Predmet:
Analiza linearnih sistemov

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Zapiski avditornih vaj

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Volja do življenja !

ANALIZA SISTEMOV

avditorne vaje

skripta Teorija lin. sistemov
rešeni izpiti -II-

ponovi matrike

Sistem enačb

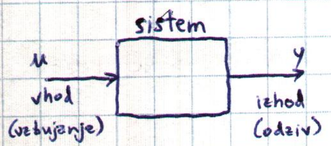
$$\begin{aligned}
10 &= x_1 + 2x_2 - 3x_3 + 1\\
5 &= 2x_1 - x_2 - x_3 + 2\\
10 &= 3x_1 - 2x_2 - 5x_3 + 3\\
\hline
20 &= 4x_1 - 8x_3 + 4 \quad /\cdot 5\\
20 &= 5x_1 - 5x_3 + 5 \quad /\cdot 4\\
\hline
20 &= -20x_3
\end{aligned}$$

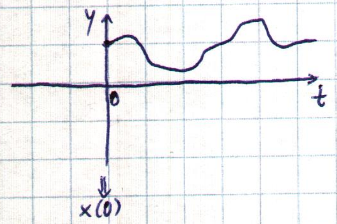
$x_3 = -1,\ x_1 = 2,\ x_2 = 2$

$$y_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1$$
$$y_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2$$
$$y_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_3u_3$$

$$\underset{3\times1}{\begin{bmatrix} y_1\\ y_2\\ y_3\end{bmatrix}} = \underset{3\times3}{\begin{bmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}} \underset{3\times1}{\begin{bmatrix} x_1\\ x_2\\ x_3\end{bmatrix}} + \underset{3\times3}{\begin{bmatrix} b_1 & 0 & 0\\ 0 & b_2 & 0\\ 0 & 0 & b_3\end{bmatrix}} \cdot \underset{3\times1}{\begin{bmatrix} u_1\\ u_2\\ u_3\end{bmatrix}}$$

$3\times3 = 3\times1$





število x-ov določa red sistema, A je kvadratna matrika

lin. algebr. enačbe

$$\boxed{y = Ax + Bu} \Rightarrow Ax = y - Bu$$
$$x = A^{-1}(y - Bu)$$

$$x = \begin{bmatrix} 1 & 2 & -3\\ 2 & -1 & -1\\ 3 & -2 & -5\end{bmatrix}^{-1}\left(\begin{bmatrix}10\\5\\10\end{bmatrix} - \begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = *$$

$$A^{-1} = \frac{1}{|A|}C_A^T$$

$$|A| = 1\cdot3 - 2(-7) - 3(-1) = 20$$

zamenjaš vrstice in stolpce

$$* = \frac{1}{20}\begin{bmatrix}3 & 7 & -1\\ 16 & 4 & 8\\ -5 & -5 & -5\end{bmatrix}^T\left(\begin{bmatrix}10\\5\\10\end{bmatrix} - \begin{bmatrix}1\\2\\3\end{bmatrix}\right) = \frac{1}{20}\begin{bmatrix}3 & 16 & -5\\ 7 & 4 & -5\\ -1 & 8 & 5\end{bmatrix}\begin{bmatrix}9\\3\\7\end{bmatrix} = \frac{1}{20}\begin{bmatrix}40\\40\\-20\end{bmatrix} = \begin{bmatrix}2\\2\\-1\end{bmatrix} = x = \begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

sistem lin. dif. enačb

$$\boxed{\dot{x} = Ax + Bu}$$

matrika vzbujanja

vektor vhodnih spr. (vhod, vzbujanje)

odvod spremenljivk stanj

x vektor spr. stanj sistema (stanje)

OSNOVNA MATRIKA SISTEMA

vh-izh. matrika

$$\boxed{y = Cx + Du}$$

vektor izhodnih spr.

izhodna matrika

MODELIRANJE VEZIJ S TOPOLOŠKIMI POSTOPKI

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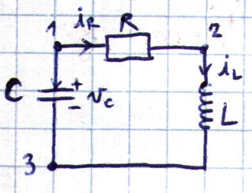


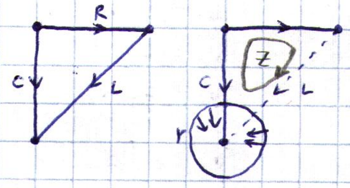
veljа za RL, RC, RLC

$$\dot{x} = Ax + Bu$$

$$x = \begin{bmatrix} i_L \\ v_c \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix}$$

1. Narišemo usmerjeni graf vezja:

veje dopolnilnega zvezja so KITE



→ vsa vozlišča in nobene zanke
→ vsebovati mora vse veje grafa, na kateri so kapacitivnosti, če s kapac. ne moremo zajeti vseh vozlišč, uporabimo veje, kjer so upornosti

več kapac. → več rezov

2. Določimo zvezje in dopolnilno vezje:

- Zvezje je tisti podgraf grafa vezja, ki vsebuje vsa vozlišča grafa, ne vsebuje pa nobene zanke. Vsebovati mora vse veje grafa, v katerih so kond., ter nobene veje, v katerih so tuljave. Če z vejami s kond. ne moremo sestaviti zvezja, si pomagamo z vejami, kjer so upornosti.

- Dopolnilno zvezje ali sistem neodvisnih vej je tisti podgraf grafa vezja, ki predstavlja komplement zvezja. Dopolnilno zvezje ter zvezje skupaj sestavljata graf. Veje dop. zv. se imenujejo kite (prekinjene črte).

3. Določimo spremenljivke stanj. $x(t) = \begin{bmatrix} i_L(t) \\ u_c(t) \end{bmatrix}$

Vektor spremenljivk stanj je sestavljen iz tokov preko tuljav v dop. v. ter napetosti na kond. v zvezju.

4. Vsako vejo zvezja, ki vsebuje kond., prerežemo z osnovnim rezom r. Vsaki kiti iz sistema neodv. vej, ki vsebuje tuljavo, priredimo osnovno zanko z:

- Osnovni rez r je sestavljen iz samo ene veje zvezja in iz poljubnega števila kit dopolnilnega zvezja. Z osn. rezom razpade graf na dva dela. Smer osn. reza je enaka smeri referenčnega toka v veji, ki določa osnovni rez.

- Osnovna zanka z je enolično določena z vsako kito dopol. zvezja ter z ustreznimi vejami zvezja, ne vsebuje pa ostalih kit dopol. zvezja. Smer osn. zanke je enaka smeri refer. toka v kiti, ki določa zanko.

5. Zapišemo enačbe stanja sistema:

enačbe osn. rezov — $r: \; i_c + i_L = 0$

enačbe osn. zank — $z: \; v_L - v_c + v_R = 0$

$$C\frac{dv_c}{dt} = -i_L$$

$$L\frac{di_L}{dt} = -v_R + v_c = -Ri_L + v_c$$

$$\dot{v}_c = -\frac{1}{C}\, i_L$$

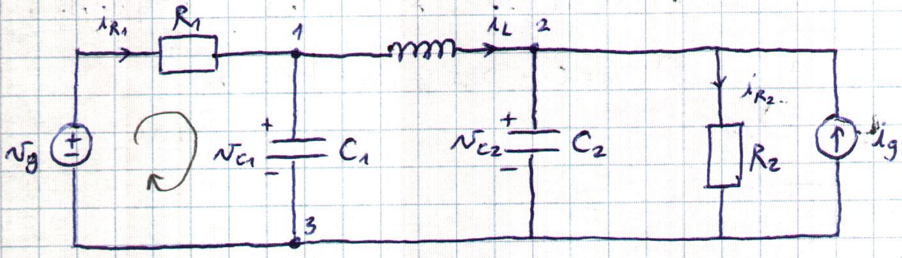
$$\dot{i}_L = -\frac{R}{L}\, i_L + \frac{1}{L}\, v_c$$

$p: \; i_R = i_L \;\Rightarrow\; v_R = Ri_L$

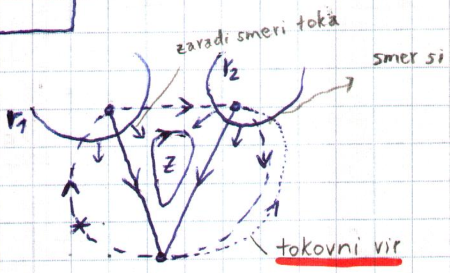
$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} -\frac{R}{L} & \frac{1}{L} \\ -\frac{1}{C} & 0 \end{bmatrix} \cdot x$$

Primer:



○ → prerežem le eno vejo



$x = [i_L \ v_{C1} \ v_{C2}]^T$

$u = [v_g \ i_g]^T$ $\qquad \dot{x} = Ax + Bu \Rightarrow x(t) = ?$

Koliko je vozlišč? 3

$r_1: \ i_{C1} - i_{R1} + i_{R2} = 0$ $\qquad v_{R1} + v_{C1} - v_g = 0$

$r_2: \ i_{C2} - i_L + i_{R2} - i_g = 0$

$z: \ v_L + v_{C2} - v_{C1} = 0$ $\qquad i_{R1} = -\frac{1}{R_1} v_{C1} + \frac{1}{R_1} v_g$

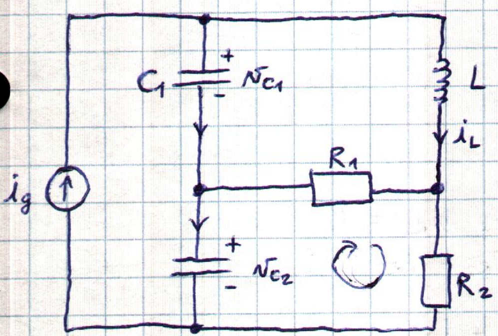
$$\left.\begin{aligned} C_1 \dot{v}_{C1} &= -i_L - \frac{1}{R_1} v_{C1} + \frac{1}{R_1} v_g \\ C_2 \dot{v}_{C2} &= i_L - \frac{1}{R_2} v_{C2} + i_g \\ L \dot{i}_L &= v_{C1} - v_{C2} \end{aligned}\right\} *$$

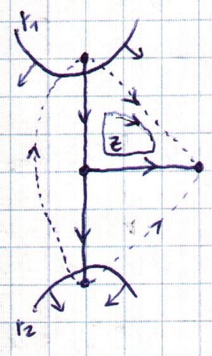
$v_{R2} = v_{C2}$

$i_{R2} = \frac{1}{R_2} v_{C2}$

$$* \quad \dot{x} = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{R_1 C_1} & 0 \\ \frac{1}{C_2} & 0 & -\frac{1}{R_2 C_2} \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ \frac{1}{R_1 C_1} & 0 \\ 0 & \frac{1}{C_2} \end{bmatrix} u$$

Primer:





