



Predmet:
Osnove elektrotehnike 2

Izvajalec:
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Vrsta gradiva:
Zapiski avditornih vaj

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Študijsko leto:
2015/16



Avditorne vaje (B)

⊛

AMP. SILA

BIOT-SAVART

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I d\vec{l}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

FORMULE:

premice

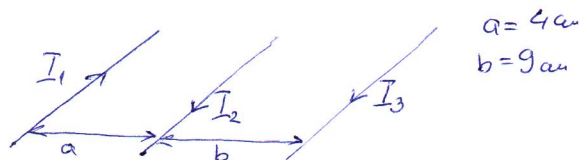
$$\vec{B}(r) = \vec{e}_\phi \frac{\mu_0 I}{2\pi r}$$

daljica:

$$\vec{B}(r) = \vec{e}_\phi \frac{\mu_0 I}{4\pi r} (\cos \alpha_1 - \cos \alpha_2)$$

$$\textcircled{2} \quad \text{---} \quad \text{---} \quad \textcircled{1}$$

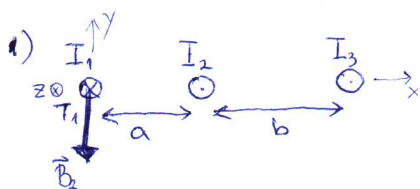
① Trije vzporedni vodniki s tokovi $I_1 = 10 \text{ A}$, $I_2 = 20 \text{ A}$ in $I_3 = 30 \text{ A}$ ležijo v isti ravnini, kot kaže slika



1) Določi $\vec{B}$ na mestu levega.

2) Določi $\vec{F}_{\text{silo}}$ na srednjega.

3) Pri katerem I_1 je sila na desnega = 0?



enota:

$$\frac{Vs}{Am} \cdot \frac{A}{m} = \frac{Vs}{m^2} = T$$

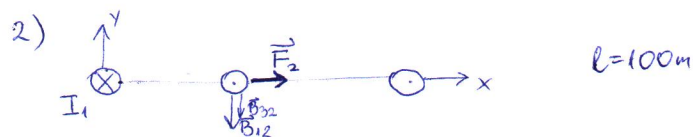
$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am} = \frac{1}{\epsilon_0 c^2}$$

* Zemljino mag. polje

* SLO $\sim 45 \mu T$ $\rightarrow 45^\circ$

Rio $\sim 22 \mu T$ $\rightarrow 0^\circ$

Polj $\sim 150 \mu T$ $\downarrow \uparrow 90^\circ$
S N



$$\vec{F}_2 = I_2 \vec{l} \times (\vec{B}_{12} + \vec{B}_{32}) = I_2 \cdot \vec{e}_z \cdot l \times \left[(-\vec{e}_y) \frac{\mu_0 I_1}{2\pi a} + (-\vec{e}_y) \frac{\mu_0 I_3}{2\pi b} \right] =$$

$$= I_2 \cdot \vec{e}_z \cdot l \times (-\vec{e}_y) \frac{\mu_0 I_1}{2\pi a} \left[1 + \frac{I_3/I_1}{b/a} \right] =$$

$$= \vec{e}_z \times (-\vec{e}_y) \cdot \frac{I_2 \mu_0 I_1 l}{2\pi a} \left[1 + \frac{I_3/I_1}{b/a} \right] =$$

$$= \vec{e}_x \cdot \frac{4\pi \cdot 10^{-7} \cdot 10^5 \cdot 20 \cdot 100}{2\pi \cdot 4 \cdot 10^{-2}} \left[1 + \frac{30/10}{9 \cdot 10^{-2} / 4 \cdot 10^{-2}} \right] = \vec{e}_x \frac{7}{30} \text{ N}$$

3) Sila na tretji vodnik



$$\vec{F}_3 = I_3 \cdot \vec{l} \times \vec{B}(T_3) = 0$$

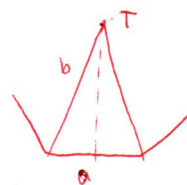
$$\vec{B}_1(T_3) + \vec{B}_2(T_3) = 0$$

$$-\vec{e}_y \frac{\mu_0 I_1}{2\pi(a+b)} + \vec{e}_y \frac{\mu_0 I_2}{2\pi b} = 0$$

$$-\frac{I_1}{a+b} + \frac{I_2}{b} = 0$$

$$I_1 = I_2 \cdot \frac{a+b}{b} = I_2 \left(1 + \frac{a}{b}\right) = 20 \left(1 + \frac{4}{9}\right) = 20 \cdot \frac{13}{9} \approx \underline{\underline{27.8 A}}$$

③ n-utornik



$$\vec{B}(T_0) = n \cdot (\vec{B}_d(T_0))$$

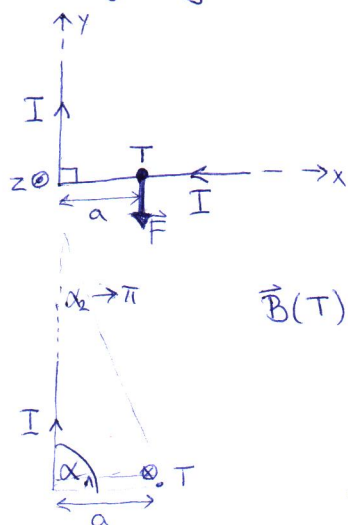
v limiti $n \rightarrow \infty$

$$\vec{B}(T_0) = \vec{e}_z \frac{\mu_0 I}{2b}$$



② Predloga naloga 4.4.

Določite vektor sile na dolžinsko enoto v točki T pravokotno linijsnega vodnika!



$$\vec{f} = \frac{d\vec{F}}{dl}$$

$$\vec{F} = I \vec{l} \times \vec{B}$$

$$\vec{F} = I \cdot \vec{e}_l \cdot l \times \vec{B}$$

$$d\vec{F} = I \cdot \vec{e}_l \cdot dl \times \vec{B}$$

$$\frac{d\vec{F}}{dl} = I \cdot \vec{e}_l \times \vec{B}$$

$$\vec{B}(T) = \vec{e}_\phi \frac{\mu_0 I}{4\pi r} (\cos \alpha_1 - \cos \alpha_2) =$$

$$= (-\vec{e}_z) \frac{\mu_0 I}{4\pi a} (\cos \frac{\pi}{2} - \cos \pi) =$$

$$= -\vec{e}_z \frac{\mu_0 I}{4\pi a}$$

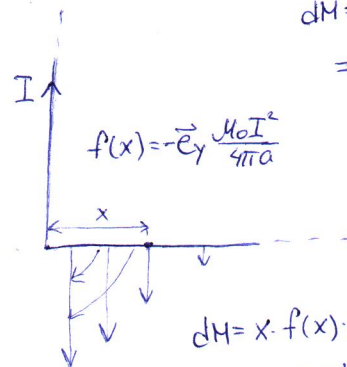
$$\Rightarrow \vec{f}(T) = I (-\vec{e}_x) \times (-\vec{e}_z) \frac{\mu_0 I}{4\pi a} = \underline{\underline{+\vec{e}_y \frac{\mu_0 I^2}{4\pi a}}}$$

$$\vec{f}(T) = \frac{d\vec{F}}{dl}(T) = I \cdot \vec{e}_l \times \vec{B}(T) = I \cdot (-\vec{e}_x) \times \vec{B}(T)$$

$$\text{Navor: } \vec{M} = \vec{r} \times \vec{F}$$

$$d\vec{M} = \vec{r} \times d\vec{F}$$

$$= -\vec{e}_z \cdot r \cdot dF$$



$$f(x) = -\vec{e}_y \frac{\mu_0 I^2}{4\pi a}$$

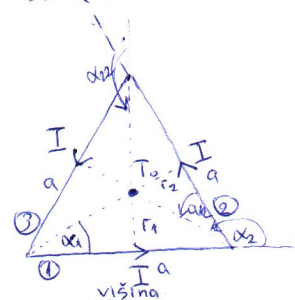
$$dM = x \cdot f(x) \cdot dx =$$

$$= x \frac{\mu_0 I^2}{4\pi a} \cdot dx$$

$$M = \frac{\mu_0 I^2}{4\pi} \int_0^a dx = \underline{\underline{\frac{\mu_0 I^2 L}{4\pi}}}$$

③ P.N. 4.3

Doloži $\vec{B}$ v težišču trikotne zanke s tokom I (ENAKOSTRANIČNE)



$$\vec{B}(T_0) = \vec{B}_1(T_0) + \vec{B}_2(T_0) + \vec{B}_3(T_0) =$$

$$\vec{B}_1(T_0) = \frac{\mu_0 I}{4\pi r_1} \cdot \vec{e}_z \cdot (\cos \alpha_{11} - \cos \alpha_{21}) = \vec{e}_z \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{6} a} (\cos 30^\circ - \cos 150^\circ)$$

$$\vec{B}_2(T_0) = \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{6} a} \cdot \vec{e}_z (\cos \alpha_{12} - \cos \alpha_{22}) = \vec{e}_z \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{6} a} (\cos 30^\circ - \cos 150^\circ)$$

$$\vec{B}_3(T_0) = \dots$$

$$r_1 = \frac{1}{3} \frac{\sqrt{3}}{2} a$$

$$r_2 = \frac{1}{3} \frac{\sqrt{3}}{2} a$$

$$\alpha_{11} = 30^\circ$$

$$\alpha_{12} = 30^\circ$$

$$\alpha_{21} = 150^\circ$$

$$\alpha_{22} = 150^\circ$$

$$\Rightarrow \vec{B}(T_0) = 3 \cdot \vec{B}_1(T_0) = \vec{e}_z \cdot 3 \cdot \frac{\mu_0 I}{4\pi \frac{\sqrt{3}}{6} a} \cdot 2 \cdot \frac{\sqrt{3}}{2} = \underline{\underline{\vec{e}_z \frac{9\mu_0 I}{2\pi a}}}$$

Avditorne vaje (5)

Tuljane
Krožni

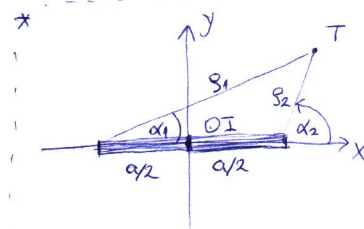
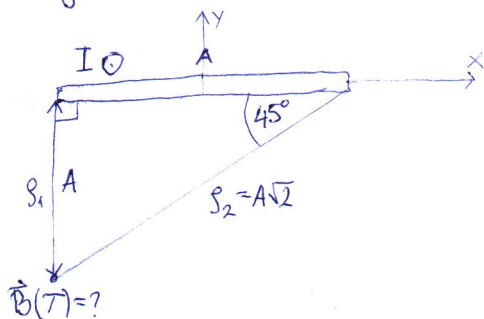
① Določite magnetno polje v točki T. (tračni vodnik)

$$I_1 = 1 \text{ A}$$

$$A = 2 \text{ cm}$$

$$K_2 = \frac{1}{0,02}$$

$$K = \frac{I}{2a}$$

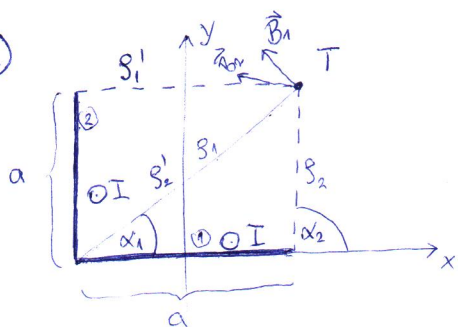


$$\vec{B} = \frac{\mu_0 K}{2\pi} (\vec{e}_x (\alpha_1 - \alpha_2) + \vec{e}_y \ln \frac{s_1}{s_2})$$

$$\vec{B} = \frac{\mu_0 K}{2\pi} (\vec{e}_x (\alpha_1 - \alpha_2) + \vec{e}_y \ln \frac{s_1}{s_2})$$

$$\vec{B} = \frac{4\pi \cdot 10^{-7} \cdot 1}{2\pi \cdot 2 \cdot 10^{-2}} (\vec{e}_x \frac{\pi}{4} + \vec{e}_y \ln \frac{1}{\sqrt{2}}) = 10^{-5} (\vec{e}_x \frac{\pi}{4} + \vec{e}_y \ln \frac{1}{\sqrt{2}}) = \vec{e}_x$$

②



(prvi vodnik)

$$K_2 = \frac{I}{2a}$$

$$s_1 = \sqrt{x^2 + y^2} = \sqrt{2a^2} = a\sqrt{2}$$

$$s_2 = a$$

$$\alpha_1 = \frac{\pi}{4}$$

$$\alpha_2 = \frac{\pi}{2}$$

$$\vec{B}_1 = \frac{\mu_0 K}{2\pi} (-\vec{e}_x \frac{\pi}{4} + \vec{e}_y \ln \sqrt{2})$$

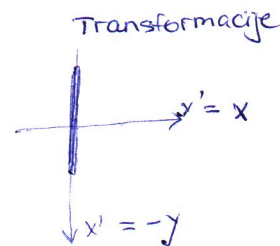
(drugi vodnik)

$$s'_1 = a$$

$$s'_2 = a\sqrt{2}$$

$$\alpha_1 = \frac{\pi}{2}$$

$$\alpha_2 = \frac{\pi}{4}$$

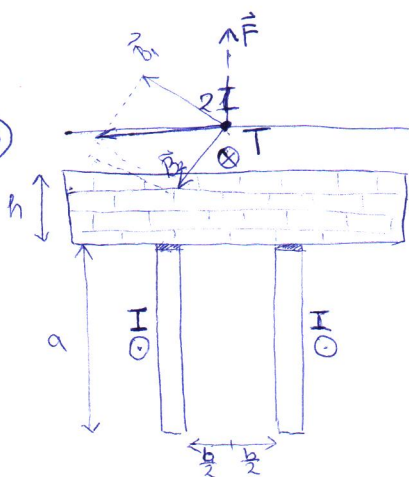


$$\vec{B}_2 = \frac{\mu_0 K}{2\pi} (-\vec{e}_x \ln \sqrt{2} + \vec{e}_y (\frac{\pi}{2} - \frac{\pi}{4}))$$

$$\vec{B}_2 = \frac{\mu_0 K}{2\pi} (-\vec{e}_x \ln \sqrt{2} + \vec{e}_y \frac{\pi}{4})$$

$$\vec{B}_2 = \frac{4\pi \cdot 10^{-7} \cdot 1}{2\pi \cdot 2} \cdot \frac{1}{a} (-\vec{e}_x \ln \sqrt{2} + \vec{e}_y \frac{\pi}{4}) \frac{V_s}{A \cdot m} \quad [T]$$

③



$$\vec{F} = I \vec{L} \times \vec{B}$$

Avditorne vaje (Bulič)

①

$$Q = 1 \mu C$$

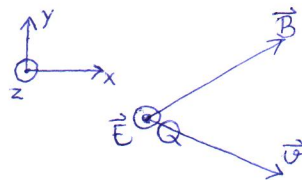
$$\vec{v} = (2\vec{e}_x - 3\vec{e}_y) \frac{m}{s}$$

$$\vec{E} = \vec{e}_z 3 V/m$$

$$\vec{B} = (\vec{e}_x + \frac{1}{2}\vec{e}_y) T$$

$$\vec{F}_L = ?$$

$Q \cdot \vec{v}$ - moment
tokovnega
elementa



$$\vec{F}_L = Q(\vec{E} + \vec{v} \times \vec{B}) = \underbrace{Q\vec{E}}_{F_e} + \underbrace{Q\vec{v} \times \vec{B}}_{F_m}$$

$$\vec{F}_e = Q \cdot \vec{E} = 10^{-6} C \cdot \vec{e}_z 3 V/m = \vec{e}_z \cdot 3 \cdot 10^{-6} \frac{CV}{m} = \vec{e}_z 3 \mu N$$

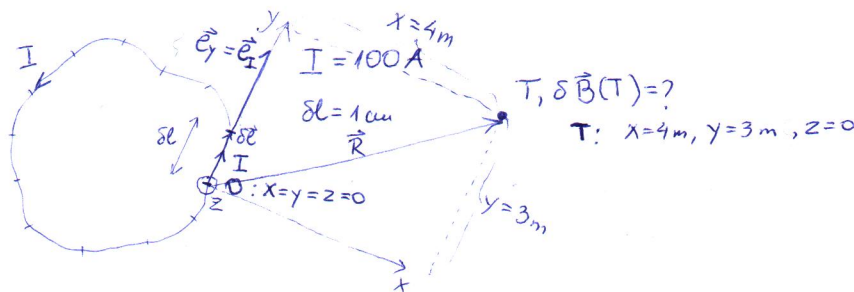
$$\vec{F}_m = Q \vec{v} \times \vec{B} = 10^{-6} C \cdot (2\vec{e}_x - 3\vec{e}_y) \times (\vec{e}_x + \frac{1}{2}\vec{e}_y) = \vec{e}_z 4 \mu N$$

$(\vec{v} \times \vec{B})$ - gibalna količina

$$\vec{v} \times \vec{B} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 2 & -3 & 0 \\ 1 & \frac{1}{2} & 0 \end{vmatrix} \frac{m}{s} = \vec{e}_x \cdot (-3 \cdot 0 - \frac{1}{2} \cdot 0) - \vec{e}_y (2 \cdot 0 - 1 \cdot 0) + \vec{e}_z (2 \cdot \frac{1}{2} - (-3) \cdot 1) = 4\vec{e}_z \frac{m}{s} \frac{Vs}{m^2} = \frac{Vs}{m}$$

$$\Rightarrow \vec{F}_L = F_e + F_m = \vec{e}_z 3 \mu N + \vec{e}_z 4 \mu N = \vec{e}_z 7 \mu N$$

②



$$R = \sqrt{(4m-0)^2 + (3m-0)^2 + (0-0)^2} = 5m \gg \delta l = 1cm$$

B.S. zakon

$$d\vec{B}(T) = \frac{\mu_0 I d\vec{l} \times \vec{R}}{4\pi R^3}$$

$$\delta\vec{B}(T) = \frac{\mu_0 I \delta\vec{l} \times \vec{R}}{4\pi R^3}$$

$$\delta\vec{l} = \vec{e}_y \cdot \delta l = \delta l \cdot \vec{e}_y$$

smernik
toka



$$\vec{R} = (4m-0)\vec{e}_x + (3m-0)\vec{e}_y + (0-0)\vec{e}_z = (4\vec{e}_x + 3\vec{e}_y)m$$

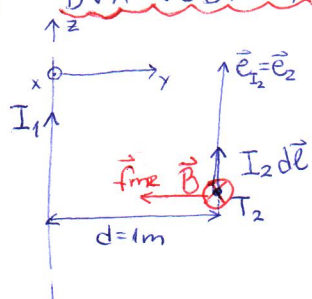
$I d\vec{l}$ - moment
tokovnega
elementa

$$\delta\vec{B}(T) = \frac{\mu_0 I \delta l \vec{e}_y \times \vec{R}}{4\pi R^3} = \frac{4 \cdot \pi \cdot 10^{-7} \frac{Vs}{Am} \cdot 100 A \cdot 10^{-2} m \cdot (-4)\vec{e}_z}{4\pi \cdot 5^3 m^3} = -\vec{e}_z \frac{4 \cdot 10^{-7}}{125}$$

$$\vec{e}_y \times \vec{R} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 1 & 0 \\ 4 & 3 & 0 \end{vmatrix} m = (-4)\vec{e}_z m$$

$$\Rightarrow \delta\vec{B}(T) = -\vec{e}_z \frac{4 \cdot 10^{-7}}{125} \cdot \frac{Vs}{m^2} = -\vec{e}_z 3,2 nT$$

③ DVA VODNIKA



$$\frac{d\vec{F}_{m2}}{dl} = \vec{f}_{m2} = ?$$

$$\vec{e}_{I_2} = \vec{e}_2$$

$$d\vec{F}_{m2} = I_2 d\vec{l} \times \vec{B}(T_2) \quad / : dl$$

$$\vec{f}_{m2} = I_2 \frac{d\vec{l}}{dl} \times \vec{B}(T_2)$$

$$\vec{f}_{m2} = I_2 \vec{e}_{I_2} \times \vec{B}(T_2)$$

$$\vec{B} = \vec{B}_{I_1} + d\vec{B}_{I_2}$$

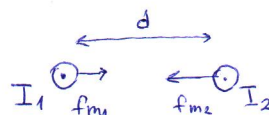
$$\vec{B}(T_2) = \vec{B}_{I_1}(T_2) = \frac{\mu_0 I_1}{2\pi d} (-\vec{e}_x)$$

$$d\vec{B}_{I_2}(T_2) = \vec{0}$$

ko merimo B v točki T2 prispeva le polje B_{I1}!

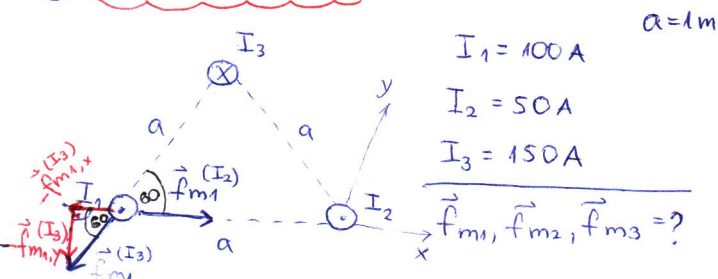
$$\vec{f}_{m2} = I_2 \vec{e}_2 \times (-\vec{e}_x) \frac{\mu_0 I_1}{2\pi d} = -\vec{e}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$

$$\vec{f}_{m2} = -\vec{e}_y \frac{\mu_0 I_1 I_2}{2\pi d}$$



$$\Rightarrow \vec{f}_{m2} = -\vec{e}_y \frac{\mu_0 \cdot 10 \text{ A} \cdot 2 \text{ A}}{2\pi \cdot 1 \text{ m}} = -\vec{e}_y 4 \mu\text{N/m}$$

④ TRI VODNIKI



$$I_1 = 100 \text{ A}$$

$$I_2 = 50 \text{ A}$$

$$I_3 = 150 \text{ A}$$

$$\vec{f}_{m1}, \vec{f}_{m2}, \vec{f}_{m3} = ?$$

$$\vec{f}_{m1} = \vec{f}_{m1}^{(I_2)} + \vec{f}_{m1}^{(I_3)}$$

$$1) \vec{f}_{m1}^{(I_2)} = \frac{\mu_0 I_1 I_2}{2\pi a} \vec{e}_x$$

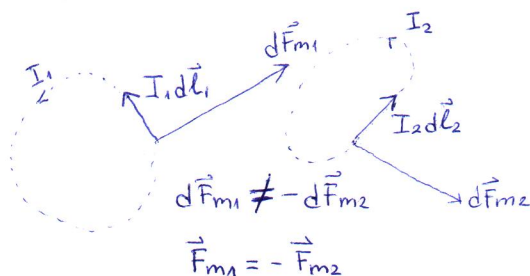
$$2) \vec{f}_{m1}^{(I_3)} = \frac{\mu_0 I_1 I_3}{2\pi a} \left(\cos 60^\circ (-\vec{e}_x) + \sin 60^\circ (-\vec{e}_y) \right)$$

$$\Rightarrow \vec{f}_{m1} = \frac{\mu_0 I_1 I_2}{2\pi a} \vec{e}_x + \frac{\mu_0 I_1 I_3}{2\pi a} (\cos 60^\circ (-\vec{e}_x) + \sin 60^\circ (-\vec{e}_y))$$

$$\vec{f}_{m1} = \frac{\mu_0 I_1}{2\pi a} \left[I_2 \vec{e}_x + I_3 \left(-\frac{1}{2} \vec{e}_x - \frac{\sqrt{3}}{2} \vec{e}_y \right) \right]$$

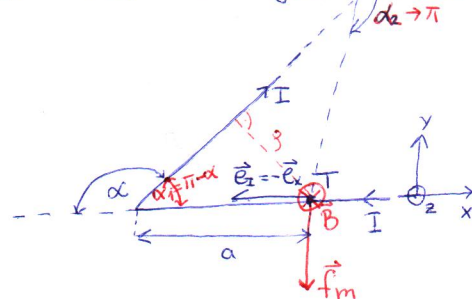
$$\vec{f}_{m1} = \frac{\mu_0 I_1}{2\pi a} \left(\left(I_2 - \frac{I_3}{2} \right) \vec{e}_x - \frac{\sqrt{3}}{2} I_3 \vec{e}_y \right) = (-0,5 \vec{e}_x - 2,6 \vec{e}_y) \cdot 10^{-3} \text{ N/m}$$

$$\vec{f}_{m1} + \vec{f}_{m2} + \vec{f}_{m3} = \vec{0}$$



$$\vec{F}_{m1} = -\vec{F}_{m2}$$

⑤ 4.6) Lomljeni vodnik



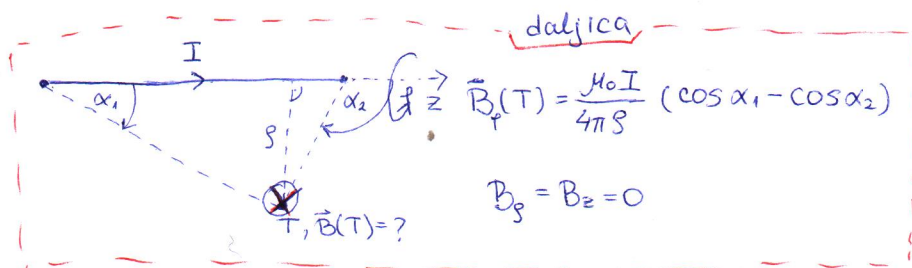
$$\vec{f}_m(T) = ?$$

$$\vec{f}_m(T) = I \vec{e}_1 \times \vec{B}(T)$$

$$\vec{B} = \vec{B}^{(spod.)} + \vec{B}^{(zgor.)}$$

$$\vec{B}(T) = \vec{B}^{(zgor.)}(T)$$

$$\vec{B}^{(spod.)}(T) = \vec{0}$$



$$\vec{B}_f(T) = \frac{\mu_0 I}{4\pi s} (\cos \alpha_1 - \cos \alpha_2) \vec{e}_z$$

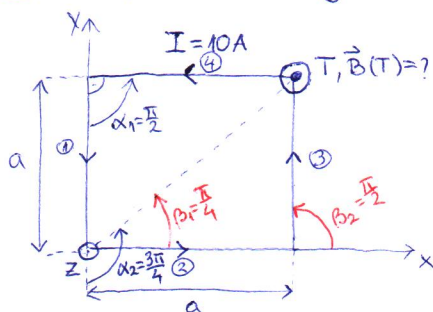
$$B_y = B_z = 0$$

$$\Rightarrow \vec{B}(T) = \vec{B}^{(zgor.)}(T) = \frac{\mu_0 I}{4\pi s} (\cos \alpha_1 - \cos \alpha_2) (-\vec{e}_z) = (-\vec{e}_z) \frac{\mu_0 I}{4\pi a \sin(\pi - \alpha)} \left(\frac{\cos(\pi - \alpha)}{-\cos \alpha} - \frac{\cos \pi}{-1} \right)$$

$$s = a \sin \alpha$$

$$\vec{f}_m(T) = I \vec{e}_x \times \vec{B}(T) = I (-\vec{e}_x) \times (-\vec{e}_z) \frac{\mu_0 I}{4\pi a \sin \alpha} (1 - \cos \alpha) = -\vec{e}_y \frac{\mu_0 I^2}{4\pi a} \cdot \frac{1 - \cos \alpha}{\sin \alpha}$$

⑥ Izpitna naloga



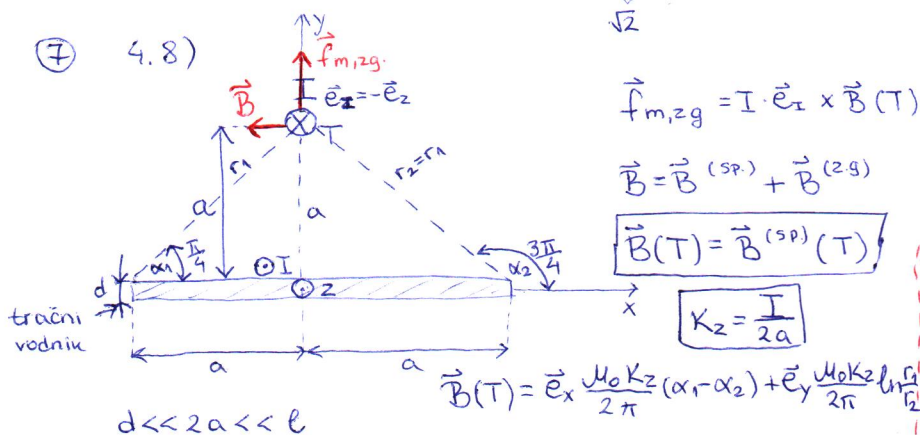
$$\vec{B}(T) = \vec{B}_0(T) + \vec{B}_2(T) + \vec{B}_3(T) + \vec{B}_4(T)$$

$$\vec{B}_0(T) = \frac{\mu_0 I}{4\pi a} (\cos \alpha_1 - \cos \alpha_2) \vec{e}_z$$

$$\vec{B}_3(T) = \frac{\mu_0 I}{4\pi a} (\cos \beta_1 - \cos \beta_2) \vec{e}_z$$

$$\Rightarrow \vec{B}(T) = \frac{\mu_0 I}{4\pi a} \cdot \vec{e}_z \left[\underbrace{\cos \frac{\pi}{2}}_0 - \underbrace{\cos \frac{3\pi}{2}}_{-\sqrt{2}/2} + \underbrace{\cos \frac{\pi}{4}}_{\sqrt{2}/2} - \underbrace{\cos \frac{\pi}{2}}_0 \right] = \vec{e}_z \frac{\mu_0 I}{4\pi a} \cdot \sqrt{2} = \vec{e}_z 28,3 \mu T$$

⑦ 4.8)



$$\vec{f}_{m,zg} = ?$$

$$\vec{f}_{m,sp} = -\vec{f}_{m,zg}$$

$$\vec{B}(T) = \frac{\mu_0 I}{2a \cdot 2\pi} \left[\vec{e}_x \left(\frac{\pi}{4} - \frac{3\pi}{4} \right) + \vec{e}_y \ln \frac{r_1}{r_2} \right]$$

$$\vec{B}(T) = -\vec{e}_x \frac{\mu_0 I}{8a}$$

$$\vec{f}_{m,zg} = I \vec{e}_x \times \vec{B}(T)$$

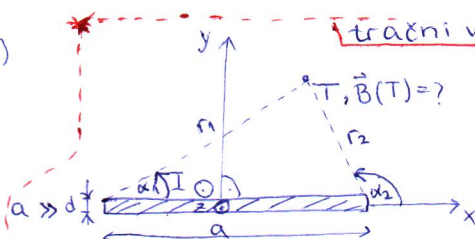
$$\vec{B} = \vec{B}^{(sp.)} + \vec{B}^{(zg.)}$$

$$\vec{B}(T) = \vec{B}^{(sp.)}(T)$$

$$K_z = \frac{I}{2a}$$

$$\vec{B}(T) = \vec{e}_x \frac{\mu_0 K_z}{2\pi} (\alpha_1 - \alpha_2) + \vec{e}_y \frac{\mu_0 K_z}{2\pi} \ln \frac{r_1}{r_2}$$

tračni vodnik



$$B_x(T) = \frac{\mu_0 K_z}{2\pi} (\alpha_1 - \alpha_2)$$

$$B_y(T) = \frac{\mu_0 K_z}{2\pi} \left(\ln \frac{r_1}{r_2} \right)$$

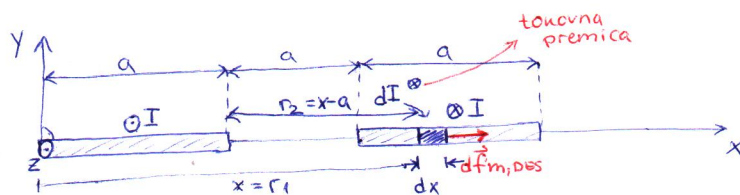
$$B_z(T) = 0$$

$$\vec{K} = \vec{e}_z K_z (+\vec{e}_x \cdot 0 + \vec{e}_y \cdot 0)$$

$$K_z = \frac{I}{a}$$

$$\vec{f}_{m,zg} = I \cdot \underbrace{(-\vec{e}_z) \times (-\vec{e}_x)}_{\vec{e}_y} \frac{\mu_0 I}{8a} = \vec{e}_y \frac{\mu_0 I^2}{8a}$$

⑧ DVOUOD (OBA VODNIKA STA TRACNA) 4.9)



$$\vec{f}_{m,DES} = -\vec{f}_{m,LEV} = ?$$

$$d\vec{f}_{m,DES} = d\vec{f}_{m,DES}^{(lev)} + \underbrace{d\vec{f}_{m,DES}^{(des)}}_{\neq 0}$$

$$\vec{f}_{m,DES} = \int_{x=2a}^{3a} d\vec{f}_{m,DES} = \int_{x=2a}^{3a} d\vec{f}_{m,DES}^{(lev)}$$

$$d\vec{f}_{m,DES} = dI (-\vec{e}_z) \times \vec{B}(x,0), \quad d\vec{f}_{m,DES}^{(lev)} = dI (-\vec{e}_z) \times \vec{B}^{(lev)}(x,0)$$

$$\vec{B} = \vec{B}^{(lev)} + \vec{B}^{(des)}$$

$$\vec{B}^{(lev)}(x,0) = \frac{\mu_0 K_z}{2\pi} \left[\vec{e}_x \underbrace{(x_1 - x_2)}_0 + \vec{e}_y \ln \frac{r_1}{r_2} \right]$$

$$\vec{f}_{m,DES} = \vec{f}_{m,DES}^{(lev)} + \underbrace{\vec{f}_{m,DES}^{(des)}}_{\vec{0} = \int_{x=2a}^{3a} d\vec{f}_{m,DES}^{(des)}}$$

$$\vec{B}^{(lev)}(x,0) = \frac{\mu_0 I/a}{2\pi} \vec{e}_y \ln \frac{x}{x-a}$$

$$K_z = \frac{I}{a}, \quad x_1 = x_2 = 0$$

$$d\vec{f}_{m,DES}^{(lev)} = \frac{I}{a} dx (-\vec{e}_z) \times (\vec{e}_y) \frac{\mu_0 I}{2\pi a} \ln \frac{x}{x-a} = \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} [\ln x - \ln(x-a)] dx$$

$$dI = \frac{I}{a} dx$$

$$\vec{f}_{m,DES} = \int_{x=2a}^{3a} d\vec{f}_{m,DES}^{(lev)} = \int_{2a}^{3a} \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} [\ln x - \ln(x-a)] dx$$

$$\vec{f}_{m,DES} = \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} \int_{2a}^{3a} [\ln x - \ln(x-a)] dx$$

Integral

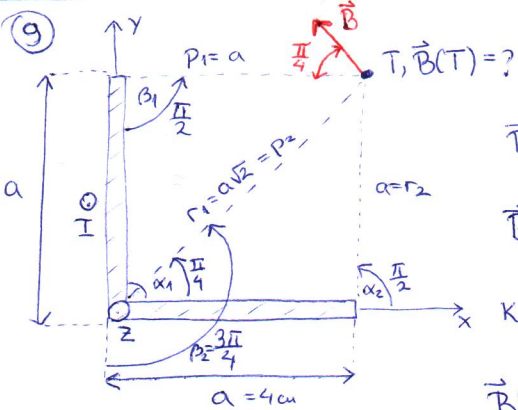
$$\int \ln x \cdot dx = x \cdot \ln x - x + C$$

$$\vec{f}_{m,DES} = \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} \left[x \cdot \ln x - x - ((x-a) \cdot \ln(x-a) - (x-a)) \right]_{2a}^{3a} =$$

$$= \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} \left[\underbrace{(3a \ln 3a - 3a - (3a-a) \ln(3a-a) + (3a-a))}_{= 3a \ln 3a - 3a - 2a \ln 2a + 2a} + a(3 \ln 3a - 2 \ln 2a - 2 \ln 2a + \ln a) \right] =$$

$$= \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} \left[a(\ln(3a)^3 - \ln(2a)^2 - \ln(2a)^2 + \ln a) \right] = \vec{e}_x \frac{\mu_0 I^2}{2\pi a^2} \ln \frac{27}{16}$$

$$\Rightarrow \vec{f}_{m,DES} = \vec{e}_x \frac{\mu_0 I^2}{2\pi a} \ln \frac{27}{16}$$



$$\vec{B} = \vec{B}^{(\text{HOR})} + \vec{B}^{(\text{VER})}$$

$$\vec{B}^{(\text{HOR})}(T) = \frac{\mu_0 K_z}{2\pi} \left[\vec{e}_x (\alpha_1 - \alpha_2) + \vec{e}_y \ln \frac{r_1}{r_2} \right] = \frac{\mu_0 I}{4\pi a} \left[\vec{e}_x \left(\frac{\pi}{4} - \frac{\pi}{2} \right) + \vec{e}_y \ln \frac{\sqrt{2}a}{a} \right]$$

$$K_z = \frac{I/2}{a} = \frac{I}{2a}$$

$$\vec{B}^{(\text{VER})}(T) = \frac{\mu_0 K_z}{2\pi} \left[\vec{e}_x \ln \frac{r_1}{r_2} - \vec{e}_y (\beta_1 - \beta_2) \right]$$

$$K_z = \frac{I/2}{a} = \frac{I}{2a}$$

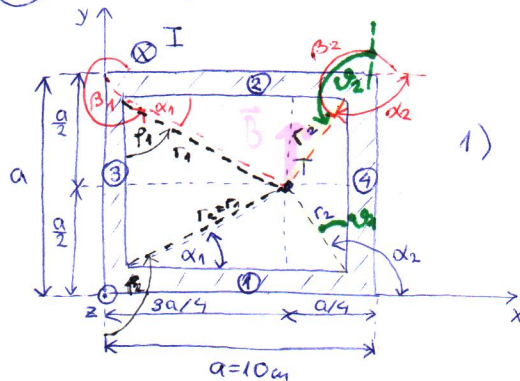
$$\vec{B}^{(\text{VER})}(T) = \frac{\mu_0 I}{4\pi a} \left[\vec{e}_x \ln \frac{a}{\sqrt{2}a} - \vec{e}_y \left(\frac{\pi}{2} - \frac{3\pi}{4} \right) \right]$$

$$\vec{B}(T) = \vec{B}^{(\text{HOR})}(T) + \vec{B}^{(\text{VER})}(T) = \frac{\mu_0 I}{4\pi a} \left[\vec{e}_x \left(\ln \frac{1}{\sqrt{2}} + \left(-\frac{\pi}{4} \right) \right) + \vec{e}_y \left(-\frac{\pi}{4} + \ln \sqrt{2} \right) \right]$$

$$\vec{B}(T) = \frac{\mu_0 I}{4\pi a} \cdot \frac{1}{4} \left[-\vec{e}_x (2 \ln 2 + \pi) + \vec{e}_y (2 \ln 2 + \pi) \right] \doteq 566 \mu\text{T} (-\vec{e}_x + \vec{e}_y)$$

$$|\vec{B}(T)| = \sqrt{B_x^2(T) + B_y^2(T)} = 800 \mu\text{T}$$

10 4.10)



$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

$$1) B_x = B_{x,1} + B_{x,2} + B_{x,3} + B_{x,4}$$

$$B_x(T) = \frac{\mu_0 K_z}{2\pi} (\alpha_1 - \alpha_2) + \frac{\mu_0 K_z}{2\pi} (\beta_1 - \beta_2) + \frac{\mu_0 K_z}{2\pi} \ln \frac{r_1}{r_1} - \frac{\mu_0 K_z}{2\pi} \ln \frac{r_2}{r_2}$$

$$K_z = \frac{-I/4}{a} = -\frac{I}{4a}$$

$$\beta_1 = 2\pi - \alpha_1$$

$$\beta_2 = 2\pi - \alpha_2$$

$$B_x(T) = \frac{\mu_0 K_z}{2\pi} \left[\alpha_1 - \alpha_2 + 2\pi - \alpha_1 - 2\pi + \alpha_2 \right]$$

$$B_x(T) = 0$$

$$2) B_y(T) = \frac{\mu_0 K_z}{2\pi} \left[\ln \frac{r_1}{r_2} + \ln \frac{r_1}{r_2} - (\phi_1 - \phi_2) + (\psi_1 - \psi_2) \right] = \frac{\mu_0 K_z}{2\pi} \left[\ln \frac{r_1^2}{r_2^2} + \pi - 2\phi_1 + 2\psi_1 - \pi \right]$$

$$r_1 = \sqrt{\left(\frac{3a}{4} \right)^2 + \left(\frac{a}{2} \right)^2} =$$

$$r_2 = \sqrt{\left(\frac{a}{4} \right)^2 + \left(\frac{a}{2} \right)^2} =$$

$$\tan \phi_1 = \frac{3a/4}{a/2} = \frac{3}{2}$$

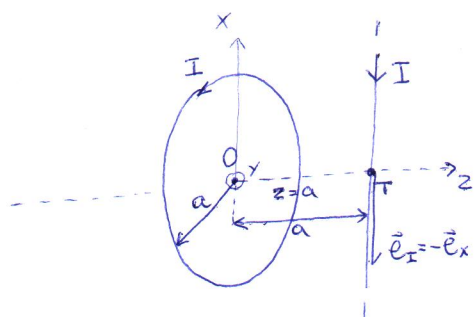
$$\tan \psi_1 = \frac{a/4}{a/2} = \frac{1}{2}$$

$$B_y(T) = \frac{\mu_0 (-I/4a)}{2\pi} \left[\ln \frac{\left(\frac{3a}{4} \right)^2 + \left(\frac{a}{2} \right)^2}{\left(\frac{a}{4} \right)^2 + \left(\frac{a}{2} \right)^2} - 2\phi_1 + 2\psi_1 \right] =$$

$$= -\frac{\mu_0 I}{8\pi a} \left[\ln \frac{13}{5} - 2 \arctan \frac{3}{2} + 2 \arctan \frac{1}{2} \right] \doteq 4,14 \mu\text{T} > 0$$

$$\vec{B}(T) = \vec{e}_y 4,14 \mu\text{T}$$

(11) Kolonvijska naloga (krožna zanka z tokom in ravni tanji vodniku)



$\vec{f}_m(T) = ?$

$$\vec{f}_m(T) = I \cdot \vec{e}_x \times \vec{B}(T)$$

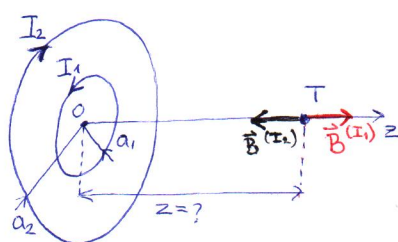
$$\vec{B} = \vec{B}^{(LEV)} + \vec{B}^{(DES)}$$

$$\vec{B}(T) = \vec{B}^{(LEV)}(T) = \vec{e}_z \frac{\mu_0 I}{2} \cdot \frac{a^2}{(a^2 + z^2)^{3/2}} = \vec{e}_z \frac{\mu_0 I}{2} \cdot \frac{a^2}{(a^2 + a^2)^{3/2}}$$

$$\vec{B}(T) = \vec{e}_z \frac{\mu_0 I}{\sqrt{2} \cdot 4 \cdot a}$$

$$\vec{f}_m(T) = I \underbrace{(\vec{e}_x \times \vec{e}_z)}_{\vec{e}_y} \frac{\mu_0 I}{4\sqrt{2}a} = \vec{e}_y \frac{\mu_0 I^2}{4\sqrt{2}a}$$

(12) Izpitna naloga (dve koncentrični krožni zanki)



$I_1 = 10A$ $a_1 = 10cm$
 $I_2 = 15A$ $a_2 = 20cm$

$$\vec{B} = \vec{B}^{(I_1)} + \vec{B}^{(I_2)}$$

$$z = ? \text{ da bo } \vec{B} = 0, \vec{B}_z(z) = 0$$

$$B_z(z) = \underbrace{\frac{\mu_0 I_1}{2} \cdot \frac{a_1^2}{(a_1^2 + z^2)^{3/2}}}_{B_z^{(I_1)}(z) > 0} - \underbrace{\frac{\mu_0 I_2}{2} \cdot \frac{a_2^2}{(a_2^2 + z^2)^{3/2}}}_{B_z^{(I_2)}(z) < 0} = 0 \quad / : \frac{\mu_0}{2}$$

$$\frac{I_1 a_1^2}{(a_1^2 + z^2)^{3/2}} = \frac{I_2 a_2^2}{(a_2^2 + z^2)^{3/2}}$$

$$I_1 a_1^2 (a_2^2 + z^2)^{3/2} = I_2 a_2^2 (a_1^2 + z^2)^{3/2} \quad / : I_1 a_1^2 \quad /^{2/3}$$

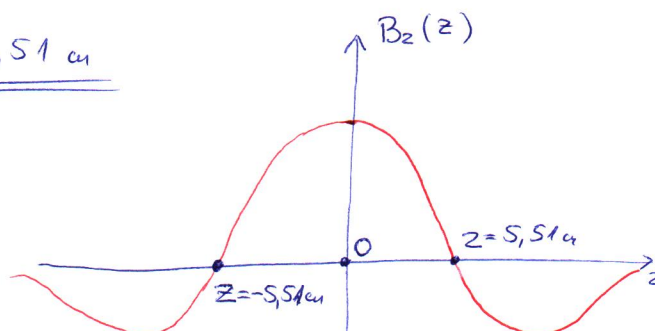
$$a_2^2 + z^2 = \underbrace{\left(\frac{I_2 a_2^2}{I_1 a_1^2} \right)^{2/3}}_{1,5 \cdot 2^2 = 6} (a_1^2 + z^2) = 6 a_1^2 + 6^{2/3} z^2$$

$$6^{2/3} z^2 - z^2 = a_2^2 - 6^{2/3} a_1^2$$

$$z^2 (6^{2/3} - 1) = 400cm^2 - 6^{2/3} \cdot 100cm^2 \quad / : 6^{2/3} - 1$$

$$z^2 = 100cm^2 \frac{4 - 6^{2/3}}{6^{2/3} - 1} \quad / \sqrt{\quad}$$

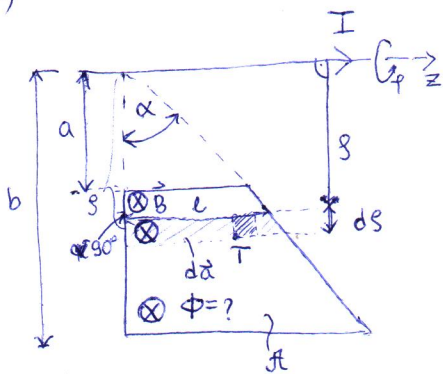
$$\underline{\underline{z = \pm 5,51cm}}$$



Vse razen
 krožnih
 zank
 - tou elem
 mag polje in sile
 - tou daljica
 - premera
 - dražni vodniku

4.13)

a)



$$\Phi = \int_A \vec{B} \cdot d\vec{a}$$

$$\vec{B} = \vec{e}_r B_r = \vec{e}_r \frac{\mu_0 I}{2\pi s} = \vec{B}(s)$$

$$d\vec{a} = \vec{e}_r da$$

$$da = l \cdot ds = \tan \alpha \cdot s \cdot ds$$

$$l(s) = \tan \alpha \cdot s$$

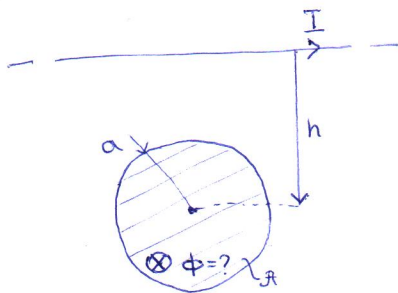
$$\tan \alpha = \frac{l(s)}{s}$$

$$\vec{B} \cdot d\vec{a} = \frac{\mu_0 I}{2\pi s} \underbrace{\vec{e}_r \cdot \vec{e}_r}_{=1} \tan \alpha \cdot s \cdot ds$$

$$\vec{B} \cdot d\vec{a} = \frac{\mu_0 I}{2\pi} \tan \alpha \cdot ds$$

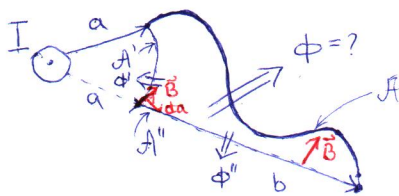
$$\Rightarrow \Phi = \int_A \vec{B} \cdot d\vec{a} = \int_a^b \frac{\mu_0 I}{2\pi} \tan \alpha \cdot ds = \frac{\mu_0 I}{2\pi} \tan \alpha \cdot \int_a^b ds = \frac{\mu_0 I}{2\pi} \tan \alpha \cdot (b-a)$$

b)



$$\Phi = \int_A \vec{B} \cdot d\vec{a} = \mu_0 I (b - \sqrt{b^2 - a^2})$$

c)



neizvornost $\vec{B}$

$$\oint \vec{B} \cdot d\vec{a} = 0 = \Phi + \Phi' + \Phi'' \Rightarrow \Phi = -(\Phi' + \Phi'')$$

$\nabla \cdot \vec{B} = 0$

$$\text{na } A': \vec{B} \perp d\vec{a} \Rightarrow \vec{B} \cdot d\vec{a} = 0$$

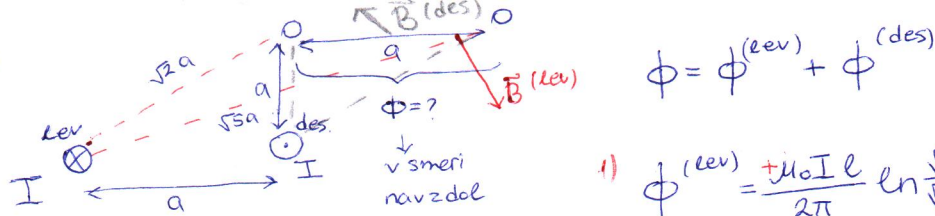
$$\Phi' = \int_{A'} \vec{B} \cdot d\vec{a} = 0$$

$$\text{mag. pretok skozi pravokotnik} \Rightarrow \Phi'' = \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}$$

$$\Phi = \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}$$

zaradi nasprotni smer $\vec{B}$ od Φ''

