Predmet:
Signali

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SIGNALI

laboratorijske in avditorne

ENERGIJSKI IN MOČNOSTNI SIGNALI

MATLAB tutorial

1. Močnostne

$P_f < \infty \quad P_f \neq 0$

$$\boxed{P_f = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} f(t)\overline{f(t)}\,dt}$$ spl. enačba

$$\boxed{P_f = \frac{1}{T} \int_{t_1}^{t_1+T} f(t)\overline{f(t)}\,dt}$$ za periodične signale

2. Energijski (če je njegova energija končna)

$E_f < \infty$

$$\boxed{E_f = \lim_{T\to\infty} \int_{-T}^{T} f(t)\overline{f(t)}\,dt}$$

WOLFRAMALPHA

P(x)

- Dan je signal. Ker je periodičen, določi moč na dva načina (po spl. def., po zvezi za period. s.)

$\underline{x(t) = \cos(t)}$

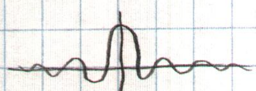
$P_x = ?$

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos^2 t\,dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left(\frac{1}{2} + \frac{\cos 2t}{2}\right) dt =$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\frac{1}{2}t \Big|_{-T}^{T} + \frac{\sin 2t}{4} \Big|_{-T}^{T} \right) =$$

$$\cos^2 t = \frac{1+\cos 2t}{2}$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left(\frac{1}{2}(T+T) + \frac{\sin 2t}{4} - \frac{\sin(-2t)}{4} \right) =$$



$$= \frac{1}{2} + \underbrace{\lim_{T\to\infty} \frac{1}{2T} \left(\frac{2\sin 2t}{4} \right)}_{0} = \underline{\underline{\frac{1}{2}}}$$

$T = 2\pi$

$$P_x = \frac{1}{2\pi} \int_0^{2\pi} \cos^2 t\,dt = \frac{1}{2\pi} \left(\frac{1}{2}t \Big|_0^{2\pi} + \frac{\sin 2t}{4} \Big|_0^{2\pi} \right) = \frac{1}{2\pi}\left(\frac{1}{2}2\pi + 0\right) = \underline{\underline{\frac{1}{2}}}$$

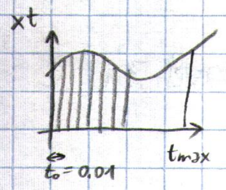
- Signal $x(t) = \begin{cases} 5e^{-2t}; & t \geq 0 \\ 0; & \text{sicer} \end{cases}$. Izr. energ. signala $x(t)$.

$$E_f = \lim_{T\to\infty} \int_{-T}^{T} f(t)\overline{f(t)}\,dt = \lim_{T\to\infty} \int_{-T}^{T} 25e^{-4t}\,dt = \lim_{T\to\infty} \left(25 \frac{e^{-4t}}{-4} \Big|_0^{T} \right) = \lim_{T\to\infty} \left(-\frac{25}{4}(e^{-4T} - 1) \right) = \frac{25}{4}$$

```
function narisi (xt,t)      (ime dat. enako!)
  plot (t,xt);
end

[xt2,t2] = narisi (xt,t)
```

```
M=randn(3,4)
M(1,2)
M(3,:)
t=0:10    ali t=0:0.1:10
xt=sin(t);
whos
plot(xt,t)
xlabel ('Cas')
ylabel ('Sin')
title('Funkcija')
subplot(1,2,1)
```



$t = [0 \ 0.01 \ 0.02 \ldots \ t_{max}]$
$xt = [\qquad]$

$T = \infty \rightarrow T$ je velik

$$E_f = \lim_{T \to \infty} \int_{-T}^{T} x(t) \, x(t) \, dt$$

v matlabu: $E_f = \sum xt * xt \cdot t_0$
sum
↳ v for zanko

$t = 0 : t_0 : T$
0.0001 ↗ ↗ 1000 000

// rač. zmrzne!

ne več kot 10.000

vsota = 0
for (cez xt)

DN (pregleda prof., oddaj prof. ali asist.)

IZRAŽANJE SIGNALOV S TEMELJNIMI FUNKCIJAMI

$f(x) \approx \tilde{f}(x)$ aproksimacija signala

$$\tilde{f}(t) = C_0 \phi_0(t) + C_1 \phi_1(t) + \ldots + C_n \phi_n(t)$$

↳ lin. kombinacija temeljnih funkcij

Koliko sta si signala podobna?

$$\boxed{\overline{\varepsilon}^2 = \frac{1}{T} \int_{t_0}^{t_0+T} (f(t) - \tilde{f}(t)) \overline{(f(t) - \tilde{f}(t))} \, dt} = *$$

ortogonalnost

$$\phi_i \phi_j = \begin{cases} K_i & i = j \\ 0 & i \neq j \end{cases} \quad \text{ortogonalnost}$$

$K_i = 1$ ortonormalnost

$$\boxed{C_i = \frac{1}{K_i} \int_{t_0}^{t_0+T} f(t) \overline{\phi_i(t)} \, dt}$$

$$\boxed{* = \frac{1}{T} \left(\int f(t) \tilde{f}(t) \, dt - \sum_{i=0}^{n} K_i \cdot C_i \cdot C_i \right)}$$

$\lim_{n \to \infty} \overline{\varepsilon}^2 = 0$ govorimo o polnem sistemu temeljnih funkcij

WALSHEVE TEMELJNE FUNKCIJE

- ortonormalne; $K_i = 1$
- zavzamejo 2 vrednosti: $1, -1$
- definirane med 0 in 1

rekurzivna formula:

$$H(0) = 1$$

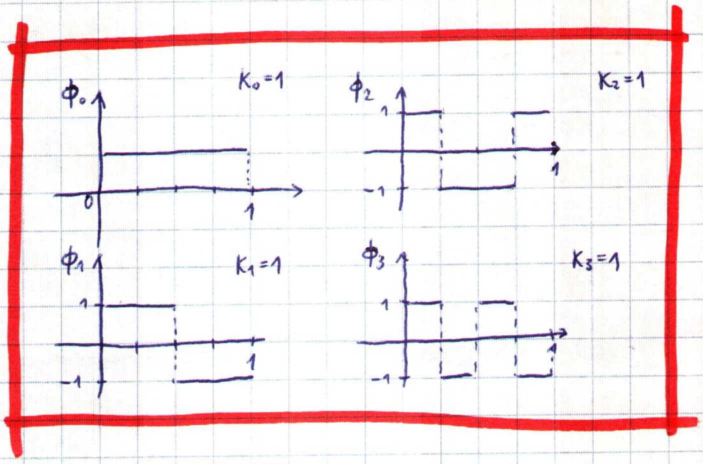
$$H(m) = \begin{bmatrix} H(m-1) & H(m-1) \\ H(m-1) & -H(m-1) \end{bmatrix}$$

2^m Walshevih funkcij

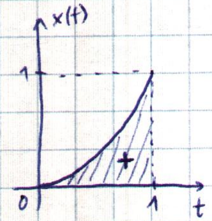
Primer:

$$H(0) = 1$$
$$H(1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
$$H(2) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$\phi_0 \quad \phi_3 \quad \phi_1 \quad \phi_2$ kokrat gremo skoz ničlo



Signal $x(t)=t^2$ med $[0,1]$. Izrazite s prbl. $\tilde{x}(t)$, ki ga tvorijo prve 4 Walsheve funkcije! Določite sr. kvadr. napako in skicirajte približek.



$$\tilde{x}(t) = C_0\phi_0(t) + C_1\phi_1(t) + C_2\phi_2(t) + C_3\phi_3(t) = \frac{1}{3}\phi_0(t) - \frac{1}{4}\phi_1(t) + \frac{1}{16}\phi_2(t) - \frac{1}{8}\phi_3(t)$$

$$C_i = \frac{1}{K_i}\int_{t_0}^{t_0+T} x(t)\,\phi_i(t)\,dt$$

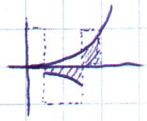
$$C_0 = \frac{1}{1}\int_0^1 t^2 \cdot 1\,dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$$

$C_0 = \frac{1}{3}$, $C_1 = -\frac{1}{4}$, $C_2 = \frac{1}{16}$, $C_3 = -\frac{1}{8}$

$$C_1 = \frac{1}{1}\left(\int_0^{\frac{1}{2}} t^2\,dt - \int_{\frac{1}{2}}^1 t^2\,dt\right) = $$
$$= \frac{t^3}{3}\Big|_0^{\frac{1}{2}} - \frac{t^3}{3}\Big|_{\frac{1}{2}}^1 = \frac{1}{3}\left(\frac{1}{8} - \left(\frac{8}{8} - \frac{1}{8}\right)\right) = -\frac{1}{4}$$

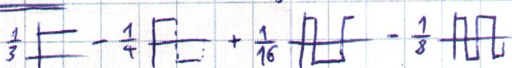
$\cdot(-1) \rightarrow$ zrc. čez x-os

$$C_2 = \frac{1}{1}\left(\int_0^{\frac{1}{4}} t^2\,dt - \int_{\frac{1}{4}}^{\frac{3}{4}} t^2\,dt + \int_{3/4}^{4/4} t^2\,dt\right) = \frac{t^3}{3}\Big|_0^{\frac{1}{4}} - \frac{t^3}{3}\Big|_{\frac{1}{4}}^{\frac{3}{4}} + \frac{t^3}{3}\Big|_{\frac{3}{4}}^{\frac{4}{4}} = $$
$$= \frac{1}{3}\left(\frac{1}{64} - \left(\frac{27}{64} - \frac{1}{64}\right) + \left(\frac{64}{64} - \frac{27}{64}\right)\right) = \frac{1}{3\cdot 64}(1 - 26 + 37) = \frac{1}{16}$$

