Predmet:
Osnove elektrotehnike 2

Izvajalec:
izr. prof. ddr. Iztok Humar

Vrsta gradiva:
Zapiski avditornih vaj

Avtor:
Angela Divanisova

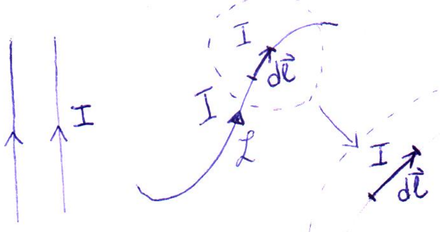
Študijsko leto:
2015/16

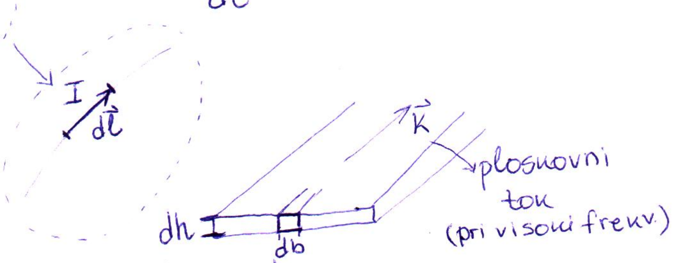
Magnetostatika

El. statika $\rightarrow dQ;\ \overbrace{q, \sigma, \rho}^{\text{porazdelitve}} \rightarrow q\,dl = \sigma \cdot dA = \rho\, dV_{ol}$

potrebujemo "ekvivalente" v magnetiki: $\boxed{dQ \cdot \vec{v} = \underbrace{I \cdot d\vec{l}}_{\text{tokovni element}} = \vec{K} \cdot da = \vec{J} \cdot dV_{ol}}$

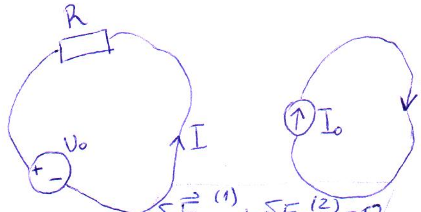
$$\frac{dQ \cdot \vec{v}\, dt}{dt} = I \cdot d\vec{l} = \vec{J}\, da \cdot dl = \vec{J} \cdot dV_{ol} = K \cdot db \cdot dl$$





"koščen toka" = tokovni element

Amperova sila

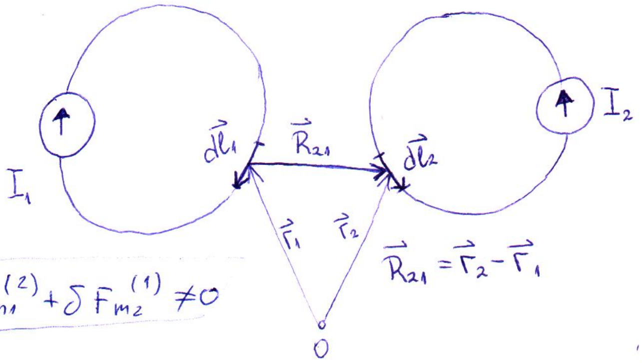


$\delta \vec{F}_e^{(1)} + \delta F_e^{(2)} = 0$

$$d\vec{F}_e = \frac{dQ_1 \cdot dQ_2}{4\pi\varepsilon_0 R^2} \cdot \vec{e}_R$$

$$d\vec{F}_e = \frac{dQ_1 \cdot dQ_2}{4\pi\varepsilon_0 \underbrace{R^3}_{\text{skalarni produkt}}} \vec{R}$$

- sil vzajemni
- sorazmerni z noski naboj
- obratnosorazmerna z R^2



$\delta \vec{F}_{m1}^{(2)} + \delta F_{m2}^{(1)} \neq 0$

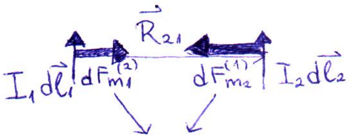
Sile 1. vodnika na 2. vodnik:

$$d\vec{F}_{m2}^{(1)} = \frac{\mu_0}{4\pi R_{21}^3} I_2 d\vec{l}_2 \times \underbrace{(I_1 d\vec{l}_1 \times \vec{R}_{21})}_{\text{dvojni vektorski produkt}}$$

$$d\vec{F}_{m1}^{(2)} = \frac{\mu_0}{4\pi R_{12}^3} I_1 d\vec{l}_1 \hat{\times} (I_2 d\vec{l}_2 \hat{\times} \vec{R}_{12})$$

→ obratnosorazmerna s kvadratom oddaljenosti tokovnih elementov
→ sorazmerna s produktom jakosti tokov
→ razlika je v smeri!

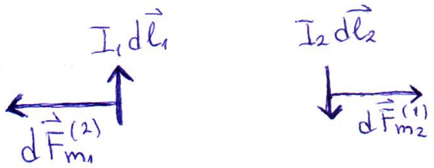
a)



Za vzporedna toka: sili privlačni

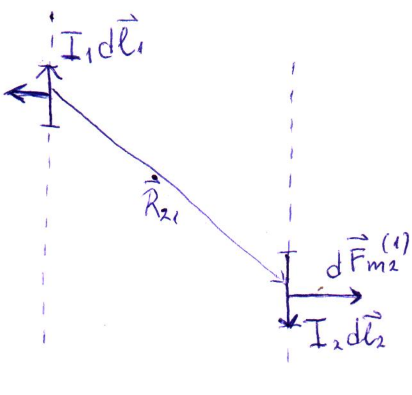
v tablo
$(I d\vec{l}_1 \times \vec{R}_{21}) \times I_2 d\vec{l}_2$
$(I_2 d\vec{l} \times \vec{R}_{12}) \times I_1 d\vec{l}_1$

b)



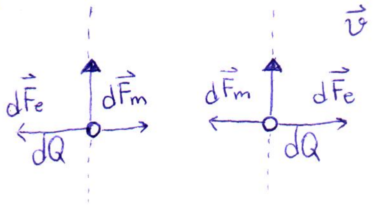
za zaporedna toka: sili odbojni

c)



Primerjava Coulombove in Amperove sile:

1)



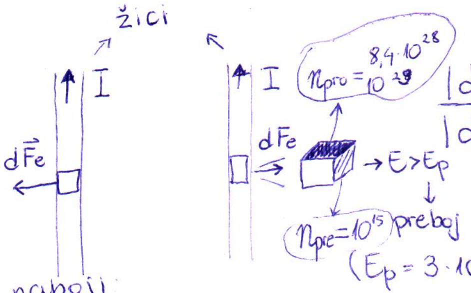
$$\frac{|dF_m|}{|dF_e|} = \frac{\frac{\mu_0}{4\pi R^2}(dQ\cdot v)^2}{\frac{dQ^2}{4\pi\varepsilon_0 R^2}} = \frac{v^2}{C_0^2} = 10^{-25}$$

v: $< mm/s$; C_0: $3\cdot10^8 m/s$

$\mu_0 \cdot \varepsilon_0 = \frac{1}{C_0^2} \rightarrow$ svetlobna hitrost

*Magnetna sila je 10^{25} krat manjša od el. sile.

2)



$$\frac{|dF_m|}{|dF_e|} = \frac{v^2\, dQ_{pro}^2}{C_0^2\, dQ_{pre}^2} = 10^{-25}\cdot(10^{14})^2 = \underline{\underline{10^3}}$$

$\downarrow$ magnetna sila je 10^3 večja

$(E_p = 3\cdot10^6\, V/m)$

$\sigma_p = \varepsilon_0 \cdot E_p = 25\cdot10^{-6}\, C/m$

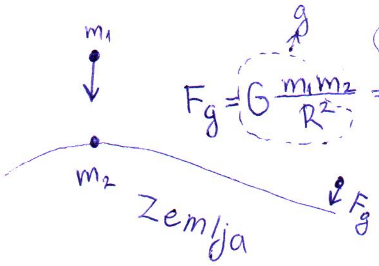
$n_{pre} = \frac{\sigma}{e} = 1,6\cdot10^{14}$

naboji:

- vezani
- prosti $\boxed{Q_{pro} \rightarrow \vec{F}_m}$
- presezni: $\boxed{Q_{pre} \rightarrow \vec{F}_e}$

P3 (1.3.2016)

Biot-Savartov zakon



$$F_g = \underbrace{G\frac{m_1 m_2}{R^2}}_{g} = \underbrace{m}_{\text{objekt}}\cdot g \rightarrow \text{polje}$$

"Stanje prostora"

elektrostatika

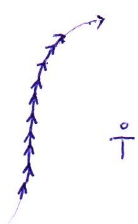
$$\vec{F}_e = \frac{dQ_1\, dQ_2}{4\pi\varepsilon_0 R^3}\vec{R} \;;\; \vec{F}_e = \underbrace{\vec{E}}_{\text{polje}} \underbrace{dQ}_{\text{objekt}} \qquad \vec{E} = \frac{\vec{F}}{Q}$$

$$\boxed{d\vec{F}_m = \underbrace{I\,d\vec{l}}_{\text{objekt tok. elem.}} \times \underbrace{\vec{B}}_{\text{mag. polje}}} = dQ\cdot\vec{v}\times\vec{B}$$

$$d\vec{F}_m = \underbrace{I\cdot d\vec{l}}_{\text{objekt}} \times \underbrace{(I'\cdot d\vec{l}' \times \vec{R})\cdot\frac{\mu_0}{4\pi R^3}}_{d\vec{B}}$$

I': povzročitelj polja

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi R^3}(I\cdot d\vec{l}\times\vec{R})}$$ B. S. zakon



$B(T) = \int d\vec{B}$

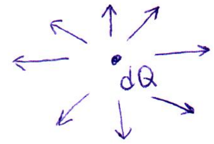
Vpelje mag. polje za tokovni element

Lorentzova sila

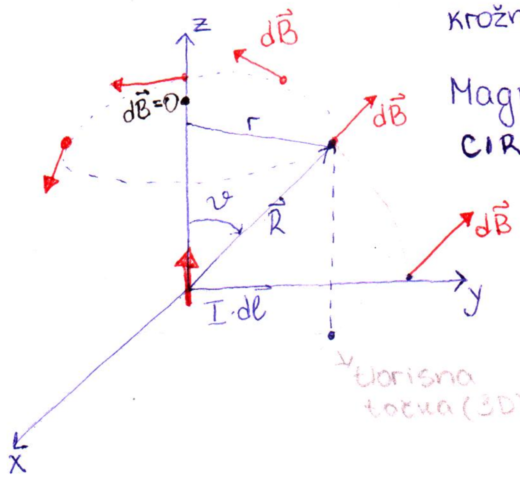
$$\vec{F}_L = \vec{F}_e + \vec{F}_m = dQ\,\vec{E} + dQ\cdot\vec{v}\times\vec{B} = \underline{dQ(\vec{E} + \vec{v}\times\vec{B})}$$

Magnetno polje tokovnega elementa

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\vec{e}_R$$



El polje je RADIALNO



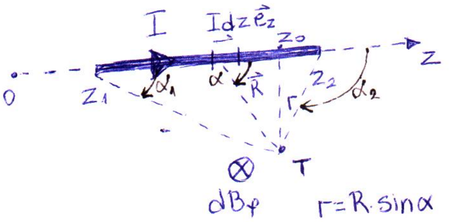
Krožna smer

Magnetno polje je **CIRKULARNO**

$$\vec{e}_{dl} \times \vec{e}_R = \sin\vartheta$$

$$d\vec{B} = \mu_0 \frac{I\,d\vec{l}}{4\pi R^2}\sin\vartheta$$

Magnetno polje daljice



$$d\vec{B} = \mu_0 \frac{I\,d\vec{l}\times\vec{R}}{4\pi R^3}$$

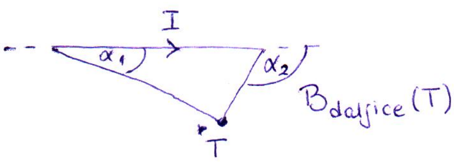
$$dB = \frac{\mu_0 I\,dz\cdot\sin\alpha}{4\pi R^2}$$

$r = R\cdot\sin\alpha \;\rightarrow\; R = \frac{r}{\sin\alpha}$

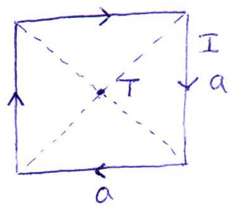
$\mathrm{tg}\,\alpha = \frac{r}{z_0 - z} \;\rightarrow\; z_0 - z = r\cdot\mathrm{ctg}\,\alpha \;/\; \frac{dz}{d\alpha}$; $\underline{dz} = r\cdot\frac{1}{\sin^2\alpha}\cdot d\alpha$

$$B_{daljice} = \int_{z_1}^{z_2} dB_\varphi = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2}\frac{\sin\alpha}{R^2}\,dz = \frac{\mu_0 I\cdot \cancel{r}}{4\pi r^{\cancel{2}}}\int_{\alpha_1}^{\alpha_2}\frac{\cancel{\sin^2\alpha}\cdot\sin\alpha}{\cancel{\sin^2\alpha}}\,d\alpha \Rightarrow$$

$$\boxed{\vec{B}_{daljice} = \vec{e}_\varphi\,\frac{\mu_0 I}{4\pi r}(\cos\alpha_1 - \cos\alpha_2)}$$



Magnetno polje zanke



$I = 100\,A$

$a = 10cm = 0{,}1\,m$

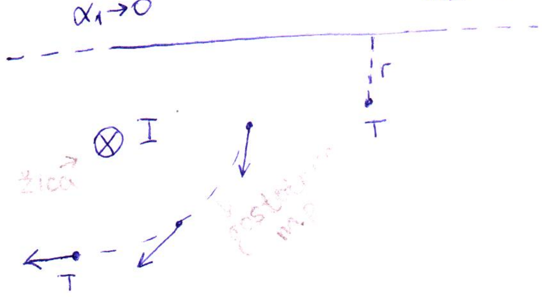
$$\boxed{B_{\square} = 4 \cdot B_{daljice} \quad \alpha_1 = 45° = \frac{\pi}{4} \quad \alpha_2 = 135° = 3\frac{\pi}{4}}$$

$$\Rightarrow B_{\square} = \frac{\mu_0 \cdot 100}{4\pi \cdot 0{,}05}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

$$B_{\square} = 8 \cdot \sqrt{2} \cdot 10^{-4}\ T$$

(Tesla)

Magnetno polje premice



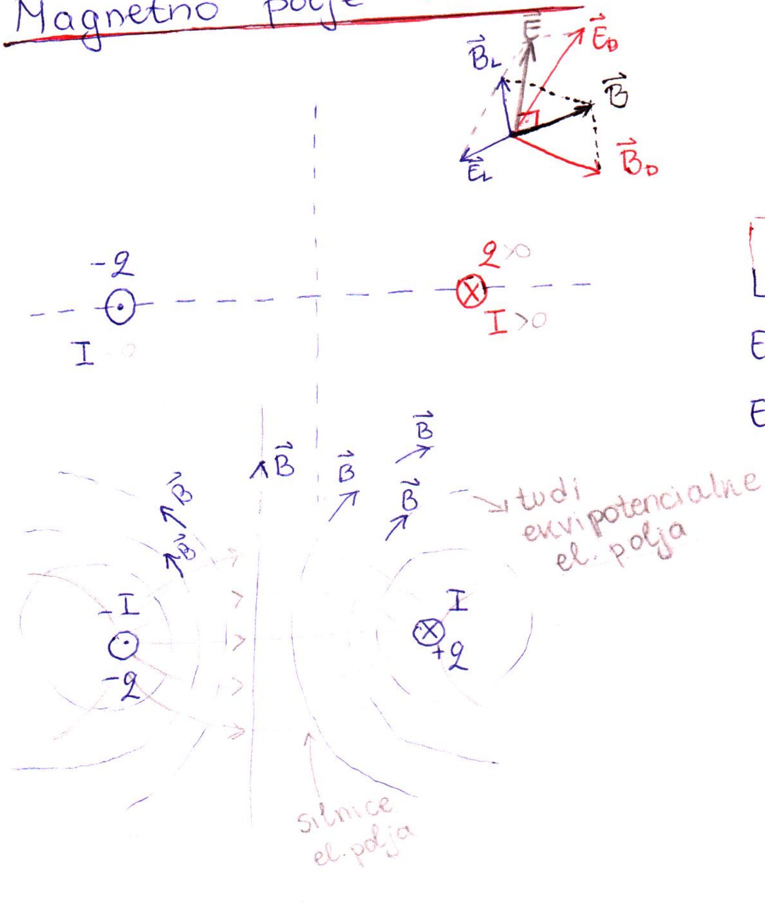
$$\boxed{\vec{B}_{premice} = \vec{e}_\varphi \frac{\mu_0 I}{2\pi r}}$$

cirkularna smer

$$\left\{ \vec{E}_p = \frac{q \cdot \vec{e}_r}{2\pi \varepsilon_0 r} \right\}$$

radialna smer

Magnetno polje dvovoda



$\vec{E}_L \perp \vec{B}_L$

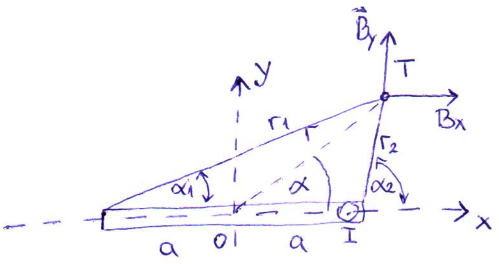
$\vec{E}_D \perp \vec{B}_D$

$$\boxed{\vec{E} \perp \vec{B}}$$

Ekvipotencialne $\perp \vec{E}$

Ekvipotencialne $\parallel \vec{B}$

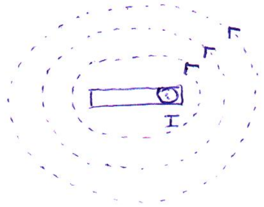
Magnetno polje tračnega vodnika

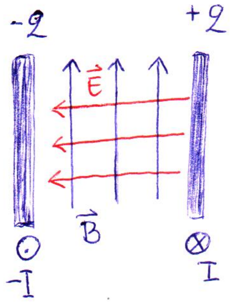


$$B_x(T) = \frac{\mu_0 K}{2\pi}(\alpha_1 - \alpha_2)$$

$$B_y(T) = \frac{\mu_0 K}{2\pi} \ln \frac{r_1}{r_2}$$

$$K = \frac{I}{2a}$$

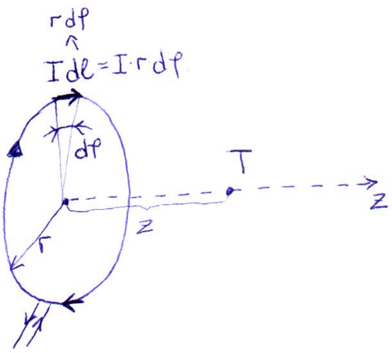


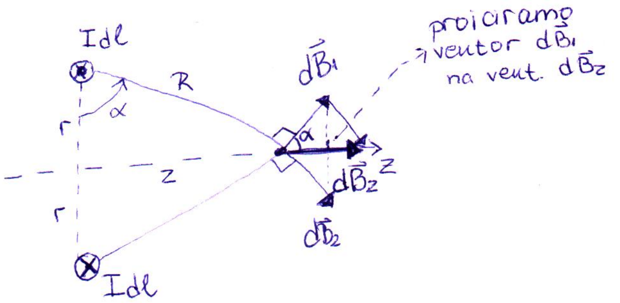


$$B = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K$$

P4 (4.3.2016)

Magnetno polje (v osi) krožnega ovoja



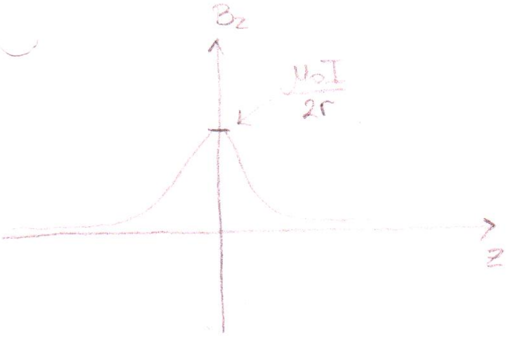


$$dB_z = 2 \cdot dB_{1z}$$

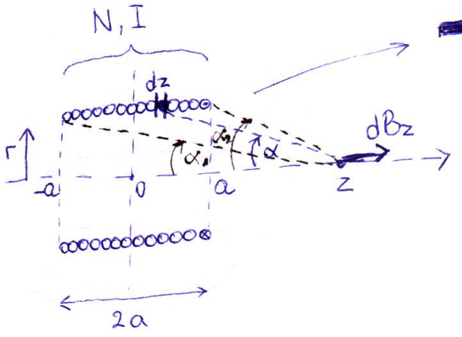
$$dB_{1z} = \underbrace{\frac{\mu_0 I\,dl}{4\pi R^2}}_{\text{B.S.z.}} \cdot \cos\alpha = \frac{\mu_0 I}{4\pi} \cdot \frac{r^2\,d\varphi}{(r^2+z^2)^{3/2}} \qquad \cos\alpha = \frac{r}{R}$$

$$B_z = \int dB_z = \frac{\mu_0 I r^2}{2\pi (r^2+z^2)^{3/2}} \int_0^{\pi} d\varphi = \frac{\mu_0 I r^2}{2(r^2+z^2)^{3/2}}$$

magnetno polje v osi krožnega ovoja



Magnetno polje v osi tanke tuljave



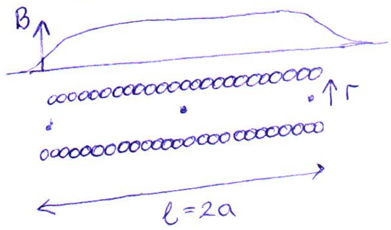
$$K = \frac{N \cdot I}{2a}$$

$$dI = K \cdot dz = \frac{N \cdot I}{2a} \cdot dz$$

$$dB_z = \ldots$$

$$B_z = \frac{\mu_0 I \cdot N}{4a} (\cos\alpha_1 - \cos\alpha_2)$$

Dolga tanka tuljava



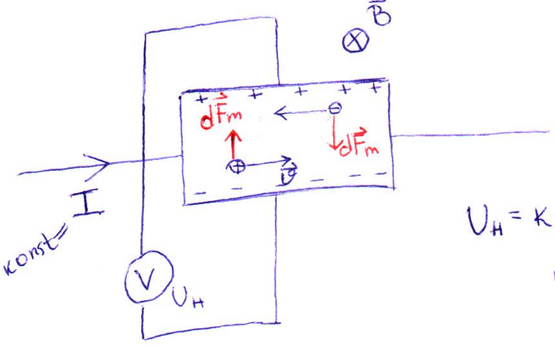
$l >> r$

$$B_{središče}\Big|_{\substack{\alpha_1 \Rightarrow 0 \\ \alpha_2 \Rightarrow \pi}} = \frac{\mu_0 I N}{2a} = \frac{\mu_0 I N}{l}$$

$$B_{robu}\Big|_{\substack{\alpha_1 \Rightarrow 0 \\ \alpha_2 \Rightarrow \pi/2}} = \frac{\mu_0 I N}{4a} = \frac{1}{2} B_{središča} = \frac{\mu_0 I N}{2l}$$

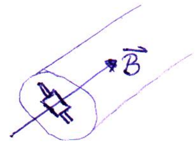
Hallov pojav in merilnik mag. polja - Teslameter

* "pravilo leve roke"



$$d\vec{F}_m = I d\vec{l} \times \vec{B}$$

$$= dQ \vec{v} \times \vec{B}$$



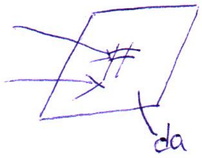
$$U_H = \underbrace{K \cdot I}_{konst} \cdot B = K \cdot B$$

P5 (8.3.2016)

Magnetni pretok

OE1: $\Phi_e = \int \vec{D}\cdot d\vec{a} \rightarrow \oint_A \vec{D}\cdot d\vec{a} = Q_{notr.\ prosti}$

Φ_e ↓ električni pretok; $\vec{D}$ ↘ gostota el. pretoka

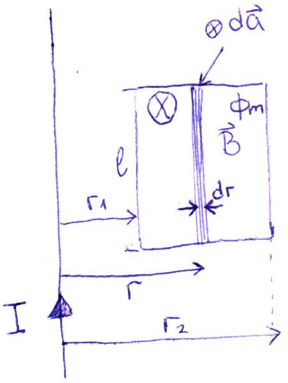


$$\boxed{\Phi_m = \int \vec{B}\cdot d\vec{a}}$$

; $\vec{B}$ - vektor gostote magnetnega pretoka

$d\vec{a}$ ↓ vektor

Magnetni pretok ravnega vodnika skozi pravokotno zanko

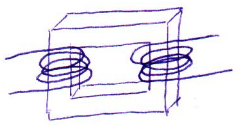


$B_\varphi = \frac{\mu_0 I}{2\pi r}$

$\Phi_m = \int \underbrace{\vec{B}\cdot d\vec{a}}_{sk.\ prod.} = \int B_\varphi \cdot \overset{=l\cdot dr}{da} = \frac{\mu_0 I l}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} \Rightarrow$

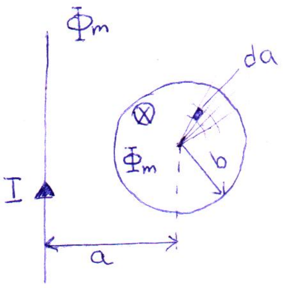
$B_\varphi \parallel da$ ↓ $\cos 0° = 1$

transformator



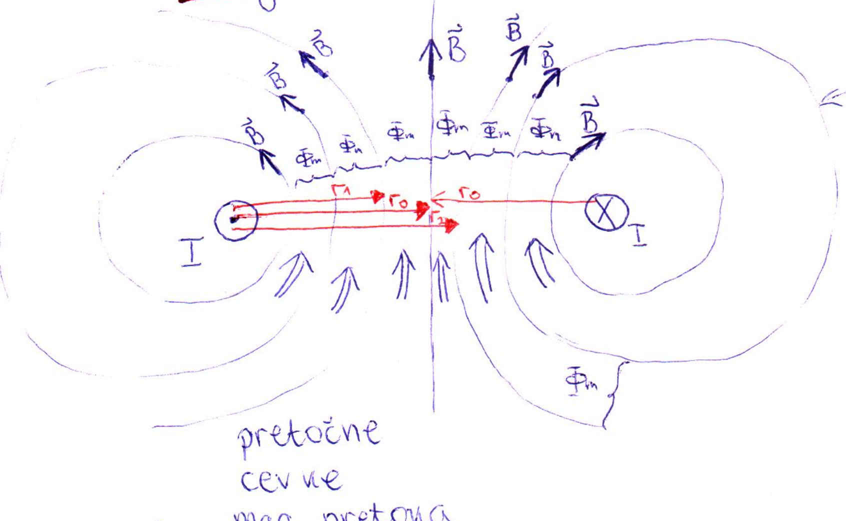
$$\Rightarrow \boxed{\Phi_m = \frac{\mu_0 I l}{2\pi}\cdot \ln\frac{r_2}{r_1}}$$

Magnetni pretok ravnega vodnika skozi okroglo zanko



$$\boxed{\Phi_m = \mu_0 I\left(a - \sqrt{a^2-b^2}\right)}$$

Magnetni pretok dvovoda in pretočne cevke



gostotnice mag. polja ≡ ekvipotencialke ⊥ silnice $\vec{E}$

$$\boxed{\Phi_m = \frac{\mu_0 I l}{2\pi}\left(\ln\frac{r_0}{r_1} + \ln\frac{r_2}{r_0}\right) = \frac{\mu_0 I l}{2\pi}\ln\frac{r_2}{r_1}}$$

pretočne cevke mag. pretoka

Značilnosti magnetnega pretoka

1) Izvornost $\vec{B}$ (Gaussov stavek)

$$\oint_A \vec{J}\cdot d\vec{a} = -\frac{dQ_{not}}{dt}$$

$$\oint_A \vec{E}\cdot d\vec{a} = +\frac{Q_{not}}{\varepsilon_0}$$

$$\oint_A \vec{D}\cdot d\vec{a} = +Q_{not.\,prosti}$$

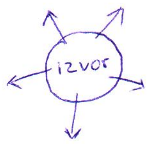
$$\oint_A \vec{P}\cdot d\vec{a} = -Q_{not.\,polarizacijski}$$

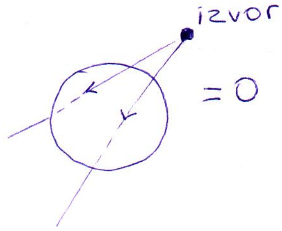
$$\oint_A \vec{B}\cdot d\vec{a} = 0$$

↗ III Maxwellova enačba

$? = \oint_A \vec{x}\cdot d\vec{a}$

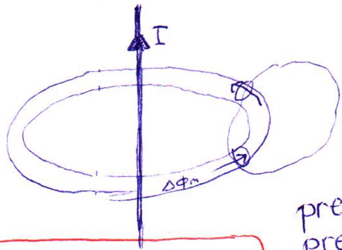
$\neq 0$





$= 0$

$$\oint_A \vec{B}\cdot d\vec{a} = ?$$

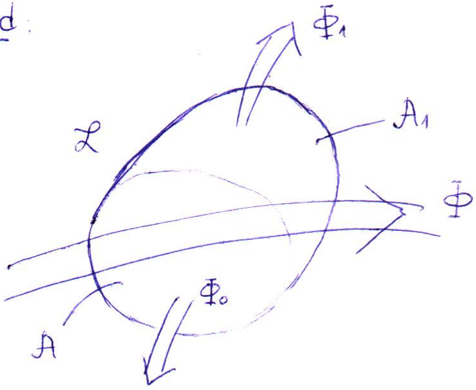


pretočna cevka prebada dvakrat

$$\boxed{\oint_A \vec{B}\cdot d\vec{a} = \sum \int_A \underbrace{\vec{B}\cdot d\vec{a}}_{\Delta\Phi_m} = 0}$$

Magnetno polje ni izvorno!