$r_1: \ i_{C1} + i_L - i_g = 0$ $\qquad v_R = R \cdot i_R$

$r_2: \ i_{C2} - i_{R2} - i_g = 0$

$z: \ v_L - v_{R1} - v_{C1} = 0$

$(v_{R1} - v_{R2} - v_{C2} = 0)$

$i_{R1} + i_{R2} + i_L = 0$

$R_1 i_{R1} - R_2 i_{R2} - v_{C2} = 0$

$(R_1 + R_2) i_{R2} + R_1 i_L + v_{C2} = 0$

$i_{R2} = -\frac{R_1}{R_1 + R_2} i_L - \frac{1}{R_1 + R_2} v_{C2}$

$v_{R1} = -\frac{R_1 R_2}{R_1 + R_2} i_L + \frac{R_1}{R_1 + R_2} v_{C2}$

$i_{R1} = -\frac{R_2}{R_1 + R_2} i_L + \frac{1}{R_1 + R_2} v_{C2}$

$x = [i_L \ v_{C1} \ v_{C2}]^T$ $\qquad \dot{x} = Ax + Bu$

$u = i_g$

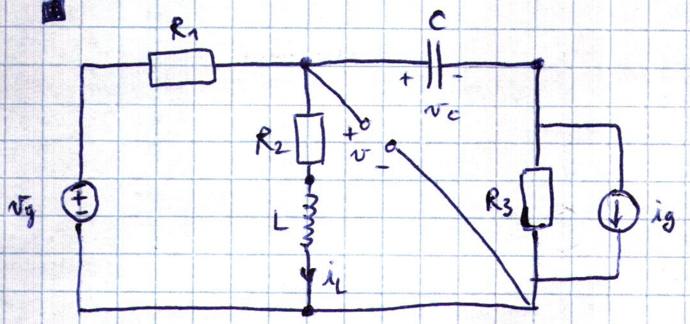
$$\left.\begin{aligned} C_1 \dot{v}_{C1} &= -i_L + i_g \\ C_2 \dot{v}_{C2} &= -\frac{R_1}{R_1 + R_2} i_L - \frac{1}{R_1 + R_2} v_{C2} + i_g \\ L \dot{i}_L &= -\frac{R_1 R_2}{R_1 + R_2} i_L + v_{C1} + \frac{R_1}{R_1 + R_2} v_{C2} \end{aligned}\right\}$$

$$\dot{x} = \begin{bmatrix} -\frac{R_1 R_2}{L(R_1 + R_2)} & \frac{1}{L} & \frac{R_1}{L(R_1 + R_2)} \\ -\frac{1}{C_1} & 0 & 0 \\ -\frac{R_1}{C_2 (R_1 + R_2)} & 0 & -\frac{1}{C_2 (R_1 + R_2)} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix} u$$

$y = [i_{R1} \ u_{C2}]^T$

$y = Cx + Du$

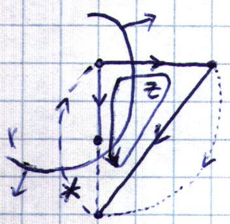
$$y = \begin{bmatrix} -\frac{R_2}{R_1 + R_2} & 0 & \frac{1}{R_1 + R_2} \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$



a) $\dot{x} = Ax + Bu$; $x = [i_L \ u_C]^T$, $u = [i_g \ v_g]^T$

b) $y = Cx + Du$; $y = [v \ v_C]^T$

4 vozlišča



$$r: \ i_C + i_L - i_{R1} = 0$$
$$z: \ v_L - v_{R3} - v_C + v_{R2} = 0$$

$$v_{R2} = R_2 i_{R2} = R_2 i_L$$

$$v_{R1} + v_C + v_{R3} = v_g$$
$$i_{R1} - i_L - i_{R3} = i_g \quad / \cdot R_3$$

$$(R_1+R_3) i_{R1} + v_C - R_3 i_L = v_g + R_3 i_g$$

$$i_{R1} = \frac{R_3}{R_1+R_3} i_L - \frac{1}{R_1+R_3} v_C + \frac{R_3}{R_1+R_3} i_g + \frac{1}{R_1+R_3} v_g$$

$$v_{R3} = -\frac{R_1R_3}{R_1+R_3} i_L - \frac{R_3}{R_1+R_3} v_C - \frac{R_1R_3}{R_1+R_3} i_g + \frac{R_3}{R_1+R_3} v_g$$

$$C\dot{v}_C = i_{R1} - i_L = -\frac{R_1}{R_1+R_3} i_L - \frac{1}{R_1+R_3} v_C + \frac{R_3}{R_1+R_3} i_g + \frac{1}{R_1+R_3} v_g$$

$$L\dot{i}_L = v_{R3} + v_C - R_2 i_L = \left(-R_2 - \frac{R_1R_3}{R_1+R_3}\right) i_L + \frac{R_1}{R_1+R_3} v_C - \frac{R_1R_3}{R_1+R_3} i_g + \frac{R_3}{R_1+R_3} v_g$$

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} = [2\times2]x + [2\times2]u$$

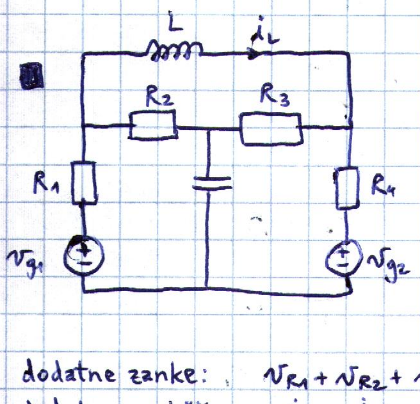
$$\dot{x} = \begin{bmatrix} -\frac{R_2}{L} - \frac{R_1R_3}{(R_1+R_3)L} & \frac{R_1}{(R_1+R_3)L} \\ -\frac{R_1}{(R_1+R_3)C} & -\frac{1}{(R_1+R_3)C} \end{bmatrix} x + \begin{bmatrix} -\frac{R_1R_3}{(R_1+R_3)L} & \frac{R_3}{(R_1+R_3)L} \\ \frac{R_3}{(R_1+R_3)C} & \frac{1}{(R_1+R_3)C} \end{bmatrix} u$$

$$y_1 = v = v_{R2} + v_L = R_2 i_L + L\dot{i}_L$$

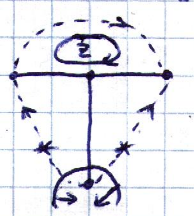
$$v = v_C + v_{R3} = -\frac{R_1R_3}{R_1+R_3} i_L + \frac{R_1}{R_1+R_3} v_C - \frac{R_1R_3}{R_1+R_3} i_g + \frac{R_3}{R_1+R_3} v_g$$

$$y_2 = v_C = v$$

$$y = \begin{bmatrix} -\frac{R_1R_3}{R_1+R_3} & \frac{R_1}{R_1+R_3} \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} -\frac{R_1R_3}{R_1+R_3} & \frac{R_3}{R_1+R_3} \\ 0 & 0 \end{bmatrix} u$$



$x = [i_L, v_C]^T$, $u = [v_{g1}, v_{g2}]$



$$r: \ i_C - i_{R1} - i_{R4} = 0$$
$$z: \ v_L - v_{R2} - v_{R3} = 0$$

$$C\dot{v}_C = i_{R1} + i_{R4}$$
$$L\dot{i}_L = v_{R2} + v_{R3}$$

dodatne zanke: $v_{R1} + v_{R2} + v_C = v_{g1}$ $\quad -v_{R3} + v_{R4} + v_C = v_{g2}$

dodatna vozlišča: $i_{R1} - i_{R2} - i_L = 0$ $\quad i_{R3} + i_{R4} + i_L = 0$

$$i_{R1} = \frac{R_2}{R_1+R_2} i_L - \frac{1}{R_1+R_2} v_C + \frac{1}{R_1+R_2} v_{g1}$$

$$v_{R2} = -\frac{R_1R_2}{R_1R_2} i_L - \frac{R_2}{R_1+R_2} v_C + \frac{R_2}{R_1+R_2} v_{g1}$$

$$i_{R4} = -\frac{R_3}{R_3+R_4} i_L - \frac{1}{R_3+R_4} v_C + \frac{1}{R_3+R_4} v_{g2}$$

$$v_{R3} = -\frac{R_3R_4}{R_3+R_4} i_L + \frac{R_3}{R_3+R_4} v_C - \frac{R_3}{R_3+R_4} v_{g2}$$

$$\dot{x} = \begin{bmatrix} -\frac{R_1R_2}{(R_1+R_2)L} - \frac{R_3R_4}{(R_3+R_4)L} & -\frac{R_2}{(R_1+R_2)L} + \frac{R_3}{(R_3+R_4)L} \\ \frac{R_2}{(R_1+R_2)C} - \frac{R_3}{(R_3+R_4)C} & -\frac{1}{(R_1+R_2)C} - \frac{1}{(R_3+R_4)C} \end{bmatrix} x +$$

$$+ \begin{bmatrix} \frac{R_2}{(R_1+R_2)L} & -\frac{R_3}{(R_3+R_4)L} \\ \frac{1}{(R_1+R_2)C} & \frac{1}{(R_3+R_4)C} \end{bmatrix} u$$

$$y = \begin{bmatrix} u_{R1} \\ u_{R4} \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

smo že poračunali i_{R1} in i_{R4}

$$y = \begin{bmatrix} \frac{R_1R_2}{R_1+R_2} & -\frac{R_1}{R_1+R_2} \\ -\frac{R_3R_4}{R_3+R_4} & -\frac{R_4}{R_3+R_4} \end{bmatrix} x + \begin{bmatrix} \frac{R_1}{R_1+R_2} & 0 \\ 0 & \frac{R_4}{R_3+R_4} \end{bmatrix}$$

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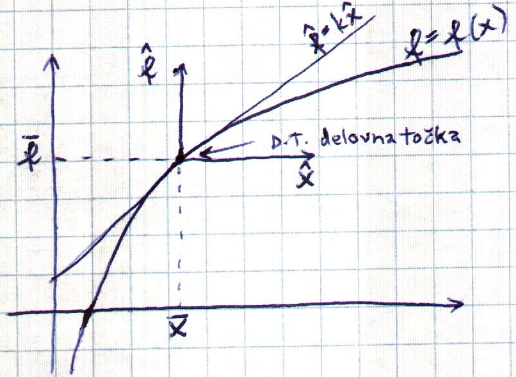
$$\left.\begin{aligned} f &= f(t) \\ x &= x(t) \end{aligned}\right\} \quad f(t) = f(x(t))$$

$f = f(x)$ ← nelin. odvisnost



D.T. $(\bar{x}, \bar{f})$ nominalni vrednosti
↳ konst.

