

Ploskve v prostoru

Osnovna ukaza za risanje ploskev v prostoru :

Plot3D	ce je enacba ploskve ... eksplicitna	$z=f(x,y)$
ParametricPlot3D	parametrična	$\vec{r}=\{x(u,v),y(u,v),z(u,v)\}$

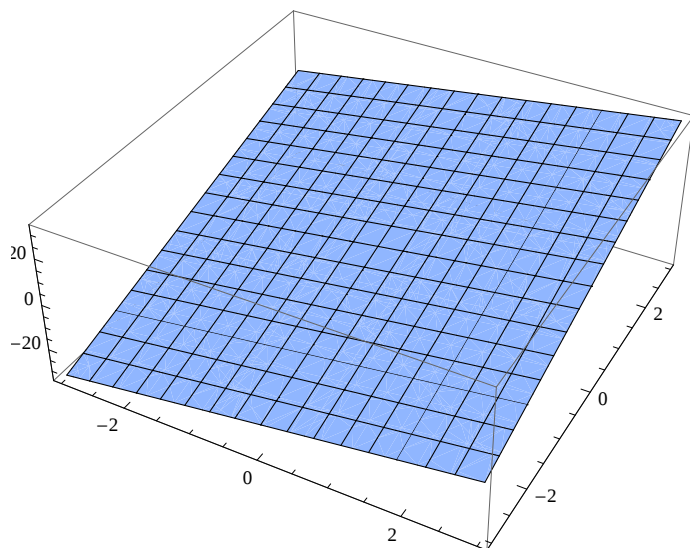
? Plot3D

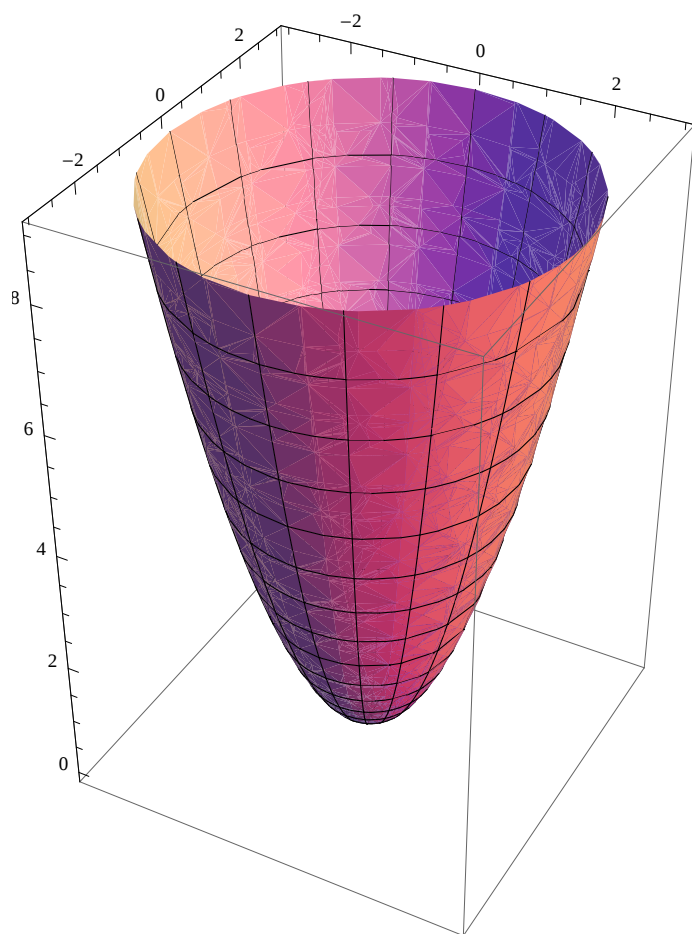
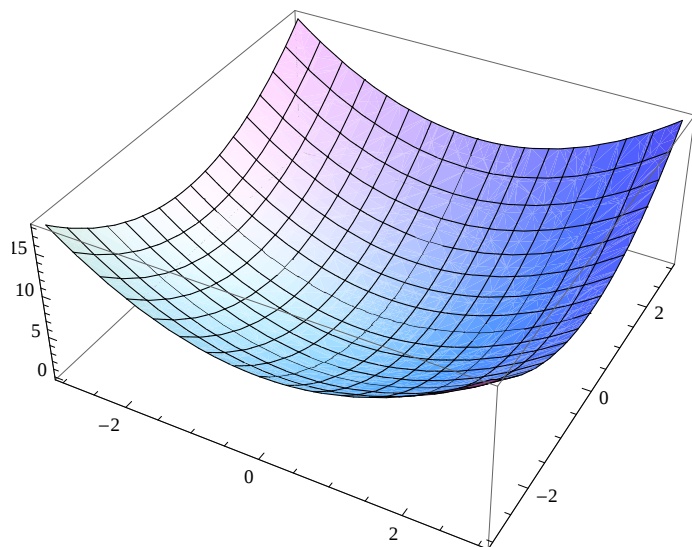
? ParametricPlot3D

