3.10.2012

SIGNALI IN SISTEMI

(3 - 1 - 2) — P A L

tonek 13 - 14

pon 8 - 12 (odvisno od skupine)

vaje začetek 15.10

zimska: 31.1, 5.2

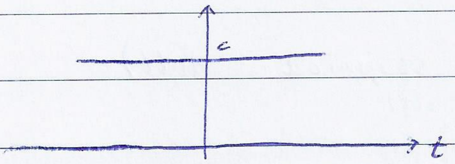
poletni: 17.6

Časovno zvezni osnovni signali

- konstanta
- eksponentni
- sinusni signal
- dušeni sinusni signal

$x(t) = C e^{at}$, $C > 0$

1) $C, a = 0 \rightarrow x(t) = C$

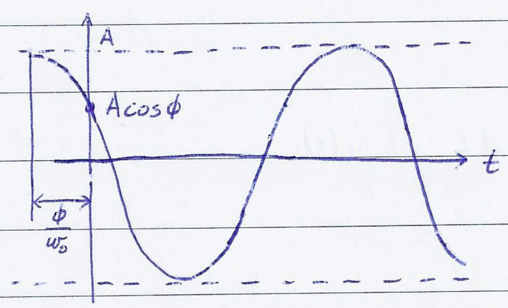


2) $C = A e^{j\phi} = A \angle \phi$, $a = j\omega_0$

$\rightarrow x(t) = A e^{j\phi} e^{j\omega_0 t} = A e^{j(\omega_0 t + \phi)}$

$= x(t) = \underbrace{A \cos(\omega_0 t + \phi)}_{\text{realni}} + j \underbrace{A \sin(\omega_0 t + \phi)}_{\text{imaginarni}}$

$x(t) = \mathrm{Re}\left[A e^{j(\omega_0 t + \phi)}\right]$

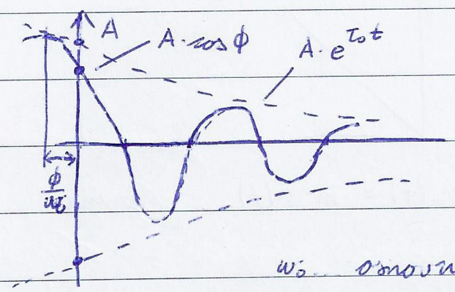


3) $C = A e^{j\phi}$, $a = \tau_0 + j\omega_0$

$\rightarrow x(t) = A e^{j\phi} e^{(\tau_0 + j\omega_0)}$

$\rightarrow = A e^{\tau_0 t} \cos(\omega_0 t + \phi) + j A e^{\tau_0 t} \sin(\omega_0 t + \phi)$

$\tau_0 < 0$

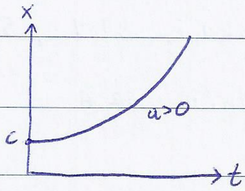


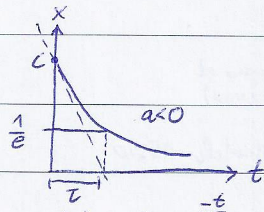
ω_0 ... osnovna frekvenca

τ_0 ... dušenje

1a) $a>0$, $a<0$

$\rightarrow x(t) = Ce^{at}$





	$\frac{x(t)}{C}$
τ	0,368
2τ	0,135
3τ	0,05
4τ	0,02
5τ	0,0067

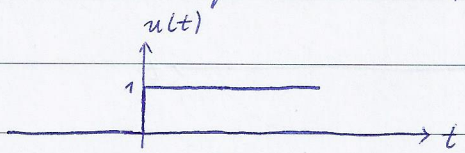
$x(t) = C e^{\frac{-t}{\tau}}$

$a = -\frac{1}{\tau}$ τ... časovna konstanta

Singularne funkcije

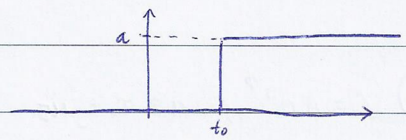
Nekateri aperiodični signali imajo preprosto mat obliko. Imajo <u>nezveznost</u> ali <u>nezvezne odvode</u> ali eno ali oboje.

1) enotina stopnica $u(t)$

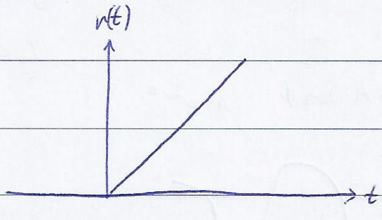


$x(t) = a\, u(t-t_0)$

zamik



2) enotina strmina $r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = t \cdot u(t)$

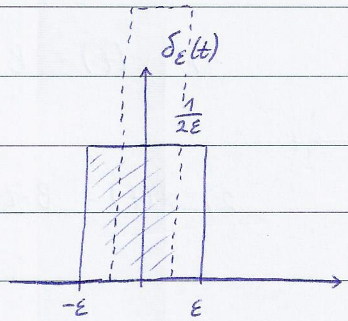


$r_m(t) = m \cdot r(t)$ ← strmina

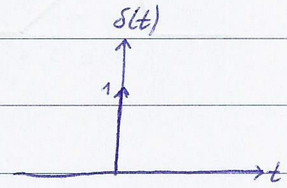
3) enotin impulz (delta f, Diracov impulz)

$$\delta(t) = \frac{du(t)}{dt}$$

ali $$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$



$\Rightarrow$



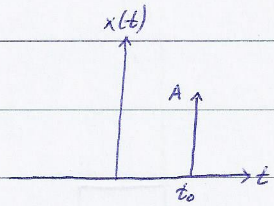
$\rightarrow \delta(t) = 0 \quad , t \neq 0$

$\rightarrow \int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$\varepsilon \rightarrow 0$

$\delta_\varepsilon(t) \rightarrow \delta(t)$

$x(t) = A\, \delta(t - t_0)$



uporaba sita:

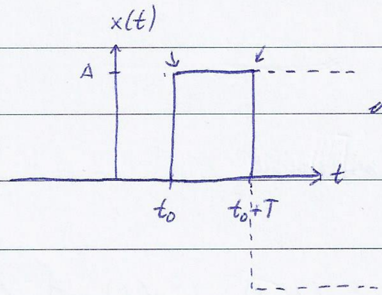
$$f(t_1) = \int_{-\infty}^{\infty} f(t)\, \delta(t - t_1)\, dt$$

vrednost f pri t_1

časovno invertiranje $u(t) \rightarrow u(-t)$

Primer

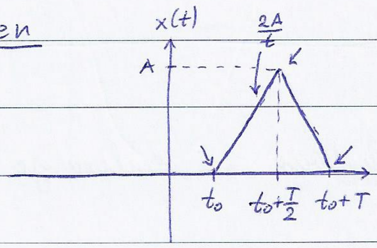
Pravokotni pulz štarta pri t_0, čas trajanja T in višina A.



dve stopnici

$$x(t) = A\, u(t - t_0) - A\, u(t - (t_0 + T))$$

Primer



$\cancel{x(t) = A \cdot r(t - t_0)}$

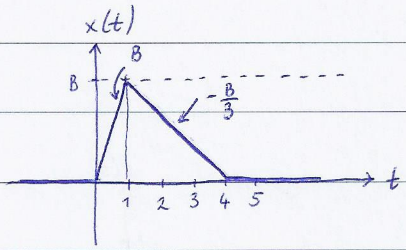
$$x(t) = \frac{2A}{T} r(t - t_0) - \frac{4A}{T} r\left(t - t_0 - \frac{T}{2}\right) + \frac{2A}{T} r(t - t_0 - T)$$

drugi način

$$x(t) = \frac{2A}{T}(t - t_0) \cdot \left[u(t - t_0) - u\left(t - t_0 - \frac{T}{2}\right)\right] - \frac{2A}{T}(t - t_0 - T) \cdot$$

$$\cdot \left[u\left(t - t_0 - \frac{T}{2}\right) - u(t - t_0 - T)\right]$$

Primer

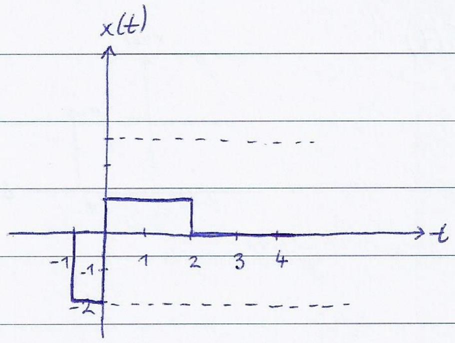


1) $x(t) = B\cdot r(t) - \frac{4}{3}B\cdot r(t-1) + \frac{B}{3}\, r(t-4)$

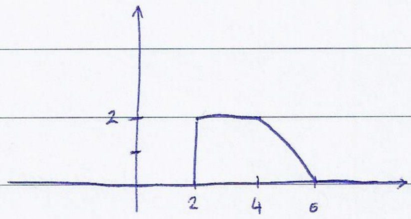
2) $x(t) = B\cdot t\,[u(t) - u(t-1)] - \frac{B}{3}\underset{t-4}{(t-4)}\,[u(t-1) - u(t-4)]$