②



$I = 10 \text{ A}$
 $a = 0,5 \text{ m}$
 $l = 10 \text{ cm}$

1) $\phi^{(lev)} = \frac{\mu_0 I l}{2\pi} \ln \frac{\sqrt{5}a}{\sqrt{2}a}$

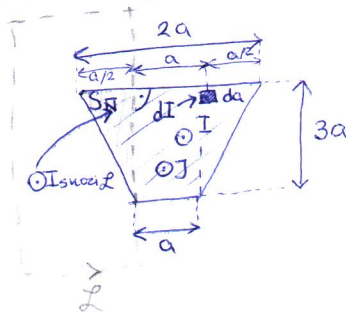
2) $\phi^{(des)} = -\frac{\mu_0 I l}{2\pi} \ln \frac{\sqrt{2}a}{a}$

$\Rightarrow \phi = \frac{\mu_0 I l}{2\pi} \left[\ln \frac{\sqrt{5}}{\sqrt{2}} - \ln \left(\frac{1}{\sqrt{2}} \right) \right] = \frac{\mu_0 I l}{2\pi} \ln \frac{\sqrt{5}}{2} = 2,23 \text{ mWb}$

veber

③

(Kolokvij) Vodnik ima trapezni preseku



$\oint \vec{B} \cdot d\vec{l} = ?$; $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{suzozl}$

$J = \frac{I}{S} = \frac{2I}{9a^2}$

$I_{suzozl} = J \cdot S_{\nabla} = \frac{2I}{9a^2} \cdot \frac{3a^2}{4} = \frac{I}{6}$

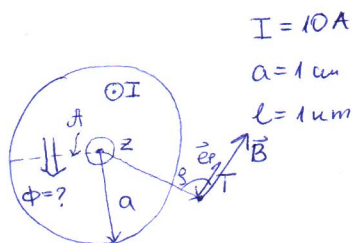
$(S = \frac{2a+a}{2} \cdot 3a = \frac{9a^2}{2})$

$S_{\nabla} = \frac{(a/2) \cdot 3a}{2} = \frac{3a^2}{4}$

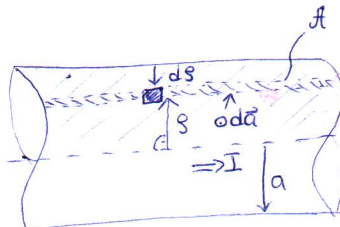
$\Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{suzozl} = \frac{I}{6} \cdot \mu_0$

Amperov zakon toka (zakon o vrtinosti)

④ Vodnik ima okrogli preseku:



$I = 10 \text{ A}$
 $a = 1 \text{ cm}$
 $l = 1 \text{ cm}$



$B_r = \begin{cases} \frac{\mu_0 I}{2\pi a^2} s, & s \leq a \\ \frac{\mu_0 I}{2\pi s}, & s > a \end{cases}$

$\vec{B} = \vec{e}_r \cdot B_r(s)$

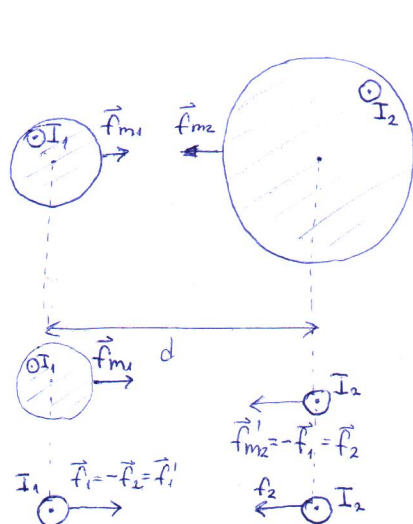
$\phi = \int \vec{B} \cdot d\vec{a} = \int_0^a \frac{\mu_0 I l}{2\pi a^2} \cdot s \cdot ds = \frac{\mu_0 I l}{2\pi a^2} \cdot \frac{s^2}{2} \Big|_0^a = \frac{\mu_0 I l}{2\pi a^2} \cdot \frac{a^2}{2} = \frac{\mu_0 I l}{4\pi} = \phi = 1 \text{ mWb} //$

$d\vec{a} = \vec{e}_r \cdot l \cdot ds$

$s \leq a: \vec{B} \cdot d\vec{a} = \frac{\mu_0 I}{2\pi a^2} s \cdot \underbrace{\vec{e}_r \cdot \vec{e}_r}_1 \cdot l \cdot ds = \frac{\mu_0 I l}{2\pi a^2} s \cdot ds$

$\phi = \frac{\mu_0 I l}{4\pi}$

⑤



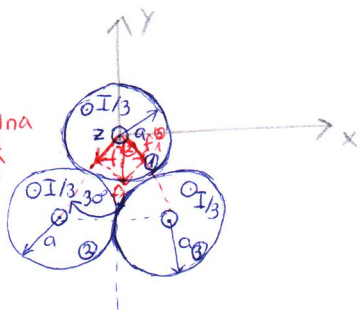
$\vec{f}_{m1} = -\vec{f}_{m2}$
 $f_{m1}, f_{m2} = ?$

$f_1 = f_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$

⑥ Snop 3 vodnikov

$$\vec{f} = ?$$

* horizontalna komponenta (x) se izniči



$$\vec{f}_1 = \vec{f}_1^{(2)} + \vec{f}_1^{(3)} = \vec{e}_y f_{1,y} = \vec{e}_y \cdot 2 \cdot \left(-\frac{\mu_0 I^2}{36\pi a} \cdot \frac{\sqrt{3}}{2} \right) = -\vec{e}_y \frac{\mu_0 I^2}{12\sqrt{3}\pi a}$$

$$\vec{f}_{1,x} = 0$$

$$\vec{f}_1^{(2)} = -\vec{f}_1^{(3)}$$

$$f_{1,y} = f_{1,y}^{(2)} = f_{1,y}^{(3)}$$

$$f_{1,y}^{(2)} = f_1^{(2)} \cdot \cos 30^\circ = -\frac{\mu_0 I^2}{36\pi a} \cdot \frac{\sqrt{3}}{2}$$

$$f_1^{(2)} = |\vec{f}_1^{(2)}| = \frac{\mu_0 I_1 I_2}{2\pi(2a)} = \frac{\mu_0 I^2}{36\pi a}$$

2. Test

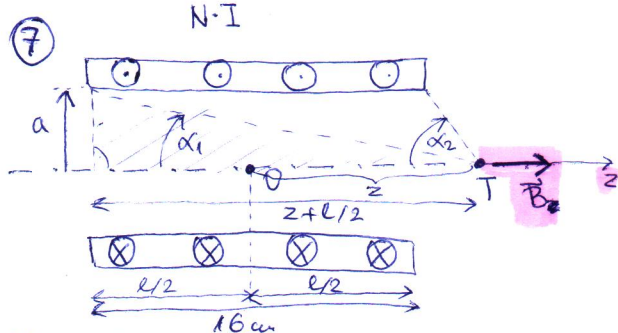
- krožni tokovi + I
(na osi B)

- tuljava B

- neizvarnost

- vrtilščnost (A, z)

- B, f, pri vodniku črtnatega preseka



$$N = 2000$$

$$I = 0,5 \text{ A}$$

$$a = 1 \text{ cm}$$

$$B_z = \frac{\mu_0 NI}{2l} [\cos \alpha_1 - \cos \alpha_2]$$

$$\cos \alpha_1 = \frac{z + l/2}{\sqrt{(z + l/2)^2 + a^2}}, \quad \cos \alpha_2 = \frac{z - l/2}{\sqrt{(z - l/2)^2 + a^2}}$$

$$B_z = \frac{\mu_0 NI}{2l} \left[\frac{z + l/2}{\sqrt{(z + l/2)^2 + a^2}} - \frac{z - l/2}{\sqrt{(z - l/2)^2 + a^2}} \right]$$

z/cm	0	0,5	1	2	4	7	8	9	12
Bz/mT	7,793	7,793	7,790	7,781	7,723	6,695	3,919	1,143	0,112
							$\approx \frac{B_z(z=0)}{2}$		

$$1) z = 0$$

$$B_z(z=0) = \frac{\mu_0 NI}{2l} \left[\frac{l/2}{\sqrt{(l/2)^2 + a^2}} - \frac{-l/2}{\sqrt{(-l/2)^2 + a^2}} \right] = \frac{\mu_0 NI}{2l} \cdot \frac{l}{\sqrt{(l/2)^2 + a^2}} \approx 7,793 \text{ mT}$$

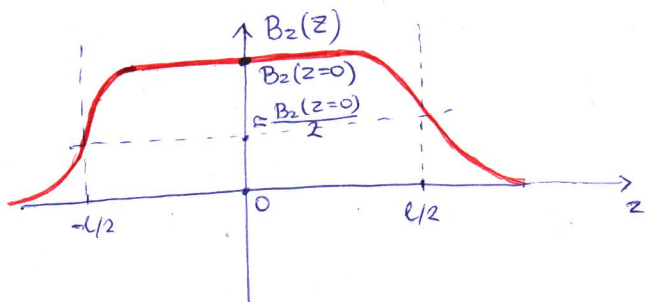
pri $l \gg 2a$: (za zelo dolga tuljavo)

$$B_z(z=0) \approx \frac{\mu_0 NI}{2} \cdot \frac{1}{l/2} = \frac{\mu_0 NI}{l} \approx 7,854 \text{ mT}$$

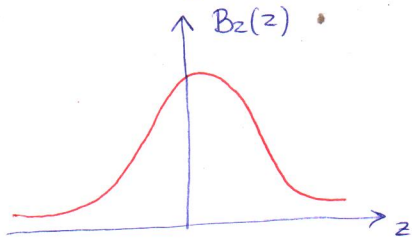
$$\Delta_{rel} = \frac{7,854 - 7,793}{7,793} \cdot 100\% = 0,78\% < 1\%$$

2) $z = 0,5 \text{ cm}$

$$B_z(z=0,5\text{ m}) = \frac{\mu_0 N I}{2l} \left[\frac{0,5\text{ m} + l/2}{\sqrt{(0,5\text{ m} + l/2)^2 + a^2}} - \frac{0,5\text{ m} - l/2}{\sqrt{(0,5\text{ m} - l/2)^2 + a^2}} \right] = 7,793\text{ mT}$$



kratka tuljava: $l \ll 2a$



⑧ 4.17)

$$l = 2 \text{ m}$$

$$2a = 1\text{m}$$

$$N = 100$$

$$B(z=0) = 1,8 \text{ T}$$

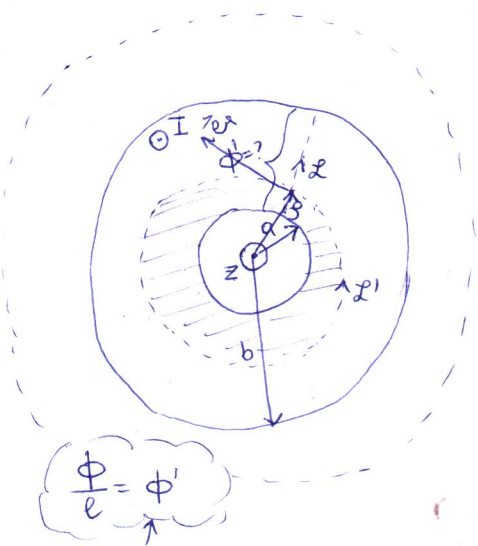
$$I = ?$$

(super prevodna tuljava)

$$B(z=0) = \frac{\mu_0 N I}{2} \cdot \frac{1}{\sqrt{(r/2)^2 + a^2}}$$

$$I = \frac{2 \cdot B(z=0)}{\mu_0 N} \cdot \sqrt{(l/2)^2 + a^2} = 32 \mu A$$

(g) 4.23)



$$\phi = \int_A \vec{B} \cdot d\vec{a} \quad /: l$$

$$\frac{\Phi}{\ell} = \Phi' = \int_{\mathcal{L}} \vec{B} \cdot \frac{d\vec{s} \cdot \vec{e}_r}{d\vec{r}/\ell}$$

$$d\vec{a} = l \cdot dS \cdot \vec{e}_r \quad /: l$$

$$\frac{d\vec{a}}{l} = d\varphi \cdot \vec{e}_\varphi$$

magnetni
pretok
na enoto
dolžine

Magnetno polje cevastega vodnika: $\vec{B}=?$

osna simetrija $\Rightarrow \vec{B} = \vec{B}(s) = \vec{e}_\rho B_\rho(s)$

$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{skozi } \mathcal{L}'}$$

$$\oint_{\mathcal{L}'} B_\rho(s) \vec{e}_\rho \cdot d\vec{\ell} = \oint_{\mathcal{L}'} B_\rho(s) \underbrace{\vec{e}_\rho \cdot \vec{e}_\rho}_1 d\ell = B_\rho(s) \oint_{\mathcal{L}'} d\ell = 2\pi s B_\rho(s)$$

$d\vec{\ell} = \vec{e}_\rho d\ell$

$I_{\text{skozi } \mathcal{L}'} = ?$ otštejemo (unjo)

$$J = \frac{I}{\pi b^2 - \pi a^2} = \frac{I}{\pi(b^2 - a^2)}$$

1) $a < s < b$

$$I_{\text{skozi } \mathcal{L}'} = J \cdot (\pi s^2 - \pi a^2) = \frac{I}{\pi(b^2 - a^2)} (\pi(s^2 - a^2)) = \frac{I(s^2 - a^2)}{b^2 - a^2}$$

2) $s > b$:

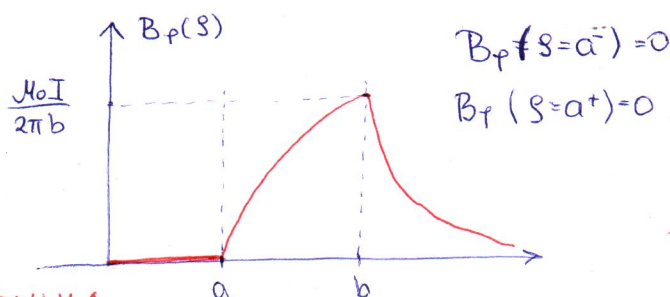
$$I_{\text{skozi } \mathcal{L}'} = I$$

3) $s < a$

$$I_{\text{skozi } \mathcal{L}'} = 0$$

$$2\pi s \cdot B_\rho(s) = \begin{cases} 0, & s < a \\ \mu_0 \frac{I}{b^2 - a^2} (s^2 - a^2), & a < s < b \\ \mu_0 I, & s > b \end{cases} \quad / : 2\pi s$$

$$B_\rho(s) = \begin{cases} 0, & s \leq a \\ \frac{\mu_0 I}{2\pi(b^2 - a^2)} (s - \frac{a^2}{s}), & a < s \leq b \\ \frac{\mu_0 I}{2\pi s}, & s > b \end{cases}$$



MAGNETNO POLJE CEVASTEGA VODNIKA

$$\vec{B} = \vec{e}_\rho B_\rho(s)$$

$$\Phi' = \int_{\mathcal{L}} \vec{B} \cdot \vec{e}_\rho ds = \int_a^b \underbrace{\frac{\mu_0 I}{2\pi(b^2 - a^2)} (s - \frac{a^2}{s})}_{\vec{B}} \cdot \vec{e}_\rho \cdot \vec{e}_\rho ds = \frac{\mu_0 I}{2\pi(b^2 - a^2)} \int_a^b (s - \frac{a^2}{s}) ds$$

$$\left[\frac{s^2}{2} - a^2 \ln s \right]_a^b = \frac{b^2 - a^2}{2} - a^2 \ln \frac{b}{a}$$

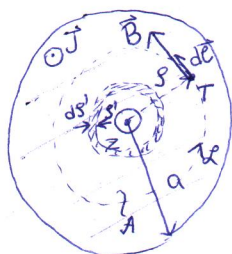
$$\Phi' = \frac{\mu_0 I}{2\pi(b^2 - a^2)} \left(\frac{b^2 - a^2}{2} - a^2 \ln \frac{b}{a} \right)$$

$$\Phi = \Phi' \cdot \ell$$

Magnetni pretok na enoto dolžine
cevastega vodnika

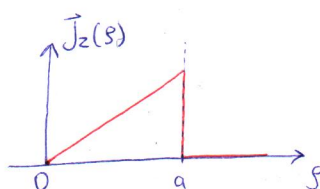
(10)

4.26)

 $\vec{B} = ?$ 

$$\vec{J} = \vec{e}_z \frac{3I_0}{2\pi a^3} s = \vec{e}_z J_z(s), \quad s \leq a \Rightarrow \vec{B} = \vec{e}_\varphi B_\varphi(s)$$

$$\vec{J}(s > a) = \vec{0}$$



$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{suozil}} L$$

$$d\vec{l} = \vec{e}_\varphi dl$$

$$\oint_L \vec{B} \cdot d\vec{l} = \oint_L B_\varphi(s) dl = B_\varphi(s) \cdot 2\pi s$$

2) $s > a$:

$$I_{\text{suozil}} L = \int_0^a \frac{3I_0}{a^3} s'^2 ds' = I_0$$

$$\oint_L \vec{B} \cdot d\vec{l} = \begin{cases} \frac{\mu_0 I_0}{a^3} s^3, & s < a \\ \mu_0 I, & s > a \end{cases} \quad /: 2\pi s$$

$$\underline{B_\varphi(s)} = \begin{cases} \frac{\mu_0 I_0}{2\pi a^3} s^2, & s \leq a \\ \frac{\mu_0 I_0}{2\pi s}, & s > a \end{cases}$$

1) $s < a$

$$I_{\text{suozil}} L = \int_V \vec{J} \cdot d\vec{a} =$$

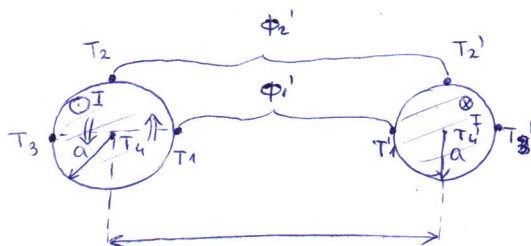
$$d\vec{a} = 2\pi s' ds' \vec{e}_z$$

$$I_{\text{suozil}} L = \int_0^s \frac{3I_0}{2\pi a^3} s' \cdot \vec{e}_z \cdot \vec{e}_z \cdot 2\pi s' ds' = \frac{I_0}{a^3} s^3$$

$$I_{\text{suozil}} L = \frac{3I_0}{a^3} \int_0^s s'^2 ds' = \frac{I_0}{a^3} s^3$$

Video Sinigol: $a_1, a_2 \ll \delta$

$$L = \frac{\mu_0 I}{\pi} \left(\frac{1}{2} + \ln \frac{d}{\sqrt{a_1 a_2}} \right)$$

 $\phi' = ?$ $\frac{\phi}{L}$ za preboj suozil pravouga.

$$\frac{\phi_1}{L} = \phi_1' = \phi_1'^{(\text{lev})} + \underbrace{\phi_1'^{(\text{des})}}_{\phi_1'^{(\text{lev})}} = 2 \cdot \phi_1'^{(\text{lev})}$$

$$\underline{\phi_1'} = 2 \cdot \frac{\mu_0 I}{8\pi} \ln \frac{d-a}{a}$$

$$\underline{\phi_2'} = 2 \cdot \phi_2'^{(\text{lev})} = 2 \cdot \frac{\mu_0 I}{8\pi} \ln \frac{\sqrt{d^2 + a^2}}{a}$$

$$\underline{\phi_3'} = 2 \cdot \phi_3'^{(\text{lev})} = 2 \cdot \frac{\mu_0 I}{8\pi} \ln = 2 \left[\phi_1'^{(\text{lev})}(T_3, T_1) + \phi_1'^{(\text{lev})}(T_1, T_3') \right] = 2 \cdot \frac{\mu_0 I}{8\pi} \ln \frac{d+a}{a}$$

$$\underline{\phi_4'} = 2 \cdot \phi_4'^{(\text{lev})} = 2 \cdot \left[\phi_1'^{(\text{lev})}(T_4, T_1) + \phi_1'^{(\text{lev})}(T_1, T_4') \right] = 2 \left[\frac{\mu_0 I}{4\pi} + \frac{\mu_0 I}{2\pi} \ln \frac{d}{a} \right]$$

$$\underline{\phi_4'} = \frac{\mu_0 I}{\pi} \left[\frac{1}{2} + \ln \frac{d}{a} \right]$$

$$L = \frac{\langle \phi \rangle}{I} = \frac{\mu_0 L}{\pi} \left[\frac{1}{2} + \ln \frac{d}{a} \right]$$

$$\langle \phi' \rangle = \frac{\mu_0 I}{\pi} \left[\frac{1}{2} + \ln \frac{d}{a} \right]$$

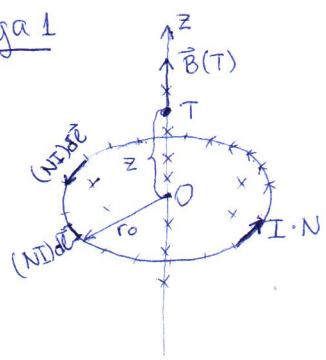
Induktivnost
dvostrana

www.stromar.si

Avditorne vaje (18.3.2016)

kratka tuljava - krožna tokovna zanka

Naloga 1

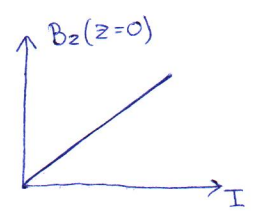


$N=100$
 $I=1A$
 $N \cdot I = 100A$

$$B_z(z) = \frac{\mu_0(NI)}{2} \cdot \frac{r_0^2}{[r_0^2 + z^2]^{3/2}}$$

③

$I(A)$	0	0,1	...	0,9	1
$B_z(z=0)$	0			365 μT	



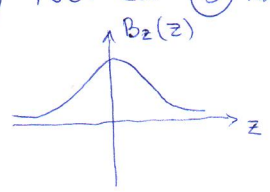
$I=1A$:

z/cm	-13	-12	...	27
$B_z(z)/\mu T$		200 μT		

④ Fakultativno ($\oint \vec{B} \cdot d\vec{l} = \mu_0(NI)$) s B.S. zakon

* Računalnik (Dodatni gradiva: • Izračun magnetnega polja v ravnini kratke tuljave oz. krožne tokovne zanke) Mathematica (Numerična integracija)

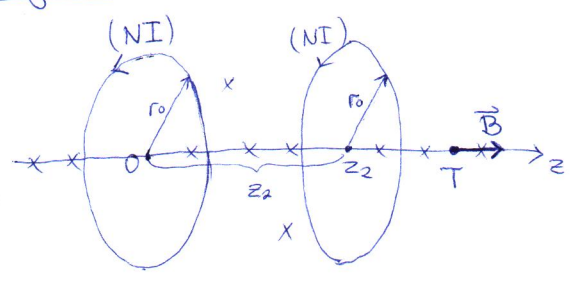
③ Tudi za ③ vprašanje: • Uporaba računalnika za risanje grafov



(www.torata.forum.fe.uni-lj.si)

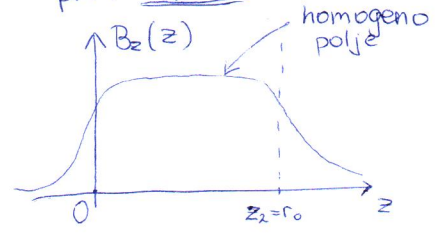
OE2: Priprave na 1. lab vajo

Naloga 2



z/cm	-13	-12	...	27
$B_z(z)/\mu T$		212 μT		

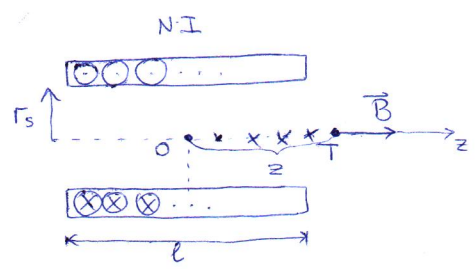
pri: $z_2 = r_0$ (Helmholtzov par)



$l: 5cm, 10cm, 20cm$

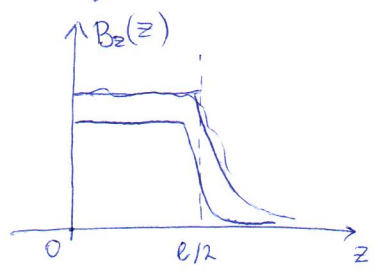
• Tudi z računalnikom

Naloga 3



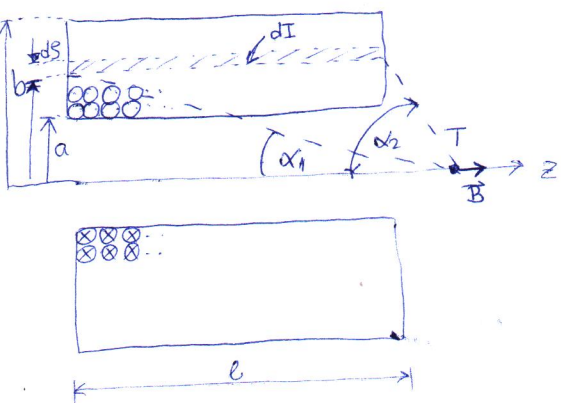
* Formula za dolga tuljava

z/cm	0	1	2	...	15
$B_z(z)/\mu T$		9,29			



4) Fakultativna

$$N = 2000$$



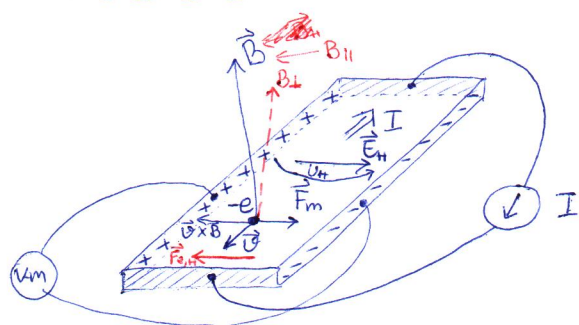
$$dB_z = \frac{\mu_0 \cdot dI}{2 \cdot l} (\cos \alpha_1 - \cos \alpha_2)$$

$$dI = \frac{N \cdot I}{b-a} \cdot ds$$

$$B_z(T) = \int_{s=a}^b dB_z$$

Zbirna nalog: 4.16

Hallov pojav



$$\vec{F}_m = (-e) \vec{v} \times \vec{B}$$

$$\vec{F}_m + (-e) \vec{E}_H = \vec{0}$$

$$U_H \propto F_m$$

$$|\vec{F}_m| = F_m = e |\vec{v} \times \vec{B}| = e \cdot v \cdot B_{\perp}$$

$$U_H \propto B_{\perp}$$

$$U_H \propto I$$

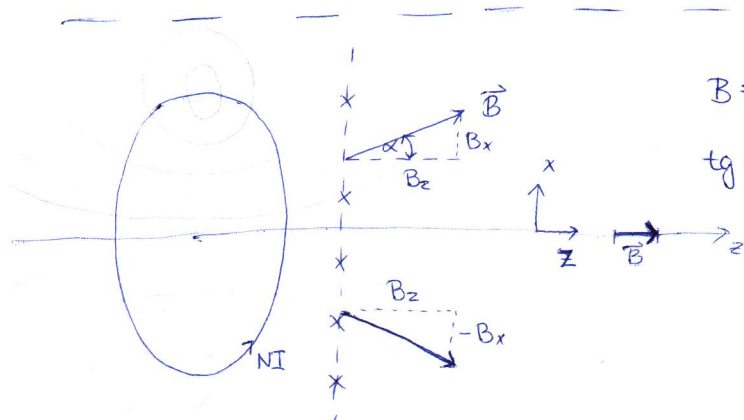
Tesla-meter

Kalibracija

$$\vec{B} = \vec{B}_{izm}^{(I)} + \vec{B}_{smog}$$

zemljeno polje
u in druga
polja

pri: $I=0$: $B^{(I)}=0 \Rightarrow \vec{B}_{izm} = \vec{B}_{smog} \rightarrow$ nastavitel
ničle

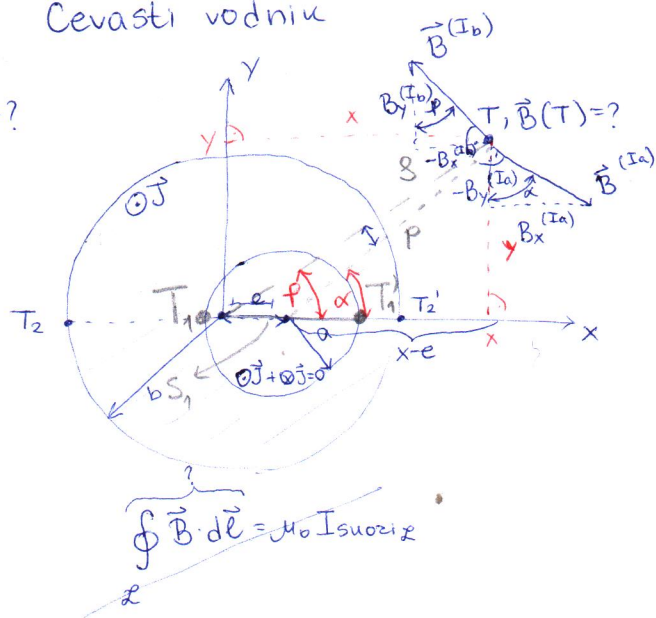


$$B = \sqrt{B_x^2 + B_z^2}$$

$$\tan \alpha = \frac{B_x}{B_z} \Rightarrow \alpha = \arctan \frac{B_x}{B_z}$$

4.20)

Čevasti vodnik

 $B=?$ 

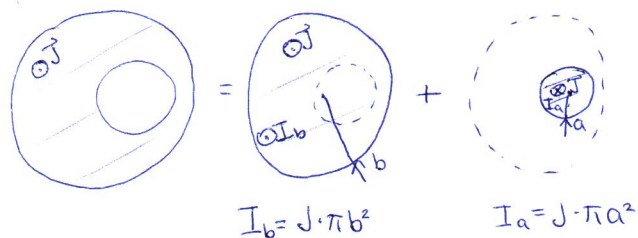
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

$$\underline{B^{(I_b)}} = \begin{cases} \frac{\mu_0 I_b}{2\pi b^2} s, & s \leq b \\ \frac{\mu_0 I_b}{2\pi s}, & s > b \end{cases} =$$

$$= \begin{cases} \frac{\mu_0 J \cdot \pi b^2}{2\pi b^2} s, & s \leq b \\ \frac{\mu_0 J \cdot \pi b^2}{2\pi s}, & s > b \end{cases} \Rightarrow$$

$$= \begin{cases} \frac{\mu_0 J}{2} \cdot s, & s \leq b \\ \frac{\mu_0 J \cdot b^2}{2s}, & s > b \end{cases}$$

$$\underline{B^{(I_a)}} = \begin{cases} \frac{\mu_0 J}{2} \cdot p, & p \leq b \\ \frac{\mu_0 J a^2}{2p}, & p > a \end{cases}$$



$$\vec{B} = \vec{B}^{(I_b)} + \vec{B}^{(I_a)}$$

$$B_x = B_x^{(I_b)} + B_x^{(I_a)}, \quad B_y = B_y^{(I_b)} + B_y^{(I_a)}$$

$$B_x^{(I_b)} = -B^{(I_b)} \sin \phi, \quad B_x^{(I_a)} = B^{(I_a)} \sin \alpha$$

$$B_y^{(I_b)} = B^{(I_b)} \cos \phi, \quad B_y^{(I_a)} = -B^{(I_a)} \cos \alpha$$

$$\sin \phi = \frac{y}{s}, \quad \cos \phi = \frac{x}{s}, \quad s = \sqrt{x^2 + y^2}$$

$$p = \sqrt{(x-e)^2 + y^2}$$

$$\sin \alpha = \frac{y}{p}, \quad \cos \alpha = \frac{x-e}{p}$$

1) LUKNJI: $p < a$

$$B_x = -B^{(I_b)} \sin \phi + B^{(I_a)} \sin \alpha =$$

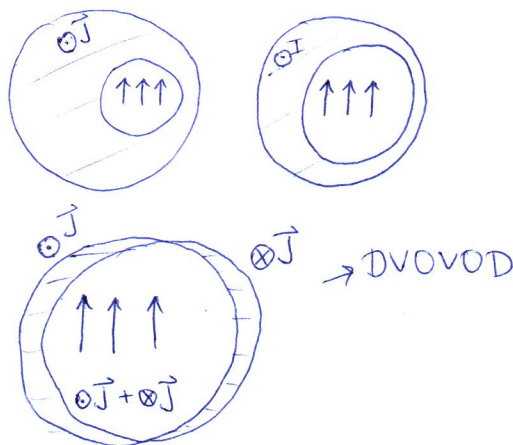
$$= -\frac{\mu_0 J}{2} \cdot s \cdot \frac{y}{s} + \frac{\mu_0 J}{2} \cdot p \cdot \frac{y}{p} = 0$$

$$\underline{B_y} = B^{(I_b)} \cos \phi - B^{(I_a)} \cos \alpha =$$

$$= \frac{\mu_0 J}{2} \cdot s \cdot \frac{x}{s} - \frac{\mu_0 J}{2} \cdot p \cdot \frac{x-e}{p} = \underline{\underline{\frac{\mu_0 J}{2} e}}$$

$$\underline{B_y = \frac{\mu_0 J}{2} \cdot e = \text{konst}}$$

(homogeno polje)



2) V VODNIKU: $p > a$ in $s < b$

$$B_x = -\frac{\mu_0 J}{2} s \cdot \frac{y}{s} + \frac{\mu_0 J \cdot a^2}{2p} \cdot \frac{y}{p} = \frac{\mu_0 J}{2} \cdot y \left[-1 + \frac{\frac{a^2/p^2}{(x-e)^2+y^2}} \right] = B_x(x, y)$$

B je funkcija
koord. x in y
↑ (polje se spreminja)

$$B_y = \frac{\mu_0 J}{2} s \cdot \frac{x}{s} - \frac{\mu_0 J \cdot a^2}{2p} \cdot \frac{x-e}{p} = \frac{\mu_0 J}{2} \left[x - \frac{a^2(x-e)}{p^2} \right] = B_y(x, y)$$

3) IZVEN VODNIKA: $p < a$

$$B_x = \dots$$

$$B_y = \dots$$

Magnetni pretok

$$\Phi(T_1, T_1') = \int_{S_1} \vec{B} \cdot d\vec{s} = \int_{-(a-e)}^{a+e} \underbrace{\frac{\mu_0 J}{2} \cdot e \cdot \vec{e}_y}_{\vec{e}_y \frac{\mu_0 J}{2} \cdot e = \vec{B}} \cdot \underbrace{\vec{e}_y \cdot \vec{e}_y \cdot l \cdot dx}_{d\vec{s}} = \frac{\mu_0 J}{2} \cdot e \cdot l \left[x \right]_{-(a-e)}^{a+e} = \frac{\mu_0 J}{2} \cdot e \cdot l \cdot 2a \Rightarrow$$

$$\Phi = \mu_0 J \cdot l \cdot a \cdot e \rightarrow \text{magnetni pretok v lučki (polje je homogeno)}$$

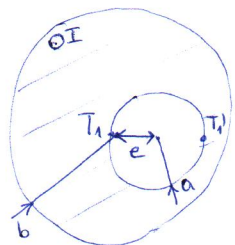
$$\Phi(T_2, T_2') = \underbrace{\Phi^{(I_b)}(T_2, T_2')}_0 + \underbrace{\Phi^{(I_a)}(T_2, T_2')}_{\frac{\mu_0 I_a \cdot l}{2\pi} \ln \frac{b+e}{b-e}} = \frac{\mu_0 J \cdot \pi \cdot a^2 l}{2\pi} \ln \frac{b+e}{b-e} = \frac{\mu_0 J a^2 l}{2} \ln \frac{b+e}{b-e}$$

(pretok skozi pravokotnik)

$$\Phi = \frac{\mu_0 J a^2 l}{2} \ln \frac{b+e}{b-e} \rightarrow \text{magnetni pretok v ekscentrični cevasti vodniku}$$

4.21)

Pretou na enoto dolžine



$$\frac{\Phi(T_1, T_1')}{l} = \Phi'(T_1, T_1') = ?$$

$$\Phi'(T_1, T_1') = \mu_0 J \cdot a \cdot e = \frac{\mu_0 I a \cdot e}{\pi(b^2 - a^2)} = 0,2 \mu Wb/m$$

$$I = 3A$$

$$a = 2cm$$

$$b = 4cm$$

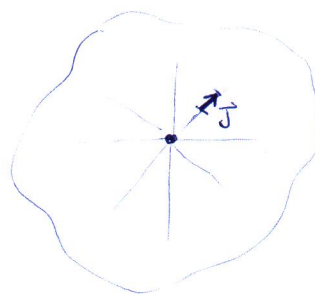
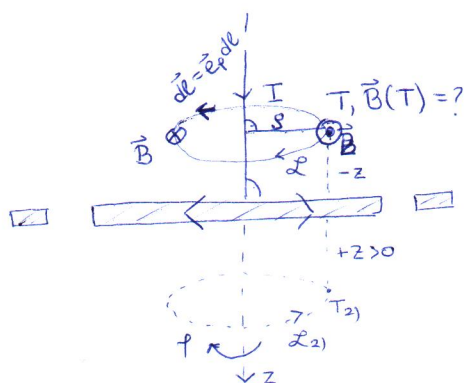
$$e = 1cm$$

* V lunjji
NITOKA
!

$$J = \frac{I}{\pi b^2 - \pi a^2}$$

Naloga 1

(točkovna premica in zelo rasežna plošča)

radialni
planarni
tok

• zakon o vrtinčnosti:

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{suozl } L}$$

$$\vec{B} = \vec{e}_\varphi \cdot B_\varphi(s, z)$$

$$\oint_L \underbrace{B_\varphi(s, z)}_{\text{konst}} \underbrace{\vec{e}_\varphi \cdot \vec{e}_\varphi}_1 d\ell = B_\varphi(s, z) \int_L d\ell = B_\varphi(s, z) \cdot 2\pi s = \begin{cases} \mu_0 I, & z < 0 \\ 0, & z > 0 \end{cases} : 2\pi s$$

na krožnici L : $s = \text{konst}$, $z = \text{konst}$.

$$I_{\text{suozl } L} = ?$$

1) nad ploščo: $z < 0$

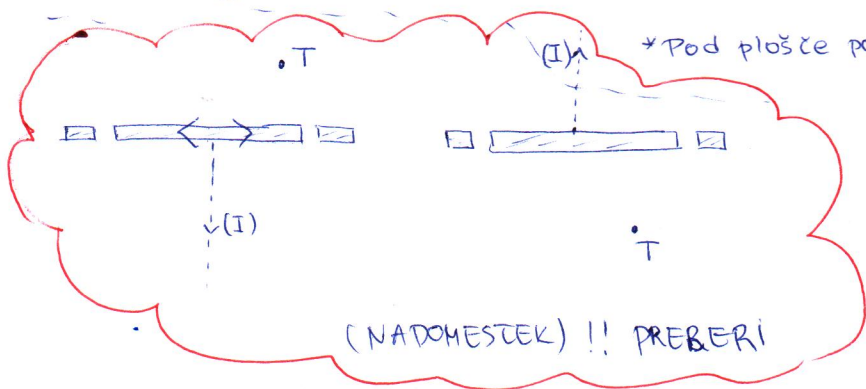
$$I_{\text{suozl } L} = I$$

2) pod ploščo: $z > 0$

$$I_{\text{suozl } L} = 0$$

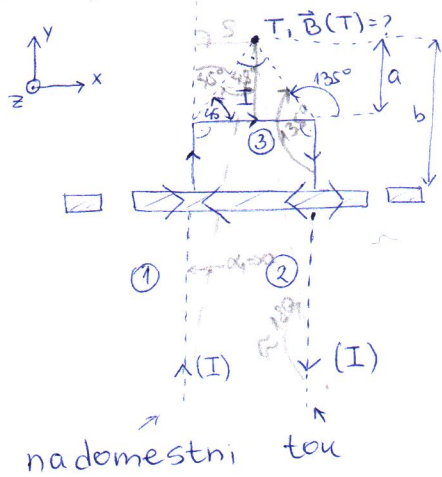
$$B_\varphi(s, z) = \begin{cases} \frac{\mu_0 I}{2\pi s}, & z < 0 \\ 0, & z > 0 \end{cases}$$

* Pod plošče poljani!



(NADOMESTEK) !! PREBERI

4.28) Razsežna plošča (radialni ploskovni tok)

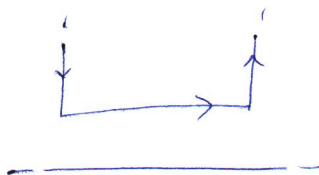
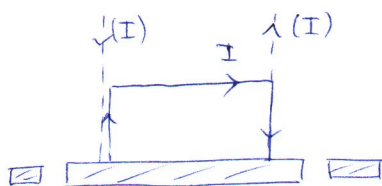


$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\vec{B}(T) = -\vec{e}_z \frac{\mu_0 I}{4\pi a} \left(\cos 0^\circ - \cos 45^\circ \right) - \vec{e}_z \frac{\mu_0 I}{4\pi a} \left(\cos 135^\circ - \cos 180^\circ \right) + \vec{e}_z \frac{\mu_0 I}{4\pi a} \left(\cos 45^\circ - \cos 135^\circ \right)$$

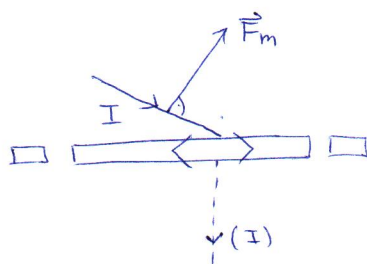
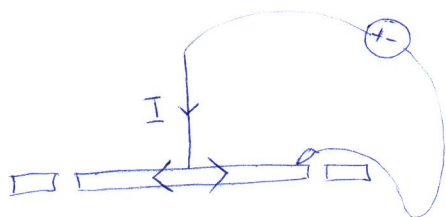
$$\vec{B}(T) = \vec{e}_z \frac{\mu_0 I}{4\pi a} \left[-\left(1 - \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} + 1\right) + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \right] =$$

$$\vec{B}(T) = \vec{e}_z \frac{\mu_0 I}{2\pi a} 2(\sqrt{2} - 1) = \vec{e}_z \frac{\mu_0 I}{2\pi a} (\sqrt{2} - 1)$$

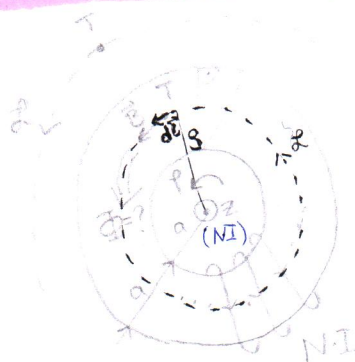


$$T, B(T) = ?$$

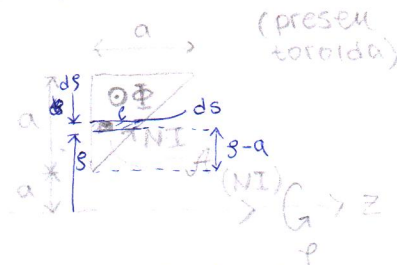
PRAKSA (VARJENJE)



① Toroidno navitje (tuljava)



$$I_{\text{skozil}} = \begin{cases} NI, & \text{znotraj toroida} \\ 0, & \text{zunaj toroida.} \end{cases}$$



$$\begin{aligned} dS &= l(s)ds = (s-a) \cdot ds \\ l(s) &= s-a \\ d\vec{S} &= \vec{e}_r (s-a) \cdot ds \end{aligned}$$

$$\Phi = \int_A \vec{B} \cdot d\vec{a}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{skozil}}$$

$$\vec{B} = \vec{e}_r B_r$$

$$\oint B_r \vec{e}_r \cdot \frac{1}{d\vec{l}} d\vec{l} = B_r(s) \oint d\vec{l} = 2\pi s B_r(s)$$

$$2\pi s B_r(s) = \begin{cases} \mu_0 NI, & \text{znotraj t.} \\ 0, & \text{izven t.} \end{cases} \quad / : 2\pi s$$

$$B_r(s) = \begin{cases} \frac{\mu_0 NI}{2\pi s}, & \text{znotraj toroidne tuljave} \\ 0, & \text{izven t.t.} \end{cases} \quad \vec{B} = \vec{e}_r B_r(s)$$

$$\Phi = \int_A \vec{B} \cdot d\vec{a} = \int_a^{2a} \frac{\mu_0 NI}{2\pi s} \vec{e}_r \cdot \vec{e}_r (s-a) \cdot ds = \frac{\mu_0 NI}{2\pi} \int_a^{2a} \left(1 - \frac{a}{s}\right) \cdot ds = \frac{\mu_0 NI}{2\pi} \left[s - a \ln s \right]_a^{2a}$$

$$\Phi = \frac{\mu_0 NI}{2\pi} \left[2a - a - a \ln \frac{2a}{a} \right] = \frac{\mu_0 NI}{2\pi} a \left(1 - \ln 2 \right) = \frac{\mu_0 NI a}{2\pi} \ln \frac{e}{2}$$

$e \approx 2,718...$

TEST 2

Hydrogen

Cerovnik (tudi avtoritativni)

avtoritativni

podatki: avtoritativni

3)

$$A_m = ?$$

$$A_m = I (\Phi_{\text{uon.}} - \Phi_{\text{zač.}}) = I (B \cdot S - \underbrace{\vec{B} \cdot \vec{S}}_{-\vec{B}_z S}) = I (B \cdot S - (-\vec{B}_z S)) = I \cdot S \cdot (B + B_z) \doteq 2,42 \text{ J}$$

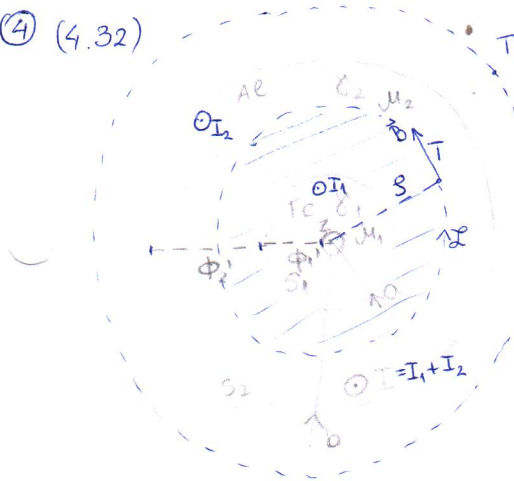
$$\Phi = \int_A \vec{B} \cdot d\vec{S}$$

$$B = \sqrt{20^2 + 30^2 + 50^2} \text{ mT}$$

$$A_m = W_{\text{mp, zač.}} - W_{\text{mp, uon.}} = -I \underbrace{\vec{S} \cdot \vec{B}}_{-B_z S} - (-I S B) = I (S B - (-B_z S))$$

$$W_{\text{mp}} = -\vec{m} \cdot \vec{B}$$

④ (4.32)



$$I = 400 \text{ A}$$

$$S_1 = 20 \text{ mm}^2$$

$$S_2 = 120 \text{ mm}^2$$

$$\delta_2 / \delta_1 = 4$$

$$\mu_1 / \mu_2 = 500$$

$$\mu_2 \approx \mu_0$$

$$\mu_1 \approx 500 \mu_0$$

$$\Phi_1' = ?$$

$$\Phi_2' = ?$$

$$\vec{B} = \vec{e}_\varphi B_\varphi(s)$$

$$\pi a^2 = S_1 \Rightarrow a = \sqrt{S_1 / \pi} \doteq 2,52 \text{ mm}$$

$$\pi b^2 = S_1 + S_2 \Rightarrow b = \sqrt{(S_1 + S_2) / \pi} \doteq 6,68 \text{ mm}$$

$$I_1, I_2 = ?$$

$$I = I_1 + I_2$$

$$U = R_1 I_1 = R_2 I_2 \quad \frac{I_2}{I_1} = \frac{R_1}{R_2} = \frac{\frac{\delta_1 \cdot S_1}{\delta_2 \cdot S_2}}{\frac{\delta_1 \cdot S_1}{\delta_2 \cdot S_2}} = \frac{\delta_2}{\delta_1} \cdot \frac{S_2}{S_1} = 20$$

$$I_2 = 24 I_1$$

$$I = I_1 + 24 I_1 = 25 I_1$$

$$I_1 = 16 \text{ A} ; I_2 = 384 \text{ A}$$

$$P = I^2 \cdot R \quad \text{manjše izgube (AL) (moč je večja)}$$

$$\Phi_1' = \int_0^a B_\varphi(s) \cdot dS$$

$$\Phi_2' = \int_a^b B_\varphi(s) \cdot dS$$

Na mesto:

$$\oint_L \vec{B} \cdot d\vec{e} = \mu_0 I_{\text{suozil}} \quad ?! ? (\mu_A \text{ tou zank.})$$

Uporabimo:

$$\oint_L \vec{H} \cdot d\vec{e} = I_{\text{prosti suozil}} ; \quad \vec{H} = \vec{e}_\varphi H_\varphi(s) \Rightarrow \oint_L \vec{H} \cdot d\vec{e} = 2\pi s \cdot H_\varphi(s) = \dots \quad /: 2\pi s$$

$$I_{\text{prosti suozil}} = \begin{cases} \frac{I_1}{\pi a^2} \pi \cdot s^2, & s < a \\ I_1 + \frac{I_2}{\pi(b^2 - a^2)} \pi(s^2 - a^2), & a < s < b \\ I, & s > b \end{cases}$$

$$H_\varphi(s) = \begin{cases} \frac{I_1}{2\pi a^2} s, & s < a \\ \frac{I_1}{2\pi s} + \frac{I_2}{2\pi(b^2 - a^2)} (s - \frac{a^2}{s}), & a < s < b \\ \frac{I}{2\pi s}, & s > b \end{cases}$$

$$\vec{B} = \mu \vec{H}$$

$$\mu = \begin{cases} \mu_1, & s < a \\ \mu_2 = \mu_0, & a < s < b \\ \mu_0, & s > b \end{cases}$$

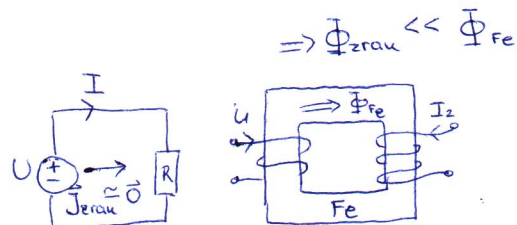
$$B_p(s) = \begin{cases} \frac{\mu_1 I_1}{2\pi a^2} s, & s < a \\ \frac{\mu_0 I_1}{2\pi s} + \frac{\mu_0 I_2}{2\pi(b^2 - a^2)} \left(s - \frac{a^2}{s}\right), & a < s < b \\ \frac{\mu_0 I}{2\pi s}, & s > b \end{cases}$$

$$\Phi_1' = \int_0^a \frac{\mu_0 I_1}{2\pi a^2} s \cdot ds = \frac{\mu_0 I_1}{2\pi a^2} \cdot \frac{a^2}{2} = \frac{\mu_0 I_1}{4\pi} = 800 \mu \text{ Wb/m}$$

