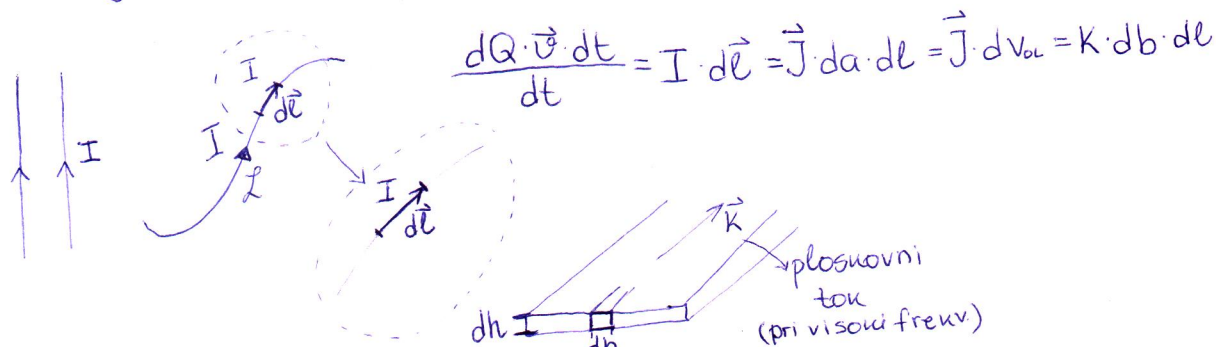


Magnetostatika

El. statika $\rightarrow dQ$; $\overbrace{q, \epsilon, \rho}^{\text{porazdelitve}} \rightarrow q dl = \epsilon dA = \rho dV_{\text{vol}}$

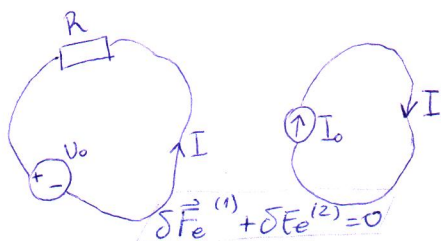
tokovni element

potrebujemo "ekvivalente" v magnetiki: $dQ \cdot \vec{v} = \vec{I} \cdot d\vec{l} = \vec{K} \cdot d\vec{a} = \vec{J} \cdot dV_{\text{vol}}$



"košček toka" = tokovni element

Amperova sila



$$d\vec{F}_e = \frac{dQ_1 \cdot dQ_2}{4\pi\epsilon_0 R^2} \cdot \vec{e}_R$$

- sil vzajemni

- sorazmerni z noski naboja

- obratnosorazmerna z R^2

$$d\vec{F}_e = \frac{dQ_1 \cdot dQ_2}{4\pi\epsilon_0 R^3} \cdot \vec{R}$$

skalarni produkt

Sile 1. vodnika na 2. vodnik:

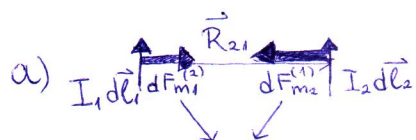
$$d\vec{F}_{m2}^{(1)} = \frac{\mu_0}{4\pi R_{21}^3} I_2 d\vec{l}_2 \times (I_1 d\vec{l}_1 \times \vec{R}_{21})$$

dvojni vektorski produkt

$$d\vec{F}_{m1}^{(2)} = \frac{\mu_0}{4\pi R_{12}^3} I_1 d\vec{l}_1 \times (I_2 d\vec{l}_2 \times \vec{R}_{12})$$

$\rightarrow$ obratnosorazmerna s kvadratom oddaljenosti tokovnih elementov
 $\rightarrow$ sorazmerna s produktom jakosti tokov

$\rightarrow$ razlika je v smeri!

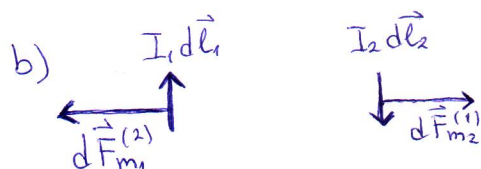


Za vzporedna toka:
sile privlačni

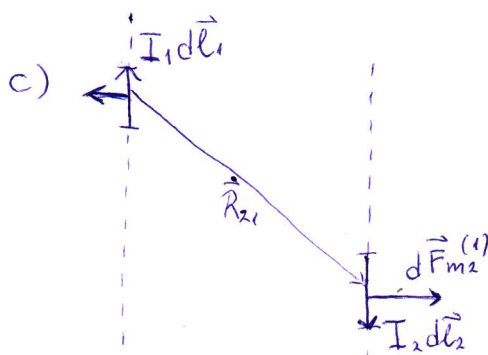
v tablo

$$(I_1 d\vec{l}_1 \times \vec{R}_{21}) \times I_2 d\vec{l}_2$$

$$(I_2 d\vec{l}_2 \times \vec{R}_{12}) \times I_1 d\vec{l}_1$$

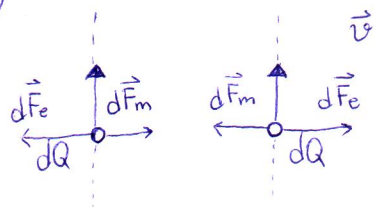


za zaporedna toka:
sile odbojni



Primerjava Coulombove in Amperove sile:

1)

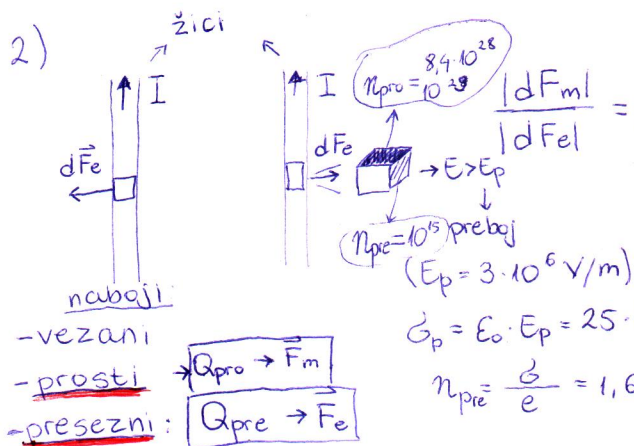


$$\frac{|dF_m|}{|dF_e|} = \frac{\frac{\mu_0}{4\pi R^2} (dQ \cdot v)^2}{\frac{dQ^2}{4\pi \epsilon_0 R^2}} = \frac{v^2}{c^2} = 10^{-25}$$

$\mu_0 \cdot \epsilon_0 = \frac{1}{c^2} \rightarrow$ svetlobna hitrost

*Magnetna sila je 10^{25} krat manjša od el. sile.

2)



$$\frac{|dF_m|}{|dF_e|} = \frac{v^2 \cdot dQ_{pro}^2}{c^2 \cdot dQ_{pre}^2} = 10^{-25} \cdot (10^{14})^2 = 10^3$$

magnetna sila je 10^3 večja

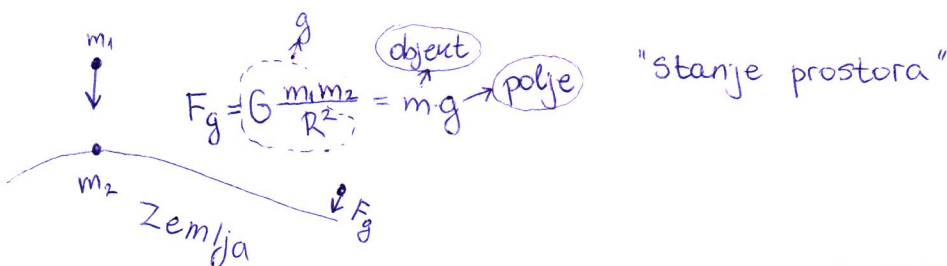
$$E_p = 3 \cdot 10^6 \text{ V/m}$$

$$G_p = \epsilon_0 \cdot E_p = 25 \cdot 10^{-6} \text{ C/m}$$

$$n_{pre} = \frac{G}{e} = 1,6 \cdot 10^{14}$$

p3 (1.3.2016)

Biot - Savartov zakon



elektrostatika

$$\vec{F}_e = \frac{dQ_1 dQ_2}{4\pi \epsilon_0 R^2} \cdot \vec{R} ; \vec{F}_e = \vec{E} \cdot dQ \rightarrow \text{objekt}$$

$$\vec{E} = \frac{\vec{F}}{Q}$$



$$d\vec{F}_m = I d\vec{l} \times \vec{B} = dQ \cdot \vec{v} \times \vec{B} \quad \left\{ \begin{array}{l} \text{objekt} \\ \text{tok. elem.} \end{array} \right. \quad \left\{ \begin{array}{l} \text{polje} \\ \text{mag.} \end{array} \right.$$

$$d\vec{F}_m = \underbrace{I \cdot d\vec{l}}_{\text{objekt}} \times \underbrace{(I' \cdot d\vec{l}' \times \vec{R})}_{\text{povzročitelj polja}} \cdot \frac{\mu_0}{4\pi R^3}$$

$$d\vec{B} = \frac{\mu_0}{4\pi R^3} (I \cdot d\vec{l} \times \vec{R}) \quad \text{B. S. zakon}$$

Vpelje mag. polje za tokovni element

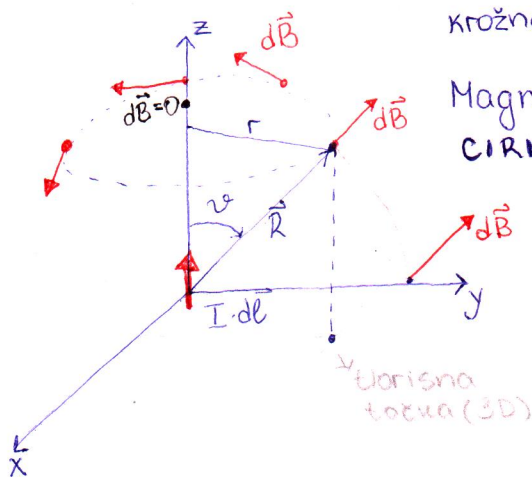


$$B(T) = \int d\vec{B}$$

Lorentzova sila

$$\vec{F}_L = \vec{F}_e + \vec{F}_m = dQ \vec{E} + dQ \cdot \vec{v} \times \vec{B} = dQ (\vec{E} + \vec{v} \times \vec{B})$$

Magnetno polje tokovnega elementa



krožna smer

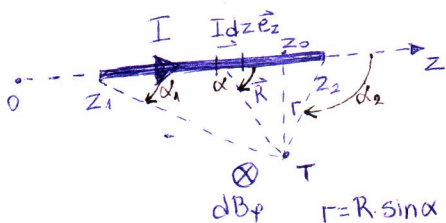
Magnetno polje je
CIRKULARNO

$$\vec{e}_{de} \times \vec{e}_R = \sin \vartheta$$

$$d\vec{B} = \mu_0 \frac{I d\vec{l}}{4\pi R^2} \sin \vartheta$$

$$\left\{ \begin{array}{l} d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \vec{e}_R \\ \text{El polje je RADIALNO} \end{array} \right.$$

Magnetno polje daljice



$$d\vec{B} = \mu_0 \frac{I d\vec{l} \times \vec{R}}{4\pi R^3}$$

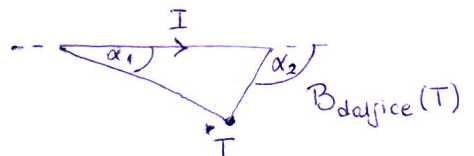
$$dB = \frac{\mu_0 I dz \sin \alpha}{4\pi R^2}$$

$$r = R \sin \alpha \rightarrow R = \frac{r}{\sin \alpha}$$

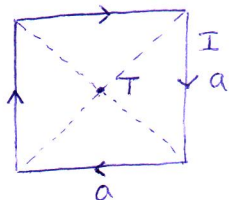
$$\tan \alpha = \frac{r}{z_0 - z} \rightarrow z_0 - z = r \cdot \cot \alpha \quad \left| \frac{dz}{d\alpha} \right| \quad ; \quad dz = r \cdot \frac{1}{\sin^2 \alpha} \cdot d\alpha$$

$$B_{daljice} = \int_{z_1}^{z_2} dB_p = \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{\sin \alpha}{R^2} dz = \frac{\mu_0 I}{4\pi r^2} \int_{\alpha_1}^{\alpha_2} \frac{\sin^3 \alpha \cdot \sin \alpha}{\sin^2 \alpha} d\alpha \Rightarrow$$

$$\vec{B}_{daljice} = \vec{e}_\varphi \frac{\mu_0 I}{4\pi r} (\cos \alpha_1 - \cos \alpha_2)$$



Magnetno polje zanke



$$I = 100 \text{ A}$$

$$a = 10 \text{ cm} = 0,1 \text{ m}$$

$$B_0 = 4 \cdot B_{\text{daljice}}$$

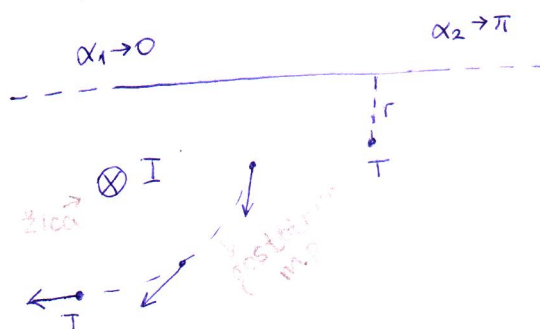
$$\alpha_1 = 45^\circ = \frac{\pi}{4}$$

$$\alpha_2 = 135^\circ = 3 \frac{\pi}{4}$$

$$\Rightarrow B_0 = \frac{\mu_0 \cdot 100}{4\pi \cdot 0,05} \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right)$$

$$B_0 = 8 \cdot \sqrt{2} \cdot 10^{-4} \text{ T} \quad (\text{Tesla})$$

Magnetno polje premice

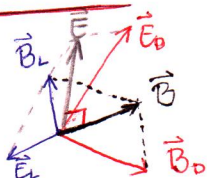


$$\vec{B}_{\text{premice}} = \vec{e}_\varphi \frac{\mu_0 I}{2\pi r}$$

↓
cirkularna smer

$$\left\{ \vec{E}_p = \frac{q \cdot \vec{e}_r}{2\pi \epsilon_0 r} \right\} \begin{matrix} \text{radialna} \\ \text{smer} \end{matrix}$$

Magnetno polje dnovoda



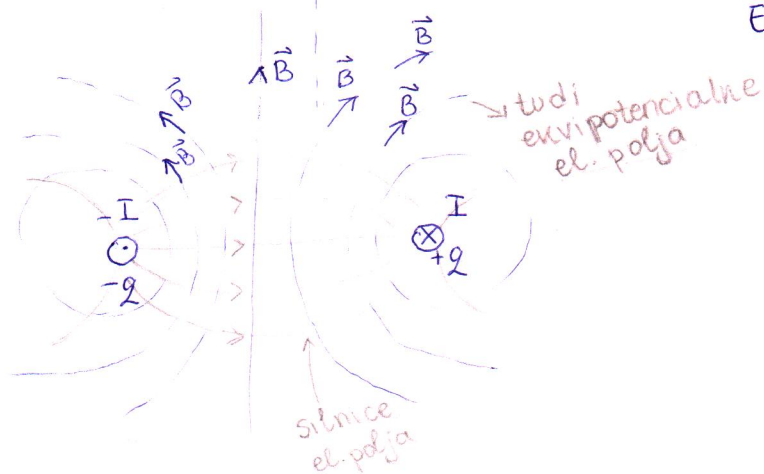
$$\vec{E}_L \perp \vec{B}_L$$

$$\vec{E}_D \perp \vec{B}_D$$

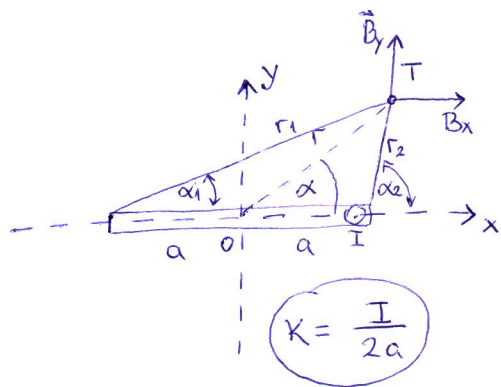
$$\vec{E} \perp \vec{B}$$

Envipotencialne $\perp \vec{E}$

Envipoten cialne $\parallel \vec{B}$

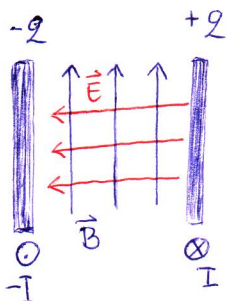
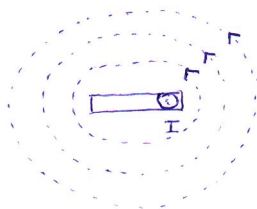


Magnetno polje tračnega vodnika



$$B_x(T) = \frac{\mu_0 K}{2\pi} (\alpha_1 - \alpha_2)$$

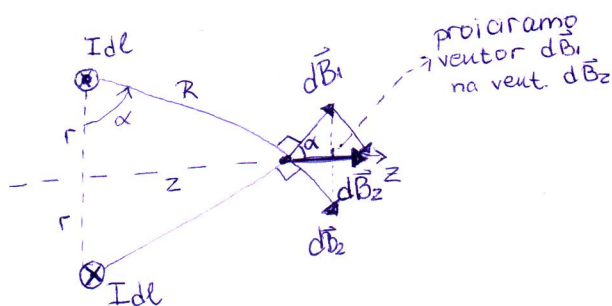
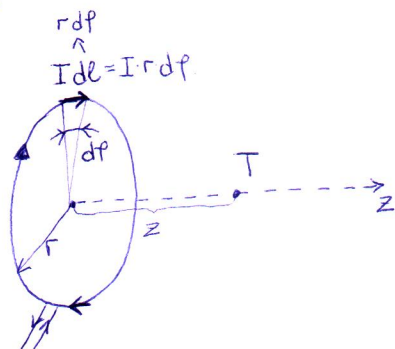
$$B_y(T) = \frac{\mu_0 K}{2\pi} \ln \frac{r_1}{r_2}$$



$$B = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K$$

p4(4.3.2016)

Magnetno polje (v osi) krožnega ovojja

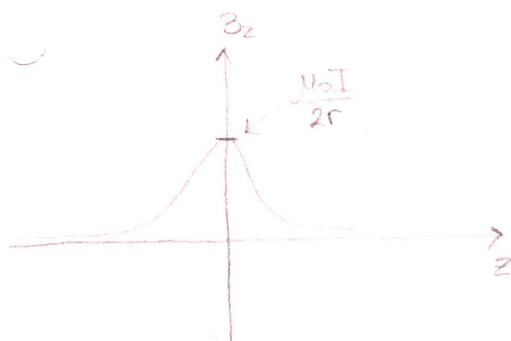


$$dB_z = 2 \cdot dB_{1z}$$

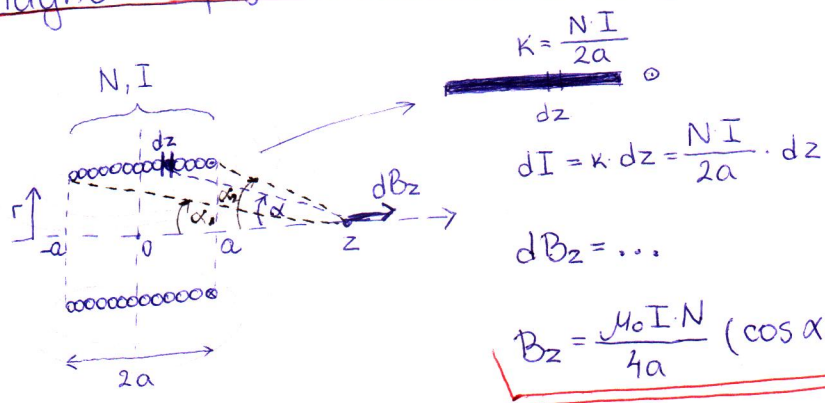
$$dB_{1z} = \underbrace{\frac{\mu_0 I dl}{4\pi R^2}}_{\text{B.S.z.}} \cdot \cos \alpha = \frac{\mu_0 I}{4\pi} \cdot \frac{r d\phi}{(r^2 + z^2)^{3/2}}$$

$$\underline{\underline{B_z = \int dB_z = \frac{\mu_0 I r^2}{2\pi (r^2 + z^2)^{3/2}} \int_0^\pi d\phi = \frac{\mu_0 I r^2}{2 (r^2 + z^2)^{3/2}}}}$$

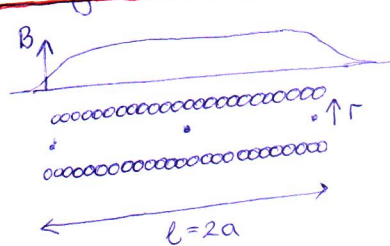
↑
magnetno polje v osi
krožnega ovojja



Magnetno polje v osi tanke tuljave



Dolga tanca tuljava



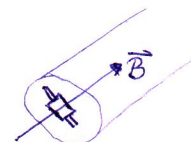
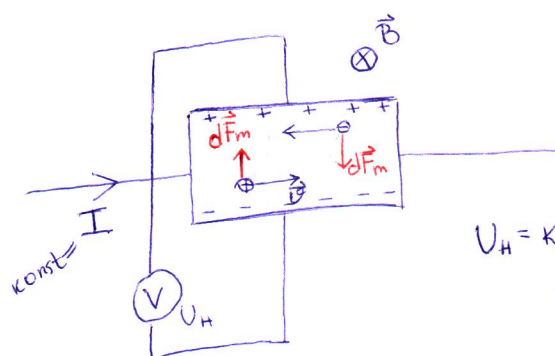
$$B_{\text{bredišče}} \Big|_{\alpha_1 \rightarrow 0, \alpha_2 \rightarrow \pi} = \frac{\mu_0 I N}{2a} = \frac{\mu_0 I N}{l}$$

$$B_{\text{robna}} \Big|_{\alpha_1 \rightarrow 0, \alpha_2 \rightarrow \pi/2} = \frac{\mu_0 I N}{4a} = \frac{1}{2} B_{\text{bredišča}} = \frac{\mu_0 I N}{2l}$$

Hallov pojav in merilnik mag. polja - Teslameter

* "pravilo leve roke"

$$\begin{aligned} d\vec{F}_m &= I d\vec{l} \times \vec{B} \\ &= dQ \vec{v} \times \vec{B} \end{aligned}$$



Magnetni pretok

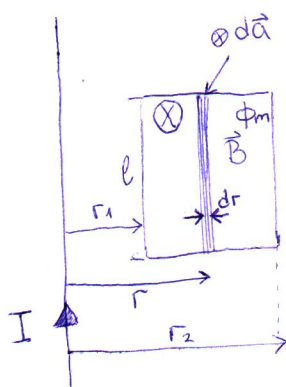
OE1: $\Phi_e = \int \vec{D} \cdot d\vec{a} \rightarrow \oint \vec{D} \cdot d\vec{a} = Q_{\text{notr. prosti}}$

$\downarrow$ električni pretok
 $\downarrow$ gostota el. pretoka



$\Phi_m = \int \vec{B} \cdot d\vec{a}$

$\downarrow$ vektor
 $\downarrow$ vektor gostote magnetnega pretoka

Magnetni pretok ravnega vodnika skozi pravokotno zanko

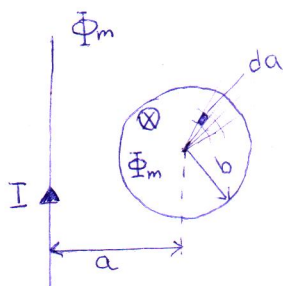
$$B_r = \frac{\mu_0 I}{2\pi r}$$

$$\Phi_m = \int \vec{B} \cdot d\vec{a} = \int B_r \cdot da = \int_{r_1}^{r_2} B_r \cdot l \cdot dr$$

sk. prod.
 $B_r \parallel da$
 $\cos 0^\circ = 1$



$$\Rightarrow \Phi_m = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$$

Magnetni pretok ravnega vodnika skozi okroglo zanko

$$\Phi_m = \mu_0 I (a - \sqrt{a^2 - b^2})$$

Magnetni pretok dvovoda in pretočne cevke

gostotnice mag. polja $\equiv$ ekvipotencialne $\perp$ silnice $\frac{1}{E}$

$$\Phi_m = \frac{\mu_0 I l}{2\pi} \left(\ln \frac{r_0}{r_1} + \ln \frac{r_2}{r_0} \right) = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$$

pretočne
cevke
mag. pretoka

Značilnosti magnetnega pretoka

1) Izvornost $\vec{B}$ (Gaussov stavek)

$$\oint_A \vec{J} \cdot d\vec{a} = - \frac{dQ_{\text{not}}}{dt}$$

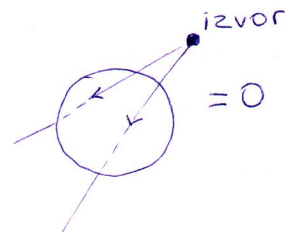
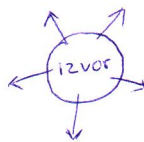
$$\oint_A \vec{E} \cdot d\vec{a} = + \frac{Q_{\text{not}}}{\epsilon_0}$$

$$\oint_A \vec{D} \cdot d\vec{a} = + Q_{\text{not. prosti}}$$

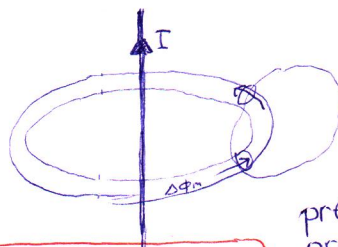
$$\oint_A \vec{P} \cdot d\vec{a} = - Q_{\text{not. polarizacijski}}$$

$$\oint_A \vec{B} \cdot d\vec{a} = 0 \quad \nearrow \quad \text{III Maxwellova enačba}$$

$$? = \oint_A \vec{x} \cdot d\vec{a} \neq 0$$



$$\oint_A \vec{B} \cdot d\vec{a} = ?$$

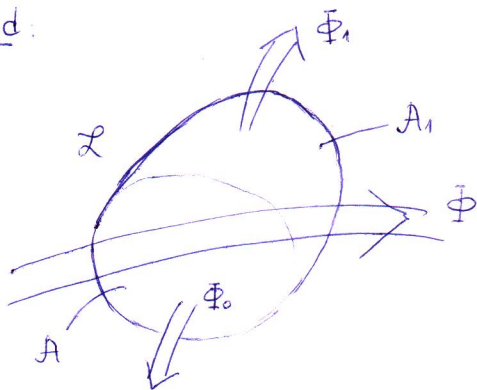


pretočna cevka prebada dvakrat

$$\oint_A \vec{B} \cdot d\vec{a} = \sum_A \int_{\Delta\Phi_m} \vec{B} \cdot d\vec{a} = 0$$

Magnetno polje ni izvorno!

Zgled:



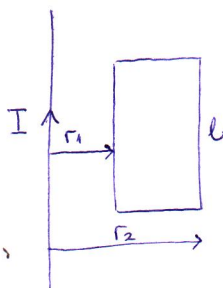
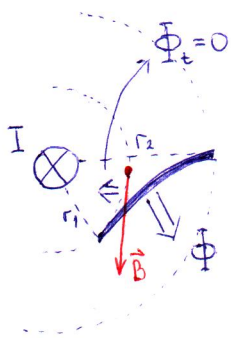
$$\Phi = \int_{A_1} \vec{B} \cdot d\vec{a}$$

$$\Phi = - \int_A \vec{B} \cdot d\vec{a} \quad / \cdot (-1)$$

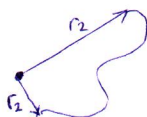
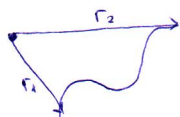
$$\int_{A_1} \vec{B} \cdot d\vec{a} + \int_A \vec{B} \cdot d\vec{a} = 0 \Rightarrow \Phi_1 = -\Phi_0$$

$$\Rightarrow \Phi_0 = -\Phi_1$$

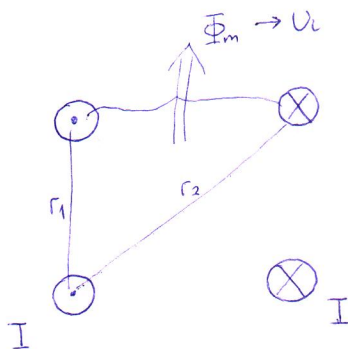
Φ_m skozi "zvitugan trak" v okolici premega vodnika



$$\Phi_m = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$$



Zgled: DVA DVOVODA



$$\Phi_m = \Phi_{mL} + \Phi_{mD} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1} \cdot 2$$

$$\Phi_m = \frac{\mu_0 I l}{\pi} \ln \frac{r_2}{r_1}$$

*recipročnost

*spodnji dvovod na zgornjega (in obratno)

P6 (11.3.2016)

Vrtinčnost vektorja magnetnega pretoka

Značilnosti $\vec{B}$ → Gaussov stavek

-(Ne)izvornost $\vec{B}$: $\oint_A \vec{B} \cdot d\vec{a} = 0 \rightarrow$ računanje Φ_m

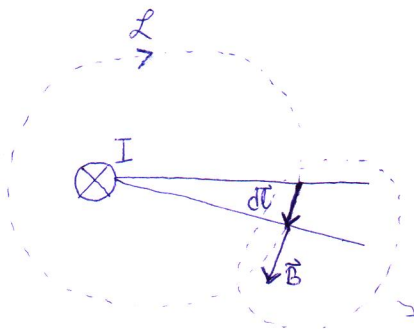
-Vrtinčnost $\vec{B}$

↓
Stokesov stavek: $\oint_L \vec{X} \cdot d\vec{l} = ?$

$\oint_L \vec{E} \cdot d\vec{l} = 0$ (elektrostatično polje je nevrtinčno)

$$\oint_L \vec{B} \cdot d\vec{l} = ?$$

1) Vrtinčnost $\vec{B}$ premega toka

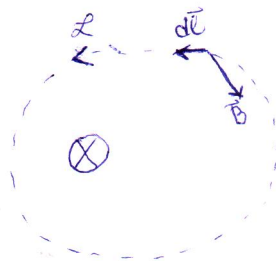


$$\oint_L \vec{B} \cdot d\vec{l} = \oint_L \frac{\mu_0 I}{2\pi a} \cdot dl_B = \oint_L \frac{\mu_0 I}{2\pi a} a d\varphi = \frac{\mu_0 I}{2\pi} \oint_L \frac{d\varphi}{2\pi} = \underline{\underline{\mu_0 I}}$$

projekcija $d\vec{l}$ na $\vec{B}$:

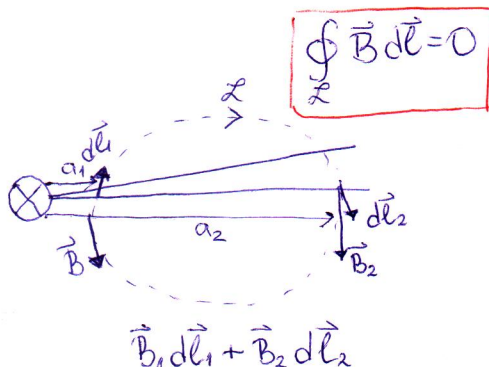
dl_B - dolžini loka: $a \cdot d\varphi$

2)



$$\oint_L \vec{B} \cdot d\vec{l} = -\mu_0 I$$

3)

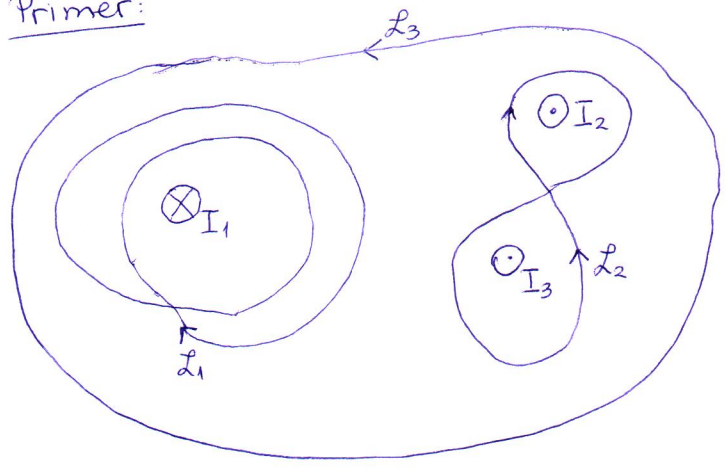


$$-\frac{\mu_0 I}{2\pi a_1} a_1 d\varphi + \frac{\mu_0 I}{2\pi a_2} a_2 d\varphi = 0$$

$$\oint_L \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{zaobjeti}}$$

$= \mu_0 N \cdot I$
 $\uparrow$
 št. obojev

Primer:

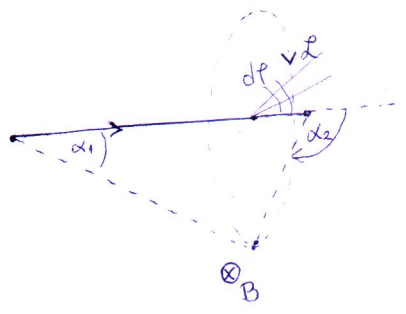


$$\oint_{L_1} \vec{B} \cdot d\vec{\ell} = 2\mu_0 I_1$$

$$\oint_{L_2} \vec{B} \cdot d\vec{\ell} = \mu_0 (I_3 - I_2)$$

$$\oint_{L_3} \vec{B} \cdot d\vec{\ell} = \mu_0 (I_2 + I_3 - I_1)$$

Vrtinčnost $\vec{B}$ tokovne daljice

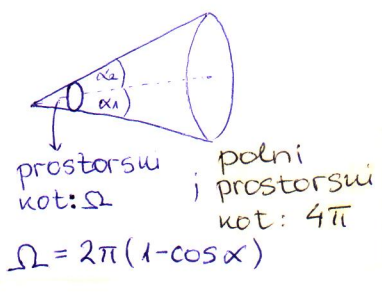


$$B_r = \frac{\mu_0 I}{4\pi r} (\cos \alpha_1 - \cos \alpha_2)$$

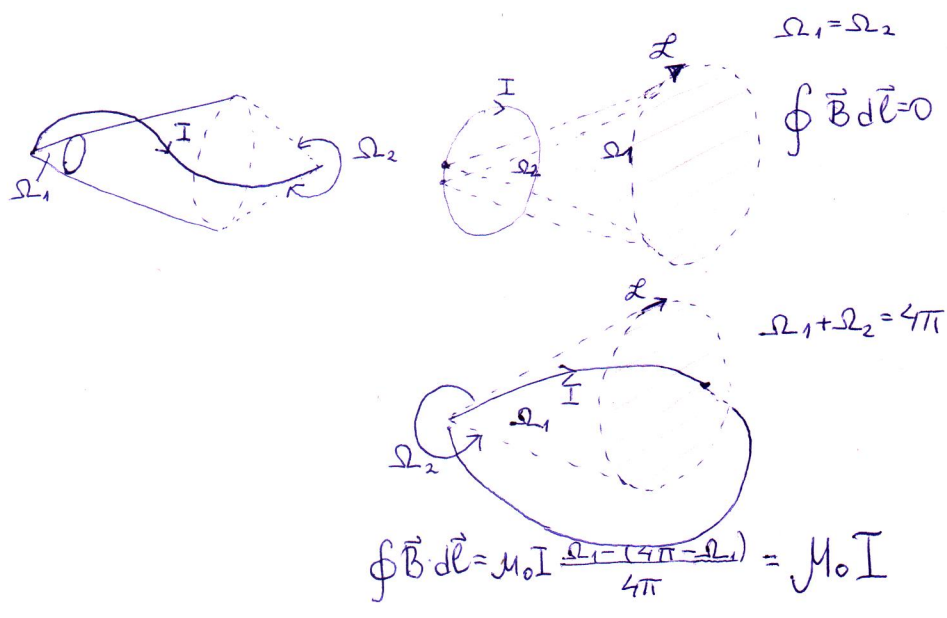
$$\oint_{r \cdot dl} B_r \cdot dl = \frac{\mu_0 I}{4\pi} (\cos \alpha_1 - \cos \alpha_2) \oint_L \frac{dl}{r}$$

$$\oint B_r dl = \frac{\mu_0 I}{2} (\cos \alpha_1 - \cos \alpha_2) / \frac{4\pi}{4\pi} = \frac{\mu_0 I}{2} (1 - \cos \alpha_2 - 1 + \cos \alpha_1) = \frac{\mu_0 I}{2} (\Omega_2 - \Omega_1)$$

$$\oint_L \vec{B} \cdot d\vec{\ell} = \mu_0 I \frac{\Omega_2 - \Omega_1}{4\pi}$$



Posplošitev vrtinčnosti $\vec{B}$

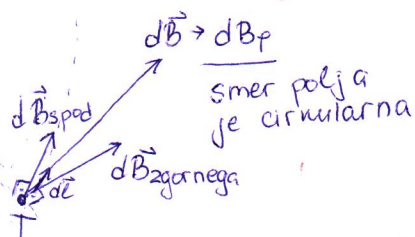
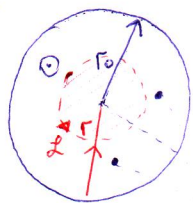


$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{zaobjeti}}$$

- premi vodniku
- vse suljenjene vodniki

$\vec{B}$ "debelega" ravnega vodnika urožnega preseka

I (znotraj so 2 žice)



zunajnost: $\oint_L \vec{B}_T \cdot d\vec{l} = \mu_0 I$

$$B_T \cdot 2\pi r = \mu_0 I$$

$$B_T = \frac{\mu_0 I}{2\pi r} \quad \Rightarrow B \text{ premice}$$

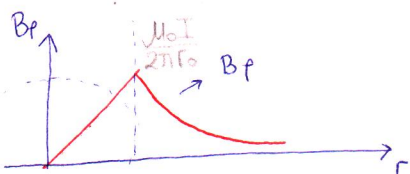
notrajnost: $\int \vec{B}_T \cdot d\vec{l} = \mu_0 I_{\text{zaobjeti}} = \mu_0 \frac{I}{\pi r_0^2} \pi r^2$

$$\int B_T \cdot dl = \mu_0 \frac{I r^2}{r_0^2} \quad \text{zaobjeti del tona}$$

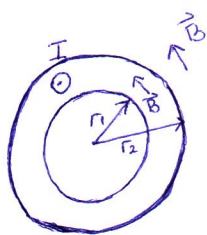
$$B_T \cdot 2\pi r = \frac{\mu_0 I r^2}{r_0^2}$$

$$B_T = \frac{\mu_0 I r}{2\pi r_0^2}$$

$$B_T = \begin{cases} \frac{\mu_0 I}{2\pi r} & , r \geq r_0 \\ \frac{\mu_0 I r}{2\pi r_0^2} & , r \leq r_0 \end{cases}$$

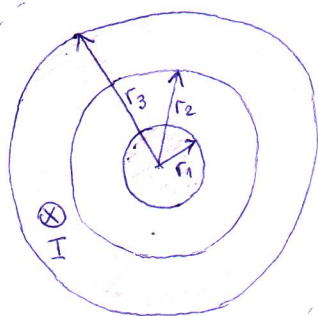


$\vec{B}$ "mavaroni"



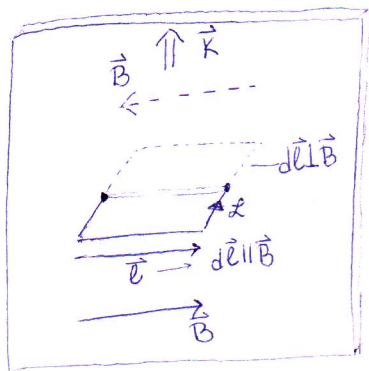
$$B = \begin{cases} 0 & , r < r_1 \\ \mu_0 I \frac{r^2 - r_1^2}{r_2^2 - r_1^2} & , r_1 < r < r_2 \\ \frac{\mu_0 I}{2\pi r} & , r_2 < r \end{cases}$$

$\vec{B}$ koaksialnega kabla



$$B_T = \begin{cases} \frac{\mu_0 I r}{2\pi r_1^2} & , r < r_1 \\ \frac{\mu_0 I}{2\pi r} & , r_1 < r < r_2 \\ \dots & , r_2 < r < r_3 \rightarrow \frac{\mu_0 I}{2\pi(r_3 - r_2)} \left(\frac{r_3^2}{r} - r \right) \\ 0 & , r_3 < r \end{cases}$$

$\vec{B}$ v ovdici stene s ploskovnim tokom - folijski tok



$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{zaobjeti}}$$

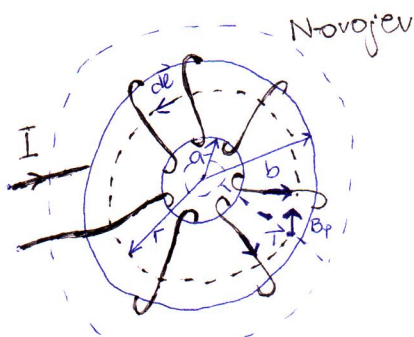
$$= B \cdot l + 0 + B \cdot l + 0 = K \cdot l$$

$$\boxed{\vec{B} = \frac{\mu_0 \vec{K}}{2}} \quad (E = \frac{\phi}{2\epsilon_0})$$

P7 (15.3.2016)

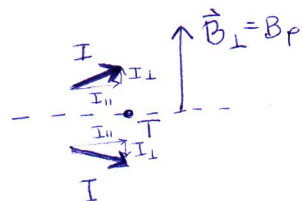
$$\boxed{\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{zaobjeti}}} \quad \text{Amperov zakon (Zakon o vrtinčnosti)}$$

$\vec{B}$ toroidnega navitja



$$\underline{B} = \begin{cases} 0 & , \quad r < a \\ \frac{\mu_0 N I}{2\pi r} & , \quad a < r < b \\ 0 & , \quad b < r \end{cases}$$

$$B \cdot \oint dl = 0$$

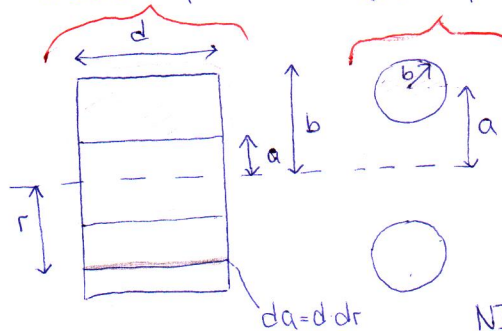


$$\oint_L \vec{B} \cdot d\vec{l} = B_\phi \int dl = B_\phi \cdot 2\pi r = \mu_0 N I$$

Preseči toroida, magnetni pretok toroida

uvadratni presek

krožni presek

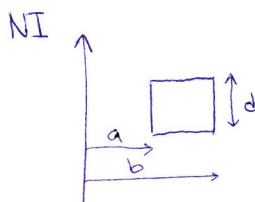


$$B_\phi 2\pi r = \mu_0 N I$$

$$\Phi_{m\Box} = \int \vec{B} \cdot d\vec{a} = \frac{\mu_0 N I d}{2\pi} \int_a^b \frac{1}{r} dr$$

$$\boxed{\Phi_{m\Box} = \frac{\mu_0 N I d}{2\pi} \ln \frac{b}{a}}$$

magnetni pretok toroida (uvadratni presek)



$$\boxed{L_{\Box} = \frac{\mu_0 N^2 d}{2\pi} \ln \frac{b}{a}}$$

Induktivnost toroida ($\Box$)

$$\Phi_{mo} = \mu_0 N I (a - \sqrt{a^2 - b^2})$$

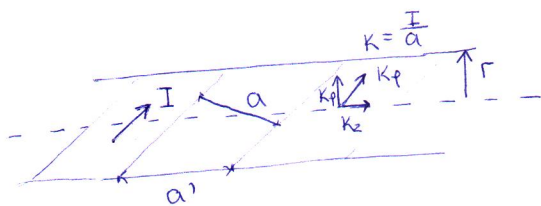
→ magnetni pretok toroida (krožnega preseka)

$$L = N \frac{\Phi}{I}$$

$$L_0 = \mu_0 N^2 (a - \sqrt{a^2 - b^2})$$

→ induktivnost toroida (0)

Spiralno navit ovlop

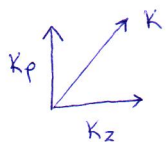


a - širina traku

a' - korak navijanja

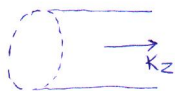
r - polmer valja

$$(a')^2 + (2\pi r)^2 = a^2$$



- dekompozicija toka

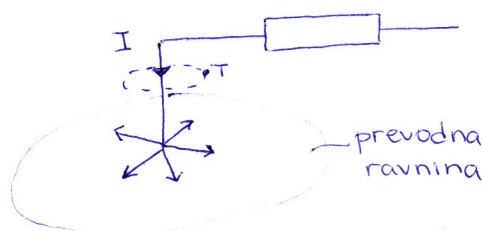
- ① K_z - povzroča B kot "tanja cev"



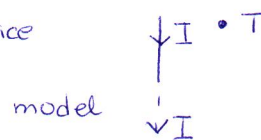
- ② K_p - povzroča B kot solenoid



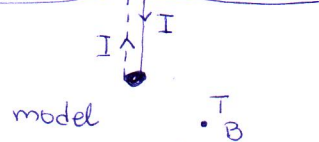
Radialni tok



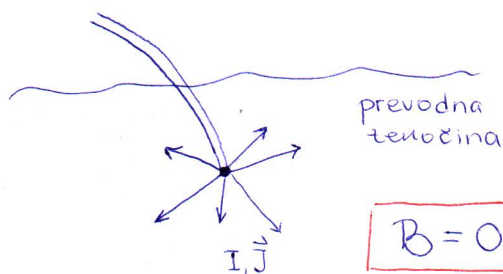
zgoraj: B premice



spodaj



Sferični tok



$$B = 0$$

Gibanje delca v $\vec{E}$ in $\vec{B}$

$t=0$ $\vec{v}_0$ $\xrightarrow{\Delta t}$ $t=t_1$ $\vec{v}_1$
 $T_0, \vec{r}_0, \vec{v}_0$ $T_1, \vec{r}_1, \vec{v}_1$

$$\vec{a} = \dot{\vec{v}} = \frac{\vec{v}_1 - \vec{v}_0}{\Delta t}$$

$$\vec{v}_1 = \vec{v}_0 + \dot{\vec{v}} \Delta t$$

$$\vec{r}_1 = \vec{r}_0 + \vec{v}_0 \Delta t + \frac{1}{2} \dot{\vec{v}} \Delta t^2$$

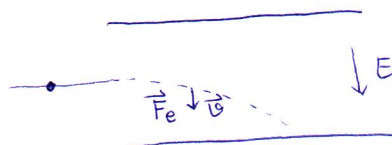
$$\dot{\vec{v}} = \vec{a} = \frac{\vec{F}_L}{dm}; \quad \vec{F}_L = dQ(\vec{E} + \vec{v} \times \vec{B})$$

Gibanje delca v $\vec{E}$

$$\dot{\vec{v}} = \frac{dQ}{dm} \vec{E}$$

pospešek je konstanten

pospešek je usmerjen v smeri polja



trajektorija gibanja je parabola

Gibanje delca v $\vec{B}$

$$\dot{\vec{v}} = \frac{dQ}{dm} (\vec{v} \times \vec{B}) / \vec{v}$$

$$\vec{v} \cdot \dot{\vec{v}} = \frac{dQ}{dm} (\vec{v} \times \vec{B}) \cdot \vec{v} = \frac{dQ}{dm} \underbrace{(\vec{v} \times \vec{v})}_0 \cdot \vec{B}$$

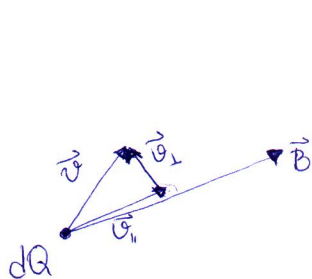
$$*(y^2)' = 2y \cdot y' \quad v \cdot v \cdot \sin \varphi = 0$$

$$\frac{1}{2} (y^2)' = y \cdot y'$$

$$* \frac{1}{2} (\vec{v}^2)' = 0$$

$$(\vec{v}^2)' = 0$$

$$v^2 = \text{konst} \rightarrow \underline{v = \text{konst}}$$



$$\dot{\vec{v}}_{\perp} + \dot{\vec{v}}_{\parallel} = \frac{dQ}{dm} (\vec{v}_{\perp} \times \vec{B}) + \frac{dQ}{dm} \underbrace{(\vec{v}_{\parallel} \times \vec{B})}_0$$

$\downarrow$
 $\perp$ na $\vec{v}_{\perp}$ in $\vec{B}$

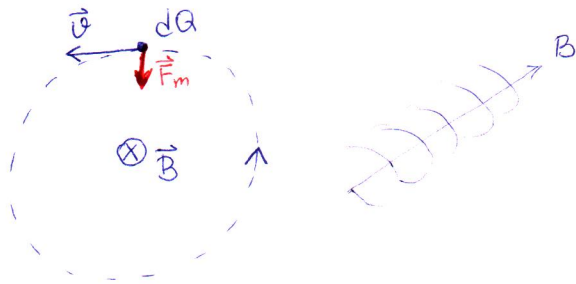
$$\left. \begin{array}{l} v = \text{konst} \\ \dot{\vec{v}}_{\parallel}, \dot{\vec{v}}_{\parallel} = 0 \end{array} \right\} \underline{\dot{\vec{v}}_{\perp} = \text{konst}} \Rightarrow \underline{\text{enakomerno kroženje}}$$

$$\dot{\vec{v}}_{\perp} = a_r = \frac{v_{\perp}^2}{R} = \frac{dQ}{dm} (|\vec{v}_{\perp}| \cdot B)$$

$$R = \frac{dm \cdot v}{dQ \cdot B}$$

Kroženje delca v $\vec{B}$

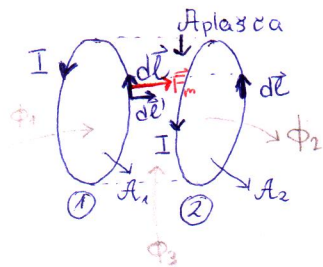
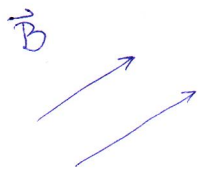
$$T = \frac{2\pi r}{v} = 2\pi \cdot \frac{dm}{dQ \cdot B}$$



P8(18.3.2016) Interakcija magnetnih sil

- med dvema ^{premima} vodnikoma: sili nista vzajemni
- med dvema zankama: sili sta vzajemni

Delo magnetne sile tokovne zanke



$$d\vec{F}_m = I d\vec{l} \times \vec{B}$$

$$dA = \int_L d\vec{F}_m \cdot d\vec{l}' = I \int_L (d\vec{l} \times \vec{B}) \cdot d\vec{l}' = I \int_L \underbrace{(d\vec{l}' \times d\vec{l})}_{d\vec{a}} \cdot \vec{B}$$

(usmerjena v notr. plasca)

$$dA = I \int \vec{B} \cdot d\vec{a}$$

$A_1 \cup A_2 \cup A_{\text{plazca}}$

$$A = \int_{\text{od ① lege}}^{\text{do ② lege}} dA = I \int \vec{B} \cdot d\vec{a} = I \cdot \Phi_{\text{v notranjost skozi plasc}}$$

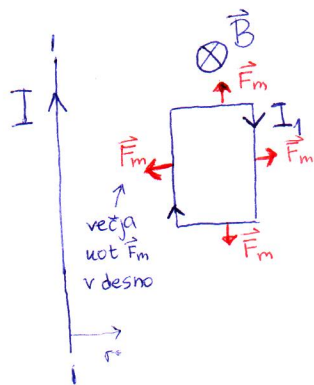
$$\oint_A \vec{B} \cdot d\vec{a} = 0$$

$$A = I \cdot (\Phi_2 - \Phi_1) \quad \left\{ \begin{array}{l} A = U \cdot Q \end{array} \right.$$

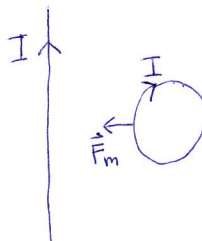
$$-\Phi_1 - \Phi_{\text{v notr. plasca}} + \Phi_2 = 0$$

$$\Phi_3 = \Phi_{\text{v notr. pl.}} = \Phi_2 - \Phi_1$$

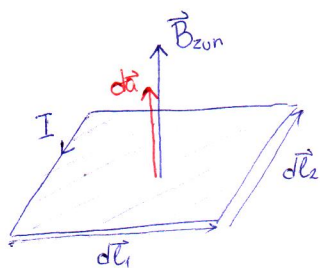
Zgled: Smer premija



$$B = \frac{\mu_0 I}{2\pi r}$$



Navor na tokovno zanko v $\vec{B}$

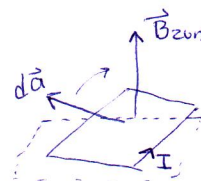


$$d\vec{M} = \vec{r} \times d\vec{F} = -\frac{d\vec{l}_2}{2} \times (I d\vec{l}_1 \times \vec{B}) + \frac{d\vec{l}_1}{2} \times (I d\vec{l}_2 \times \vec{B}) + \frac{d\vec{l}_2}{2} \times (-I d\vec{l}_1 \times \vec{B}) - \frac{d\vec{l}_1}{2} \times (-I d\vec{l}_2 \times \vec{B})$$

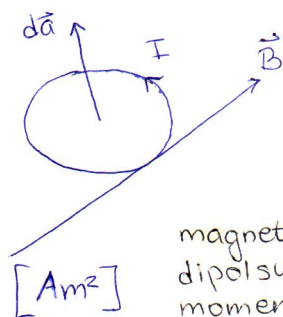
$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{a} \cdot \vec{b}) \cdot \vec{c}$$

$$d\vec{M} = I (\underbrace{d\vec{l}_1 \times d\vec{l}_2}_{d\vec{a}}) \times \vec{B} = I (d\vec{a} \times \vec{B})$$

* Navor zasuka zanko v smeri polja: $\vec{B} \parallel d\vec{a}$:
 $f=0, \sin f=0, M=0$



Magnetni dipol



$$d\vec{m} = I \cdot d\vec{a}$$

$$d\vec{M}_m = I d\vec{a} \times \vec{B}$$

navor na zanko

Elektrostatika

$$d\vec{p} = \vec{r} \cdot dQ$$

$$\vec{M}_e = d\vec{p} \times \vec{E}$$

elektrostat. dipolski moment

Snov v magnetnem polju

$\vec{B}, \vec{M}, \vec{H} \rightarrow$ magnetna poljsna jakost
 $\uparrow$
 magnetizacija

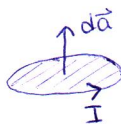
prevodniki: influencia $\vec{E}=0$
 izolanti: polarizacija
 $\chi_e \rightarrow \epsilon_r$ (susceptibilnost)
 $\vec{P}, \vec{D} = \epsilon_0 \epsilon_r \vec{E}$

Vektor magnetizacije

def. polarizacija:
 $\vec{P} = \lim_{dV_{ol} \rightarrow 0} \frac{\sum d\vec{p}}{dV_{ol}}$

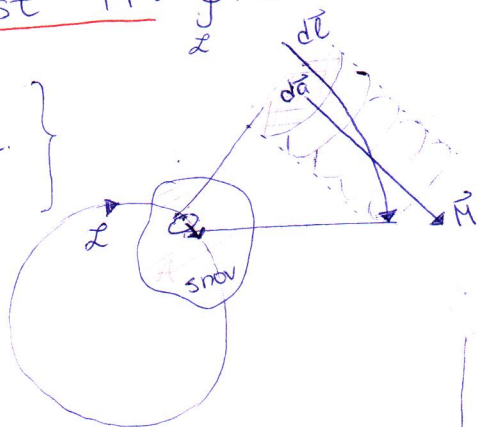


$$\vec{M} = \lim_{dV_{ol} \rightarrow 0} \frac{\sum d\vec{m}}{dV_{ol}} \quad [A/m]$$



Vrtinčnost $\vec{M}$: $\oint_L \vec{M} d\vec{l}$

$$\left\{ \oint_A \vec{P} d\vec{a} = -Q_{not. pol.} \right\}$$



$$\vec{M} \cdot d\vec{l} = \frac{I_{mag} d\vec{a} \cdot d\vec{l}}{dV_{ol}}$$

($dV_{ol} = d\vec{a} \cdot d\vec{l}$)

$$\vec{M} \cdot d\vec{l} = dI_{mag}$$

$$\oint_L \vec{M} \cdot d\vec{l} = I_{mag}$$

Vektor magnetne poljsne jakosti: $\vec{H}$

$$\left\{ \begin{array}{l} \oint_A \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\epsilon_0} \quad / \cdot \epsilon_0 \\ \oint_A \vec{P} \cdot d\vec{a} = -Q_{not. pol.} \end{array} \right. +$$

$$\oint_A (\epsilon_0 \vec{E} + \vec{P}) d\vec{a} = Q_{not. prosti}$$

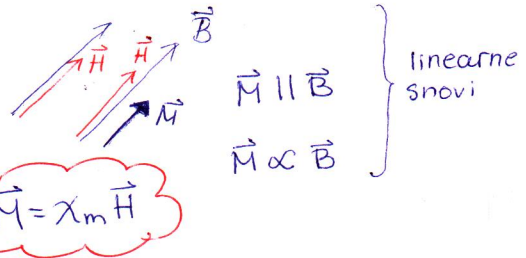
$\vec{D}$

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I \quad \nearrow \text{vsi toki} \quad / : \mu_0$$

$$\oint_L \vec{M} \cdot d\vec{l} = I_{mag.} \quad \nearrow \text{tok magnetizacije}$$

$$\oint_L \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M} \right)}_{\vec{H}} d\vec{l} = I_{prosti}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} \quad \text{vektor magnetne poljske jakosti [A/m]}$$



$$\mu_0(\vec{H} + \chi_m \vec{H}) = \vec{B}$$

↑
magnetna susceptibilnost

$$\mu_0(1 + \chi_m) \vec{H} = \vec{B}$$

μ_r

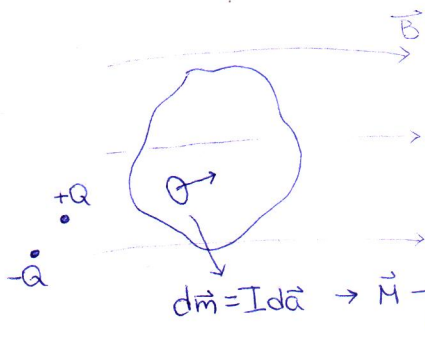
$$\vec{B} = \mu_0 \mu_r \vec{H}$$

↑
relativna permeabilnost snovi

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J}_{\text{prosti}} \cdot d\vec{a}$$

PG(22.3.2016)

Snov v magnetnem polju



$$\vec{B} = \mu_0(1 + \chi_m) \vec{H} = \mu_0 \mu_r \vec{H}$$

$\{ \vec{D} = \epsilon_0 \epsilon_r \vec{E} \}$ vpliv snovi

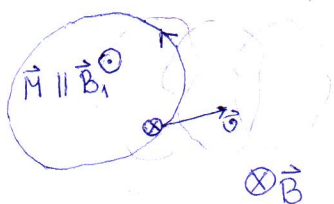
magnetna poljska jakost

Načini magnetenja snovi

- diamagnetne snovi (Cu)
- paramagnetne snovi (Al)
- feromagnetne snovi (Fe, Ni, Co, Gd, Dy)

① Diamagnetizem (dia = prečno)

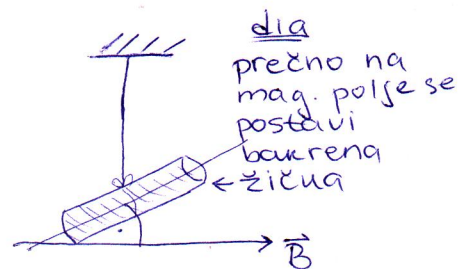
- nimajo izražene mag. dipolov
- ošibijo zunanje mag. polje



$$\chi_m \doteq -10^{-5} \rightarrow \mu_r \doteq 0,99999$$

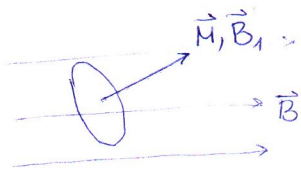
$$\vec{M} \cdot \vec{B} < 0$$

↑ ↓



② Paramagnetne snovi (para = vzporedno)

-imajo šibko izražene mag. dipole

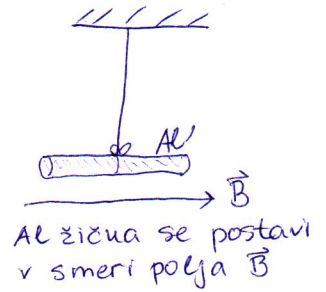


$$\chi_m = +10^{-5}$$

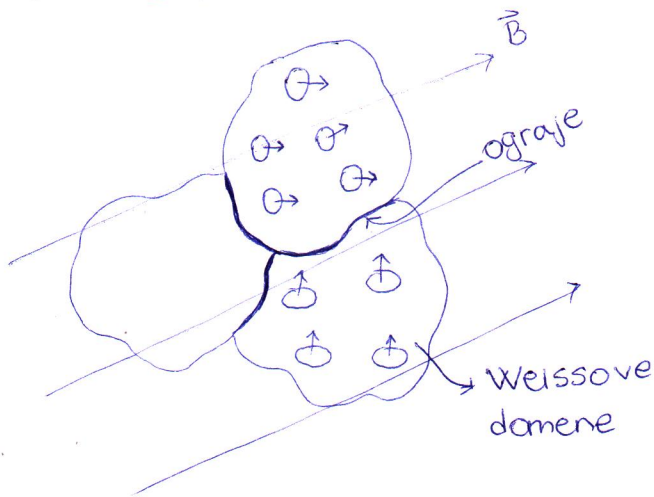
$$\mu_r = 1,00001$$

$$\vec{M} \parallel \vec{B}$$

$$\vec{M} \cdot \vec{B} > 0$$

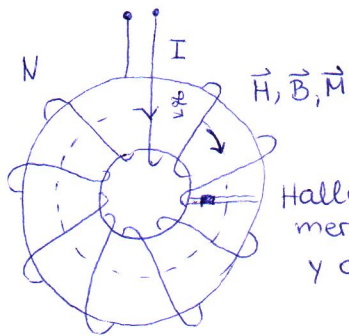


③ Feromagnetizem

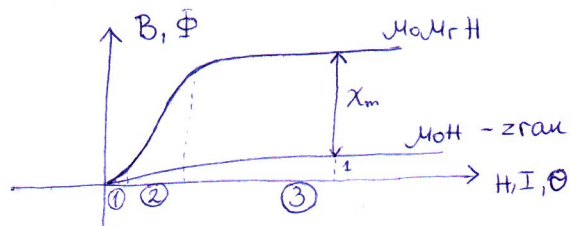


- Weissove domene se orientirajo v smeri $\vec{B}$
- stapljanje domen
- Nelinearen odziv

B-H diagram



$$\oint \vec{H} d\vec{\ell} = I_{\text{zaobjeti}} = NI \rightarrow H = \frac{NI}{2\pi r} = f(r) \text{ na x-osi}$$



- ① cona je reverzibilno področje obračanja domen
- ② nereverzibilno področje močne ojačanja
- ③ nasičenje

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

↓
zveza med $\vec{B}$, $\vec{H}$, $\vec{M}$

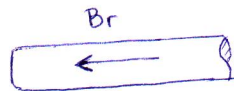
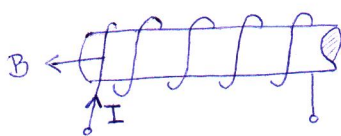
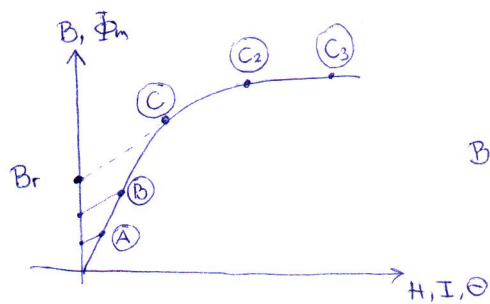
$$\chi_m = 10^{-10^6}$$

$$\mu_r = 10^{-10^6}$$

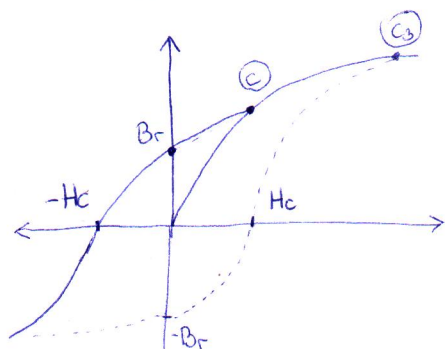
$$(\mu_r = 1 + \chi_m)$$

* zaradi nelinearnega odziva, μ_r ni konstantna vrednost.