Primer

$-2u(t+1) + 3u(t) - u(t-2)$



Primer

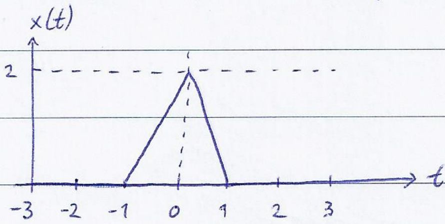


$x(t) = 2u(t-2) - t$

$ = 2u(t-2) - r(t-4) + r(t-6)$

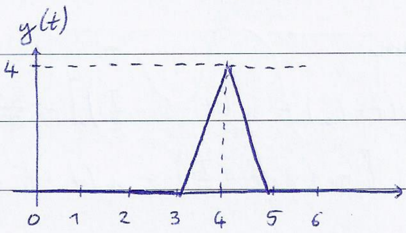
Primer

signal na sliki opišite s funkcijami strmine

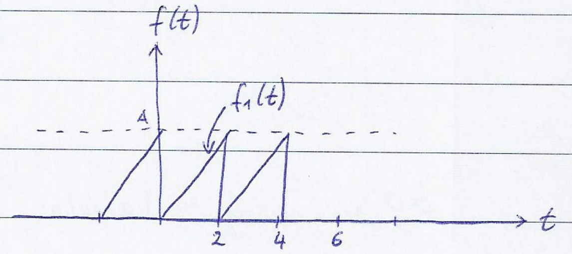


$x(t) = 2r(t+1) - 4r(t) + 2r(t-1)$

signal na sliki opišite s časovno premaknitvijo in skaliranjem



$y(t) = 2x(t-4)$



vklop izklop

$$f_1(t) = \frac{A}{2}\, t\, [u(t) - u(t-2)] \quad \text{prva perioda}$$

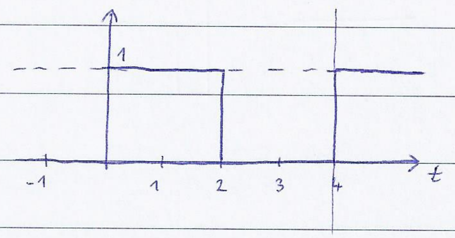
$$f_2(t) = \frac{A}{2}(t-2)[u(t-2) - u(t-2-2)]$$

2 za naš primer

$$\boxed{f_p(t) = \sum_{m=-\infty}^{\infty} f_1(t-mT)}$$

9.10.2012

Primer zapiši analitične ~~signale~~ izraze za signale

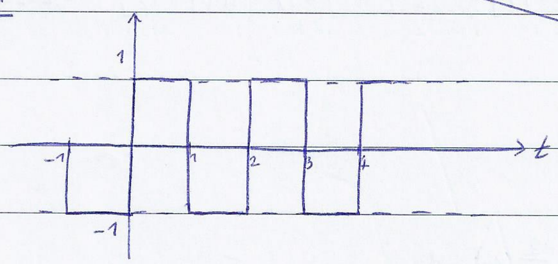


$T=4$

$$f_1(t) = u(t) - u(t-2) \quad \rightarrow \quad \sum [u(t-4m) - u(t-2-4m)]$$

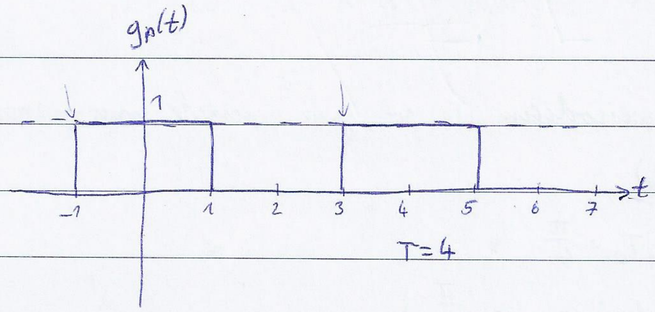
$$f_p(t) = \sum_{m=-\infty}^{\infty} f_1(t-4m)$$

Primer



$$f_p(t) = 2d_p(t) - 1$$

Primer

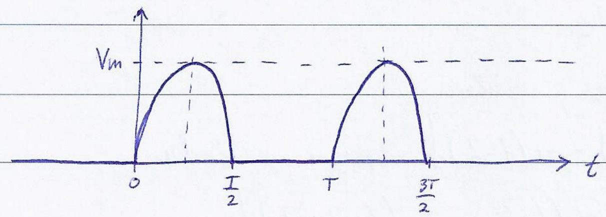


$$g_1(t) = u(t+1) - u(t-1)$$

$$g_p(t) = \sum_{m=-\infty}^{\infty} [u(t+1-4m) - u(t-1-4m)]$$

Primer

polvalni sinusni signal



$t=0$ in naprej → kavzalni signal

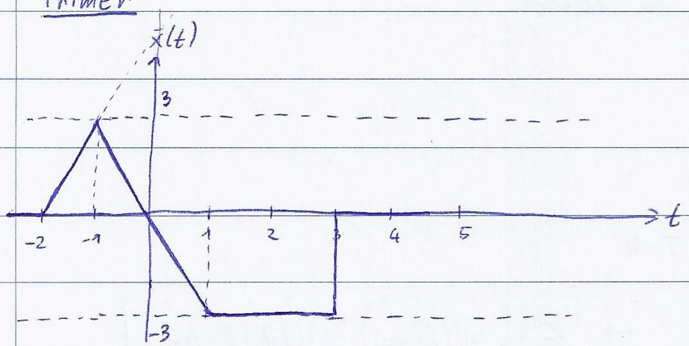
$T=T \qquad \omega = 2\pi f = \frac{2\pi}{T}$

$$v_1(t) = f_1(t) = V_m \sin(\omega t)\left[u(t) - u\left(t-\tfrac{T}{2}\right)\right]$$

$$v_p(t) = f_p(t) = v_1(t) + v_1(t-T) + v_1(t-2T)$$

$$\underline{v_p(t) = \sum_{k=0}^{\infty} v_1(t-kT)}$$

Primer



$$x(t) = 3\cdot r(t+2) - 6r(t+1) + 3r(t-1) + 3u(t-3)$$

$$x(t) = \underbrace{(3t+6)}_{3(t+2)}\left[u(t+2) - u(t+1)\right] - 3t\left[u(t+1) - u(t-1)\right] - 3\left[u(t-1) - u(t-3)\right]$$

Primer

določite periodo T

$$x(t) = 4\cdot\cos(\underbrace{5\pi}_{\omega} t)$$

$$T = \frac{2\pi}{\omega} = \frac{2}{5} \text{ sekunde}$$

Primer

ugotovite, ali so signali periodični, če so, jim poiščite osn. periodo

$$x(t) = \cos(\underbrace{4}_{\omega} t) + 2\sin(8t)$$

$$T_1 = \frac{\pi}{2} \qquad T_2 = \frac{\pi}{4}$$

(skupna perioda pomoje $\underline{\underline{\frac{\pi}{2}}}$)

$\underbrace{\frac{T_1}{T_2} = 2 = \frac{2}{1}}$ kvocient period

$\downarrow$

signal periodičen (če je $\frac{T_1}{T_2}$ racionalno število, je signal periodičen)

$T_1 : \frac{\pi}{2} \quad \frac{2\pi}{2} \quad \frac{3\pi}{2} \quad \frac{4\pi}{2} \quad \dots$

$T_2 : \frac{\pi}{4} \quad \frac{2\pi}{4} \quad \frac{3\pi}{4} \quad \frac{4\pi}{4} \quad \dots$

najmanjši skupni večkratnik ! (LCM)

osnovna perioda $\frac{\pi}{2}$

__Primer__ $\quad x(t) = 3 \cdot \cos(4t) + \sin(\pi t) \qquad \left(T = \frac{2\pi}{\omega}\right)$

$T_1 = \frac{\pi}{2} \qquad T_2 = 2$

$\frac{T_1}{T_2} = \frac{\pi}{4} \rightarrow$ signal ni periodičen !