Zgled:



$$\Phi = \int_{A_1} \vec{B}\, d\vec{a}$$

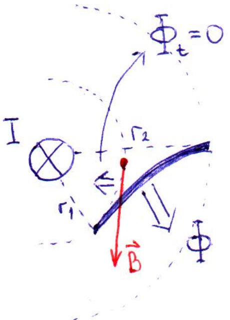
$$\Phi = -\int_{A} \vec{B}\, d\vec{a} \quad / \cdot (-1)$$

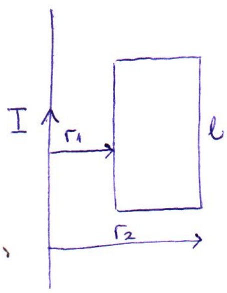
Σ

$$\underbrace{\int_{A_1} \vec{B}\cdot d\vec{a}}_{\Phi_1} + \underbrace{\int_{A} \vec{B}\cdot d\vec{a}}_{\Phi_2} = 0 \Rightarrow \Phi_1 = -\Phi_0$$

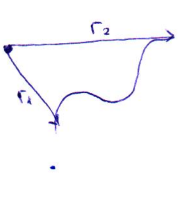
$$\Rightarrow \Phi_0 = -\Phi_1$$

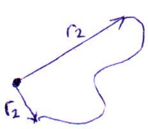
Φ_m skozi "zvijugan trak" v okolici premega vodnika



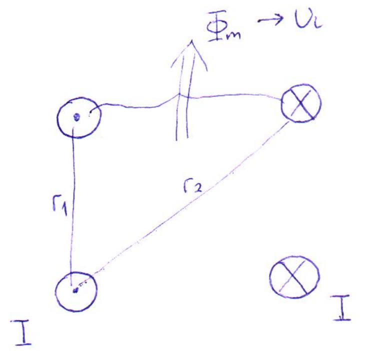


$$\boxed{\Phi_m = \frac{\mu_0 I l}{2\pi} \ln\frac{r_2}{r_1}}$$





Zgled: DVA DVOVODA



$$\Phi_m = \Phi_{mL} + \Phi_{mD} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1} \cdot 2$$

$$\boxed{\Phi_m = \frac{\mu_0 I l}{\pi} \ln \frac{r_2}{r_1}}$$

*recipročnost

*spodnji dvovod na zgornjega (in obratno)

P6 (11.3.2016)

Vrtinčnost vektorja magnetnega pretoka

Značilnosti $\vec{B}$

Gaussov stavek

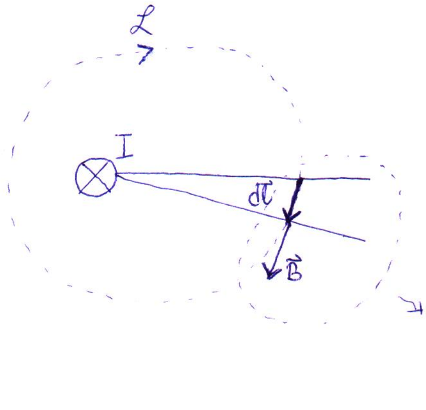
-(Ne)izvornost $\vec{B}$: $\oint_A \vec{B} \cdot d\vec{a} = 0$ → računanje Φ_m

-Vrtinčnost $\vec{B}$

Stokesov stavek: $\oint_{\mathcal{L}} \vec{X} \cdot d\vec{l} = ?$

$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0$ (elektrostatično polje je nevrtinčno) ; $\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = ?$

1) Vrtinčnost $\vec{B}$ premega toka



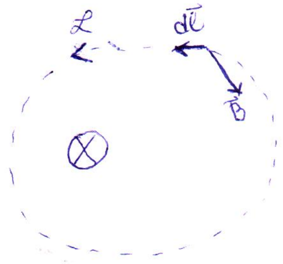
$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = \oint_{\mathcal{L}} \frac{\mu_0 I}{2\pi a} \cdot dl_B = \oint_{\mathcal{L}} \frac{\mu_0 I}{2\pi a} a \, d\varphi = \frac{\mu_0 I}{2\pi} \underbrace{\oint_{\mathcal{L}} d\varphi}_{2\pi} = \mu_0 I$$

↓

projekcija $d\vec{l}$ na $\vec{B}$:

dl_B - dolžini loka: $a \cdot d\varphi$

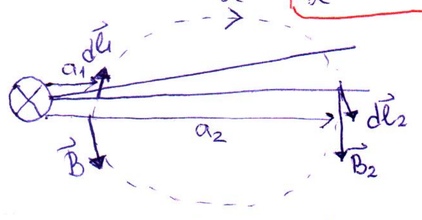
2)



$$\boxed{\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = -\mu_0 I}$$

3)

$$\boxed{\oint_{\mathcal{L}} \vec{B} d\vec{l} = 0}$$



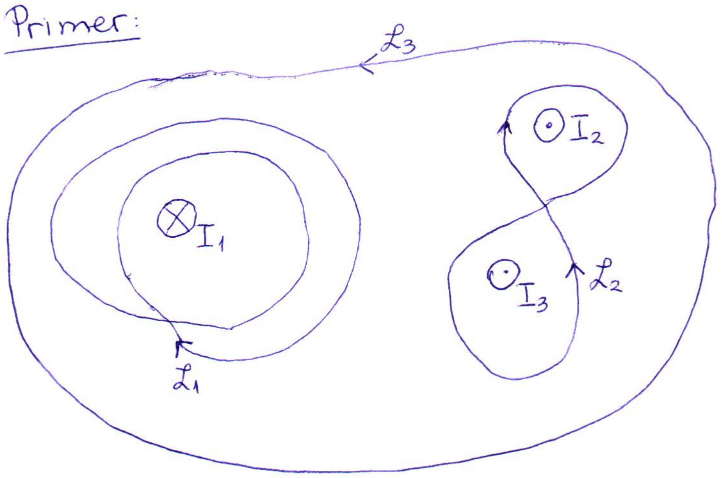
$$\vec{B}_1 d\vec{l}_1 + \vec{B}_2 d\vec{l}_2$$

$$-\frac{\mu_0 I}{2\pi a_1} a_1 d\varphi + \frac{\mu_0 I}{2\pi a_2} a_2 d\varphi = 0$$

$$\boxed{\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{zaobjeti}} \;=\; \mu_0 N \cdot I}$$

N ↑ št. ovojev

Primer:

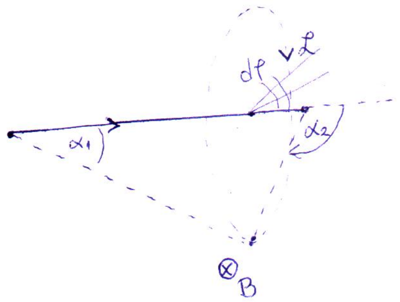


$$\oint_{\mathcal{L}_1} \vec{B}\cdot d\vec{l} = 2\mu_0 I_1$$

$$\oint_{\mathcal{L}_2} \vec{B}\cdot d\vec{l} = \mu_0 (I_3 - I_2)$$

$$\oint_{\mathcal{L}_3} \vec{B}\cdot d\vec{l} = \mu_0 (I_2 + I_3 - I_1)$$

Vrtinčnost $\vec{B}$ tokovne daljice

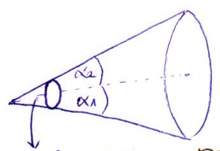


$$B_\varphi = \frac{\mu_0 I}{4\pi r}(\cos\alpha_1 - \cos\alpha_2)$$

$$\oint B_\varphi \cdot \underbrace{dl}_{r\cdot d\varphi} = \frac{\mu_0 I}{4\pi}(\cos\alpha_1 - \cos\alpha_2)\oint_{\mathcal{L}} \underbrace{d\varphi}_{2\pi}$$

$$\oint B_\varphi dl = \frac{\mu_0 I}{2}\underbrace{(\cos\alpha_1 - \cos\alpha_2)}_{(1-\cos\alpha_2 - 1 + \cos\alpha_1) = \Omega_2 - \Omega_1} \Big/ \cdot \frac{4\pi}{4\pi}$$

$$\boxed{\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \mu_0 I \frac{\Omega_2 - \Omega_1}{4\pi}}$$

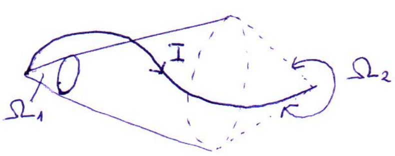


prostorski kot: Ω

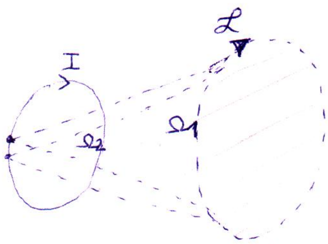
polni prostorski kot: 4π

$$\Omega = 2\pi(1-\cos\alpha)$$

Posplošitev vrtinčnosti $\vec{B}$



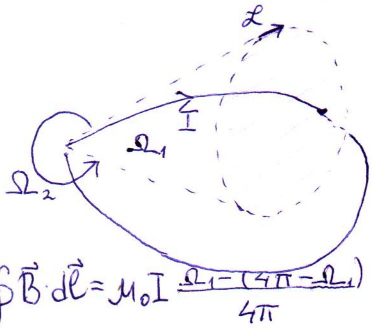
$\Omega_1 = \Omega_2$



$$\oint \vec{B} d\vec{l} = 0$$

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{zaobjeti}}}$$

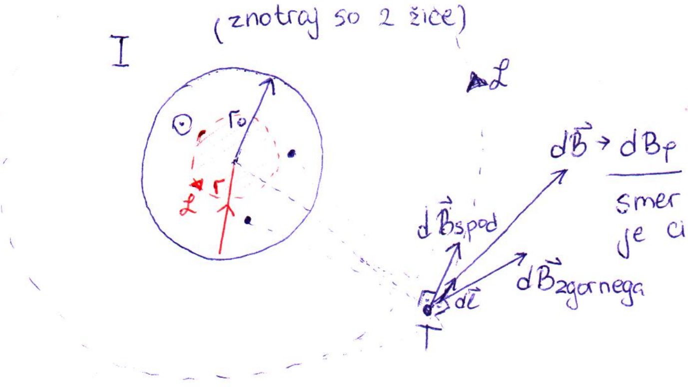
- premi vodnik
- vse skljenjene vodniki



$\Omega_1 + \Omega_2 = 4\pi$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \frac{\Omega_1 - (4\pi - \Omega_1)}{4\pi} = \mu_0 I$$

$\vec{B}$ "debelega" ravnega vodnika okroglega preseka



zunanjost: $\oint_{\mathcal{L}} B_\varphi \cdot dl = \mu_0 I$

$B_\varphi \cdot 2\pi r = \mu_0 I$

$$\boxed{B_\varphi = \frac{\mu_0 I}{2\pi r}} \Rightarrow \vec{B} \text{ premice}$$

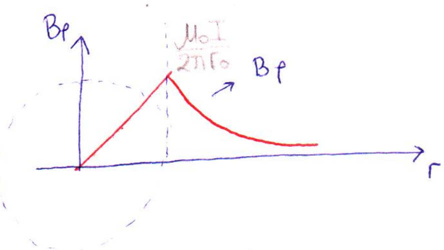
notranjost: $\int B_\varphi \cdot dl = \mu_0 \cdot I_{zaobjeti} = \mu_0 \frac{I}{\cancel{\pi} r_0^2} \cancel{\pi} r^2$

$\int B_\varphi \cdot dl = \mu_0 \frac{I r^2}{r_0^2}$ zaobjeti del toka

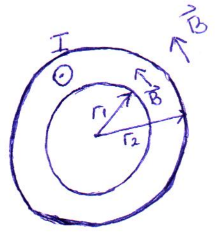
$B_\varphi \cdot 2\pi r = \frac{\mu_0 I r^2}{r_0^2}$

$$\boxed{B_\varphi = \frac{\mu_0 I r}{2\pi r_0^2}}$$

$$B_\varphi = \begin{cases} \frac{\mu_0 I}{2\pi r}, & r \geq r_0 \\ \frac{\mu_0 I r}{2\pi r_0^2}, & r \leq r_0 \end{cases}$$

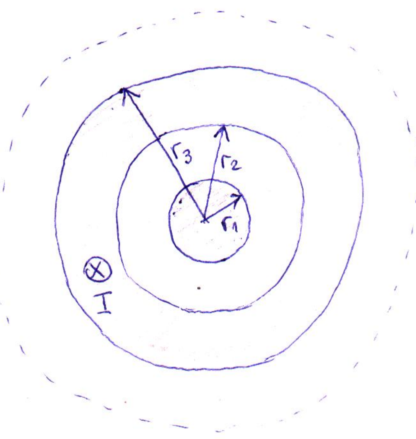


$\vec{B}$ "makarone"



$$B = \begin{cases} 0, & r < r_1 \\ \mu_0 I \frac{r^2 - r_1^2}{r_2^2 - r_1^2}, & r_1 < r < r_2 \\ \frac{\mu_0 I}{2\pi r}, & r_2 < r \end{cases}$$

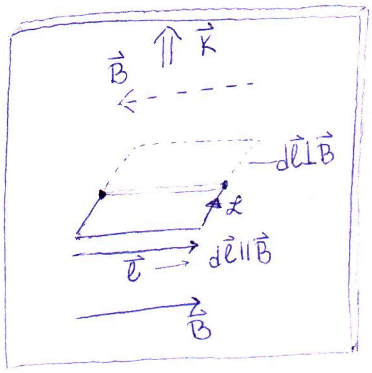
$\vec{B}$ koaksialnega kabla



$$B_\varphi = \begin{cases} \frac{\mu_0 I r}{2\pi \cdot r_1^2}, & r < r_1 \\ \frac{\mu_0 I}{2\pi r}, & r_1 < r < r_2 \\ \dots, & r_2 < r < r_3 \\ 0, & r_3 < r \end{cases}$$

$\dots, \quad r_2 < r < r_3 \longrightarrow \frac{\mu_0 I}{2\pi(r_3 - r_2)}\left(\frac{r_3^2}{r} - r\right)$

$\vec{B}$ v okolici stene s ploskovnim tokom - folijski tok



$$\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \mu_0 I_{zaobjeti}$$

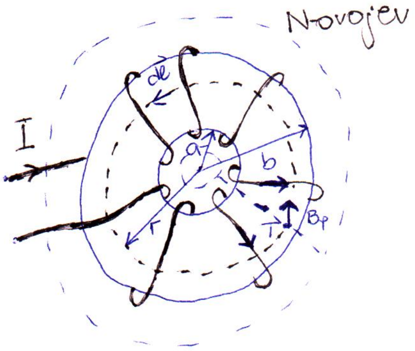
$$= B\cdot l + 0 + B\cdot l + 0 = K\cdot l$$

$$\boxed{\vec{B} = \frac{\mu_0 \vec{K}}{2}} \qquad \left(E = \frac{\sigma}{2\varepsilon_0}\right)$$

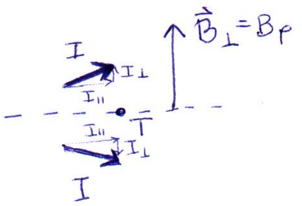
P7 (15.3.2016)

$$\boxed{\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \mu_0 I_{zaobjeti}}$$ Amperov zakon (Zakon o vrtinčnosti)

$\vec{B}$ toroidnega navitja



$$B = \begin{cases} 0 & , \quad r<a \\ \dfrac{\mu_0 N I}{2\pi r} & , \quad a<r<b \\ 0 & , \quad b<r \end{cases}$$



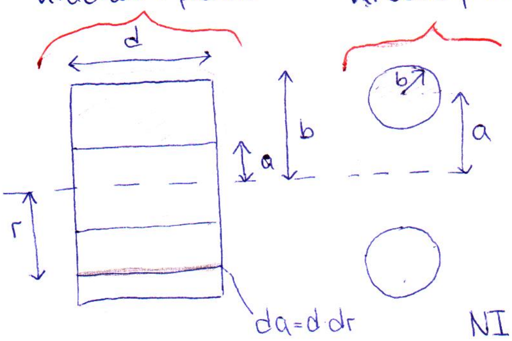
$$B\cdot \oint dl = 0$$

$B_\varphi \parallel dl$

$$\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = B_\varphi \int dl = B_\varphi \cdot 2\pi r = \mu_0 N I$$

Preseki toroida, magnetni pretok toroida

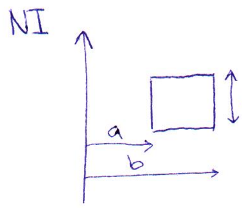
kvadratni prerez   krožni prerez



$$\Phi_{m\square} = \int \vec{B}\cdot d\vec{a} = \frac{\mu_0 N I d}{2\pi}\int_a^b \frac{1}{r}\,dr$$

$$\boxed{\Phi_{m\square} = \frac{\mu_0 I N d}{2\pi}\ln\frac{b}{a}}$$ → magnetni pretok toroida (kvadratni presek)

$$B_\varphi 2\pi r = \mu_0 N I$$



$$\boxed{L_\square = \frac{\mu_0 N^2 d}{2\pi}\ln\frac{b}{a}}$$

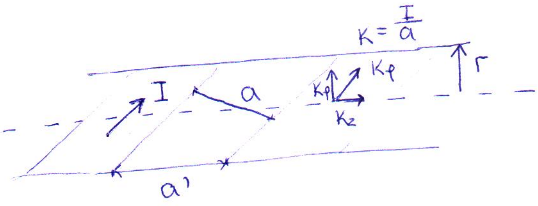
induktivnost toroida (□)

$$\boxed{\Phi_{m0} = \mu_0 N I \left(a - \sqrt{a^2 - b^2} \right)}$$ → magnetni pretok toroida (krožnega preseka)

$$\boxed{L_0 = \mu_0 N^2 \left(a - \sqrt{a^2 - b^2} \right)}$$ → induktivnost toroida (o)

$$L = N \frac{\Phi}{I}$$

Spiralno navit oklop

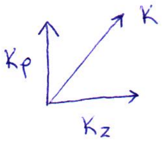


a – širina traku

a' – korak navijanja

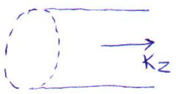
r – polmer kabla

$$(a')^{-2} + (2\pi r)^{-2} = a^{-2}$$

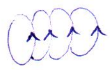


– dekompozicija toka

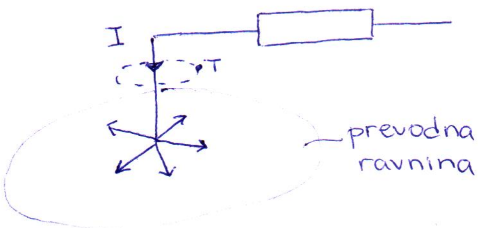
1. K_z – povzroča B kot "tanka cev"

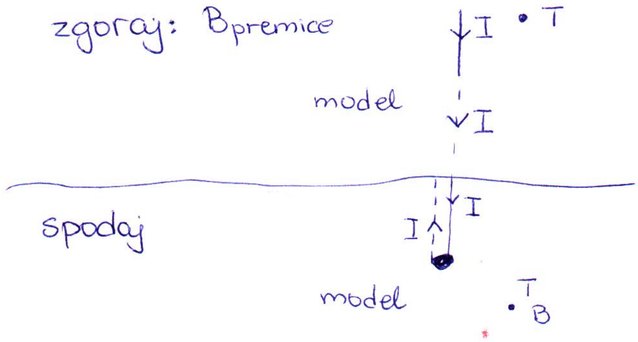


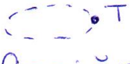
2. K_φ – povzroča B kot solenoid



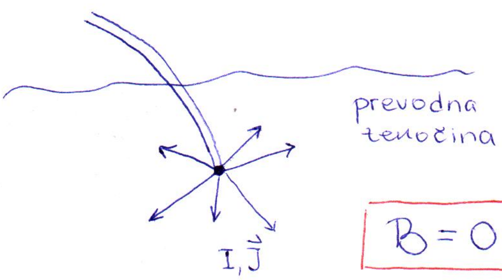
Radialni tok







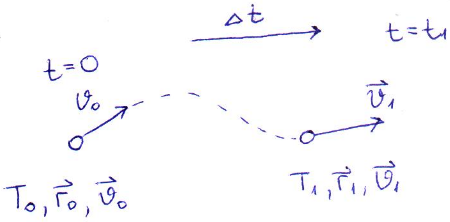
Sferični tok



$$\boxed{B = 0}$$

www.stromar.si

Gibanje delca v $\vec{E}$ in $\vec{B}$



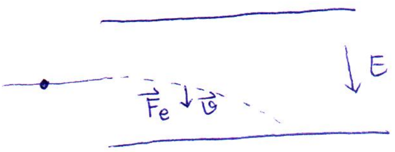
$$\vec{a} = \dot{\vec{v}} = \frac{\vec{v}_1 - \vec{v}_0}{\Delta t}$$

$$\vec{v}_1 = \vec{v}_0 + \dot{\vec{v}}\Delta t$$

$$\vec{r}_1 = \vec{r}_0 + \vec{v}_0 \Delta t + \frac{1}{2}\dot{\vec{v}} t^2$$

$$\dot{\vec{v}} = \vec{a} = \frac{\vec{F}_L}{dm} \quad ; \quad \vec{F}_L = dQ(\vec{E} + \vec{v} \times \vec{B})$$

Gibanje delca v $\vec{E}$



$$\dot{\vec{v}} = \underbrace{\frac{dQ}{dm}}\vec{E}$$

pospešek je konstanten
pospešek je usmerjen v smeri polja

} trajektorija gibanja je parabola

Gibanje delca v $\vec{B}$

$$\dot{\vec{v}} = \frac{dQ}{dm}(\vec{v} \times \vec{B}) \quad / \cdot \vec{v}$$

$$\vec{v}\dot{\vec{v}} = \frac{dQ}{dm}(\vec{v} \times \vec{B}) \cdot \vec{v} = \frac{dQ}{dm}\overbrace{(\vec{v} \times \vec{v})}^{0} \cdot \vec{B}$$

$^*(y^2)' = 2y \cdot y'$

$\frac{1}{2}(y^2)' = y \cdot y'$

$v \cdot v \cdot \sin\varphi = 0$

$*$

$$\frac{1}{2}(\vec{v}^2)' = 0$$

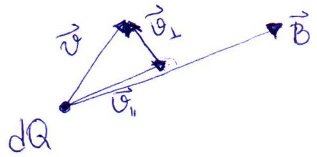
$$(\vec{v}^2)' = 0$$

$$v^2 = konst \rightarrow \underline{\underline{v = konst}}$$

$$\dot{\vec{v}}_\perp + \overset{\,0}{\cancel{\dot{\vec{v}}_\parallel}} = \frac{dQ}{dm}(\vec{v}_\perp \times \vec{B}) + \frac{dQ}{dm}\underbrace{(\vec{v}_\parallel \times \vec{B})}_{0}$$

$\downarrow$

$\perp$ na $v_\perp$ in $\vec{B}$



$v = konst$
$\dot{\vec{v}}_\parallel, \vec{v}_\parallel = 0$

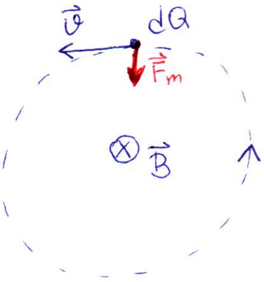
} $\underline{\underline{\vec{v}_\perp = konst}}$ => enakomerno kroženje

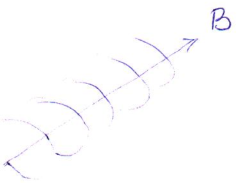
$$\dot{v}_\perp = a_r = \frac{v_\perp^2}{R} = \frac{dQ}{dm}(|v_\perp| \cdot B)$$

$$\boxed{R = \frac{dm \cdot v}{dQ \cdot B}}$$

Kroženje delca v $\vec{B}$

$$T = \frac{2\pi r}{v} = 2\pi \cdot \frac{dm}{dQ \cdot B}$$



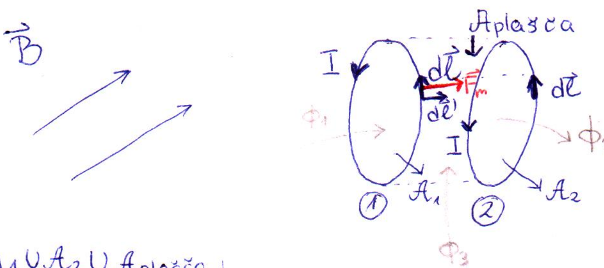


P8(18.3.2016)

Interakcija magnetnih sil

- med dvema premima vodnikoma: sili nista vzajemni
- med dvema zankama: sili sta vzajemni

Delo magnetne sile tokovne zanke



$A_1 \cup A_2 \cup A_{plašča}$

$$d\vec{F}_m = I d\vec{l} \times \vec{B}$$

(usmerjena v notr. plašča)

$$dA = \int_{\mathcal{L}} d\vec{F}_m \cdot d\vec{l}' = I \int (d\vec{l} \times \vec{B}) \cdot d\vec{l}' = I \int \overbrace{(d\vec{l}' \times d\vec{l})}^{d\vec{a}} \cdot \vec{B}$$

$$dA = I \int \vec{B} \cdot d\vec{a}$$

$$A = \int_{\text{od (1) lege}}^{\text{do (2) lege}} dA = I \int \vec{B}\, d\vec{a} = I \cdot \Phi_{\substack{\text{v notranjost} \\ \text{skozi plašč}}}$$

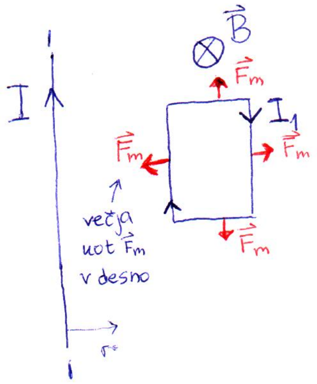
$$\oint_{\mathcal{A}} \vec{B} \cdot da = 0$$

$$\boxed{A = I \cdot (\Phi_2 - \Phi_1)} \quad \{ A = U \cdot Q$$

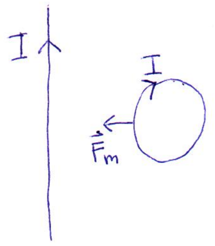
$$-\Phi_1 - \Phi_{\substack{\text{v notr.} \\ \text{plašča}}} + \Phi_2 = 0$$

$$\Phi_3 = \Phi_{\substack{\text{v notr.} \\ \text{pl.}}} = \Phi_2 - \Phi_1$$

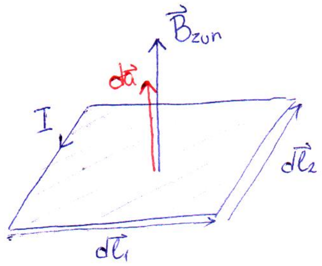
Zgled: Smer premika



$$B=\frac{\mu_0 I}{2\pi r}$$



Navor na tokovno zanko v $\vec{B}$



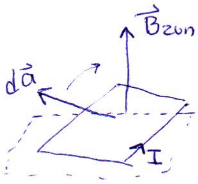
$$d\vec{M}=\vec{r}\times d\vec{F}=-\frac{d\vec{l}_2}{2}\times\left(I\,d\vec{l}_1\times\vec{B}\right)+\frac{d\vec{l}_1}{2}\times\left(I\,d\vec{l}_2\times\vec{B}\right)+$$

$$+\frac{d\vec{l}_2}{2}\times\left(-I\,d\vec{l}_1\times\vec{B}\right)-\frac{d\vec{l}_1}{2}\times\left(-I\,d\vec{l}_2\times\vec{B}\right)$$

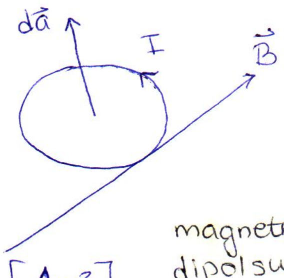
$$\boxed{\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\cdot\vec{b}-(\vec{a}\cdot\vec{b})\cdot\vec{c}}$$

$$d\vec{M}=I\,(\underbrace{d\vec{l}_1\times d\vec{l}_2}_{d\vec{a}})\times\vec{B}=I\cdot(d\vec{a}\times\vec{B})$$

* Navor zasuka zanko v smeri polja: $\vec{B}\parallel d\vec{a}$: $\varphi=0$, $\sin\varphi=0$, $M=0$



Magnetni dipol



Elektrostatika

d, dQ, $\vec{E}_{zun}$, $-dQ$

$d\vec{p}=\vec{d}\cdot dQ$ — elektrostat. dipolski moment

$\vec{M}_e=d\vec{p}\times\vec{E}$

$$d\vec{m}=I\cdot d\vec{a}$$

magnetni dipolski moment $[Am^2]$

$$d\vec{M}_m=I\,d\vec{a}\times\vec{B}$$

→ navor na zanko

Snov v magnetnem polju

prevodniki: influenca $\vec{E}=0$

izolanti: polarizacija

$\chi_e \to \varepsilon_r$ (susceptibilnost)

$\vec{P}, \vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

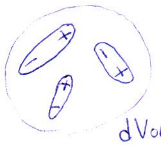
$\vec{B}, \vec{M}, \vec{H} \to$ magnetna poljska jakost

$\uparrow$ magnetizacija

Vektor magnetizacije

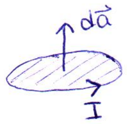
def. polarizacija:

$$\vec{P} = \lim_{dV_{ol} \to 0} \frac{\sum d\vec{p}}{dV_{ol}}$$



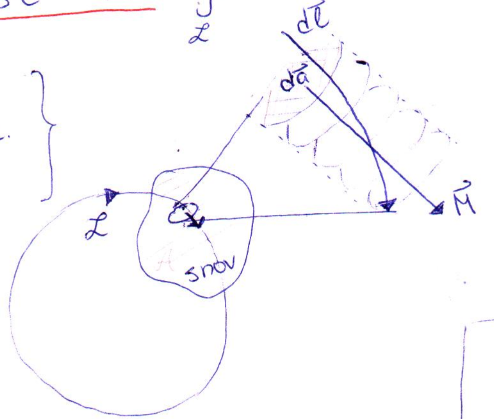
$$\vec{M} = \lim_{dV_{ol} \to 0} \frac{\sum d\vec{m}}{dV_{ol}}$$

[A/m]



Vrtinčnost $\vec{M}$: $\oint_{\mathcal{L}} \vec{M} d\vec{l}$

$$\oint_{\mathcal{A}} \vec{P} d\vec{a} = -Q_{not.\,pol.}$$



$$\vec{M} \cdot d\vec{l} = \frac{I_{mag} \cdot d\vec{a}}{dV_{ol}} \cdot d\vec{l}$$

$$(dV_{ol} = d\vec{a} \cdot d\vec{l})$$

$$\vec{M} \cdot d\vec{l} = dI_{mag}$$

$$\boxed{\oint_{\mathcal{L}} \vec{M} \cdot d\vec{l} = I_{mag}}$$

Vektor magnetne poljske jakosti : $\vec{H}$

$$\oint_{\mathcal{A}} \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\varepsilon_0} \quad / \cdot \varepsilon_0$$

$$\oint_{\mathcal{A}} \vec{P} \cdot d\vec{a} = -Q_{not.\,pol}$$

$+$

$$\oint_{\mathcal{A}} \underbrace{(\varepsilon_0 \vec{E} + \vec{P})}_{\vec{D}} d\vec{a} = Q_{not.\,prosti}$$

$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = \mu_0 I^{\text{vsi toki}} \quad / : \mu_0$$

$$\oint_{\mathcal{L}} \vec{M} \cdot d\vec{l} = I_{mag.} \to \text{tok magnetizacije}$$

$-$

$$\oint_{\mathcal{L}} \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{\vec{H}} d\vec{l} = I_{prosti}$$

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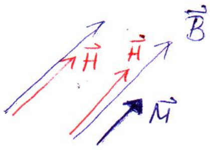
$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}}$$ vektor magnetne poljske jakosti $[A/m]$



$\vec{M} \parallel \vec{B}$

$\vec{M} \propto \vec{B}$

} linearne snovi

$$\vec{M} = \chi_m \vec{H}$$

$$\mu_0(\vec{H} + \chi_m \cdot \vec{H}) = \vec{B}$$

χ_m — magnetna susceptibilnost

$$\boxed{\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \int_{A} \vec{J}_{prosti} \cdot d\vec{a}}$$

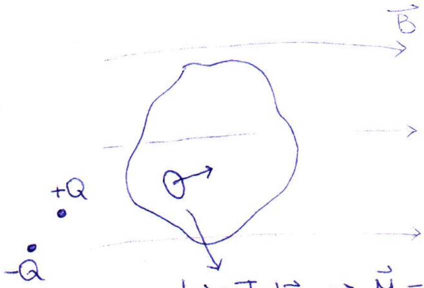
$$\mu_0(\underbrace{1 + \chi_m}_{\mu_r})\vec{H} = \vec{B}$$

$$\vec{B} = \mu_0 \mu_r \vec{H}$$

μ_r — relativna permeabilnost snovi

P9 (22.3.2016)

Snov v magnetnem polju



$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu_0 \mu_r \vec{H}$$

$$\{\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}\}$$

μ_r, ε_r — vpliv snovi

$\vec{H}$ — magnetna poljska jakost

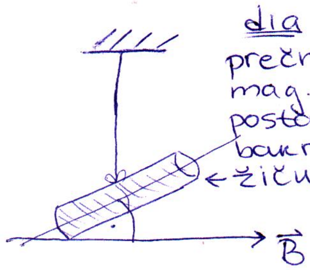
$d\vec{m} = I d\vec{a} \rightarrow \vec{M}$ – volumska gostota $d\vec{m}$

Načini magnetenja snovi

- diamagnetne snovi (Cu)
- paramagnetne snovi (Al)
- feromagnetne snovi (Fe, Ni, Co, Gd, Dy)

① Diamagnetizem (dia = prečno)

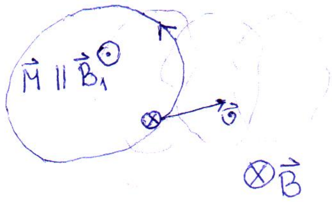
- nimajo izražene mag. dipolov
- ošibijo zunanje mag. polje



dia

prečno na mag. polje se postavi

$$\chi_m \doteq -10^{-5} \rightarrow \mu_r \doteq 0{,}99999$$

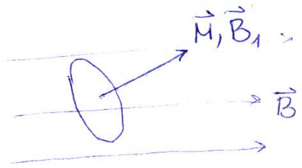


$$\vec{M} \cdot \vec{B} < 0$$

$\uparrow \downarrow$

② Paramagnetne snovi (para = vzporedno)

- imajo šibko izražene mag. dipole

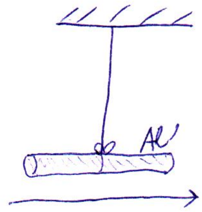


$\chi_m = +10^{-5}$

$\mu_r = 1{,}00001$

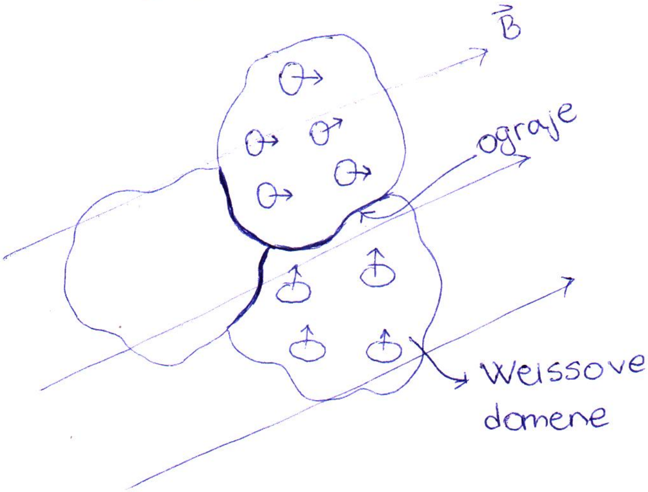
$\vec{M} \parallel \vec{B}$

$\vec{M} \cdot \vec{B} > 0$



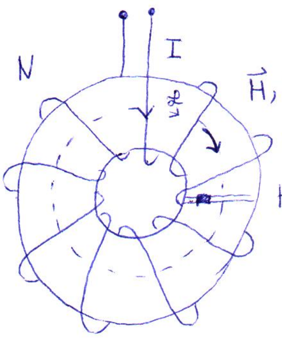
Al žička se postavi v smeri polja $\vec{B}$

③ Feromagnetizem



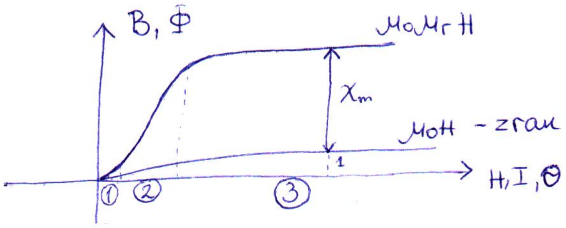
- Weissove domene se orientirajo v smeri $\vec{B}$
- Stapljenje domen
- Nelinearen odziv

B-H diagram



Hallov senzor: merimo B na y osi

$$\oint \vec{H}\, d\vec{l} = I_{zaobjeti} = NI \rightarrow H = \frac{NI}{2\pi r} = f(r) \text{ na x-osi}$$



① cona je reverzibilno področje obračanja domen

② nereverzibilno področje močne ojačanja

③ nasičenje

$\chi_m = 10 - 10^6$

$\mu_r = 10 - 10^6$

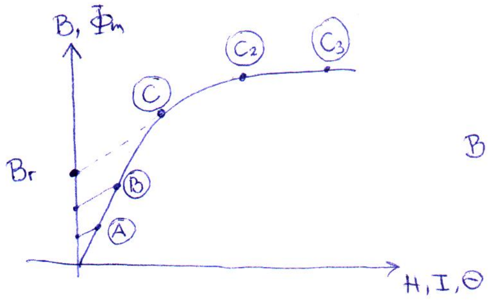
$(\mu_r = 1 + \chi_m)$

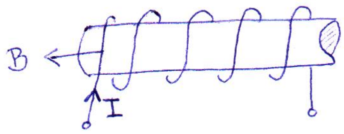
* zaradi nelinearnega odziva, μ_r ni konstantna vrednost.

