



Predmet:
Osnove robotike

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Vrsta gradiva:
Zapiski predavanj

Avtor:
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Študijsko leto:
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OSNOVE ROBOTIKE

lab. vaje TOR 11-14h

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vaje so obvezne, štejejo 50% → asistent sprašuje 1 ura

e-fe → navodila za vaje, pripravi se (prvič demonstracija, drugič 3 ure - vaja)

če te ni, se prej opraviči in nadomestiš

sprintej skripto (oz. beri PDF)

izp. vpr. iz predavanj (tudi drugih predavateljev)

če pišeš več kot 90%, ni ustnega (vpraša, kar na pisnem nisi znal)

na lab. vajah se ocenjuje preštudirana navodila, kok sprašuješ...

na e-fe učbenik Osnove robotike

do 22.1. faks, do 19.2. izpitno obd.

PHP, dnevi ind. rob,
delo, CIBER,

Razdelitev robotov

- robotski manipulatorji
- robotska vozila
- sistemi človek-robot (haptični, telemanipulatorji, eksoskeleti)
- biološko zasnovani roboti

ROBOTSKI MANIPULATORJI

serijski, paralelni, mikro in nano

- robotska roka (omogoča zahtevano pozicijo prijemala)
- robotsko zapestje (-11- orientacijo prijemala)
- robotsko prijemalo

^{6D}
Pozicija (iz koordinat), orientacija (spr. kotov)

Prostostne stopnje togega telesa

1 masni delec	2	3	
3 prost. st.	+ 2	+ 1	= 6 prostostnih st.
3 translacije	2 rotaciji	1 rotacija	
<u>POZICIJA</u>	<u>ORIENTACIJA</u>		
<u>LEGA</u>			

Robotski sklepi

Imajo eno prost. st.

ROTACIJSKI SKLEP

motorji (el., hidravlični)

TRANSLACIJSKI SKLEP

senzorji (pretvorniki kota, hitrosti)

antropomorfna roka RRR, scara

Robotsko zapestje

Predstavlja zadnje 3 sklepe (rotacijski).

Prijemala z dvema prstoma

Delovni prostor robota

- kartezični
- izsek valja

ROBOTSKA VOZILA

kopno, zrak (ETH Zurich, flight arena)

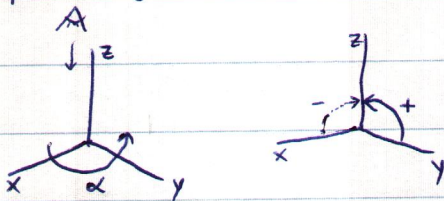
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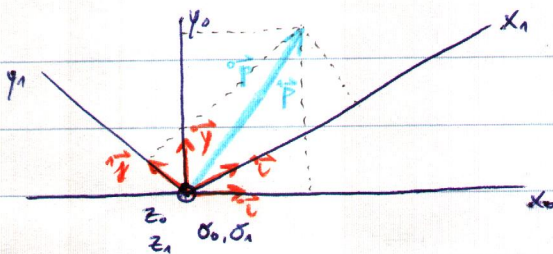
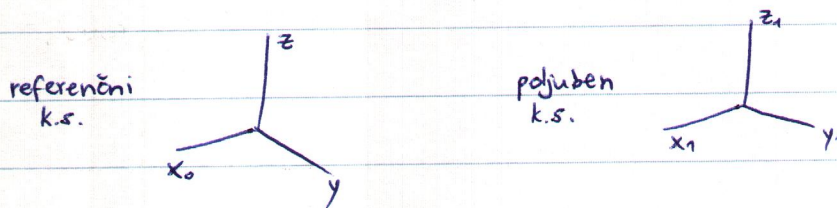
Matjaž Mihelj

ORIENTACIJA

predstavlja zasuk med dvema telesoma (goba in okolica)



0. opisuje kako je koord. sistem pripet, zasukan glede na telo.



lega in premik → homogena

transformacijska matrika



LEGA • orientacija
■ pozicija

PREMIK • rotacija
■ translacija

□ perspektiva

$$O_1: \vec{p} = {}^1p_x \vec{x} + {}^1p_y \vec{y} + {}^1p_z \vec{z}$$

$$O_0: {}^0\vec{p} = {}^0p_x \vec{x} + {}^0p_y \vec{y} + {}^0p_z \vec{z} \quad / \vec{x} \quad / \vec{y}$$

$${}^0\vec{p} \vec{x} = {}^0p_x = {}^1p_x \vec{x} = {}^1p_x \vec{x} + {}^1p_y \vec{y} + {}^1p_z \vec{z}$$

$${}^0\vec{p} \vec{y} = {}^0p_y = {}^1p_y \vec{y} = -11 - \vec{y} + \dots$$

$${}^0\vec{p} \vec{z} = {}^0p_z = {}^1p_z \vec{z} = -11 - \vec{z} + \dots$$

$$\begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix} = \begin{bmatrix} {}^0R_1 \end{bmatrix} \begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix}$$

$${}^0R_1 = \begin{bmatrix} {}^1i^0i & {}^1j^0i & {}^1k^0i \\ {}^1i^0j & {}^1j^0j & {}^1k^0j \\ {}^1i^0k & {}^1j^0k & {}^1k^0k \end{bmatrix} \quad \text{rotacijska matrika} \quad \left. \vphantom{\begin{bmatrix} {}^1i^0i & {}^1j^0i & {}^1k^0i \\ {}^1i^0j & {}^1j^0j & {}^1k^0j \\ {}^1i^0k & {}^1j^0k & {}^1k^0k \end{bmatrix}} \right\} O_0$$

(kako starik vpliva na nove osi)

$${}^0R_1 = \begin{bmatrix} \cos \varphi_{i^0i} & & \\ & & \\ & & \cos \varphi_{k^0k} \end{bmatrix}$$

orientacija O_1 glede na O_0

$$\begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix} = \underline{{}^1R_0} \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix}$$

$$\underline{{}^1R_0} = \begin{bmatrix} {}^0i^1i & {}^0j^1i & {}^0k^1i \\ & {}^0j^1j & {}^0k^1j \\ & & {}^0k^1k \end{bmatrix} \left\{ \begin{matrix} {}^1i \\ {}^1j \\ {}^1k \end{matrix} \right.$$

Lastnosti: $A^T = -A$, $\det(A^T) = \det(-A)$

$${}^1i^0j = {}^0j^1i$$

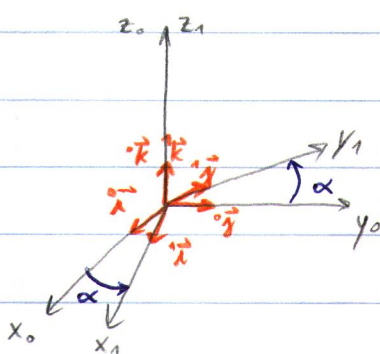
$$\underline{{}^1R_0} = (\underline{{}^0R_1})^{-1} = (\underline{{}^0R_1})^T$$

antisimetrična matrika

rotacijska matrika je ortogonalna matrika ($\det = 1$)

R - naenkrat zna popisati zasuk okrog vseh treh osi

V robotiki običajno potrebujemo le eno rotacijo naenkrat.



glejamo samo rotacijo okrog z-osi

$${}^0k^1k = 1$$

$${}^0i^1j = \cos(90+\alpha) = -\sin\alpha$$

$${}^0i^1i = \cos\alpha$$

$${}^0j^1i = \cos(90-\alpha) = \sin\alpha$$

$${}^0j^1j = \cos\alpha$$

ostali so enaki 0

$$\underline{\underline{R_{z,\alpha}}} = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

to je kul
za izpit ☺

$$\underline{\underline{R_{x,\alpha}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

rotacija okrog x

$$\underline{\underline{R_{y,\alpha}}} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

$${}^0\hat{k} {}^1\hat{k} = 1$$

$${}^0\hat{j} {}^1\hat{j} = 1$$

$$\underline{{}^0\hat{i} {}^1\hat{i} = 1}$$

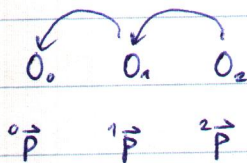
$$\underline{\underline{R}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Lastnosti asim.m.

$$R_{x,\alpha_1} \cdot R_{x,\alpha_2} = R_{x,\alpha_1+\alpha_2}$$

$$R_{x,\alpha}^{-1} = R_{x,-\alpha} = (\quad)^T$$

Izraz za zaporedne rotacije



vsi k.s. imajo isto izhodišče

$$\begin{aligned} {}^0\vec{P} &= {}^0\underline{R}_1 {}^1\vec{P} && \text{rotacija glede na } O_0 \text{ k.s.} \\ &\uparrow && \text{rot. glede na } O_1 \text{ k.s.} \\ {}^1\vec{P} &= {}^1\underline{R}_2 {}^2\vec{P} \end{aligned}$$

$${}^0\vec{P} = {}^0\underline{R}_1 {}^1\underline{R}_2 {}^2\vec{P}$$

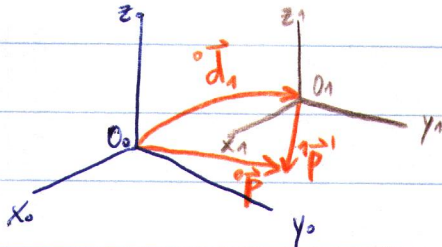
$${}^0\underline{R}_2 = {}^0\underline{R}_1 {}^1\underline{R}_2$$

$$\underline{\underline{{}^0R_n}} = {}^0\underline{R}_1 {}^1\underline{R}_2 \dots {}^{n-1}\underline{R}_n$$

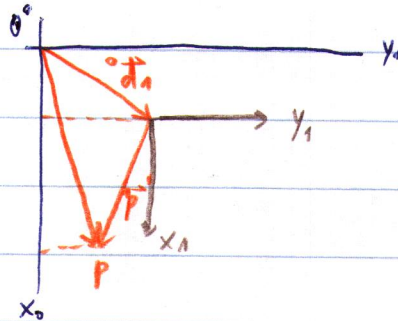
LEGA

Lega = pozicija + orientacija

$${}^0\vec{p} = {}^0R_1 \cdot {}^1\vec{p} + {}^0\vec{d}_1$$



Horis:



$$\underline{{}^0\vec{p} = {}^1\vec{p} + {}^0\vec{d}_1}$$

$${}^0p_x = {}^1p_x + {}^0d_{1x}$$

$${}^0p_y = {}^1p_y + {}^0d_{1y}$$

$${}^0p_z = {}^1p_z + {}^0d_{1z}$$

$${}^1\vec{p} = {}^0R_1 \cdot {}^0\vec{p}$$

$$\underline{{}^0\vec{p} = {}^0R_1 \cdot {}^1\vec{p} + {}^0\vec{d}_1}$$

V primeru treh k.s.