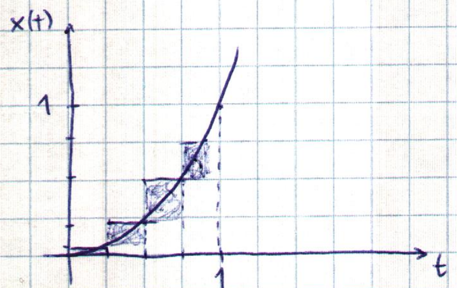


$$C_3 = \frac{1}{1}(\ldots) = \frac{t^3}{3}\Big|_0^{\frac{1}{4}} - \frac{t^3}{3}\Big|_{1/4}^{2/4} + \frac{t^3}{3}\Big|_{2/4}^{3/4} - \frac{t^3}{3}\Big|_{3/4}^{4/4} = $$
$$= \frac{1}{3}\left(\frac{1}{64} - \left(\frac{8}{64} - \frac{1}{64}\right) + \left(\frac{27}{64} - \frac{8}{64}\right) - \left(\frac{64}{64} - \frac{27}{64}\right)\right) = \frac{1}{3\cdot 64}(1 - 7 + 19 - 37) = -\frac{1}{8}$$

$$\overline{\varepsilon^2} = \frac{1}{T}\left(\int_{t_0}^{t_0+T} x(t)\,x(t)\,dt - \sum_{i=0}^{n} K_i C_i C_i\right) = 1\left(\int_0^1 t^2 t^2\,dt - \left(\frac{1}{9} + \frac{1}{16} + \frac{1}{256} + \frac{1}{64}\right)\right) = $$
$$= \frac{t^5}{5}\Big|_0^1 - (-\text{"}-) = \frac{79}{11520} \approx \underline{0{,}00686} \quad \rightarrow \text{ srednja kvadratna napaka}$$



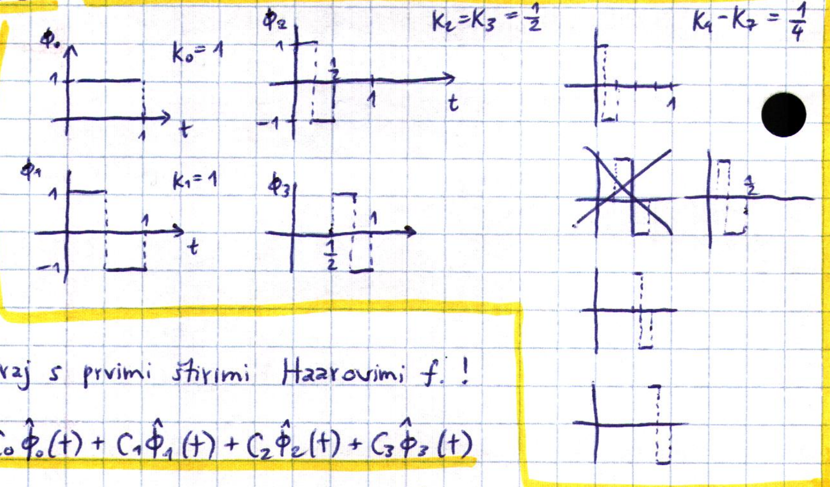
$$\frac{1}{3}\cdot 1 - \frac{1}{4}\cdot 1 + \frac{1}{16}\cdot 1 - \frac{1}{3}\cdot 1 = \frac{1}{48}$$
$$\frac{1}{3}\cdot 1 - \frac{1}{4}\cdot 1 - \frac{1}{16} + \frac{1}{8} = \frac{9}{48}$$
$$\frac{1}{3} + \frac{1}{4} - \frac{1}{16} - \frac{1}{8} = \frac{19}{48}$$
$$\frac{1}{3} + \frac{1}{4} + \frac{1}{16} + \frac{1}{8} = \frac{27}{48}$$



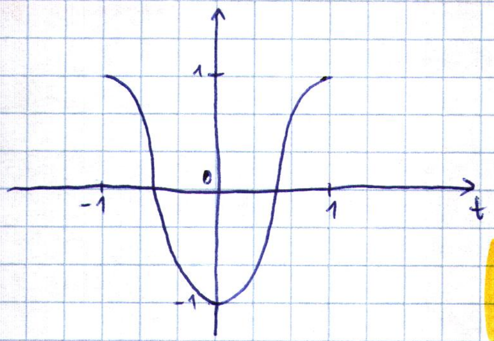
HAAROVE TEMELJNE FUNKCIJE

(lahko imaš na izpitu)

- ortogonalne
- zavzemajo 3 vrednosti $0, 1, -1$
- definirane med 0 in 1



Signal $x(t) = -\cos \pi t$ na $[-1,1]$ aproksimiraj s prvimi štirimi Haarovimi f. !



$$\tilde{x}(t) = C_0 \hat{\phi}_0(t) + C_1 \hat{\phi}_1(t) + C_2 \hat{\phi}_2(t) + C_3 \hat{\phi}_3(t)$$

$$\hat{\phi}_i(t) = \phi_i(u(t)) \qquad \hat{K}_i = \frac{k_i}{a} = 2K_i$$

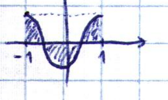
$$u(t) = at + b \qquad \hat{K}_0 = \hat{K}_1 = 2$$

$$\hat{K}_2 = \hat{K}_3 = 1$$

$$u(t): \overset{t\ \ t}{[-1,1]} \rightarrow \overset{u(t)\ u(t)}{[0,1]}$$

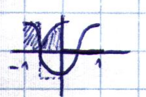
$$\left.\begin{aligned} 0 &= -a + b \\ 1 &= a + b \end{aligned}\right\} +$$

$$1 = 2b \Rightarrow b = \frac{1}{2},\ a = \frac{1}{2}$$



$$C_0 = \underset{K_0}{\frac{1}{2}} \int_{-1}^{1} -\cos \pi t \, dt = -\frac{1}{2}\left(\frac{\sin \pi t}{\pi}\Big|_{-1}^{1}\right) = 0$$

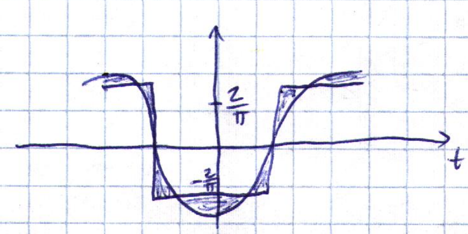
$$C_1 = 0$$



$$C_2 = \frac{1}{\hat{K}_2} \int_{-1}^{1} x(t) \hat{\phi}_2(t) \, dt = 1\left(\int_{-1}^{-\frac{1}{2}} (-\cos \pi t) \, dt + \int_{-\frac{1}{2}}^{0} \cos \pi t \, dt\right) = -\frac{\sin \pi t}{\pi}\Big|_{-1}^{-\frac{1}{2}} + \frac{\sin \pi t}{\pi}\Big|_{-\frac{1}{2}}^{0} = \frac{1}{\pi} - (-1-0) + (0-(-1)) = \frac{2}{\pi}$$

$$C_3 = -\frac{2}{\pi}$$

$$\tilde{x}(t) = \frac{2}{\pi} \hat{\phi}_2(t) - \frac{2}{\pi} \hat{\phi}_3(t)$$



FOURIERJEVA VRSTA

• $f(t) = f(t+T)$

1.) realna FV

$$\boxed{f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t}$$ $$\omega = \frac{2\pi}{T}$$

2.) kompleksna FV

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} F(n)\, e^{jn\omega t}}$$ $$e^{j\theta} = \cos\theta + j\sin\theta$$

Dirichletovi pogoji:

1.) $\int_{t_0}^{t_0+T} |f(t)|\, dt < \infty$

2.) končno št. končnih max in min

3.) končno število nezveznosti

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t) \cos n\omega t\, dt \qquad b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t) \sin n\omega t\, dt$$

$$F(n) = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-jn\omega t}\, dt$$

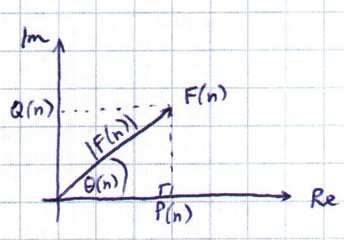
$a_n = F(n) + \overline{F(n)}$;
$b_n = j\left(F(n) - \overline{F(n)}\right)$

$$F(n) = \frac{a_n - j b_n}{2}$$

Kompleksni spekter

$$F(n) = \underbrace{P(n)}_{\text{realni spekter}} + j\,\underbrace{Q(n)}_{\text{imaginarni spekter}} =$$

$$= \underbrace{|F(n)|}_{\text{amplitudni spekter}}\, e^{\overbrace{j\theta(n)}^{\text{fazni spekter}}}$$