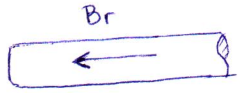
$$\boxed{\vec{B} = \mu_0 (\vec{H} + \vec{M})}$$

zveza med $\vec{B}, \vec{H}, \vec{M}$

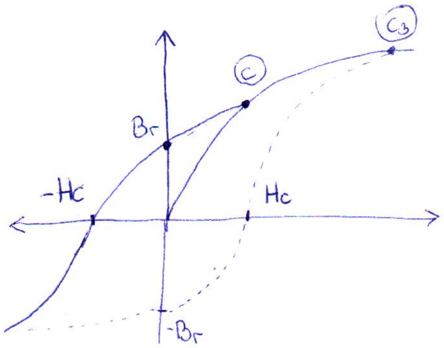
Remenentna gostota B_r







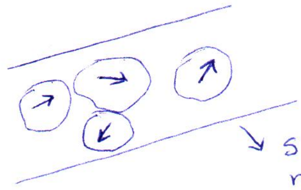
Koercitivna poljsua jauost H_c



-razmagnetenje

Curiejeva temperatura

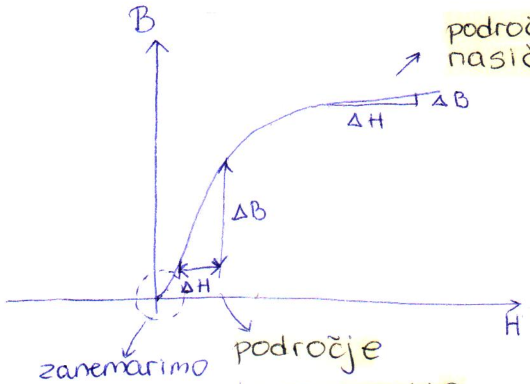
Fe : 770°C



snov izgubi magnetne lastnosti

Udarci in razmagnetenje

Magnetilno krivulje



področje nasičenja $\mu_0 = \frac{\Delta B}{\Delta H}$

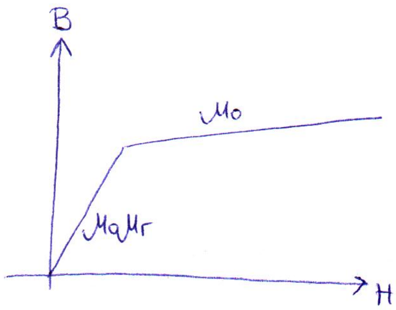
zanemarimo

področje linearizacije

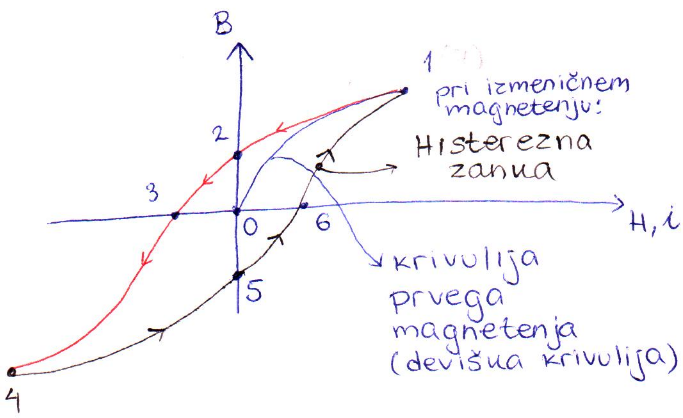
$\mu_r \mu_0 = \frac{\Delta B}{\Delta H}$

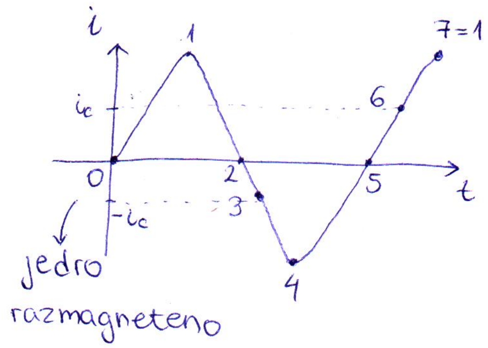
$\vec{B} = \mu_0 \mu_r \vec{H}$

Linearizacija mag. krivulje



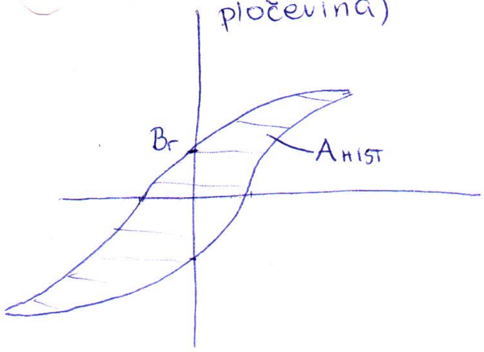
Histerezna zanka





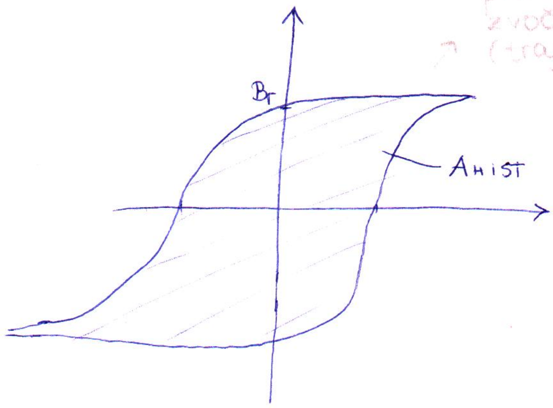
Dve gružini feromag. materialov

Mehkomagnetne snovi ($H_c \sim 10 A/m$)
(transformatorska pločevina)

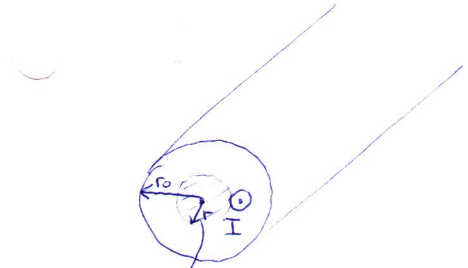


Trdomagnetne snovi ($H_c \sim 10^5 A/m$)

primerni za zvočniki (trajne magnete)



Zgled: B v in ob jeklenem vodniku (feromagnetnem)



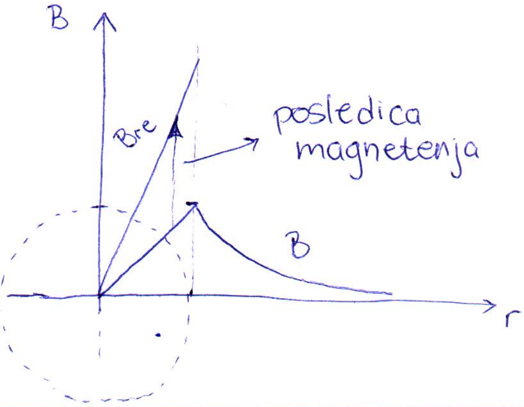
$$I' = \frac{I}{\pi r_0^2} \cdot \pi r^2$$

$$\oint \vec{H} \cdot d\vec{l} = 2\pi r \cdot H_\varphi = I_{zaobjet}$$

$$H_\varphi = \begin{cases} \frac{I \cdot r}{2\pi r_0^2} & \text{, znotraj} \\ \frac{I}{2\pi r} & \text{, zunaj} \end{cases}$$

$\mu_r = 3$

$$B_\varphi = \mu_0 \mu_r H = \begin{cases} \frac{\mu_0 \mu_r I r}{2\pi r_0^2} & \text{, znotraj} \\ \frac{\mu_0 \mu_r I}{2\pi r} & \text{, zunaj} \end{cases}$$



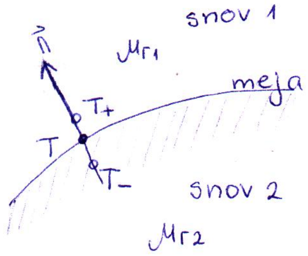
P10 (25.3.2016)

Mejna pogoja $\vec{B}, \vec{H}$

$$\oint_A \vec{B} \cdot d\vec{a} = 0 \quad : \quad \boxed{\vec{e}_n(\vec{B}(T_+) - \vec{B}(T_-)) = 0 \qquad B_{n1} = B_{n2}}$$

$$\oint_A \vec{J}\, d\vec{a} = 0 \rightarrow J_n \text{ - zvezna}$$

mejni pogoj za gostoto $\vec{B}$.



$$\oint_L \vec{H} \cdot d\vec{l} = I$$

$$H_x(T_+) \cdot (-\Delta x) + H_x(T_-) \cdot (\Delta x) = K_z \cdot \Delta x = I$$

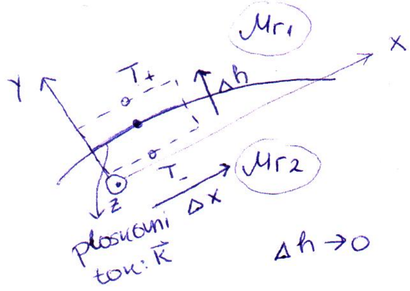
$$H_x(T_-) - H_x(T_+)$$

H_y ..

H_z ..

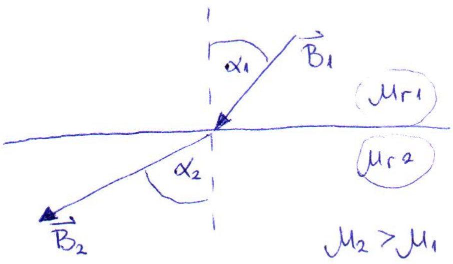
$$\Rightarrow \vec{e}_n \times (\vec{H}(T_+) - \vec{H}(T_-)) = \vec{K}$$ nezveznost

mejni pogoj za vektor $\vec{H}$



Lomni zakon za mag. polje $(\vec{K} = 0)$

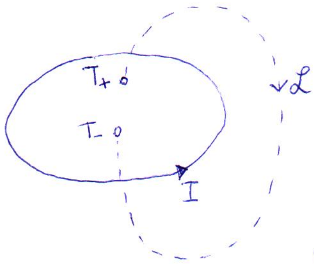
$$\boxed{\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\mu_{r1}}{\mu_{r2}} = \frac{\mu_1}{\mu_2}} \rightarrow \mu_1 = \mu_0 \cdot \mu_{r1}$$



Magnetna napetost in potencial

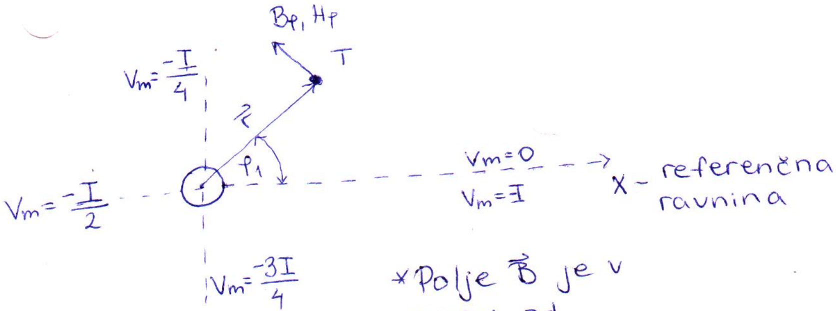
$$\left\{ U = \frac{A_e}{Q} \; ; \; V = \frac{W_e}{Q} \right\} \qquad \left\{ \oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0 \; ; \; \int_{T_1}^{T_2} \vec{E} \cdot d\vec{l} = U_{12} \; ; \; V = \int_{T}^{T_\infty} \vec{E} \cdot d\vec{l} \right\}$$

$$\left\{ \oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = N \cdot I \neq 0 \right\} \qquad \left\{ \begin{array}{l} \vec{E} = -\mathrm{grad}(V) \\ \mathrm{grad}(V) = \left(\frac{dV}{dx}, \frac{dV}{dy}, \frac{dV}{dz}\right) \end{array} \right\}$$



$$\boxed{\underbrace{\Theta_{T_+T_-}}_{\text{mag. napetost}} = \underbrace{V_m(T_+) - V_m(T_-)}_{\text{skalarni magnetni potencial}} = \int_{T_-}^{T_+} \vec{H}\, d\vec{l} \;\rightarrow\; \vec{H} = -\mathrm{grad}(V_m)}$$

Zgled: Skalarni mag. potencial premega vodnika



$$H_\varphi = \frac{I}{2\pi r}$$

$$(H_r, H_\varphi, H_z) = -\left(\frac{dV_m}{dr}, \frac{dV_m}{r\,d\varphi}, \frac{dV_m}{dz}\right)$$

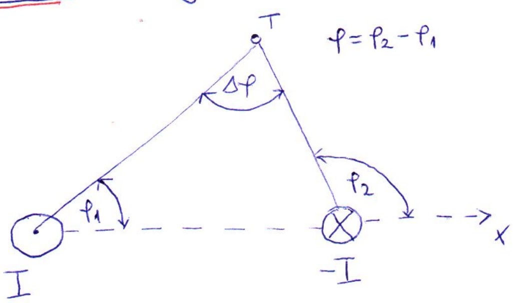
Lamejev koeficient

$$\boxed{H_\varphi = \frac{dV_m}{r\,d\varphi} = \frac{-I}{2\pi r}}$$

$$\boxed{V_m = -\int_0^{\varphi_1} \frac{I}{2\pi}\, d\varphi = \frac{-I \cdot \varphi_1}{2\pi}}$$

* Polje $\vec{B}$ je v smeri od visjega potenciala (V_m) prema nižjega.

Zgled: Magnetni skalarni potencial dvovoda



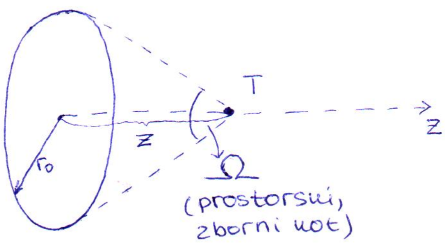
$$\varphi = \varphi_2 - \varphi_1$$

$$V_m = -\frac{I}{2\pi} \cdot \varphi_1 + \frac{I}{2\pi} \cdot \varphi_2$$

$$V_m = \frac{I}{2\pi}(\varphi_2 - \varphi_1) = \frac{I}{2\pi} \Delta\varphi$$

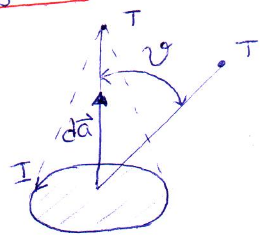
$$\boxed{V_m = \frac{I}{2\pi} \cdot \Delta\varphi}$$

Zgled: Vm v osi obroča



$$\boxed{V_m(T) = \frac{I}{4\pi} \cdot \Omega}$$

Zgled: V_m magnetnega dipola



$$d\vec{m} = I \cdot d\vec{a}$$

$$V_m = \frac{I \cdot d\Omega}{4\pi} \cdot \cos\vartheta \qquad \left(d\Omega = \frac{da}{r^2} \right)$$

$$V_m = \frac{I\, da}{4\pi r^2} \cdot \cos\vartheta$$

$$\boxed{V_m = \frac{dm}{4\pi r^2} \cdot \cos\vartheta} \qquad \left\{ V_{dipola} = \frac{dp}{4\pi\varepsilon_0 r^2} \cos\vartheta \right\}$$

Magnetna vezja

tokovna polja	magnetna polja
$\vec{J}: \int \vec{J}\, d\vec{a} = I$	$\vec{B}: \int \vec{B}\, d\vec{a} = \Phi_m$
$\oint \vec{J}\, d\vec{a} = 0$ (v spojiščih) $\sum I_k = 0$	$\oint \vec{B}\, d\vec{a} = 0$ $\sum \Phi_{mk} = 0$
$J_n(T_+) - J_n(T_-) = 0$	$B_n(T_+) - B_n(T_-) = 0$
$\vec{J} = \gamma \vec{E}$	$\vec{B} = \mu \vec{H}$; $\vec{B} \overset{\mu}{\Longleftrightarrow} \vec{H}$
$\oint \vec{E}\, d\vec{l} = 0$ $\sum U_k = 0$	$\oint \vec{H}\, d\vec{l} = 0$ (če s krivuljo ne prestopimo ref. ravn.) $\Theta = NI$ (generat. v mag. vezjih) $\sum \Theta_k = 0$
$V = \int_T^{T_\infty} \vec{E}\, d\vec{l}$; $\vec{E} = -\mathrm{grad}\, V$	$V_m = \int_T^{T_\infty} \vec{H} \cdot d\vec{l}$; $\vec{H} = -\mathrm{grad}\, V_m$
$E_T(T_+) - E(T_-) = 0$	$H_T(T_+) - H(T_-) = 0$
$\gamma_1 < \gamma_2$ slab prevodnik izolant; dober prevodnik	$\mu_{r1} < \mu_{r2}$
EL. vezja	MAG. vezja

$$U = R I$$

$$\Theta = R_m \cdot \Phi_m$$

Θ → mag. gen.

R_m → mag. bremena

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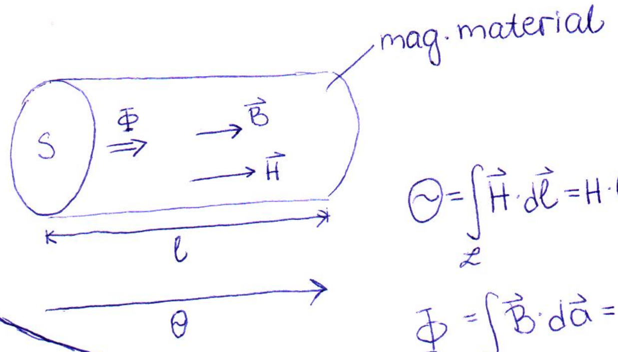
P11 (29.3.2016)

Elementi magnetnih vezij

Magnetni upor

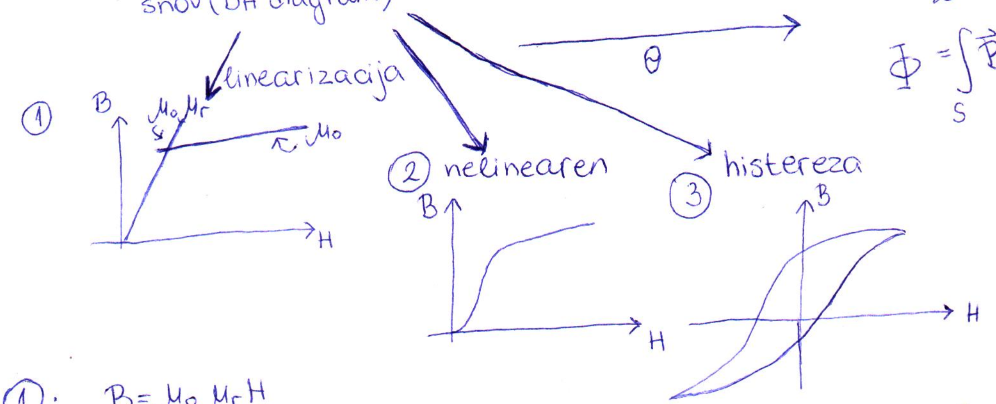


$$R_m = \frac{\Theta}{\Phi} = \frac{H \cdot l}{B \cdot S} = \boxed{\frac{H}{B}} \cdot \boxed{\frac{l}{S}}$$ — geometrija

snov (BH diagram)

$$\Theta = \int_{\mathcal{L}} \vec{H} \cdot d\vec{l} = H \cdot l$$

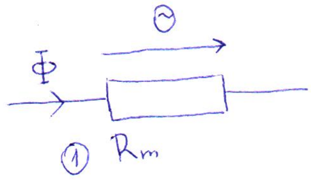
$$\Phi = \int_S \vec{B} \cdot d\vec{a} = B \cdot S$$



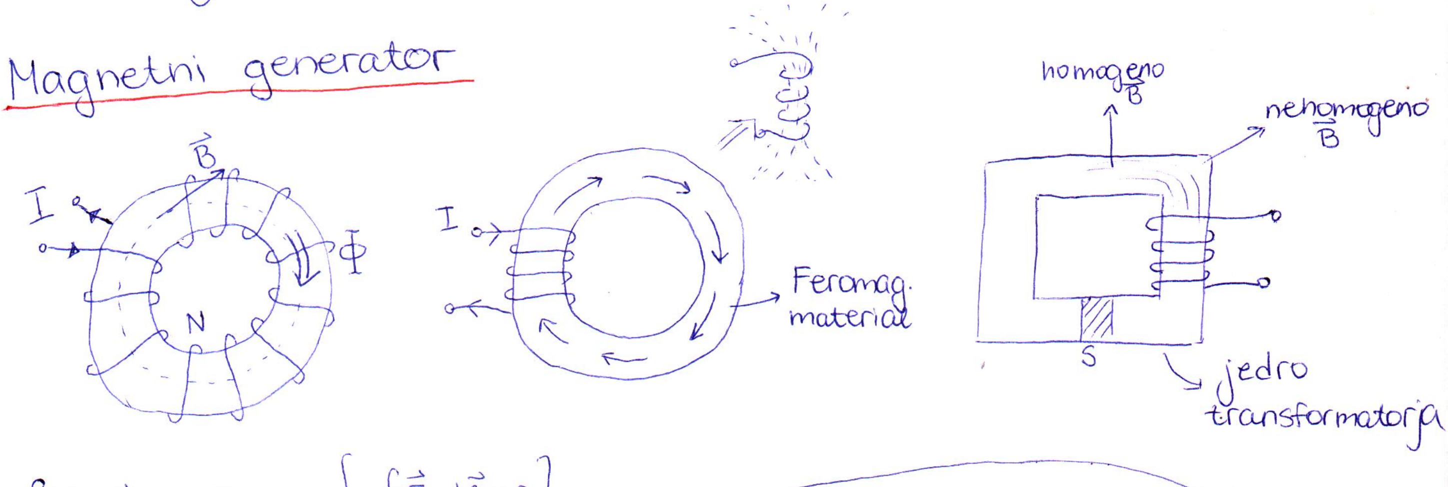
za ①: $B = \mu_0 \mu_r H$

$$R_m = \frac{\cancel{H} \cdot l}{\mu_0 \mu_r \cancel{H} \cdot S} = \frac{l}{\mu_0 \mu_r S}$$ → snovno-geometrijska lastnost $\left\{ R = \frac{l}{\gamma \cdot S} \right\}$

simbol:

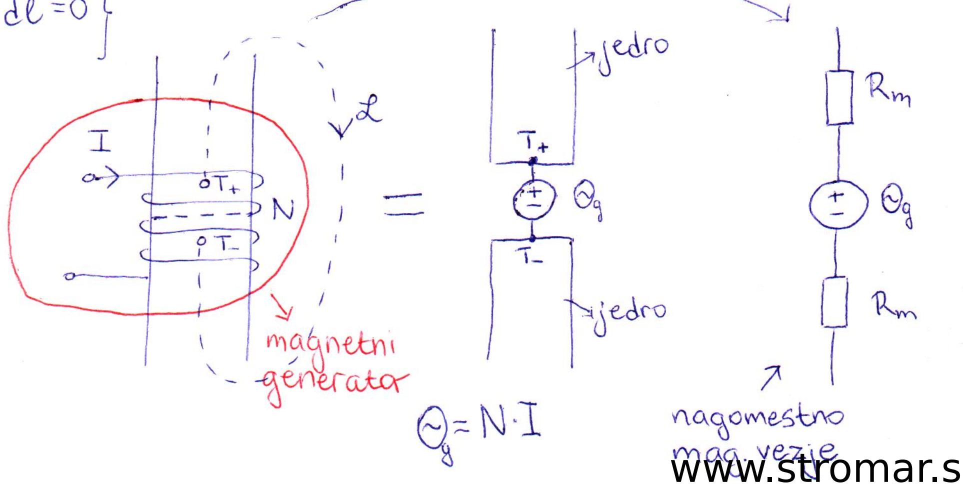


Magnetni generator



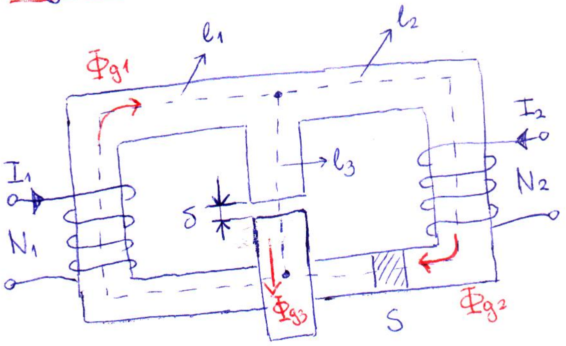
$$\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = N \cdot I = \quad \left\{ \oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0 \right\}$$

$$= V_m(T_+) - V_m(T_-) = \Theta_g$$

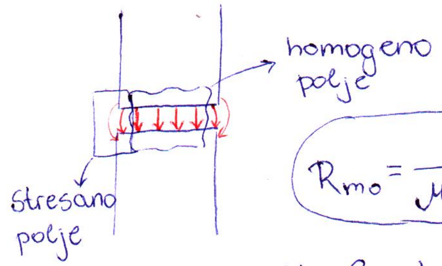


$$\Theta_g = N \cdot I$$

Zgled: Varilni transformator



Zračna reža



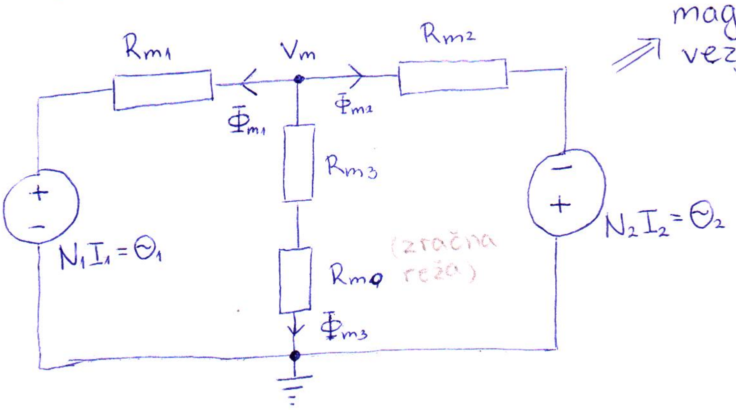
$$R_{m0} = \frac{\delta}{\mu_0 \cdot S \cdot K}$$

K – Carterjev faktor

povečanje preseka zračne reže zaradi stresanja

$S_0 = S \cdot K$, $K > 1$

Nadomestno vezje



$\Rightarrow$ magnetno vezje

$$\Phi_{m1} = \frac{V_m - \Theta_1}{R_1}$$

$$\Phi_{m2} = \frac{V_m + \Theta_2}{R_2}$$

$$\Phi_{m3} = \frac{V_m}{R_{m3} + R_0}$$

$$\Phi_{m1} + \Phi_{m2} + \Phi_{m3} = 0 \quad \ldots \Rightarrow V_m = \ldots$$

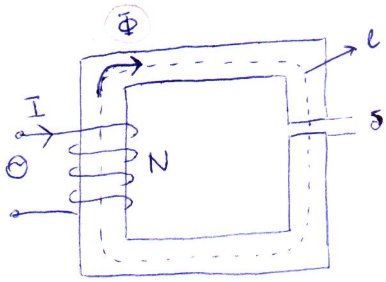
$$\Rightarrow B, H \ldots$$

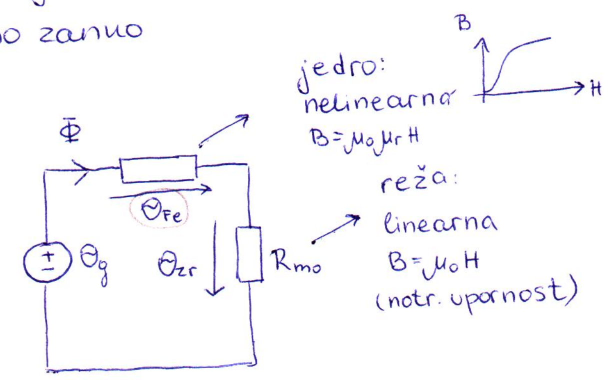
$$\oint_A \vec{B} \cdot d\vec{a} = 0$$

Nelinearna mag. vezja

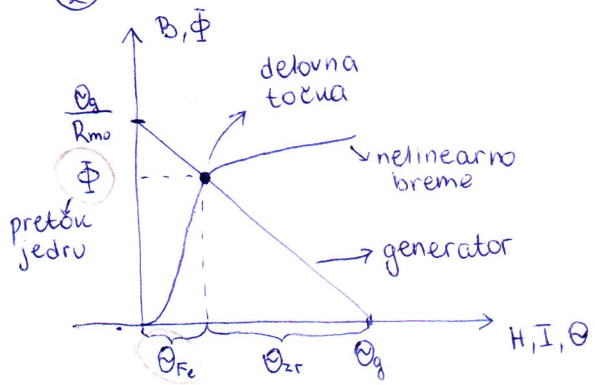
(2) magnetik z nelin. krivuljo

(3) magnetik s histerezno zanko

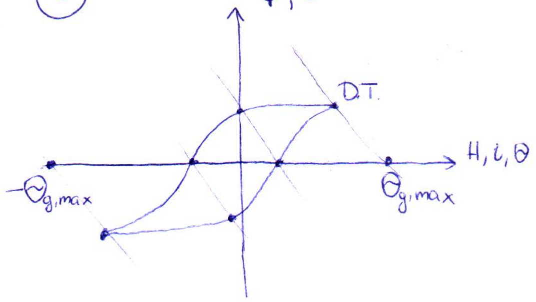




(2) Grafično reševanje

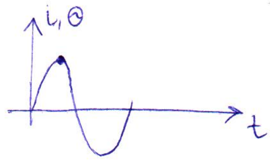


③ histereza

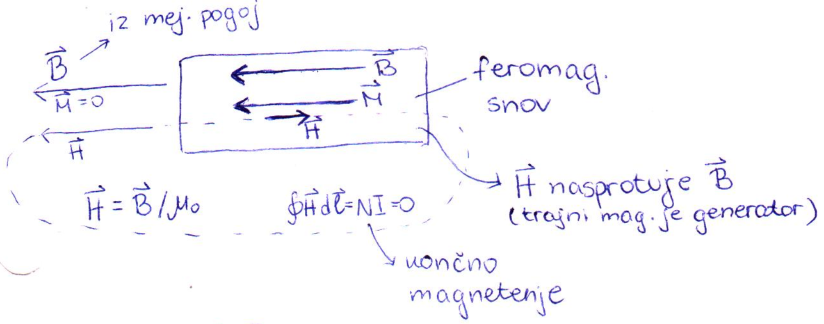


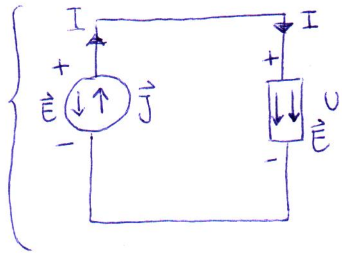
i - harmoničen

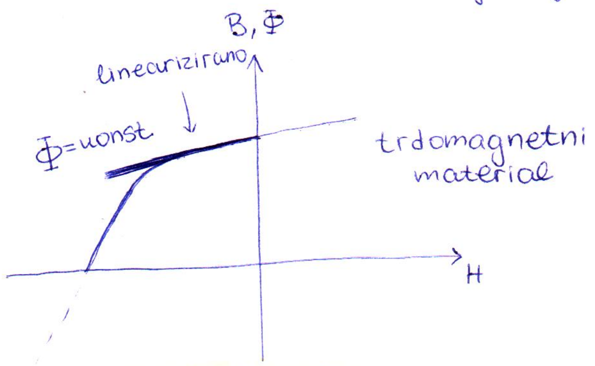
H, Θ - harmoničen

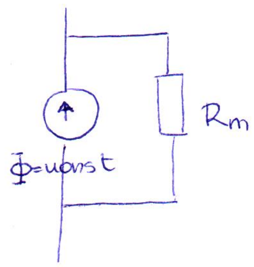


Trajni magneti







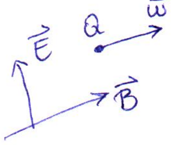


P12 (1.4.2016)