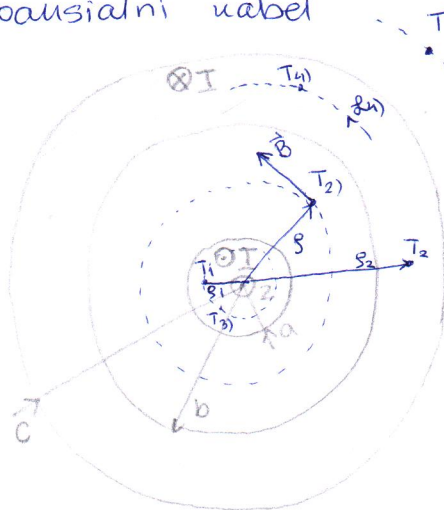
$$\Phi_2' = \int_a^b \left[\frac{\mu_0 I_1}{2\pi s} + \frac{\mu_0 I_2}{2\pi(b^2 - a^2)} \left(s - \frac{a^2}{s}\right) \right] ds = \dots \approx 29,1 \mu \text{ Wb/m}$$

$$\frac{\Phi_1'}{\Phi_2'} \approx 27,5$$

Feromagnetni so zelo dobri prevodniki za magnetno polje (magnetni pretoki)



5. Koaksialni kabel



$$I = 400 \text{ A}$$

$$a = 1 \text{ cm}$$

$$b = 2 \text{ cm}$$

$$c = 2,5 \text{ cm}$$

lito jenko

$$\vec{B}(T_1), \vec{B}(T_2) = ?$$

$$s_1 = \frac{a}{2}; \quad s_2 = \frac{b+c}{2}$$

$$\oint_L \vec{B} \cdot d\vec{l} = \mu_0 \frac{I_{\text{suozil}}}{2} \quad \text{?!? (} \mu \text{A tou. zanu.)}$$

$$\oint_L \vec{H} \cdot d\vec{l} = I_{\text{prosti suozil}}$$

$$\vec{H} = \vec{e}_r H_p(s)$$

$$\text{v nemagnetni: } \vec{B} \approx \mu_0 \vec{H}$$

$$\text{v magnetni: } \vec{B} \propto \vec{H} \text{ (ni vedno)}$$

$$\oint_L \vec{H} \cdot d\vec{l} = \oint_L H_p(s) \underbrace{\vec{e}_r \cdot \vec{e}_r}_1 dl = H_p(s) \oint_L dl = 2\pi s H_p(s)$$

$$I_{\text{prosti suozil}} = ?$$

$$1) \underline{s > c}$$

$$I_{\text{prosti suozil}} = I + (-I) = 0$$

$$2) \underline{a < s < b}$$

$$I_{\text{prosti suozil}} = I$$

$$3) \underline{s < a}$$

$$J_{\text{žile}} = \frac{I}{\pi a^2}$$

$$I_{\text{prosti suozil}} = J_{\text{žile}} \cdot \pi s^2 = \frac{I}{a^2} s^2$$

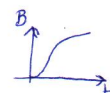
4) $b < s < c$

$$J_{\text{plaz.}} = \frac{I}{\pi(c^2 - b^2)}$$

$$I_{\text{prosti suozil}} = I + (-J_{\text{plaz.}}) \cdot \pi(s^2 - b^2) = I - \frac{I}{c^2 - b^2} (s^2 - b^2) = \frac{c^2 - b^2 - s^2 + b^2}{c^2 - b^2} I = \frac{c^2 - s^2}{c^2 - b^2} I$$

$$2\pi s H_f(s) = \begin{cases} \frac{I}{a^2} s^2, & s < a \\ I, & a < s < b \\ \frac{I}{c^2 - b^2} (c^2 - s^2), & b < s < c \\ 0, & s > c \end{cases} : 2\pi s$$

$$H_f(s) = \begin{cases} \frac{I}{2\pi a^2} s, & s < a \\ \frac{I}{2\pi s}, & a < s \leq b \\ \frac{I}{2\pi(c^2 - b^2)} \left(\frac{c^2}{s} - s \right), & b < s < c \\ 0, & s > c \end{cases}$$



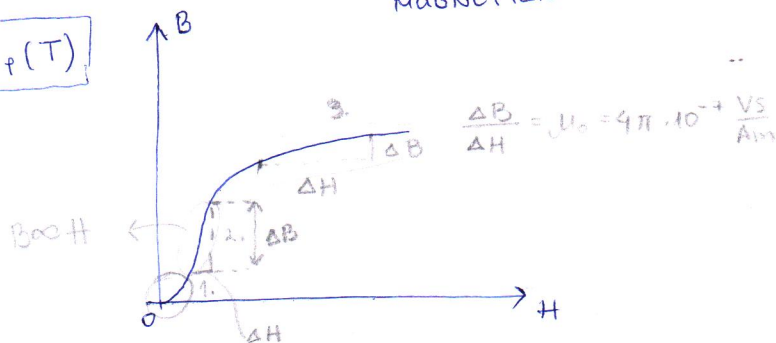
$$H_f(T_1) = H_f(s_1) = H_f\left(\frac{a}{2}\right) = \frac{I}{2\pi a^2} \cdot \frac{a}{2} = \frac{I}{4\pi a} = 3,2 \mu\text{A/m} \longrightarrow B_f(T_1) = 1,52 \text{ T}$$

$$H_f(T_2) = H_f(s_2) = H_f\left(\frac{b+c}{2}\right) = \frac{I}{2\pi(c^2 - b^2)} \cdot \left(\frac{c^2}{(b+c)/2} - \frac{b+c}{2} \right) = 1,5 \mu\text{A/m} \longrightarrow B_f(T_2) = 1,28 \text{ T}$$

$$B_f(T) = \mu(T) \cdot H_f(T)$$

$$\mu_{Fe} \neq \text{const}$$

$$\mu_{Fe} = \mu_{Fe}(H)$$



$$\vec{B}(T_1) = \vec{e}_r 1,52 \text{ T}$$

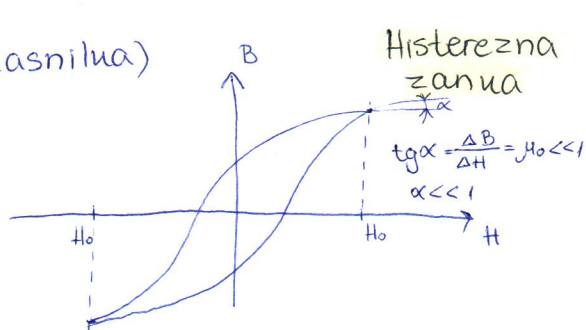
$$\vec{B}(T_2) = \vec{e}_r 1,28 \text{ T}$$

$$H_f(s=a^-) = \frac{I}{2\pi a^2} a$$

$$H_f(s=a^+) = \frac{I}{2\pi a}$$

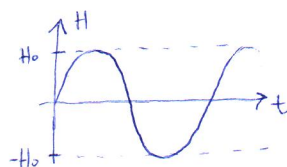
$$B_f(s=a^-) \gg B_f(s=a^+)$$

(nasnilua)



(pri sinusno magnetenje)

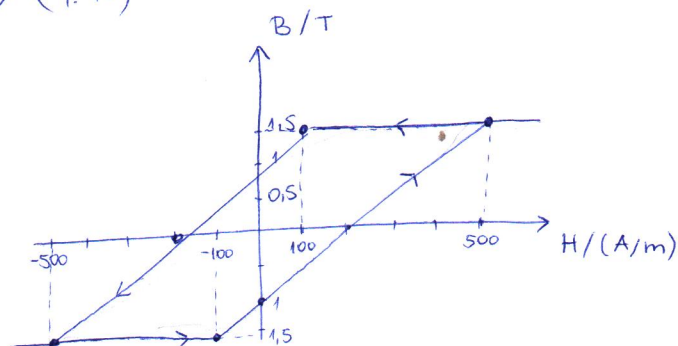
$$H(t) = H_0 \cdot \sin(\omega t)$$



$$B(t) \neq f(H(t))$$

$$B(t_0) = f(H(t \leq t_0))$$

⑥ (4.41)



$$B = B(t) = ?$$

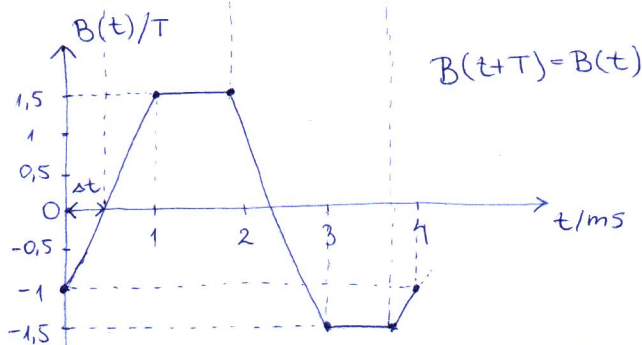
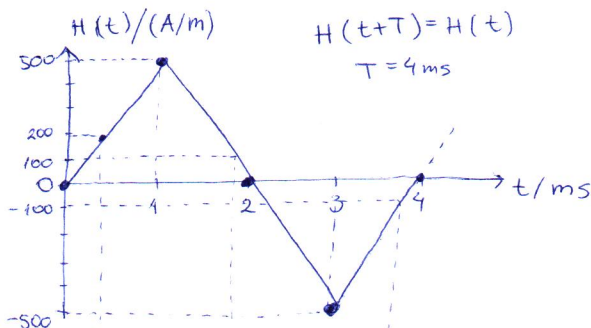
$$H(t=0) = 0, B(t=\Delta t) = 0$$

$$\Delta t = ?$$

$$\frac{500 \text{ A/m}}{1 \text{ ms}} = \frac{200 \text{ A/m}}{\Delta t}$$

$$\Delta t = \frac{2}{5} \text{ ms} = 0,4 \text{ ms}$$

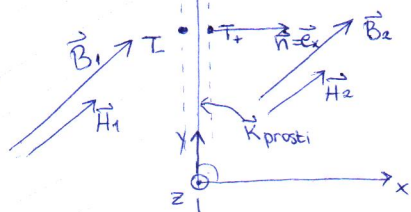
Prejeto na red 20
spomin
H. 10.



⑦ (4.34)

$$\mu_1 = 0,1 \frac{\text{Vs}}{\text{A} \cdot \text{m}}$$

$$\mu_2 = 0,01 \frac{\text{Vs}}{\text{A} \cdot \text{m}}$$



$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = (1, 1, 1) \text{ A/m}$$

$$-(H_{2z} - H_{1z}) = 0 \Rightarrow H_{2z} = H_{1z} = 1 \text{ A/m}$$

$$H_{2y} - H_{1y} = 0 \Rightarrow H_{2y} = H_{1y} = 1$$

$$H_{2x} = \frac{B_{2x}}{\mu_2} = 10 \text{ A/m}$$

$$\vec{H}_2 = (10, 1, 1) \text{ A/m}$$

$$\vec{B}_1 = (100, 100, 100) \text{ mT}$$

$$1) \vec{H}_2 = ? , \vec{K}_{\text{prosti}}$$

$$B_x = \text{konst}$$

$$B_y = \text{konst}$$

$$B_z = \text{konst}$$

$$\vec{n} \cdot [\vec{B}(T_+) - \vec{B}(T_-)] = 0$$

$$B_n(T_+) - B_n(T_-) = 0$$

$$B_n(T_+) = B_n(T_-)$$

$$B_{2x} = B_{1x} = 100 \text{ mT}$$

$$\vec{n} \times \left[\frac{\vec{H}(T_+) - \vec{H}(T_-)}{\vec{H}_2} \right] = \frac{\vec{K}_{\text{prosti}}(T)}{\vec{0}}$$

$$\vec{e}_x \times [\vec{H}_2 - \vec{H}_1] = \vec{0}$$

$$\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ H_{2x} - H_{1x} & H_{2y} - H_{1y} & H_{2z} - H_{1z} \end{vmatrix} = \vec{e}_x \cdot 0 - \vec{e}_y (H_{2z} - H_{1z}) + \vec{e}_z (H_{2y} - H_{1y}) = \vec{0} =$$

$$= \vec{e}_x \cdot 0 + \vec{e}_y \cdot 0 + \vec{e}_z \cdot 0$$

2) $\vec{K}_{prosti} = ?$, $H_{2y} = H_{2z} = 0$

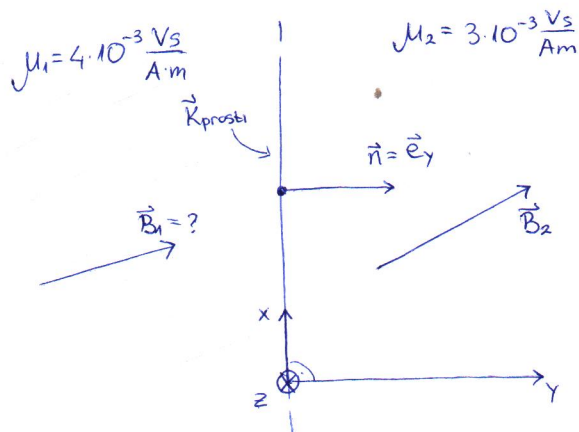
$\vec{H}_1 = (1, 1, 1) \text{ A/m}$

$\vec{H}_2 = (10, 0, 0) \text{ A/m}$

$\vec{e}_x \times (\vec{H}_2 - \vec{H}_1) = \vec{K}_{prosti} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ 9 & -1 & -1 \end{vmatrix} \text{ A/m} = [\vec{e}_x \cdot 0 - \vec{e}_y \cdot 1 \cdot (-1) + \vec{e}_z \cdot 1 \cdot (-1)] \text{ A/m} =$

$\vec{K}_{prosti} = (\vec{e}_y 1 - \vec{e}_z 1) \text{ A/m}$

⑧ Kolonvijsua



$\vec{B}_2 = (9, 5, 3) \text{ mT}$

$\vec{K}_{prosti} = (3, 0, -3) \text{ A/m}$

$\vec{B}_1 = ?$

$B_{1n} = B_{2n} \Rightarrow B_{1y} = B_{2y} = \underline{5 \text{ mT}}$

$\vec{n} \times [\vec{H}_2 - \vec{H}_1] = \vec{K}_{prosti}$

$\vec{e}_y \times [\vec{H}_2 - \vec{H}_1] = \vec{K}_{prosti} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 1 & 0 \\ H_{2x} - H_{1x} & H_{2y} - H_{1y} & H_{2z} - H_{1z} \end{vmatrix} =$

$= \vec{e}_x (H_{2z} - H_{1z}) - \vec{e}_y \cdot 0 + \vec{e}_z (- (H_{2x} - H_{1x})) =$
 $= (\vec{e}_x \cdot 3 + \vec{e}_y \cdot 0 - \vec{e}_z 3) \text{ A/m}$

$H_{2z} - H_{1z} = 3 \text{ A/m} \Rightarrow H_{1z} = H_{2z} - 3 \text{ A/m} = -2 \text{ A/m}$
 $-(H_{2x} - H_{1x}) = -3 \text{ A/m} \Rightarrow H_{1x} = H_{2x} - 3 \text{ A/m} = 0$

$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = \frac{(9, 5, 3) \text{ mT}}{3 \cdot 10^{-3} \frac{\text{Vs}}{\text{Am}}} = (3, \frac{5}{3}, 1) \text{ A/m}$

$B_{1z} = \mu_1 \cdot H_{1z} = 4 \cdot 10^{-3} \cdot (-2) \text{ T} = \underline{-8 \text{ mT}}$

$B_{1x} = \mu_1 \cdot H_{1x} = \underline{0}$

$\Rightarrow \boxed{\vec{B}_1 = (0, 5, -8) \text{ mT}}$

⑨ Tokovna premica (napetost)

$U_{AB} = \int_A^B \vec{E} \cdot d\vec{l} = V(A) - V(B)$

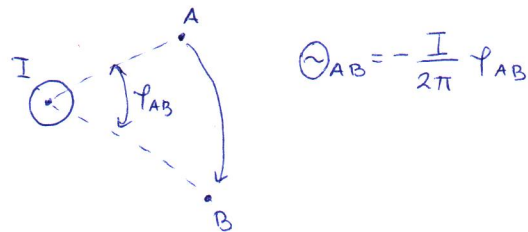
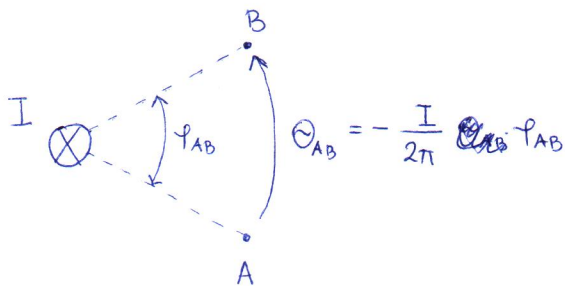
$\mathcal{O}_{AB} = \int_A^B \vec{H} \cdot d\vec{l} = V_m(A) - V_m(B) = \int_A^C \frac{\vec{H} \cdot d\vec{l}}{H \cdot dl} + \int_C^B \frac{\vec{H} \cdot d\vec{l}}{H \cdot dl}$

$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{1}{\mu_0} \cdot \frac{\mu_0 I}{2\pi s} \cdot \vec{e}_\varphi = \frac{I}{2\pi s} \cdot \vec{e}_\varphi$

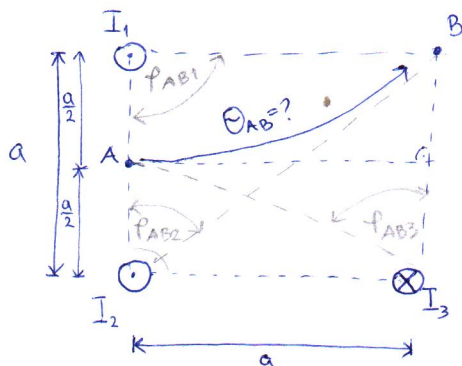
$\mathcal{O}_{AB} = \int_A^C \vec{H} \cdot d\vec{l} = \int_A^C \frac{I}{2\pi s_A} \vec{e}_\varphi \cdot s_A \cdot d\varphi \cdot \vec{e}_\varphi = \int_A^C \frac{I}{2\pi} d\varphi = \frac{I}{2\pi} \int_{\varphi_A}^{\varphi_B} d\varphi = \frac{I}{2\pi} (\varphi_B - \varphi_A) = \frac{I}{2\pi} \varphi_{AB} = \mathcal{O}_{AB}$

$d\vec{l} = s_A \cdot d\varphi \cdot \vec{e}_\varphi$

$\mathcal{O}_{AB} = \frac{I}{360^\circ} \cdot \varphi_{AB}$
 ali
 $\mathcal{O}_{AB} = \frac{I}{360^\circ} \cdot \varphi_{AB}$



9



$$\vec{B}_{AB} = \vec{B}_{AB}^{(I_1)} + \vec{B}_{AB}^{(I_2)} + \vec{B}_{AB}^{(I_3)}$$

$$\vec{B}_{AB} = \frac{I_1}{2\pi} \frac{\varphi_{AB1}}{\pi/2} + \frac{I_2}{2\pi} \frac{\varphi_{AB2}}{\pi/4} + \frac{I_3}{2\pi} \frac{\varphi_{AB3}}{\arctan \frac{1}{2}} = 47,7 \text{ A}$$

$$\tan \varphi_{AB3} = \frac{a}{a/2} = \frac{1}{2}$$

$$\Phi [\text{Vs}], \quad \Theta = U_m [\text{A}]$$

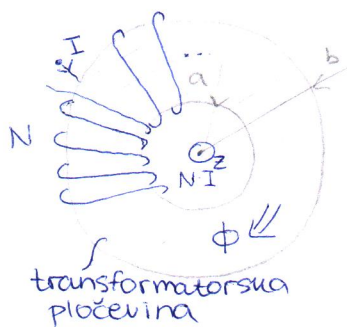
$$U [\text{V}], \quad I [\text{A}]$$

$$I_2 = I_1 = 100 \text{ A}$$

$$I_3 = 200 \text{ A}$$

Avditorne vaje (8.4.2016)

1 4.40)



$$a = 10 \text{ mm}$$

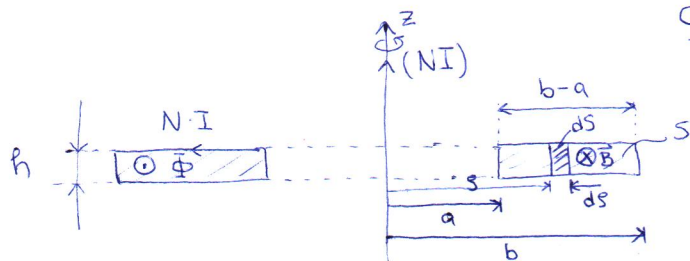
$$b = 30 \text{ mm}$$

$$I = 0,1 \text{ A}$$

$$N = 200$$

$$h = 3 \text{ mm}$$

$$\Phi = ?$$



$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \int B_r(s) \vec{e}_r \cdot \vec{e}_r \cdot h \cdot ds = h \int_a^b B_r(s) ds$$

$$d\vec{S} = \vec{e}_r \cdot h \cdot ds$$

$$\vec{B} = \frac{\mu(NI)}{2\pi s} \vec{e}_r \quad (\text{not polje okrog premice})$$

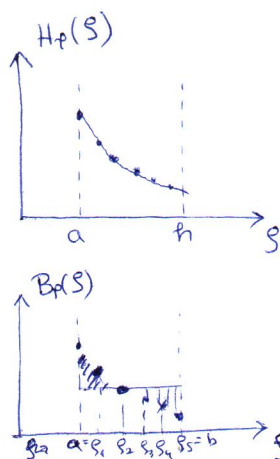
(znotraj toroidne tuljave)

$\mu = ?$

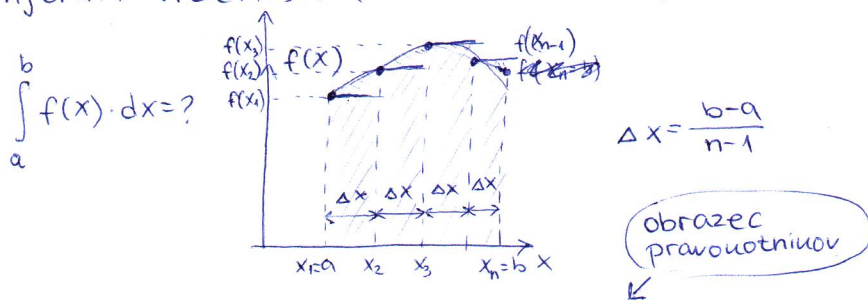
$$\vec{H} = \frac{(NI)}{2\pi s} \cdot \vec{e}_r; \quad \vec{B} = \vec{e}_r B_r(s)$$

$$H_r(s) = \frac{NI}{2\pi s}$$

Ne poznamo μ , zato ne moremo izračunati $\vec{B}$. Lahko izračunamo $\vec{B}$ pa v neki točki.



- Lahko izračunamo integral neke funkcije, ki jo ne poznamo, ampak poznamo vse njene vrednosti (v neki točki)



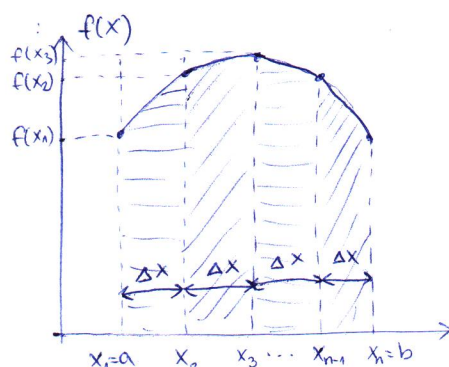
Zaključek:

Bomo računali srednjo vrednost polja.

Napaka bo le 2%.

(Ne računamo po Simpsonu, zato ker je časovno več zahtevno)

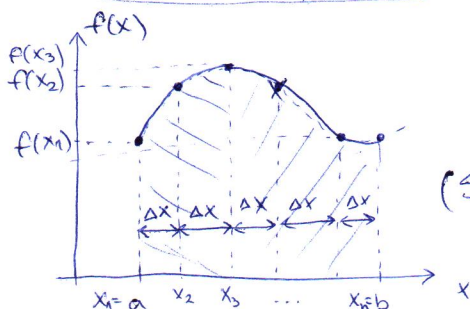
- Boljša aproksimacija (trapezni stolpci) manjše napake pri oceni integrala



$$\int_a^b f(x) \cdot dx \cong \Delta x \cdot \frac{f(x_1) + f(x_2)}{2} + \Delta x \cdot \frac{f(x_2) + f(x_3)}{2} + \dots + \Delta x \cdot \frac{f(x_{n-2}) + f(x_{n-1})}{2} + \Delta x \cdot \frac{f(x_{n-1}) + f(x_n)}{2}$$

$$\cong \frac{\Delta x}{2} [f(x_1) + 2f(x_2) + 2f(x_3) + \dots + 2f(x_{n-1}) + f(x_n)]$$

Obrazec parabole



$$mx^2 + kx + c$$

rabimo parabola skozi 3 točke

(Simpsonova integralska formula)

$$\Delta x = \frac{b-a}{n-1}$$

(n-1) število pod integralov mora biti sodo

$$\int_a^b f(x) dx = \frac{\Delta x}{3} [f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

menjamo faktorje 1, 2, 4, 2, 4, ..., 1

VELIKO SE UPORABLJA V PRAKSI

Nazaj na naloga ① :

izberemo 5 točk

$$\Delta s = \frac{b-a}{4} = \frac{20\text{mm}}{4} = 5\text{mm}$$

$$s_1 = a = 10\text{mm}$$

$$s_2 = s_1 + \Delta s = 15\text{mm}$$

$$s_3 = s_2 + \Delta s = 20\text{mm}$$

$$s_4 = s_3 + \Delta s = 25\text{mm}$$

$$s_5 = s_4 + \Delta s = 30\text{mm} = b$$

$$H_p(s_1) = \frac{NI}{2\pi s_1} = 318\text{A/m}$$

$$H_p(s_2) = \frac{NI}{2\pi s_2} = 212\text{A/m}$$

$$H_p(s_3) = 159\text{A/m}$$

$$H_p(s_4) = 127\text{A/m}$$

$$H_p(s_5) = 106\text{A/m}$$

Očitamo od magnetilno krivuljio
 $B_p(s_1) = 1,11\text{T}$

$$B_p(s_3) = 0,92\text{T}$$

$$B_p(s_4) = 0,88\text{T}$$

$$B_p(s_5) = 0,8\text{T}$$

Vrednosti B_p vstavimo v Simpsonovo integralno f-b.

$$\Phi = h \int_a^b B_p(s) ds \approx \Phi^{(n)} = h \cdot \frac{\Delta s}{3} [B_p(s_1) + 4B_p(s_2) + 2B_p(s_3) + 4B_p(s_4) + B_p(s_5)]$$

$$\Phi^{(4)} = 56,35\text{ }\mu\text{Vs} \approx 56\text{ }\mu\text{Vs} \text{ (4 integrali)}$$

$$\frac{\Phi^{(4)} - \Phi^{(8)}}{\Phi^{(8)}} \cdot 100\% \approx 0,13\%$$

$$\Phi^{(8)} = 56,275\text{ }\mu\text{Vs} \approx 56\text{ }\mu\text{Vs} \text{ (8 podintegrali)}$$

$$\Phi^{(2)} = h \cdot \underbrace{\frac{1}{3} \cdot \frac{b-a}{2}}_{\Delta s \text{ nam se spreminja}} [B_p(s_1) + 4B_p(s_3) + B_p(s_5)] = 55,9\text{ }\mu\text{Vs} \approx 56\text{ }\mu\text{Vs}$$

Napaka:

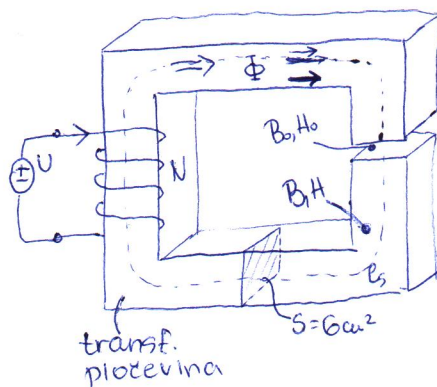
$$\frac{\Phi^{(2)} - \Phi^{(4)}}{\Phi^{(4)}} \cdot 100\% = -0,8\%$$

→ če je z samo en podinterval

$$\Phi^{(1)} = h B_p\left(\frac{a+b}{2}\right) (b-a) = h (b-a) \underbrace{B_p(s_3)}_{\approx 0,925} = 55,2\text{ }\mu\text{Vs}$$

$$\frac{\Phi^{(1)} - \Phi^{(8)}}{\Phi^{(8)}} \cdot 100\% \approx -1,91\% \text{ napaka}$$

2. 4.39)



$$U = 0,5 \text{ V}$$

$$N = 150$$

Polje v reži je različno kot v transf. pločevina

$$\Delta = 0,5 \text{ mm} \ll l_{ov} = 10 \text{ cm}$$

$$S_{žice} = 2,5 \text{ mm}^2$$

$$\delta_{žice} = 56 \text{ MS/m}$$

$$\Delta = 0,5 \text{ mm}$$

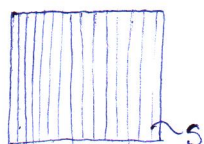
$$I = \frac{U}{R_{žice}} = \frac{U}{\frac{N \cdot l_{ov}}{\delta_{žice} \cdot S_{žice}}} = 4,67 \text{ A}$$

$$l_{srednja} = l_s = 0,5 \text{ m}$$

$$S = 6 \text{ cm}^2$$

$$f = 0,95$$

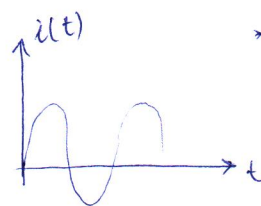
$$S_{ef} = f \cdot S = 5,7 \text{ cm}^2$$



Lamelejevo jedro

95% feromagnetika

5% izolacijski lak med lamelama



* spremenljiv tok povzroči indukcijo v jedru (poravilo se vrtilne tone)

$$I \approx 4,67 \text{ A}$$

$$B, H = ?$$

$$B_0, H_0 = ?$$

$$M = ?$$

$$\Phi = ?$$

$$\oint \vec{H} \cdot d\vec{l} = N \cdot I = 700 \text{ A}$$

$$H \cdot l_s + H_0 \cdot \Delta = N I = H \cdot l_s + \frac{B_0}{\mu_0} \cdot \Delta = H l_s + B \frac{S_{ef} \cdot \Delta}{S \cdot \mu_0} = N I \Rightarrow$$

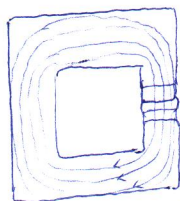
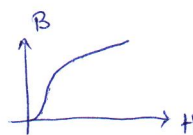
$$H = \frac{N I}{l_s} - B \frac{\Delta \cdot S_{ef}}{\mu_0 l_s S}$$

$$\Delta = 0,5 \text{ mm} \ll l_s = 0,5 \text{ m}$$

$$H_0 = \frac{B_0}{\mu_0}$$

v zračni

v transf. jedro H ni linearno rabimo:



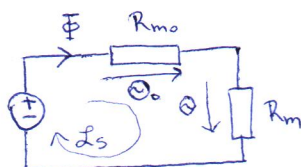
na reži pride do stresanje

$$\oint \vec{B} \cdot d\vec{a} = 0$$

* v našem primeru zanemarimo to stresanje

$$\Phi = B_0 \cdot S = B \cdot S_{ef} \Rightarrow B_0 = B \cdot \frac{S_{ef}}{S}$$

$$\mathcal{O}_g = N \cdot I$$



$$\mathcal{O}_{12} = \int_{T_1}^{T_2} \vec{H} \cdot d\vec{l} \rightarrow T_2$$

$$\mathcal{L}_s: \mathcal{O}_g = \mathcal{O}_0 + \mathcal{O}_H$$

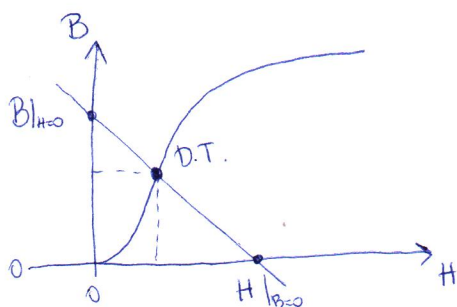
$$\mathcal{O}_0 = R_{mo} \cdot \Phi = \frac{\Delta}{\mu_0 S} \cdot B \cdot S_{ef}$$

$$R_{mo} = \frac{\Delta}{\mu_0 S}$$

$$\Rightarrow \mathcal{L}_s: N I = \frac{\Delta}{\mu_0 S} B S_{ef} + H l_s$$

$$H = \frac{NI}{l_s} - \frac{\Delta \cdot S_{ef}}{\mu_0 \cdot l_s \cdot S} \cdot B$$

Do isto enačbo
pridemo za dva
načina (Amp. z. ali mag. vezje)



$$H|_{B=0} = \frac{N \cdot I}{l_s} = 1400 \text{ A/m}$$

$$B|_{H=0} = \frac{N \cdot I \cdot \mu_0 \cdot S}{\Delta \cdot S_{ef}} = 1,85 \text{ T}$$

$$H = 500 \text{ A/m}$$

$$B = 1,23 \text{ T}$$

} očitamo od
mag. krivulje
no dobimo delovno točko

$$B_0 = B \frac{S_{ef}}{S} = 1,17 \text{ T} \approx B$$

$$H_0 = \frac{B_0}{\mu_0} = 930 \cdot 10^3 \text{ A/m} \gg H$$

$$\Theta = H \cdot l_s = 250 \text{ A}$$

$$\Theta_0 = H_0 \cdot \Delta = 465 \text{ A} > \Theta$$

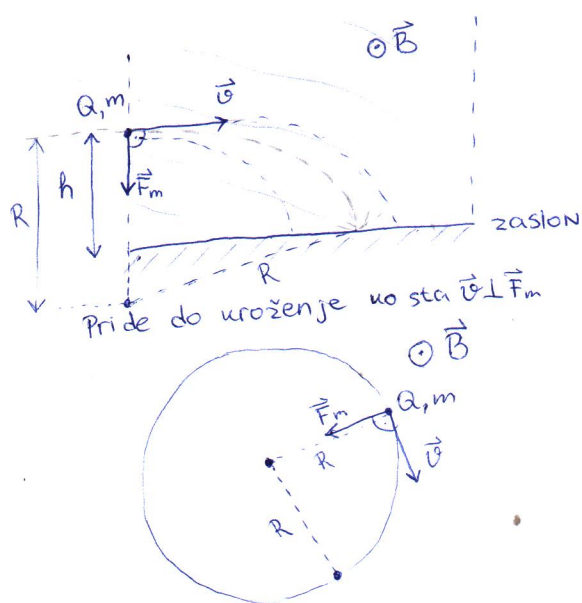
$$R_{m0} = \frac{\Delta}{\mu_0 S} = 663 \text{ kA/Vs (za zraun)}$$

$$R_m = R_m(\Phi) = \frac{\Theta}{\Phi} = 357 \frac{\text{kA}}{\text{Vs}} < R_{m0} \text{ (transf. plošč.)}$$

$$M = \frac{B}{\mu_0} - H = 978 \text{ kA/m}$$

$$M_0 = \frac{B_0}{\mu_0} - H_0 = 0 \rightarrow \underline{\text{zraun se ne namagnetil!}}$$

3) HASNI SPEKTROGRAF



$$B = 1 \text{ mT}$$

$$h = 20 \text{ cm}$$

$$Q = 1,602 \cdot 10^{-19} \text{ C}$$

$$v = 6265 \text{ m/s}$$

$$x = 0,469 \text{ m}$$

$$m = ?$$

$$a_R = \frac{F_m}{m} = \frac{Q \cdot v \cdot B}{m} = \frac{v^2}{R} \Rightarrow R = \frac{m \cdot v}{Q \cdot B}$$

$$\vec{v} \perp \vec{B} \Rightarrow |\vec{v} \times \vec{B}| = v \cdot B$$

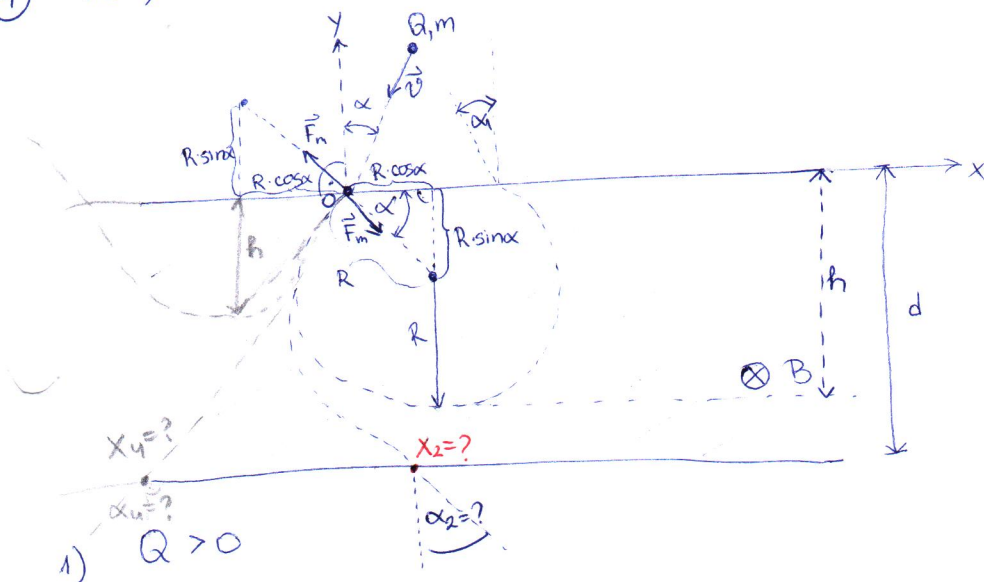
* Različne ione imajo različne mase in s temu različne radije krožnic gibanja

$$R^2 = (R-h)^2 + x^2 = R^2 - 2Rh + h^2 + x^2$$

$$2hR = h^2 + x^2 \Rightarrow R = \frac{h^2 + x^2}{2h} = 64,9 \text{ cm}$$

$$R = \frac{m \cdot v}{Q \cdot B} \Rightarrow m = \frac{Q \cdot B \cdot R}{v} = 1,66 \cdot 10^{-26} \text{ kg (ION BDR)}$$

4) 4.30)



$$\vec{F}_m = Q \vec{v} \times \vec{B}$$

$$1) Q > 0$$

$$h = R \cdot \sin \alpha + R < d$$

$$R(1 + \sin \alpha) = \frac{m \cdot v}{Q \cdot B} (1 + \sin \alpha) < d$$

$$\alpha_1 = \alpha, \quad x_1 = 2 \cdot R \cdot \cos \alpha$$

razdalja od vstopnega mesta do izstopnega

$$2) Q < 0$$

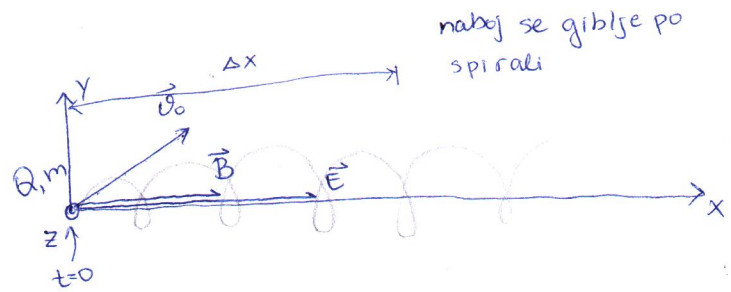
$$h = R - R \cdot \sin \alpha = R \cdot (1 - \sin \alpha) < d$$

$$R = \frac{m \cdot v}{|Q| \cdot B}, \quad \frac{m \cdot v}{|Q| \cdot B} (1 - \sin \alpha) < d$$

$$\alpha_3 = \alpha, \quad x_3 = -2 \cdot R \cdot \cos \alpha$$

$$R = \frac{m \cdot v}{|Q| \cdot B}$$

5.



naboj se giblje po spirali

$$Q = 1,602 \cdot 10^{-19} \text{ C}$$

$$m = 1,15 \cdot 10^{-26} \text{ ug}$$

$$\vec{v}_0 = \vec{v}(t=0) = (1 \cdot \vec{e}_x + 5 \vec{e}_y + 3 \vec{e}_z) \cdot 10^5 \frac{\text{m}}{\text{s}}$$

$$\vec{E} = \vec{e}_x \cdot 120 \text{ V/m}$$

$$\vec{B} = \vec{e}_x \cdot 20 \text{ mT}$$

$$\vec{F}_L = Q(\vec{E} + \vec{v} \times \vec{B}) = \vec{F}_e + \vec{F}_m$$

$$\vec{F}_e = Q\vec{E} \parallel \vec{e}_x$$

$\vec{F}_e$ vpliva le na v_x

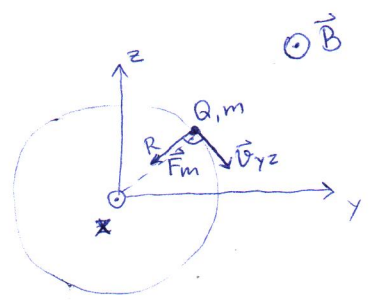
$$\vec{F}_m = Q\vec{v} \times \vec{B} \perp \vec{B} \parallel \vec{e}_x \Rightarrow \vec{F}_m \perp \vec{e}_x$$

$\vec{F}_m$ vpliva le na $\vec{v}_{yz} = \vec{e}_y v_y + \vec{e}_z v_z$

$$\vec{F}_m \perp \vec{F}_e$$

1) $\vec{F}_m$: kroženje

$$R = \frac{m \cdot v}{|Q| \cdot B} =$$



$$\vec{v}_{yz} \perp \vec{B}$$

$$v_{yz} = \sqrt{v_y^2 + v_z^2} = \sqrt{v_{0y}^2 + v_{0z}^2} = 5,83 \cdot 10^5 \text{ m/s}$$

$$v_y = v_y(t), v_z = v_z(t)$$

$$v_{yz}(t) = \text{const.}$$

$$R = \frac{m \cdot v_{yz}}{|Q| \cdot B} = 2,09 \text{ m}$$

$$T = \frac{2\pi R}{v_{yz}} = 22,55 \mu\text{s}$$

2) $\vec{F}_e$: enakomerno pospeš. premo črtno gibanje

$$F_{e,x} = Q \cdot E_x = \text{const} = m \cdot a_x \Rightarrow a_x = \frac{Q \cdot E_x}{m} = \text{const.}$$

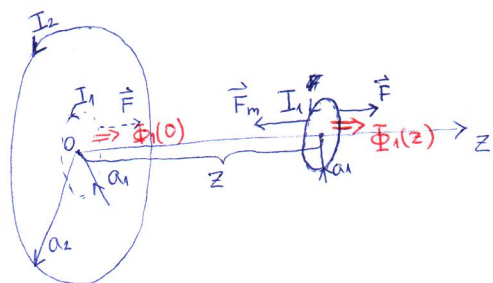
$$\Delta t = 4T, \Delta x = ?$$

$$v_x(t) = v_{0x} + a_x t$$

$$\Delta x = \int_0^{\Delta t} v_x(t) \cdot dt = \int_0^{\Delta t} [v_{0x} + a_x t] \cdot dt = v_{0x} \cdot \Delta t + a_x \frac{\Delta t^2}{2} = 15,8 \text{ m}$$

$$v_x(t = \Delta t) = v_{0x} + a_x \cdot \Delta t = 2,51 \cdot 10^5 \text{ m/s} > v_{0x} = 1 \cdot 10^5 \text{ m/s}$$

⑥ 4.37)



$$a_1 \ll a_2$$

$$F = ?$$

$$\vec{F} = -\vec{F}_m \Rightarrow F_z = -F_{m,z}$$

$$A = \int \vec{F}_m \cdot d\vec{\ell} = I_1 (\Phi_1(z) - \Phi_1(0))$$

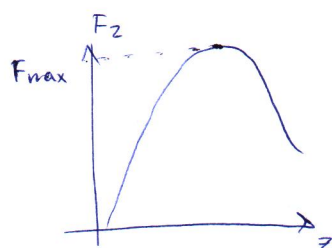
$$dA_m = \vec{F}_m \cdot d\vec{\ell} = \vec{F}_m \cdot \vec{e}_z dz = F_{m,z} dz \Rightarrow F_{m,z} = \frac{dA_m}{dz}, F_z = -F_{m,z} = -\frac{dA_m}{dz} = -\frac{d(I_1(\Phi_1(z) - \Phi_1(0)))}{dz}$$

$$F_z = -I_1 \frac{d\Phi_1(z)}{dz} = -I_1 \frac{d[\Phi_1^{(I_1)} + \Phi_1^{(I_2)}(z)]}{dz} = -I_1 \frac{d\Phi_1^{(I_2)}(z)}{dz} = -I_1 \cdot \frac{\mu_0 I_2 a_2^2}{2} \left(-\frac{3}{2}\right) (a_2^2 + z^2)^{-3/2} \cdot 2z$$

$$\Phi_1^{(I_2)}(z) \equiv B_z^{(I_2)}(z) \cdot \pi a_1^2 = \frac{\mu_0 I_2}{2} \cdot \frac{a_2^2}{(a_2^2 + z^2)^{3/2}} \cdot \pi a_1^2$$

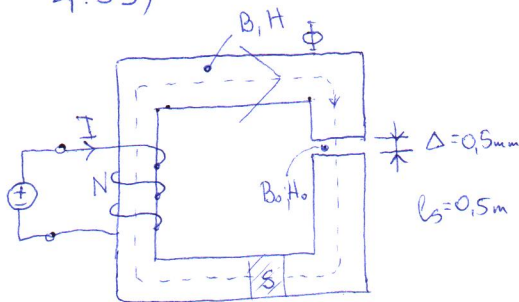
$$\Phi_1 = \underbrace{\Phi_1^{(I_1)}}_{\text{const}} + \Phi_1^{(I_2)}$$

$$F_z = \frac{3\mu_0 I_1 I_2 \pi a_1^2 a_2^2}{2} \cdot \frac{z}{(a_2^2 + z^2)^{5/2}}$$



Avditarne vaje (13.4.2016)

① 4.39)



$$I = 4.67 \text{ A}$$

$$\oint_{L_s} \vec{H} \cdot d\vec{\ell} = N \cdot I = H(l_s - \Delta) + H_0 \cdot \Delta$$

$$\oint_A \vec{B} \cdot d\vec{a} = 0 \Rightarrow \Phi = \Phi_0$$

$$\Phi = B \cdot S_{\text{ef}} = \Phi = B_0 \cdot S \Rightarrow B_0 = B \cdot \frac{S_{\text{ef}}}{S}$$

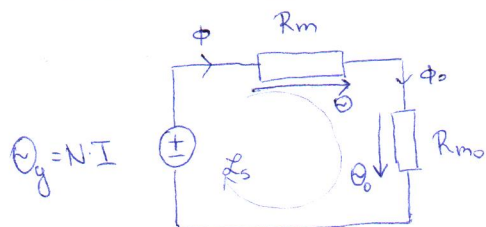
$$B_0 = \mu_0 H_0 \Rightarrow H_0 = \frac{B_0}{\mu_0} = \frac{B \cdot S_{\text{ef}}}{\mu_0 \cdot S}$$

ena enačba nam je podana grafično (moramo vse reševati + grafično)

$$\oint_{L_s} \vec{H} \cdot d\vec{\ell} = N \cdot I = H(l_s) + H_0 \cdot \Delta = H \cdot l_s + B \cdot \frac{\Delta}{\mu_0} \cdot \frac{S_{\text{ef}}}{S} = N \cdot I$$

$$\mathcal{C}_g = N \cdot I$$

MAGNETNO VEZJE



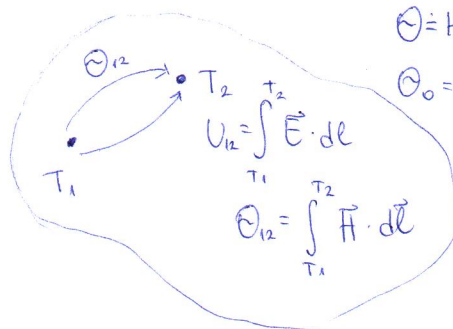
$$\mathcal{Q}_g = NI$$

$$\mathcal{L}_s: -\mathcal{Q}_g + \mathcal{Q} + \mathcal{Q}_0 = 0$$

$$\mathcal{Q}_g = \mathcal{Q} + \mathcal{Q}_0 \Rightarrow \mathcal{Q} = \mathcal{Q}_g - \mathcal{Q}_0$$

$$R_{mo} = \frac{\Delta}{\mu_0 \cdot S} \doteq 663 \frac{\text{kA}}{\text{Vs}}$$

$$R_m = \frac{l_s}{\mu \cdot S_{ef}} = ???$$



$$\mathcal{Q} \doteq H \cdot l_s$$

$$\mathcal{Q}_0 = R_{mo} \cdot \Phi$$

$$H \cdot l_s = N \cdot I - R_{mo} \cdot \Phi = N \cdot I - B \frac{\Delta \cdot S_{ef}}{\mu_0 \cdot S} = H l_s \quad \left\{ \begin{array}{l} \text{dobilo smo} \\ \text{ista ena\u010dba} \\ \text{!} \end{array} \right.$$

$$H = \frac{N \cdot I}{l_s} - B \frac{\Delta \cdot S_{ef}}{\mu_0 l_s \cdot S}$$

$$\Rightarrow \left. \begin{array}{l} B \doteq 1,23 \text{ T} \\ H \doteq 500 \text{ A/m} \end{array} \right\} \text{v JEDRU}$$