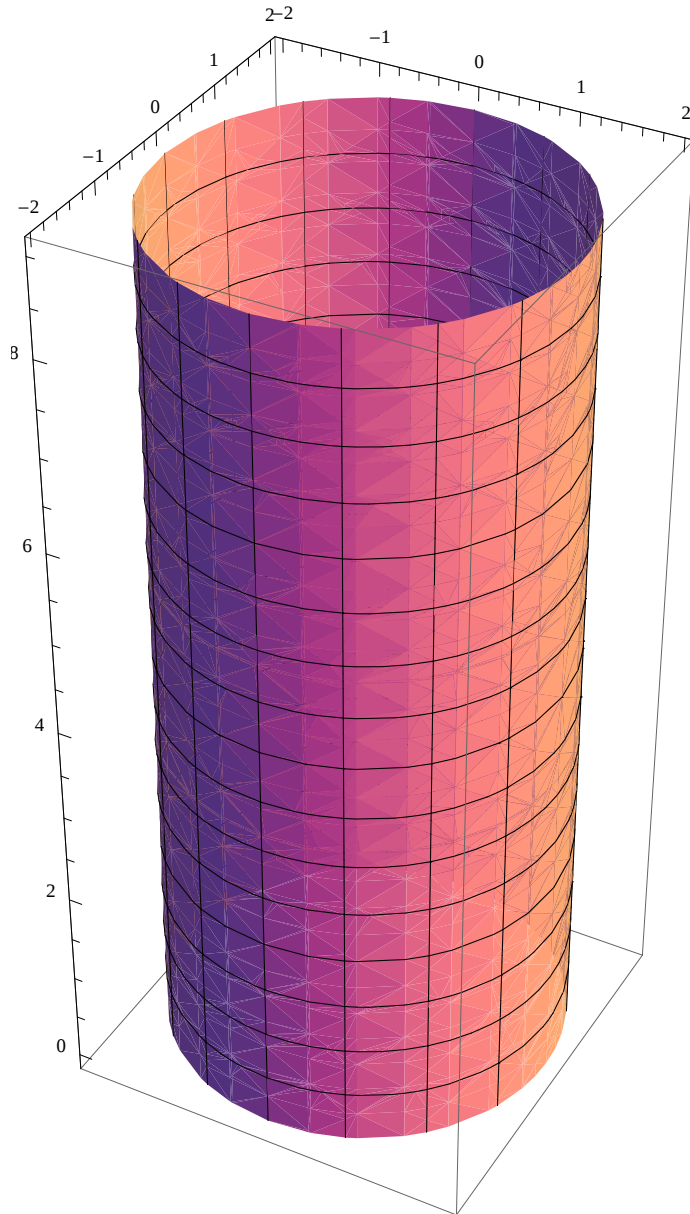
Primeri osnovnih geometrijskih oblik :