$O_1 \quad O_2 \quad O_3$

	P	
${}^0\vec{p}$	${}^1\vec{p}$	${}^2\vec{p}$

$${}^0\vec{p} = {}^0R_1 \cdot {}^1\vec{p} + {}^0\vec{d}_1$$

$${}^1\vec{p} = {}^1R_2 \cdot {}^2\vec{p} + {}^1\vec{d}_2$$

$${}^0\vec{p} = {}^0R_1 \cdot {}^1R_2 \cdot {}^2\vec{p} + {}^0R_1 \cdot {}^1\vec{d}_2 + {}^0\vec{d}_1$$

$$\underline{{}^0R_2 = {}^0R_1 \cdot {}^1R_2}$$

$$\underline{{}^0\vec{d}_2 = {}^0R_1 \cdot {}^1\vec{d}_2 + {}^0\vec{d}_1}$$

$${}^0\vec{p} = {}^0R_2 \cdot {}^2\vec{p} + {}^0\vec{d}_2$$

zaporedne rotacije

zaporedne lege

translac. matrike smemo sešteti, če so v istem k.s.

2 k.s.

$${}^0\vec{p} = {}^0R_1 \cdot {}^1\vec{p} + {}^0\vec{d}_1$$

$$\begin{bmatrix} {}^0\vec{p} \\ 1 \end{bmatrix} = H \cdot \begin{bmatrix} {}^1\vec{p} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^0\vec{p} \\ 1 \end{bmatrix} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^1\vec{p} \\ 1 \end{bmatrix}$$

H - homogena matrika

↓ info. kako je sestavljena matrika (info. o orientaciji in poziciji)

3 k.s.
$$\begin{bmatrix} {}^0R_1 & {}^0d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^1R_2 & {}^1d_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^0R_1 {}^1R_2 & {}^0R_1 {}^1d_2 + {}^0d_1 \\ 0 & 1 \end{bmatrix}$$

$${}^0H_1 \cdot {}^1H_2 = {}^0H_2$$

$${}^0H_n = {}^0H_1 \cdot {}^1H_2 \dots {}^{n-1}H_n$$

geometrijski zapis modela robota

Inverzna matrika H^{-1}

$${}^0\vec{p} = R {}^1\vec{p} + \vec{d} \quad / \cdot R^T \quad \text{premultiplicirana}$$

$$R^T {}^0\vec{p} = (R^T R) {}^1\vec{p} + R^T \vec{d}$$

$$\vec{p} = R^T {}^0\vec{p} - R^T \vec{d}$$

$$H^{-1} = \begin{bmatrix} R^T & -R^T \vec{d} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^1\vec{p} \\ 1 \end{bmatrix} = \begin{bmatrix} R^T & -R^T \vec{d} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^0\vec{p} \\ 1 \end{bmatrix}$$

OPIS LEGE KOORDINATNEGA SISTEMA

homogena transformacija opisuje orientacijo in pozicijo.

Primer:

R $\vec{p}$

$$H = \text{Trans}(4, -3, 7), \text{Rot}(y, 90), \text{Rot}(z, 90)$$

$$\text{Rot}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos & -\sin & 0 \\ 0 & \sin & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

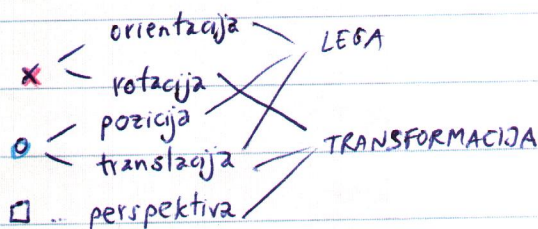
$$H = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \text{Rot}(z, 90) = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \text{Rot}(y) = \begin{bmatrix} \cos & 0 & \sin & 0 \\ 0 & 1 & 0 & 0 \\ -\sin & 0 & \cos & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(z) = \begin{bmatrix} \cos - \sin & 0 & 0 \\ \sin & \cos & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

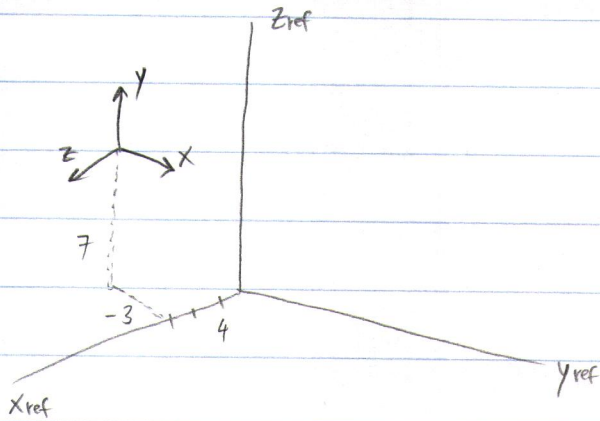
novi
trenutni

$$\begin{matrix} x_{\text{ref}} \\ y_{\text{ref}} \\ z_{\text{ref}} \end{matrix} \begin{bmatrix} \times & \times & \times & \circ \\ \times & \times & \times & \circ \\ \times & \times & \times & \circ \\ \square & \square & \square & 1 \end{bmatrix}$$



$$H = \begin{bmatrix} R & \vec{d} \\ 0 & 1 \end{bmatrix}$$

razloži pomen
H matrike
(izpit)



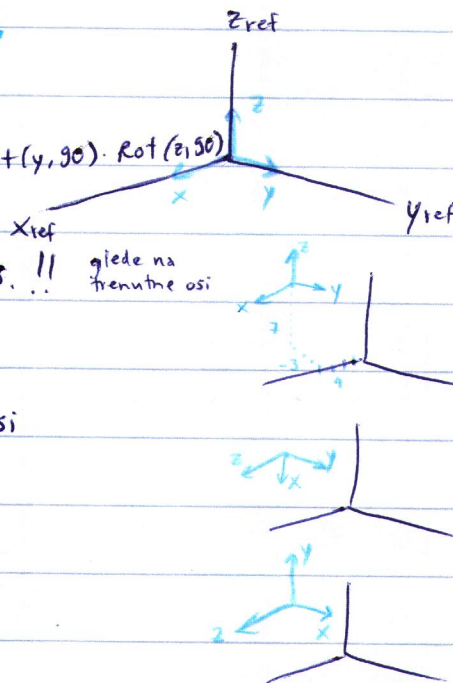
TRANSFORMACIJA

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Trans(4, -3, 7) · Rot(y, 90) · Rot(z, 90)

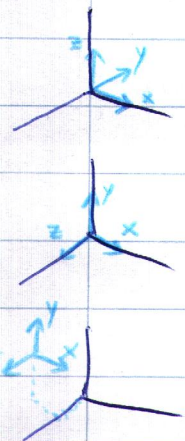
⇒ relativni k.s. !! glede na trenutne osi

- translacijski premik [4, -3, 7]
- zavrtimo za 90° okrog y osi
- -||- okrog z osi



$$\text{Trans}(4, -3, 7) \cdot \text{Rot}(y, 90) \cdot \text{Rot}(z, 90) \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⇒ referenčni / absolutni / globalni
(množenje od desne proti levi)



OPIS TRANSFORMACIJE KOORDINATNEGA SISTEMA

$$\begin{bmatrix} \text{transformacija} \\ \underline{P} \end{bmatrix} \cdot \begin{bmatrix} \text{lega} \\ \text{LEGA} \\ \text{k.s.} \\ \underline{L} \end{bmatrix}$$

sestavljena iz orientacije in pozicije

premultiplikacija

dogodek velja za $\underline{L}$ glede na refer. k.s.

$$\begin{bmatrix} \text{LEGA} \\ \underline{L} \end{bmatrix} \cdot \begin{bmatrix} \text{transform.} \\ \underline{P} \end{bmatrix}$$

postmultiplikacija

dogodek gledamo glede na relativni k.s.

primer: transformacija k.s. v referenčnem k.s.

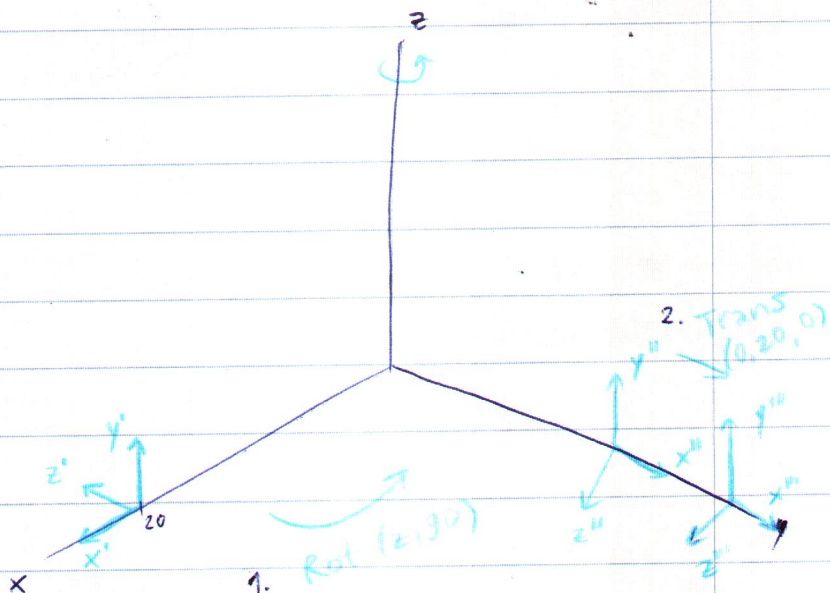
$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

transf. naj bo sestavljena iz translacij in rotacij

$$\underline{P} = \text{Trans}(0, 20, 0) \cdot \text{Rot}(z, 90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

kakšna bo nova lega k.s.?

$$\underline{X} = \underline{P} \cdot \underline{L} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



isto v relativnem k.s.

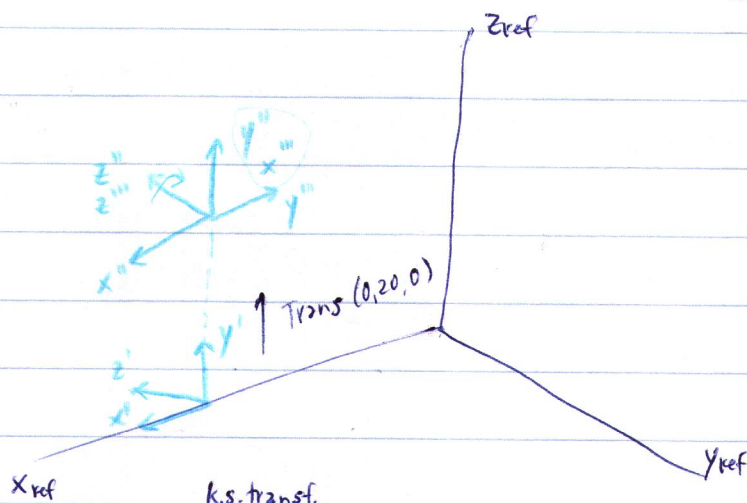
$\underline{L}, \underline{P}$

$$\underline{P} = \text{Trans}(0, 20, 0) \text{ Rot}(z, 90)$$

$$\underline{Y} = \underline{L} \cdot \underline{P}$$

$$\underline{Y} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{Y} = \begin{bmatrix} 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



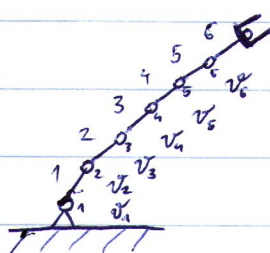
Ugotovitev: ni isto!

k.s. transf.