Remenentna gostota B_r



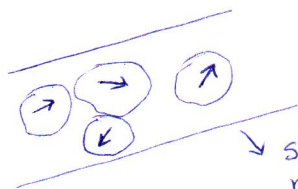
Koercitivna poljsna jakost H_c



-razmagnetenje

Curiejeva temperatura

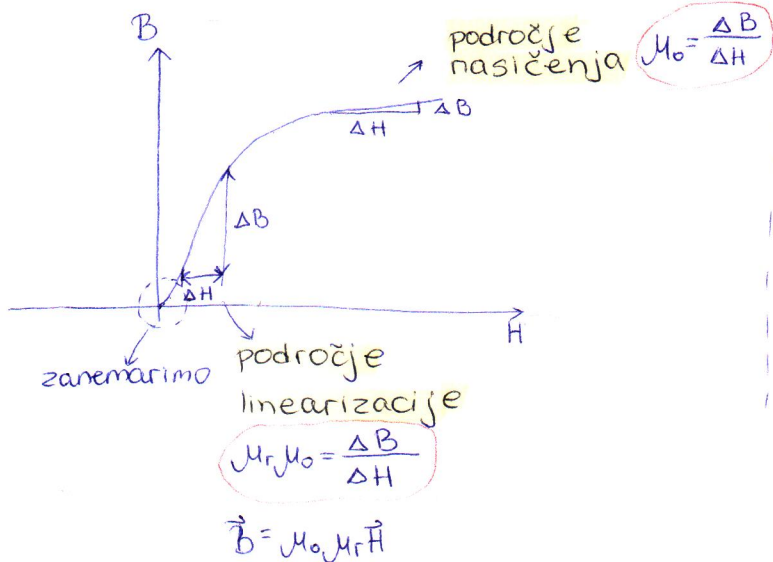
$Fe: 770^\circ C$



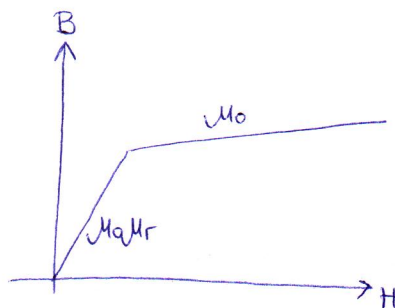
snov izgubi magnetne lastnosti

Udarci in razmagnetenje

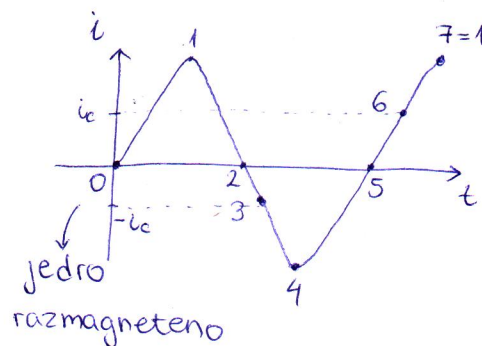
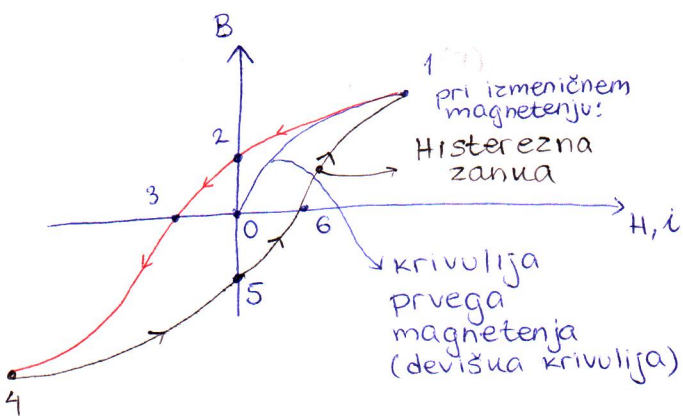
Magnetilno krivulje



Linearizacija mag. krivulje

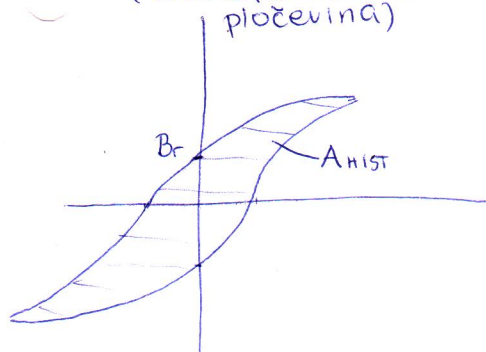


Histerezna zanka

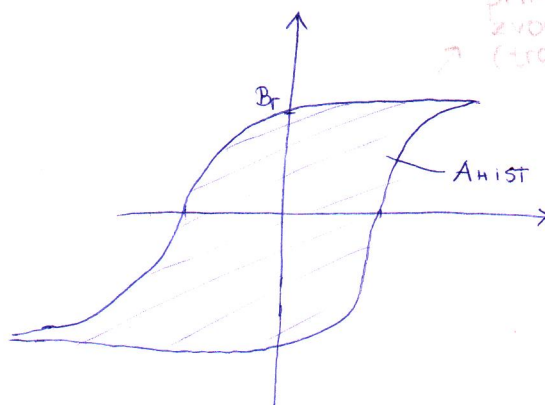


Dve gružini feromag. materialov

Mehuromagnetne snovi ($H_0 \sim 10^4 \text{ A/m}$)
(transformatorska pločevina)

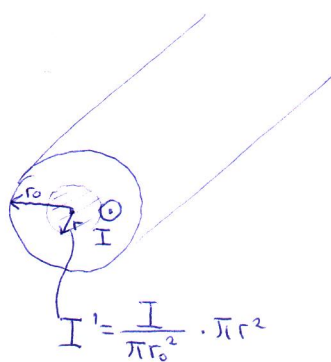


Trdomagnetne snovi ($H_0 \sim 10^5 \text{ A/m}$)



primerni za zvočni magneti

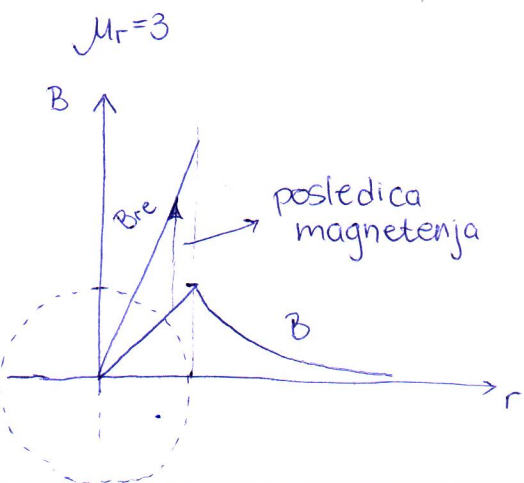
Zgled: B v in ob jeuklenem vodniku (feromagnetnem)



$$\int \vec{H} \cdot d\vec{l} = 2\pi r \cdot H_p = I_{\text{zaobjet}}$$

$$H_p = \begin{cases} \frac{I \cdot r}{2\pi r_0^2} & , \text{ znotraj} \\ \frac{I}{2\pi r} & , \text{ zunaj} \end{cases}$$

$$B_p = \mu_0 \mu_r H = \begin{cases} \frac{\mu_0 \mu_r I r}{2\pi r_0^2} & , \text{ znotraj} \\ \frac{\mu_0 \mu_r I}{2\pi r} & , \text{ zunaj} \end{cases}$$



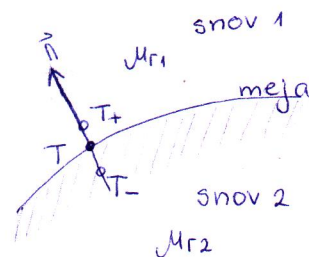
Mejna pogoja $\vec{B}$, $\vec{H}$

$$\oint_A \vec{B} \cdot d\vec{a} = 0 : \boxed{\vec{e}_n (\vec{B}(T_+) - \vec{B}(T_-)) = 0}$$

$$B_{n1} = B_{n2}$$

$$\oint_A \vec{J} \cdot d\vec{a} = 0 \rightarrow J_n - \text{zvezna}$$

mejni pogoj za gostoto $\vec{B}$.



$$\oint_L \vec{H} \cdot d\vec{l} = I$$

$$H_x(T_+) \cdot (-\Delta x) + H_x(T_-) \cdot (\Delta x) = K_z \Delta x = I$$

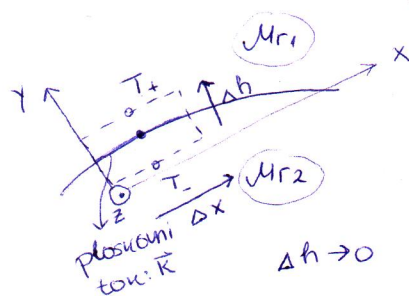
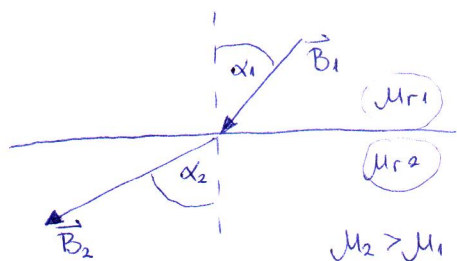
$$H_x(T_-) - H_x(T_+)$$

$$H_y \dots$$

$$H_z \dots$$

$$\Rightarrow \vec{e}_n \times (\vec{H}(T_+) - \vec{H}(T_-)) = \vec{K} \quad \leftarrow \text{nezveznost}$$

mejni pogoj za vektor $\vec{H}$

Lomni zakon za mag. polje ($\vec{K} = 0$)

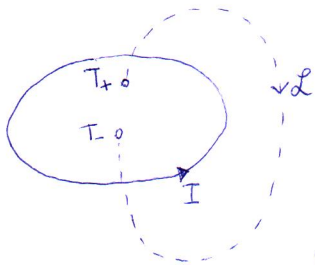
$$\boxed{\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_{r1}}{\mu_{r2}} = \frac{\mu_1}{\mu_2}}$$

$$\rightarrow \mu_1 = \mu_0 \cdot \mu_{r1}$$

Magnetna napetost in potencial

$$\left\{ U = \frac{Ae}{Q} ; V = \frac{We}{Q} \right\} \quad \left\{ \oint_L \vec{E} \cdot d\vec{l} = 0 ; \int_{T_1}^{T_2} \vec{E} \cdot d\vec{l} = U_{12} ; V = \int_T^{T_0} \vec{E} \cdot d\vec{l} \right\}$$

$$\left\{ \oint_L \vec{H} \cdot d\vec{l} = N \cdot I \neq 0 \right\} \quad \left\{ \vec{E} = -\text{grad}(V) \right. \\ \left. \text{grad}(V) = \left(\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz} \right) \right\}$$

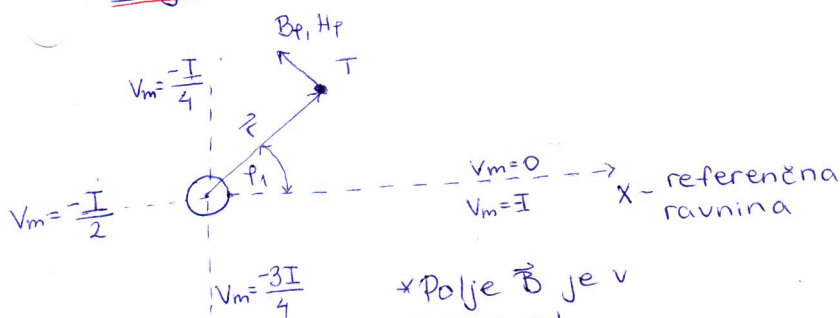


$$\mathcal{Q}_{T_+T_-} = V_m(T_+) - V_m(T_-) = \int_{T_-}^{T_+} \vec{H} \cdot d\vec{l} \rightarrow \vec{H} = -\text{grad}(V_m)$$

mag. napetost

sualarni magnetni potencial

Zgled: Sualarni mag. potencial premega vodnika



$$H_p = \frac{I}{2\pi r}$$

$$(H_r, H_p, H_z) = -\left(\frac{dV_m}{dr}, \frac{dV_m}{r dp}, \frac{dV_m}{dz} \right)$$

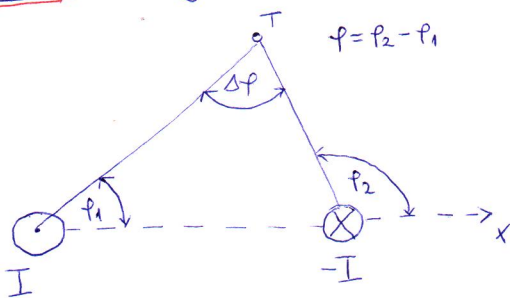
↑ ↑
Lamejev koeficient

* Polje $\vec{B}$ je v smeri od visjega potenciala (V_m) prema nižjega.

$$H_p = \frac{dV_m}{r dp} = \frac{-I}{2\pi r}$$

$$V_m = \int_0^{p_1} \frac{I}{2\pi} dp = \frac{-I \cdot p_1}{2\pi}$$

Zgled: Magnetni sualarni potencial dvovoda

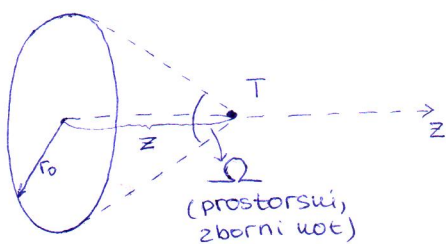


$$V_m = -\frac{I}{2\pi} \cdot p_1 + \frac{I}{2\pi} \cdot p_2$$

$$V_m = \frac{I}{2\pi} (p_2 - p_1) = \frac{I}{2\pi} \Delta p$$

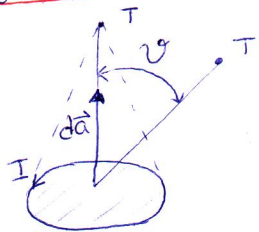
$$V_m = \frac{I}{2\pi} \cdot \Delta p$$

Zgled: Vm v osi obroča



$$V_m(T) = \frac{I}{4\pi} \cdot \Omega$$

Zgled: V_m magnetnega dipola



$$d\vec{m} = I \cdot d\vec{a}$$

$$V_m = \frac{I \cdot d\Omega}{4\pi} \cdot \cos \vartheta \quad \left(d\Omega = \frac{da}{r^2} \right)$$

$$V_m = \frac{I \cdot da}{4\pi r^2} \cdot \cos \vartheta$$

$$\boxed{V_m = \frac{dm}{4\pi r^2} \cdot \cos \vartheta} \quad \left\{ V_{\text{dipola}} = \frac{dp}{4\pi \epsilon_0 r^2} \cos \vartheta \right\}$$

Magnetna vezja

tokovna polja	magnetna polja
$\vec{J} = \int \vec{J} \cdot d\vec{a} = I$	$\vec{B}: \int \vec{B} \cdot d\vec{a} = \Phi_m$
$\oint \vec{J} \cdot d\vec{a} = 0$ (v spojitih) $\sum I_k = 0$	$\oint \vec{B} \cdot d\vec{a} = 0$ $\sum \Phi_{mk} = 0$
$J_n(T_+) - J_n(T_-) = 0$	$B_n(T_+) - B_n(T_-) = 0$
$\vec{J} = \gamma \vec{E}$	$\vec{B} = \mu \vec{H}; \quad \vec{B} \propto \vec{H}$
$\oint \vec{E} \cdot d\vec{l} = 0$ $\sum U_k = 0$	$\oint \vec{H} \cdot d\vec{l} = 0$ (če s ukrivljeno ne prestopimo ref. ravn.) $\Theta = NI$ (generat. v mag. vezjih) $\sum Q_k = 0$
$V = \int_T^\infty \vec{E} \cdot d\vec{l}; \quad \vec{E} = -\text{grad } V$	$V_m = \int_T^\infty \vec{H} \cdot d\vec{l}; \quad \vec{H} = -\text{grad } V_m$
$E_T(T_+) - E(T_-) = 0$	$H_T(T_+) - H(T_-) = 0$
$\gamma_1 < \gamma_2$ slab prevodnik izolant	$\mu_{r1} < \mu_{r2}$ dobro prevodnik
EL. vezja	MAG. vezja

$$U = R \cdot I$$

$$\Theta = R_m \cdot \Phi_m$$

↓
mag.
gen.

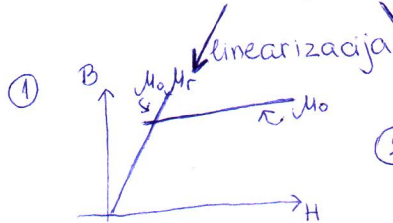
↓
mag.
bremena

Elementi magnetnih vezij

Magnetni upor

$$R_m = \frac{\Theta}{\Phi} = \frac{H \cdot l}{B \cdot S} = \frac{H}{B} \frac{l}{S} \text{ geometrija}$$

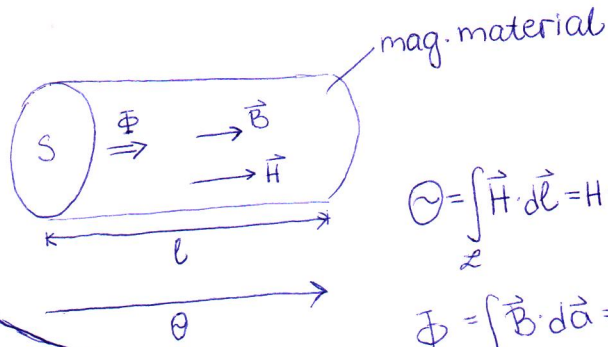
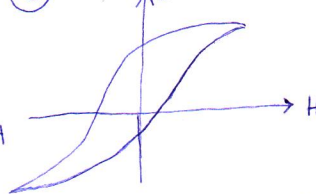
snov (BH diagram)



② nelinearen



③ histereza



$$\Theta = \int_L \vec{H} \cdot d\vec{l} = H \cdot l$$

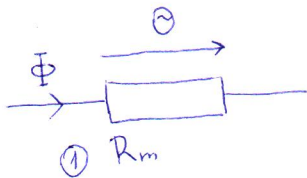
$$\Phi = \int_S \vec{B} \cdot d\vec{a} = B \cdot S$$

za ①: $B = \mu_0 \mu_r H$

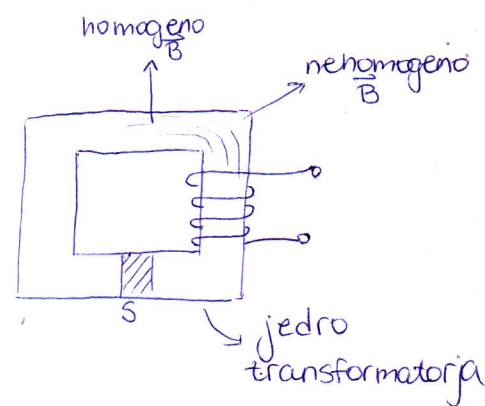
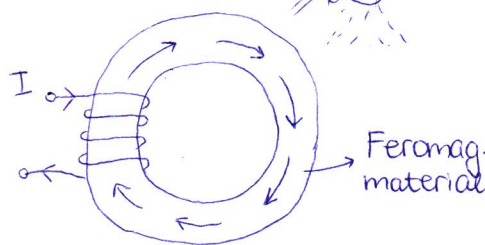
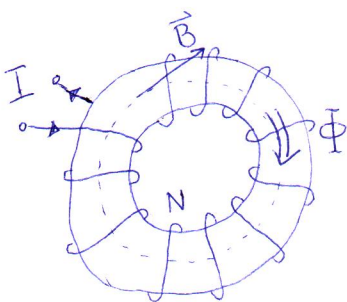
$$R_m = \frac{H \cdot l}{\mu_0 \mu_r H \cdot S} = \frac{l}{\mu_0 \mu_r S} \rightarrow \text{snovno-geometrijska lastnost}$$

$$\left\{ R = \frac{l}{\mu_r S} \right\}$$

simbol:

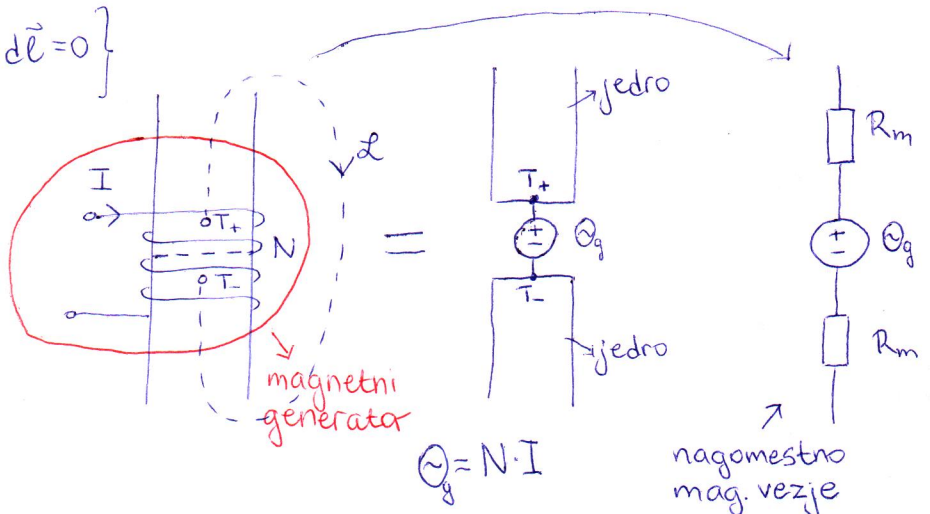


Magnetni generator



$$\oint_L \vec{H} \cdot d\vec{l} = N \cdot I = \left\{ \oint_L \vec{E} \cdot d\vec{l} = 0 \right\}$$

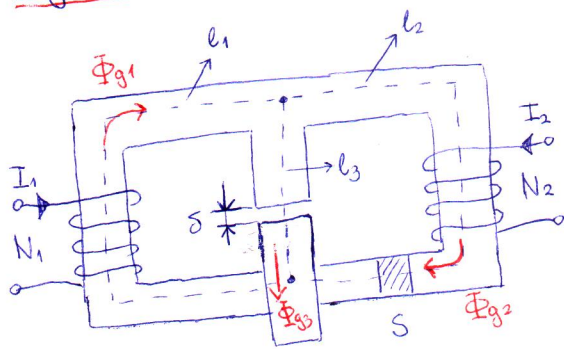
$$= V_m(T_+) - V_m(T_-) = Q_g$$



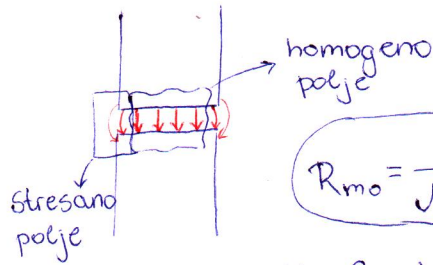
$$Q_g = N \cdot I$$

nagomestno mag. vezje

Zgled: Variabilni transformator



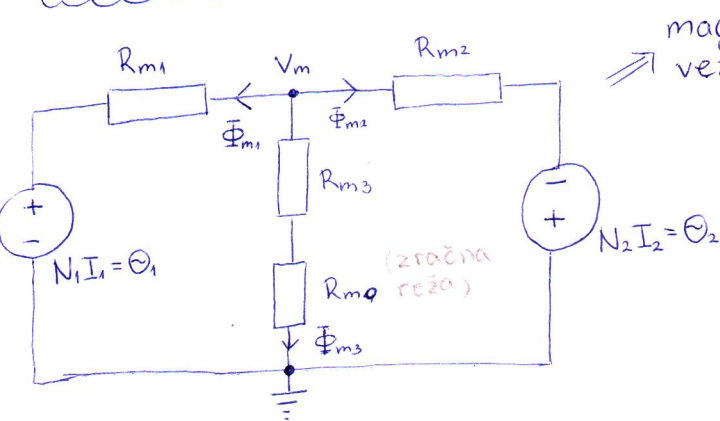
Zračna reža



$$R_{mo} = \frac{\delta}{\mu_0 \cdot S \cdot K}$$

K - Carterjev faktor
 - povečanje preseka
 - zračne reže zaradi
 - stresa
 - $S_0 = S \cdot K, K > 1$

Nadomestno vezje



magnetno vezje

$$\Phi_{m1} = \frac{V_m - \Theta_1}{R_1}$$

$$\Phi_{m2} = \frac{V_m + \Theta_2}{R_2}$$

$$\Phi_{m3} = \frac{V_m}{R_{m3} + R_0}$$

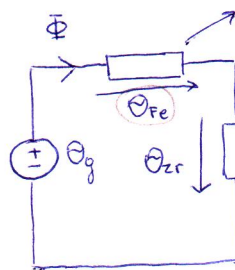
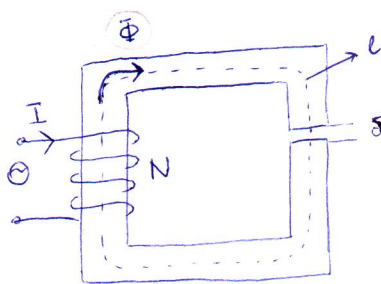
$$\Phi_{m1} + \Phi_{m2} + \Phi_{m3} = 0 \dots \Rightarrow V_m = \dots$$

$$\Rightarrow B, H \dots$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

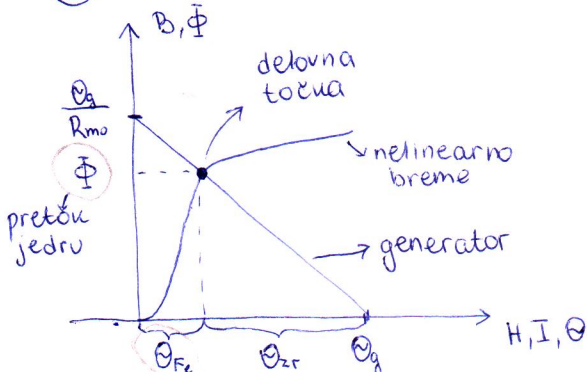
Nelinearna mag. vezja

- ② magnetik z nelin. krivuljo
- ③ magnetik s histerezo zanko

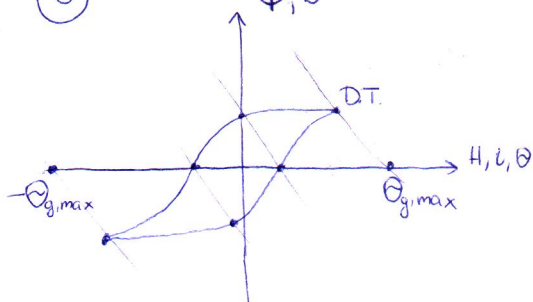


jedro:
 nelinearna
 $B = \mu_0 \mu_r H$
 reža:
 linearna
 $B = \mu_0 H$
 (notr. upornost)

② Grafično reševanje

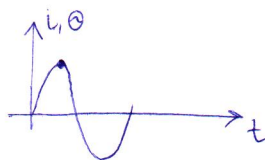


③ histereza Φ, B

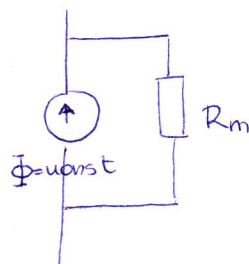
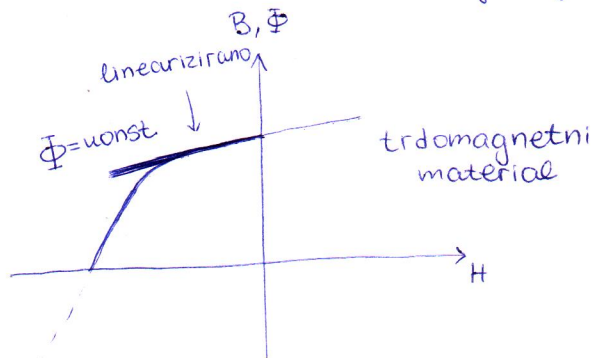
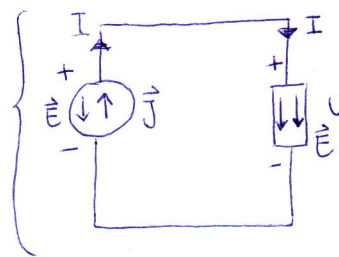
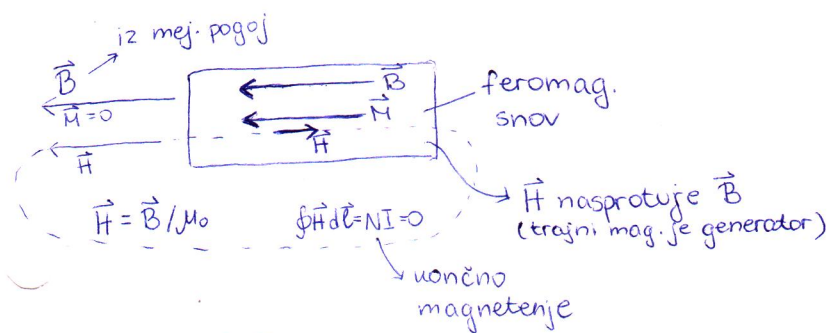


i - harmoničen

H, θ - harmoničen



Trajni magneti



P12 (1.4.2016)

Dinamično elektromagnetno polje

$$\oint_L \vec{E} \cdot d\vec{l} = 0 \Rightarrow$$

$$= - \frac{d\Phi_L}{dt} \quad \left(\begin{array}{l} \text{Farad.} \\ \text{Maxwell} \end{array} \right)$$

Amper

$$\vec{F}_L = Q(\vec{E} + \vec{\omega} \times \vec{B})$$

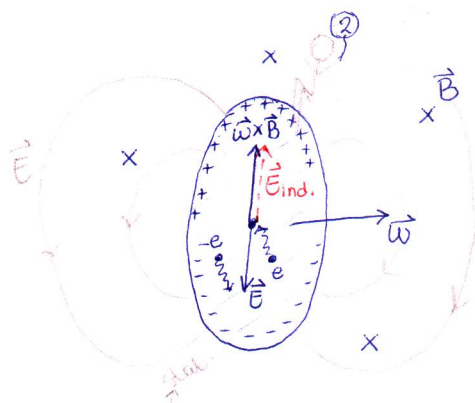
$\vec{E}_{\text{Coul.}}$ (Coulomb) $\vec{E}_{\text{Farad.}}$ (Farad)

$$\oint_L \vec{H} \cdot d\vec{l} = \int_A \vec{J} \cdot d\vec{a} + \int_A \frac{\partial \vec{D}}{\partial t} \cdot d\vec{a}$$

$$\oint_A \vec{D} \cdot d\vec{a} = Q_{\text{not}}$$

$$\oint_A \vec{B} \cdot d\vec{a} = 0$$

Gibanje prevodnika v magnetnem polju



(1)

$$-e\vec{\omega} \times \vec{B}$$

$$-e(\vec{E} + \vec{\omega} \times \vec{B}) = \vec{0}$$

$$\vec{E} + \vec{\omega} \times \vec{B} = \vec{0}$$

(ravnovesje)

Influenca

$$\vec{E}_0 \text{ (primarno)}$$

$$\vec{E} = \vec{E}_0 + \vec{E}_\lambda = \vec{0}$$

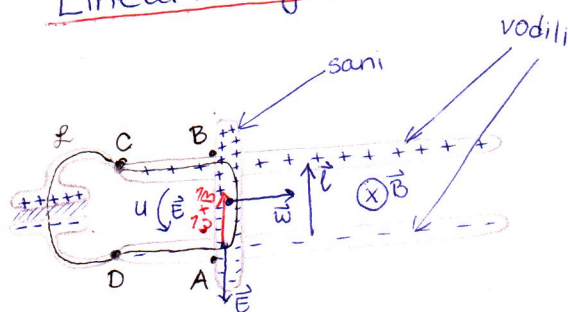
$$\vec{E}_\lambda \text{ (reakcijsko)}$$