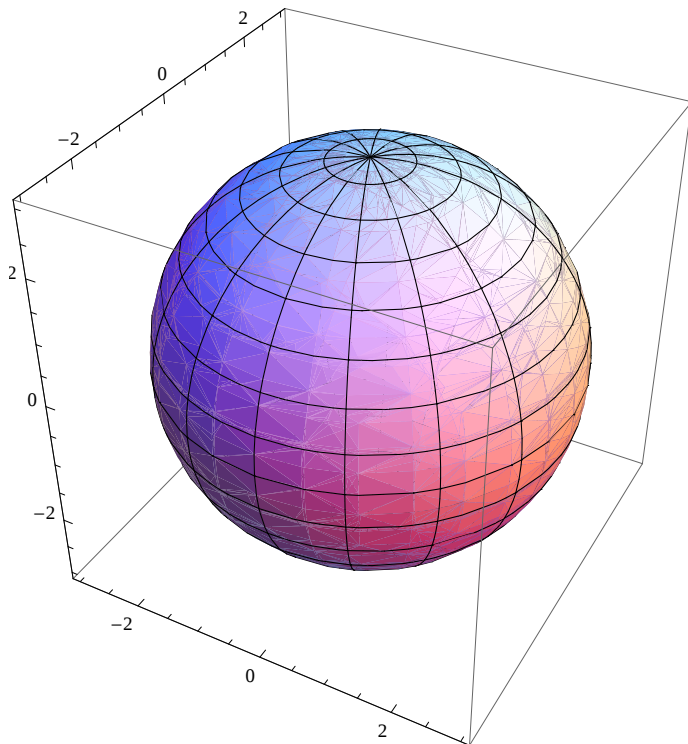
ravnina, paraboloid, valj, sfera

```
ravnina = Plot3D[4 x + 7 y, {x, -3, 3}, {y, -3, 3}]
paraboloid = Plot3D[x^2 + y^2, {x, -3, 3}, {y, -3, 3}]
paraboloid = ParametricPlot3D[{ρ Cos[φ], ρ Sin[φ], ρ^2}, {φ, 0, 2 π}, {ρ, 0, 3}]
valj = ParametricPlot3D[{2 Cos[φ], 2 Sin[φ], z}, {φ, 0, 2 π}, {z, 0, 9}]
sfera =
  ParametricPlot3D[{3 Cos[θ] Cos[φ], 3 Cos[θ] Sin[φ], 3 Sin[θ]}, {φ, 0, 2 π}, {θ, -π/2, π/2}]
```









Veliko ploskev se najbolj naravno parametrizira, če za parametra izberemo cilindrični koordinati (ρ, φ) , oziroma sferični (φ, θ) . Za risanje takih ploskev lahko uporabimo ukaza:

RevolutionPlot3D	vrtenina okrog z-osi	$z=f(\rho)$, $0 < \varphi < 2\pi$
SphericalPlot3D	središnja ploskev	$r=f(\varphi, \theta)$

? RevolutionPlot3D

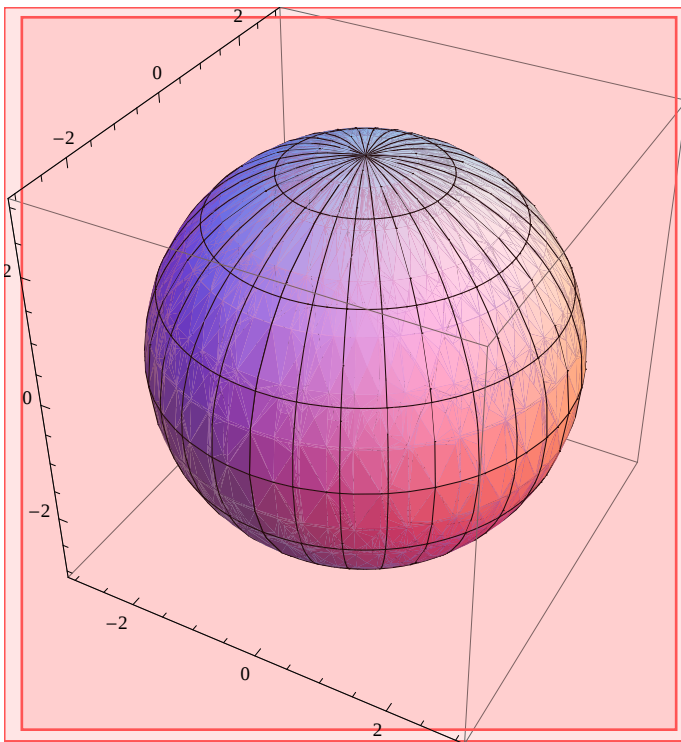
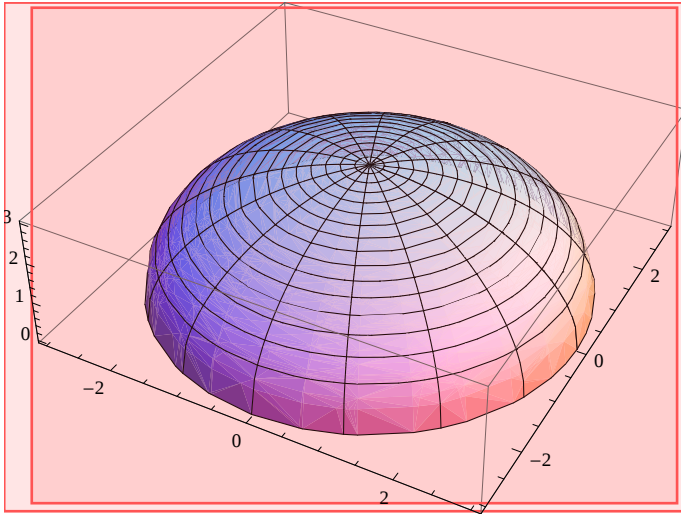
? SphericalPlot3D

