

13.10.2008

Linearnost

$$\mathcal{H}(a_1 x_1 + a_2 x_2) = \mathcal{H}(a_1 x_1) + \mathcal{H}(a_2 x_2)$$

Časovna invariantnost

$$\mathcal{H}(x(t-\tau)) = y(t-\tau)$$

$$\mathcal{H}(x(t)) = y(t)$$

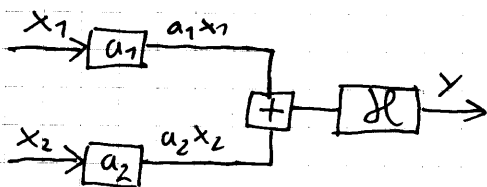
1. Ali je tuljava linearna in ali je časovno invariantna?

$$\begin{array}{c} i(t) \quad u(t) \\ \xrightarrow{\quad L \quad} \end{array}$$

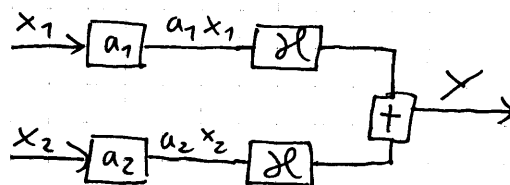
$$u(t) = L \frac{d}{dt} i(t)$$

Linearna? $\mathcal{H}$

Levi del enačbe:



Desni del enačbe:



$$L \frac{d}{dt} (a_1 i_1(t) + a_2 i_2(t))$$

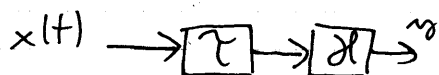
$$L \frac{d}{dt} a_1 i_1(t) + L \frac{d}{dt} a_2 i_2(t)$$

$$L \frac{d}{dt} a_1 i_1(t) + L \frac{d}{dt} a_2 i_2(t)$$

Tuljava je linearen element.

Časovna inu?

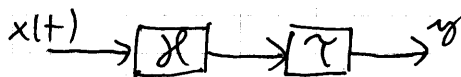
Leva stran



$$i(t) \rightarrow i(t-\tau)$$

$$\underline{u(t-\tau) = L \frac{d}{dt} i(t-\tau)}$$

Desna stran



$$i(t)$$

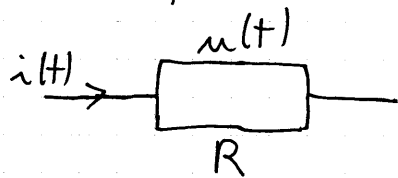
$$u(t) = L \frac{d}{dt} i(t)$$

$$u(t-\tau) = L \frac{d}{dt} i(t) \Big|_{t-\tau}$$

$$\underline{u(t-\tau) = L \frac{d}{dt} i(t-\tau)}$$

Tuljava je časovna invariantna.

2.) Ali je temperaturno odvisni upor R časovno invarianten?



$$u(t) = R \cdot i(t)$$

$$\Downarrow$$

$$u(t) = R(t) \cdot i(t)$$

$$\underline{T \rightarrow T(t)}$$

$$T(t) = T_2$$

$$T(t-\tau) = T_1$$

L

$$i(t) \rightarrow i(t-\tau)$$

$$u(t-\tau) = R(T(t-\tau)) \cdot i(t-\tau)$$

$$u(t-\tau) = R(T_1) \cdot i(t-\tau)$$

D

$$u(t) = R(T(t)) \cdot i(t)$$

$$u(t-\tau) = R(T(t)) \cdot i(t-\tau)$$

$$u(t-\tau) = R(T_2) \cdot i(t-\tau)$$

Če $T_1 = T_2$ je sistem časovno invarianten.

Če $T_1 \neq T_2$ ni sistem časovno invarianten.

3.) Ugotovi ali je sistem

• linearen? ✓

• časovno invarianten? ✗

$$\int_0^+ x(u) du = \frac{y(t)}{2}$$

$$y(t) = 2 \int_0^+ x(u) du$$

$$\begin{array}{l} \text{L} \\ 2 \cdot \int_0^+ (a_1 x_1(u) + a_2 x_2(u)) du \\ = 2 \cdot \int_0^+ a_1 x_1(u) du + 2 \cdot \int_0^+ a_2 x_2(u) du \end{array} \quad \begin{array}{l} \text{D} \\ 2 \cdot \int_0^+ a_1 x_1(u) du + 2 \cdot \int_0^+ a_2 x_2(u) du \end{array}$$

$L = D$

$$\begin{array}{l} \text{L} \\ x(u) \rightarrow x(u-\tau) \\ 2 \int_0^+ x(u-\tau) du \end{array} \quad \begin{array}{l} \text{D} \\ 2 \int_0^+ x(u) du = ; \\ = 2 \int_{\tau}^+ x(v-\tau) dv \end{array}$$

$L \neq D$

$v = u + \tau; u = v - \tau$
 $dv = du$

4.) Ugotovi ali je sistem linearen in časovno invarianten?

$$y(t) = \frac{d}{dt} x(t) + 2x(t)$$

Linearnost:

L:

$$\frac{d}{dt} (a_1 x_1(t) + a_2 x_2(t)) + 2(a_1 x_1(t) + a_2 x_2(t))$$

$$\frac{d}{dt} a_1 x_1(t) + 2a_1 x_1(t) + \frac{d}{dt} a_2 x_2(t) + 2a_2 x_2(t)$$

D:

$$\frac{d}{dt} a_1 x_1(t) + 2a_1 x_1(t) + \frac{d}{dt} a_2 x_2(t) + 2a_2 x_2(t)$$

Časovna invariantnost:

L:

$$x(t) \rightarrow x(t - \tau)$$

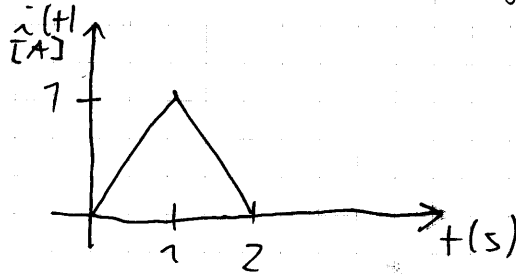
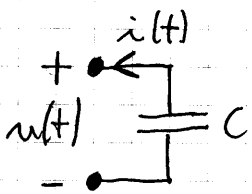
$$\frac{d}{dt} x(t - \tau) + 2x(t - \tau)$$

D:

$$\left. \frac{d}{dt} x(t) + 2x(t) \right|_{t-\tau}$$

$$\frac{d}{dt} x(t - \tau) + 2x(t - \tau)$$

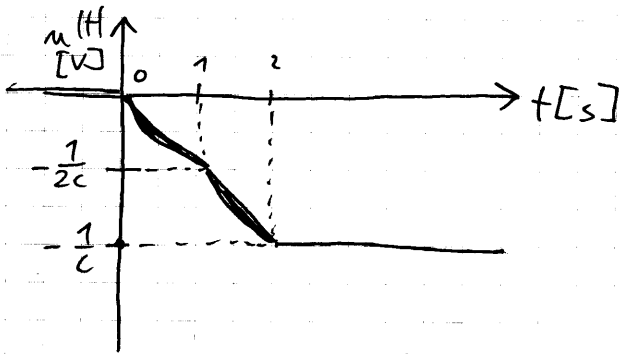
5.) Skiciraj napetost na kondenzatorju $u(t)$



$$u(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

$$u(t) = -\frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = -\frac{1}{C} \int_{-\infty}^0 i(\tau) d\tau - \frac{1}{C} \int_0^t i(\tau) d\tau$$

$= 0$



$$i(t) = \begin{cases} 0 & t < 0 \text{ s} \\ t & t \in [0, 1] \text{ s} \\ 2-t & t \in [1, 2] \text{ s} \\ 0 & t > 2 \text{ s} \end{cases}$$

$$u(t) = -\frac{1}{C} \int_0^t t dt = -\frac{t^2}{2C} \Big|_0^t = -\frac{t^2}{2C}$$

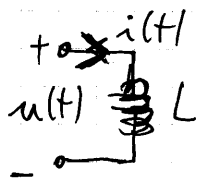
$$u(t) = -\frac{1}{C} \int_0^t i(t) dt = -\frac{1}{C} \int_0^1 t dt - \frac{1}{C} \int_1^t (2-t) dt =$$

$$= -\frac{1}{2C} - \frac{1}{C} \int_1^t (2-t) dt = -\frac{1}{2C} - \frac{1}{C} \left(2t - \frac{t^2}{2} \right) \Big|_1^t$$

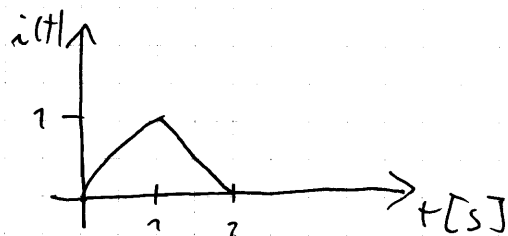
$$= \cancel{\frac{1}{2C}} - \frac{1}{C} + \frac{1}{2C} (t-2)^2$$

$$u(t) = -\frac{1}{C} \int_0^t i(t) dt = -\frac{1}{C} \int_0^1 t dt - \frac{1}{C} \int_1^2 (2-t) dt - \frac{1}{C} \int_2^t 0 dt$$

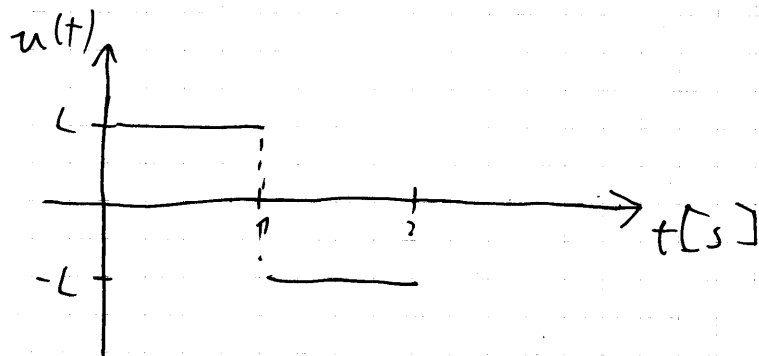
6.1 Skiciraj $u(t)$



$$u(t) = L \frac{d}{dt} i(t)$$



$$i(t) = \begin{cases} 0 & t < 0 \\ t & t \in [0, 1] \\ 2-t & t \in [1, 2] \\ 0 & t > 2 \end{cases}$$



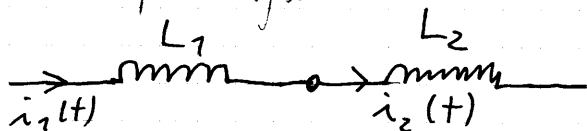
$$t \in [0, 1]$$

$$u(t) = L \frac{d}{dt} t = L$$

$$t \in [1, 2]$$

$$u(t) = L \frac{d}{dt} (2-t) = -L$$

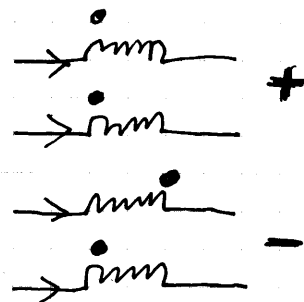
Sklop tuljau



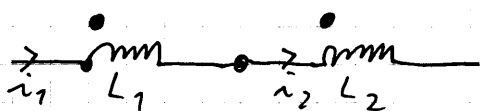
$$u_1(t) = L_1 \frac{d}{dt} i_1(t) \pm M \frac{d}{dt} i_2(t)$$

$$u_2(t) = L_2 \frac{d}{dt} i_2(t) \pm M \frac{d}{dt} i_1(t)$$

$$M = k \cdot \sqrt{L_1 L_2}$$



7.) Izračunaj nadomestno induktivnost - L_s



$$i_1(t) = i_2(t) = i(t)$$

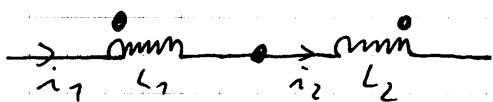
$$u(t) = u_1(t) + u_2(t) = L_1 \frac{d}{dt} i(t) + M \frac{d}{dt} i(t) + L_2 \frac{d}{dt} i(t) + M \frac{d}{dt} i(t)$$

$$u(t) = (L_1 + L_2 + 2M) \frac{d}{dt} i(t)$$

$$L_s$$

$$\begin{array}{l} L_1 = L_2 = L \\ k = 1 \end{array} \quad \left. \begin{array}{l} M = \sqrt{L^2} = L \\ \Rightarrow u(t) = (L + L + 2L) \frac{d}{dt} i(t) \\ u(t) = 4L \frac{d}{dt} i(t) \end{array} \right\}$$

$$u(t) = ?$$



$$u(t) = u_1(t) + u_2(t) = L_1 \frac{d}{dt} i(t) - M \frac{d}{dt} i(t) + L_2 \frac{d}{dt} i(t) - M \frac{d}{dt} i(t)$$

$$u(t) = (L_1 + L_2 - 2M) \frac{d}{dt} i(t)$$

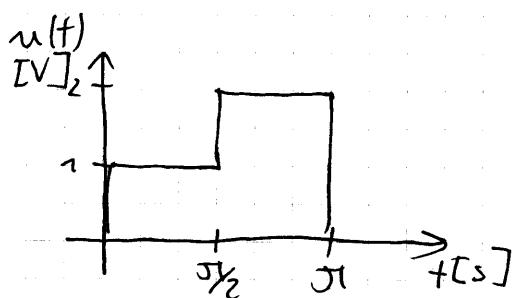
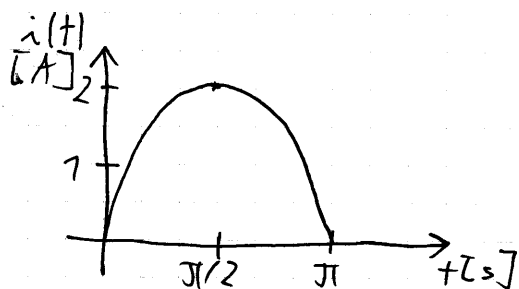
$$L_1 = L_2 = L$$

$$k = 1$$

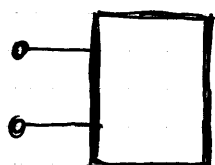
$$u(t) = ?$$

$$u(t) = (L + L - 2L) \frac{d}{dt} i(t) = 0$$

8.) Izračunaj energijo W , ki se je v času $0 \leq t \leq \pi$ stekla v duopol.



$$i(t) = 2 \sin(t)$$

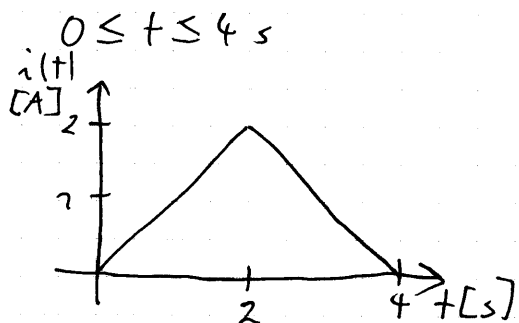


$$W = \int_0^{\pi} i(t) \cdot u(t) dt = \int_0^{\pi/2} 2 \sin(t) dt + \int_{\pi/2}^{\pi} 4 \sin(t) dt$$

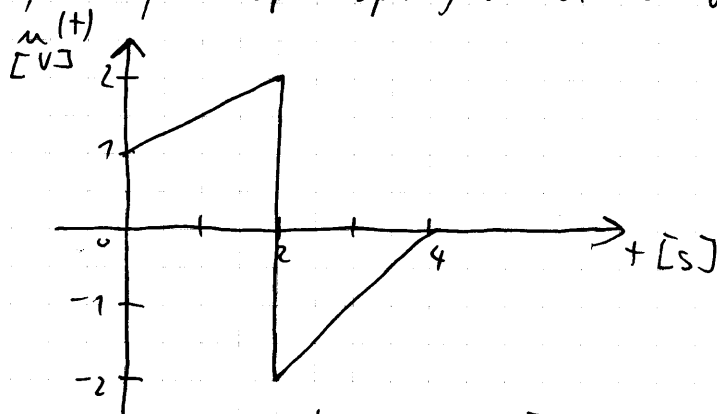
$$W = 2(-\cos(t)) \Big|_0^{\pi/2} + 4(-\cos(t)) \Big|_{\pi/2}^{\pi}$$

$$W = 2(0+1) + 4(1-0) = \underline{\underline{6J}}$$

9.) Izračunaj energijo W , ki jo duopol sprejme od 0 do 4 s.



$$i(t) = \begin{cases} t & t \in [0, 2]s \\ 4-t & t \in [2, 4]s \end{cases}$$



$$u(t) = \begin{cases} \frac{t}{2} + 1 & t \in [0, 2]s \\ t-4 & t \in [2, 4]s \end{cases}$$

$$p(t) = \begin{cases} \frac{t^2}{2} + t & t \in [0, 2] \\ (4-t)(t-4) & t \in [2, 4] \end{cases}$$

$$W = \int_0^4 p(t) \cdot dt = \int_0^2 \left(\frac{t^2}{2} + t \right) dt + \int_2^4 (t-4)(4-t) dt = \left(\frac{t^3}{6} + \frac{t^2}{2} \right) \Big|_0^2 + \left(\frac{t^3}{3} - \frac{8t^2}{2} + 16t \right) \Big|_2^4$$

$$8W = \underline{\underline{\frac{2}{3}J}}$$

$W(t_0) + W(t_0, t) \geq 0$ je pasivno.

Ali je kondenzator pasiven element?

$$\begin{aligned} W(t_0) &= W(-\infty, t_0) \\ W(t_0, t) &= \int_{t_0}^t u(t) \cdot i(t) dt \end{aligned}$$

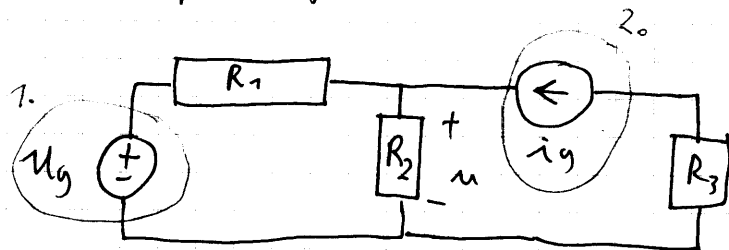
$$\begin{aligned} W(t_0) + W(t_0, t) &= \int_{-\infty}^{t_0} u(t) i(t) dt + \int_{t_0}^t u(t) i(t) dt = \\ &= \int_{-\infty}^{t_0} u(t) \frac{du(t)}{dt} \cdot C \cdot dt + \int_{t_0}^t u(t) \cdot C \frac{du(t)}{dt} \cdot dt = \\ &= \int_{-\infty}^{t_0} C \cdot u(t) du(t) + \int_{t_0}^t C u(t) du(t) = \frac{C}{2} (\cancel{u^2(t_0)} - \underbrace{u^2(-\infty)}_{=0, \text{ ker ni bil nabit}}) + \\ &+ \frac{C}{2} (\cancel{u^2(t)} - \cancel{u^2(t_0)}) = \underline{\underline{\frac{C}{2} u^2(t) \geq 0}} \end{aligned}$$

Dokaži, da je upor pasiven.

$$\begin{aligned} W &= \int_{-\infty}^{t_0} u(t) i(t) dt + \int_{t_0}^t u(t) i(t) dt = \\ &= \int_{-\infty}^{t_0} R i^2(t) dt + \int_{t_0}^t R i^2(t) dt \geq 0 \end{aligned}$$

Upor je pasiven.

Superpozicija

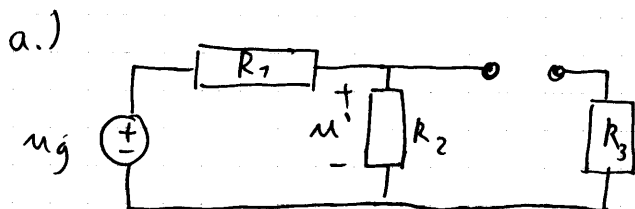


1. Označi neodvisne viře

2. Izberes enega, ostale izklopiš



3. Seštej delne rezultate



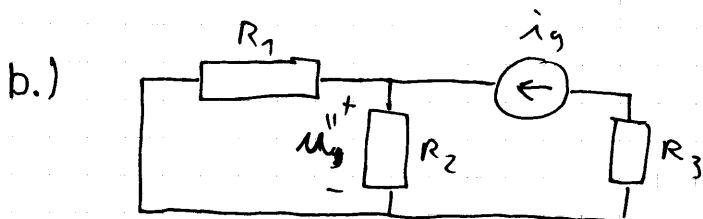
$$u' = \frac{R_2}{R_1 + R_2} \cdot u_g$$

$$\begin{aligned} i_{R_1} &= i_{R_2} \quad , \quad u_{R_1} + u_{R_2} = u_g \\ \frac{u_{R_1}}{R_1} &= \frac{u_{R_2}}{R_2} \\ u_{R_1} &= \frac{R_1}{R_2} \cdot u_{R_2} \end{aligned}$$

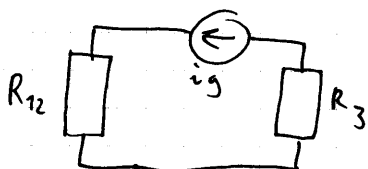
$$\frac{R_1}{R_2} u_{R_2} + u_{R_2} = u_g$$

$$\frac{R_1 + R_2}{R_2} \cdot u_{R_2} = u_g$$

$$u_{R_2} = \frac{R}{R_1 + R_2} u_g$$

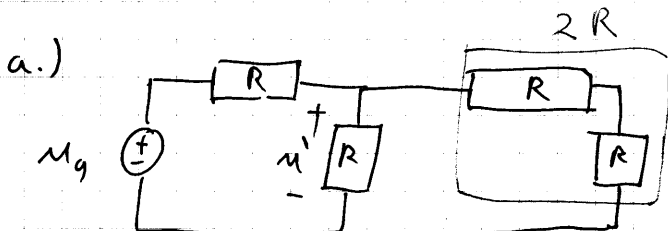
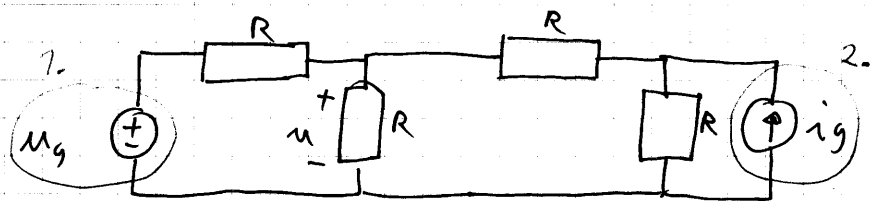


$$R_{12} = \frac{R_1 R_2}{R_1 + R_2}$$

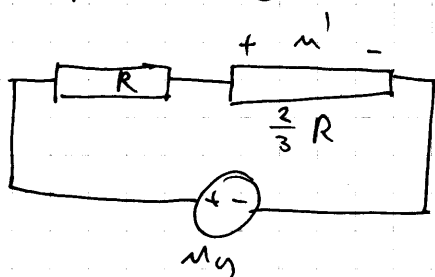


$$u'' = \frac{R_1 R_2}{R_1 + R_2} \cdot i_g$$

Izračunaj napetost $u(t)$.