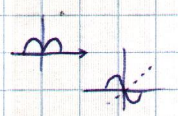
$$|F(n)| = \sqrt{P(n)^2 + Q(n)^2} = \sqrt{F(n)\,\overline{F(n)}} =$$ tako izr. ampl spekter

$$= \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

$$\theta(n) = \begin{cases} \operatorname{arctg} \frac{Q(n)}{P(n)} & ; P(n) > 0 \\ \operatorname{arctg} \frac{Q(n)}{P(n)} \pm \pi & ; P(n) < 0 \end{cases}$$

Lastnosti:

1.) $P(n) = P(-n)$ sod
2.) $Q(n) = -Q(-n)$ lih
3.) $|F(n)| = |F(-n)|$ sod
4.) $\theta(n) = -\theta(-n)$ lih



- $f(t)$ je realen, sod (se poenostavi)

$$F(n) = P(n) + \overbrace{jQ(n)}^{0} = P(n)$$

$$\boxed{F(n) = P(n) = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega t \, dt}$$

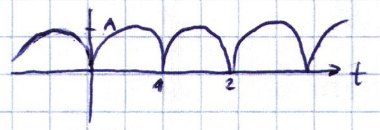
- $f(t)$ je lih, realen

$$F(n) = \overbrace{P(n)}^{0} + jQ(n) = jQ(n)$$

$$\boxed{F(n) = jQ(n) = -j\frac{2}{T}\int_0^{T/2} f(t)\sin n\omega t \, dt}$$

■ Izrazi period. f. $f(t)$ s kompl. FV! Določi koef. realne FV ter zapiši realno FV. Določi in nariši ampl. in fazni spekter

$$f(t) = \begin{cases} A\sin\pi t \ ; \ 0<t<1; \ A>0 \\ f(t) = f(t+n) \ ; \ n \in \mathbb{Z} \end{cases}$$



$$F(n) = P(n) = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega t \, dt = 2A\int_0^{1/2} \sin\pi t \cos n2\pi t \, dt =$$

$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$

$\alpha+\beta = \pi t + n2\pi t = (1+2n)\pi t$

$\alpha-\beta = \pi t - n2\pi t = (1-2n)\pi t$

$$= A\left(\int_0^{1/2} \sin(1+2n)\pi t \, dt + \int_0^{1/2} \sin(1-2n)\pi t \, dt\right) =$$

$$= A\left(-\frac{\cos(1+2n)\pi t}{(1+2n)\pi}\Big|_0^{1/2} - \frac{\cos(1-2n)\pi t}{(1-2n)\pi}\Big|_0^{1/2}\right) = \frac{A}{\pi}\left(\frac{(0+1)(1-2n)}{(1+2n)(1-2n)} + \frac{(0+1)(1+2n)}{(1-2n)(1+2n)}\right) =$$

$$= \frac{A}{\pi}\left(\frac{1-2n+1+2n}{1-4n^2}\right) = \frac{2A}{\pi(1-4n^2)}$$

$$f(t) = \frac{2A}{\pi}\sum_{n=-\infty}^{\infty} \frac{e^{jn2\pi t}}{1-4n^2}$$

$$a_n = F(n) + \overline{F(n)} = \frac{2A+2A}{\pi(1-4n^2)} = \frac{4A}{\pi(1-4n^2)}$$

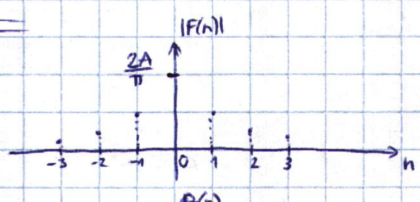
$$a_0 = \frac{4A}{\pi}$$

$$b_n = j(F(n) - \overline{F(n)}) = \underline{0}$$

R.F.V.

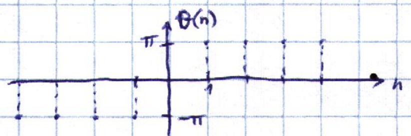
$$f(t) = \frac{2A}{\pi} + \sum_{n=1}^{\infty} \frac{4A}{\pi(1-4n^2)} \cos n2\pi t$$

■ $$|F(n)| = \begin{cases} \frac{2A}{\pi} \ ; \ n=0 \\ \frac{2A}{\pi(4n^2-1)} \ ; \ n\neq 0 \end{cases}$$



$$P(n) = \begin{cases} \frac{2A}{\pi} > 0 & n=0 \\ \frac{2A}{\pi(1-4n^2)} < 0 & n\neq 0 \end{cases}$$

$$\theta(n) = \begin{cases} \pi \ ; \ n>0 \\ 0 \ ; \ n=0 \\ -\pi \ ; \ n<0 \end{cases}$$



$$\theta(n) = \mathrm{arctg}\frac{Q(n)}{P(n)} = \underline{0} \quad n=0$$

$$\theta(n) = \underbrace{\mathrm{arctg}\frac{Q(n)}{P(n)}}_{0} \pm \pi = \underline{\underline{\pm\pi}}$$

$$\theta(n) = \begin{cases} \mathrm{arctg}\frac{Q}{P} \ ; \ P>0 \\ \mathrm{arctg}\frac{Q}{P} \pm \pi \ ; \ P<0 \end{cases}$$

FOURIERJEVA TRANSFORMACIJA

$f(t)$ ... neperiodični

Dirichletovi pogoji

1.) $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$

2.) max, min

3.) končno $\neq$ nezveznost

FT: $\boxed{\mathcal{F}(f(t)) = F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt}$

IFT: $\mathcal{F}^{-1}(F(\omega)) = f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, dt$

$f(t) \leftrightarrow F(\omega)$

$F(\omega) = P(\omega) + jQ(\omega) = |F(\omega)|\, e^{j\theta(\omega)}$

komp. sp. signala — spekter amplitudne gostote — fazni spekter

$|F(\omega)|$ ... prepiši od FV

$\theta(\omega)$ ... -||-

Lastnosti:

1.) $P(\omega)$ sod

2.) $Q(\omega)$ lih

3.) $|F(\omega)|$ sod

4.) $\theta(\omega)$ lih

- $f(t)$ je realen, sod

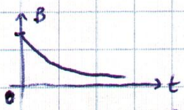
$$F(\omega) = P(\omega) = 2\int_0^{\infty} f(t) \cos \omega t\, dt$$

- $f(t)$ je realen, lih

$$F(\omega) = jQ(\omega) = -2j \int_0^{\infty} f(t) \sin \omega t\, dt$$

■ Izr. kompl. spekter, spekter ampl. in faz. gostote za signal

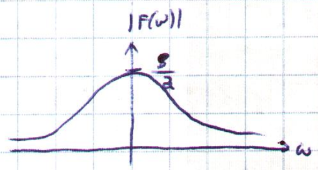
$$f(t) = \begin{cases} Be^{-at}; & t \geq 0,\ B, a > 0 \\ 0; & \text{sicer} \end{cases}$$



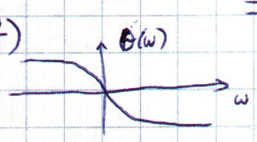
$$F(\omega) = B\int_0^{\infty} e^{-at} e^{-j\omega t}\, dt = B\int_0^{\infty} e^{-(a+j\omega)t}\, dt = B \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_0^{\infty} = \frac{B}{-(a+j\omega)}(0-1) = \frac{B\,(a-j\omega)}{(a+j\omega)(a-j\omega)} =$$

$$= \frac{Ba}{a^2+\omega^2} - j\frac{B\omega}{a^2+\omega^2}$$

$$|F(\omega)| = \sqrt{P(\omega)^2 + Q(\omega)^2} = \sqrt{\frac{B^2a^2 + B^2\omega^2}{(a^2+\omega^2)^2}} = \sqrt{\frac{B^2(a^2+\omega^2)}{(a^2+\omega^2)^2}} = \frac{B}{\sqrt{a^2+\omega^2}}$$



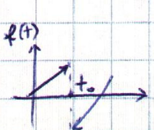
$$\theta(\omega) = \text{arctg}\left(\frac{-\frac{B\omega}{a^2+\omega^2}}{\frac{Ba}{a^2+\omega^2}} \right) = \text{arctg}\left(-\frac{\omega}{a}\right) = -\text{arctg}\left(\frac{\omega}{a}\right)$$



Lastnosti:

1.) linearnost

$$f_2(t) = af_a(t) + bf_b(t)$$
$$F_2(\omega) = aF_a(\omega) + bF_b(\omega)$$



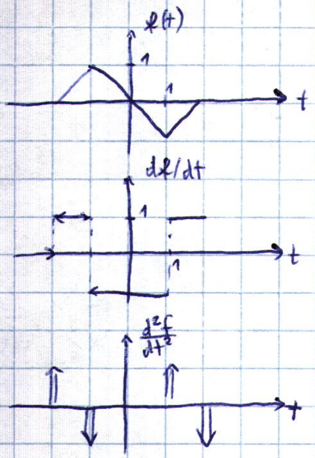
$$f(t_0) = \frac{1}{2}\left[f^+ + f^-\right]$$

$$\left.\frac{df(t)}{dt}\right|_{t=t_0} = \delta(t-t_0)\left[f^+(t_0) - f^-(t_0)\right]$$

