Predmet:

Industrijski krmilni in regulacijski sistemi

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Vrsta gradiva:

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Študijsko leto:

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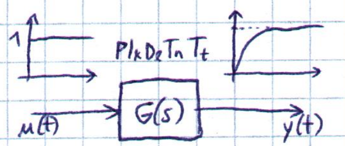


karakterizacija sistema

1. Zapis prenosne funkcije v faktorizirani obliki
2. Krajšanje polov in ničel
3. Multiplikacija števca
4. Deljenje števca s k-kratnim polom v k.i. (in imenovalca)
5. Ali ima števec konstantni člen? Če da $\Rightarrow P$
6. Poiščemo člen $\frac{c}{s^k}$ z najvišjo potenco k $\Rightarrow I_k$
7. Poiščemo člen cs^ℓ z najvišjo potenco ℓ $\Rightarrow D_\ell$
8. Označimo red imenovalca brez s^k z n $\Rightarrow T_n$
9. Ali ima sistem mrtvi čas? $\Rightarrow T_n$ (je člen e^{-sT_t}?)

Primer:

oblika dinamike

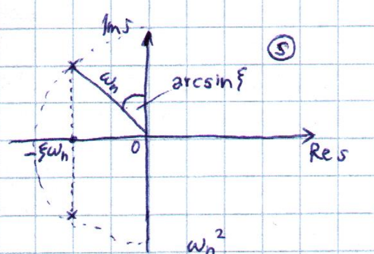
$$G(s) = \frac{s^3+3s^2+2s}{s^2(s+1)(s+3)(s+5)} = \frac{s(s+1)(s+2)}{s^2\cancel{(s+1)}(s+3)(s+5)} = \frac{1+\frac{2}{s}}{(s+3)(s+5)} \quad \boxed{PI_1T_2}$$

$$G(s) = \frac{3}{s+5} = PT_1$$

$$G(s) = \frac{2}{s^2+3s+2} = PT_2 = \frac{2}{(s+1)(s+2)} = \frac{1}{(s+1)(\frac{1}{2}s+1)} \qquad G(s) = \frac{1}{(T_1s+1)(T_2s+1)}$$

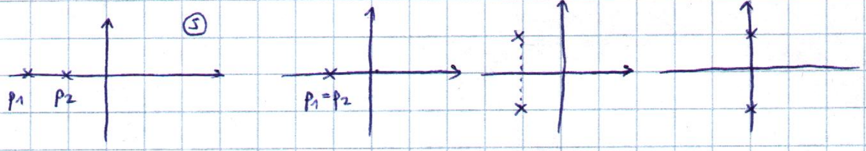
poli: $s_{1,2} = \{-1,-2\}$
ničle: $\{\ \}$
$T: \{1, \frac{1}{2}\}$
0-ta vrsta (ni polov v k.i.)
stabilen (leva stran)
$k = 1$

$$\boxed{K = \lim_{s\to 0} G(s)}$$



$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$T_i = \frac{1}{|Re(p_i)|}$$



$\xi > 1$ $\qquad \xi = 1 \qquad 0 < \xi < 1 \qquad \xi = 0$

$$G(s) = \frac{k}{(s+p_1)(s+p_2)}$$

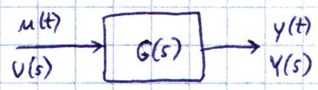
Primer: $G(s) = \frac{2s+6}{s^2+2s+2} = \frac{\overbrace{2s+6}^{2(s+3)}}{(s+1+j)(s+1-j)}$ $\quad PD_1T_2$

$p: \{-1+j, -1-j\}$
$n: \{-3\}$
$k = 3$
$\omega_n = \sqrt{2}$
$\xi = \frac{1}{\sqrt{2}}$
$T = \{1, 1\}$
stabilen

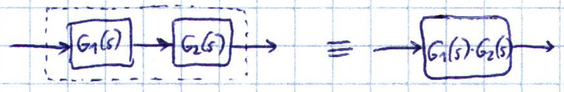
$G(s) = \frac{1}{(s+1)(s-3)}$ $\quad PT_2 \quad$ nestabilen

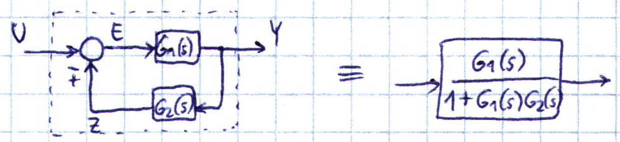
$p: \{-1, 3\}$
$n: \{\}$
$(k = -\frac{1}{3})$
$T = \{1, -\frac{1}{3}\}$

PREDSTAVITEV SISTEMA Z BLOČNIMI DIAGRAMI



$Y(s) = G(s) \cdot U(s)$

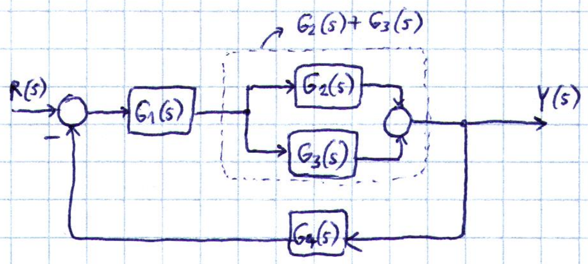




$Y = G_1(s) \cdot E = G_1(s) \cdot (U \mp Z) = G_1(s) \cdot (U \mp G_2(s) \cdot Y) =$

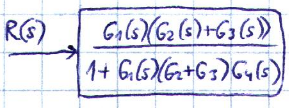
$Y(1 \pm G_1(s) G_2(s)) = G_1(s) \cdot U$

$$G(s) = \frac{Y}{U} = \frac{G_1(s)}{1 \pm G_1(s) G_2(s)}$$



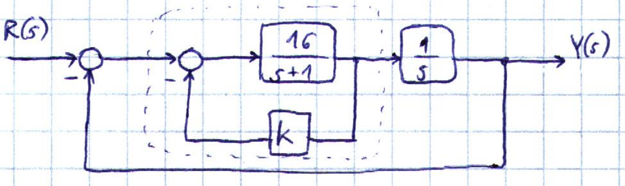
$G_1(s) = \frac{1}{s}$, $G_2(s) = \frac{1}{s+1}$, $G_3(s) = \frac{2}{s+2}$, $G_4(s) = \frac{1}{3s+4}$

$$G(s) = \frac{Y(s)}{R(s)} = ?$$



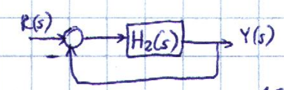
$$\frac{Y(s)}{R(s)} = G(s) = \frac{G_1(s)(G_2(s) + G_3(s))}{1 + G_1(s)(G_2(s) + G_3(s)) \cdot G_4(s)} = \frac{\frac{1}{s}\left(\frac{1}{s+1} + \frac{2}{s+2}\right)}{1 + \frac{1}{s}\left(\frac{1}{s+1} + \frac{2}{s+2}\right) \cdot \frac{1}{3s+4}} = \frac{\frac{1}{s}\left(\frac{s+2+2s+2}{(s+1)(s+2)}\right)}{1 + \frac{1}{s} \cdot \frac{\cancel{3s+4}}{(s+1)(s+2)} \cdot \frac{1}{\cancel{3s+4}}} =$$

$$= \frac{3s+4}{s(s+1)(s+2)+1} = \frac{3s+4}{s^3 + 3s^2 + 2s + 1} \qquad PD_1 T_3$$



$$H(s) = \frac{\frac{16}{s+1}}{1 + k \frac{16}{s+1}} = \frac{16}{s+1+16k}$$

$$H_2(s) = H_1(s) \cdot \frac{1}{s} = \frac{16}{s(s+1+16k)}$$



a) $\frac{Y(s)}{R(s)} = ?$

b) ojačenje, red, vrsto p.f., karakterizacija dinamike

c) Določite k tako, da bo ZS kritično dušen! Določite pole in ničle!

$$\frac{Y(s)}{R(s)} = \frac{H_2(s)}{1 + H_2(s)} = \frac{16}{s^2 + (1+16k)s + 16}$$