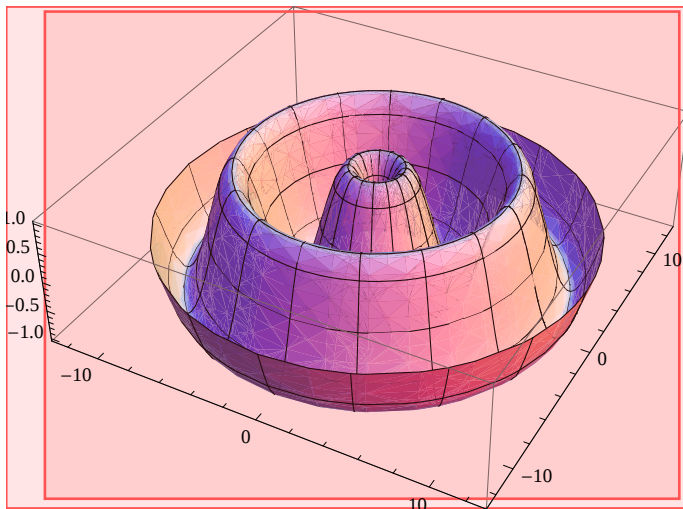
(* Primeri *)

```
polsfera = RevolutionPlot3D[Sqrt[9 - ρ^2], {ρ, 0, 3}]
```

```
sfera = SphericalPlot3D[3, {φ, 0, 2 π}, {θ, -π / 2, π / 2}]
```

```
RevolutionPlot3D[Sin[ρ], {ρ, 0, 4 π}]
```





1. a) Zapisi parametrično enačbo stožca $z = \sqrt{x^2 + y^2}$, $0 < z < 3$.

Parametra naj bosta polarni koordinati (ρ, φ) .

b) Narisi stožec z ukazom **ParametricPlot3D**.

c) Narisi naslednje koordinatne krivulje:

$$\rho = 1, 2, 3; \quad \varphi = 0, \frac{\pi}{4}, \frac{\pi}{2}.$$

Stožec in koordinatne krivulje prikazi v isti sliki !

```

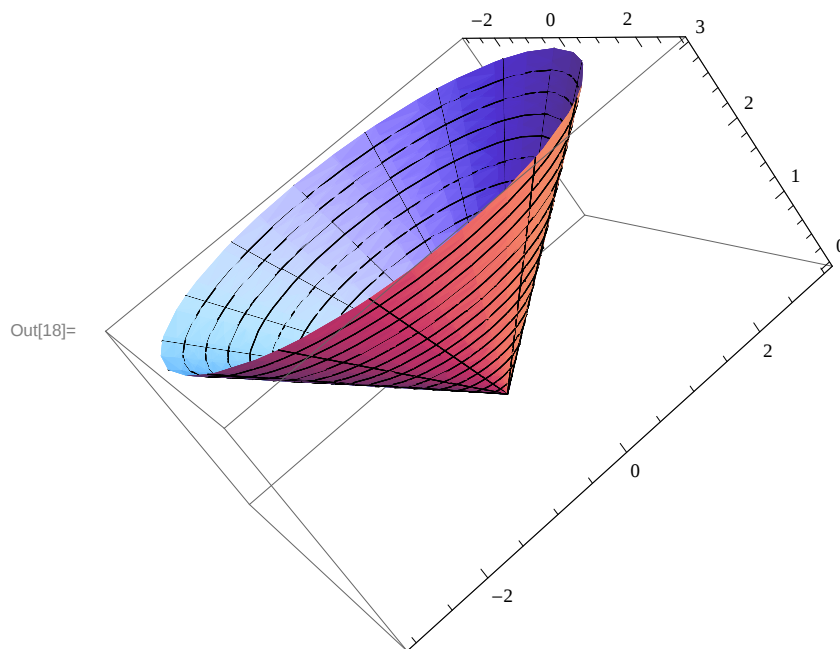
In[10]:= Clear[x, y, z, r]
          x = r Cos[fi]
          y = r Sin[fi]
          z = r
          stozec = ParametricPlot3D[{x, y, z}, {fi, 0, 2 π}, {r, 0, 3}];
          kr1 = ParametricPlot3D[{x, y, z}, {fi, 0, 2 π}, {r, 0, 3}] /. r -> 1;
          kr2 = ParametricPlot3D[{x, y, z}, {fi, 0, 2 π}, {r, 0, 3}] /. r -> 2;
          kr3 = ParametricPlot3D[{x, y, z}, {fi, 0, 2 π}, {r, 0, 3}] /. r -> 3;
          Show[stozec, kr1, kr2, kr3]