$\hat{x}, \hat{f}$ inkrementalni spremenljivki

$x(t) = \bar{x} + \hat{x}(t)$ $\qquad x = \bar{x} + \hat{x}$

$f(t) = \bar{f} + \hat{f}(t)$ $\qquad f = \bar{f} + \hat{f}$

razvijemo v Tay. vrsto: $f(x) = f(\bar{x}) + \left.\frac{df(x)}{dx}\right|_{x=\bar{x}} (x-\bar{x}) + \frac{1}{2!} \left.\frac{d^2f(x)}{dx^2}\right|_{x=\bar{x}} (x-\bar{x})^2 + \dots$

$$f = \bar{f} + \left.\frac{df}{dx}\right|_{\bar{x}} (x-\bar{x}) + \frac{1}{2!} \left.\frac{d^2f}{dx^2}\right|_{\bar{x}} (x-\bar{x})^2 + \dots$$

↳ x

$$f \approx \bar{f} + \left.\frac{df}{dx}\right|_{\bar{x}} (x-\bar{x})$$

$$f - \bar{f} \approx \left.\frac{df}{dx}\right|_{\bar{x}} (x-\bar{x})$$

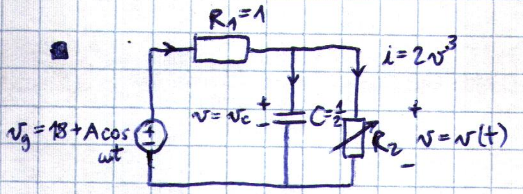
$$\hat{f} \approx \underbrace{\left.\frac{df}{dx}\right|_{\bar{x}}}_{k} \hat{x}$$

$$\underline{\underline{\hat{f} \approx k\hat{x}}}$$

$x(0), f(0)$

$x(0) = \bar{x} + \hat{x}(0)$
$\hat{x}(0) = x(0) - \bar{x}$

$\hat{f}(0) = f(0) - \bar{f}$



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$$i_{R1} - i_c - i = 0$$

$$\frac{v_g - v_c}{R_1} - C\dot{v}_c - 2v_c^3 = 0$$

$$\dot{v}_c = -\frac{1}{R_1 C} v_c - \frac{2}{C} v_c^3 + \frac{1}{R_1 C} v_g$$

$$\left.\begin{array}{l}\underset{\text{LIN}}{\dot{v}} = \underset{\text{LIN}}{-2v} - 4v^3 + 2\underset{18+A\cos\omega t}{v_g}\end{array}\right\}\ \text{NELIN. MODEL}$$

Postopek linearizacije:

① določitev D.T. $(\dot{x} = 0;\ u = \bar{u})$

$$0 = -2\bar{v} - 4\bar{v}^3 + 2\cdot 18$$
$$2\bar{v}^3 + \bar{v} = 18$$
$$\text{D.T.: } \underline{\underline{\bar{v} = 2}}$$

② zamenjava linearnih členov

$$v \to \bar{v} + \hat{v}$$
$$\dot{v} \to \underbrace{\dot{\bar{v}}}_{\text{odvod konst. je 0}} + \dot{\hat{v}} = \dot{\hat{v}}$$

$$\dot{\hat{v}} = -2(\bar{v}+\hat{v}) - 4v^3 + 2v_g$$

③ zamenjava nelinearnih členov

$$i = i(v) = \bar{i} + \hat{i}$$
$$i = \bar{i} + \left.\frac{di}{dv}\right|_{\bar{v}} (v - \bar{v})$$
$$i = \bar{i} + \left.6v^2\right|_{\bar{v}} \underbrace{(v - \bar{v})}_{\hat{v}}$$
$$i = 16 + 24\hat{v}$$

$$4v^3 = 2\cdot 2\bar{v}^3 = 2\cdot i = 32 + 48\hat{v}$$

$$\dot{\hat{v}} = -2(2+\hat{v}) - 32 - 48\hat{v} + 36 + 2A\cos\omega t$$

④ preoblikovanje DE

$$\dot{\hat{v}} = -50\hat{v} + 2A\cos\omega t \qquad \text{LIN. MODEL } \hat{v}$$

⑤ določitev začetnih vrednosti

$$v(0) = 0$$
$$\hat{v}(0) = v(0) - \bar{v} \Rightarrow \underline{\underline{\hat{v}(0) = -2}}$$

$\dot{x}_1 = x_2$

$\dot{x}_2 = -|x_1|x_1 - 2x_1 - 2x_2^3 - 3 + A\sin t$

$x_1(0) = x_2(0) = 0 \Rightarrow x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

① D.T.: $0 = \bar{x}_2$

$(\bar{x}_1, \bar{x}_2) = (-1, 0)$

$0 = -|\bar{x}_1| \cdot \bar{x}_1 - 2\bar{x}_1 - 2\bar{x}_2^3 - 3 + 0$

$|\bar{x}_1| \cdot \bar{x}_1 + 2\bar{x}_1 + 3 = 0$

$x_1 \geq 0 \rightarrow \bar{x}_1^2 + 2\bar{x}_1 + 3 = 0$

$\bar{x}_1 = \frac{-2 \pm \sqrt{4-12}}{2} = -1 \pm \sqrt{-2}$

$\bar{x}_1 < 0 \quad -\bar{x}_1^2 + 2\bar{x}_1 + 3 = 0$

$\bar{x}_1 = \frac{-2 \pm \sqrt{4+12}}{-2} = 1 \pm 2$

$\Downarrow$

$\boxed{\bar{x}_1 = -1}$

$x_1 = 3$

② zamenjava lin. členov

$\dot{\hat{x}}_1 = \bar{x}_2 + \hat{x}_2$

$\dot{\hat{x}}_2 = -|x_1|x_1 - 2(\bar{x}_1 + \hat{x}_1) - 2x_2^3 - 3 + A\sin t$

③ zamenjava nelin. členov

$f(x_1) = |x_1|x_1$

$f(x_1) = f(\bar{x}_1) + \left.\frac{df(x_1)}{dx_1}\right|_{\bar{x}_1} \overbrace{(x_1 - \bar{x}_1)}^{\hat{x}_1}$

$f(x_1) = |\bar{x}_1| \cdot \bar{x}_1 + 2|\bar{x}_1|\hat{x}_1$

$f(x_1) = -1 + 2\hat{x}_1$

$\frac{d|x|}{dx} = \frac{x}{|x|}$

$\frac{d(|x_1| \cdot x_1)}{dx_1} = \frac{x_1}{|x_1|} \cdot x_1 + |x_1| = \frac{x_1^2 + x_1^2}{|x_1|}$

$= \frac{2x_1^2}{|x_1|} = 2|x_1|$

④ preoblikovanje

$g(x_2) = x_2^3$

$g(x_2) = g(\bar{x}_2) + \left.\frac{dg(x_2)}{dx_2}\right|_{\bar{x}_2} (x_2 - \bar{x}_2)$

$g(x_2) = \bar{x}_2^3 + 3\bar{x}_2^2 \hat{x}_2$

$g(x_2) = 0$

$\dot{\hat{x}}_1 = 0 + \hat{x}_2$

$\dot{\hat{x}}_2 = 1 - 2\hat{x}_1 - 2(-1 + \hat{x}_1) - 2 \cdot 0 - 3 + A\sin t$

$\dot{\hat{x}}_1 = \hat{x}_2$

$\dot{\hat{x}}_2 = -4\hat{x}_1 + A\sin t$

$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$

$u = \sin t$

$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ A \end{bmatrix} u$

$\hat{x}(0) = x(0) - \bar{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} \hat{x}_1(0) = 1 \\ \hat{x}_2(0) = 0 \end{matrix}$

nadomeščanje torek 14.00 - 16.00

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \dots + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_0 u(t)$$ 1 dif. enačba n-tega reda

$x_1(t) = y(t)$ $\quad \dot{x}_1(t) = \frac{dy(t)}{dt} = x_2(t)$

$x_2(t) = \frac{dy(t)}{dt}$ $\quad \dot{x}_2(t) = \frac{d^2y(t)}{dt} = x_3(t)$

$\vdots$

$x_n(t) = \frac{d^{n-1}y(t)}{dt}$ $\quad \dot{x}_n(t) = \frac{d^n y(t)}{dt^n} = -a_0 y(t) - a_1\frac{dy(t)}{dt} - \dots - a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + b_0 u(t)$

$$= -a_0 x_1(t) - a_1 x_2(t) - \dots - a_{n-1}x_n(t) + b_0 u(t)$$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & & \ddots & \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1}\end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(t)$$ n dif. enačb 1. reda

Primer:

$$\dddot{y} + \frac{3}{2}\ddot{y} + 2\dot{y} + y = u$$

$x_1 = y \quad \dot{x}_1 = \dot{y}$

$x_2 = \dot{y} \quad \dot{x}_2 = \ddot{y}$

$x_3 = \ddot{y} \quad \dot{x}_3 = \dddot{y} = -y - 2\dot{y} - \frac{3}{2}\ddot{y} + u$

$= -x_1 - 2x_2 - \frac{3}{2}x_3 + u$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -\frac{3}{2}\end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1\end{bmatrix} u$$

$x_1 = \dot{y} \quad \dot{x}_1 = \ddot{y} = x_3$

$x_2 = y \quad \dot{x}_2 = \dot{y} = x_1$

$x_3 = \ddot{y} \quad \dot{x}_3 = \dddot{y} = -x_2 - 2x_1 - \frac{3}{2}x_3 + u$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ -2 & -1 & -\frac{3}{2}\end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1\end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x$$

$$x(t) = \underbrace{e^{At}x(0)}_{\text{odziv na začetno stanje}} + \underbrace{\int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau}_{\text{odziv na vzbujanje}}$$

$e^{At} = \Phi(t)$ matrika prehajanja stanj

$$A = \begin{bmatrix} -2 & 0 \\ -1 & -1\end{bmatrix} \quad e^{At} = \begin{bmatrix} e^{-2t} & 0 \\ -e^{-t}+e^{-2t} & e^{-t}\end{bmatrix}, \quad x(0) = \begin{bmatrix} 1 \\ 1\end{bmatrix}$$

$B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $u(t)$ ... enotina stopnica

$$x(t) = \begin{bmatrix} e^{-2t} & 0 \\ -e^{-t}+e^{-2t} & e^{-t}\end{bmatrix}\begin{bmatrix} 1 \\ 1\end{bmatrix} + \int_0^t \begin{bmatrix} e^{-2(t-\tau)} & 0 \\ -e^{-(t-\tau)}+e^{-2(t-\tau)} & e^{-(t-\tau)}\end{bmatrix}\cdot\begin{bmatrix} 2 \\ 1\end{bmatrix}\cdot 1 \cdot d\tau =$$

$$= \begin{bmatrix} e^{-2t} \\ e^{-2t}\end{bmatrix} + \int_0^t \begin{bmatrix} 2e^{-2(t-\tau)} \\ -e^{-(t-\tau)} + 2e^{-2(t-\tau)}\end{bmatrix} d\tau = \begin{bmatrix} e^{-2t} \\ e^{-2t}\end{bmatrix} + \begin{bmatrix} e^{-2(t-\tau)} \\ -e^{-(t-\tau)} + e^{-2(t-\tau)}\end{bmatrix}\Bigg|_0^t =$$

$$= \begin{bmatrix} \cancel{e^{-2t}} + 1 - \cancel{e^{-2t}} \\ \cancel{e^{-2t}} \cancel{-1} \cancel{+1} + e^{-t} \cancel{- e^{-2t}}\end{bmatrix} = \begin{bmatrix} 1 \\ e^{-t}\end{bmatrix} ; \quad t \geq 0$$

$\dot{x} = Ax + Bu \quad / \mathcal{L}$

$sX - x(0) = AX + BU$

$(sI - A)X = x(0) + BU$

! $$X = \underbrace{(sI-A)^{-1}x(0)}_{\text{odziv na zač. stanje}} + \underbrace{(sI-A)^{-1}BU}_{\text{odziv na vzbujanje}}$$

$$\Phi(t) = e^{At} = \mathcal{L}^{-1}\{(sI-A)^{-1}\}$$

$$\dot{x} = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \delta(t), \quad x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x(t) = ?$

inverz → člena na diag. zamenj. položaj, ostali predznak

$$(sI-A)^{-1} = \left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix}\right)^{-1} = \begin{bmatrix} s+2 & 2 \\ 1 & s+3 \end{bmatrix}^{-1} = \frac{1}{s^2+5s+4}\begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix}$$

$$X = (sI-A)^{-1}BU = \frac{1}{(s+1)(s+4)}\begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot 1 = \frac{1}{(s+1)(s+4)}\begin{bmatrix} 2s+4 \\ s \end{bmatrix} \Rightarrow \mathcal{L}^{-1} \Rightarrow$$

$$X_1 = \frac{2s+4}{(s+1)(s+4)} = \frac{\frac{2}{3}}{s+1} + \frac{\frac{4}{3}}{s+4}$$

$$X_2 = \frac{s}{(s+1)(s+4)} = \frac{-\frac{1}{3}}{s+1} + \frac{\frac{4}{3}}{s+4}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\Rightarrow x(t) = \begin{bmatrix} \frac{2}{3}e^{-t} + \frac{4}{3}e^{-4t} \\ -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t} \end{bmatrix} \quad ; t \geq 0$$

$$x^+(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\mathcal{L}\{\delta(t)\} = 1$$

na listu z enačbami

$$e^{bt}\sin at = \mathcal{L}^{-1}\left\{\frac{a}{(s-b)^2+a^2}\right\}$$

$$e^{bt}\cos at = \mathcal{L}^{-1}\left\{\frac{s-b}{(s-b)^2+a^2}\right\}$$

Primer: $\dot{x} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} x \; ; \; x(0) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