(2) $-e(\vec{E} + \vec{E}_{ind}) = \vec{0}$ (ravnovesje)

$$\vec{E} + \vec{E}_{ind} = \vec{0}$$

$$\vec{E}_{ind} \equiv \vec{\omega} \times \vec{B}$$

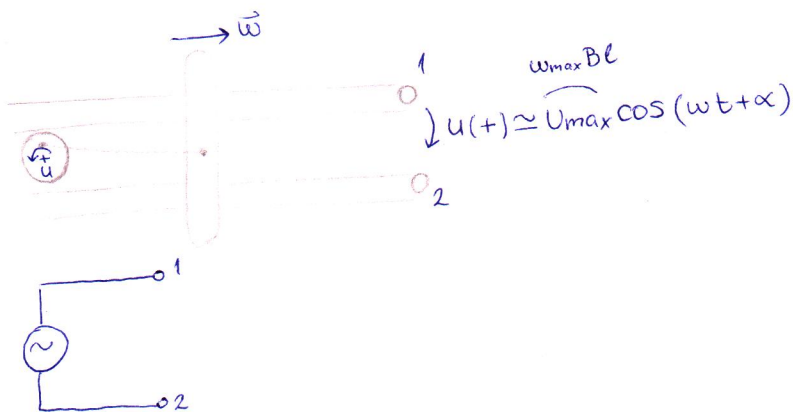
Linearni generator



$$\oint \vec{E} \cdot d\vec{l} = \int_A^B \vec{E} \cdot d\vec{l} + \int_B^C \vec{E} \cdot d\vec{l} + \int_C^D \vec{E} \cdot d\vec{l} + \int_D^A \vec{E} \cdot d\vec{l} = 0$$

$$\underbrace{-\int_A^B (\vec{\omega} \times \vec{B}) \cdot d\vec{l}}_{-wBl} = \underbrace{\int_B^C \vec{E} \cdot d\vec{l}}_{\sim \emptyset} + \underbrace{\int_C^D \vec{E} \cdot d\vec{l}}_u + \underbrace{\int_D^A \vec{E} \cdot d\vec{l}}_{\sim \emptyset}$$

$$\Rightarrow u = wBl = u_{ind.} \text{ (inducirana napetost)}$$



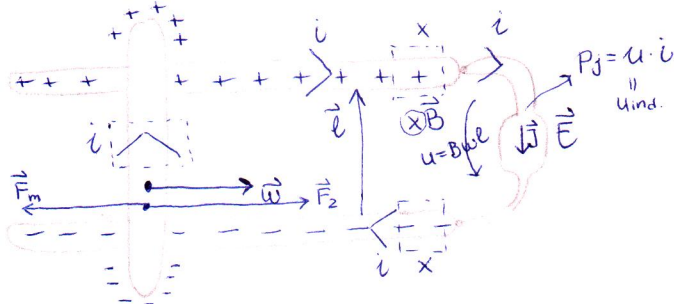
Obremenjen lin. generator

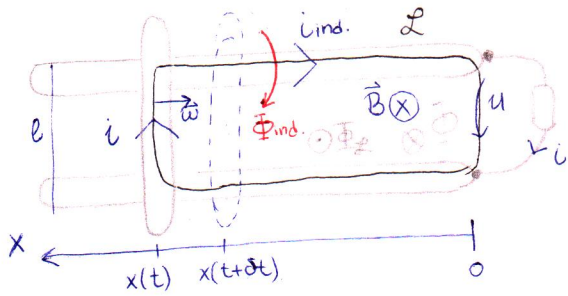
$$\vec{F}_m = i\vec{l} \times \vec{B}$$

$$\vec{F}_z + \vec{F}_m = \vec{0}$$

$$P_z = \vec{F}_z \cdot \vec{\omega} = i(lB\omega) = i \cdot u_{ind}$$

$$P_z = P_j$$





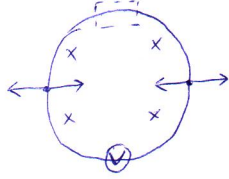
$$U = U_{\text{ind}} = w \cdot B \cdot l = \frac{x(t) - x(t + \delta t)}{\delta t} B \cdot l =$$

$$= - \left[\Phi(t + \delta t) - \Phi(t) \right] =$$

$$= \frac{x(t) B l - x(t + \delta t) B l}{\delta t} = \frac{\Phi(t) - \Phi(t + \delta t)}{\delta t}$$

$$U = - \lim_{\delta t \rightarrow 0} \frac{\Phi(t + \delta t) - \Phi(t)}{\delta t} = - \frac{d\Phi}{dt}$$

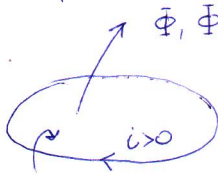
$$U_{\text{ind}} = - \frac{d\Phi_L}{dt}$$



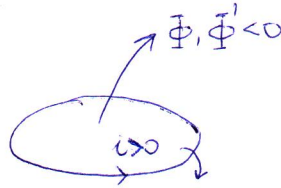
$$\Phi = \int \vec{B} \cdot d\vec{a}$$

variram (transformator sua napetost)
variram (gibalna napetost)

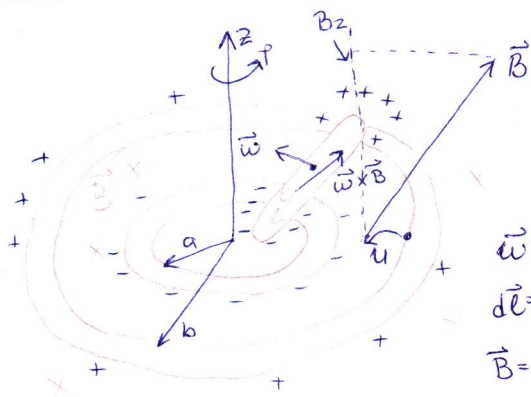
$$U_{\text{ind}} = - \frac{d\Phi}{dt} = - \underbrace{\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}}_{\text{transf.}} + \underbrace{\oint \vec{v} \times \vec{B} \cdot d\vec{\ell}}_{\text{gib.}}$$



Lenz



Faraday-ev generator



$$\vec{v} = (0, \omega r, 0)$$

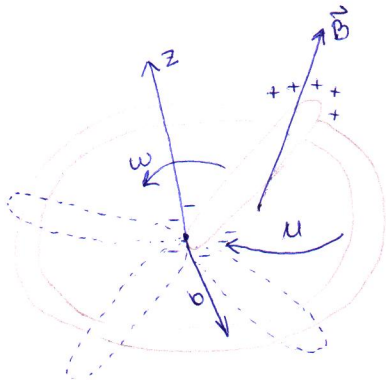
$$d\vec{\ell} = (dr, 0, 0)$$

$$\vec{B} = (B_x, B_y, B_z)$$

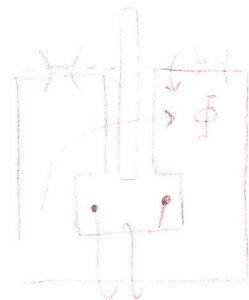
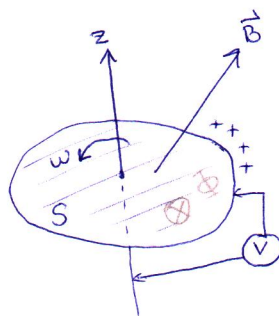
$$U = U_{\text{ind}} = \int_a^b (\vec{v} \times \vec{B}) \cdot d\vec{\ell} = \int_a^b (d\vec{\ell} \times \vec{v}) \cdot \vec{B} =$$

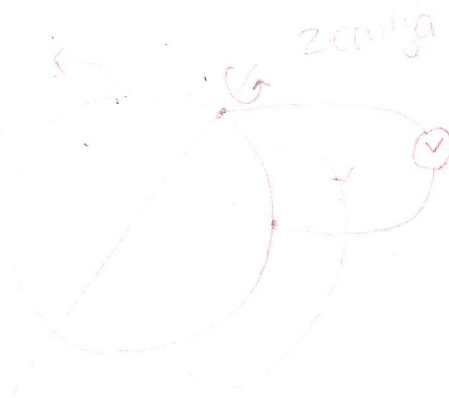
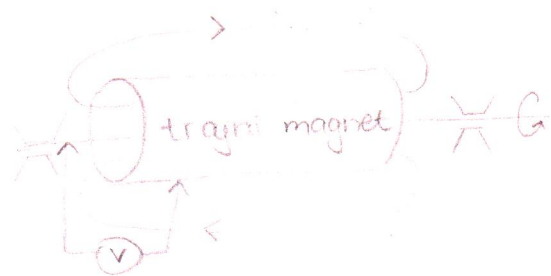
$$= \int_a^b B_z \omega r dr = \frac{\omega B_z}{2} (b^2 - a^2)$$

enosmerna

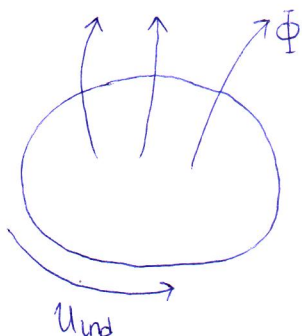


$$U = \frac{\omega B_z b^2}{2} = \frac{2\pi f B_z b^2}{2} = f \cdot \Phi_{\text{diska}}$$





P13(5.4.2016)

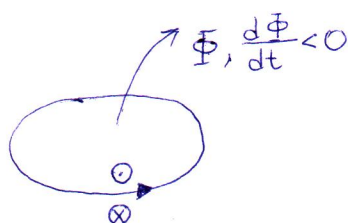
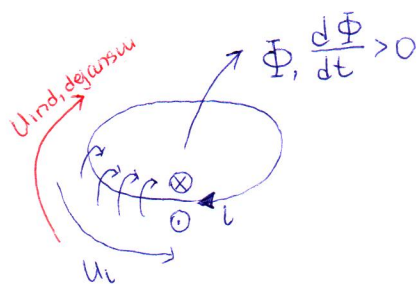


Dogovor:

Označevanje inducirane napetosti v sklenjeni zanki izvajamo v matematično pozitivnem smislu glede na smer flusa (pravilo desnega vijava)

$$U_{ind} = - \frac{d\Phi}{dt} \quad \text{--- sprememba pretoka}$$

Lenz + dogovor.



2. Maxwellova enačba

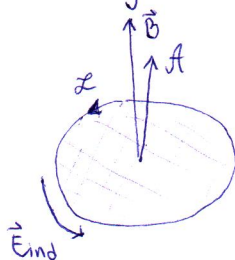
$$U_{ind} = - \frac{d\Phi}{dt}$$

↓
nanaša se
na zanko

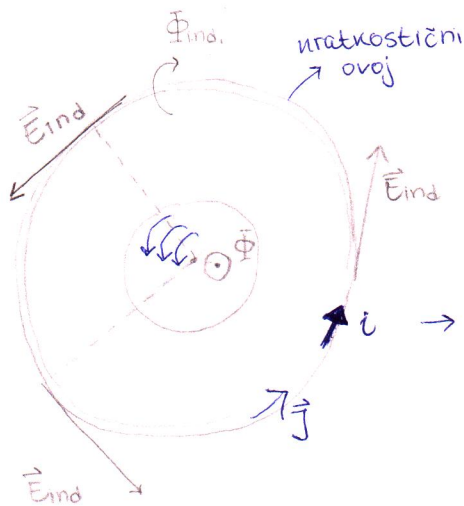
$$\oint \vec{E}_{ind} \cdot d\vec{l} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{a}$$

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

$$U_{ind} = \int \vec{E}_{ind} \cdot d\vec{l}$$

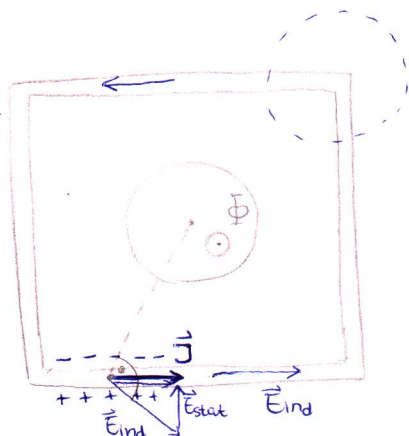


Vrtinčni toki, izgube, lameliranje



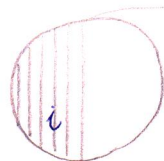
$$\vec{j} = \gamma \cdot \vec{E} = \gamma (\vec{E}_{stat} + \vec{E}_{ind})$$

$i \rightarrow$ v prevodnih materialih zaradi spreminjajočega Φ tečejo t.i. vrtinčni toki (vrtinčne izgube jedja)



Urepa za preprečevanje vrtinčnih tokov:

$\rightarrow$ Lameliranje

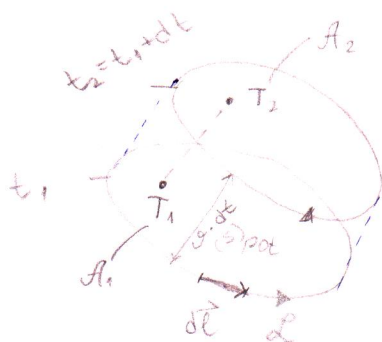


90% Fe
10% Loua } faktor polnjenja: 0,9

vrtinčni toki le znotraj lamele

$\rightarrow$ hlajenje

Transformatorska in gibalna inducirana napetost



$$U_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{a}$$

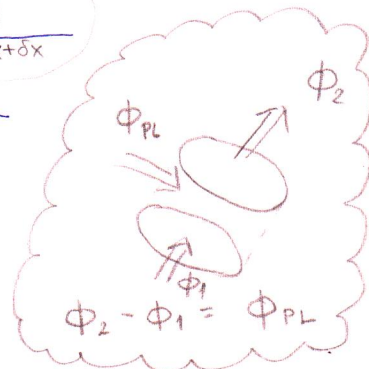
$$\frac{d}{dt} \int_A \vec{B} \cdot d\vec{a} = \lim_{dt \rightarrow 0} \frac{1}{dt} \left(\int_{A_2} \vec{B}(T_2, t_2) \cdot d\vec{a} - \int_{A_1} \vec{B}(T_1, t_1) \cdot d\vec{a} \right)$$

$\lim$ $d\Phi = \Phi_{v t_2} - \Phi_{v t_1}$

$$\vec{B}(T_2, t_2) = \vec{B}(T_2, t_1) + dt \frac{d\vec{B}(T_2, t_1)}{dt}$$

$$\Rightarrow \frac{d}{dt} \int_A \vec{B} \cdot d\vec{a} = \int_{A_2} \frac{d\vec{B}}{dt} \cdot d\vec{a} + \lim_{dt \rightarrow 0} \frac{1}{dt} \left(\int_{A_2} \vec{B}(T_2, t_1) \cdot d\vec{a} - \int_{A_1} \vec{B}(T_1, t_1) \cdot d\vec{a} \right)$$

$\Phi_{2v t_1} - \Phi_{1v t_1}$
 $d\Phi = \Phi_{PL} =$



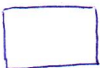
$$\Phi_{\text{PLASČA}} = \oint_{\mathcal{L}} \vec{B} (\vec{v} \cdot d\vec{t} \times d\vec{\ell}) = - \oint_{\mathcal{L}} (\vec{v} \times \vec{B}) \cdot d\vec{\ell} \cdot dt$$

$$\frac{d}{dt} \int_A \vec{B} \cdot d\vec{a} = \int_A \frac{d\vec{B}}{dt} \cdot d\vec{a} - \oint_{\mathcal{L}} (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

$$\Rightarrow U_{\text{ind}} = - \underbrace{\int_A \frac{d\vec{B}}{dt} \cdot d\vec{a}}_{\substack{\text{posledica} \\ \text{spreminjanja} \\ \text{flouasa skozi} \\ \text{zanuo}}} + \underbrace{\oint_{\mathcal{L}} (\vec{v} \times \vec{B}) \cdot d\vec{\ell}}_{\substack{\text{posledica} \\ \text{gibanja } \vec{v} \text{ } \vec{B} \\ \text{GIBALNA INDUCIRANA} \\ \text{NAPETOST}}}$$

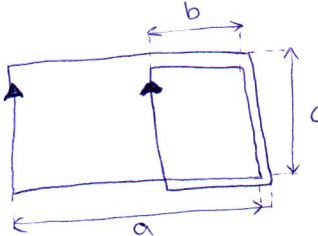
TRANSFORMATORSKA
INDUCIRANA NAPETOST

Magnetni sulep Ψ

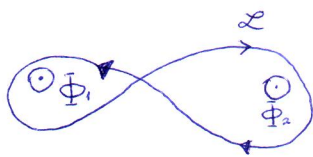
do sedaj:  
večje število ovogev



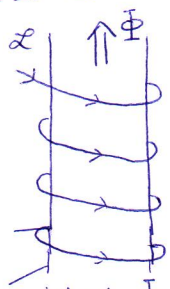
$$\Psi = N \cdot \Phi$$



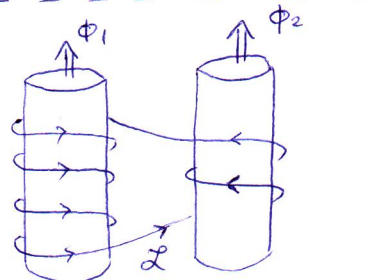
$$\Psi = \Phi_{\text{a} \times c} + 2\Phi_{\text{b} \times c}$$



$$\Psi = \Phi_1 - \Phi_2$$

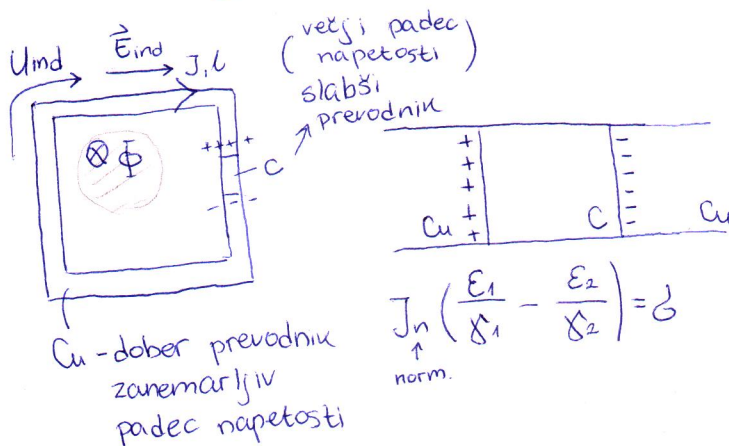


$$\Psi = 4 \Phi$$



$$\Psi = 4 \Phi_1 - 2 \Phi_2$$

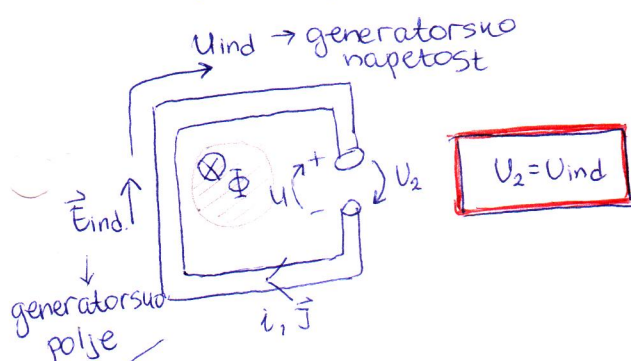
Inducirana napetost v odprtih zanihah



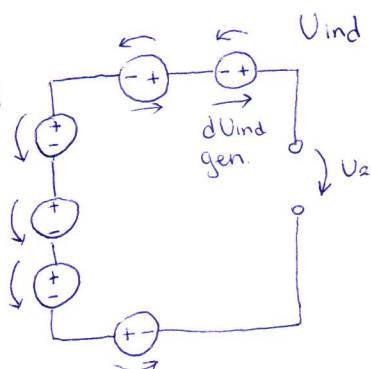
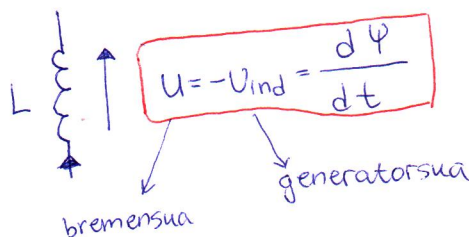
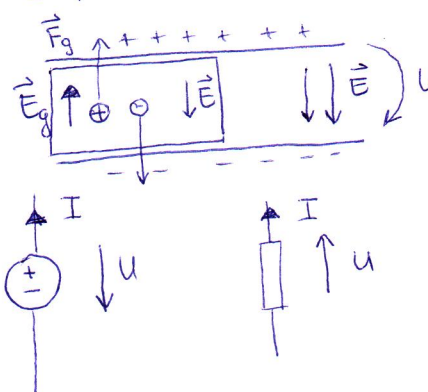
$$U_{ind} = - \frac{d\Phi}{dt}$$

$$J_n \left(\frac{E_1}{\delta_1} - \frac{E_2}{\delta_2} \right) = \sigma$$

↑
norm.

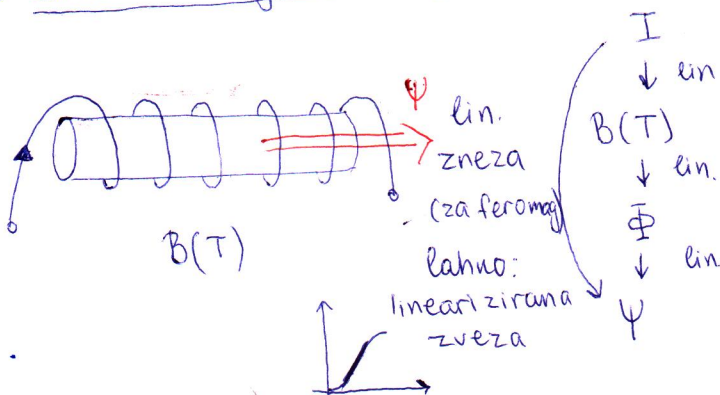


napetostni vir



Induktivnost

• Snovno geometrijska lastnost



$$L = \frac{\Phi}{I}$$

induktivnost

$$U_{ind} = L \frac{di}{dt}$$

zvezo med nap. in tokom na tuljavi

$$\left\{ i = C \frac{du}{dt} \right\}$$

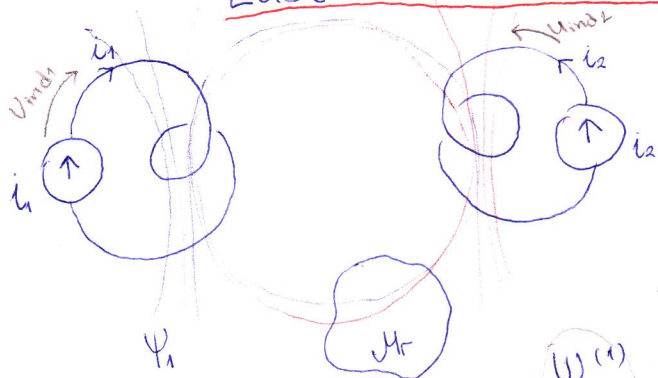
generator's EMF or voltage.

$$U = L \frac{di}{dt}$$

L → bremsna
ozn. nap.

$$\left\{ i = C \frac{du}{dt}, \quad u = R \cdot i \right\}$$

Lastna in medsebojna induktivnost



$$\Psi_1 = \Psi_1^{(1)} + \Psi_2^{(2)}$$

$$\Psi_2 = \Psi_2^{(1)} + \Psi_1^{(2)}$$

→ induksijski
koeficienti

$L [H]$
→ Henri

$$\begin{array}{cccc} \frac{\Psi_1^{(1)}}{i_1} & \frac{\Psi_1^{(2)}}{i_2} & \frac{\Psi_2^{(1)}}{i_1} & \frac{\Psi_2^{(2)}}{i_2} \\ \Downarrow & \Downarrow & \Downarrow & \Downarrow \\ L_{11} & L_{12} & L_{21} & L_{22} \end{array}$$

L_{11}, L_{22} - lastna induktivnost

L_{12}, L_{21} - Medsebojne induktivnosti

$$U_{ind1} = -L_{11} \frac{di_1}{dt} - L_{12} \frac{di_2}{dt}$$

$$U_{ind2} = -L_{21} \frac{di_1}{dt} - L_{22} \frac{di_2}{dt}$$

• Posplošitev na N zank

$$\Psi_k = \sum_{j=1}^N \Psi_k^{(j)} = \sum_{j=1}^N L_{kj} \cdot i_j$$

$$U_{indk} = - \frac{d\Psi_k}{dt} = - \sum_{j=1}^N \frac{d\Psi_k^{(j)}}{dt} = - L_{kk} \frac{di_k}{dt} - \sum_{\substack{j=1 \\ j \neq k}}^N L_{kj} \frac{di_j}{dt}$$

lastna
induktivnost

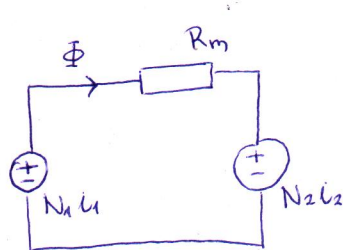
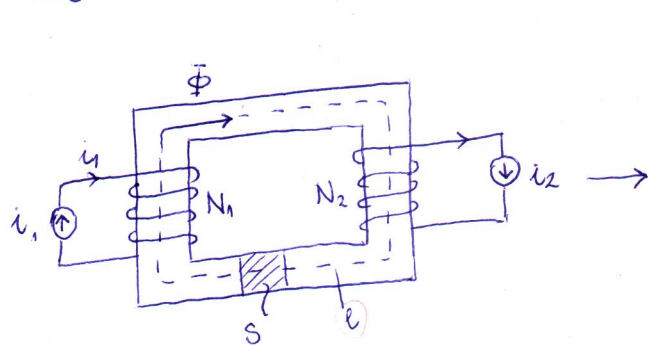
medsebojna
induktivnost

$$L = \begin{bmatrix} L_{11} & \dots & L_{1N} \\ L_{21} & & L_{2N} \\ \vdots & & \vdots \\ L_{N1} & & L_{NN} \end{bmatrix}$$

→ matrika induktivnosti

→ lastne induktivnosti

Zgled: Induktivnost navitij transformatorja



$$\Phi = \frac{N_1 i_1 - N_2 i_2}{R_m}$$

$$\Psi_1 = N_1 \Phi = \frac{N_1^2}{R_m} i_1 - \frac{N_1 N_2}{R_m} i_2$$

$$\Psi_2 = N_2 (-\Phi) = -\frac{N_1 N_2}{R_m} i_1 + \frac{N_2^2}{R_m} i_2$$

• Lastni induktivnosti

$$L_{11} = \frac{N_1^2}{R_m}; L_{22} = \frac{N_2^2}{R_m}$$

(sorazmerni s uvratom ovojev)

$$L = \frac{N^2}{R_m}$$

$$L_{ii} > 0$$

$$R_m = \frac{l}{S \mu}$$

snovno-
geometrijska
značilnost

• Medsebojne induktivnosti

$$L_{12} = -\frac{N_1 N_2}{R_m}$$

$$L_{ij} \geq 0$$

(pozitivne, če si mag. pretoka
para tuljav ^{so} podpirata)

$$L_{12} = L_{21} \rightarrow \text{Recipročnost}$$

Opazimo, da velja:

$$\{L_{11} \cdot L_{22} = L_{12} \cdot L_{21}\}$$

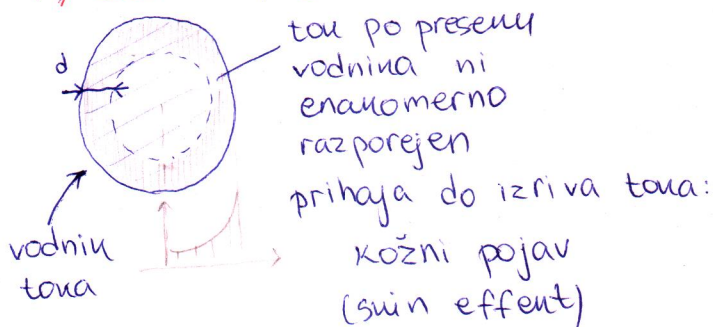
Skupni faktor

$$k^2 = \frac{L_{12} \cdot L_{21}}{L_{11} \cdot L_{22}}$$

$0 = k = 1 \rightarrow$ popoln
↓
ni vpliva sulop

Dileme pri določanju induktivnosti

1) Kožni pojav



$$d = \sqrt{\frac{2}{\omega \mu \epsilon}}$$

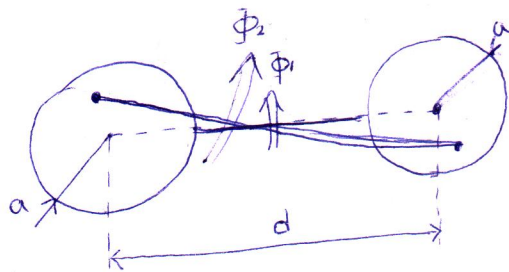
$$2\pi f$$

vdorna
globina

* pri 50 Hz $\rightarrow d = 9,6 \text{ mm}$

5 kHz $\rightarrow d = 1 \text{ mm}$

2) Dvovod



$$L = \frac{\Phi}{i} \quad \text{materi flux}$$

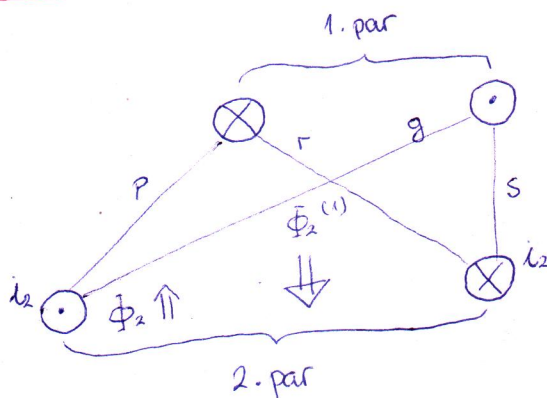
vzajemno poprečje
vseh flusov

$$L = \frac{\langle \Phi \rangle}{i}$$

lastna
induktivnost
dvovoda

$$L = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{d}{a} \right)$$

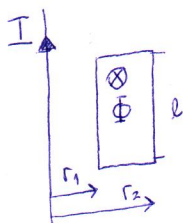
Medsebojna induktivnost dvovodov



$$L_{21} = - \frac{\Phi_2^{(1)}}{i_1} = - \frac{\mu_0 i_1 l}{2\pi i_1} \ln \frac{r \cdot g}{p \cdot s}$$

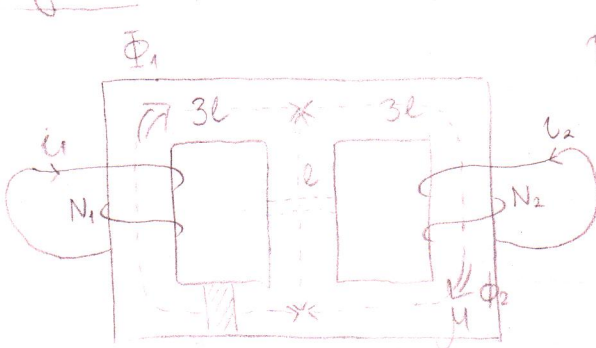
$$L_{21} = - \frac{\mu_0 l}{2\pi} \ln \frac{r \cdot g}{p \cdot s}$$

$$L_{12} = L_{21}$$



$$\Phi = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$$

Zgled: Dve navitji na tristejernem jedru



$$R_m = \frac{l}{\mu \cdot S}$$

$$-N_1 i_1 + 3 R_m \Phi_1 + R_m (\Phi_1 - \Phi_2) = 0$$

$$-N_2 i_2 + R_m (\Phi_2 - \Phi_1) + 3 R_m \Phi_2 = 0$$

$$4 R_m \Phi_1 - R_m \Phi_2 = N_1 i_1 \quad / \cdot 4$$

$$-R_m \Phi_1 + 4 R_m \Phi_2 = N_2 i_2 \quad / \cdot 4$$

$$15 R_m \Phi_1 = 4 N_1 i_1 + N_2 i_2$$

$$\Phi_1 = \frac{4 N_1 i_1 + N_2 i_2}{15 R_m}$$

$$\Phi_2 = \frac{N_1 i_1 + 4 N_2 i_2}{15 R_m}$$

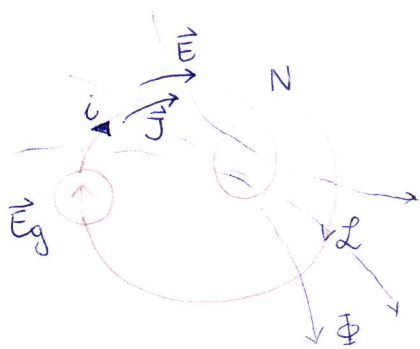
$$\Rightarrow \Psi_1 = N_1 \Phi_1 = \frac{4 N_1^2}{15 R_m} i_1 + \frac{N_1 N_2}{15 R_m} i_2$$

$$\Psi_2 = N_2 \Phi_2 = \frac{N_1 N_2}{15 R_m} i_1 + \frac{4 N_2^2}{15 R_m} i_2$$

$$L_{12} = L_{21}$$

$$L_{12} L_{21} < L_{11} L_{22} \quad | \quad k_{12} = \sqrt{\frac{L_{12} L_{21}}{L_{11} L_{22}}}$$

Tuljava, not strnjen element



$$\vec{J} = \gamma(\vec{E} + \vec{E}_g)$$

$$\frac{\vec{J}}{\gamma} = \vec{E} + \vec{E}_g$$

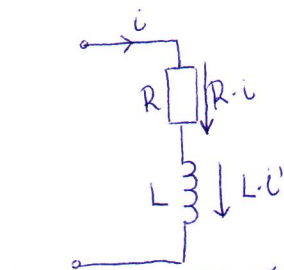
$$\oint_L \vec{E} \cdot d\vec{l} = \oint_L \frac{\vec{J}}{\gamma} \cdot d\vec{l} - \oint_L \vec{E}_g \cdot d\vec{l} = - \frac{d\psi}{dt}$$

$N \cdot \Phi$

$R \cdot i \quad U_g = -L \cdot i' \rightarrow \text{inducirana nap.}$

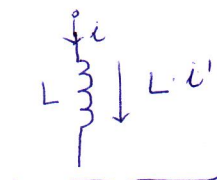
$$R \cdot i - U_g = -L \cdot i'$$

$$U_g = R \cdot i + L \cdot i'$$



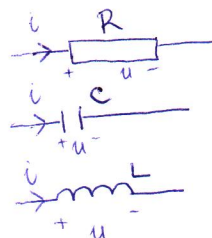
MODEL (REALNE)
TULJAVE

R - upornost
navitje
tuljave



IDEALNA
TULJAVA

$$U = L \frac{di}{dt}$$



$$u = R \cdot i$$

$$i = C \cdot u'$$

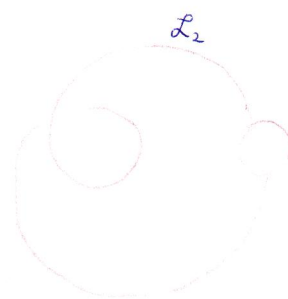
$$u = L \cdot i'$$

Modelno vezje dveh magnetno sklopljenih tuljav

1)