$L, P \rightarrow$ drugi k.s.	ref.	1
	rel.	2
$L, Y \rightarrow$ iščemo P		3
		4
$P, X \rightarrow$ začetni k.s.		5
		6

GEOMETRIJSKI MODEL ROBOTA Z DH SKALARNIMI PARAMETRI (DIREKTNA KINEMATIKA)

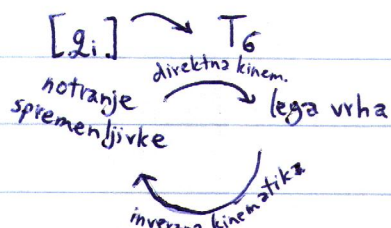
Denavit-Hartenberg



$[x, y, z]$

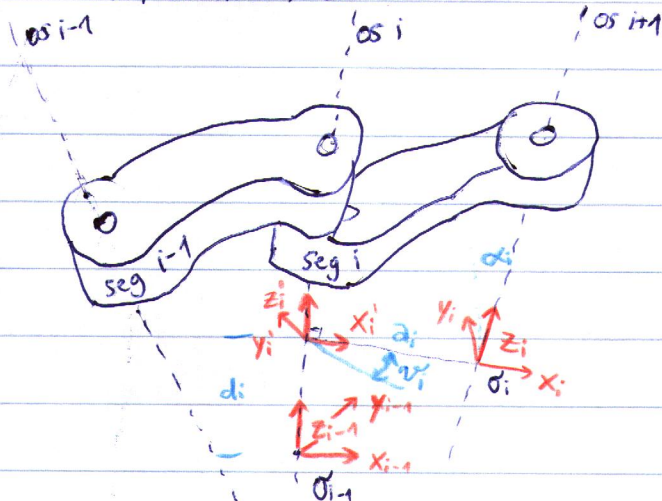
$[\phi_x, \phi_y, \phi_z]$

T_6



kako to naredimo?

kvaternioni
DH, kvaternioni, vektorski model



rekurzivno

os je vedno na začetku segmenta

I. Narišemo k.s.:

DH pravila:

1. os z_i poteka vzdolž $i+1$
2. O_i na presečišču osi z_i in skupne normale na osi z_i in z_{i-1}
3. izberem os x_i vzdolž skupne normale, tako da je usmerjena od $i \rightarrow i+1$
4. izberi y_i tako da dobim desnosučni k.s.

II. Napišemo DH parametre:

DH parametri

1. $\theta_i \neq x_{i-1}$ in x_i , okrog (z_{i-1})
 2. $d_i = \overline{O_{i-1}O_i}$ vzdolž (z_{i-1})
 3. $a_i = \overline{O_iO_i'}$ vzdolž $[x_i]$
 4. $\alpha_i \neq z_{i-1}, z_i$, okrog $[x_i]$
- } konst.

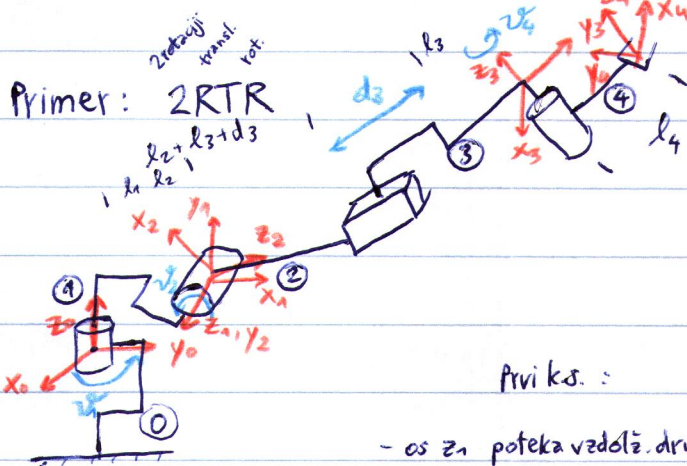
Izjeme:

1. Če sta osi z_i, z_{i-1} vzporedni, je $d_i = 0$.
2. Če se z_i, z_{i-1} sekata, je $x_i \perp z_{i-1}, z_i$, O_i je v presečišču
3. bazni k.s.
 - os z_0 je določena
 - izhodišče O_0 pomaknemo v 1. sklep
 - $x_0 \parallel x_1$
4. k.s. vrha robota
 - $x_n \perp z_{n-1}$; $z_n \parallel z_{n-1}$
 - kadar so osi sosednjih sklepov $\perp$ ali $\parallel$, je zadnji k.s. vzporeden s prejšnjim
5. translacijski sklep
 - os z_{i-1} poteka vzdolž translacije
 - izhodišče O_{i-1} postavimo na začetek gibanja sklepa

Transformacija med i in $i-1$ k.s.

$${}^{i-1}A_i(q_i) = \text{Rot}(z_{i-1}, \alpha_i) \text{Trans}(0, 0, d_i) \text{Trans}(a_i, 0, 0) \cdot \text{Rot}(x_i, \alpha_i)$$

$$\begin{bmatrix} c\alpha_i & -s\alpha_i & 0 & 0 \\ s\alpha_i & c\alpha_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\alpha_i^0 & -s\alpha_i^1 & s\alpha_i^1 & a_i c\alpha_i^0 \\ s\alpha_i^1 & c\alpha_i^0 & -c\alpha_i^0 s\alpha_i^1 & a_i s\alpha_i^1 \\ 0 & s\alpha_i^0 & c\alpha_i^0 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- oštevilčimo segmente

(fiksni na bazi je 0)

- bazi, referenčni k.s.

- os z_0 poteka vzdolž osi prvega

sklepa

- os z_1 poteka vzdolž drugega sklepa

izhodišče k.s. v presečišču z_0 in z_1

- izhodišče k.s. postavim kjerkoli vzdolž 0-tega segn. običajno izberem središče 1. sklepa

- os x_1 poteka vzdolž skupne pravokotnice

na z_0 in z_1

(včasih $x_0 \parallel x_1$)

- y_1

- os y_0 dopolnjuje desnosučni k.s.

Drugi k.s.:

- os z_2 poteka vzdolž translacije tretjega sklepa

- izhodišče k.s. postavimo na začetek translacije $d_3 = 0$

- osi z_1 in z_2 se sekata $x_2 \perp z_1, z_2$

Tretji k.s.:

- os z_3 poteka vzdolž osi vrtenja 4. sklepa

- izhodišče postavimo na presečišče z_2 in z_3

- osi z_2 in z_3 se sekata $x_3 \perp z_2, z_3$

potrebno
transl.
okrog z

Četrti ks.:

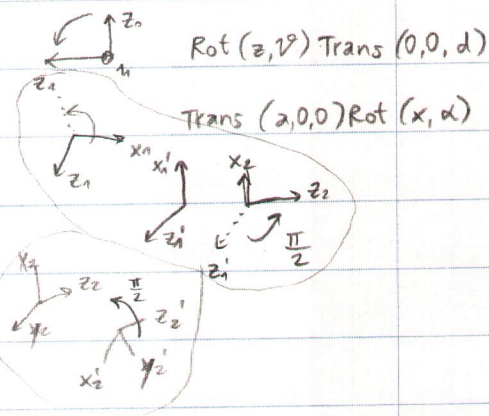
Smo $\frac{1}{3}$ naloge naredi

- $x_4 \perp z_3$

- najenostavneje $z_4 \parallel z_3$

TABELA DH parametrov

II	segment	θ_i	d_i	a_i	α_i
	1	θ_1 $\begin{matrix} c\theta_1 \\ s\theta_1 \end{matrix}$	h_1	l_1	$\pi/2$ $\begin{matrix} 0 \\ 1 \end{matrix}$
	2	θ_2 $\begin{matrix} c\theta_2 \\ s\theta_2 \end{matrix}$	0	0	$\pi/2$ $\begin{matrix} 0 \\ 1 \end{matrix}$
	3	$\pi/2$ $\begin{matrix} 0 \\ 1 \end{matrix}$	d_3	0	$\pi/2$ $\begin{matrix} 0 \\ 1 \end{matrix}$
	4	θ_4 $\begin{matrix} c\theta_4 \\ s\theta_4 \end{matrix}$	$-h_3$	l_4	0 $\begin{matrix} 1 \\ 0 \end{matrix}$



vrednosti s 1. vrstice

$${}^0A_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & l_1 c\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & l_1 s\theta_1 \\ 0 & 1 & 0 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} c\theta_2 & 0 & s\theta_2 & 0 \\ s\theta_2 & 0 & -c\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3A_4 = \begin{bmatrix} c\theta_4 & -s\theta_4 & 0 & l_4 c\theta_4 \\ s\theta_4 & c\theta_4 & 0 & l_4 s\theta_4 \\ 0 & 0 & 1 & -h_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1 - vpišemo k.s.

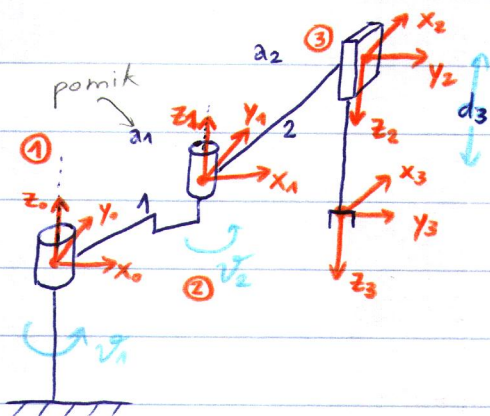
2 - napišemo tabelo

3 - napišemo matrike

za vajo: pomnoži

modra - je aktiven,
se spreminja

Primer 2: SCARA robot



(ni v knjigi)

načeloma x_0 obračamo
proti nast. elementu
čim prvega določimo...
 x_1 bi lahko gledal nast. segment
to ni "home" lega 0,0,0

$$x \perp z_1, z_2$$

$$z_1 \parallel z_2$$

ker je po z_1 še vedno enako visoko

	i	$\mathcal{V}_i$	d_i	a_i	α_i
R	1	$\mathcal{V}_1$	0	a_1	0
Ker imamo 3 prost. st.	R	$\mathcal{V}_2$	0	a_2	π — $z_1 \uparrow, z_2 \downarrow \rightarrow \pi!$
T	3	0	d_3	0	0

$${}^{i-1}A_i = \text{Trans}(0,0,\alpha_i) \cdot \text{Rot}(z_{i-1}, \mathcal{V}_i) \cdot \text{Trans}(a_i,0,0) \cdot \text{Rot}(x_i, \alpha_i)$$

zavrtitev okrog
osi vrtenja

vzdolž (dolžina segmenta)

$${}^0A_1 = \begin{bmatrix} c\mathcal{V}_1 & -s\mathcal{V}_1 & 0 & a_1 c\mathcal{V}_1 \\ s\mathcal{V}_1 & c\mathcal{V}_1 & 0 & a_1 s\mathcal{V}_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} c\mathcal{V}_2 & s\mathcal{V}_2 & 0 & +a_2 c\mathcal{V}_2 \\ s\mathcal{V}_2 & -c\mathcal{V}_2 & 0 & a_2 s\mathcal{V}_2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} x & y & z \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

na pamet, ker je samo translacija

$$T_3 = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3$$

DN

Rešitev:

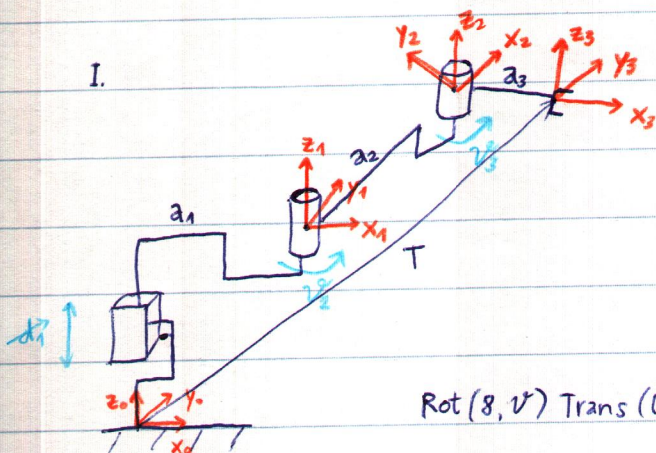
$$\begin{bmatrix} c\mathcal{V}_1\mathcal{V}_2 & s\mathcal{V}_1\mathcal{V}_2 & 0 & a_2 c\mathcal{V}_1\mathcal{V}_2 + a_1 c\mathcal{V}_1 \\ s\mathcal{V}_1\mathcal{V}_2 & -c\mathcal{V}_1\mathcal{V}_2 & 0 & a_2 s\mathcal{V}_1\mathcal{V}_2 + a_1 s\mathcal{V}_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c\theta & -s\theta c\alpha & s\theta s\alpha & a_1 c\theta \\ s\theta & c\theta c\alpha & -c\theta s\alpha & a_1 s\theta \\ 0 & s\alpha & c\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer: SCARA robot

a-dolžina segmenta

robot ni v home legi



II.

seg	θ_i	d_i	a_i	α_i
1	0	d_1	a_1	0°
2	θ_2	0	a_2	0°
3	θ_3	0	a_3	0°

Rot(0, θ) Trans(0, 0, d) Trans(a , 0, 0) Rot(x , α)

III. ${}^0A_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

enako sta obrnjena k.s. ($\rightarrow$ enotna matrika)

$${}^1A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(x, \alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix}$$

$$\text{Rot}(y, \beta) = \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}$$

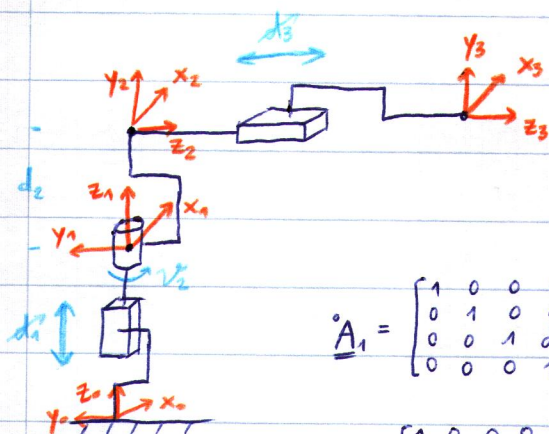
$$\text{Rot}(z, \theta) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{I} = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 + a_1 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 c_3 - s_2 s_3 & -c_2 s_3 - s_2 c_3 & 0 & c_2 a_3 c_3 - s_2 a_3 s_3 + a_2 c_2 + a_1 \\ s_2 c_3 + c_2 s_3 & -s_2 s_3 + c_2 c_3 & 0 & s_2 a_3 c_3 + c_2 a_3 s_3 + a_2 s_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 + a_1 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer: Cilindrični robot



i	$\mathcal{V}_i$	d_i	a_i	α_i
1	0	d_1	0	0
2	$\mathcal{V}_2$	d_2	0	$+\frac{\pi}{2}$
3	0	d_3	0	0

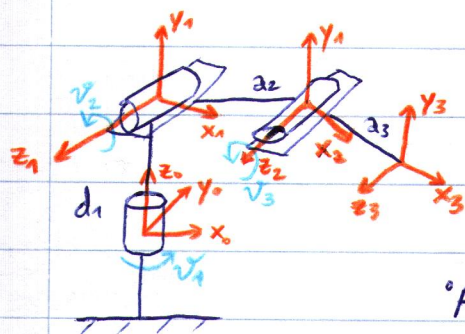
$${}^0A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = {}^0A_1 \cdot {}^1A_2 \cdot {}^2A_3 = \begin{bmatrix} c_2 & 0 & s_2 & d_3 s_2 \\ s_2 & 0 & -c_2 & -d_3 c_2 \\ 0 & 1 & 0 & d_1 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer: Antropomorfni robotski manipulator



i	$\mathcal{V}$	d	a	α
1	$\mathcal{V}_1$	d_1	0	$\frac{\pi}{2}$
2	$\mathcal{V}_2$	0	a_2	0
3	$\mathcal{V}_3$	0	a_3	0

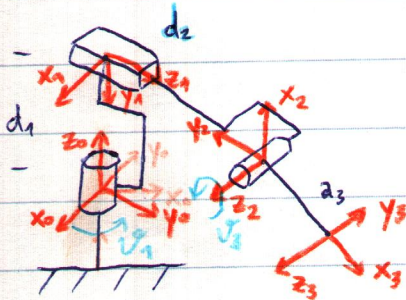
$${}^0A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix}$$

$\underline{T} =$

Primer: Ispitni robot

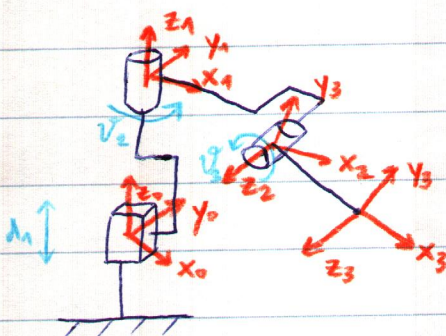


i	$\mathcal{V}_i$	d_i	a_i	α_i
1	$\mathcal{V}_1$	d_1	0	$-\frac{\pi}{2}$
2	$-\frac{\pi}{2}$	d_2	0	$-\frac{\pi}{2}$
3	$\mathcal{V}_3$	0	a_3	0

$${}^0A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} s_1 s_3 & -s_1 c_3 & c_1 & -d_2 s_1 + d_3 s_1 s_3 \\ -c_1 s_3 & -c_1 c_3 & s_1 & d_2 c_1 - d_3 c_1 s_3 \\ c_3 & -s_3 & 0 & d_1 + d_3 c_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer: Ispitni robot



i	$\mathcal{V}_i$	d_i	a_i	α_i
1	0	d_1	0	0
2	$\mathcal{V}_2$	0	a_2	$\frac{\pi}{2}$
3	$\mathcal{V}_3$	0	a_3	0

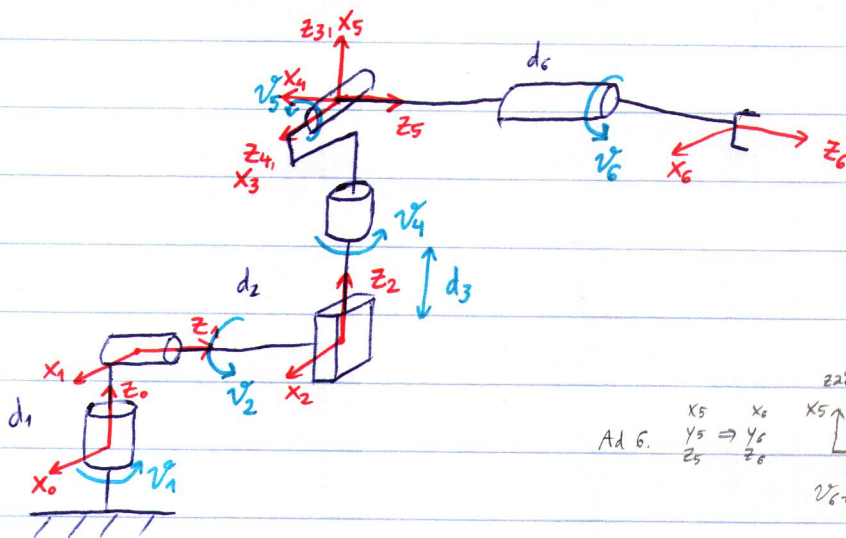
$${}^0A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^1A_2 = \begin{bmatrix} c_2 & 0 & s_2 & a_2 c_2 \\ s_2 & 0 & -c_2 & a_2 s_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2A_3 = \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^0A_1 {}^1A_2 = \begin{bmatrix} c_2 & 0 & s_2 & a_2 c_2 \\ s_2 & 0 & -c_2 & a_2 s_2 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

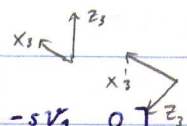
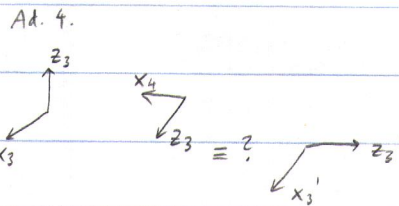
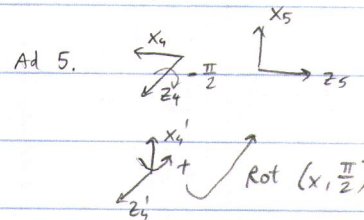
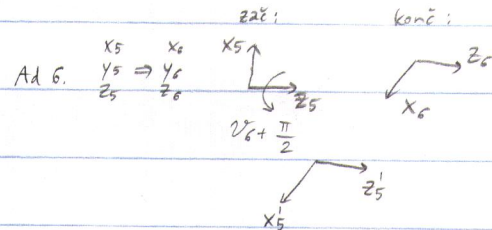
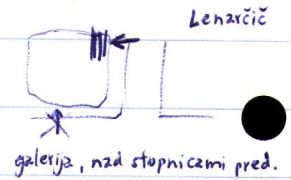
$$I = \begin{bmatrix} c_2 & 0 & s_2 & a_2 c_2 \\ s_2 & 0 & -c_2 & a_2 s_2 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 c_3 & -c_2 s_3 & s_2 & c_2 a_3 c_3 + a_2 c_2 \\ s_2 c_3 & -s_2 s_3 & -c_2 & s_2 a_3 c_3 + a_2 s_2 \\ s_3 & c_3 & 0 & a_3 s_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer: Stanfordski robotski manipulator

RRTRRR



$x_1 \parallel x_0$
 $x_2 \parallel x_3$
 $x_4 \perp z_3, z_4$
 $x_5 \perp z_4, z_5$



i	v_i	d_i	a_i	α_i
1	v_1	d_1	0	$-\frac{\pi}{2} \begin{smallmatrix} c=0 \\ s=-1 \end{smallmatrix}$
2	v_2	d_2	0	$\frac{\pi}{2} \begin{smallmatrix} s=+1 \end{smallmatrix}$
3	0	d_3	0	0
4	$v_4 - \frac{\pi}{2}$	0	0	$-\frac{\pi}{2}$
5	$v_5 - \frac{\pi}{2}$	0	0	$+\frac{\pi}{2}$
6	$v_6 + \frac{\pi}{2}$	d_6	0	0 $\begin{smallmatrix} c=1 \\ s=0 \end{smallmatrix}$

$${}^0A_1 = \begin{bmatrix} c v_1 & -s v_1 c \alpha_1 & s v_1 s \alpha_1 & a_1 c v_1 \\ s v_1 & c v_1 c \alpha_1 & -c v_1 s \alpha_1 & a_1 s v_1 \\ 0 & s \alpha_1 & c \alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c v_1 & 0 & -s v_1 & 0 \\ s v_1 & 0 & +c v_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1A_2 = \begin{bmatrix} c v_2 & 0 & s v_2 & 0 \\ s v_2 & 0 & -c v_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^2A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, {}^3A_4 = \begin{bmatrix} \text{enako ①} & 0 \\ \text{upoštevamo} & -\frac{\pi}{2} \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$${}^4A_5 = \begin{bmatrix} \text{enako ②} & 0 \\ \text{upošt.} & -\frac{\pi}{2} \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, {}^5A_6 = \begin{bmatrix} c v_6 & -s v_6 & 0 & 0 \\ s v_6 & c v_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \leftarrow + \text{upoštevam} + \frac{\pi}{2}$$

manipulator ni
home legi
(izhodišnji)

