V_A) + I_3(V_B - V_A) + I_4(V_B - V_A) =$$

$$= V_A(\underbrace{-I_2 - I_3 - I_4}_{0}) + V_B(\underbrace{I_1 + I_3 + I_4}_{0}) = 0$$

1. Kir. zakon

Vsebinska osvetlitev Tellegenovega stavka

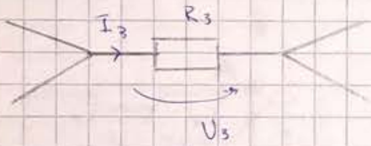




$$U_1 I_1 = U_{b1} I_1 + U_{g1} I_1$$

$$\underbrace{U_1 I_1}_{P_{v1}} = \underbrace{U_{b1} I_1}_{P_{b1}} - \underbrace{(-U_{g1} I_1)}_{P_{g1}}$$

$$U_2 I_2 = U_2 I_{b2} - (-U_2 I_{g2})$$



$U_3 \cdot I_3$

Ugotovili smo, da je moč:

* BREMENA: $P > 0$ pozitivna
* VIRI: $P < 0$ negativna

$U_1 = 84\,V$	$I_1 = -1,6\,A$	→	$P_{v1} = -134,4\,W$
$U_2 = -32\,V$	$I_2 = -1,6\,A$	→	$P_{v2} = 51,2\,W$
$U_3 = 52\,V$	$I_3 = 1,3\,A$	→	$P_{v3} = 67,6\,W$
$U_4 = 52\,V$	$I_4 = 0,3\,A$	→	$P_{v4} = 15,6\,W$

MOČI VEJ

$P = 0\,W$

$G_1 U_1^2 = 705,6\,W$

$R_2 I_2^2 = 51,2\,W$

$R_3 I_3^2 = 67,6\,W$

$R_4 I_4^2 = 5,6\,W$

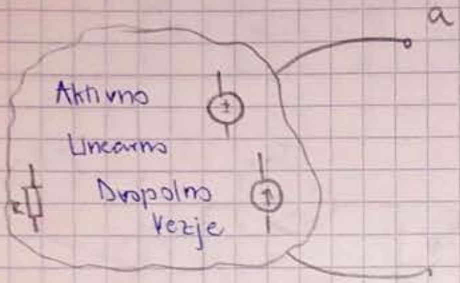
$P_{g1} = -840\,W$ DAJE MOČ

$P_{g4} = 12\,W$ PREJEMA MOČ

MOČI BREMEN

4.) Stavek THEVENINA IN NORTONA

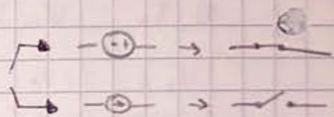
Napetostni vir — Tokovni vir

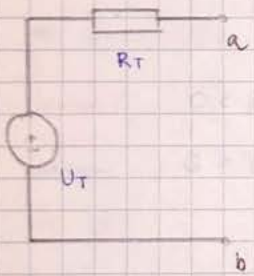


spodnja dva vezja sta ekvivalentna temu

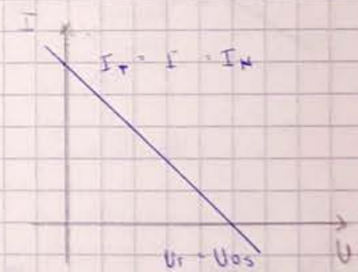
THEVENIN:

* U_T - napetost odprtih sponk ALDV
* R_T - nadomestna notr. upornost LDV, vire DEAKTIVIRAMO
* Vsako ALDV lahko nadomestimo z REALNIM NAPETOSTNIM VIROM