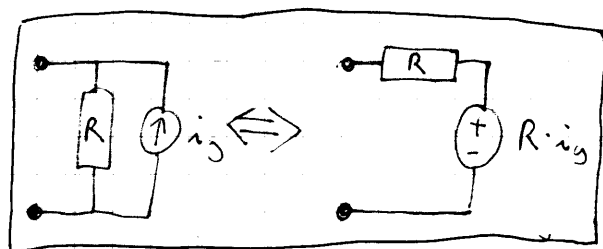
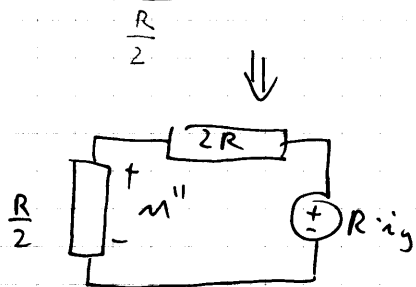
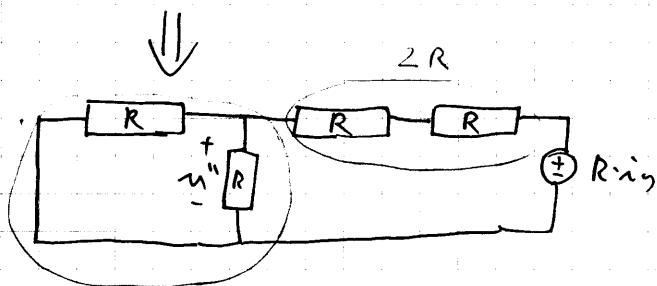
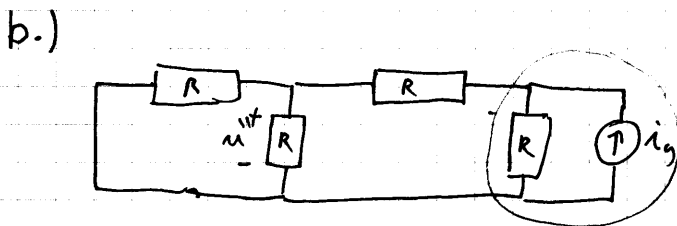


$$\left(\frac{1}{2R} + \frac{1}{R}\right)^{-1} = \frac{2}{3}R$$



$$u' = \frac{\frac{2}{3}R}{R + \frac{2}{3}R} \cdot u_g$$

$$u' = \frac{\frac{2}{3}R}{\frac{5}{3}R} = \underline{\underline{\frac{2}{5} u_g}}$$



$$i_g = \frac{1}{R} \cdot u_g$$

$$R \cdot i_g = u_g$$

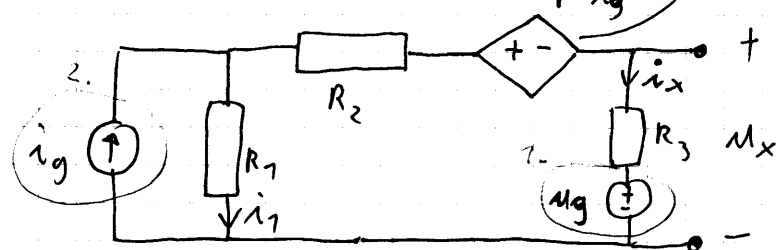
$$u'' = \frac{\frac{R}{2}}{\frac{R}{2} + 2R} \cdot R \cdot i_g = \frac{\frac{R}{2}}{\frac{5}{2}R} \cdot R \cdot i_g$$

$$u'' = \underline{\underline{\frac{1}{5} R \cdot i_g}}$$

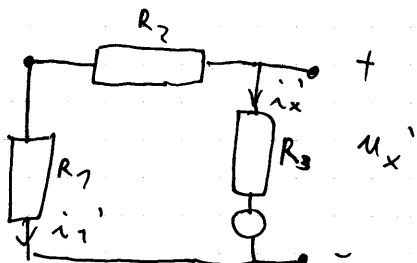
$$u(t) = u' + u'' = \boxed{\frac{2}{5} u_g + \frac{1}{5} R \cdot i_g}$$

Izračunaj napetost

tokovo kr. nap. generator



a.)



$$i_1' = \frac{u_g}{R_1 + R_2 + R_3}$$

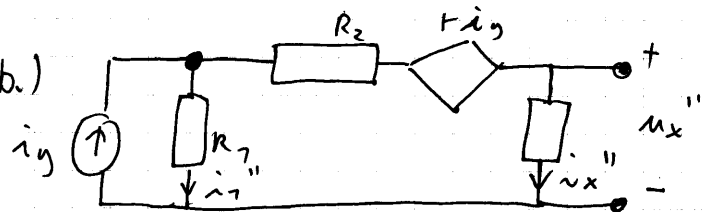
$$u_{R_3} = i_{x'} \cdot R_3$$

$$u_x = u_{R_3} + u_g = i_{x'} \cdot R_3 + u_g \quad ; \quad i_{x'} = -i_1'$$

$$u_x' = -\frac{R_3}{R_1 + R_2 + R_3} u_g + u_g =$$

$$u_x' = \frac{R_1 + R_2}{R_1 + R_2 + R_3} u_g$$

b.)



$$-i_g + i_1'' + i_{x''} = 0$$

$$i_1'' + i_{x''} = i_g$$

$$u_{R_1} = u_{R_2} + u_{R_3} + r i_g$$

$$i_1'' \cdot R_1 = i_{x''} \cdot R_2 + i_{x''} \cdot R_3 + r i_g$$

$$i_1'' = i_g - i_{x''}$$

$$(i_g - i_{x''}) \cdot R_1 = i_{x''} (R_2 + R_3) + r i_g$$

$$i_g (R_1 - r) = i_{x''} (R_1 + R_2 + R_3)$$

$$i_{x''} = \frac{i_g (R_1 - r)}{R_1 + R_2 + R_3}$$

$$u_x'' = \frac{R_3 (R_1 - r)}{R_1 + R_2 + R_3} i_g$$

$$u_x = u_x' + u_x''$$

$$u_x = \frac{R_1 + R_2}{R_1 + R_2 + R_3} u_g + \frac{R_3 (R_1 - r)}{R_1 + R_2 + R_3} i_g$$

Imamo 2. harmonična generatorja. Velja $\omega_1 \neq \omega_2$. Ali superpozicij moči velja ali ne?

$$P_{12} = P_1 + P_2$$

$$P = \overline{u \cdot i} = \frac{1}{T} \int_0^T u(t) i(t) dt$$

$$\cos(a) + \cos(b) = \frac{1}{2} (\cos(a+b) + \cos(a-b))$$

$$u_1(t) = U_{g10} \cdot \cos(\omega_1 t + \varphi_1)$$

$$i_1(t) = I_{g10} \cdot \cos(\omega_1 t + \psi_1)$$

$$u_2(t) = U_{g20} \cdot \cos(\omega_2 t + \varphi_2)$$

$$i_2(t) = I_{g20} \cdot \cos(\omega_2 t + \psi_2)$$

$$P_{12} = \overline{(u_1 + u_2) \cdot (i_1 + i_2)} = \overline{u_1 \cdot i_1} + \overline{u_1 \cdot i_2} + \overline{u_2 \cdot i_2} + \overline{u_2 \cdot i_1}$$

$$\begin{aligned} \overline{u_1 i_2} &= \frac{1}{T} \int_0^T U_{g10} \cdot \cos(\omega_1 t + \varphi_1) \cdot I_{g20} \cdot \cos(\omega_2 t + \psi_2) dt \\ &= \frac{1}{2T} \left(\int_0^T U_{g10} I_{g20} \cdot \cos((\omega_1 + \omega_2)t + \varphi_1 + \psi_2) dt + \right. \\ &\quad \left. + \int_0^T U_{g10} I_{g20} \cos((\omega_1 - \omega_2)t + \varphi_1 - \psi_2) dt \right) \end{aligned}$$

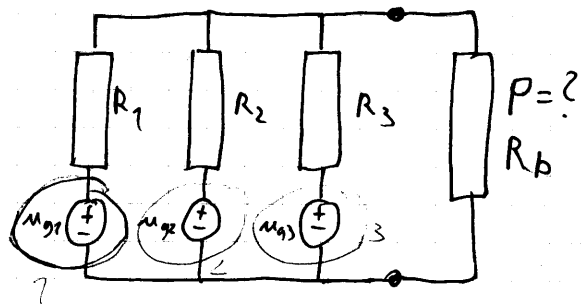
$$\int_0^T \cos(\omega t + \varphi) dt \begin{cases} 0 & \omega > 0 \\ T \cos(\varphi) & \omega = 0 \end{cases}$$

$$\overline{u_1 i_2} = \begin{cases} \frac{U_{g10} I_{g20}}{2T} \cos(\varphi_1 + \psi_2) & \omega_1 = \omega_2 \\ 0 & \omega_1 \neq \omega_2 \end{cases}$$

Če sta frekvenci enaki potem ne velja.

Če frekvenci nista enaki potem velja.

Moć na oporniku R_p ?



~~$$P = P_1 + P_2 + P_3$$~~

$$P = P_2 + P_3$$

$$R_1 = 1 \Omega$$

$$R_2 = 2 \Omega$$

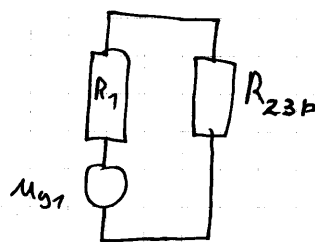
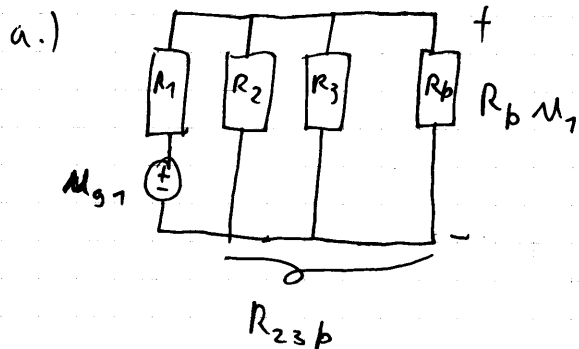
$$R_3 = 3 \Omega$$

$$R_p = 1 \Omega$$

$$u_{g1} = 10V \cdot \cos(0)$$

$$u_{g2} = 20V \cdot \cos(0)$$

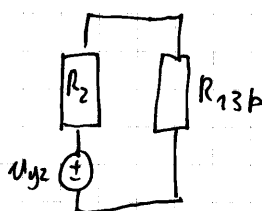
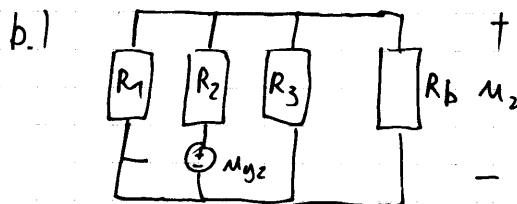
$$u_{g3} = 10 \cos(\omega t) V$$



$$u_1 = \frac{R_{23p}}{R_1 + R_{23p}} \cdot u_{g1}$$

$$R_{23p} = \frac{R_2 R_3 R_p}{R_2 R_3 + R_2 R_p + R_3 R_p} = \frac{6}{17} \Omega$$

$$u_1 = \frac{60}{17} V$$



$$u_2 = \frac{R_{13p}}{R_2 + R_{13p}} u_{g2}$$

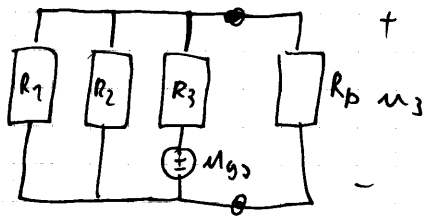
$$R_{13p} = \frac{R_1 R_3 R_p}{R_1 R_3 + R_1 R_p + R_3 R_p} = \frac{3}{7} \Omega$$

$$u_2 = \frac{60}{17} V$$

$$u_{12} = u_1 + u_2 = \frac{120}{17} V$$

$$P_{12} = u_{12}^2 / R_p = \left(\frac{120}{17} \right)^2 = 49.8 W$$

$$c.) \quad P_3 = \frac{u_3^2}{R_p}$$



$$u_3 = \frac{R_{12p}}{R_3 + R_{12p}} \cdot u_{gs}$$

$$R_{12p} = \frac{R_1 \cdot R_2 \cdot R_p}{R_1 R_2 + R_1 R_p + R_2 R_p} = \frac{2}{5} \Omega$$

$$u_3 = \frac{\frac{2}{5}}{3 + \frac{2}{5}} \cdot 10 \cos(\omega t)$$

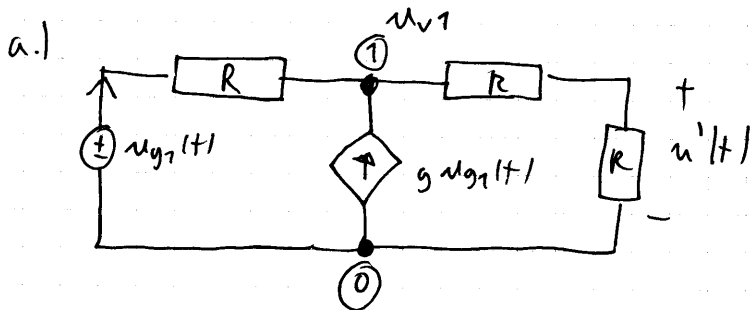
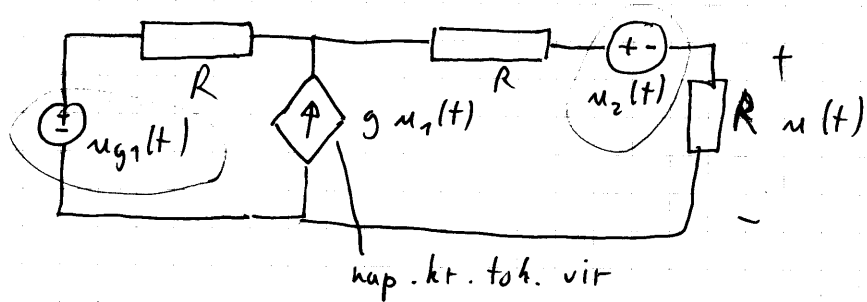
$$u_3 = \frac{20}{17} \cos(\omega t) V$$

$$P_3 = \frac{1}{2} U_{p3} \cdot I_{p3} \cdot \cos(\varphi) ; \quad i_{p3} = \frac{u_p}{R_p} = \frac{20}{17} \cdot \cos(\omega t) A$$

$$P_3 = \frac{1}{2} \cdot \frac{20}{17} \cdot \frac{20}{17} \cos(0) = \frac{1}{2} \left(\frac{20}{17} \right)^2 = 0,69 W$$

$$P = P_{12} + P_3 = 50,5 W$$

Napetost s pomočjo superpozicije!



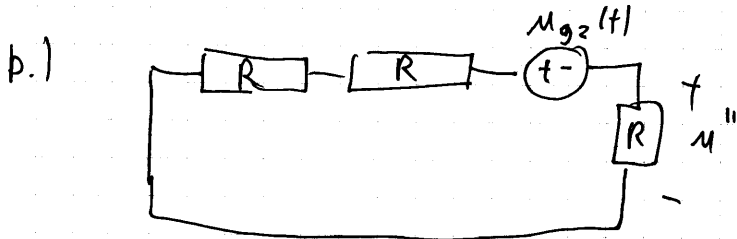
$$1: \frac{u_{v1}}{R} - \frac{u_{g1}}{R} - g u_{g1}(t) + \frac{u_{v1}}{2R} = 0$$

$$u_{v1} = \left(\frac{1}{R} + \frac{1}{2R} \right) = \frac{u_{g1}(t)}{R} + g u_{g1}(t)$$

$$u_{v1} \frac{3}{2R} = u_{g1}(t) \left(\frac{1}{R} + g \right)$$

$$u_{v1} = \frac{2}{3} (u_{g1}(t) + g R u_{g1}(t)) ; u'(t) = \frac{1}{2} u_{v1}$$

$$u'(t) = \frac{1}{3} (u_{g1}(t) + g R u_{g1}(t))$$



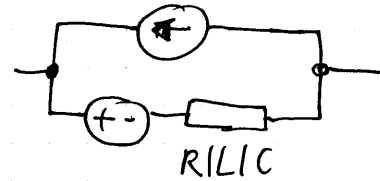
$$u'' = \frac{u_{g2}(t)}{3}$$

27.10.2008

Opis vezja

1. Izberi referenčno vozlišče

0

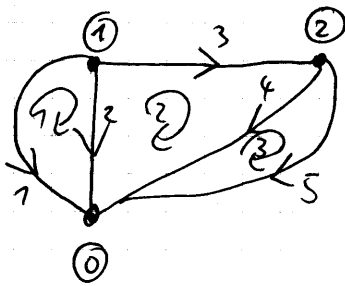


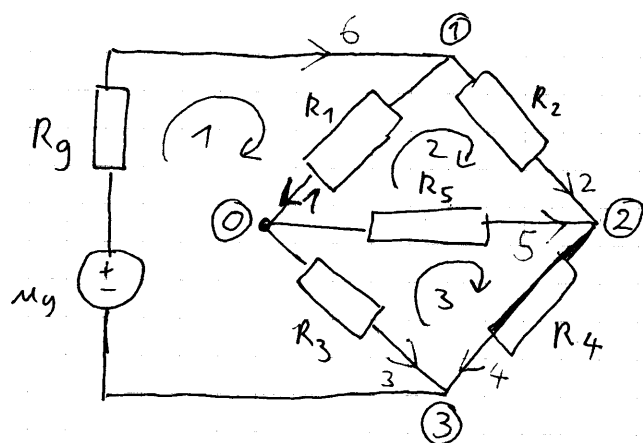
2. Označimo/oštevilčimo ostala vozlišča

3. Označi smer vej

4. Označimo okna

Graf vezja





Vejna

- samo $\ominus \oplus$
- iščemo vejne tokove $i_1, i_2, i_3, \dots, i_n$
- Kirch. tokovne enačbe $\rightarrow$ vozlišča



- Kirch. napetostne enačbe po zankah



$$\textcircled{1} \quad i_1 + i_2 - i_6 = 0$$

$$\textcircled{2} \quad -i_2 + i_4 - i_5 = 0$$

$$\textcircled{3} \quad -i_3 - i_4 + i_6 = 0$$

$$\textcircled{1} \quad R_1 \cdot i_1 + R_3 \cdot i_3 - u_g + R_g i_6 = 0$$

$$\textcircled{2} \quad -R_1 \cdot i_1 + R_2 \cdot i_2 - R_5 \cdot i_5 = 0$$

$$\textcircled{3} \quad -R_3 \cdot i_3 + R_4 \cdot i_4 + R_5 \cdot i_5 = 0$$

Zančna

- samo 
- iščemo zanke tokov $i_{z1}, i_{z2}, i_{z3}, \dots, i_{zn}$
- Kirch. napetostne enačbe po zankah



$$(1) R_1(i_{z1} - i_{z2}) + R_3(i_{z1} - i_{z3}) - u_g + R_g i_{z1} = 0$$

$i_{z1} \rightarrow i_{z2}, i_{z3}$

$$(2) R_1(i_{z2} - i_{z1}) + R_2(i_{z2}) + R_5(i_{z2} - i_{z3}) = 0$$

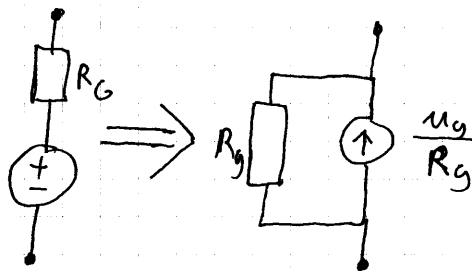
$i_{z2} \rightarrow i_{z1}, i_{z3}$

$$(3) R_3(i_{z3} - i_{z1}) + R_4 i_{z3} + R_5(i_{z3} - i_{z2}) = 0$$

$i_{z3} \rightarrow i_{z2}, i_{z1}$

Vozliščna

- samo 



- iščemo vozliščne napetosti $u_{v1}, u_{v2}, \dots, u_{vn}$
- Kirch. tokovne enačbe po vozliščih



$$(1) \frac{u_{v1}}{R_1} + \frac{u_{v1} - u_{v2}}{R_2} - \frac{u_g}{R_g} + \frac{u_{v1} - u_{v3}}{R_g} = 0$$

$u_{v1} \rightarrow u_{v2}, u_{v3}$

$$(2) \frac{u_{v2} - u_{v1}}{R_2} + \frac{u_{v2}}{R_5} + \frac{u_{v2} - u_{v3}}{R_4} = 0$$

$u_{v2} \rightarrow u_{v1}, u_{v3}$

$$(3) \frac{u_{v3}}{R_3} + \frac{u_{v3} - u_{v2}}{R_4} + \frac{u_g}{R_g} + \frac{u_{v3} - u_{v1}}{R_g} = 0$$

$u_{v3} \rightarrow u_{v1}, u_{v2}$

Alta

Enako vezje kot prej. Izračunaj tok skozi upor R_5 !

$$(1) R_1(i_{z1} - i_{z2}) + R_3(i_{z1} - i_{z3}) - u_g + R_g i_{z1} = 0$$

$$(2) R_1(i_{z2} - i_{z1}) + R_2(i_{z2}) + R_5(i_{z2} - i_{z3}) = 0$$

$$(3) R_3(i_{z3} - i_{z1}) + R_4(i_{z3}) + R_5(i_{z3} - i_{z2}) = 0$$

$$R = 1 \Omega$$

$$u_g = 10 V$$

$$i_{z1}(R_1 + R_3 + R_g) - i_{z2}R_1 - i_{z3}R_3 = u_g$$

$$-i_{z1}R_1 + i_{z2}(R_1 + R_2 + R_5) - i_{z3}R_5 = 0$$

$$-i_{z1}R_3 - i_{z2}R_5 + i_{z3}(R_3 + R_4 + R_5) = 0$$

$$3i_{z1} - i_{z2} - i_{z3} = 10$$

$$-i_{z1} + 3i_{z2} - i_{z3} = 0$$

$$-i_{z1} - i_{z2} + 3i_{z3} = 0$$

$$\left. \begin{array}{l} -i_{z1} + 3i_{z2} - i_{z3} = 0 \\ -i_{z1} - i_{z2} + 3i_{z3} = 0 \end{array} \right\} \rightarrow \begin{cases} 4i_{z2} - 4i_{z3} = 0 \\ i_{z2} = i_{z3} \end{cases}$$

$$3 \cdot i_{z1} - 2i_{z2} = 10$$

$$i_{z1} = \frac{10 + 2i_{z2}}{3}$$

$$-\frac{10 + 2i_{z2}}{3} - i_{z2} + 3i_{z2} = 0 \quad / \cdot 3$$

$$-10 - 2i_{z2} - 3i_{z2} + 9i_{z2} = 0$$

$$-10 = -4i_{z2}$$

$$i_{z2} = \frac{5}{2} A = i_{z3} \quad i_{z1} = 5 A$$

$$i_5 = i_{z3} - i_{z2} = 0 A$$

Reducirana vpadna:

Enako vezje kot prej

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

VEJE

veje v/iz vozlišča

0 - ne stik
1 - stik

Reducirana matrika mreže

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix}$$

VEJE

zanka - veja

1 - sovpada
-1 - nasprotuje

Matrika tokovih operatorjev

$$\begin{bmatrix} R_1 & & & & & \\ & R_2 & & & & \\ & & R_3 & & & \\ & & & R_4 & & \\ & & & & R_5 & \\ & & & & & R_5 \end{bmatrix}$$

Tok veje

Napetost
veje

Napetostni operatorji

$$\begin{bmatrix} \frac{1}{R_1} & & & & & \\ & \frac{1}{R_2} & & & & \\ & & \frac{1}{R_3} & & & \\ & & & \frac{1}{R_4} & & \\ & & & & \frac{1}{R_5} & \\ & & & & & \frac{1}{R_5} \end{bmatrix}$$

Napetost veje

Tok
veje

Napetostno in tokovno vzbujana vektorja

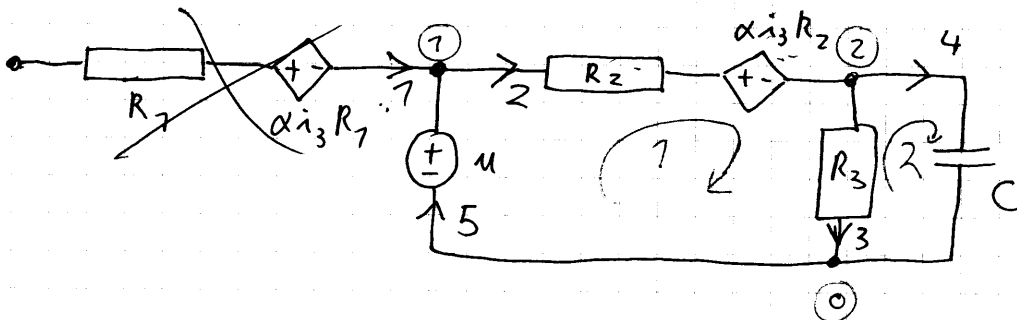
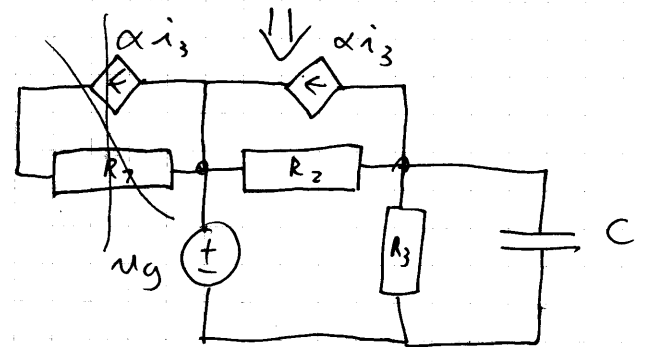
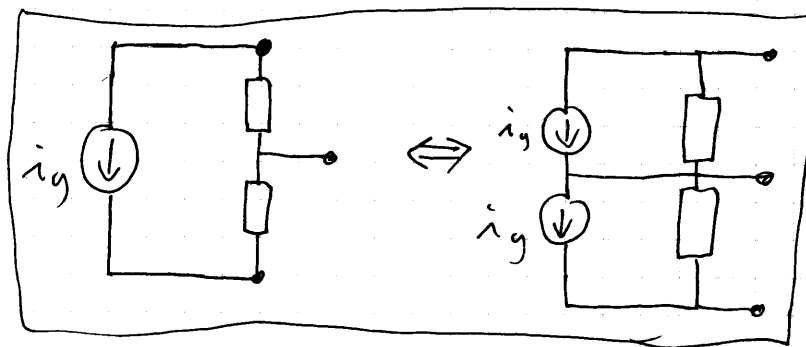
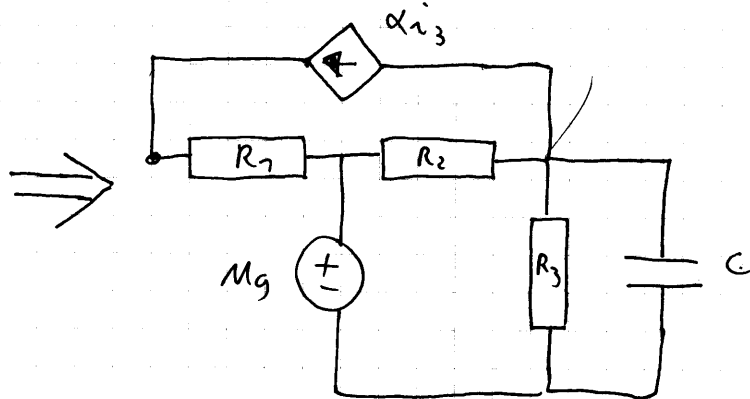
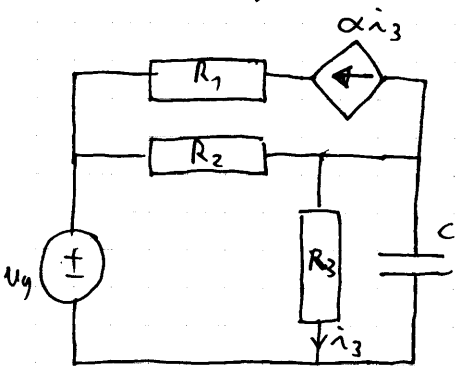
$$i_g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad u_g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ u_g \end{bmatrix}$$

$$M \cdot Z(D) \cdot M^T = \begin{bmatrix} R_1 + R_3 + R_g & -R_1 & -R_3 \\ -R_1 & R_1 + R_2 + R_5 & -R_5 \\ -R_3 & -R_5 & R_3 + R_4 + R_5 \end{bmatrix}$$

$$A \cdot Y(D) \cdot A^T = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_g} & -\frac{1}{R_2} & -\frac{1}{R_g} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & -\frac{1}{R_4} \\ -\frac{1}{R_g} & -\frac{1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_g} \end{bmatrix}$$

Dalje.