Na abscisi je $B=0$

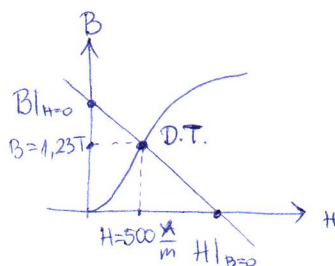
$$H|_{B=0} = \frac{N \cdot I}{l_s} = 1400 \text{ A/m}$$

$$B|_{H=0} = \frac{N \cdot I \mu_0 S}{\Delta \cdot S_{ef}} \doteq 1,85 \text{ T}$$

$$\Phi = B \cdot S \doteq 0,7 \text{ mWb}$$

$$B_0 = B \cdot \frac{S_{ef}}{S} \doteq 1,17 \text{ T}$$

$$H_0 = \frac{B_0}{\mu_0} \doteq 930 \text{ kA/m} \gg H$$



$$\mathcal{Q} \doteq H \cdot l_s = 250 \text{ A} \rightarrow \text{padec napetosti v jedru}$$

$$\mathcal{Q}_0 \doteq R_{mo} \cdot \Phi = H_0 \cdot \Delta = 465 \text{ A} > \mathcal{Q} \rightarrow \text{padec napetosti v zra\u010dni re\u017ei}$$

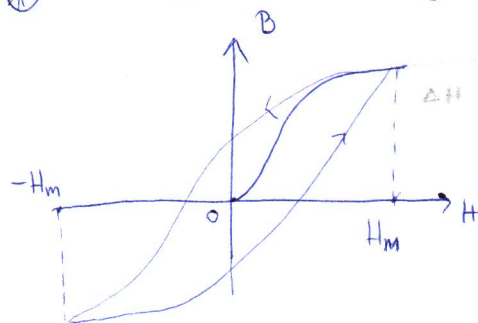
$$\Delta \ll l_s$$

$$R_m = \frac{\mathcal{Q}}{\Phi} \doteq 357 \text{ kA/(Vs)} < R_{mo}$$

$$\text{MAGNETIZACIJA: } H = \frac{B}{\mu_0} - H \doteq 978 \text{ kA/m}$$

$$M_0 = \frac{B_0}{\mu_0} - H_0 = 0 \quad \text{zrau se ne namagnetira}$$

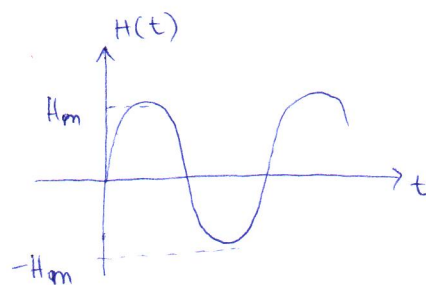
① Začetna krivulja magnetenja



$$\frac{\Delta B}{\Delta H} = \mu_0$$

dobimo
Histeretna
zanua

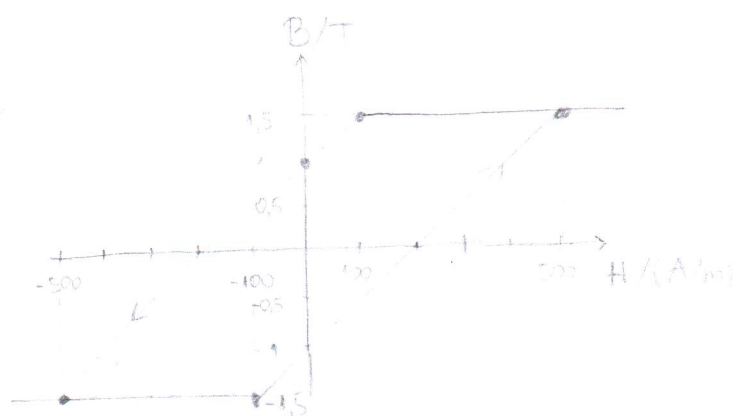
Če jedro magnetimo izmenično



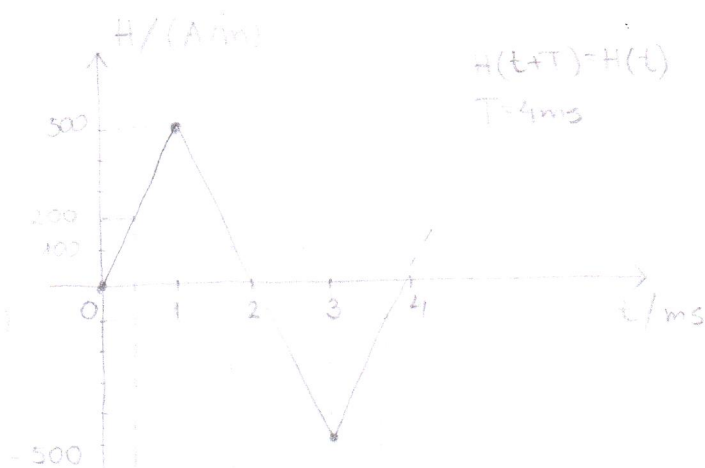
$$B(t=t_0) \neq f(H(t=t_0))$$

$$B(t=t_0) = f(H(t \leq t_0))$$

② 4.41)



$$B = B(T) = ?$$

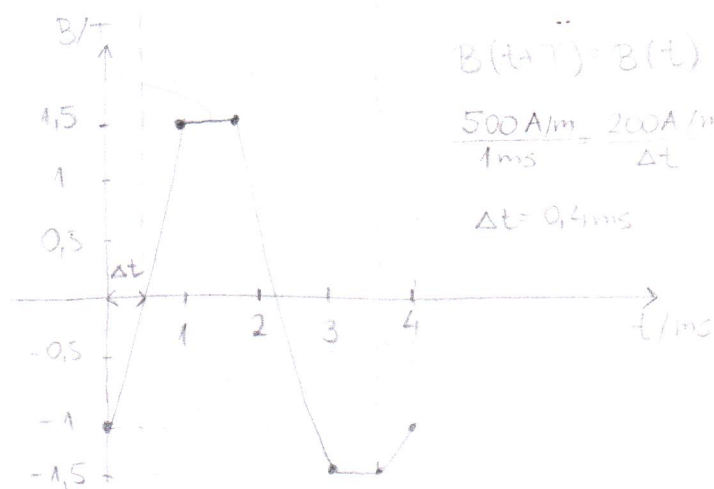


$$H(t+T) = H(t)$$

$$T = 4 \text{ ms}$$

nasičenje

nasitila

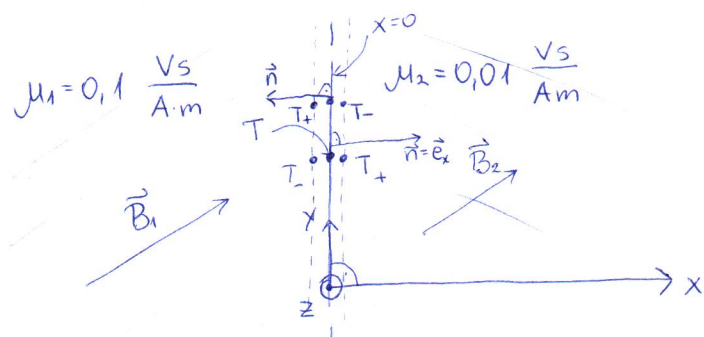


$$B(t+T) = B(t)$$

$$\frac{500 \text{ A/m}}{1 \text{ ms}} = \frac{200 \text{ A/m}}{\Delta t}$$

$$\Delta t = 0,4 \text{ ms}$$

3) 4.34)



$$\vec{B}_1 = (100, 100, 100) \text{ mT}$$

1) $\vec{H}_2 = ?$, $\vec{K}_{\text{prosti}} = \vec{0}$

* Če zamenjamo normala (smer) se nam tudi zamenjata točki T_+ in T_- .

$$\vec{n} \cdot [\vec{B}(T_+) - \vec{B}(T_-)] = B_n(T_+) - B_n(T_-) = 0$$

$B_{2x} - B_{1x} = 0$ → zato uer je polje homogeno

$$B_{2x} = B_{1x} = 100 \text{ mT}$$

$$H_{2x} = \frac{B_{2x}}{\mu_2} = 10 \text{ A/m}$$

$$\vec{n} \times [\vec{H}(T_+) - \vec{H}(T_-)] = \vec{K}_{\text{prosti}}(T)$$

$\vec{e}_x \times (\vec{H}_2 - \vec{H}_1) = 0$

$$\vec{H}_1 = \frac{\vec{B}_1}{\mu_1} = (1, 1, 1) \text{ A/m}$$

$$\vec{e}_x \times [\vec{H}_2 - \vec{H}_1] = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ H_{2x} - H_{1x} & H_{2y} - H_{1y} & H_{2z} - H_{1z} \end{vmatrix} = \vec{e}_x \cdot 0 - \vec{e}_y (H_{2z} - H_{1z}) + \vec{e}_z (H_{2y} - H_{1y})$$

$$(0, H_{2z} - H_{1z}, H_{2y} - H_{1y}) = (0, 0, 0)$$

$$H_{2z} - H_{1z} = 0 \Rightarrow H_{2z} = H_{1z} = 1 \text{ A/m}$$

$$H_{2y} - H_{1y} = 0 \Rightarrow H_{2y} = H_{1y} = 1 \text{ A/m}$$

$$\vec{H}_2 = (10, 1, 1) \text{ A/m}$$

2) $\vec{K}_{\text{prosti}} = ?$; $\vec{B}_{2y} = \vec{B}_{2z} = \vec{0}$

$$H_{2y} = \frac{B_{2y}}{\mu_2} = 0; \quad H_{2z} = \frac{B_{2z}}{\mu_2} = 0$$

$$\vec{H}_2 = (10, 0, 0) \text{ A/m}$$

$$\vec{H}_1 = (1, 1, 1) \text{ A/m}$$

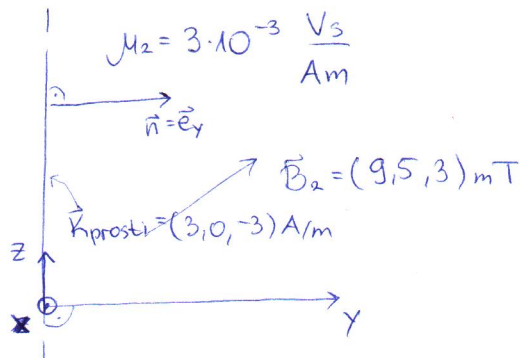
$$\vec{K}_{\text{prosti}}(\vec{n}) = \vec{n} \times [\vec{H}_2 - \vec{H}_1] = \vec{e}_x \times [(9, -1, -1) \text{ A/m}] = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 1 & 0 & 0 \\ 9 & -1 & -1 \end{vmatrix} \text{ A/m} = (0, -1, -1) \text{ A/m}$$

4

$$\mu_1 = 4 \cdot 10^{-3} \frac{\text{Vs}}{\text{Am}}$$

$$\mu_2 = 3 \cdot 10^{-3} \frac{\text{Vs}}{\text{Am}}$$

$$\vec{B}_1 = ?$$



$$\vec{n} = \vec{e}_y$$

$$\vec{n} \cdot [\vec{B}(T_+) - \vec{B}(T_-)] = 0$$

$$\vec{e}_y \cdot [\vec{B}_2 - \vec{B}_1] = B_{2y} - B_{1y} = 0 \Rightarrow B_{2y} = B_{1y} = 5 \text{ mT}$$

$$\vec{n} \times [\vec{H}(T_+) - \vec{H}(T_-)] = \vec{k}_{\text{prosti}}(T)$$

$$\vec{e}_y \times [\vec{H}_2 - \vec{H}_1] = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 0 & 1 & 0 \\ H_{2x} - H_{1x} & H_{2y} - H_{1y} & H_{2z} - H_{1z} \end{vmatrix} = \vec{e}_x (H_{2z} - H_{1z}) - \vec{e}_y \cdot 0 + \vec{e}_z (- (H_{2x} - H_{1x})) = (3, 0, -1) \frac{\text{A}}{\text{m}}$$

$$\vec{e}_x \Rightarrow H_{2z} - H_{1z} = 3 \text{ A/m} \Rightarrow H_{2z} = H_{1z} + 3 \text{ A/m} ; H_{1z} = H_{2z} - 3 \text{ A/m} = 1 \text{ A/m} - 3 \text{ A/m} = -2 \text{ A/m}$$

$$\vec{e}_y \Rightarrow 0 = 0$$

$$\vec{e}_z \Rightarrow - (H_{2x} - H_{1x}) = -3 \text{ A/m} \Rightarrow H_{1x} = H_{2x} - 3 \text{ A/m} = 3 \text{ A/m} - 3 \text{ A/m} = 0$$

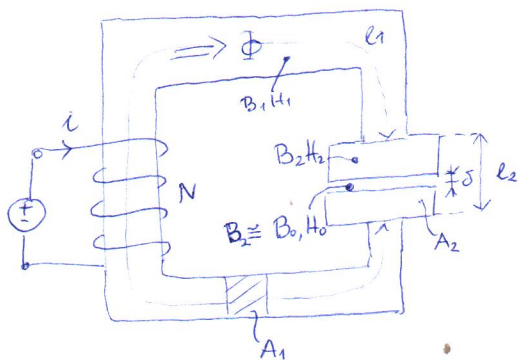
$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_2} = (3, \frac{5}{3}, 1) \text{ A/m}$$

$$B_{1z} = \mu_1 \cdot H_{1z} = -8 \text{ mT}$$

$$B_{1x} = \mu_1 \cdot H_{1x} = 0$$

$$\vec{B}_1 = (0, 5, -8) \text{ mT}$$

Prpriava za 2. Lab vaja



$B = ?$

$$\oint_{\mathcal{L}_S} \vec{H} \cdot d\vec{l} = N \cdot i \cong H_1 \cdot l_1 + H_2 \cdot l_2 + H_0 \delta$$

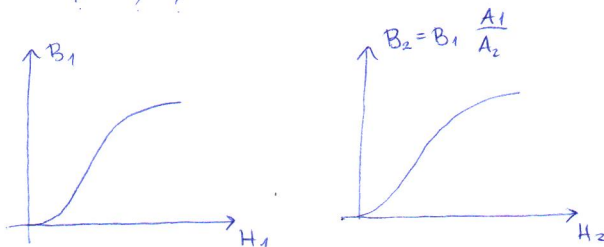
$$\Phi = \Phi_1 = \Phi_2 = \Phi_3 \Rightarrow B_1 \cdot A_1 = B_2 \cdot A_2 = B_0 \cdot A_2$$

$$\Rightarrow B_0 = B_2 = B_1 \cdot \frac{A_1}{A_2}$$

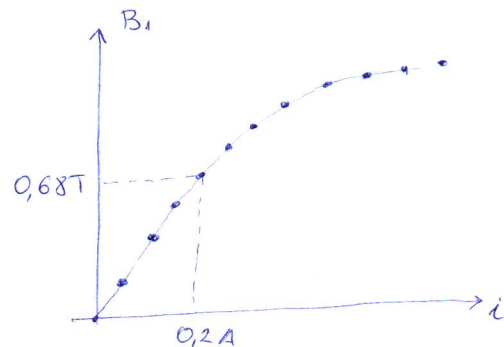
$$H_0 = \frac{B_0}{\mu_0} = \frac{B_1}{\mu_0} \cdot \frac{A_1}{A_2}$$

$$\oint_{\mathcal{L}_S} \vec{H} \cdot d\vec{l} = H_1 l_1 + H_2 l_2 + B_1 \frac{\delta}{\mu_0} \cdot \frac{A_1}{A_2} \cong N i \Rightarrow i \cong \frac{H_1 l_1 + H_2 l_2 + B_1 \frac{\delta}{\mu_0} \cdot \frac{A_1}{A_2}}{N}$$

str. 29.:



str. 28.:



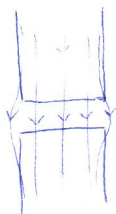
$\rightarrow B_1/T$	0	0.1	0.2	0.3	0.6	0.7	1.3
$B_0 = B_2/T$					0.15	0.175	
$H_1/(A/m)$					880	1200	
$H_2/(A/m)$					220	240	
$\rightarrow i/A$					0.16	0.21	

str. 28.

i/A	0	0.05	0.1	0.2	...	2.5
B_1/T				0.68		

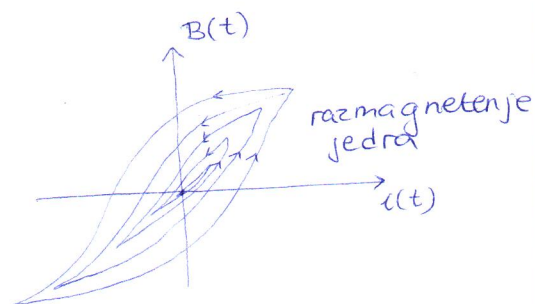
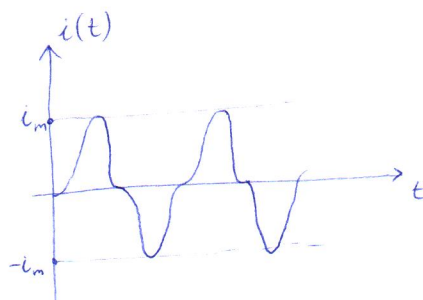
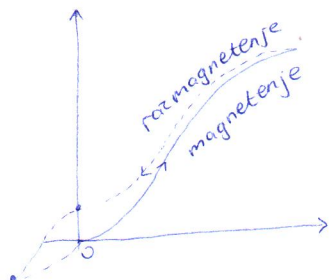
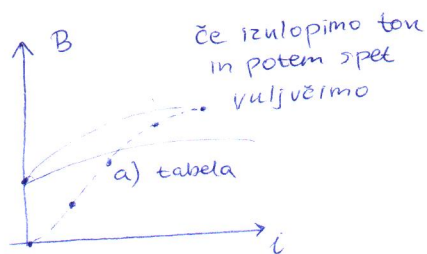
$$\Phi = B_1 \cdot A_1, \text{ pri } i = 0.2 A \Rightarrow \Phi \cong 0.68 T \cdot A_1 \cong \underline{212 \mu Wb} \quad \left(\text{vprašanje 4 (str. 28)} \right)$$

• Polje v reži



$$B_0 < B_2$$

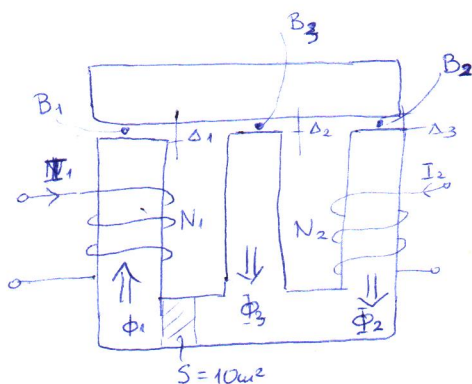
* Sondo zadržimo na mest z največje polje B_0



• OSCILOSKOP (uporaba!)

Za test:

① 4.46)



$$\Delta_1 = 1 \text{ m}$$

$$\Delta_2 = 3 \text{ mm}$$

$$\Delta_3 = 2 \text{ mm}$$

$$N_1 = 200 \text{ A}$$

$$I_1 = 2 \text{ A}$$

$$N_2 = 100$$

$$I_2 = 1 \text{ A}$$

$$B_1, B_2, B_3 = ?$$

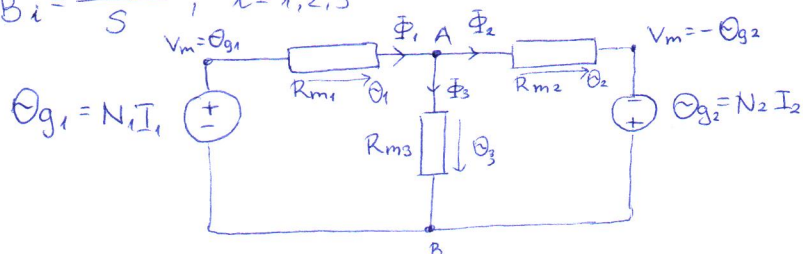
$$R_{m, Fe} \ll R_{m, z \text{ reži}}$$

$$R_{m1} = \frac{\Delta_1}{\mu_0 S} = 0,8 \frac{\text{MA}}{\text{Wb}}$$

$$R_{m2} = \frac{\Delta_2}{\mu_0 S} = 2,4 \frac{\text{MA}}{\text{Wb}}$$

$$R_{m3} = \frac{\Delta_3}{\mu_0 S} = 1,6 \frac{\text{MA}}{\text{Wb}}$$

$$B_i = \frac{\Phi_i}{S}, \quad i = 1, 2, 3$$



$$m = 2; \quad n = 3$$

$$\text{M.S.P.: } m - 1 = 1 \quad \Psi$$

$$\text{M.Z.T.: } n - (m - 1) = 2$$

$$V_{m,B} = 0, \quad V_{m,A} = ?$$

$$A: -\Phi_1 + \Phi_2 + \Phi_3 = 0$$

$$\Phi_1 = \frac{\Theta_1}{R_{m1}} = \frac{\Theta_{g1} - V_{m,A}}{R_{m1}}$$

$$\Phi_2 = \frac{\Theta_2}{R_{m2}} = \frac{V_{m,A} - (-\Theta_{g2})}{R_{m2}}$$

$$\Phi_3 = \frac{\Theta_3}{R_{m3}} = \frac{V_{m,A} - V_{m,B}}{R_{m3}}$$

$$A \Rightarrow -\frac{\Theta_{g1} - V_{m,A}}{R_{m1}} + \frac{V_{m,A} + \Theta_{g2}}{R_{m2}} + \frac{V_{m,A}}{R_{m3}} = 0$$

$$\Theta_{g1} = N_1 \cdot I_1 = 400 \text{ A}$$

$$\Theta_{g2} = N_2 \cdot I_2 = 100 \text{ A}$$

$$V_{m,A} \left(\frac{1}{R_{m1}} + \frac{1}{R_{m2}} + \frac{1}{R_{m3}} \right) = \frac{\Theta_{g1}}{R_{m1}} - \frac{\Theta_{g2}}{R_{m2}} \quad | : (..)$$

$$V_{m,A} = \frac{\frac{\Theta_{g1}}{R_{m1}} - \frac{\Theta_{g2}}{R_{m2}}}{1/R_{m1} + 1/R_{m2} + 1/R_{m3}} \doteq 200 \text{ A}$$

$$\Rightarrow \Phi_1 = 0,25 \text{ mWb}$$

$$\Phi_2 = 0,125 \text{ mWb}$$

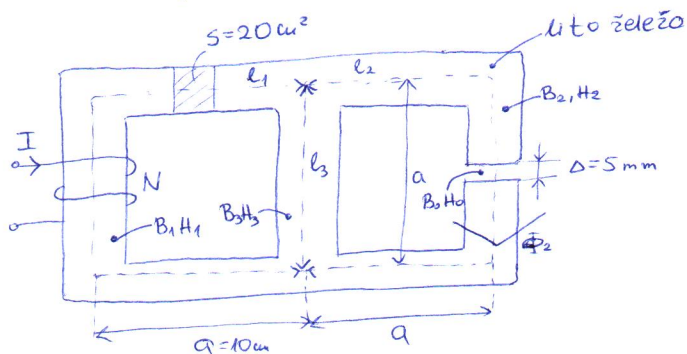
$$\Phi_3 = 0,125 \text{ mWb}$$

$$B_1 = \frac{\Phi_1}{S} = 0,25 \text{ T}$$

$$B_2 = \frac{\Phi_2}{S} = 0,125 \text{ T}$$

$$B_3 = \frac{\Phi_3}{S} = 0,125 \text{ T}$$

2. 4.45)



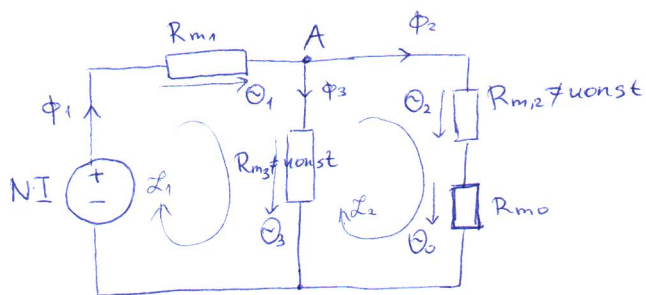
$$l_3 = a$$

$$l_1 = l_2 = 3a$$

$$\Phi_2 = 0,4 \text{ mWb}, NI = ?$$

$$B_2 = \frac{\Phi_2}{S} = 0,2 \text{ T} \rightarrow H_2 = 620 \text{ A/m}$$

$$B_0 \approx B_2 = 0,2 \text{ T} \rightarrow H_0 = \frac{B_0}{\mu_0} = 159 \text{ kA/m}$$



$$\oint \vec{H} \cdot d\vec{l} = 0 \Rightarrow \Theta_2 + \Theta_0 - \Theta_3 = 0$$

$$\Rightarrow H_2 \cdot l_2 + H_0 \cdot \Delta - H_3 \cdot l_3 = 0$$

$$H_3 = \frac{H_2 \cdot l_2 + H_0 \cdot \Delta}{l_3} = 9,82 \text{ kA/m}$$

$$B_3 = 0,93 \text{ T}$$

$$B_1 = ?; H_1 = ?$$

$$A: -\Phi_1 + \Phi_2 + \Phi_3 = 0$$

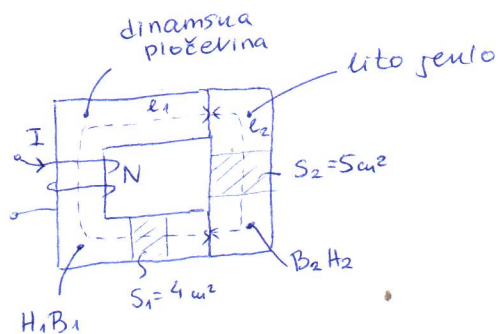
$$\Rightarrow \Phi_1 = \Phi_2 + \Phi_3 \quad | : S$$

$$B_1 = B_2 + B_3 = 1,13 \text{ T} \rightarrow H_1 = 20 \text{ kA/m}$$

$$\oint_{L_1} \vec{H} \cdot d\vec{l} = NI$$

$$NI = \Phi_1 + \Phi_3 = H_1 l_1 + H_3 l_3 = \underline{\underline{6,98 \mu A}}$$

③



$$l_1 = 30 \text{ cm}$$

$$l_2 = 10 \text{ cm}$$

$$N = 1700$$

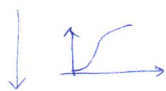
$$I = ? \quad \text{da bo} \quad H_2 = 3 \text{ A/m}$$



$$B_2 = 1,5 \text{ T}$$

$$\Phi = \Phi_1 = \Phi_2 = B_2 \cdot S_2 = 750 \mu \text{ Wb}$$

$$B_1 = \frac{\Phi}{S_1} = 1,875 \text{ T}$$



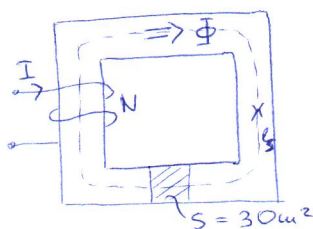
$$H_1 = 16 \text{ A/m}$$

$$\oint_{L_s} \vec{H} \cdot d\vec{l} = NI \approx H_1 l_1 + H_2 l_2 \quad / : N$$

$$I = \frac{H_1 l_1 + H_2 l_2}{N} = \underline{\underline{3 \text{ A}}}$$

④

4.47)

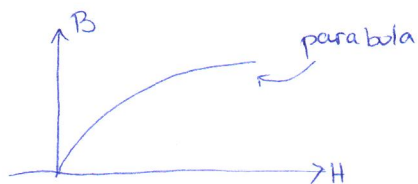


$$l_s = 0,5 \text{ m}$$

$$N = 2000$$

$$I = ? \quad \text{da bo} \quad \Phi = 3 \text{ mWb}$$

$$B = \kappa \sqrt{H}, \quad \kappa = 0,05 \text{ T} \sqrt{\frac{\text{m}}{\text{A}}}$$



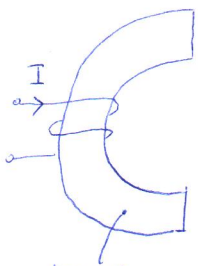
$$B = \frac{\Phi}{S} = 1 \text{ T}$$

$$H = \left(\frac{B}{\kappa} \right)^2 = 400 \text{ A/m}$$

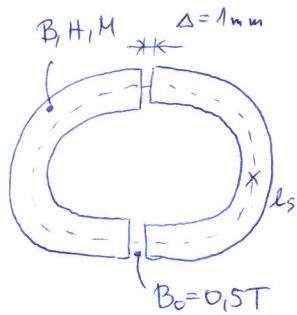
$$\oint \vec{H} \cdot d\vec{l} = NI \approx H \cdot l_s \Rightarrow I = \frac{H l_s}{N} = \underline{\underline{0,1 \text{ A}}}$$

5

5



transformatorska
ploščevina



$$l_s = 10 \text{ cm}$$

$$H, M = ?$$

$$B \approx B_0 = 0,5 \text{ T}$$

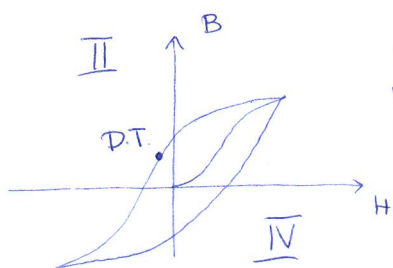
$$H \neq 0, 1 \text{ nA/m}$$

$$H_0 = \frac{B_0}{\mu_0} = 398 \text{ A/m}$$

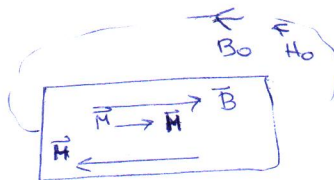
$$\oint \vec{H} \cdot d\vec{\ell} \approx H l_s + H_0 2\Delta = 0$$

$$H = -H_0 \frac{2\Delta}{\ell_s} = \underline{\underline{-7.96 \text{ uA/m}}}$$

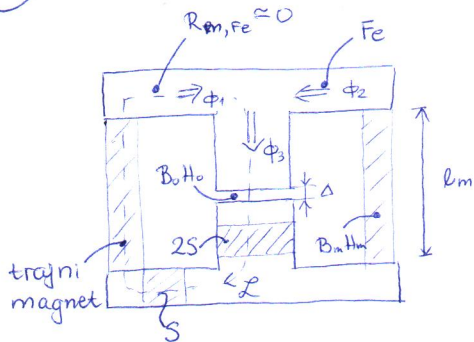
$$M = \frac{B}{\mu_0} - H = \underline{\underline{406 \text{ A/m}}}$$



↑
uvadrant v
vateri se
nahaja D.T



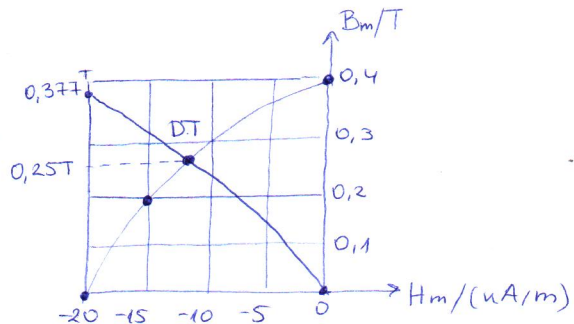
⑥



$$\Delta = 2 \text{ mm}$$

$$l_m = 30 \text{ mm}$$

$$B_0 = ?$$



Kirchoff:
Zaun o
zvornosti

$$\Phi_3 = \Phi_1 + \Phi_2 \Rightarrow B_o \cdot 2S = B_m \cdot S + B_m \cdot S / 2S$$

$$B_0 = B_m \quad ; \quad H_0 = \frac{B_0}{\mu_0} = \frac{B_m}{\mu_0}$$

Амперов закон

$$\oint_{\mathcal{L}} \vec{H} \cdot d\vec{\ell} = 0 \cong H_m \cdot l_m + \underbrace{H_{Fe} \cdot l_{Fe}}_{\approx 0} + H_0 \Delta = H_m \cdot l_m + B_m \frac{\Delta}{\mu_0} \cong 0$$

$$\Rightarrow B_m = - \frac{lm}{\Delta} \mu_0 H_m$$

pove da
sta v konsta
smeri Bin H.

12 D.T otocitamo:

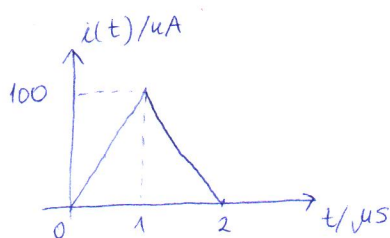
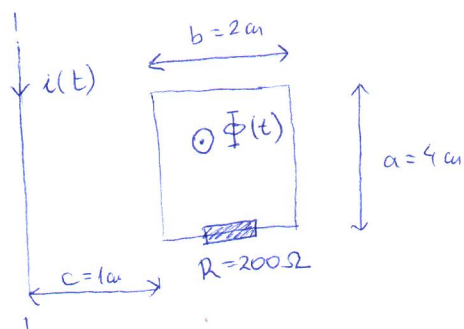
$$B_m \doteq 0,25T \doteq B_0$$

$$H_m = -13 \text{ uA/m}$$

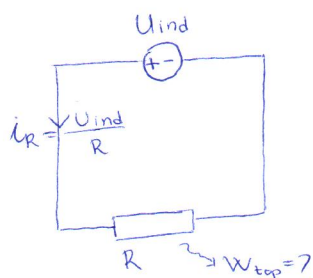
$$B_m|_{H_m=0} = 0; \quad B_u|_{H_m=-20 \text{ uA/m}} = 0,377 \text{ T}$$

7

Dinamično elektromagnetno polje



$$U_{\text{ind.}} = -\frac{d\Phi}{dt}$$



$$\Phi(t) = \Phi^{(i)}(t) + \underbrace{\Phi^{(i_R)}(t)}_{\ll \Phi^{(i)}} \approx \Phi^{(i)}(t)$$

* zanemarimo samoinducijo

$$i_R \ll i$$

$$\Phi(t) \approx \Phi^{(i)}(t) = \frac{\mu_0 \cdot i(t) \cdot a}{2\pi} \ln \frac{b+c}{c} \rightarrow \text{list z formulami}$$

$$U_{\text{ind.}} = -\frac{d\Phi}{dt} = -\frac{\mu_0 a}{2\pi} \ln \frac{b+c}{c} \cdot \frac{di}{dt} \rightarrow \text{edino tok se spreminja s časom}$$

$$\frac{di}{dt} = \begin{cases} \frac{100 \mu\text{A}}{1 \mu\text{s}} = 100 \cdot 10^3 \text{ A/s}, & 0 < t < 1 \mu\text{s} \\ -100 \cdot 10^3 \text{ A/s}, & 1 \mu\text{s} < t < 2 \mu\text{s} \\ 0, & \text{sicer} \end{cases}$$

$$0 < t < 2 \mu\text{s}:$$

$$U_{\text{ind.}} = \pm 879 \text{ V} \begin{cases} + \text{ no se zmanjšuje tok} \\ - \text{ no se povečuje} \end{cases}$$

$$p(t) = U_{\text{ind.}} \cdot i_R = \frac{U_{\text{ind.}}^2}{R} = \frac{U_{\text{ind.}}^2}{R} = 3,86 \mu\text{W} = \text{const}$$

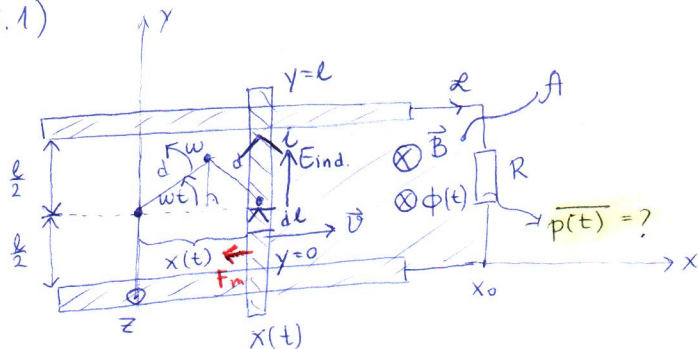
$$W_{\text{top.}} = p \cdot 2 \mu\text{s} = \underline{\underline{7,72 \text{ mJ}}}$$

tok na upor $\Rightarrow i_R = \frac{U_{\text{ind.}}}{R} = \pm 4,4 \text{ A} \ll 100 \mu\text{A}$

(zato smo zanemarili samoinducijo)

8)

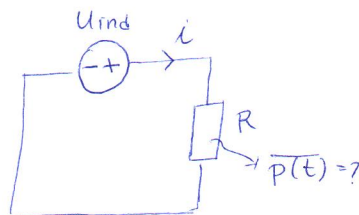
S.1)



$$x(t) = 2 \cdot d \cdot \cos(\omega t)$$

$$\vec{B} = -\vec{e}_z B$$

$$\Phi(t) = B \cdot \underbrace{l \cdot (x_0 - x(t))}_{s(t)}$$



$$U_{ind} = -\frac{d\Phi}{dt} = -B \cdot l \cdot \left(-\frac{dx}{dt}\right) = B \cdot l \cdot 2 \cdot d \cdot (-\omega \sin(\omega t)) = -2 \cdot d \cdot l \cdot B \cdot \omega \sin(\omega t)$$

List s formulami:

$$U_{ind} = \underbrace{\int_A -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}}_{\text{transformatorska ind. napetost}} + \underbrace{\oint_L (\vec{v} \times \vec{B}) \cdot d\vec{\ell}}_{\text{gibalna ind. napetost}}$$

* Ne glede zaradi česa se spreminja ~~tem~~ pretok:

$$U_{ind} = -\frac{d\Phi}{dt}$$

$$U_{ind} = \int_{y=0}^{y=l} (\vec{v} \times \vec{B}) \cdot d\vec{\ell} = \int_{y=0}^{y=l} \vec{E}_{ind} \cdot d\vec{\ell}$$

$$\vec{v} = \vec{e}_x \cdot v_x ; \quad \underline{v_x = \frac{dx}{dt} = -2d \cdot \omega \sin(\omega t)}$$

$$\underline{\vec{v} \times \vec{B} = \vec{E}_{ind}} = \vec{e}_y \cdot v_x \cdot B$$

$$d\vec{\ell} = \vec{e}_y \cdot dy$$

$$\Rightarrow U_{ind} = \int_{y=0}^{y=l} \vec{E}_{ind} \cdot d\vec{\ell} = \int_0^l v_x B \underbrace{\vec{e}_y \cdot \vec{e}_y}_{1} \cdot dy = v_x \cdot B \cdot \underbrace{\int_0^l dy}_l = v_x \cdot B \cdot l = -2d\omega \sin(\omega t) \cdot B \cdot l$$

$$\boxed{U_{ind} = -2d \cdot \omega \cdot B \cdot l \cdot \sin(\omega t)}$$

$$P(t) = \frac{U_{ind}^2}{R} = \underbrace{\frac{4d^2 \omega^2 B^2 l^2}{R}}_{P_0} \cdot \sin^2(\omega t) = P_0 \cdot \sin^2(\omega t)$$

$$\sin(\omega t + T) = \sin(\omega t)$$

$$T = \frac{1}{f} ; f = \frac{\omega}{2\pi} \Rightarrow T = \frac{2\pi}{\omega}$$

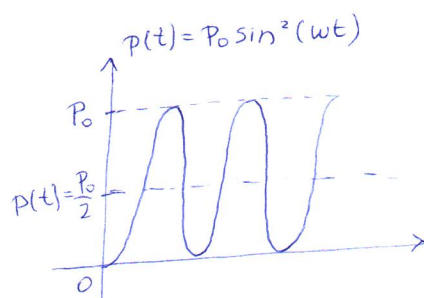
$$\overline{P(t)} = \frac{1}{T} \int_0^T P(t) \cdot dt = \frac{1}{T} \int_0^T P_0 \underbrace{\sin^2(\omega t)}_{\frac{1}{2}(1-\cos(\omega t))} \cdot dt = \frac{1}{T} \cdot \frac{P_0}{2} \left[t - \frac{\sin(2\omega t)}{2\omega} \right]_0^T$$

$$\overline{P(t)} = \frac{1}{T} \cdot \frac{P_0}{2} \left[\underbrace{\left(T - \frac{\sin(2\omega T)}{2\omega} \right)}_0 - \underbrace{\left(0 - \frac{\sin 0}{2\omega} \right)}_0 \right] = \frac{P_0}{2}$$

$$2\omega \cdot T = 2\omega \cdot \frac{2\pi}{\omega} = 4\pi ; \sin(4\pi) = 0$$

$$\boxed{\overline{P(t)} = \frac{P_0}{2} = \frac{2 \cdot d^2 \cdot l^2 \cdot B^2 \cdot \omega^2}{R}}$$

povprežje moči segrevanja

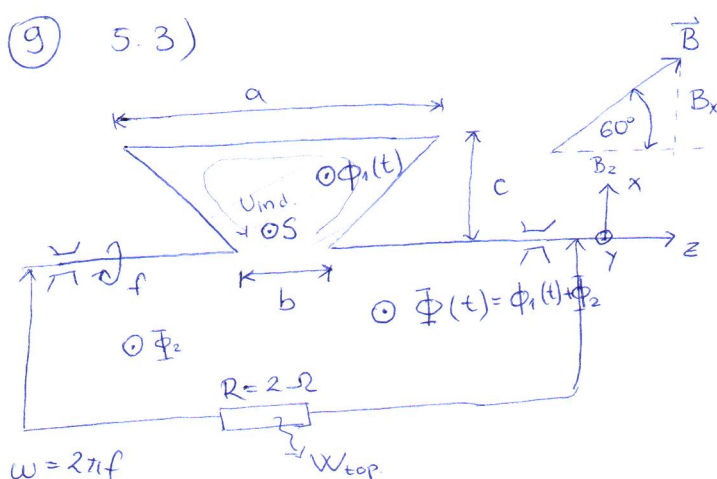


$$d\vec{F}_m = i \cdot \underbrace{d\vec{l}}_{\vec{e}_y dy} \times \underbrace{\vec{B}}_{\vec{e}_z B} = -i B \cdot dy \cdot \underbrace{(\vec{e}_y \times \vec{e}_z)}_{\vec{e}_x} = -\vec{e}_x i \cdot B \cdot dy$$

$$\vec{F}_m = \int_{y=0}^{y=l} dF_m = \boxed{-\vec{e}_x i \cdot B \cdot l = \vec{F}_m}$$

zaviralna sila

(9) 5.3)



$$a = 30 \text{ cm}$$

$$b = 10 \text{ cm}$$

$$c = 20 \text{ cm}$$

$$f = 25 \text{ Hz}$$

$$B = 100 \text{ mT}$$

$$\Delta t = 60 \text{ s}$$

$$W_{\text{top}} = ?$$

$$U_{\text{ind}} = - \frac{d\Phi}{dt}$$

$$P(t) = \frac{U_{\text{ind}}^2}{R}$$

$$\Phi(t) = \Phi_1(t) + \underbrace{\Phi_2}_{\text{const}}$$

le trapezna
zanika se
gi blje

$$\Phi_1(t) = \vec{B} \cdot \vec{S}(t)$$

samo
smer
mu se
spreminja

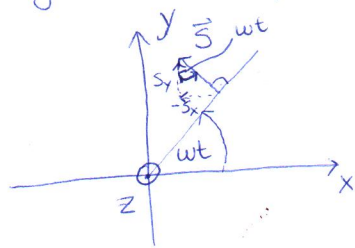
$$|\vec{S}(t)| = \frac{a+b}{2} \cdot c = \text{const}$$

$$\vec{B} = \underbrace{\vec{e}_x B \sin 60^\circ}_{B_x} + \underbrace{\vec{e}_z B \cos 60^\circ}_{B_z} = B \left(\vec{e}_x \frac{\sqrt{3}}{2} + \vec{e}_z \frac{1}{2} \right)$$

$$\vec{S} = \frac{a+b}{2} \cdot c \cdot \vec{e}_f$$

moramo izraziti
v uartež. coord. sistem

Pogledamo trapez od strana



$$\vec{S} = -\vec{e}_x \cdot S \cdot \sin(\omega t) + \vec{e}_y \cdot S \cdot \cos(\omega t)$$

$$\vec{S} = \underbrace{\frac{a+b}{2} \cdot c}_{S} \cdot \vec{e}_r = S [-\vec{e}_x \sin(\omega t) + \vec{e}_y \cos(\omega t)] = \vec{S}(t)$$

$$\Phi_1(t) = \vec{B} \cdot \vec{S}(t) = B \frac{\sqrt{3}}{2} \cdot (-S \sin(\omega t)) + 0 + 0$$

$$\Phi_1(t) = -\frac{\sqrt{3}}{2} B \cdot S \cdot \sin(\omega t)$$

$$U_{ind.} = + \frac{\sqrt{3}}{2} B \cdot S \cdot \omega \cdot \cos(\omega t) = - \frac{d\Phi_1}{dt}$$

$$P(t) = \underbrace{\frac{3 \cdot B^2 \cdot S^2 \cdot \omega^2}{4 \cdot R}}_{P_0} \cdot \cos^2(\omega t) = P_0 \cdot \cos^2(\omega t)$$

$$\overline{P}(t) = \frac{P_0}{2}$$

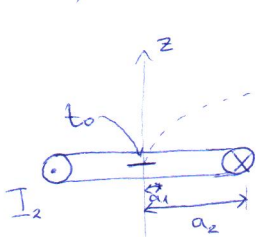
$$\omega = 2\pi f ; \quad T = \frac{1}{f} = 0,04s$$

$$\frac{\Delta t}{T} = 1500$$

$$W_{top} = \int_0^{\Delta t} P(t) \cdot dt = \overline{P}(t) \cdot \Delta t = \frac{P_0}{2} \cdot \Delta t = \frac{3 \cdot B^2 \cdot S^2 \cdot \omega^2}{2 \cdot 4R} \cdot \Delta t = \frac{3B^2 \cdot \left[\frac{a+b}{2} \cdot c\right]^2 (2\pi f)^2 \Delta t}{8 \cdot R}$$

$$W_{top} = 4,44 \text{ J}$$

10) 5.2) Dve vložni zaviti gledani od strani

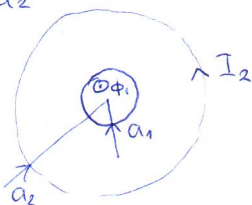


" ∞ "
manjšo
zanako
vržemo
daleč stran

$$\Phi_1(t) = \underbrace{B^{(I_2)}}_{\frac{\mu_0 I_2}{2a_2}} \cdot \pi \cdot a_1^2$$

$$Q_1 = \frac{1}{R_1} \frac{\mu_0 I_2 \pi a_1^2}{2a_2}$$

$$a_1 \ll a_2$$

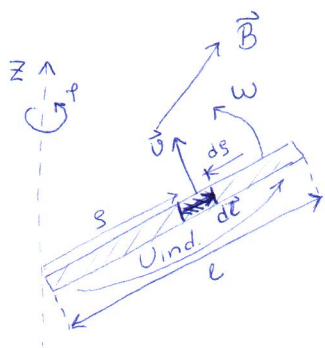


$$U_{ind,1} = - \frac{d\Phi_1}{dt}$$

$$Q_1 = ?$$

$$Q_1 = \int_{t_0}^{t_1} i_1(t) \cdot dt = \int_{t_0}^{t_1} \frac{U_{ind,1}}{R_1} \cdot dt = \frac{1}{R_1} \int_{t_0}^{t_1} - \frac{d\Phi_1}{dt} \cdot dt = - \frac{1}{R_1} \left[\Phi_1(t_1) - \Phi_1(t_0) \right]$$

①



$$U_{ind} = \int_0^l \vec{E}_{ind} \cdot d\vec{\ell}$$

$$\vec{E}_{ind} = \vec{v} \times \vec{B} = \begin{vmatrix} \vec{e}_s & \vec{e}_r & \vec{e}_z \\ 0 & s\omega & 0 \\ B_s & B_r & B_z \end{vmatrix} = \vec{e}_s \cdot s\omega \cdot B_z + \vec{e}_r \cdot 0 + \vec{e}_z \cdot (-s\omega \cdot B_s)$$

$$\vec{v} = s \cdot \omega \cdot \vec{e}_r$$

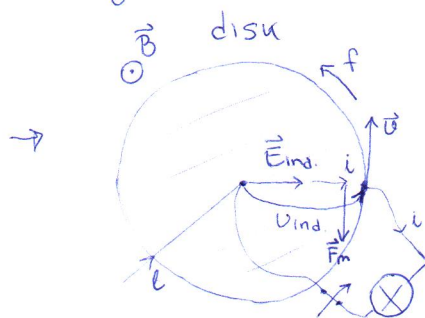
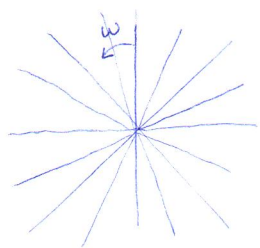
$$\vec{B} = B_s \vec{e}_s + B_r \vec{e}_r + B_z \vec{e}_z$$

$$d\vec{\ell} = \vec{e}_s ds$$

$$\omega = 2\pi f$$

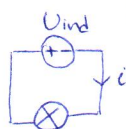
$$\vec{E}_{ind} \cdot d\vec{\ell} = s \cdot \omega \cdot B_z \cdot ds$$

$$\Rightarrow U_{ind} = \int_0^l s \cdot \omega \cdot B_z \cdot ds = \omega B_z \int_0^l s \cdot ds = \omega B_z \frac{s^2}{2} \Big|_0^l = 2\pi f \cdot B_z \cdot \frac{l^2}{2} = \pi l^2 f \cdot B_z$$



$$\vec{E}_{ind} = \vec{v} \times \vec{B}$$

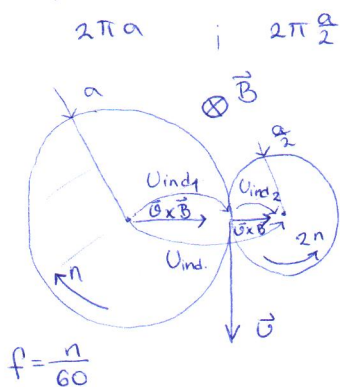
$$U_{ind} = \pi \cdot l^2 \cdot f \cdot B = S \cdot f \cdot B$$



→ Faradayev enosmerni generator.