__Primer__ $\quad x(t) = \cos(3\pi t) + 2\cos(4\pi t)$

$T_1 = \frac{2}{3} \qquad T_2 = \frac{1}{2}$

je periodičen !

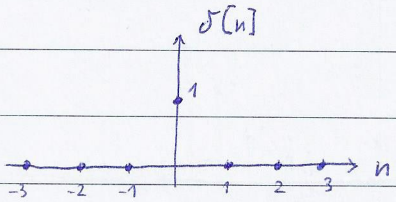
$T_1 : \quad \frac{2}{3} \quad \frac{4}{3} \quad \frac{6}{3} \qquad T = 2$

T_2

__Primer__

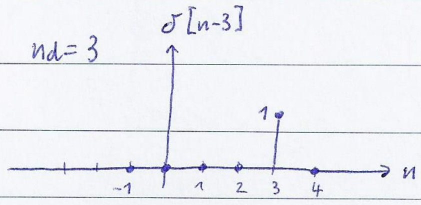
__Osnovni diskretni signali__

1) časovno diskretni enotin impulz $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$



$\delta[n - n_d] = \begin{cases} 1 & n = n_d \\ 0 & n \neq n_d \end{cases}$

$n_d = 3$



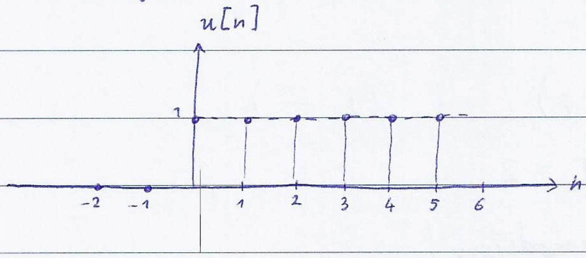
Akta®

Poljubno zaporedje

Poljubno zaporedje lahko zapišemo kot vsoto enotnih vzorcev

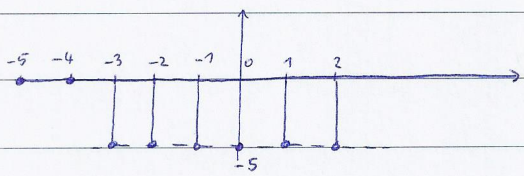
$$f(n) = \sum_{m=-\infty}^{\infty} f(m)\, \delta(n-m)$$

10.10.2012 2) zaporedje enotine stopnice $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

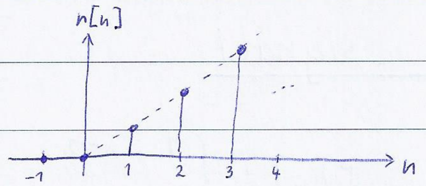


$$B\, u[n-n_0] = \begin{cases} B & n \geq n_0 \\ 0 & n < n_0 \end{cases}$$

<u>Primer</u> $x[n] = -5\, u[n+3]$

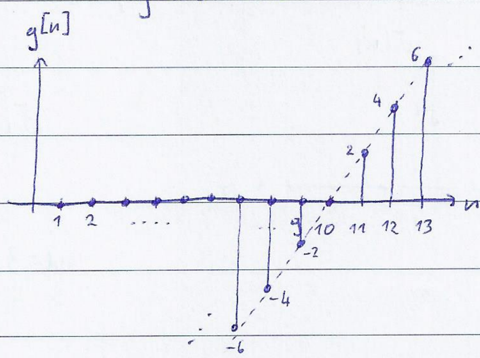


3) enotina strmina $r[n] = n\, u[n]$



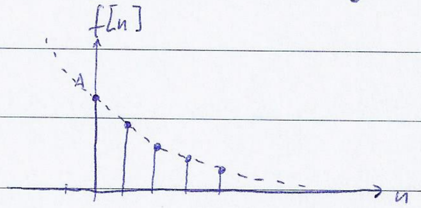
$g[n] = B(n-n_0)$

$g[n] = 2[n-10]$ ← navadna premica !



4) realni eksponentni signal $f[n] = A \cdot a^n$

ponavadi negativen



$p[n] = A \cdot a^n \cdot u[n]$

$p[n] = 0$ za $n < 0$

5) sinusno zaporedje $f[n] = A \cdot \cos\left(\frac{2\pi n}{N} + \alpha\right)$

osnovna perioda

$\Updownarrow$

$\left(f(t) = A \cdot \cos(\omega t + \alpha)\right)$

$t = n T_{vz}$ čas vzorčenja

$\omega = \frac{2\pi}{T}$, $T = N \cdot T_{vz}$

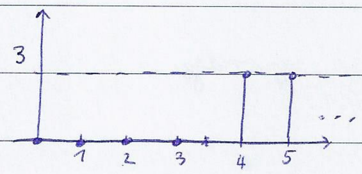
perioda

6) eksponentno modulirano sinusno zaporedje

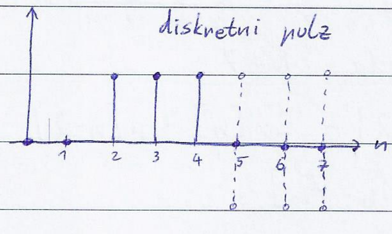
$g[n] = A \cdot a^n \cos\left(\frac{2\pi n}{N} + \alpha\right)$ ← v radianih

Primer skicirajte zaporedja po opisu

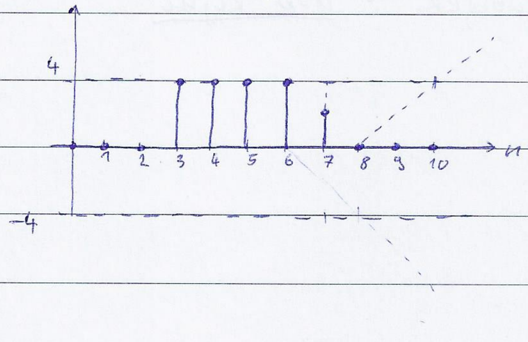
$x[n] = 3\, u[n-4]$



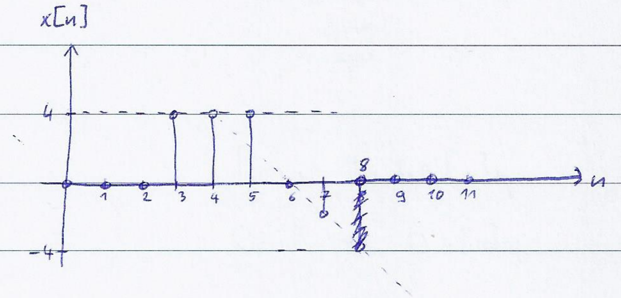
$x[n] = u[n-2] - u[n-5]$



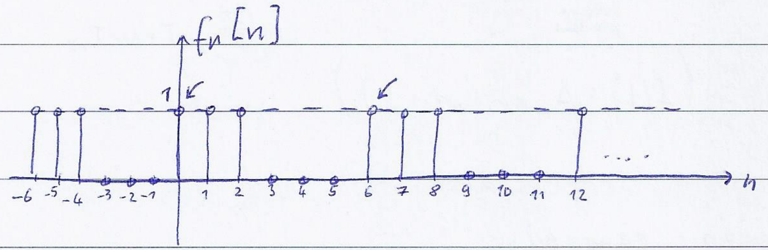
$4\, u[n-3] - 2\underbrace{(n-6)\, u[n-6]}_{r[n-6]} + 2\underbrace{(n-8)\, u[n-8]}_{r[n-8]}$