Dinamično elektromagnetno polje

$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0 \Rightarrow$$

$$= -\frac{d\Phi_{\mathcal{L}}}{dt} \quad \left(\begin{matrix}\text{Farad.} \\ \text{Maxwell}\end{matrix}\right)$$



$$\vec{F}_L = Q(\vec{E} + \overbrace{\vec{w} \times \vec{B}}^{\text{Amper}})$$

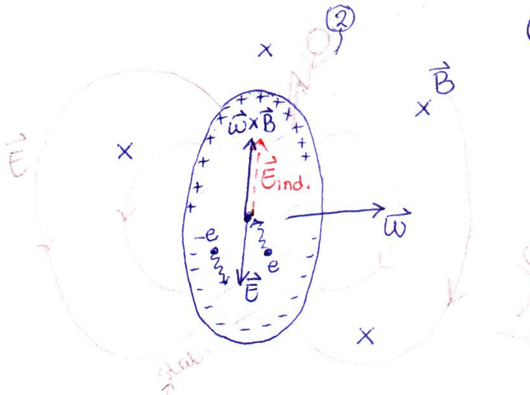
$\vec{E}_{coul.}$ (Coulomb) $\quad$ $\vec{E}_{Farad.}$ (Farad)

$$\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \int_{\mathcal{A}} \vec{J} \cdot d\vec{a} \left(+ \int_{\mathcal{A}} \frac{\partial D}{\partial t} \cdot d\vec{a} \right)$$

$$\oint_{\mathcal{A}} \vec{D} d\vec{a} = Q_{not.}$$

$$\oint_{\mathcal{A}} \vec{B} \cdot d\vec{a} = 0$$

Gibanje prevodnika v magnetnem polju

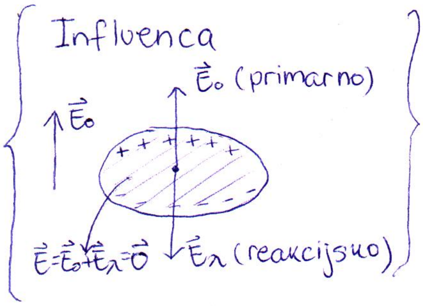


(1)

$-e\vec{w}\times\vec{B}$

$-e(\vec{E}+\vec{w}\times\vec{B})=\vec{0}$

$\vec{E}+\vec{w}\times\vec{B}=\vec{0}$ (ravnovesje)

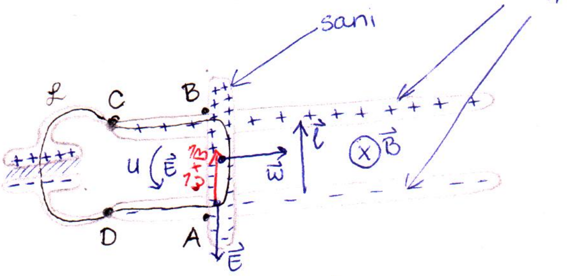


(2) $-e(\vec{E}+\vec{E}_{ind.})=\vec{0}$ (ravnovesje)

$\vec{E}+\vec{E}_{ind.}=\vec{0}$

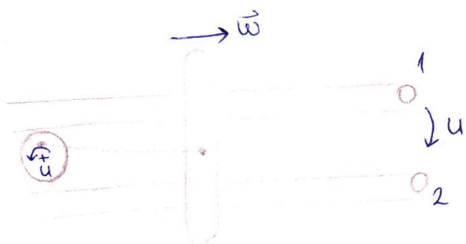
$\vec{E}_{ind.}\equiv\vec{w}\times\vec{B}$

Linearni generator

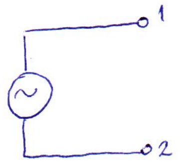


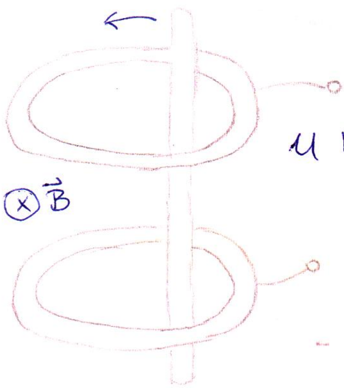
$$\oint_{\mathcal{L}}\vec{E}\cdot d\vec{l}=\underbrace{\int_A^B\vec{E}\cdot d\vec{l}}_{-\int_A^B(\vec{w}\times\vec{B})\cdot d\vec{l}}+\underbrace{\int_B^C\vec{E}\cdot d\vec{l}}_{\sim\emptyset}+\underbrace{\int_C^D\vec{E}\cdot d\vec{l}}_{u}+\underbrace{\int_D^A\vec{E}\cdot d\vec{l}}_{\sim\emptyset}=0$$

$$\underbrace{-\int_A^B(\vec{w}\times\vec{B})\cdot d\vec{l}}_{-wBl}\Rightarrow u=\underbrace{wBl}_{u_{ind.}}=u_{ind.}\ \text{(inducirana napetost)}$$



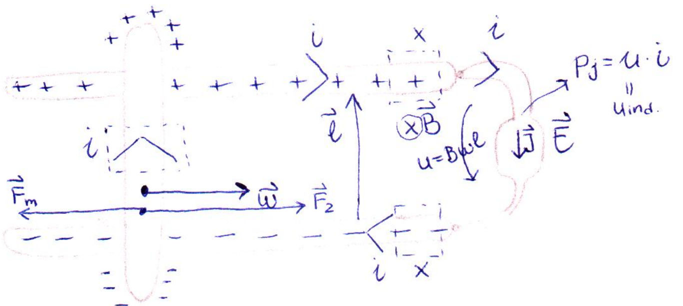
$$u(t)\simeq\overbrace{U_{max}}^{w_{max}Bl}\cos(\omega t+\alpha)$$





u harmonična napetost

Obremenjen lin. generator

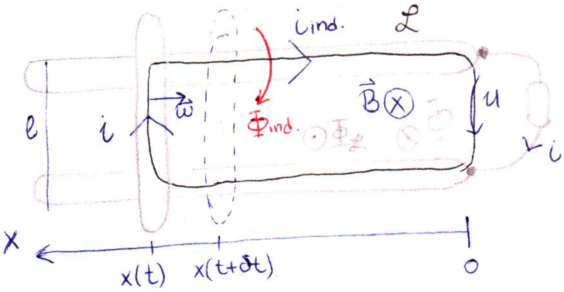


$\vec{F}_m=i\vec{l}\times\vec{B}$

$\vec{F}_z+\vec{F}_m=\vec{0}$

$P_z=\vec{F}_z\cdot\vec{w}=i\cdot l\cdot B\cdot w=i\cdot u_{ind}$

$\boxed{P_z=P_J}$



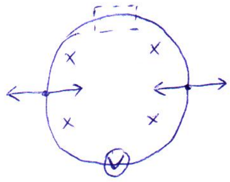
$$u = u_{ind.} = w \cdot B \cdot \ell = \underbrace{\frac{x(t) - x(t+\delta t)}{\delta t}}_{w} B \cdot \ell =$$

$$= -\left[\Phi(t+\delta t) - \Phi(t)\right] =$$

$$= \frac{x(t) B\ell - x(t+\delta t) B\ell}{\delta t} = \frac{\Phi(t) - \Phi(t+\delta t)}{\delta t}$$

$$u = -\lim_{\delta t \to 0} \frac{\Phi(t+\delta t) - \Phi(t)}{\delta t} = -\frac{d\Phi}{dt}$$

$$\boxed{u_{ind.} = -\frac{d\Phi_{\mathcal{L}}}{dt}}$$

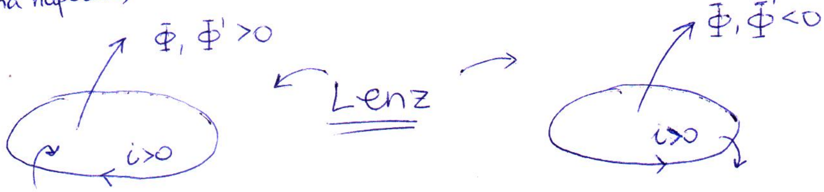


$$\Phi = \int_A \vec{B} \cdot d\vec{a}$$

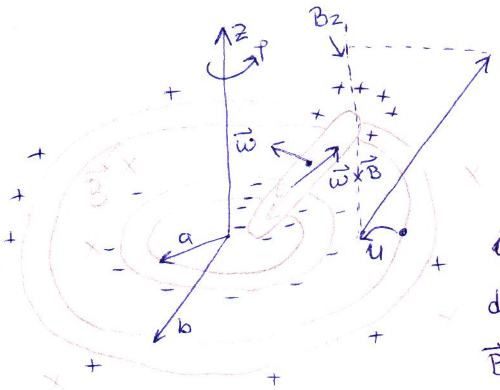
variram (transformatorska napetost)

variram (gibalna napetost)

$$u_{ind.} = -\frac{d\Phi}{dt} = -\underbrace{\int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}}_{\text{transf.}} + \underbrace{\oint_{\mathcal{L}} (\vec{w} \times \vec{B}) \cdot d\vec{\ell}}_{\text{gib.}}$$



Faraday-ev generator



$$u = u_{ind.} = \int_a^b (\vec{w} \times \vec{B}) \cdot d\vec{\ell} = \int_a^b \underbrace{(d\vec{\ell} \times \vec{w})}_{\vec{e}_z \cdot w\rho d\rho} \cdot \vec{B} =$$

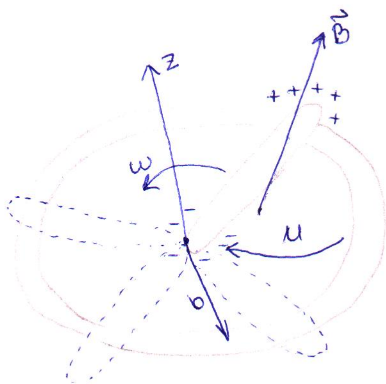
$$= \int_a^b B_z w \rho \, d\rho = \frac{w B_z}{2} (b^2 - a^2)$$

enosmerna

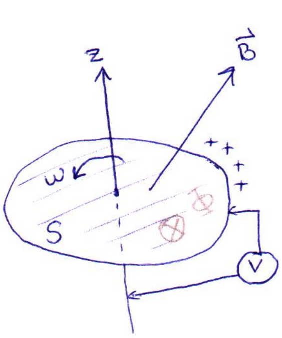
$\vec{w} = (0, w\cdot\rho, 0)$

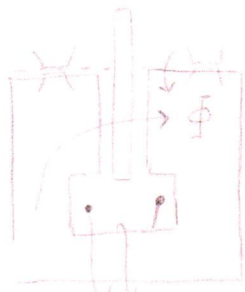
$d\vec{\ell} = (d\rho, 0, 0)$

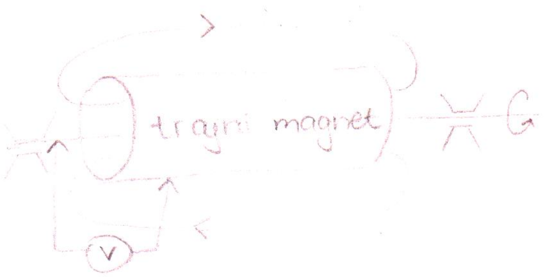
$\vec{B} = (B_x, B_y, B_z)$

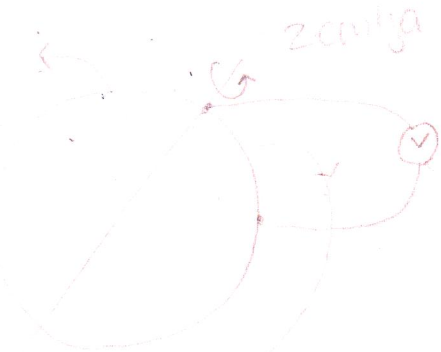


$$u = \frac{w B_z b^2}{2} = \frac{\cancel{2}\pi f B_z \cdot b^2}{\cancel{2}} = f \cdot \Phi_{diska}$$

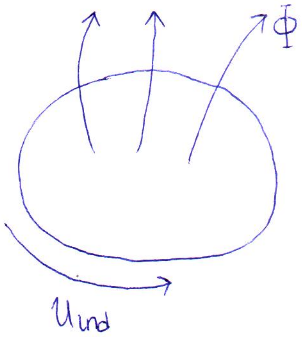








P13(5.4.2016)



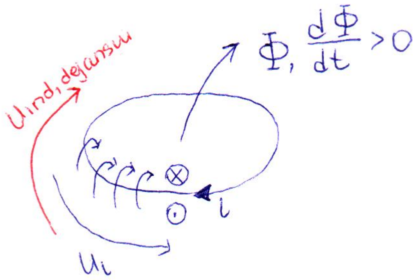
Dogovor:

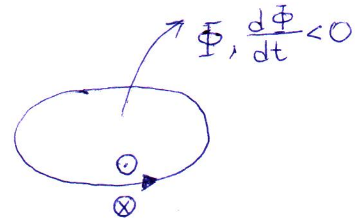
Označevanje inducirane napetosti v sklenjeni zanki izvajamo v matematično pozitivnem smislu glede na smer fluksa (pravilo desnega vijaka)

$$U_{ind} = -\frac{d\Phi}{dt}$$

$\frac{d\Phi}{dt}$ --- sprememba pretoka

$-$ → Lenz + dogovor.





2. Maxwellova enačba

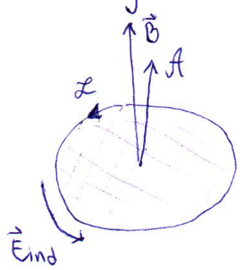
$$U_{ind} = -\frac{d\Phi}{dt}$$

↓ nanaša se na zanko

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

$$U_{ind} = \int \vec{E}_{ind} \cdot d\vec{l}$$

$$\boxed{\oint_{\mathcal{L}} \vec{E}_{ind} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{a}}$$



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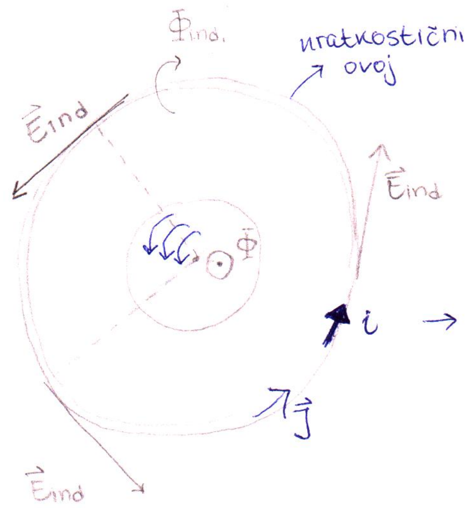
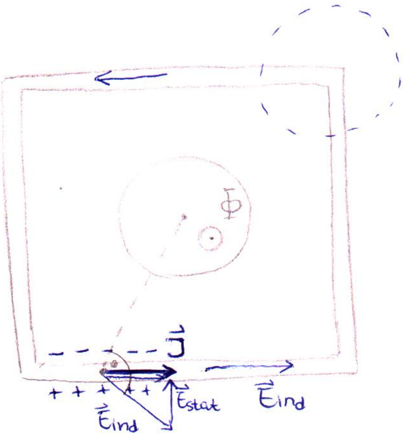
Vrtinčni toki, izgube, lameliranje



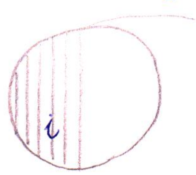
$$\vec{J} = \gamma \cdot \vec{E} = \gamma (\vec{E}_{stat.} + \vec{E}_{ind.})$$

→ v prevodnih materialih zaradi spreminjajočega Φ tečejo t.i. vrtinčni toki (vrtinčne izgube jedja)



Ukrep za preprečevanje vrtinčnih tokov:

→ Lameliranje



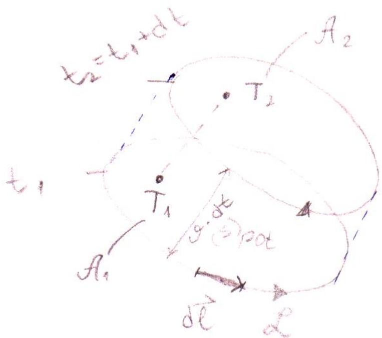
90% Fe
10% Loka

} faktor polnjenje: 0,9

vrtinčni toki le znotraj lamele

→ hlajenje

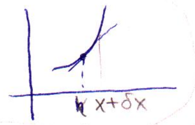
Transformatorska in gibalna inducirana napetost



$$U_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B}\cdot d\vec{a}$$

$$\underset{\downarrow\ \lim}{\frac{d}{dt}}\int \vec{B}\cdot d\vec{a} = \lim_{dt\to 0} \frac{1}{dt}\Big(\underbrace{\int_{A_2} \vec{B}(T_2,t_2)\,d\vec{a}}_{d\Phi\ =\ \Phi_v t_2} - \int_{A_1} \vec{B}(T_1,t_1)\,d\vec{a}\Big) \quad - \Phi_v t_1$$

$$y(x+h) = y(x) + h\cdot y' \quad \text{„}(t_1+dt)\text{"} \longrightarrow$$

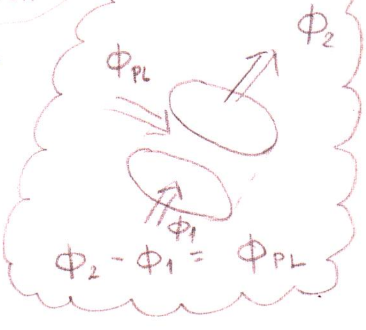


$$\vec{B}(T_2,t_2) = \vec{B}(T_2,t_1) + dt\,\frac{d\vec{B}(T_2,t_1)}{dt}$$

$$\Rightarrow \frac{d}{dt}\int \vec{B}\cdot d\vec{a} = \int_{A_2} \frac{d\vec{B}}{dt}\, d\vec{a} + \lim_{dt}^{\frac{1}{dt}}\Big(\int_{A_2} \vec{B}(T_2,t_1)\,d\vec{a} - \int_{A_1} \vec{B}(T_1,t_1)\,d\vec{a}\Big)$$

$$\underbrace{\Phi_{2vt_1} - \Phi_{1vt_1}}$$

$$d\Phi = \Phi_{PL} =$$



$$\Phi_{PLAŠČA} = \oint_{\mathcal{L}} \vec{B}(\vec{v}\cdot dt \times d\vec{l}) = -\oint_{\mathcal{L}} (\vec{v}\times\vec{B})\cdot d\vec{l}\cdot dt$$

$$\frac{d}{dt}\int_{A} \vec{B}\cdot d\vec{a} = \int_{A} \frac{d\vec{B}}{dt}\, d\vec{a} - \oint_{\mathcal{L}} (\vec{v}\times\vec{B})\, d\vec{l}$$

$$\Rightarrow U_{ind} = -\underbrace{\int_{A} \frac{d\vec{B}}{dt}\cdot d\vec{a}}_{\substack{\text{posledica spreminjanja}\\ \text{flusa skozi zanko}}} + \oint_{\mathcal{L}} \underbrace{(\vec{v}\times\vec{B})\cdot d\vec{l}}_{\text{posledica gibanja v } \vec{B}}$$

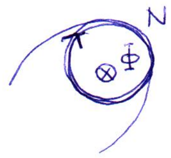
TRANSFORMATORSKA INDUCIRANA NAPETOST

GIBALNA INDUCIRANA NAPETOST

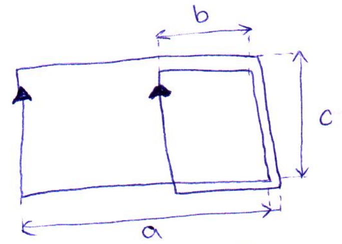
Magnetni sklep Ψ

do sedaj:

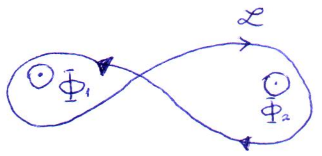
večje število ovojev



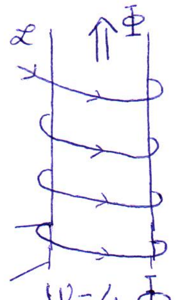
$\Psi = N\cdot\Phi$



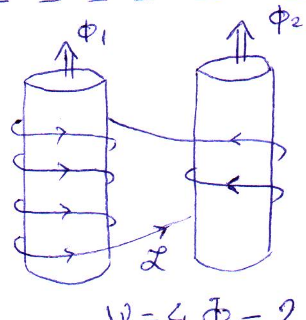
$\Psi = \Phi_{a\times c} + 2\Phi_{b\times c}$



$\Psi = \Phi_1 - \Phi_2$



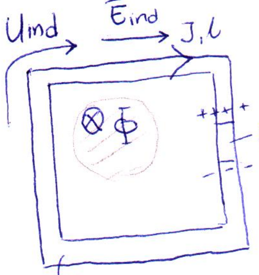
$\Psi = 4\,\Phi$



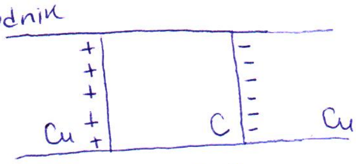
$\Psi = 4\,\Phi_1 - 2\,\Phi_2$

P14 (8.4.2016)

Inducirana napetost v odprtih zankah



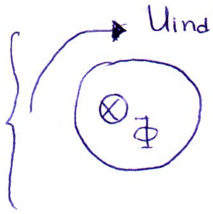
(večji padec napetosti) slabši prevodnik



Cu - dober prevodnik zanemarljiv padec napetosti

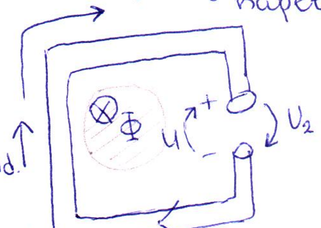
$$J_n \left(\frac{\varepsilon_1}{\gamma_1} - \frac{\varepsilon_2}{\gamma_2} \right) = \sigma$$

norm.



$$\left\{ U_{ind} = -\frac{d\Psi}{dt} \right.$$

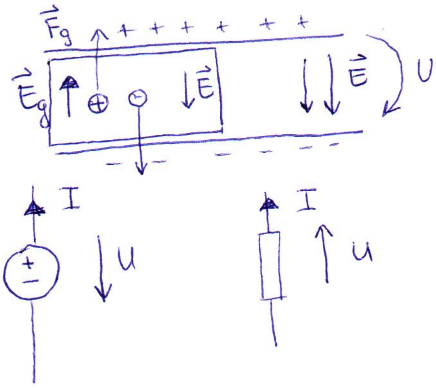
$U_{ind} \to$ generatorska napetost



$\vec{E}_{ind} \to$ generatorsko polje

$$\boxed{U_2 = U_{ind}}$$

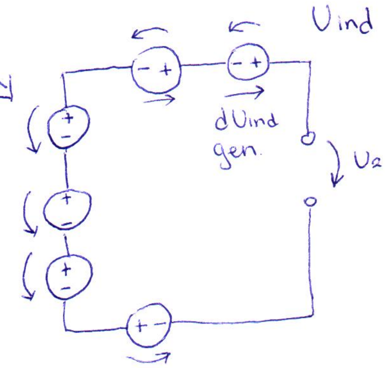
napetostni vir



$$\boxed{u = -U_{ind} = \frac{d\Psi}{dt}}$$

bremenska

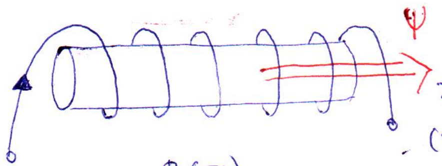
generatorska



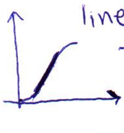
Indutivnost

- Snovno geometrijska lastnost

$$\boxed{L = \frac{\Psi}{I}}$$ indutivnost



lin. zveza (za feromag. lahko: linearizirana zveza



I
$\downarrow$ lin.
B(T)
$\downarrow$ lin.
Φ
$\downarrow$ lin.
Ψ

$$\boxed{U_{ind} = -L \frac{di}{dt}}$$

zveza med nap. in tokom na tuljavi

generatorsko ozn. nap.

$$\left\{ i = C \frac{du}{dt} \right\}$$

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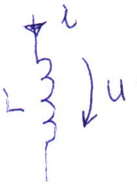
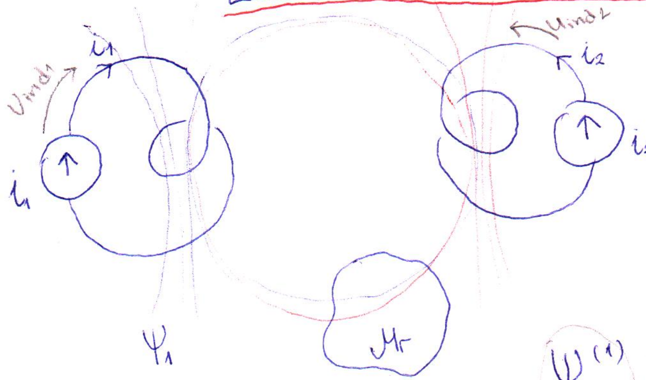
$$\boxed{u = L\frac{di}{dt}}$$



bremenska ozn. nap.

$$\left\{ i = C\frac{du}{dt}, \quad u = R \cdot i \right\}$$

Lastna in medsebojna induktivnost



$$\Psi_1 = \Psi_1^{(1)} + \Psi_1^{(2)}$$

$$\Psi_2 = \Psi_2^{(1)} + \Psi_2^{(2)}$$

$$\underbrace{\frac{\Psi_1^{(1)}}{i_1}}_{L_{11}}, \quad \underbrace{\frac{\Psi_1^{(2)}}{i_2}}_{L_{12}}, \quad \underbrace{\frac{\Psi_2^{(1)}}{i_1}}_{L_{21}}, \quad \underbrace{\frac{\Psi_2^{(2)}}{i_2}}_{L_{22}}$$

→ indukcijski koeficienti

$L\,[H]$ → Henri

L_{11}, L_{22} - lastna induktivnost

L_{12}, L_{21} - Medsebojne induktivnosti

$$\boxed{u_{ind\,1} = -L_{11}\frac{di_1}{dt} - L_{12}\frac{di_2}{dt}}$$

$$\boxed{u_{ind\,2} = -L_{21}\frac{di_1}{dt} - L_{22}\frac{di_2}{dt}}$$

- Posplošitev na N zank

$$\Psi_k = \sum_{j=1}^{N} \Psi_k^{(j)} = \sum_{j=1}^{N} L_{kj} \cdot i_j$$

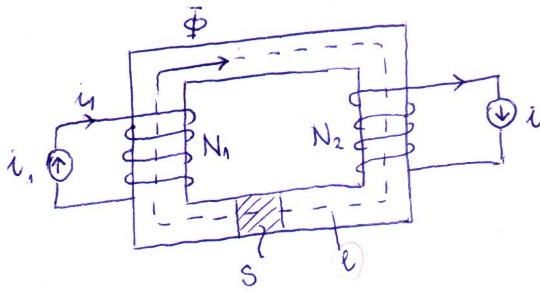
$$u_{ind\,k} = -\frac{d\Psi_k}{dt} = -\sum_{j=1}^{N} \frac{d\Psi_k(j)}{dt} = -\underbrace{L_{kk}}_{\text{lastna induktivnost}}\frac{di_k}{dt} - \sum_{\substack{j=1 \\ j\neq k}}^{N} \underbrace{L_{kj}}_{\text{medsebojna induktivnost}}\frac{di_j}{dt}$$

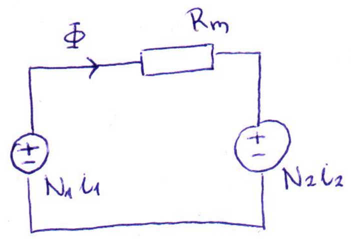
$$L = \begin{bmatrix} L_{11} & \cdots & L_{1N} \\ L_{21} & & L_{2N} \\ \vdots & & \vdots \\ L_{N1} & & L_{NN} \end{bmatrix}$$

→ matrika induktivnosti

→ lastne induktivnosti

Zgled: Induktivnost navitij transformatorja





$$\Phi = \frac{N_1 i_1 - N_2 i_2}{R_m}$$

$$\Psi_1 = N_1 \Phi = \frac{N_1^2}{R_m} \cdot i_1 - \frac{N_1 N_2}{R_m} i_2$$

$$\Psi_2 = N_2(-\Phi) = -\frac{N_1 N_2}{R_m} i_1 + \frac{N_2^2}{R_m} i_2$$

- Lastni induktivnosti

$$L_{11} = \frac{N_1^2}{R_m} \; ; \; L_{22} = \frac{N_2^2}{R_m}$$

$$R_m = \frac{l}{S\mu}$$ snovno-geometrijska značilnost

(sorazmerni s kvadratom ovojev)

$$\boxed{L = \frac{N^2}{R_m}} \qquad L_{ii} > 0$$

- Medsebojne induktivnosti

$$\boxed{L_{12} = -\frac{N_1 \cdot N_2}{R_m}} \; ; \; L_{ij} \gtrless 0$$ (pozitivne, če si mag. pretoka para tuljav se podpirata)

$L_{12} = L_{21}$ → Recipročnost

Opažemo, da velja:

$$\{L_{11} \cdot L_{22} = L_{12} \cdot L_{21}\}$$

Sklopni faktor

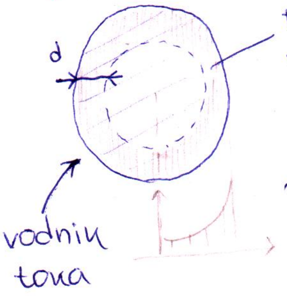
$$\boxed{k^2 = \frac{L_{12} \cdot L_{21}}{L_{11} \cdot L_{22}}}$$

$0 = k = 1$ → popoln sklop

↓ ni vpliva

Dileme pri določanju induktivnosti

1) Kožni pojav



tok po preseku vodnika ni enakomerno razporejen

prihaja do izriva toka: kožni pojav (skin effect)

$$d = \sqrt{\frac{2}{\omega \mu \varepsilon}}$$

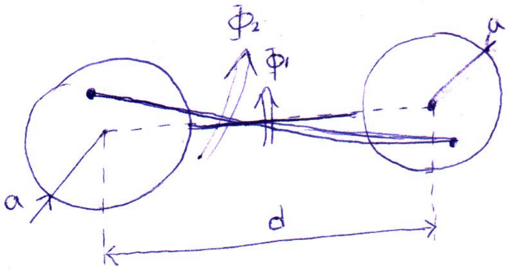
ω → $2\pi f$

d → vdorna globina

* pri 50 Hz → d = 9,6 mm

5 kHz → d = 1 mm

2) Dvovod



$$L = \frac{\Phi}{i} \rightarrow \text{vnateri fluks}$$

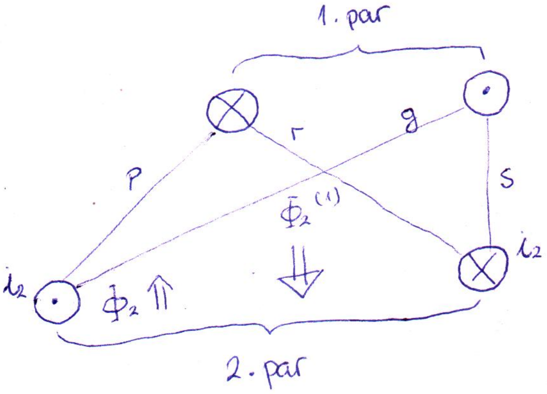
$\Downarrow$

vzamemo povprečje vseh fluksov

$$L = \frac{\langle \Phi \rangle}{i}$$

lastna induktivnost dvovoda $\boxed{L = \frac{\mu_0 l}{\pi}\left(\frac{1}{4} + \ln\frac{d}{a}\right)}$

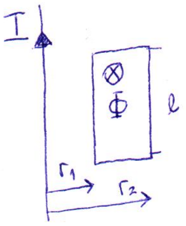
Medsebojna induktivnost dvovodov



$$L_{21} = -\frac{\Phi_2^{(1)}}{i_1} = \frac{-\mu_0 \cancel{i_1} l}{2\pi \cancel{i_1}} \ln\frac{r\cdot g}{p\cdot s}$$

$$L_{21} = -\frac{\mu_0 l}{2\pi} \ln\frac{r\cdot g}{p\cdot s}$$

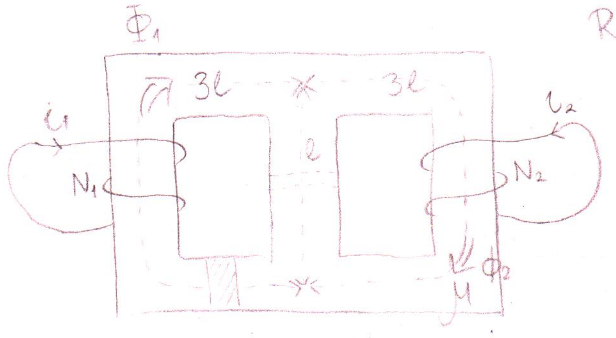
$$L_{12} = L_{21}$$

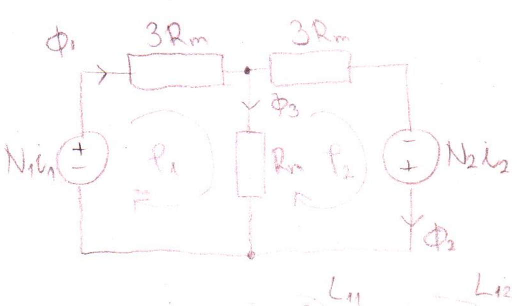


$$\Phi = \frac{\mu_0 I l}{2\pi} \ln\frac{r_2}{r_1}.$$

Zgled: Dve navitji na tristebernem jedru

$$R_m = \frac{l}{\mu S}$$





$$-N_1 i_1 + 3R_m\varphi_1 + R_m(\varphi_1 - \varphi_2) = 0$$

$$-N_2 i_2 + R_m(\varphi_2 - \varphi_1) + 3R_m\varphi_2 = 0$$

$$4R_m\varphi_1 - R_m\varphi_2 = N_1 i_1 \quad /\cdot 4$$

$$-R_m\varphi_1 + 4R_m\varphi_2 = N_2 i_2 \quad /\cdot 4$$

$$15 R_m \varphi_1 = 4N_1 i_1 + N_2 i_2$$

$$\varphi_1 = \Phi_1 = \frac{4N_1 i_1 + N_2 i_2}{15 R_m}$$

$$\varphi_2 = \Phi_2 = \frac{N_1 i_1 + 4N_2 i_2}{15 R_m}$$

$$\Rightarrow \Psi_1 = N_1\Phi_1 = \underbrace{\frac{4N_1^2}{15R_m}}_{L_{11}} i_1 + \underbrace{\frac{N_1 N_2}{15R_m}}_{L_{12}} i_2$$

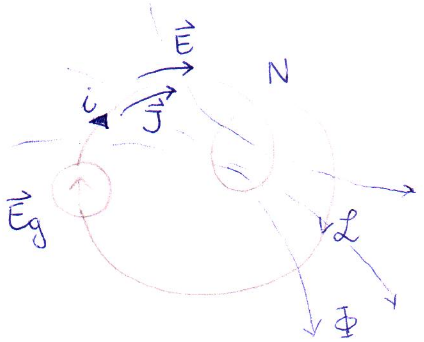
$$\Psi_2 = N_2\Phi_2 = \underbrace{\frac{N_1 N_2}{15R_m}}_{L_{21}} i_1 + \underbrace{\frac{4N_2^2}{15R_m}}_{L_{22}} i_2$$

$$L_{12} = L_{21}$$

$$L_{12}L_{21} < L_{11}L_{22}, \quad K_{12} = \sqrt{\frac{L_{12}L_{21}}{L_{11}L_{22}}}$$

P15 (12.4.2016)

Tuljava, uot strnjen element



$\vec{J} = \gamma(\vec{E} + \vec{E}_g)$

$\frac{\vec{J}}{\gamma} = \vec{E} + \vec{E}_g$

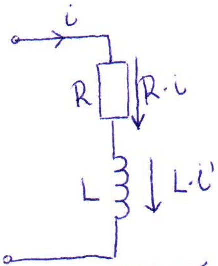
$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = \oint_{\mathcal{L}} \underbrace{\frac{\vec{J}}{\gamma} d\vec{l}}_{R \cdot i} - \oint_{\mathcal{L}} \underbrace{\vec{E}_g d\vec{l}}_{U_g} = \underbrace{-\frac{d\Psi}{dt}}_{= -L \cdot i'} \quad d\Psi \to N \cdot \Phi$$

$U_g = -L \cdot i'$ → inducirana nap.