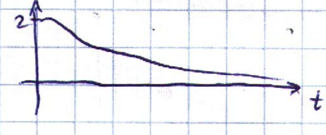
$x(t) = ?$

$$X = (sI-A)^{-1}x(0) = \begin{bmatrix} s+\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & s+\frac{1}{2} \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 2 \end{bmatrix} =$$

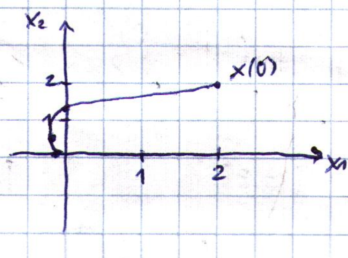
$$= \frac{1}{(s+\frac{1}{2})^2 + (\frac{1}{2})^2}\begin{bmatrix} s+\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & s+\frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \frac{1}{(s+\frac{1}{2})^2+(\frac{1}{2})^2}\begin{bmatrix} 2s \\ 2s+2 \end{bmatrix} =$$

$$= \frac{1}{(s+\frac{1}{2})^2+(\frac{1}{2})^2}\begin{bmatrix} 2(s+\frac{1}{2}) - 2(\frac{1}{2}) \\ 2(s+\frac{1}{2}) + 2(\frac{1}{2}) \end{bmatrix}$$

$$x(t) = 2e^{-\frac{1}{2}t}\begin{bmatrix} \cos\frac{1}{2}t - \sin\frac{1}{2}t \\ \cos\frac{1}{2}t + \sin\frac{1}{2}t \end{bmatrix} \quad ; t \geq 0$$



t	$x_1(t)$	$x_2(t)$
0	2	2
$\pi/2$	0	1,3
π	-0,4	+0,4
$3\pi/2$	-0,3	0
∞	0	0



prostor stanj

DOLOČANJE ODZIVA $x(t)$ S POMOČJO LASTNIH VREDNOSTI MATRIKE A

$g(\lambda) = |A - \lambda I| = 0$

značilna enačba (karak. polinom) — lastne vrednosti

PCA (analiza osnovnih komponent)

C

$w_1 v_1 + w_2 v_2 + _3 \quad + _4$

$\lambda \rightarrow \lambda_1, \lambda_2, \lambda_3, \lambda_4, | \lambda_5$

$\frac{\lambda_1^2}{\sum \lambda_i^2}$ 99%

$f(\lambda) \rightarrow f(A)$

Cayley - Hamiltonov teorem

$$e^{\lambda t} = \sum_{k=0}^{n-1} \alpha_k \lambda^k \rightarrow \alpha_k \rightarrow \boxed{e^{At}} = \sum_{k=0}^{n-1} \alpha_k A^k$$

$\frac{d}{d\lambda}$

Primer: $A = \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$

najprej izr. lastne vred.

mole enačbe iščemo

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -3-\lambda & -1 \\ 2 & -\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = (\lambda+1)(\lambda+2) = 0$$

$n=2$ $\quad \lambda_1 = -1, \ \lambda_2 = -2$

$$e^{\lambda t} = \sum_{k=0}^{1} \alpha_k \lambda^k = \alpha_0 \lambda^0 + \alpha_1 \lambda^1 = \alpha_0 + \alpha_1 \lambda$$

$$e^{-t} = \alpha_0 + \alpha_1(-1) = \alpha_0 - \alpha_1$$
$$e^{-2t} = \alpha_0 + \alpha_1(-2) = \alpha_0 - 2\alpha_1$$

$$\alpha_0 = 2e^{-t} - e^{-2t}$$
$$\alpha_1 = e^{-t} - e^{-2t}$$

$$e^{At} = \sum_{k=0}^{1} \alpha_k A^k = \alpha_0 A^0 + \alpha_1 A^1 = \alpha_0 I + \alpha_1 A$$

$$e^{At} = (2e^{-t} - e^{-2t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^{-t} - e^{-2t}) \begin{bmatrix} -3 & -1 \\ 2 & 0 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} -e^{-t} + 2e^{-2t} & -e^{-t} + e^{-2t} \\ 2e^{-t} - 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$$

Primer: $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ $\quad g(\lambda) = |A - I\lambda| = \begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^3 = 0 \rightarrow \lambda_{1,2,3} = 2$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda + \alpha_2 \lambda^2 \quad / \frac{d}{d\lambda}$$
$$t e^{\lambda t} = \alpha_1 + 2\alpha_2 \lambda \quad / \frac{d}{d\lambda}$$
$$t^2 e^{\lambda t} = 2\alpha_2$$

$$e^{2t} = \alpha_0 + 2\alpha_1 + 4\alpha_2$$
$$te^{2t} = \alpha_1 + 4\alpha_2$$
$$t^2 e^{2t} = 2\alpha_2$$

$$\alpha_2 = \frac{1}{2} t^2 e^{2t}$$
$$\alpha_1 = t e^{2t} - 2t^2 e^{2t}$$
$$\alpha_0 = e^{2t} - 2t e^{2t} + 2t^2 e^{2t}$$

$$e^{At} = \alpha_0 I + \alpha_1 A + \alpha_2 A^2$$

$$e^{At} = (e^{2t} - 2te^{2t} + 2t^2 e^{2t}) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} +$$
$$+ (te^{2t} - 2t^2 e^{2t}) \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} + \frac{1}{2} t^2 e^{2t} \begin{bmatrix} 4 & 4 & 1 \\ 0 & 4 & 4 \\ 0 & 0 & 4 \end{bmatrix} =$$
$$= e^{2t} \begin{bmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

Primer:

$$\dot{x} = \begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} \frac{2}{5} \\ -1 \end{bmatrix} u$$

a) $\Phi(t) = ?$

$$g(\lambda) = |A - I\lambda| = \begin{vmatrix} -3-\lambda & 4 \\ 0 & 2-\lambda \end{vmatrix} = (2-\lambda)(-3-\lambda) = 0$$

$$\lambda_1 = 2, \quad \lambda_2 = -3$$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda$$
$$e^{2t} = \alpha_0 + 2\alpha_1$$
$$e^{-3t} = \alpha_0 - 3\alpha_1$$
$$\alpha_0 = 3/5\, e^{2t} + 2/5\, e^{-3t}$$
$$\alpha_1 = 1/5\, e^{2t} - \frac{1}{5} e^{-3t}$$

$$e^{At} = \alpha_0 I + \alpha_1 A = \left(\frac{3}{5}e^{2t} + \frac{2}{5}e^{-3t}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{5}e^{2t} - \frac{1}{5}e^{-3t}\right)\begin{bmatrix} -3 & 4 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-3t} & \frac{4}{5}e^{2t} - \frac{4}{5}e^{-3t} \\ 0 & e^{2t} \end{bmatrix}$$

b) $x(t) = ?$; $x(0) = [2 \;\; 2]^T$; $u = 0$ → vzbujanje je 0

$$x(t) = e^{At}x(0) + \int_0^t \cancel{e^{A(t-\tau)} B u(\tau)}^{\,0}\, d\tau$$

$$x(t) = \begin{bmatrix} e^{-3t} & \frac{4}{5}e^{2t} - \frac{4}{5}e^{-3t} \\ 0 & e^{2t} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{5}e^{2t} + \frac{2}{5}e^{-3t} \\ 2e^{2t} \end{bmatrix} \quad ; \; t \geq 0$$

ustni
x je vektor
spremenljivk stanj
↑
opisujejo lastnost
sistema ?

Primer:

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \qquad y = [2, 1]\, x$$

a) $e^{At} = ?$

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 = 0 \quad ; \quad \lambda_{1,2} = 2$$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda$$
$$te^{\lambda t} = \alpha_1$$
$$e^{2t} = \alpha_0 + 2\alpha_1$$
$$te^{2t} = \alpha_1$$
$$e^{2t} - 2te^{2t} = \alpha_0$$

$$e^{At} = \alpha_0 I + \alpha_1 A = (e^{2t} - 2te^{2t})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{2t}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}$$

b) $x(0) = [2 \;\; -2]^T$, $u = u(t)$ – enotina stopnica
$x(t) = ?$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau$$

en. stopnica

$$x(t) = \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}\begin{bmatrix} 2 \\ -2 \end{bmatrix} + \int_0^t \begin{bmatrix} e^{2(t-\tau)} & 0 \\ 0 & e^{2(t-\tau)} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot 1 \cdot d\tau =$$

$$= \begin{bmatrix} 2e^{2t} \\ -2e^{2t} \end{bmatrix} + \int_0^t \begin{bmatrix} e^{2(t-\tau)} \\ e^{2(t-\tau)} \end{bmatrix} d\tau = \begin{bmatrix} 2e^{2t} \\ -2e^{2t} \end{bmatrix} + \left.\begin{bmatrix} -\frac{1}{2}e^{2(t-\tau)} \\ -\frac{1}{2}e^{2(t-\tau)} \end{bmatrix}\right|_0^t =$$

$$= \begin{bmatrix} 2e^{2t} - \frac{1}{2} + \frac{1}{2}e^{2t} \\ -2e^{2t} - \frac{1}{2} + \frac{1}{2}e^{2t} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} + \frac{5}{2}e^{2t} \\ -\frac{1}{2} - \frac{3}{2}e^{2t} \end{bmatrix} \quad ; \; t \geq 0$$

· DOLOČANJE ODZIVA $x(t)$ S POMOČJO DIAGONALIZACIJE MATRIKE A

$$\Lambda = \theta^{-1} \cdot A \cdot \theta \Rightarrow * \qquad \theta = [g_1 \vdots g_2 \vdots \dots \vdots g_n]_{n \times n}$$

A ← osnovna matrika sistema

$$\Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}_{n \times n}$$

$$\lambda_1 \leftrightarrow g_1$$
$$\vdots$$
$$\lambda_n \leftrightarrow g_n$$

$$* \Rightarrow A = \theta \cdot \Lambda \cdot \theta^{-1} \rightarrow e^{At} = \theta e^{\Lambda t} \theta^{-1}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{\lambda_1 t} & & & 0 \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n t} \end{bmatrix}$$