$k = 1$, $n = 2$, vrsta: 0, PT_2

$s^2 + (1+16k)s + 16 = (s+p)^2 = s^2 + 2ps + p^2$

$p^2 = 16$; $p_{1,2} = \pm 4$

$p = +4$

$1 + 16k = 8$

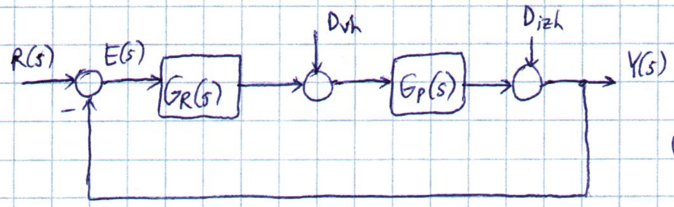
$k = \frac{7}{16}$

poli: $\{-4, 4\}$

ničle: $\{\}$

$\omega_n = 4$

$\zeta = 1$



$\frac{Y(s)}{R(s)} = ?$ $\frac{Y(s)}{D_{vh}(s)} = ?$ $\frac{Y(s)}{D_{izh}(s)} = ?$

$G_P(s) = \frac{4}{s^2+4s+2}$, $G_R(s)$ je PI regulator s $k_P = 0{,}75$ in $T_i = 1{,}5$

$$G_R(s) = k_P + k_i \frac{1}{s} = k_P\left(1 + \frac{1}{T_i s}\right) =$$
$$= 0{,}75\left(1 + \frac{1}{1{,}5s}\right) = \frac{3}{4}\left(1 + \frac{2}{3s}\right) = \frac{3s+2}{4s}$$

$$\frac{Y(s)}{R(s)} = \frac{G_R(s)\,G_P(s)}{1 + G_R(s)\,G_P(s)}$$

$$\frac{Y(s)}{D_{vh}(s)} = \frac{G_P(s)}{1 - G_P(s)\cdot(-G_R(s))} = \frac{G_P(s)}{1 + G_R(s)\cdot G_P(s)}$$

$$\frac{Y(s)}{D_{izh}(s)} = \frac{1}{1 + G_R(s)\,G_P(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{3s+2}{4s}\,\frac{4}{s^2+4s+2}}{1 + \frac{3s+2}{4s}\,\frac{4}{s^2+4s+2}} = \frac{3s+2}{s^3+4s^2+5s+2} = \frac{3s+2}{(s+1)^2(s+2)} \qquad PD_1T_3$$

$$R(s) = \frac{1}{s} \qquad Y(s) = G(s)\cdot R(s) = \frac{3s+2}{(s+1)^2(s+2)}\cdot\frac{1}{s} = \frac{A}{s+2} + \frac{B}{s} + \frac{C}{s+1} + \frac{D}{(s+1)^2} \quad *$$

$$A = \lim_{s\to -2} \frac{3s+2}{(s+1)^2\cdot s} = 2$$

$$B = \lim_{s\to 0} \frac{3s+2}{(s+1)^2(s+2)} = 1$$

$$D = \lim_{s\to -1} \frac{3s+2}{(s+2)\cdot s} = 1$$

$$C = \lim_{s\to -1} \frac{\partial}{\partial s}\left[\frac{3s+2}{(s+2)s}\right] = \lim_{s\to -1} \frac{3(s+2)s - (3s+2)(2s+2)}{(s+2)^2 s^2} = -3$$

$$f(s) = \frac{E_1}{s+s_1} + \dots + \frac{E_i}{s+s_i} + \dots + \frac{E_n}{s+s_n}$$

$$E_i = \lim_{s\to s_1} f(s)(s+s_i)$$

$$f(s) = \frac{F_1}{s+s_1} + \frac{F_2}{(s+s_1)^2} + \dots + \frac{F_i}{(s+s_1)^i} + \dots \frac{F_n}{(s+s_1)^n}$$

$$F_i = \frac{1}{(n-i)!} \lim_{s\to s_1} \frac{\partial^{n-i}}{\partial s^{n-i}}\left[f(s)\cdot(s+s_1)^n\right]$$

$$* \; Y(s) = \frac{2}{s+2} + \frac{1}{s} - 3\frac{1}{s+1} + \frac{1}{(s+1)^2}$$

$$y(t) = 2e^{-2t} + 1 - 3e^{-t} + te^{-t}\,; \quad t \geq 0$$

RUTHOV-HURWITZOV STABILNOSTNI KRITERIJ

$$P(s) = p_n s^n + p_{n-1}s^{n-1} + \dots + p_1 s + p_0$$

brez da razstavimo, bomo videli, ali je sistem stabilen

s^n	p_n	p_{n-2}	p_{n-4}
s^{n-1}	p_{n-1}	p_{n-3}	p_{n-5}
s^{n-2}	b_1	b_2	b_3
s^{n-3}	c_1	c_2	c_3
$\vdots$			
s^2			
s^1			
1			

$$b_1 = -\frac{1}{p_{n-1}}\begin{vmatrix} p_n & p_{n-2} \\ p_{n-1} & p_{n-3}\end{vmatrix}$$

$$b_2 = -\frac{1}{p_{n-1}}\begin{vmatrix} p_n & p_{n-4} \\ p_{n-1} & p_{n-5}\end{vmatrix}$$

$$c_1 = -\frac{1}{b_1}\begin{vmatrix} p_{n-1} & p_{n-3} \\ b_1 & b_2\end{vmatrix}$$

$$c_2 = -\frac{1}{b_1}\begin{vmatrix} p_{n-1} & p_{n-3} \\ b_1 & b_3\end{vmatrix}$$

$$s^3 + 4s^2 + 100s + 500 = 0$$

kot da bi iskali ničle

s^3	1	100
s^2	4	500
s	$-\frac{1}{4}\cdot 100$	0
1	500	

sistem je nestabilen

2 spr. predznaka → 2 pola

Akta

$s^3 + s^2 - s - 1 = 0$

$(s+1)(s^2-1) = 0$

$(s+1)^2(s-1) = 0$

s^3	1	-1
s^2	1	-1
s	ε	0
1	-1	

ker je 0, napišemo ε (v 1. stolpcu)

NESTABILEN

1 prehod skozi ničlo

$s^3 + s^2 + s + 1 = 0$

$(s+1)(s^2+1) = 0$

$(s+1)(s+j)(s-j) = 0$

s^3	1	1
s^2	1	1
s	ε	
1	1	

$\lim \varepsilon \to 0$ odvisno, če je $\varepsilon > 0$ ali $\varepsilon < 0$

MEJNO STABILEN

2 dotika

$s^4 + s^3 + s - 1 = 0$

s^4	1	0	-1
s^3	1	1	0
s^2	-1	-1	
s	ε	0	
1	-1		

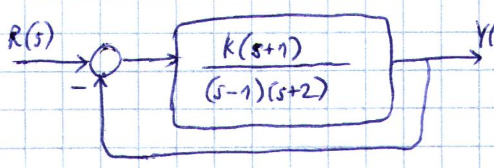
NESTABILEN

1 prehod, 2 dotika

$(s+j)(s-j)(s+\frac{1+\sqrt{3}}{2})(s+\frac{1-\sqrt{3}}{2})$

Naloge

Določi k, da bo sistem stabilen?



$$\frac{Y(s)}{R(s)} = \frac{\frac{k(s+1)}{(s-1)(s+2)}}{1 + \frac{k(s+1)}{(s-1)(s+2)}} = \frac{k(s+1)}{(s-1)(s+2) + k(s+1)} = $$

$$= \frac{k(s+1)}{s^2 + (1+k)s - 2 + k}$$

s^2	1	$k-2$
s	$1+k$	
1	$k-2$	

$1+k > 0 \qquad k > -1$

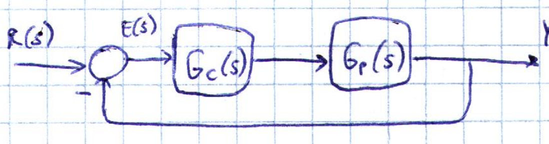
$k - 2 > 0 \qquad \boxed{k > 2}$

e_{ss} minimalen — stacionarni pogr.