2.) odvodi

$$\mathcal{F}\left(\frac{d^n f(t)}{dt^n}\right) = (j\omega)^n F(\omega)$$
$$\mathcal{F}\left(a\delta(t-t_0)\right) = a\, e^{-j\omega t_0}$$
$$\mathcal{F}\left(\left.\frac{df(t)}{dt}\right|_{t=t_0}\right) = e^{-j\omega t_0}\left[f^+(t_0) - f^-(t_0)\right]$$

- Izr. kompl. spekter z odvajanjem



$$\frac{d^2f}{dt^2} = \delta(t+2)[1] + \delta(t+1)[-1-1] + \delta(t-1)[2] + \delta(t-2)[-1]$$

$$\frac{d^2f}{dt^2} = \delta(t+2) - 2\delta(t+1) + 2\delta(t-1) - \delta(t-2) \quad / \mathcal{F}$$

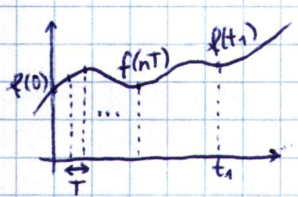
$$(j\omega)^2 F(\omega) = e^{j2\omega} - 2e^{j\omega} + 2e^{-j\omega} - e^{j2\omega}$$

$$-\omega^2 F(\omega) = 2j\sin 2\omega - 2\cdot 2j\sin\omega \quad / : -\omega^2 \neq 0$$

$$2\cos x = e^{j\omega} + e^{-j\omega}$$
$$2j\sin x = e^{jx} - e^{-j\omega}$$

$$F(\omega) = \frac{2j}{\omega^2}(2\sin\omega - \sin 2\omega)$$

Diskretni signali:



$f(nT) = \{f(0), f(T), f(2T), \ldots f((N-1)T)\} \xrightarrow{DFT} F_D(k\Omega) = \{F_D(0), F_D(1\Omega), \ldots$
$\ldots F_D((N-1)\Omega)\}$

$n = 0, 1, \ldots N-1$

N točk vzorčenja

$k = 0, 1, \ldots N-1$

$$\boxed{\Omega = \frac{2\pi}{NT}}$$

Stavek o vzorčenju: Zvezen in frekv. omejen signal $f(t)$ opazovan v času od 0 do t_1 je popolnoma določen, če poznamo njegove vzorce, ki si sledijo v enakomernih čas. presledkih $T = \frac{1}{2F}$, F - največja frekv., ki nastopa v signalu

$f = \frac{1}{T} = 2F$ Shannonova frekv. vzorčenja

$$\boxed{N = \frac{t_1}{T} = t_1 2F}$$

DFT (diskretna fourier. transf.)

$$\boxed{\mathcal{F}\{f(nT)\} = F_D(k\Omega) = \sum_{n=0}^{n-1} f(nT)e^{-jnTk\Omega}}$$

DFT je periodična f. s periodo $N\Omega$!

zveza med FT in DFT

$$\boxed{F(\omega)\Big|_{\omega=k\Omega} \doteq T F_D(k\Omega)}$$

IDFT (inverzna)

$$\boxed{\mathcal{F}^{-1}\{F_D(k\Omega)\} = f(nT) = \frac{1}{N}\sum_{k=0}^{N-1} F(k\Omega)e^{jnTk\Omega}}$$

- Naj bo DFT signala enak $\{H_D(k\Omega)\} = \{1, -1-j, -1, -1+j\}$. Določite vrednost $f(t)\big|_{t=1}$ pri čemer predpost., da smo signal vzorčili s čas. presledkom $T=1$

$\Rightarrow N=4 \qquad \Omega = \frac{2\pi}{NT} = \frac{2\pi}{4\cdot 1} = \frac{\pi}{2} = k\omega \qquad t = nT$

$1 = n\cdot 1 \rightarrow n=1$

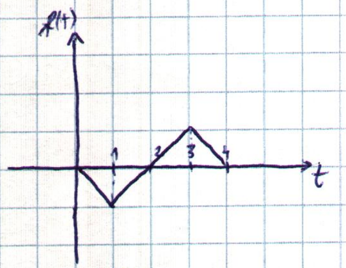
$$f(t)\Big|_{t=1} = \frac{1}{N}\sum_{k=0}^{N-1} H_D(k\Omega)\, e^{jnTk\Omega} = \frac{1}{4}\left(1e^{j0} + (-1-j)e^{j\frac{\pi}{2}} - 1e^{j2\frac{\pi}{2}} + (-1+j)e^{j\frac{3\pi}{2}}\right) =$$

$$= \frac{1}{4}\left(1 + (-1-j)j + 1 + (-1+j)(-j)\right) = \frac{1}{4}(1-j+1+1+j+1) = \underline{1}$$

$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$

$e^{j\pi} = \cos\pi + j\sin\pi = -1$

- Za signal $f(t)$ na sliki določite pribl. vrednost ampl. in faznega spektra $\omega = \frac{\pi}{2}$, če smo signal vzorčili s $T=0.5$. Določite napako ocene ampl. in faz. sp. zaradi vzorčenja!



$\varepsilon_{|F(\omega)|} = \left| |F(\omega)| - |\hat{F}(\omega)| \right|$

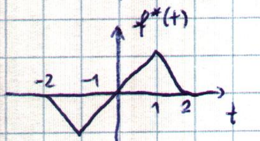
$\varepsilon_{faz} = |\theta(\omega) - \hat{\theta}(\omega)|$

$\hat{}$ – ocena

postopek

zv: $f(t) \xrightarrow{FT} F(\omega) \rightarrow |F(\omega)|, \theta(\omega)$

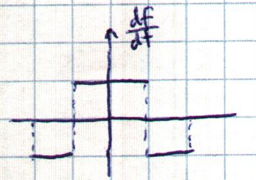
dis: $f(t) \rightarrow f(nT) \xrightarrow{DFT} F_D(k\Omega) \xrightarrow{T} \hat{F}(\omega) \rightarrow |\hat{F}(\omega)|, \hat{\theta}(\omega)$

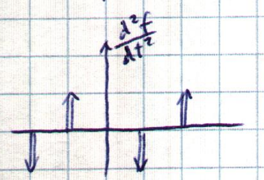


$f^*(t) = f(t+2) \;/\mathcal{F}$

$F^*(\omega)\Big|_{\omega=\frac{\pi}{2}} = F(\omega)\, e^{j2\omega}$ $\qquad e^{j2\frac{\pi}{2}} = e^{j\pi} = -1$

$F^*(\omega)\Big|_{\omega=\frac{\pi}{2}} = -F(\omega)$





limite gledamo

$$\frac{d^2f}{dt^2} = \delta(t+2)[-1] + \delta(t+1)[2] + \delta(t-1)[-2] + \delta(t-2)[1]$$

$$\frac{d^2f}{dt^2} = -\delta(t+2) + 2\delta(t+1) - 2\delta(t-1) + \delta(t-2) \quad /\mathcal{F}$$

$(j\omega)^2 F(\omega) = -e^{j2\omega} + 2e^{j\omega} - 2e^{-j\omega} + e^{-j2\omega}$

$-\omega^2 F(\omega) = -2j\sin 2\omega + 4j\sin\omega \quad /: -\omega^2 \neq 0$

$$\underline{\underline{F(\omega) = \frac{2j}{\omega^2}(\sin 2\omega - 2\sin\omega)}}$$

$F(\omega)^*\Big|_{\omega=\frac{\pi}{2}} = \frac{2j\,4}{\pi^2}(0-2) = -\underline{\underline{\frac{16j}{\pi^2}}}$

$F(\omega)\Big|_{\omega=\frac{\pi}{2}} = \frac{16j}{\pi^2} \Longrightarrow |F(\omega)|\Big|_{\omega=\frac{\pi}{2}} = \frac{16}{\pi^2}$

$\theta(\omega)\Big|_{\omega=\frac{\pi}{2}} = \frac{\pi}{2}$

$|F(\omega)| = \frac{16}{\pi^2} \doteq \underline{\underline{1.621}}$

$\theta(\omega) = \frac{\pi}{2}$

$$f(nT) = \begin{cases} 0 & ; n=0 \\ -0.5 & ; n=1 \\ -1 & ; n=2 \\ -0.5 & ; n=3 \\ 0 & ; n=4 \\ 0.5 & ; n=5 \\ 1 & ; n=6 \\ 0.5 & ; n=7 \end{cases}$$

$N=8$

$\Omega = \frac{2\pi}{NT} = \frac{2\pi\,2}{8\cdot 1} = \frac{\pi}{2}$

$\omega = k\Omega$

$\frac{\pi}{2} = k\frac{\pi}{2} \Longrightarrow \underline{k=1}$

$H_D(k\Omega) = \sum_{n=0}^{N-1} f(nT)\, e^{-jnTk\Omega} =$ $\qquad Tk\Omega = \frac{1}{2}\cdot 1\frac{\pi}{2} = \frac{\pi}{4}$

$= \left(0e^{-j0} - 0.5e^{-j\frac{\pi}{4}} - 1e^{-j\frac{2\pi}{4}} - 0.5e^{-j\frac{3\pi}{4}} + 0.5e^{-j\frac{5\pi}{4}} + \right.$

$\left. + 1e^{-j\frac{6\pi}{4}} + 0.5e^{-j\frac{7\pi}{4}}\right) = \ldots = (\sqrt{2}+2)j$

Akta®

$\hat{F}(\omega)\Big|_{\omega=\frac{\pi}{2}} = T F_D(1\Omega) = \underline{\underline{0.5(\sqrt{2}+2)j}} \rightarrow |\hat{F}(\omega)| \doteq 1.707$

$\hat{\Theta}(\omega) = \underline{\underline{\frac{\pi}{2}}}$

$\varepsilon_{|F(\omega)|} = \underline{\underline{0.086}}$

$\varepsilon_{faz} = \underline{\underline{0}}$

Korelacija periodičnih signalov

$\mathcal{F}\{f_i(t)\}$; T... enaka perioda

$$\boxed{\varphi_{ij}(\tau) = \frac{1}{T}\int_{t_0}^{t_0+T} \overline{f_i(t)}\, f_j(t+\tau)\, dt}$$

$i=j$ avtokorelacija

$i\neq j$ križna

Lastnosti: avtokor.