VEKTORSKI GEOMETRIJSKI MODEL ROBOTSKEGA MANIPULATORJA

1. rob. mehanizem postavimo v referenčno lego $\begin{matrix} z \\ x_0 \quad y_0 \end{matrix}$, $d=0$ transl. sklepi so stisnjeni
(narišemo mehanizem)

2. $[x_i, y_i, z_i]$ narišemo (izhodišča, vsaj 2 osi)

3. v središča sklepov postavimo sklepne enotske vektorje $\vec{e}_i$
potekajo vzdolž translacije ali rotacijske osi sklepa

$\downarrow d_i$ $\downarrow \dot{v}_i$

4. povežemo izhodišča koord. sistemov s segmentnimi vektorji $\vec{b}_{i-1,i}$

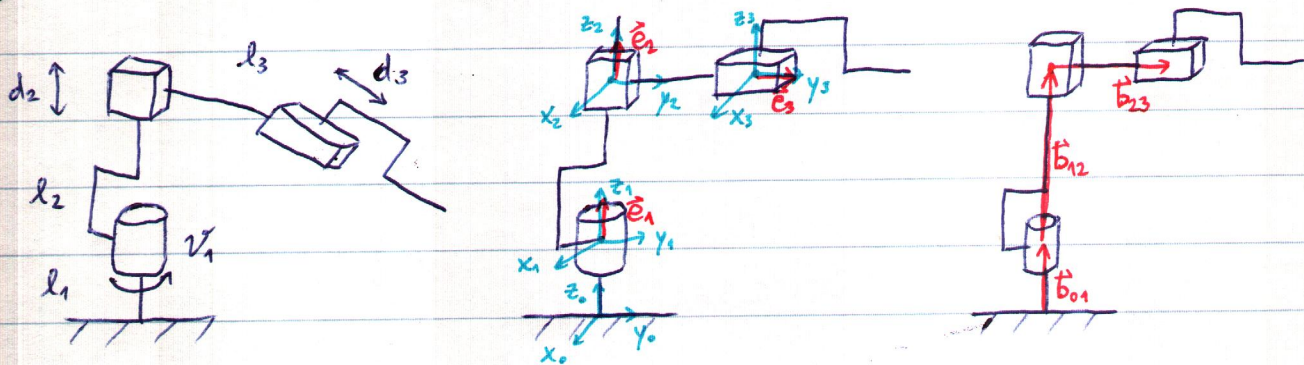
5. zapišemo tabele (tabela spremenljivk, segm. vektorjev, sklepnih vektorjev)

6. zapišemo matrice na osnovi tabel

$${}^{i-1}H_i = \begin{bmatrix} \text{rot.} & \\ R_x R_y R_z & b_{i-1,i} \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} \text{transl.} & \\ \vec{b}_{i-1,i} + d_i \vec{e}_i \\ 0 \end{bmatrix}$$

7. $I = {}^0H_1 \dots {}^{n-1}H_n$

Cilindrični robot



i	1	2	3
$\dot{v}_i$	$\dot{v}_1$	0	0
d_i	0	d_2	d_3

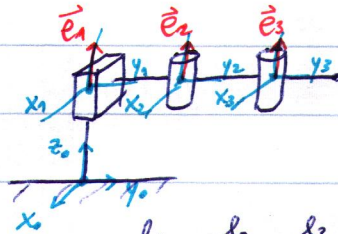
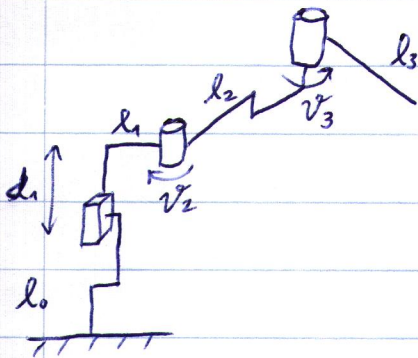
$\vec{e}_i$	$\vec{e}_1$	$\vec{e}_2$	$\vec{e}_3$
$i=1$	1	0	0
$i=2$	0	1	0
$i=3$	0	0	1

$\vec{b}_{i-1,i}$	b_{01}	b_{12}	b_{23}
$i=1$	1	0	0
$i=2$	0	1	0
$i=3$	0	0	1

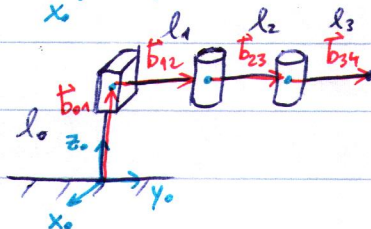
$${}^0H_1 = \begin{bmatrix} R_z & b_{01} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^1H_2 = \begin{bmatrix} \underline{I} & d_2\vec{e}_2 + \vec{b}_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 + l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3 + l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T = {}^0H_1 {}^1H_2 {}^2H_3 \stackrel{DN}{=} \begin{bmatrix} c_1 & -s_1 & 0 & -s_1(d_3 + l_3) \\ s_1 & c_1 & 0 & c_1(d_3 + l_3) \\ 0 & 0 & 1 & l_1 + l_2 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

SCARA



i	1	2	3
v_i	0	v_2	v_3
d_i	d_1	0	0



$$T = \begin{bmatrix} c_2 c_3 - s_2 s_3 & -c_2 s_3 - s_2 c_3 & 0 & l_1 c_2 s_3 - l_3 s_2 c_3 - l_2 s_2 \\ s_2 c_3 + c_2 s_3 & -s_2 s_3 + c_2 c_3 & 0 & -l_1 s_2 s_3 + l_3 c_2 c_3 + l_2 c_2 \\ 0 & 0 & 1 & l_0 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	$\vec{e}_1$	$\vec{e}_2$	$\vec{e}_3$
1	0	0	0
2	0	0	0
3	1	1	1
$\vec{b}_{0i}$	$\vec{b}_{12}$	$\vec{b}_{23}$	$\vec{b}_{34}$
1	2	3	4
$b_{i-1,i}$	0	0	0
	0	l_1	l_2
	l_0	0	0

VAJE OR

str. 64 (primeri v učbeniku)

1. Premiki v referenčnem sistemu

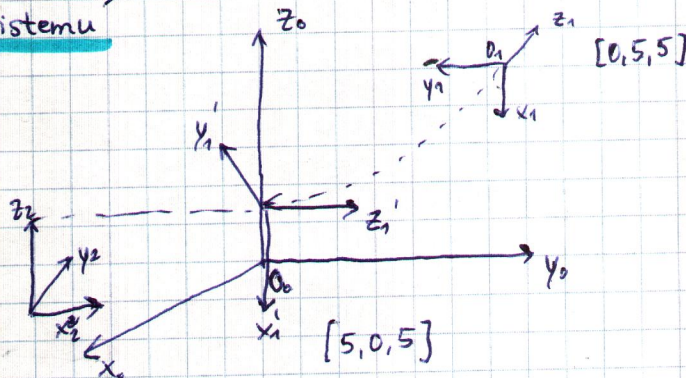
najprej transformacija

①

$$H_2 = T \cdot H_1$$

pozicija

$$H_1 = \begin{bmatrix} x_1 & y_1 & z_1 & p \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



negat. mat. smer

1. 0, za $\frac{\pi}{2}$, z_0 osi, ②

2. y_0 -3

3. $\frac{\pi}{2}$, x_0 osi, ③

$$\begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

obratni vrstni red

$$H_2 = \text{Rot}(x, \frac{\pi}{2}) \cdot \text{Trans}([0, -3, 0]) \cdot \text{Rot}(y, -\frac{\pi}{2}) \cdot H_1$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

nova lega

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & -3 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

②

$$H_2 = T \cdot H_1 \quad / \cdot H_1^{-1}$$

$$H_2 \cdot H_1^{-1} = T \cdot \underbrace{H_1 \cdot H_1^{-1}}_I$$

$$T = H_2 \cdot H_1^{-1}$$

v REF. KS

$$H = \begin{bmatrix} \underline{R} & \underline{d} \\ 0 & 1 \end{bmatrix}$$

$$H^{-1} = \begin{bmatrix} \underline{R}^T & -\underline{R}^T \cdot \underline{d} \\ 0 & 1 \end{bmatrix}$$

$$H_1^{-1} = \begin{bmatrix} 0 & 0 & -1 & 5 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-\underline{R}^T \cdot \underline{d} = - \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

rešitev

$$T' = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⑥

VI. primer

$$H_2 = []$$

$$H_2 = H_1 \cdot T'$$

$$T' = []$$

$$H_2 \cdot (T')^{-1} = H_1 \cdot \overbrace{T' \cdot (T')^{-1}}^I$$

$$H_1 = ? = H_2 \cdot (T')^{-1}$$

$$\text{rešitev: } (T')^{-1} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HUMANOIDNA ROBOTIKA

Aleš Vde

asimo.honda.com

problemi: napajanje (zadostuje le 1h)

Wabian II, 2006

Cog, MIT (1996)

DB SARCOS (1999)

ARMAR, Karlsruhe

i-Cub, Geneva

Sony QRIO

COMAN (mekkost je pomembna lastnost)

HRP-2

dolina nelagodja

humanoidne prijemalke, glave (umetni vs. človeški vid)
razpoznavanje objektov

Baxter DARPA, tekmovalje

manjka mu splošno znanje

20.11. POLNOČNI KLUB

AVTOPILOT

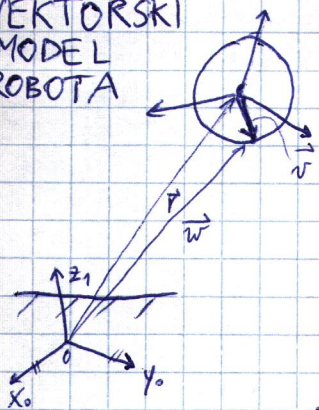
se še ne izognejo oviram (pomembno načrtovanje leta)
natančnost pristajanja
GPS