Določite $G_c(s) = k$, ki bo zagotavljal min. stac. pogrešek

$G_P(s) = \frac{2}{s^3+4s^2+5s+2}$

$G_c(s) = k$



Še en pogoj: sistem stabilen

$e_{ss} = \lim_{t \to \infty} e(t)$

$$\frac{Y(s)}{R(s)} = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) G_p(s)} = \frac{k \cdot \frac{2}{s^3+4s^2+5s+2}}{1 + \frac{2k}{s^3+4s^2+5s+2}} = \frac{2k}{s^3+4s^2+5s+2+2k}$$

s^3	1	5
s^2	4	$2+2k$
s	b	
1	$2+2k$	

$b = -\frac{1}{4}(2+2k-20) = \frac{18-2k}{4}$

$2+2k > 0 \qquad \frac{18-2k}{4} > 0$

$k > -1 \qquad k < 9$

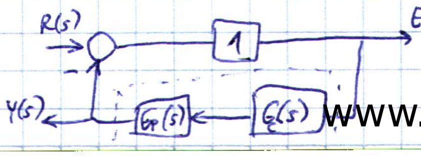
stabilen za $-1 < k < 9$

$$G_E(s) = \frac{E(s)}{R(s)} = \frac{1}{1 + G_c(s) \cdot G_P(s)}$$

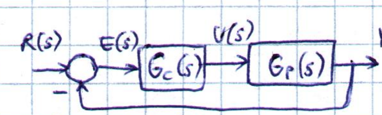
$$E(s) = G_E(s) \cdot R(s) = \frac{1}{1 + G_c(s) \cdot G_P(s)} \cdot \frac{1}{s}$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s E(s) = \lim_{s \to 0} \frac{1}{1 + G_c(s) G_P(s)} = \lim_{s \to 0} \frac{1}{1 + \frac{2k}{s^3+4s^2+5s+2}} =$$

$$= \lim_{s \to 0} \frac{s^3+4s^2+5s+2}{s^3+4s^2+5s+2+2k} = \frac{1}{1+k}$$



DN $G_P(s) = \frac{2}{s^3+4s^2+5s+2}$



$G_{ZZ}(s) = \frac{G_P(s)\,G_c(s)}{1+G_P(s)\,G_c(s)} = *$

$G_c(s) = k_P + \frac{k_i}{s} = \frac{K_P s + K_i}{s}$

$* \ \frac{\frac{2}{s^3+4s^2+5s+2}\cdot\frac{K_P s+K_i}{s}}{1+\frac{2}{s^3+4s^2+5s+2}\cdot\frac{K_P s+K_i}{s}} = \frac{2(K_P s+K_i)}{s^4+4s^3+5s^2+(2+2K_P)s+2K_i} \rightarrow$ tabela

ZZ stabilen

$K_P > 9$

$K_i < 0$

+ neenačba

s^4	1	5	$2K_i$
s^3	4	$2+2K_P$	
s^2	b		
s^1	c		
1	$2K_i$		

$b = -\frac{1}{4}(2+2K_P-20) = \frac{9-K_P}{2}$

$c = -\frac{2}{9-K_P}\left(8K_i - 2(1+K_P)\frac{9-K_P}{2}\right) = 2(1+K_P) - \frac{16K_i}{9-K_P}$

pogoji: vsi v 1. stolpcu pozitivni!

$2K_i > 0$

$\boxed{K_i > 0}$

$9 - K_P > 0$

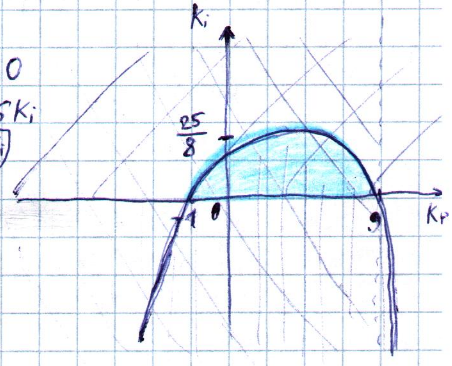
$\boxed{K_P < 9}$

$2(1+K_P) - \frac{16K_i}{9-K_P} > 0$

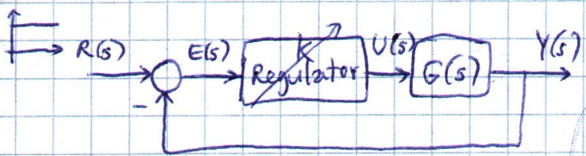
$2(1+K_P)(9-K_P) > 16K_i$

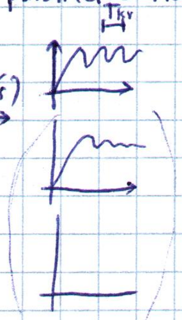
$\boxed{(1+K_P)(9-K_P) > 8K_i}$

rešitev je presek



Z metodo nihajnega preizkusa določite parametre PID regulatorja!





$G(s) = \frac{1}{(s+1)(s+3)(s+4)}$

$k_{kr},\ T_{kr}$

kritično

P

PI

PID

$\frac{Y(s)}{R(s)} = \frac{k\cdot G(s)}{1+k\cdot G(s)} = \frac{\frac{k}{(s+1)(s+3)(s+4)}}{1+\frac{k}{(s+1)(s+3)(s+4)}} = \frac{k}{(s^2+4s+3)(s+4)+k} = \frac{k}{s^3+8s^2+19s+12+k}$

s^3	1	19
s^2	8	$12+k$
s	b	
1	$12+k$	

$b = -\frac{1}{8}(12+k-8\cdot 19) = 19 - \frac{12+k}{8}$

$12+k=0$

$k=-12$ //

$19 - \frac{12+k}{8} = 0$

$12+k = 19\cdot 8$

$k_{kr} = 19\cdot 8 - 12 = \boxed{140}$ ✓

$8s^2 + 12 + k = 0$

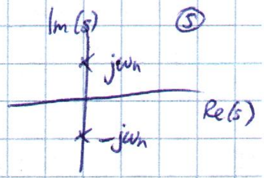
$s^2 + \frac{12+k_{kr}}{8} = 0$

$s^2 + \frac{152}{8} = 0$

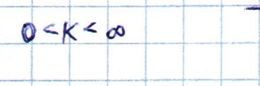
$s^2 + 19 = 0$

$s_{1,2} = \pm j\sqrt{19} = \pm j\omega_n$

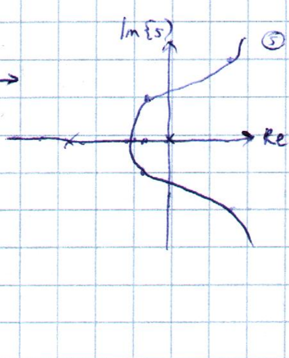
$\omega_n = \sqrt{19}$ *



* $\omega_n = 2\pi\frac{1}{T_{kr}} \Leftrightarrow T_{kr} = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{19}} = \boxed{1{,}44}$



$0 < K < \infty$



LAB.VAJA

③

$\boxed{1.}$ $\dfrac{\frac{k\cdot 12}{(s+1)(s+2)(s+3)}}{1+\frac{12k}{(s+1)(s+2)(s+3)}} = \dfrac{12k}{(s+1)(s+2)(s+3)+12k} =$

$= \dfrac{12k}{(s^2+3s+2)(s+3)+12k} = \dfrac{12k}{s^3+3s^2+2s+3s^2+9s+6+12k} =$

$= \dfrac{12k}{s^3+6s^2+11s+6+12k}$

s^3	1	11
s^2	6	$6+12k$
s	b	
1	$6+12k$	

$6+12k>0$

$k>-\frac{1}{2}$

$b=-\frac{1}{6}(6+12k-66)$

$=-1+11-2k>0$

$10>2k$

$k<5$

$-\frac{1}{2}<k<5$

$\boxed{4.}$ $G(s)=\dfrac{2(5-s)}{(s+1)(s-1)}$

$\dfrac{\frac{2k(5-s)}{(s+1)(s-1)}}{1+\frac{2k(5-s)}{(s+1)(s-1)}} = \dfrac{2k(5-s)}{(s+1)(s-1)+2k(5-s)} = \dfrac{2k(5-s)}{s^2-s-1+10k-2ks} = \dfrac{2k(5-s)}{s^2+s(-1-2k)+10k-1}$

s^2	1	$10k-1$
s	$-1-2k$	0
1	b	

$b=\dfrac{-1}{-1-2k}\cdot\overbrace{-(10k-1)(-1-2k)}^{(10k-1)(1+2k)}=$

$=\dfrac{1}{1+2k}(10k+20k^2-1-2k) > 0$

$\dfrac{1}{1+2k}(20k^2+8k-1) > 0$

$-1-2k>0$

$1+2k<0$

$2k<-1$

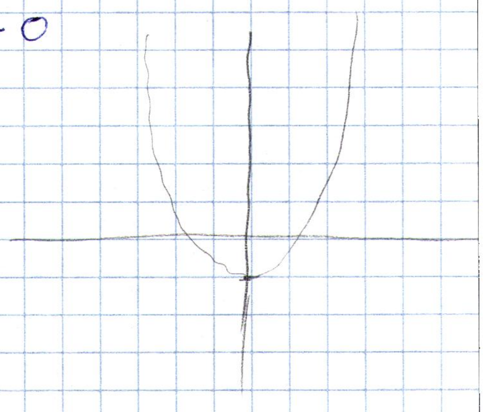
$k<-\frac{1}{2}$

?