$\vec{F}_m \uparrow \downarrow \vec{v}$ (zaviralna sila)

② 5.4)



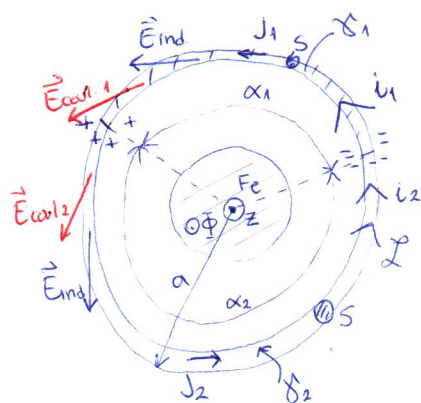
$$U_{ind} = U_{ind1} + U_{ind2} = \pi a^2 \cdot \frac{n}{60} \cdot B + \pi \left(\frac{a}{2}\right)^2 \cdot \frac{2n}{60} B$$

$$U_{ind} = \pi a^2 n B \left(\frac{1}{60} + \frac{1}{42} \cdot \frac{2}{60} \right) = \frac{\pi a^2 n B}{40}$$

$$\frac{2+1}{120} = \frac{1}{40}$$

n - št. obratov
levega diska
v 1 min.

③ 5.9)



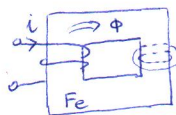
$$\gamma_1 / \gamma_2 = b$$

$$\alpha_1 / \alpha_2 = c$$

$$\phi(t) = \phi_0 \cos(\omega t)$$

$$E_{ind} = ?$$

$$E_{coul} = ?$$



$\vec{E}_{ind}$ je povsod vzdolž obroča enaka

$$U_{ind} = \oint_L \vec{E}_{ind} \cdot d\vec{l} = - \frac{d\phi}{dt} = \phi_0 \omega \sin(\omega t) = E_{ind} \cdot 2\pi a \Rightarrow E_{ind} = \frac{\phi_0 \omega}{2\pi a} \sin(\omega t)$$

$$\oint_L \vec{E}_{coul} \cdot d\vec{l} = 0 = E_{coul,1} \cdot a \cdot \alpha_1 + E_{coul,2} \cdot a \cdot \alpha_2 \quad / : a \cdot \alpha_2$$

$$E_{coul,2} = - E_{coul,1} \cdot \frac{\alpha_1}{\alpha_2} = - E_{coul,1} \cdot c$$

$$\boxed{i_1 = i_2} \quad \text{1. K. Z.} \quad (J_1 = J_2)$$

$$\Rightarrow \gamma_1 (E_{ind} + E_{coul,1}) = \gamma_2 (E_{ind} + E_{coul,2}) \quad / : \gamma_2$$

$$\frac{\gamma_1}{\gamma_2} (E_{ind} + E_{coul,1}) = E_{ind} + E_{coul,2} = E_{ind} - E_{coul,1} \cdot c$$

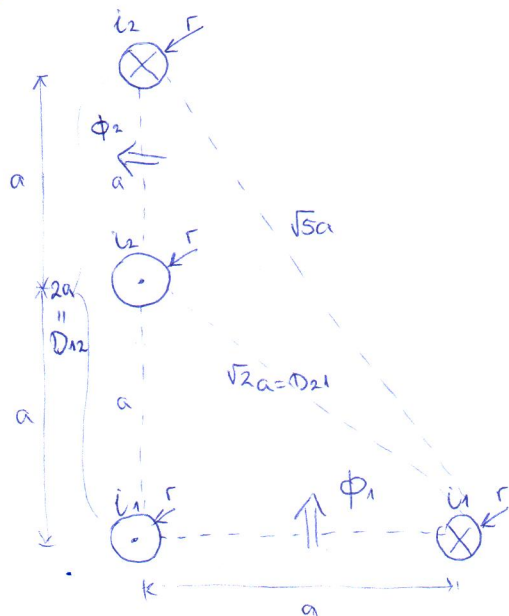
$$(b+c) E_{coul,1} = (1-b) E_{ind}$$

$$E_{coul,1} = \frac{1-b}{b+c} E_{ind} = \frac{1-b}{b+c} \cdot \frac{\phi_0 \omega}{2\pi a} \sin(\omega t)$$

$$E_{coul,2} = -c \frac{1-b}{b+c} \cdot \frac{\phi_0 \omega}{2\pi a} \sin(\omega t)$$

$E_{coul,1}$

④ Dva dvovoda



$$r = \frac{a}{10}$$

$$L_{12}, L_{21} = ?$$

$$L_{11}, L_{22} = ?$$

$$L_{12} = \frac{\Phi_1^{(i_2)}}{i_2}$$

$$\Phi_1^{(i_2)} = \frac{\mu_0 i_2 l}{2\pi} \left[\ln \frac{\sqrt{2}a}{r} + \ln \frac{\sqrt{5}a}{2a} \right]$$

$$\Phi_1^{(i_2)} = \frac{\mu_0 i_2 l}{2\pi} \ln \frac{2\sqrt{2}}{\sqrt{5}}$$

- predenani zaradi smerjo toka !!! desnega vijaka

(preton suozni pravouotniu)

$$L_{21} = \frac{\Phi_2^{(i_1)}}{i_1} = \frac{1}{\mu_0} \left[\frac{\mu_0 i_1 l}{2\pi} \ln \frac{2}{\sqrt{5} a} - \frac{\mu_0 i_1 l}{2\pi} \ln \frac{\sqrt{5} a}{\sqrt{2} a} \right]$$

$$L_{21} = \frac{\mu_0 l}{2\pi} \ln \frac{2\sqrt{2}}{\sqrt{5}} = L_{12} = L_{21} \quad ! \quad \text{Recipročnost medsebojnih induktivnosti}$$

List s formulami:

$$L_{12} = \frac{\mu_0 l}{2\pi} \ln \frac{D_{12} \cdot D_{21}}{d_{12} \cdot d_{21}}$$

$$2a = D_{12}$$

$$\sqrt{5}a = d_{21}$$

$$\sqrt{2}a = D_{21}$$

$$a = d_{12}$$

$$L_{12} = \frac{\mu_0 l}{2\pi} \ln \frac{2\sqrt{2}}{\sqrt{5}}$$

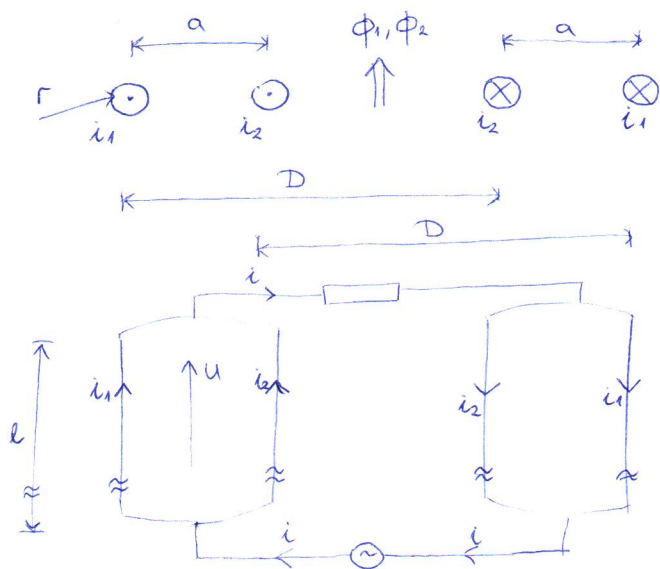
Lastna induktivnost: (List s formulami)

$$L_1 = \frac{\Phi_1^{(i_1)}}{i_1} = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{a}{r} \right) = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln 10 \right)$$

Dva enaka dvovoda imata enaki lastni induktivnosti

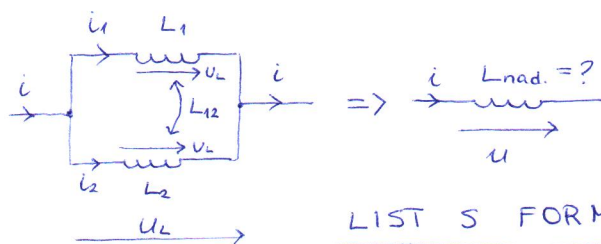
$$L_2 = \frac{\Phi_2^{(i_2)}}{i_2} = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{a}{r} \right) = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln 10 \right)$$

5.14.) Zbirna Sepljen dvovod



$$L_1 = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{D+a}{r} \right); \quad L_2 = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{D-a}{r} \right)$$

$$L_{12} = L_{21} = \frac{\mu_0 l}{2\pi} \ln \frac{D \cdot D}{a \cdot a} = \frac{\mu_0 l}{\pi} \ln \frac{D}{a}$$



LIST S FORM.

$$U_L = L_{nad} \cdot \frac{di}{dt} = L_{nad} \cdot i'$$

$$i = i_1 + i_2 \quad / : \frac{d}{dt}$$

$$U_L = L_1 \cdot i_1' + L_{12} \cdot i_2' = L_2 \cdot i_2' + L_{12} \cdot i_1' \Rightarrow (L_2 - L_{12}) i_2' = (L_1 - L_{12}) i_1' / : L_2 - L_{12}$$

$$i_2' = \frac{L_1 - L_{12}}{L_2 - L_{12}} i_1'$$

$$i' = i_1' + i_2' = i_1' + \frac{L_1 - L_{12}}{L_2 - L_{12}} i_1'$$

$$i' = \frac{L_2 - L_{12} + L_1 - L_{12}}{L_2 - L_{12}} i_1' = \frac{L_1 + L_2 - 2L_{12}}{L_2 - L_{12}} i_1'$$

$$i_1' = \frac{L_2 - L_{12}}{L_1 + L_2 - 2L_{12}} i'$$

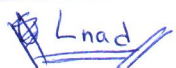
|||

$$U_L = L_1 \cdot i_1' + L_{12} \cdot \frac{L_1 - L_{12}}{L_2 - L_{12}} i_1' = \dots =$$

$$U_L = \frac{L_1 L_2 - L_{12}^2}{L_2 - L_{12}} i_1' = U_L$$

$$U_L = \frac{L_1 L_2 - L_{12}^2}{L_2 - L_{12}} \cdot \frac{L_2 - L_{12}}{L_1 + L_2 - 2L_{12}} \cdot i'$$

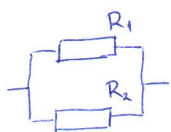
$$U_L = \frac{L_1 L_2 - L_{12}^2}{L_1 + L_2 - 2L_{12}} i'$$



$$\Rightarrow L_{nad} = \frac{L_1 L_2 - L_{12}^2}{L_1 + L_2 - 2L_{12}}$$

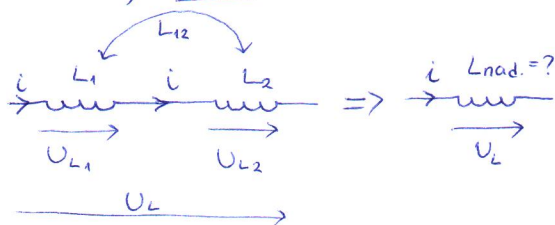
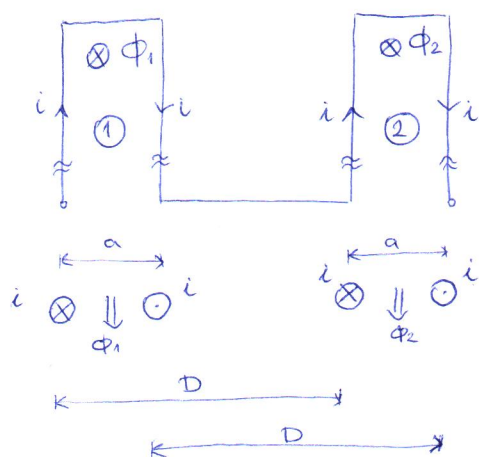
Pri
vzporedna
vezava

npr. $L_{12} = 0$: $L_{nad} = \frac{L_1 L_2}{L_1 + L_2}$



$$R_{nad} = \frac{R_1 R_2}{R_1 + R_2}$$

Dvojna zanka (zaporedna vezava) toja sta enaka!



$$U_L = U_{L1} + U_{L2} = (L_1 i' + L_{12} i') + (L_2 i' + L_{12} i')$$

$$U_L = (L_1 + L_2 + 2L_{12}) i'$$

$$L_{nad} = L_1 + L_2 + 2L_{12}$$

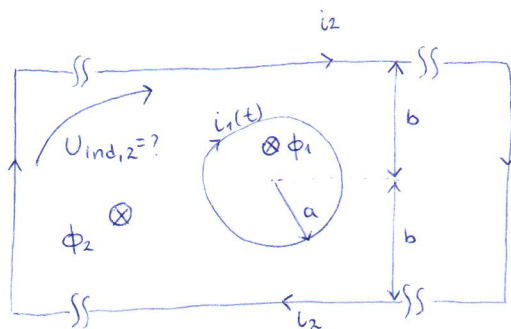
Pri
zaporedna
vezava

pri $L_{12} = 0$; $L_{nad} = L_1 + L_2$

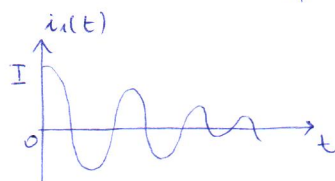
$$L_1 = L_2 = \frac{\mu_0 l}{\pi} \left(\frac{1}{4} + \ln \frac{a}{r} \right)$$

$$L_{12} = L_{21} = \frac{\mu_0 l}{2\pi} \ln \frac{(D+a)(D-a)}{D \cdot D} = \frac{\mu_0 l}{2\pi} \ln \underbrace{\frac{D^2 - a^2}{D^2}}_{< 1} < 0 !$$

⑥ 5.16)



$$i_1(t) = I e^{-\alpha t} \cos(\beta t + \gamma)$$



$$U_{ind,2} = - \frac{d\Phi_2(i_1)}{dt} = -L_{21} \cdot \frac{di_1}{dt}$$

$$\frac{\Phi_2(i_1)}{i_1} = L_{21} \Rightarrow \Phi_2(i_1) = L_{21} \cdot i_1(t)$$

$$L_{21} = L_{12} = \frac{\Phi_1(i_2)}{i_2}$$

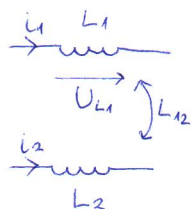
$$\phi_1^{(i_2)} = \underbrace{\phi_{1,ZG}^{(i_2)}}_{\mu_0 i_2 (b - \sqrt{b^2 - a^2})} + \underbrace{\phi_{1,SP}^{(i_2)}}_{\phi_{1,ZG}^{(i_2)}} + \underbrace{\phi_{1,LEV}^{(i_2)}}_{\approx 0} + \underbrace{\phi_{1,DES}^{(i_2)}}_{\approx 0}$$

$$L_{21} = L_{12} = \frac{\phi_1^{(i_2)}}{i_2} \Rightarrow L_{12} = 2 \mu_0 (b - \sqrt{b^2 - a^2})$$

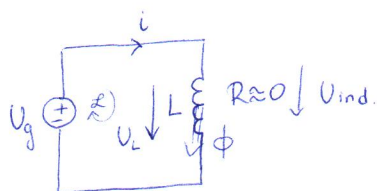
$$U_{ind,2} = -2 \mu_0 (b - \sqrt{b^2 - a^2}) \cdot I \left[\alpha \cdot e^{-\alpha t} \cos(\beta t + \gamma) + e^{-\alpha t} (-\beta \sin(\beta t + \gamma)) \right]$$

$$U_{ind,2} = 2 \mu_0 (b - \sqrt{b^2 - a^2}) \cdot I \cdot e^{-\alpha t} [\alpha \cdot \cos(\beta t + \gamma) + \beta \cdot \sin(\beta t + \gamma)]$$

$$U_{ind,2} = -L_{21} \cdot \frac{di_1}{dt}$$



$$U_{L1} = L_1 \cdot \frac{di_1}{dt} + L_{12} \frac{di_2}{dt}$$

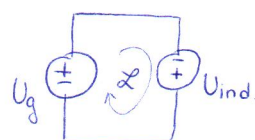


$$U_L = +L \cdot \frac{di}{dt}$$

$$U_{ind} = -\frac{d\phi}{dt} = -L \cdot \frac{di}{dt}$$

$$L = \frac{\phi}{i} \Rightarrow \phi = L \cdot i$$

$$\oint \vec{E}_{coul} \cdot d\vec{l} = 0 \Rightarrow -U_g + \underbrace{U_{coul}}_{U_g = U_L} = 0$$

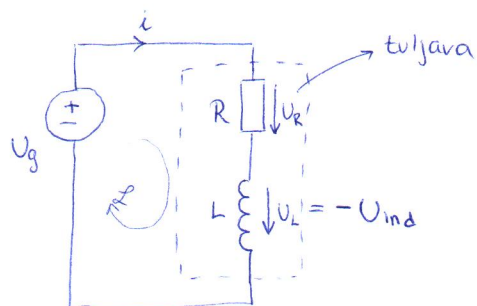


$$\mathcal{L}: -U_g - U_{ind} = 0$$

$$U_{ind} = -U_g = -U_L$$

$$U_L = -U_{ind}$$

$$U_{tuj. cel.} = U_{coul.} + U_{ind.} = U_L + U_{ind} = L \frac{di}{dt} - L \frac{di}{dt} = 0 ; i = \frac{U_{tujave}}{R}$$

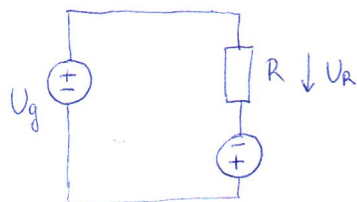


$$\mathcal{L}: -U_g + U_R + U_L = 0$$

$$U_g = U_R + U_L = R \cdot i + L \cdot \frac{di}{dt}$$

$$U_{tujave} = \underbrace{U_R + U_L + U_{ind}}_{U_{coul.}} = U_R$$

$$U_{ind} = -U_L = -L \frac{di}{dt}$$



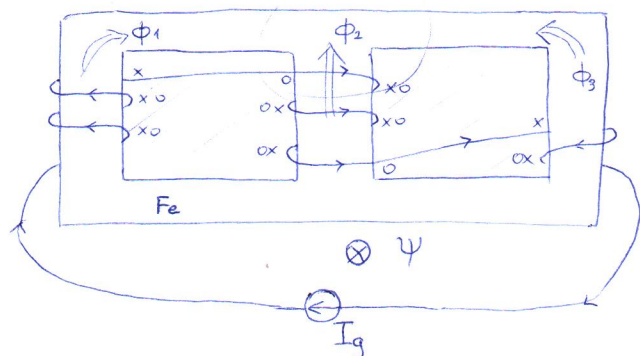
$$\mathcal{L}: -U_g + U_R - U_{ind} = 0$$

$$U_g = U_R - U_{ind} = R \cdot i + L \frac{di}{dt}$$

Inducirana napetost upoštevamo z nasprotnim predznakom

7) 5.13)

$$\Psi = ?$$



Kolimo urat žica prebada opno

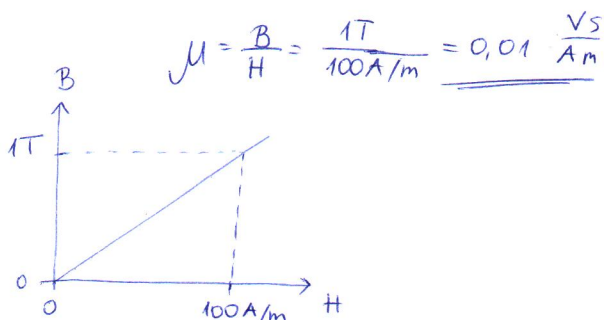
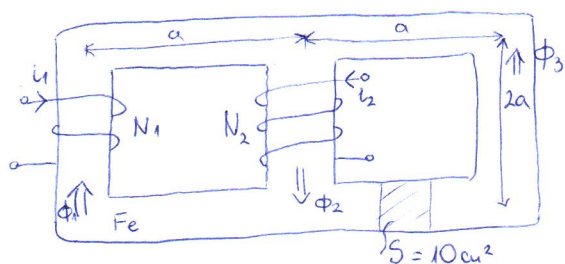
$$\Psi_a = -3\Phi_1 + 2\Phi_2 - 2\Phi_3 \quad \checkmark$$

$$\Psi_b = -2\Phi_1 + 3\Phi_2 - 1\Phi_3 \quad \checkmark$$

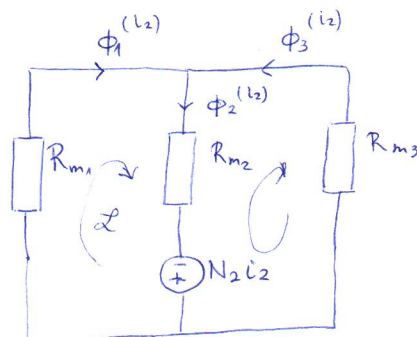
$$\Psi_b - \Psi_a = \Phi_1 + \Phi_2 + \Phi_3 = 0$$

oba sta prov. $\checkmark$

8) 5.23)



permeabilnost
stalnega
jedra



$$a = 10 \text{ cm}$$

$$N_1 = 200$$

$$N_2 = 400$$

$$L_{12} = ?$$

$$L_{12} = \frac{\Psi_1(i_2)}{i_2}$$

$$\Psi_1(i_2) = N_1 \cdot \Phi_1(i_2)$$

$$R_{m1} = \frac{4a}{\mu \cdot S}$$

$$R_{m2} = \frac{2a}{\mu \cdot S}$$

$$R_{m3} = \frac{4a}{\mu S} = R_{m1} \Rightarrow \Phi_1(i_2) = \Phi_3(i_2)$$

$$A: -\Phi_1(i_2) - \underbrace{\Phi_3(i_2)}_{\Phi_1(i_2)} + \Phi_2(i_2) = 0 \Rightarrow \Phi_2(i_2) = 2 \cdot \Phi_1(i_2) \Rightarrow \Phi_1(i_2) = \frac{\Phi_2(i_2)}{2}$$

$$\mathcal{L}: -N_2 \cdot i_2 + R_{m1} \cdot \underbrace{\Phi_1(i_2)}_{\Phi_2(i_2)/2} + R_{m2} \cdot \Phi_2(i_2) = 0$$

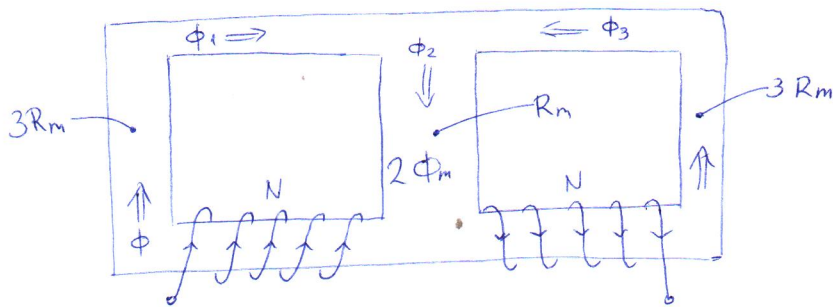
$$\left(\frac{R_{m1}}{2} + R_{m2} \right) \Phi_2(i_2) = N_2 \cdot i_2 \quad / : () \Rightarrow \Phi_2(i_2) = \frac{N_2 \cdot i_2}{\frac{R_{m1}}{2} + R_{m2}}$$

$$\underline{\Phi_2(i_2)} = \frac{N_2 \cdot i_2}{\frac{2a}{\mu S} + \frac{2a}{\mu S}} = \frac{N_2 \cdot i_2 \cdot \mu \cdot S}{4a} \quad / \quad \Phi_1(i_2) = \frac{\Phi_2(i_2)}{2} = \frac{N_2 \cdot i_2 \cdot \mu \cdot S}{8a}$$

$$L_{12} = \frac{N_1 \phi_1^{(i_2)}}{i_2} = \frac{N_2 N_1 \mu S}{8a} = \underline{1 \text{ H}}$$

$$L_2 = \frac{\psi_2^{(i_2)}}{i_2} = \frac{N_2 \cdot \phi_2^{(i_2)}}{i_2} = \frac{N_2^2 \mu \cdot S}{4a} = \underline{4 \text{ H}}$$

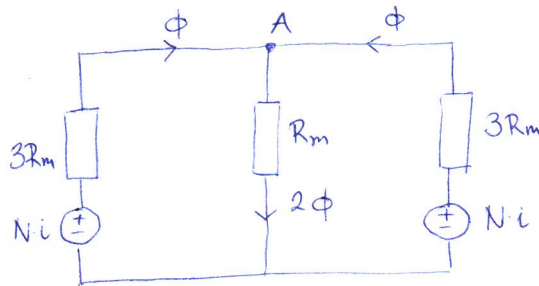
9) 5.21)



$L = ?$

$$\phi_2 = \phi_1 + \phi_3 = 2\phi_1$$

$$L = \frac{\psi}{i} = \frac{N\phi + N\phi}{i} = 2N \frac{\phi}{i}$$

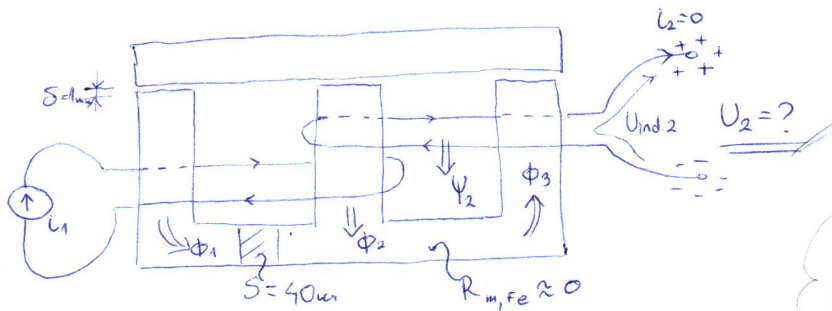


$$\mathcal{L}: -N \cdot i + 3R_m \cdot \phi + R_m \cdot 2\phi = 0$$

$$5R_m \phi = N \cdot i \Rightarrow \phi = \frac{N \cdot i}{5R_m}$$

$$L = 2 \dots$$

10) 5.19)

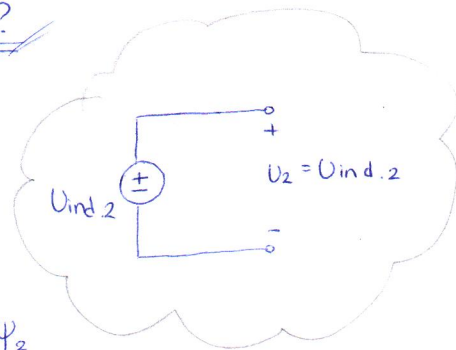


$$i_1(t) = 10 \cdot \cos(\omega t) \cdot \text{A}$$

$$\omega = 400 \text{ Hz}$$

$$\psi_2 = \phi_2 - \phi_3$$

$$U_{\text{ind},2} = - \frac{d\psi_2}{dt}$$



$$A: \phi_3 = \phi_1 + \phi_2 = 2\phi_1$$

$$\mathcal{L}: -i_1 + R_S \cdot \phi_3 + R_S \cdot \phi_1 = 0$$

$$3R_S \cdot \phi_1 = i_1$$

$$\phi_1 = \frac{i_1}{3R_S}$$

$$R_S = \frac{\delta}{\mu_0 \cdot S} = 200 \frac{\mu\text{A}}{\text{Vs}}$$

$$\psi_2 = \phi_2 - \phi_3 = \phi_1 - 2\phi_1 = -\phi_1 = -\frac{i_1}{3R_S}$$

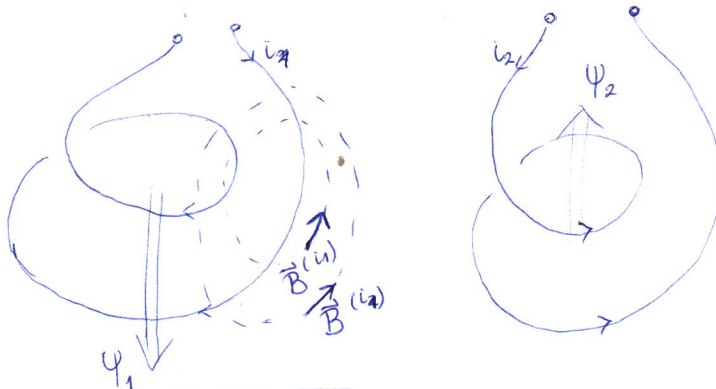
$$L_{12} = \frac{\psi_2^{(i_1)}}{i_1} = \frac{\psi_2}{i_1} = -\frac{1}{3R_S} = -1,67 \mu\text{H}$$

$$U_2 = U_{\text{ind},2} = -\frac{d\psi_2}{dt} = \frac{1}{3R_8} \cdot \frac{d\psi_1}{dt}$$

$$U_2 = \frac{1}{3R_8} \cdot (-10A \cdot \omega) \cdot \sin(\omega t) = \underline{\underline{-6,67mV \cdot \sin(\omega t)}}$$

$$R_8 = \frac{5}{10^{-5}} = 200 \frac{\mu A}{V s}$$

⑩ 5.27)



$$40\% \cdot \psi_1(i_1) = \psi_2(i_1)$$

$$90\% \cdot \psi_2(i_2) = \psi_1(i_2)$$

$$K_{12} = ?$$

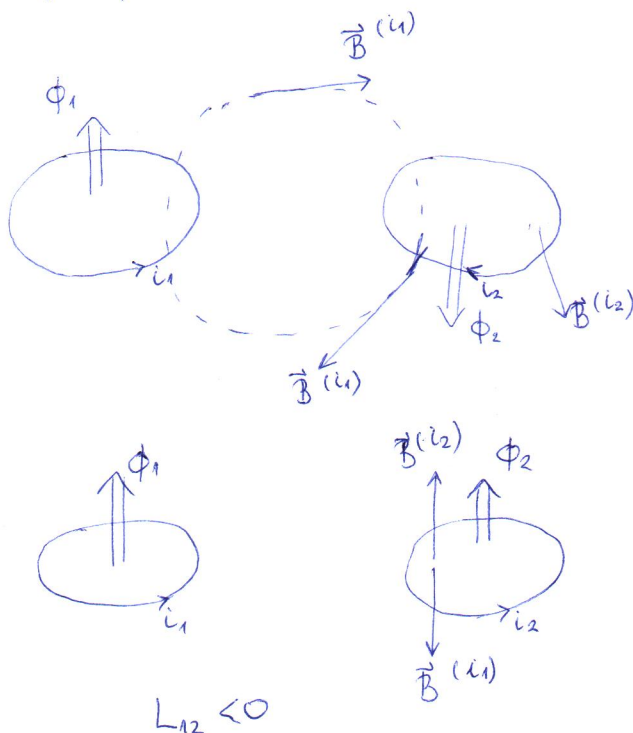
$$K_{12} = \sqrt{\frac{\psi_2(i_1)}{\psi_1(i_1)} \cdot \frac{\psi_1(i_2)}{\psi_2(i_2)}} = \sqrt{\underbrace{0,4}_{40\%} \cdot \underbrace{0,9}_{90\%}} = 0,6$$

$$K_{12}^2 = \frac{(\psi_2(i_1)/i_1) \cdot (\psi_1(i_2)/i_2)}{(\psi_1(i_1)/i_1) \cdot (\psi_2(i_2)/i_2)} = \frac{L_{21} \cdot L_{12}}{L_1 \cdot L_2} = \frac{L_{12}^2}{L_1 \cdot L_2}$$

$$L_{12}^2 = K_{12}^2 \cdot L_1 \cdot L_2 \quad / \sqrt{\quad}$$

$$L_{12} = \pm K_{12} \sqrt{L_1 \cdot L_2}$$

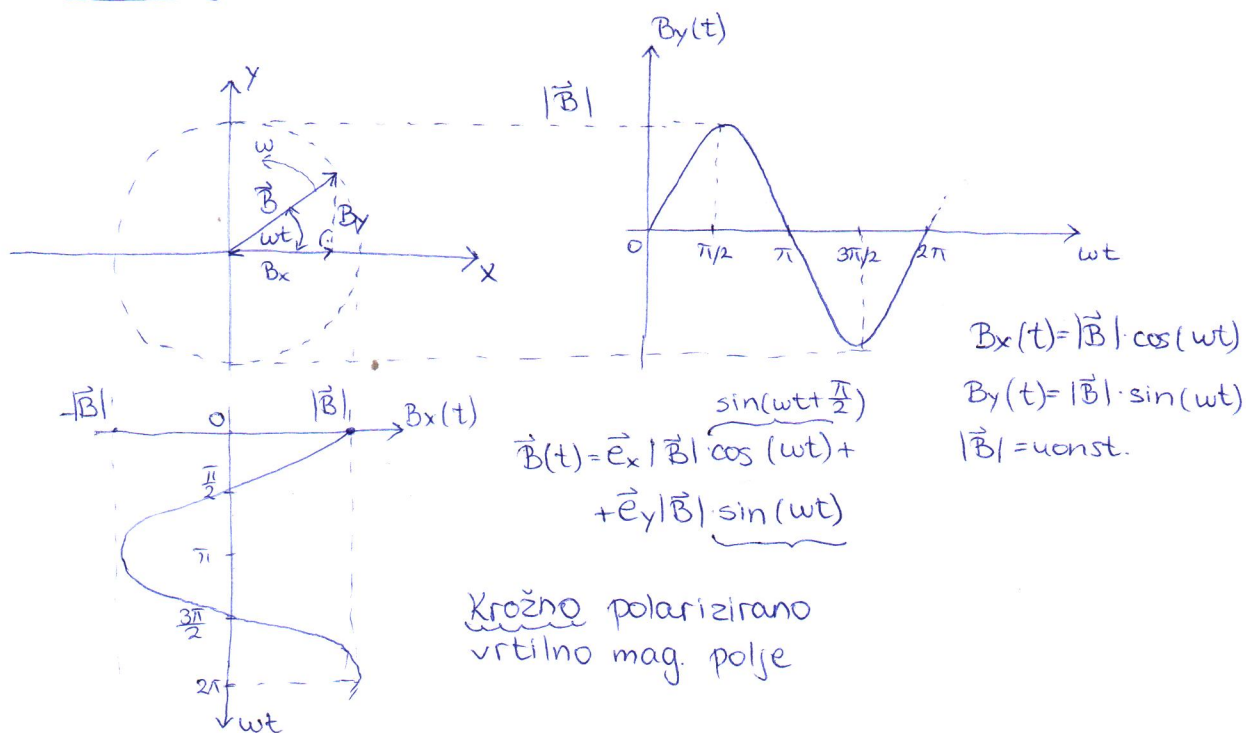
$|L_{12}| = M$



$$L_{12} < 0$$

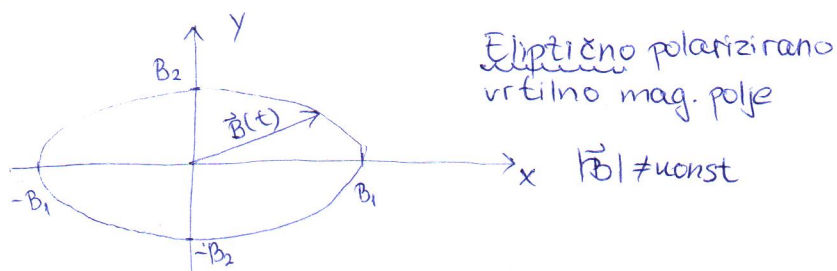
3 LAB VAJA

Vrtilno mag. polje



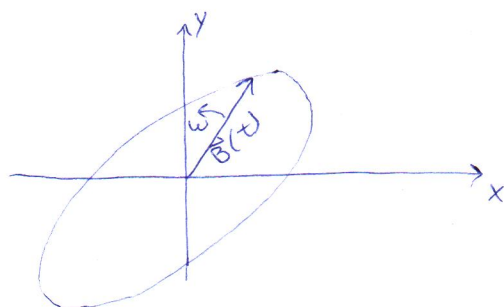
$$B_x = B_1 \cos(\omega t) = B_1 \sin(\omega t + \frac{\pi}{2})$$

$$B_y = B_2 \sin(\omega t)$$

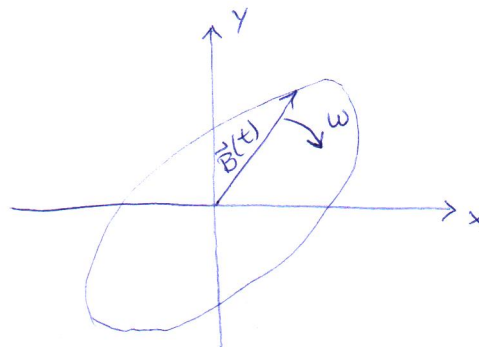


$$B_x = B_0 \sin(\omega t + \varphi), \quad 0 < \varphi < \frac{\pi}{2}$$

$$B_y = B_0 \sin(\omega t)$$



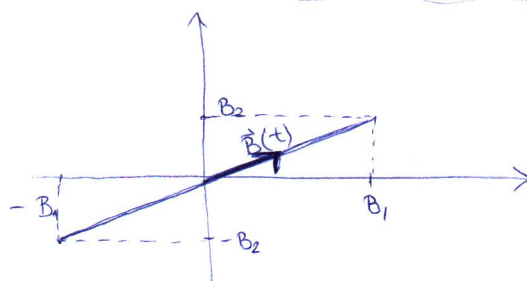
$$-\frac{\pi}{2} < \varphi < 0$$



$$\varphi = 0$$

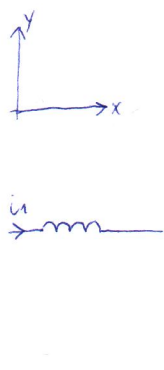
$$B_x(t) = B_1 \sin(\omega t)$$

$$B_y(t) = B_2 \sin(\omega t)$$



Limitni primer:

Linearno polarizirano mag. polje



$$i_1(t) = I_0 \cdot \cos(\omega t)$$

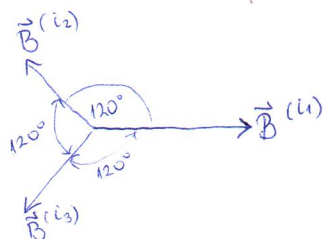
$$i_2(t) = I_0 \cdot \sin(\omega t)$$

$$\vec{B} = \vec{B}^{(i1)} + \vec{B}^{(i2)} = \vec{e}_x B_0 \cos(\omega t) + \vec{e}_y B_0 \sin(\omega t)$$

$$B^{(i1)}(t) \propto i_1(t) \Rightarrow \vec{B}^{(i1)} = \vec{e}_x B_0 \cos(\omega t)$$

$$B^{(i2)}(t) \propto i_2(t) \Rightarrow \vec{B}^{(i2)} = \vec{e}_y B_0 \sin(\omega t)$$

2 para tuljave

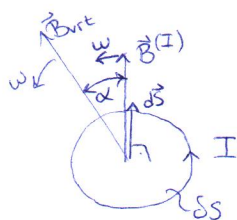
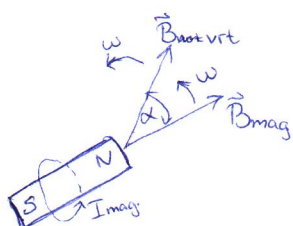


$$B^{(i1)}(t) = B_0 \cdot \sin(\omega t)$$

$$B^{(i2)}(t) = B_0 \cdot \sin(\omega t + 120^\circ)$$

$$B^{(i3)}(t) = B_0 \cdot \sin(\omega t + 240^\circ)$$

$$\vec{B}(t) = \vec{B}^{(i1)}(t) + \vec{B}^{(i2)}(t) + \vec{B}^{(i3)}(t) \Rightarrow \text{vrtilno mag. polje}$$



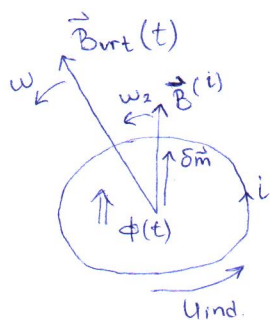
$$\vec{\delta m} = I \cdot d\vec{S}$$

$$\vec{\delta M} = \vec{\delta m} \times \vec{B}_{vrt} = I \cdot \delta \vec{S} \times \vec{B}_{vrt}$$

$$|\delta \vec{M}| = I \delta S \cdot B_{vrt} \cdot \sin \alpha$$

Sinhrono se vrtila.

Sinhroni elektromotor:



$$U_{ind.} = - \frac{d\phi}{dt}$$

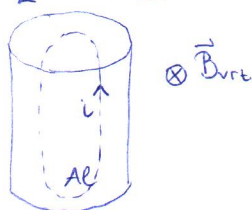
$$\omega_2 \neq \omega$$

$$\omega_2 < \omega$$

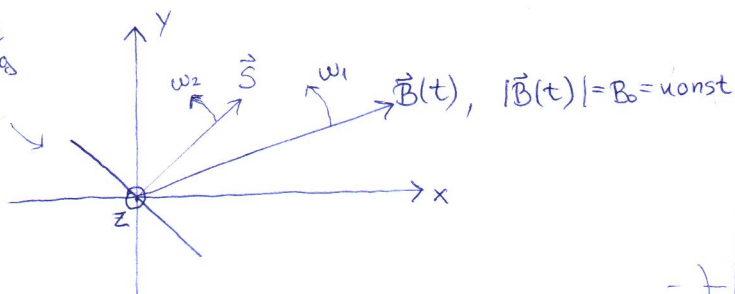
Zanua in polje se vrtila asinhrono.
asinhroni elektromotor.

vafj: (se bo vrtil)

$$\omega_2 \neq \omega$$

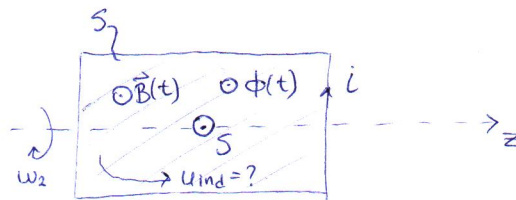


5.7)

zanika se
vrtili okrog
osi z

$$\vec{B}(t) = \vec{e}_x \cdot B_0 \cos(\omega_1 t) + \vec{e}_y \cdot B_0 \sin(\omega_1 t)$$

$$\vec{S}(t) = \vec{e}_x \cdot S \cos(\omega_2 t) + \vec{e}_y \cdot S \sin(\omega_2 t)$$



$$U_{ind} = - \frac{d\Phi}{dt}$$

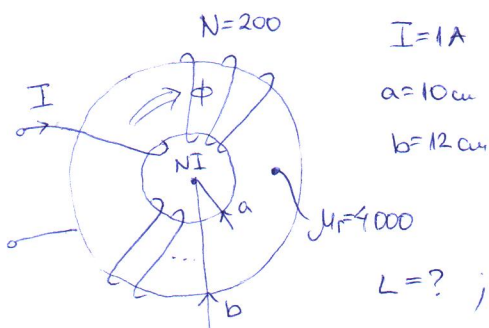
$$\Phi(t) = \vec{B}(t) \cdot \vec{S}(t) = B_0 \cdot S \cos(\omega_1 t) \cos(\omega_2 t) + B_0 \cdot S \sin(\omega_1 t) \sin(\omega_2 t)$$

$$\Phi(t) = B_0 \cdot S \cos(\omega_1 t - \omega_2 t) = B_0 S \cos((\omega_1 - \omega_2)t)$$

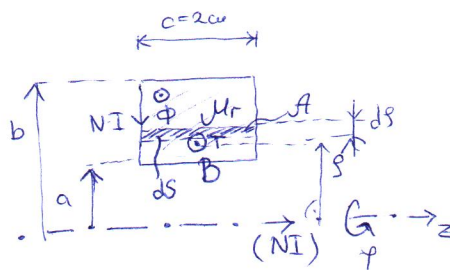
$$U_{ind}(t) = B_0 \cdot S \cdot (\omega_1 - \omega_2) \cdot \sin((\omega_1 - \omega_2)t)$$

①

Magnetna energija



$$L = ? ; W_m = ?$$



1)

$$L = \frac{\Psi}{I} = \frac{N \cdot \Phi}{I} = \frac{\mu_0 \mu_r N^2 c}{2\pi} \ln \frac{b}{a} = 117 \text{ mH}$$

$$\Phi = \int_A \vec{B} \cdot d\vec{S} = \int_a^b \frac{\mu_0 \mu_r (NI)}{2\pi s} \vec{e}_\phi \cdot \vec{e}_\phi c ds = \frac{\mu_0 \mu_r (NI) c}{2\pi} \int_a^b \frac{ds}{s} = \frac{\mu_0 \mu_r NI c}{2\pi} \ln \frac{b}{a}$$

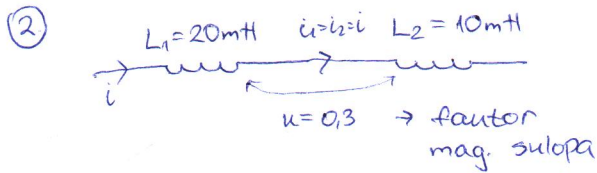
$$\vec{B} = \vec{e}_\phi \frac{\mu_0 \mu_r (NI)}{2\pi s} = \vec{B}(r) = \vec{e}_\phi B_\phi(s)$$

$$d\vec{S} = c ds \cdot \vec{e}_\phi$$

$$2) W_m = \frac{1}{2} L I^2 = 58 \text{ mJ}$$

$$\left\{ W_e = \frac{1}{2} C U^2 \right\} \quad c = \frac{Q}{U} \Rightarrow W_e = \frac{1}{2} Q \cdot U$$

$$W_m = \frac{1}{2} \frac{\Psi}{I} \cdot I^2 = \frac{1}{2} \Psi \cdot I$$



$$i(t) = I_0 \cdot \cos(\omega t)$$

$$I_0 = \sqrt{2} \cdot 10\text{A}$$

$$L_{12} < 0 \quad (\text{megsebojna ind. je negativna})$$

$$|L_{12}| = M = k \sqrt{L_1 \cdot L_2}$$

$$L_{12} = -k \sqrt{L_1 \cdot L_2}$$

LIST S FORMULAMI:

$$W_m(t) = \frac{1}{2} \sum_{j=1}^2 \sum_{k=1}^2 L_{jk} \cdot i_1(t) \cdot i_2(t) =$$

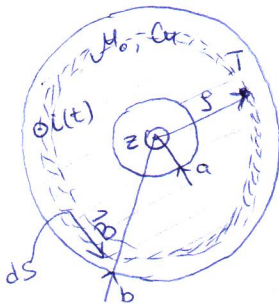
$$= \frac{1}{2} \left[\underbrace{L_{11}}_{L_1} \cdot i^2(t) + \underbrace{L_{12}}_{L_{12}} i^2(t) + \underbrace{L_{21}}_{L_{12}} i^2(t) + \underbrace{L_{22}}_{L_2} i^2(t) \right] = W_m(t) = ?$$

$$= \frac{1}{2} \underbrace{(L_1 + L_2 + 2L_{12})}_{L_{\text{nad.}}} \cdot i^2(t) = \frac{1}{2} L_{\text{nad.}} \cdot i^2(t)$$

$$L_{\text{nad.}} = L_1 + L_2 - 2k \sqrt{L_1 L_2} = 21,5\text{mH}$$

$$W_m(t) = \frac{1}{2} L_{\text{nad.}} \cdot I_0^2 \cdot \cos^2(\omega t) \Rightarrow \overline{W_m(t)} = \frac{1}{2} L_{\text{nad.}} \cdot I_0^2 \cdot \underbrace{\overline{\cos^2(\omega t)}}_{1/2} = \frac{1}{4} L_{\text{nad.}} \cdot I_0^2 = 1,08\text{J}$$

③ 5.28)



$$a = 2\text{cm}$$

$$b = 2,5\text{cm}$$

$$i(t) = I_0 \cdot \cos(\omega t)$$

$$I_0 = 3\text{mA}$$

$$\omega = 2\pi f$$

$$f = 50\text{Hz}$$

$$\overline{W_{m,a < s < b}(t)} = \overline{W'_{m,a < s < b}(t)} = ?$$

$$w_m = \frac{W_m}{V} = \frac{1}{2} \mu_0 \cdot H^2$$

$$B = \mu_0 H \Rightarrow H = \frac{B}{\mu_0}$$

$$W'_{m,a < s < b} = \int_s \underbrace{w_m}_{\frac{dV}{l}} ds$$

$$a < s < b:$$

$$w_m = \frac{1}{2} \mu_0 \frac{i^2}{4\pi^2 (b^2 - a^2)} \left(s - \frac{a^2}{s}\right)^2 = w_m(s)$$

$$ds = 2\pi s \cdot ds$$

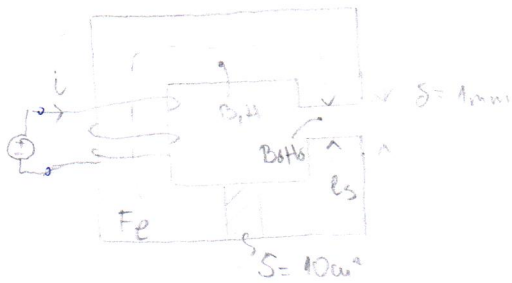
$$\Rightarrow W'_{m,a < s < b} = \int_a^b \underbrace{\frac{\mu_0 i^2}{8\pi^2 (b^2 - a^2)^2} \left(s^2 - 2s \frac{a^2}{s} + \frac{a^4}{s^2}\right)}_{W_m} \cdot 2\pi s ds = \frac{\mu_0 i^2(t)}{4\pi (b^2 - a^2)^2} \int_a^b \left(s^2 - 2a^2 + \frac{a^4}{s^2}\right) ds$$

$$W'_{m,a < s < b} = \frac{\mu_0}{4\pi (b^2 - a^2)^2} \left[\frac{b^4 - a^4}{4} - 2a^2 \cdot \frac{b^2 - a^2}{2} + a^4 \ln \frac{b}{a} \right] \cdot i^2(t) = K \cdot I_0^2 \cdot \cos^2(\omega t)$$

$$K = 6,64 \cdot 10^{-9} \frac{\text{Vs}}{\text{Am}}$$

$$\overline{W'_{m,a < s < b}} = 29,9 \cdot 10^{-3} \text{J/m}$$

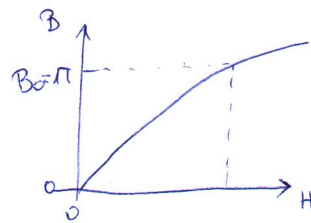
④ 5.29)



$$l_s = 30 \text{ cm}$$

$$H(B) = \mu \cdot B^2$$

$$\mu = 900 \frac{\text{A}}{\text{T}^2 \cdot \text{m}}$$



$$W_{\text{mag}} = ? , B_0 = 1 \text{ T}$$

LIST S FORMULAMI:

$$W_{\text{mag, Fe}} = \frac{W_{\text{mag, Fe}}}{V} = \int_0^{B_0} H \cdot dB = \int_0^{B_0} \mu \cdot B^2 \cdot dB = \mu \cdot \frac{B^3}{3} \Big|_0^{B_0} = \mu \cdot \frac{B_0^3}{3}$$