2)



$$\vec{J}_1 = \gamma_1(\vec{E}_1 + \vec{E}_{g1})$$

$$\vec{J}_2 = \gamma_2(\vec{E}_2 + \vec{E}_{g2})$$

$$\oint_{L_1} \vec{E}_1 \cdot d\vec{l} = \oint_{L_1} \frac{\vec{J}_1}{\gamma_1} \cdot d\vec{l} - \oint_{L_1} \vec{E}_{g1} \cdot d\vec{l} = - \frac{d\psi}{dt}$$

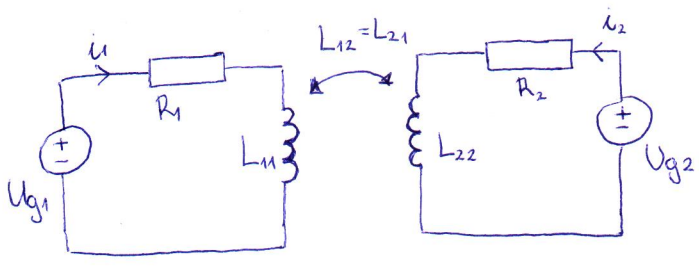
$R_1 i_1 - U_{g1} = -L_{11} i_1' - L_{12} i_2'$

$$R_1 \cdot i_1 + L_{11} \cdot i_1' + L_{12} \cdot i_2' = U_{g1}$$

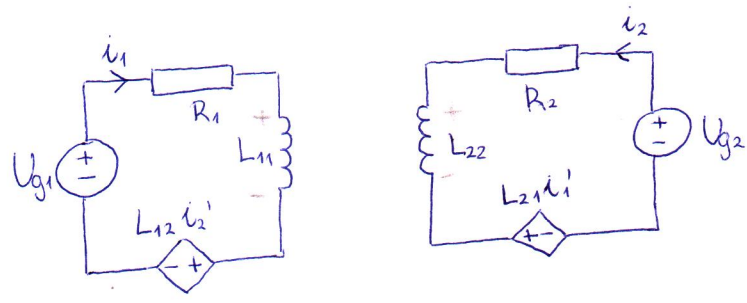
$$R_2 \cdot i_2 + L_{21} \cdot i_1' + L_{22} \cdot i_2' = U_{g2}$$



1) Modelno vezje 1:



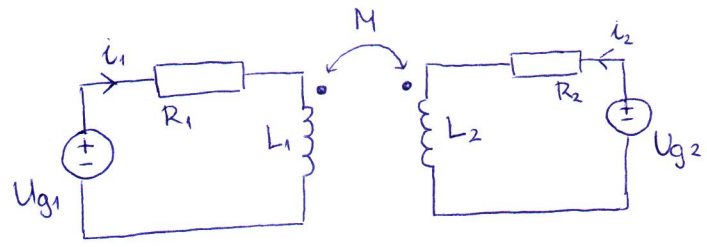
2) Modelno vezje 2:



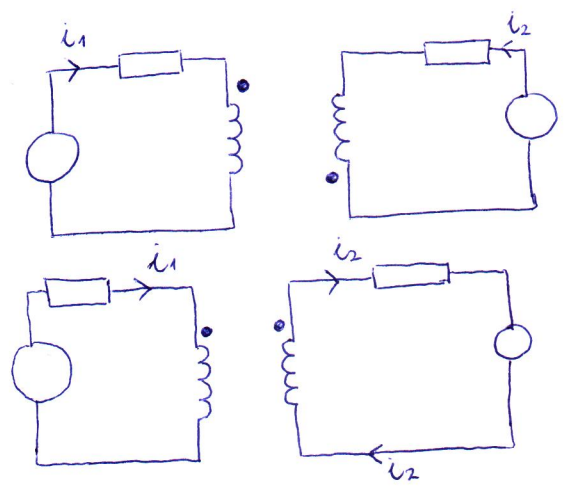
$L_{12} = L_{21} > 0$; podpirata
 < 0 ; nasprotujeta

$L_{11} = L_1$
 $L_{22} = L_2$
 $|L_{12}| = |L_{21}| = M$

Dogovor o pikah - 3:



$R_1 i_1 + L_1 \cdot i_1' + M i_2' = U_{g1}$
 $R_2 i_2 + M \cdot i_1' + L_2 i_2' = U_{g2}$

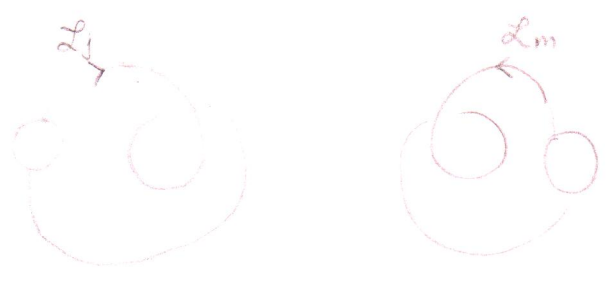


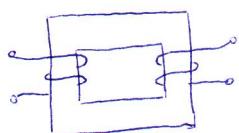
$R_1 i_1 + L_1 \cdot i_1' - M i_2' = U_{g1}$

Sklopni faktor

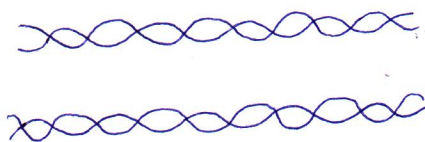
$K_{jm}^2 = \frac{\psi_m^{(j)} \cdot \psi_j^{(m)}}{\psi_m^{(m)} \cdot \psi_j^{(j)}}$

$K_{jm} = \frac{|L_{mj}|}{\sqrt{L_{mm} \cdot L_{jj}}}$



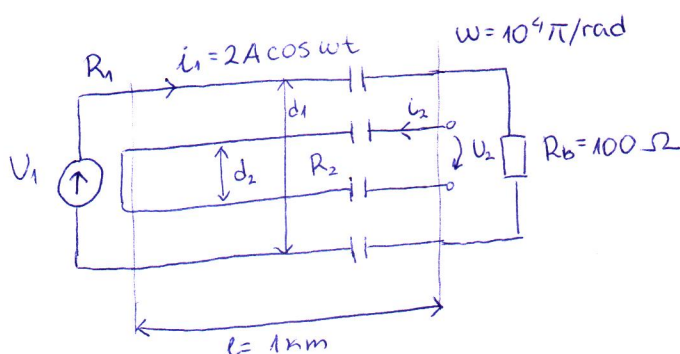


$K=1$



$K=0$

Zgled: DVOVODA



$r_0 = 1 \text{ mm}$

$\gamma_{cu} = 56 \cdot 10^6 \text{ S/m}$

$$R_1 = R_2 = \frac{2l}{\gamma_{cu} \cdot \pi r_0^2} = 11,4 \Omega$$

→ upornost žice

$$L_{11} = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{d_1}{r_0} \right) = 2,5 \text{ mH}$$

$$L_{22} = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{d_2}{r_0} \right) = 2,22 \text{ mH}$$

lastni induktivnosti
dvojvoda

$$L_{12} = L_{21} = \frac{\mu_0}{\pi} \ln \frac{(d_1 + d_2)/2}{(d_1 - d_2)/2} = 0,44 \text{ mH}$$

→ medsebojna
induktivnost

$$U_1 = (R_1 + R_b) i_1 + L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

$$U_2 = R_2 i_2 + L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt}$$

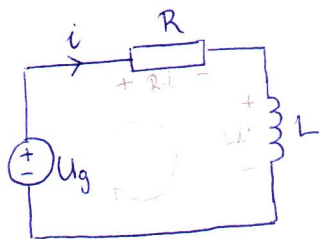
$$U_1 = (R_1 + R_b) i_1 + L_{11} \frac{di_1}{dt}$$

$$U_2 = L_{21} \frac{di_1}{dt}$$

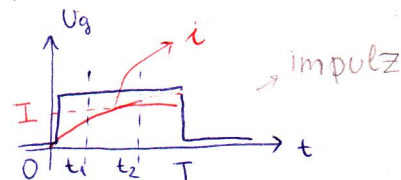
$$U_1 = \dots$$

$$U_2 = -L_{21} \cdot \omega \cdot 2A \sin \omega t = \underline{\underline{27,6 \text{ V} \cdot \sin \omega t}}$$

Energija magnetnega polja



1) $0 < t < T$



$$U_g = R \cdot i + L \cdot i'$$

$$U_g \cdot i = R \cdot i^2 + L \cdot i \cdot i' \quad \bigg/ \int_{t_1}^{t_2} dt$$

$$\int_{t_1}^{t_2} U_g \cdot i \cdot dt = \int_{t_1}^{t_2} R \cdot i^2 dt + \int_{t_1}^{t_2} L \cdot i \cdot \frac{di}{dt} dt$$

delo vira sproščena toplota žica energija mag. polja tuljave

$$A_g(t_1, t_2) = A_J(t_1, t_2) + \frac{L \cdot i^2(t_2)}{2} - \frac{L \cdot i^2(t_1)}{2}$$

(Jouleske izgube)

$$A_g(0, T) = A_J(0, T) + \underbrace{L \cdot \frac{I^2}{2}}_{\text{ENERGIJA V TULJAVI}}$$

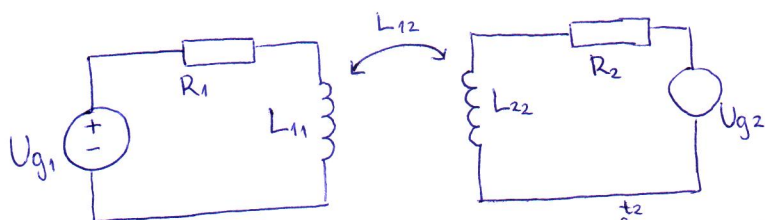
2) $t > T$ (izulop)

$$R \cdot i + L \cdot i' = 0$$

$$A_J = \frac{L}{2} I^2$$

Vsa v tuljavi akumulirana mag. energija se sprosti ob izulopu.

Energija mag. polja sklopljenih tuljav



$$A_{g1}(t_1, t_2) = A_{J1}(t_1, t_2) + \int_{t_1}^{t_2} (L_{11} \cdot i_1' + L_{12} \cdot i_2') i_1 \cdot dt$$

desni člen per partes
 $\int u dv = u \cdot v - \int v du$

$$\oplus \quad A_{g2}(t_1, t_2) = A_{J2}(t_1, t_2) + \dots$$

$$A_g(t_1, t_2) = A_J(t_1, t_2) + \frac{L_{11} i_1^2}{2} + \frac{L_{12} i_1 i_2}{2} + \frac{L_{21} i_1 i_2}{2} + \frac{L_{22} i_2^2}{2}$$

$$A_g(0, T) = A_J(0, T) + \frac{L_{11} I_1^2}{2} + L_{12} I_1 I_2 + \frac{L_{22} I_2^2}{2}$$

Energije magnetenja

$$W_m = \frac{1}{2} (L_{11} i_1^2 + L_{12} i_1 i_2 + L_{21} i_1 i_2 + L_{22} i_2^2)$$

$$W_m = \frac{1}{2} i_1 (\underbrace{L_{11} i_1 + L_{12} i_2}_{\Psi_1}) + \frac{1}{2} i_2 (\underbrace{L_{21} i_1 + L_{22} i_2}_{\Psi_2})$$

$$W_m = \frac{1}{2} i_1 \Psi_1 + \frac{1}{2} i_2 \Psi_2 \quad \rightarrow \text{za dve tuljavi}$$

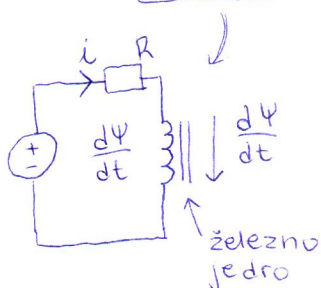
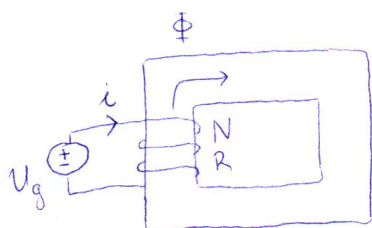
• Posplošitev za N-tuljav

$$W_m = \frac{1}{2} \sum_{k=1}^N i_k \Psi_k = \frac{1}{2} \sum_{k=1}^N \sum_{j=1}^N L_{kj} i_j i_k$$

$$\Psi_k = \sum_{j=1}^N \underbrace{\Psi_k^{(j)}}_{L_{kj} i_j}$$

$$L = \frac{\Psi}{I} \quad ; \quad L - \text{tudi kot energijski koeficient}$$

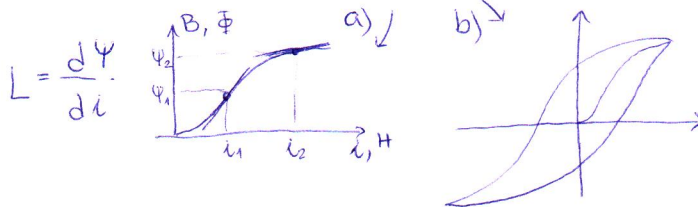
Magnetenje nelinearnih struktur



→ Linearost: linearno zvezo med:
 B, Φ, Ψ in i, Θ, H

-zrau: $B = \mu_0 H$

→ Nelinearna odvisnost



$$U_g = R \cdot i + \frac{d\Psi}{dt} \quad / \cdot i$$

$$U_g \cdot i = R \cdot i^2 + i \frac{d\Psi}{dt} \quad / \int_{t_1}^{t_2} dt$$

moč vira Joulske izgube navitja moč magnetenja

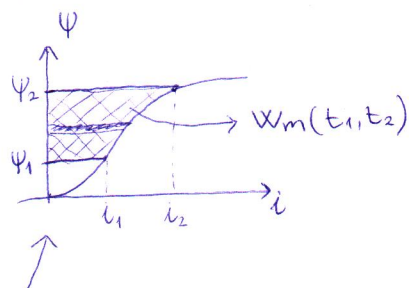
Energijski vloženi za magnetenje

$$\int_{t_1}^{t_2} i \frac{d\psi}{dt} dt = \int_{\psi_1}^{\psi_2} i d\psi = W_m(t_1, t_2)$$

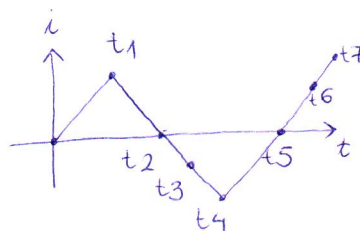
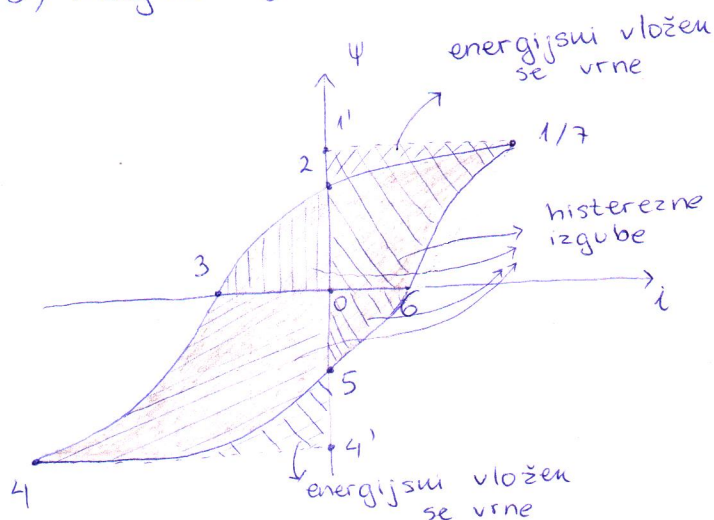
$$i_1 = i(t_1) ; \psi_1 = \psi(t_1)$$

$$i_2 = i(t_2) ; \psi_2 = \psi(t_2)$$

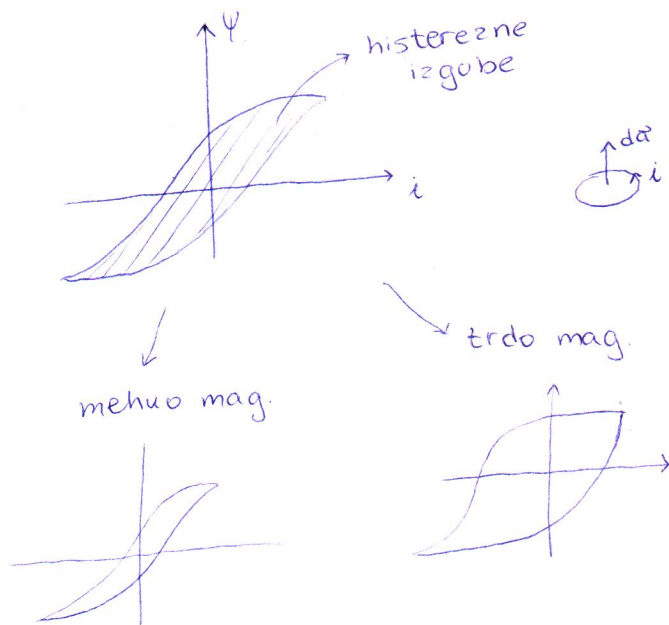
← a) Deležno magnetenje



b) magnetenju histereze



$\int_{t_1}^{t_2} i d\psi \propto 1, 1', 2$ $\downarrow \quad \downarrow$ $\oplus \cdot \ominus < 0$; generator gedro vrača energijo	$\int_{t_4}^{t_5} i \cdot d\psi \propto 4, 4', 5$ $\downarrow \quad \downarrow$ $\ominus \cdot \oplus < 0$; generator
$\int_{t_2}^{t_3} i \cdot d\psi \propto 2, 3, 0$ $\downarrow \quad \downarrow$ $\ominus \cdot \ominus > 0$; breme	$\int_{t_5}^{t_6} i \cdot d\psi \propto 5, 6, 0$ $\downarrow \quad \downarrow$ $\oplus \cdot \oplus > 0$; breme
$\int_{t_3}^{t_4} i \cdot d\psi \propto 3, 4, 4', 0$ $\downarrow \quad \downarrow$ $\ominus \cdot \ominus > 0$; breme	$\int_{t_6}^{t_7} i \cdot d\psi \propto 6, 7, 1', 0$ $\downarrow \quad \downarrow$ $\oplus \cdot \oplus > 0$; breme



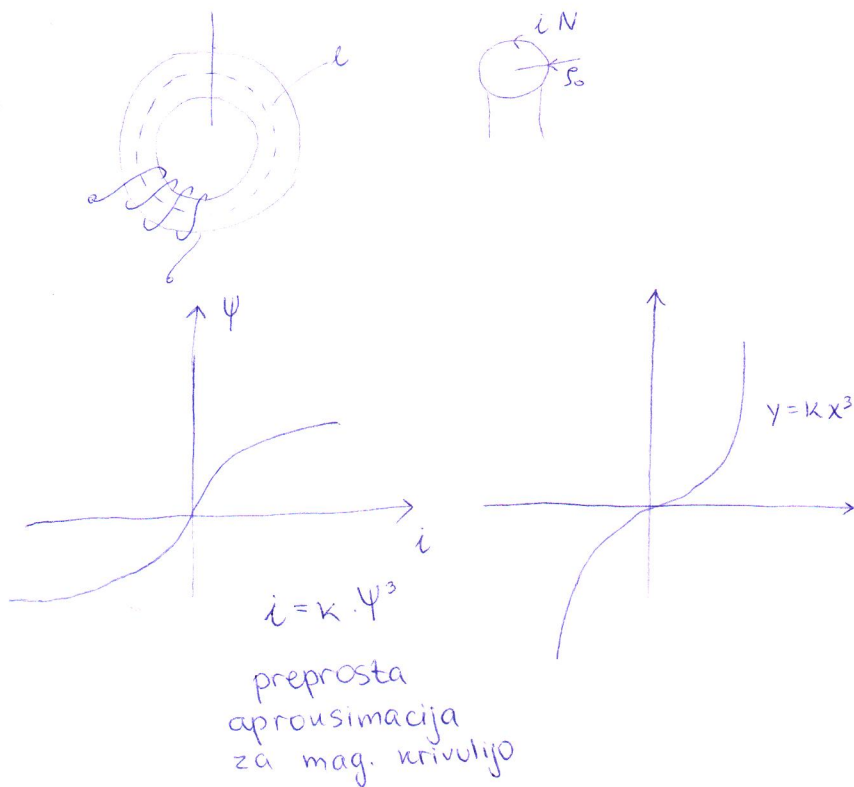
Mož histereznih izgub

$$P_h = \frac{1}{T} \oint_{\mathcal{H}} i \, d\Psi$$

$$P_h = f \oint_{\mathcal{H}} i \, d\Psi$$

pri $f = 50 \text{ Hz} \Rightarrow P_h = 0,5 \text{ W/kg}$

Posledica nelinearnosti



$$R = 0$$

$$u = \frac{d\Psi}{dt} = U_{gm} \cos \omega t$$

$$\Psi = \frac{U_{gm}}{\omega} \sin \omega t$$

$$i = \kappa \left(\frac{U_{gm}}{\omega} \right)^3 \sin^3 \omega t = C \cdot \sin^3 \omega t$$

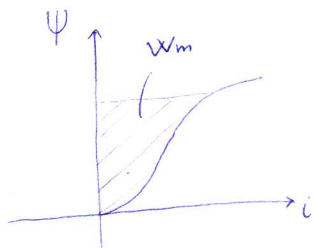
$$\left\{ \sin^3 x = \frac{3}{4} \sin x - \frac{1}{4} \sin 3x \right\}$$

$$\Rightarrow i = \frac{3}{4} C \sin \omega t - \frac{1}{4} C \sin 3\omega t$$

↑
osnovna
komponenta

↑
višje harmonike
→ posledica popačenj
zaradi nelinearnosti

(Volumsna) Gostota magnetne energije w_m



$$W_m = \int_0^t i \cdot d\psi$$

↓
 $Ni = \oint_L \vec{H} d\vec{l}$

↘ $d\psi = N \cdot d\Phi = N \cdot d\int_C \vec{B} \cdot d\vec{a}$

$$W_m = \int_0^t N i d\vec{B} \cdot d\vec{a} = \oint \int_V \vec{H} \cdot d\vec{B} \cdot dV$$

Vol celotne
strukture,
kjer je polje

↓
volumen

$$W_m = \int_0^t \vec{H} \cdot d\vec{B} \quad \rightarrow \text{volumsna gostota mag. energije}$$

$\int_V w_m \cdot dV$
 ↓
 volumen

• Za linearne strukture:

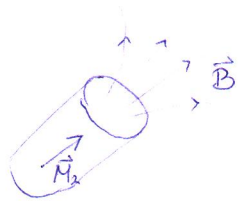
$$B = \mu_0 \mu_r H \rightarrow H = \frac{B}{\mu_0 \mu_r}$$

$$w_m = \int H dB = \int \frac{B \cdot dB}{\mu_0 \mu_r} = \frac{B^2}{2 \mu_0 \mu_r} = \frac{\mu_0 \mu_r H^2}{2}$$

$$\left\{ w_e = \frac{\epsilon_0 \epsilon E^2}{2} \right\}$$

Gibalni procesi v magnetizmu

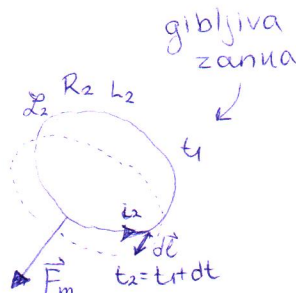
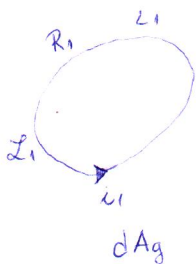
1) avtonomni sistemi = sistem brez virov



$$W_m = \frac{1}{2} \int \vec{B} \cdot \vec{H} dv \quad \text{volumen}$$

$$d\vec{F}_m = -\left(\frac{dW_m}{dx}, \frac{dW_m}{dy}, \frac{dW_m}{dz}\right)$$

2) sistem z viri



gibljiva zanka

$$Ag(t_1, t_1 + dt) = W_J(t_1, t_1 + dt) + \underbrace{\frac{1}{2} \sum_{k=1}^N I_k \Psi_k}_{dW_m} + \underbrace{\vec{F}_m \cdot d\vec{L}}_{\text{delo za premik}} \quad (*)$$

z vidna vezja:

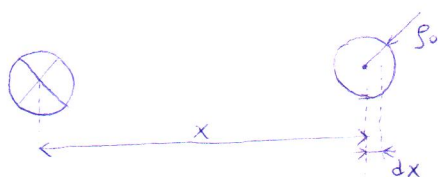
$$dAg = W_J(t_1, t_1 + dt) + \underbrace{\int_{t_1}^{t_1 + dt} \sum_{k=1}^N I_k d\Psi_k}_{\sum_{k=1}^N I_k d\Psi_k} = W_J(t_1, t_1 + dt) + \sum_{k=1}^N I_k d\Psi_k \quad (**)$$

$$\frac{1}{2} \sum_{k=1}^N I_k \cdot \Psi_k + \vec{F}_m \cdot d\vec{L} = \sum_{k=1}^N I_k \cdot \Psi_k$$

$$\vec{F}_m \cdot d\vec{L} = \frac{1}{2} \sum_{k=1}^N I_k \cdot \Psi_k = dW_m$$

$$\Rightarrow \vec{F}_m = \left(\frac{dW_m}{dx}, \frac{dW_m}{dy}, \frac{dW_m}{dz}\right)$$

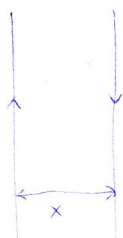
Zgled: Sila (dvoval d)



$$L = \frac{\mu_0 \cdot l}{\pi} \left(\frac{1}{4} + \ln \frac{x}{\rho_0}\right)$$

$$W_m = L \cdot \frac{i^2}{2} = \frac{\mu_0 \cdot l \cdot i^2}{2\pi} \left(\frac{1}{4} + \ln \frac{x}{\rho_0}\right)$$

$$F_{mx} = \frac{dW_m}{dx} = \frac{\mu_0 \cdot l \cdot i^2}{2\pi x}$$

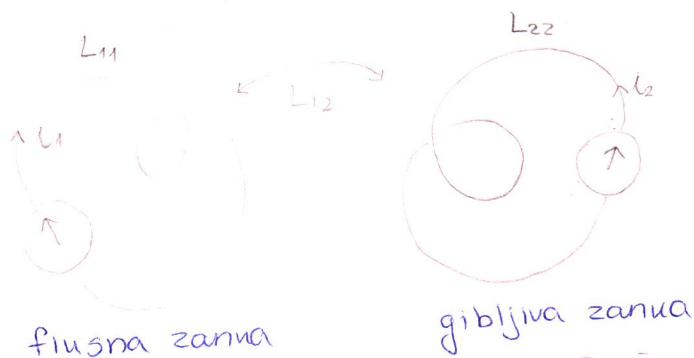


$$B = \frac{\mu_0 I}{2\pi x}$$

$$F_m = B \cdot i \cdot l$$

$$d\vec{F}_m = i d\vec{L} \times \vec{B}$$

Sistem dveh togih zank

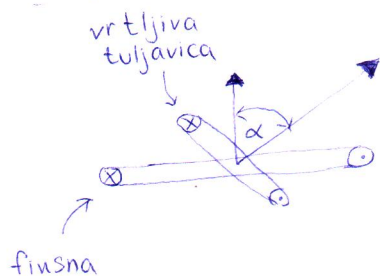


$$W_m = L_{11} \frac{i_1^2}{2} + L_{12} i_1 i_2 + L_{22} \frac{i_2^2}{2}$$

L_{12} se spreminja ob premiku
 $L_{11} = L_{22} = \text{const}$ (njuni odvod je = 0)
 le L_{12} imamo

$$\vec{F}_m = \left(\frac{dL_{12}}{dx}, \frac{dL_{12}}{dy}, \frac{dL_{12}}{dz} \right) \cdot i_1 i_2$$

odvod medsebojne induktivnosti



$$L_{12} = f(\alpha)$$

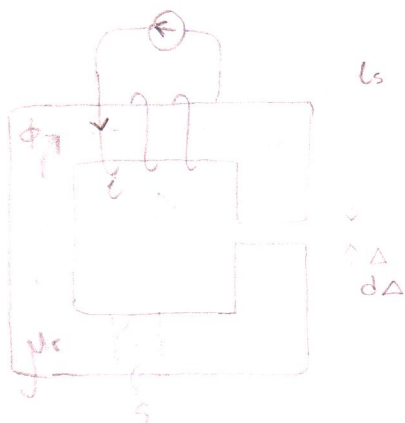
$$\alpha = 0: L_{12} = \max$$

$$\alpha = 90: L_{12} = 0$$

$$M = \frac{dL_{12}}{d\alpha} \cdot i_1 i_2$$

navor

Elektromagneti - ploskovne sile



$$W_m = \frac{1}{2} L \cdot i^2 = \frac{N^2 i^2}{2 \cdot R_m}$$

$$L = \frac{N^2}{R_m}$$

$$R_m = \frac{l_s}{\mu_0 \cdot \mu_r \cdot S} + \frac{\Delta}{\mu_0 \cdot S}$$

- pomeni privlek

$$F_m = \frac{dW_m}{d\Delta} = \frac{dW_m}{dR_m} \cdot \frac{dR_m}{d\Delta} = \frac{-N^2 i^2}{2 \cdot R_m^2} \cdot \frac{1}{\mu_0 S} = \frac{-\Phi^2}{2 \mu_0 S} = \frac{-B^2 \cdot S^2}{2 \mu_0 S} = \frac{-B^2 S}{2 \mu_0} = F_m$$

$$\Theta = N \cdot i$$

$$\Phi_m = \frac{\Theta}{R_m} = \int \vec{B} \cdot d\vec{a} = B \cdot S$$

homogeno polje v zračni reži

$$F_m = W_m \cdot S$$

tlau mag. sile

$$P = \frac{F}{S}$$

$$W_m = \frac{B^2}{2 \mu_0}$$

Zgled: (magnetna)

$$B_{\text{reže}} = 1 \text{ T}$$

$$w_m = \frac{1}{2 \cdot 4\pi \cdot 10^{-7}} \cdot \frac{\text{N}}{\text{m}^2} = 4 \cdot 10^5 \text{ Pa}$$

$$S = 1 \text{ dm}^2 = 10^{-2} \text{ m}^2$$

$$F_m = 400 \text{ N} \rightarrow 400 \mu\text{g}$$

$$w_m = \frac{B^2}{2\mu_0}$$

$$\frac{F_m}{F_e} = 10^4$$

(elektrostatna)

$$w_e = \frac{\epsilon_0 E^2}{2}$$

$$E = 3 \text{ MV/m}$$

$$\epsilon_0 = 8,85 \cdot 10^{-12}$$

$$w_e = 45 \text{ Pa}$$

p18(22.4.2016)

Revizija četrte Maxwellove enačbe

Razširjen Amperov zakon vrtinčnosti
magnetnega polja

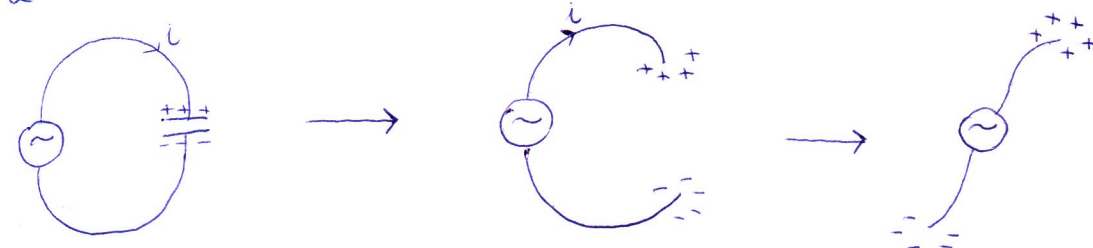
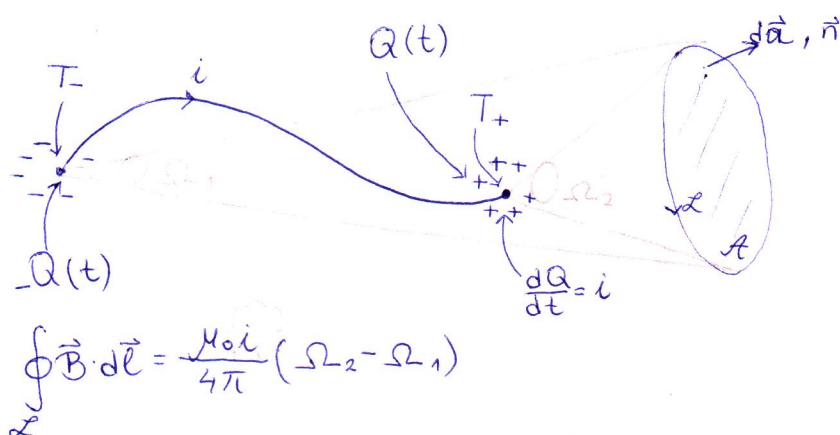
INDUKCIJA

$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0 + u_i \rightarrow \frac{d\psi}{dt}$$

MAXWELLOV TOK (preidualni tok)

$$\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \int_{\mathcal{A}} \vec{J}_{\text{prosti}} \cdot d\vec{a} + \dots$$

Vrtinčnost tokovne niti (posplošitev iz tokovne daljice)



+ kontinuitetna enačba

$$\oint_A \vec{J} \cdot d\vec{a} = -\frac{dQ_{\text{not}}}{dt} = i$$

tok ni odteka je enak zmanjšanja notranjega naboja

→ v naši primer tok priteka

i vstavimo v enačbo:

$$\oint_L \vec{B} \cdot d\vec{l} = \frac{\mu_0 \epsilon_0}{dt} \frac{d}{dt} \frac{Q}{4\pi \epsilon_0} (\Omega_2 - \Omega_1) =$$

$$= \mu_0 \epsilon_0 \frac{d}{dt} (\Phi_{e_2} - \Phi_{e_1}) = \mu_0 \epsilon_0 \left(\frac{d}{dt} \right) \int \vec{E} \cdot d\vec{a} = \mu_0 \epsilon_0 \left(\frac{d \Phi_e}{dt} \right)$$

$$\left\{ \oint_L \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt} \right\} \text{ indukcija (simetrija enačb)}$$

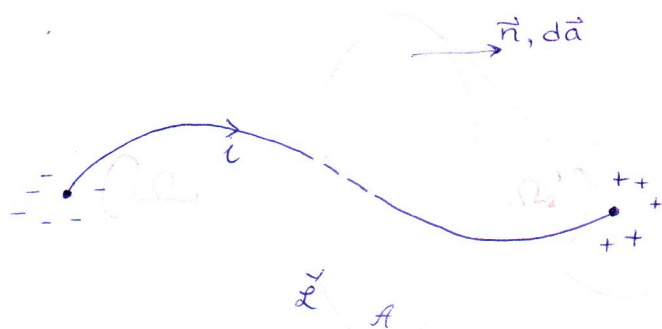
obstaja kadar imamo časovno spremembo (dinamika)

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_e}{dt}$$

vstavimo OE I

$$\Phi_e = \int_A \vec{E} \cdot d\vec{a} = \frac{Q}{4\pi \epsilon_0} \Omega$$

- Konstelacija kjer tok prebode A :



$$\Omega_2' = 4\pi - \Omega_2$$

$$\oint_L \vec{B} \cdot d\vec{l} = \frac{\mu_0 i}{4\pi} (\Omega_2 - \Omega_1) = \mu_0 i - \frac{\mu_0 i}{4\pi} (\Omega_2' + \Omega_1) = \mu_0 i + \mu_0 \epsilon_0 \left(\frac{d}{dt} \right) \int \vec{E} \cdot d\vec{a}$$

$$\frac{d}{dt} \mu_0 \epsilon_0 \int_A \vec{E} \cdot d\vec{a} \quad \leftarrow \vec{E}_+ + \vec{E}_-$$

$$\boxed{\int \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \epsilon_0 \int \frac{d\vec{E}}{dt} \cdot d\vec{a}}$$

IV Maxwellova enačba

magnetostatična

dodaten člen (dinamika)

enležen tok → ni Q → ni $\vec{E}$...