Opiši vezje po vseh treh metodah.



$$i_2 = i_5$$

Vejna

$$\alpha i_3 R_2$$

$$(1) i_2 - i_5 = 0$$

$$(2) -i_2 + i_3 + i_4 = 0$$

$$(1^v) -u_g + R_2 i_2 + \alpha i_3 R_2 + R_3 i_3 = 0$$

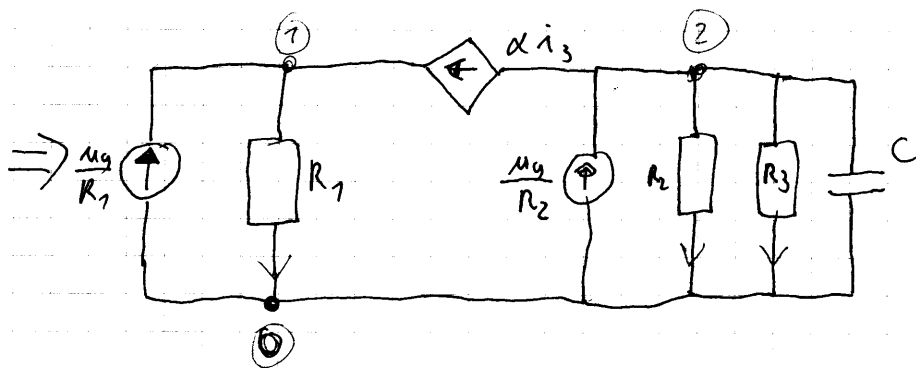
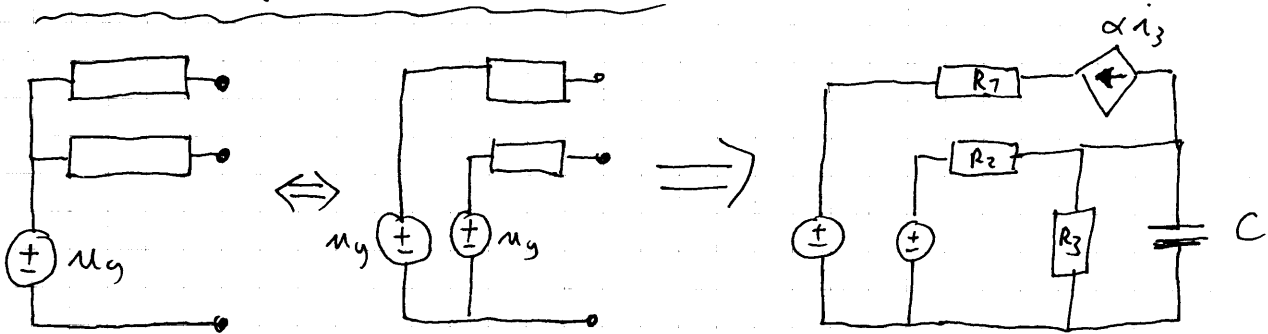
$$(2^v) -R_3 i_3 + \frac{1}{CD} i_4 = 0$$

Začinn

$$\alpha i_3 R_2 = \alpha R_2 (i_{z1} - i_{z2})$$

$$(1^v) -u_g + R_2 i_{z2} + \alpha R_2 (i_{z1} - i_{z2}) + R_3 (i_{z1} - i_{z2})$$

$$(2^v) R_3 (i_{z2} - i_{z1}) + \frac{1}{CD} i_{z2} = 0$$



$$i_3 = \frac{u_{v2}}{R_3} \quad \alpha i_3 \Rightarrow \alpha \frac{u_{v2}}{R_3}$$

$$(1) -\frac{u_g}{R_1} + \frac{u_{v1}}{R_1} - \alpha \frac{u_{v2}}{R_3} = 0$$

$$(2) \alpha \frac{u_{v2}}{R_3} - \frac{u_g}{R_2} + \frac{u_{v2}}{R_2} + \frac{u_{v2}}{R_3} + CD u_{v2} = 0$$

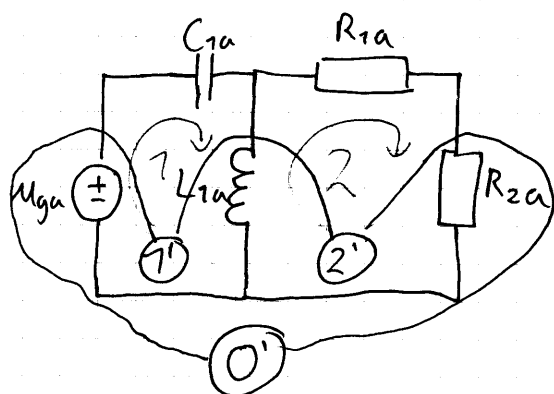
$$R \Leftrightarrow G$$

$$C \Leftrightarrow L$$

$$u \Leftrightarrow i$$

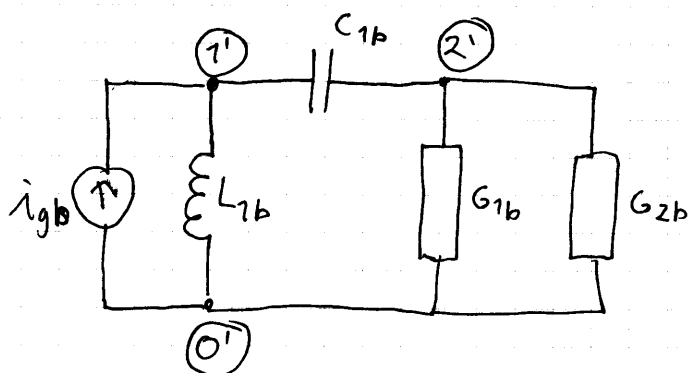
okno $\Leftrightarrow$ vozlišče

zaporedna $\Leftrightarrow$ vzporedna



$$(1') \quad -u_{ga} + \frac{1}{C_{1a}} i_{z1} + L_{1a} D(i_{z1} - i_{z2}) = 0$$

$$(2') \quad L_{1a} D(i_{z2} - i_{z1}) + R_{1a} i_{z2} + R_{2a} i_{z1} = 0$$



$$R_{1a} \Leftrightarrow G_{1b}$$

$$R_{2a} \Leftrightarrow G_{2b}$$

$$L_{1a} \Leftrightarrow C_{1b}$$

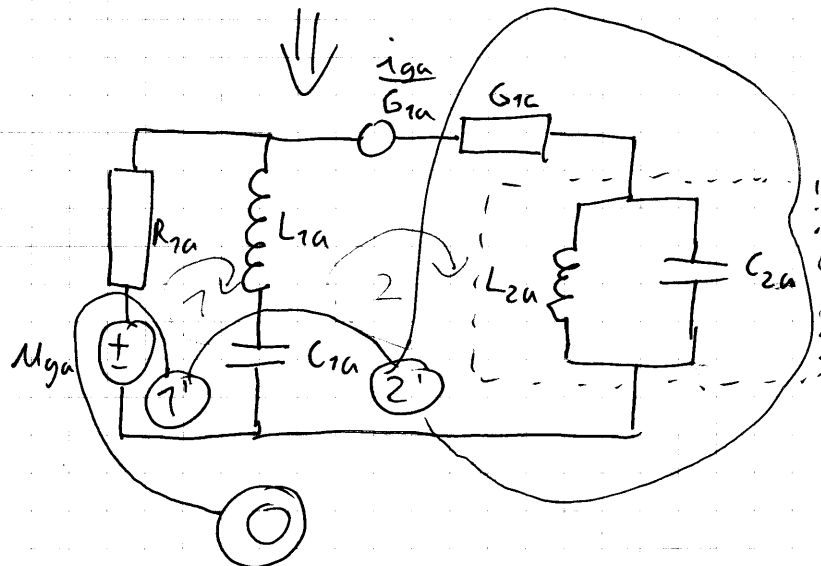
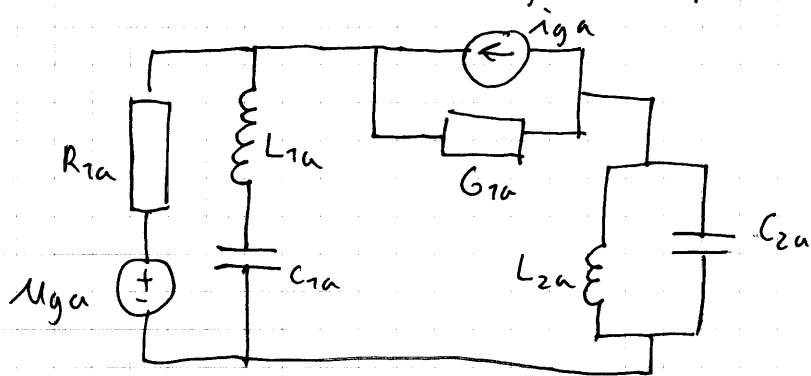
$$C_{1a} \Leftrightarrow L_{1b}$$

$$u_{ga} \Leftrightarrow i_{gb}$$

$$(1') \quad -i_{gb} + \frac{1}{L_{1b}} u_{v1} + C_{1b} D(u_{v1} - u_{v2}) = 0$$

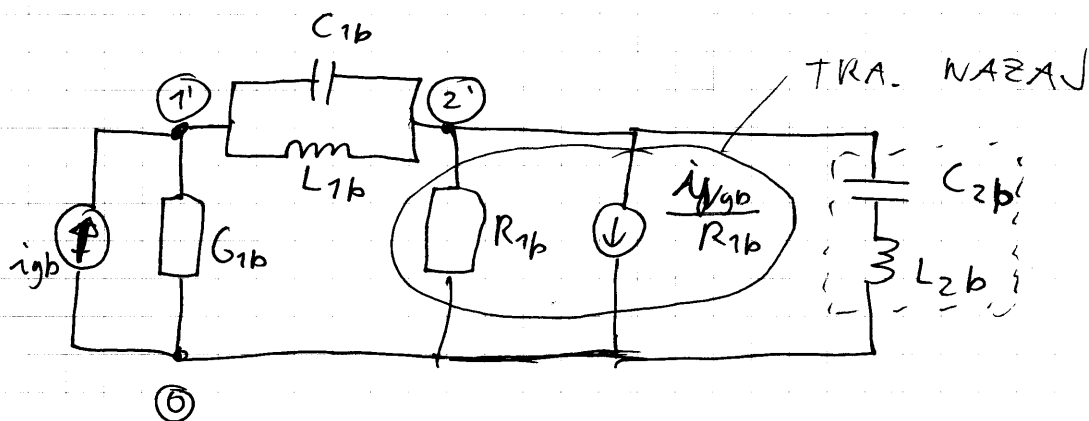
$$(2') \quad C_{1b} D(u_{v2} - u_{v1}) + G_{1b} u_{v2} + G_{2b} u_{v2} = 0$$

Poišči dualno vezje in poišči dualnost sistema enačb.



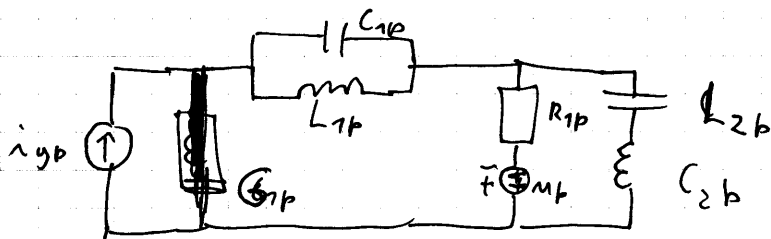
$$\textcircled{1}: -U_{ga} + R_{1a} i_{z1} + L_{1a} D i_{z1} - i_{z2} + \frac{1}{C_{1a} D} (i_{z1} - i_{z2}) = 0$$

$$\textcircled{2}: L_{1a} D (i_{z2} - i_{z1}) + \frac{1}{C_{1a} D} (i_{z2} - i_{z1}) + \frac{i_{ga}}{G_{1a}} + \frac{i_{z2}}{G_{1a}} + \cancel{Z} i_{z2} = 0$$



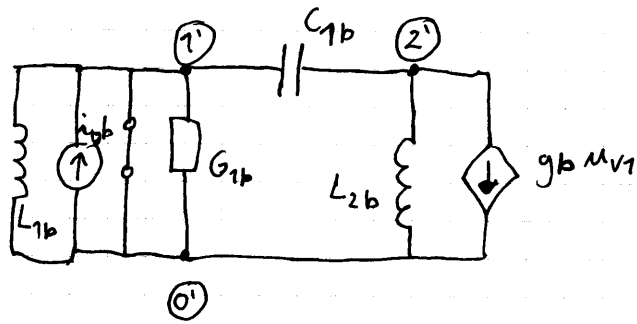
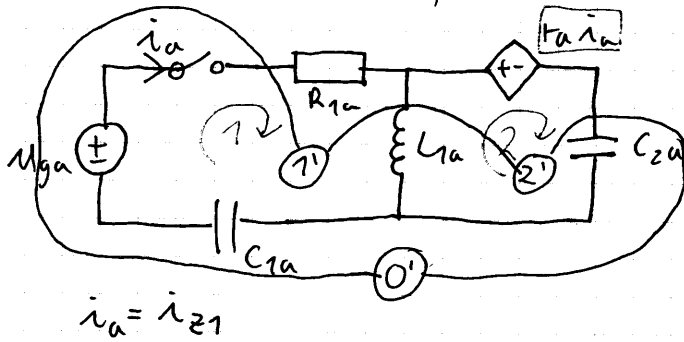
$$\textcircled{1}: -i_{gb} + U_{v1} G_{1b} + C_{1b} D (U_{v1} - U_{v2}) + \frac{1}{L_{1b} D} (U_{v1} - U_{v2}) = 0$$

$$\textcircled{2}: C_{1b} D (U_{v2} - U_{v1}) + \frac{1}{L_{1b} D} (U_{v2} - U_{v1}) + \frac{U_{gb}}{R_{1b}} + \frac{U_{v2}}{R_{1b}} + Y \cdot U_{v2} = 0$$



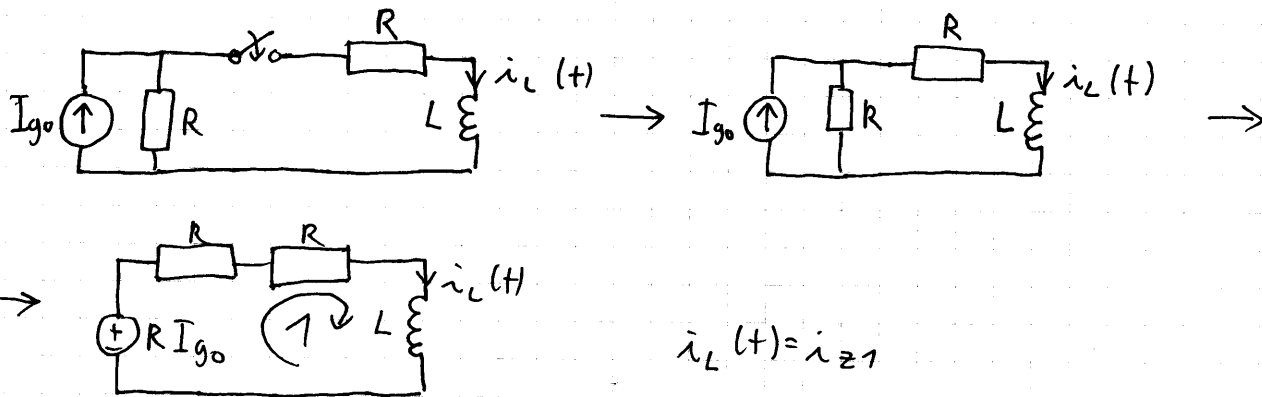
03.11.2008

Poisci dualno vezje.



Klasicna analiza

Zapiši DE za $t \geq 0_s$

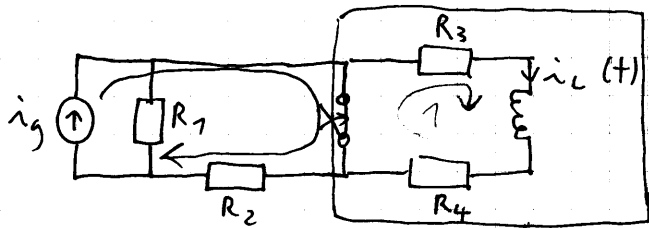


$$\textcircled{1} - RI_{g0} + Ri_{z1} + Ri_{z1} + LD \dot{i}_{z1} = 0$$

$$LD \dot{i}_{z1} + 2Ri_{z1} = RI_{g0}$$

Zapisi DE

$t \geq 0s$



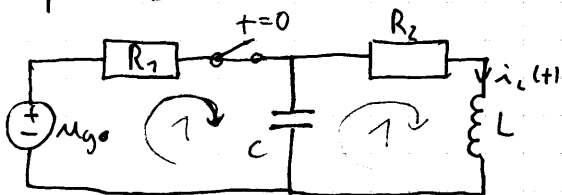
$$\textcircled{1} R_3 \dot{i}_{z1} + R_4 \dot{i}_{z1} + L D \dot{i}_{z1} = 0$$

$$i_L(t) = \dot{i}_{z1}$$

$$L D \dot{i}_{z1} + (R_3 + R_4) \cdot \dot{i}_{z1} = 0$$

Zapisi DE

$t \geq 0s$



$$i_L(t) = \dot{i}_{z2}$$

$$\textcircled{1} -u_{g0} + R_1 \dot{i}_{z1} + \frac{1}{CD} (\dot{i}_{z1} - \dot{i}_{z2}) = 0$$

$$\textcircled{2} \frac{1}{CD} (\dot{i}_{z2} - \dot{i}_{z1}) + R_2 \dot{i}_{z2} + L D \dot{i}_{z2} = 0$$

$$\dot{i}_{z1} \left(R_1 + \frac{1}{CD} \right) = u_{g0} + \dot{i}_{z2} \cdot \frac{1}{CD} \quad / \cdot CD$$

$$\dot{i}_{z1} (R_1 CD + 1) = CD u_{g0} + \dot{i}_{z2}$$

$$\dot{i}_{z1} = \frac{CD u_{g0} + \dot{i}_{z2}}{1 + R_1 CD}$$

$$\frac{1}{CD} (\dot{i}_{z2}) + \dot{i}_{z2} (R_2 + LD) - \frac{1}{CD} \left(\frac{CD u_{g0} + \dot{i}_{z2}}{1 + R_1 CD} \right) = 0$$

$$\frac{1}{CD} \dot{i}_{z2} - \frac{1}{CD} \frac{\dot{i}_{z2}}{1 + R_1 CD} + \dot{i}_{z2} (R_2 + LD) = \frac{1}{CD} \frac{CD u_{g0}}{1 + R_1 CD}$$

$$\frac{\dot{i}_{z2} (1 + R_1 CD) - \dot{i}_{z2}}{CD (1 + R_1 CD)} + \dot{i}_{z2} (R_2 + LD) = \frac{u_{g0}}{1 + R_1 CD}$$

$$\frac{\dot{i}_{z2} + \dot{i}_{z2} R_1 CD - \dot{i}_{z2}}{CD (1 + R_1 CD)} + \dot{i}_{z2} (R_2 + LD) = \frac{u_{g0}}{1 + R_1 CD}$$

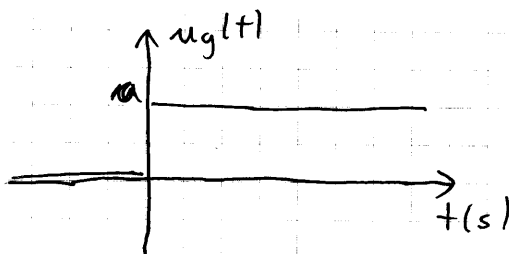
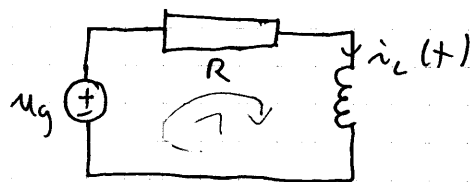
$$\frac{\dot{i}_{z2} R_1}{1 + R_1 CD} + \dot{i}_{z2} (R_2 + CD) = \frac{u_{g0}}{1 + R_1 CD} \quad / \cdot (1 + R_1 CD)$$

$$\dot{i}_{z2} R_1 + \dot{i}_{z2} (R_2 + CD) (1 + R_1 CD) = u_{g0}$$

$$R_1 L CD^2 \dot{i}_{z2} + (L + R_1 R_2 C) D \dot{i}_{z2} + (R_1 + R_2) \dot{i}_{z2} = u_{g0}$$

Alketa

Izračunaj in nariši $i_L(t)$



$$\textcircled{1} -u_g + R i_{z1} + L D i_{z1} = 0$$

$$i_L = i_{z1}$$

$$L D i_L + R i_L = u_g$$

$$t < 0 \quad i_L(t) = 0$$

a) Homogena

$$L D i_L + R i_L = 0$$

$$L p + R = 0$$

$$p = -\frac{R}{L}$$

$$i_H(t) = K \cdot e^{-\frac{R}{L}t}$$

$$i_H(t) = K \cdot e^{pt}$$

-krat. pol.

b) Partikularni del

$$i_p(t) = k$$

$$L D i_p(t) + R i_p(t) = a$$

$$L D k + R k = a$$

$$0 \quad R k = a$$

$$k = \frac{a}{R}$$

$$i_p(t) = \frac{a}{R}$$

c) Določi konstanto

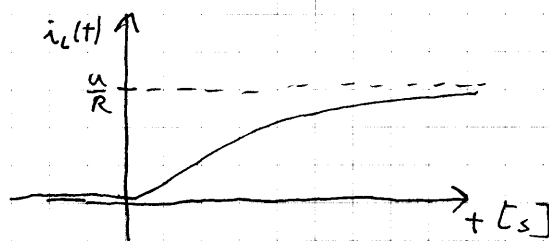
$$i_L(t) = i_H(t) + i_p(t) = K e^{-\frac{R}{L}t} + \frac{a}{R}$$

$$\text{začetni pogoji: } i_L(t < 0) = 0 \text{ A} \quad i_L(t < 0_s) = i_L(0_s)$$

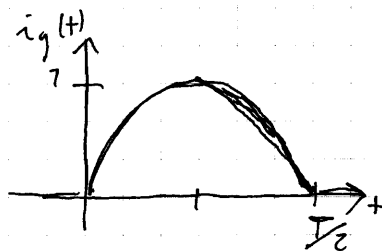
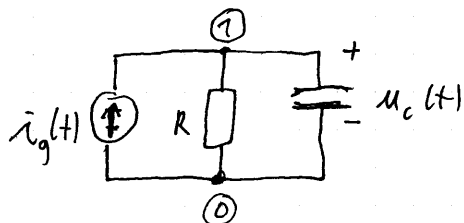
$$i_L(0) = K + \frac{a}{R} = 0 \quad ; \quad K = -\frac{a}{R}$$

$$i_L(t) = \frac{a}{R} - \frac{a}{R} e^{-\frac{R}{L}t}$$

$$i_C(t) = \frac{a}{R} (1 - e^{-\frac{R}{L}t})$$



Izračunaj $u_C(t)$.



$$i_g(t) = \begin{cases} 0, & t < 0 \\ \sin(\omega t), & t \in [0, T/2] \\ 0, & t > T/2 \end{cases}$$

$$\textcircled{1} -i_g(t) + \frac{u_C}{R} + C \frac{du_C}{dt} = 0$$

$$u_C(t) = u_{v1}$$

$$t < 0 \quad u_C(t) = 0$$

$$C \frac{du_C(t)}{dt} + \frac{1}{R} u_C(t) = i_g(t)$$

a) Homogeni del

$$C \frac{du_C(t)}{dt} + \frac{1}{R} u_C(t) = 0$$

$$u_C(t) = K e^{pt}; \quad Cp + \frac{1}{R} = 0; \quad p = -\frac{1}{RC}$$

$$u_C(t) = K e^{-\frac{t}{RC}}$$

b) Partikularni del $t \in [0, T/2] \Rightarrow i_g \neq 0$

$$i_g(t) = \sin(\omega t); \quad u_{cp}(t) = a \sin(\omega t) + b \cos(\omega t)$$

$$C \frac{du_{cp}(t)}{dt} + \frac{1}{R} u_{cp}(t) = \sin(\omega t)$$

$$C \cdot (a\omega \cos(\omega t) - b\omega \sin(\omega t)) + \frac{1}{R} (a \sin(\omega t) + b \cos(\omega t)) = \sin(\omega t)$$

$$(a\omega + \frac{b}{RC}) \cos(\omega t) + (\frac{a}{RC} - b\omega) \sin(\omega t) = \frac{1}{C} \sin(\omega t)$$

$$a\omega + \frac{b}{RC} = 0 \quad \frac{a}{RC} - b\omega = \frac{1}{C}$$

$$a\omega = -\frac{b}{RC}$$

$$a = -\frac{b}{\omega RC}$$

$$-\frac{\frac{b}{\omega RC}}{RC} - b\omega = \frac{1}{C} \Rightarrow -\frac{b}{\omega R^2 C^2} - b\omega = \frac{1}{C}$$

$$b \left(-\frac{1}{\omega R^2 C^2} - \omega \right) = \frac{1}{C}$$

$$b = \frac{-\omega R^2 C}{1 + \omega^2 R^2 C^2}$$

$$a = -\frac{-\frac{\omega R^2 C}{1 + \omega^2 R^2 C^2}}{\omega RC} = \frac{R}{1 + \omega^2 R^2 C^2}$$

$$u_{cp}(t) = \frac{R}{1+\omega^2 R^2 C^2} \sin(\omega t) - \frac{\omega R^2 C}{1+\omega^2 R^2 C^2} \cos(\omega t)$$

$$t \in [0, T/2]$$

$$u_M(0) + u_p(0) = 0$$

$$K = \frac{\omega R^2 C}{1+\omega^2 R^2 C^2} = 0$$

$$\underline{K = \frac{\omega R^2 C}{1+\omega^2 R^2 C^2}}$$

$$t > T/2 \rightarrow u_p = 0$$

$$u_M(T/2) = K e^{-\frac{T}{2RC}} = \frac{\omega R^2 C}{1+\omega^2 R^2 C^2} (\omega R C e^{-\frac{T}{2RC}} - 1)$$