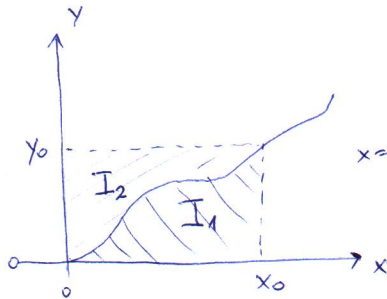
$$W_{\text{mag, Fe}} = W_{\text{mag, Fe}} \cdot \frac{S \cdot l_s}{V_{\text{Fe}}} = \mu \cdot \frac{B_0^3}{3} \cdot S \cdot l_s = 90 \text{ mJ}$$

$$W_{\text{mag}} = W_{\text{mag, Fe}} + W_{m, \delta} = 488 \text{ mJ}$$

$$W_{m, \delta} = W_{m, \delta} \cdot \frac{S \cdot \delta}{V_{\delta}} = \frac{B_{\delta}^2}{2\mu_0} \cdot S \cdot \delta = \frac{B_0^2}{2\mu_0} \cdot S \cdot \delta = 398 \text{ mJ}$$

$$B \propto B_{\delta}$$

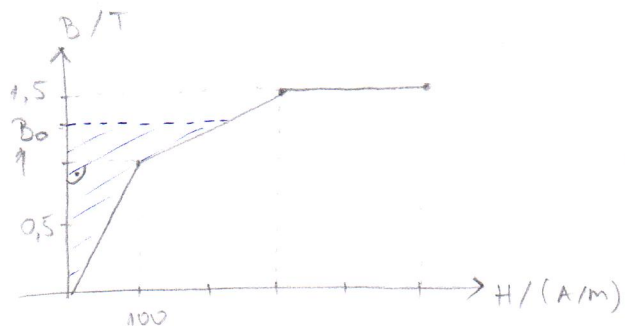
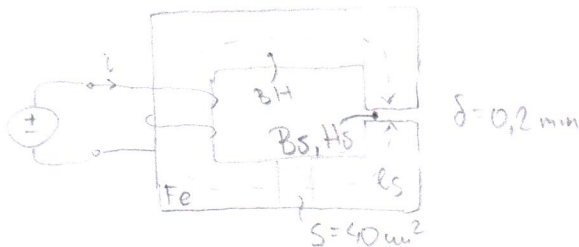
$$y = y(x)$$



$$I_1 = \int_0^{x_0} y(x) dx$$

$$x = x(y) \quad I_2 = \int_0^{y_0} x(y) dy$$

⑤



$$l_s = 0.5 \text{ m}$$

$$B_0 = 1.25 \text{ T}$$

$$W_{\text{mag}} = ? , B_0 = 1.25 \text{ T}$$

$$W_{\text{mag}} = W_{\text{mag, Fe}} + W_{m, \delta} = W_{\text{mag, Fe}} \cdot S \cdot l_s + W_{m, \delta} \cdot S \cdot \delta$$

$$W_{m, \delta} = \frac{B_0^2}{2\mu_0} = 622 \text{ uJ/m}^2 ; W_{\text{mag, Fe}} = \int_0^{B_0} H \cdot dB = \frac{1}{2} \cdot 100 \frac{\text{A}}{\text{m}} \cdot 1 \text{ T} = 50 \text{ J/m}^2$$

$$W_{\text{mag, Fe}} = 87.5 \text{ J/m}^2$$

$$\Rightarrow W_{\text{mag}} = 0.672 \text{ J}$$

Trapez
ploščim

⑥

$$l_s = 25 \text{ cm}$$

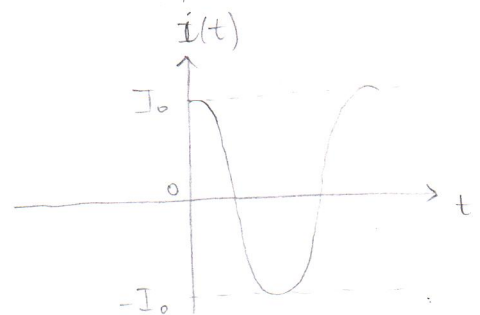
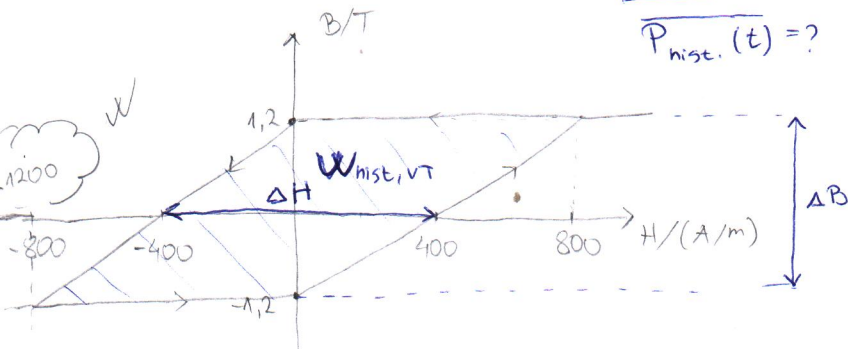
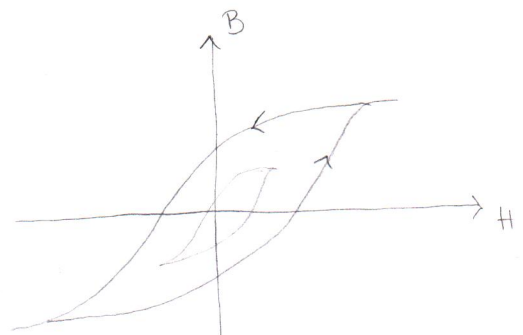
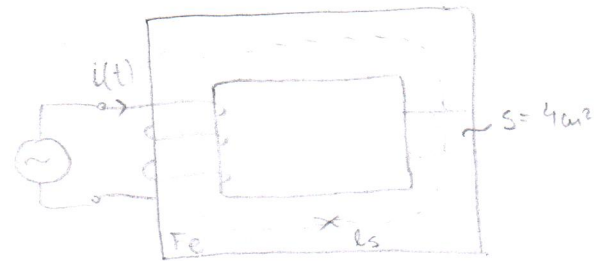
$$N = 100$$

$$i(t) = I_0 \cdot \cos(\omega t)$$

$$I_0 = 3 \text{ A}$$

$$\omega = 2\pi f, f = 50 \text{ Hz}$$

$$\overline{P}_{\text{hist.}}(t) = ?$$



$$\oint_L \vec{H} \cdot d\vec{l} = N \cdot i \equiv H \cdot l_s \Rightarrow H(t) = \frac{N}{l_s} i(t)$$

Preverimo ali je
podana hist.
zanua ou

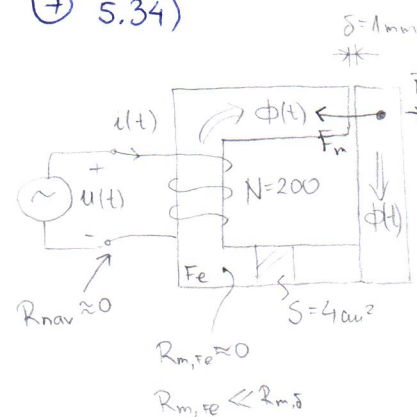
$$l_{\text{max}} = I_0 = 3 \text{ A}, H_{\text{max}} = \frac{N}{l_s} \cdot i_{\text{max}} = 1200 \text{ A/m} > 800 \text{ A/m} \quad \text{u je ou}$$

$$\overline{P}_{\text{hist.}}(t) = \frac{W_{\text{hist, VT}}}{T} = f \cdot W_{\text{hist, VT}} = f \cdot S \cdot l_s \cdot W_{\text{hist, VT}} = 9,6 \text{ W}$$

$$T = \frac{1}{f}; W_{\text{hist}} = \int_0^T H \cdot dB = \oint_L H \cdot dB = \Delta H \cdot \Delta B = 800 \frac{\text{A}}{\text{m}} \cdot 2,4 \text{ T} = 1920 \frac{\text{J}}{\text{m}^2}$$

po eni periodi
je polje enako
kot na zač.

⑦ 5.34)



$$u(t) = U_0 \cdot \cos(\omega t)$$

$$F_m = F_v, \vec{F}_m + \vec{F}_v = \vec{0}$$

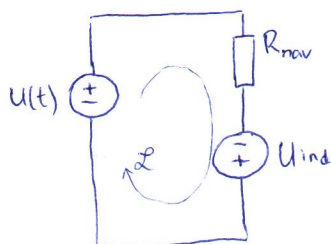
$$U_0 = 10 \text{ V}, \omega = 400 \text{ Hz}$$

$$F_v(t) = F_m(t) = ?$$

FORMELE: (tlan na)
reži

$$f_m = \frac{F_m}{S} = \frac{B^2}{2\mu_0}$$

$$\Rightarrow F_m = \frac{B^2}{2\mu_0} \cdot S$$



$$\mathcal{L}: -u(t) + \underbrace{U_{R_{\text{nov}}}}_{\approx 0} - U_{\text{ind}} = 0 \Rightarrow U_{\text{ind}} = -u(t)$$

$$U_{\text{ind.}} = -u(t) = -\frac{d\psi(t)}{dt} = -\frac{d(N\phi(t))}{dt} = -N \frac{d\phi(t)}{dt}$$

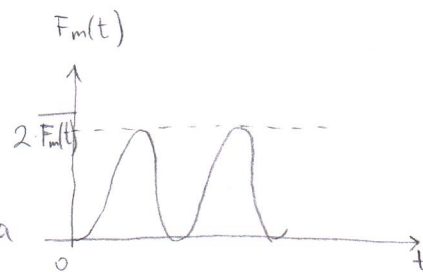
$$\frac{d\phi}{dt} = \frac{u(t)}{N} = \frac{U_0}{N} \cdot \cos(\omega t) \Rightarrow \phi(t) = \frac{U_0}{N} \cdot \frac{\sin(\omega t)}{\omega} = B(t) \cdot S$$

$$B(t) = \frac{U_0}{N \cdot w \cdot S} \sin(\omega t)$$

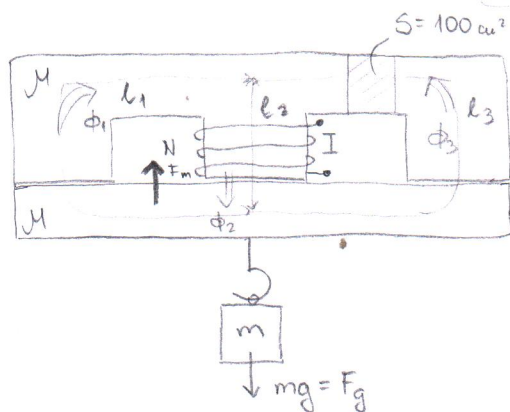
$$F_m(t) = \frac{1}{2\mu_0} \cdot S \cdot \left(\frac{U_0}{N \cdot w \cdot S} \sin(\omega t) \right)^2 = \frac{U_0^2}{2\mu_0 \cdot N^2 \cdot w^2 \cdot S} \sin^2(\omega t)$$

$$\overline{F_m(t)} = \frac{U_0^2}{4\mu_0 \cdot N^2 \cdot w^2 \cdot S} = 7,8 \text{ N}$$

Sila je vedno privlačna
zato ker ih magnetimo
v isto smer (notro in iz)



8.



$$N = 1000$$

$$\mu = 10^{-3} \frac{\text{Vs}}{\text{Am}}$$

$$l_1 = l_3 = 1 \text{ m}$$

$$l_2 = 0,25 \text{ m}$$

$$m = 1500 \mu\text{g}$$

$$I = ?$$

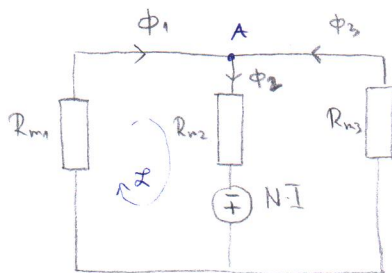
$$g = 9,81 \text{ m/s}^2$$

$$F_m = F_g = mg$$

$$F_m = F_{m1} + F_{m2} + F_{m3} = \frac{B_1^2}{2\mu_0} \cdot S + \frac{B_2^2}{2\mu_0} S + \frac{B_3^2}{2\mu_0} S = \underbrace{(B_1^2 + B_2^2 + B_3^2)}_{\phi_i^2/S^2} \cdot \frac{S}{2\mu_0}$$

$$B_i = \frac{\phi_i}{S}, \quad F_m = (\phi_1^2 + \phi_2^2 + \phi_3^2) \frac{1}{2\mu_0 S}$$

$$i = 1, 2, 3$$



$$R_{m1} = \frac{l_1}{\mu \cdot S} = R_{m3} = \frac{l_3}{\mu \cdot S}$$

$$R_{m2} = \frac{l_2}{\mu \cdot S}$$

$$A: \quad \phi_2 = \phi_1 + \phi_2 = 2\phi_1$$

$$\mathcal{L}: \quad R_{m1} \cdot \phi_1 + R_{m2} \cdot \frac{\phi_2}{2\phi_1} - NI = 0$$

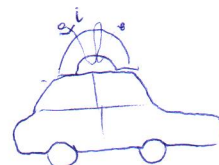
$$\phi_1 (R_{m1} + 2R_{m2}) = NI$$

$$\phi_1 = \frac{N \cdot I}{R_{m1} + 2R_{m2}} = \frac{N \cdot I}{\frac{l_1}{\mu \cdot S} + \frac{2l_2}{\mu \cdot S}} = \frac{N \cdot I \cdot \mu \cdot S}{l_1 + 2l_2}$$

$$\Rightarrow F_m = (\phi_1^2 + (2\phi_1)^2 + \phi_1^2) \frac{1}{2\mu_0 S} = \frac{3\phi_1^2}{\mu_0 S} = \frac{3}{\mu_0 S} \left(\frac{N I \mu S}{l_1 + 2l_2} \right)^2 = \frac{3N^2 I^2 \mu^2 S}{(l_1 + 2l_2)^2 \mu_0} = mg$$

$$\Rightarrow I^2 = \frac{m \cdot g (l_1 + 2l_2)^2 \mu_0}{3N^2 \cdot \mu^2 S} \Rightarrow$$

$$I = \frac{l_1 + 2l_2}{\mu N} \sqrt{\frac{m \cdot g \cdot \mu_0}{3 \cdot S}} = 1,19 \text{ A}$$

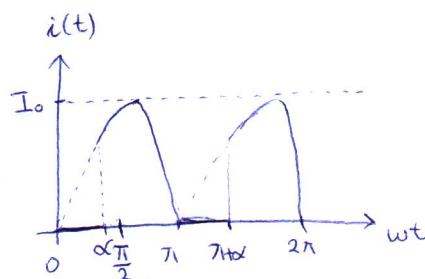


9) VEZJA ČASOVNO SPREMENLJIVIH TOKOV

6.1)

$$i(t) = \begin{cases} 0, & 0 \leq \omega t < \alpha, & 0 \leq t \leq \frac{\alpha}{\omega} \\ I_0 |\sin(\omega t)|, & \alpha \leq \omega t < \pi, & \frac{\alpha}{\omega} \leq t < \frac{\pi}{\omega} \end{cases}$$

$$i(t+T) = i(t), \quad \omega \cdot T = \pi \Rightarrow T = \frac{\pi}{\omega}$$



$$I_{sr} = ? ; I_{ef} = ?$$

$$I_{sr} = \frac{1}{T} \int_0^T i(t) \cdot dt = \frac{1}{\pi/\omega} \int_0^{\pi/\omega} i(t) \cdot dt = \frac{\omega}{\pi} \left[\underbrace{\int_0^{\alpha/\omega} 0 \cdot dt}_0 + \underbrace{\int_{\alpha/\omega}^{\pi/\omega} I_0 \sin(\omega t) \cdot dt}_{\geq 0} \right]$$

$$I_{sr} = \frac{\omega}{\pi} \cdot I_0 \cdot \left[-\frac{\cos(\omega t)}{\omega} \right]_{\alpha/\omega}^{\pi/\omega} = I_0 \cdot \frac{1}{\pi} \left[\cos\left(\omega \cdot \frac{\alpha}{\omega}\right) - \underbrace{\cos\left(\omega \cdot \frac{\pi}{\omega}\right)}_{-1} \right]$$

$$I_{sr} = I_0 \cdot \frac{1 + \cos \alpha}{\pi}$$

npr: $\boxed{\alpha=0}$ $i(t) = I_0 |\sin(\omega t)|$, $I_{sr} = I_0 \cdot \frac{1 + \cos 0}{\pi} = I_0 \cdot \frac{2}{\pi}$

$$I_{ef} = \sqrt{\frac{1}{T} \int_0^T i^2(t) \cdot dt} \Rightarrow I_{ef}^2 = \frac{1}{\pi/\omega} \int_0^{\pi/\omega} i^2(t) \cdot dt = \frac{\omega}{\pi} \int_{\alpha/\omega}^{\pi/\omega} I_0^2 \sin^2(\omega t) \cdot dt =$$

R.M.S

$$= \frac{\omega}{\pi} I_0^2 \frac{1}{2} \left[t - \frac{\sin(2\omega t)}{2\omega} \right]_{\alpha/\omega}^{\pi/\omega}$$

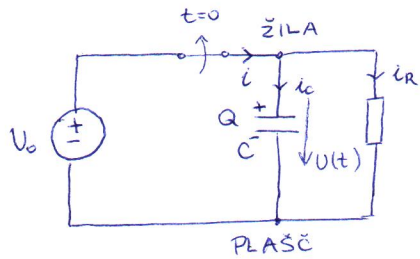
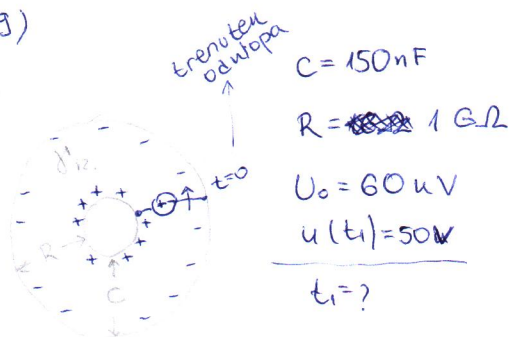
$$I_{ef}^2 = \frac{I_0^2}{2} \cdot \frac{\omega}{\pi} \left[\frac{\pi}{\omega} - \frac{\alpha}{\omega} - \frac{\sin(2\omega \frac{\pi}{\omega}) - \sin(2\omega \frac{\alpha}{\omega})}{2\omega} \right] = \frac{I_0^2}{2\pi} \cdot [\pi - \alpha + \sin(2\alpha)]$$

$$I_{ef} = I_0 \sqrt{\frac{\pi - \alpha + \sin(2\alpha)}{2\pi}}$$

npr: $\boxed{\alpha=0}$: $i(t) = I_0 |\sin(\omega t)|$, $I_{ef} = I_0 \frac{1}{\sqrt{2}}$, $I_{sr} = 0$

↑
če ne bi bil odrezan tok na začetni ~~amplitude~~ periode

① 5.9)



1. K. Z: $i = i_C + i_R$

$t > 0$: $i = 0$

$i_C + i_R = 0 \Rightarrow C \frac{du}{dt} + \frac{1}{R} u(t) = 0 \quad (*)$

$i_R = \frac{u}{R}$, $i_C = \frac{dQ}{dt} = C \frac{du}{dt}$

$(*) \frac{du}{dt} + \frac{1}{R \cdot C} u(t) = 0$

$R \cdot C = \tau = 10^9 \Omega \cdot 150 \cdot 10^{-9} \text{ F} = 150 \frac{\Omega}{\frac{V}{A}} \cdot \frac{As}{V} = 150 \text{ s}$; τ - časovna konstanta

$\Rightarrow \frac{du}{dt} + \frac{1}{\tau} u(t) = 0 \rightarrow u(t) = ?$

$\frac{du}{dt} + \frac{1}{\tau} u(t) = 0 \Rightarrow \frac{du}{dt} = -\frac{1}{\tau} u(t) \quad | \cdot dt / : u(t)$

$\frac{du}{u(t)} = -\frac{dt}{\tau} \quad | \int_0^t$

$\int_{t_0}^t \frac{du}{u(t)} = -\frac{1}{\tau} \int_0^t dt$

$\ln u(t) \Big|_{t_0}^t = -\frac{1}{\tau} t$

$\ln \frac{u(t)}{u(0)} = -\frac{t}{\tau} \quad | e^{\dots}$

$e^{\ln \frac{u(t)}{u(0)}} = e^{-t/\tau}$

$\frac{u(t)}{u(0)} = e^{-t/\tau} \quad | \cdot u(0)$

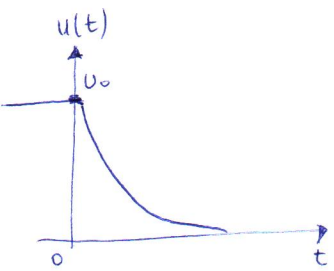
$u(t) = u(0) \cdot e^{-t/\tau}$
 $u(t) = U_0 \cdot e^{-t/\tau}$

rabimo začetni pogoj $u(0) = ?$

$u(t < 0) = U_0$

$U = \frac{Q}{C}$

$u(0^-) = u(0^+) = U_0$



t	$e^{t/\tau}$	$\frac{u(t)}{U_0}$
0	1	100%
$t = \tau$	$e^1 = 2,7$	$1/e \approx 36,8\%$
$t = 2\tau$	$e^2 = 7,4$	$1/e^2 \approx 13,5\%$
$t = 3\tau$	$e^3 = 20,1$	$1/e^3 \approx 5\%$
$t = 4\tau$	54,6	1,8%
$t = 5\tau$	148	0,67%

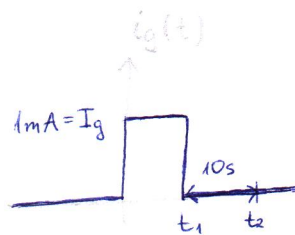
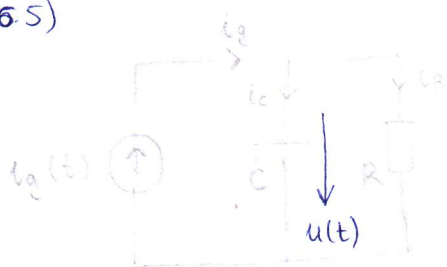
* Rešitev linearne dif. enačbe prvega reda (homogena / ~~nehomogena~~) je eksponentna f-ja

$$u(t_1) = U_0 e^{-t_1/\tau} \Rightarrow e^{t_1/\tau} = \frac{U_0}{u(t_1)} \quad / \ln \dots$$

$t_1 = ?$

$$t_1/\tau = \ln \frac{U_0}{u(t_1)} \Rightarrow t_1 = \tau \ln \frac{U_0}{u(t_1)} = 1064s = \underline{\underline{17,7 \text{ min}}}$$

2) 6.5)



$$t_1 = 10 \text{ ms}$$

$$C = 1 \mu\text{F}$$

$$R = 10 \text{ M}\Omega$$

$$t_2 = t_1 + 10 \text{ s}$$

$$u(t_2) = ?$$

$0 < t < t_1$:

$$i_g = I_g = i_c + i_R = C \frac{du}{dt} + \frac{u(t)}{R} \quad / : C \Rightarrow \frac{du}{dt} + \frac{1}{R \cdot C} u(t) = \frac{I_g}{C}$$

$$R \cdot C = \tau = 10 \cdot 10^6 \Omega \cdot 10^{-6} \text{ F} = 10 \text{ s}$$

$$1 \cdot \dot{u} + \frac{1}{\tau} u(t) = \frac{I_g}{C}$$

substitucija:

$$v = u - R \cdot I_g \Rightarrow u = v + R \cdot I_g$$

$$\frac{d}{dt} \left(v + \frac{R \cdot I_g}{\text{const.}} \right) + \frac{1}{\tau} (v + R \cdot I_g) = \frac{I_g}{C}$$

$$\frac{dv}{dt} + \frac{1}{\tau} v + \frac{R}{\tau} I_g = \frac{I_g}{C} \Rightarrow \frac{dv}{dt} + \frac{1}{\tau} v = 0 \Rightarrow v(t) = \frac{V_0}{?} e^{-t/\tau} = u(t) - R \cdot I_g \Rightarrow$$

$$\frac{R}{\tau} I_g = \frac{R}{R \cdot C} I_g = \frac{I_g}{C}$$

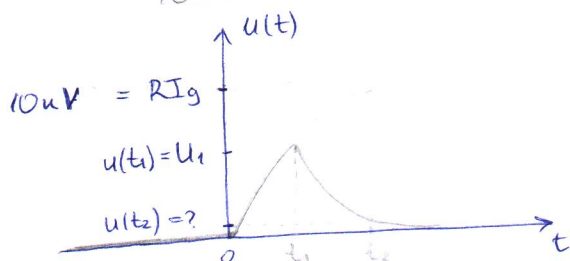
$$\Rightarrow u(t) = V_0 e^{-t/\tau} + R \cdot I_g$$

začetni pogoji

$$u(0^-) = u(0^+) = 0$$

$$u(0) = 0 = V_0 \underbrace{e^{-0/\tau}}_1 + R \cdot I_g \Rightarrow \underline{\underline{V_0 = -R \cdot I_g}}$$

$$\Rightarrow \underline{\underline{u(t) = -R \cdot I_g \cdot e^{-t/\tau} + R \cdot I_g = R \cdot I_g (1 - e^{-t/\tau})}}$$



$$u_1 = u(t_1) = R \cdot I_g (1 - e^{-t_1/\tau}) = 10 \cdot 10^6 \cdot 10^{-3} (1 - e^{-\frac{10 \cdot 10^{-3}}{10}}) \text{ V}$$

$$u_1 = 10 \text{ V}$$

$t > t_1$

$$i_g = 0$$

$$i_c + i_R = 0 = C \frac{dv}{dt} + \frac{1}{R} u(t) = 0 \quad / : C$$

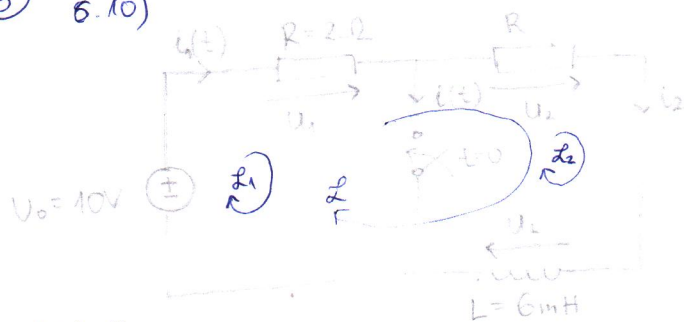
$$\frac{dv}{dt} + \frac{1}{\tau} u(t) = 0 \Rightarrow u(t) = \frac{A}{?} e^{-t/\tau}$$

$$u(t_1) = ? \quad ; \quad u(t_1^-) = u_1 = u(t_1^+) = A \cdot e^{-t_1/\tau} \Rightarrow A = u_1 \cdot e^{t_1/\tau}$$

$$u(t) = u_1 e^{t_1/\tau} \cdot e^{-t/\tau} = u_1 e^{\frac{t_1}{\tau} - \frac{t}{\tau}} = u_1 e^{-\frac{t-t_1}{\tau}}$$

$$u(t_2) = u_1 \cdot e^{-\frac{t_2 - t_1}{\tau}} = 10V \cdot e^{-\frac{10s}{10s}} = \underline{\underline{3,68V}}$$

③ 6.10)



$$i(t_1) = ?$$

$$t_1 = 1ms$$

$$t < 0:$$

$$i = 0$$

$$i_1 = i_2$$

$$\frac{d}{dt} = 0$$

$$u_1 = u_2 = R \cdot i = R \cdot i_2$$

$$u_L = L \frac{di}{dt} = 0$$

$$\mathcal{L}: U_0 = u_1 + u_2 + u_L = 2R \cdot i$$

$$i_1(t < 0) = i_2(t < 0) = \frac{U_0}{2R} = 2,5A$$

$$t > 0:$$

$$1KZ: i_1 = i + i_2 \Rightarrow i = i_1 - i_2$$

$$\mathcal{L}_1: U_0 = u_1 = R \cdot i_1 \Rightarrow i_1 = \frac{U_0}{R} = 5A$$

$$\mathcal{L}_2: u_2 + u_L = 0 = R i_2 + L \frac{di_2}{dt} \quad | :L$$

$$\frac{di_2}{dt} + \frac{R}{L} i_2(t) = 0 \Rightarrow \frac{di_2}{dt} + \frac{1}{\tau} i_2(t) = 0 \Rightarrow i_2(t) = A \cdot e^{-t/\tau}$$

$$\frac{L}{R} = \frac{6mH}{2\Omega} = 3 \cdot 10^{-3} \frac{Vs/A}{V/A} = \underline{\underline{3ms = \tau}}$$

začetni
pogoji za A
(za drugi del toka)

$$i_2(0) = ?$$

$$i_2(0^-) = \frac{U_0}{2R} = \underline{\underline{2,5A = i_2(0^+)}}$$

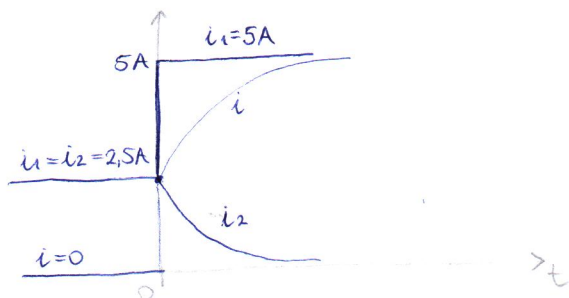
$$i_2(0) = A e^{-0/\tau} = \boxed{A = \frac{U_0}{2R}}$$

$$i_2(t) = \frac{U_0}{2R} e^{-t/\tau}$$

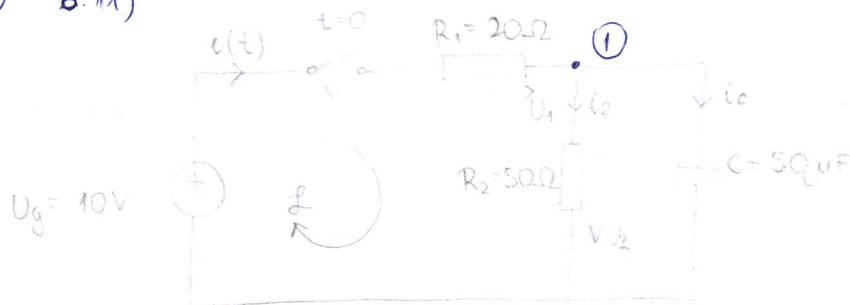
$$u_L = L \frac{di_2}{dt}$$

$$i_C = C \frac{du_C}{dt}$$

$$i = i_1 - i_2 = \frac{U_0}{R} - \frac{U_0}{2R} e^{-t/\tau} = 5A - 2,5A e^{-t/\tau}$$



4) 5.11)



$i(t) = ?$

$t > 0$

$\mathcal{L}: U_g = U_1 + U_2 \Rightarrow U_1 = U_g - U_2$

①: $i = i_2 + i_C \Rightarrow \frac{U_g - U_2}{R_1} = \frac{U_2}{R_2} + C \frac{dU_2}{dt} \quad | : C$

$i = \frac{U_1}{R_1} = \frac{U_g - U_2}{R_1}$

$i_2 = \frac{U_2}{R_2} ; i_C = C \frac{dU_2}{dt}$

$\frac{dU_2}{dt} + \frac{1}{\tau} U_2(t) = \frac{U_g}{C \cdot R_1} \Rightarrow U_2(t) = A e^{-t/\tau} + B$

$A \cdot \left(-\frac{1}{\tau}\right) e^{-t/\tau} + 0 + \frac{1}{\tau} [A e^{-t/\tau} + B] = \frac{U_g}{C \cdot R_1}$

$\frac{B}{\tau} = \frac{U_g}{C \cdot R_1} \quad | \cdot \tau$

$B = \frac{U_g}{C \cdot R_1} \cdot \tau = \frac{U_g}{R_1} \cdot R_{12} = \underline{\underline{\frac{U_g}{R_1} R_{12}}}$

$U_2(0) = ?$

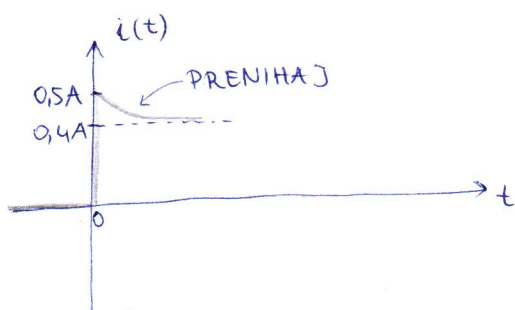
$U_2(0^-) = 0 = U_2(0^+)$

$U_2(0) = A \cdot \underbrace{e^{-0/\tau}}_1 + B \Rightarrow A = -B$

$U_2(t) = B(1 - e^{-t/\tau}) = \underbrace{\frac{U_g}{R_1} \cdot R_{12}}_{2V} (1 - e^{-t/\tau})$

$i = \frac{U_1}{R_1} = \frac{U_g - U_2}{R_1} = \frac{10V - 2V(1 - e^{-t/\tau})}{20\Omega} = \frac{8V + 2V \cdot e^{-t/\tau}}{20\Omega}$

$i(t) = 0,4A + 0,1A e^{-t/\tau}$



PREHODNI
POJAV

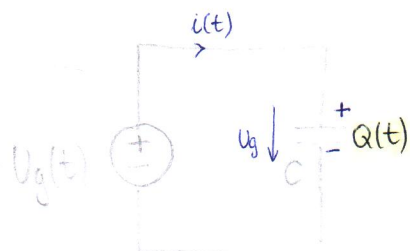
$\frac{dU_2}{dt} + \frac{1}{C} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) U_2(t) = \frac{U_g}{C R_1}$

$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2} = 4\Omega$

$\frac{dU_2}{dt} + \frac{1}{C \cdot R_{12}} U_2(t) = \frac{U_g}{C \cdot R_1}$

$C \cdot R_{12} = \tau = 50\mu F \cdot 4\Omega = 200\mu s$

5) PREHODNI POJAVI



$$U_g(t) = 10V \cdot \sin(\omega t) = 10V \cdot \cos(\omega t - \pi/2)$$

$$\omega = 1000 \frac{\text{rad}}{\text{s}}$$

$$C = 10 \mu\text{F}$$

$$i(t) = ? ; Q_{\text{max}} = ? ; W_e(t) = ? ; P_g(t) = ?$$

$$i = C \frac{dU_g}{dt} = \underbrace{C \cdot 10V \cdot \omega}_{100 \text{ mA}} \cos(\omega t) = \underline{100 \text{ mA} \cdot \cos(\omega t)}$$

$$Q(t) = C \cdot U_g(t) = \underbrace{C \cdot 10V}_{100 \mu\text{C}} \cdot \sin(\omega t) = \underbrace{100 \mu\text{C}}_{Q_{\text{max}}} \cdot \sin(\omega t) \Rightarrow \underline{Q_{\text{max}} = 100 \mu\text{C}}$$

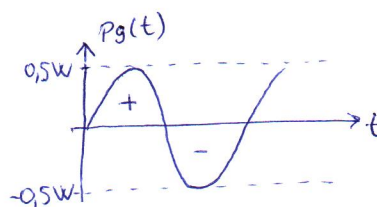
$$W_e(t) = \frac{1}{2} C U_g^2(t) = \underbrace{\frac{1}{2} C (10V)^2}_{500 \mu\text{J}} \sin^2(\omega t)$$

$$\overline{W_e(t)} = 500 \mu\text{J} \cdot \frac{1}{2} = \underline{250 \mu\text{J}}$$

$$P_g(t) = U_g(t) i(t) = 10V \cdot \sin(\omega t) \cdot 100 \text{ mA} \cdot \cos(\omega t) = 1W \cdot \frac{\sin(2\omega t)}{2}$$

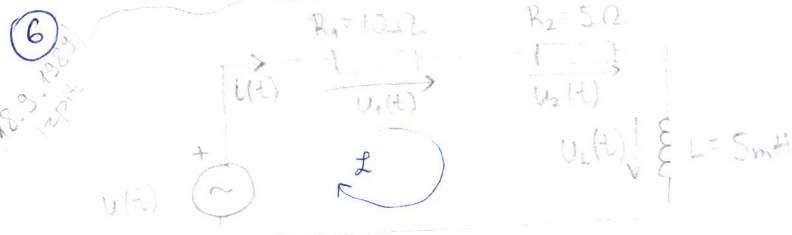
$$2) \underline{P_g(t) = 0,5W \cdot \sin(2\omega t)}$$

* moč niha
z dvojno
frekvenco



- 1) NAVIDEZNA MOČ
- 2) DELOVNA MOČ

$$1) \underline{\overline{P_g(t)} = 0,5W \cdot \underbrace{\overline{\sin(2\omega t)}}_0 = 0}$$



$$u(t) = 10\sqrt{2} V \cdot \sin(\omega t)$$

$$U_{\text{ef.}} = \frac{10\sqrt{2} V}{\sqrt{2}} = 10V$$

$$\omega = 1000 \frac{\text{rad}}{\text{s}}$$

$$u_L(t_1) = 0 ; u_1(t_1) = ?$$

$$u_L(t) = U_L \cdot \cos(\omega t + \varphi_{uL}) \rightarrow \underline{U_L = U_L e^{j\varphi_{uL}}}$$

$$u_1(t) = U_1 \cdot \cos(\omega t + \varphi_1) \rightarrow \underline{U_1 = U_1 e^{j\varphi_1}}$$

$$j = \sqrt{-1}$$

$$u(t) = 10\sqrt{2} V \cdot \sin(\omega t) = 10\sqrt{2} V \cdot \cos(\omega t - \frac{\pi}{2}) \rightarrow \underline{U = 10\sqrt{2} V \cdot \underbrace{e^{j(-\pi/2)}}_{\cos(-\frac{\pi}{2}) + j\sin(-\frac{\pi}{2})} = 10\sqrt{2} V \cdot (-j)}$$

$$\mathcal{L}: \underline{U} = \underline{U_1} + \underline{U_2} + \underline{U_L} = R_1 \underline{I} + R_2 \underline{I} + j\omega L \cdot \underline{I} = (R_1 + R_2 + j\omega L) \underline{I}$$

$$\underline{U} = \underline{Z} \underline{I} ; \underline{Z} \triangleq \frac{\underline{U}}{\underline{I}}$$

$$\Rightarrow \underline{I} = \frac{\underline{U}}{R_1 + R_2 + j\omega L} = \frac{-j 10\sqrt{2} V}{10\Omega + 5\Omega + j5\Omega} \cdot \frac{1:5}{1:5} = \frac{-j 2\sqrt{2}}{3+j} A \cdot \frac{3-j}{3-j} = \frac{-j 2\sqrt{2} (3-j)}{(3+j)(3-j)} A$$

$$\underline{I} = \frac{2\sqrt{2} (-j3 - 1)}{10} A = \frac{\sqrt{2} (-1 - j3)}{5} A = \underline{\underline{\frac{\sqrt{2}}{5} (-1 - j3) A}}$$

$$\underline{U}_L = j\omega L \cdot \underline{I} = j5\Omega \cdot \frac{\sqrt{2}}{5}(-1-j3)A = \sqrt{2}(-j+3)V$$

→ Gremo nazaj
v originalni
časovni prostor

$$\underline{U}_1 = R_1 \underline{I} = 10\Omega \cdot \frac{\sqrt{2}}{5}(-1-j3)A = 2\sqrt{2}(-1-j3)V$$

$$\underline{U}_L = \sqrt{2} \cdot \sqrt{3^2 + (-1)^2} \cdot e^{j \arctan \frac{-1}{3}} V = \frac{\sqrt{2} \cdot \sqrt{10}}{2\sqrt{5}} e^{-j18,4^\circ} V \rightarrow \underline{U}_L(t) = 2\sqrt{5} V \cos(\omega t - 18,4^\circ)$$

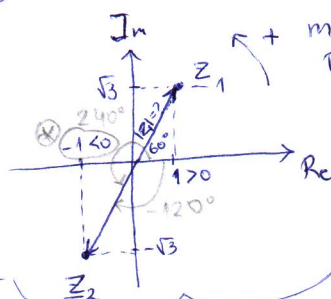
$$\arctan \frac{-1}{3} = -18,4^\circ$$

$$\underline{U}_1 = 2\sqrt{2} \sqrt{(-1)^2 + (-3)^2} \cdot e^{j \arctan \frac{-3}{-1} \pm 180^\circ} V = \frac{2\sqrt{2} \cdot \sqrt{10}}{4\sqrt{5}} e^{-j108,4^\circ} V \rightarrow \underline{U}_1(t) = 4\sqrt{5} V \cos(\omega t - 108,4^\circ)$$

$$\arctan \frac{-3}{-1} =$$

$$\underline{Z}_1 = 1 + j\sqrt{3} = 2 \cdot e^{j \arctan \frac{\sqrt{3}}{1}} = 2 \cdot e^{j60^\circ}$$

$$\underline{Z}_2 = -1 - j\sqrt{3} = 2 \cdot e^{j \arctan \frac{-\sqrt{3}}{-1} \pm 180^\circ} = 2e^{j240^\circ} = 2e^{-j120^\circ}$$

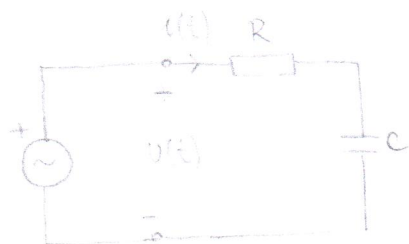


$$U_L(t_1) = 0 \Rightarrow \omega t_1 - 18,4^\circ = \pm 90^\circ \Rightarrow \omega t_1 = \begin{matrix} 108,4^\circ \\ -71,6^\circ \end{matrix}$$

$$u_1(t) = 4\sqrt{5} V \begin{cases} \cos(108,4^\circ - 108,4^\circ) \\ \cos(\underbrace{-71,6^\circ - 108,4^\circ}_{\omega t_1 - 180^\circ}) \end{cases} = 4\sqrt{5} V \begin{cases} +1 \\ -1 \end{cases} = \pm 4\sqrt{5} V = \pm 8,94 V$$

$$f_{u_L} - f_1 = -18,4^\circ - (-108,4^\circ) = 90^\circ$$

7



$$u(t) = 100V \cdot \cos(\omega t - \pi/6) \rightarrow \underline{U} = 100V e^{-j\pi/6}$$

$$\omega = 1000 \frac{\text{rad}}{\text{s}}$$

$$i(t) = 10A \cdot \cos(\omega t + \pi/6) \rightarrow \underline{I} = 10A \cdot e^{j\pi/6}$$

$$R = ? ; C = ?$$

$$\frac{\underline{U}}{\underline{I}} = \underline{Z} = \underline{Z}_R + \underline{Z}_C = R + \frac{1}{j\omega C} = \frac{100V e^{-j\pi/6}}{10A \cdot e^{j\pi/6}} = 10\Omega \cdot e^{-j\pi/3}$$

$$\underline{Z} = R - j\frac{1}{\omega C} = 10\Omega \cdot e^{-j\pi/3} = 10\Omega \left(\underbrace{\cos(-\pi/3)}_{1/2} + j \underbrace{\sin(-\pi/3)}_{-\sqrt{3}/2} \right) = 5\Omega - j\sqrt{3} \cdot 5\Omega$$

Re:

$$R = 5\Omega$$

Im:

$$-\frac{1}{\omega C} = -5\sqrt{3}\Omega \Rightarrow C = \frac{1}{\omega \cdot 5\sqrt{3}\Omega} = 115 \mu F$$

$$f_z = -\frac{\pi}{3} = f_u - f_i = -\frac{\pi}{6} - \frac{\pi}{6} = -\frac{\pi}{3}$$

$$\underline{S} = P + jQ = \frac{1}{2} \underline{U} \cdot \underline{I}^* \quad (\text{kompleksor} \\ \text{navidezne moči})$$

$$|\underline{S}| = S ; \text{Re}[\underline{S}] = P ; \text{Im}[\underline{S}] = Q$$

$$\underline{S} = \frac{1}{2} 100V \cdot e^{-j\pi/6} \cdot 10A \cdot e^{-j\pi/6} = 500VA \cdot e^{-j\pi/3} = 500VA \left(\underbrace{\cos(-\frac{\pi}{3})}_{1/2} + j \underbrace{\sin(-\frac{\pi}{3})}_{-\sqrt{3}/2} \right) =$$

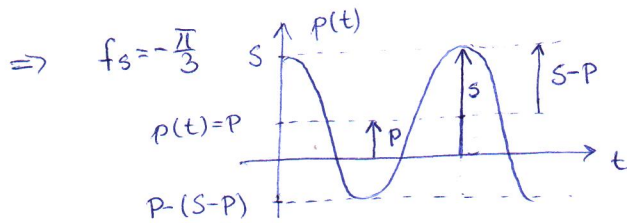
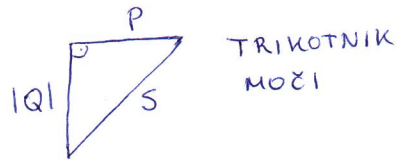
$$|\underline{S}| = S = 500VA$$

$$\operatorname{Re}[\underline{S}] = P = 250W \quad (V \cdot A = W)$$

$$\operatorname{Im}[\underline{S}] = Q = -\sqrt{3} \cdot 250 \text{ VAR}$$

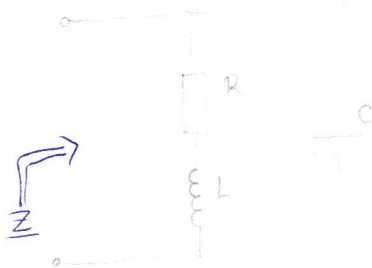
$$\underline{S} = (250 - j\sqrt{3} \cdot 250) VA$$

$$|\underline{S}| = S = \sqrt{P^2 + Q^2}$$



$$Q = \pm \sqrt{S^2 - P^2}$$

⑧ 6.21)



$$\omega L = 5\Omega; \quad \underline{Z}_L = j\omega L = j5\Omega$$

$$\frac{1}{\omega C} = 10\Omega, \quad \underline{Z}_C = \frac{1}{j\omega C} = -j10\Omega$$

$$\underline{Z} \in \mathbb{R} \rightarrow \operatorname{Im}[\underline{Z}] = 0$$

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{\operatorname{Re}[\underline{Z}]} \in \mathbb{R} \Rightarrow \operatorname{Im}[\underline{Y}] = 0$$

$$(G = \frac{1}{R})$$

$$\underline{Z} = \operatorname{Re}[\underline{Z}] + j \operatorname{Im}[\underline{Z}]$$

$$\underline{Y} = \underline{Y}_{RL} + \underline{Y}_C$$

$$\underline{Y}_C = \frac{1}{\underline{Z}_C} = j\omega C = \frac{j}{10\Omega}$$

$$\underline{Y}_{RL} = \frac{1}{\underline{Z}_{RL}} = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + (\omega L)^2}$$

$$\operatorname{Im}[\underline{Y}_{RL} + \underline{Y}_C] = 0 = \underbrace{\operatorname{Im}[\underline{Y}_{RL}]}_{-\frac{\omega L}{R^2 + (\omega L)^2}} + \underbrace{\operatorname{Im}[\underline{Y}_C]}_{\frac{1}{10\Omega}}$$

$$\Rightarrow -\frac{\omega L}{R^2 + (\omega L)^2} + \frac{1}{10\Omega} = 0 = -\frac{5\Omega}{R^2 + (5\Omega)^2} + \frac{1}{10\Omega} = 0$$

$$\Rightarrow R^2 + (5\Omega)^2 = 50\Omega^2$$

$$R^2 = 25\Omega^2 / \sqrt{\quad}$$

$$\boxed{R = \pm 5\Omega}$$

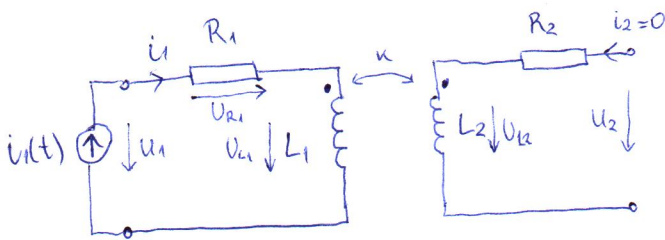
uporabno (ohmsko)
režje

pri tej frekvenci!

$$\underline{Y} = \operatorname{Re}[\underline{Y}] = \frac{R}{R^2 + (\omega L)^2} = 0,1 \frac{\Omega^{-1}}{S}$$

$$\underline{Z} = \frac{1}{\underline{Y}} = 10\Omega$$

Inducirano električno polje

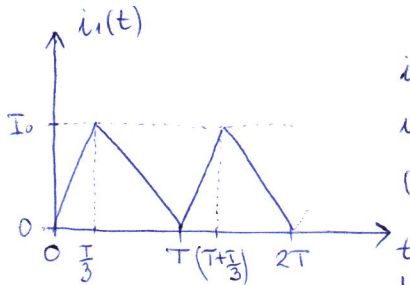


$$M = k \sqrt{L_1 L_2}$$

$$L_{12} = \oplus M$$

$$k = 0,95$$

SE TOLJAVI
PODPIRATA

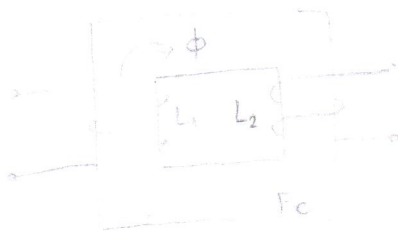


$$i_1(t+T) = i_1(t) \quad U_{L1}$$

$$u_1(t) = L_1 \frac{di_1}{dt} \oplus M \frac{di_2}{dt} + R_1 i_1 = R_1 L_1 \frac{di_1}{dt} + R_1 i_1 \quad (i_2=0)$$