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \left(\underbrace{\vec{J}}_{\text{tok}} + \underbrace{\frac{d(\epsilon \vec{E})}{dt}}_{\text{"Maxwellov tok" (ni pravi tok)}} \right) d\vec{a}$$

Teorija Etra

tok

"Maxwellov tok"
(ni pravi tok)

$\vec{J}_{\text{max}}$ (ni gibanja nabojev)
(premikalni tok, tok odmika)

tok: gibanje nabojev

$\vec{J}$ - gostota toka

$\vec{J}_{\text{pe}}$ = polarizacijski tok; $\vec{J}_{\text{mag}}$ = magnetizacijski tok

$$\int_A \underbrace{\vec{J} \cdot d\vec{a}}_i = \int_A \vec{J}_{\text{prosti}} \cdot d\vec{a} + \underbrace{\int_A \frac{d\vec{P}}{dt} \cdot d\vec{a}}_{\vec{J}_{\text{pl.}}} + \underbrace{\int_L \vec{M} \cdot d\vec{l}}_{\text{v magnetnih}}$$

v diel. v magnetnih

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \left(\vec{J}_{\text{prosti}} + \frac{d\vec{P}}{dt} + \frac{d\epsilon_0 \vec{E}}{dt} \right) d\vec{a} + \mu_0 \oint \vec{M} \cdot d\vec{l} \quad /: \mu_0$$

$$\oint \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M} \right)}_{\vec{H}} \cdot d\vec{l} = \int_A \left(\vec{J}_{\text{prosti}} + \frac{d}{dt} (\epsilon_0 \vec{E} + \vec{P}) \right) d\vec{a}$$

$$\oint_L \vec{H} \cdot d\vec{l} = \int_A \left(\vec{J}_{\text{prosti}} + \frac{d\vec{D}}{dt} \right) d\vec{a}$$

P19 (3.5.2016)

ELEKTRIČNA VEZJA ČASOVNO SPREMENLJIVIH TOKOV

Označevanje veličin/veličin:

- časovno nespremenljive: $\vec{E}, \vec{B}, \phi, \theta, U, I$

- časovno spremenljive: $\vec{E}(t), \vec{B}(t), \phi(t), \theta(t)$

$$u = u(t) = f(t)$$

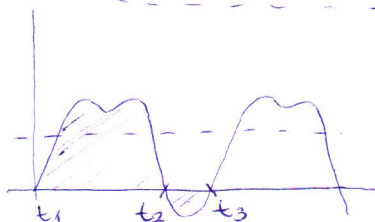
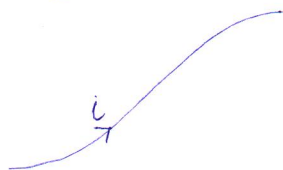
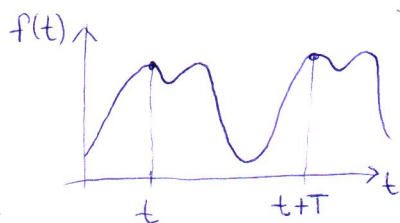
* neperiodične funkcije:

- prehodni pojavi
- šumi (bel šum)

* periodične funkcije:

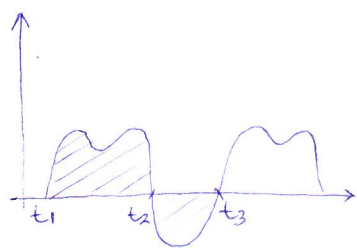
$f(t) = f(t + nT)$ → imamo novo funkcijo ki jo premaunimo
 neZ ← perioda za periodo

• pulzirajoče veličine



$$\int_{t_1}^{t_2} f(t) dt \neq \int_{t_2}^{t_3} f(t) dt$$

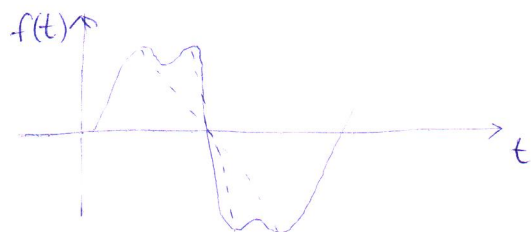
• izmenične veličine



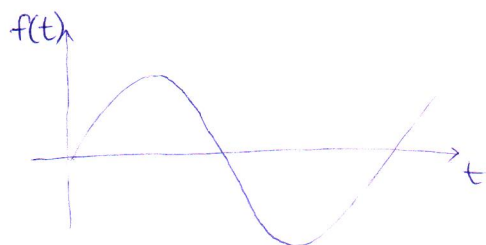
$$\left| \int_{t_1}^{t_2} f(t) dt \right| = \left| \int_{t_2}^{t_3} f(t) dt \right|$$

• izmenične

→ simetrične izmenične veličine



harmonične veličine: sinusne oz. kosinusne



freuvenca: f [Hz]

$$f = \frac{1}{T}$$

kotna hitrost: ω [rd/s]

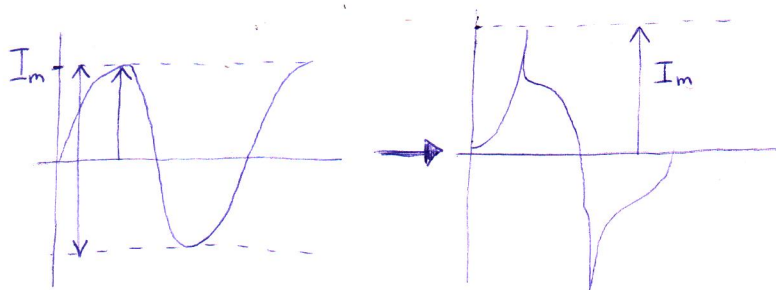
$$\omega = 2\pi \cdot f$$

Srednja vrednost: I_{sr}

$$I_{sr} = \frac{1}{T} \int_t^{t+T} i(t) dt = 0$$

(za izmenične količine,
harmonične)

Maksimalna vrednost: I_m



Prehodni pojavi

Odzivi na vstop/izstop

Elementi: -aktivne U_g, I_g

-pasivne K

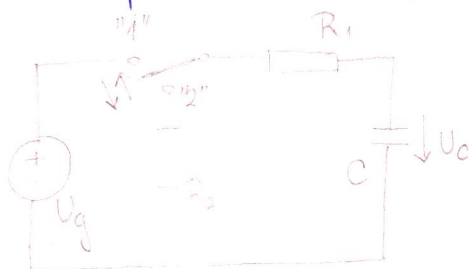
-reaktivni: $C, L \rightarrow$ "akumulabilnost" elementov

$$U = R \cdot i$$

$$i = C \cdot U_c'$$

$$U_L = L \cdot i'$$

1) Vstop in izstop RC vezja



Začetni pogoj:

$U_c(0) = U_0 \rightarrow$ začetna (pred)napetost
na kondenzatorju

vstop - C se polni

izstop - C se prazni

a) situacijo v položaju "1" $0 < t < t_1$

Kirchoffova napetostna enačba:

$$-U_g + R_1 \cdot i + U_c = 0$$

$$i = C \cdot U_c'$$

$$\underline{R_1 C \cdot U_c' + U_c = U_g} \rightarrow \text{Diferencialna enačba I reda}$$

$$R_1 C \cdot U_c' + U_c = 0 \rightarrow \text{homogen del}$$

+
particularna/konkretna rešitev

vedno rabimo začetni
pogoj v dif. enačbo

$$\text{Rešujemo: } R_1 C \cdot U_c' + U_c = U_g \text{ in } U_c(0) = U_0$$

homogen del:

$$R_1 C \cdot U_{ch}' + U_{ch} = 0$$

$$R_1 C \cdot A \lambda e^{\lambda t} + A e^{\lambda t} = 0$$

$$\underbrace{(R_1 C \lambda + 1)}_{=0} \underbrace{A e^{\lambda t}}_{\neq 0} = 0$$

$$R_1 C \lambda = -1$$

$$\lambda = -\frac{1}{R_1 C} = -\frac{1}{\tau} \rightarrow \text{časovna konst. } \tau$$

$$U_{ch} = A \cdot e^{\lambda t} \quad \begin{matrix} A=? \\ \lambda=? \end{matrix}$$

$$U_{ch} = A e^{-\frac{t}{R_1 C}}$$

particularna rešitev

po dolgem času $i \rightarrow 0$
 $u_c' \rightarrow 0$

$$u_{cp} = +U_g$$

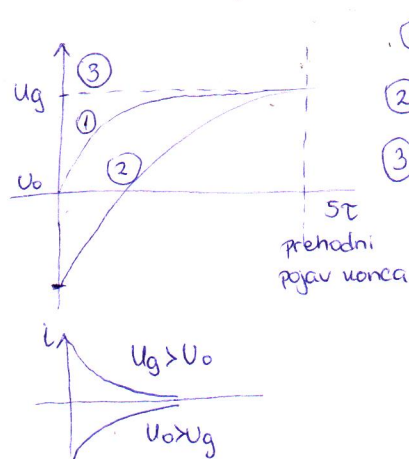
$$u_c = u_{ch} + u_{cp} = A e^{-\frac{t}{\tau_{1c}}} + U_g$$

uporabimo začetni pogoj

$$u_c(0) = U_0 = A e^0 + U_g \rightarrow A = U_0 - U_g$$

$$u_c = (U_0 - U_g) e^{-\frac{t}{\tau_{1c}}} + (U_g - U_0) + U_0$$

$$u_c = U_0 + (U_g - U_0)(1 - e^{-\frac{t}{\tau_{1c}}})$$



① $U_0 = 0$

② $U_0 = -U_g$

③ $U_0 = U_g$

$$i = C \cdot u_c' = C \frac{U_g - U_0}{\tau} e^{-t/\tau}$$

$$= \frac{U_g - U_0}{R_1} e^{-t/\tau}$$

2) stikalo v položaj "2" $t \gg t_1$

$$u_1 = u_c(t_1 - 0) = U_0 + (U_g - U_0) e^{-t_1/\tau_1}$$

$$(R_1 + R_2) i + u_c = 0, \quad i = C \cdot u_c'$$

$$(R_1 + R_2) C \cdot u_c' + u_c = 0$$

$$R_{12} C \lambda + 1 = 0$$

$$\lambda = -\frac{1}{R_{12} C}, \quad R_{12} C = \tau_{12}$$

$$u_c = B e^{-t/\tau_{12}}$$

$$u_c(t_1) = u_1 = B e^{-t_1/\tau_{12}}$$

$$B = u_1 e^{t_1/\tau_{12}} \Rightarrow u_c(t \gg t_1) = u_1 e^{-(t-t_1)/\tau_{12}}$$

Efektivna vrednost veličine I_{ef}

$$P = \frac{1}{T} \int_t^{t+T} p(t) dt = R \cdot \frac{1}{T} \int_t^{t+T} i^2(t) dt = R I_{ef}^2; \quad I_{ef} = \sqrt{\frac{1}{T} \int_t^{t+T} i^2(t) dt}$$

za harmonične:

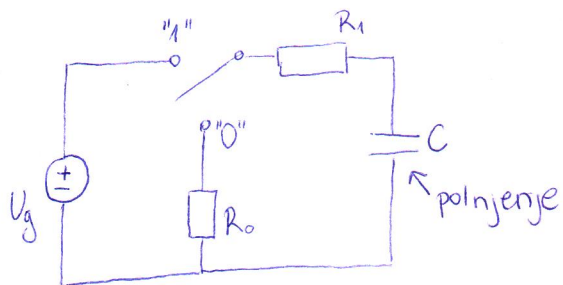
$$I_{ef} = \frac{I_m}{\sqrt{2}}$$

Faktor oblike, terenski faktor

$$\frac{I}{I_{gr}}$$

$$\frac{I_m}{I}$$

za harmonične $\sqrt{2}$



||

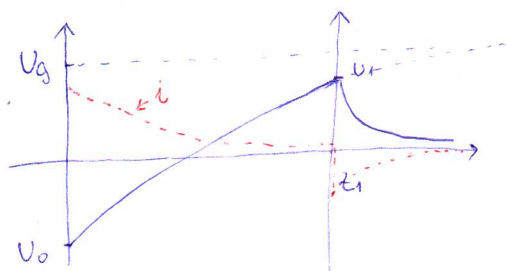
$$U_{ch}(t_1) = U_1 = A \cdot e^{-\frac{t_1}{R_{10} \cdot C}}$$

$$A = U \cdot e^{\frac{t}{R_{10} \cdot C}}$$

||

$$U_{ch} = U_1 \cdot e^{-\frac{(t-t_1)}{R_{10} \cdot C}}$$

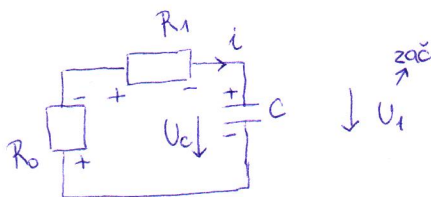
časovni zamik
praznjenja
za čas t_1



b) Položaj "0" $t_1 < t$

$$U_c(t_1) = (U - U_g) e^{-\frac{t_1}{C}} + U_g = U_1 < U_g$$

↑ začetna vrednost
(po preklopa)



$$R_1 \cdot i + U_g + R_0 \cdot i = 0; \quad i = C \cdot U_c'$$

$$(R_1 + R_0) \cdot C \cdot U_c' + U_c = 0$$

$$R_{10} \cdot C \cdot U_c' + U_c = 0 \rightarrow \text{lim. dif. en. I reda homogena}$$

$$U_{ch} = A \cdot e^{\lambda t} = A \cdot e^{-\frac{t}{R_{10} \cdot C}}$$

$$U_{ch}' = A \lambda e^{\lambda t}$$

$$(R_{10} \cdot C \cdot \lambda + 1) A e^{\lambda t} = 0$$

$$\lambda = -\frac{1}{R_{10} \cdot C}$$

$$i = C \cdot U_c'$$

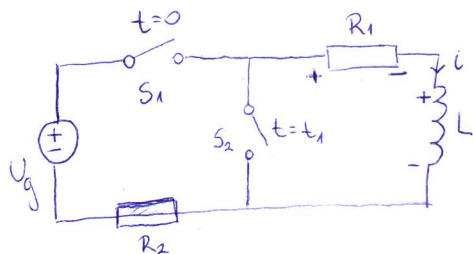
$$i = C \cdot \left(-\frac{1}{R_{10} \cdot C}\right) U_1 e^{-\dots} = -\frac{1}{R_{10}} \cdot U_1 \cdot e^{-\frac{(t-t_1)}{R_{10} \cdot C}}$$

Vklon in izklop RL vezga

$$W_e = \frac{C \cdot U^2}{2} \rightarrow \text{napetost na kond. je zvezna f.}$$

$$W_m = \frac{L \cdot i^2}{2} \rightarrow \text{tok skozi tuljavo} \quad -||-$$

↑
zvezna f.



1) $0 < t < t_1$ vnaprej S_1

$$R_1 i + U_L + R_2 i = U_g \quad ; \quad U_L = L \cdot i'$$

$$(R_1 + R_2) i + L i' = U_g$$

$$R_{12} i + L i' = U_g \quad \text{lin. dif. en. I reda, nehomog.}$$

$$+ i(0) = 0 \quad \text{zač. pogoji}$$

homogen del:

$$(R_{12} \lambda + L) \cdot A \cdot e^{\lambda t} = 0$$

$$i = A \cdot e^{\lambda t} \Rightarrow A e^{-\frac{L}{R_{12}} t}$$

$$i' = A \cdot \lambda e^{\lambda t}$$

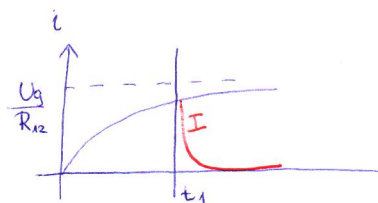
$$\lambda = -\frac{L}{R_{12}} = -\frac{1}{\tau}$$

$$\tau = \frac{R_{12}}{L} [s] \quad \text{prehodni čas v času } 5\tau$$

particularna rešitev:

$$i(\infty) = \dot{I}_p = \frac{U_g}{R_{12}}$$

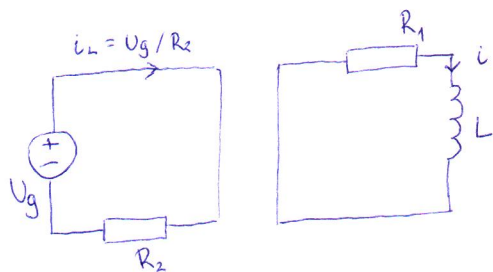
$$i = \underbrace{A \cdot e^{-\frac{L}{R_{12}} t}}_{\text{homogena}} + \underbrace{\frac{U_g}{R_{12}}}_{\text{partiu.}} = \frac{U_g}{R_{12}} \left(1 - e^{-\frac{L}{R_{12}} t} \right)$$



$$i(0^-) = i(0^+) = 0 = A e^0 + \frac{U_g}{R_{12}}$$

$$A = -\frac{U_g}{R_{12}}$$

2) izključitev tuljave



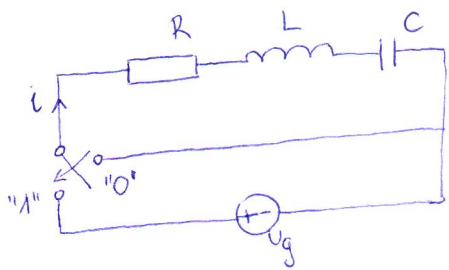
$$R_1 i + L \cdot i' = 0$$

lin. dif. en. I reda (homog.)

$$+ i(t_1) = I_1$$

$$i(t > t_1) = I_1 e^{-\frac{t-t_1}{\tau_1}} \quad ; \quad \tau = \frac{R_1}{L}$$

Vulop in izulop RLC vezja



$$\left. \begin{array}{l} U_C(0^-) = 0 \\ i(0^-) = 0 \end{array} \right\} \begin{array}{l} \text{dva} \\ \text{zacetna} \\ \text{pogoja} \end{array}$$

1) $0 < t < t_1$ (polodny "1")

$$V_R + V_L + V_C = V_g \quad / \frac{d}{dt}$$

$$R \cdot i' + L \cdot i'' + \frac{i}{C} = 0$$

$$i'' + \frac{R}{L} i + \frac{i}{LC} = 0$$

$$U_R = R \cdot i$$

$$V_L = L \cdot i'$$

$$V_c = \frac{1}{C} \int i dt \quad (i = C \cdot V_c')$$

$$U_c' = \frac{i}{c}$$

Lin. dif. en. II reda
(homog.)

$$i = A e^{\lambda t}$$

$$i' = A\lambda e^{\lambda t}$$

$$i'' = A\lambda^2 e^{\lambda t}$$

$$\left(\lambda^2 + \frac{R}{L} \lambda + \frac{1}{LC}\right) A e^{\lambda t} = 0$$

karan. polinom

$$\lambda_{1/2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$P = \frac{R}{2L} \quad ; \quad Q^2 = \left(\frac{R}{2L} \right)^2 - \frac{1}{LC}$$

$$\left. \begin{array}{l} 1) \quad g^2 > 0: \quad \lambda_1 = -p - g \\ \quad \quad \quad \lambda_2 = -p + g \end{array} \right\} \quad \left. \begin{array}{l} 2) \quad g^2 = 0: \quad \lambda_1 = \lambda_2 = -p \end{array} \right\} \quad \left. \begin{array}{l} 3) \quad g^2 < 0: \\ \quad \quad \quad \lambda_1 = -p - ig \\ \quad \quad \quad \lambda_2 = -p + ig \end{array} \right\}$$

$$1) \quad i = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \quad + \quad i(0^-) = 0 \quad + \quad V_c(0^-) = 0$$

$$i = \frac{U_0/L}{\lambda_1 - \lambda_2} (e^{\lambda_1 t} + e^{\lambda_2 t}) = -\frac{U_0/L}{2 \cdot g} (e^{-pt-2t} - e^{-pt+2t}) =$$

$$= -\frac{V_0}{2L} e^{-pt} \left(\frac{e^{-2t} - e^{2t}}{2} \right) = -\frac{V_0}{2L} e^{-pt} \operatorname{sh}(2t)$$

2)

$$i = \frac{V_g}{L} \cdot t \cdot e^{-pt}$$

(rešitev dobimo s L'Hospitalovo pravilo)

$$\lim_{t \rightarrow 0} \frac{\sinh t}{t} = 1$$

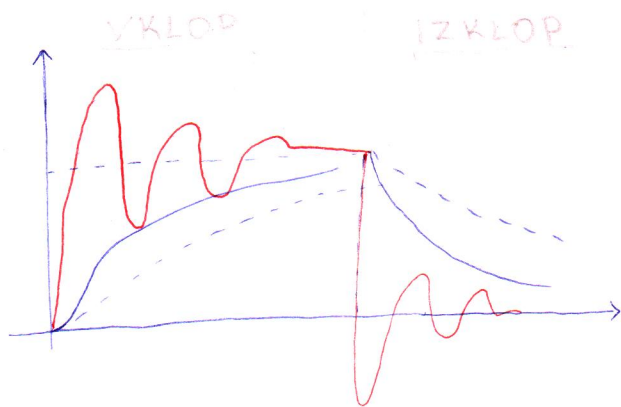
3)

$$i = \frac{U_0 e^{-pt}}{|q|L} \sin |q|t = \frac{U_0}{|q|L} e^{-pt} \sin(qt)$$



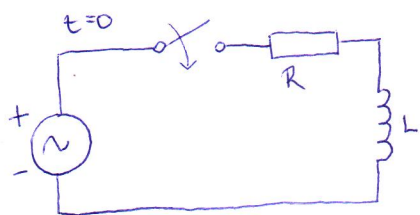
RLC - nihajni krog

$$\left\{ \begin{aligned} \sinh x &= \frac{e^{ix} - e^{-ix}}{2i} \cdot i = i \sin x \end{aligned} \right\}$$



P21 (10.5.2016)

Vulop RL vezja na harmoničen vir



$$U_g = U_{gm} \cdot \cos(\omega t + \varphi_u)$$

$$U_L = L \cdot i'$$

$$U_R = R \cdot i$$

začetni
fazni uot

nap: φ_u

tok: φ_i

razlika faz:

$$\varphi_u - \varphi_i = \varphi$$

$$\varphi_i = \varphi_u - \varphi$$

1) Vulop ($t > 0$)

$$U_R + U_L = U_g$$

→

$$R \cdot i + L \cdot i' = U_{gm} \cos(\omega t + \varphi_u) \rightarrow \text{Nehomogena lin. dif. en. I reda}$$

homogen del:

$$(i' + \frac{R}{L} i) = 0 \rightarrow i = A e^{\lambda t} = A e^{-\frac{R}{L} t}$$

$$(\lambda + \frac{R}{L}) A e^{\lambda t} = 0$$

$$\lambda = -\frac{R}{L} = -\frac{1}{\tau}$$

$$i' = A \lambda e^{\lambda t}$$

partikularna rešitev:

amplituda

$$i_p = I_m \cos(\omega t + \varphi_i)$$

fazni zamik

$$i_p = I_m \cos(\omega t + \varphi_u - \varphi)$$

$$i_p' = -\omega I_m \sin(\omega t + \varphi_u - \varphi)$$

$$R \cdot I_m \cos(\omega t + \varphi_u - \varphi) - \omega L I_m \sin(\omega t + \varphi_u - \varphi) = U_{gm} \cos(\omega t + \varphi_u)$$

$$\begin{cases} \sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y \\ \cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y \end{cases}$$

$$R I_m \cos \varphi \cdot \cos(\omega t + \varphi_u) + R I_m \sin \varphi \sin(\omega t + \varphi_u) = \dots$$

$$\dots - \omega L I_m \cos \varphi \sin(\omega t + \varphi_u) + \omega L I_m \sin \varphi \cos(\omega t + \varphi_u) =$$

$$= U_{gm} \cos(\omega t + \varphi_u) + 0 \cdot \sin(\omega t + \varphi_u)$$

$$- \omega L I_m \cos \varphi + R \cdot I_m \sin \varphi = 0 \quad : (-\cos \varphi)$$

$$\omega L I_m = R I_m \cdot \tan \varphi$$

$$\omega L I_m \sin \varphi + R \cdot I_m \cos \varphi = U_{gm}$$

$$\tan \varphi = \frac{\omega L}{R}$$

fazni zamik φ !

obe enačbi kvadriramo in seštejemo:

$$\omega^2 L^2 I_m^2 + R^2 I_m^2 = U_{gm}^2$$

amplituda

$$I_m = \frac{U_{gm}}{\sqrt{R^2 + \omega^2 L^2}} \rightarrow \text{abs. vrednost impedance}$$

$$\Rightarrow i_p = \frac{U_{gm}}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t + \varphi_u - \arctg \frac{\omega L}{R})$$

$$i = i_h + i_p$$

za določitev A:

$$i(0^+) = i(0^-) = 0 = A e^{-\frac{R}{L}t} + I_m \cos(\omega t + \varphi_u - \varphi)$$

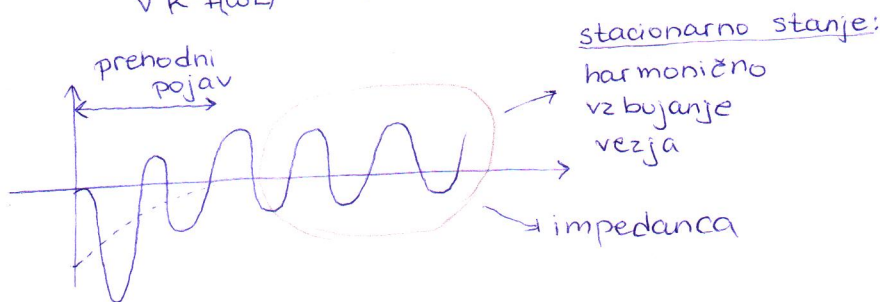
$$= A + I_m \cos(\varphi_u - \varphi) = 0$$

$$A = -I_m \cos(\varphi_u - \varphi)$$

rešitev:

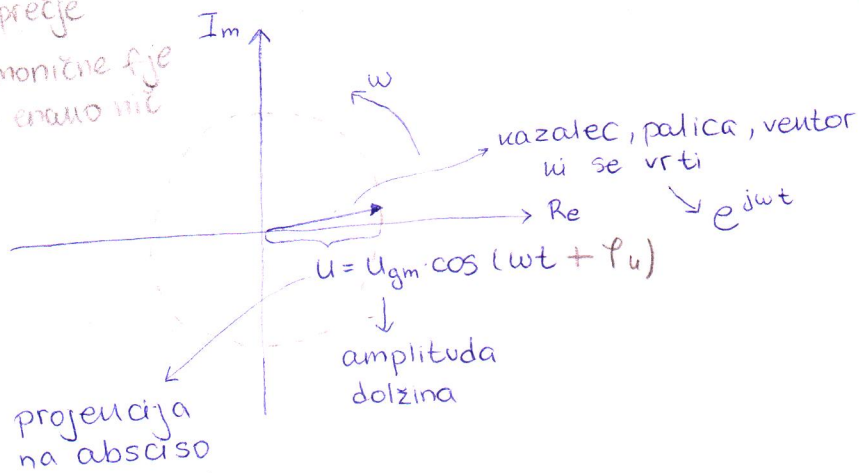
$$i(t) = -I_m \cos(\varphi_u - \varphi) e^{-\frac{R}{L}t} + I_m \cos(\omega t + \varphi_u - \varphi)$$

$$I_m = \frac{U_{gm}}{\sqrt{R^2 + (\omega L)^2}} ; \varphi = \arctg \frac{\omega L}{R} \quad (\text{fazni zamik})$$



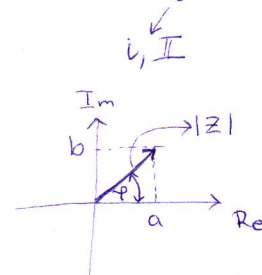
Harmonično vzbujana el. vezja

povprečje
harmonične fje
je enako nič



• vstopamo v cplx prostor

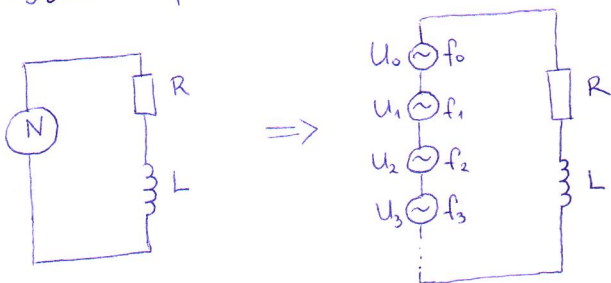
$$\underline{Z} = a + jb = |Z| \cdot e^{j\varphi}$$



2) Odvod in integral sta harmonični funkciji

$$u = L \cdot i' \quad i = C \cdot u'$$

3) Vsota periodičen signal je vsota harmoničnih



Uporaba cplx računa pri harmoničnih vezjih

$$e^{j\alpha} = \cos \alpha + j \sin \alpha$$

$$\underline{U} = U_m \cdot e^{j(\omega t + \varphi_u)} = U_m \cdot \cos(\omega t + \varphi_u) + j U_m \cdot \sin(\omega t + \varphi_u)$$

kompleksni zapis
veličine (napetosti)

funkcija v časovnem
prostoru $u(t) = \text{Re}(\underline{U})$ (1)