1) Hermitska simetrija $\varphi_{ii}(\tau) = \overline{\varphi_{ii}(-\tau)}$

2) periodičnost $\varphi_{ii}(\tau) = \varphi_{ii}(\tau+T)$

3.) $\varphi_{ii}(0) \geq |\varphi_{ii}(\tau)|$; $\varphi_{ii}(0)$ je moč signala

4.) zveznost

5.) $f_2(t) = f_1(t-t_0)$ $\varphi_{22}(\tau) = \varphi_{11}(\tau)$

6.) FV avtokorelacije

$$f_i(t) = \sum_{n=-\infty}^{\infty} F(n)\, e^{jn\omega t}$$

$$\varphi_{ii}(\tau) = \sum_{n=-\infty}^{\infty} \phi_{ii}(n) e^{jn\omega t} \underset{f_i\in\mathbb{R}}{=} \phi_{ii}(0) + 2\sum_{n=1}^{\infty} \phi_{ii}(n)\cos n\omega t$$

↑ močnostni spekter signala

$$\boxed{\phi_{ii}(n) = F_i(n)\overline{F_i(n)} = |F_i(n)|^2}$$

Lastnosti: križna

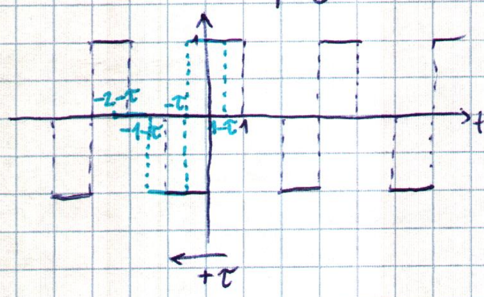
1.) antisimetričnost $\varphi_{ij}(\tau) = \overline{\varphi_{ji}(-\tau)}$

2.) periodičnost ; T

3.) zveznost

Za period. signal $f(t)$ določi avtokorelacijo in skiciraj njen potek!

$T=3$



$$\varphi_{11}(\tau) = \frac{1}{T}\int_{t_0}^{t_0+T} f_1(t)\, f_1(t+\tau)\,dt$$

sv. modr, temno modr

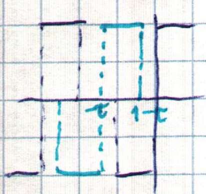
$0<\tau<1$:

$$\varphi_{11}(\tau) = \frac13\left(\int_{-2-\tau}^{-1-\tau} 0\,dt + \int_{-1-\tau}^{-1}(-1)\cdot 0\,dt + \int_{-1}^{-\tau}(-1)(-1)\,dt + \int_{-\tau}^{0}1(-1)\,dt + \int_0^{1-\tau}1\,dt\right) = \frac13\left(t\Big|_{-1}^{-\tau} - t\Big|_{-\tau}^{0} + t\Big|_0^{1-\tau}\right) =$$

$$= \frac13(-\tau+1-(0+\tau)+(1-\tau)) = \frac13(-\tau+1-\tau+1-\tau) =$$

$$= \frac13(-3\tau+2) = \underline{\underline{-\tau+\frac23}}$$

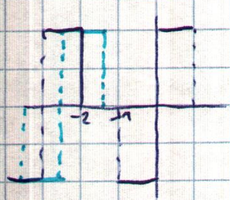
$1<\tau<2$:



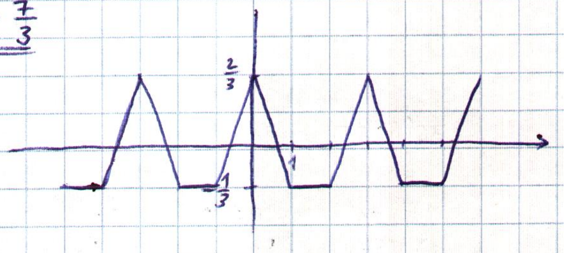
$$\varphi_{11}(\tau) = \frac13\left(\int_{-1-\tau}^{-2}(-1)\,dt + \int_{1}^{1-\tau}(-1)\,dt\right) = \frac13\left(-t\Big|_{-1-\tau}^{-2} - t\Big|_{1}^{1-\tau}\right) = \frac13\left(-(-2+1+\tau)-(1-\tau-1)\right) =$$

$$= \frac13(\mp 1-\tau+\tau) = \underline{\underline{-\frac13}} \quad - ?$$

$2<\tau<3$:



$$\varphi_{11}(\tau) = \frac13\left(\int_{-1-\tau}^{-3} 1\,dt + \int_{-3}^{-\tau}(-1)\,dt + \int_{-\tau}^{-2}(1)\,dt\right) = \underline{\underline{\tau - \frac73}}$$



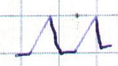
- Za signal na sliki določi Four. vrsto avtokorelacije!

Opcije:

1.) $f_1(t) \to F_1(n) \to |F_1(n)| \to \Phi_{11}(n)$

2.) $f_1(t) \to \varphi_{11}(\tau) \to \Phi_{11}(n)$

Kaj bo lažje spremenit v frekv. prostor?

 ali 

$T=3,\ \omega=\frac{2\pi}{3}$

$$F_1(n) = \frac1T\int_{t_0}^{t_0+T} f_1(t)\,e^{-jn\omega t}\,dt =$$

$$= \frac13\left[\int_{-1}^{0}(-1)\,e^{-jn\frac{2\pi}{3}t}\,dt + \int_0^1 1\,e^{-jn\frac{2\pi}{3}t}\,dt\right] =$$

$$= \frac13\left[-\frac{1\cdot3}{jn2\pi}e^{-jn\frac{2\pi}{3}t}\Big|_{-1}^{0} - \frac{3}{jn2\pi}e^{-jn\frac{2\pi}{3}t}\Big|_0^1\right] = \frac{1}{jn2\pi}\left[1-e^{jn\frac{2\pi}{3}} - \left(e^{-jn\frac{2\pi}{3}}-1\right)\right] =$$

$$= \frac{1}{jn2\pi}\left[1-\cos n\frac{2\pi}{3} - j\sin n\frac{2\pi}{3} - \left(\cos n\frac{2\pi}{3} - j\sin n\frac{2\pi}{3} - 1\right)\right] = \frac{1}{jn2\pi}\left[2-2\cos n\frac{2\pi}{3}\right] =$$

$$= \frac{1\cdot2}{jn\pi}\sin^2\frac{n\pi}{3} = \underline{-\frac{2j}{n\pi}\sin^2\frac{n\pi}{3}}$$

$n=0 \quad F_1(n) = \frac13\left[\int_{-1}^{0}(-1)\,dt + \int_0^1(1)\,dt\right] = 0$

$\Rightarrow \Phi_{11}(0) = 0$

avtokorel. v obliki vrste

$$\Phi_{11}(n) = \frac{4}{n^2\pi^2}\sin^4\frac{n\pi}{3}, \quad \varphi_{11}(\tau) = \sum_{n=-\infty}^{\infty}\Phi_{11}(n)\,e^{jn\omega\tau} = \Phi_{11}(0) + 2\sum_{n=1}^{\infty}\Phi_{11}(n)\cos n\omega\tau =$$

$$= \underline{2\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\sin^4\frac{n\pi}{3}\cos n\frac{2\pi}{3}\tau}$$

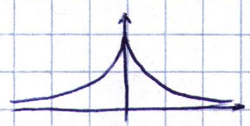
KORELACIJA NEPERIODIČNIH SIGNALOV

$$\varphi_{ij}(\tau) = \int_{-\infty}^{\infty} \overline{f_i(t)}\, f_j(t+\tau)\, dt$$

$i=j$ avtokorelacija
$i \neq j$ križna korelacija

Lastnosti : avtokorelacije

1.) Hermitska simetrija $\varphi_{ii}(\tau) = \overline{\varphi_{ii}(-\tau)}$

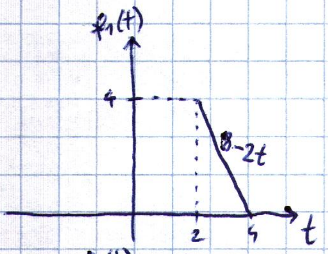


2.) $\varphi_{ii}(0) = E_{f_i}$ energija signala
$\varphi_{ii}(0) \geq |\varphi_{ii}(\tau)|$
3.) $\varphi_{ii}(\pm\infty) = 0$
4.) Frekv. prostor :
$\varphi_{ii}(\tau) \Leftrightarrow \Phi_{ii}(\omega)$ spekter energ. gostote signala f_i
$\Phi_{ii}(\omega) = |F(\omega)|^2$ nenegativen
$\Theta(\omega) = 0$
5.) zvezna
6.) neodvisna od premika $f_2(t) = f_1(t-t_0)$
$\varphi_{22}(\tau) = \varphi_{11}(\tau)$

Lastnosti križne k. :

1.) antisimetričnost $\varphi_{ij}(\tau) = \overline{\varphi_{ji}(-\tau)}$
2.) Frekv. predstavitev : $\varphi_{ij}(\tau) \Leftrightarrow \Phi_{ij}(\omega)$
3.) zveznost

- Določi avtokor. in energ. neperiod. signala $f_1(t)$ na sliki. Avtok. skiciraj!