$\Lambda \subseteq J$ ← Jordanova kanonična forma

$$A = \begin{bmatrix} -5 & 2 \\ -1 & -2 \end{bmatrix} \qquad |A - \lambda I| = \begin{vmatrix} -5-\lambda & 2 \\ -1 & -2-\lambda \end{vmatrix} = \lambda^2 + 7\lambda + 12 = (\lambda+3)(\lambda+4) = 0 \qquad \lambda_1 = -3,\ \lambda_2 = -4$$

$$f(\lambda, \mu) = \frac{g(\lambda) - g(\mu)}{\lambda - \mu} = \frac{\lambda^2 + 7\lambda + 12 - (\mu^2 + 7\mu + 12)}{\lambda - \mu}$$

$$f(\lambda, \mu) = \frac{(\lambda+\mu)(\lambda-\mu) + 7(\lambda-\mu)}{\lambda - \mu} = \lambda + \mu + 7$$

$$f(\lambda, \mu) \rightarrow C(\lambda);\ \lambda \rightarrow \lambda I;\ \mu \rightarrow A$$

$$C(\lambda) = \lambda I + A + 7I = (\lambda + 7) I + A$$

$$C(\lambda) = \begin{bmatrix} \lambda+7 & 0 \\ 0 & \lambda+7 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} \lambda+2 & 2 \\ -1 & \lambda+5 \end{bmatrix}$$

$$C(\lambda_1 = -3) = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \qquad g_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C(\lambda_2 = -4) = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} \qquad g_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$e^{\Lambda t} = \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-4t} \end{bmatrix}, \qquad \theta = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow e^{At} = \theta e^{\Lambda t} \theta^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-4t} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} e^{-3t} & 2e^{-4t} \\ e^{-3t} & e^{-4t} \end{bmatrix} \cdot \frac{1}{+1} \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} -e^{-3t} + 2e^{-4t} & 2e^{-3t} - 2e^{-4t} \\ -e^{-3t} + e^{-4t} & 2e^{-3t} - e^{-4t} \end{bmatrix}$$

Primer: $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ $\quad g(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & -1 \\ 1 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{vmatrix} =$

$$= (1-\lambda)\big((2-\lambda)(3-\lambda)-2\big) - 1\big(2-2(2-\lambda)\big) = (1-\lambda)(\lambda^2-5\lambda+4) + 2(1-\lambda) =$$

$$= \underbrace{(1-\lambda)(\lambda-2)(\lambda-3)}_{\downarrow} = 0 \Rightarrow \begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 2 \\ \lambda_3 = 3 \end{array}$$

$$\lambda^2 - 5\lambda + 6 - \lambda^3 + 5\lambda^2 - 6\lambda = -\lambda^3 + 6\lambda^2 - 11\lambda + 6$$

$$f(\lambda,\mu) = \frac{g(\lambda)-g(\mu)}{\lambda-\mu} = \frac{-(\lambda^3-\mu^3)+6(\lambda^2-\mu^2)-11(\lambda-\mu)}{\lambda-\mu} = -(\lambda^2+\lambda\mu+\mu^2)+6(\lambda+\mu)-11$$

$$C(\lambda) = -(\lambda^2 I + \lambda A + A^2) + 6(\lambda I + A) - 11 I$$

$$C(\lambda) = (-\lambda^2+6\lambda-11) I + (-\lambda+6) A - A^2$$

$$C(\lambda) = (-\lambda^2+6\lambda-11) I + (-\lambda+6) \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & -4 \\ 5 & 6 & 4 \\ 10 & 10 & 9 \end{bmatrix}$$

$$C(\lambda) = \begin{bmatrix} -\lambda^2+5\lambda-4 & 2 & \lambda-2 \\ -\lambda+1 & -\lambda^2+4\lambda-5 & -\lambda+2 \\ -2\lambda+2 & -2\lambda+2 & -\lambda^2+3\lambda-2 \end{bmatrix}$$

$$C(\lambda_1=1) = \begin{bmatrix} 0 & 2 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad q_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$C(\lambda_2=2) = \begin{bmatrix} 2 & 2 & 0 \\ -1 & -1 & 0 \\ -2 & -2 & 0 \end{bmatrix} \qquad q_2 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

$$C(\lambda_3=3) = \begin{bmatrix} 2 & 2 & 1 \\ -2 & -2 & -1 \\ -4 & -4 & -2 \end{bmatrix} \qquad q_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

$$e^{At} = Q e^{\Lambda t} Q^{-1} = \begin{bmatrix} -1 & 2 & 1 \\ 1 & -1 & -1 \\ 0 & -2 & -2 \end{bmatrix} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{3t} \end{bmatrix} \frac{1}{(-1)\cdot 0-(-2)} \begin{bmatrix} 0 & 2 & -2 \\ 2 & 2 & -2 \\ -1 & 0 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} -e^{+t} & 2e^{2t} & e^{3t} \\ e^t & -e^{2t} & -e^{3t} \\ 0 & -2e^{2t} & -2e^{3t} \end{bmatrix} \begin{bmatrix} 0 & 1 & -\frac{1}{2} \\ 1 & 1 & 0 \\ -1 & -1 & -\frac{1}{2} \end{bmatrix} = \ldots$$

$\lambda_{1,2,\ldots,m}$ $\nearrow$ $q_{1,2,\ldots,m}$ v $C(\lambda)$ $\Rightarrow$ algebrajska večkratnost = geometrijska v.

$\lambda_{1,2,\ldots,m}$ $\searrow$ $q_{1,2,\ldots,m}$ v $C(\lambda)$, $C'(\lambda)$, ... $\Rightarrow$ alg. $\neq$ geom.

$$\downarrow$$

$$\lambda_1 \; ; \; \overset{a\neq g}{\lambda_{2,3}} \; ; \; \overset{a\neq g}{\lambda_{4,5,6}} \; ; \; \overset{a=g}{\lambda_{7,8}}$$

Primer: $\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad , \quad y = [-1 \ \ 2]\, x$

a) $w = Tx \Rightarrow A$ diagonalna

$T = ?$

$\dot{w} = \overset{diag}{A'} w + B' u$

$\dot{x} = Ax + Bu \quad ; \quad A = \Theta \Lambda \Theta^{-1}$

$\dot{x} = \Theta \Lambda \Theta^{-1} x + Bu$

$\Theta^{-1} \dot{x} = \Lambda \Theta^{-1} x + \Theta^{-1} B u \quad ; \quad \boxed{w = \Theta^{-1} x}$

$\dot{w} = \Lambda w + \Theta^{-1} B u \qquad \dot{w} = \Theta^{-1} \dot{x}$

$\dot{w} = A' w + B'' u \qquad T = \Theta^{-1}$

$$g(\lambda) = |A + \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -\frac{1}{6} & \frac{5}{6} - \lambda \end{vmatrix} = \lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = (\lambda - \tfrac{1}{2})(\lambda - \tfrac{1}{3}) = 0$$

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = \frac{1}{3}$$

$f(\lambda, \mu) = \lambda + \mu - \frac{5}{6}$

$C(\lambda) = (\lambda - \frac{5}{6}) I + A$

$$C(\lambda) = \begin{bmatrix} \lambda - \frac{5}{6} & 1 \\ -\frac{1}{6} & \lambda \end{bmatrix}$$

$$C(\lambda_1 = \tfrac{1}{2}) = \begin{bmatrix} -\frac{1}{3} & 1 \\ -\frac{1}{6} & \frac{1}{2} \end{bmatrix} \qquad g_1 = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

$$C(\lambda_2 = \tfrac{1}{3}) = \begin{bmatrix} -\frac{1}{2} & 1 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix} \qquad g_2 = \begin{bmatrix} 1 \\ \frac{1}{3} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix}, \quad \Theta^{-1} = -6 \begin{bmatrix} \frac{1}{3} & -1 \\ -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ 3 & -6 \end{bmatrix} = T$$

preverimo: $A = \Theta \Lambda \Theta^{-1}$

$$\begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{9} \end{bmatrix} \begin{bmatrix} -2 & 6 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix}$$

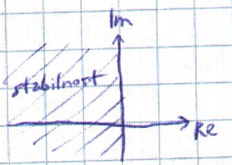
LASTNOSTI SISTEMOV

- STABILNOST

  če njegov odziv po zač. stanju izzveni oz. ostane omejen
  ↳ ne narašča preko vseh meja

  $g(\lambda) = |A - \lambda I| = 0 \Rightarrow \lambda_i$

  $\forall \lambda_i : \ Re\{\lambda_i\} \leq 0$



- VODLJIVOST

  če ga je mogoče z ustrez. vzbujanjem v končnem času privesti iz zač. v kon.

  $M = [B \ \vdots \ AB \ \vdots \ A^2 B \ \vdots \ \dots \ \vdots \ A^{n-1} B]$

  nesingularna: $\underset{\text{rang}}{\rho}(M) = n$

  $(\det(M) \neq 0)$

- SPOZNAVNOST

  kadar je mogoče določiti stanje na osnovi končno dolgega opazovanja njegovega izhoda

  $N = [C^T \ \vdots \ A^T C^T \ \vdots \ (A^T)^2 C^T \ \vdots \ \dots \ | \ (A^T)^{n-1} C^T]$

  nesingularna: $\rho(N) = n$

  $(\det(N) \neq 0)$

- $\dot{x}_1 = x_1 + 2x_2 + u$
  $\dot{x}_2 = 3x_2 + u$
  $y = x_1 - x_2$

$g(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda) = 0$

$\lambda_1 = 1\ ,\ \lambda_2 = 3$

zapis v matrični obliki:

$\dot{x} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & -1 \end{bmatrix} x$

Sistem ni stabilen.

$M = \begin{bmatrix} 1 & 3 \\ 1 & 3 \end{bmatrix}$ Sistem ni vodljiv.

$N = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ Sistem ni spoznaven.

pravilo: $A^T C^T = (CA)^T$

- Satelit v orbiti. Določanje položaja – 2 senzorja.

$J\ddot{\theta} = M$

J – vztraj moment (konst.)

M – vrtilni moment (vzbujanje)

1) $y_1 = \theta$

2) $y_2 = \dot{\theta} + \varepsilon$

ε – napaka senzorja (konst.)

a) ~~Opišite~~ Za enačbo $\dot{x} = Ax + Bu$, če je $x = [\underset{x_1}{\theta}\ \underset{x_2}{\dot{\theta}}\ \underset{x_3}{\varepsilon}]^T$, $u = M$

$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{1}{J}M \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/J \\ 0 \end{bmatrix} u$

b) $y = Cx + Du$

$y = [y_1\ y_2]^T$

$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}}_{2\times 3} x + \cancel{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} M$

$A^2B = A \cdot (AB)$

$A^3B = A \cdot (A^2B)$

c) $M = [B \,\vdots\, AB \,\vdots\, A^2B]$ $\quad N = [C^T \,\vdots\, A^TC^T \,\vdots\, (A^T)^2 C^T]$

$M = \begin{bmatrix} 0 & 1/J & 0 \\ 1/J & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad N = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

Sistem ni vodljiv.

$\rho(N) = 3 = n$

Sistem je spoznaven.
(ker imamo 3 neodvisne stolpce)

$(CA)^T = A^T C^T$

$A^T \cdot A^T \cdot C^T = (CAA)^T =$
$= ((CA) \cdot A)^T$

$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $C = [-2\ 0\ 0]$

$\dot{x} = Ax + Bu$

$y = Cx$

a) Določi matriko preh. stanj!