★ če je vzbujanje harmonična funkcija z frekvenco ω ,
je tudi odziv harmonična funkcija z frekvenco ω .

$$\underline{U} = U_m \cdot e^{j\varphi_u} \cdot e^{j\omega t} \rightarrow \text{opuščamo pisanje vrtenja kompleksorjev}$$

$$(1) \quad \underline{U} = U_m \cdot e^{j\varphi_u}$$

Kirchoffova zakona v Cplx

$$\sum_{k=1}^n I_k = 0$$

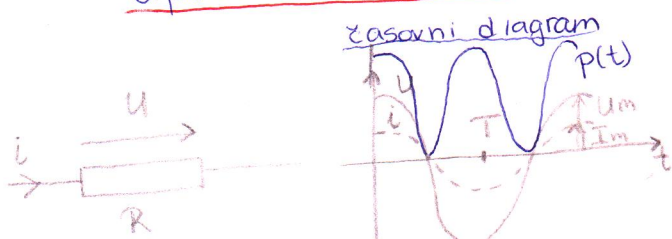
$$\sum_{k=1}^n i_k = 0 \Rightarrow \sum_{k=1}^n I_k \cdot \cos(\omega t + \varphi_k) = \operatorname{Re} \sum_{k=1}^n \vec{I}_k e^{j\omega t} = \sum_{k=1}^n \operatorname{Re}(I_k \cdot \cos(\omega t + \varphi_k)) \Rightarrow$$

$$\sum_{k=1}^n \underline{I}_k = 0$$

$$\Rightarrow \sum_{k=1}^n \underline{U}_k = 0$$

P22 (13.5.2016)

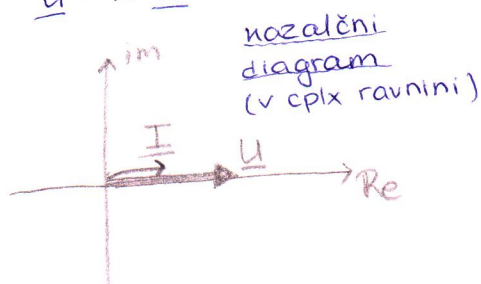
Upornost priključena na harmonični vir



$$u = U_m \cos \omega t \rightarrow \underline{u} = U_m e^{j\varphi_u} = 0$$
$$i = \frac{u}{R} = I_m \cos \omega t \quad \underline{I} = I_m e^{j\varphi_i} = 0$$

za upor velja
 $\varphi = \varphi_u - \varphi_i = 0$

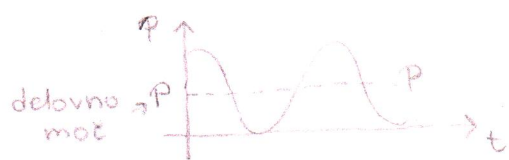
$$\underline{U} = R \cdot \underline{I}$$



Trenutno moč

$$p = u \cdot i = U_m \cdot I_m \cdot \cos^2 \omega t = U_m \cdot I_m \cdot \frac{1 + \cos 2\omega t}{2} = \frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} (1 + \cos 2\omega t)$$

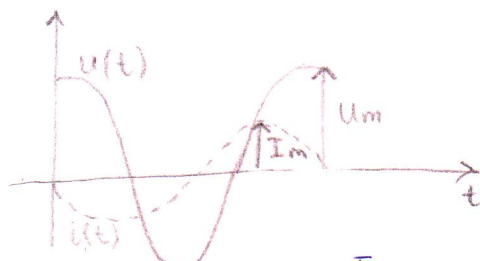
dvojno frekvenco



- energija prihaja "v paketih"
- frekvenca utripanja 100 Hz

$$\bar{P} = \frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = U_{\text{ef}} \cdot I_{\text{ef}} = P$$

Kondenzator, priključen na harm. vir



za C: $\varphi = \varphi_u - \varphi_i = -\frac{\pi}{2}$

tou prehiteva napetost
za 90°

$$u = U_m \cdot \cos \omega t \Rightarrow \underline{u} = U_m e^{j\varphi_u = 0}$$

$$i = \frac{dQ}{dt} = C \frac{du}{dt}$$

$$= -\omega C U_m \sin \omega t$$

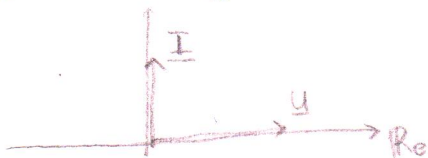
$$= \omega C U_m \cdot \cos(\omega t + \frac{\pi}{2})$$

$$\underline{I} = \omega C \cdot U_m \cdot e^{j(\varphi_u + \frac{\pi}{2})} = \omega C \cdot \underbrace{e^{j\frac{\pi}{2}}}_j \cdot \underbrace{U_m e^{j\varphi_u}}_{\underline{u}}$$

$$\underline{I} = j\omega C \cdot \underline{u}$$

namesto odvoda, množenje z "jw"

Kazalčni diagram:



$$p(t) = U_m \cdot \cos \omega t \cdot (-I_m) \cdot \sin \omega t =$$

$$= -\frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \sin 2\omega t$$

↳ niha z dvojno frekvenco
↳ povp. vrednost enaka 0

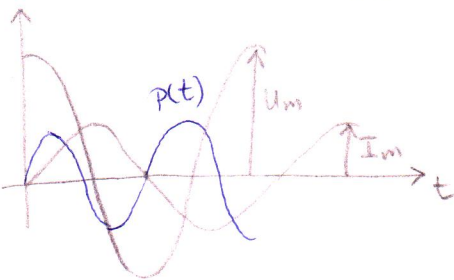
Tuljava, priključena na harm. vir



za L velja:

$$\varphi = \varphi_u - \varphi_i = \frac{\pi}{2}$$

napetost prehiteva
tou za 90°



$$u = U_m \cdot \cos \omega t \Rightarrow \underline{u} = U_m e^{j\varphi_u = 0}$$

$$u = L \frac{di}{dt} \rightarrow i = \frac{1}{L} \int u dt = \frac{U_m}{\omega L} \sin \omega t =$$

$$= \frac{U_m}{\omega L} \cdot \cos(\omega t - \frac{\pi}{2})$$

$$\underline{I} = \frac{U_m}{\omega L} e^{j(\varphi_u - \frac{\pi}{2})} \rightarrow \varphi_i = -\frac{\pi}{2}$$

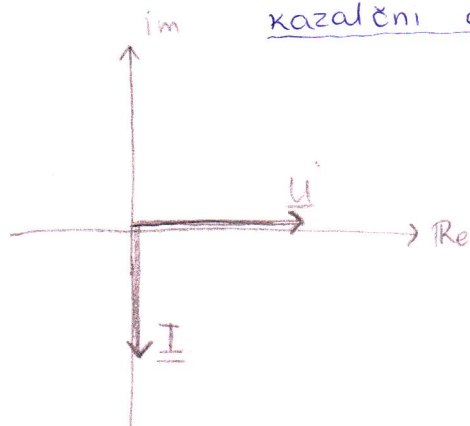
$$\underline{I} = \frac{U_m \cdot e^{j\varphi_u}}{\omega L} \underbrace{e^{-j\frac{\pi}{2}}}_{-j = \frac{1}{j}} = \frac{\underline{u}}{j\omega L}$$

$$\underline{I} = \frac{\underline{u}}{j\omega L}$$

$$\underline{u} = j\omega L \cdot \underline{I}$$

integral v časovnem prostoru
→ deljenje z "jw" v cplx prostoru

kazalčni diagram:



Sulopljeni tuljavi priključeni na harm. vir

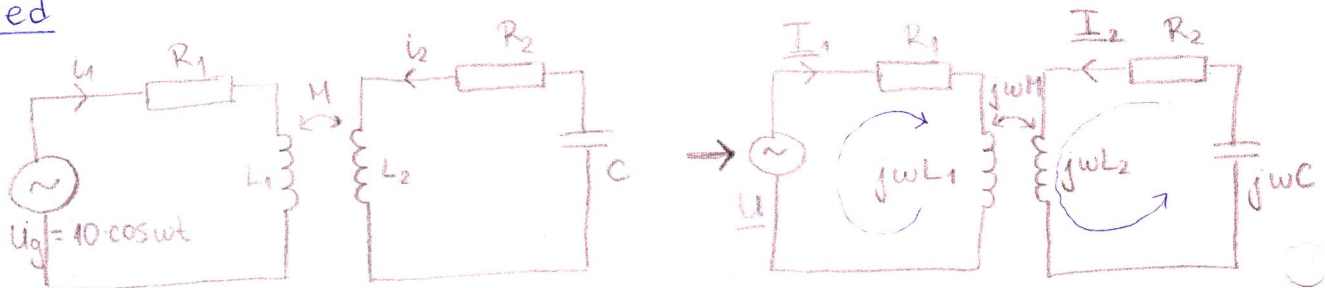
$$U_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \rightarrow \underline{U}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$$

$$U_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \rightarrow \underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$$

velja za vse časovne oblike

velja za harmonične veličine

Zgled



$$U_g = 10V$$

$$L_1 = 4mH$$

$$\gamma_u = 0$$

$$\omega = 1kHz$$

$$L_2 = 1mH$$

$$R_1 = 4\Omega$$

$$K = 1; M = 2mH$$

$$R_2 = 1\Omega$$

$$C = 1mF$$

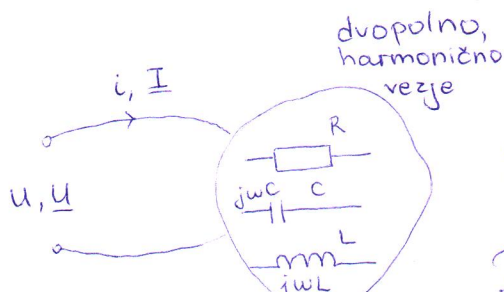
$$\begin{cases} (R_1 + j\omega L_1) \underline{I}_1 + j\omega M \underline{I}_2 = \underline{U} \\ j\omega M \underline{I}_1 + (R_2 + j\omega L_2 + \frac{1}{j\omega C}) \underline{I}_2 = 0 \end{cases}$$

$$(4 + j4) \underline{I}_1 + j2 \underline{I}_2 = 10$$

$$j2 \underline{I}_1 + \underline{I}_2 = 0$$

$$\Rightarrow \begin{aligned} \underline{I}_1 &= 1 - \frac{j}{2} = 1,12 A e^{-j26,6^\circ} \rightarrow \underline{i}_1 = 1,12 \cos(\omega t - 0,46) A \\ \underline{I}_2 &= -1 - j2 = 2,24 A e^{j243^\circ} \rightarrow \underline{i}_2 = 2,24 \cos(\omega t + 4,25) A \end{aligned}$$

Impedanca oz. admitanca



N časovnem prostoru

$\frac{U(t)}{i(t)}$ - ne pomeni ničesar

Def:

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} [\Omega] \text{ Impedanca}$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{\underline{I}}{\underline{U}} \text{ Admitanca}$$

$$\underline{Z} = \text{Re}(\underline{Z}) + j\text{Im}(\underline{Z})$$

	$\underline{Z}$	$\underline{Y}$
upor	R	$\frac{1}{R} = G$
kond.	$\frac{1}{j\omega C}$	$j\omega C$
tuljava	$j\omega L$	$\frac{1}{j\omega L}$

Zaporedna in vzporedna vezava impedance

Zaporedna

1) $\underline{Z}_{nad} = \sum_{i=1}^n \underline{Z}_i$



$$\underline{Z} = R + j\omega L$$

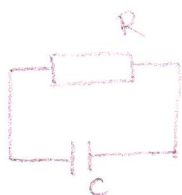


$$\underline{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Vzporedna

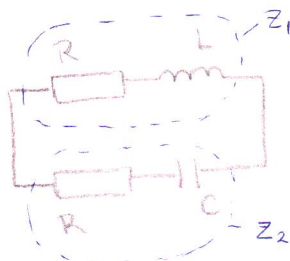
1) $\underline{Y}_{nad} = \underline{Y}_1 + \underline{Y}_2 \Rightarrow \frac{1}{\underline{Z}_{nad}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} \Rightarrow \underline{Z}_{nad} = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$

2)



$$\underline{Y}_{nad} = \frac{1}{R} + j\omega C$$

3)



$$\underline{Y}_{nad} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}$$

$$\underline{Y}_{nad} = \frac{1}{R + j\omega L} + \frac{1}{R + \frac{1}{j\omega C}}$$

$$\frac{1}{a + jb} = \frac{a - jb}{(a + jb)(a - jb)}$$

$$i \quad \frac{1}{Z \cdot e^{j\varphi}} = \frac{1}{Z} e^{-j\varphi}$$

impedanca: $\underline{Z} = R + jX$

resistanca: R

reaktanca: X

$$\begin{aligned} +jX_L &= j\omega L \\ -jX_C &= \frac{1}{j\omega C} = -j\frac{1}{\omega C} \end{aligned}$$

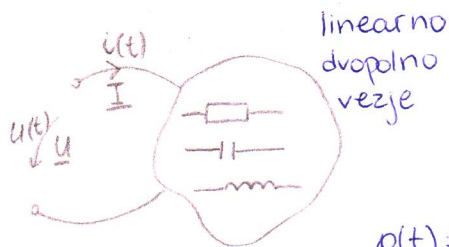
$$\underline{Y} = G + jB$$

admitanca

konduktanca

susceptanca

($\neq$ susceptibilnost λ_e, λ_m)

Kompleksor moči

$$p(t) = u(t) \cdot i(t) = U_m \cos(\omega t + \varphi_u) \cdot I_m \cos(\omega t + \varphi_i)$$

$$\{\cos \alpha \cdot \cos \beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))\}$$

$$p(t) = \frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} (\underbrace{\cos(\varphi_u - \varphi_i)}_{\varphi} + \underbrace{\cos(2\omega t + \varphi_u + \varphi_i)}_{*})$$

→ niha z dvojno frekvenco

moč razdelimo na $p_1(t)$ - nenegativni del
 $p_2(t)$ - jalov del

razvijmo * izraz:

$$\frac{U_m I_m}{2} \cos(2\omega t + \varphi_u + \varphi_i) = \frac{U_m I_m}{2} \cos(2\omega t + 2\varphi_u - \varphi) =$$

$$\varphi_u - \varphi_i = \varphi \Rightarrow \varphi_i = \varphi_u - \varphi$$

$$= \frac{U_m I_m}{2} \cos(2(\omega t + \varphi_u) - \varphi) = \frac{U_m I_m}{2} \cos \varphi \cdot \cos 2(\omega t + \varphi_u) + \frac{U_m I_m}{2} \sin \varphi \cdot \sin 2(\omega t + \varphi_u)$$

$$p(t) = \underbrace{\frac{U_m I_m}{2} \cos \varphi (1 + \cos 2(\omega t + \varphi_u))}_{p_1(t)} + \underbrace{\frac{U_m I_m}{2} \sin \varphi \cdot \sin 2(\omega t + \varphi_u)}_{p_2(t)}$$

p_1 - nenegativni del

$p_2(t)$

$$\boxed{P = \overline{P}_1 = \frac{U_m I_m}{2} \cos \varphi} \rightarrow \text{delovna moč (aktivna)}$$

$$\boxed{\overline{P}_2 = 0}$$

$$\boxed{\frac{U_m I_m}{2} \sin \varphi = Q} \rightarrow \text{jalova (reaktivna) moč}$$

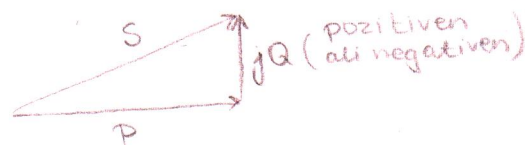
• Definirajmo kompleksor moči

$$\boxed{\underline{S} = P + jQ}$$

navidezna moč

[VA] (kompleksna)

→ jalova moč [VAR], [var]
 → delovna moč [W]



$$\underline{S} = P + jQ = \frac{U_m I_m}{2} \cos \varphi + j \frac{U_m I_m}{2} \sin \varphi =$$

$\varphi = \varphi_u - \varphi_i$

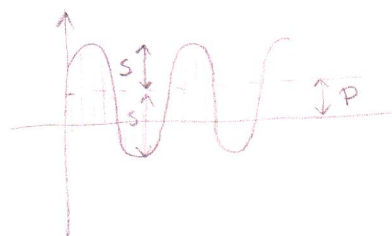
$$= \underbrace{\frac{1}{2} U_m e^{j\varphi_u}}_{\underline{U}} \cdot \underbrace{\left(\frac{1}{2} I_m e^{-j\varphi_i} \right)}_{\underline{I}^*}$$

konjugirano

$$\begin{cases} \underline{z} = a + jb \\ \overline{\underline{z}} = a - jb \end{cases}$$

$$\underline{S} = \frac{1}{2} \underline{U} \cdot \underline{I}^* = \frac{1}{2} \underline{z} \cdot \underline{I}^2 = \frac{1}{2} \underline{Y} \cdot \underline{U}^2$$

$$S = \frac{U_m I_m}{2}$$

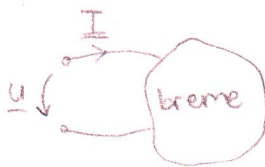


Zgled:

$$\begin{aligned} U &= 250V \\ I &= 4A \end{aligned} \quad \left. \begin{array}{l} \text{max} \\ \text{vrednosti} \end{array} \right\}$$

$$\varphi = 30^\circ \rightarrow \cos 30^\circ = \frac{\sqrt{3}}{2} \quad (*)$$

faktor delavnosti



če je $\cos \varphi = 1 \Rightarrow$ ohmsko breme

$\cos \varphi > 0 \Rightarrow$ INDUKTIVNO BREME

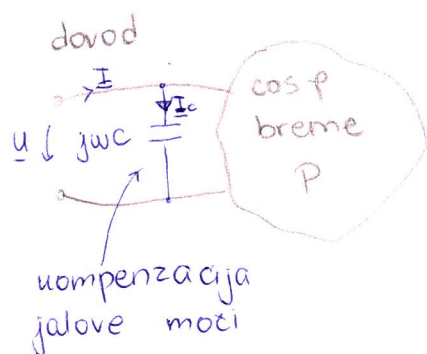
$$P = \frac{U \cdot I}{2} \cdot \cos \varphi = 433W$$

$$S = P + jQ = (433 + j250)VA$$

$$Q = \frac{U \cdot I}{2} \sin \varphi = 250var$$

$$S = \frac{U \cdot I}{2} = 500VA$$

Kompenzacija jalove moči



$$\begin{aligned} P &= 2kW \\ \cos \varphi &= 0,8 \\ U &= 230V \\ f &= 50Hz \end{aligned}$$

$$\rightarrow S = \frac{P}{\cos \varphi} = 2,5kVA$$

$$Q = S \cdot \sin \varphi = \sqrt{S^2 - P^2} = 1,5kvar$$

* prenos jalove moči:
izmenjava energije med virom in (L/C) in nazaj.

* To niso izgube!

* Zaradi tega v dovodu teče večji tok ki mu rečemo (I_c);

* Dovod ima upornost: R
zaradi $R \cdot I_c^2 \rightarrow$ dodatne izgube na dovodu.

$$I_c = j\omega C \cdot U$$

$$S = \frac{1}{2} U \cdot \bar{I} = \frac{1}{2} U (\bar{I} - j\omega C \bar{U})$$

$$S = P + (jQ - j\omega C U^2) \Rightarrow \text{kompenzacija}$$

$\downarrow$
jalova moč bremena

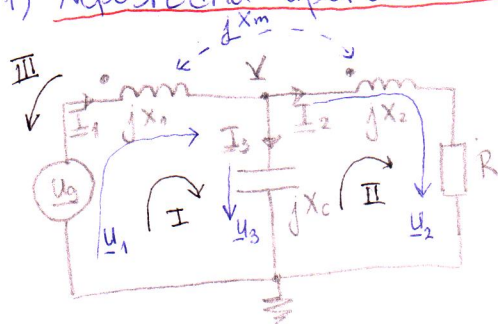
$\rightarrow Q=0$

$$Q = \omega C U^2$$

$$C = \frac{Q}{\omega U^2} = 98,6 \mu F$$

jw-metoda

količina operacija	čas prostor	cpłx prostor
tok	$i(t)$	$\underline{I}$
napetost	$u(t)$	$\underline{U}$
impedanca	$\underline{\quad}^*$	$\underline{Z}$
moč	$p(t) = u(t) \cdot i(t)$	$S = \frac{1}{2} \underline{U} \bar{\underline{I}} = P + jQ$
produkt	$a \cdot i(t)$	$a \cdot \underline{I}$
odvod	di/dt	$j\omega \underline{I}$
integral	$\int i dt$	$\underline{I}/j\omega$
časovni zamik	$i(t+t_1)$	$\underline{I} \cdot e^{j\omega t_1}$

1) Neposredna uporaba K.Z.

podatki vezja

$x_1 = 20 \Omega$

$x_2 = 20 \Omega$

$x_m = 10 \Omega$

$x_c = 20 \Omega$

$R = 6 \Omega$

$U_g = 150 \text{ V}$

točnovni KZ: $-\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$

napetostni KZ: $\underline{U}_1 + \underline{U}_3 = 0$

$\underline{U}_2 - \underline{U}_3 = 0$

$$\begin{cases} \underline{U}_1 = -U_g + jX_1 \underline{I}_1 + jX_m \underline{I}_2 \\ \underline{U}_2 = +jX_2 \underline{I}_2 + jX_m \underline{I}_1 + R \underline{I}_2 \\ \underline{U}_3 = -jX_c \underline{I}_3 \end{cases} \rightarrow \text{lahko izrazimo tudi tako}$$

$\underline{I}_1 = \frac{\underline{U}_1 + U_g - jX_m \underline{I}_2}{jX_1}$

$\underline{I}_2 = \frac{\underline{U}_2 - jX_m \underline{I}_1}{jX_2}$

$\underline{I}_3 = \frac{jU_g}{X_c}$

$-\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$

$jX_1 \underline{I}_1 + jX_m \underline{I}_2 - jX_c \underline{I}_3 = U_g$

$jX_m \underline{I}_1 + (R + jX_2) \underline{I}_2 + jX_c \underline{I}_3 = 0$

$$\rightarrow \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix} = \begin{bmatrix} \dots \\ \dots \\ \dots \end{bmatrix} \rightarrow \text{dobimo } \underline{I}_1, \underline{I}_2, \underline{I}_3$$

2) Zančni toki (isto vezje)

$\underline{I}_I$ in $\underline{I}_{II}$ (dve neznanu) $j(X_1 - X_c) \underline{I}_I + jX_c \underline{I}_{II} = U_g - jX_m \underline{I}_{II}$

$\underline{I}_1 = \underline{I}_I$

$jX_c \underline{I}_I + (R + j(X_2 - X_c)) \underline{I}_{II} = -jX_m \underline{I}_I$

$\underline{I}_2 = \underline{I}_{II}$

$\underline{I}_3 = \underline{I}_I - \underline{I}_{II}$

$0 \cdot \underline{I}_I + j30 \underline{I}_{II} = 150$

$j30 \underline{I}_I + 6 \underline{I}_{II} = 0$

$\underline{I}_I = 1 \text{ A}$

$\underline{I}_{II} = -j5 \text{ A}$

3) Spojiščni potenciali

(ena sama enačba, neposredno izrazimo V)

$V =$

Stavki (teoremi) za reševanje harmoničnih vezij

1) Staveu superpozicije

a) koherentni vir $\rightarrow$ v cplx prostor

b) nekoherentni viri $\rightarrow$ v časovnem prostoru

- vire vezja deaktiviramo



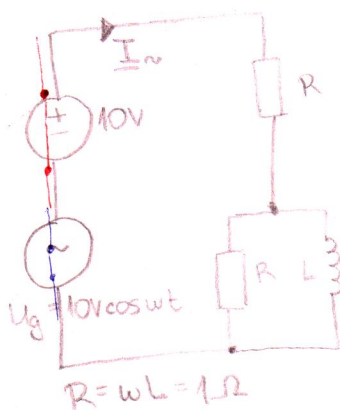
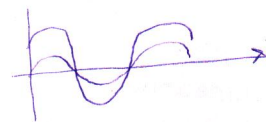
- po en vir vložimo, rešimo, iskana veličina je vsota analiziranih veličin



$$\underline{I} = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 + \dots$$

• harmonična napetost z''

$\rightarrow$ dodana enosmerna komponenta



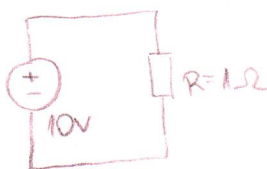
I) Za harmonični del:

$$\underline{Z} = R + \frac{R \cdot j\omega L}{R + j\omega L} = \frac{3 + j}{2}$$

$$\underline{I}_n = \frac{\underline{U}_g}{\underline{Z}} = 2 \cdot \frac{10e^{j0}}{3 + j} = (6 + j2) \text{ A}$$

$$i_n = \sqrt{40} \cos(\omega t - \arctan \frac{1}{3}) = 6,32 \text{ A} \cdot \cos(\omega t - 0,32)$$

II za enosmerno komponento



$$I_+ = \frac{10\text{V}}{1\Omega} = 10 \text{ A}$$

$$i = I_+ + i_n = (10 + 6,32 \cos(\omega t - 0,32)) \text{ A}$$

$$P = \bar{P} = \overline{(U_0 + U_g)(I_+ + i_n)} = \underbrace{\overline{U_0 I_+}}_{\substack{\downarrow \\ \text{enosm.} \\ \text{moč}}} + \underbrace{\overline{U_0 i_n}}_{\substack{\downarrow \\ \cos x = 0 \text{ !!}}} + \underbrace{\overline{I_+ U_g}}_0 + \underbrace{\overline{U_g i_n}}_{\substack{\downarrow \\ \text{izmenična} \\ \text{moč}}}$$

$$P = P_+ + P_n \Rightarrow \frac{U_0^2}{R} = 100 \text{ W} = P_+$$

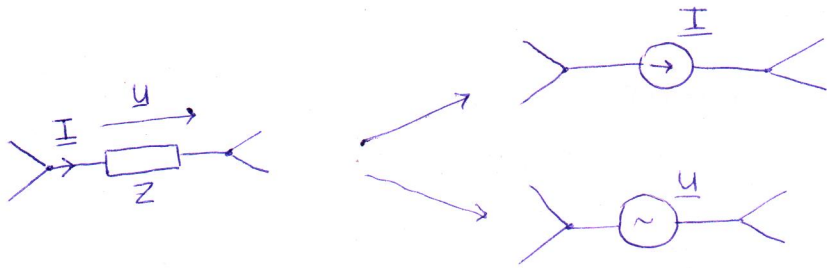
$$\Rightarrow P = 130 \text{ W}$$

$$\frac{1}{2} U_{gm} I_{nm} \cos \varphi = \frac{1}{2} 10\sqrt{20} = 30 \text{ W} = P_n$$

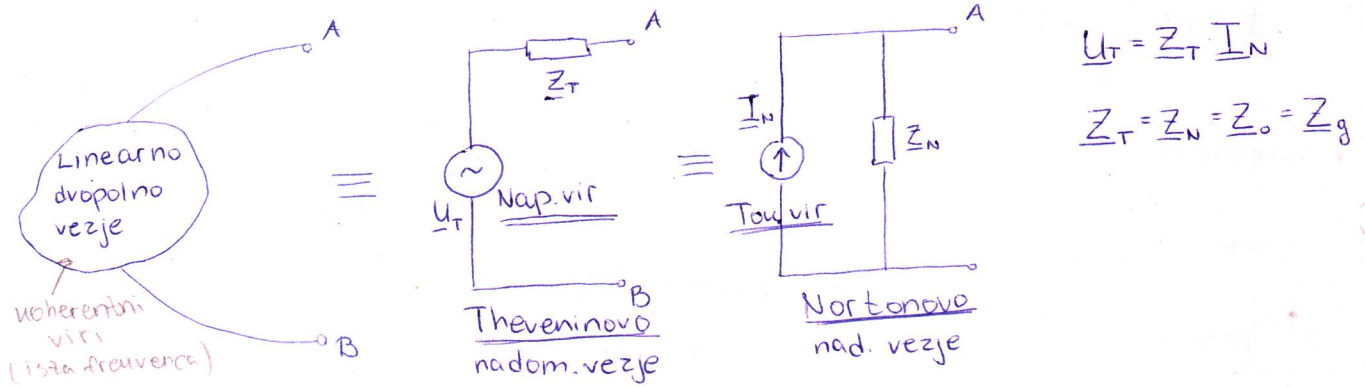
$$P = \text{Re}(\underline{S})$$

$$\rightarrow \frac{1}{2} \underline{U} \cdot \underline{I}^*$$

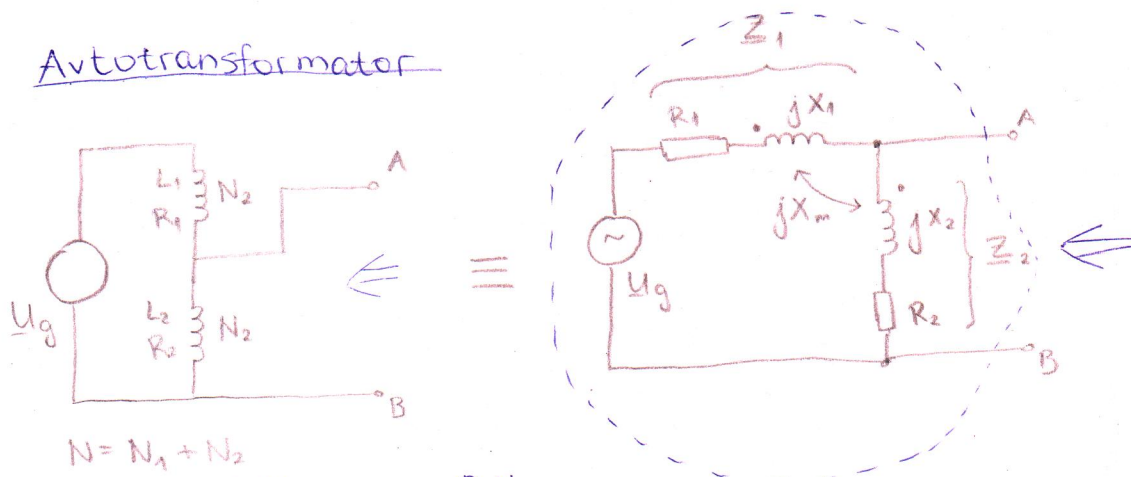
2) Staveu o nadomestitvi



Staveu Theveninga in Nortona



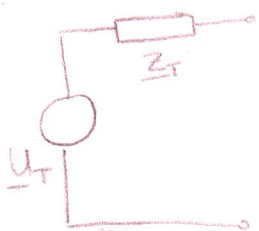
Autotransformator



$$N = N_1 + N_2$$

$$R \rightarrow R_1 = \frac{R \cdot N_1}{N} ; R_2 = \frac{R \cdot N_2}{N}$$

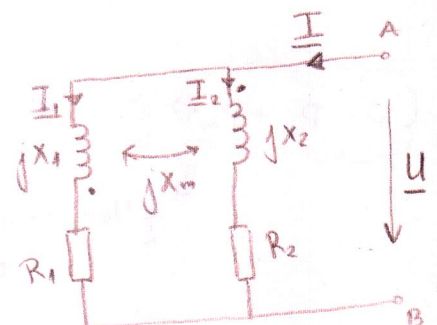
$$L \rightarrow L_1 = \frac{L N_1^2}{N^2} ; L_2 = \frac{L N_2^2}{N^2} ; K \rightarrow 1 ; M = \sqrt{L_1 L_2} = L \frac{N_1 N_2}{N^2}$$



$$\underline{U}_T = \frac{\underline{U}_g}{\underline{Z}_1 + \underline{Z}_2 + jX_m} (\underline{Z}_2 + jX_m)$$

$\underline{Z} \rightarrow$ deaktiviramo vire

$$\underline{Z}_T = \frac{\underline{U}}{\underline{I}}$$



$$\underline{I} = \underline{I}_1 + \underline{I}_2$$

leva veja: $\underline{U} = \underline{Z} \cdot \underline{I}_1 - jX_m \underline{I}_2$

desna veja: $\underline{U} = -jX_m \underline{I}_1 + \underline{Z}_2 \underline{I}_2$

$\Rightarrow$

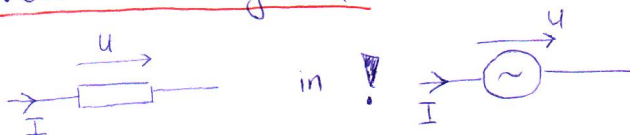
$$\underline{I}_1 = \frac{\underline{Z}_2 + jX_m}{\underline{Z}_1 \underline{Z}_2 + X_m^2} \underline{U}$$

$$\underline{I}_2 = \frac{\underline{Z}_1 + jX_m}{\underline{Z}_1 \underline{Z}_2 + X_m^2} \underline{U}$$

rešitev:

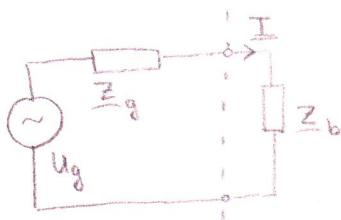
$$\underline{Z}_T = \frac{\underline{Z}_1 \underline{Z}_2 + X_m^2}{\underline{Z}_1 + \underline{Z}_2 - 2jX_m}$$

4) Staven Tellegena



$$\sum_{j=0}^N \frac{1}{2} \underline{U}_j \underline{I}_j^* = 0$$

5) Staven največje moči



$$\underline{I} = \frac{\underline{U}_g}{\underline{Z}_g + \underline{Z}_b} = \frac{\underline{U}_g}{(R_g + R_b) + j(X_g + X_b)}$$

$$P = \operatorname{Re} \left[\frac{1}{2} \underline{Z}_b \underline{I}^2 \right] = \frac{\underline{U}_g}{2} \cdot \frac{R_b}{(R_g + R_b)^2 + (X_g + X_b)^2}$$

$$\frac{\partial P}{\partial R_b} = \dots = 0 \rightarrow R_b = R_g$$

$$\frac{\partial P}{\partial X_b} = \dots = 0 \rightarrow X_b = -X_g$$

$$\underline{Z}_b = \overline{\underline{Z}_g}$$

$$P_{b, \max} = \frac{\underline{U}_{gm}^2}{8 \cdot R_g}$$

- Poseben primer - realno breme

$$R_b = |\underline{Z}_g| = \sqrt{R_g^2 + X_g^2}$$

$$P_{b, \max} = \frac{\underline{U}_{g \max}^2}{4(R_g + |\underline{Z}_g|)}$$

6) Staven recipročnosti

$$\left. \frac{\underline{U}_1}{\underline{I}_2} \right|_{\underline{I}_1=0} = \left. \frac{\underline{U}_2}{\underline{I}_1} \right|_{\underline{I}_2=0}$$

$$\begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$

P25(24.5.2016)

Transformator

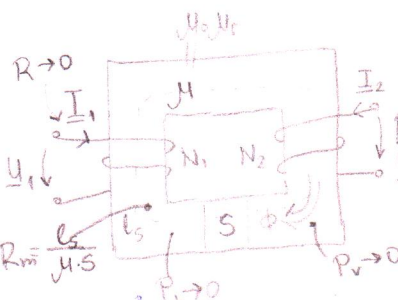
A) idealiziran (brez izgub), $k=1$

B) idealni ($\mu \rightarrow \infty$)

c) realni

Jouleske izgube (upornosti)
Kožni efekt (časovne dinamice)
Histerezne izgube
Vrtinčne izgube

A)



$$L_1 = \frac{N_1^2}{R_m}; L_2 = \frac{N_2^2}{R_m}$$

$$M = \frac{N_1 N_2}{R_m}; M = \sqrt{L_1 L_2}$$

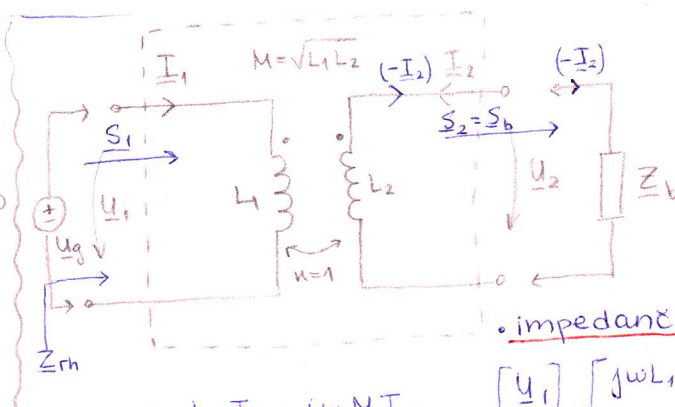
• verizna oblika (matrika)

$$\underline{I}_1 = \frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M} \underline{I}_2$$

$$\underline{U}_1 = j\omega \cdot L_1 \left(\frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M} \underline{I}_2 \right) + j\omega M \underline{I}_2$$

$$\underline{U}_1 = \frac{L_1}{M} \underline{U}_2$$

$$\underline{I}_1 = \frac{\underline{U}_2}{j\omega M} + \frac{L_2}{M} (-\underline{I}_2)$$



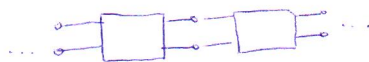
$$\underline{U}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$$

$$\underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$$

• impedančna matrika

$$\begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} L_1/M & 0 \\ 1/j\omega M & L_2/M \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ (-\underline{I}_2) \end{bmatrix}$$



$$1) \underline{U}_1 = \frac{L_1}{M} \underline{U}_2 = \frac{N_1}{N_2} \underline{U}_2 = n \underline{U}_2 \rightarrow \text{predstava razmerje}$$

$$2) |Z_b| \rightarrow \infty \text{ (odprta spona)}, \underline{I}_2 = 0$$

$$\underline{I}_1 = \frac{\underline{U}_2}{j\omega M} = \frac{\underline{U}_1}{j\omega L_1} = \underline{I}_{1m} \text{ (magnetilni tok)}$$

3) $0 < |Z_b| < \infty$ (breme)

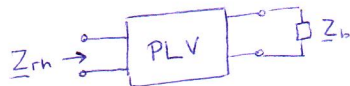
$$\underline{I}_1 = \underline{I}_{1m} + \frac{N_2}{N_1} (-\underline{I}_2) = \underline{I}_{1m} + \underline{I}_{1r} = \underline{I}_{1m} + \frac{1}{n} (-\underline{I}_2)$$

$$\underline{I}_{1r} \text{ (reducijski)} \quad \underline{I}_{1r} = -\frac{1}{n} \underline{I}_2$$

$$\underline{I}_1 = \underline{I}_{1m} + \underline{I}_{1r}$$

4) $\underline{\Phi} = N_1 \underline{I}_1 + N_2 \underline{I}_2 = N_1 \underline{I}_{1m} + N_1 \frac{N_2}{N_1} (-\underline{I}_2) + N_2 \underline{I}_2 = N_1 \underline{I}_{1m}$; $\underline{\Phi} = N_1 \underline{I}_{1m}$

5) $\underline{Z}_{ch} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{n \cdot \underline{U}_2 : (-\underline{I}_2)}{\frac{\underline{U}_2}{j\omega M} + \frac{1}{n} (-\underline{I}_2) : (-\underline{I}_2)}$



$$\underline{Z}_{ch} = \frac{n \underline{Z}_b}{\frac{\underline{Z}_b}{j\omega M} + \frac{1}{n}}$$

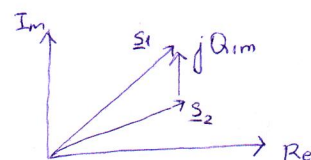
6) $\underline{S}_1 = \frac{1}{2} \underline{U}_1 \underline{I}_1 = \frac{1}{2} \underline{U}_1 \underline{I}_{1m} + \frac{1}{2} \underline{U}_1 \frac{1}{n} (-\underline{I}_2)$

$$\underline{S}_1 = \frac{1}{2} \underline{U}_1 \underline{I}_{1m} + \frac{1}{2} \underline{U}_1 \frac{1}{n} (-\underline{I}_2)$$

$$\underline{S}_1 = \underline{S}_1 + \underline{S}_2$$

$$\underline{S}_1 = jQ_{1m} + \underline{S}_2$$

$$P_1 = P_2, \quad Q_1 = Q_{1m} + Q_2$$



B) $\mu \rightarrow \infty, L_1, L_2, M \rightarrow \infty$

1) $\underline{U}_1 = n \underline{U}_2$

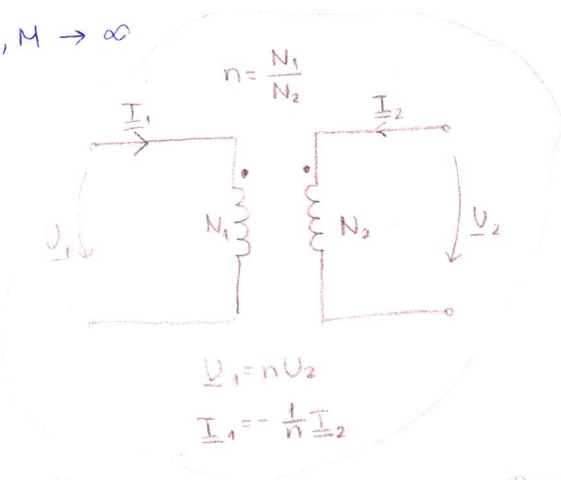
2) $\underline{I}_{1m} \rightarrow 0$

3) $\underline{I}_1 = -\frac{1}{n} \underline{I}_2$

4) $\underline{\Phi} \rightarrow 0$

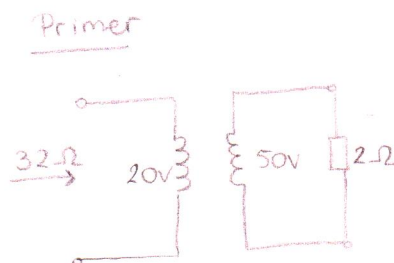
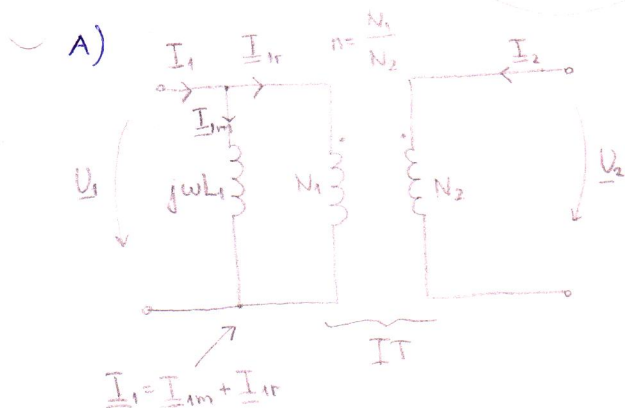
5) $\underline{Z}_{ch} = n^2 \underline{Z}_b$

6) $\underline{S}_1 = \underline{S}_2$



$$\underline{U}_1 = n \underline{U}_2$$

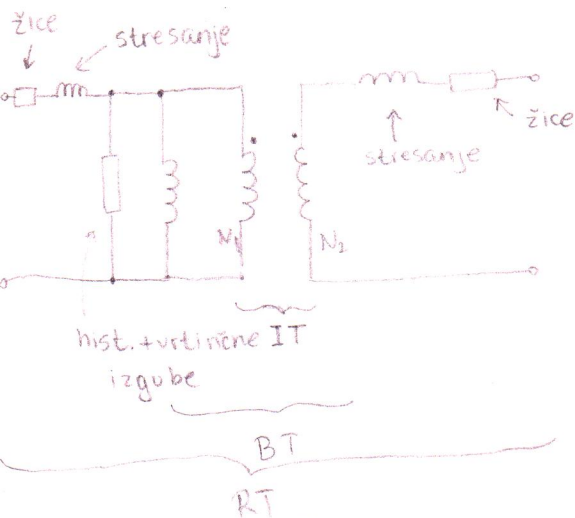
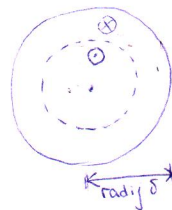
$$\underline{I}_1 = -\frac{1}{n} \underline{I}_2$$



c) vdorna globina

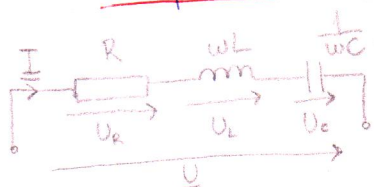
$$\delta = \sqrt{\frac{2}{\omega \mu \gamma}}$$

50Hz, Cu
 $\delta \approx 9,6 \text{ mm}$



P26 (27.5.2016)

Zaporedna vezava RLC



nihajni urog, filter

$$i = I_m \cos \omega t$$

$$U = U_R + U_L + U_C = R \cdot i + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

$$U = U_m \cos(\omega t + \varphi) = R \cdot I_m \cos \omega t - \omega L I_m \sin \omega t + \frac{I_m}{\omega C} \sin \omega t$$

$$\left\{ \begin{array}{l} \text{neznani} \end{array} \right. \quad \left\{ \begin{array}{l} \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta \end{array} \right.$$

$$U_m (\cos \omega t \cdot \cos \varphi - \sin \omega t \cdot \sin \varphi) = R \cdot I_m \cos \omega t + \left(\frac{1}{\omega C} - \omega L \right) I_m \sin \omega t$$

$$U_m \cos \varphi = R \cdot I_m$$

$$U_m \sin \varphi = \left(\omega L - \frac{1}{\omega C} \right) I_m$$

1) kvadriramo in seštejemo

$$U_m^2 (\sin^2 \varphi + \cos^2 \varphi) = U_m^2 = I_m^2 \left(R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right)$$

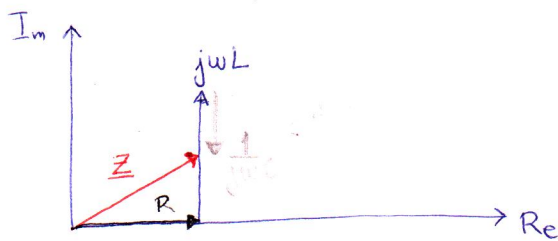
$$U_m = I_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}$$

2) enačbi delimo

$$\frac{U_m \sin \varphi}{U_m \cos \varphi} = \tan \varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$u(t) = I_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \cdot \cos \left(\omega t + \arctan \frac{\omega L - \frac{1}{\omega C}}{R} \right)$$

kazalčni diagram zap. RLC vezja nih. uroga, filtra



$$\underline{Z} = R + j(\omega L - \frac{1}{\omega C})$$

$$U = \underline{Z} \cdot \underline{I}$$

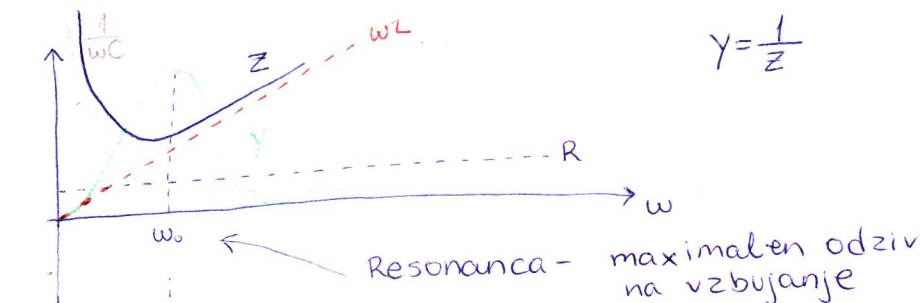
$$\tan \varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

a) $\omega L > \frac{1}{\omega C}$ induktivnosti
lastnosti

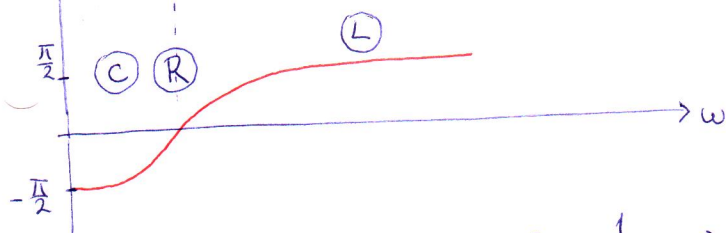
b) $\omega L = \frac{1}{\omega C}$ ohmski značaj
(popolna kompenzacija
jalove moći)
resonanca vezja

c) $\omega L < \frac{1}{\omega C}$
 $\varphi < 0$ kapacitivni
značaj vezja

Frekvenčna karakteristika



$$Y = \frac{1}{Z}$$

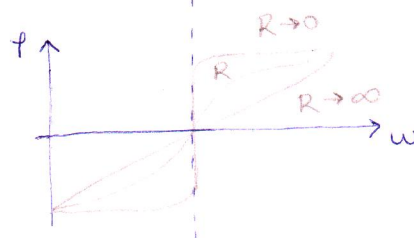
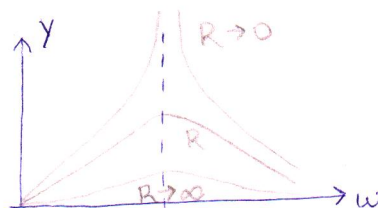
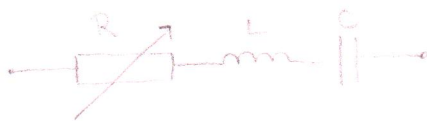


Resonanca:

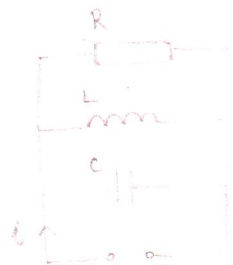
$$\omega_0^2 = \frac{1}{LC} \rightarrow$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

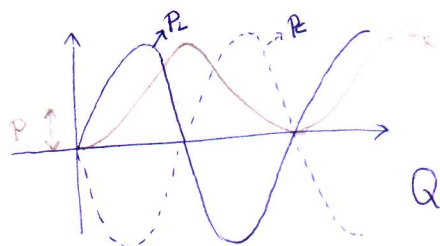
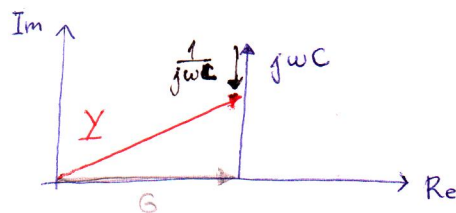


Vzporedna vezava RLC



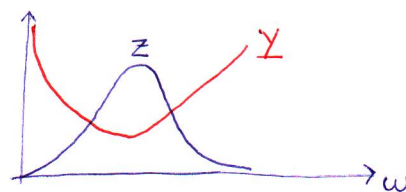
$$Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$\tan \varphi = \frac{\omega C - \frac{1}{\omega L}}{\frac{1}{R}}$$



Faktor delavnosti: Q

$$Q = 2\pi \frac{\text{energija v sistemu v resonanci}}{\text{energija izgubljena v sistemu v resonanci}}$$



impedanca na L oz. C $\omega_0 L = \frac{1}{\omega_0 C}$

$$Q = \frac{\omega_0 L}{R} \rightarrow \text{ohmska upornost}$$

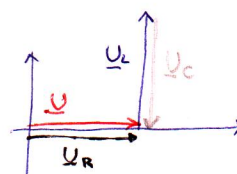
$$Q = 2\pi \frac{L \cdot I_m^2 / 2}{R \left(\frac{I_m}{\sqrt{2}}\right)^2 \cdot T_0} = 2\pi f \cdot \frac{L}{R} = \frac{\omega_0 L}{R}$$

perioda $T_0 = \frac{1}{f_0}$

$$Q = \frac{U_L}{U} = \frac{I_L}{I}$$

v resonanci:

$$U = U_R$$



Pasovna širina filtra, razglašenos, kvaliteta

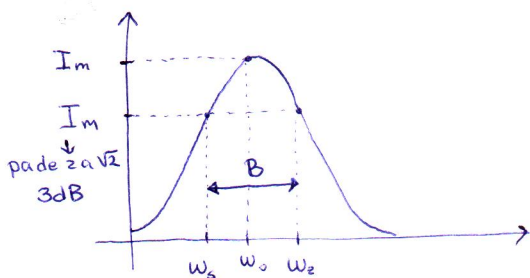
$$\left(\omega L - \frac{1}{\omega C}\right) = \omega_0 L \left(\frac{\omega}{\omega_0} - \frac{1}{\omega \omega_0 LC}\right) = \omega_0 L \underbrace{\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}_{\nu - \text{razglašenos}}$$

$\omega_0^2 = \frac{1}{LC}$

$$\nu = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}$$

$$\frac{I_{\omega_0}}{I_{\omega}} = \frac{U/R}{U/Z} = \frac{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}{R} = \sqrt{2}$$

Bočna frekvenca



ω_s - spodnja } bočna
 ω_z - zgornja } frekvenca

$$B = \omega_z - \omega_s$$

$$B = f_z - f_s \text{ [Hz]}$$

Pasovna širina filtra

$$\Rightarrow \frac{I_{\omega_0}}{I_{\omega}} = \sqrt{1 + \left(\frac{\omega_0 L}{R} \cdot \nu\right)^2} = \sqrt{1 + 1^2} \quad ; \quad \frac{\omega_0 L}{R} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \pm 1$$

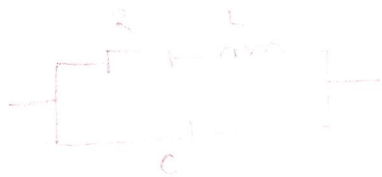
Q - kvaliteta nih. uroga

$$Q\left(\frac{\omega_z}{\omega_0} - \frac{\omega_0}{\omega_z}\right) = 1$$

$$Q\left(\frac{\omega_s}{\omega_0} - \frac{\omega_0}{\omega_s}\right) = -1$$

$$\omega_0 = \sqrt{\omega_s \cdot \omega_z}$$

Splošno o resonanci



$$\text{Im}(\underline{Z}) = 0$$

$$\underline{Y} = j\omega C + \frac{1}{R + j\omega L}$$

$$\text{Im}(\underline{Y}) = 0$$

$$\text{Im}(\underline{Y}) = 0$$



$$\underline{Y} = j\omega C_2 + \frac{1}{j\omega L + \frac{1}{j\omega C_1}}$$

$$\underline{Y} = j\left(\omega C_2 + \frac{\omega C_1}{1 - \omega^2 L C_1}\right)$$

$$\text{Im}(\underline{Y}) = 0 = \frac{\omega C_2 - \omega^3 L C_1 C_2 + \omega C_1}{1 - \omega^2 L C_1} \rightarrow$$

Nicle: napetostna resonanca

$\omega_1 = 0$ - ni zanimiva

$$\omega_2 = \sqrt{(C_1 + C_2) / L C_1 C_2}$$

Pol:

$$\omega_1 = \sqrt{L C_1}$$

točkovna resonanca

p27(31.5.2016)

Trifazni sistemi

Lastnosti: (prednosti)

- 1) Manjša poraba materiala
- 2) Skupna moč je konstantna
- 3) Vrtilno magnetno polje

Generatorski del



$$U_{\text{eff}} = \frac{U_m}{\sqrt{2}}$$

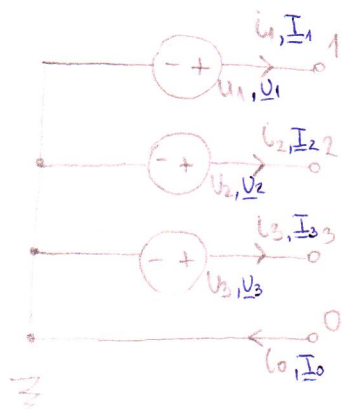
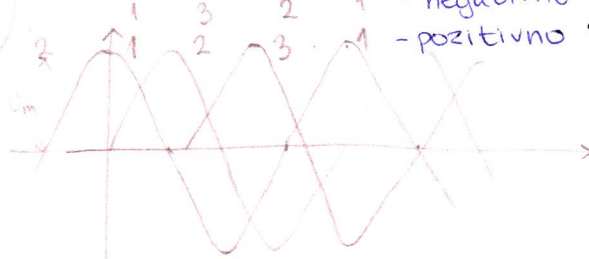
$$u_1 = U_m \cdot \cos(\omega t + \alpha) \rightarrow \underline{U}_1 = U_{\text{eff}} \cdot e^{j\alpha}$$

$$u_2 = U_m \cdot \cos(\omega t + \alpha \pm \frac{2\pi}{3}) \rightarrow \underline{U}_2 = U_{\text{eff}} \cdot e^{j(\alpha \pm \frac{2\pi}{3})}$$

$$u_3 = U_m \cdot \cos(\omega t + \alpha \pm \frac{4\pi}{3}) \rightarrow \underline{U}_3 = U_{\text{eff}} \cdot e^{j(\alpha \pm \frac{4\pi}{3})}$$

- negativno zaporedje faz

- pozitivno zaporedje faz



$$i_1 + i_2 + i_3 = i_0$$

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = \underline{I}_0$$

Simetrično vs. asimetrično (nesimetrično)

breme, viri napetosti

Simetrične vire napetosti

- amplitude med seboj enake
- fazni zamiki 120°

Označevanje kazalcev v energetiki

1) "Amplitudni kazalci"

$$u(t) = U_m \cdot \cos(\omega t + \alpha_u) \rightarrow \underline{U} = U_m \cdot e^{j\alpha_u}$$

vedno enaka
 $\downarrow$
 $u = \operatorname{Re}(\underline{U} e^{j\omega t})$

2) "Efektivni" kazalci

$$u(t) = \underbrace{\sqrt{2}}_{U_m} \cdot U_{ef} \cdot \cos(\omega t + \alpha_u) \rightarrow \underline{U} = U_{ef} \cdot e^{j\alpha_u}$$

$$u = \operatorname{Re}(\sqrt{2} \cdot \underline{U} e^{j\omega t})$$

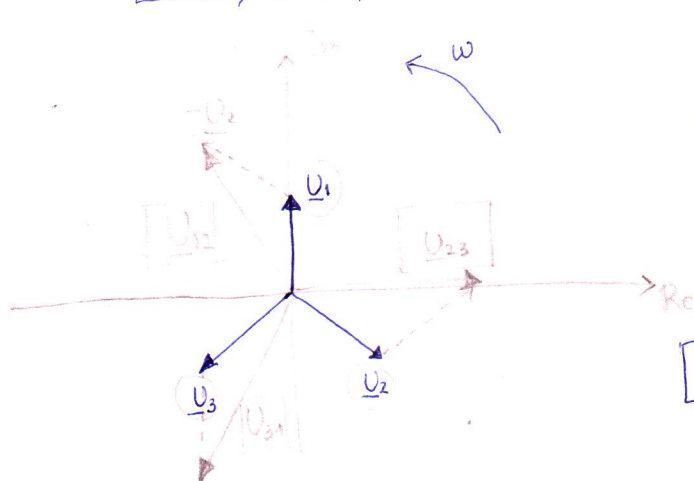
$$i(t) = \sqrt{2} \cdot I_{ef} \cdot \cos(\omega t + \alpha_i) \rightarrow \underline{I} = I_{ef} \cdot e^{j\alpha_i}$$

$$i = \operatorname{Re}(\sqrt{2} \cdot \underline{I} e^{j\omega t})$$

$\underline{Z}$ - ostaja enaka

$$\underline{S} = \underline{U} \cdot \underline{I}^*$$

$\downarrow$ brez polovičue



$$\alpha = \frac{\pi}{2}$$

$U_f: \underline{U}_1, \underline{U}_2, \underline{U}_3$: fazne

$$\boxed{U_{mf}: \underline{U}_{12} = \underline{U}_1 - \underline{U}_2}$$

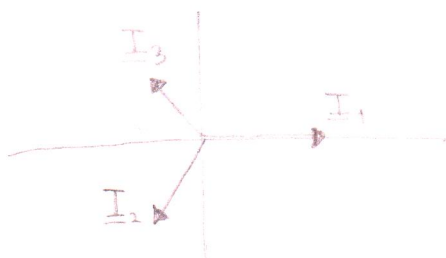
medfazna
napetost

$$U_{mf} = \sqrt{3} \cdot U_f$$

400V

230V

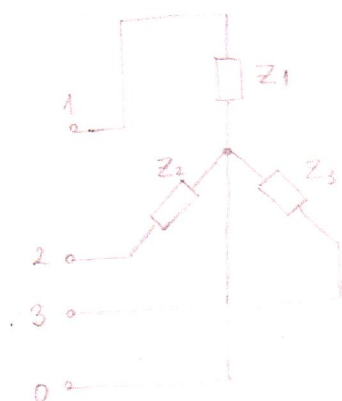
Simetričen sistem: generator, breme



$$\underline{I}_0 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

Bremensui del trifaznega sistema

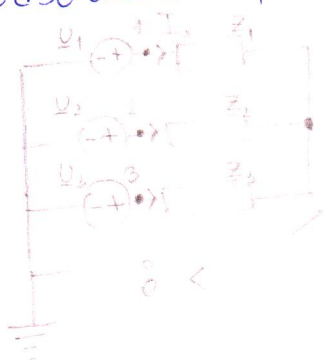
a) Breme v zvezda vezavi s pov



$$\underline{I}_u = \underline{U}_u \cdot \underline{Y}_u, \quad u=1,2,3$$

$$\underline{I}_0 = \sum_{u=1}^3 \underline{I}_u$$

b) Odsotnost povratnega vodnika



↙ potencial zvezdišča

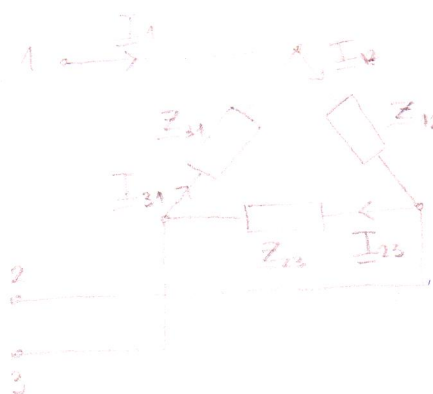
$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$(\underline{U}_1 - \underline{V}_0) \underline{Y}_1 + (\underline{U}_2 - \underline{V}_0) \underline{Y}_2 + (\underline{U}_3 - \underline{V}_0) \underline{Y}_3 = 0$$

$$\underline{U}_1 \underline{Y}_1 + \underline{U}_2 \underline{Y}_2 + \underline{U}_3 \underline{Y}_3 = \underline{V}_0 (\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3) = 0$$

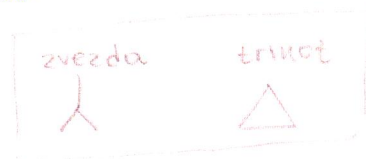
$$\underline{V}_0 = \frac{\frac{\underline{U}_1}{\underline{Z}_1} + \frac{\underline{U}_2}{\underline{Z}_2} + \frac{\underline{U}_3}{\underline{Z}_3}}{\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3}}$$

c) Vezava v trikot



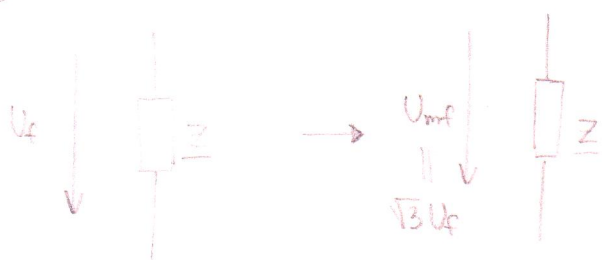
↙ medfazne napetosti

$$\underline{I}_{uL} = \frac{\underline{U}_{uL}}{\underline{Z}_{uL}}$$



$$\underline{I}_1 = \underline{I}_{12} - \underline{I}_{31}$$

⋮



$$S_\Delta \cdot 3 = S_\Delta$$

Konstanten prenos moči

$$p(t) = p_1(t) + p_2(t) + p_3(t) = u_1 i_1 + u_2 i_2 + u_3 i_3$$

----- simetrični trifazni sistem

$$p(t) = U \cdot I \cdot \cos(\omega t) \cdot \cos(\omega t - \varphi) +$$

$$+ U I \cdot \cos(\omega t + \frac{2\pi}{3}) \cdot \cos(\omega t + \frac{2\pi}{3} - \varphi) +$$

$$+ U I \cdot \cos(\omega t - \frac{2\pi}{3}) \cdot \cos(\omega t - \frac{2\pi}{3} - \varphi) =$$

$$= 3 \cdot \frac{UI}{2} \cdot \cos \varphi + \underbrace{\frac{UI}{2} \cos(2\omega t - \varphi) + \frac{UI}{2} \cos(2\omega t + \frac{4\pi}{3} - \varphi) + \frac{UI}{2} \cos(2\omega t - \frac{4\pi}{3} - \varphi)}_{\text{0}}$$



$$p(t) = \frac{3UI}{2} \cos \varphi$$

Vrtilno magnetno polje



$$i_1 = I \cdot \cos \omega t$$

$$i_2 = I \cdot \cos(\omega t + \frac{2\pi}{3})$$

$$i_3 = I \cdot \cos(\omega t - \frac{2\pi}{3})$$

$$B_x = B_{2x} + B_{3x} =$$

$$B_x = B_0 \cdot \cos(\omega t + \frac{2\pi}{3}) \cdot \cos \frac{\pi}{6} - B_0 \cdot \cos(\omega t - \frac{2\pi}{3}) \cdot \cos \frac{\pi}{6}$$

$$B_x = -\frac{3}{2} \cdot B_0 \cdot \sin \omega t$$

$$B_y = \frac{3}{2} B_0 \cos \omega t$$

} vrtenje
z ω