$R \cdot i$ ↳ padec napetosti - žica

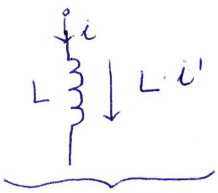
$R \cdot i - U_g = -L \cdot i'$

$U_g = R \cdot i + L i'$



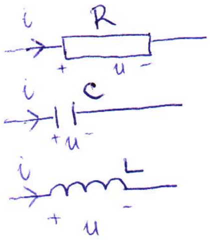
MODEL (REALNE) TULJAVE

R - upornost navitje tuljave



IDEALNA TULJAVA

$$\boxed{u = L \frac{di}{dt}}$$

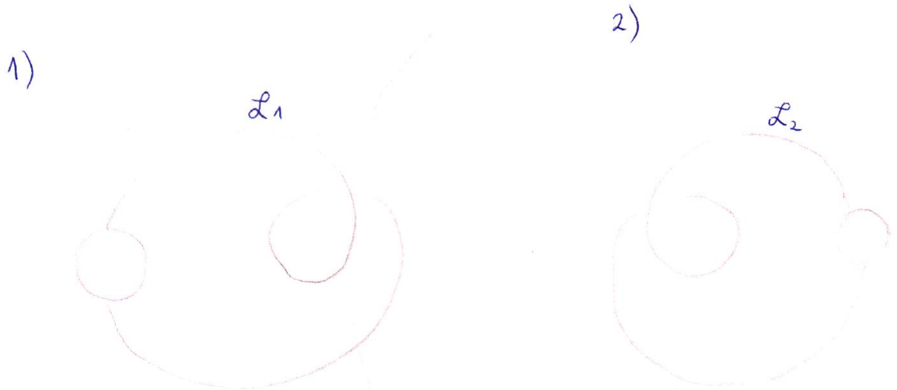


$u = R \cdot i$

$i = C \cdot u'$

$u = L \cdot i'$

Modelno vezje dveh magnetno sklopljenih tuljav

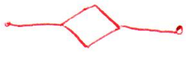


$\vec{J}_1 = \gamma_1(\vec{E}_1 + \vec{E}_{g1})$

$\vec{J}_2 = \gamma_2(\vec{E}_2 + \vec{E}_{g2})$

$$\oint_{\mathcal{L}_1} \vec{E}_1 \cdot d\vec{l} = \oint_{\mathcal{L}_1} \underbrace{\frac{\vec{J}_1}{\gamma_1} d\vec{l}}_{R_1 i_1} - \oint_{\mathcal{L}_1} \underbrace{\vec{E}_{g1} d\vec{l}}_{U_{g1}} = \underbrace{-\frac{d\Psi}{dt}}_{= -L_{11} i_1' - L_{12} i_2'}$$

$R_1 \cdot i_1 + L_{11} \cdot i_1' + L_{12} \cdot i_2' = U_{g1}$



$R_2 \cdot i_2 + L_{21} \cdot i_1' + L_{22} \cdot i_2' = U_{g2}$

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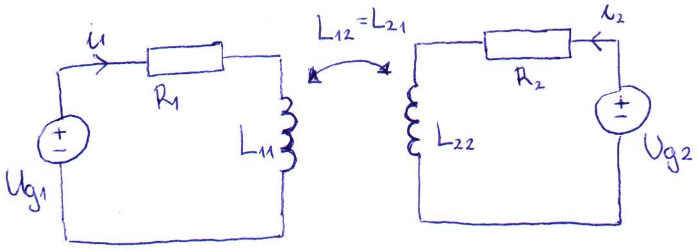
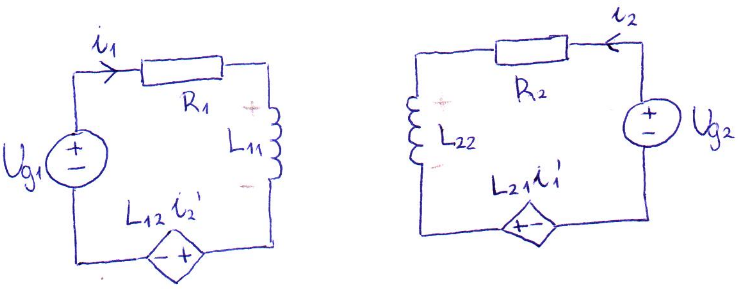
1) Modelno vezje 1:



$L_{12}=L_{21}>0$; podpirata
<0; nasprotujeta

2) Modelno vezje 2:

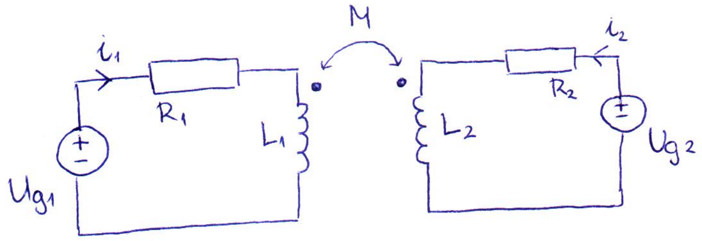


$L_{11}=L_1$

$L_{22}=L_2$

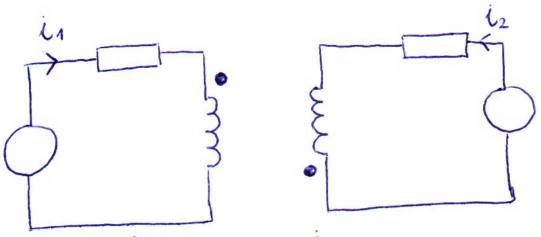
$|L_{12}|=|L_{21}|=M$

Dogovor o pikah - 3:

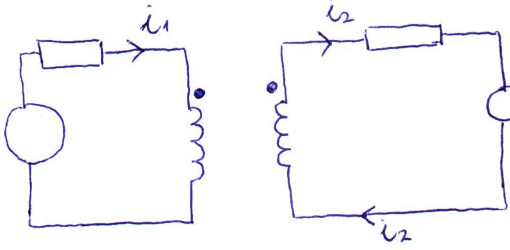


$R_1 i_1 + L_1 \cdot i_1' + M i_2' = U_{g1}$

$R_2 \cdot i_2 + M \cdot i_1' + L_2 i_2' = U_{g1}$



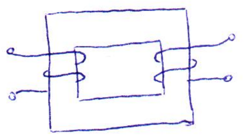
$R_1 i_1 + L_1 \cdot i_1' - M i_2' = U_{g1}$



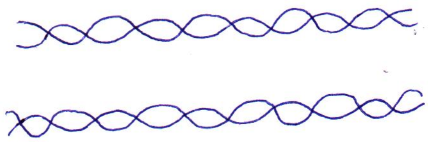
Sklopni faktor

$$K^2_{jm} = \frac{\psi_m^{(j)} \cdot \psi_j^{(m)}}{\psi_m^{(m)} \cdot \psi_j^{(j)}}$$

$$K_{jm} = \frac{|L_{mj}|}{\sqrt{L_{mm} \cdot L_{jj}}}$$

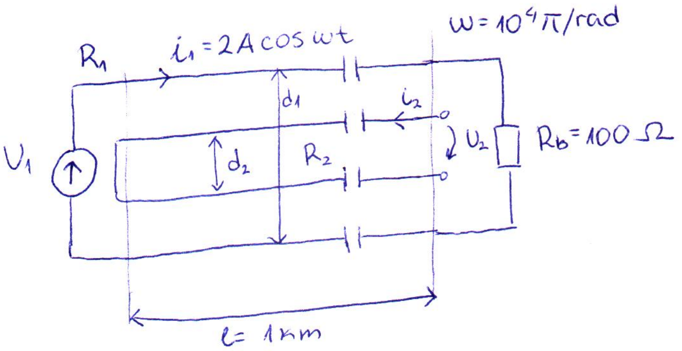


$K = 1$



$K = 0$

Zgled: DVOVODA



$r_0 = 1mm$

$\gamma_{Cu} = 56 \cdot 10^6 \; S/m$

$$R_1 = R_2 = \frac{2l}{\gamma_{Cu} \cdot \pi r_0^2} = 11{,}4\,\Omega$$ → upornost žice

$$L_{11} = \frac{\mu_0 l}{\pi}\left(\frac{1}{4} + \ln\frac{d_1}{r_0}\right) = 2{,}5\,mH$$

$$L_{22} = \frac{\mu_0 l}{\pi}\left(\frac{1}{4} + \ln\frac{d_2}{r_0}\right) = 2{,}22\,mH$$

} lastni induktivnosti dvovoda

$$L_{12} = L_{21} = \frac{\mu_0}{\pi} \ln\frac{(d_1+d_2)/\cancel{2}}{(d_1-d_2)/\cancel{2}} = 0{,}44\,mH$$ → medsebojna induktivnost

$$u_1 = (R_1 + R_b)\,i_1 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

$$u_2 = R_2 i_2 + L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

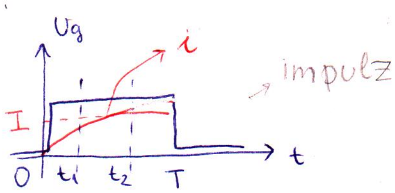
$$u_1 = (R_1 + R_b)\,i_1 + L_{11}\frac{di_1}{dt}$$

$$u_2 = L_{21}\frac{di_1}{dt}$$

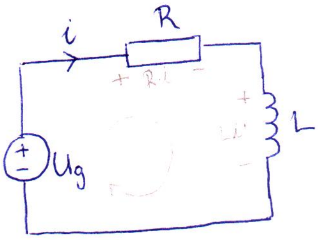
$u_1 = \ldots$

$$u_2 = -L_{21} \cdot \omega \cdot 2A \sin\omega t = \underline{\underline{27{,}6\,V \cdot \sin\omega t}}$$

Energija magnetnega polja



1) $0 < t < T$



$$U_g = R\cdot i + L\cdot i' \;/\cdot i$$

$$U_g\cdot i = R\cdot i^2 + L\cdot i\cdot i' \;/\int_{t_1}^{t_2} dt$$

$$\int_{t_1}^{t_2} U_g\cdot i\cdot dt = \int_{t_1}^{t_2} R\cdot i^2\, dt + \int_{t_1}^{t_2} L\cdot i\,\frac{di}{\cancel{dt}}\,\cancel{dt}$$

delo vira — sproščena toplota žica — energija mag. polja tuljave

$$Ag(t_1,t_2) = A_J(t_1,t_2) + \frac{L\cdot i^2(t_2)}{2} - \frac{L\cdot i^2(t_1)}{2}$$

(Joulske izgube)

$$Ag(0,T) = A_J(0,T) + \underbrace{L\cdot\frac{I^2}{2}}_{\text{ENERGIJA V TULJAVI}}$$

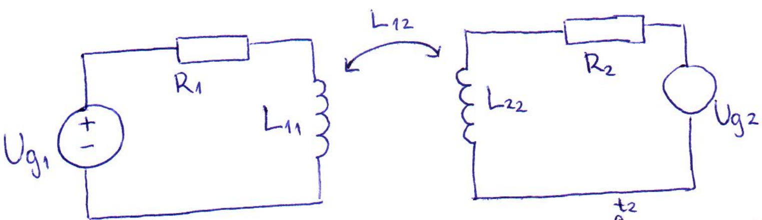
2) $t > T$ (izklop)

$$R\cdot i + L\cdot i' = 0$$

$$A_J = \frac{L}{2} I^2$$

Vsa v tuljavi akumulirana mag. energija se sprosti ob izklopu.

Energija mag. polja sklopljenih tuljav



$$Ag_1(t_1,t_2) = A_{J1}(t_1,t_2) + \int_{t_1}^{t_2}\left(L_{11}\, i_1' + L_{12}\, i_2'\right) i_1\, dt$$

desni člen per partes

$$\int u\,dv = u\cdot v - \int v\,du$$

$$Ag_2(t_1,t_2) = A_{J2}(t_1,t_2) + \ldots$$

$\oplus$

$$Ag(t_1,t_2) = A_J(t_1,t_2) + \frac{L_{11}\, i_1^2}{2} + \frac{L_{12}\, i_1\, i_2}{2} + \frac{L_{21}\cdot i_1\, i_2}{2} + \frac{L_{22}\, i_2^2}{2}$$

$$Ag(0,T) = A_J(0,T) + \frac{L_{11} I_1^2}{2} + L_{12} I_1 I_2 + \frac{L_{22} I_2^2}{2}$$

P16 (15.4.2016)

Energije magnetenja

$$W_m = \frac{1}{2}\left(L_{11} i_1^2 + L_{12} i_1 i_2 + L_{21} i_1 i_2 + L_{22} i_2^2\right)$$

$$W_m = \frac{1}{2} i_1 \underbrace{\left(L_{11} i_1 + L_{12} i_2\right)}_{\Psi_1} + \frac{1}{2} i_2 \underbrace{\left(L_{21} i_1 + L_{22} i_2\right)}_{\Psi_2}$$

$$\boxed{W_m = \frac{1}{2} i_1 \Psi_1 + \frac{1}{2} i_2 \Psi_2}$$ → za dve tuljavi

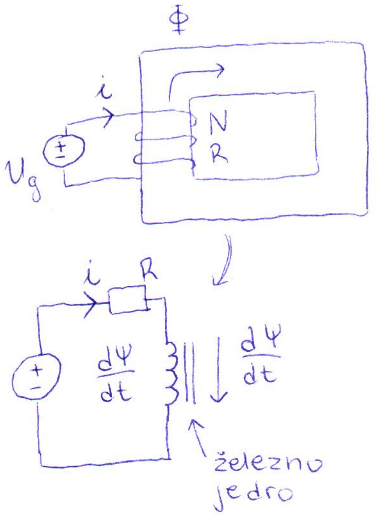
· Posplošitev za N-tuljav

$$\boxed{W_m = \frac{1}{2}\sum_{k=1}^{N} i_k \underbrace{\Psi_k} = \frac{1}{2}\sum_{k=1}^{N}\sum_{j=1}^{N} L_{jk} \cdot i_j \cdot i_k}$$

$$\Psi_k = \sum_{j=1}^{N} \underbrace{\Psi_k^{(j)}}_{L_{kj} \cdot i_j}$$

$$L = \frac{\Psi}{I}$$; L - tudi koeficient energijski koeficient

Magnetenje nelinearnih struktur



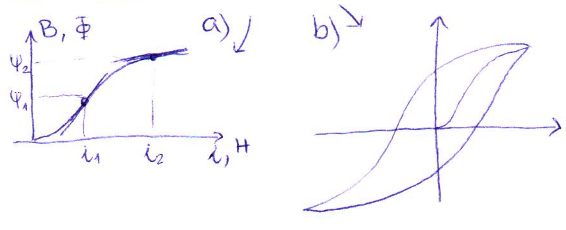
→ Linearnost: linearno zvezo med:

B, Φ, Ψ in i, Θ, H

-zrak: $B = \mu_0 H$

→ Nelinearna odvisnost

$$L = \frac{d\Psi}{di}$$



$$U_g = R \cdot i + \frac{d\Psi}{dt} \quad / \cdot i$$

$$U_g \cdot i = R \cdot i^2 + i\frac{d\Psi}{dt} \quad / \int_{t_1}^{t_2} dt$$

$U_g \cdot i$ → moč vira

$R \cdot i^2$ → Joulske izgube navitja

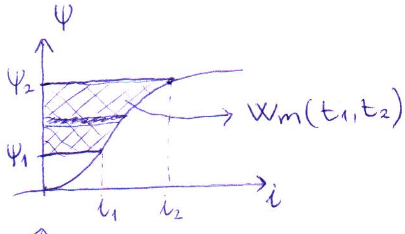
$i\frac{d\Psi}{dt}$ → moč magnetenja

Energijski vložek za magnetenje

$$\boxed{\int_{t_1}^{t_2} i \frac{d\psi}{dt} \cdot dt = \int_{\psi_1}^{\psi_2} i\, d\psi = W_m(t_1, t_2)}$$

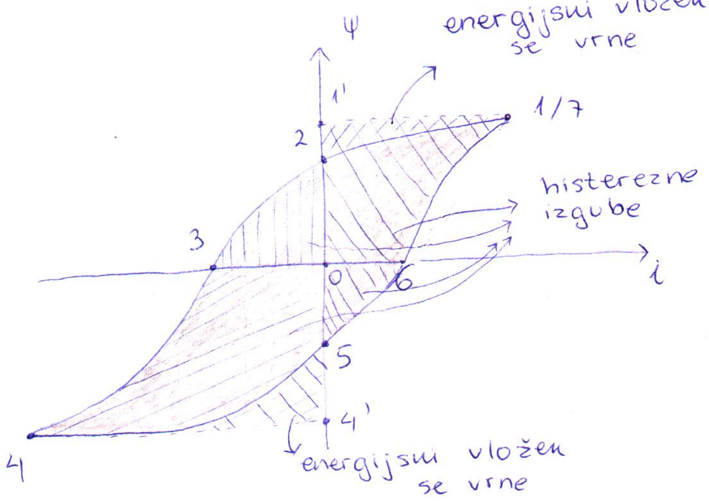
$i_1 = i(t_1)$; $\psi_1 = \psi(t_1)$

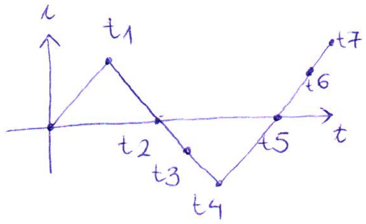
$i_2 = i(t_2)$; $\psi_2 = \psi(t_2)$



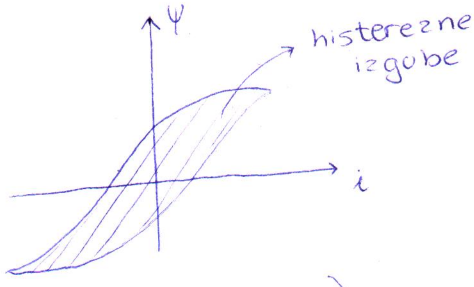
a) Deviško magnetenje

b) magnetenju histereze



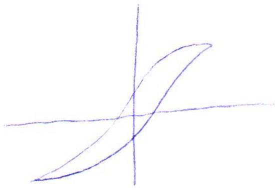


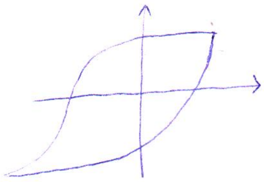
$\int_{t_1}^{t_2} i\, d\psi \propto 1, 1', 2$ $\oplus \cdot \ominus < 0$; generator, jedro vrača energijo	$\int_{t_4}^{t_5} i \cdot d\psi \propto 4, 4', 5$ $\ominus \cdot \oplus < 0$; generator
$\int_{t_2}^{t_3} i \cdot d\psi \propto 2, 3, 0$ $\ominus \cdot \ominus > 0$; breme	$\int_{t_5}^{t_6} i \cdot d\psi \propto 5, 6, 0$ $\oplus \cdot \oplus > 0$; breme
$\int_{t_3}^{t_4} i \cdot d\psi \propto 3, 4, 4', 0$ $\ominus \cdot \ominus > 0$; breme	$\int_{t_6}^{t_7} i \cdot d\psi \propto 6, 7, 1', 0$ $\oplus \cdot \oplus > 0$; breme



trdo mag.

mehko mag.





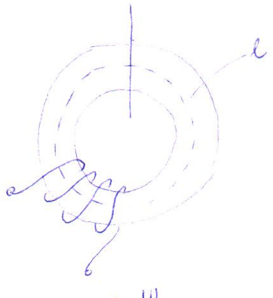
Moč histereznih izgub

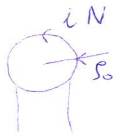
$$\boxed{P_h = \frac{1}{T} \cdot \oint_{\mathcal{H}} i \, d\Psi}$$

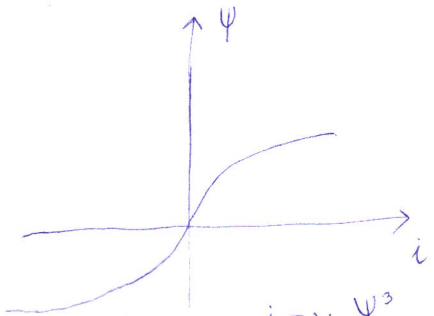
$$\boxed{P_h = f \cdot \oint_{\mathcal{H}} i \, d\Psi}$$

pri f50 Hz => $P_h = 0{,}5$ W/kg

Posledica nelinearnosti

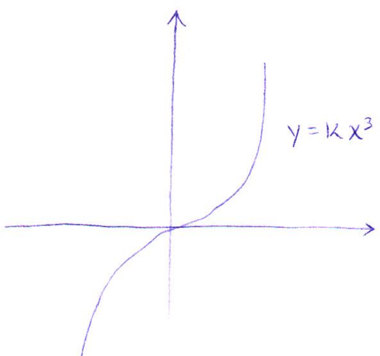






$i = k \cdot \Psi^3$

preprosta aproksimacija za mag. krivuljo



$R = 0$

$$u = \frac{d\Psi}{dt} = U_{gm} \cdot \cos \omega t$$

$$\Psi = \frac{U_{gm}}{\omega} \cdot \sin \omega t$$

$$i = k \left(\frac{U_{gm}}{\omega}\right)^3 \sin^3 \omega t = C \cdot \sin^3 \omega t$$

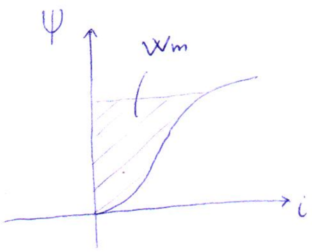
$$\left\{ \sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x \right\}$$

$$\Rightarrow i = \frac{3}{4}C\cdot\sin\omega t - \frac{1}{4}C\cdot\sin 3\omega t$$

osnovna komponenta

višje harmonik → posledica popačenj zaradi nelinearnosti

(Volumsua) Gostota magnetne energije w_m



$$W_m = \int_0^t i \cdot d\Psi$$

$d\Psi = N\cdot d\Phi = N\cdot d_t\vec{B}\cdot da$

$N\cdot i = \oint_{\mathcal{L}} \vec{H} d\vec{l}$

$$W_m = \int_0^t N\cdot i\, d\vec{B}\cdot d\vec{a} = \oint \int_0^t \vec{H}\cdot d\vec{B}\cdot dV$$

Vol celotne strukture, kjer je polje

volumen

$$w_m = \int_0^t \vec{H}\cdot d\vec{B}$$

→ volumsua gostota mag. energije

$$\oint_{V} w_m \cdot dV$$

volumen

- Za linearne strukture:

$$B = \mu_0\mu_r H \rightarrow H = \frac{B}{\mu_0\mu_r}$$

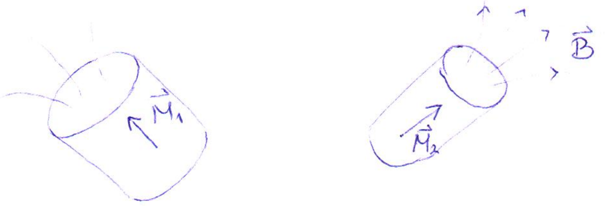
$$w_m = \int H\, dB = \int \frac{B\cdot dB}{\mu_0\cdot\mu_r} = \frac{B^2}{2\mu_0\mu_r} = \frac{\mu_0\mu_r H^2}{2}$$

$$\left\{ w_e = \frac{\varepsilon_0\varepsilon E^2}{2} \right\}$$

P17 (19.4.2016)

Gibalni procesi v magnetiki

1) avtonomni sistemi = sistem brez virov



$$W_m = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,dv \rightarrow \text{volumen}$$

$$d\vec{F}_m = -\left(\frac{dW_m}{dx}, \frac{dW_m}{dy}, \frac{dW_m}{dz}\right)$$

2) sistem z viri

$$\overbrace{Ag(t_1, t_1+dt)}^{dAg} = \underbrace{W_J(t_1, t_1+dt)}_{\text{Joulske izgube navitja}} + \underbrace{\frac{1}{2}\sum_{k=1}^{N} i_k \Psi_k}_{dW_m} + \underbrace{\vec{F}_m\cdot d\vec{l}}_{\text{delo za premik}} \quad ⊛$$

z vidika vezja:

$$dAg = W_J(t_1, t_1+dt) + \underbrace{\int_{t_1}^{t_1+dt}\sum_{k=1}^{N} i_k\, d\Psi_k}_{\sum_{k=1}^{N} i_k\, d\Psi_k} = W_J(t_1, t_1+dt) + \sum_{k=1}^{N} i_k\, d\Psi_k \quad ⊛$$

$$\frac{1}{2}\sum_{k=1}^{N} i_k\cdot\Psi_k + \vec{F}_m\cdot d\vec{l} = \sum_{k=1}^{N} i_k\cdot\Psi_k$$

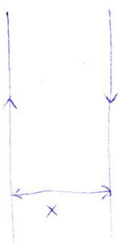
$$\boxed{\vec{F}_m\cdot d\vec{l} = \frac{1}{2}\sum_{k=1}^{N} i_k\cdot\Psi_k = dW_m} \Rightarrow \boxed{\vec{F}_m = \left(\frac{dW_m}{dx}, \frac{dW_m}{dy}, \frac{dW_m}{dz}\right)}$$

Zgled: Sila (dvovod)

$$L = \frac{\mu_0\cdot l}{\pi}\left(\frac{1}{4} + \ln\frac{x}{\rho_0}\right)$$

$$W_m = L\cdot\frac{i^2}{2} = \frac{\mu_0\cdot l\cdot i^2}{2\pi}\left(\frac{1}{4} + \ln\frac{x}{\rho_0}\right)$$

$$F_{mx} = \frac{dW_m}{dx} = \frac{\mu_0\cdot l\cdot i^2}{2\pi x}$$

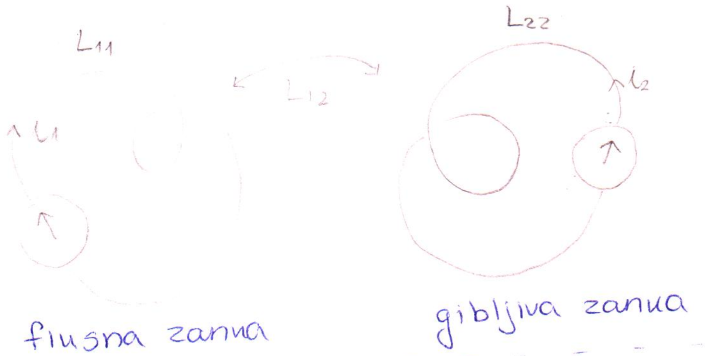


$$B = \frac{\mu_0 i}{2\pi x}$$

$$F_m = B\cdot i\cdot l$$

$$d\vec{F}_m = i\,d\vec{l}\times\vec{B}$$

Sistem dveh togih zanu



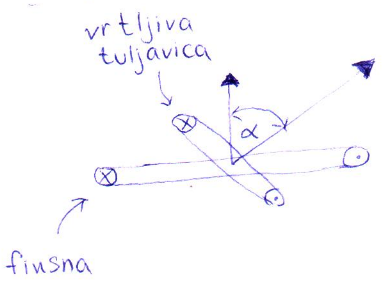
$$W_m = L_{11} \cdot \frac{i_1^2}{2} + L_{12} i_1 i_2 + L_{22} \frac{i_2^2}{2}$$

L_{12} se spreminja ob premiuu

$L_{11} = L_{22} = $ uonst (njuni odvod je $=0$ le L_{12} imamo)

$$\vec{F}_m = \left(\frac{dL_{12}}{dx}, \frac{dL_{12}}{dy}, \frac{dL_{12}}{dz} \right) \cdot i_1 \cdot i_2$$

odvod medsebojne induktivnosti



$$L_{12} = f(\alpha)$$

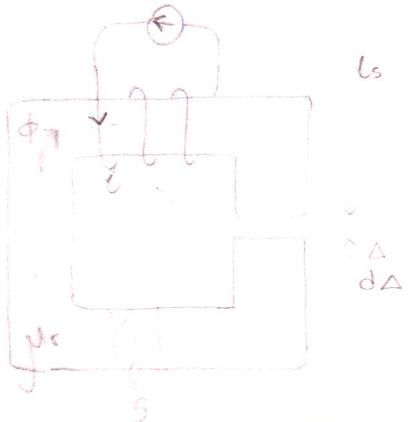
$$M = \frac{dL_{12}}{d\alpha} \cdot i_1 \cdot i_2$$

navor

$\alpha = 0: \quad L_{12} = max$

$\alpha = 90: \quad L_{12} = 0$

Eleutromagneti - plosuovne sile



$$W_m = \frac{1}{2} L \cdot i^2 = \frac{N^2 \cdot i^2}{2 \cdot R_m}$$

$$L = \frac{N^2}{R_m}$$

$$R_m = \frac{l_s}{\mu_0 \cdot \mu_r \cdot S} + \frac{\Delta}{\mu_0 \cdot S}$$

pomeni privlak

$$F_m = \frac{dW_m}{d\Delta} = \frac{dW_m}{dR_m} \cdot \frac{dR_m}{d\Delta} = \frac{-N^2 \cdot i^2}{2 \cdot R_m^2} \cdot \frac{1}{\mu_0 \cdot S} = \frac{-\Phi^2}{2\mu_0 S} = \frac{-B^2 \cdot S^2}{2\mu_0 S} = \frac{-B^2 S}{2\mu_0} = F_m$$

$$\Theta = N \cdot i$$

$$\Phi_m = \frac{\Theta}{R_m} = \int \vec{B} \cdot d\vec{a} = B \cdot S$$

homogeno polje v zračni reži

$F_m = w_m \cdot S$

↳ tlau mag. sile

$$p = \frac{F}{S}$$

$$w_m = \frac{B^2}{2\mu_0}$$

Zgled: (magnetika)

Breže = 1 T

$$w_m = \frac{1}{2 \cdot 4\pi \cdot 10^{-7}} \cdot \frac{N}{m^2} = 4 \cdot 10^5 \text{ Pa}$$

$S = 1 \text{ dm}^2 = 10^{-2} \text{ m}^2$

$F_m = 400 \text{ N} \rightarrow 400 \text{ kg}$

$$w_m = \frac{B^2}{2\mu_0}$$

(elektrostatika)

$$w_e = \frac{\varepsilon_0 E^2}{2}$$

$E = 3 \text{ MV/m}$

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}$

$w_e = 45 \text{ Pa}$

$$\boxed{\frac{F_m}{F_e} = 10^4}$$

P18 (22.4.2016)

Revizija četrte Maxwellove enačbe
Razširjen Amperov zakon vrtinčnosti magnetnega polja