Onedrone

Janez Nebec

041 667 742

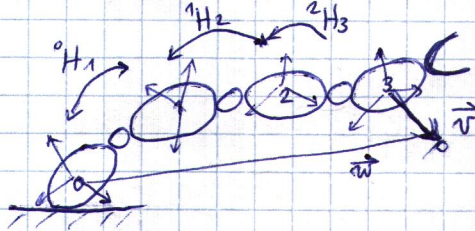
VEKTORSKI MODEL ROBOTA



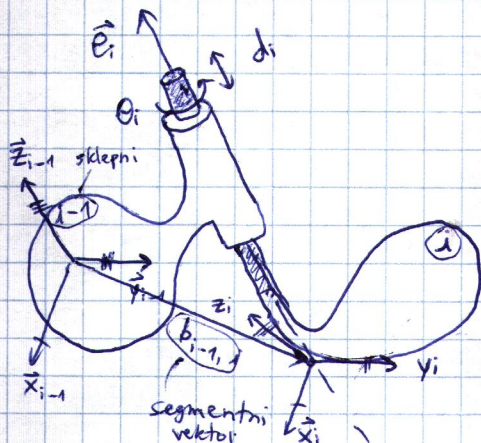
$$\vec{w} = \vec{r} + {}^0R_1 \vec{v}$$

v rotaciji
spremenjena
pozicija

$${}^0H_1 = \begin{bmatrix} {}^0R_1 & \vec{r} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \vec{w} \\ 1 \end{bmatrix} = {}^0H_1 \cdot \begin{bmatrix} \vec{v} \\ 1 \end{bmatrix} = \begin{bmatrix} \vec{r} + {}^0R_1 \vec{v} \\ 1 \end{bmatrix}$$



KINEMATIČNI PAR



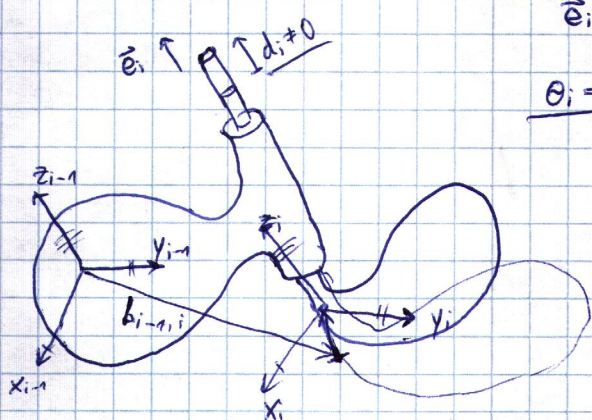
Referenčna lega
začetna

$$d_i = 0, \theta_i = 0$$

določiti središče sklepa (pogoj, da je na osi $\vec{e}_i$)

$${}^{i-1}H_i = \begin{bmatrix} I & b_{i-1,i} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

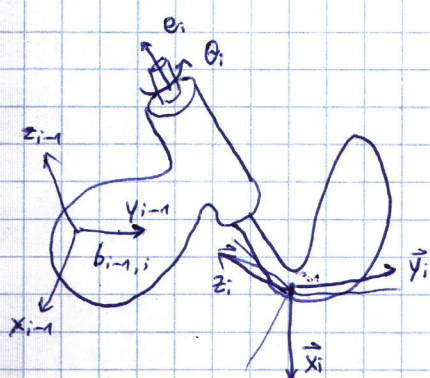
TRANSLACIJA



$$\vec{e}_i, b_{i-1,i}$$

$$\theta_i = 0 \quad {}^{i-1}H_i = \begin{bmatrix} I & b_{i-1,i} + d_i \vec{e}_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ROTACIJA



$${}^{i-1}H_i = \begin{bmatrix} {}^{i-1}R_i & b_{i-1,i} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \Delta \sin \theta + (I - \Delta) \cos \theta + \Delta$$

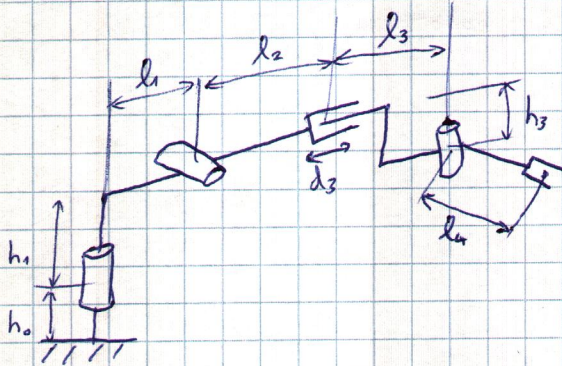
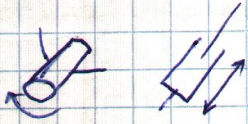
$$\Delta = \vec{e} \vec{e}^T$$

$$\Delta = \vec{e} \times \vec{I}$$

TRANSLACIJA in ROTACIJA

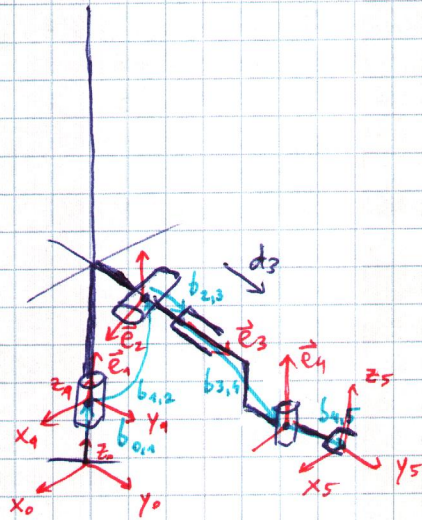
$$d_i \neq 0, \theta_i \neq 0$$

$${}^{i-1}H_i = \begin{bmatrix} {}^{i-1}R_i & b_{i-1,i} + d_i \vec{e}_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



1. postavite mehanizem v ref. lego
2. izberite središče sklepov

	e_1	e_2	e_3	e_4
e_i	0	1	0	0
	0	0	1	0
	1	0	0	1



e naj bo usmerjen
v x

vsi K.S. so vzporedni

$b_{i-1,i}$	$b_{0,1}$	$b_{1,2}$	$b_{2,3}$	$b_{3,4}$	$b_{4,5}$
	0	0	0	0	0
	0	l_1	l_2	l_3	l_4
	h_0	h_1	0	$-h_3$	0

	1	2	3	4
d_i	0	0	d_3	0
θ_i	θ_1	θ_2	0	θ_4

$${}^0H_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & h_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & l_1 \\ 0 & s_2 & c_2 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2H_3 = \begin{bmatrix} 1 & 0 & 0 & l_2 + d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3H_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & l_3 \\ 0 & 0 & 1 & -h_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4H_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PRODOR ROBOTIZACIJE NA NOVA APLIKATIVNA PODROČJA

Doc. dr. Igor Kovač

Problem: kontakt z obdelovancem (problematika toleranc, operacije v periferiji

↑ kar je okoli robota

Argumenti vpeljave: ekonomski vidiki, pri 3-izmenjskem delu je ind. robot ekonomsko upravičen (1x), hitrost! (prednost je v zanesljivosti in stalnosti dela)

Izdelava pettonovih turbin (ročno brušenje nadomestiti z robotskim?)

Montaža železniških zavornih kolotov

Prenašanje rezervoarjev za gorivo

Prenašanje delov za kompresorje

Avtomobilska industrija

fleksibilnost ✓
jigless

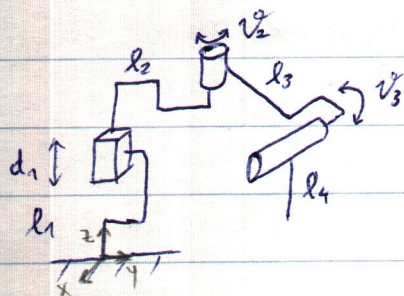
Železniška industrija

Letalska industrija

Gradbeništvo

paketiranje tlakovcev

DN: Izpiti

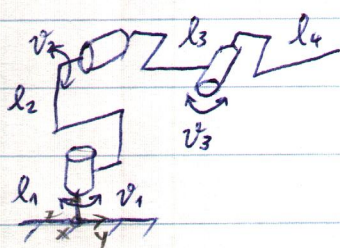


e_i	1	2	3
x	0	0	-1
y	0	0	0
z	1	1	0

i	1	2	3
v_i	0	v_2	v_3
d_i	d_1	0	0

$b_{i-1,i}$	b_{01}	b_{12}	b_{23}	b_{34}
x	0	0	0	0
y	0	l_2	l_3	l_4
z	l_1	0	0	0

DN: Antropomorfni



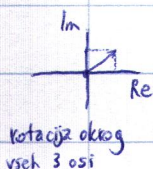
e_i	1	2	3
x	0	-1	-1
y	0	0	0
z	1	0	0

i	1	2	3
v_i	v_1	v_2	v_3
d_i	0	0	0

$b_{i-1,i}$	b_{01}	b_{12}	b_{23}	b_{34}
x	0	0	0	0
y	0	0	l_3	l_4
z	l_1	l_2	0	0

KVATERNIONI

Orientacijo lahko opišemo z rotacijskimi matrikami, RPY koti, Eulerjevimi koti, kvaternioni (za opis in računanje).



$$q = q_0 \cdot 1 + q_1 \cdot \vec{i} + q_2 \cdot \vec{j} + q_3 \cdot \vec{k}$$

q_0 realna št.
 $\vec{i}, \vec{j}, \vec{k}$ enotski vektorji

Vsota kvaternionov: $p + q = (p_0 + q_0) + (p_1 + q_1)\vec{i} + (p_2 + q_2)\vec{j} + (p_3 + q_3)\vec{k}$

Produkt s skalarjem: $wq = wq_0 + wq_1\vec{i} + wq_2\vec{j} + wq_3\vec{k}$

Konjugiran kv.: $q^* = q_0 \cdot 1 - q_1\vec{i} - q_2\vec{j} - q_3\vec{k}$

$$\vec{i} \cdot \vec{i} = \vec{i}^2 = \vec{j}^2 = \vec{k}^2 = -1 = ijk$$

Množenje kv. ni komutativno.