```

Out[11]= $r \cos[\text{fi}]$

Out[12]= $r \sin[\text{fi}]$

Out[13]= r



2. Hiperbolični paraboloid $z=xy$, $-2 < x < 2$, $-2 < y < 2$

ima obliko sedla.

a) Narisi ploskev z ukazom **Plot3D**.

b) Izboljšaj sliko z opcijama

BoxRatios-> in **ViewPoint**->

c) Narisi ploskev, spodaj in zgoraj naj bo bela,

v sredini (pri $z=0$) naj bo modra,

vmes naj se barva zvezno spreminja od bele do modre.

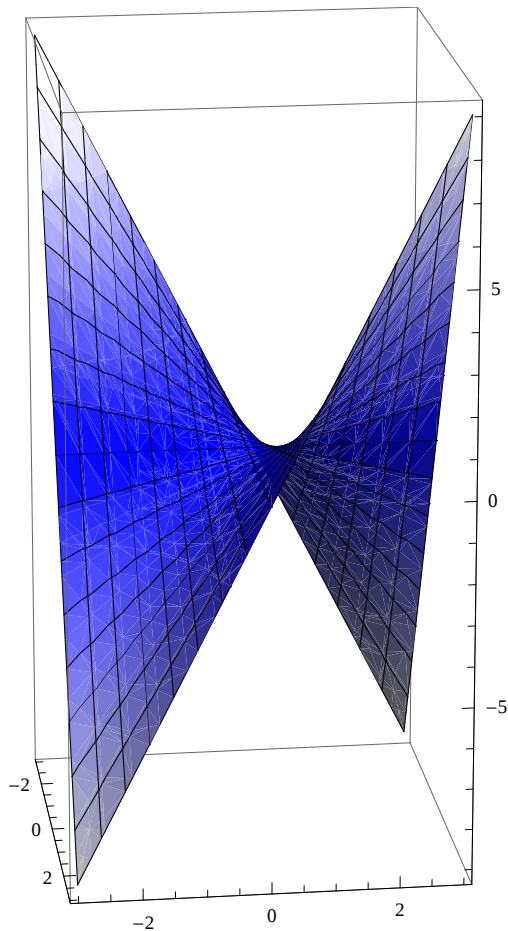
Za barvanje (in osvetlevanje, senčenje) ploskev uporabi opcijo

ColorFunction->

```

Clear[x, y, z, r]
sedlo = Plot3D[z = x y, {x, -3, 3}, {y, -3, 3},
  BoxRatios → {1, 1, 2}, ViewPoint → {2 Pi, -Pi / 4, 2},
  ColorFunction → Function[{x, y, z}, RGBColor[Abs[2 z - 1], Abs[2 z - 1], 1]]]

```



? BoxRatios
 ? ViewPoint
 ? ColorFunction

BoxRatios is an option for Graphics3D which gives the ratios of side lengths for the bounding box of the three-dimensional picture. >>

ViewPoint is an option for Graphics3D and related functions which gives the point in space from which three-dimensional objects are to be viewed. >>

ColorFunction is an option for graphics functions which specifies a function to apply to determine colors of elements. >>