$x[n] = 4[u[n-3] - u[n-6]] - 2(n-6)[u[n-6] - u[n-8]]$



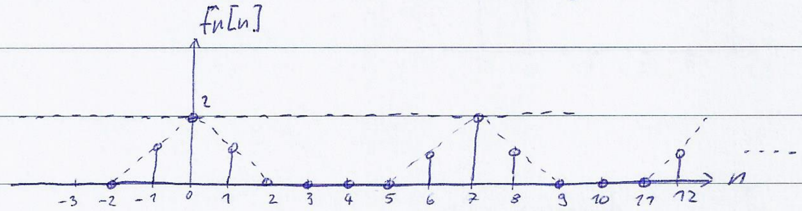
7) periodično pulzno zaporedje



$f_1[n] = u[n] - u[n-3]$ $\qquad$ $f_n[n] = \sum_{m=-\infty}^{+\infty} f_1[n - mN]$

$f_n[n] = \sum_{m=-\infty}^{\infty} f_1[n-6m]$

Primer zapišite izraz za zaporedje



perioda $N = 7$

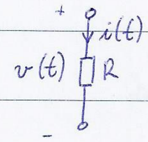
$f_1[n] = r[n+2] - 2r[n] + r[n-2]$

$f_n[n] = \sum_{m=-\infty}^{\infty} f_1[n-7m]$

Lab vaje → ponedeljek → usb ključ

Elementi in sistemi

1)

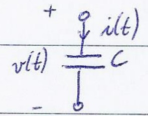


$G = \frac{1}{R}$

$v(t) = R\, i(t)$

$i(t) = G\, v(t)$

2)

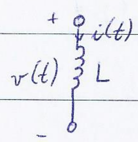


naboj
↓
$q(t) = C \cdot v(t)$ začetna napetost $v(0)$ ali $v(t_0)$

$$i(t) = \frac{dq(t)}{dt} = C\,\frac{dv(t)}{dt}\,,\quad v(t_0)$$

$$v(t) = \frac{1}{C}\int_{t_0\,(0)}^{t} i(\tau)\,d\tau + v(t_0)$$

3)



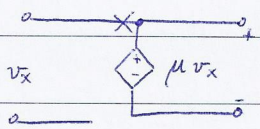
$\phi(t) = L \cdot i(t)$ začetni tok $i(0)$ ali $i(t_0)$

$$v(t) = \frac{d\phi(t)}{dt} = L\,\frac{di(t)}{dt}\,,\quad i(t_0)$$

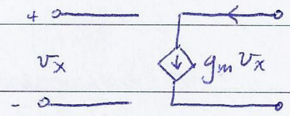
$$i(t) = \frac{1}{L}\int_{t_0\,(0)}^{t} v(\tau)\,d\tau + i(t_0)$$

odvisni viri:

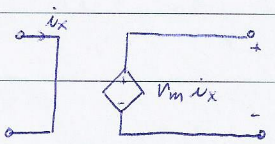
1) NNV



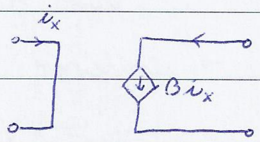
2) NTV



3) TNV

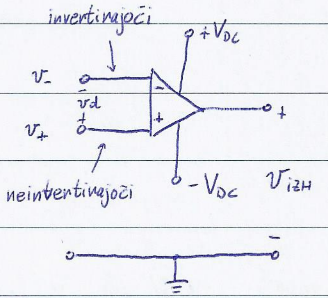


4) TTV



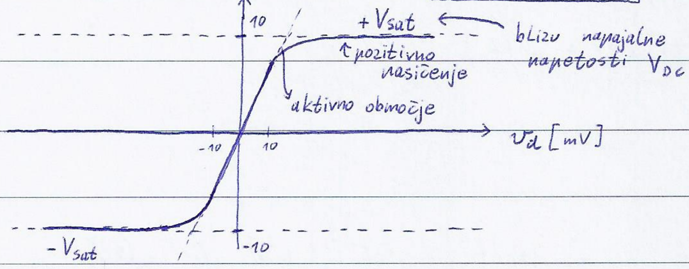
operacijski ojačevalnik (OPAMP)

integrirano vezje iz tranzistorjev in uporov



zanima nas v_{izH} v odvisnosti od vhodnih napetostih pri odprtih sponkah

$$\boxed{\begin{aligned} v_d &= v_+ - v_- \\ v_{izH} &= f(v_d) \end{aligned}}$$



A... strmina $(\sim 10^5 - 10^6)$

Osnovna analiza OPAMP temelji na aproksimaciji z linearnimi segmenti. Segment skozi izhodišče imenujemo aktivno območje. Strmina (A) se imenuje ojačanje odprte zanke (brez povratne vezave).

$|V_{sat}| = 15\ V$

$A = 10^5$

$v_d = \pm 0{,}15\ mV$

→ v linearnem območju:

$$v_{izH} = A \cdot v_d\ , \quad |v_{izH}| < V_{sat} \quad \text{ali} \quad |v_d| < \frac{V_{sat}}{A}$$

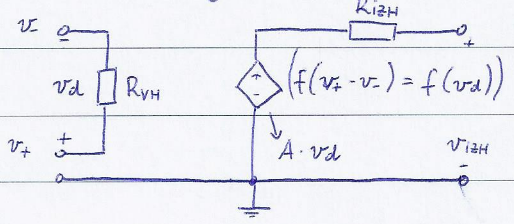
→ pozitivno nasičenje

$$v_{izH} = +V_{sat}\ , \text{ za } v_d > \frac{V_{sat}}{A}$$

→ negativno nasičenje

$$v_{izH} = -V_{sat}\ , \text{ za } v_d < -\frac{V_{sat}}{A}$$

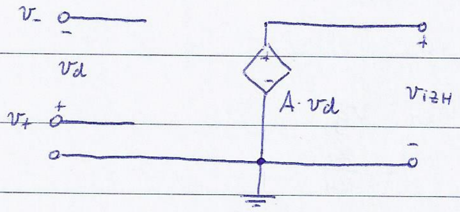
Model za operacijski ojačevalnik:



$R_{VH} \to \infty$ (za $R_G \ll R_{VH}$)

$R_{izH} \to 0$ (za $R_B \gg R_{izH}$)

$(A \to \infty)$ idealni opamp



OPAMP izvaja številne operacije

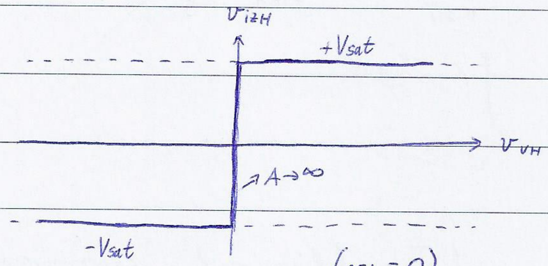
Analiza osnovnih vezij OPAMPa pri virtualnem kratkem stiku

predpostavimo:

$R_{VH} \to \infty$

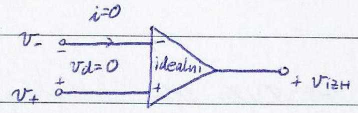
$R_{izH} \to 0$

$A \to \infty$



($v_d = 0$)

na vhodu virtualen kratek stik



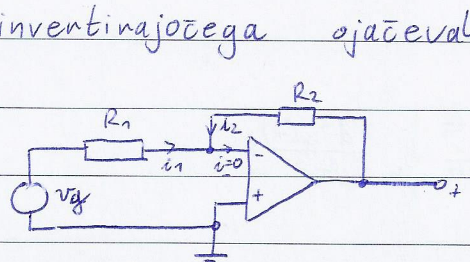
$v_d = v_+ - v_- = 0$ (1. zlato pravilo)

$i_- = 0,\ i_+ = 0$ (2. zlato pravilo)

$v_d = 0$ dejansko ne pomeni, da sta vhodna priključka kratko sklenjena, le stanje na priključkih je takšno, kot da bi imeli na vhodu kratek stik → virtualen kratek stik

Če je eden od vhodnih priključkov ozemljen, pravimo da deluje OPAMP v načinu z virtualno maso