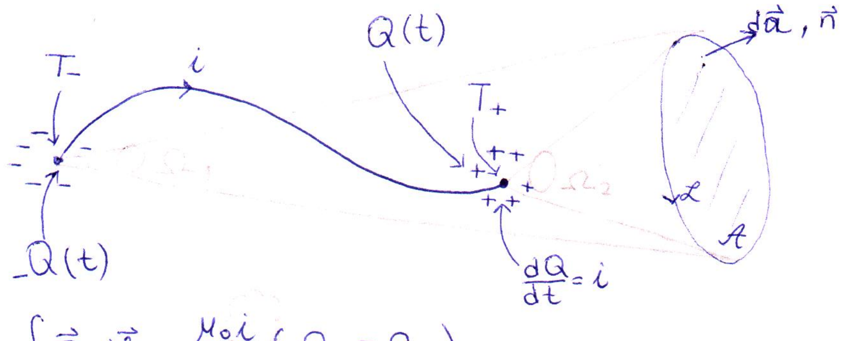
INDUKCIJA

$$\oint_{\mathcal{L}} \vec{E} \cdot d\vec{l} = 0 + u_i \quad \frac{d\Psi}{dt}$$

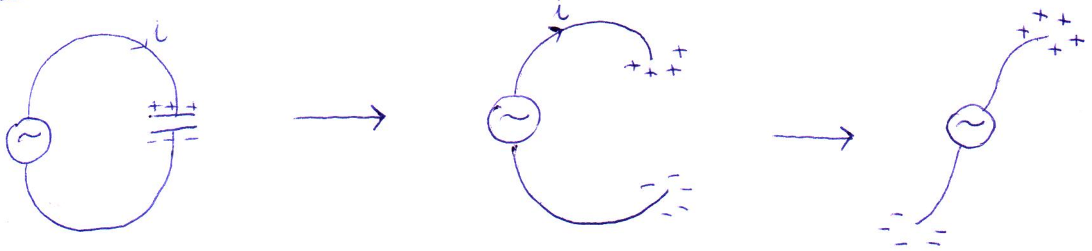
MAXWELLOV ~~TOK~~ (premikalni ~~tok~~)

$$\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \int_{\mathcal{A}} \vec{J}_{prosti} \cdot d\vec{a} + \ldots$$

Vrtinčnost tokovne niti (posplošitev iz tokovne daljice)



$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{l} = \frac{\mu_0 i}{4\pi} (\Omega_2 - \Omega_1)$$



+ kontinuitetna enačba

$$\oint_A \vec{J}\cdot d\vec{a} = -\frac{dQ_{not}}{dt} = i$$

tok ni odtoka je enak zmanjšanjega notranjega naboja

↳ v naši primer tok pritoka

i vstavimo v enačbo:

vstavimo ← OE I

$$\Phi_e = \int_A \vec{E}\cdot d\vec{a} = \frac{Q}{4\pi\varepsilon_0}\cdot\Omega$$

$$\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \frac{\mu_0\varepsilon_0 d}{dt}\cdot\frac{Q}{4\pi\varepsilon_0}(\Omega_2-\Omega_1) =$$

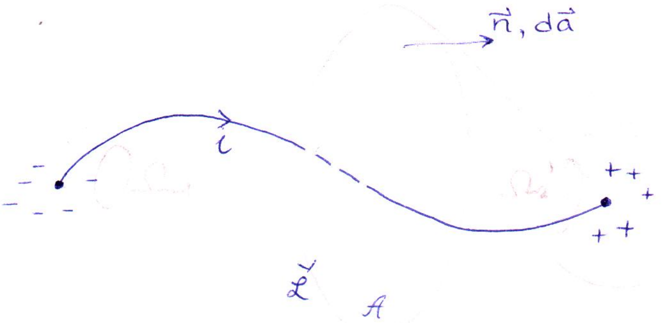
$$= \mu_0\varepsilon_0\frac{d}{dt}(\Phi_{e_2}-\Phi_{e_1}) = \mu_0\varepsilon_0\frac{d}{dt}\int \vec{E}\cdot d\vec{a} = \mu_0\varepsilon_0\frac{d\Phi_e}{dt}$$

→ obstaja kadar imamo časovno spremembo (dinamika)

$$\left\{\begin{array}{l} \oint_{\mathcal{L}} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_m}{dt} \quad \text{indukcija} \\[2ex] \boxed{\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_e}{dt}} \end{array}\right.$$

(simetrija enačb)

- Konstelacija kjer tok prebode A:



$$\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \frac{\mu_0 i}{4\pi}(\Omega_2-\Omega_1) = \mu_0 i \underbrace{- \frac{\mu_0 i}{4\pi}(\Omega_2'+\Omega_1)}_{\frac{d}{dt}\mu_0\varepsilon_0\int_A \vec{E}\cdot d\vec{a}} = \mu_0 i + \mu_0\varepsilon_0\frac{d}{dt}\int\vec{E}\cdot d\vec{a}$$

$\vec{E} = \vec{E}_+ + \vec{E}_-$

$$\boxed{\int \vec{B}\cdot d\vec{l} = \mu_0 i + \mu_0\varepsilon_0\int\frac{d\vec{E}}{dt}\cdot d\vec{a}}$$

$\underline{\mathrm{IV}}$ Maxwellova enačba

$\mu_0 i$ → magnetostatika

$\mu_0\varepsilon_0\int\frac{d\vec{E}}{dt}\cdot d\vec{a}$ → dodaten člen (dinamika)

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \Big(\underbrace{\vec{J}}_{\text{tok}} + \underbrace{\frac{d(\varepsilon \vec{E})}{dt}}_{\substack{\text{"Maxwellov tok"} \\ \text{(ni pravi tok)}}}\Big)\, d\vec{a}$$

Teorija Extra

$\vec{J}_{max}$ (ni gibanja nabojev) (premikalni tok, tok odmika)

tok: gibanje nabojev

$\vec{J}$ - gostota toka

$\vec{J}_{pe}$ = polarizacijski tok ; $\vec{J}_{mag}$ = magnetizacijski tok

$$\int_A \underbrace{\vec{J}\cdot d\vec{a}}_{i} = \int_A \vec{J}_{prosti}\, d\vec{a} + \underbrace{\underbrace{\int_A \frac{d\vec{P}}{dt}\cdot d\vec{a}}_{\vec{J}_{pe.}}}_{\text{v diel.}} + \underbrace{\int_{\mathcal{L}} \vec{M}\cdot d\vec{l}}_{\text{v magnetikih}}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \Big(\vec{J}_{prosti} + \frac{d\vec{P}}{dt} + \frac{d\varepsilon_0 \vec{E}}{dt}\Big)\, d\vec{a} + \mu_0 \cdot \oint \vec{M}\cdot d\vec{l} \quad /:\mu_0$$

$$\oint_{\mathcal{L}} \underbrace{\Big(\frac{\vec{B}}{\mu_0} - \vec{M}\Big)}_{\vec{H}}\cdot d\vec{l} = \int_A \Big(\vec{J}_{prosti} + \frac{d}{dt}(\varepsilon_0 \vec{E} + \vec{P})\Big)\, d\vec{a}$$

$$\oint_{\mathcal{L}} \vec{H}\cdot d\vec{l} = \int \Big(\vec{J}_{prosti} + \frac{d\vec{D}}{dt}\Big)\, d\vec{a}$$

P19 (3.5.2016)

ELEKTRIČNA VEZJA ČASOVNO SPREMENLJIVIH TOKOV

Označevanje veličin/količin:

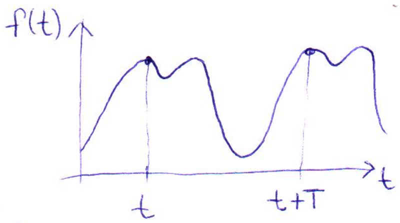
- časovno nespremenljive: $\vec{E}, \vec{B}, \phi, \Theta, U, I$
- časovno spremenljive: $\vec{E}(t), \vec{B}(t), \phi(t), \Theta(t)$

  $u = u(t) = f(t)$

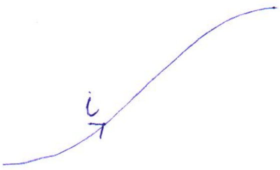
  - * neperiodične funkcije:
    - prehodni pojavi
    - šumi (bel šum)
  - * periodične funkcije:

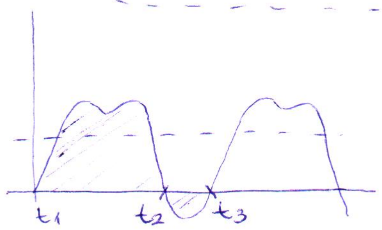
$f(t) = f(t + nT)$ → imamo neuo funucijo ki jo premaunimo za periodo

$n \in \mathbb{Z}$; T — perioda



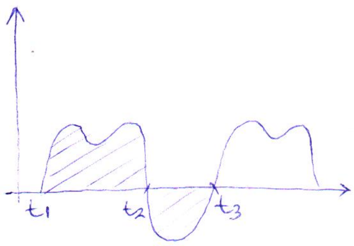
- pulzirajoče veličine





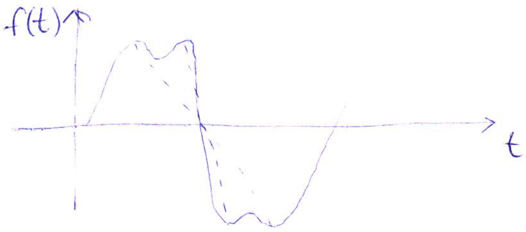
$$\int_{t_1}^{t_2} f(t)dt \neq \int_{t_2}^{t_3} f(t)dt$$

- izmenične veličine

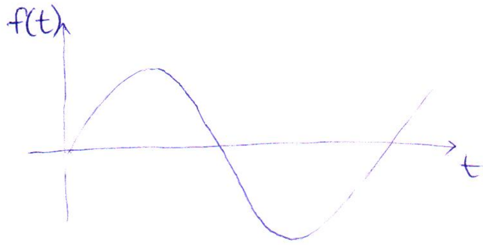


$$\left|\int_{t_1}^{t_2} f(t)dt\right| = \left|\int_{t_2}^{t_3} f(t)dt\right|$$

- izmenične
  → simetrične izmenične veličine



harmonične veličine: sinusne oz. kosinusne



freuvenca: f [Hz]

$$\boxed{f = \frac{1}{T}}$$

kotna hitrost: ω [rd/s]

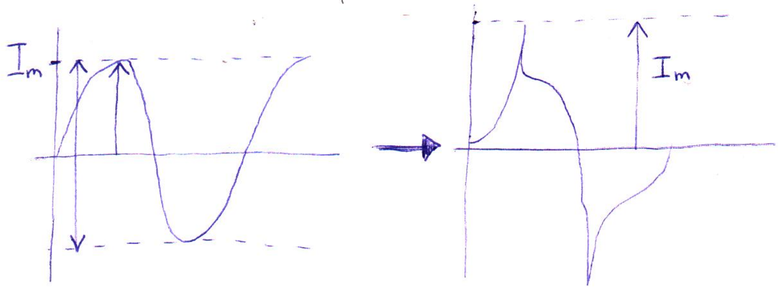
$$\boxed{\omega = 2\pi \cdot f}$$

Srednja vrednost: I_{sr}

$$I_{sr} = \frac{1}{T}\int_{t}^{t+T} i(t)\,dt = 0$$

(za izmenične količine, harmonične)

Maksimalna vrednost: I_m



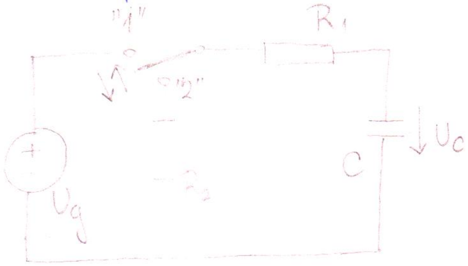
Prehodni pojavi

$$\boxed{\begin{aligned} U &= R\cdot i \\ i &= C\cdot U_c' \\ U_i &= L\cdot i' \end{aligned}}$$

Odzivi na vklop/izklop

Elementi:
- aktivne U_g, I_g
- pasivne R
- reaktivni: C, L → "akumulabilnost" elementov

1) Vklop in izklop RC vezja



Začetni pogoj:

$U_c(0) = U_0$ → začetna (pred)napetost na kondenzatorju

vklop – C se polni
izklop – C se prazni

a) stikalo v položaj "1" $0 < t < t_1$

Kirchoffova napetostna enačba:

$$-U_g + R_1 i + U_c = 0$$

$$i = C\cdot U_c'$$

$\underline{R_1 C\cdot U_c' + U_c = U_g}$ → Diferencialna enačba I reda

$R_1 C\cdot U_c' + U_c = 0$ → homogen del
+
partikularna/konkretna rešitev

vedno rabimo začetni pogoj v dif. enačbo

Rešujemo: $R_1 C U_c' + U_c = U_g$ in $U_c(0) = U_0$

homogen del:

$$\boxed{U_{ch} = A\cdot e^{\lambda t}} \quad A = ? \quad \lambda = ?$$

$$R_1 C\cdot U_{ch}' + U_{ch} = 0$$

$$R_1 C\cdot A\lambda e^{\lambda t} + A e^{\lambda t} = 0$$

$$\underbrace{(R_1 C\lambda + 1)}_{=0}\underbrace{A e^{\lambda t}}_{\neq 0} = 0$$

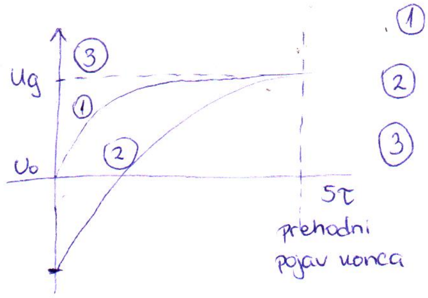
$$R_1 C\lambda = -1$$

$$\lambda = -\frac{1}{R_1 C} = -\frac{1}{\tau} \rightarrow \text{časovna konst. } \tau$$

$$U_{ch} = A e^{-\frac{t}{R_1 C}}$$

partikularna rešitev

po dolgem času $i \to 0$
$u_c' \to 0$



① $U_0 = 0$

② $U_0 = -U_g$

③ $U_0 = U_g$

$$U_{cp} = +U_g$$

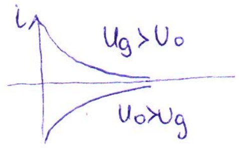
$$U_c = U_{ch} + U_{cp} = A e^{-\frac{t}{R_1 C}} + U_g$$

uporabimo začetni pogoj

$$U_c(0) = U_0 = A e^0 + U_g \rightarrow A = U_0 - U_g$$

$$U_c = (U_0 - U_g)\, e^{-\frac{t}{R_1 C}} + (U_g - U_0) + U_0$$

$$\boxed{U_c = U_0 + (U_g - U_0)\left(1 - e^{-\frac{t}{R_1 C}}\right)}$$



$$i = C \cdot U_c' = C\, \frac{U_g - U_0}{\tau}\, e^{-t/\tau}$$

$$= \frac{U_g - U_0}{R_1}\, e^{-t/\tau}$$

2) stikalo v položaj "2" $t \geq t_1$

$$U_1 = U_c(t_1 - 0) = U_0 + (U_g - U_0)\, e^{-t_1/\tau_1}$$

$$(R_1 + R_2)\, i + U_c = 0 \quad , \quad i = C \cdot U_c'$$

$$\underbrace{(R_1 + R_2)}_{R_{12}}\, C \cdot U_c' + U_c = 0$$

$$R_{12} C \cdot \lambda + 1 = 0$$

$$\lambda = -\frac{1}{R_{12} C} \quad , \quad R_{12} C = \tau_{12}$$

$$U_c = B \cdot e^{-t/\tau_{12}}$$

$$U_c(t_1) = U_1 = B e^{-t_1/\tau_{12}}$$

$$B = U_1 e^{t_1/\tau_{12}} \implies U_c(t \geq t_1) = U_1 e^{-(t - t_1)/\tau_{12}}$$

Efektivna vrednost veličine I_{ef}

$$P = \frac{1}{T}\int_t^{t+T} p(t)\,dt = R \cdot \underbrace{\frac{1}{T}\int_t^{t+T} i^2(t)\,dt}_{I_{ef}^2} = R I_{ef}^2 \quad ; \quad I_{ef} = \sqrt{\frac{1}{T}\int_t^{t+T} i^2(t)\,dt}$$

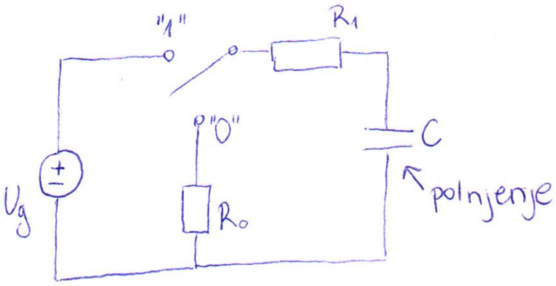
za harmonične:

$$\boxed{I_{ef} = \frac{I_m}{\sqrt{2}}}$$

Faktor oblike, temenski faktor

$$\frac{I}{I_{sr}} \qquad \frac{I_m}{I} \qquad \text{za harmonične } \sqrt{2}$$

P20 (6.5.2016)



$\Downarrow$

$$U_{ch}(t_1) = U_1 = A\cdot e^{-\frac{t_1}{R_{10}\cdot C}}$$

$$A = U\cdot e^{\frac{t}{R_0\cdot C}}$$

časovni zamik praznenja za čas t_1

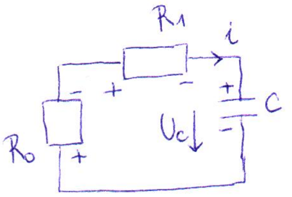
$\Downarrow$

$$U_{ch} = U_1\, e^{-\frac{(t-t_1)}{R_0\cdot C}}$$

b) Položaj "0" $t_1 < t$

$$U_c(t_1) = (U_0 - U_g)\, e^{-\frac{t_1}{\tau}} + U_g = U_1 < U_g$$

začetna vrednost (po preklopa)



$\downarrow U_{1\,zač.}$

$$R_1\cdot i + U_{gc} + R_0\cdot i = 0\,; \qquad i = C\cdot U_c'$$

$$(R_1 + R_0)\, C\cdot U_c' + U_c = 0$$

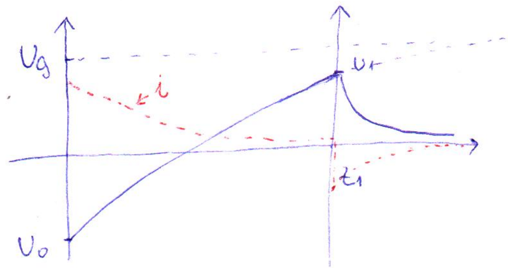
$$R_{10}\cdot C\cdot U_c' + U_c = 0 \rightarrow \text{lin. dif. en. I reda homogena}$$

$$U_{ch} = A\cdot e^{\lambda t} = A\cdot e^{-\frac{t}{R_{10}\cdot C}}$$

$$U_{ch}' = A\lambda e^{\lambda t}$$

$$(R_{10}\cdot C\cdot\lambda + 1)\underbrace{A\, e^{\lambda t}}_{\neq 0} = 0$$

$$\lambda = -\frac{1}{R_{10}\, C}$$



$$i = C\cdot U_c'$$

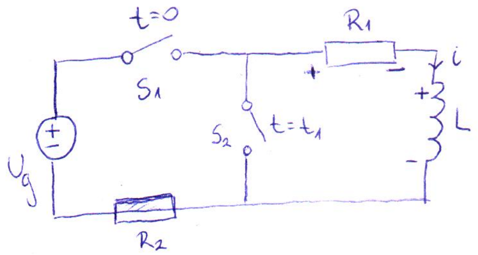
$$i = \cancel{C}\cdot\left(-\frac{1}{R_{10}\cancel{C}}\right)U_1 e^{-\cdots} = -\frac{1}{R_{10}}\cdot U_1\cdot e^{-\frac{(t-t_1)}{R_{10}\cdot C}}$$

Vklop in izklop RL vezja

$$W_e = \frac{C\cdot u^2}{2} \rightarrow \text{napetost na kond. je zvezna f.}$$

$$W_m = \frac{L\cdot i^2}{2} \rightarrow \text{tok skozi tuljavo} \quad -\text{''}-$$

zvezna f.



1) $0 < t < t_1$ vklop S_1

$R_1 i + U_L + R_2 \cdot i = U_g$; $U_L = L \cdot i'$

$(R_1 + R_2) i + L i' = U_g$

$\boxed{R_{12} \cdot i + L i_1' = U_g}$ lin. dif. en. I reda, nehomog.

$+ \boxed{i(0) = 0}$ zač. pogoj

homogen del

$(R_{12} \cdot \lambda + L) \cdot A \cdot e^{\lambda t} = 0$

$i = A \cdot e^{\lambda t} \Rightarrow A e^{-\frac{L}{R_{12}} \cdot t}$

$i' = A \cdot \lambda e^{\lambda t}$

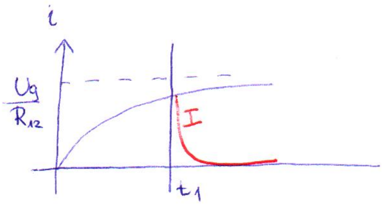
$\lambda = -\frac{L}{R_{12}} = -\frac{1}{\tau}$

$\tau = \frac{R_{12}}{L}$ [s] prehodni pojav v času 5τ

partikularna rešitev:

$i(\infty) = i_p = \frac{U_g}{R_{12}}$

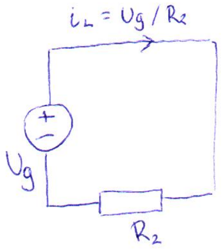
$i = \underbrace{A \cdot e^{-\frac{L}{R_{12}} t}}_{\text{homogena}} + \underbrace{\frac{U_g}{R_{12}}}_{\text{partiku.}} = \frac{U_g}{R_{12}} \left(1 - e^{-\frac{L}{R_{12}} \cdot t}\right)$

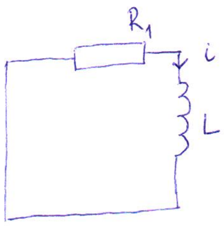


$i(0^-) = i(0^+) = 0 = A e^0 + \frac{U_g}{R_{12}}$

$A = -\frac{U_g}{R_{12}}$

2) izklop tuljave



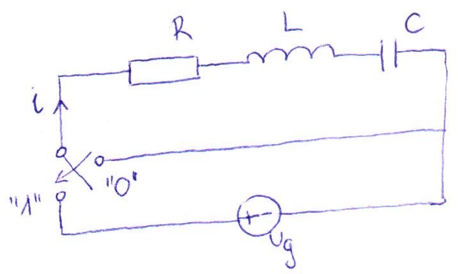


$\boxed{R_1 i + L \cdot i' = 0}$ lin. dif. en. I reda (homog.)

$+ \boxed{i(t_1) = I_1}$

$i(t > t_1) = I_1 e^{-\frac{t - t_1}{\tau_1}}$; $\tau = \frac{R_1}{L}$

Vklop in izklop RLC vezja



$U_C(0^-)=0$, $i(0^-)=0$ } dva začetna pogoja

1) $0 < t < t_1$ (položaj "1")

$U_R + U_L + U_C = U_g \quad / \frac{d}{dt}$

$\boxed{U_R = R\cdot i}$
$U_L = L\cdot i'$
$U_C = \frac{1}{C}\int i\,dt \quad (i = C\cdot U_C')$
$U_C' = \frac{i}{C}$

$$\boxed{R i' + L i'' + \frac{i}{C} = 0}$$

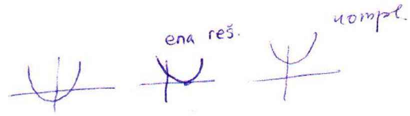
$$i'' + \frac{R}{L} i' + \frac{i}{LC} = 0$$

Lin. dif. en. II reda (homog.)

$i = A e^{\lambda t}$
$i' = A\lambda e^{\lambda t}$
$i'' = A\lambda^2 e^{\lambda t}$

$$\left(\lambda^2 + \frac{R}{L}\lambda + \frac{1}{LC}\right) A\cdot e^{\lambda t} = 0$$

karak. polinom



$$\lambda_{1/2} = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$p = \frac{R}{2L}\ ; \quad q^2 = \left(\frac{R}{2L}\right)^2 - \frac{1}{LC}$$

1) $q^2 > 0$: $\lambda_1 = -p - q$, $\lambda_2 = -p + q$

2) $q^2 = 0$: $\lambda_1 = \lambda_2 = -p$

3) $q^2 < 0$: $\lambda_1 = -p - iq$, $\lambda_2 = -p + iq$

1) $i = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 \cdot t}$, $+\ i(0^-)=0$, $+\ U_C(0^-) = 0$

$$i = \frac{U_g/L}{\lambda_1 - \lambda_2}\left(e^{\lambda_1 t} + e^{\lambda_2 t}\right) = -\frac{U_g/L}{2\cdot q}\left(e^{-pt-qt} - e^{-pt+qt}\right) =$$

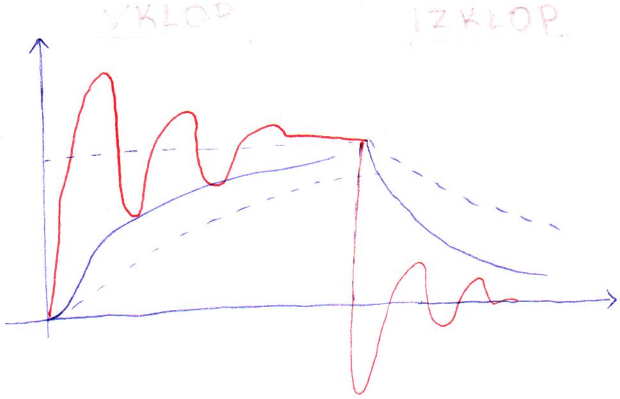
$$= -\frac{U_g}{q\cdot L} e^{-pt}\left(\frac{e^{-qt} - e^{qt}}{2}\right) = -\frac{U_g}{q\cdot L} e^{-pt}\,\mathrm{sh}(qt)$$

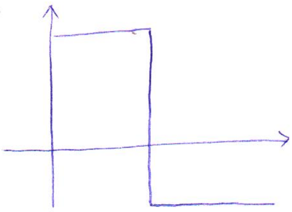
2) $i = \frac{U_g}{L}\, t\, e^{-pt}$ (rešitev dobimo s L'Hospitalovo pravilo) $\left(\lim_{q\to 0} \frac{\mathrm{sh}\, qt}{q} = t\right)$

3) $i = \frac{U_g e^{-pt}}{i|q|L}\,\mathrm{sh}\, i|q|t = \frac{U_g}{|q|L} e^{-pt}\sin(qt)$

RLC - nihajni krog

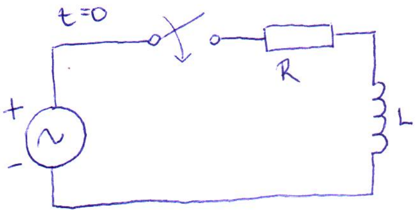
$\mathrm{sh}\, ix = \frac{e^{ix} - e^{-ix}}{2} = i\sin x$?





P21 (10.5.2016)

Vklop RL vezja na harmoničen vir



$u_g = U_{gm} \cdot \cos(\omega t + \varphi_u)$ ← začetni fazni kot

$u_L = L \cdot i'$

$u_R = R \cdot i$

nap: φ_u

tok: φ_i

razlika faz:

$\varphi_u - \varphi_i = \varphi$

$\varphi_i = \varphi_u - \varphi$

1) Vklop $(t>0)$

$$\boxed{u_R + u_L = u_g} \rightarrow R \cdot i + L \cdot i' = U_{gm} \cos(\omega t + \varphi_u)$$ → Nehomogena lin. dif. en. I reda

homogen del:

$$\left(i' + \frac{R}{L} i\right) = 0 \rightarrow i = Ae^{\lambda t} = Ae^{-\frac{R}{L}t}$$

$$i' = A\lambda e^{\lambda t}$$

$$\underbrace{\left(\lambda + \frac{R}{L}\right)}_{\lambda = -\frac{R}{L} = -\frac{1}{\tau}} \underbrace{Ae^{\lambda t}}_{\neq 0} = 0$$

partikularna rešitev:

$i_p = I_m \cdot \cos(\omega t + \varphi_i)$ (amplituda: I_m; fazni zamik: φ_i)

$i_p = I_m \cdot \cos(\omega t + \varphi_u - \varphi)$

$i_p' = -\omega I_m \sin(\omega t + \varphi_u - \varphi)$

$$R \cdot I_m \cdot \cos(\underbrace{\omega t + \varphi_u}_{x} - \underbrace{\varphi}_{y}) - \omega L I_m \cdot \sin(\underbrace{\omega t + \varphi_u}_{x} - \underbrace{\varphi}_{y}) = U_{gm} \cos(\underbrace{\omega t + \varphi_u}_{x})$$

$$\left[\begin{array}{l} \sin(x \pm y) = \sin x \cdot \cos y \pm \cos x \cdot \sin y \\ \cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y \end{array}\right]$$

$R I_m \cos\varphi \cdot \cos(\omega t + \varphi_u) + R I_m \sin\varphi \sin(\omega t + \varphi_u) - \ldots$

$\ldots - \omega L I_m \cos\varphi \sin(\omega t + \varphi_u) + \omega L I_m \sin\varphi \cdot \cos(\omega t + \varphi_u) =$

$= U_{gm} \cos(\omega t + \varphi_u) + 0 \cdot \sin(\omega t + \varphi_u)$

$$\begin{cases} -\omega L I_m \cdot \cos\varphi + R \cdot I_m \cdot \sin\varphi = 0 \quad \xrightarrow{:(-\cos\varphi)} \\ \omega L I_m \cdot \sin\varphi + R \cdot I_m \cdot \cos\varphi = U_{gm} \end{cases}$$

$\omega L I_m = R I_m \cdot tg\,\varphi$

$$\boxed{tg\,\varphi = \frac{\omega L}{R}}$$

fazni zamik φ !

obe enačbi kvadriramo in seštejemo:

$$\omega^2 L^2 I_m^2 + R^2 I_m^2 = U_{gm}^2$$

amplituda

$$I_m = \frac{U_{gm}}{\sqrt{R^2 + \omega^2 L^2}}$$

abs. vrednost impedance

$$\Rightarrow \quad i_p = \frac{U_{gm}}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\omega t + \varphi_u - \operatorname{arctg} \frac{\omega L}{R}\right)$$

$$i = i_h + i_p$$

za določitev A:

$$i(0^+) = i(0^-) = 0 = A e^{-\frac{R}{L}t} + I_m \cdot \cos(\omega t + \varphi_u - \varphi)$$

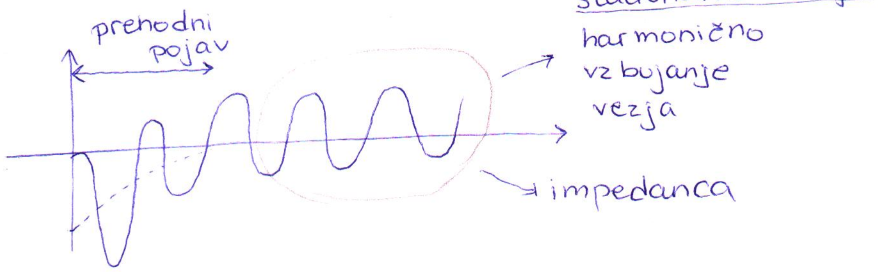
$$= A + I_m \cdot \cos(\varphi_u - \varphi) = 0$$

$$\boxed{A = -I_m \cos(\varphi_u - \varphi)}$$

rešitev:

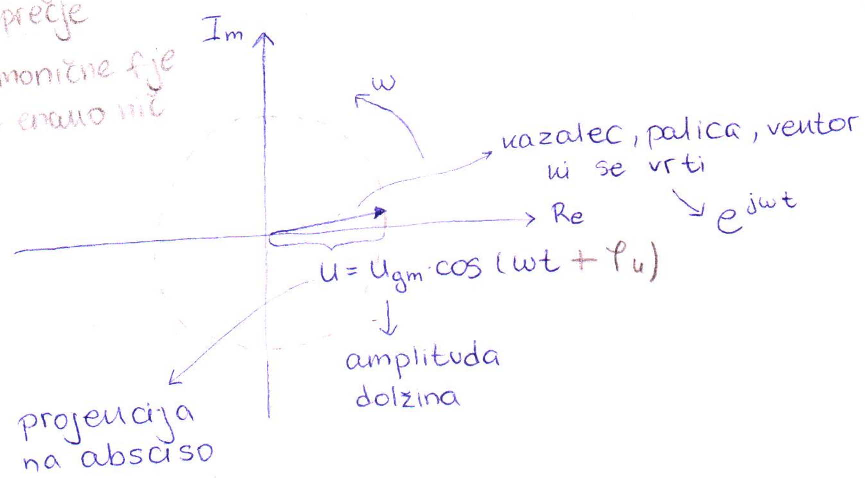
$$i(t) = -I_m \cdot \cos(\varphi_u - \varphi) \cdot e^{-\frac{R}{L}t} + I_m \cos(\omega t + \varphi_u - \varphi)$$

$$I_m = \frac{U_{gm}}{\sqrt{R^2 + (\omega L)^2}} \quad ; \quad \varphi = \operatorname{arc\,tg} \frac{\omega L}{R} \quad \text{(fazni zamik)}$$



Harmonično vzbujana el. vezja

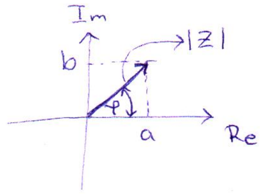
povprečje harmonične f-je je enako nič



· vstopamo v cplx prostor

$\underline{Z} = a + jb = |Z| \cdot e^{j\varphi}$

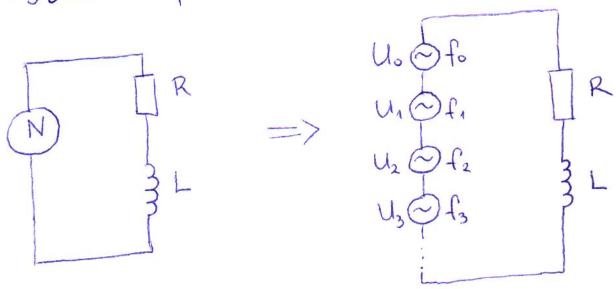
i, II



2) Odvod in integral sta harmonični funkciji

$u = L \cdot i'$ $\qquad i = C \cdot u'$

3) Vsak periodičen signal je vsota harmoničnih



Uporaba cplx računa pri harmoničnih vezjih

$e^{j\alpha} = \cos\alpha + j\sin\alpha$

$\underbrace{\underline{U} = U_m \cdot e^{j(\omega t + \varphi_u)}}_{\text{kompleksni zapis veličine (napetosti)}} = \underbrace{U_m \cdot \cos(\omega t + \varphi_u)}_{\text{funkcija v časovnem prostoru}} + jU_m \cdot \sin(\omega t + \varphi_u)$

$\boxed{u(t) = \mathrm{Re}(\underline{U})}$ (1)

☆ če je vzbujanje harmonična funkcija z frekvenco ω, je tudi odziv harmonična funkcija z frekvenco ω.