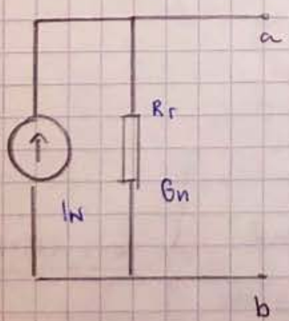


$$\boxed{U_T = R_T \cdot I_N}$$



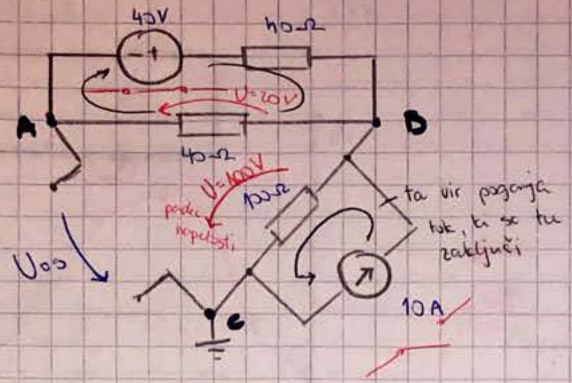
NORTON

* I_N - tok kratkega stika ALDV
* G_N - nadomestna prevodnost LDV, vire DEAKTIVIRAMO
* Vsako ALDV lahko nadomestimo z REALNIM TOKOVNIM VIROM



$$\boxed{G_N = \frac{1}{R_T}}$$

Izgled



* Kaj bo lažje dobiti?

- nadomestno upornost?
- napetost med sponkami?

Theveninov nad. vir:

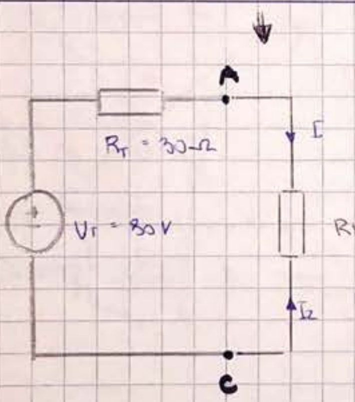
I. Vire deaktiviramo, dobimo R_T

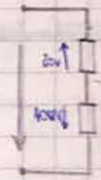
$$40 \| 40 = \frac{40 \cdot 40}{40+40}$$

$$R_T = R_{nad} = 40 \| 40 + 10 = \underline{30\,\Omega}$$

II. Dobimo napetost odprtih sponk: Theveninova napetost

$$U_T = U_{os} = 100 - 20 = 80\,V$$





* vežemo breme

$$R_b = R = 20\,\Omega$$

$$(I_2) = I = \frac{80}{50} = \underline{1,6\,A}$$

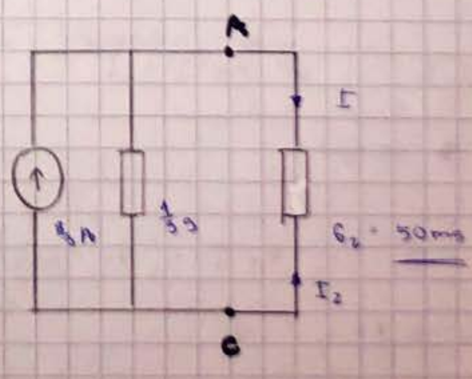
$$I_2 = \underline{-1,6\,A}$$

Kako po najlažji poti do Nortonovega?

Nortonov nad. vir:

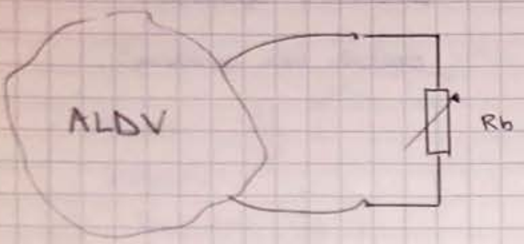
$$G_N = \frac{1}{R_N} = \underline{\frac{1}{30}}\,S$$