$f_1(t), f_2(t) \Rightarrow$ imata isto korelacijo $\varphi_{11}(\tau) = \varphi_{22}(\tau)$

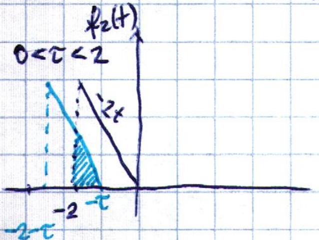
$$\varphi_{22}(\tau) = \int_{-2}^{-\tau} (-2t)(-2(t+\tau))\, dt = 4\int_{-2}^{-\tau} (t^2 + t\tau)\, dt = 4\left[\frac{t^3}{3}\Big|_{-2}^{-\tau} + \frac{t^2\tau}{2}\Big|_{-2}^{-\tau}\right] =$$

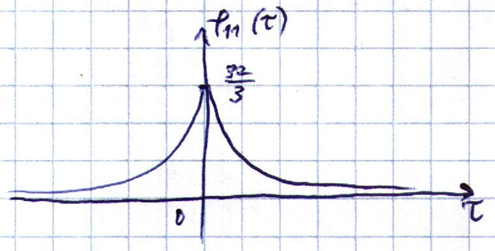
int. le na področju prekrivanja

$$= 4\left[\frac{1}{3}(-\tau^3+8) + \frac{\tau}{2}(\tau^2-4)\right] = \frac{4}{3}\left(-\tau^3+8+\frac{4\pi}{2}(\tau^2-4)\right) =$$

$$= \underline{\underline{\frac{2\tau^3 - 24\tau + 32}{3}}}$$

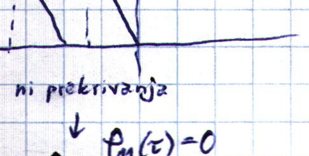
$$\varphi_{22}(0) = \varphi_{11}(0) = \frac{32}{3}$$





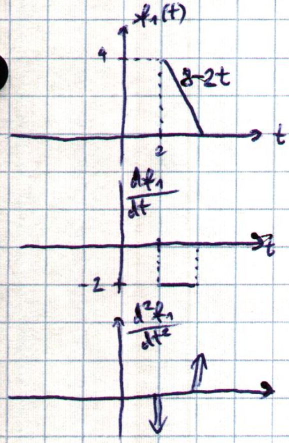
vedno bo špica okoli izhodišča

$\tau > 2$:



ni prekrivanja
$\downarrow$
$\varphi_{11}(\tau) = 0$

Izr. še spekter energ. gostote !



$$\frac{d^2f_1}{dt^2} = \delta(t-2)[-2] + \delta(t-4)[2]$$

$$\frac{d^2f_1}{dt^2} = -2\delta(t-2) + 2\delta(t-4) \quad / \mathcal{F}$$

$$(j\omega)^2 F_1(\omega) = -2e^{-j2\omega} + 2e^{-j4\omega}$$

$$-\omega^2 F_1(\omega) = -2e^{-j2\omega} + 2e^{-j4\omega} \quad / : -\omega^2 \neq 0$$

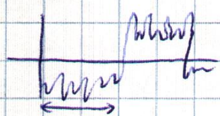
$$F_1(\omega) = \frac{2}{\omega^2}\left(e^{-j2\omega} - e^{-j4\omega}\right)$$

$$\Rightarrow |F_1(\omega)| = \frac{2}{\omega^2}\left|e^{-j2\omega} - e^{-j4\omega}\right|$$

$$\underline{\underline{\phi_{11}(\omega) = \frac{4}{\omega^4}\left|e^{-j2\omega} - e^{-j4\omega}\right|^2}}$$

NAKLJUČNI SIGNALI

- stacionarnost



- ergodičnost
- Dirichletovi pogoji ne veljajo

$$\lim \frac{1}{2T}\int_{-T}^{T} |f(t)|^2\, dt < \infty$$

$$\varphi_{11}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \overline{f_1(t)}\, f_1(t+\tau)\, dt = \iint_{-\infty\,-\infty}^{\infty\,\infty} x_1 x_2\, p(x_1, x_2, \tau)\, dx_1\, dx_2$$

Lastnosti ak. naklj. signalov :

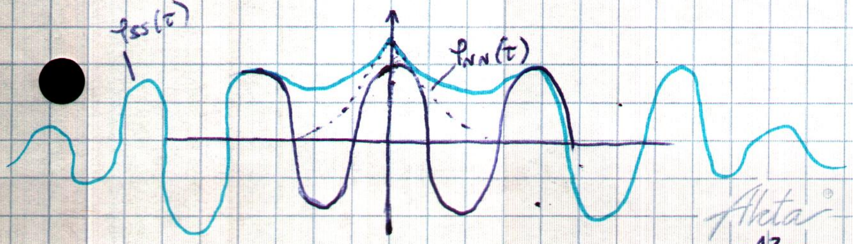
1.) Hermitska simetrija $\varphi_{ii}(\tau) = \overline{\varphi_{ii}(-\tau)}$
2.) $\varphi_{ii}(0)$ – povpr. moč signala
3.) maksimalnost $\varphi_{ii}(0) \geq |\varphi(\tau)|$
4.) $\varphi(\pm\infty) = 0$
5.) Wienerjev stavek

- Odkrijmo periodično komponento! Imamo period. signal $S(t)$ in motilni naklj. signal $N(t)$. Oba imata sr. vred. enako 0. $f(t)$ je vsota obeh.

$\overline{S(t)} = 0$, $\overline{N(t)} = 0$, $\underset{\text{meritve}}{f_1(t)} = S(t) + N(t)$

$$\varphi_{11}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\overline{f_1(t)}\, f_1(t+\tau)\, dt = \lim_{T\to\infty}\frac{1}{2T}\left(\int_{-T}^{T}(S(t)+N(t))(S(t+\tau)+N(t+\tau))\, dt\right) =$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left(\int_{-T}^{T}(S(t)S(t+\tau) + S(t)N(t+\tau) + N(t)S(t+\tau) + N(t)N(t+\tau))\, dt\right) = \varphi_{SS}(\tau) + \underbrace{\overset{=0}{\varphi_{SN}(\tau)} + \overset{=0}{\varphi_{NS}(\tau)}}_{C} + \varphi_{NN}(\tau)$$



LINEARNI STACIONARNI SISTEMI

$u(t) \longrightarrow \boxed{LSS} \longrightarrow y(t)$

1.) - linearnost

$u_1(t) \to y_1(t)$

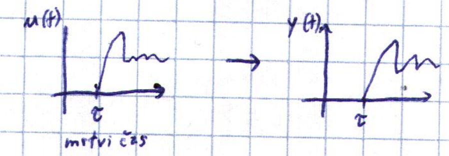
$u_2(t) \to y_2(t)$

$a u_1(t) + b u_2(t) \to a y_1(t) + b y_2(t)$

- stacionarnost (ni izhoda pred vhodom)

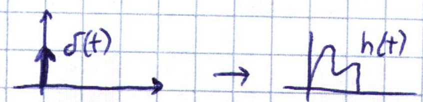
$u(t) \to y(t)$

$u(t-\tau) \to y(t-\tau)$



2.) $\delta(t) \to h(t)$

↖ odziv na enotin impulz



3.) KONVOLUCIJA

$$\boxed{y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau} \quad / \mathcal{F}$$

$t - \tau = \tau'$

$\tau = t - \tau'$

$d\tau = -d\tau'$

$$\boxed{Y(\omega) = \underbrace{U(\omega)}_{u(\tau)\ \downarrow FFT} \cdot \underbrace{H(\omega)}_{h(t-\tau)\ \downarrow FFT}} \longrightarrow \boxed{H(\omega) = \frac{Y(\omega)}{U(\omega)}}$$

$\downarrow$ IFFT

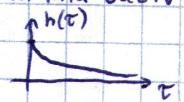
$y(t)$

↑ prenosna funkcija lin. stacionarnega sistema (LSS)

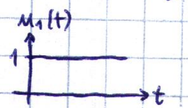
prevajalna f.