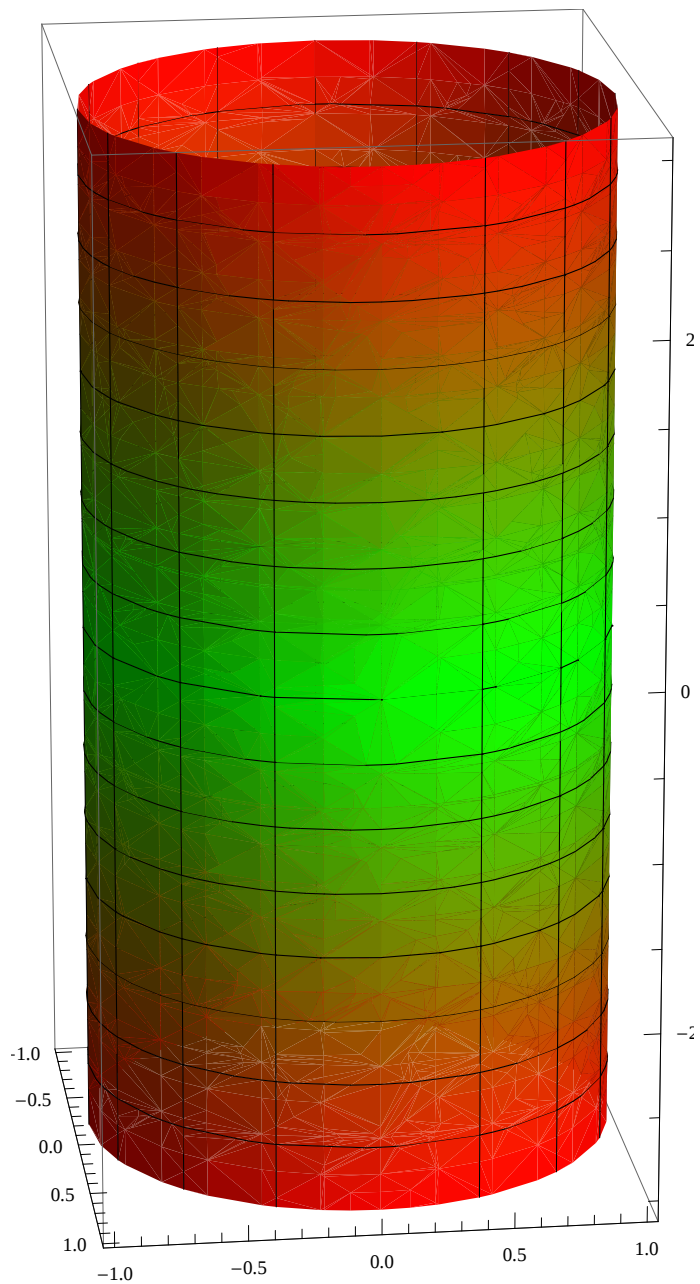
3. Dan je valj $x^2 + y^2 = 1$, $-5 < z < 5$.

Narisi valj z ukazom **ParametricPlot3D**. V sredini naj bo zelene barve, zgoraj in spodaj naj bo rde■, vmes naj se barva zvezno spreminja.

```

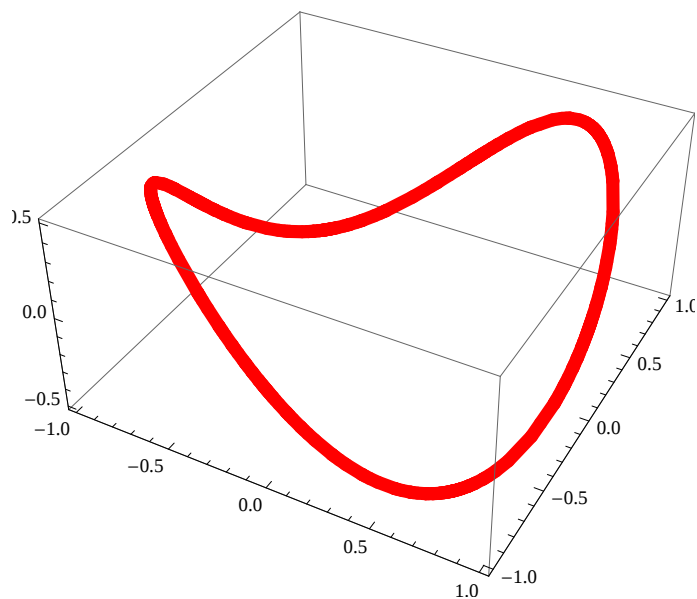
Clear[x, y, z, r, fi]
x = Cos[fi]
y = Sin[fi]
z = u
valj = ParametricPlot3D[{x, y, z}, {fi, 0, 2 Pi},
  {u, -3, 3}, BoxRatios -> {1, 1, 2}, ViewPoint -> {2 Pi, -Pi / 4, 2},
  ColorFunction -> Function[{x, y, z}, RGBColor[Abs[2 z - 1], 1 - Abs[2 z - 1], 0]]]
Cos[fi]
Sin[fi]
u