$$I_N = \frac{U_T}{R_T} = \underline{\frac{8}{3}}\,A$$

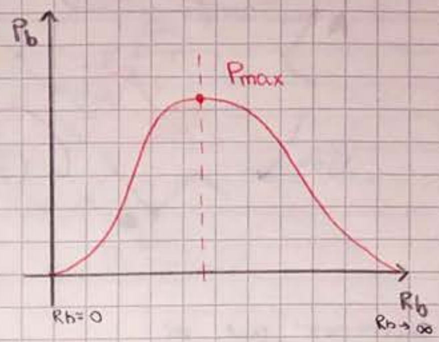


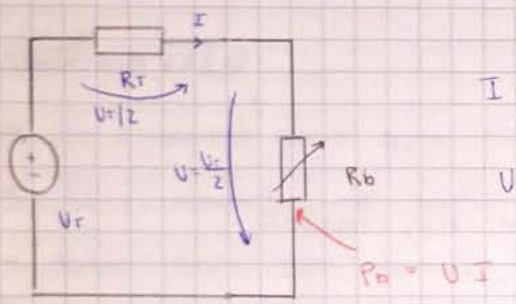
5.) STAVEK MAKSIMALNE MOČI

* Na ALDV bomo priključili breme



⇒ graf moči





$$I = \frac{U_T}{R_T + R_b}$$

$$U = U_T - R_T I$$

Če spravimo upornost na 0:

$R_b \to 0$: $U = 0$, $I = I_k$ → $P_b = 0$

Če spravimo upornost proti ∞:

$R_b \to \infty$: $U = U_{os}$, $I = 0$ → $P_b = 0$

torej pri grafu dobimo vmes nek EKSTREM

PRILAGOJENO BREME - breme, ki bo imelo max moč

$$P_b = U I = (U_T - R_T I) I$$

↓ odvajamo

$$\frac{dP_b}{I} = 0 = -R_T I + (U_T - R_T I)$$

$$2 R_T I = U_T$$

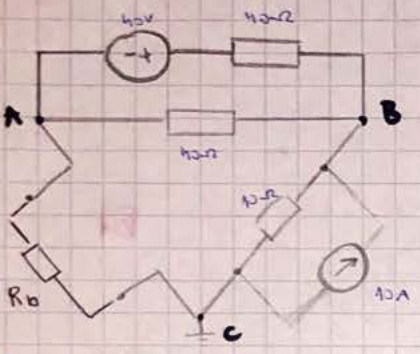
$I = \frac{U_T}{2 R_T}$ ⇒ primerjamo $I = \frac{U_T}{R_T + R_b}$ ⇒ zaključek

Če želimo P_{max}: $\boxed{R_b = R_T}$ PRILAGOJENO BREME

$$P_b = \frac{U^2}{R_b} = \frac{\left(\frac{U_T}{2}\right)^2}{R_T} = \frac{U_T^2}{4R_b}$$

$$\boxed{P_{bmax} = \frac{U_T^2}{4R_b}}$$

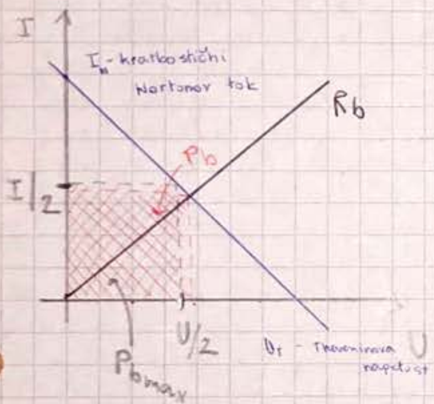
Zgled



priključimo prilagojeno breme

P_{bmax}: $R_b = 30\,\Omega$

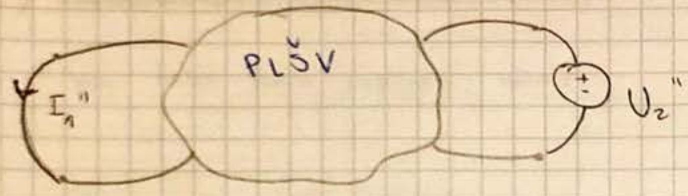
$$P_{bmax} = \frac{80^2}{4\cdot 30} = 53,3\,W$$



* Izkaže se, da je P_{max}, ko sta I_N in U_T za pol manjša.

6.) RECIPROČNOST

* Imamo Pasivno, Linearno, Štiripolno (dvovhodno) vezje



Velja: ZA NAP gen

$$\boxed{\frac{U_1'}{I_2'} = \frac{U_2''}{I_1''}}$$

Velja: ZA TOK gen

$$\frac{U_2'}{I_1'} = \frac{U_1''}{I_2''}$$

IZPITI / KOLOKVIJI