(LIST 5 FORMULAMI)

$$u_2(t) = U_{R2} + U_{L2} = R_2 i_2 + L_2 \frac{di_2}{dt} \oplus M \frac{di_1}{dt} =$$



$$L_1 \neq \text{const}$$

$$L_2 \neq \text{const}$$

$$L_{12} \neq \text{const}$$

$$i_1(T/3) = I_0$$

$$i_1(T) = 0$$

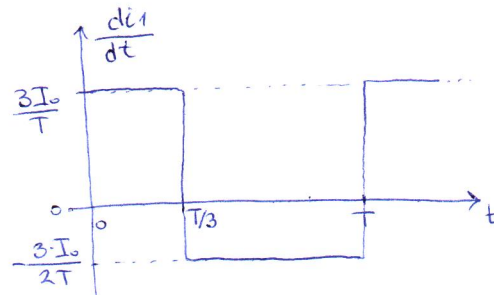
$$i_1(t) = \frac{I_0 - 0}{T - T/3} \left(t - \frac{T}{3} \right) + I_0$$

$$i_1(t) = I_0 \left[1 - \frac{t - T/3}{2T/3} \right]$$

$$i_1(t) = \begin{cases} I_0 \cdot \frac{t}{T/3} & , 0 < t < \frac{T}{3} \\ I_0 \left[1 - \frac{t - T/3}{2T/3} \right] & , \frac{T}{3} < t < \frac{2T}{3} \end{cases}$$

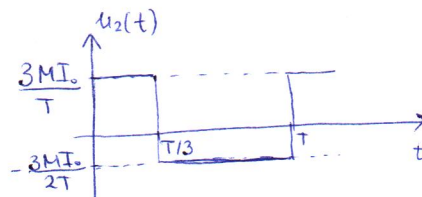
ČASOVNI ODVOD TOLA:

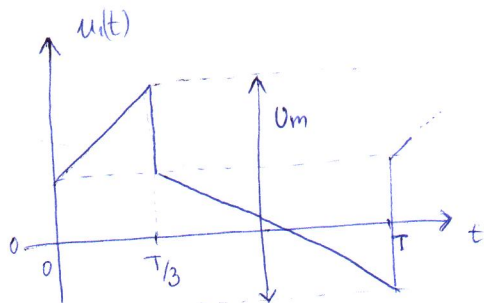
$$\frac{di_1}{dt} = \begin{cases} \frac{3I_0}{T} & , 0 < t < \frac{T}{3} \\ -\frac{3}{2} \cdot \frac{I_0}{T} & , \frac{T}{3} < t < T \end{cases}$$



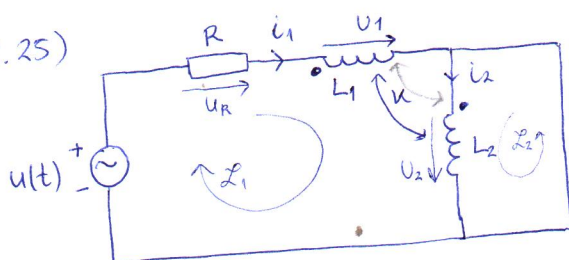
$$u_1(t) = \begin{cases} R_1 \cdot \frac{3I_0 t}{T} + L_1 \frac{3I_0}{T} & , 0 < t < \frac{T}{3} \\ R_1 \cdot I_0 \left(1 - \frac{t - T/3}{2T/3} \right) - L_1 \frac{3I_0}{2T} & , \frac{T}{3} < t < T \end{cases}$$

$$u_2(t) = \begin{cases} M \cdot \frac{3I_0}{T} & , 0 < t < \frac{T}{3} \\ -M \cdot \frac{3I_0}{2T} & , \frac{T}{3} < t < T \end{cases}$$





① 6.25)



$$u(t) = 1V \cdot \cos(\omega t + 0) \quad L_1 = 40 \text{ mH}$$

$$\omega = 1000 \frac{\text{rad}}{\text{s}} \quad L_2 = 10 \text{ mH}$$

$$R = 10 \Omega \quad k = 0,5$$

a) $M = k \sqrt{L_1 \cdot L_2} = 10 \text{ mH} = L_2$

b) $W_m = W_m(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) \pm M \cdot i_1(t) \cdot i_2(t)$

$W_m(t_0) = ? \quad i_1, i_2 = ?$

$|u(t_0)| = 1V \Rightarrow |\cos(\omega t_0)| = 1 \Rightarrow \omega t_0 = \begin{matrix} 0 \\ 180^\circ \end{matrix}$

$\underline{U} = 1V \cdot \frac{e^{j0}}{1} = 1V$

$\mathcal{L}_2: \underline{U}_2 = 0 = j\omega L_2 \cdot \underline{I}_2 \pm j\omega M \cdot \underline{I}_1 \Rightarrow j\omega L_2 \underline{I}_2 = -j\omega M \underline{I}_1 \quad / : j\omega \quad / : L_2$

$\underline{I}_2 = -\underline{I}_1$

$\mathcal{L}_1: \underline{U} = \underline{U}_R + \underline{U}_1 + \underline{U}_2 = R \cdot \underline{I}_1 + j\omega L_1 \cdot \underline{I}_1 + j\omega M \cdot \frac{-\underline{I}_1}{1} = [R + j\omega(L_1 - M)] \underline{I}_1 \quad / : (R + j\omega(L_1 - L_2))$

$\underline{I}_1 = \frac{\underline{U}}{R + j\omega(L_1 - M)} = \frac{1V}{(10 + j30) \Omega} = \frac{(10 - j30)}{1000} \text{ A} = (10 - j30) \text{ mA}$

$\underline{I}_2 = -\underline{I}_1 = (-10 + j30) \text{ mA}$

IZ KOMPLEKSNEM NAZRAJ V ČASOVNI PROSTOR:

$\underline{I}_1 = (10 - j30) \text{ mA} = \frac{\sqrt{1000}}{10\sqrt{10}} \text{ mA} \cdot e^{j \arctg(-30/10)} = 10\sqrt{10} \text{ mA} \cdot e^{-j72^\circ}$

$i_1(t) = 10\sqrt{10} \text{ mA} \cos(\omega t - 72^\circ)$

$\underline{I}_2 = (-10 + j30) \text{ mA} = \sqrt{1000} \text{ mA} \cdot e^{j(\arctg \frac{30}{-10} \pm 180^\circ)} = 10\sqrt{10} \text{ mA} \cdot e^{j108^\circ}$

$i_2(t) = 10\sqrt{10} \text{ mA} \cos(\omega t + 108^\circ)$

$\varphi_{i_2} - \varphi_{i_1} = 108^\circ + 72^\circ = 180^\circ \Rightarrow i_2(t) = -i_1(t)$

$\cos(x + 180^\circ) = -\cos x$

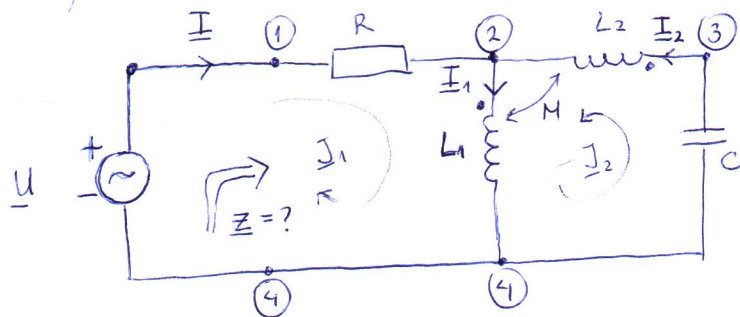
$i_1(t_0) = 10\sqrt{10} \text{ mA} \cos(\omega t_0 - 72^\circ) = 10\sqrt{10} \text{ mA} \begin{cases} \cos(0^\circ - 72^\circ) = \pm 10 \text{ mA} = i_1(t_0) \\ \cos(180^\circ - 72^\circ) = -\cos(-72^\circ) \end{cases} \quad i_2(t_0) = \mp 10 \text{ mA}$

$W_m(t_0) = \frac{1}{2} L_1 (\pm 10 \text{ mA})^2 + \frac{1}{2} L_2 (\mp 10 \text{ mA})^2 + M (\pm 10 \text{ mA}) (\mp 10 \text{ mA})$

$\underbrace{\quad\quad\quad}_{i_1(t_0)} \quad \underbrace{\quad\quad\quad}_{i_2(t_0)}$

$W_m(t_0) = (10 \text{ mA})^2 \left[\frac{L_1}{2} + \frac{L_2}{2} - M \right] = 1,5 \mu\text{J}$

② 6.32)



* Smer toka postavimo v pivo (pozitiven predznak)

$$R = 20 \Omega$$

$$j\omega L_1 = j20 \Omega$$

$$j\omega L_2 = j10 \Omega$$

$$j\omega M = j5 \Omega$$

$$\frac{1}{j\omega C} = \frac{-j}{\omega C} = -j10 \Omega$$

$$Z = ?$$

$$Z \triangleq \frac{U}{I} \quad ; \quad n=5$$

$$m=4$$

$$\text{HSP: } (m-1)=3$$

$$\text{MZT: } n - (m-1) = 2$$

$$\text{D.K.M: } n=5$$

$$I = I_1$$

$$I_1 = I_1 + I_2$$

$$I_2 = I_2$$

$$\mathcal{L}_1: -U + R \cdot I + (j\omega L_1 I_1 + j\omega M I_2) = 0$$

$$\mathcal{L}_2: (j\omega L_1 I_1 + j\omega M I_2) + \frac{j}{\omega C} I_2 + (j\omega L_2 I_2 + j\omega M I_1) = 0$$

$$\mathcal{L}_1: R \cdot I_1 + j\omega L_1 (I_1 + I_2) + j\omega M I_2 = U$$

$$\mathcal{L}_2: j\omega L_1 (I_1 + I_2) + j\omega M I_2 - \frac{j}{\omega C} I_2 + j\omega L_2 I_2 + j\omega M (I_1 + I_2) = 0$$

$$\mathcal{L}_2: (j\omega L_1 + j\omega M - \frac{j}{\omega C} + j\omega M) \cdot I_2 = -(j\omega L_1 + j\omega M) \cdot I_1$$

$$(j \cdot 20 \Omega + j5 \Omega - j10 \Omega + j5 \Omega) \cdot I_2 = -(j20 \Omega + j5 \Omega) \cdot I_1$$

$$j30 \Omega \cdot I_2 = -j25 \Omega \cdot I_1 \quad / : j30 \Omega$$

$$\boxed{I_2 = -\frac{5}{6} I_1}$$

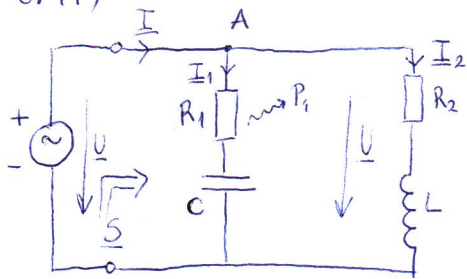
$$\mathcal{L}_1: R \cdot I_1 + j\omega L_1 \cdot I_1 + j\omega L_1 \cdot (-\frac{5}{6} I_1) + j\omega M \cdot (-\frac{5}{6} I_1) = U$$

$$(20 + j20 + j20(-\frac{5}{6}) + j5(-\frac{5}{6})) \cdot I_1 = U$$

$$20 + j \frac{120 - 100 - 25}{6} \Omega \cdot I_1 = (20 - j \frac{5}{6}) \Omega \cdot \underline{I} \quad / : \underline{I}$$

$$\Rightarrow \frac{U}{I} = Z = (20 - j \frac{5}{6}) \Omega$$

3) 6.47)



$$R_1 = 2\Omega$$

$$R_2 = 1\Omega$$

$$\frac{1}{\omega C} = 5\Omega \Rightarrow Z_C = -\frac{j}{\omega C} = -j5\Omega$$

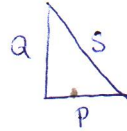
$$\omega L = 1\Omega \Rightarrow Z_L = j\omega L = j1\Omega$$

$$P_1 = 20W$$

$$S = ?$$

$$S = \frac{1}{2} U \cdot I^* = P + jQ$$

$$|S| = S$$



$$P_1 = \text{Re}[S_1] = \text{Re}\left[\frac{1}{2} U_1 \cdot I_1^*\right] = \text{Re}\left[\frac{1}{2} R_1 \underbrace{I_1 \cdot I_1^*}_{I_1^2}\right] = \text{Re}\left[\frac{1}{2} R_1 I_1^2\right] = \frac{1}{2} R_1 I_1^2 = P_1$$

deležna
moč na
uporu

$$\Rightarrow I_1 = \sqrt{\frac{2P_1}{R_1}} = 2\sqrt{5} A$$

$$|I_1| = I_1, \quad \underline{I}_1 = I_1 \cdot e^{j\varphi_{I_1}} = 2\sqrt{5} A \cdot e^{j\varphi_{I_1}}$$

$$\underline{U} = R_1 \cdot \underline{I}_1 + \frac{-j}{\omega C} \underline{I}_1 = \left(R_1 - \frac{j}{\omega C}\right) \underline{I}_1 = (2 - j5)\Omega \cdot 2\sqrt{5} A \cdot e^{j\varphi_{I_1}} = 2\sqrt{5} (2 - j5) V e^{j\varphi_{I_1}} = \underline{U}$$

$$\underline{I}_2 = \frac{\underline{U}}{R_2 + j\omega L} = \frac{2\sqrt{5} (2 - j5) V e^{j\varphi_{I_1}}}{(1 + j)\Omega} = 2\sqrt{5} (2 - j5) \frac{1 - j}{2} A e^{j\varphi_{I_1}} = \sqrt{5} \underbrace{(2 - j2 - j5 - 5)}_{-3 - 7j} A e^{j\varphi_{I_1}}$$

$$\underline{I}_2 = \sqrt{5} (-3 - 7j) A \cdot e^{j\varphi_{I_1}}; \quad \underline{A}: \text{spojis\c{c}e}$$

$$\underline{A}: \underline{I} = \underline{I}_1 + \underline{I}_2 = \sqrt{5} \cdot 2 \cdot A e^{j\varphi_{I_1}} + \sqrt{5} (-3 - 7j) A e^{j\varphi_{I_1}} = (-1 - 7j) \sqrt{5} A e^{j\varphi_{I_1}}$$

$$S = \frac{1}{2} U \cdot \underline{I}^* = P + jQ = \frac{1}{2} \cdot 2\sqrt{5} (2 - j5) V e^{j\varphi_{I_1}} \cdot \underbrace{(-1 + j7) \sqrt{5} A e^{-j\varphi_{I_1}}}_{\underline{I}_1^*} =$$

$$S = 5(2 - j5)(-1 + j7) VA = \underbrace{e^{j\varphi_{I_1}} \cdot e^{-j\varphi_{I_1}}}_1$$

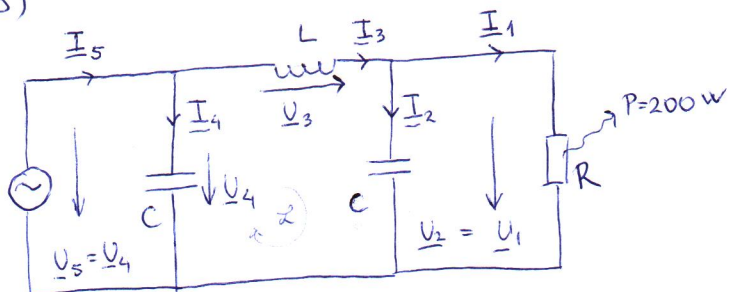
* Ko računamo moč faze niti ne potrebujemo (se porajšata!)

$$S = 5(-2 + j14 + j5 + 35) VA = 5(33 + j19) VA = (165 + j95) VA$$

$$S = 165W + j95VAR$$

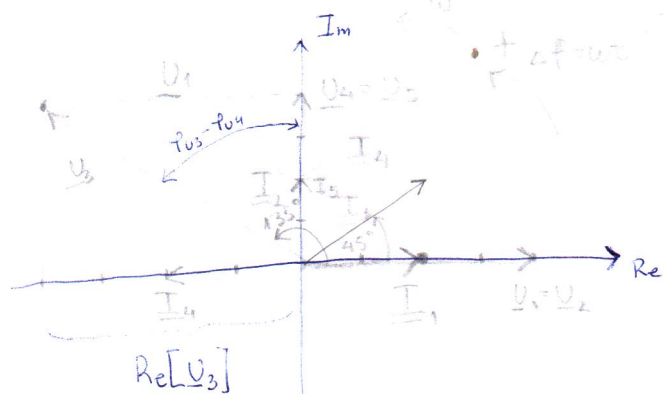
$$S = \sqrt{P^2 + Q^2} = 190 VA$$

④ 6.28)



$$R = \omega L = \frac{1}{\omega C} = 100 \Omega$$

$I_1 = \sqrt{\frac{2P}{R}} = 2A$; $\underline{I}_1 = 2A e^{j\varphi_{i1}}$, $\varphi_{i1} = ?$ (npr. $\varphi_{i1} = 0$): $\underline{I}_1 = 2A (\underbrace{e^{j0}}_1)$
 $\underline{U}_1 = R \cdot \underline{I}_1 = 200V$



$$m_I = 1A / \text{razdelen}$$

$$m_U = 50V / \text{razd}$$

$$\underline{I}_2 = \frac{\underline{U}_2}{\frac{-j}{\omega C}} = j \frac{200V}{100\Omega} = j2A$$

$$\underline{I}_3 = \underline{I}_1 + \underline{I}_2 = 2A + j2A = 2\sqrt{2}A e^{j45^\circ}$$

$$\underline{U}_3 = j\omega L \cdot \underline{I}_3 = j \cdot 100\Omega \cdot (2A + j2A) = -200V + j200V$$

$$\underline{U}_3 = 200\sqrt{2}V e^{j135^\circ}$$

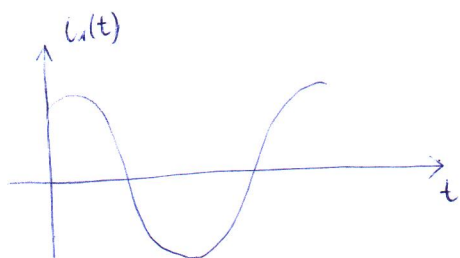
$$\underline{U}_4 = \underline{U}_3 + \underline{U}_2 = (-200 + j200)V + 200V = j200V$$

$$\underline{I}_4 = \frac{\underline{U}_4}{\frac{-j}{\omega C}} = j \frac{j200V}{100\Omega} = -2A$$

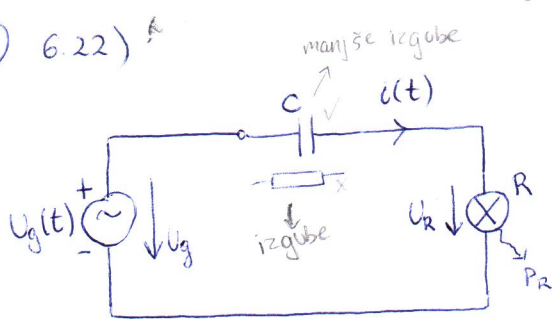
$$\underline{I}_5 = \underline{I}_3 + \underline{I}_4 = (2 + j2)A - 2A = j2A$$

$$i_1(t) = \text{Re}[\underline{I}_1 \cdot e^{j\omega t}] = \text{Re}[\underline{I}_1 \cdot e^{j\varphi_{i1}} \cdot e^{j\omega t}] = I_1 \cdot \cos(\omega t + \varphi_{i1})$$

$$e^{j(\omega t + \varphi_{i1})} = \cos(\omega t + \varphi_{i1}) + j\sin(\omega t + \varphi_{i1})$$



5) 6.22)



$$U_{g,ef} = 220V$$

$$f = 50Hz$$

$$U_{R,ef} = 110V$$

$$P_R = 60W$$

$$1) C = ?$$

$$\frac{P_R}{R} = \frac{U_{R,ef}^2}{R^2} \Rightarrow R = \frac{U_{R,ef}^2}{P_R} = 202 \Omega$$

$$I_{ef} = 545mA$$

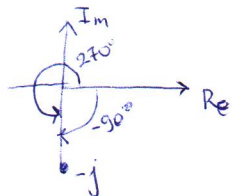
$$\frac{U_g}{I} = Z = \frac{-j}{\omega C} + R \quad |1...1|$$

$$\frac{U_g / \sqrt{2}}{I / \sqrt{2}} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \frac{U_{g,ef}}{I_{ef}} = |Z| = Z = 403 \Omega \quad |^2 \Rightarrow R^2 + \frac{1}{\omega^2 C^2} = Z^2 \Rightarrow$$

$$\omega = 2\pi f$$

$$2) U_R(t_0) = ? , U_C(t_0) = 0$$

$$\underline{U}_C = -\frac{j}{\omega C} \cdot \underline{I} = (-j) \frac{1}{\omega C} I e^{j\varphi_I} = \frac{I}{\omega C} e^{j\varphi_I} e^{-j90^\circ} = \frac{I}{\omega C} e^{j(\varphi_I - 90^\circ)}$$



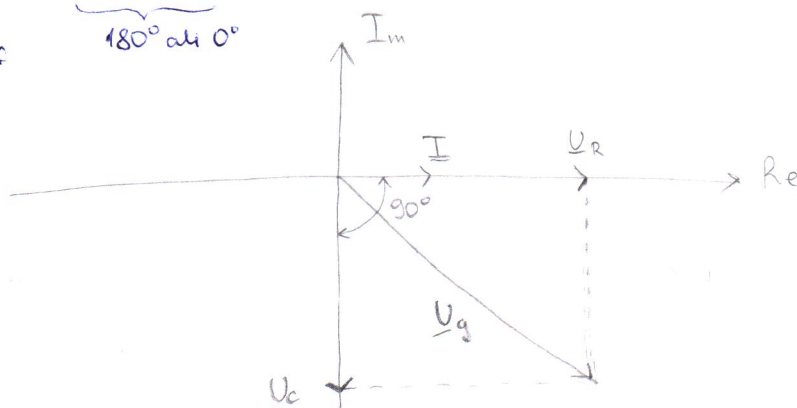
$$U_C(t) = \frac{I}{\omega C} \cos(\omega t + \varphi_I - 90^\circ)$$

$$U_C(t_0) = \frac{I}{\omega C} \cos(\omega t_0 + \varphi_I - 90^\circ) = 0 \Rightarrow \omega t_0 + \varphi_I - 90^\circ = \begin{cases} 90^\circ \\ -90^\circ \end{cases}$$

$$\omega t_0 + \varphi_I = \begin{cases} 180^\circ \\ 0^\circ \end{cases}$$

$$\underline{U}_R = R \cdot \underline{I} = R \cdot I e^{j\varphi_I} \rightarrow U_R(t) = R \cdot I \cdot \cos(\omega t + \varphi_I)$$

$$U_R(t_0) = R \cdot I \cdot \cos(\omega t_0 + \varphi_I) = R \cdot \underbrace{I}_{\sqrt{2} U_{R,ef}} \cdot \underbrace{\cos(\omega t_0 + \varphi_I)}_{180^\circ \text{ ali } 0^\circ} = \pm \sqrt{2} \cdot U_{R,ef} = \pm 156V$$



$$\underline{U}_g = \underline{U}_C + \underline{U}_R$$

$$U_{g,ef} = 220V$$

$$U_R = 110V$$

$$U_{C,ef} = 190,5 \approx \sqrt{U_{g,ef}^2 - U_{R,ef}^2}$$

$$I = \sqrt{\frac{2 P_R}{R}}$$

$$\underline{U}_R = R \cdot \underline{I} \quad |1...1|$$

$$U_R = R \cdot I \quad |:\sqrt{2}|$$

$$U_{R,ef} = R \cdot I_{ef} \Rightarrow I_{ef} = \frac{U_{R,ef}}{R}$$

$$I = \sqrt{2} \cdot I_{ef} = \sqrt{2} \frac{U_{R,ef}}{R}$$

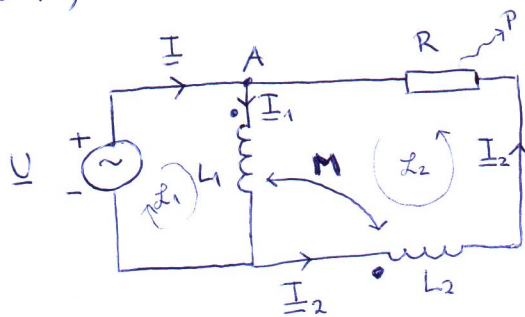
$$I = \sqrt{2} \cdot \frac{U_{R,ef}}{R} = \sqrt{\frac{2 P_R}{R}} \quad |^2$$

$$\frac{1}{\omega^2 C^2} = Z^2 - R^2 \quad | \cdot \omega^2$$

$$\frac{1}{C^2} = \omega^2 (Z^2 - R^2) \quad |^{-\frac{1}{2}}$$

$$C = \frac{1}{\omega \sqrt{Z^2 - R^2}} = 9,11 \mu F$$

⑥ 6.42)



$$U_{ef} = 220V$$

$$R = 10\Omega$$

$$\omega L_1 = 10\Omega = \omega L_2$$

$$\omega M = 5\Omega$$

$$P = ?$$

$$P = \frac{1}{2} R \cdot I_2^2 = R \cdot I_{ef,2}^2$$

$$A: -I + I_1 - I_2 = 0 \Rightarrow I = I_2 + I_1$$

$$L_1: U = j\omega L_1 I_1 + j\omega M I_2$$

$$L_2: (j\omega L_1 I_1 + j\omega M I_2) + (j\omega L_2 I_2 + j\omega M I_1) + R I_2 = 0$$

$$(j\omega L_1 + j\omega M) I_1 = -(R + j\omega M + j\omega L_2) I_2 \quad / : (j\omega L_1 + j\omega M)$$

$$I_1 = -\frac{R + j\omega M + j\omega L_2}{j\omega L_1 + j\omega M} I_2 = -\frac{10 + j5 + j10}{j10 + j5} I_2 = -\frac{10 + j15}{j15} I_2 = (j\frac{2}{3} - 1) I_2 = I_1$$

$$U = j\omega L_1 (-1 + j\frac{2}{3}) I_1 + j\omega M I_2 = (j10\Omega (-1 + j\frac{2}{3}) + j5\Omega) I_2 = (-\frac{20}{3} - j5)\Omega I_2 = U$$

$$-j10\Omega \frac{20}{3} \Omega + j5\Omega$$

$$I_2 = \frac{U}{(-\frac{20}{3} - j5)\Omega} \quad / |...| \quad / : \sqrt{2}$$

$$I_{2,ef} = \frac{U_{ef}}{\sqrt{(-\frac{20}{3})^2 + (-5)^2} \Omega} = 26,4A \quad \Rightarrow P = 6,97 \mu W$$

npr $M=0$ (Toljavi nista mag. sklopljeni)

$$I_1 = -\frac{R + j\omega L_2}{j\omega L_1} I_2 = -\frac{10 + j10}{j10} I_2 = j(1+j) I_2 = (j-1) I_2 = I_1$$

$$L_1: U = j\omega L_1 I_1 = j\omega L_1 (-1+j) I_2 = j10\Omega (-1+j) I_2 = 10(-j-1)\Omega I_2$$

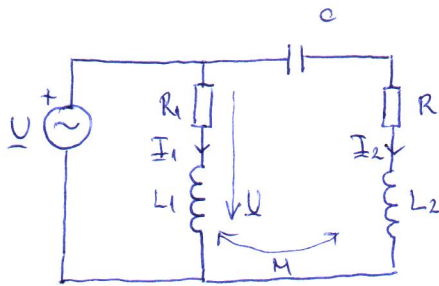
$$I_2 = \frac{U}{10(-1-j)\Omega} \quad / |...| \quad / : \sqrt{2} \Rightarrow I_{2,ef} = \frac{U_{ef}}{10\sqrt{(-1)^2 + (-1)^2} \Omega} = 15,56A$$

$$P' = R \cdot I_{2,ef}^2 = 2,42 \mu W$$

$$P - P' = \Delta P = 4,55 \mu W$$

→ Tolino moči se preneša preno mag. polja (sklopci)

① 6.45)



$$U_{ef} = 220V$$

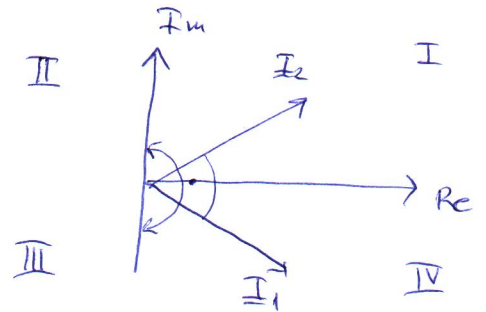
$$f = 50Hz$$

$$R_1 = 20\Omega$$

$$L_1 = 0,2H$$

$$L_2 = 0,1H$$

$$C, R, = ?$$



$$|I_1| = |I_2|, I_1 = I_2, \varphi_{I1} = \varphi_{I2} \pm \frac{\pi}{2}$$

$$I_2 = I_1 \cdot e^{j\frac{\pi}{2}}$$

$$I_1 = \frac{U}{R_1 + j\omega L_1}, I_2 = \frac{U}{R_2 + j\omega L_2 + \frac{1}{j\omega C}} = \frac{U}{R_1 + j\omega L_1} \cdot \frac{e^{j\frac{\pi}{2}}}{\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}} = j$$

$$\frac{1}{R_2 + j\omega L_2 - \frac{1}{j\omega C}} = \frac{j}{R_1 + j\omega L_1} \quad /^{-1}$$

$$R_2 + j\omega L_2 - \frac{1}{j\omega C} = \frac{R_1 + j\omega L_1}{j} = -jR_1 + \omega L_1$$

$$Re: R = \omega L_1 = 2\pi f \cdot L_1 = 62,8\Omega$$

$$Im: \omega L_2 - \frac{1}{\omega C} = -R_1 \Rightarrow \frac{1}{\omega C} = \omega L_2 + R_1 \quad /^{-1}$$

$$\Rightarrow \omega C = \frac{1}{\omega L_2 + R_1} \quad / : \omega$$

$$C = \frac{1}{2\pi f \cdot (2\pi f \cdot L_2 + R_1)} = 61,9\mu F$$

• če sta stavljeni mag. sklopjenca:

$$U = R_1 \cdot I_1 + (j\omega L_1 I_1 + j\omega M I_2) = \frac{1}{j\omega C} \frac{I_2}{I_1} + R_1 \frac{I_2}{I_1} + (j\omega L_2 \frac{I_2}{I_1} + j\omega M \frac{I_2}{I_1}) \quad / : I_1$$

② KOMPENZACIJA JALOVE MOČI

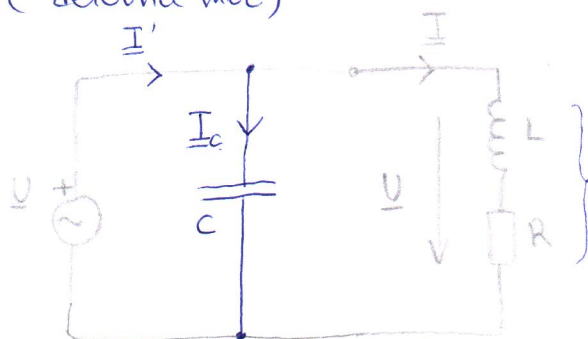
* če piše w (delovna moč)

$$P = 2 \text{ kW}$$

$$\cos \varphi = 0,75$$

$$U_{\text{ef}} = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

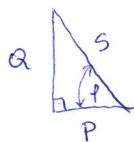


→ w je kondenzator vzporeden motorju, se napetost motorja nič ne spremeni.

$$Z = R + jwL = |Z| \cdot e^{j\varphi}$$

$$P_u - P_c = P$$

1) trikotni moči



$$\underline{S} = P + jQ = S \cdot e^{j\varphi} = \underbrace{S \cdot \cos \varphi}_P + j \underbrace{S \cdot \sin \varphi}_Q$$

$$S = |\underline{S}| = \sqrt{P^2 + Q^2}$$

$$P = S \cdot \cos \varphi$$

faktor delavnosti

$$S = \frac{P}{\cos \varphi} = 2,67 \text{ kVA}$$

$$Q = \pm \sqrt{S^2 - P^2} \quad \left(\begin{array}{l} \text{pri tuljavi} + \\ \text{pri kondenz.} - \end{array} \right)$$

$$Q = 1,76 \text{ kVAR}$$

2) $C = ?$ ($Q' = 0$)

$$\underline{S}' = \frac{1}{2} \underline{U} (\underline{I}')^* = \frac{1}{2} \underline{U} \cdot (\underline{I}_c + \underline{I})^* = \underbrace{\frac{1}{2} \underline{U} \cdot \underline{I}_c^*}_{\underline{S}_c} + \underbrace{\frac{1}{2} \underline{U} \cdot \underline{I}^*}_S \Rightarrow$$

$$\underline{I}' = \underline{I}_c + \underline{I}$$

$$\underline{I}_c = \underbrace{jwC \cdot \underline{U}}_Y$$

$$\Rightarrow \underline{S}' = \frac{1}{2} \underline{U} \cdot (-jwC \cdot \underline{U}^*) + (P + jQ)$$

$$\underline{S}_c' = -\frac{1}{2} jwC \cdot U^2 = -j \frac{1}{2} wC \cdot (\sqrt{2} \cdot U_{\text{ef}})^2$$

$$\underline{S}_c = -jwC \cdot U_{\text{ef}}^2 = \underbrace{P_c}_0 + j \underbrace{Q_c}_{-wC \cdot U_{\text{ef}}^2}$$

$$Q_c = -wC \cdot U_{\text{ef}}^2$$

$$\underline{S}' = jQ_c + P + jQ = P + j(Q_c + Q)$$

$$Q' = 0$$

$$Q' = Q_c + Q = 0 \Rightarrow Q_c = -Q$$

(jalova moč uon denzatorja mora biti ravno nasprotna tista od motorja)

$$Q_c = -Q = -wC \cdot U_{\text{ef}}^2 \Rightarrow C = \frac{Q}{w \cdot U_{\text{ef}}^2} = \frac{Q}{2\pi f \cdot U_{\text{ef}}^2} = 116 \mu\text{F}$$

- 3) Kompenzacija jalove moči nič ne vpliva na delovanje motorja ampak vpliva na to, da ni ga vlečemo iz omrežja.

$$\Delta I_{ef} = I_{ef} - I_{ef}'$$

$$S' = \frac{1}{2} U (\underline{I}')^* = P + jQ' = P \quad / \quad 1 \dots 1$$

$$S = \frac{1}{2} U \cdot \underline{I}^* \quad / \quad 1 \dots 1$$

$$S' = P = U_{ef} \cdot I_{ef}'$$

$$S = \frac{1}{2} U \cdot I = U_{ef} \cdot I_{ef} \quad / \quad \because U_{ef}$$

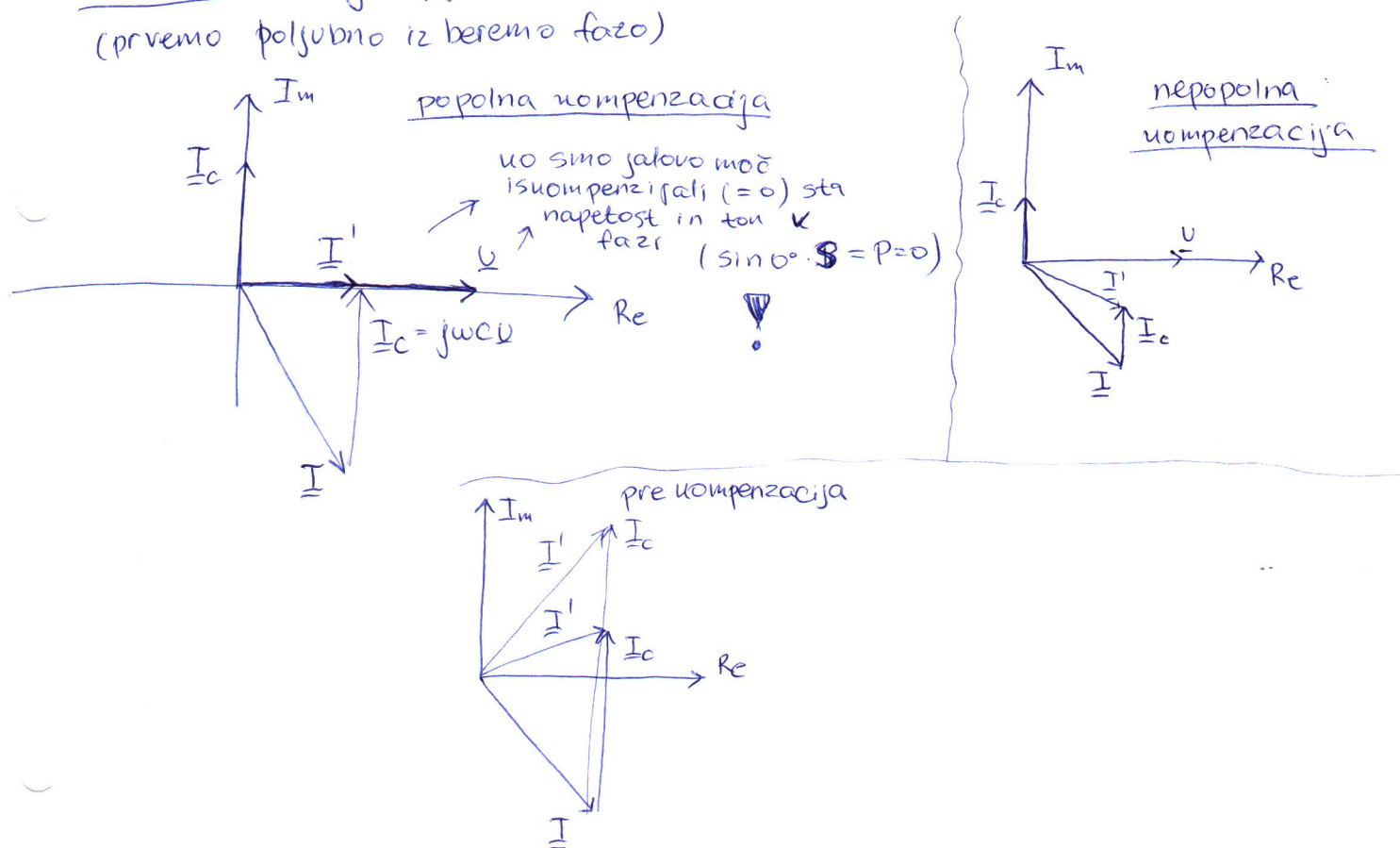
$$I_{ef}' = \frac{P}{U_{ef}} = \underline{9,1 \text{ A}}$$

$$I_{ef} = \frac{S}{U_{ef}} = \underline{12,1 \text{ A}}$$

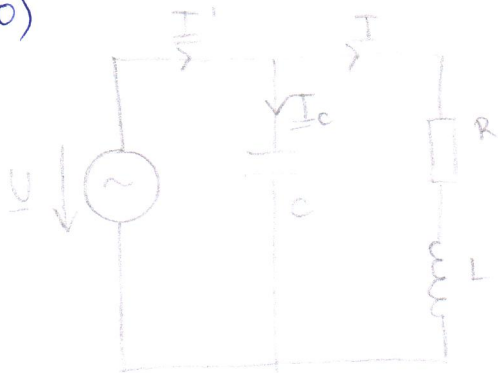
$$\Delta I_{ef} = 12,1 \text{ A} - 9,1 \text{ A} = 3 \text{ A} \rightarrow \text{manj energije vlečemo iz dalnovoda}$$

* Kazalčni diagram

(prvemu poljubno iz beremo fazo)



6.40)



$$P = 2,2 \text{ kW}$$

$$\cos \varphi = 0,75$$

$$U_{\text{ef}} = 220 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$\cos \varphi' > 0,95$$

* Ne išče se
popolna uompenzacija

$$\underline{S} = P + jQ = S \cdot \cos \varphi + j \cdot S \cdot \sin \varphi$$

$$P = S \cdot \cos \varphi, \quad Q = S \cdot \sin \varphi = \frac{P}{\cos \varphi} \cdot \sin \varphi = P \cdot \tan \varphi$$

$$S = \frac{P}{\cos \varphi}$$

induktivno breme
(motor)

$$\cos \varphi = 0,75 \Rightarrow \varphi = \arccos 0,75 = 41,41^\circ \Rightarrow \tan \varphi = 0,88$$

$$\cos \varphi' = 0,95 \Rightarrow \varphi' = \arccos 0,95 = \pm 18,2^\circ \Rightarrow \tan \varphi' = \pm 0,33$$

$$\underline{S}' = P + j(Q + Q_c) \Rightarrow \underbrace{Q + Q_c}_{P \cdot \tan \varphi'} = P \cdot \tan \varphi' \Rightarrow Q_c = P(\tan \varphi' - \tan \varphi)$$

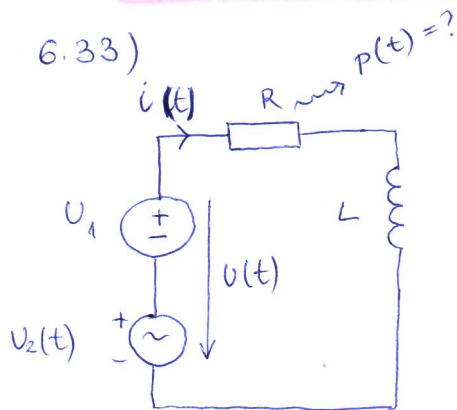
$$Q_c = -\omega C \cdot U_{\text{ef}}^2 = P(\tan \varphi' - \tan \varphi) / (-\omega U_{\text{ef}}^2)$$

$$C = \frac{P}{\omega U_{\text{ef}}^2} (\tan \varphi - \tan \varphi') = \frac{2,2 \text{ kW}}{2\pi \cdot 50 \text{ Hz} \cdot (220 \text{ V})^2} (0,88 - (\pm 0,33)) = \begin{matrix} 80 \mu\text{F} \\ 175 \mu\text{F} \end{matrix}$$

$$80 \mu\text{F} < C < 175 \mu\text{F} \Rightarrow -18,2^\circ < \varphi' < 18,2^\circ \Rightarrow \cos \varphi' > 0,95$$

TEOREMI (SUPERPOZICIJA)

6.33)



$$U_1 = 100V$$

$$U_2(t) = 100V \cdot \cos(\omega t)$$

$$\omega = 300 \frac{\text{rad}}{\text{s}}$$

$$R = 8\Omega$$

$$L = 20\text{mH}$$

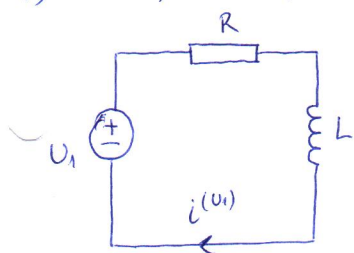
$$\overline{P(t)} = ?$$

$$u(t) = U_1 + U_2(t) \rightarrow \text{ZAPOREDNA VEZAVA}$$

* Superpozicija za napetost in tok v časovnem velja vedno, v uimpl. pa samo uo sta viri noherentni.

* Superpozicija moči pa (v časovnem) lahko seštejemo taurat uo sta vira NEKOHERENTNA.

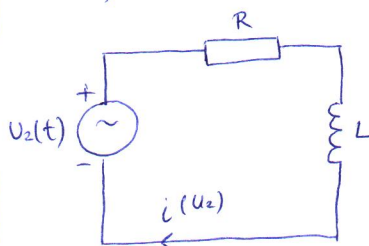
1) $U(t) = U_1, U_2 = 0$



$$\underline{i(u_1)} = \frac{U_1}{R} = I_1 = 12,5A$$

$$U_L = L \cdot \frac{di}{dt}$$

2) $U(t) = U_2(t), U_1 = 0$



$$U_2 = 100V \cdot e^{j0} = 100V$$

$$\underline{I(u_2)} = \frac{U_2}{R + j\omega L} = \frac{100V}{(8 + j6)\Omega}$$

$$\underline{I(u_2)} = \frac{100V(8 - j6)\Omega}{8^2 + 36} = (8 - j6)A$$

$$\underline{I(u_2)} = \sqrt{8^2 + 6^2} A \cdot e^{j \arctg \frac{-6}{8}} = 10A \cdot e^{-j37^\circ}$$

$$\underline{i(u_2)(t)} = \frac{10A \cdot \cos(\omega t - 37^\circ)}{I_2}$$

z superpozicijo dobimo vsoto obeh prispevkov vira (oba dva toka)

$$i(t) = i(u_1) + i(u_2) = I_1 + I_2 \cdot \cos(\omega t - 37^\circ) \rightarrow \text{CELOTEN TOK SLOZI UPOR}$$

MOČ

$$P(t) = R \cdot i^2(t) = R \cdot (I_1 + I_2 \cdot \cos(\omega t - 37^\circ))^2 = RI_1^2 + 2RI_1I_2 \cos(\omega t - 37^\circ) + RI_2^2 \cos^2(\omega t - 37^\circ)$$

$$\overline{P(t)} = R \cdot I_1^2 + 2RI_1I_2 \underbrace{\cos(\omega t - 37^\circ)}_0 + RI_2^2 \underbrace{\cos^2(\omega t - 37^\circ)}_{\frac{1}{2}} = RI_1^2 + \frac{1}{2} RI_2^2$$

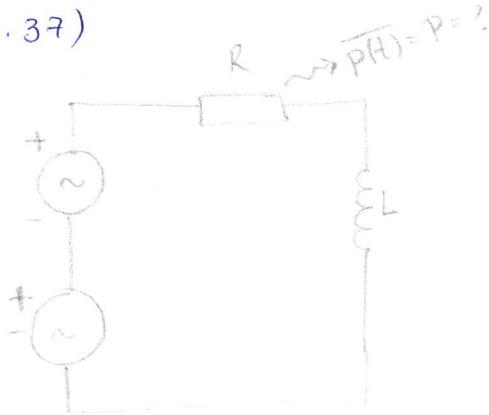
$$P(u_1) = R \cdot I_1^2$$

SUPERPOZICIJA
=> VELJA TUDI ZA
MOČ

$$P(u_2) = \frac{1}{2} \cdot \underbrace{U_R}_{R \cdot I(u_2)} \cdot \left(\frac{I(u_2)}{I_R} \right)^* = \frac{1}{2} R \cdot |I(u_2)|^2 = \frac{1}{2} R \cdot I_2^2$$

$$\overline{P(t)} = P = P(u_1) + P(u_2) = R \cdot I_1^2 + \frac{1}{2} R \cdot I_2^2 = 1250W + 400W = 1650W$$

6.37)



$$U_{1,ef} = 100V$$

$$f_1 = 50Hz$$

$$U_{2,ef} = 200V$$

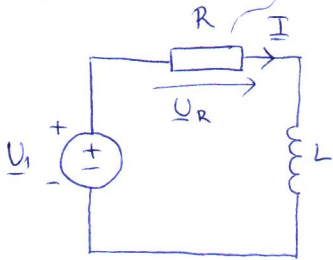
$$f_2 = 150Hz$$

$$R = 20\Omega$$

$$L = 50\mu H$$

$$f_1 \neq f_2 \Rightarrow P = P(u_1) + P(u_2)$$

$$1) P(u_1) = ? \quad \overline{P^{(u_1)}(t)} = P^{(u_1)} = ?$$



$$P^{(u_1)} = \frac{1}{2} \underbrace{U_R \cdot I}_{R \cdot I} = \frac{1}{2} R I^2 = R \cdot I_{ef}^2$$

$$I = \frac{U_1}{R + j\omega_1 L} \quad | \dots | \Rightarrow I = \frac{U_1}{\sqrt{R^2 + (\omega_1 L)^2}} \quad | : \sqrt{2}$$

$$I_{ef} = \frac{U_{1,ef}}{\sqrt{R^2 + (\omega_1 L)^2}}$$

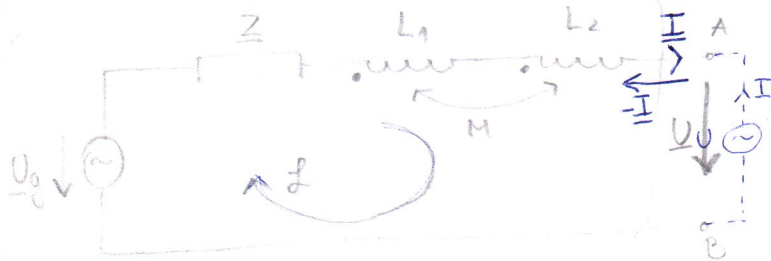
$$P^{(u_1)} = R \cdot \frac{U_{1,ef}^2}{R^2 + (\omega_1 L)^2} \doteq 309W$$

$$\Rightarrow P = 309W + 305W = \underline{\underline{614W}}$$

$$2) P^{(u_2)} = ?$$

$$P^{(u_2)} = R \frac{U_{2,ef}^2}{\sqrt{R^2 + (\omega_2 L)^2}} \doteq 305W$$

5



$$U_g = 50V, \quad \omega = 100\pi \frac{\text{rad}}{\text{s}}$$

$$Z = (100 - j100) \Omega$$

$$L_1 = 0,1 \text{ H}$$

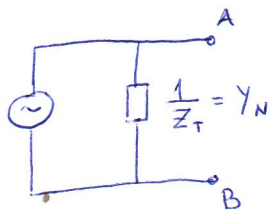
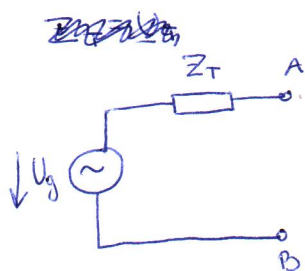
$$L_2 = 0,2 \text{ H}$$

$$M = 9 \text{ mH}$$

Norton in Thevenin ?

$$U_T = ?$$

$$Z_T = ?$$



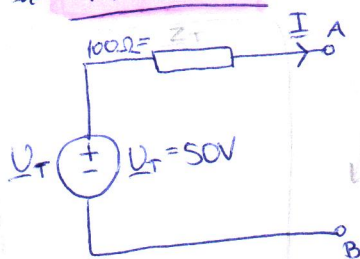
$$Z = \frac{U}{I}$$

$$U = Z \cdot I - j\omega L_1 I + j\omega M I - j\omega L_2 I + j\omega M I$$

$$U = I (Z - j\omega L_1 - j\omega L_2 + 2j\omega M)$$

$$Z = \frac{I (Z - j\omega L_1 - j\omega L_2 + 2j\omega M)}{I} = Z - j\omega L_1 - j\omega L_2 + 2j\omega M \Rightarrow$$