$\Phi(t) = e^{At}$

$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -\lambda & 0 & 0 \\ 1 & -1-\lambda & 1 \\ 1 & 0 & -1-\lambda \end{vmatrix} =$

$= -\lambda(-1-\lambda)(-1-\lambda) = 0$

$\lambda_1 = 0\ ,\ \lambda_{2,3} = -1$

$e^{\lambda t} = \sum_{k=0}^{2} \alpha_k \lambda^k = \alpha_0 + \alpha_1 \lambda + \alpha_2 \lambda^2$

$te^{\lambda t} = \alpha_1 + 2\alpha_2 \lambda$ (ne vstavljat 0! samo kar je večkratna)

$1 = \alpha_0$

$e^{-t} = \alpha_0 - \alpha_1 + \alpha_2$

$te^{-t} = \alpha_1 - 2\alpha_2$ $\quad)+$

$\alpha_2 = 1 - e^{-t} - te^{-t}$

$\alpha_1 = 2 - 2e^{-t} - te^{-t}$

$e^{At} = \alpha_0 I + \alpha_1 A + \alpha_2 A^2 =$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + (2 - 2e^{-t} - te^{-t}) \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} +$

$+ (1 - e^{-t} - te^{-t}) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix} =$

$= \begin{bmatrix} 1 & 0 & 0 \\ 2 - 2e^{-t} - te^{-t} & e^{-t} & te^{-t} \\ 1 - e^{-t} & 0 & e^{-t} \end{bmatrix}$

b) $M = [B \;\; AB \;\; A^2B]$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -2 \\ 1 & 0 & 0 \end{bmatrix}$$

$\rho(M) = 2 \neq 3 = n$

Ni vodljiv.

c) $N = [C^T \;\; A^T C^T \;\; (A^T)^2 C^T]$

$$N = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(N) = 1 \neq 3 = n$

Ni spoznaven.

d) Določite odziv sistema na vzbujanje z enotino stopnico, če so zač. stanja 0.

$y(t) = ?$ $\quad \hookrightarrow u = u(t)$ $\quad x(0) = 0$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = x(t) = \cancel{e^{At} x(0)}^{\,0} + \int_0^t e^{A(t-\tau)} B\, u(\tau)\, d\tau$$

$y = Cx = [-2 \;\; 0 \;\; 0]\, x = -2x_1(t)$ $\quad \leftarrow$ čas prišparamo, če je ta korak tu

$$x_1(t) = \int_0^t 1 \cdot 1 \, d\tau = t$$

$$\underline{\underline{y(t) = -2t \quad ; \; t \geq 0}}$$

$\boxed{\text{Matlab}}$

skripta 77

$R_1 = 1 \quad R_2 = 2 \quad L = 1 \quad C = 1$

$u = u_g(t) = 4 \cdot u(t)$

$\dot{x} = Ax + Bu$

$$\dot{x} = \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \overbrace{4 \cdot u(t)}^{4}$$

$v_c = x_1(t) = \frac{8}{3}\left(1 - e^{-\frac{3}{2}t} \cos\frac{\sqrt{3}}{2}t\right) \quad t \geq 0$

$i_c = x_2(t) = \frac{4}{3}\left(1 - e^{-\frac{3}{2}t}\left(\cos\frac{\sqrt{3}}{2}t + \sqrt{3}\sin\frac{\sqrt{3}}{2}t\right)\right), \quad t \geq 0$

R1=1; R2=2; L=1; C=1;

$$A = \begin{bmatrix} -\frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_2}{L} \end{bmatrix} x + \begin{bmatrix} \frac{1}{R_1 C} \\ 0 \end{bmatrix} u$$

```
global A B u;
A= [-1/(R1*C), -1/C ; 1/L, -R2/L];
B = [1/(R1*C); 0];
u = 4*1;
x0 = [0; 0];

% num rešitev
t1=0; t2=100;
[t_NUM, x_NUM] = ode45 (@desna_stran, [t1,t2], x0);
plot(t_NUM, x_NUM(:,1), 'b-', t_NUM, x_NUM(:,2), 'r-');
xlabel('Čas t(s)'); ylabel('Spremenljivke stanj');
title('Numerična rešitev - ode45');
legend('x_1(t)= v_C(t) (V)', 'x_2(t) =i_L(t) (A)');
```

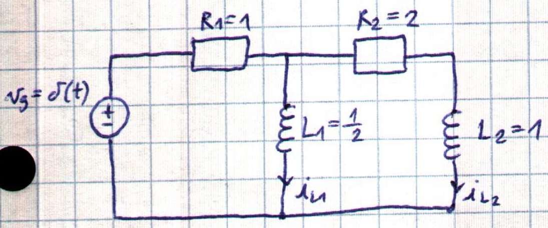
```
plot(x_NUM(:,1), x_NUM(:,2), 'k-');
grid on;
xlabel('x_1(t)'); ylabel('x_2(t)');
title('Num reš - prostor stanj');

% analit. rešitev z Lapl. transf.

  syms t s;
  u_L = 4*heaviside(t);                    heaviside - stopnica
U_s = laplace(u_t);
 X_s = inv(s*eye(size(A)) - A)*(x0 + B*U_s);
 x_t = ilaplace(X_s);
 t_LAP = [t1:dt:t2]';
 x_LAP = subs(x_t, t, t_LAP);
eval(x_LAP


% anal. reš. z integracijo v čas. prostoru
 syms t tau;
 u_t = 4*1;
 x_t = expm(A*t)*x0 + int(expm(A*(t-tau))*B*u_t, tau, 0, t);
```

PDF! Naloge! Nal. 15
Oddaj mu 21.1.!

(naris potek z Matlabom, fazni potek...)

POROČILO

spenjač



a) $\dot{x} = Ax + Bu$

$$x = \begin{bmatrix} i_{L1} \\ i_{L2} \end{bmatrix}$$

$u = v_g$

$v_{R1} + v_{R2} = v_g$

$v_{L2} + v_{R1} + v_{R2} = v_g$

$L_1 \dot{i}_{L1} + R_1 (i_{L1} + i_{L2}) = v_g$

$L_2 \dot{i}_{L2} + R_1 (i_{L1} + i_{L2}) + R_2 i_{L2} = v_g$

$$\dot{x} = \begin{bmatrix} -\frac{R_1}{L_1} & -\frac{R_1}{L_1} \\ -\frac{R_1}{L_2} & -\frac{R_1+R_2}{L_2} \end{bmatrix} x + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} u = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

b) $i_{L1}(t) = ?$ $\quad i_{L2}(t) = ?$ $\quad i_{L1}(0) = 0, \quad i_{L2}(0) = 0$

$$X = \cancel{(sI-A)^{-1} x(0)}^{\,0} + (sI-A)^{-1} \cdot B \cdot U$$

$$X = \begin{bmatrix} s+2 & 2 \\ 1 & s+3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot 1$$

$$X = \frac{1}{s^2+5s+4} \begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{(s+1)(s+4)} \begin{bmatrix} 2s+4 \\ s \end{bmatrix} = \begin{bmatrix} \frac{\frac{2}{3}}{s+1} + \frac{\frac{4}{3}}{s+4} \\ \frac{-\frac{1}{3}}{s+1} + \frac{\frac{4}{3}}{s+4} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} i_{L1}(t) \\ i_{L2}(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{3} e^{-t} + \frac{4}{3} e^{-4t} \\ -\frac{1}{3} e^{-t} + \frac{4}{3} e^{-4t} \end{bmatrix} \quad ; \ t \geq 0$$

c) Izr. ravnotežno stanje sistema ! $\quad \bar{x} = ?$ $\qquad$ (ko vse spremembe izzvenijo)

$$\bar{x} = \begin{bmatrix} \bar{i}_{L1} \\ \bar{i}_{L2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\int_0^t f(t-\tau)\, \delta(\tau)\, d\tau = f(t)$$

■ $\dot{x} = \begin{bmatrix} -2 & -1 \\ -3 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

a) C.H. teorem $\rightarrow$ $\phi(t) = ? = e^{At} = ?$

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -2-\lambda & -1 \\ -3 & -\lambda \end{vmatrix} = \lambda^2 + 2\lambda - 3 = (\lambda-1)(\lambda+3) = 0$$

$\lambda_1 = 1, \ \lambda_2 = -3$

$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda$ $\quad$ (tok kot je red sistema)

$$\left.\begin{aligned} e^{t} &= \alpha_0 + \alpha_1 \\ e^{-3t} &= \alpha_0 - 3\alpha_1 \end{aligned}\right\} -$$

$\alpha_1 = \frac{1}{4} e^{t} - \frac{1}{4} e^{-3t}$

$\alpha_0 = \frac{3}{4} e^{t} + \frac{1}{4} e^{-3t}$

$e^{At} = \alpha_0 I + \alpha_1 A$

$$e^{At} = \left(\frac{3}{4} e^{t} + \frac{1}{4} e^{-3t}\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\frac{1}{4} e^{t} - \frac{1}{4} e^{-3t}\right) \begin{bmatrix} -2 & -1 \\ -3 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{4} e^{t} + \frac{3}{4} e^{-3t} & -\frac{1}{4} e^{t} + \frac{1}{4} e^{-3t} \\ -\frac{3}{4} e^{t} + \frac{3}{4} e^{-3t} & \frac{3}{4} e^{t} + \frac{1}{4} e^{-3t} \end{bmatrix}$$

b) $x(t) = ?$

$u = u(t)$ enotina stopnica

$x(0) = [1 \;\; 0]^T$

$$\int e^{t-\tau} d\tau = -e^{t-\tau}$$

$$\int e^{-3(t-\tau)} d\tau = = \frac{1}{3} e^{-3(t-\tau)}$$

$$x(t) = e^{At} \cdot x(0) + \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau$$

$$x(t) = \begin{bmatrix} \frac{1}{4}e^{t} + \frac{3}{4}e^{-3t} & -\frac{1}{4}e^{t} + \frac{1}{4}e^{-3t} \\ -\frac{3}{4}e^{t} + \frac{3}{4}e^{-3t} & \frac{3}{4}e^{t} + \frac{1}{4}e^{-3t} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \int_0^t \begin{bmatrix} -\frac{1}{4}e^{t-\tau} + \frac{1}{4}e^{-3(t-\tau)} \\ \frac{3}{4}e^{t-\tau} + \frac{1}{4}e^{-3(t-\tau)} \end{bmatrix} d\tau$$

$$x(t) = \begin{bmatrix} \frac{1}{4}e^{t} + \frac{3}{4}e^{-3t} \\ -\frac{3}{4}e^{t} + \frac{3}{4}e^{-3t} \end{bmatrix} + \begin{bmatrix} \frac{1}{4} - \frac{1}{4}e^{t} + \frac{1}{12} - \frac{1}{12}e^{-3t} \\ -\frac{3}{4} + \frac{3}{4}e^{t} + \frac{1}{12} - \frac{1}{12}e^{-3t} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} + \frac{2}{3}e^{-3t} \\ -\frac{2}{3} + \frac{2}{3}e^{-3t} \end{bmatrix}, \quad t \geq 0$$

c) Je sistem stabilen ? Ni. (lande morajo bit na levi

■ $A = \begin{bmatrix} -5 & 2 \\ -1 & -2 \end{bmatrix}$

a) $\lambda, q = ?$ lastni vekt.