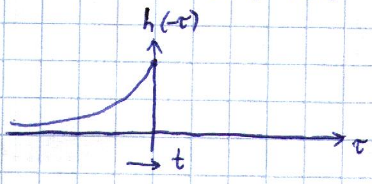
- Določi in skiciraj odziv LSS, ki ima odziv na enotin impulz h(t) enak:

$$h(t) = \begin{cases} 0 & ; t < 0 \\ e^{-t} & ; t \geq 0 \end{cases}$$

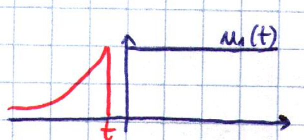


a) $$u_1(t) = \begin{cases} 0 & ; t < 0 \\ 1 & ; t \geq 0 \end{cases}$$





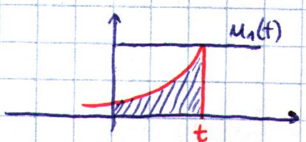
I) $t < 0$



$y(t) = 0$

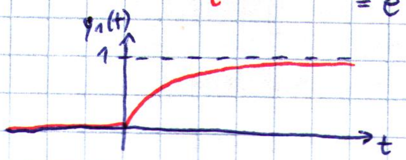
izhod je 0, ker ni prekrivanja

II) $t > 0$

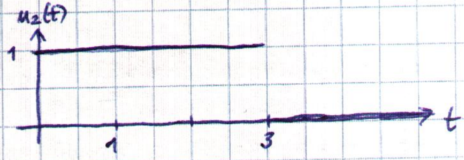


$$y_1(t) = \int_0^t 1 \cdot e^{-(t-\tau)}\, d\tau = \int_0^t e^{-t} \cdot e^{+\tau}\, d\tau =$$

$$= e^{-t} \int_0^t e^{\tau}\, d\tau = e^{-t} \cdot \left(e^{\tau}\Big|_0^t\right) = e^{-t}(e^{t} - 1) = \underline{\underline{1 - e^{-t}}}$$



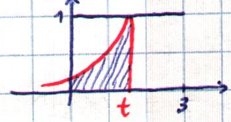
b) $u_2(t) = \begin{cases} 0 ; & t < 0 \\ 1 ; & 0 \le t \le 3 \\ 0 ; & t > 3 \end{cases}$



- $t < 0$

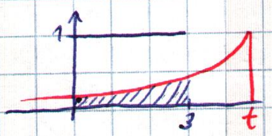
$y_2(t) = 0$

- $0 < t < 3$

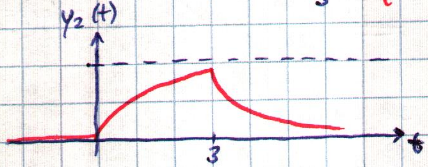


$y_2(t) = 1 - e^{-t}$

- $t > 3$

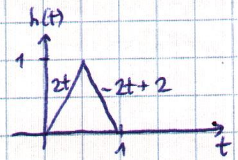


$$y_2(t) = \int_0^3 1 \cdot e^{-(t-\tau)} d\tau = e^{-t}(e^3 - 1) = \underline{\underline{19{,}08 \cdot e^{-t}}}$$

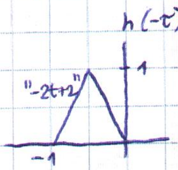


Določi in skiciraj odziv sistema LSS, ki ima odziv na enotin impulz $h(t)$ enak:

$$h(t) = \begin{cases} 2t & ; \ 0 \le t \le \frac{1}{2} \\ -2t+2 & ; \ \frac{1}{2} \le t \\ 0 & ; \ \text{sicer} \end{cases}$$

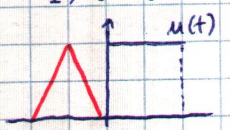


$\Rightarrow$



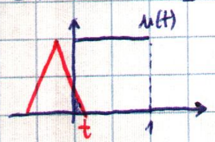
$$u(t) = \begin{cases} 1 ; & 0 \le t \le 1 \\ 0 ; & \text{sicer} \end{cases}$$

I) $t < 0$



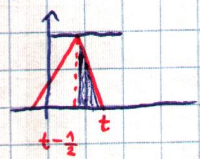
$y(t) = 0$

II) $0 \le t \le \frac{1}{2}$



$$y(t) = \int_0^t 1 \cdot 2(t-\tau)\, d\tau = 2 \cdot \left(t\tau - \frac{\tau^2}{2} \Big|_0^t\right) = \underline{\underline{t^2}}$$

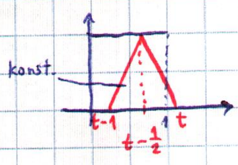
III) $\frac{1}{2} \le t \le 1$



$$y(t) = \underbrace{\int_0^{t-\frac{1}{2}} (-2(t-\tau)+2)\, d\tau + \int_{t-\frac{1}{2}}^{t} 2(t-\tau)\, d\tau}_{\frac{1}{4}} = \underline{\underline{-t^2 + 2t - \frac{1}{2}}}$$

površina Δ je konst.

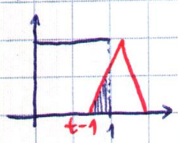
IV) $1 \le t \le \frac{3}{2}$



$$y(t) = \overbrace{\int_{t-1}^{t-\frac{1}{2}} (-2(t-\tau)+2)\, d\tau + \int_{t-\frac{1}{2}}^{1} 2(t-\tau)\, d\tau}^{\frac{1}{4}} =$$

$$= \underline{\underline{-t^2 + 2t - \frac{1}{2}}}$$

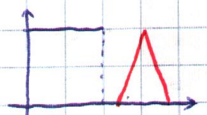
V) $\frac{3}{2} \le t \le 2$



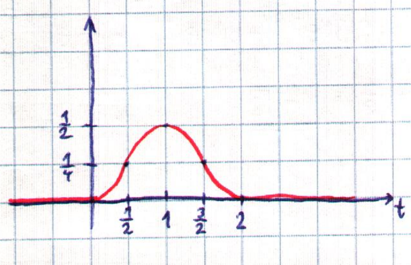
$$y(t) = \int_{t-1}^{1} (-2(t-\tau)+2)\, d\tau =$$

$$= \underline{\underline{t^2 - 4t + 4}}$$

VI) $t \ge 2$



$y(t) = 0$



■ Diskretna Four. transformacija vhodnega signala $u(t)$ v LSS je $\{U_D(k\Omega)\} = \{2, 1+j, 1, 1-j\}$. DFT odziva sistema $y(t)$ na ta vhodni signal pa je $\{Y_D(k\Omega)\} = \{2, -2j, -1, 2j\}$. Pri določitvi obeh DFT smo signala vzorčili s časovnim presledkom $T=1$. Določite pribl. vrednost ampl. in faz. spektra prevajalne funkcije pri $\omega = \frac{\pi}{2}$.

$\{U_D(k\Omega)\} = \{2, 1+j, 1, 1-j\}$ $\quad N=4$

$\{Y_D(k\Omega)\} = \{2, -2j, 1, 2j\}$

$T=1$

$\omega = \frac{\pi}{2}$

$|H(\omega)|\big|_{\omega=\frac{\pi}{2}} = ?$ $\quad \Theta(\omega)\big|_{\omega=\frac{\pi}{2}} = ?$

$U_D, Y_D \to H_D \xrightarrow{T} H(\omega)$

$\Omega = \frac{2\pi}{NT} = \frac{2\pi}{4\cdot 1} = \frac{\pi}{2}$

$\omega = k\Omega \to \underline{k=1}$

$H_D(k\Omega) = \frac{1}{T}\cdot\frac{Y_D(k\Omega)}{U_D(k\Omega)} = \left\{\frac{2}{2}, \frac{-2j}{1+j}, -1, \frac{2j}{1-j}\right\} = \{\overset{k=0}{1}, \overset{k=1}{-1-j}, -1, -1+j\}$

$*$

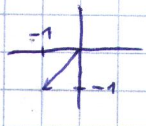
$* \quad \frac{-2j(1-j)}{(1+j)(1-j)} = \frac{-2j+2j^2}{2} = -1-j$

$\frac{2j(1+j)}{(1-j)(1+j)} = \frac{2j+2j^2}{2} = -1+j$

$H(\omega)\big|_{\omega=\frac{\pi}{2}} \doteq T\cdot H_D(1\cdot\Omega) = \underline{\underline{-1-j}}$

$|H(\omega)|\big|_{\omega=\frac{\pi}{2}} = \underline{\underline{\sqrt{2}}}$

$\Theta(\omega)\big|_{\omega=\frac{\pi}{2}} = \underline{\underline{-\frac{3\pi}{4}}}$



$Y(\omega) \doteq T\cdot Y_D(k\Omega)$

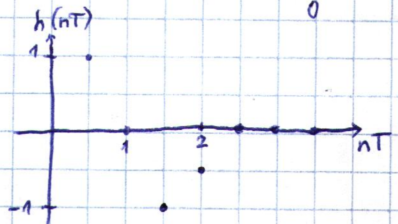
$U(\omega) \doteq T\cdot U_D(k\Omega)$

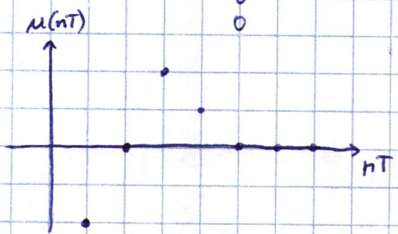
■ Podana sta odziv na enotin impulz $h(t)$ in vhodni signal $u(t)$. Vzorčena sta s časovnim razmakom $T=0{,}5$. Določi približno vrednost amplit. spektra $Y(\omega)$ pri $\omega = \frac{\pi}{2}$.