Primer invertirajočega ojačevalnika



$i_1 + i_2 = 0$

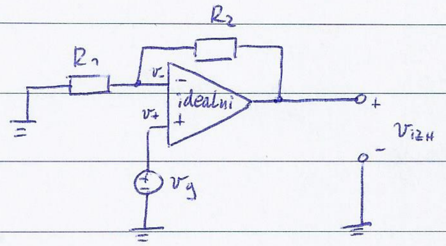
$i_1 = \frac{v_g}{R_1}$

$i_2 = \frac{v_{izH}}{R_2}$

$\frac{v_g}{R_1} + \frac{v_{izH}}{R_2} = 0$

$$\boxed{A_u = \frac{v_{izH}}{v_{VH}} = -\frac{R_2}{R_1}}$$

16.9.2012



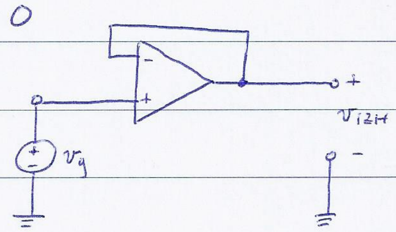
$$v_d = v_+ - v_- = 0$$

$$v_- = \frac{R_1}{R_1+R_2} v_{izh} = v_g$$

$$A_u = \frac{v_{izh}}{v_g} = 1 + \frac{R_2}{R_1}$$

$R_1 \to \infty$

$R_2 = 0$

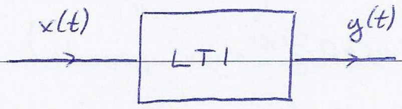


napetostni sledilnik (buffer)

$v_{izh} = v_g$

Analiza LTI (linearnih časovno ne-spremenljivih) sistemov v t prostoru

Diferencialne enačbe so osnova za analizo sistemov



$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_1 \frac{dy(t)}{dt} + a_0 y(t) =$$

$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^m} + \dots$$

pri fizikalno realnih sistemih so a_n konstante (pri pasivnih še pozitivni)

n... red sistema

ponavadi $n \geq m$

$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{m} b_k \frac{d^k y(t)}{dt^k}$$

še druga oblika $\frac{d}{dt} = p \qquad \frac{d^n}{dt^n} = p^n$

$$\underbrace{(a_n p^n + a_{n-1} p^{n-1} + \ldots + a_1 p + a_0)}_{Q(p)} \cdot y(t) = \underbrace{(b_m p^m + b_{m-1} p^{m-1} + \ldots + b_1 p + b_0)}_{P(p)} x(t)$$

$$\Downarrow$$

$$Q(p)\, y(t) = P(p)\, x(t)$$

opomba:

$$p\, y(t) \neq y(t)\, p$$

$$y(t) = \frac{P(p)}{Q(p)} x(t) = H(p)\, x(t)$$

$\downarrow$

sistemski operator (prenosna fja)

Odziv LTI sistemov na začetno stanje

$x(t) \equiv 0$

$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = 0$$

reševanje s pomočjo nastavka $\quad y(t) = Ce^{st}$

$\Downarrow$

$$\underbrace{(a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0)}_{Q(s)=0} \underbrace{Ce^{st}}_{\text{ne more biti } 0} = 0$$

karakteristična enačba

$\downarrow$

$s_1, s_2, \ldots, s_n$ karakteristični koreni $\rightarrow \sum_{k=0}^{n} a_k s^k = a_n (s-s_1)(s-s_2)\ldots(s-s_n) = 0$ v drugi obliki

začetni pogoji

$$\sum_{k=0}^{n} a_k \frac{d^k y(t)}{dt^k} = 0$$

$y(0), y'(0), \ldots, y^{(n-1)}(0) \qquad y(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} + \ldots + C_n e^{s_n t} = \sum_{k=1}^{n} C_k e^{s_k t}$

Če je lakšen koren $(s-s_k)^q$ večkraten:

$$y(t) = \dots + (C_1 + C_2 t + \dots + C_q t^{q-1}) e^{s_k t} + \dots$$

množenje s t!

Iskanje odziva sistema prvega reda

$$a_1 \frac{dy}{dt} + a_0 y = 0 \quad , \quad y(0)$$

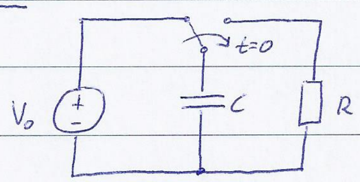
karakter. enačba:

$$a_1 s + a_0 = 0$$

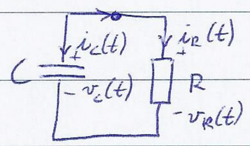
$$s = -\frac{a_0}{a_1} \qquad y(t) = C e^{-\frac{a_0}{a_1}t}$$ za več potrebujemo začetni pogoj

$$\boxed{y(t) = y(0)\, e^{-\frac{a_0}{a_1}t}}$$

Primer



$v_C(0) = V_0$ (takoj ob preklopu)



kirchoff:

$$v_C(t) - v_R(t) = 0 \qquad R: \; v_R = R\, i_R \; , \; i_R = \frac{v_R}{R}$$

$$i_C(t) + i_R(t) = 0 \qquad C: \; i_C = C\frac{dv_C}{dt} \; , \; v_C(t) = v_0(t) + \frac{1}{C}\int_0^t i_C(\tau)d\tau$$

$$C\frac{dv_C}{dt} + \frac{v_R}{R} = 0 \quad , \quad t \geq 0 \; , \; v_C(0) = V_0$$

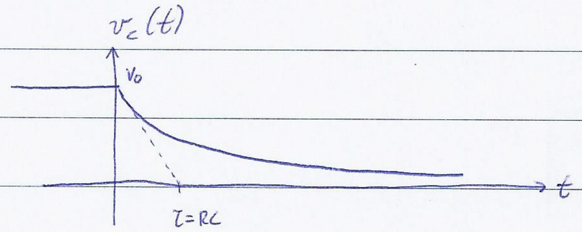
$$\frac{dv_C}{dt} + \frac{v_C}{RC} = 0 \qquad v_C(t) = Ae^{st}$$

$$s + \frac{1}{RC} = 0 \qquad v_C(0) = A = V_0$$

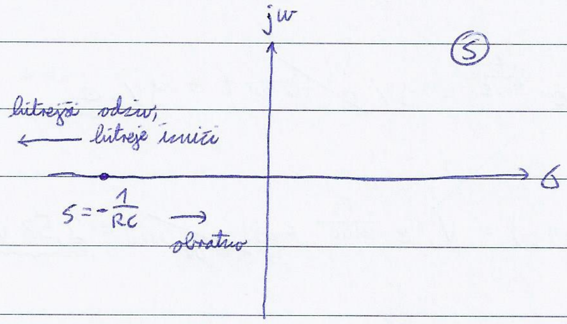
$$s = -\frac{1}{RC} \qquad \underline{\underline{v_C(t) = V_0 e^{-\frac{t}{RC}}}}$$

$$v_R(t) = v_C(t)$$

$$i_C = C\frac{dv_C}{dt} = -\frac{V_0}{R} e^{-\frac{t}{RC}}$$



S-ravnina

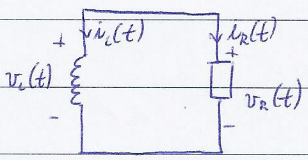


sistem stabilen, če odziv slabi, ko $t \to \infty$ $(v_c(t) \to 0)$

lega na levi polovici v S-ravnini

vsi koreni negativne realne korene $Re[s] < 0$

Primer 17.10.2012

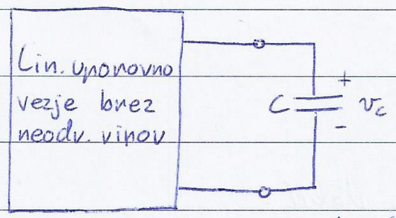


$s = -\frac{R}{L}$

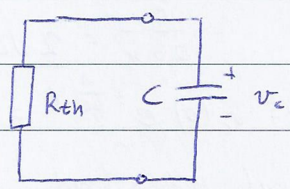
$i_L(t) = I_0 \, e^{-\frac{R}{L} t}$

$i_L(0) = I_0$

Thevenin



$\Rightarrow$



pomik / delay

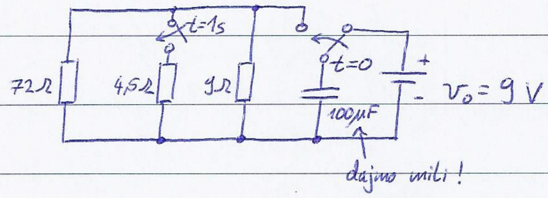
$v_c(t) = v_c(t_0) \cdot e^{-\frac{1}{R_{th}C}(t - t_0)}$