```



4. Z ukazom **ParametricPlot3D** narisi krivuljo, ki je prese█is█e ploskev iz nalog 2. in 3. Krivuljo poudari z debelino in rde█o barvo.

```
Clear[x, y, z, r, fi]
x = Cos[fi]
y = Sin[fi]
z = x y
presek = ParametricPlot3D[{x, y, z}, {fi, 0, 2 Pi}, PlotStyle -> {Red, Thickness[0.02]}]
Cos[fi]
Sin[fi]
Cos[fi] Sin[fi]
```

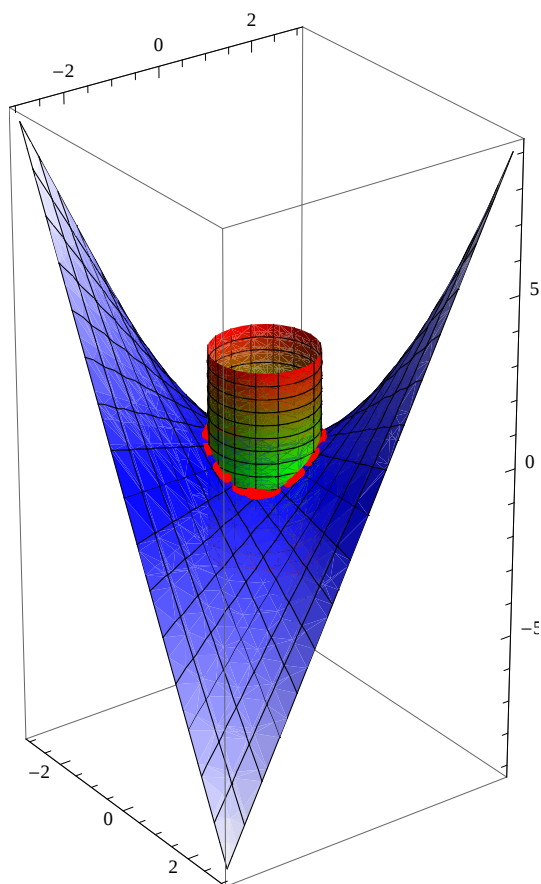


? Thickness

Thickness[r] is a graphics directive which specifies that lines which follow are to be drawn with thickness r . The thickness r is given as a fraction of the horizontal plot range. >>

5. Rezultat nalog 2.3. in 4. prikazi v isti sliki z ukazom **Show**.

Show[sedlo, valj, presek]



6. Izračunaj, pod kakšnim kotom se v točki z največjo koordinato z sekata ploskvi $x^2 + y^2 = 1$ in $z = xy$.

Rez.: $\frac{3\pi}{4}$

```

Clear[x, y, z, r, fi]
x = Cos[fi]
y = Sin[fi]
z = x y /. {{fi -> - 3 Pi / 4}, {fi -> - Pi / 4}, {fi -> Pi / 4}, {fi -> 3 Pi / 4}}
F = x^2 + y^2 - 1
Solve[D[z, fi] == 0, fi]
n1 =
Cos[fi]
Sin[fi]
{1/2, -1/2, 1/2, -1/2}
-1 + Cos[fi]^2 + Sin[fi]^2
{{}}

Clear[x, y, z, r, fi]
z = x y
x = Cos[fi]
y = Sin[fi]
F = x^2 + y^2 - 1
ns = {D[z, x], D[z, y], -1}
nv = {D[F, x], D[F, y], 0}
kot = VectorAngle[ns, nv] /. {fi -> Pi / 4}
x y
Cos[fi]
Sin[fi]
-1 + Cos[fi]^2 + Sin[fi]^2
{Sin[fi], Cos[fi], -1}
{2 Cos[fi], 2 Sin[fi], 0}
Pi / 4

```



Integrali s parametrom

7. Narisi graf funkcije $f(x) = \int_0^1 \text{sign}(x - y) \, dy$.

Najprej narisi graf funkcije **sign(x)**.

Ta enostavna funkcija je vgrajena v paket *Mathematica*.

```
Clear[x, y, z, r, fi]
```

```
Plot[Integrate[Sign[x - y], {y, 0, 1}], {x, -5, 5}]
```