$k<0 \wedge k>\frac{1}{10}$

ni možno

$k=\{\}$



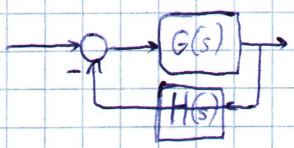
$\boxed{3.}$ $k>\frac{3}{2}$, $-\frac{1}{5}<k$

$\boxed{2.}$ $\frac{1}{2}<k<\frac{5}{6}$

④ rlocus
rlocfind

nekje postane sistem nestabilen

PRAVILA ZA RISANJE DLK



$$1 + G(s)H(s) = 1 + k \cdot \frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)} = 0$$

① $\lim\limits_{k\to 0} \left| \frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)} \right| = \lim\limits_{k\to 0} \frac{1}{k} = \infty$ ✕→   pole označujemo z X (križci)

$\lim\limits_{k\to \infty} \left| \frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_n)} \right| = \lim\limits_{k\to \infty} \frac{1}{k} = 0$ O←   ničle s krogci O←

② DLK na realni osi

Točka pripada DLK, če je desno od nje liho število realnih polov & ničel.

③ $n-m$ je število asimptot

$$B_K = \frac{\pm 180°(2K+1)}{n-m} \quad ; \quad k = 0, 1, \ldots (n-m-1)$$

$$\sigma_x = -\frac{\sum\limits_{k=1}^{n} p_i - \sum\limits_{i=1}^{n} z_i}{n-m}$$

④ razcepišča

$$\left.\frac{dK}{ds}\right|_{s=\sigma_n} = 0 \Rightarrow \sigma_b$$

⑤ $\theta_{out} = 180° + \sum\limits_i \phi_i - \sum\limits_i \theta_i$

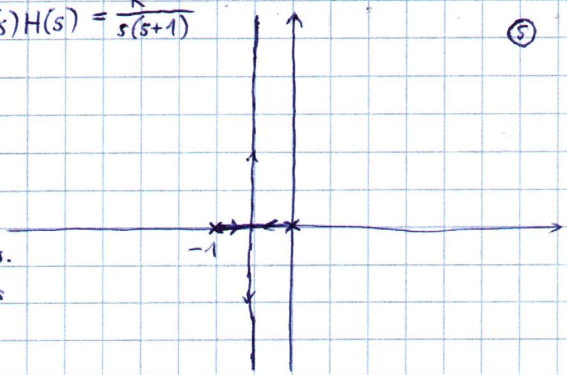
izstopni koti

koti od odprtozančnih ničel

koti od odprtozančnih polov

vstopni koti $\phi_i = 180° - \sum\limits_i \phi_i + \sum\limits_i \theta_i$

$G(s)H(s) = \frac{K}{s(s+1)}$



DLK vedno simetr. glede na absc. os

⑤

$B_0 = 90°$

$B_1 = 270°$

$\sigma_x = -\frac{0+1}{2} = -\frac{1}{2}$

$1 + G(s)H(s) = 0$

$1 + \frac{K}{s(s+1)} = 0$

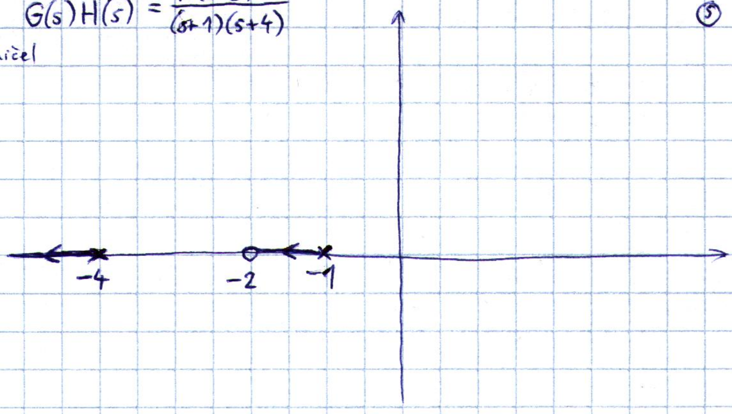
$K = -s(s+1)$ $\left.\frac{dK}{ds}\right|_{s=\sigma_b} = \left.(-2s-1)\right|_{s=\sigma_b} = 0$

$2\sigma_b + 1 = 0$

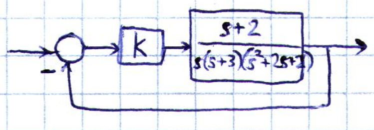
$\sigma_b = -\frac{1}{2}$

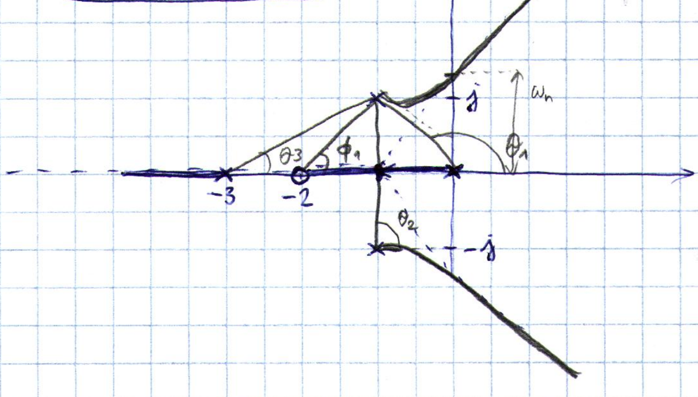
$G(s)H(s) = \frac{k(s+2)}{(s+1)(s+4)}$ ⑤

liho št. ničel



$$G(s)H(s) = k\frac{s+2}{s(s+3)(s^2+2s+2)} = \frac{k(s+2)}{s(s+3)(s+1+j)(s+1-j)}$$





odprtozančni poli

3 asimptote

$\beta_0 = 60^\circ$

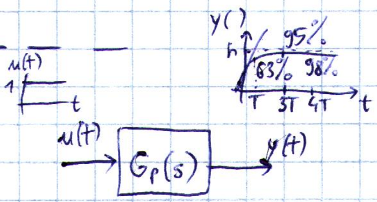
$\beta_1 = 180^\circ$

$\beta_2 = 60 \cdot 5 = 300^\circ$

$$\sigma_x = -\frac{0+3+1+j+1-j-2}{3} = -1$$

$$\Theta_{out} = 180^\circ + 45^\circ - 135^\circ - 90^\circ - 27^\circ = -27^\circ$$

na desni polravnini sistem postane nestabilen

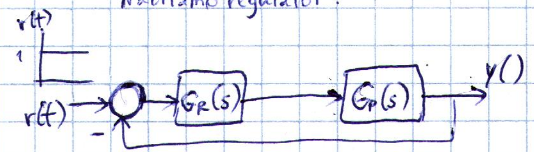


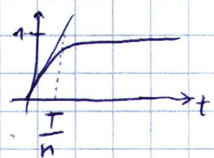
T - čas. konst. sistema

n - pohitritev

$$G_P(s) = \frac{K}{Ts+1}$$

Načrtamo regulator!





$$\frac{Y(s)}{R(s)} = \frac{1}{\frac{T}{n}\cdot s + 1} = G_{zz}(s)$$ zaprtozančna prenosna f.