R_{th}... Theveninova nadomestna upornost

podobno za tuljavo

$v_L(t) = v_L(t_0) \cdot e^{-\frac{R_{th}}{L}(t - t_0)}$

Primer



1) $0s \le t_1 \le 1s$ $\qquad v_c(t_0) = 9V$

$R_{th} = 72\Omega \parallel 9\Omega = \underline{8\Omega}$

$$v_c(t) = V_0\, e^{-\frac{t}{R_{th}C}} = 9V\, \cancel{e^{-\frac{1}{125\cdot 10^4}}}\, t = 9V \cdot e^{-1{,}25t}$$

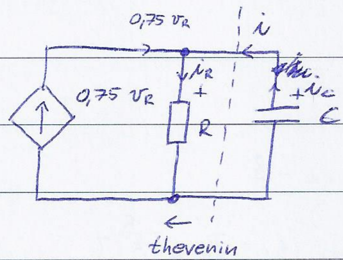
2) $t > 1s$

$R_{th} = 8\Omega \parallel 4{,}5\Omega = \underline{2{,}88\Omega}$ $\qquad v_c(t{=}1s) = V_0\, e^{-\frac{1s}{R_{th}C}} = \cancel{\text{skoraj nič}} = 2{,}58\,V$

skupaj potem

↳ $\underline{v_c(t) = 9\, e^{-1{,}25t}\,[u(t) - u(t-1)] + 2{,}58\, e^{-\frac{t-1}{0{,}288}}\,[u(t-1)] \quad V}$

Primer



$v_c(t) = ?$

$v_c(0) = 10V$

$R = 4\Omega$

$C = 250\,mF$

$R_{TH} = \frac{v}{i}$

$\cancel{v_{th}} =$

$\cancel{i_{th}} =$

$$i_c = 0{,}75\, v_R - i_R = 0{,}75\, v_R - \frac{v_R}{R} = v_R(0{,}75 - 0{,}25) = \underline{0{,}5\, v_R}$$

$$= \frac{v}{0{,}5\, v} = 2\Omega$$

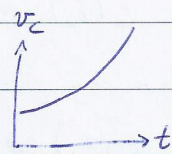
$$v_c(t) = v_c(0)\, e^{-\frac{1}{RC}t} = 10\, e$$

Če bi tok definiral noter (kot porabnik), bi dobil:

$$i_c = i_R - 0{,}75\, v_R = -0{,}5\, v_R$$

$$R_{th} = \frac{v_R}{i_c} = \frac{v_R}{-0{,}5\, v_R} = \underline{-2\Omega}$$

$$v_c(t) = v_c(0)\, e^{-\frac{1}{RC}t} = \underline{10\, e^{1t}\ V}$$



Odziv na začetno stanje sistema drugega reda

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = 0 \quad , \quad y(0), \; y'(0)$$

nastavek $y(t) = Ce^{st}$

karakteristična enačba $a_2 s^2 + a_1 s + a_0 = 0$

$$s^2 + \frac{a_1}{a_2} s + \frac{a_0}{a_2} = s^2 + 2\sigma s + \omega_n^2 = s^2 + 2\xi \omega_n s + \omega_n^2 + s^2 + \underset{\uparrow ?}{\frac{\omega_n}{\sigma}} s + \omega_n^2 =$$

$$= (s - s_1)(s - s_2) = 0$$

vsi možni zapisi (^^)

$\sigma = \xi \omega_n$ ... dušilni koeficient sistema

ξ ... stopnja dušenja

ω_n ... lastna frekvenca nedušenega nihanja

koreni: $s_{1,2} = -\frac{a_1}{2a_2} \pm \sqrt{\left(\frac{a_1}{2a_2}\right)^2 - \frac{a_0}{a_2}} = -\sigma \pm \sqrt{\sigma^2 - \omega_n^2} =$

$$= \omega_n \left(-\xi \pm \sqrt{\xi^2 - 1}\right) = \frac{\omega_n}{2Q}\left(1 \pm \sqrt{1 - 4Q^2}\right)$$

1) $\sigma > \omega_n$ oz. $\xi > 1$ oz. $0 < Q < 0{,}5$

korena sta realna

$$y(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$y(0) = C_1 + C_2$$

$$y'(0) = s_1 C_1 + s_2 C_2 \quad \Rightarrow \quad \begin{bmatrix} y(0) \\ y'(0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \frac{1}{\underset{\uparrow\, s_2 - s_1}{D}} \underbrace{\begin{bmatrix} s_2 & -1 \\ -s_1 & 1 \end{bmatrix}}_{\text{inverz}} \begin{bmatrix} y(0) \\ y'(0) \end{bmatrix}$$

vstavimo ...

$$C_1 = \frac{y'(0) - s_2 y(0)}{s_1 - s_2} \quad , \quad C_2 = -\frac{y'(0) - s_1 y(0)}{s_1 - s_2}$$

$$s_{1,2} = -\sigma \pm \underbrace{\sqrt{\sigma^2 - \omega_n^2}}_{\sigma_D} = -\sigma \pm \sigma_D$$

$$y(t) = \frac{y'(0) - (-\sigma-\sigma_D)\, y(0)}{2\sigma_D} e^{(-\sigma-\sigma_D)t} - \frac{y'(0) - (-\sigma+\sigma_D)\, y(0)}{2\sigma_D} e^{\overset{?}{(-\sigma+\sigma_D)t}} =$$

$$= e^{-\sigma t}\left(y(0)\cosh(\sigma_D t) + y'(0) + \frac{\sigma(y(0))}{\sigma_D}\sinh(\sigma_D t)\right)$$

Podkritično dušenje

2) $0 \leq \sigma \leq \omega_n$ oz. $0 \leq \xi \leq 1$ ali $Q = 0{,}5$

$$s_{1,2} = -\frac{a_1}{2a_2} \pm j\sqrt{\frac{a_0}{a_2} - \left(\frac{a_1}{2a_2}\right)^2} = -\sigma \pm j\sqrt{\omega_n^2 - \sigma^2} = \omega_n\left(-\xi \pm j\sqrt{1-\xi^2}\right) =$$

$$= \frac{\omega_n}{2Q}\left(-1 \pm j\sqrt{4Q^2-1}\right)$$

$\omega_0 = \sqrt{\omega_n^2 - \sigma^2}$ ... Lastna frekvenca dušenega nihanja

$$\omega_D = \omega_n\sqrt{1-\xi^2} = \omega_n\sqrt{1-\frac{1}{4Q^2}}$$

$$y(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

s_1 in s_2 sta konj. kompleksna, tudi $e^{s_1 t}$ in $e^{s_2 t}$ sta, ker je izhodni signal realen, morata biti C_1 in C_2 konj. kompleksna

$$C_1 = C_2^*$$

$$y(t) = C_1 e^{(-\sigma + j\omega_0)t} + C_2 e^{(-\sigma - j\omega_0)t} = C_1 e^{-\sigma t}\left(\cos(\omega_0 t) + j\sin(\omega_0 t)\right) +$$

$$+ C_2 e^{-\sigma t}\left(\cos(\omega_0 t) - j\sin(\omega_0 t)\right) = e^{-\sigma t}\left(A\cos(\omega_0 t) + B\sin(\omega_0 t)\right)$$

$$\boxed{A = C_1 + C_2 \qquad B = j(C_1 - C_2)}$$

$$y(t) = Ce^{-\sigma t}\cos(\omega_0 t + \Phi)$$

$$C = \sqrt{A^2 + B^2}$$

$$\Phi = -\arctan\left(\frac{B}{A}\right)$$

začetni pogoji

$$y(0) = A$$

$$y'(0) = -\sigma A + \omega_0 B$$

$$B = \frac{y'(0) + \sigma A}{\omega_0}$$

rešitev potem:

$$y(t) = e^{-\sigma t}\left[y(0)\cos(\omega_0 t) + \frac{y'(0) + \sigma y(0)}{\omega_0}\sin(\omega_0 t)\right]$$

3) $\omega_0 = 0$ <u>Kritično dušenje</u>

$$\frac{a_0}{a_2} = \left(\frac{a_1}{2a_2}\right)^2 \quad \text{oz.} \quad \sigma = \omega_n \quad \text{oz.} \quad \xi = 1 \quad \text{oz} \quad Q = \frac{1}{2}$$

$$s_{1,2} = \underset{?}{s} = -\frac{a_1}{2a_2} = -\sigma = -\omega_n \xi = -\frac{\omega_n}{2Q}$$

$$y(t) = (C_1 + C_2 t)\, e^{st} \qquad C_1 = y(0)$$
$$C_2 = y'(0) - s\,y(0)$$

$$y(t) = \left[y(0) + \left(y'(0) - s\,y(0)\right) t\right] e^{st} =$$

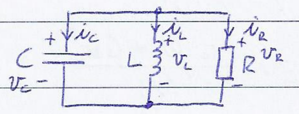
$$= \left[y(0) + \left(y'(0) + \sigma y(0)\right) t\right] e^{-\sigma t}$$

$\xi = 1$ je meja kritičnosti

<u>Lega v ravnini</u>

na listu ...