$\underline{U} = U_m \cdot e^{j\varphi_u} \cdot \cancel{e^{j\omega t}}$ opuščamo pisanje vrtenja kompleksorjev

(1) $\boxed{\underline{U} = U_m \cdot e^{j\varphi_u}}$

Kirchoffova zakona v Cplx

$$\sum_{k=1}^{n} I_k = 0$$

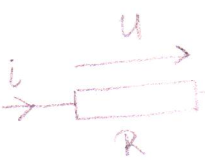
$$\sum_{k=1}^{n} i_k = 0 \Rightarrow \sum_{k=1}^{n} I_k \cdot \cos(\omega t + \varphi_k) = Re \sum \dots = \sum_{k=1}^{n} Re(\underline{I}_k \cdot \cos(\omega t + \varphi_k)) \Rightarrow$$

$$\sum_{k=1}^{n} \underline{I}_k = 0$$

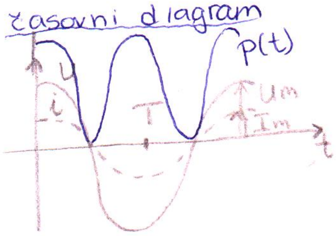
$$\Rightarrow \sum_{k=1}^{n} \underline{U}_k = 0$$

P22 (13.5.2016)

Upornost priuljučena na harmonični vir



časovni diagram



$u = U_m \cos \omega t \rightarrow \underline{U} = U_m e^{j\varphi_u}$, $\varphi_u = 0$

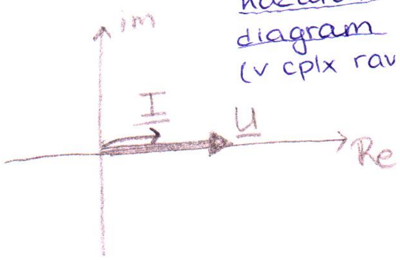
$i = \frac{u}{R} = I_m \cos \omega t$ $\quad \underline{I} = I_m e^{j\varphi_i}$, $\varphi_i = 0$

za upor velja

$\varphi = \varphi_u - \varphi_i = 0$

$$\underline{U} = R \cdot \underline{I}$$

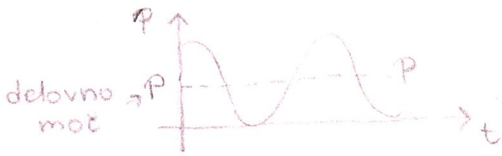
nazalčni diagram (v cplx ravnini)



Trenutno moč

$$p = u \cdot i = U_m \cdot I_m \cdot \cos^2 \omega t = U_m \cdot I_m \frac{1 + \cos 2\omega t}{2} = \frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} (1 + \cos 2\omega t)$$

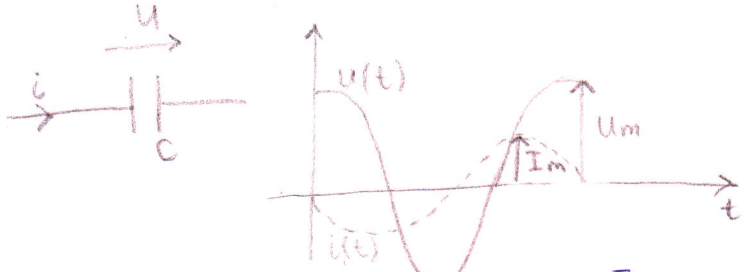
dvojno frekvenco



- energija prihaja "v paketih"
- frekvenca utripanja 100 Hz

$$\bar{p} = \frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = U_{ef} \cdot I_{ef} = P$$

Kondenzator, priključen na harm. vir



$u = U_m \cdot \cos\omega t \Rightarrow \underline{U} = U_m e^{j\varphi_u}$, $\varphi_u = 0$

$$\boxed{i = \frac{dQ}{dt} = C\frac{du}{dt}}$$

$= -\omega C U_m \sin\omega t$

$= \omega C U_m \cdot \cos(\omega t + \frac{\pi}{2})$

za C: $\varphi = \varphi_u - \varphi_i = -\frac{\pi}{2}$

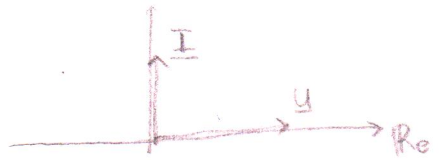
tok prehiteva napetost za 90°

$$\underline{I} = \omega C \cdot U_m \cdot e^{j(\varphi_u + \frac{\pi}{2})} = \omega C \cdot \underbrace{e^{j\frac{\pi}{2}}}_{j} \cdot \underbrace{U_m \cdot e^{j\varphi_u}}_{\underline{U}}$$

$$\boxed{\underline{I} = j\omega C \cdot \underline{U}}$$

namesto odvoda, množenje z "jω"

Kazalčni diagram:



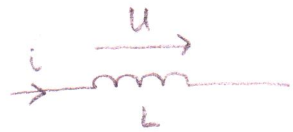
$p(t) = U_m \cdot \cos\omega t \cdot (-I_m) \cdot \sin\omega t =$

$= -\frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cdot \sin 2\omega t$

↳ niha z dvojno frekvenco

↳ povp. vrednost enaka 0

Tuljava, priključena na harm. vir



za L velja:

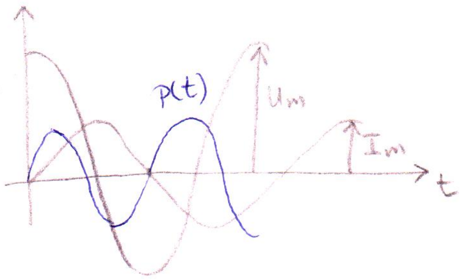
$\varphi = \varphi_u - \varphi_i = \frac{\pi}{2}$

napetost prehiteva tok za 90°

$u = U_m \cdot \cos\omega t \Rightarrow \underline{U} = U_m e^{j\varphi_u}$, $\varphi_u = 0$

$u = L\frac{di}{dt} \rightarrow i = \frac{1}{L}\int u\,dt = \frac{U_m}{\omega L}\sin\omega t =$ ($\frac{U_m}{\omega L} = I_m$)

$= \frac{U_m}{\omega L} \cdot \cos(\omega t - \frac{\pi}{2})$



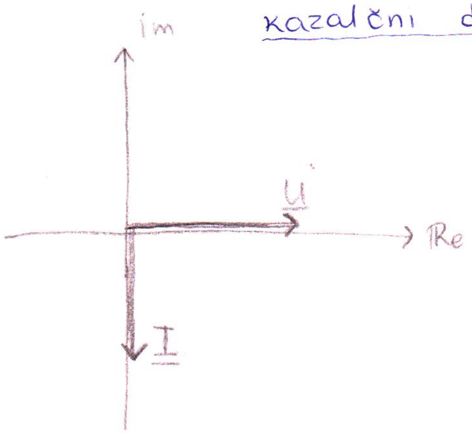
$\underline{I} = \frac{U_m}{\omega L} e^{j(\varphi_u - \frac{\pi}{2})} \rightarrow \varphi_i = -\frac{\pi}{2}$

$\underline{I} = \frac{U_m \cdot e^{j\varphi_u}}{\omega L} \underbrace{e^{-j\frac{\pi}{2}}}_{-j = \frac{1}{j}} = \frac{\underline{U}}{j\omega L}$

$$\boxed{\underline{I} = \frac{\underline{U}}{j\omega L}} \quad \boxed{\underline{U} = j\omega L \cdot \underline{I}}$$

integral v časovnem prostoru → deljenje z "jω" v cplx prostoru

Kazalčni diagram:



Sklopljeni tuljavi priključeni na harm. vir

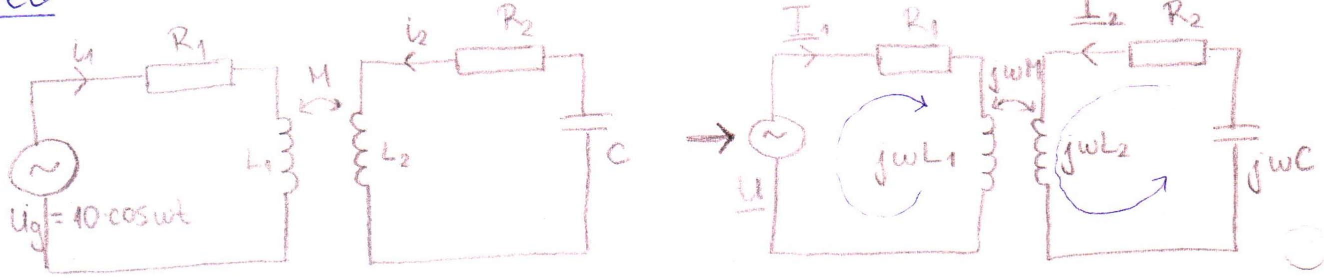
$$u_1 = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \rightarrow \underline{U}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$$

$$u_2 = M\frac{di_1}{dt} + L_2 \frac{di_2}{dt} \rightarrow \underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$$

velja za vse časovne oblike

velja za harmonične veličine

Zgled



$u_g = 10V$ $\quad L_1 = 4mH \quad \varphi_u = 0$

$\omega = 1kHz$ $\quad L_2 = 1mH$

$R_1 = 4\Omega$ $\quad K = 1 \; ; \; M = 2mH$

$R_2 = 1\Omega$ $\quad C = 1mF$

$$\left[\begin{array}{l}(R_1 + j\omega L_1)\underline{I}_1 + j\omega M \underline{I}_2 = \underline{U} \\ j\omega M \underline{I}_1 + \left(R_2 + j\omega L_2 + \frac{1}{j\omega C}\right)\underline{I}_2 = 0\end{array}\right.$$

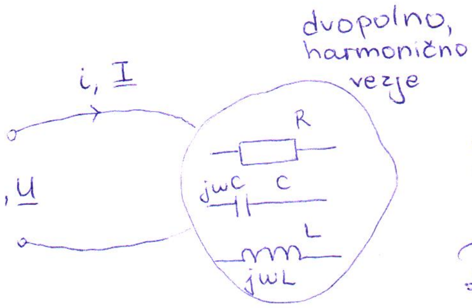
$$(4 + j4)\underline{I}_1 + j2\underline{I}_2 = 10$$
$$j2\underline{I}_1 + \underline{I}_2 = 0$$

$\Rightarrow$

$$\underline{I}_1 = 1 - \frac{j}{2} = 1{,}12A\, e^{-j26{,}6^\circ} \rightarrow \underline{i_1 = 1{,}12\cos(\omega t - 0{,}46)A}$$

$$\underline{I}_2 = -1 - j2 = 2{,}24A\, e^{j243^\circ} \rightarrow \underline{i_2 = 2{,}24\cos(\omega t + 4{,}25)A}$$

Impedanca oz. admitanca



V časovnem prostoru

$\frac{u(t)}{i(t)}$ - ne pomeni ničesar

ne obstaja v čas prost.

Def:

$$Z = \frac{\underline{U}}{\underline{I}} \; [\Omega] \quad \text{Impedanca}$$

$$Y = \frac{1}{Z} = \frac{\underline{I}}{\underline{U}} \quad \text{Admitanca}$$

$Z = Re(Z) + jIm(Z)$

	Z	Y
upor	R	$\frac{1}{R} = G$
kond.	$\frac{1}{j\omega C}$	$j\omega C$
tuljava	$j\omega L$	$\frac{1}{j\omega L}$

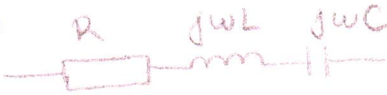
Zaporedna in vzporedna vezava impedance

zaporedna

1) $\underline{Z}_{nad} = \sum_{i=1}^{n} \underline{Z}_i$

$\underline{Z} = R + j\omega L$

2)

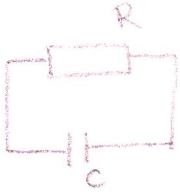


$\underline{Z} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$

Vzporedna

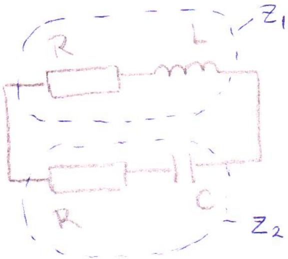
1) $\underline{Y}_{nad} = \underline{Y}_1 + \underline{Y}_2 \Rightarrow \frac{1}{\underline{Z}_{nad}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} \Rightarrow \underline{Z}_{nad} = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$

2)



$\underline{Y}_{nad} = \frac{1}{R} + j\omega C$

3)



$\underline{Y}_{nad} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}$

$\underline{Y}_{nad} = \frac{1}{R + j\omega L} + \frac{1}{R + \frac{1}{j\omega C}}$

$$\boxed{\frac{1}{a+jb} = \frac{a-jb}{(a+jb)(a-jb)} \quad ; \quad \frac{1}{Z \cdot e^{j\varphi}} = \frac{1}{Z} e^{-j\varphi}}$$

impedanca: $\underline{Z} = R + jX$

resistanca: R

reaktanca: X

$+jX_L = j\omega L$

$-jX_C = \frac{1}{j\omega C} = -j\frac{1}{\omega C}$

$\underline{Y} = G + jB$

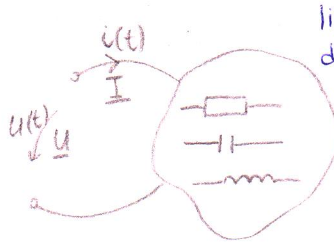
admitanca

konduktanca

susceptanca

($\neq$ susceptibilnost χ_e, χ_m)

Kompleksor moči



linearno dvopolno vezje

$$p(t) = u(t)\cdot i(t) = U_m \cos(\omega t + \varphi_u) \cdot I_m \cos(\omega t + \varphi_i)$$

$$\left\{ \cos\alpha \cdot \cos\beta = \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right) \right\}$$

$$p(t) = \frac{U_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \left(\cos(\underbrace{\varphi_u - \varphi_i}_{\varphi}) + \overbrace{\cos(2\omega t + \varphi_u + \varphi_i)}^{*} \right)$$

↳ niha z dvojno frekvenco

moč razdelimo na $p_1(t)$ - nenegativni del
$p_2(t)$ - jalov del

razvijmo * izraz:

$$\frac{U_m I_m}{2} \cos(2\omega t + \varphi_u + \varphi_i) = \frac{U_m I_m}{2} \cos(2\omega t + 2\varphi_u - \varphi) =$$

↳ $\varphi_u - \varphi_i = \varphi \Rightarrow \varphi_i = \varphi_u - \varphi$

$$= \frac{U_m I_m}{2} \cos\left(2(\omega t + \varphi_u) - \varphi\right) = \frac{U_m I_m}{2} \cos\varphi \cdot \cos 2(\omega t + \varphi_u) + \frac{U_m I_m}{2} \sin\varphi \cdot \sin 2(\omega t + \varphi_u)$$

$$p(t) = \underbrace{\frac{U_m I_m}{2} \cos\varphi \left(1 + \cos 2(\omega t + \varphi_u)\right)}_{p_1 \text{ - nenegativni del}} + \underbrace{\frac{U_m I_m}{2} \sin\varphi \cdot \sin 2(\omega t + \varphi_u)}_{p_2(t)}$$

$$\boxed{P = \overline{p_1} = \frac{U_m I_m}{2} \cos\varphi} \rightarrow$$ delovna moč (aktivna)

$$\boxed{\overline{p_2} = 0}$$

$$\boxed{\frac{U_m I_m}{2} \sin\varphi = Q} \rightarrow$$ jalova (reaktivna) moč

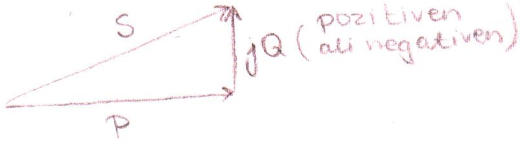
- Definirajmo kompleksor moči

$$\boxed{\underline{S} = P + jQ}$$

$\underline{S}$ ↓ navidezna moč [V·A] (kompleksna)

P ↳ delovna moč [W]

Q ↳ jalova moč [VAr], [var]



$$\underline{S} = P + jQ = \frac{U_m I_m}{2} \cos\varphi + j \frac{U_m I_m}{2} \sin\varphi =$$

$\varphi = \varphi_u - \varphi_i$

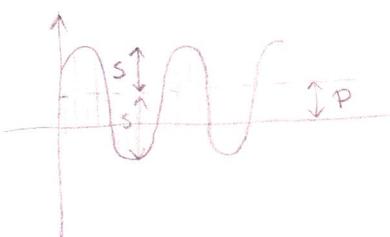
konjugirano

$$\left\{ \begin{array}{l} \underline{z} = a + jb \\ \overline{\underline{z}} = a - jb \end{array} \right\}$$

$$= \underbrace{\frac{1}{2} U_m \cdot e^{j\varphi_u}}_{\underline{U}} \underbrace{\overline{\left(\frac{1}{2} I_m e^{j\varphi_i}\right)}}_{\overline{\underline{I}}}$$

$$\underline{S} = \frac{1}{2} \underline{U}\, \overline{\underline{I}} = \frac{1}{2} \underline{Z} \cdot I^2 = \frac{1}{2} \overline{\underline{Y}} \cdot U^2$$

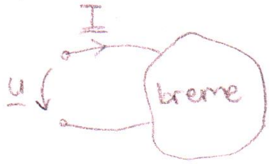
$$S = \frac{U_m I_m}{2}$$



Zgled:

$U = 250V$, $I = 4A$ } max vrednosti

$\varphi = 30° \rightarrow \underbrace{\cos 30°}_{\text{faktor delavnosti}} = \frac{\sqrt{3}}{2}$ ⊛



Če je $\cos\varphi = 1 \Rightarrow$ ohmsko breme

⊛ $\cos\varphi > 0 \Rightarrow$ INDUKTIVNO BREME

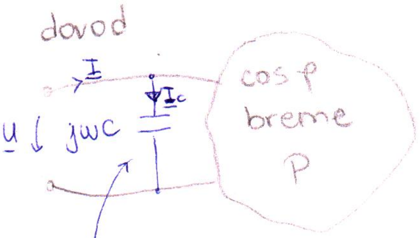
$P = \frac{U \cdot I}{2} \cdot \cos\varphi = \underline{433\,W}$

$\underline{S} = P + jQ = \underline{(433 + j250)\,VA}$

$Q = \frac{U \cdot I}{2} \sin\varphi = \underline{250\,var}$

$S = \frac{U \cdot I}{2} = \underline{500\,V \cdot A}$

Kompenzacija jalove moči



kompenzacija jalove moči

$P = 2kW$

$\cos\varphi = 0{,}8$

$U = 230V$

$f = 50\,Hz$

$\rightarrow S = \frac{P}{\cos\varphi} = 2{,}5\,kVA$

$Q = S \cdot \sin\varphi = \sqrt{S^2 - P^2} = 1{,}5\,kvar$

* prenos jalove moči: izmenjava energije med virom in (L/C) in nazaj.
* To niso izgube!
* Zaradi tega v dovodu teče večji tok ki mu rečemo (I_c);
* Dovod ima upornost: R zaradi $R \cdot I_c^2 \rightarrow$ dodatne izgube na dovodu.

$\underline{I}_c = j\omega C \cdot \underline{U}$

$S = \frac{1}{2} \underline{U}\,\overline{\underline{I}} = \frac{1}{2} U (\overline{I} - j\omega C \overline{\underline{U}})$

$S = P + (\underbrace{jQ}_{\text{jalova moč bremena}} - j\omega C U^2) \Rightarrow$ kompenzacija, $\rightarrow Q = 0$

$Q = \omega C U^2$

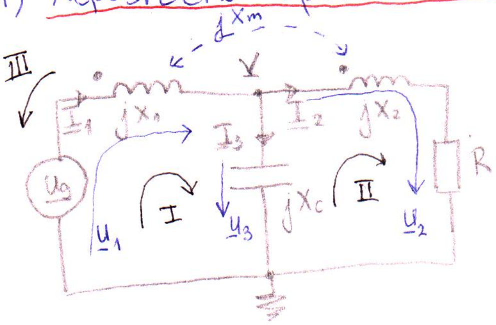
$C = \frac{Q}{\omega U^2} = \underline{\underline{98{,}6\,\mu F}}$

jω - metoda

količina operacija	čas prostor	cplx prostor
tok	$i(t)$	$\underline{I}$
napetost	$u(t)$	$\underline{U}$
impedanca	___ *	$\underline{Z}$
moč	$p(t) = u(t) \cdot i(t)$	$S = \frac{1}{2} \underline{U}\,\overline{\underline{I}} = P + jQ$
produkt	$a \cdot i(t)$	$a \cdot \underline{I}$
odvod	di/dt	$j\omega \underline{I}$
integral	$\int i \cdot dt$	$\underline{I}/j\omega$
časovni zamik	$i(t + t_1)$	$\underline{I} \cdot e^{j\omega t_1}$

P24(20.5.2016) Metode za reševanje

1) Neposredna uporaba K.Z.



podatki vezja

$X_1 = 20\Omega$

$X_2 = 20\Omega$

$X_m = 10\Omega$

$X_c = 20\Omega$

$R = 6\Omega$

$U_g = 150V$

tokovni KZ: $-\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$

napetostni KZ: $\underline{U}_1 + \underline{U}_3 = 0$

$\underline{U}_2 - \underline{U}_3 = 0$

$$\begin{cases} \underline{U}_1 = -U_g + jX_1 \underline{I}_1 + jX_m \underline{I}_2 \\ \underline{U}_2 = +jX_2 \underline{I}_2 + jX_m \underline{I}_1 + R\underline{I}_2 \\ \underline{U}_3 = -jX_c \cdot \underline{I}_3 \end{cases}$$

lahko izrazimo tudi takole

$\rightarrow \underline{I}_1 = \frac{U_1 + U_g - jX_m \underline{I}_2}{jX_1}$

$\rightarrow \underline{I}_2 = \frac{\underline{U}_2 - jX_m \underline{I}_1}{jX_2}$

$\underline{I}_3 = \frac{j U_3}{X_c}$

$-\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$

$jX_1 \underline{I}_1 + jX_m \underline{I}_2 - jX_c \underline{I}_3 = U_g$

$jX_m \underline{I}_1 + (R + jX_2)\underline{I}_2 + jX_c \underline{I}_c = 0$

$$\rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \cdot \begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \rightarrow$$

dobimo $\underline{I}_1, \underline{I}_2, \underline{I}_3$

2) Zančni toki (isto vezje)

J_I in J_{II} (dve neznanki, dve enačbi)

$\underline{I}_1 = J_I$

$\underline{I}_2 = J_{II}$

$\underline{I}_3 = J_I - J_{II}$

$j(X_1 - X_c) J_I + jX_c \cdot J_{II} = U_g - jX_m \underline{I}_{II}$

$jX_c \cdot J_I + (R + j(X_2 - X_c)) J_{II} = -jX_m J_I$

$$\begin{rcases} 0 \cdot J_I + j30 J_{II} = 150 \\ j30 J_I + 6 J_{II} = 0 \end{rcases} \begin{matrix} J_I = 1A \\ J_{II} = -j5A \end{matrix}$$

3) Spojiščni potenciali

(ena sama enačba, neposredno izrazimo V)

V =

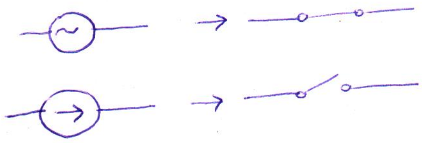
Stavki (teoremi) za reševanje harmoničnih vezij

1) Stavek superpozicije

a) koherentni vir → v cplx prostor

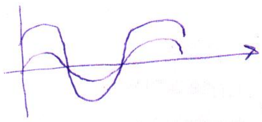
b) nekoherentni viri → v časovnem prostoru

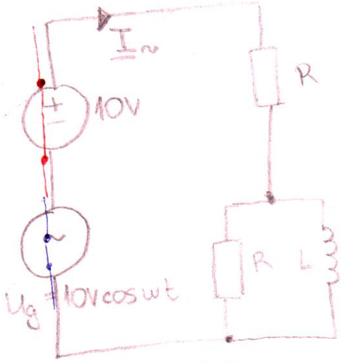
- vire vezja deaktiviramo
- po en vir vklopljamo, rešimo, iskana veličina je vsota analiziranih veličin



$\underline{I} = \underline{I}_1' + \underline{I}_1'' + \underline{I}_1''' + \dots$

• harmonična napetost z " " → dodana enosmerna komponenta





$R = \omega L = 1\Omega$

I) Za harmonični del:

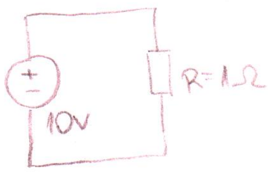
$$\underline{Z} = R + \frac{R \cdot j\omega L}{R + j\omega L} = \frac{3+j}{2}$$

$$\underline{I_\sim} = \frac{\underline{U_g}}{\underline{Z}} = 2 \cdot \frac{10e^{j0}}{3+j} = (6+j2)\,A$$

$$\downarrow$$

$$i_\sim = \sqrt{40}\cos(\omega t - \mathrm{arctg}\tfrac{1}{3}) = 6{,}32\,A \cdot \cos(\omega t - 0{,}32)$$

II za enosmerno komponento



$$I_= = \frac{10V}{1\Omega} = 10\,A$$

$$i = I_= + i_\sim = (10 + 6{,}32\cos(\omega t - 0{,}32))\,A$$

$$P = \bar{p} = \overline{(u_0 + u_g)(I_= + i_\sim)} = \overline{U_0 I_=} + \underbrace{\overline{u_0 i_\sim}}_{0} + \underbrace{\overline{I_= u_g}}_{0} + \overline{u_g i_\sim}$$

enosm. moč — $\cos x = 0$!! — izmenična moč

$$\boxed{P = P_= + P_\sim} \Rightarrow \frac{U_0^2}{R} = 100W = P_=$$

$$\Downarrow$$

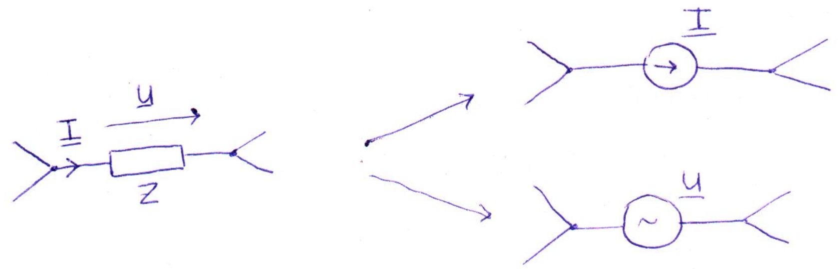
$$\tfrac{1}{2} U_{gm} \cdot I_{\sim m} \cos\varphi = \tfrac{1}{2}\, 10\sqrt{40} \doteq 30\,W = P_\sim$$

$$\Rightarrow P = 130W$$

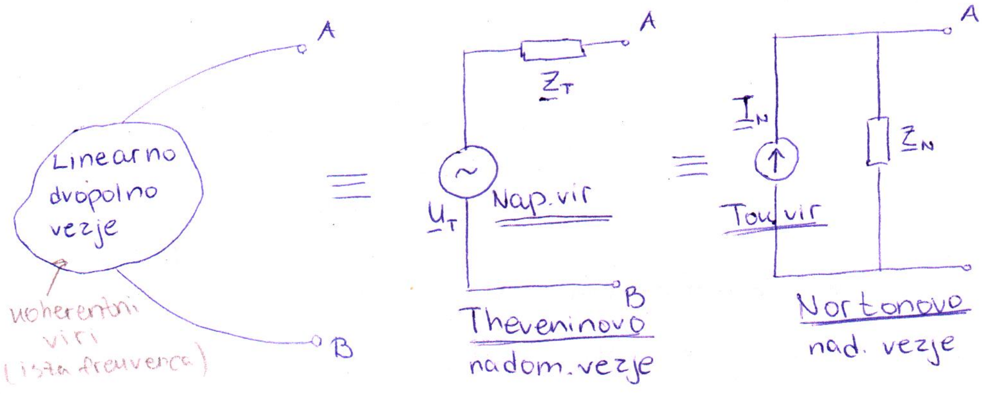
$$P = \mathrm{Re}(\underline{S})$$

$$\searrow \tfrac{1}{2} U \cdot I^*$$

2) Stavek o nadomestitvi



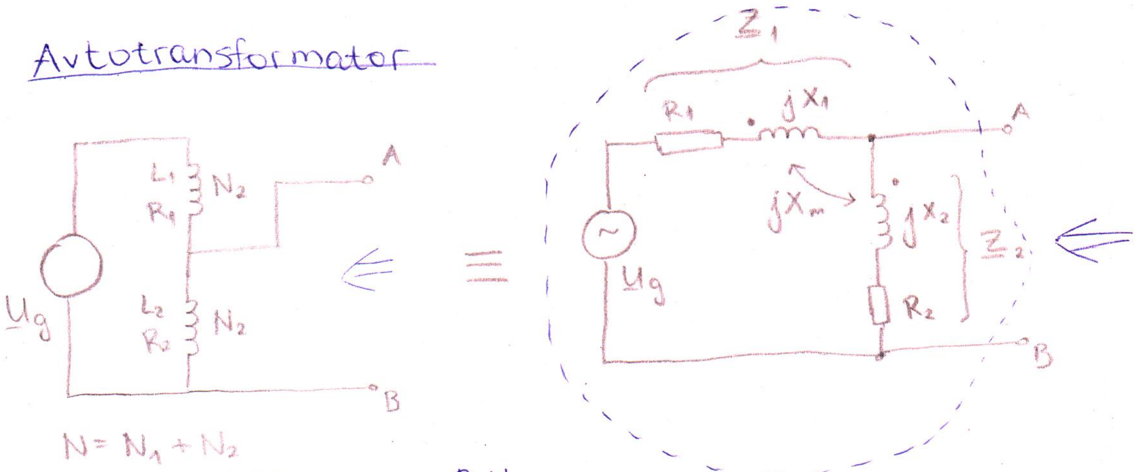
Stavek Thevenina in Nortona



$\underline{U}_T = \underline{Z}_T \underline{I}_N$

$\underline{Z}_T = \underline{Z}_N = \underline{Z}_0 = \underline{Z}_g$

Avtotransformator

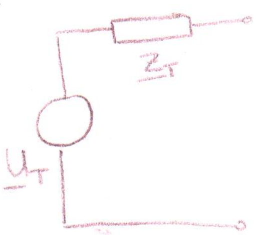


$N = N_1 + N_2$

$R \rightarrow R_1 = \frac{R \cdot N_1}{N}$; $R_2 = \frac{R N_2}{N}$

premosorazm. z N

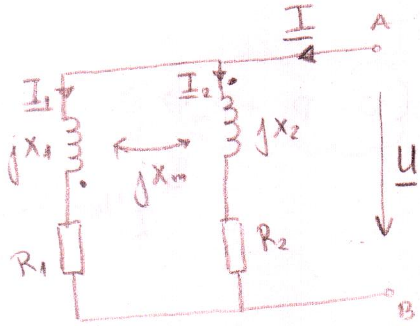
$L \rightarrow L_1 = \frac{L N_1^2}{N^2}$; $L_2 = \frac{L N_2^2}{N^2}$, $K \rightarrow 1$; $M = \sqrt{L_1 L_2} = L\frac{N_1 N_2}{N^2}$



$$U_T = \underbrace{\frac{U_g}{Z_1 + Z_2 + 2jX_m}}_{I} (Z_2 + jX_m)$$

$\underline{Z} \rightarrow$ deaktiviramo vire

$$\boxed{Z_T = \frac{U}{I}}$$



$\underline{I} = \underline{I}_1 + \underline{I}_2$

leva veja: $\underline{U} = \underline{Z} \cdot \underline{I}_1 - jX_m \underline{I}_2$

desna veja: $\underline{U} = -jX_m \underline{I}_1 + \underline{Z}_2 \underline{I}_2$

$\Rightarrow$

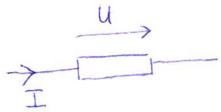
$$\underline{I}_1 = \frac{\underline{Z}_2 + jX_m}{\underline{Z}_1 \cdot \underline{Z}_2 + X_m^2} \cdot \underline{U}$$

$$\underline{I}_2 = \frac{\underline{Z}_1 + jX_m}{\underline{Z}_1 \cdot \underline{Z}_2 + X_m^2} \underline{U}$$