$$G_{zz}(s) = \frac{G_R(s)\cdot G_P(s)}{1+G_R(s)\,G_P(s)}$$

$$(1+G_R(s)\,G_P(s))\,G_{zz}(s) = G_R(s)\cdot G_P(s)$$

$$G_R(s)\left(G_P(s)\cdot G_{zz}(s) - G_P(s)\right) = -G_{zz}(s)$$

$$G_R(s) = \frac{1}{G_P(s)}\cdot\frac{+G_{zz}(s)}{-G_{zz}(s)+1} = \frac{Ts+1}{k}\cdot\frac{\frac{1}{\frac{T}{n}s+1}}{1-\frac{1}{\frac{T}{n}s+1}} = \frac{Ts+1}{k(\frac{T}{n}s+\cancel{1-1})} = \frac{Ts+1}{k\frac{T}{n}s} = \underbrace{\frac{n}{k}}_{P} + \underbrace{\frac{n}{kT}\frac{1}{s}}_{=G_R^I(s)} =$$

$\boxed{PI}$

DN $\quad G_P(s) = \frac{k}{(Ts+1)s}$

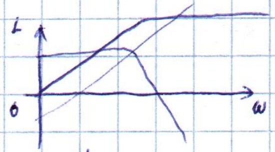
$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$

$$G(s)\Big|_{s \to j\omega} = G(j\omega) = \underbrace{|G(j\omega)|}_{\text{amplitudni potek}} e^{j \underbrace{\measuredangle G(j\omega)}_{\text{fazni potek}}}$$

$$G(s) = K \frac{\left(1+\frac{s}{z_1}\right) \cdots \left(1+\frac{s}{z_m}\right) s^d}{\left(1+\frac{s}{p_1}\right) \cdots \left(1+\frac{s}{p_n}\right) s^i \left(1+\frac{2\xi}{\omega_n} s + \left(\frac{s}{\omega_n}\right)^2\right)}$$

$$L[dB] = 20 \log \frac{|G(j\omega)|}{1} = 20 \log|K| + \sum_{i=1}^{n} 20 \log\left|1+\frac{j\omega}{z_i}\right| - \left(\sum_{i=1}^{n} 20 \log\left|1+\frac{j\omega}{p_i}\right|\right) + d\, 20 \log|j\omega| - i\, 20 \log|j\omega| +$$
$$+ 20 \log\left|1+\frac{2\xi'}{\omega_n'} j\omega + \left(\frac{j\omega}{\omega_n'}\right)^2\right| - 20 \log\left|1+\frac{2\xi}{\omega_n} j\omega + \left(\frac{j\omega}{\omega_n}\right)^2\right|$$

ampl potek

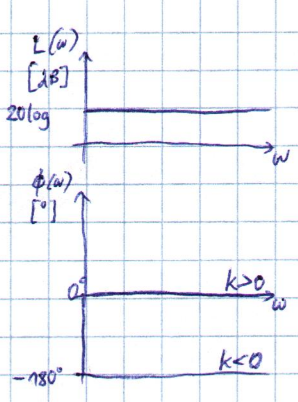


$$\phi(\omega) = \measuredangle K + \sum_{i=1}^{n} \measuredangle\left\{1+\frac{j\omega}{z_i}\right\} - \sum_{i=1}^{n} \measuredangle\left\{1+\frac{j\omega}{p_i}\right\} + \measuredangle\{(j\omega)^d\} - \measuredangle\{(j\omega)^i\} + \measuredangle\left\{1+\frac{2\xi'}{\omega_n'} j\omega + \left(\frac{j\omega}{\omega_n'}\right)^2\right\} -$$
$$- \measuredangle\left\{1+\frac{2\xi}{\omega_n} j\omega + \left(\frac{j\omega}{\omega_n}\right)^2\right\}$$

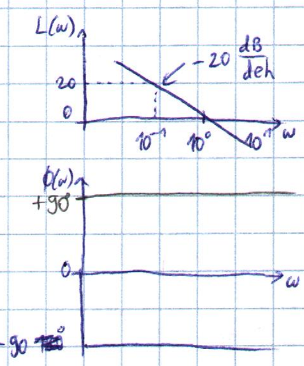
① Ojačenje K

$$L(\omega)\,[dB] = 20 \log|K|$$

$$\phi(\omega) = \begin{cases} 0^\circ & ; k>0 \\ \pm 180^\circ & ; k<0 \end{cases}$$



② Integrator



$$G(j\omega) = \frac{1}{j\omega}$$

$$L(\omega) = 20 \cdot \log\left|\frac{1}{j\omega}\right| = 20 \cdot \log\frac{1}{\omega} = -20 \log \omega$$

$$\phi(\omega) = \measuredangle\left\{\frac{1}{j\omega}\right\} = \arctan\frac{-\omega}{0} = -90^\circ$$

③ Diferenciator

$$G(j\omega) = j\omega$$
$$L(\omega) = 20 \cdot \log \omega$$
$$\phi(\omega) = 90^\circ$$

člen 1. reda

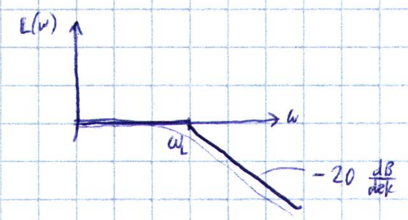
$$G(j\omega) = \frac{1}{1+\frac{j\omega}{\omega_L}}$$

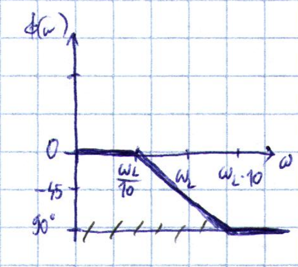
$$L(\omega) = 20\log|G(j\omega)| = 20\log\left|\frac{1}{1+\frac{j\omega}{\omega_L}}\right| = -20\log\left|1+\frac{j\omega}{\omega_L}\right| = -20\log\sqrt{1+\left(\frac{\omega}{\omega_L}\right)^2}$$

$$L(\omega) = -20\cdot\log\sqrt{1+\left(\frac{\omega}{\omega_L}\right)^2}$$

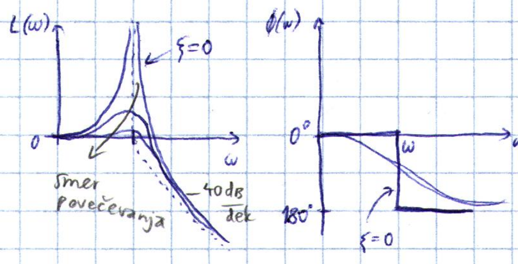
$$\phi(\omega) = -\arctan\frac{\omega}{\omega_L}$$

$$\phi(\omega) = \measuredangle\{G(j\omega)\} = \measuredangle\left\{\frac{1}{1+\frac{j\omega}{\omega_L}}\right\} = -\measuredangle\left\{1+\frac{j\omega}{\omega_L}\right\} = -\arctan\frac{\frac{\omega}{\omega_L}}{1} = -\arctan\frac{\omega}{\omega_L}$$

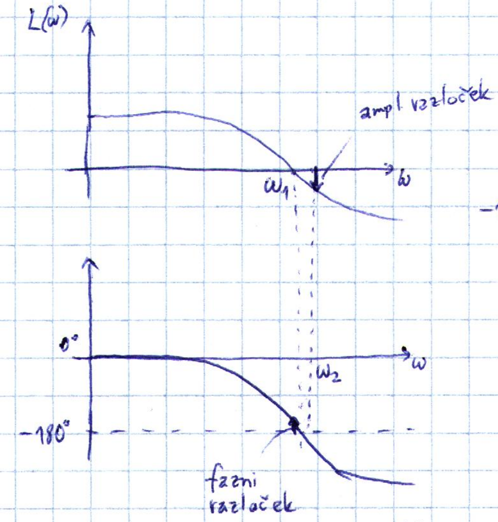


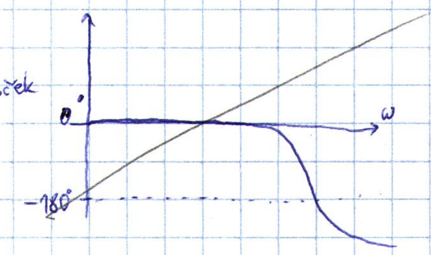


$$G(j\omega) = \frac{1}{1+\frac{2\xi}{\omega_n}j\omega+\left(\frac{j\omega}{\omega_n}\right)^2}$$



fazni in amplitudni razloček





za stabilnost morata bit pozitivna

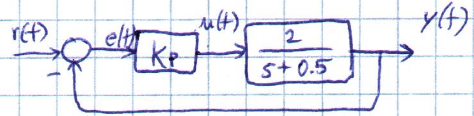
NASTAVLJANJE PID REGULATORJEV

tabelce na echo

$$\boxed{G_{PID}(s) = K_P\left(1 + \frac{1}{T_i s} + \frac{T_D \cdot s}{T_f \cdot s + 1}\right)}$$

- $G(s) = \dfrac{2}{s+0.5}$ proces 1. reda $= \dfrac{4}{2s+1}$ (4 $= K$, 2 $= T$)

  $G(s) = \dfrac{K}{Ts+1}$



$$G_{zz}(s) = \frac{K_P \cdot \frac{2}{s+0.5}}{1 + K_P \cdot \frac{2}{s+0.5}} = \frac{2K_P}{s + 0.5 + 2K_P} = \frac{\frac{2K_P}{0.5+2K_P}}{\frac{1}{0.5+2K_P}\, s + 1}$$

K_{zz} ojač. — to člen gre proti 1

T_{zz} zaprtozančna čas. konst.

hitr. sistema se povečuje

pogrešek v ustaljenem stanju ?