$|Y(\omega)|\big|_{\omega=\frac{\pi}{2}} = ?$

$$h(nT) = \begin{cases} 0.5 & ; n=0 \\ 1 & ; n=1 \\ 0 & ; n=2 \\ -1 & ; n=3 \\ -0.5 & ; n=4 \\ 0 & \\ 0 & \\ 0 & \end{cases} \qquad u(nT) = \begin{cases} -0.5 & ; n=0 \\ -1 & ; n=1 \\ 0 & ; n=2 \\ 1 & ; n=3 \\ 0.5 & ; n=4 \\ 0 & \\ 0 & \\ 0 & \end{cases} \qquad N=5$$

zraven dodamo še 3 ničle





$\omega = k\Omega$

$\Omega = \frac{2\pi}{NT} = \frac{2\pi\cdot 2}{5\cdot 1} = \frac{4\pi}{5} = k\cdot\frac{\pi}{2}$

$N=8$

$\Omega = \frac{2\pi\cdot 2}{8\cdot 1} = \frac{\pi}{2} \to \underline{\underline{k=1}}$

$\left.\begin{array}{l} h(nT) \to H_D(k\Omega) \xrightarrow{T} H(\omega) \\ u(nT) \to U_D(k\Omega) \xrightarrow{T} U(\omega) \end{array}\right\rangle Y(\omega) \to |Y(\omega)|$

$H_D(k\Omega) = \sum_{n=0}^{7} h(nT)e^{-jnTk\Omega} = 0.5\,e^{-j0} + 1\cdot e^{-j\frac{\pi}{4}} - 1\cdot e^{-j\frac{3\pi}{4}} - 0.5\,e^{-j\pi} =$

$= \frac{1}{2} + \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) + \frac{1}{2} = \underline{\underline{1+\sqrt{2}}}$

$Tk\Omega = \frac{1}{2}\cdot 1\cdot\frac{\pi}{2} = \frac{\pi}{4}$

$$e^{-j\frac{\pi}{4}} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$$
$$e^{-j\frac{3\pi}{4}} = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$$
$$e^{-j\pi} = -1$$

$$H(\omega)\Big|_{\omega=\frac{\pi}{2}} \doteq \frac{1}{2}(1+\sqrt{2}) = \frac{1+\sqrt{2}}{2}$$

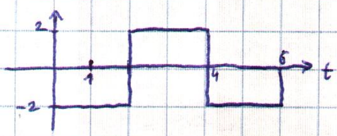
$$U_D(k\Omega) = \sum_{n=0}^{7} u(nT)e^{-jnTk\Omega} = -0.5e^{-j0} - 1\cdot e^{-j\frac{\pi}{4}} + 1e^{-j\frac{3\pi}{4}} + 0.5e^{-j\pi} =$$
$$= -\frac{1}{2} - \left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) - \frac{1}{2} = \underline{\underline{-1-\sqrt{2}}}$$
$$U(\omega)\Big|_{\omega=\frac{\pi}{2}} \doteq \frac{1}{2}(-1-\sqrt{2}) = \frac{-1-\sqrt{2}}{2}$$
$$Y(\omega)\Big|_{\omega=\frac{\pi}{2}} = H(\omega)\cdot U(\omega)\Big|_{\omega=\frac{\pi}{2}} = \frac{(1+\sqrt{2})(-1-\sqrt{2})}{4} \doteq \underline{\underline{-1{,}457}} \quad \rightarrow \quad |Y(\omega)|\Big|_{\omega=\frac{\pi}{2}} = \underline{\underline{1{,}457}}$$

LINEARNI STACIONARNI SISTEMI ZA NAKLJUČNE SIGNALE

$$u(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$\varphi_{yy}(\tau) = \varphi_{hh}(\tau) * \varphi_{uu}(\tau)$$
$$\downarrow \mathcal{F}$$
$$\boxed{\Phi_{yy}(\omega) = \Phi_{hh}(\omega)\cdot\Phi_{uu}(\omega) = |H(\omega)|^2\,\Phi_{uu}(\omega)}$$

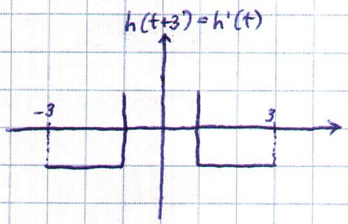
- Linearni stacionarni sistem ima vpliv na enotin impulz enak signalu $h(t)$ na sliki: Določite spekter močnostne gostote odziva tega sistema, če je vhodni signal beli šum $f_a(t)$, katerega avtokorelacija $\varphi_{aa}(\tau) = k\delta(t)$; $k>0$.



$\Phi_{yy}(\omega) = ?$

$f_a(t) \leftrightarrow \varphi_{aa}(\tau) = k\delta(t)$; $k>0$

$$\Phi_{aa}(\omega) = \mathcal{F}\{k\underset{1}{\delta(t)}\} = k$$



$$\Phi_{yy}(\omega) = \Phi_{hh}(\omega)\,\Phi_{aa}(\omega) = k\Phi_{hh}(\omega)$$

$$\frac{dh'(t)}{dt} = \delta(t+3)(-1) + \delta(t+1)\cdot 3 + \delta(t-1)(-3) + \delta(t-3)\cdot 1$$
$$\frac{dh'(t)}{dt} = -\delta(t+3) + 3\delta(t+1) - 3\delta(t-1) + \delta(t-3) \quad / \mathcal{F}$$

$$j\omega H'(\omega) = -e^{-j3\omega} + 3e^{-j\omega} - 3e^{-j\omega} + e^{3j\omega}$$
$$j\omega H'(\omega) = 2j\sin 3\omega - 6j\sin\omega \quad /: j\omega \neq 0$$

$$H'(\omega) = \frac{2}{\omega}(\sin 3\omega - 3\sin\omega) \Rightarrow |H(\omega)| = \frac{2}{\omega}|\sin 3\omega - 3\sin\omega|$$

$$\Phi_{yy}(\omega) = \Phi_{hh}(\omega)\Phi_{aa}(\omega) = k\Phi_{hh}(\omega) = k|H(\omega)|^2 = \underline{\underline{\frac{k\cdot 4}{\omega^2}(\sin 3\omega - \sin\omega)^2}}$$

Naj bo $e(t)$ poissonov naključni signal. To je signal, ki je predstavljen kot vsota naključno pojavljajočih se motenj enake oblike $g(t)$.

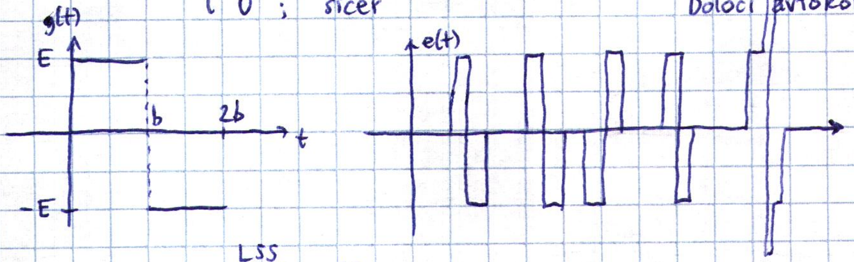
$$e(t) = \sum_{i=-\infty}^{\infty} \pm g(t-t_i)$$

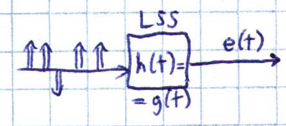
Naklj. tok dogodkov množice $\{t_i\}$ naj izpolnjuje lastnosti pois. procesa.

$$g(t) = \begin{cases} E \ ; & 0<t\le b \ ; E>0 \\ -E \ ; & b<t\le 2b \\ 0 \ ; & \text{sicer} \end{cases}$$

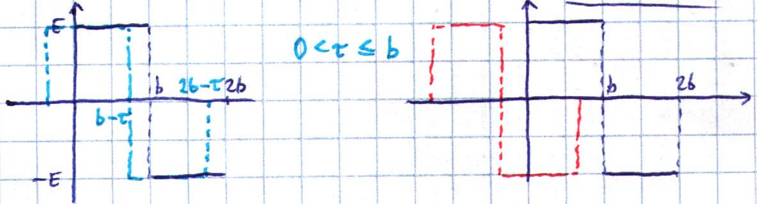
Motnja $g(t)$ se lahko z enako verjetnostjo pojavlja s pozit. ali neg. predznakom.

Določi avtokor. $g(t)$!





$$\varphi_{ee}(\tau) = \varphi_{hh}(\tau) * \varphi_{uu}(\tau) = k\varphi_{hh}(\tau) = k\varphi_{gg}(\tau)$$



$0<\tau\le b$ $\quad$ $b<\tau\le 2b$

$0<\tau<b$:

$$\varphi_{gg}(\tau) = \int_0^{b-\tau} E^2\,dt + \int_{b-\tau}^{b} (-E^2)\,dt + \int_b^{2b-\tau} E^2\,dt = E^2(b-\tau) - E^2(b-b+\tau) + E^2(2b-\tau-b) =$$

$$= E^2b - E^2\tau - E^2\tau + E^2b - E^2\tau = 2E^2b - 3E^2\tau = E^2(2b-3\tau)$$

$b<\tau\le 2b$:

$$\varphi_{gg}(\tau) = \int_0^{2b-\tau} -E^2\,dt = -E^2(2b-\tau)$$

$$\varphi_u(\tau) = \begin{cases} kE^2(2b-3\tau) \ ; & 0<\tau\le b \\ -kE^2(2b-\tau) \ ; & b<\tau\le 2b \\ 0 \ ; & \text{sicer} \end{cases}$$