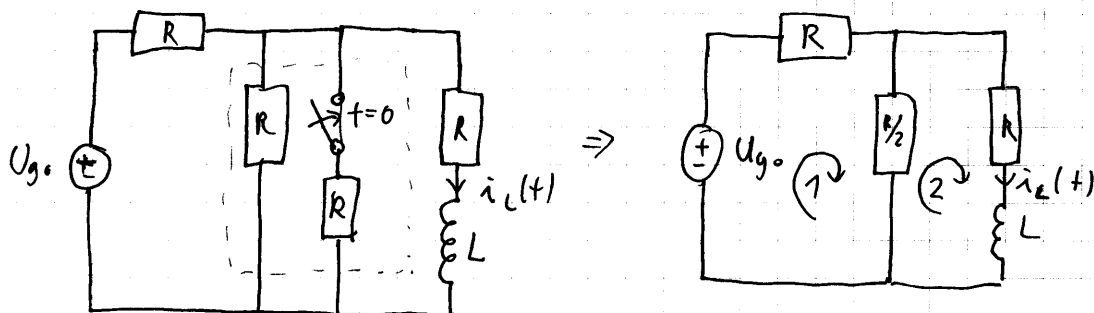
$$K = \frac{\omega^2 R^2 C}{1+\omega^2 R^2 C^2} (1 - e^{-\frac{T}{2RC}})$$

$$t < 0, \quad u_c(t) = 0$$

$$t \in [0, T/2] \quad u_c(t) = \frac{R}{1+\omega^2 R^2 C^2} (\omega R C e^{-\frac{t}{RC}} + \sin(\omega t) - \omega R C \cdot \cos(\omega t))$$

$$t > T/2 \quad u_c(t) = \frac{\omega R^2 C}{1+\omega^2 R^2 C^2} (1 - e^{-\frac{T}{2RC}}) \cdot e^{-\frac{t}{RC}}$$

Kakšen je tok skozi tuljavo



$$\begin{aligned} (1) \quad -U_{g0} + R i_{z1} + \frac{R}{2} (i_{z1} - i_{z2}) &= 0 \\ (2) \quad \frac{R}{2} (i_{z2} - i_{z1}) + R i_{z2} + L D i_{z2} &= 0 \end{aligned}$$

$$i_L(t) = i_{z2}$$

$$\rightarrow \frac{3R}{2} i_{z1} - \frac{R}{2} i_{z2} - U_{g0} = 0$$

$$\frac{3R}{2} i_{z1} = \frac{R}{2} i_{z2} + U_{g0} \quad / \cdot \frac{2}{3R}$$

$$i_{z1} = \frac{1}{3} i_{z2} + \frac{2}{3R} U_{g0}$$

$$\rightarrow \frac{3R}{2} i_{z2} + L D i_{z2} - \frac{R}{2} i_{z1} = 0$$

$$\frac{3R}{2} i_{z2} + L D i_{z2} - \frac{R}{2} \left(\frac{1}{3} i_{z2} + \frac{2}{3R} U_{g0} \right) = 0$$

$$\frac{3R}{2} i_{z2} + L D i_{z2} - \frac{R}{6} i_{z2} - \frac{U_{g0}}{3} = 0$$

$$\frac{9R-R}{6} i_{z2} + L D i_{z2} = \frac{U_{g0}}{3}$$

$$\frac{4}{3} \frac{8R}{6} i_{z2} + L D i_{z2} = \frac{U_{g0}}{3} \quad / \cdot 3$$

$$3 \cdot L D i_{z2} + 4R i_{z2} = U_{g0}$$

a) Homogeni

$$3L D i_L + 4R i_L = 0 \quad ; \quad i_H(t) = K \cdot e^{pt}$$

$$3Lp + 4R = 0$$

$$p = -\frac{4R}{3L}$$

b) Partikularni

$$3L \dot{i}_L + 4R i_L = U_{g0} \quad ; \quad \dot{i}_p(t) = n$$

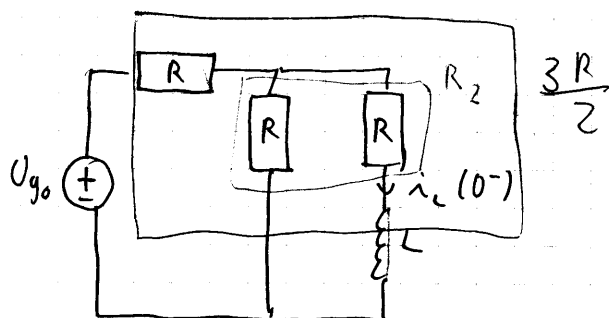
$$\underbrace{3L \dot{n}}_{=0} + 4R n = U_{g0}$$

$$n = \frac{U_{g0}}{4R}$$

$$i_L(t) = K \cdot e^{-\frac{4R}{3L}t} + \frac{U_{g0}}{4R}$$

c) Posebna rešitev - začetni pogoj

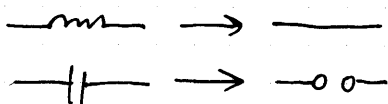
$$i_L(0) = i_L(0^-)$$



$$\tilde{i} = \frac{U_{g0}}{\frac{3R}{2}} = \frac{2}{3R} U_{g0}$$

$$i_L(0^-) = \frac{1}{3R} U_{g0} \quad (\text{tok razpolovi})$$

Zač. pogoj / stacionarno stanje

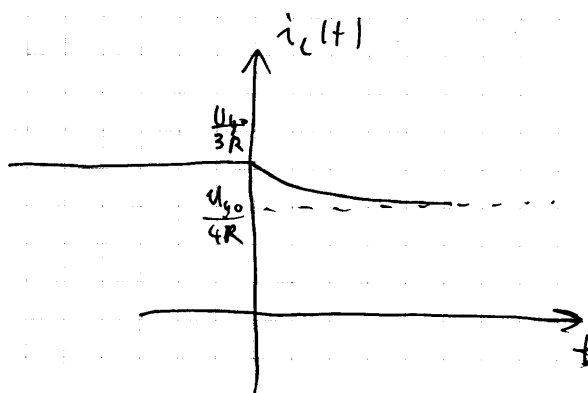


$$i_L(0^-) = \tilde{i}_L(0)$$

$$\frac{U_{g0}}{3R} = K \cdot e^0 + \frac{U_{g0}}{4R}$$

$$\frac{4U_{g0} - 3U_{g0}}{12R} = K \Rightarrow K = \frac{U_{g0}}{12R}$$

$$i_L(t) = \frac{U_{g0}}{12R} \cdot e^{-\frac{4R}{3L}t} + \frac{U_{g0}}{4R}$$

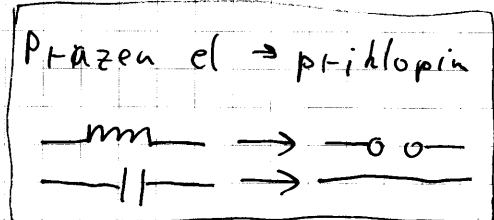
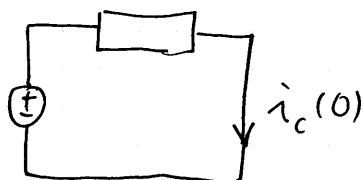
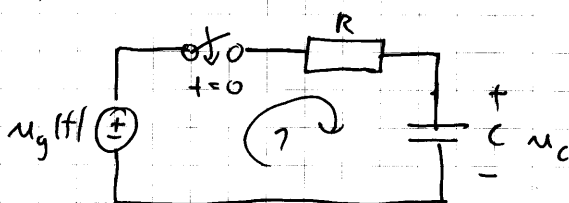


Določiti začetne pogoje $u_c(0)$, $i_c(0)$, $D i_c(0)$. $u_g(t) = U_{g0} \cdot \cos(\omega t)$

$$u_c(0) = u_c(0^-) = \underline{0V}$$

$$i_c(0) =$$

$$D i_c(0) =$$



$$i_c(0) = \frac{u_g(t)}{R} = \underline{\underline{\frac{U_{g0}}{R} \cos(\omega \cdot 0)}}$$

$$\textcircled{12} -u_g(t) + R i_{z1} + \frac{1}{C0} i_{z1} = 0 \quad / \cdot \frac{D}{R}$$

$$-\frac{D}{R} u_g(t) + D i_{z1} + \frac{1}{R^2 C} i_{z1} = 0$$

$$t=0$$

$$-\frac{D}{R} u_g(0) + D i_{z1}(0) + \frac{1}{R^2 C} \underbrace{i_{z1}(0)}_{\frac{U_{g0}}{R}} = 0$$

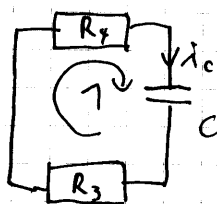
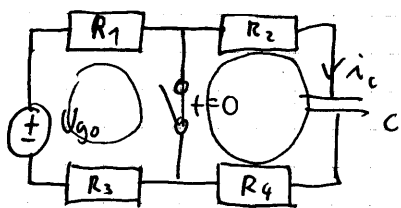
$$-\frac{U_{g0}}{R} \omega \sin(0) + D i_{z1}(0) + \frac{1}{R^2 C} \cdot U_{g0} = 0$$

$$D i_{z1}(0) = -\frac{1}{R^2 C} U_{g0}$$

Izračunaj tok kondenzatorju!

$$i_c(t) = ?$$

$$t \geq 0$$



$$\textcircled{1} R_3 i_z + R_4 i_z + \frac{1}{C} \int i_z dt = 0 \quad i_c(t) = i_z$$

$$(R_3 + R_4) \cdot C \dot{i}_c + i_c = 0$$

a.) Homogeni

$$(R_3 + R_4) \cdot C \dot{i}_c + i_c = 0$$

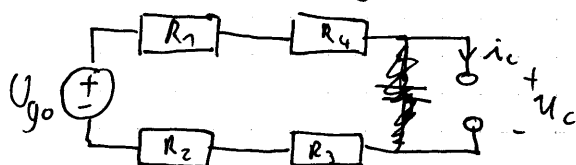
$$i_H(t) = k \cdot e^{pt}$$

$$(R_3 + R_4) C p + 1 = 0$$

$$p = \frac{-1}{(R_3 + R_4) \cdot C}$$

$$i_c(t) = k \cdot e^{-\frac{t}{(R_3 + R_4) \cdot C}}$$

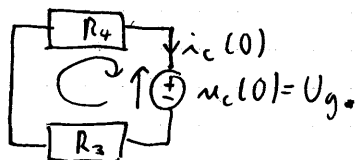
Začetni pogoji



$$i_c(0^-) = 0 \neq i_c(0)$$

$$u_c(0^-) = U_{g0} = u_c(0)$$

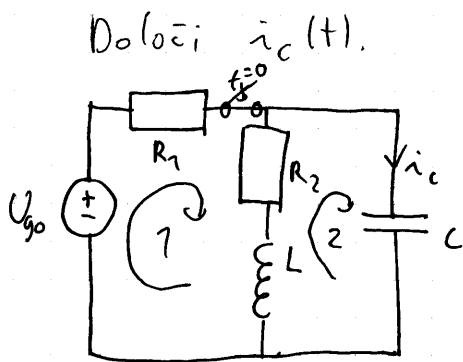
$$t = 0$$



$$i_{z1} = \frac{U_{g0}}{R_3 + R_4}$$

$$i_c(0) = -\frac{U_{g0}}{R_3 + R_4}$$

$$i_c(0) = k \cdot e^0 = -\frac{U_{g0}}{R_3 + R_4} \Rightarrow i_c(t) = -\frac{U_{g0}}{R_3 + R_4} \cdot e^{-\frac{t}{(R_3 + R_4) \cdot C}}$$



$$u_c(0) = 0V$$

$$i_c(0) = 0A$$

$$R_1 = 500\Omega$$

$$R_2 = 7,5 k\Omega$$

$$C = 5pF$$

$$L = 10\mu H$$

$$U_g = 1V$$

$$(1) -U_{g0} + R_1 i_{z1} + R_2 (i_{z1} - i_{z2}) + L D(i_{z1} - i_{z2}) = 0$$

$$(2) L D(i_{z2} - i_{z1}) + R_2 (i_{z2} - i_{z1}) + \frac{1}{CD} i_{z2} = 0$$

$$\underline{i_c(t) = i_{z2}}$$

$$(1) + (2) -U_{g0} + R_1 i_{z1} + \frac{1}{CD} i_{z2} = 0$$

$$R_1 i_{z1} = U_{g0} - \frac{1}{CD} i_{z2}$$

$$i_{z1} = \frac{U_{g0}}{R_1} - \frac{1}{R_1 CD} i_{z2}$$

$$i_{z2} (LD + R_2 + \frac{1}{CD}) - i_{z1} (R_2 + LD) = 0$$

$$i_{z2} (LD + R_2 + \frac{1}{CD}) - \left(\frac{U_{g0}}{R_1} - \frac{i_{z2}}{R_1 CD} \right) (R_2 + LD) = 0$$

$$i_{z2} (LD + R_2 + \frac{1}{CD}) - \frac{R_2}{R_1} U_{g0} - \frac{LD U_{g0}}{R_1} + \frac{R_2}{R_1 CD} i_{z2} + \frac{LD}{R_1 CD} i_{z2} = 0$$

$$i_{z2} \left(\frac{LCD^2 + R_2 CD + 1}{CD} + \frac{R_2}{R_1 CD} + \frac{LD}{R_1 CD} \right) = \frac{R_2}{R_1} U_{g0} / R_1 CD$$

$$i_{z2} (R_1 LCD^2 + R_1 R_2 CD + R_1 + R_2 + LD) = R_2 CD U_{g0}$$

$$\underbrace{R_1 LCD^2}_{a_2} i_{z2} + \underbrace{(R_1 R_2 C + L)}_{a_1} D i_{z2} + \underbrace{(R_1 + R_2)}_{a_0} i_{z2} = 0$$

$$a_2 = 2,5 \cdot 10^{-14} \frac{Vs^2}{A}$$

$$a_1 = 13,75 \cdot 10^{-6} \frac{Vs}{A}$$

$$a_0 = 2 k\Omega$$

a) Homogeni

$$a_2 p^2 + a_1 p + a_0 = 0$$

$$\hat{i}_h(t) = K_1 e^{p_1 t} + K_2 e^{p_2 t}$$

$$p_{1,2} = \frac{a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}{2a_2}$$

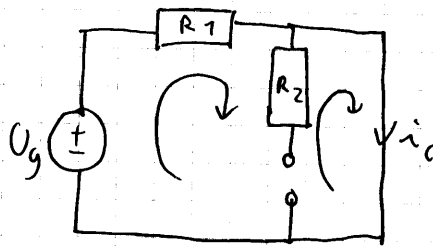
$$p_1 = -275 \cdot 10^6 + j 66,14 \cdot 10^7 \text{ 1/s}$$

$$p_2 = \underbrace{-275 \cdot 10^6}_{\omega} - j \underbrace{66,14 \cdot 10^7}_{\omega} \text{ 1/s}$$

b) Posebna rešitev

$$\hat{i}_c(0) = \frac{U_{g0}}{R_1}$$

$$D\hat{i}_c(0) = -\frac{U_{g0}}{R_1^2 C}$$



$\mathcal{R} + \mathcal{C}$

$$-U_g + R \cdot \hat{i}_{z1} + \frac{1}{C\mathcal{D}} \hat{i}_{z2} = 0 \quad / \cdot C\mathcal{D}$$

$$\underbrace{-C\mathcal{D}U_g}_{=0} + R C\mathcal{D} \hat{i}_{z1} + \hat{i}_{z2} = 0$$

$$D\hat{i}_{z1} = -\frac{\hat{i}_{z2}}{R_1 C} \quad ; \quad \hat{i}_{R_2} = 0 = \hat{i}_{z1} - \hat{i}_{z2} \Rightarrow \hat{i}_{z1} = \hat{i}_{z2}$$

$$D\hat{i}_{z2}(0) = -\frac{\hat{i}_{z2}(0)}{R_1 C} = -\frac{U_{g0}}{R_1^2 C}$$

$$\hat{i}_c(0) = K_1 e^0 + K_2 e^0 = \frac{U_{g0}}{R_1} \quad ; \quad K_1 + K_2 = \frac{U_{g0}}{R_1} \quad ; \quad K_2 = \frac{U_{g0}}{R_1} - K_1$$

$$D\hat{i}_c(0) = K_1 p_1 e^0 + K_2 p_2 e^0 = -\frac{U_{g0}}{R_1^2 C}$$

$$K_1 \cdot p_1 + p_2 \left(\frac{U_{g0}}{R_1} - K_1 \right) = -\frac{U_{g0}}{R_1^2 C}$$

$$K_1 (p_1 - p_2) = -\frac{U_{g0}}{R_1^2 C} - \frac{p_2}{R_1} U_{g0}$$

$$K_1 = 0,001 + j 0,019$$

$$K_2 = K_1^* = 0,001 - j 0,019$$

$$\hat{i}_L(t) = k_1 e^{p_1 t} + k_2 e^{p_2 t}$$

$$= e^{\delta t} (k_1 e^{j\omega t} + k_1^* e^{-j\omega t}) \quad \text{Re}(p)$$

$$= e^{\delta t} \cdot 2 \operatorname{Re}(k_1 e^{j\omega t}) = 2 \cdot \underbrace{e^{\delta t}}_{\text{amplituda}} \cdot \underbrace{\operatorname{Re}(k_1 e^{j\arg(k_1)} e^{j\omega t})}_{\text{krožená frekvence u harmon.}} \quad \text{Im}(p)$$

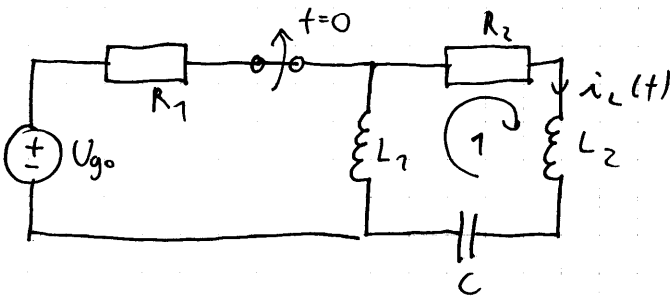
$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$\arg = \arctg \frac{\operatorname{Im}}{\operatorname{Re}}$$

$$-\operatorname{Re}, -\operatorname{Im} = -\pi \quad \swarrow$$

$$-\operatorname{Re}, +\operatorname{Im} = +\pi \quad \searrow$$

$$\hat{i}_L(t) \stackrel{?}{,}$$



$$L_1 D i_{z1} + L_2 D i_{z1} + R i_{z1} + \frac{1}{CD} i_{z1} = 0 \quad / \cdot CD$$

$$(L_1 + L_2) CD^2 i_L(t) + R CD i_L(t) + i_L(t) = 0$$

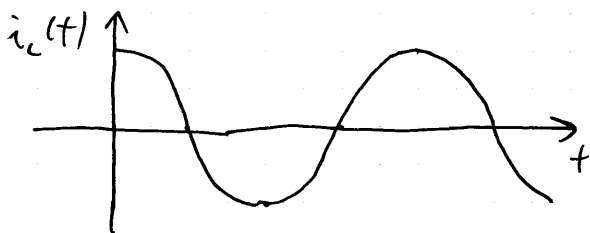
$$(L_1 + L_2) C p^2 + R C p + 1 = 0$$

$$p_{1,2} = \frac{-RC \pm \sqrt{(RC)^2 - 4(L_1 + L_2)C}}{2 \cdot (L_1 + L_2)C}$$

$$-\frac{RC}{2 \cdot (L_1 + L_2)C} = \delta$$

$$(RC)^2 - 4(L_1 + L_2)C < 0$$

$$a) \text{ nedušená } , p_{1,2} \in \mathbb{C} \text{ in } \delta = 0$$

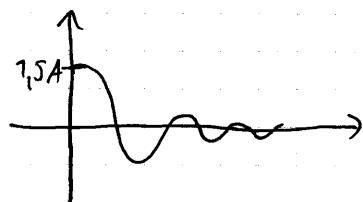


$$p_1 = j^2 \quad p_2 = j^4$$

$$Re = 0$$

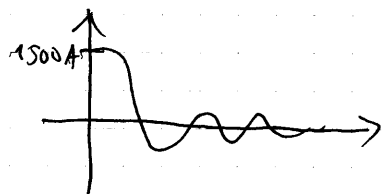
$$p_{1,2}, \text{ kuzlični, čisto imaginární}$$

b) podkritično $D < 0, \delta > 0$; $D = (RC)^2 - 4(L_1 + L_2)C < 0$



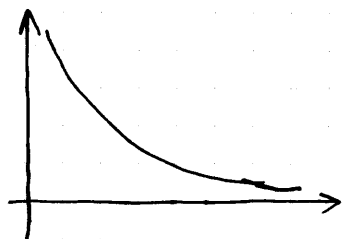
$$p_1 = p_2^* \quad \text{Re} \neq 0$$

c) kritično



$$I_m = 0 \quad p_1 = p_2$$

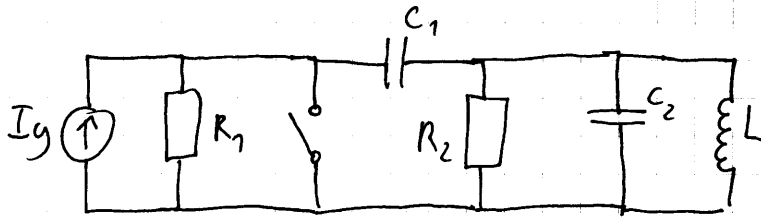
d) nadkritično



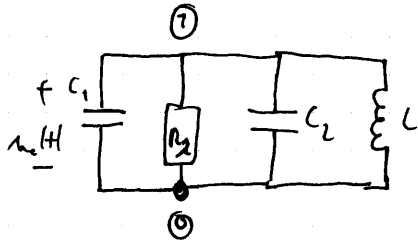
$$I_m = 0 \quad p_1 \neq p_2$$

17.11.2008

$u_{C1}(t)!$



$$\begin{aligned} R_1 &= 1 \Omega \\ R_2 &= 1 \Omega \\ C_1 &= C_2 = 1 \mu F \\ L &= 160 \mu H \\ I_{g0} &= 5 A \end{aligned}$$



$$C_1 D u_{v1} + \frac{u_{v1}}{R_2} + C_2 D u_{v1} + \frac{1}{L D} u_{v1} = 0 \quad / \cdot R_2 L D$$

$$\begin{aligned} R_2 C_1 L D^2 u_{v1} + R_2 C_2 L D^2 u_{v1} + L D u_{v1} + R_2 u_{v1} &= 0 \\ \underbrace{(R_2 C_1 L + R_2 C_2 L)}_{a_2} D^2 u_{v1} + \underbrace{L D}_{a_1} u_{v1} + \underbrace{R_2}_{a_0} u_{v1} &= 0 \end{aligned}$$

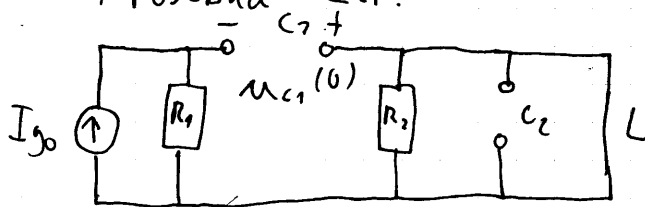
a) Homogeni

$$u_H(t) = K_1 e^{p_1 t} + K_2 e^{p_2 t}$$

$$a_2 p^2 + a_1 p + a_0 = 0$$

$$p_{1,2} = (-2,5 \pm j 17,5) \cdot 10^2 \text{ A/s} \quad \text{podkritično dušeno uihanje}$$

b.) Posebna z.p.



$$u_{C1}(0) = -I_{g0} R_1$$

$$D u_{C1}(0) =$$

$$u_{C1}(0) = -I_{g0} \cdot R_1 \quad \text{samo u stat. stanju}$$

$$D u_{C1}(0) = \frac{i_C(0)}{C} = \frac{i_C(0)}{C} = 0$$

$$u_{C1}(0) = -I_{g0} R_1 = K_1 e^0 + K_2 e^0$$

$$D u_{C1}(0) = 0 = K_1 p_1 e^0 + K_2 p_2 e^0$$

$$\left. \begin{aligned} K_1 &= -\frac{5}{2} - \frac{5}{14} j \\ K_2 &= -\frac{5}{2} + \frac{5}{14} j \end{aligned} \right\}$$

$$u_c(t) = 2|k_1| \cos(\omega t + \arg(k_1)) \quad ; \quad \arg(k_1) = \arctan\left(\frac{\frac{5}{24}}{\frac{5}{2}}\right) - \pi$$

$$u_c(t) = -5,05 \cdot e^{-2,5 \cdot 10^2 t} \cdot \cos(17,5 \cdot 10^2 t + 8,13^\circ) \text{ V}$$