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \frac{1}{1 + K_P \cdot \frac{2}{s+0.5}} = \frac{1}{1 + 4K_P}$$

$$E(s) = G_E(s) \cdot R(s) = \frac{1}{1 + K_P \cdot \frac{2}{s+0.5}} \cdot \frac{1}{s}$$

kako pogrešek izničimo ?

- $G_{PI}(s) = K_P + \dfrac{K_i}{s} = K_P\left(1 + \dfrac{1}{T_i s}\right)$

  $K_i = \dfrac{K_P}{T_i}$

$$G_{zz}(s) = \frac{\frac{K_P \cdot s + K_i}{s} \cdot \frac{2}{s+0.5}}{1 + \frac{K_P s + K_i}{s} \cdot \frac{2}{s+0.5}} = \frac{2(K_P \cdot s + K_i)}{s^2 + \underbrace{(0.5 + K_P)}_{2\xi\omega_n} \cdot s + \underbrace{2K_i}_{\omega_n^2}}$$

ω_n lastna frekv.

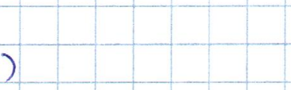
$$\omega_n = +\sqrt{2K_i}$$

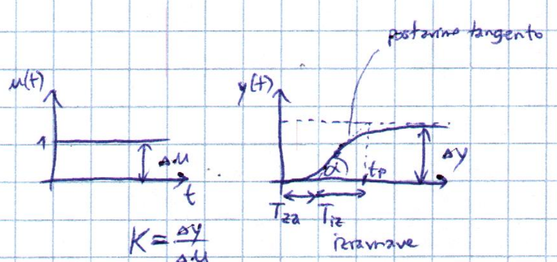
$$\xi = \frac{0.5 + K_P}{2\sqrt{2K_i}}$$

pogr.: $\cancel{E(s)}\ e_{ss} = \lim_{s\to 0} \dfrac{1}{1 + \frac{K_P \cdot s + K_i}{s} \cdot \frac{2}{s+0.5}} = 0$

- uporaba 1. Zi-Nich. metode (tabela 1)

$$G(s) = \frac{1}{(s+1)(s+2)}$$





$K = \dfrac{\Delta y}{\Delta u}$

izravnave

$$U(s) = \frac{1}{s}$$

$$Y(s) = G(s) \cdot U(s) = \frac{1}{s(s+1)(s+2)} \overset{\text{parc.}}{=} \ldots = \frac{\frac{1}{2}}{s} - \frac{1}{s+1} + \frac{\frac{1}{2}}{s+2}$$

inverzni Laplace

$$y(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}$$

$$K = \frac{\lim_{t\to\infty} y(t)}{1} = \frac{\lim_{s\to 0} s \cdot Y(s)}{1} = \underline{\underline{\frac{1}{2}}}$$

$$\dot{y}(t) = e^{-t} - e^{-2t}$$

$$\ddot{y}(t) = -e^{-t} + 2e^{-2t}$$

$$\ddot{y}(t_P) = 0 = -e^{-t_P} + 2e^{-2t_P} = e^{-t_P}(-1 + 2e^{-t_P}) = 0$$

$$-1 + 2e^{-t_P} = 0$$

$$e^{-t_P} = \frac{1}{2}$$

$$t_P = -\log_e \frac{1}{2} = \log_e 2 = 0.69$$

$$\tan\alpha = \frac{\Delta y}{T_{iz}}$$

$$T_{iz} = \frac{\Delta y}{\tan\alpha} = \frac{\frac{1}{2}}{\frac{1}{4}} = \underline{\underline{2}}$$

$$\tan\alpha = \dot{y}(t_P) = e^{-t_P} - e^{-2t_P} = e^{-t_P} - (e^{-t_P})^2 = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

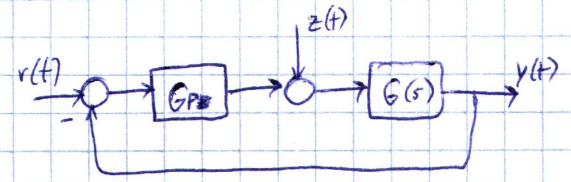
$$\tan\alpha = \frac{y(t_p)}{t_p - T_{za}} \Rightarrow T_{za} = \frac{-y_{t_p}}{\tan\alpha} + t_p = -\frac{\frac{1}{8}}{\frac{1}{4}} + 0.69 = \underline{\underline{0.19}}$$

$$y(t_p) = \frac{1}{2} - e^{-t_p} + \frac{1}{2}\left(e^{-t_p}\right)^2 = \frac{1}{2} - \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

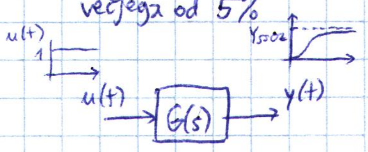
PI – reg.

$$K_p = 0.9 \cdot \frac{T_{iz}}{K_s \cdot T_{za}} = 18.33$$

$$T_i = 3.3 \cdot T_{za} = 0.637$$



Določi ojač. P-regulatorja na način, da pri stopničasti motnji sistem ne bo mel pogr. v ust. stanju večjega od 5%



odprta zanka

$$Y_{ssoz} \cdot 0.05 \geq |e_{ss}|$$

$$G_z(s) = \frac{Y(s)}{Z(s)} = \frac{G(s)}{1 + G_p \cdot G(s)} = \frac{\frac{1}{(s+1)(s+2)}}{1 + K_p \cdot \frac{1}{(s+1)(s+2)}} = \frac{1}{(s+1)(s+2) + K_p}$$

$$Y_{ssoz} = \lim_{s\to 0} s \cdot Y(s) = \lim_{s\to 0} s \cdot G(s) \frac{1}{s} = \underline{\underline{\frac{1}{2}}}$$

$$E(s) = R(s) - Y(s) = -G_z(s) \cdot Z(s) = -G_z(s) \cdot \frac{1}{s} = -\frac{1}{((s+1)(s+2)+K_p) \cdot s}$$

$$e_{ss} = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} \frac{-1}{(s+1)(s+2)+K_p} = -\frac{1}{2+K_p}$$

$$Y_{ssoz} \cdot 0{,}05 \geq \frac{1}{2+K_p}$$

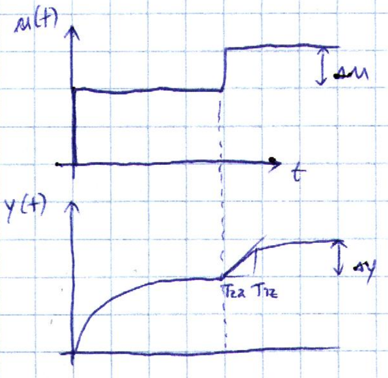
$$\underset{\frac{1}{2}}{\overset{\|}{}} \quad 0{,}025 \geq \frac{1}{2+K_p}$$

$$\boxed{K_p \geq \frac{1}{0.025} - 2 = 38}$$

~~Zegel-Ni., P-reg.~~

tabela 2

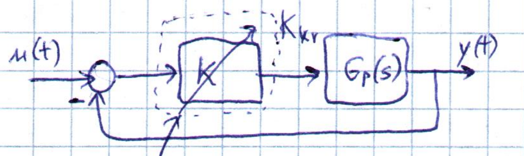
$$G(s) = \frac{1}{(s+1)(5s+1)(10s+1)}$$



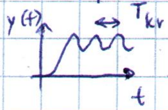
2. Z-N metoda – metoda z nihajnim preizkusom

$$G(s) = \frac{1}{s(s^2+4s+5)}$$

kritična perioda nihanja



zamenjamo z regulatorjem



meja nestabilnosti

Določi kr. ojač. in kritično periodo!