	1	$\vec{i}$	$\vec{j}$	$\vec{k}$
1	1	$\vec{i}$	$\vec{j}$	$\vec{k}$
$\vec{i}$	$\vec{i}$	-1	$\vec{k}$	$-\vec{j}$
$\vec{j}$	$\vec{j}$	$-\vec{k}$	-1	$\vec{i}$
$\vec{k}$	$\vec{k}$	$\vec{j}$	$-\vec{i}$	-1



$$q = q_0 + \vec{q}$$

$$\vec{q} = q_1\vec{i} + q_2\vec{j} + q_3\vec{k}$$

dva kv. p, q

način ①
 $pq = (p_0 + p_1\vec{i} + p_2\vec{j} + p_3\vec{k})(q_0 + q_1\vec{i} + q_2\vec{j} + q_3\vec{k}) =$
 $= p_0q_0 + q_0(p_1\vec{i} + p_2\vec{j} + p_3\vec{k}) + p_0(q_1\vec{i} + q_2\vec{j} + q_3\vec{k}) + p_1q_1\vec{i}^2 + p_2q_1\vec{j}\vec{i} + p_3q_1\vec{k}\vec{i} +$
 $+ p_1q_2\vec{i}\vec{j} + p_2q_2\vec{j}^2 + p_3q_2\vec{k}\vec{j} + p_1q_3\vec{i}\vec{k} + p_2q_3\vec{j}\vec{k} + p_3q_3\vec{k}^2 = \dots =$
 $= p_0q_0 + p_0\vec{k} + q_0\vec{p} + (p_1q_1 - p_2q_2 - p_3q_3) + (p_2q_3 - p_3q_2)\vec{i} + (p_3q_1 - p_1q_3)\vec{j} + (p_1q_2 - p_2q_1)\vec{k}$
 ② $pq = p_0q_0 + p_0\vec{k} + q_0\vec{p} - \vec{p} \cdot \vec{q} + \vec{p} \times \vec{q}$

$$\textcircled{3} r = p\bar{q} = \overset{\text{realna}}{p_0 q_0} - \vec{p} \cdot \vec{q} + \overset{\text{img.}}{p_0 \vec{q} + q_0 \vec{p} + \vec{p} \times \vec{q}}$$

$$r_0 = p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 q_1 \\ p_0 q_2 \\ p_0 q_3 \end{bmatrix} + \begin{bmatrix} q_0 p_1 \\ q_0 p_2 \\ q_0 p_3 \end{bmatrix} + \begin{bmatrix} p_2 q_3 - p_3 q_2 \\ p_3 q_1 - p_1 q_3 \\ p_1 q_2 - p_2 q_1 \end{bmatrix}$$

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Primer:

$$\begin{aligned} (2+3\hat{i}-\hat{j}+5\hat{k}) \cdot (3-4\hat{i}+2\hat{j}+\hat{k}) &= 6+9\hat{i}-3\hat{j}+15\hat{k}-8\hat{i}-12\hat{i}^2+4\hat{j}\hat{i}-20\hat{k}\hat{i}+ \\ &+ 4\hat{j}+6\hat{i}\hat{j}-2\hat{j}^2+10\hat{k}\hat{j}+2\hat{k}+3\hat{i}\hat{k}-\hat{j}\hat{k}+5\hat{k}^2 = 6+9\hat{i}-3\hat{j}+15\hat{k}-8\hat{i}+12-4\hat{k}-20\hat{j}+ \\ &+ 4\hat{j}+6\hat{k}+2-10\hat{i}+2\hat{k}-3\hat{j}-\hat{i}-5 = 15-10\hat{i}-22\hat{j}+19\hat{k} \end{aligned}$$

$$r = p\bar{q} = p_0 q_0 + p_0 \vec{q} + q_0 \vec{p} - \vec{p} \cdot \vec{q} + \vec{p} \times \vec{q}$$

$$r = (2 + \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}) \cdot (3 + \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}) = 6 - [3 \ -1 \ 5] \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} + 2 \cdot \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} + 3 \cdot \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} + \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 5 \\ -4 & 2 & 1 \end{bmatrix} =$$

$$= 6+9 + \begin{bmatrix} 1 \\ 1 \\ 17 \end{bmatrix} + \begin{bmatrix} -11 \\ -23 \\ 2 \end{bmatrix} = 15 + \begin{bmatrix} -10 \\ -22 \\ 19 \end{bmatrix}$$

$$r = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 3 & 2 & -5 & -1 \\ -1 & 5 & 2 & -3 \\ 5 & 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \\ -22 \\ 19 \end{bmatrix}$$

$$\vec{r}_2 = \underline{R} \cdot \vec{r}_1$$

$\vec{r}_1$ zavrtimo v novo lego/orientacijo

$$r_2 = q \cdot r_1 \cdot q^*$$

* - konjugirano

$$q = q_0 + \vec{q}$$

$$r_1 = 0 + \vec{r}_1, \quad r_2 = 0 + \vec{r}_2$$

$$q^* = q_0 - \vec{q}$$

$$r_2 = (q_0 + \vec{q}) \cdot \underset{r_1=0}{(0 + r_1)} \cdot q^*$$

$$r_2 = \underbrace{(-\vec{q}\vec{r}_1)}_{\text{skalar}} + \underbrace{q_0 \vec{r}_1 + \vec{q} \times \vec{r}_1}_{\text{vektor}} \cdot (q_0 - \vec{q})$$

$$\begin{aligned} \vec{r}_2 = & -\vec{q} \cdot \vec{r}_1 \vec{q}_0 + \vec{q}_0 \vec{r}_1 \vec{q} + (\vec{q} \times \vec{r}_1) \cdot \vec{q} + (\vec{q} \cdot \vec{r}_1) \vec{q} + q_0^2 \vec{r}_1 + \vec{q}_0 (\vec{q} \times \vec{r}_1) - \\ & - \vec{q}_0 (\vec{r}_1 \times \vec{q}) - \underbrace{\vec{q} \times \vec{r}_1 \times \vec{q}}_{\substack{q(q \cdot r_1) \\ = (q \cdot q)r_1 + (r_1 \cdot q)\vec{q}}} \end{aligned}$$

$$\vec{r}_2 = q_0^2 \vec{r}_1 + (q \cdot q) \vec{r}_1 + 2q_0 (\vec{q} \times \vec{r}_1) + 2q(q \cdot \vec{r}_1)$$

$$(\vec{q} \times \vec{r}_1) = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ q_1 & q_2 & q_3 \\ r_{1x} & r_{1y} & r_{1z} \end{bmatrix}$$

$$2(\vec{q} \cdot \vec{r}_1) \vec{r}_1 = 2r_1 \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$$

$$\vec{r}_2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) \vec{I} + 2q_0 \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_3 & q_1 & 0 \end{bmatrix} + 2 \begin{bmatrix} q_1^2 & q_1 q_2 & q_1 q_3 \\ q_1 q_2 & q_1^2 & q_2 q_3 \\ q_1 q_3 & q_2 q_3 & q_3^2 \end{bmatrix}$$

$$\underline{\underline{R}} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

pretvorba iz kvatern.
v R matriko

$$\vec{r}_2 = \underline{\underline{R}} \vec{r}_1 \quad ; \quad \vec{r}_2 = \vec{q} \cdot \vec{r}_1 \cdot \vec{q}^*$$

$\vec{q} \mapsto \underline{\underline{R}}$

Kako izrazimo kvat. iz znanih členov rot. matrike?

$$\rightarrow q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$$

podana
je $\underline{\underline{R}}$
matrika

$$\underline{\underline{R}} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$q_0 = \frac{1}{2} \sqrt{1 + r_{11} + r_{22} + r_{33}}$$

$$q_1 = \frac{1}{2} \sqrt{1 + r_{11} - r_{22} - r_{33}}$$

$$q_2 = \frac{1}{2} \sqrt{1 - r_{11} + r_{22} - r_{33}}$$

$$q_3 = \frac{1}{2} \sqrt{1 - r_{11} - r_{22} + r_{33}}$$

$$\text{trace } \underline{\underline{R}} = r_{11} + r_{22} + r_{33} = 3q_0^2 - q_1^2 - q_2^2 - q_3^2 = 4q_0^2 - 1$$

zapis krat. primeren za opis rotacije

$$\vec{q} = \cos \frac{\psi}{2} + \sin \frac{\psi}{2} \cdot \vec{s}$$

$\vec{s}$ - enotski vektor 1×3

$$\vec{q} = \cos \frac{\psi}{2} + s_x \sin \frac{\psi}{2} \hat{x} + s_y \sin \frac{\psi}{2} \hat{y} + s_z \sin \frac{\psi}{2} \hat{z}$$

vstavimo v rot. matriko

$$r_{11} = q_0^2 + q_1^2 - q_2^2 - q_3^2 = q_0^2 + 2q_1^2 - q_2^2 - q_3^2 = q_0^2 + 2q_1^2 - 1 + q_0^2 =$$

$$= 2q_0^2 + 2q_1^2 - 1 = 2\cos^2 \frac{\psi}{2} + 2s_x^2 \sin^2 \frac{\psi}{2} - 1 =$$

$$= 1 + \cos \psi + s_x^2 \text{vers} \psi - 1 = \cos \psi + s_x^2 \text{vers} \psi$$

Bronstajn

$$\cos^2 \frac{\psi}{2} - \sin^2 \frac{\psi}{2} = \cos \psi$$

$$\cos^2 \frac{\psi}{2} + \sin^2 \frac{\psi}{2} = 1$$

$$2\cos^2 \frac{\psi}{2} = 1 + \cos \psi$$

$$2\sin^2 \frac{\psi}{2} = 1 - \cos \psi = \text{vers} \psi$$

$$\underline{R} = \begin{bmatrix} s_x^2 \text{vers} \psi + c\psi & s_x s_y \text{vers} \psi - s_z s\psi & s_x s_z \text{vers} \psi + s_y s\psi \\ s_x s_y \text{vers} \psi + s_z s\psi & s_y^2 \text{vers} \psi + c\psi & s_y s_z \text{vers} \psi - s_x s\psi \\ s_x s_z \text{vers} \psi - s_y s\psi & s_y s_z \text{vers} \psi + s_x s\psi & s_z^2 \text{vers} \psi + c\psi \end{bmatrix}$$

Primer: $\vec{s} = [1 0 0]^T$; $q = \cos \frac{\psi}{2} + \sin \frac{\psi}{2} \hat{x}$

ta krat. ima takšno rot. matriko

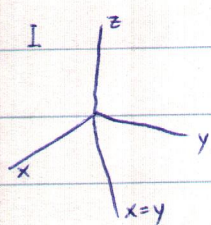
$$\underline{R}_x = \begin{bmatrix} \text{vers} \psi + c\psi & 0 & 0 \\ 0 & c\psi & -s\psi \\ 0 & s\psi & c\psi \end{bmatrix}$$

I.

1. Zapiši s kvaternionom rot. za $\frac{\pi}{2}$ okrog osi, kijo opisujeta enačbi $x=y$ in $z=0$,

ter rot. za $\frac{2\pi}{3}$ okrog osi, kijo opisujeta enačbi $x=y$ in $z=x$. S produktom obeh

kvaternionov določi še celokupno rotacijo.