Izmenična analiza

$$u(t) = 2 \cdot \sin(\omega t) - 3 \cos(\omega t - \frac{\pi}{4}) \text{ V} \quad \sin \rightarrow \cos$$

$$= 2 \cdot \cos(\omega t - (\frac{\pi}{2})) - 3 \cos(\omega t - (\frac{\pi}{4})) \text{ V}$$

$$U = 2 \cdot e^{j(-\frac{\pi}{2})} - 3 \cdot e^{j(-\frac{\pi}{4})} = *$$

$$u(t) = \operatorname{Re}[U \cdot e^{j\omega t}] = \operatorname{Re}[2 \cdot e^{j(-\frac{\pi}{2})} e^{j\omega t}] = \operatorname{Re}[2 \cdot e^{j(\omega t - \frac{\pi}{2})}] =$$

$$= \operatorname{Re}[2(\cos(\omega t - \frac{\pi}{2}) + j \sin(\omega t - \frac{\pi}{2}))]$$

$$* = \underbrace{2 \cdot (\cos(-\frac{\pi}{2}) + j \sin(-\frac{\pi}{2}))}_{-2j} - \underbrace{3 \cdot (\cos(-\frac{\pi}{4}) + j \sin(-\frac{\pi}{4}))}_{-3 \cdot (\frac{1-j}{\sqrt{2}})} =$$

$$= \frac{-2 \cdot \sqrt{2}}{\sqrt{2}} j - \frac{3-3j}{\sqrt{2}} = -\frac{3}{\sqrt{2}} + \frac{-2\sqrt{2}-3}{\sqrt{2}} j =$$

$$= \underbrace{\sqrt{(\frac{-3}{\sqrt{2}})^2 + (\frac{-2\sqrt{2}-3}{\sqrt{2}})^2}}_{\substack{\text{mod} \\ |k|}} \cdot e^{j \arctan(\frac{\frac{-2\sqrt{2}-3}{\sqrt{2}}}{\frac{-3}{\sqrt{2}}})} = \sqrt{a^2 + b^2} e^{j \arctan(\frac{b}{a})} - \text{pari an} \pm \pi$$

$$1) \cos \rightarrow |k| \cdot e^{j\varphi}$$

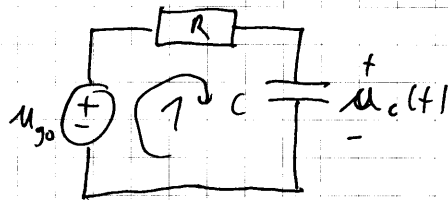
$$2) |k| \cdot e^{j\varphi} \rightarrow |k| \cdot (\cos(\varphi) + j \sin(\varphi))$$

3.) izvrši operaciju

$$4.) a + jb = \sqrt{a^2 + b^2} \cdot e^{j \arctan(\frac{b}{a})}$$

$$u_c(t) = ?$$

$$u_g(t) = U_{g0} \sin(\omega t)$$



$$u_g(t) \rightarrow U_g$$

$$u_g(t) = U_{g0} \cdot \cos(\omega t - \frac{\pi}{2})$$

$$U_g = U_{g0} \cdot e^{j(\omega t - \frac{\pi}{2})} = -j U_{g0}$$

$$-U_g + R I_z + U_c = 0$$

$$U_c + R I_z = U_g$$

systemshaf
ned. v.h.
in izh.

$$H(j\omega) = \frac{U_c}{U_g} = \frac{U_c}{U_c + R I_z} = \frac{U_c}{U_c + R \cdot j\omega C U_c} = \frac{U_c}{U_c (1 + R j\omega C)} =$$

$$i_c = C \frac{dU_c}{dt}$$

$$I_c = C j\omega U_c$$

$$= \frac{1}{1 + j\omega RC}$$

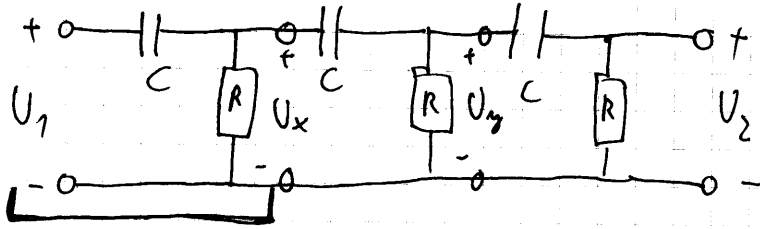
$$H(j\omega) = \frac{U_c}{U_g} \Rightarrow U_c = H(j\omega) \cdot U_g$$

$$U_c = \frac{1}{1 + j\omega RC} \cdot (-j U_g) = \frac{-j U_g}{1 + j\omega RC} = \frac{-j U_g (1 - j\omega RC)}{1 + (\omega RC)^2} =$$

$$= \frac{-U_g \omega RC - j U_g}{1 + (\omega RC)^2} = \underbrace{\sqrt{\left(\frac{-U_g \omega RC}{1 + (\omega RC)^2}\right)^2 + \left(\frac{-U_g}{1 + (\omega RC)^2}\right)^2}}_{|U_c|} \cdot e^{j(\arctan \frac{-U_g}{-U_g \omega RC} - \pi)}$$

$$u_c(t) = |U_c| \cdot \cos(\omega t - \pi + \arctan \frac{-U_g}{-U_g \omega RC}) = -|U_c| \cdot \cos(\omega t + \arctan \frac{1}{\omega RC})$$

U_1 in U_2 v profi fazi!

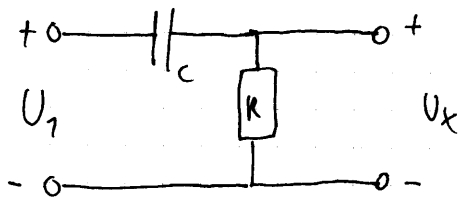


$$U_1 = |U_1| \cdot e^{j\varphi_1}$$

$$U_2 = |U_2| \cdot e^{j\varphi_2}$$

$$H(j\omega) = \frac{U_2}{U_1} = \frac{|U_2|}{|U_1|} \cdot e^{j(\varphi_2 - \varphi_1)}$$

$$\varphi_2 - \varphi_1 = \arg\left(\frac{|U_2|}{|U_1|}\right) = \pi$$



$$U_x = \frac{R}{R + j\omega C} U_1 = \frac{j\omega CR}{1 + j\omega CR} U_1$$

$$z_C = \frac{1}{j\omega C}$$

$$z_L = j\omega L$$

$$H'(j\omega) = \frac{U_x}{U_1} = \frac{j\omega CR}{1 + j\omega CR}$$

$$U_y = \underbrace{U_x}_{\text{from previous stage}} \cdot H'(j\omega) = U_1 \cdot H'(j\omega) \cdot H'(j\omega) = U_1 \cdot (H'(j\omega))^2$$

$$U_2 = (H'(j\omega))^3 \cdot U_1$$

$$H(j\omega) = \frac{U_2}{U_1} = \underbrace{\left(\frac{j\omega CR}{1 + j\omega CR}\right)^3}_{|H| \cdot e^{j\varphi}} ; \arg(H(j\omega)) = \pi$$

$$(|H| \cdot e^{j\varphi})^3 = |H|^3 \cdot e^{j3\varphi}$$

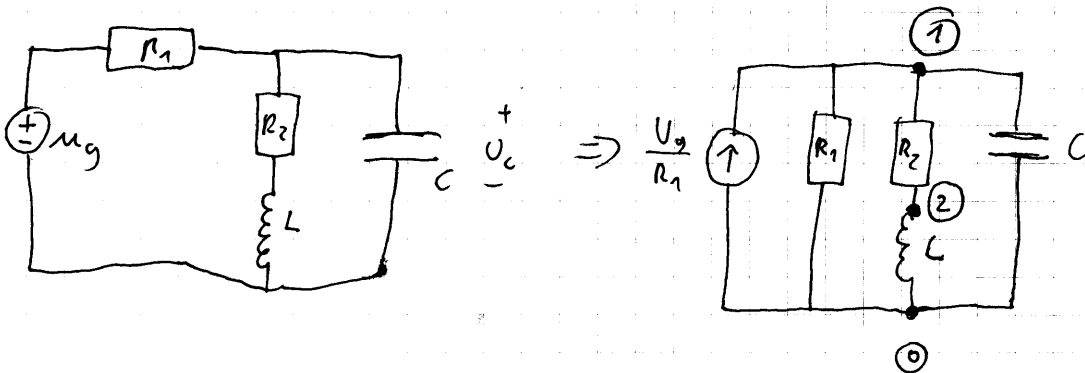
$$\arg\left(\frac{j\omega CR}{1 + j\omega CR}\right) = \frac{\pi}{3} ; \frac{j\omega CR (1 - j\omega CR)}{1 + (\omega CR)^2} = \frac{(\omega CR)^2 + j\omega CR}{1 + (\omega CR)^2}$$

$$\arg(M'(j\omega)) = \arctan \left(\frac{\frac{\omega L R}{1 + \omega^2 R^2 C^2}}{\frac{\omega C R}{1 + \omega^2 R^2 C^2}} \right) = \arctan \left(\frac{1}{\omega C R} \right) = \frac{\pi}{3}$$

$$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$$\frac{1}{\omega C R} = \sqrt{3} \Rightarrow \underline{\omega = \frac{1}{\sqrt{3} R C}}$$

U_C ?



$$\textcircled{1} -\frac{U_g}{R_1} + \frac{U_{V2}}{R_1} + \frac{U_{V2} - U_{V1}}{R_2} + j\omega C U_{V1} = 0$$

$$\textcircled{2} \frac{U_{V2} - U_{V1}}{R_2} + \frac{U_{V2}}{j\omega L} \quad U_C = U_{V1}$$

$$U_{V2} \left(\frac{1}{R_2} + \frac{1}{j\omega L} \right) = \frac{U_{V1}}{R_2}$$

$$U_{V2} \frac{j\omega L + R_2}{R_2 j\omega L} = \frac{U_{V1}}{R_2} \Rightarrow U_{V2} = \frac{j\omega L}{R_2 + j\omega L} \cdot U_{V1}$$

$$U_{V1} \left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C \right) - \frac{1}{R_2} U_{V2} = \frac{U_g}{R_1}$$

$$U_{V1} \frac{R_2 + R_1 + R_1 R_2 j\omega C}{R_1 R_2} - \frac{1}{R_2} \frac{j\omega L}{R_2 + j\omega L} U_{V1} = \frac{U_g}{R_1} \quad / \cdot R_2$$

$$U_{V1} \frac{R_2 + R_1 + R_1 R_2 j\omega C}{R_1} - \frac{j\omega L}{R_2 + j\omega L} U_{V1} = \frac{R_2}{R_1} U_g$$

$$U_{V1} \frac{R_1 + R_2 + j\omega L + j\omega R_1 R_2 - \omega^2 L C R_1}{R_1 (R_2 + j\omega L)} = U_g$$

$$U_{V1} = \frac{R_2 + j\omega L}{|R_1 + R_2 - \omega^2 R_1 L C + j\omega L + j\omega R_1 R_2 C|} \cdot U_g$$

$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

$H(j\omega) \cdot |U_g| \cdot e^{j\varphi}$ - v splosuen

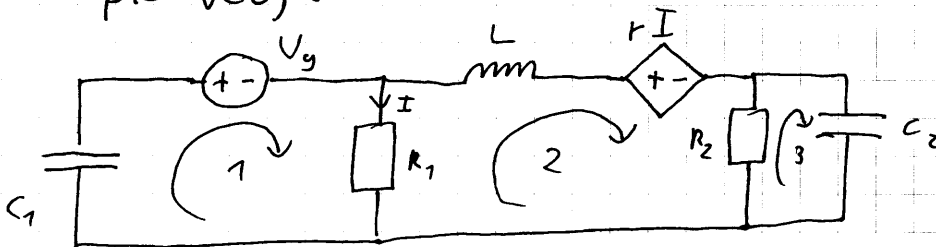
$$U_{V1} = \frac{\sqrt{R_2^2 + (\omega L)^2}}{\sqrt{(R_1 + R_2 - \omega^2 R_1 L C)^2 + (\omega L + \omega R_1 R_2 C)^2}} \cdot |U_g| \cdot e^{j \arctg\left(\frac{\omega L}{R_2}\right)} \cdot e^{j \arctg\left(\frac{\omega L + \omega R_1 R_2 C}{R_1 + R_2 - \omega^2 R_1 L C}\right)}$$

$$e^{j \arctg\left(\frac{\omega L}{R_2}\right)} \cdot e^{-j \arctg\left(\frac{\omega L}{R_2}\right)}$$

$$e^{j\varphi}$$

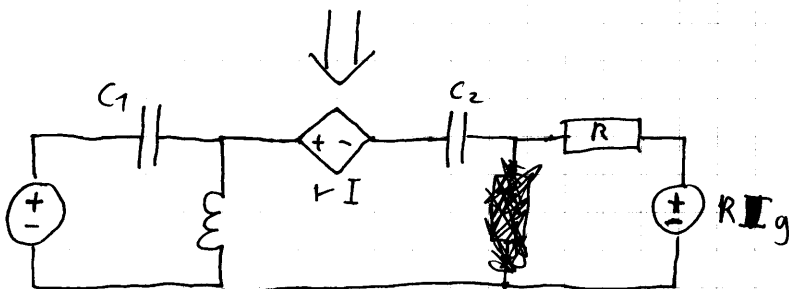
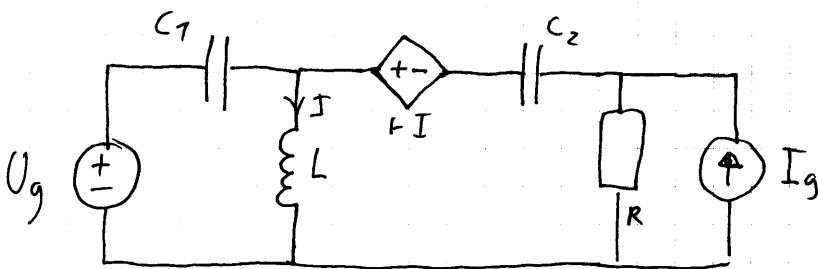
24.11.2008

Opis vezja



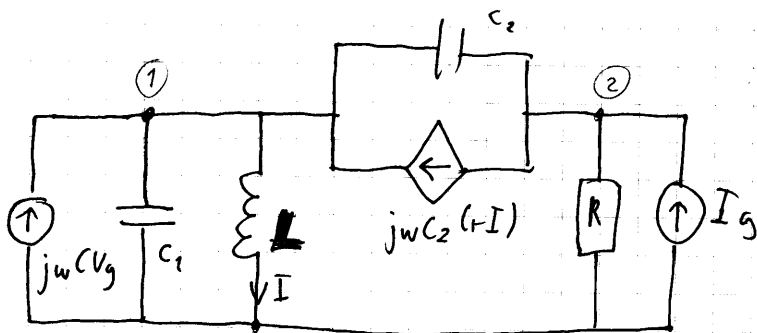
$$I = \dot{I}_{z1} - \dot{I}_{z2}$$

$$\begin{bmatrix} \frac{1}{j\omega C_1} + R_1 & -R_1 & 0 \\ -R_1 & R_1 + R_2 + j\omega L & -R_2 \\ 0 & -R_2 & R_2 + \frac{1}{j\omega C_2} \end{bmatrix} \begin{bmatrix} \dot{I}_{z1} \\ \dot{I}_{z2} \\ \dot{I}_{z3} \end{bmatrix} = \begin{bmatrix} -U_g \\ 0 \\ 0 \end{bmatrix}$$



$$I = \dot{I}_{z1} - \dot{I}_{z2}$$

$$\begin{bmatrix} \frac{1}{j\omega C_1} + j\omega L & -j\omega L \\ -j\omega L & R + j\omega L + \frac{1}{j\omega C_2} \end{bmatrix} \begin{bmatrix} \dot{I}_{z1} \\ \dot{I}_{z2} \end{bmatrix} = \begin{bmatrix} U_g \\ -RI_g \end{bmatrix}$$



$$I = \frac{V}{j\omega L} = \frac{U_{K1}}{j\omega L}$$

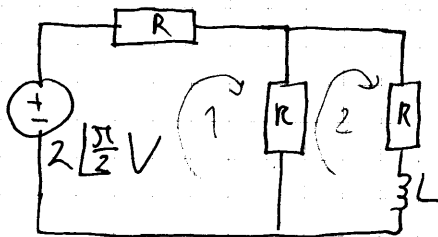
$$j\omega C_2 \cdot \frac{U_{K1}}{j\omega L} = \frac{C_2 r}{L} U_{V1}$$

$$\begin{bmatrix} j\omega C_1 + j\omega C_2 + \frac{1}{j\omega L} & -j\omega C_2 \\ -j\omega C_2 & \frac{1}{R} + j\omega C_2 \end{bmatrix} \begin{bmatrix} U_{V1} \\ U_{V2} \end{bmatrix} = \begin{bmatrix} j\omega C_1 U_g \\ I_g \end{bmatrix}$$

Izračunaj tok skozi tuljavo $R = 100 \Omega$.

$$R = 100 \Omega$$

$$L = 10 \mu H$$



$$2 \sqrt{\frac{\pi}{2}} V = 2 \cdot \cos(\omega t - \frac{\pi}{2}) V = u_g(t)$$

$$\begin{aligned} \textcircled{1} -U_g + R I_{z1} + R (I_{z1} - I_{z2}) &= 0 \\ \textcircled{2} R (I_{z2} - I_{z1}) + R I_{z2} + j\omega L I_{z2} &= 0 \end{aligned}$$

$$2 R I_{z1} = U_g + R \cdot I_{z2}$$

$$I_{z1} = \frac{U_g + R I_{z2}}{2 R}$$

$$I_L = I_{z2}$$

$$\begin{aligned} 2 R I_{z2} + j\omega L I_{z2} - R I_{z1} &= 0 \\ 2 R I_{z2} + j\omega L I_{z2} - R \left(\frac{U_g + R I_{z2}}{2 R} \right) &= 0 \\ 2 R I_{z2} + j\omega L I_{z2} - \frac{R I_{z2}}{2} &= \frac{U_g}{2} \quad / \cdot 2 \\ 3 R I_{z2} + 2 j\omega L I_{z2} &= U_g \\ I_{z2} (3 R + 2 j\omega L) &= U_g \end{aligned}$$

$$U_g = 2 \cdot e^{j(-\frac{\pi}{2})} = 2 \cdot (-j)$$

$$I_L = I_{z2} = \frac{U_g}{3 R + 2 j\omega L} = \frac{-2j}{300 + 2 \cdot 10^{-5} \omega j} = \frac{-2j (300 - 2 \cdot 10^{-5} \omega j)}{300^2 + (2 \cdot 10^{-5} \omega)^2}$$

$$I_L = \frac{2}{\sqrt{(300)^2 + (2 \cdot 10^{-5} \omega)^2}} \cdot e^{j(\arctan(\frac{600}{-4 \cdot 10^{-6} \omega}) - \pi)}$$

$$i_L(t) = \text{Re}[I_L \cdot e^{j\omega t}] = \frac{2}{\sqrt{300^2 + (2 \cdot 10^{-5} \omega)^2}} \cdot \cos(\omega t + \arctan(\frac{-600}{-4 \cdot 10^{-6} \omega}) - \pi) \text{ A}$$

Fazni zasuk!

$$(D^2 + aD + b) y(t) = D^2 x(t)$$

↓

$$(j\omega^2 + a \cdot j\omega + b) Y = (j\omega^2) X$$

$$(-\omega^2 + a j\omega + b) Y = -\omega^2 X$$

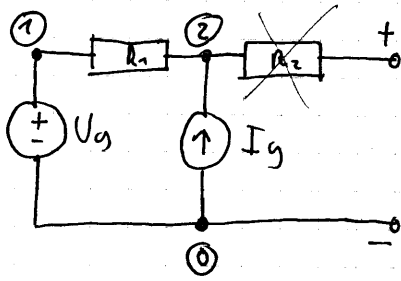
$$H = \frac{Y}{X} = \frac{-\omega^2}{b - \omega^2 + a j\omega}$$

$$\text{fazni zasuk} = \arg(H(j\omega))$$

$$H(j\omega) = \frac{-\omega^2 (1b - j\omega - a j\omega)}{(b - \omega^2)^2 + (a\omega)^2} = \frac{-\omega^2 (b - \omega^2) + a j\omega^3}{(b - \omega^2)^2 + (a\omega)^2}$$

$$\arg(H(j\omega)) = \arctan\left(\frac{a j\omega^3}{-\omega^2(b - \omega^2)}\right) = \arctan\left(\frac{a\omega}{-(b - \omega^2)}\right)$$

$$\arg(H(j\omega)) = \begin{cases} b < \omega^2 \rightarrow \arctan\left(\frac{a\omega}{-(b - \omega^2)}\right) \\ b = \omega^2 \rightarrow \frac{\pi}{2} \\ b > \omega^2 \rightarrow \arctan\left(\frac{a\omega}{-(b - \omega^2)}\right) \pm \pi \end{cases}$$



Thevenin

U_{os}, Z_n

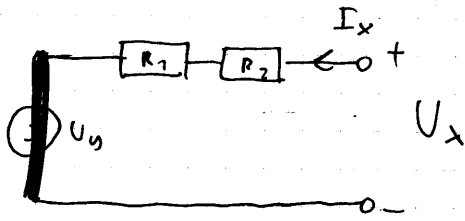
U_{os} - odstrani el. brez I
- vozljska metoda

$$U_{v1} = U_g$$

$$\textcircled{2} \quad \frac{U_{v2} - U_g}{R_1} - I_g = 0 \quad / \cdot R_1$$

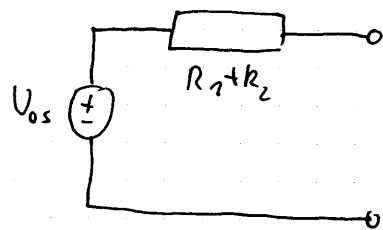
$$U_{v2} = R_1 I_g + U_g = \underline{U_{os}}$$

Linearni del



$$Z_L = \frac{U_x}{I_x} = \frac{R_1 I_x + R_2 I_x}{I_x} = R_1 + R_2$$

veže se prelevi v Theveninov vir:



Linearni del

- izklopi vse vire

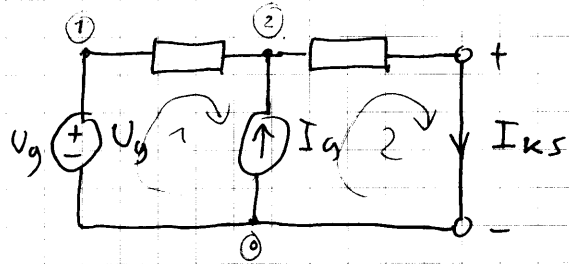


- izračunaj Z_L

$$Z_L = \frac{U_x}{I_x}$$

Norton

- Na vhod postavi kratak stih



$$I_g = I_{z2} - I_{z1} \Rightarrow I_{z1} = I_{z2} - I_g$$

$$(1,2) -U_g + R_1 I_{z1} + R_2 I_{z2} = 0$$

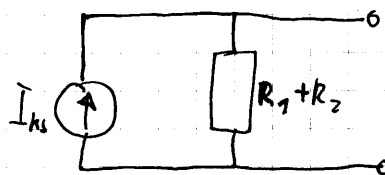
$$I_{k5} = I_{z2}$$

$$R_2 I_{z2} + R_1 I_{z1} = U_g$$

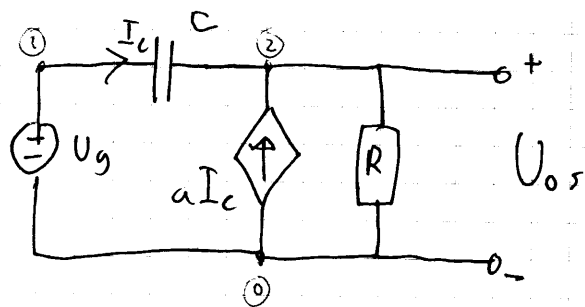
$$R_2 I_{z2} + R_1 (I_{z2} - I_g) = U_g$$

$$I_{z2} (R_2 + R_1) = U_g + R_1 I_g$$

$$I_k = \frac{U_g + R_1 I_g}{R_2 + R_1}$$



Dođoži Theveninov in Nortanov vir.