$$G_{zz}(s) = \frac{K \cdot \frac{1}{s(s^2+4s+5)}}{1+K\frac{1}{s(s^2+4s+5)}} = \frac{k}{s^3+4s^2+5s+k}$$

s^3	1	5
s^2	4	k
s	$-\frac{1}{4}(k-20)$	
1	k	

$k > 0$

$-\frac{1}{4}(k-20) > 0$

$k-20 < 0$

$k < 20$

$\underline{\underline{K_{kr} = 20}}$

$$4s^2 + k = 0$$

$$s^2 = -\frac{K_{kr}}{4}$$

$$s = \pm j\frac{\sqrt{K_{kr}}}{2} = \pm j\omega_n$$

$$\omega_n = \frac{\sqrt{K_{kr}}}{2}$$

$$2\pi\frac{1}{T_{kr}} = \omega_n$$

$$T_{kr} = \frac{2\pi}{\omega_n}$$

$$T_{kr} = \frac{4\pi}{\sqrt{K_{kr}}} = \frac{2\pi}{\sqrt{5}} = 2{,}8$$

$K_P = 0{,}6\,K_{kr} = 12$

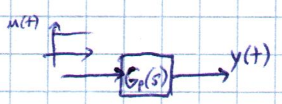
$T_i = 0.5 \cdot T_{kr} = 1{,}4$

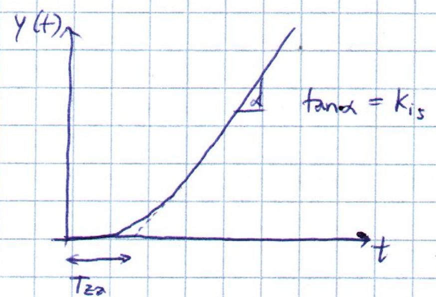
$T_D = 0{,}125 \cdot T_{kr} = 0.35$

$$G_{PID}(s) = K_P\left(1 + \frac{1}{T_i s} + \frac{T_D \cdot s}{T_k s + 1}\right)$$

$T_k = 0{,}1 \cdot T_D = 0{,}035$

$$G_P(s) = \frac{1}{s(s+1)(s+5)}$$





$\tan\alpha = K_{is}$

$$Y(s) = \frac{1}{s^2(s+1)(s+5)} = \frac{0.25}{s+1} + \frac{-0.01}{s+5} + \frac{-0.24}{s} + \frac{0.2}{s^2}$$

$$y(t) = 0.25\,e^{-t} - 0.01\,e^{-5t} - 0.24 + 0.2\,t$$

$y_{ss}(t) = -0.24 + 0.2\,t$

$k_{is} = 0.2$

$$y_{ss}(t)\Big|_{t=T_{za}} = 0 = -0.24 + 0.2\,T_{za}$$

$T_{za} = 1{,}2$

$K_P = \frac{0.4}{k_{is} \cdot T_{za}} = 1{,}667$

$T_i = 3.2 \cdot T_{za}$ 3,84

$T_D = 0.8 \cdot T_{za}$ 0.96

$$G_P(s) = \frac{1}{(s+1)^2} \qquad G_R(s) = \frac{K_D s^2 + K_P s + K_i}{s}$$

$\omega_n = 10\ \frac{rad}{s}$

$\boxed{PID}$

$k = \omega_n$

višji redi od 2:

za 1. red = $k = \omega_L$

$$G_{zz}(s) = \frac{G_P(s)\cdot G_R(s)}{1+G_P(s)\cdot G_R(s)} = \frac{\frac{1}{(s+1)^2}\cdot\frac{K_D s^2 + K_P s + K_i}{s}}{1+\frac{1}{(s+1)^2}\cdot\frac{K_D s^2 + K_P \cdot s + K_i}{s}} =$$

$$= \frac{K_D s^2 + K_P s + K_i}{s^3 + (2+K_D)s^2 + (1+K_P)s + K_i}$$

$$s^3 (2+K_D)s^2 + (1+K_P)s + K_i = s^3 + 1.75\,ks^2 + 2.15\,k^2 s + k^3 =$$

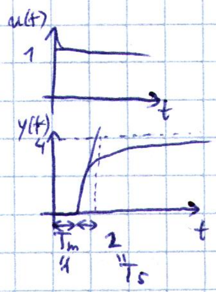
$$= s^3 + 17.5\,s^2 + 215\,s + 1000$$

$2 + K_D = 17.5 \qquad 1 + K_P = 215 \qquad K_i = 1000$

$K_D = 15.5 \qquad K_P = 214$

$$G_P(s) = \frac{\overset{K}{4}\,e^{-1 s}}{\underset{T}{2}s+1}$$

$1 = T_m$ mrtvi čas



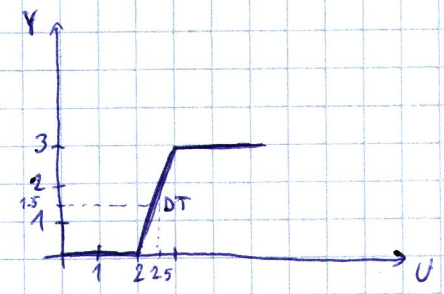
$K_s = 4$

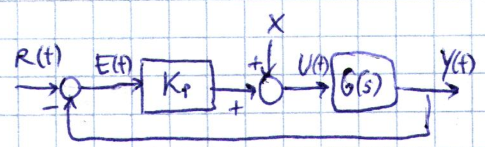
PI-reg

$K_p = 0{,}9 \cdot \frac{T_s}{K_s \cdot T_m} = 0.45$

$T_i = 3{,}3 \cdot T_m = 3.3$

Statična karakteristika



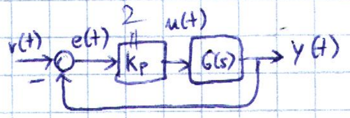


a) Določite vrednost konst. X & ref. sig. R(t), da doseže proces DT brez pogreška v ust. st.

b) Določite E(t) v u.s., če je R(t) = 3 ; ojač. regulatorja $K_p = 2$ (reg. sistem mora biti stabilen)

a) $X = 2{,}5$, $R(t) = 1{,}5$

b)

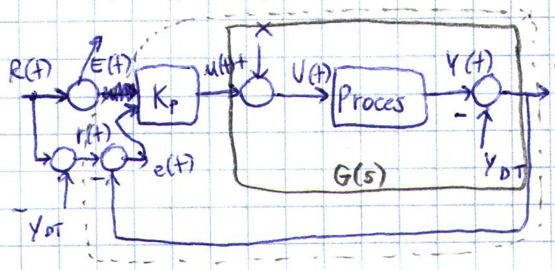


$$\frac{e(s)}{r(s)} = \frac{1}{1+K_P\,G(s)}$$

$$e(s) = \frac{1}{1+K_P \cdot G(s)} \cdot r(s)$$

$G(0) = \frac{\Delta y}{\Delta u} = 3$ (pogledamo naklon premice)

$$e_{ss} = \lim_{s\to 0} s\cdot e(s) = \lim_{s\to 0} \frac{1}{1+K_P \cdot G(s)} = \frac{1}{1+K_P\cdot G(0)} = \frac{1\cdot 1{,}5}{1+2\cdot 3} = \frac{1\cdot 3}{7\cdot 2} = \frac{3}{14}$$