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -5-\lambda & 2 \\ -1 & -2-\lambda \end{vmatrix} = \lambda^2 + 7\lambda + 12 = (\lambda+3)(\lambda+4) = 0$$

$$\lambda_1 = -3, \quad \lambda_2 = -4$$

$$f(\mu, \lambda) = \frac{g(\lambda) - g(\mu)}{\lambda - \mu} = \lambda + \mu + 7$$

$$C(\lambda) = (\lambda + 7)I + A = \begin{bmatrix} \lambda+2 & 2 \\ -1 & \lambda+5 \end{bmatrix}$$

$$C(\lambda_1 = -3) = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \qquad q_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C(\lambda_2 = -4) = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} \qquad q_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$A q = \lambda q$$

$$\begin{bmatrix} -5 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{12} \end{bmatrix} = -3 \begin{bmatrix} q_{11} \\ q_{12} \end{bmatrix} \qquad \begin{bmatrix} -5 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} q_{21} \\ q_{22} \end{bmatrix} = -4 \begin{bmatrix} q_{21} \\ q_{22} \end{bmatrix}$$

$$-5q_{11} + 2q_{12} = -3q_{11} \rightarrow q_{11} = q_{12} \qquad -5q_{21} + 2q_{22} = -4q_{21} \rightarrow 2q_{22} = q_{21}$$

$$-q_{11} - 2q_{12} = -3q_{12} \rightarrow q_{11} = q_{12} \qquad -q_{21} - 2q_{22} = -4q_{22} \rightarrow q_{21} = 2q_{22}$$

$$e^{At} = Q \cdot e^{\Lambda t} \cdot Q^{-1}$$

$$Q = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

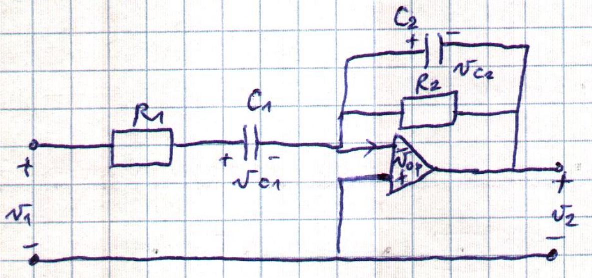
$$e^{\Lambda t} = \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-4t} \end{bmatrix}$$

ne smemo jih obrnit!

$$e^{At} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-3t} & 0 \\ 0 & e^{-4t} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} e^{-3t} & 2e^{-4t} \\ e^{-3t} & e^{-4t} \end{bmatrix} \cdot \frac{1}{+1} \begin{bmatrix} -1 & +2 \\ +1 & -1 \end{bmatrix} = \begin{bmatrix} -e^{-3t} + 2e^{-4t} & 2e^{-3t} - 2e^{-4t} \\ -e^{-3t} + e^{-4t} & 2e^{-3t} - e^{-4t} \end{bmatrix}$$

c) Je stabilen ? ✓, realna dela sta negat.

$\lambda_1 = -3, \; \lambda_2 = -4$



a) $\dot{x} = Ax + Bu$

$x = \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix}; \quad u = v_1$

$i_{C1} = i_{C2} + i_{R2}$

$v_{C2} = v_{R2}$

$v_{C1} + v_{R1} = v_1$

$v_{C1} + R_1 i_{C1} = v_1$

$v_{C1} + R_1 C_1 \dot{v}_{C1} = v_1$

$\dot{x}_1 = -\frac{1}{R_1 C_1} x_1 + \frac{1}{R_1 C_1} u$

$i_{C2} = i_{C1} - i_{R2} = i_{R1} - i_{R2}$

$i_{C2} = \frac{v_1 - v_{C1}}{R_1} - \frac{v_{C2}}{R_2}$

$\dot{x}_2 = -\frac{1}{R_1 C_2} x_1 - \frac{1}{R_2 C_2} x_2 + \frac{1}{R_1 C_2} u$

$\dot{x} = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 \\ -\frac{1}{R_1 C_2} & -\frac{1}{R_2 C_2} \end{bmatrix} x + \begin{bmatrix} \frac{1}{R_1 C_1} \\ \frac{1}{R_1 C_2} \end{bmatrix} u$

b) $y = Cx + Du$

$y = v_2$

$v_{R1} + v_{C1} + v_{C2} + v_2 = v_1$

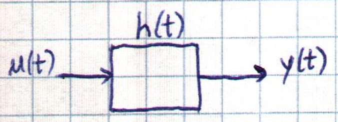
$v_1 - v_{C1}$

$v_{C2} + v_2 = 0$

$y = -x_2$

$y = [0 \ \ -1] \, x$

PREVAJALNA (PRENOSNA) FUNKCIJA SISTEMA



$\dot{x} = Ax + Bu, \quad y = Cx + Du \ ; \quad x(0) = 0$

$X(s) = (sI - A)^{-1} B U(s)$

$Y(s) = C X(s) + D U(s)$

$\boxed{H(s) = C(sI - A)^{-1} B + D}$

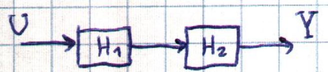
na listu z enačbami

$y(t) = h(t) * u(t)$

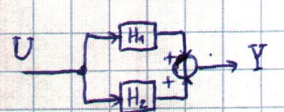
$y(t) = \int_0^t h(t-\tau)\, u(\tau)\, d\tau$

$Y(s) = H(s) \cdot U(s)$

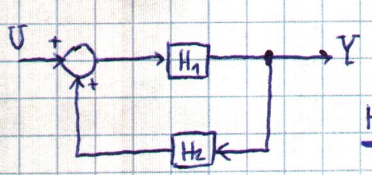
$H(s) = \frac{Y(s)}{U(s)} \ ; \ h(t) = \mathcal{L}^{-1}\{H(s)\}$



$\underline{H = H_2 \cdot H_1}$



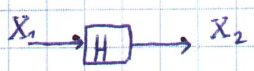
$\underline{H = H_1 + H_2}$



povratna zanka

$\underline{H = (I \mp H_1 H_2)^{-1} H_1}$

$\left\{ H = \frac{H_1}{1 \mp H_1 H_2} \right\}$



$X_2 = H X_1$

! od izhoda proti vhodu

$H = s$ → to pomeni odvajanje (diferenciator)

$X_2 = s X_1$

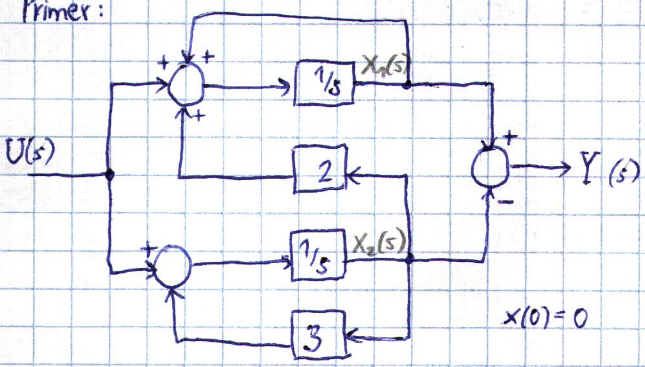
$x_2(t) = \dot{x}_1(t)$

$H = \frac{1}{s}$ (integrator)

$X_2 = \frac{1}{s} X_1$

$\dot{x}_2(t) = x_1(t) \rightarrow x_2(t) = \int x_1(t)\, dt$

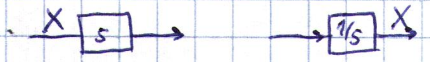
Primer:



$x(0)=0$

Določi prevajalno f. !

<u>spr. stanj vedno na - izhode integratorjev in vhode diferenciatorjev</u>



$$\underline{Y = X_1 - X_2}$$

$$X_1 = \frac{1}{s}(X_1 + 2X_2 + U)$$

$$\underline{X_2 = \frac{1}{s}(3X_2 + U)}$$

$y = x_1 - x_2$

$\dot{x}_1 = x_1 + 2x_2 + u$

$\dot{x}_2 = 3x_2 + u$

$y = [1 \ -1]\, x$

$\dot{x} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$

$$H = C(sI - A)^{-1}B + D$$

$$H = [1 \ -1]\begin{bmatrix} s-1 & -2 \\ 0 & s-3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [1 \ -1]\frac{1}{(s-1)(s-3)}\begin{bmatrix} s-3 & 2 \\ 0 & s-1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{[1 \ -1]}{(s-1)(s-3)}\begin{bmatrix} s-1 \\ s-1 \end{bmatrix} = \underline{\underline{0}}$$

nič ne pride na izhod

Izr. prev. f. $h'(t) = ?$, če iz sistema odstranimo blok, ki predstavlja ojačanje s faktorjem 2 ?

$$H = [1 \ -1]\begin{bmatrix} s-1 & 0 \\ 0 & s-3 \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = [+1 \ -1]\frac{1}{(s-1)(s-3)}\begin{bmatrix} s-3 & 0 \\ 0 & s-1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{[1 \ -1]}{(s-1)(s-3)}\begin{bmatrix} s-3 \\ s-1 \end{bmatrix} =$$

$$= \frac{-2}{(s-1)(s-3)} = H'(s)$$

$$H'(s) = \frac{1}{s-1} + \frac{-1}{s-3}$$

$$h'(t) = e^{t} - e^{3t}$$

$$H(s)=\frac{3s+7}{(s+1)(s+2)(s+5)} = \frac{1}{s+1} + \frac{-\frac{1}{3}}{s+2} + \frac{-\frac{2}{3}}{s+5} = \frac{Y}{U}$$

PARC. ULOMKI

$\dot{x} = Ax + Bu$

$x(0) = 0$

u je skalar (vzbujanje je eno samo)

$$Y = X_1 + X_2 + X_3 \Rightarrow y = x_1 + x_2 + x_3$$

$$X_1 = \frac{1}{s+1} U \rightarrow sX_1 = -X_1 + U$$

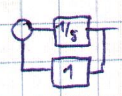
$$X_2 = \frac{-1/3}{s+2} U \rightarrow sX_1 = -2X_2 - \frac{1}{3} U \quad \Rightarrow \text{inv. Lap. tr.}$$

$$X_3 = \frac{-2/3}{s+5} \rightarrow sX_3 = -5X_3 - \frac{2}{3} U$$

$$\begin{aligned} \dot{x}_1 &= -x_1 + u \\ \dot{x}_2 &= -2x_2 - \tfrac{1}{3} u \\ \dot{x}_3 &= -5x_3 - \tfrac{2}{3} u \end{aligned} \quad \Rightarrow \quad \dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -5 \end{bmatrix} x + \begin{bmatrix} 1 \\ -\frac{1}{3} \\ -\frac{2}{3} \end{bmatrix} u$$

$$y = [1 \;\; 1 \;\; 1]\, x$$

lahko tud

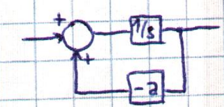


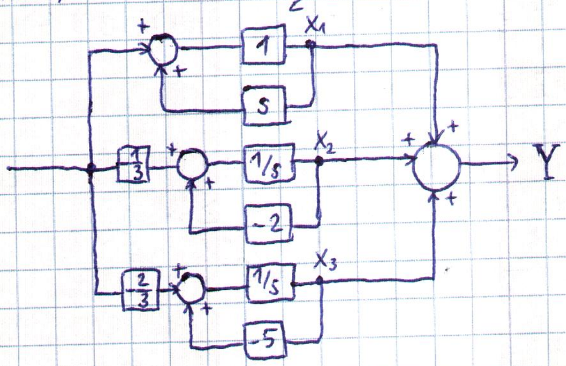
bločna shema sistema v obliki (največji blok) $\frac{1}{s} / s / k$

$$\frac{1}{s+1} = \frac{1}{1+1\cdot s} \left(\frac{H_1}{1+H_1H_2}\right) = \frac{\frac{1}{s}}{1+1\cdot\frac{1}{s}}$$