Thevenin



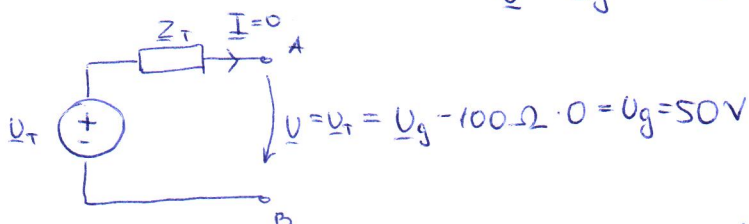
$$\mathcal{L}: -U_g + Z \cdot I + (j\omega L_1 I + j\omega M I) + (j\omega L_2 I + j\omega M I) + U = 0$$

$$-U_g + \left(\frac{Z}{(100 - j100) \Omega + j100\pi (0,318) \Omega} \right) \cdot I + U = 0 \quad \doteq 100 \Omega$$

$$-U_g + 100 \Omega \cdot I + U = 0$$

$$U = U_g - 100 \Omega \cdot I$$

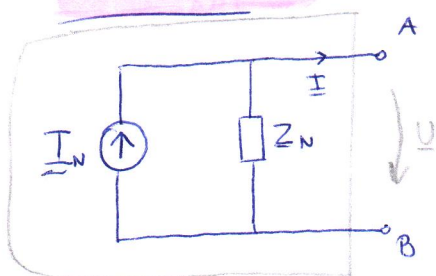
1) $U_T = ?$



2) $Z_T = ?$ (uraden stin = vir se deaktiviran (=0))

$$Z_T = \frac{U}{-I} \Big|_{-U_g=0} = \frac{0 - 100 \Omega \cdot I}{-I} = \underline{\underline{100 \Omega}}$$

Norton

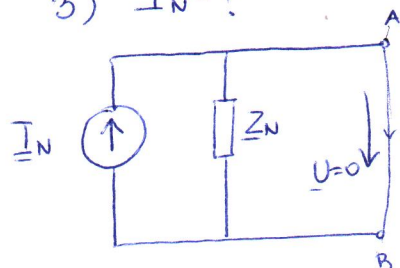


$$I_N = ? ; Z_N = ?$$

deaktiviramo vire

$$Z_N = Z_T = 100 \Omega$$

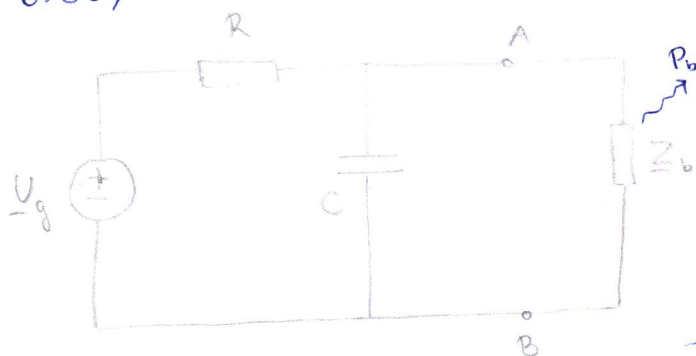
$$3) I_N = ?$$



$$I = I_N = \frac{U_g - 0}{100 \Omega} = \frac{50V}{100 \Omega} = \underline{\underline{\frac{1}{2} A}}$$

$$Z_T = Z_N = \frac{U_T}{I_N} \Rightarrow 100 \Omega = \frac{50V}{1/2 A} \checkmark$$

⑥ 6.36)



$$U_g = 4V$$

$$R = 1 \Omega$$

$$\omega C = 0,5 S$$

$$Z_c = \frac{1}{j\omega C} = -j2 \Omega$$

$$Z_b = ? \quad \text{da bi} \quad P_b = P_{b, \max}$$

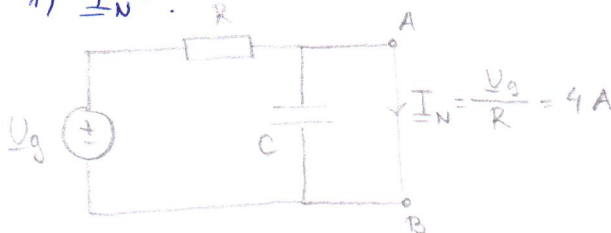
$$Z_b = Z_{Th}^*$$

$$P_{b, \max} = \frac{|U_{Th}|^2}{8 R_T}$$

$$R_T = \text{Re}[Z_T]$$

Najlažje bi bilo izračunati tou kratkgastru

$$1) I_N = ?$$

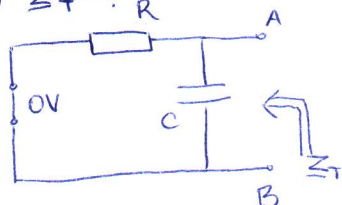


$$I_N = \frac{U_g}{R} = 4 A$$

$$Z_T = \frac{R \cdot Z_c}{R + Z_c} = \frac{1 \cdot (-j2)}{1 - j2} \Omega$$

$$Z_T = (0,8 - j0,4) \Omega$$

$$2) Z_T = ? R$$



3) $\underline{U}_T = ?$

$$\underline{U}_T = \underline{Z}_T \cdot \underline{I}_N = 4 \cdot (0,8 - j0,4) \text{ V}$$

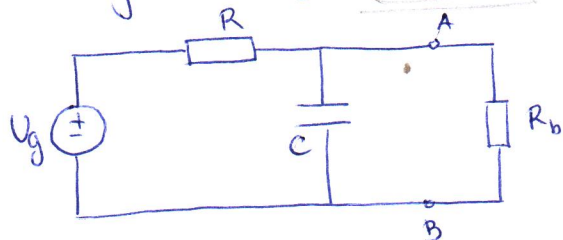
$$|\underline{U}_T| = U_T = 4 \cdot \sqrt{0,8^2 + (-0,4)^2} \text{ V} = 4 \cdot \sqrt{\frac{4}{5}} \text{ V} = 8 \frac{1}{\sqrt{5}} \text{ V}$$

$\sqrt{0,8} = \sqrt{4/5}$

$$\underline{Z}_b = \underline{Z}_T^* = (0,8 + j0,4) \Omega$$

$$\Rightarrow P_{bmax} = \frac{U_T^2}{8 R_T} = \frac{\frac{64}{5} \text{ V}^2}{8 \cdot 0,8 \Omega} = \underline{\underline{2 \text{ W}}}$$

* Če je breme REALNO ($I_m = 0$)

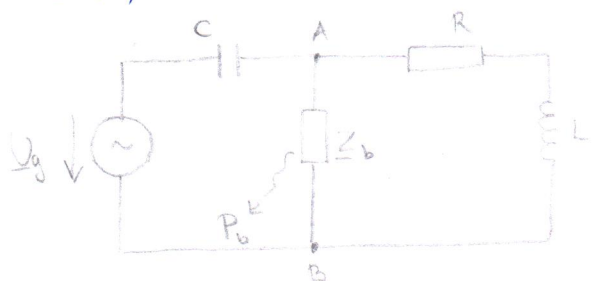


$$R_b = |\underline{Z}_T| = \sqrt{0,8^2 + (-0,4)^2} \Omega = \sqrt{\frac{4}{5}} \Omega = \frac{2}{\sqrt{5}} \Omega$$

$$P_{bmax} = \frac{U_T^2}{4(R_T + Z_T)}$$

$$P_{bmax} = \frac{(8/\sqrt{5})^2}{4(0,8 + \frac{2}{\sqrt{5}})} \text{ W} = \underline{\underline{1,89 \text{ W}}}$$

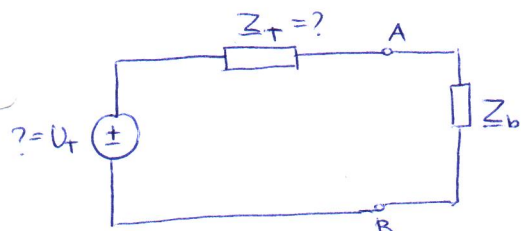
7) 6.46)



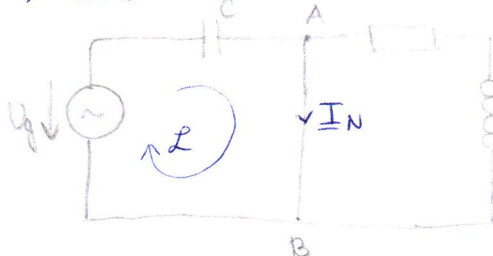
$$U_{g,ref} = 50 \text{ V}$$

$$R = \omega L = \frac{1}{\omega C} = 5 \Omega$$

$$P_{b,max} = ?$$

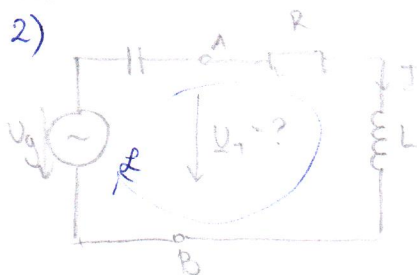


1) $\underline{I}_N = ?$



$$-U_g = \underline{Z}_C \cdot \underline{I}_N = \frac{1}{j\omega C} \underline{I}_N = (-j5 \Omega) \cdot \underline{I}_N$$

$$\underline{I}_N = \frac{50 \text{ V}}{-j5 \Omega} = j10 \text{ A} \quad \underline{I}_N = \frac{U_g}{j5 \Omega} = j \frac{U_g}{5 \Omega}$$



$$\mathcal{L}: U_g = (\underline{Z}_C + \underline{Z}_R + \underline{Z}_L) \underline{I} = (-j5 \Omega + 5 \Omega + j5 \Omega) \underline{I}$$

$$U_g = 5 \Omega \cdot \underline{I} \Rightarrow \underline{I} = \frac{U_g}{5 \Omega} ; \underline{U}_T = \underline{I} (R + j\omega L) = \frac{U_g}{5 \Omega} (5 + j5) \Omega = U_g \cdot (1 + j)$$

$$U_T = U_g \sqrt{1^2 + 1^2} = \sqrt{2} \cdot |U_g| = \sqrt{2} (\sqrt{2} \cdot U_{g,ref}) = 2 \cdot U_{g,ref} = \underline{\underline{100 \text{ V}}}$$

3) $Z_T = ?$

$$Z_T = \frac{U_T}{I_N} = \frac{U_g(1+j)}{jU_g/5\Omega} = 5\Omega \cdot \frac{1+j}{j} = 5\Omega(1-j)$$

$$Z_b = Z_T^* = 5\Omega(1+j)$$

$$\operatorname{Re}[Z_T] = R_T = 5\Omega$$

$$P_{bmax} = \frac{U_T^2}{8 \cdot R_T} = \frac{100^2}{8 \cdot 5} = \underline{\underline{250W}}$$

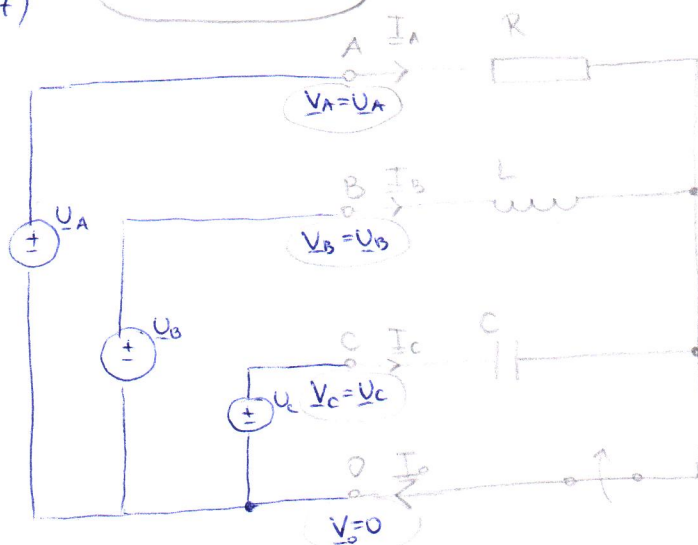
* Če je $Z_b = R_b$:

$$R_b = |Z_T| = 5 \cdot \sqrt{2} \Omega = 7,07 \Omega$$

$$P_{bmax} = \frac{U_T^2}{4(R_T + Z_T)} = \frac{100^2}{4(5\Omega + 7,07\Omega)} = \underline{\underline{207W}}$$

⑧ 6.57)

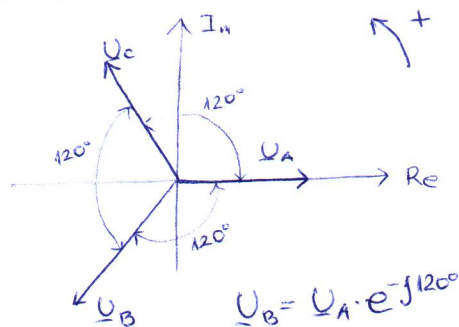
ZVEZDA



$$U_f = 230V = U_{f,ef}$$

$$\omega L = 2\Omega$$

V_2, V_3 pozit. simetr. 3-fazni sistem nap:



$$U_B = U_A \cdot e^{-j120^\circ}$$

$$U_C = U_A \cdot e^{j120^\circ}$$

$$|U_A| = |U_B| = |U_C| = U_f$$

$$R = ? ; \frac{1}{\omega C} = ? \text{ da bo } I_o = 0$$

zvezdišče: $-I_A - I_B - I_C + I_o = 0$

$$I_o = I_A + I_B + I_C = \frac{U_A}{R} + \frac{U_B}{j\omega L} + j\omega C \cdot U_C = 0$$

$$I_A = \frac{V_A - V_z}{R} = \frac{U_A}{R}$$

$$I_B = \frac{V_B - V_z}{j\omega L} = \frac{U_B}{j\omega L}$$

$$I_C = \frac{V_C - V_z}{\frac{1}{j\omega C}} = j \frac{U_C}{1/(\omega C)}$$

$$\underline{U}_B = \underline{U}_A \cdot e^{-j120^\circ}$$

$$\underline{U}_C = \underline{U}_A e^{j120^\circ}$$

$$0 = \underline{I}_0 = \frac{\underline{U}_A}{R} + \frac{\underline{U}_A \cdot e^{-j120^\circ}}{\omega L \cdot e^{j90^\circ}} + e^{j90^\circ} \omega C \cdot \underline{U}_A e^{j120^\circ} \quad / : \underline{U}_A$$

$$j = 1 \cdot e^{j90^\circ}$$

$$\frac{1}{R} + \frac{e^{-j210^\circ}}{\omega L} + \omega C \cdot e^{j210^\circ} = 0$$

$$\frac{1}{R} + \frac{1}{\omega L} \left(\underbrace{\cos(-210^\circ)}_{-\frac{\sqrt{3}}{2}} + j \underbrace{\sin(-210^\circ)}_{\frac{1}{2}} \right) + \omega C \left(\underbrace{\cos(210^\circ)}_{-\frac{\sqrt{3}}{2}} + j \underbrace{\sin(210^\circ)}_{-\frac{1}{2}} \right) = 0$$

$$\frac{1}{R} + \frac{1}{\omega L} \left(-\frac{\sqrt{3}}{2} + j \frac{1}{2} \right) + \omega C \left(-\frac{\sqrt{3}}{2} - j \frac{1}{2} \right) = 0$$

$$\left(\frac{1}{R} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\omega L} - \frac{\sqrt{3}}{2} \omega C \right) + j \left(\frac{1}{2} \frac{1}{\omega L} - \frac{1}{2} \omega C \right) = 0 + j \cdot 0$$

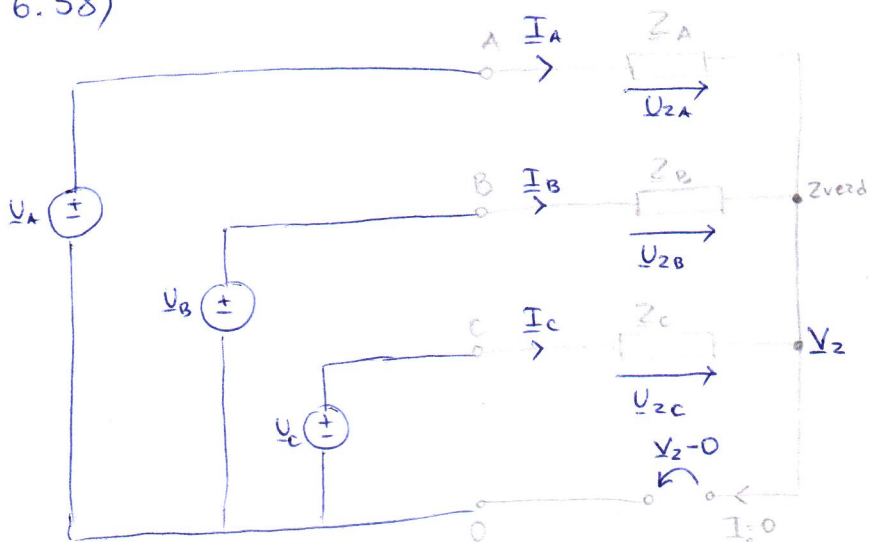
$$\text{Re: } \frac{1}{R} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\omega L} - \frac{\sqrt{3}}{2} \omega C = \frac{1}{R} - \frac{\sqrt{3}}{2} \left(\frac{1}{\omega L} + \omega C \right) = 0 \Rightarrow$$

$$\left\{ \begin{aligned} \frac{1}{R} &= \frac{\sqrt{3}}{2} \left(\frac{1}{2\Omega} + \frac{1}{2\Omega} \right) = \frac{\sqrt{3}}{2\Omega} \\ \underline{R} &= \frac{2}{\sqrt{3}} \Omega \end{aligned} \right.$$

$$\text{Im: } \frac{1}{2} \left(\frac{1}{\omega L} - \omega C \right) = 0 \Rightarrow \frac{1}{\omega L} = \omega C \quad / \cdot$$

$$\omega L = \frac{1}{\omega C} = \underline{\underline{2\Omega}}$$

⑨ 6.58)



$$U_f = 220V$$

$$Z_A = 100\Omega \cdot e^{j0^\circ}$$

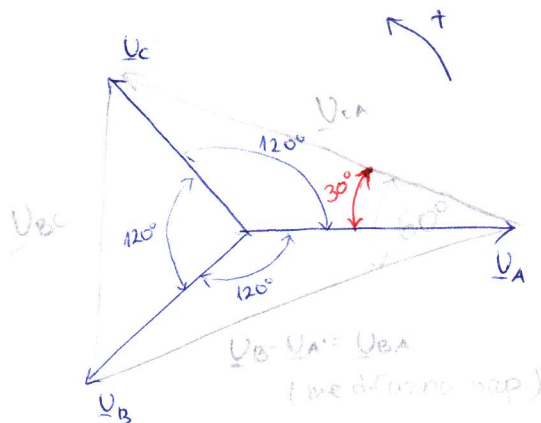
$$Z_B = 100\Omega \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) = 100\Omega e^{j60^\circ}$$

$$Z_C = 100\Omega \left(\frac{1}{2} + j \frac{\sqrt{3}}{2} \right) = 100\Omega e^{-j60^\circ}$$

$$\underline{S}_A, \underline{S}_B, \underline{S}_C = ?$$

pozitivni simetrični trifazni sistem:

BREZ POLOVIČKE



$$\underline{S}_A = \frac{\underline{U}_{ZA} \cdot \underline{I}_A^*}{\underline{Z}_A \cdot \underline{I}_A} = \underline{Z}_A \cdot \underline{I}_A \cdot \underline{I}_A^* = \underline{Z}_A \cdot \underline{I}_A^2$$

$$\underline{S}_B = \underline{Z}_B \cdot \underline{I}_B^2$$

$$\underline{S}_C = \underline{Z}_C \cdot \underline{I}_C^2$$

$$\underline{U}_B = A \underline{U}_A e^{-j120^\circ}$$

$$\underline{U}_C = \underline{U}_A e^{j120^\circ}$$

VELIKOSTI
MEDFAZNIH NAP:

$$|\underline{U}_{BA}| = |\underline{U}_{CA}| = |\underline{U}_{BC}| = U_m$$

$$\frac{U_m}{2} = U_f \cdot \frac{\cos 30^\circ}{\sqrt{3/2}} \cdot 2$$

$$U_m = \sqrt{3} U_f$$

$$\underline{I}_A = \frac{\underline{U}_{ZA}}{\underline{Z}_A} = \frac{\underline{U}_A - \underline{V}_Z}{\underline{Z}_A} = \frac{\underline{U}_A - \underline{V}_Z}{\underline{Z}_A} = \frac{\underline{U}_A - \underline{V}_A}{\underline{Z}_A} = 0$$

$$\underline{I}_B = \frac{\underline{U}_B - \underline{V}_Z}{\underline{Z}_B} = \frac{\underline{U}_B - \underline{V}_A}{\underline{Z}_B} = \frac{\underline{U}_{BA}}{\underline{Z}_B}$$

$$\underline{I}_C = \frac{\underline{U}_C - \underline{V}_Z}{\underline{Z}_C} = \frac{\underline{U}_{CA}}{\underline{Z}_C}$$

$$\underline{V}_Z \neq 0$$

$$\underline{I}_0 = \underline{I}_A + \underline{I}_B + \underline{I}_C = 0 = \frac{\underline{U}_A - \underline{V}_Z}{\underline{Z}_A} + \frac{\underline{U}_B - \underline{V}_Z}{\underline{Z}_B} + \frac{\underline{U}_C - \underline{V}_Z}{\underline{Z}_C}$$

$$\underline{V}_Z \left(\frac{1}{\underline{Z}_A} + \frac{1}{\underline{Z}_B} + \frac{1}{\underline{Z}_C} \right) = \frac{\underline{U}_A}{\underline{Z}_A} + \frac{\underline{U}_B}{\underline{Z}_B} + \frac{\underline{U}_C}{\underline{Z}_C}$$

$$\underline{V}_Z = \frac{\frac{\underline{U}_A}{\underline{Z}_A} + \frac{\underline{U}_B}{\underline{Z}_B} + \frac{\underline{U}_C}{\underline{Z}_C}}{\frac{1}{\underline{Z}_A} + \frac{1}{\underline{Z}_B} + \frac{1}{\underline{Z}_C}}$$

→ potencial
zvezdišča
u o povratnega
vodnika ni.

$$\underline{V}_Z = \frac{\frac{\underline{U}_A}{100\Omega} + \frac{\underline{U}_A \cdot e^{-j120^\circ}}{100\Omega e^{-j60^\circ}} + \frac{\underline{U}_A e^{j120^\circ}}{100\Omega e^{j60^\circ}}}{\frac{1}{100\Omega} + \frac{1}{100\Omega \cdot e^{-j60^\circ}} + \frac{1}{100\Omega e^{j60^\circ}}} \cdot 100\Omega$$

$$\underline{V}_Z = \frac{\underline{U}_A (1 + e^{-j60^\circ} + e^{j60^\circ})}{1 + e^{j60^\circ} + e^{-j60^\circ}} = \underline{U}_A$$

$$I_A = 0$$

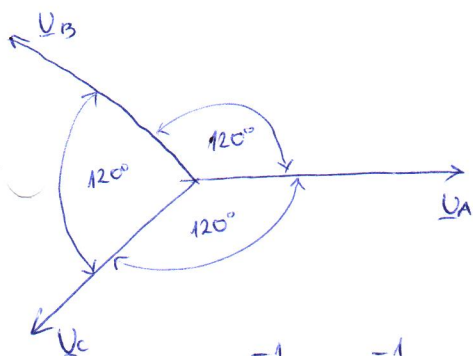
$$I_B = \frac{U_m}{|Z_B|} = \frac{U_m}{100\Omega}$$

$$I_C = \frac{U_m}{Z_C} = \frac{U_m}{100\Omega}$$

$$\Rightarrow \begin{cases} S_A = 0 \\ S_B = 100\Omega \cdot e^{-j60^\circ} \left(\frac{U_m}{100\Omega} \right)^2 = 14.52 e^{-j60^\circ} \text{ VA} \\ S_C = 100\Omega \cdot e^{j60^\circ} \left(\frac{U_m}{100\Omega} \right)^2 = 14.52 e^{j60^\circ} \text{ VA} \end{cases}$$

$$\underline{S_A + S_B + S_C} = 0 + 14.52 \text{ VA} \left[\underbrace{\cos(-60^\circ)}_{1/2} + j \underbrace{\sin(-60^\circ)}_{1/2} + \underbrace{\cos 60^\circ}_{1/2} + j \underbrace{\sin 60^\circ}_{1/2} \right] = \underline{14.52 \text{ W}} = \underline{P_A + P_B + P_C}$$

negativni sim. trif. sistem



$$U_B = U_A \cdot e^{j120^\circ}$$

$$U_C = U_A \cdot e^{-j120^\circ}$$

$$V_z = \frac{\frac{U_A}{100\Omega} + \frac{U_A \cdot e^{j120^\circ}}{100\Omega \cdot e^{-j60^\circ}} + \frac{U_A \cdot e^{-j120^\circ}}{100\Omega \cdot e^{j60^\circ}}}{\frac{1}{100\Omega} + \frac{1}{100\Omega \cdot e^{-j60^\circ}} + \frac{1}{100\Omega \cdot e^{j60^\circ}}} \cdot 100\Omega$$

$$V_z = \frac{U_A \left(1 + e^{j180^\circ} + e^{-j180^\circ} \right)}{1 + e^{j60^\circ} + e^{-j60^\circ}} = \frac{U_A \cdot (-1)}{1 + \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) + \left(\frac{1}{2} - j\frac{\sqrt{3}}{2} \right)} = -\frac{1}{2} U_A = -\frac{U_A}{2}$$

$$I_A = \frac{U_A - \left(-\frac{U_A}{2} \right)}{Z_A} = \frac{3}{2} \frac{U_A}{Z_A}$$

$$|I_A| = I_A = \frac{3}{2} \frac{U_f}{100\Omega}$$

$$I_B = \frac{U_B - \left(-\frac{U_A}{2} \right)}{Z_B} = \frac{U_A \cdot e^{j120^\circ} + \frac{1}{2} U_A}{Z_B} = \frac{U_A}{Z_B} \left(\underbrace{-\frac{1}{2} + j\frac{\sqrt{3}}{2} + \frac{1}{2}}_{e^{j120^\circ}} \right) = j\frac{\sqrt{3}}{2} \cdot \frac{U_A}{Z_B}$$

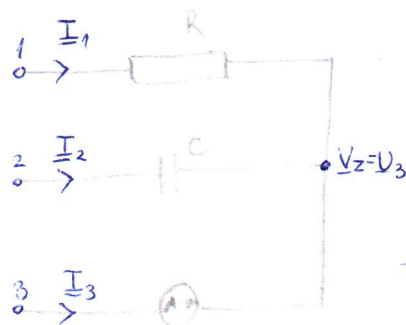
$$I_B = |I_B| = \frac{\sqrt{3}}{2} \cdot \frac{U_f}{|Z_B|} = \frac{\sqrt{3}}{2} \cdot \frac{U_f}{100\Omega}$$

$$I_C = \frac{U_C - \left(-\frac{U_A}{2} \right)}{Z_C} = \frac{U_A \cdot e^{-j120^\circ} + \frac{1}{2} U_A}{Z_C} = \frac{U_A}{Z_C} \left(\underbrace{-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2}}_{e^{-j120^\circ}} \right), I_C = \frac{\sqrt{3}}{2} \cdot \frac{U_f}{100\Omega}$$

$$\Rightarrow \begin{cases} S_A = Z_A \cdot I_A^2 = 100\Omega \cdot \left(\frac{3}{2} \cdot \frac{U_f}{100\Omega} \right)^2 = 1089 \text{ VA} \\ S_B = \underbrace{100\Omega}_{Z_B} \cdot e^{-j60^\circ} \cdot \underbrace{\left(\frac{\sqrt{3}}{2} \cdot \frac{U_f}{100\Omega} \right)^2}_{I_B^2} = 363 e^{-j60^\circ} \text{ VA} \\ S_C = \underbrace{100\Omega}_{Z_C} \cdot e^{j60^\circ} \cdot \underbrace{\left(\frac{\sqrt{3}}{2} \cdot \frac{U_f}{100\Omega} \right)^2}_{I_C^2} = 363 e^{j60^\circ} \text{ VA} \end{cases} \quad + = \underline{14.52 \text{ W}} = P_A + P_B + P_C$$

10) 6.59)

U_f	220 V	230 V
U_m	380 V	400 V



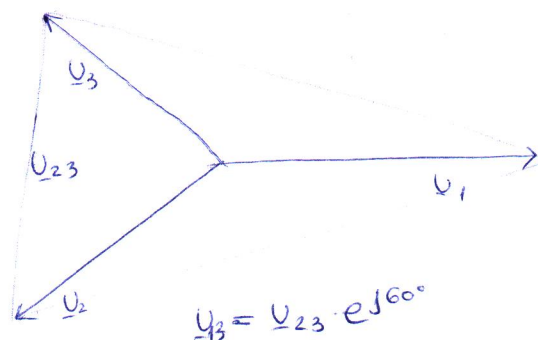
$$R = \frac{1}{\omega C} = 100 \Omega$$

$$R_{A-m} = 0$$

$$U_m = 380 V$$

$$I_3 = ? \quad (\text{koliko kaže AMPERMETER})$$

pozitivni simetr. 3-fazni sistem.



$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$\underline{I}_3 = -\underline{I}_1 - \underline{I}_2$$

$$\underline{I}_1 = \frac{U_1 - V_2}{R} = \frac{U_1 - U_3}{R} = \frac{U_{13}}{R}$$

$$\underline{I}_2 = \frac{U_2 - V_2}{-\frac{j}{\omega C}} = \frac{U_2 - U_3}{-jR} = \frac{U_{23}}{-jR}$$

$V_2 =$ ne bomo računali
zato ker moramo deliti z $R=0$
 $\Rightarrow$ kraten stik

$$\underline{I}_3 = -\frac{U_{13}}{R} + j \frac{U_{23}}{R} = \frac{1}{R} (-U_{13} + j U_{23})$$

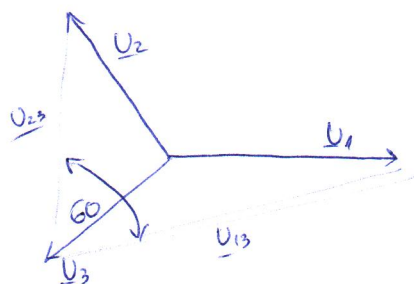
$$\underline{Z}_C = \frac{1}{j\omega C} = \frac{-j}{j\omega C} = -\frac{j}{\omega C} = -jR$$

$$\underline{I}_3 = -\frac{1}{R} (U_{23} e^{j60^\circ} + j U_{23}) = -\frac{U_{23}}{R} \left(\left(\frac{1}{2} + j \frac{\sqrt{3}}{2} \right) + j \right) = -\frac{U_{23}}{R} \left(\frac{1}{2} + j \left(\frac{\sqrt{3}}{2} + 1 \right) \right)$$

$$I_3 = \frac{U_m}{100 \Omega} \sqrt{\left(\frac{1}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} + 1 \right)^2} = 7,34 A \rightarrow \text{Ampermetar kaže } 7,34 A$$

če je nesimetrični
(moramo podati vse)

negativno fazno zaporedje



$$U_{13} = U_{23} e^{-j60^\circ}$$

$$\underline{I}_3 = -\frac{1}{R} [U_{23} e^{-j60^\circ} + j U_{23}] = -\frac{U_{23}}{R} \left[\frac{1}{2} - j \frac{\sqrt{3}}{2} + j \right]$$

$$I_3 = \frac{U_m}{R} \sqrt{\left(\frac{1}{2} \right)^2 + \left(-\frac{\sqrt{3}}{2} + 1 \right)^2} = 1,97 A$$

6.61) / 3a
шрениш

(11)

VEZAVA TRIKOT

$$\frac{U_m}{U_f} = 380V / 220V$$

$$Z = (30 + j30) \Omega = 30(1 + j) \Omega$$

$$\frac{P}{P_i} = ?$$

Sprememba pri
obratovanju, pri
ispadu faze C.

1) vse tri faze:

$$S_{AC} = Z \cdot I_{AC}^2$$

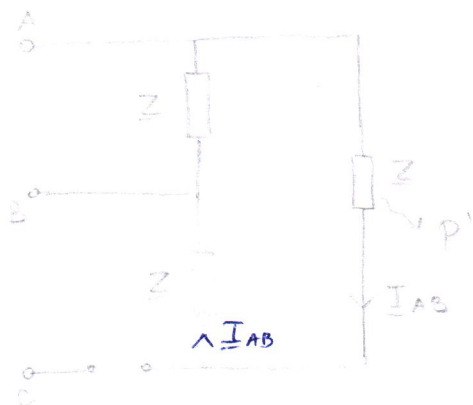
$$P = \operatorname{Re}[S_{AC}] = \operatorname{Re}[Z \cdot I_{AC}^2] = \underbrace{\operatorname{Re}[Z]}_{30\Omega} \cdot I_{AC}^2$$

$$P = 30\Omega \cdot I_{AC}^2 = 30\Omega \cdot \frac{U_m^2}{|Z|^2} = \underline{\underline{2407W}}$$

$$I_{AC} = \frac{U_{AC}}{Z} \Rightarrow I_{AC} = \frac{U_m}{|Z|} = \frac{U_m}{\sqrt{2} \cdot 30\Omega}$$

$$|Z| = 30\sqrt{2}\Omega$$

2) odulop faze C



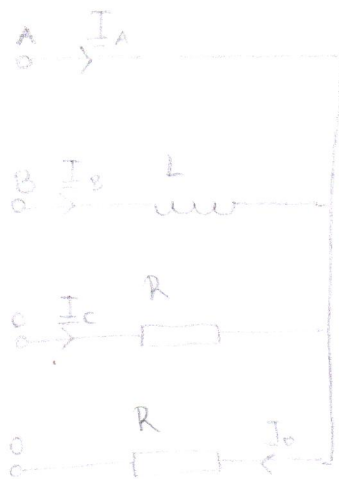
$$P' = \operatorname{Re}[Z] \cdot I_{AB}^2 = 30\Omega \cdot \frac{U_m^2}{4 \cdot |Z|^2} = \underline{\underline{\frac{1}{4} P}}$$

$$I_{AB} = \frac{U_{AB}}{2 \cdot Z} \Rightarrow I_{AB} = \frac{U_m}{2 \cdot |Z|}$$

$$P' = \underline{\underline{602W}}$$

$$\frac{P}{P'} = \frac{P}{\frac{1}{4}P} = \underline{\underline{4}}$$

12) 6.65)



$$R = \omega L = 10 \Omega$$

$$U_f = 230 V$$

$$U_m = 400 V$$

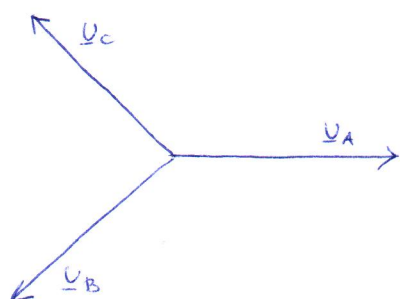
$$I_A = ?$$

$$\text{zvezd. } -I_A - I_B - I_C + I_0 = 0$$

$$I_A = I_0 - I_B - I_C$$

$$I_0 = \frac{U_A - 0}{R} ; I_B = \frac{U_B - U_A}{j\omega L} ; I_C = \frac{U_C - U_A}{R}$$

$$I_A = \frac{U_A}{R} + \frac{U_A - U_B}{jR} + \frac{U_A - U_C}{R} = \frac{1}{R} \left[U_A + j(U_A - U_A e^{-j120}) + (U_A - U_A e^{j120}) \right] =$$



$$U_B = U_A e^{-j120}$$

$$U_C = U_A e^{j120}$$

$$= \frac{U_A}{R} \left[1 - j(1 - e^{-j120}) + 1 - e^{j120} \right] =$$

$$= \frac{U_A}{R} \left[1 - \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) + 1 - \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2} \right) \right] =$$

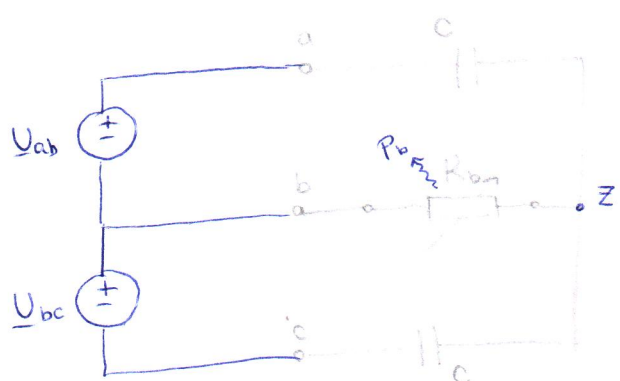
$$1 - j\left(\frac{3}{2} + j\frac{\sqrt{3}}{2} \right) + \frac{3}{2} + j\frac{\sqrt{3}}{2} =$$

$$= 1 - j\frac{3}{2} + \frac{\sqrt{3}}{2} + \frac{3}{2} + j\frac{\sqrt{3}}{2}$$

$$= \frac{U_A}{R} \left[\frac{5 + \sqrt{3}}{2} - j\frac{3 + \sqrt{3}}{2} \right]$$

$$I_A = \frac{U_f}{R} \sqrt{\left(\frac{5 + \sqrt{3}}{2} \right)^2 + \left(-\frac{3 + \sqrt{3}}{2} \right)^2} = \underline{\underline{9.46 A}}$$

13) 6.69)



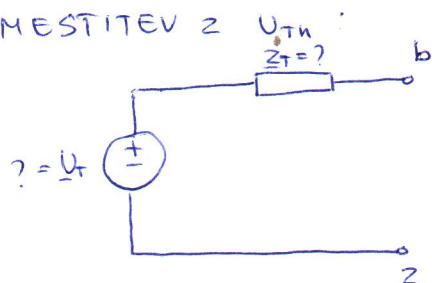
$$C = 100 \mu F$$

$$U_m = 400 V$$

$$f = 50 Hz$$

$$P_{o,max} = ?$$

NADOMEŠTITEV Z

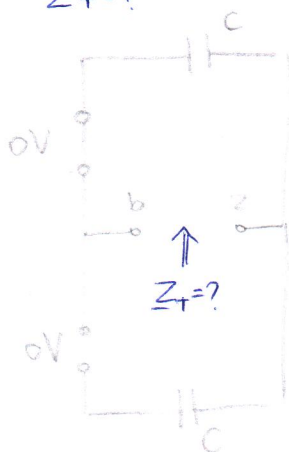


$$R_b = |Z_T| \quad ; \quad U_{T,ref} = U_T$$

$$P_{o,max} = \frac{(U_{T,ref} \sqrt{2})^2}{4(R_T + Z_T)} = \frac{U_T^2}{2(R_T + Z_T)}$$

$$R_T = 0 \quad Z_T = 15,9 \Omega$$

1) $Z_T = ?$



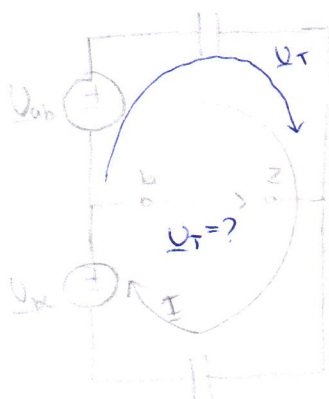
$$Z_T = \frac{1}{j\omega C} \parallel \frac{1}{j\omega C}$$

$$Z_T = \frac{1}{2} \frac{1}{j\omega C}$$

$$Z_T = -j \frac{1}{2(2\pi f \cdot C)}$$

$$Z_T = -j 15,9 \Omega$$

2) $U_T = ?$



$$-U_{bc} - U_{ab} + I \cdot \frac{1}{j\omega C} + I \cdot \frac{1}{j\omega C} = 0$$

$$I \cdot \frac{2}{j\omega C} = U_{bc} + U_{ab} \Rightarrow$$

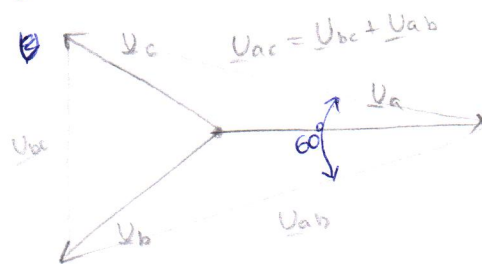
$$I = \frac{j\omega C}{2} (U_{bc} + U_{ab})$$

$$I = \frac{j\omega C}{2} U_{ac}$$

$$U_T = -U_{ab} + I \cdot \frac{1}{j\omega C} = -U_{ab} + j \frac{\omega C}{2} \cdot U_{ac} \cdot \frac{1}{j\omega C}$$

$$U_T = -U_{ab} + \frac{U_{ac}}{2} = -U_{ab} \cdot \frac{e^{j60^\circ}}{\frac{1}{2} + j\frac{\sqrt{3}}{2}} + \frac{1}{2} U_{ac}$$

$$U_{ab} = U_{ac} e^{j60^\circ}$$



$$\underline{U}_T = \underline{U}_{ac} \left(-\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) + \frac{1}{2} \underline{U}_{ac} = -j \frac{\sqrt{3}}{2} \underline{U}_{ac}$$

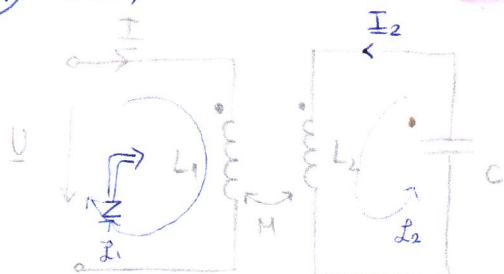
$$U_T = \frac{\sqrt{3}}{2} \cdot U_m$$

$$R_b = |\underline{Z}_T| = 15,9 \Omega$$

$$P_{b, \max} = \frac{\left(\frac{\sqrt{3}}{2} \cdot 400V \right)^2}{2(0 + 15,9 \Omega)} = \underline{\underline{3,77 \text{ nW}}}$$

RESONANCA

14) 6.50)



$$\omega_0 = ?$$

$$\omega_{\pm} = ?$$

$$\frac{U}{I} = \underline{Z} = ?$$

BREZIZGUBNO
VEZJE

$$\text{Re}[\underline{Z}] = 0$$

$$P = \text{Re}[\underline{S}] = 0$$

$$\mathcal{L}_1: -U + (j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2) = 0$$

$$\mathcal{L}_2: (j\omega L_2 \underline{I}_2 + j\omega M \underline{I}_1) + \frac{\underline{I}_2}{j\omega C} = 0$$

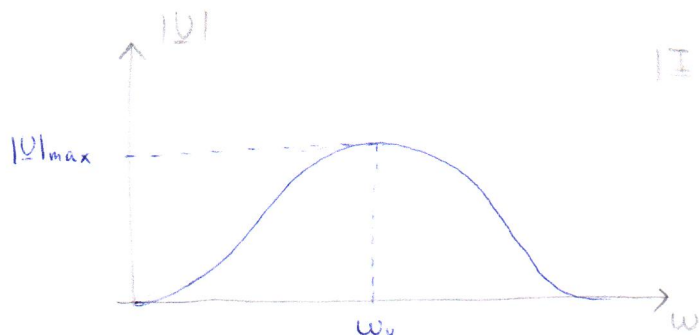
$$(j\omega L_2 - \frac{1}{\omega C}) \underline{I}_2 = -j\omega M \underline{I}_1 \quad / : j(\omega L_2 - \frac{1}{\omega C})$$

$$\underline{I}_2 = - \frac{\omega M \underline{I}_1}{(\omega L_2 - \frac{1}{\omega C})} \cdot \frac{1}{\omega C} = - \frac{\omega^2 M C}{(\omega^2 L_2 C - 1)} \underline{I}_1 = \frac{\omega^2 M C}{(1 - \omega^2 L_2 C)} \underline{I}_1 = \underline{I}_2$$

$$U = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2 = \underline{I}_1 j\omega \left[L_1 + \frac{\omega^2 M^2 C}{1 - \omega^2 L_2 C} \right] \quad / : \underline{I}_1$$

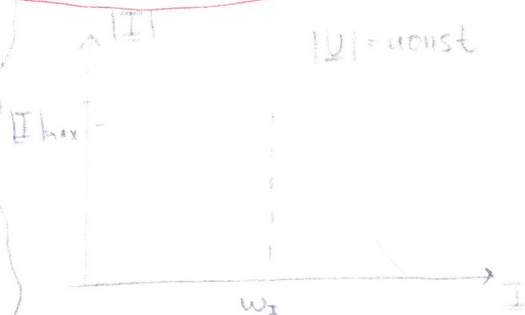
$$\frac{U}{I} = \underline{Z} = j\omega \frac{L_1 - \omega^2 L_2 L_1 C + \omega^2 M^2 C}{1 - \omega^2 L_2 C} = j\omega \frac{L_1 - \omega^2 C (L_1 L_2 - M^2)}{1 - \omega^2 L_2 C}$$

→ odvajemo in
izenačimo z 0
(min in max)



$$\left. \frac{|U|}{|I|} \right|_{\max \text{ pri } \omega = \omega_0}$$

$|I| = \text{const}$



$$\left. \frac{|U|}{|I|} \right|_{\min \text{ pri } \omega = \omega_{\pm}}$$

$$|Z| = \frac{|U|}{|I|} = \omega \frac{|L_1 - \omega^2 C(L_1 L_2 - M^2)|}{|1 - \omega^2 L_2 C|} = |Z(\omega)|$$

$$|Z|_{\max} \rightarrow \infty, \quad 1 - \omega_0^2 L_2 C = 0$$

$$\omega_0^2 L_2 C = 1 \Rightarrow \omega_0^2 = \frac{1}{L_2 C}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{L_2 C}}} \quad \text{max}$$

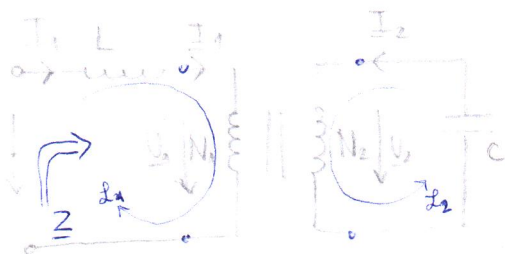
$$|Z|_{\min} \rightarrow 0, \quad \omega = 0 \quad \text{trivialna rezitev (X)}$$

$$L_1 \omega_1^2 C (L_1 L_2 - M^2) = 0$$

$$\omega_1^2 = \frac{L_1}{(L_1 L_2 - M^2) C}, \quad \boxed{\omega_1 = \sqrt{\frac{L_1}{C(L_1 L_2 - M^2)}}}$$

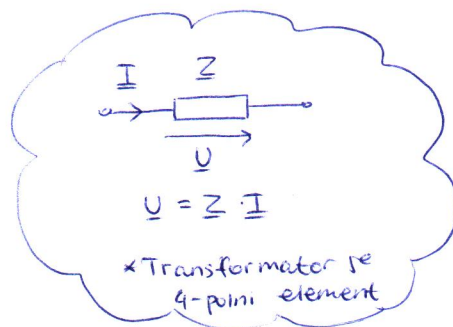
(15) 6.52)

IDEALNI TRANSFORMATOR



$$\omega_1 = ?$$

$$|Z(\omega = \omega_1)| = |Z|_{\min}$$



$$\mathcal{L}_1: -\underline{U} + j\omega L \underline{I}_1 + \underline{U}_1 = 0$$

$$\mathcal{L}_2: \underline{U}_2 + \frac{1}{j\omega C} \underline{I}_2 = 0$$

$$\frac{\underline{U}_1}{\underline{U}_2} = \frac{N_1}{N_2} = n \Rightarrow \underline{U}_1 = n \cdot \underline{U}_2 \Rightarrow \underline{U}_2 = \frac{1}{n} \underline{U}_1$$

$$\frac{\underline{I}_1}{\underline{I}_2} = -\frac{1}{n} \Rightarrow \underline{I}_1 = -\frac{1}{n} \underline{I}_2 \Rightarrow \underline{I}_2 = -n \underline{I}_1$$

$$\mathcal{L}_2: n^2 \underline{U}_1 + \frac{1}{j\omega C} (-n \underline{I}_1) = 0$$

$$\frac{\underline{U}_1}{n} = \frac{n}{j\omega C} \underline{I}_1 \quad / : n \Rightarrow \underline{U}_1 = \frac{n^2}{j\omega C} \underline{I}_1$$

$$\mathcal{L}_1: -\underline{U} + j\omega L \underline{I}_1 + \frac{n^2}{j\omega C} \underline{I}_1 = 0 \quad / : \underline{I}_1$$

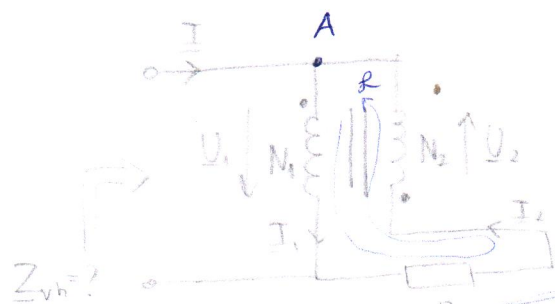
$$\frac{U}{I_1} = Z = j\omega L + \frac{n^2}{j\omega C} = j\omega L - j \frac{n^2}{\omega C} = j \frac{\omega^2 LC - n^2}{\omega C}, \quad \text{Re}[Z] = 0$$

$$|Z|_{\min} = 0 = \frac{\omega^2 LC - n^2}{\omega \cdot C} \Rightarrow \omega^2 LC - n^2 = 0$$

$$\boxed{\omega = \frac{n}{\sqrt{LC}}}$$

Napetostna
resonanca pri
nimu.

(16) 6,65)



$$Z_{vh} = \frac{U_1}{I} \quad ; \quad \frac{N_1}{N_2} = n$$

$$\mathcal{L}: U_1 + R \cdot I_2 + U_2 = 0$$

$$A: I = I_1 - I_2$$

$$U_1 = n \cdot U_2 \quad ; \quad I_2 = -n I_1$$

$$A: I = I_1 - (-n I_1) = (1+n) I_1 = I$$

$$I_1 = \frac{I}{(1+n)}$$

$$\mathcal{L}: U_1 + R(-n I_1) + \frac{U_2}{n} = 0$$

$$U_1 \left(1 + \frac{1}{n}\right) = n \cdot R \frac{I}{n+1}$$

$$\frac{n+1}{n} U_1 = \frac{n}{n+1} R \cdot I \quad / \cdot \frac{n}{n+1}$$

$$U_1 = \frac{n^2}{(n+1)^2} R \cdot I \quad / : I$$

$$Z_m = \frac{U_1}{I} = \frac{n^2}{(n+1)^2} R = R_{vh}$$