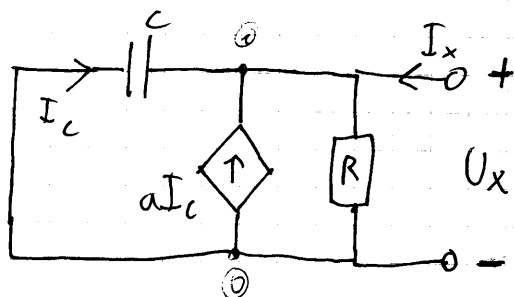
$$U_{v1} = U_g ; I_c = j\omega C (U_{v1} - U_{v2}) = j\omega C (U_g - U_{v2})$$

$$\textcircled{2} \quad j\omega C (U_{v2} - U_g) - a j\omega C (U_g - U_{v2}) + \frac{U_{v2}}{R} = 0$$

$$j\omega C (U_{v2} - U_g) + a j\omega C (U_{v2} - U_g) + \frac{U_{v2}}{R} = 0$$

$$U_{v2} \cdot (j\omega C (1+a) + \frac{1}{R}) = U_g \cdot j\omega C (1+a)$$

$$U_{v2} = \frac{U_g \cdot j\omega C (1+a)}{j\omega C (1+a) + \frac{1}{R}} = U_{o5}$$



$$I_c = j\omega C \cdot U_{v1} = j\omega C (0 - U_{v1})$$

$$U_x = U_{v1}$$

$$\textcircled{1} \quad j\omega C U_{v1} - a (-j\omega C U_{v1}) + \frac{U_x}{R} - I_x = 0$$

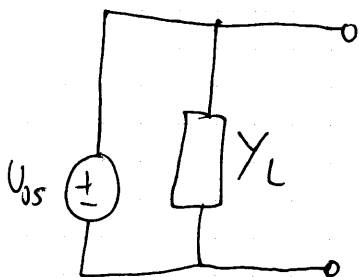
$$U_x (j\omega C (1+a) + \frac{1}{R}) - I_x = 0$$

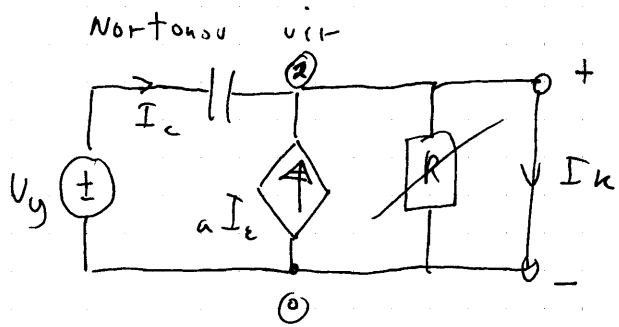
$$Z_L = \frac{U_x}{I_x}$$

$$Y_L = \frac{I_x}{U_x}$$

$$U_x (j\omega C (1+a) + \frac{1}{R}) = I_x$$

$$\frac{I_x}{U_x} = j\omega C (1+a) + \frac{1}{R}$$





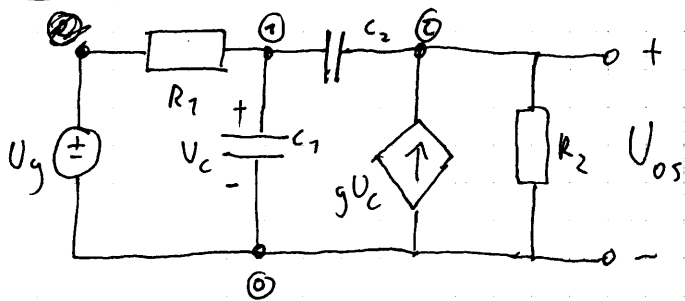
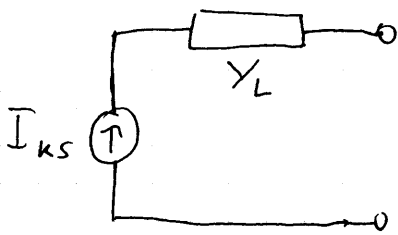
$$U_c = U_g$$

$$I_c = j\omega C U_g$$

$I_{k.s.}$

$$(2) -I_c - aI_c + I_{k.s.} = 0$$

$$I_{k.s.} = I_c (1+a) = j\omega C (1+a) \cdot U_g$$



$$(1) \frac{U_{v1} - U_g}{R_1} + j\omega C_1 U_{v1} + j\omega C_2 (U_{v1} - U_{v2}) = 0$$

$$(2) j\omega C_2 (U_{v2} - U_{v1}) - gU_{v2} + \frac{U_{v2}}{R_2} = 0$$

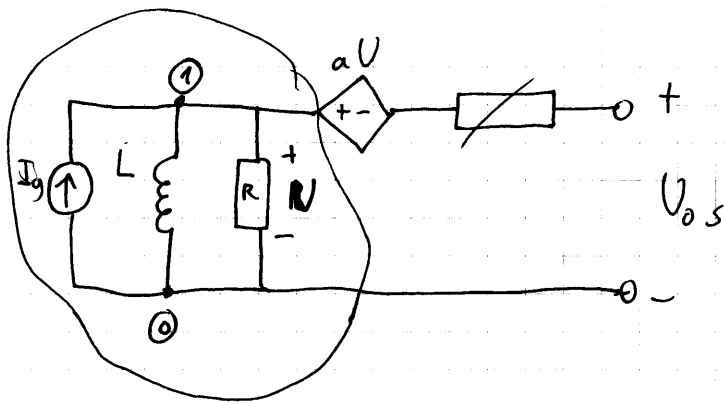
$$U_{o.s.} = U_{v2}$$

$$U_{v1} \left(\frac{1}{R_1} + j\omega C_1 + j\omega C_2 \right) = \frac{U_g}{R_1} + j\omega C_2 U_{v2} \quad U_{v1} = \frac{j\omega C_2 U_{v2} + \frac{U_g}{R_1}}{\left(\frac{1}{R_1} + j\omega C_1 + j\omega C_2 \right)}$$

$$U_{v2} (j\omega C_2 + \frac{1}{R_2}) - U_{v1} (j\omega C_2 + g) = 0$$

$$U_{v2} (j\omega C_2 + \frac{1}{R_2}) - (j\omega C_2 + g) \cdot \left(\frac{j\omega C_2 U_{v2} + \frac{U_g}{R_1}}{\left(\frac{1}{R_1} + j\omega C_1 + j\omega C_2 \right)} \right) = 0$$

$$U_{o.s.} = \frac{g + j\omega C_2}{1 + j\omega R_2 (C_1 + C_2) \left(\frac{1}{R_1} + j\omega C_2 \right) - j\omega C_2 (g + j\omega C_2)} \cdot U_g$$



$$U = U_{V1}$$

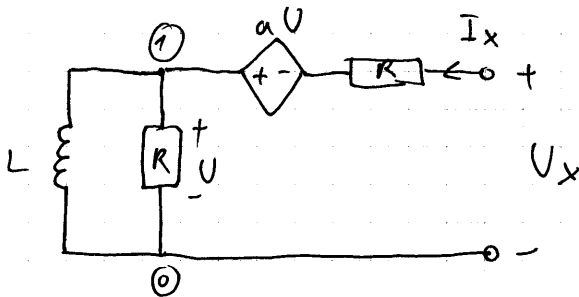
$$\textcircled{1} -I_g + \frac{U_{V1}}{j\omega L} + \frac{U_{V1}}{R} = 0$$

$$U_{V1} \left(\frac{1}{j\omega L} + \frac{1}{R} \right) = I_g$$

$$U_{V1} \frac{j\omega L + R}{j\omega L R} = I_g$$

$$\underline{U_{V1} = I_g \frac{j\omega L R}{j\omega L + R}}$$

$$U_{os} = U_{V1} - a U_{V1} = \frac{j\omega L R}{j\omega L + R} (1-a) I_g$$



$$\textcircled{1} \frac{U_{V1}}{j\omega L} + \frac{U_{V1}}{R} - I_x = 0$$

$$U_{V1} \left(\frac{1}{j\omega L} + \frac{1}{R} \right) = I_x$$

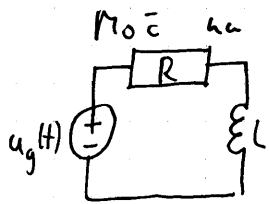
$$U_{V1} \frac{j\omega L + R}{j\omega L R} = I_x$$

$$\begin{aligned} U_x &= R \cdot I_x - a U + U \\ &= R \cdot I_x + (1-a) \cdot \frac{j\omega L R}{j\omega L + R} \cdot I_x \end{aligned}$$

$$U_{V1} = \frac{j\omega L R}{j\omega L + R} \cdot I_x$$

$$\underline{\underline{Z_c = \frac{U_x}{I_x} = R + (1-a) \frac{j\omega L R}{j\omega L + R}}}$$

01.12.2008



Moč na uporah (delovna)?

$$u_g(t) = U_{g0} \sin(\omega t)$$

$$u_g(t) = U_{g0} \cos(\omega t - \frac{\pi}{2})$$

$$P = \frac{1}{2} \operatorname{Re} [U \cdot I^*]$$

$$P = \frac{1}{2} \operatorname{Re} [R \cdot I \cdot I^*]$$

$$P = \frac{R}{2} \operatorname{Re} [I I^*]$$

$$P = \frac{R}{2} \cdot |I|^2$$

$$\textcircled{1} -U_g + R I_{z1} + j\omega L I_{z1} = 0$$

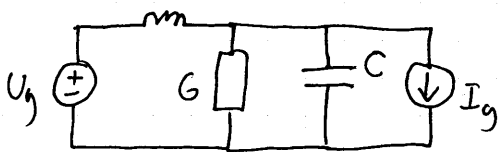
$$I_{z1} (R + j\omega L) = U_g$$

$$I = \frac{U_g}{R + j\omega L} = -j \frac{U_{g0}}{R + j\omega L}$$

$$U_g = U_{g0} \cdot e^{j(-\frac{\pi}{2})} = -j U_{g0}$$

$$P = \frac{R}{2} \cdot \frac{U_{g0}^2}{R^2 + (\omega L)^2}$$

Moč na prevodnosti (delovna)?

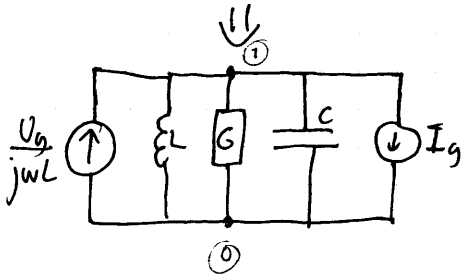


$$u_g(t) = U_{g0} \cos(\omega t) \quad i_g = I_{g0} \sin(\omega t) = I_{g0} \cos(\omega t - \frac{\pi}{2})$$

$$P = \frac{1}{2} \operatorname{Re} [U \cdot G^* \cdot U^*]$$

$$P = \frac{G}{2} \operatorname{Re} [U \cdot U^*]$$

$$P = \frac{G}{2} |U|^2$$



$$\textcircled{1} -\frac{U_g}{j\omega L} + \frac{U_{v1}}{j\omega L} + G \cdot U_{v1} + j\omega C U_{v1} + I_g = 0 \quad / \cdot j\omega L$$

$$U_{v1} + j\omega L G U_{v1} - \omega^2 C L U_{v1} = U_g - j\omega L I_g$$

$$U_{v1} = \frac{U_g - j\omega L I_g}{1 - \omega^2 C L + j\omega L G}$$

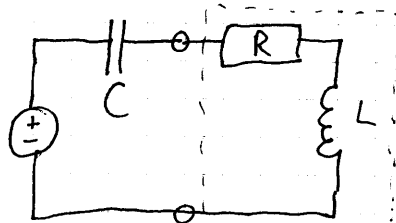
$$U_g = U_{g0} \cdot e^{j0} = U_{g0}$$

$$I_g = I_{g0} \cdot e^{j(-\frac{\pi}{2})} = -j I_{g0}$$

$$U_{v1} = \frac{U_{g0} - \omega L I_{g0}}{1 - \omega^2 C L + j\omega L G}$$

$$P = \frac{(U_{g0} - \omega L I_{g0})^2}{(1 - \omega^2 C L)^2 + (\omega L G)^2}$$

Določí vrednost kondenzatorja za maksimalno delovno moč!



$C = ?$

$$Z_g = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$Z_b = R + j\omega L$$

$$\omega L = -\left(-\frac{1}{\omega C}\right)$$

$$\omega L = \frac{1}{\omega C}$$

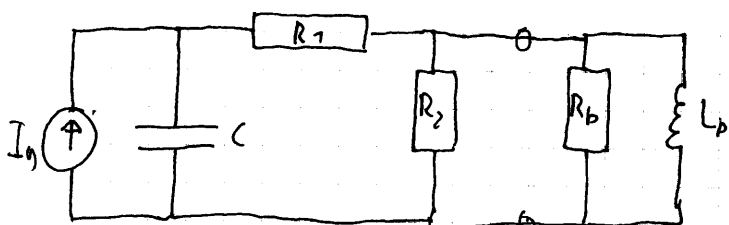
$$C = \frac{1}{\omega^2 L}$$

$$Z_b = Z_g^*$$

$$\operatorname{Re}[Z_b] = \operatorname{Re}[Z_g]$$

$$\operatorname{Im}[Z_b] = -\operatorname{Im}[Z_g]$$

Določí vrednost za max. delovno moč.



$$Y_b = \frac{1}{R_b} + \frac{1}{j\omega L_b} = \frac{1}{R_b} - \frac{j}{\omega L_b}$$

$$Y_g = \frac{1}{R_2} + \frac{1}{R_1 + \frac{1}{j\omega C}} =$$

$$Y_g = \frac{1}{R_2} + \frac{1}{1 + j\omega C R_1} = \frac{1}{R_2} + \frac{j\omega C (1 - j\omega R_1 C)}{1 + (\omega R_1 C)^2} =$$

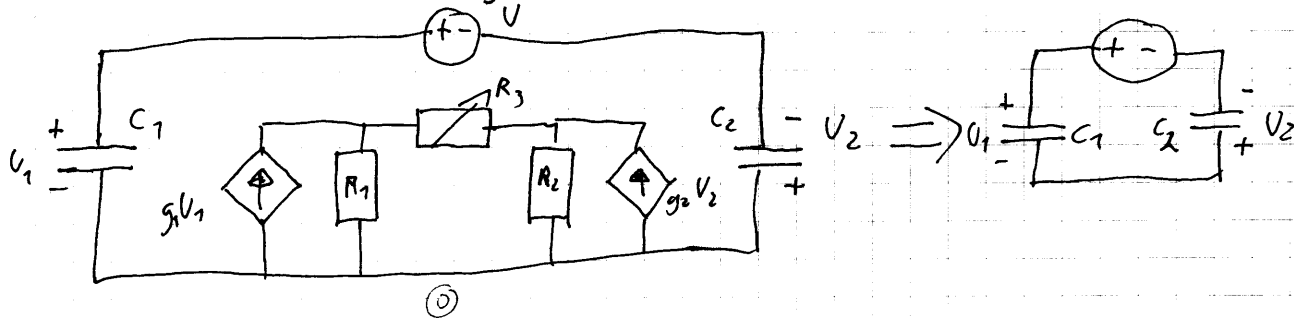
$$Y_g = \frac{1}{R_2} + \frac{\omega^2 C^2 R_1 + j\omega C}{1 + (\omega R_1 C)^2}$$

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{\omega^2 C^2 R_1}{1 + (\omega R_1 C)^2}$$

$$-\frac{1}{\omega L_p} = \frac{-\omega C}{1 + (\omega R_1 C)^2}$$

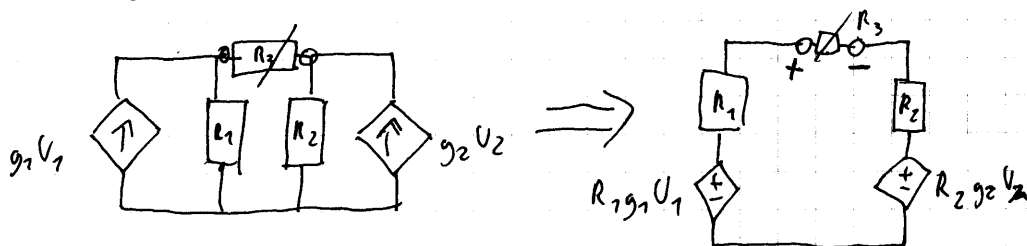
$$\frac{1}{\omega L_p} = \frac{\omega C}{1 + (\omega R_1 C)^2}$$

Določiti vrednost R_3 za maks. moč.



$$U_1 = \frac{\frac{1}{j\omega C_1}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} U = \frac{C_2}{C_1 + C_2} U$$

$$U_2 = \frac{C_1}{C_1 + C_2} U$$



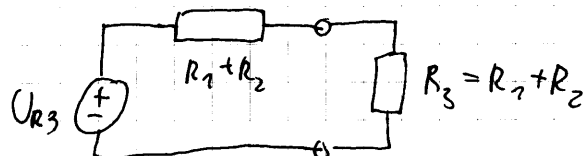
$$U_{R_3} = R_1 g_1 U_1 - R_2 g_2 U_2$$

$$Z_L = R_1 + R_2$$

$$Z_p = R_3$$

$$Z_g = R_1 + R_2$$

$$R_3 = R_1 + R_2$$



$$U = \frac{1}{2} (R_1 g_1 U_1 - R_2 g_2 U_2)$$

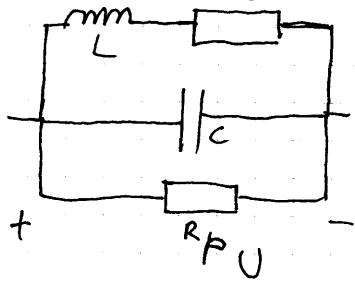
$$I = \frac{U}{R_3} = \frac{1}{2R_3} (R_1 g_1 U_1 - R_2 g_2 U_2)$$

$$P = \frac{1}{2} \operatorname{Re} [U \cdot I^*] = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2R_3} \cdot \operatorname{Re} [(R_1 g_1 U_1 - R_2 g_2 U_2) (R_1 g_1 U_1 - R_2 g_2 U_2)^*]$$

$$= \frac{1}{8R_3} \left[(R_1 g_1 \frac{C_2}{C_1 + C_2} U - R_2 g_2 \frac{C_1}{C_1 + C_2} U) (R_1 g_1 \frac{C_2}{C_1 + C_2} U^* - R_2 g_2 \frac{C_1}{C_1 + C_2} U^*) \right]$$

$$P = \frac{1}{8R_3} \cdot \frac{(R_1 g_1 C_2 - R_2 g_2 C_1)^2}{(C_1 + C_2)^2} \cdot |U|^2$$

Izračunaj R_s resonančno frekvenco in kvaliteto vezja.



$$Y = \frac{1}{R_p} + j\omega C + \frac{1}{R_s + j\omega L}$$

$$= \frac{1}{R_p} + j\omega C + \frac{R_s - j\omega L}{R_s^2 + (\omega L)^2}$$

ADMITANCA,
KO JE VEČ
VZPOREDNO VEŽANIH.

$$\text{Im}(Y) = 0$$

$$j\omega C - \frac{\omega L}{R_s^2 + (\omega L)^2} = 0$$

$$j\omega C \cdot (R_s^2 + (\omega L)^2) - \omega L = 0$$

$$R_s^2 C + \omega^2 L^2 C = L$$

$$\omega^2 L^2 C = L - R_s^2 C$$

$$\omega^2 = \frac{L - R_s^2 C}{L^2 C} = \sqrt{\frac{L - R_s^2 C}{L^2 C}}$$

$$Q_c = \frac{B_c(\omega_0)}{G(\omega_0)} = \frac{B_c(\omega_0)}{R(\omega_0)}$$

1. var.

$$Q_c = \frac{\omega_0 C}{\frac{1}{R_p} + \frac{R_s}{R_s^2 + (\omega_0 L)^2}}$$

2. var.

$$Q = \omega_0 \frac{\sum W_c}{\sum P} = \omega_0 \frac{\sum W_c}{\sum P}$$

$$P_p = \frac{1}{2R_p} |U|^2; \quad P_s = \frac{1}{2R_s} \left| \frac{R_s}{R_s + j\omega L} U \right|^2$$

$$W_c = \frac{1}{2} C |U|^2$$

$$Q = \frac{\frac{1}{2} C |U|^2 \cdot \omega_0}{\frac{1}{2R_p} |U|^2 + \frac{1}{2R_s} \left| \frac{R_s}{R_s + j\omega L} U \right|^2}$$

Izračunaj resonančno frekvenco in kval. vezja.



$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\omega L - \frac{j}{\omega C}$$

$$\operatorname{Im}(Z) = 0$$

$$\omega L - \frac{1}{\omega C} = 0; \quad \omega L = \frac{1}{\omega C}; \quad \omega^2 = \frac{1}{LC}; \quad \underline{\omega_0 = \sqrt{\frac{1}{LC}}}$$

1. var.

$$Q = \frac{\omega_0 L}{R}$$

2. var.

$$Q = \omega_0 \frac{W_L(\omega_0)}{P}; \quad P = \frac{1}{2} R |I|^2; \quad W_L = \frac{1}{2} L |I|^2$$

$$Q = \omega_0 \cdot \frac{\frac{1}{2} L |I|^2}{\frac{1}{2} R |I|^2} = \frac{\omega_0 L}{R}$$

Tokova resonanca $\rightarrow Z$

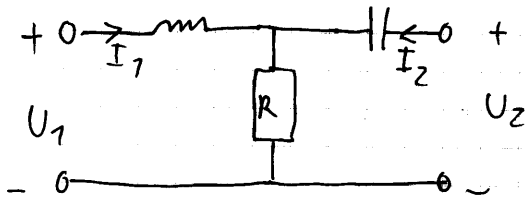
Napetostna resonanca $\rightarrow Y$

Četveropoli

Impedančni parametri

$$U_1 = z_{11} I_1 + z_{12} I_2$$

$$U_2 = z_{21} I_1 + z_{22} I_2$$



a) $I_2 = 0$ (K)

$$z_{11} = \frac{U_1}{I_1} = \frac{j\omega L I_1 + R I_1}{I_1} = \underline{j\omega L + R}$$

$$z_{21} = \frac{U_2}{I_1} = \frac{R \cdot I_1}{I_1} = \underline{R}$$

KO NI KRMILJENIH VIROV
JE RECIPROČEN.

b) $I_1 = 0$ (K)

$$z_{22} = R + \frac{1}{j\omega C}$$

$$z_{12} = \frac{U_1}{I_2} = \frac{R I_2}{I_2} = \underline{R}$$

SIMETRIČNOST

$$z_{11} = z_{22}$$

RECIPROČNOST

$$z_{12} = z_{21}$$

$$j\omega L + R = R + \frac{1}{j\omega C}$$

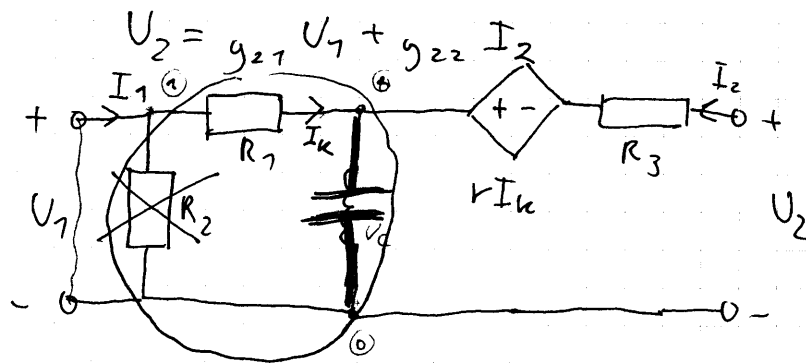
$$\underline{-\omega^2 LC = 1}$$

ni simetrično

Hibridni g parametri

$$I_1 = g_{11} U_1 + g_{12} I_2$$

$$U_2 = g_{21} U_1 + g_{22} I_2$$



RECIPROČNOST

$$g_{21} = -g_{12}$$

SIMETRIČNOST

$$\Delta g \stackrel{\text{def}}{=} 1$$

$$g_{11} g_{22} - g_{12} g_{21} = 1$$

a) $I_2 = 0$

$$g_{11} = \frac{I_1}{U_1} = \frac{1}{R_2} + \frac{1}{R_1 + \frac{1}{j\omega C}} = \frac{1}{R_2} + \frac{j\omega C}{1 + j\omega R_1 C}$$

$$g_{21} = \frac{U_2}{U_1} ; U_{U2} = U_C ; \frac{U_C - U_1}{R_1} + j\omega C U_C = 0$$

$$U_C - U_1 + j\omega R_1 C U_C = 0$$

$$(j\omega R_1 C + 1) U_C = U_1$$

$$U_C = \frac{U_1}{j\omega R_1 C + 1}$$

$$U_2 = U_C - r I_K$$

$$= U_C - r \frac{U_1 - U_C}{R_1}$$

$$= U_C \left(1 + \frac{r}{R_1}\right) - \frac{r}{R_1} U_1$$

$$= \frac{U_1 \left(1 + \frac{r}{R_1}\right)}{j\omega R_1 C + 1} - \frac{r}{R_1} U_1$$

$$g_{21} = \frac{U_2}{U_1} = \frac{1 + \frac{r}{R_1}}{1 + j\omega R_1 C} - \frac{r}{R_1}$$

b) $U_1 = 0$ (kratak stik + R_2 zagine)

$$g_{12} = \frac{I_1}{I_2} ; I_2 = \frac{U_C}{R_1} = \frac{1}{R_1} \frac{I_1 + I_2}{j\omega C} = \frac{I_1}{j\omega R_1 C} + \frac{I_2}{j\omega R_1 C}$$

$$I_1 = \frac{I_2}{j\omega R_1 C - 1}$$

$$g_{12} = \frac{1}{j\omega R_1 C - 1}$$

$$g_{22} = ?$$

$$U_2 = R_3 I_2 - r I_K + R_1 I_1$$

$$U_2 = R_3 I_2 - I_1 (r + R_1) = R_3 I_2 - \frac{I_2}{j\omega R_1 C - 1} (r + R_1) \quad / : I_2$$

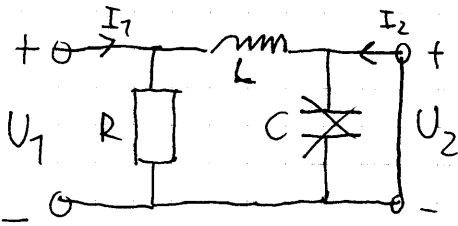
$$g_{22} = \frac{U_2}{I_2} = R_3 - \frac{r + R_1}{j\omega R_1 C - 1}$$

08.12.2008

Hibridni h parametri

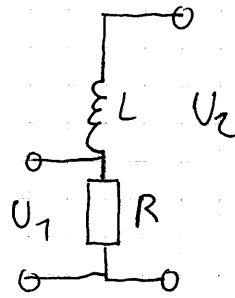
$$U_1 = h_{11} I_1 + h_{12} U_2$$

$$I_2 = h_{21} I_1 + h_{22} U_2$$



a) $I_2 = 0$

$$h_{12} = \frac{U_1}{U_2} = \frac{R}{R + j\omega L}$$



$$h_{22} = \frac{I_2}{U_2} = j\omega C + \frac{1}{R + j\omega L} \text{ admittance}$$

b) $U_2 = 0$ kratak stih (C odpade)

$$h_{11} = \frac{U_1}{I_1} = \left(\frac{1}{R} + \frac{1}{j\omega L} \right)^{-1} = \frac{j\omega L R}{j\omega L + R}$$

$$h_{21} = \frac{I_2}{I_1} \quad I_2 = -\frac{U_1}{j\omega L} \quad I_1 = \frac{j\omega L + R}{j\omega L R} U_1$$

$$h_{21} = \frac{-\frac{U_1}{j\omega L}}{\frac{j\omega L + R}{j\omega L R} U_1} = \frac{-R}{j\omega L + R}$$

reci. $y_{12} = y_{21}$
 sim. $y_{12} = y_{21}, y_{11} = y_{22}$

$$I_1 = y_{11} U_1 + y_{12} U_2$$

$$I_2 = y_{21} U_1 + y_{22} U_2$$

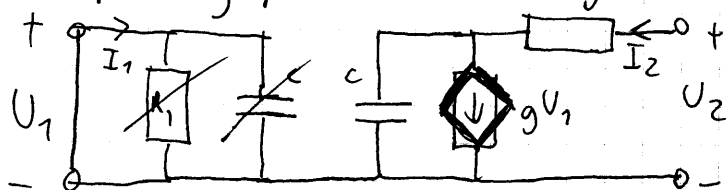
$$I_1 = g_{11} U_1 + g_{12} I_2$$

$$U_2 = g_{21} U_1 + g_{22} I_2$$

$$\rightarrow I_2 = \frac{U_2 - g_{21} U_1}{g_{22}} = - \underbrace{\frac{g_{21}}{g_{22}}}_{y_{21}} U_1 + \underbrace{\frac{1}{g_{22}}}_{y_{22}} U_2$$

$$\rightarrow I_1 = g_{11} U_1 + \frac{g_{12}}{g_{22}} U_2 - \frac{g_{12} g_{21}}{g_{22}} U_1 = \underbrace{\frac{g_{11} g_{22} - g_{12} g_{21}}{g_{22}}}_{y_{11}} U_1 + \underbrace{\frac{g_{12}}{g_{22}}}_{y_{12}} U_2$$