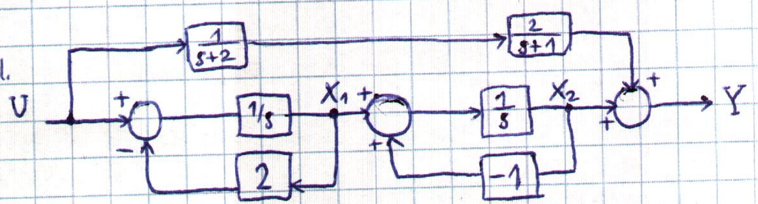
$$\frac{1}{s+2} = \frac{\frac{1}{s}}{1+\frac{1}{s}\cdot 2}$$

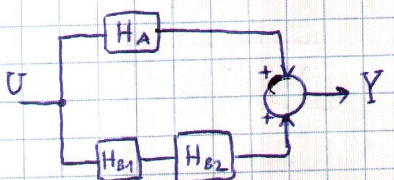
$\frac{1}{s+a}$:





3. nal.





$$H_A = \frac{2}{s+1} \cdot \frac{1}{s+2}$$

$$H_{B1} = \frac{\frac{1}{s}}{1+2\cdot\frac{1}{s}} = \frac{1}{s+2}$$

$$H_{B2} = \frac{\frac{1}{s}}{1-(-1)\frac{1}{s}} = \frac{1}{s+1}$$

$$H_B = H_{B1} \cdot H_{B2} = \frac{1}{s+2} \cdot \frac{1}{s+1}$$

$$\underline{H} = H_A + H_B = \underline{\underline{\frac{3}{(s+2)(s+1)}}} = \frac{-3}{s+2} + \frac{3}{s+1}$$

$$h(t) = 3e^{-t} - 3e^{-2t}$$

$$\dot{x} = Ax + Bu \rightarrow X_1 = \frac{1}{s}(-2X_1 + U) \rightarrow \dot{x}_1 = -2x_1 + u$$

$$X_2 = \frac{1}{s}(X_1 - X_2) \rightarrow \dot{x}_2 = x_1 - x_2$$

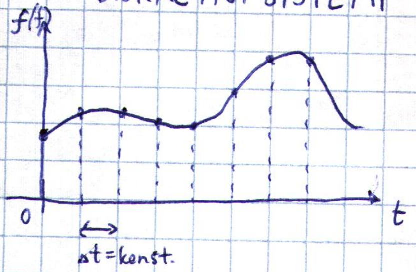
$$\dot{x} = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

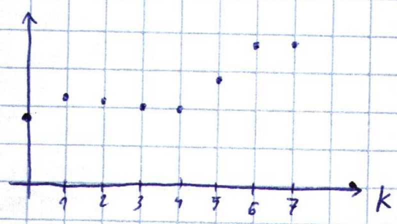
$$Y = X_2 + \left(\frac{2}{(s+1)(s+2)}\right) U$$

$$y(t) = \int_0^t h(t-\tau)\, u(\tau)\, d\tau \xrightarrow{\text{če je } u(\tau) \text{ enotin impulz}} h(t) = y(t)\Big|_{u(t)=\delta(t)}$$

izhod = prevajalni funkciji

DISKRETNI SISTEMI





$$y(k+n) + a_{n-1}\,y(k+n-1) + \ldots + a_1\,y(k+1) + a_0\,y(k) = b_0\,u(k)$$

$x_1(k) = y(k)$ | $x_1(k+1) = y(k+1) = x_2(k)$

$x_2(k) = y(k+1)$ | $x_2(k+1) = y(k+2) = x_3(k)$

$\vdots$

$x_n(k) = y(k+n-1)$ | $x_n(k+1) = y(k+n) = \ldots$ iz enačbe

$$\boxed{x(k+1) = Ax(k) + Bu(k)}$$

$k=0:\quad x(1) = Ax(0) + Bu(0)$

$k=1:\quad x(2) = Ax(1) + Bu(1) = A^2x(0) + ABu(0) + Bu(1)$

$k=2:\quad x(3) = Ax(2) + Bu(2) = A^3x(0) + A^2Bu(0) + ABu(1) + A^0Bu(2)$

$\vdots$

$$k=k:\quad x(k) = \underbrace{A^k x(0)}_{x_z(k)} + \underbrace{\sum_{m=0}^{k-1} A^{k-m-1} Bu(m)}_{x_v(k)} \qquad \boxed{\Phi(k) = A^k}$$

$$x(k+1) = Ax(k) + Bu(k)$$

$\downarrow \mathcal{Z}$ to je razlika, dodaš z pri zač. stanjih

$$zX(z) - z\,x(0) = AX(z) + BU(z)$$

$$(zI - A)X(z) = z\,x(0) + BU(z)$$

$$X(z) = z(zI-A)^{-1}x(0) + (zI-A)^{-1}BU(z)$$

$$A^k = \mathcal{Z}\{z(zI-A)^{-1}\}$$

$$\lambda^k = \sum_{m=1}^{n-1} \alpha_m \lambda^m \quad \longleftrightarrow \quad A^k = \sum_{m=1}^{n-1} \alpha_m \lambda^m$$

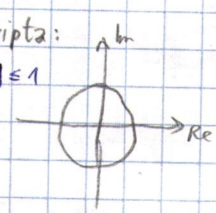
$\longrightarrow \alpha_m \longrightarrow$

$$A = \Theta \Lambda \Theta^{-1}$$

$$A^2 = \Theta\Lambda\underbrace{\Theta^{-1}\Theta}_{I}\Lambda\Theta^{-1} = \Theta\Lambda^2\Theta^{-1}$$

$$\boxed{A^k = \Theta\Lambda^k\Theta^{-1}}$$

skripta: $|\lambda| \le 1$



BODI POZOREN NA NAVODILA (zvezni, diskr..)

Primer: Dana je diferenčna enačba, zač. stanji. Rešite diferenčno ...

$$y(k+2) - 2y(k+1) + y(k) = 0 \quad , \quad y(0) = y(1) = 1$$

$$z^2Y - z^2y(0) - zy(1) - 2(zY - zy(1)) + Y = 0$$

$$(z^2 - 2z + 1)\,Y = z^2 - z$$

$$Y = \frac{z^2 - z}{z^2 - 2z + 1} = \frac{z\cancel{(z-1)}}{(z-1)\cancel{(z-1)}} = \frac{z}{z-1}$$

$$\underline{y(k) = u(k)\,;\quad k \ge 0}$$

b) Zapišite v matrični obliki!

$x_1(k) = y(k)$

$\underline{x_2(k) = y(k+1)}$

$x_1(k+1) = y(k+1) = x_2(k)$

$x_2(k+1) = y(k+2) = -y(k) + 2y(k+1) = -x_1(k) + 2x_2(k)$

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x(k)$$

c) Določi vektor zač. stanja $x(0)$!

$$x(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(0+1) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -8 & 6 \end{bmatrix} x(k) + \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} k \\ 1 \end{bmatrix}$$ str. 61

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

Določite $\phi(k)$ z metodo last. vred. in H. teorem!

$$g(\lambda) = |A-\lambda I| = \begin{vmatrix} -\lambda & 1 \\ -8 & 6-\lambda \end{vmatrix} = \lambda^2 - 6\lambda + 8 = (\lambda-2)(\lambda-4) = 0 \rightarrow \lambda_1 = 2, \lambda_2 = 4$$

$$2^k = \alpha_0 + 2\alpha_1$$
$$4^k = \alpha_0 + 4\alpha_1$$
$$\alpha_1 = -\tfrac{1}{2} 2^k + \tfrac{1}{2} 4^k$$
$$\alpha_2 = 2 \cdot 2^k - 4^k$$

$$A^k = \alpha_0 I + \alpha_1 A = (2 \cdot 2^k - 4^k)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (-\tfrac{1}{2} 2^k + \tfrac{1}{2} 4^k)\begin{bmatrix} 0 & 1 \\ -8 & 6 \end{bmatrix} =$$
$$= \begin{bmatrix} 2 \cdot 2^k - 4^k & -\tfrac{1}{2} \cdot 2^k + \tfrac{1}{2} 4^k \\ 4 \cdot 2^k - 4 \cdot 4^k & -2^k + 2 \cdot 4^k \end{bmatrix}$$

$$\left(\tfrac{1}{2} \cdot 2^k = 2^{k-1} \quad 4^k = 2^{2k} \quad -2^k = \alpha_0 + 2\alpha_1 \quad \vdots \right.$$
$$\left. 2 \cdot 2^k = 2^{k+1} \right)$$

- Dif. enačba, ki povezuje y z vzbujanjem? ?

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = -8x_1(k) + 6x_2(k) + k - 1 \longrightarrow x_1(k+2) = -8x_1(k) + 6x_1(k+1) + k - 1$$
$$y(k) = x_1(k) \qquad y(k+2) - 6y(k+1) + 8y(k) = k - 1$$

Sistem ni stabilen (λ sta pozit.)

- vsako leto proda 80% vseh novih vozil iz zaloge
vsako leto kupci vrnejo 12,5% vseh v prejšnjem letu prodanih novih vozil ⇒ rabljena postanejo
trgovec vsako leto proda 10% rabljenih vozil iz zaloge
vsako leto dobi 100 novih in uničil 10 rabljenih
modelirajte št. novih (x_1), rabljenih (x_2) na zalogi ter opišite sistem z diferenč. e.

$x_1(k)$ ← leto $\qquad x(k+1) = Ax(k) + Bu(k)$
$x_2(k)$

$$x_1(k+1) = (1-0.8)x_1(k) + 100\,u(k) = 0{,}2\,x_1(k) + 100\,u(k)$$
$$x_2(k+1) = 0{,}125 \cdot 0{,}8\,x_1(k) + (1-0{,}1)x_2(k) - 10\,u(k)$$

$$x(k+1) = \begin{bmatrix} 0{,}2 & 0 \\ 0{,}1 & 0{,}9 \end{bmatrix} x(k) + \begin{bmatrix} 100 \\ -10 \end{bmatrix} u(k) \qquad *$$

začetna stanja!

b) Določite enačbo, ki podaja št. novih v k-letu, če upošt., da ima letos na zalogi 100 novih in 20 rablj.:

$$x(0) = \begin{bmatrix} 100 \\ 20 \end{bmatrix} \qquad \text{(iščemo spl. rešitev)}$$

$$* \quad zX_1 - 100z = 0{,}2X_1 + \frac{100z}{z-1}$$

$y_n = 150$?

$$X_1 = \frac{100z}{z-0{,}2} + \frac{100z}{(z-1)(z-0{,}2)} = 100z\left(\frac{1}{z-0{,}2} + \frac{\frac{5}{4}}{z-1} + \frac{-\frac{5}{4}}{z-0{,}2}\right) =$$
$$= 100\left(-\frac{1}{4}\frac{z}{z-1/3} + \frac{5}{4}\frac{z}{z-1}\right)$$

$$x_1(k) = \cancel{100} - 25\left(\tfrac{1}{5}\right)^k + 125, \quad k \geq 0$$

Koliko mest mora imeti parkirišče, da bo vedno dovolj prostora?

$$\bar{x}_1 = 0{,}2\bar{x}_1 + 100 \longrightarrow \underline{\bar{x}_1 = 125}$$
$$\bar{x}_2 = 0{,}1\bar{x}_1 + 0{,}9\bar{x}_2 - 10$$
$$0{,}1\bar{x}_2 = 12{,}5 - 10 = 2{,}5 \longrightarrow \underline{\bar{x}_2 = 25}$$