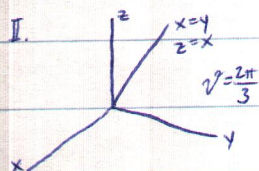


$$\vec{q} = \cos \frac{\psi}{2} + \sin \frac{\psi}{2} \vec{s}$$

$$\vec{q} = q_0 + q_1 \hat{x} + q_2 \hat{y} + q_3 \hat{z}$$

$$\vec{s} = \frac{1}{\sqrt{2}} [1, 1, 0]$$

$$\vec{q} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \cdot \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{\sqrt{2}}{2} + \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$



$$\vec{s} = \frac{1}{\sqrt{3}} [1, 1, 1]^T$$

$$\vec{p} = \cos \frac{\psi}{2} + \sin \frac{\psi}{2} \vec{s}$$

$$\vec{p} = \cos \left(\frac{2\pi}{3 \cdot 2} \right) + \sin \left(\frac{2\pi}{3 \cdot 2} \right) \vec{s}$$

$$\vec{p} = \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{2} + \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{aligned}
 \text{III. } \vec{q} \cdot \vec{p} &= \vec{q} \otimes \vec{p} = \frac{1}{4} (\sqrt{2} + \vec{i} + \vec{j}) (1 + \vec{i} + \vec{j} + \vec{k}) = \\
 &= \frac{1}{4} (\sqrt{2} + \sqrt{2}\vec{i} + \sqrt{2}\vec{j} + \sqrt{2}\vec{k} + \vec{i} - 1 + \cancel{\vec{i}\vec{j}} + \cancel{\vec{i}\vec{k}} + \cancel{\vec{j}\vec{i}} + \cancel{\vec{j}\vec{k}} - 1 + \vec{j}\vec{k}) = \\
 &= \frac{1}{4} ((\sqrt{2}-2) + (\sqrt{2}+2)\vec{i} + \sqrt{2}\vec{j} + \sqrt{2}\vec{k})
 \end{aligned}$$

Rotacija s kvaternionom

$$\vec{r}_2 = \vec{q} \cdot \vec{r}_1 \cdot \vec{q}^* \quad \vec{r}_2 = 0 + r_2$$

$$\vec{r}_1 = 0 + r_1$$

Primer: Z uporabo kvat. zasuci vektor $r_1 = [1, 1, 0]^T$ za 90° v pozit. smeri okrog osi z.

$$\vec{q} = \cos \frac{\psi}{2} + \sin \frac{\psi}{2} \vec{s} \quad , \quad \vec{s} = [0, 0, 1]^T$$

$$q = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$p \otimes q = p_0 q_0 - \vec{p} \cdot \vec{q} - \vec{p}_0 \vec{q} + q_0 \vec{p} + \vec{p} \times \vec{q}$$

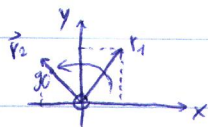
$$r_2 = q \cdot r_1 \cdot q^*$$

$$r_2 = \frac{\sqrt{2}}{2} (1 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}) (0 + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}) \frac{\sqrt{2}}{2} (1 - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})$$

$$r_2 = \frac{1}{2} (-[0, 0, 1] \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}) (1 - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})$$

$$r_2 = \frac{1}{2} (0 + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}) (1 - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix})$$

$$\begin{aligned}
 \vec{r}_2 &= \frac{1}{2} (0 + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}) (1 - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}) = \frac{1}{2} ([0, 2, 0] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}) = \\
 &= \frac{1}{2} (0 + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}) = (0 + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix})
 \end{aligned}$$



Primer: Z uporabo kvat. zavrtite enotski vektor $\vec{j}$ za 180° okrog osi, določene z $x=y$ in $z=0$.

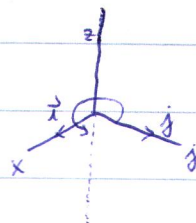
Kateri vektor dobimo?

$$q = \cos \frac{\psi}{2} + \sin \frac{\psi}{2} \vec{s} \quad \frac{180}{2}$$

$$q = 0 + 1 \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{\sqrt{2}}{2} (\vec{i} + \vec{j})$$

$$q^* = \frac{\sqrt{2}}{2} (-\vec{i} - \vec{j})$$

$$r_2 = q \cdot r_1 \cdot q^* = \frac{1}{2} (\vec{i} + \vec{j}) \cdot \vec{j} \cdot (-\vec{i} - \vec{j}) = \frac{1}{2} (\vec{k} - 1) \cdot (-\vec{i} - \vec{j}) = \vec{x}$$



GEOMETRIJA ROBOTSKIH MEHANIZMOV

Tadej Bajd

direktna, inverzna geometrija

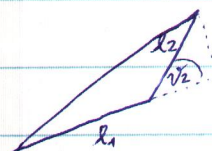
Euklidov Δ

Dvosegmentni ravninski robot

$$\theta_2 = \arccos \frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$$\theta_1 = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{l_2 \sin \theta_2}{l_1 + l_2 \cos \theta_2}\right)$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$



elbow up/down

Trisegmentni ravninski robot

$$p_1 = l_1 c_1 + l_2 c_{12} + l_3 c_{123}$$

$$p_2 = l_1 s_1 + l_2 s_{12} + l_3 s_{123}$$

orient. zaporedje $p_3 = q_1 + q_2 + q_3 \rightarrow q_3 = p_3 - q_1 - q_2$

$$\alpha = \arccos\left(\frac{l_1^2 + l_2^2 - l^2}{2 l_1 l_2}\right)$$

$$q_2 = \pm (\pi - \alpha)$$

singularna lega
(ni dveh rešitev)

Antropomorfni robotski manipulator

same rotac. prost. stopnje (stičnili)

$${}^0H_6 = \begin{bmatrix} n & s & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q = \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} - d_6 \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \quad \theta_1 = \arctan_2 \frac{q_y}{q_x}$$

↑
pri kvocienitih

$$\alpha = \arccos\left(1 - \frac{1}{2} \left(\frac{s}{a_z}\right)^2\right), \quad r = \sqrt{q_x^2 + q_y^2 + (q_z - d_1)^2}$$

$$\theta_3 = \alpha + \frac{\pi}{2}$$

$$\theta_3 = \frac{\pi}{2} - \alpha$$

$$\theta = \frac{(\pi - \alpha)}{2}$$

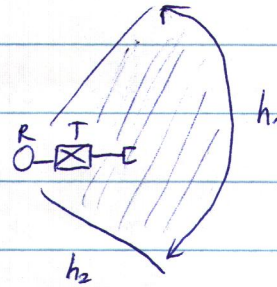
$$\theta = \arctan_2 \frac{q_z - d_1}{\sqrt{q_x^2 + q_y^2}}$$

Altaar
-27-

$$\vartheta_4 = \arctan_2 \frac{a_{y0}}{a_{x0}}, \quad \vartheta_5 = \arctan_2 \frac{\sqrt{a_{x0}^2 + a_{y0}^2}}{a_{z0}}, \quad \vartheta_6 = \dots$$

Dvodimenzionalni delovni prostor

odvisen od prost. stopenj, kako so razporejeni,
omejitve v sklepih



Dvosegmentni manipulator

mreža, liki, ploščina

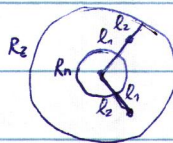
Vpliv dolžine segmentov

l_1 in l_2 vplivata na delovni prostor

$$0^\circ \leq \vartheta_1, \vartheta_2 \leq 360^\circ$$

$$A = \pi R_z^2 - \pi R_n^2 \dots$$

$$l_1 = l_2$$



dosegljivi in priročni delovni prostor

inv. model mora biti v vsakem robota

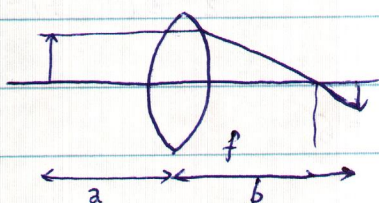
del. prostor pomemben že ob nakupu robota

PERSPEKTIVISTIČNA TRANSFORMACIJA

vodi do skaliranja prostorov

preslikava predmeta preko leče

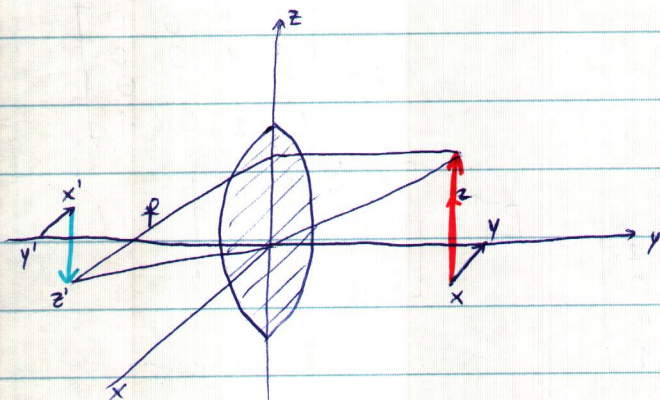
f goriščna razdalja



enačba leče

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

Lečo postavimo v x - z ravnino.



uklonjeni: $\frac{1}{y} - \frac{1}{y'} = \frac{1}{f} \rightarrow \frac{1}{y'} = \frac{1}{y} - \frac{1}{f} = \frac{f-y}{yf}$

neuklonjeni: $\frac{x}{y} = \frac{x'}{y'} ; y' = \frac{x'y}{x}$

$$y' = \frac{yf}{f-y}$$

$$\frac{x'}{x} \cdot y' = \frac{yf}{f-y}$$

$$x' = \frac{x}{1 - \frac{y}{f}}$$

$$y' = \frac{y}{1 - \frac{y}{f}}$$

$$z' = \frac{z}{1 - \frac{y}{f}}$$

Isto dosežemo s homogeno transf. matriko.

brez rot.
transf.

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 - \frac{y}{f} \end{bmatrix} = \begin{bmatrix} \frac{x}{1 - \frac{y}{f}} \\ \frac{y}{1 - \frac{y}{f}} \\ \frac{z}{1 - \frac{y}{f}} \\ 1 \end{bmatrix}$$

matrika povzroči isto preslikavo

Numerični primer:

$f=2$, v ravnini $x-z$, središče leče sovpada z izhodiščem k.s.,

T se preslika v $T' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}$. Zanima nas T .

$$y' = \frac{y}{1 - y/f}$$

$$x = x'(1 - y'/f) = -1(1 - \frac{6}{2}) = 2$$

$$y' - (1 - \frac{y'}{f}) = y$$

$$z = z'(1 - y'/f) = -2(1 - \frac{6}{2}) = 4$$

$$y' = y(1 + \frac{y'}{f})$$

$$y = \frac{y'}{1 + y'/f} = \frac{-3}{1 - \frac{3}{2}} = 6$$

z matrikami (s perspektivistično matriko)

$$\underline{T'} = \underline{P} \cdot \underline{T}$$

$$\checkmark \quad \checkmark \quad ?$$

$$\underline{T} = \underline{P}^{-1} \cdot \underline{T'} \quad ; \quad \underline{P} \cdot \underline{P}^{-1} = \underline{I}$$

$$\underline{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix}$$

$$\underline{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{f} & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -2 \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

$\cdot (-2)$
 $\curvearrowright$

ROBOTIKA - BIOMEHANIKA - ŠPORT

Roman Kamnik

↳ proučuje strukture in funkcije bioloških sistemov z znanjem mehanike

področja uporabe: medicina (kirurgija, umetni organi...), humanooidna rob.,
protetika, ortotika, rehabilitacija, eksoskeleti

EU projekt Cyberlegs (rob. proteza)

šport (razvoj sodobnih naprav za vadbo - magnetni princip, uporaba magnetoreološke tekočine, ^{spreminj. lastnost upor.}
naprava gnana s hidravličnimi aktuatorji, haptično), veslanje

Sistemi za merjenje sil (princip deformacije upor. listčev, eno- ali večosni senzor sil)

-11- kinematike gibanja (3D inercialni senzor)

DIGITALNO VODENJE

kodiranje odprtozaprte sisteme

dif. kinematika in dinamika robotov

Jacob matrika (transf. hitrosti)

$$\dot{p} = J \cdot \dot{q}$$

$$J^* = J^T \cdot (J \cdot J^T)^{-1}$$

$$\tau = J^T \cdot F$$

↳ ko ima robot več prost. stopenj, kot jih rabi

pri transf. sil in navorov

||
redundanten

$$f_i = R_i^{im} f_{im} + m_i a_{ci} - m_i g_i$$

roboti in merilni vgrajeni sistemi

načrtovanje elektromehanskih izdelkov

roboti vid (prepoznavanje značilk...)

vodenje robotov

mero.si

roboti mehanizmi

roboti v stiku s človekom

haptični roboti (sila na oblogo $F = ma + bv + kx$)

seminar iz robotike in merjenj (industrijski problem)