Zapiši g parametre in ugotovi sim. ter reci.!



a) $U_1 = 0$ (kratk stik)

$$g_{12} = \frac{I_1}{I_2} = 0$$

$$g_{22} = \frac{U_2}{I_2} = R_2 + \frac{1}{j\omega C}$$

b) $I_2 = 0$

$$g_{11} = \frac{I_1}{U_1} = \frac{1}{R_1} + j\omega C$$

$$g_{21} = \frac{U_2}{U_1} \quad U_2 = - \frac{g V_1}{j\omega C}$$

$$g_{21} = - \frac{g}{j\omega C}$$

reci. $0 = -(-\frac{g}{j\omega C})$
 ($g_{12} = -g_{21}$) $0 = \frac{g}{j\omega C}$
 $\downarrow$
 $g = 0$

sim. $g_{11} g_{22} - g_{12} g_{21} = 1$

$$(R_2 + \frac{1}{j\omega C}) (\frac{1}{R_1} + j\omega C) = 1$$

$$\frac{R_2}{R_1} + \frac{1}{j\omega R_1 C} + j\omega R_2 C + 1 = 1$$

$$\frac{R_2}{R_1} + \frac{1}{j\omega R_1 C} + j\omega R_2 C = 0 \Rightarrow R_1 = \infty$$

$$R_2 = 0$$

$C \rightarrow 0$ (kratk st.)
 $L \rightarrow 0$
 $R \rightarrow 0$
 $R \rightarrow \infty$

odprto
spoonke

Akta

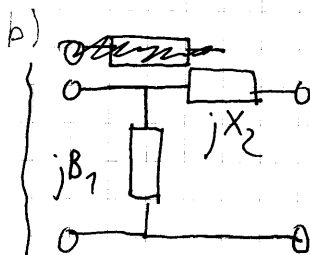
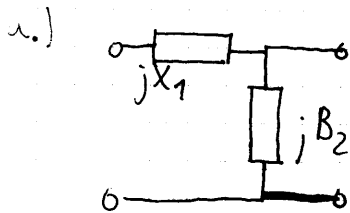
C, L prilagoditvenega četveropola

$$Z_p = (30 + j40) \Omega$$

$$Z_g = 50 \Omega$$

$$G_g = \frac{1}{50} S =$$

$$Y_p = \frac{1}{30 + j40} S = \frac{3 - 4j}{250} S$$



$$B_2 = \pm \sqrt{\frac{G_p}{R_g} (1 - G_p R_g)} - B_p$$

$$X_2 = \pm \sqrt{\frac{R_p}{G_g} (1 - R_p G_g)} - X_p$$

$$X_1 = \pm \sqrt{\frac{R_g}{G_p} (1 - G_p R_g)} - X_g$$

$$B_1 = \pm \sqrt{\frac{G_g}{R_b} (1 - R_b G_g)} - B_g$$

$$\boxed{\begin{matrix} G_p R_g < 1 \\ R_b G_g < 1 \end{matrix}}$$

če je to < 1
uporabimo a
če je to < 1
uporabimo b

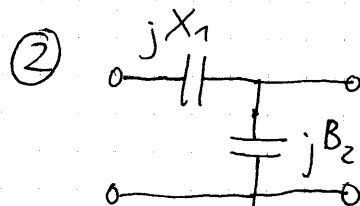
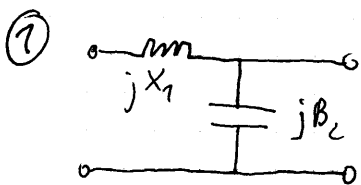
če oboje < 1
determiniramo oboje

$$B_2 = \left\{ \frac{2}{125} + \frac{\sqrt{3}}{125}, \frac{2}{125} - \frac{\sqrt{3}}{125} \right\}$$

$$X_1 = \left\{ 50 \sqrt{\frac{2}{3}}, -50 \sqrt{\frac{2}{3}} \right\}$$

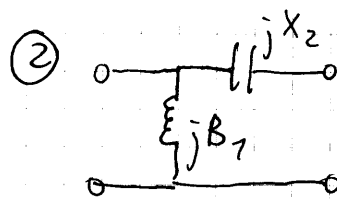
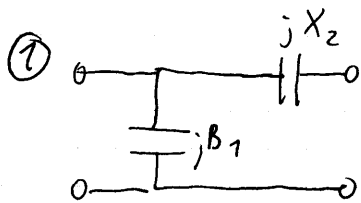
$$X \begin{cases} + \rightarrow \text{---} \\ - \rightarrow \text{---} \end{cases}$$

$$B \begin{cases} + \rightarrow \text{---} \\ - \rightarrow \text{---} \end{cases}$$



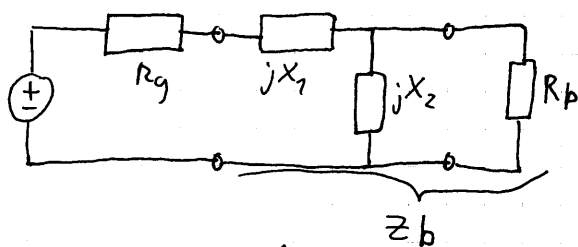
$$X_2 = \left\{ -40 + 10\sqrt{6} \quad -40 - 10\sqrt{6} \right\}$$

$$B_1 = \left\{ \frac{1}{25\sqrt{6}} \quad -\frac{1}{25\sqrt{6}} \right\}$$



Izračunaj vrednost X_1 in X_2 .

$$X_1, X_2 = ?$$



$$R_g = 70 \Omega$$

$$R_p = 600 \Omega$$

$$f = 1 \text{ kHz}$$

X
$+X = \omega L$
$-X = -\frac{1}{\omega C}$
B
$X = \omega C$
$-X = -\frac{1}{\omega L}$

$$Z_b = jX_1 + \frac{1}{\frac{1}{jX_2} + \frac{1}{R_p}} = jX_1 + \frac{X_2^2 R_p + jX_2 R_p^2}{R_p^2 + X_2^2}$$

$$Z_g = R_g$$

$$Z_b = Z_g^*$$

$$R_g = \frac{X_2^2 R_p}{R_p^2 + X_2^2}$$

$$0 = X_1 + \frac{X_2 R_p^2}{R_p^2 + X_2^2}$$

$$R_p^2 R_g + R_g X_2^2 = X_2^2 R_p$$

$$X_2^2 (R_p - R_g) = R_p^2 R_g$$

$$X_2^2 = \frac{R_p^2 R_g}{R_p - R_g}$$

$$X_2 = \pm \sqrt{\frac{R_p^2 R_g}{R_p - R_g}}$$

$$X_2 = \pm 218,05 \Omega$$

$$X_1 = -\frac{X_2 R_p^2}{R_p^2 + X_2^2} = \pm 192,6 \Omega$$

$$\textcircled{1} X_1 = 192,6 \Omega$$

$$\textcircled{2} X_1 = -192,6 \Omega$$

$$X_2 = -218,05 \Omega$$

$$X_2 = 218,05 \Omega$$

$$\frac{j\omega L}{-\frac{1}{\omega C}}$$

$$+X = \omega L$$

$$-X = -\frac{1}{\omega C}$$

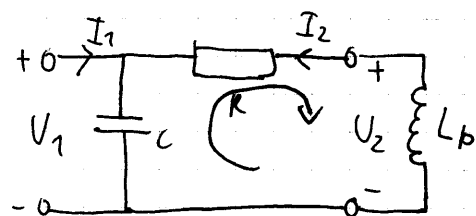
$$\textcircled{1} 192,6 = \omega L$$

$$L = \frac{192,6}{\omega} = 0,0314$$

$$\textcircled{2} -218,05 = -\frac{1}{\omega C}$$

$$C = \frac{1}{218,05 \omega} = 7,29 \cdot 10^{-7} \text{ F}$$

Določiti sistemsko prevažalno funkcijo $Z(j\omega) = \frac{I_2}{I_1}$.



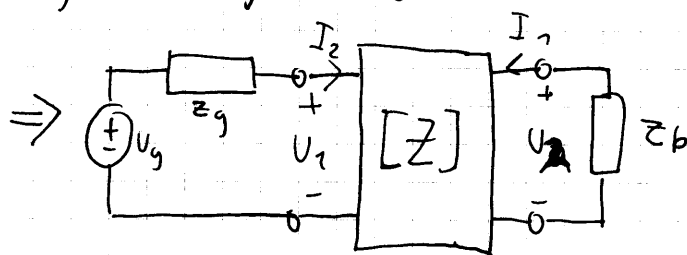
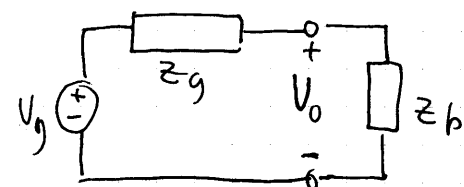
$$\textcircled{1} -\frac{I_1 + I_2}{j\omega C} + RI_2 - j\omega L_p I_2 = 0$$

$$I_2 \left(\frac{1}{j\omega C} + R + j\omega L_p \right) = -\frac{I_1}{j\omega C}$$

$$\frac{1 + j\omega RC - \omega^2 LC}{j\omega C} I_2 = -\frac{I_1}{j\omega C}$$

$$\frac{I_2}{I_1} = \frac{-1}{1 - \omega^2 LC + j\omega RC}$$

Določiti ustavitveno funkcijo $H(j\omega) = \frac{V_2}{V_0}$.



$$V_0 = \frac{Z_b}{Z_b + Z_g} V_g$$

$$V_1 = V_g - Z_g I_1$$

$$Z_b I_2 = -V_2$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

$$2. \quad I_1 = \frac{V_2 - z_{22} I_2}{z_{21}}$$

$$1. \quad z_{11} I_1 + z_{12} I_2 = V_g - Z_g I_1$$

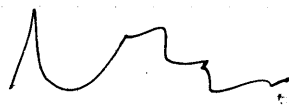
$$I_1 (z_{11} + Z_g) = V_g - z_{12} I_2$$

$$I_2 = -\frac{V_2}{Z_b}$$

$$3. \quad \frac{V_2 - z_{22} I_2}{z_{21}} \cdot (z_{11} + Z_g) = V_g - z_{12} I_2$$

$$\frac{(V_2 - z_{22} I_2)(z_{11} + Z_g)}{z_{21}} + z_{12} I_2 = V_g$$

$$\frac{V_2 z_{11} Z_b + z_{22} z_{11} V_2 + V_2 Z_g Z_b + z_{22} Z_g V_2 - z_{12} z_{21} V_2}{z_{21} Z_b} = V_g$$



15.12.2008

$$\frac{\Delta z U_2 + z_{11} z_b U_2 + z_g z_b U_2 + z_{22} z_g U_2}{z_{11} z_b} = U_g$$

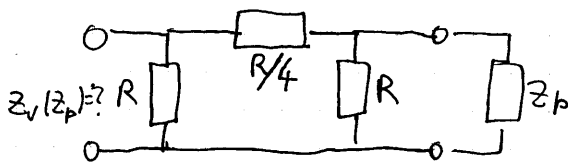
$$U_2 = \frac{z_{21} z_b}{\Delta z + z_{11} z_b + z_g z_b + z_{22} z_g} U_g$$

$$H(j\omega) = \frac{U_2}{U_0} = \frac{z_{21} (z_b + z_g)}{\Delta z + z_{11} z_b + z_g z_b + z_{22} z_g}$$

Kakšno je karakteristična impedanca od četrupola?

$$Z_K = ?$$

Na izhod moramo dati breme!



$$1) \left(\frac{1}{R} + \frac{1}{Z_b} \right)^{-1} = \frac{R Z_b}{R + Z_b}$$

$$2) \frac{R}{4} + \frac{R Z_b}{R + Z_b} = \frac{R^2 + R Z_b + 4 R Z_b}{4(R + Z_b)} = \frac{R^2 + 5 R Z_b}{4R + 4 Z_b}$$

$$3) \left(\frac{1}{R} + \frac{4R + 4Z_b}{R^2 + 5RZ_b} \right)^{-1} = \left(\frac{R + 5Z_b + 4R + 4Z_b}{R^2 + 5RZ_b} \right)^{-1} = \left(\frac{5R + 9Z_b}{R^2 + 5RZ_b} \right)^{-1}$$

$$Z_v(Z_b) = \frac{R^2 + 5RZ_b}{5R + 9Z_b}$$

$$\boxed{Z_v(Z_K) = Z_K}$$

$$\frac{R^2 + 5RZ_K}{5R + 9Z_K} = Z_K$$

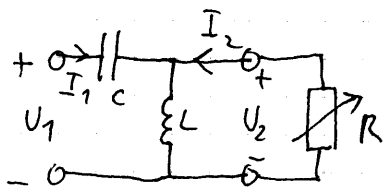
$$R^2 + 5RZ_K = 5RZ_K + 9Z_K^2$$

$$Z_K = \frac{R^2}{9}$$

$$Z_K = \sqrt{\frac{R^2}{9}} = \frac{R}{3}$$

Nariši potek od Z_V kot funkcije upora. ($\omega C=1$, $\omega L=2$)

$$Z_V(R) \quad \omega C=1 \quad \omega L=2$$



$$U_1 = Z_{11} I_1 + Z_{12} I_2$$

$$U_2 = Z_{21} I_1 + Z_{22} I_2$$

a) $I_2 = 0$

$$Z_{11} = \frac{U_1}{I_1} = \frac{1}{j\omega C} + j\omega L = \frac{1}{j} + 2j = j$$

$$Z_{21} = \frac{U_2}{I_1} = j\omega L = 2j$$

b) $I_1 = 0$

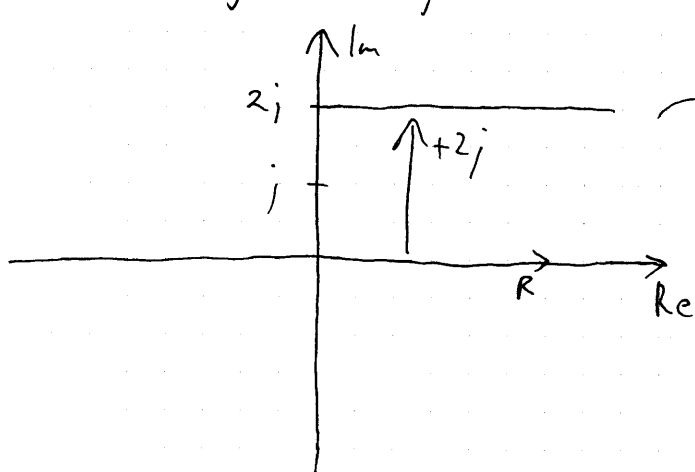
$$Z_{12} = \frac{U_1}{I_2} = j\omega L = 2j$$

$$Z_{22} = \frac{U_2}{I_2} = j\omega L = 2j$$

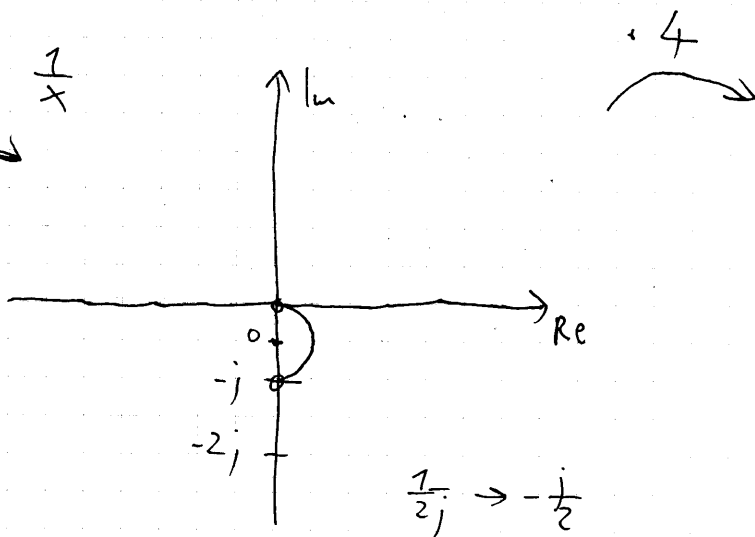
$$Z_V(Z_P) = Z_{11} - Z_{12} Z_{21} \frac{1}{Z_{22} + Z_P}$$

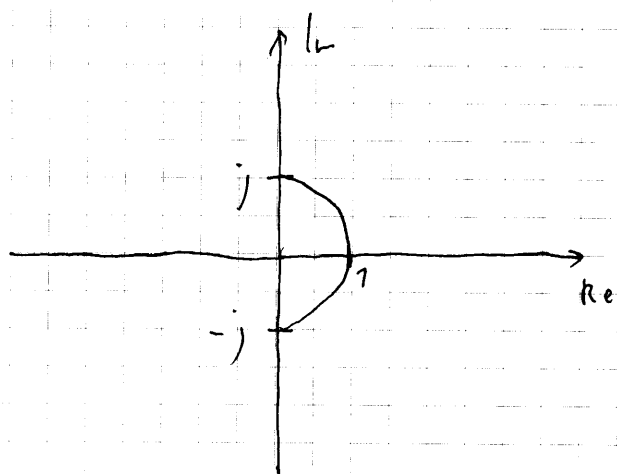
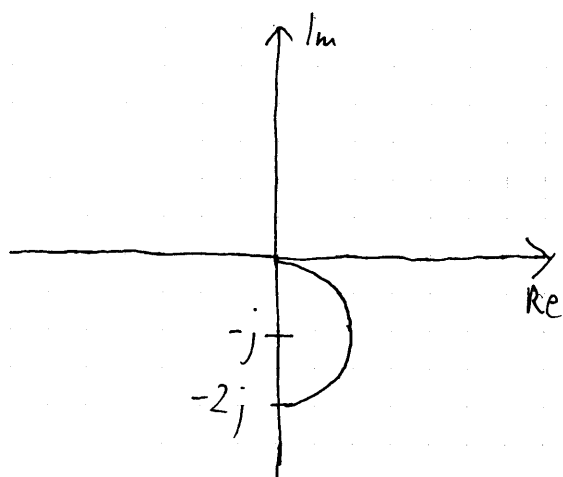
$$Z_V(R) = j - 2j \cdot 2j \frac{1}{2j + R}$$

$$= j + 4 \cdot \frac{1}{2j + R}$$



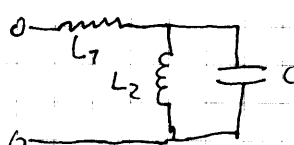
$$\frac{1}{x}$$





Izračunaj resonančno frekvenco, kvaliteto in napiši potek vhodne impe.

ω_0 , Q , potek $Z_V(\omega)$



a) Tokovna

$$Z_V = j\omega L_1 + \frac{1}{\frac{1}{j\omega L_2} + j\omega C} = j\omega L_1 + \frac{j\omega L_2}{1 - \omega^2 L_2 C}$$

$$\text{Im}(Z_V) = 0$$

$$\omega L_1 + \frac{\omega L_2}{1 - \omega^2 L_2 C} = 0$$

$$L_1 + L_2 - \omega^2 L_2 C L_1 = 0$$

$$\omega^2 L_2 C L_1 = L_1 + L_2$$

$$\omega^2 = \frac{L_1 + L_2}{L_1 L_2 C} \Rightarrow \omega_0 = \sqrt{\frac{L_1 + L_2}{L_1 L_2 C}}$$

~~$\omega = 0$~~ resonance
ni brez freh.

$$Q_L = \frac{X_L}{R} = \infty$$

b) Napetostna

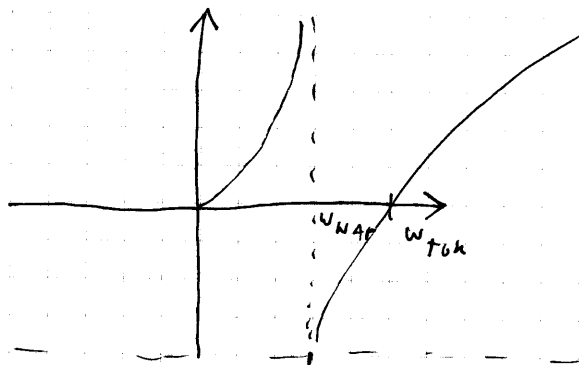
$$Y_V = (Z_V)^{-1} = \left(\frac{j\omega L_1 - j\omega^3 L_1 L_2 C + j\omega L_2}{1 - \omega^2 L_2 C} \right)^{-1} = \frac{1 - \omega^2 L_2 C}{j\omega(L_1 + L_2) - j\omega^3 L_1 L_2 C} =$$

$$= -j \frac{1 - \omega^2 L_2 C}{\omega(L_1 + L_2) - \omega^3 L_1 L_2 C}$$

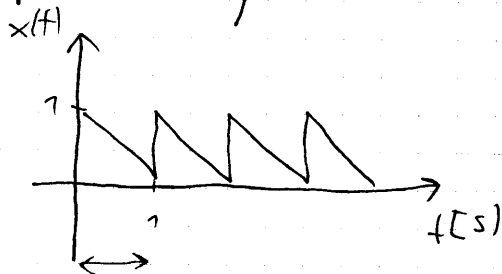
$$\text{Im}(Y_V) = 0 \Rightarrow \frac{1 - \omega^2 L_2 C}{\omega(L_1 + L_2) - \omega^3 L_1 L_2 C} = 0 \Rightarrow 1 - \omega^2 L_2 C = 0$$

$$\omega^2 L_2 C = 1 \Rightarrow \omega = \sqrt{\frac{1}{L_2 C}}$$

$\omega_{\text{otok}} = \text{ničla}$
 $\omega_{\text{onap}} = \text{pol}$
 $\omega = 0, \text{ ničla}$



Fourierjeva vrsta



$$T = 1s \quad t \in [0, 1]$$

$$\omega = 2\pi \quad f(t) = 1 - t$$

$$a_0 = \frac{1}{T} \int_0^T (1-t) dt = \left(t - \frac{t^2}{2} \right) \Big|_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\begin{aligned}
 a_n &= \frac{2}{T} \int_0^T (1-t) \cos(n \cdot 2\pi \cdot t) dt = 2 \cdot \underbrace{\int_0^1 \cos(2\pi n t) dt}_{=0} - 2 \int_0^1 t \cos(2\pi n t) dt \\
 &= -2 \cdot \left(\frac{t}{2\pi n} \sin(2\pi n t) + \frac{1}{(2\pi n)^2} \cos(2\pi n t) \right) \Big|_0^1 = \\
 &= -2 \cdot \left[\left(0 + \frac{1}{(2\pi n)^2} \right) - \left(0 + \frac{1}{(2\pi n)^2} \right) \right] = 0
 \end{aligned}$$

$$\begin{aligned}
 b_n &= \frac{2}{T} \int_0^T (1-t) \sin(2\pi n t) dt = 2 \cdot \underbrace{\int_0^1 \sin(2\pi n t) dt}_{=0} - 2 \int_0^1 t \sin(2\pi n t) dt = \\
 &= -2 \left(-\frac{t}{2\pi n} \cos(2\pi n t) + \frac{1}{(2\pi n)^2} \sin(2\pi n t) \right) \Big|_0^1 \\
 &= -2 \left(\left(-\frac{1}{2\pi n} + 0 \right) - (0 + 0) \right) = \frac{2}{2\pi n} = \frac{1}{\pi n}
 \end{aligned}$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{\pi n} \cdot (\sin(2\pi n t))$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega t) + b_n \sin(n\omega t))$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) dt$$