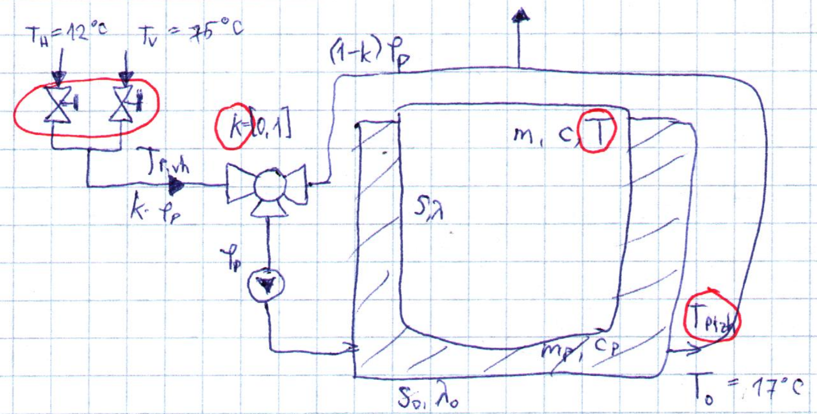


$$E(t) = R(t) - Y(t) = Y_{DT} + r(t) - (Y_{DT} + y(t)) =$$

$$= r(t) - y(t) = e(t)$$

$$\frac{6}{14} + \frac{37}{14} - \left(\frac{41}{14} - \frac{28}{14}\right)\cdot 3 \neq \frac{39}{14} - \frac{21}{14} = \frac{18}{14}$$

LAB. VAJA



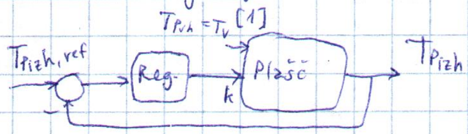
$$\frac{dQ}{dt} = \sum P_{vh} - \sum P_{izh}$$

$$m_p \cdot c_p \cdot \frac{dT_{pizh}}{dt} = k \cdot \varphi_p \cdot c_p \cdot T_{pizh} + (1-k)\, \varphi_p \cdot c_p \cdot T_{pizh} - \varphi_p \cdot c_p \cdot T_{pizh} - \lambda_0 S_0 (T_{pizh} - T_0) - \lambda \cdot S (T_{pizh} - T)$$

$$m \cdot c \cdot \frac{dT}{dt} = \lambda \cdot S \cdot (T_{pizh} - T)$$

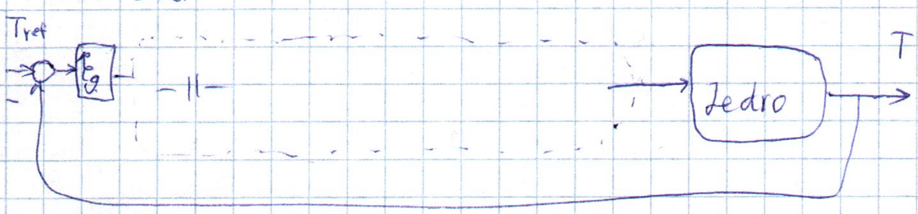
X: /IKRS / vaja 11

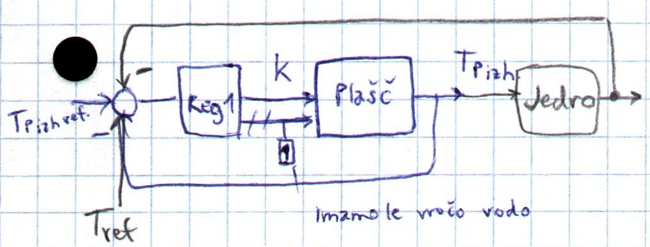
Kaskadna regulacija



— naredimo, da imamo le za toplo vodo (1. del)

2. del:



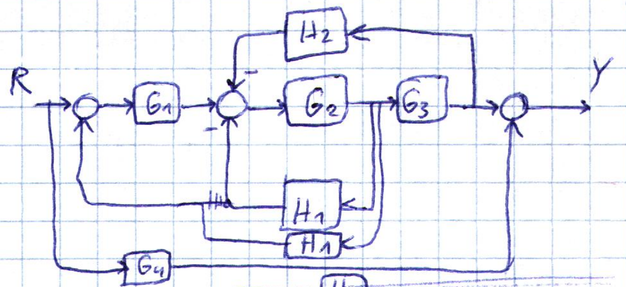


Imamo le vročo vodo

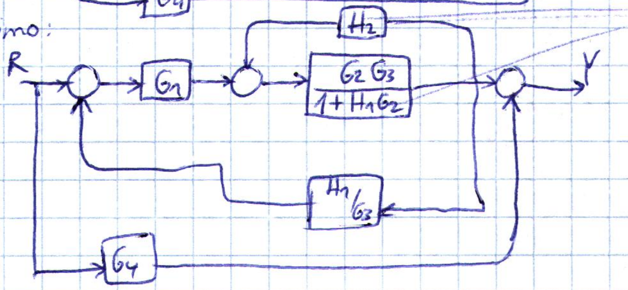
$$G(s) = \frac{K_s}{T_s \cdot s + 1} \qquad G_z(s) = \frac{1}{\frac{T_s}{n} \cdot s + 1}$$

T $\quad T_z$

Določi $\frac{Y(s)}{R(s)}$!



pocnostavimo:

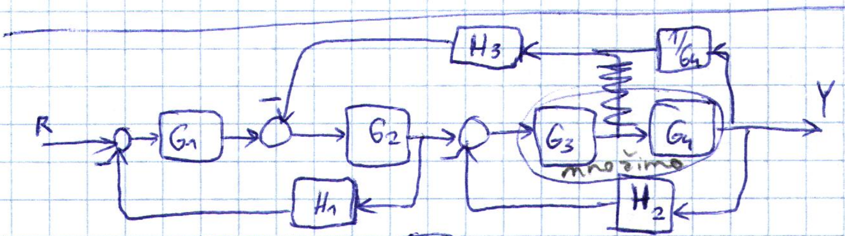


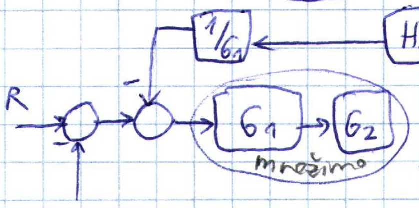
$$\left.\boxed{\frac{\frac{G_2G_3}{1+H_1G_2}}{1+\frac{G_2G_3H_2}{1+H_1G_2}}}\right\}$$

$$\left.\begin{array}{l}\boxed{\frac{G_1G_2G_3}{1+G_1H_1+G_2G_3H_2}}\\ \boxed{H_1/G_3}\\ \boxed{G_4}\end{array}\right\}$$

$$S=\frac{\frac{G_1G_2G_3}{1+G_2H_1+G_2H_2G_3}}{1-\frac{H_1}{G_3}\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2}}+G_4=$$

$$=\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-H_1G_1G_2}+G_4$$

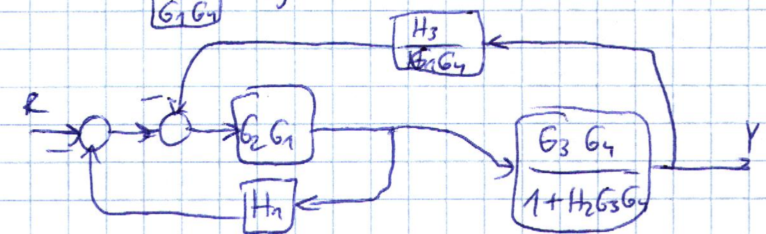


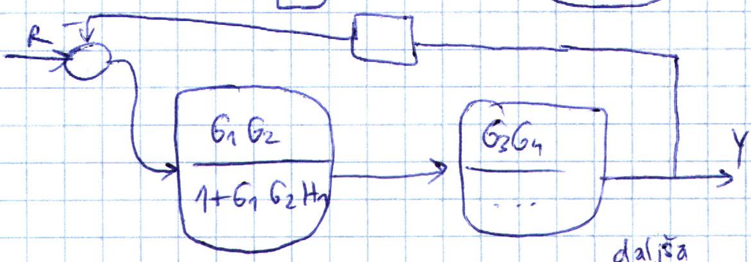


— || —

$\boxed{\frac{H_3}{G_1G_4}}$ zg. bloke množimo

2 povr. vezavi





$$\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+H_2G_3G_4)+G_2G_3H_3}$$

$$\frac{\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+H_2G_3G_4)}}{1+\frac{G_1G_2G_3G_4}{(1+G_1G_2H_1)(1+H_2G_3G_4)}\cdot\frac{H_3}{G_1G_4}}$$