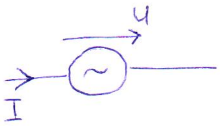
rešitev:

$$\underline{Z}_T = \frac{\underline{Z}_1 \cdot \underline{Z}_2 + X_m^2}{\underline{Z}_1 + \underline{Z}_2 - 2jX_m}$$

4) Stavek Tellegena

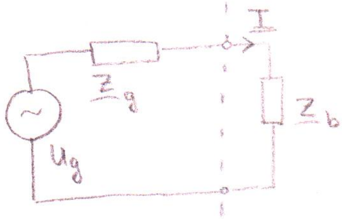


in !



$$\boxed{\sum_{j=0}^{N} \frac{1}{2} \underline{U}_j \underline{I}_j^* = 0}$$

5) Stavek največje moči



$$\underline{I} = \frac{\underline{U}_g}{\underline{Z}_g + \underline{Z}_b} = \frac{\underline{U}_g}{(R_g + R_b) + j(X_g + X_b)}$$

$$P = \mathrm{Re}\left[\frac{1}{2} \cdot \underline{Z}_b \cdot \underline{I}^2\right] = \frac{U_g}{2} \cdot \frac{R_b}{(R_g + R_b)^2 + (X_g + X_b)^2}$$

$$\frac{\partial P}{\partial R_b} = \ldots = 0 \rightarrow R_b = R_g$$

$$\frac{\partial P}{\partial X_b} = \ldots = 0 \rightarrow X_b = -X_g$$

$$\underline{Z}_b = \overline{\underline{Z}_g}$$

$$\boxed{P_{b,max} = \frac{U_{gm}^2}{8 \cdot R_g}}$$

- Poseben primer – realno breme

$$R_b = |Z_g| = \sqrt{R_g^2 + X_g^2}$$

$$P_{b,max} = \frac{U_{gmax}^2}{4(R_g + |Z_g|)}$$

6) Stavek recipročnosti

$$\left.\frac{\underline{U}_1}{\underline{I}_2}\right|_{\underline{I}_1=0} = \left.\frac{\underline{U}_2}{\underline{I}_1}\right|_{\underline{I}_2=0}$$

$$\begin{bmatrix}\underline{U}_1\\ \underline{U}_2\end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M\\ j\omega M & j\omega L_2\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}$$

P25 (24.5.2016)

Transformator

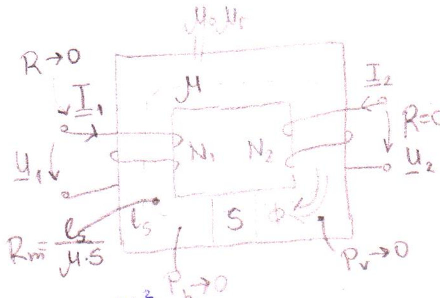
A) idealiziran (brez izgub), $k=1$

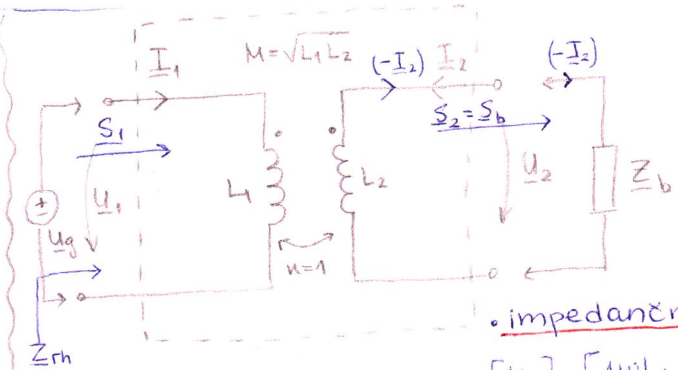
B) idealni ($\mu \to \infty$)

c) realni

Joulske izgube (upornosti)
Kožni efekt (časovne dinamike)
Histerezne izgube
Vrtinčne izgube

A)





$L_1 = \frac{N_1^2}{R_m}$; $L_2 = \frac{N_2^2}{R_m}$

$M = \frac{N_1 N_2}{R_m}$; $M = \sqrt{L_1 L_2}$

$\underline{U}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$

$\underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$

• impedančna matrika

$$\begin{bmatrix}\underline{U}_1\\ \underline{U}_2\end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M\\ j\omega M & j\omega L_2\end{bmatrix}\cdot\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}$$

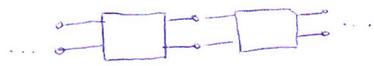
• verižna oblika (matrika)

$$\underline{I}_1 = \frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M}\underline{I}_2$$

$$\underline{U}_1 = j\omega \cdot L_1\left(\frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M}\underline{I}_2\right) + j\omega M \underline{I}_2$$

$$\boxed{\begin{aligned}\underline{U}_1 &= \frac{L_1}{M}\cdot\underline{U}_2\\ \underline{I}_1 &= \frac{\underline{U}_2}{j\omega M} + \frac{L_2}{M}(-\underline{I}_2)\end{aligned}}$$

$$\begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix}\underline{U}_1\\ \underline{I}_1\end{bmatrix} = & \begin{bmatrix} L_1/M & 0\\ 1/j\omega M & L_2/M\end{bmatrix}\cdot\begin{bmatrix}\underline{U}_2\\ (-\underline{I}_2)\end{bmatrix} \\ & \begin{array}{cc} c & d \end{array}\end{array}$$



1) $\underline{U}_1 = \frac{L_1}{M}\underline{U}_2 = \frac{N_1}{N_2}\underline{U}_2 = n\,\underline{U}_2$ → predstava razmerje

2) $|\underline{Z}_b| \to \infty$ (odprte sponke), $\underline{I}_2 = 0$

$\underline{I}_1 = \frac{\underline{U}_2}{j\omega M} = \frac{\underline{U}_1}{j\omega L_1} = \underline{I}_{1m}$ (magnetilni tok)

3) $0 < |\underline{Z}_b| < \infty$ (breme)

$$\underline{I}_1 = \underline{I}_{1m} + \underbrace{\frac{N_2}{N_1}(-\underline{I}_2)}_{\underline{I}_{1r}\ (\text{reakcijski tok})} = \underline{I}_{1m} + \underline{I}_{1r} = \underline{I}_{1m} + \frac{1}{n}(-\underline{I}_2)$$

$$\underline{I}_{1r} = -\frac{1}{n}\underline{I}_2$$

$$\boxed{\underline{I}_1 = \underline{I}_{1m} + \underline{I}_{1r}}$$

4) $$\underline{\Theta} = N_1\underline{I}_1 + N_2\underline{I}_2 = N_1\underline{I}_{1m} + \underbrace{N_1\frac{N_2}{N_1}(-\underline{I}_2) + N_2\underline{I}_2}_{\emptyset} = N_1\underline{I}_{1m} \quad ; \quad \boxed{\underline{\Theta} = N_1\underline{I}_{1m}}$$

5) $$\underline{Z}_{rh} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{n\cdot\underline{U}_2 \quad :(-\underline{I}_2)}{\frac{\underline{U}_2}{j\omega M} + \frac{1}{n}(-\underline{I}_2) : (-\underline{I}_2)}$$

$$\underline{Z}_{rh} = \frac{n\underline{Z}_b}{\frac{\underline{Z}_b}{j\omega M} + \frac{1}{n}}$$

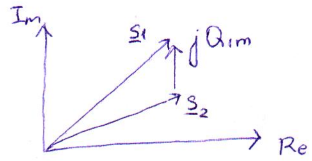
6) $$\underline{S}_1 = \frac{1}{2}\underline{U}_1\overline{\underline{I}}_1 = \underbrace{\frac{1}{2}\underline{U}_1\overline{\underline{I}}_{1m}}_{jQ_{1m}} + \underbrace{\frac{1}{2}\underline{U}_1\frac{1}{n}(-\overline{\underline{I}}_2)}_{\underline{S}_2 = \underline{S}_b}$$

U_2

vhodni

$$\boxed{\underline{S}_1 = jQ_{1m} + \underline{S}_2}$$

$P_1 = P_2$, $Q_1 = Q_{1m} + Q_2$



B) $\mu \to \infty$, $L_1, L_2, M \to \infty$

1) $\underline{U}_1 = n\underline{U}_2$

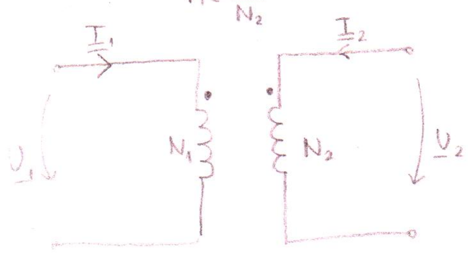
2) $\underline{I}_{1m} \to 0$

3) $\underline{I}_1 = -\frac{1}{n}\underline{I}_2$

4) $\underline{\Theta} \to 0$

5) $\underline{Z}_{rh} = n^2\underline{Z}_b$

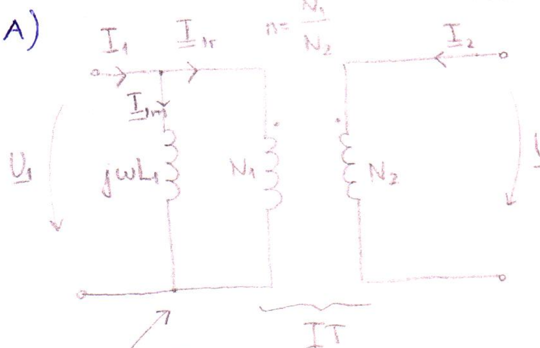
6) $\underline{S}_1 = \underline{S}_2$



$\underline{U}_1 = n\underline{U}_2$

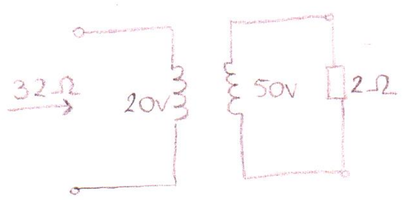
$\underline{I}_1 = -\frac{1}{n}\underline{I}_2$

A)



$\underline{I}_1 = \underline{I}_{1m} + \underline{I}_{1r}$

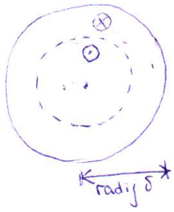
Primer

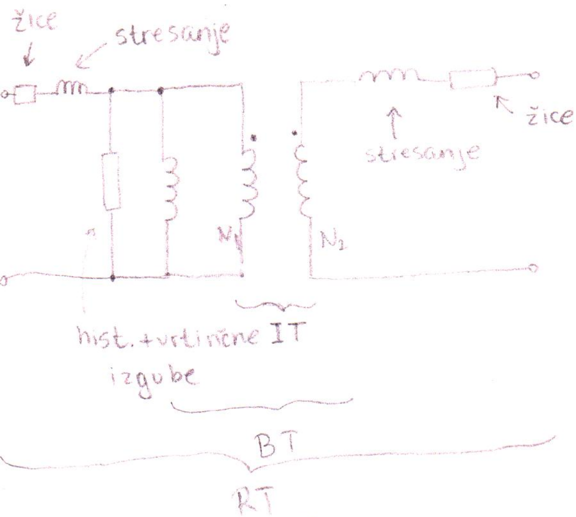


c) vdorna globina $\underbrace{\delta = \sqrt{\frac{2}{\omega \mu \gamma}}}$

50 Hz, Cu

$\delta \simeq 9{,}6\,mm$

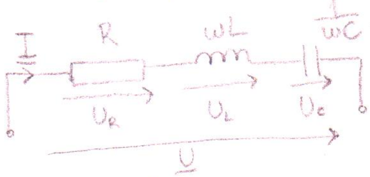




P26 (27.5.2016)

Zaporedna vezava RLC

nihajni urog, filter



$i = I_m \cos \omega t$

$u = u_R + u_L + u_C = R \cdot i + L \frac{di}{dt} + \frac{1}{C} \int i\,dt$

$u = U_m \cos(\omega t + \varphi) = R \cdot I_m \cdot \cos \omega t - \omega L I_m \sin \omega t + \frac{I_m}{\omega C} \sin \omega t$

neznanki

$\{\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta\}$

$U_m (\cos \omega t \cdot \cos\varphi - \sin\omega t \cdot \sin\varphi) = R I_m \cos \omega t + \left(\frac{1}{\omega C} - \omega L\right) I_m \sin \omega t$

$U_m \cos\varphi = R \cdot I_m$

$U_m \sin\varphi = \left(\omega L - \frac{1}{\omega C}\right) I_m$

1) kvadriramo in seštejemo

$U_m^2 (\sin^2\varphi + \cos^2\varphi) = U_m^2 = I_m^2 \left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \right)$

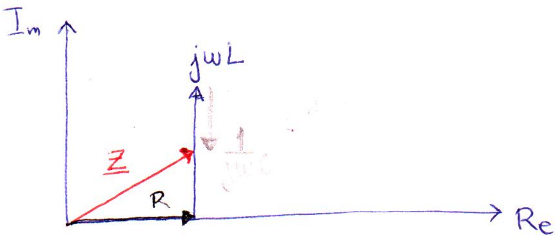
$$\boxed{U_m = I_m \underbrace{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}_{Z}}$$

2) enačbi delimo

$$\frac{\cancel{U_m} \cdot \sin\varphi}{\cancel{U_m} \cdot \cos\varphi} = \boxed{\operatorname{tg}\varphi = \frac{\omega L - \frac{1}{\omega C}}{R}}$$

$$u(t) = I_m \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \cdot \cos\left(\omega t + \operatorname{arctg} \frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

kazalčni diagram zap. RLC vezja nih. kroga, filtra

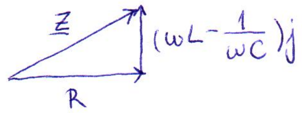


$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\underline{U} = \underline{Z} \cdot \underline{I}$$

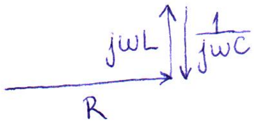
$$\operatorname{tg}\varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

a)



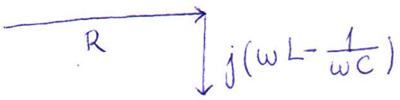
$\omega L > \frac{1}{\omega C}$ induktivnosti lastnosti

b)



$\omega L = \frac{1}{\omega C}$ ohmski značaj (popolna kompenzacija jalove moči) resonanca vezja

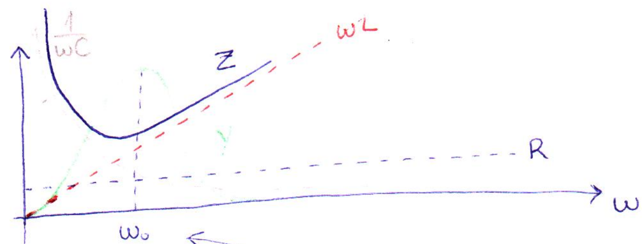
c)



$\omega L < \frac{1}{\omega C}$ kapacitivni značaj vezja

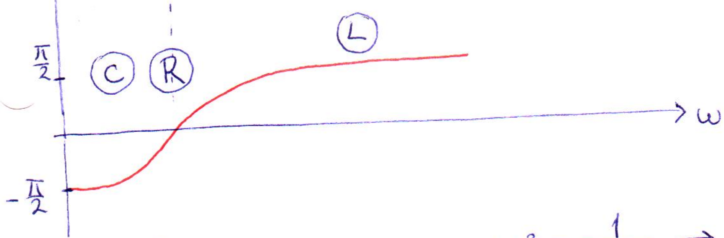
$\varphi < 0$

Frekvenčna karakteristika



$Y = \frac{1}{Z}$

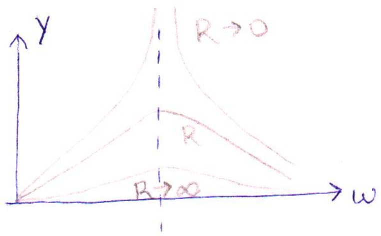
Resonanca - maximalen odziv na vzbujanje

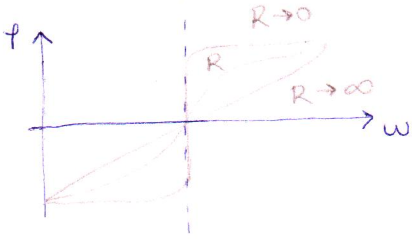


Resonanca: $\omega_0^2 = \frac{1}{LC} \rightarrow$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$





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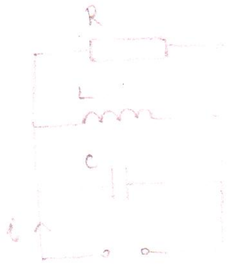
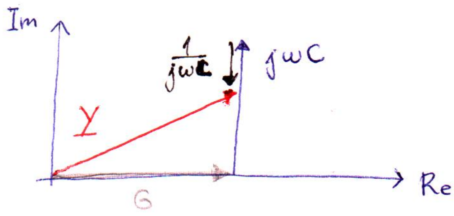
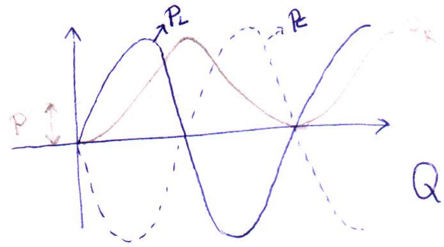
Vzporedna vezava RLC



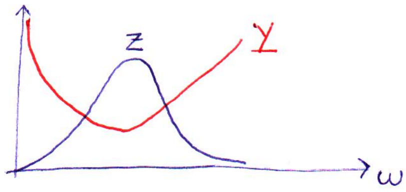
$$Y=\sqrt{\left(\frac{1}{R}\right)^2+\left(\omega C-\frac{1}{\omega L}\right)^2}$$

$$tg\,\varphi=\frac{\omega C-\frac{1}{\omega L}}{\frac{1}{R}}$$





Fautor delavnosti: Q



$$Q=2\pi\frac{\text{energija v sistemu v resonanci}}{\text{energija izgubljena v sistemu v resonanci}}$$

$$Q=2\pi\frac{L\cdot I_m^2/2}{\underbrace{R\left(\frac{I_m}{\sqrt{2}}\right)^2}_{P}\cdot T_0}=2\pi f\cdot\frac{L}{R}=\frac{\omega_0 L}{R}$$

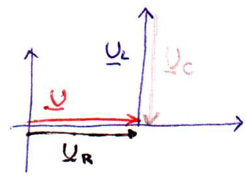
→ perioda $T_0=\frac{1}{f_0}$

impedanca na L oz. C $\quad \omega_0 L=\frac{1}{\omega_0 C}$

$$Q=\frac{\omega_0 L}{R}$$ → ohmska upornost

$$Q=\frac{U_L}{U}=\frac{I_L}{I}$$

v resonanci:
$U=U_R$



Pasovna širina filtra, razglašenost, kvaliteta

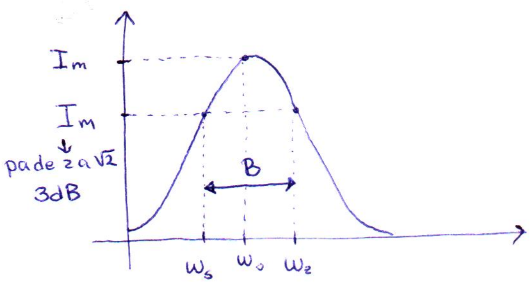
$$\left(\omega L-\frac{1}{\omega C}\right)=\omega_0 L\left(\frac{\omega}{\omega_0}-\frac{1}{\omega\omega_0 LC}\right)=\omega_0 L\underbrace{\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}_{\nu\text{ - razglašenost}}$$

$$\omega_0^2=\frac{1}{LC}$$

$$\boxed{\nu=\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}}$$

Bočna freuvenca

$$\frac{I_{\omega_0}}{I_\omega}=\frac{U/R}{U/Z}=\frac{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}{R}=\sqrt{2}$$



ω_s - spodnja
ω_z - zgornja
bočna freuvenca

$B=\omega_z=\omega_s$

$$\boxed{B=f_z-f_s\quad[Hz]}$$

Pasovna širina filtra

$$\Rightarrow\frac{I_{\omega_0}}{I_\omega}=\sqrt{1+\left(\frac{\omega_0 L}{R}\cdot\nu\right)^2}=\sqrt{1+1^2}\quad;\quad\frac{\omega_0 L}{R}\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)=\pm 1$$

Q - kvaliteta nih. kroga

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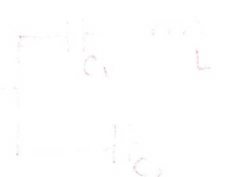
$$Q\left(\frac{\omega_z}{\omega_0} - \frac{\omega_0}{\omega_z}\right) = 1 \qquad i \qquad Q\left(\frac{\omega_s}{\omega_0} - \frac{\omega_0}{\omega_s}\right) = -1$$

$$\omega_0 = \sqrt{\omega_s \cdot \omega_z}$$

Splošno o resonanci

$$\mathrm{Im}(\underline{Z}) = 0 \qquad \underline{Y} = j\omega C + \frac{1}{R + j\omega L}$$

$$\mathrm{Im}(\underline{Y}) = 0 \qquad \mathrm{Im}(\underline{Y}) = 0$$



$$\underline{Y} = j\omega C_2 + \frac{1}{j\omega L + \frac{1}{j\omega C_1}}$$

$$\underline{Y} = j\left(\omega C_2 + \frac{\omega C_1}{1 - \omega^2 L \cdot C_1}\right)$$

$$\mathrm{Im}(\underline{Y}) = 0 = \frac{\omega C_2 - \omega^3 L C_1 C_2 + \omega C_1}{1 - \omega^2 L C_1} \rightarrow$$

Ničle: napetostna resonanca

$\omega_1 = 0$ – ni zanimiva

$\omega_2 = \sqrt{(C_1 + C_2)/L C_1 C_2}$

Pol: tokovna resonanca

$\omega_I = \sqrt{LC_1}$

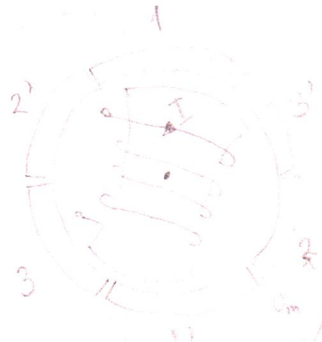
P27 (31.5.2016)

Trifazni sistemi

Lastnosti: (prednosti)

1) Manjša poraba materiala
2) Skupna moč je konstantna
3) Vrtilno magnetno polje

Generatorski del



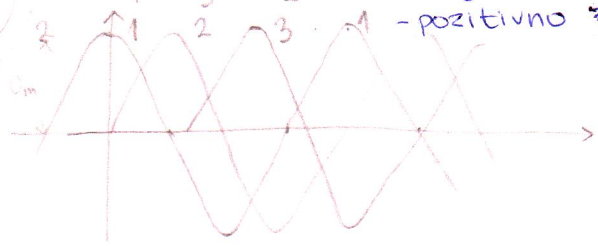
$$U_{ef} = \frac{U_m}{\sqrt{2}}$$

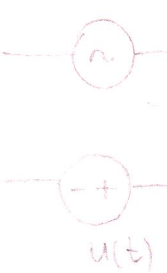
$$u_1 = U_m \cdot \cos(\omega t + \alpha) \longrightarrow \underline{U}_1 = U_{ef} \cdot e^{j\alpha}$$

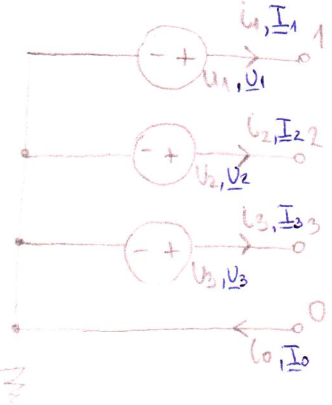
$$u_2 = U_m \cdot \cos\left(\omega t + \alpha \pm \frac{2\pi}{3}\right) \longrightarrow \underline{U}_2 = U_{ef} \cdot e^{j\left(\alpha \pm \frac{2\pi}{3}\right)}$$

$$u_3 = U_m \cdot \cos\left(\omega t + \alpha \pm \frac{4\pi}{3}\right) \longrightarrow \underline{U}_3 = U_{ef} \cdot e^{j\left(\alpha \pm \frac{4\pi}{3}\right)}$$

1 3 2 1 – negativno zaporedje faz
1 2 3 1 – pozitivno zaporedje faz







$$i_1 + i_2 + i_3 = i_0$$

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = \underline{I}_0$$

Simetrično vs. asimetrično (nesimetrično)

breme, viri napetosti

Simetrične vire napetosti
- amplitude med seboj enake
- fazni zamiki 120°

Označevanje kazalcev v energetiki

1) "Amplitudni kazalci"

$u(t) = U_m \cdot \cos(\omega t + \alpha_u) \rightarrow \underline{U} = U_m \cdot e^{j\alpha_u}$ $u = Re(\underline{U} \overset{\text{vedno enak}}{\overbrace{e^{j\omega t}}})$

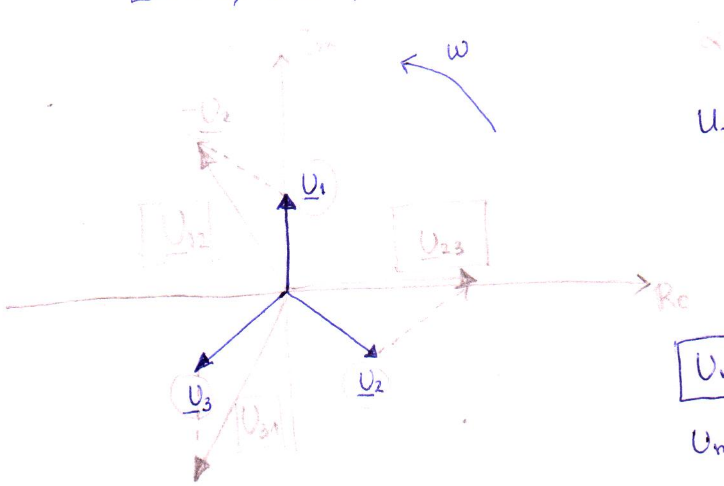
2) "Efektivni" kazalci

$u(t) = \underbrace{\sqrt{2} \cdot U_{ef}}_{U_m} \cdot \cos(\omega t + \alpha_u) \rightarrow \underline{U} = U_{ef} \cdot e^{j\alpha_u}$ $u = Re(\sqrt{2}\,\underline{U}\, e^{j\omega t})$

$i(t) = \sqrt{2} \cdot I_{ef} \cdot \cos(\omega t + \alpha_i) \rightarrow \underline{I} = I_{ef} \cdot e^{j\alpha_i}$ $i = Re(\sqrt{2}\,\underline{I}\, e^{j\omega t})$

$\underline{Z}$ - ostaja enaka

$\underline{S} = \underline{U} \cdot \underline{I}^*$ → brez polovičke



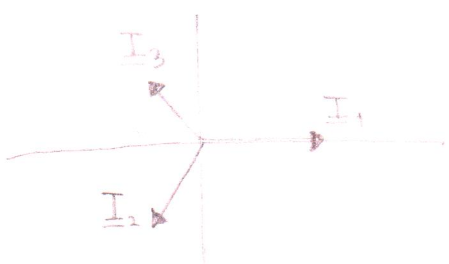
U_f: $\underline{U}_1$, $\underline{U}_2$, $\underline{U}_3$: fazne

$\boxed{U_{mf}}$: $\underline{U}_{12} = \underline{U}_1 - \underline{U}_2$ medfazna napetost

$U_{mf} = \sqrt{3} \cdot U_f$

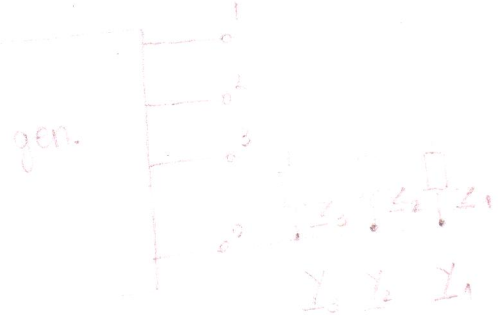
400V 230V

Simetričen sistem: generator, breme

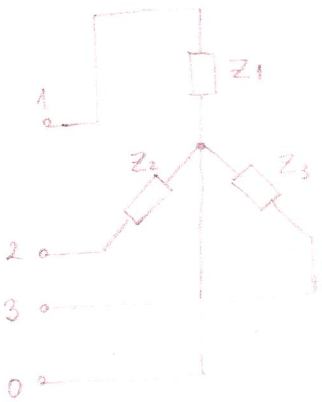


$\underline{I}_0 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$

Bremenski del trifaznega sistema



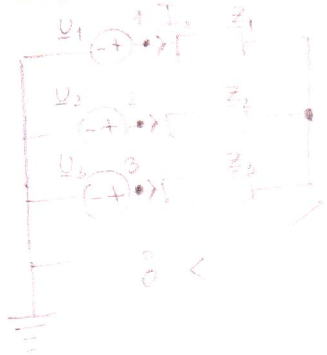
a) Breme v zvezda vezavi s pov



$\underline{I}_k = \underline{U}_k \cdot \underline{Y}_k \quad , \; k=1,2,3$

$\underline{I}_0 = \sum_{k=1}^{3} \underline{I}_k$

b) Odsotnost povratnega vodnika



$\underline{V}_0 = V_{VZ}$ potencial zvezdišča

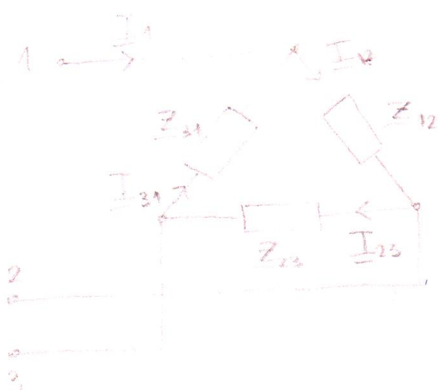
$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$

$(\underline{U}_1 - \underline{V}_0)\underline{Y}_1 + (\underline{U}_2 - \underline{V}_0)\underline{Y}_2 + (\underline{U}_3 - \underline{V}_0)\underline{Y}_3 = 0$

$\underline{U}_1 \underline{Y}_1 + \underline{U}_2 \underline{Y}_2 + \underline{U}_3 \underline{Y}_3 - V_0(\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3) = 0$

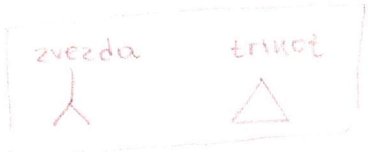
$$\underline{V}_0 = \frac{\frac{\underline{U}_1}{\underline{Z}_1} + \frac{\underline{U}_2}{\underline{Z}_2} + \frac{\underline{U}_3}{\underline{Z}_3}}{\frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} + \frac{1}{\underline{Z}_3}}$$

c) vezava v trikot



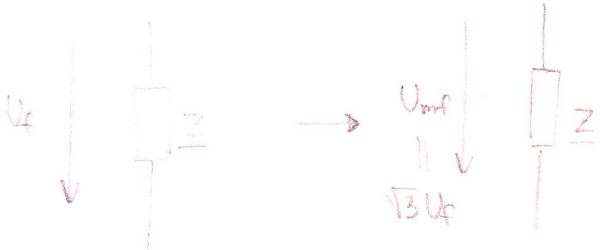
medfazne napetosti

$\underline{I}_{kL} = \frac{\underline{U}_{kL}}{\underline{Z}_{kL}}$



$\underline{I}_1 = \underline{I}_{12} - \underline{I}_{31}$

⋮



$S_\lambda \cdot 3 = S_\Delta$

Konstanten prenos moči

$$p(t) = p_1(t) + p_2(t) + p_3(t) = u_1 i_1 + u_2 i_2 + u_3 i_3$$

- - - - - - - simetrični trifazni sistem

$$p(t) = U \cdot I \cdot \cos(\omega t) \cdot \cos(\omega t - \varphi) +$$

$$+ UI \cdot \cos\left(\omega t + \frac{2\pi}{3}\right) \cdot \cos\left(\omega t + \frac{2\pi}{3} - \varphi\right) +$$

$$+ UI \cdot \cos\left(\omega t - \frac{2\pi}{3}\right) \cdot \cos\left(\omega t - \frac{2\pi}{3} - \varphi\right) =$$

$$= 3 \cdot \frac{UI}{2} \cdot \cos\varphi + \underbrace{\frac{UI}{2}\cos(2\omega t - \varphi) + \frac{UI}{2}\cos\left(2\omega t + \frac{4\pi}{3} - \varphi\right) + \frac{UI}{2}\cos\left(2\omega t - \frac{4\pi}{3} - \varphi\right)}_{0}$$

$$\boxed{p(t) = \frac{3UI}{2}\cos\varphi}$$

Vrtilno magnetno polje

$$i_1 = I \cdot \cos \omega t$$

$$i_2 = I \cdot \cos\left(\omega t + \frac{2\pi}{3}\right)$$

$$i_3 = I \cdot \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$B_x = B_{2x} + B_{3x} =$$

$$B_x = B_0 \cdot \cos\left(\omega t + \frac{2\pi}{3}\right) \cdot \cos\frac{\pi}{6} - B_0 \cdot \cos\left(\omega t - \frac{2\pi}{3}\right) \cdot \cos\frac{\pi}{6}$$

$$\left.\begin{aligned} B_x &= -\frac{3}{2} \cdot B_0 \cdot \sin\omega t \\ B_y &= \frac{3}{2} B_0 \cos\omega t \end{aligned}\right\} \text{vrtenje z } \omega$$