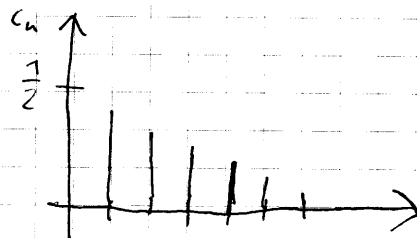
$$\text{LIHA: } a_n = 0$$

$$\text{SODA: } b_n = 0$$

amplitudni spekter

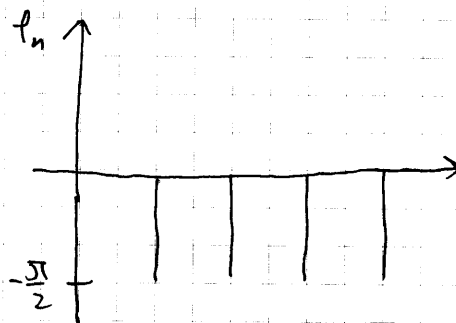
$$C_0 = |a_0| = \frac{1}{2}$$

$$C_n = \sqrt{a_n^2 + b_n^2} = \sqrt{b_n^2} = |b_n| = \frac{1}{n\pi}$$

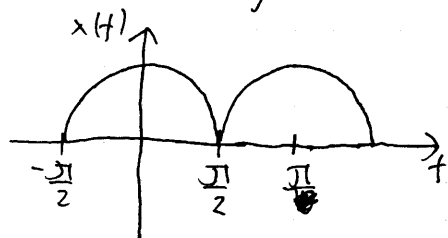


fazni spekter

$$\varphi_n = -\arctg\left(\frac{b_n}{a_n}\right) = -\frac{\pi}{2}$$



Izračunaj F vrsto, spektra, delovno ω, F eksponentno vrsto



$$x(t) = |\cos(t)|$$

$$x(t) = \text{sodna}$$

$$T = \pi$$

$$t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\omega = \frac{2\pi}{T} = 2$$

$$a_0 = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) dt = \frac{1}{\pi} \sin(t) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{\pi} (1 - (-1)) = \frac{2}{\pi}$$

$$\begin{aligned} a_n &= \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) \cdot \cos(2nt) dt = \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} (\cos((2n-1)t) + \cos((2n+1)t)) dt = \\ &= \frac{1}{\pi} \left(\frac{\sin((2n-1)t)}{2n-1} + \frac{\sin((2n+1)t)}{2n+1} \right) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \\ &= \frac{1}{\pi} \left(\frac{(-1)^{n+1}}{2n-1} + \frac{(-1)^n}{2n+1} - \left(\frac{(-1)^n}{2n-1} + \frac{(-1)^{n+1}}{2n+1} \right) \right) \end{aligned}$$

$$\begin{aligned} \sin\left(\frac{\pi}{2}(2n-1)\right) &\rightarrow (-1)^{n+1} & \sin\left(-\frac{\pi}{2}(2n+1)\right) &\rightarrow (-1)^{n+1} \\ \sin\left(\frac{\pi}{2}(2n+1)\right) &\rightarrow (-1)^n & \sin\left(-\frac{\pi}{2}(2n-1)\right) &\rightarrow (-1)^n \end{aligned}$$

$$a_n = \frac{2}{\pi} (-1)^{n+1} \left[\frac{1}{2n-1} - \frac{1}{2n+1} \right] = \frac{2}{\pi} (-1)^{n+1} \frac{2n+1-2n-1}{(2n-1)(2n+1)} = \frac{4(-1)^{n+1}}{\pi(4n^2-1)}$$

$$f(t) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(4n^2-1)} \cos(2nt)$$

$$x(t) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} A_n e^{j\omega_n t} + A_0$$

exponentiell-Funktion

$$A_0 = a_0 \quad A_n = \frac{1}{T} \int_0^T x(t) e^{-j\omega_n t} dt$$

$$A_0 = \frac{2}{\pi}$$

$$A_n = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t) e^{-2jnt} dt$$

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$\cos(t) = \frac{e^{jt} + e^{-jt}}{2}$$

$$= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{jt} + e^{-jt}}{2} e^{-2jnt} dt = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{+(1-2n)t} + e^{+(-1-2n)t} dt$$

$$= \frac{1}{2\pi} \left(\frac{e^{j(1-2n)t}}{j(1-2n)} + \frac{e^{-j(1+2n)t}}{-j(1+2n)} \right) \Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2\pi} \left(\left(\frac{(-1)^n j}{j(1-2n)} + \frac{(-1)^{n+1} j}{-j(1+2n)} \right) - \left(\frac{(-1)^{n+1} j}{j(1-2n)} + \frac{(-1)^n j}{-j(1+2n)} \right) \right) =$$

$$\begin{aligned} e^{-j(1-2n)\frac{\pi}{2}} &= -(-1)^n j & e^{j(1+2n)\frac{\pi}{2}} &= j(-1)^n \\ e^{j(1-2n)\frac{\pi}{2}} &= j(-1)^n & e^{-j(1+2n)\frac{\pi}{2}} &= -j(-1)^n \end{aligned}$$

$$= \frac{2(-1)^{n+1}}{\pi(4n^2-1)}$$

$$f(t) = \frac{2}{\pi} + \frac{2}{\pi} \sum_{n \neq 0} \frac{(-1)^{n+1}}{4n^2-1} e^{j2nt}$$

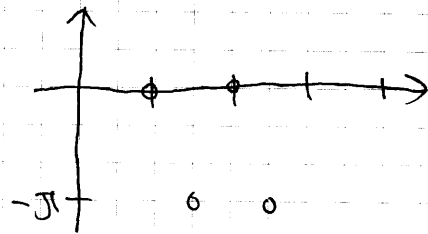
amplitudni spekter

$$c_0 = |A_0| = \frac{2}{\pi}$$

$$c_n = 2|A_n| = \frac{4}{\pi(4n^2-1)}$$

fazni spekter

$$\varphi_n = \arg(A_n) = \begin{cases} 0 & n=2k+1 \\ -\pi & n=2k \end{cases}$$

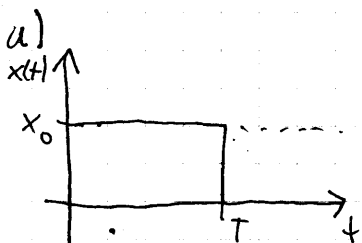


osnovni prostor

$$P = \frac{1}{T} \int_0^T x^2(t) dt = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2(t) dt = \underline{\underline{\frac{1}{2}}}$$

frekvenci prostora

$$P = c_0^2 + \sum_{n=1}^{\infty} \frac{c_n^2}{2} = \frac{4}{\pi^2} + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2}$$



$$x(t) = x_0 \cdot (1(t) - 1(t - \tau))$$

$$X(\omega) = x_0 (\pi \delta(\omega) + \frac{1}{j\omega})$$

$$-x_0 (\pi \delta(\omega) + \frac{1}{j\omega}) \cdot e^{-j\omega\tau}$$

$$1(t) \leftrightarrow \pi \delta(\omega) + \frac{1}{j\omega}$$

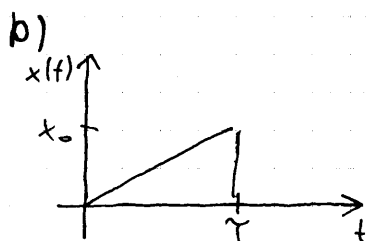
$$f(t - \tau) \leftrightarrow F(\omega) \cdot e^{-j\omega\tau}$$

$$= x_0 (\pi \delta(\omega) - \pi \delta(\omega) \cdot e^{-j\omega\tau}) +$$

$$+ x_0 \left(\frac{1}{j\omega} + \frac{e^{-j\omega\tau}}{j\omega} \right) =$$

$$= x_0 \cdot \frac{1 + e^{-j\omega\tau}}{j\omega} + x_0 \underbrace{\pi \delta(\omega) \cdot (1 - e^{-j\omega\tau})}_{=0}$$

$$= x_0 \cdot \frac{1 + e^{-j\omega\tau}}{j\omega}$$



$$x(t) = \frac{x_0}{\tau} \cdot t \cdot (1(t) - 1(t - \tau))$$

$$t \cdot f(t) \leftrightarrow \frac{\partial F(\omega)}{\partial \omega}$$

$$x(t) = \frac{x_0}{\tau} (t \cdot 1(t) - t \cdot 1(t - \tau)) =$$

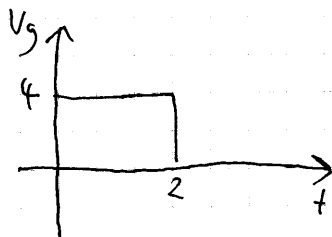
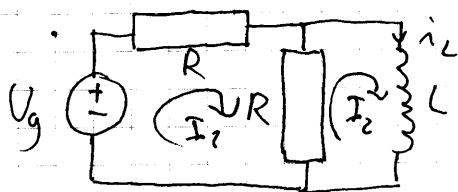
$$= \frac{x_0}{\tau} (t \cdot 1(t) - (t - \tau + \tau) \cdot 1(t - \tau)) =$$

$$= \frac{x_0}{\tau} (t \cdot 1(t) - (t - \tau) 1(t - \tau) - \tau \cdot 1(t - \tau))$$

$$X(\omega) = \frac{x_0}{\tau} \left[(-j\pi \frac{\delta(\omega)}{\omega} - \frac{1}{\omega^2}) \cdot (1 - e^{-j\omega\tau}) - \tau (\pi \delta(\omega) + \frac{1}{j\omega}) \cdot e^{-j\omega\tau} \right]$$

Izračunaj tok skozi taljavo.

$$i_L(t) = ?$$



$$u_g(t) = 4(1(t) - 1(t-2)) \text{ V}$$

$$H(j\omega) = \frac{I_L}{U_g}$$

$$\begin{aligned} U_g(\omega) &= 4 \cdot ((\pi\delta(\omega) + \frac{1}{j\omega}) - (\pi\delta(\omega) + \frac{1}{j\omega}) \cdot e^{j\omega 2}) = \\ &= 4 \left(\frac{1-e^{-2j\omega}}{j\omega} + \underbrace{(\pi\delta(\omega) \cdot (1-e^{-2j\omega}))}_{=0} \right) \\ &= 4 \frac{1-e^{-2j\omega}}{j\omega} \end{aligned}$$

$$\textcircled{I_1} \quad -U_g + RI_1 + R(I_1 - I_2) = 0$$

$$\textcircled{I_2} \quad R(I_2 - I_1) + j\omega L I_2 = 0$$

$$H(j\omega) = \frac{1}{R+2j\omega L}$$

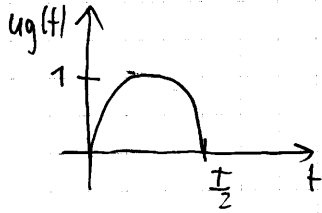
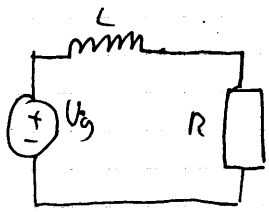
$$U_g(\omega) = 4 \cdot \frac{1-e^{-2j\omega}}{j\omega}$$

$$I_L = H(j\omega) \cdot U_g(\omega)$$

$$\begin{aligned} I_L &= 4 \cdot \frac{1-e^{-2j\omega}}{j\omega(R+2j\omega L)} = \frac{4}{R} (1-e^{-2j\omega}) \frac{1}{j\omega - \frac{2L}{R} \cdot \omega^2} = \\ &= \frac{4}{R} (1-e^{-2j\omega}) \left[\frac{1}{j\omega} - \frac{1}{j\omega + \frac{R}{2L}} \right] \end{aligned}$$

$$\begin{aligned} i_L(t) &= \frac{4}{R} \left[\frac{1}{2} \text{sign}(t) - 1(t) \cdot e^{-\frac{R}{2L}t} \right] \\ &\quad - \left[\frac{1}{2} \text{sign}(t-2) - 1(t-2) e^{-\frac{R}{2L}(t-2)} \right] \end{aligned}$$

$\frac{1}{j\omega} \leftrightarrow \frac{1}{2} \text{sign}(t)$
$\frac{1}{j\omega + \frac{R}{2L}} \leftrightarrow 1(t) e^{-\frac{R}{2L}t}$

$U_R(\omega)$ 

$$\sin(\omega_0 t) \cdot 1(t) \leftrightarrow \frac{\omega_0}{\omega_0^2 - \omega^2}$$

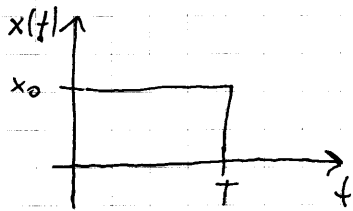
$$\begin{aligned} u_g(t) &= \sin(\omega_0 t) \left(1(t) - 1\left(t - \frac{T}{2}\right) \right) ; \quad \omega_0 = \frac{2\pi}{T} \Rightarrow \pi = \frac{\omega_0 T}{2} \\ &= \sin(\omega_0 t) \cdot 1(t) - \sin(\omega_0 t) \cdot 1\left(t - \frac{T}{2}\right) \\ &= \sin(\omega_0 t) \cdot 1(t) + \sin\left(\omega_0 t - \pi\right) \cdot 1\left(t - \frac{T}{2}\right) \\ &= \sin(\omega_0 t) \cdot 1(t) + \sin\left(\omega_0 \left(t - \frac{T}{2}\right)\right) \cdot 1\left(t - \frac{T}{2}\right) \end{aligned}$$

$$U_g(\omega) = \frac{\omega_0}{\omega_0^2 - \omega^2} \left(1 + e^{-j\frac{T}{2}\omega} \right)$$

$$H(j\omega) = \frac{R}{R + j\omega L}$$

$$U_R(\omega) = H(j\omega) \cdot U_g(\omega) = \underline{\underline{\frac{R \omega_0}{(R + j\omega L)(\omega_0^2 - \omega^2)} \left(1 + e^{-j\frac{T}{2}\omega} \right)}}$$

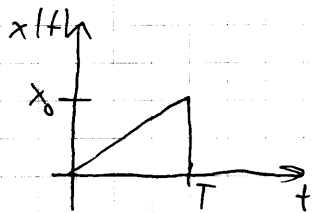
Laplaceova trans.



$$x(t) = x_0 (1(t) - 1(t - T))$$

$$X(s) = x_0 \frac{1}{s} - x_0 \frac{1}{s} \cdot e^{-Ts}$$

Veduo mora biti enotna stopnica.



$$x(t) = \frac{x_0}{T} t (1(t) - 1(t - T))$$

$$= \frac{x_0}{T} (t 1(t) - t \cdot 1(t - T))$$

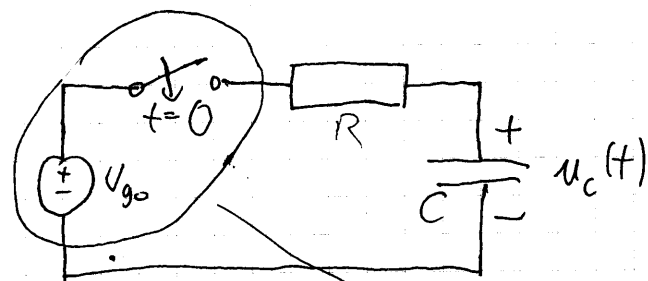
$$= \frac{x_0}{T} (t \cdot 1(t) - (t - T + T) \cdot 1(t - T))$$

$$= \frac{x_0}{T} (t \cdot 1(t) - (t - T) \cdot 1(t - T) - T 1(t - T))$$

$$X(s) = \frac{x_0}{T} \left(\frac{1}{s^2} - \frac{1}{s^2} \cdot e^{-Ts} - T \frac{1}{s} \cdot e^{-Ts} \right)$$

$$\begin{aligned} 1(t) &\leftrightarrow \frac{1}{s} \\ x(t - \tau) 1(t - \tau) &\leftrightarrow x(s) e^{-\tau s} \\ t f(t) &\leftrightarrow - \frac{\partial F(s)}{\partial s} \end{aligned}$$

Položi napetost na kondenzatorju!



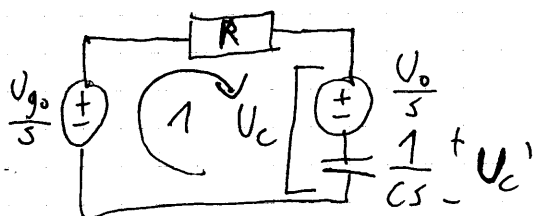
$$u_c(t) = ?$$

$$u_c(0) = U_0$$

$$U_g = 5V$$

$$U_{g0} \cdot 1(t) \leftrightarrow \frac{U_{g0}}{s}$$

$$u_c(0) = U_0 \cdot 1(t) \leftrightarrow \frac{U_0}{s}$$



$$U_c = \frac{U_0}{s} + U_c'$$

$$\textcircled{1} - \frac{U_{g0}}{s} + R I(s) + \frac{U_0}{s} + \frac{I(s)}{Cs} = 0$$

$$I(s) \left(R + \frac{1}{Cs} \right) = \frac{U_{g0} - U_0}{s}$$

$$I(s) = \frac{U_{g0} - U_0}{s \left(R + \frac{1}{Cs} \right)} = \frac{U_{g0} - U_0}{R Cs + 1} C$$

$$\begin{aligned} U_c &= \frac{U_0}{s} + \frac{I(s)}{Cs} = \frac{U_0}{s} + \frac{1}{Cs} \frac{(U_{g0} - U_0) C}{R Cs + 1} = \frac{U_0}{s} + \frac{U_{g0} - U_0}{s(1 + RCs)} = \\ &= \frac{U_0}{s} + \frac{(U_{g0} - U_0)}{RC} \cdot \frac{1}{s(s + \frac{1}{RC})} \end{aligned}$$

$$\frac{1}{(s-s_1)(s-s_2)} = \frac{K_1}{s-s_1} + \frac{K_2}{s-s_2}$$

$$s_1 = 0 \quad s_2 = -\frac{1}{RC}$$

$$K_1 = \left. \frac{1}{s-s_2} \right|_{s=s_1}$$

$$K_2 = \left. \frac{1}{s-s_1} \right|_{s=s_2}$$

$$\frac{1}{s(s + \frac{1}{RC})}$$

$$k_1 = \frac{1}{s + \frac{1}{RC}} \Big|_{s=0} = \frac{1}{\frac{1}{RC}} = RC$$

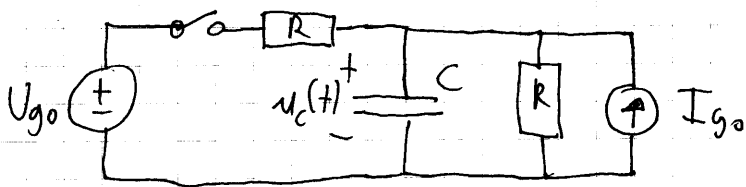
$$k_2 = \frac{1}{s} \Big|_{s=-\frac{1}{RC}} = \frac{1}{-\frac{1}{RC}} = -RC$$

$$U_C = \frac{U_0}{s} + \frac{U_0 - U_0}{RC} \cdot \left(\frac{RC}{s} - \frac{RC}{s + \frac{1}{RC}} \right)$$

$$\boxed{\frac{1}{s-a} \leftrightarrow e^{at}}$$

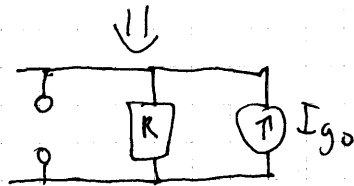
$$u_c(t) = U_0 \cdot 1(t) + (U_0 - U_0) (1(t) - e^{-\frac{t}{RC}} 1(t))$$

Doloci napetost na kondenzatorja!

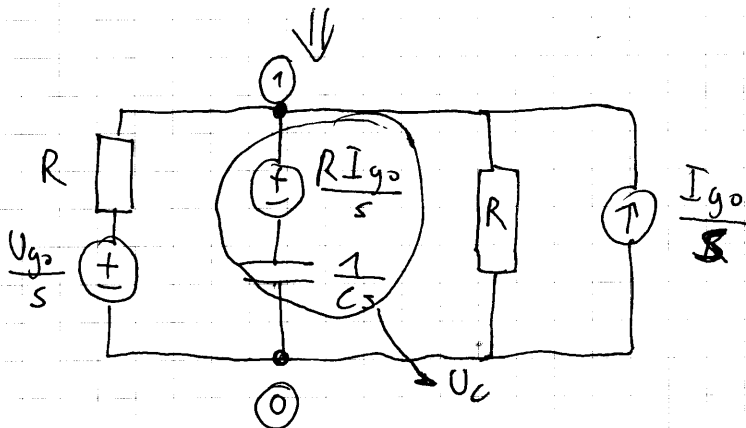


$$U_{g0} \cdot 1(t) \leftrightarrow \frac{U_{g0}}{s}$$

$$I_{g0} \cdot 1(t) \leftrightarrow \frac{I_{g0}}{s}$$



$$u_c(0) = R I_{g0} \leftrightarrow \frac{R I_{g0}}{s}$$



$$\textcircled{1} \frac{U_{V1}(s) - \frac{U_{g0}}{s}}{R} + C_s (U_{V1}(s) - \frac{I_{g0} R}{s}) + \frac{U_{V1}(s)}{R} - \frac{I_{g0}}{s} = 0$$

$$U_{V1}(s) \left(\frac{1}{R} + C_s + \frac{1}{R} \right) = \frac{U_{g0}}{R s} + \frac{I_{g0} C_s R}{s} + \frac{I_{g0}}{s}$$

$$U_{V1}(s) \frac{2 + RCs}{R} = \frac{U_{g0} + I_{g0} R^2 C_s + I_{g0} R}{R s}$$

$$U_C = \frac{U_{g0} + I_{g0} R^2 C_s + I_{g0} R}{s (2 + RCs)} = \frac{U_{g0} + R I_{g0}}{RCs (s + \frac{2}{RC})} + \frac{I_{g0} R^2 C_s}{RCs (s + \frac{2}{RC})}$$

Alketa

$$\frac{1}{s(s + \frac{2}{RC})}$$

$$s_1 = 0$$

$$s_2 = -\frac{2}{RC}$$

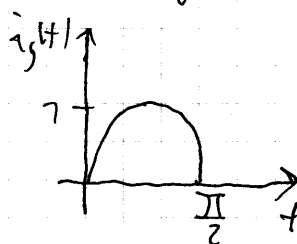
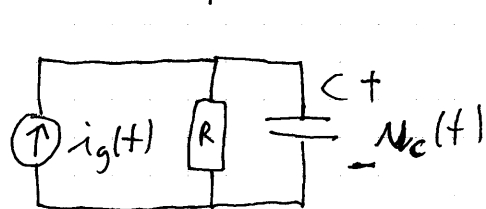
$$K_1 = \frac{1}{s + \frac{2}{RC}} \Big|_{s=0} = \frac{RC}{2}$$

$$K_2 = \frac{1}{s} \Big|_{s=-\frac{2}{RC}} = -\frac{RC}{2}$$

$$U_C = \frac{U_{g0} + R I_{g0}}{RC} \left(\frac{RC}{2} + \frac{-RC}{s + \frac{2}{RC}} \right) + \frac{I_{g0} R}{s + \frac{2}{RC}}$$

$$u_C(t) = \frac{U_{g0} + R I_{g0}}{2} (1(t) - e^{-\frac{2t}{RC}} \cdot 1(t)) + I_{g0} R \cdot e^{-\frac{2t}{RC}} 1(t)$$

Določí napetost na kondenzatorju. $u_C(t) = ?$



$$R = 1 \Omega$$

$$C = 1 F$$

$$u_C(0) = 0 V$$

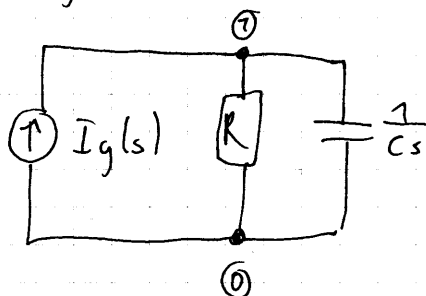
$$i_g(t) = \sin(\omega_0 t) \left(1(t) - 1\left(t - \frac{\pi}{2}\right) \right) = \sin(2t) \cdot 1(t) - \sin\left(2t\right) \cdot 1\left(t - \frac{\pi}{2}\right)$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = \underline{2}$$

$$= \sin(2t) \cdot 1(t) + \sin(2t - \pi) \cdot 1\left(t - \frac{\pi}{2}\right) =$$

$$= \sin(2t) \cdot 1(t) + \sin\left(2 \cdot \left(t - \frac{\pi}{2}\right)\right) \cdot 1\left(t - \frac{\pi}{2}\right)$$

$$I_g(s) = \frac{2}{4+s^2} + \frac{2}{4+s^2} e^{-\frac{\pi}{2} \cdot s}$$



$$\textcircled{1} -I_g(s) + \frac{U_{V1}(s)}{R} + U_{V1}(s) \cdot Cs = 0$$

$$U_{V1}(s) \cdot \left(\frac{1}{R} + Cs \right) = I_g(s)$$

$$U_{V1}(s) \cdot \frac{1 + CRs}{R} = I_g(s)$$

$$U_C = U_{V1}(s) = \frac{I_g(s) \cdot R}{1 + RCs}$$

$$= \frac{R}{1 + RCs} \cdot \frac{2}{4+s^2} (1 + e^{-\frac{\pi}{2}s})$$

$$U_c = \frac{2R}{RC} (1 + e^{-\frac{\pi}{2}s}) \cdot \frac{1}{(s + \frac{1}{RC})(4 + s^2)}$$

$$= 2 (1 + e^{-\frac{\pi}{2}s}) \cdot \frac{1}{(s+1)(s^2+4)}$$

$$= 2 (1 + e^{-\frac{\pi}{2}s}) \cdot \frac{1}{(s+1)(s+2j)(s-2j)}$$

$$K_1 = \left. \frac{1}{(s+2j)(s-2j)} \right|_{s=-1} = \left. \frac{1}{s^2+4} \right|_{s=-1} = \frac{1}{5}$$

$$K_2 = \left. \frac{1}{(s+1)(s-2j)} \right|_{s=-2j} = \frac{-2+j}{20}$$

$$K_3 = \left. \frac{1}{(s+1)(s+2j)} \right|_{s=2j} = \frac{-2-j}{20}$$

$$U_c = 2 \cdot (1 + e^{-\frac{\pi}{2}s}) \left(\frac{1}{5} \frac{1}{s+1} + \frac{-2+j}{20} \frac{1}{s+2j} + \frac{-2-j}{20} \frac{1}{s-2j} \right)$$

$$\mathcal{L}^{-1}[A] = \left(\frac{1}{5} \cdot e^{-t} + \frac{-2+j}{20} e^{-2jt} + \frac{-2-j}{20} e^{2jt} \right) \cdot 1(t) =$$

$$= \left(\frac{1}{5} e^{-t} + \frac{-2+j}{20} (\cos(-2t) + j \sin(-2t)) + \frac{-2-j}{20} (\cos(2t) + j \sin(2t)) \right) \cdot 1(t)$$

$$= \frac{1}{5} \left(e^{-t} - \frac{\cos(2t)}{2} - \frac{\cos(2t)}{2} + \frac{\sin(2t)}{4} + \frac{\sin(2t)}{4} \right) \cdot 1(t)$$

$$= \frac{1}{5} \left(e^{-t} - \cos(2t) + \frac{1}{2} \sin(2t) \right) 1(t)$$

$$u_c(t) = 2 \left[\frac{1}{5} \left(e^{-t} - \cos(2t) + \frac{1}{2} \sin(2t) \right) \cdot 1(t) + \frac{1}{5} \left(e^{-t-\frac{\pi}{2}} - \cos(2t - \frac{\pi}{2}) + \frac{1}{2} \sin(2(t - \frac{\pi}{2})) \right) \cdot 1(t - \frac{\pi}{2}) \right]$$

Tuning trik!

L^{-1} :

$$\frac{s-2}{2s^2+s+2}$$

$$\frac{s-2}{(s-s_1)(s-s_2)}$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$