<u>Primer</u> določi odziv RLC vezja



$v_C(0) = V_0$, $i_L(0) = I_0$

$v_C = v_L = v_R$ | $v_R = i_R R$ $\quad v_C = v(0) + \int_0^t \frac{i_C(\tau)}{C}\, d\tau$ $\quad v_L = L\frac{di_L}{dt}$, $i_L(0)$

$i_C + i_L + i_R = 0$ | $i_R = \frac{v_R}{R}$ $\quad i_C = C\frac{dv_C}{dt}$, $v_C(0)$ $\quad i_L = i_L(0) + \int_0^t \frac{v_L(\tau)}{L}\, d\tau$

↓

$$C\frac{dv_C}{dt} + \frac{v_C}{R} + i_L(0) + \frac{1}{L}\int_0^t v_C(\tau)\,d\tau = 0 \quad , v_C(0) = V_0,\ t \geq 0$$

$$C\, v_C'' + \frac{1}{R} v_C' + \frac{1}{L} v_C = 0 \quad , v_C(0) = V_0 \ , \frac{dv_C(0)}{dt} = ?$$

$$\frac{dv_C(0)}{dt} = \frac{i_C(0)}{C} = \frac{-i_R(0) - i_L(0)}{C} = \frac{-\frac{V_R(0)}{R} - I_0}{C} = -\frac{V_0}{RC} - \frac{I_0}{C}$$

$$Cs^2 + \frac{1}{R}s + \frac{1}{L} = 0$$

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad \sigma = \frac{1}{2RC} \quad \omega_n^2 = \frac{1}{LC} \rightarrow \omega_n = \sqrt{\frac{1}{LC}}$$

1) $\sigma > \omega_n$ oz $\xi > 1$

$$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad v_c(0) = A_1 + A_2$$

$$v_c'(0) = A_1 s_1 + A_2 s_2$$

$$v_c(t) = \frac{v_c'(0) - s_2 v_c(0)}{s_1 - s_2} - \frac{v_c'(0) - s_1 v_c(0)}{s_1 - s_2}$$

$$v_c(0) = V_0$$

$$v_c'(0) = \frac{V_0}{RC} - \frac{I_0}{C}$$

Poli, ki so bolj oddaljeni od im. osi predstavljajo prehodni pojav, ki hitreje izzveni. Pol, ki je bližje im. osi dominantno vpliva na časovni potek. Taki poli so dominantni poli.

2) $0 \leq \sigma \leq \omega_n$ oz $0 \leq \xi \leq 1$ oz $Q < 0{,}5$

$$\sigma = \frac{1}{2RC} \qquad \omega_0 = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$s_{1,2} = -\sigma \pm j\omega_0$$

$$v_c(t) = e^{-\sigma t}\left[A\cos(\omega_0 t) + B\sin(\omega_0 t)\right] \qquad A = v_c(0)$$

?

$$B = \frac{v_c'(0) + \sigma v_c}{\omega_0}$$

3) $\sigma = \omega_n$ $\qquad s_{1,2} = s$

$$v_c(t) = (C_1 + C_2 t)\, e^{st} \qquad v_c(0) = -\frac{V_0}{RC} - \frac{I_0}{C}$$

Odziv na vhodni signal

$y(0)=0,\ y'(0)=0, \dots\ y^{(n-1)}(0)=0$

$t=0^+$

$y(0^+),\ y'(0^+), \dots\ y^{(n-1)}(0^+)$

$$y(t) = \underset{\substack{\uparrow \\ \text{prehodni} \\ \text{pojav}}}{y_o(t)} + \underset{\substack{\uparrow \\ \text{stacionarno} \\ \text{stanje}}}{y_s(t)}$$

$$y(t) = \sum_{k=1}^{n} C_k e^{s_k t} + y_s(t)$$

$$Q(p)\, y_s(t) = P(p)\, x(t)$$

$$y_s(t) = \frac{P(p)}{Q(p)} x(t) = H(p)\, x(t)$$

Partikularni del $y_s(t)$ je linearna kombinacija različnih odvodov vhodne funkcije $x(t)$. Odvode določa sistemski operator $h(p)$, ki ga lahko razvijemo v pot. vrsto glede na p.

$$H(p)\, x(t) = y_s(t)$$

Izračun je enostaven za vhodne signale oblike $x(t)=Ae^{st}$. Odvodi so množenje z s.

vzbujanje $x(t)=Ae^{st}$ 23.10.2012

$$y_s(t) = \underbrace{\frac{P(s)}{Q(s)}}_{H(s)} Ae^{st}$$ tudi če je s kompleksen

$$x(t) = A\cos\omega t = \mathrm{Re}\left[Ae^{j\omega t}\right] = \mathrm{Re}\left[Ae^{st}\right]_{s=j\omega}$$

$\frac{dy}{dt} = p = \frac{d}{dt}$

<u>Primer</u>

$$y''' + 8y'' + 37y' + 50y = \underbrace{4e^{-3t}}_{\text{eksponentno vzbujanje}} \qquad s=-3$$

$$(p^3 + 8p^2 + 37p + 50)\,y(t) = 4e^{-3t}$$

$$y(t) = \left[\frac{1}{p^3+8p^2+37p+50}\right]_{p=-3} 4e^{-3t} = -0{,}25\,e^{-3t}$$

<u>Primer</u> vzbujanje $x(t) = 4\cos 3t$

$$(p^3 + 8p^2 + 37p + 50)\,y(t) = 4\cdot\cos 3t = \mathrm{Re}\left[4e^{st}\right]_{s=j3}$$

$$y_s(t) = \mathrm{Re}\left[\frac{4e^{st}}{s^3+8s^2+37s+50}\right]_{s=j3} = \mathrm{Re}\left[\frac{4(\cos 3t + j\sin 3t)}{-22+j84}\right] =$$

$$= -\frac{22}{1885}\cos 3t + \frac{84}{1885}\sin 3t$$

Poseben primer nastopi, ko je eksponent s eksponentnega vhoda Ae^{st} tudi koren polinoma $Q(p)$ oz. $Q(s)$. Kot v primeru komplementarne fje, bo tudi partikularni integral oblike $B\cdot te^{st}$ oziroma pomnožen s t. Vrednost konstante B določimo s substitucijo kot prej po predhodni odstranitvi pripadajočega faktorja iz $Q(p)$.

$$\frac{d^3y}{dt^3} + 8\frac{d^2y}{dt^2} + 37\frac{dy}{dt} + 50y = 4e^{-2t}$$

$$p^3 + 8p^2 + 37p + 50 = 0$$

koreni: $p_1 = -2$

$p_{2,3} = -3 \pm j4$

$p+2 \ldots$ izločimo iz $Q(p)$

$$y_s(t) = \left[\frac{1}{p^2+6p+25}\right]_{p=-2} t4e^{-2t} =$$

$$= \frac{4}{17}\,t\,e^{-2t}$$

Dn.

a) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 25y = 50$ $\longrightarrow$ $(p^2 + 6p + 25 = 50)\, y(t) = 50$, $p=0$

$$\underline{\underline{y(t)}} = \left[\frac{50}{p^2+6p+25}\right]_{p=0} = \underline{\underline{2}}$$

b) $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 24y = 50e^{-2t}\cos 3t$

c) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 10e^{-4t}$ $\longrightarrow$ $(p^2+6p+8)\, y(t) = 10e^{-4t}$, $p=-4$

$$\underline{\underline{y(t)}} = \left[\frac{10e^{-4t}}{p^2+6p+8}\right]_{p=-4} = \left[\frac{10e^{-4t}}{p+2}\right]$$

$$\begin{array}{r|rrr} & 1 & 6 & 8 \\ -4 & \downarrow & -4 & -8 \\ \hline & 1 & 2 & 0 \end{array}$$

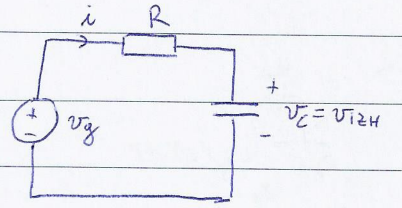
$(p+2)(p+4)$

$$\begin{array}{r|rrr} & 1 & 6 & 8 \\ -2 & \downarrow & -2 & -8 \\ \hline & 1 & 4 & 0 \end{array}$$

$\underline{\underline{= -5e^{-4t}\, t}}$

množimo s t, ker korena enaka

Primer določite V_c pri času $t>0$



$v_c(0) = 0$

$v_g(t) = u(t)$

napetosti $\quad Ri + v_c = v_g \qquad i = C\frac{dv_c}{dt}$

$$\boxed{RC\frac{dv_{izh}}{dt} + v_{izh} = v_g}$$

$$v_{izh}(t) = \underset{\text{odziv}}{v_o} + \underset{\text{ustaljeno st.}}{v_s}$$

$v_s(t) = 1V \leftarrow$ ustaljeno stanje

$$RC\frac{dv_{izh}}{dt} + v_o = 0$$

$$v_o = Ae^{st}$$

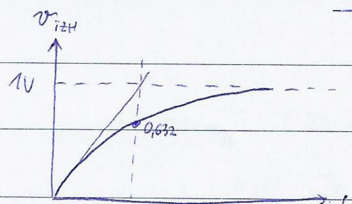
$$(RCs + 1)Ae^{st} = 0$$

$$\underline{\underline{s = -\frac{1}{RC}}}$$

$$v_{izh} = Ae^{-\frac{t}{RC}} + 1V$$

$$v_{izh}(0) = v_c(0) = 0$$

$$0 = A + 1V \rightarrow A = -1V$$



skupaj $\quad \underline{\underline{v_{izh} = (1 - e^{-\frac{t}{RC}})}} \quad t \geq 0$

$$y(t)(p^2+10p+24) = 50e^{-2t}\cos 3t$$

$$= Re[50e^{pt}] \qquad p = -2+j3$$

$$y(t) = Re\left[\frac{50e^{pt}}{p^2+10p+24}\right]_{p=-2+j3} =$$

$$= Re\left[\frac{50e^{-2t}(\cos 3t + j\sin 3t)}{p^2+10p+24}\right]$$

$$= Re\left[\frac{50e^{-2t}(\cos 3t + j\sin 3t)}{-1+18i}\right]$$

glej naslednjo stran

$$RC\frac{dv_{izh}}{dt} + \cancel{v_{izh}} = v_g \qquad t=0 \quad v_{izh} = v_c = 0$$

$$RC\frac{dv_{izh}}{dt} = v_g$$

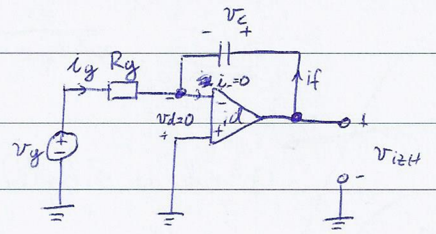
za majhne t $\quad v_c \approx 0 \approx v_{izh}$

$$\int_{v_{izh}(0)}^{v_{izh}(t)} dv_{izh} = \int_0^t \frac{v_g(\tau)}{RC}\, d\tau$$

$$\underline{v_{izh}(t) = \frac{1}{RC} \cdot t \cdot u(t)}$$

to vezje je tudi integrator (za majhne t), iz stopnice $\rightarrow$ rampa!

Primer določite izhodno napetost in ugotovite funkcijo, OP-AMP je idealen, deluje v aktivnem obm.



$v_d=0 \;\ldots\; v_c = v_{izH}$

$i_-=0 \;\ldots\; i_g + i_f = 0 \longrightarrow C\dfrac{dv_{izH}}{dt} = -\dfrac{v_g}{R_g}$

$i_g = \dfrac{v_g}{R_g}$

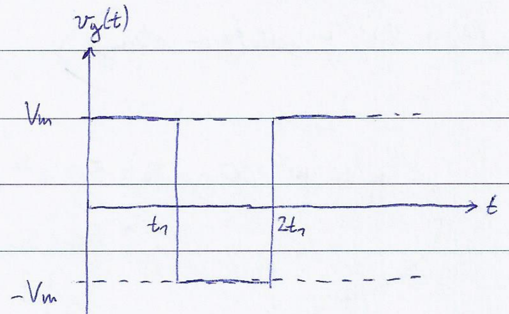
$i_f = i_c = C \cdot \dfrac{dv_{izH}}{dt}$

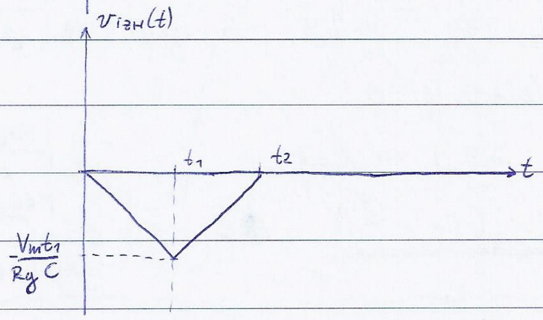
$$\int_{v_{izH}(t_0)}^{v_{izH}(t)} dv_{izH} = -\frac{1}{R_gC}\int_{t_0}^{t} v_g(\tau)\,d\tau$$

$$v_{izH}(t) = -\frac{1}{R_gC}\int_{t_0}^{t} v_g(\tau)\,d\tau + v_{izH}(t_0)$$

idealni integrator, invertira

$\Rightarrow$ müllerjev integrator





$0 < t < 2t_1$

$$v_{izH} = +\frac{1}{R_gC}\int_{t_1}^{t}(+V_m)\,d\tau - \frac{1}{R_gC}V_m t_1$$
$$= \frac{1}{R_gC}(t-t_1) - \frac{1}{R_gC}V_m t_1$$

Nadaljevanje domače naloge

signali in sistemi 23.10.2012

b) primer $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 24y = 50e^{-2t}\cos 3t$

$\vdots$

$$y(t) = \mathrm{Re}\left[\frac{50e^{-2t}(\cos 3t + j\sin 3t)}{-1+18j}\right] =$$

$$= \mathrm{Re}\left[50e^{-2t}\frac{(\cos 3t + j\sin 3t)(-1-18j)}{1+324}\right] =$$

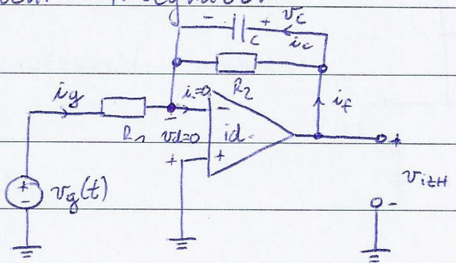
$$= \mathrm{Re}\left[\frac{2}{13}e^{-2t}\left(-\cos 3t - 18j\cos 3t - j\sin 3t + 18\sin 3t\right)\right] =$$

$$= \frac{2}{13}e^{-2t}(-\cos 3t + 18\sin 3t) =$$

$$= \underline{e^{-2t}\left(\frac{-2\cos 3t}{13} + \frac{36\sin 3t}{13}\right)}$$ recimo, menda...

$$y_s(t) = \frac{P(s)}{Q(s)}Ae^{st}$$

Primer Realni integrator 24.10.2012



$$i_g = \frac{v_g}{R_1}$$

$$i_f = \frac{v_{izh}}{R_2} + C\frac{dv_{izh}}{dt} \qquad (v_c = v_{izh})$$

$$i_g + i_f = 0$$

$$\frac{dv_{izh}}{dt} + \frac{v_{izh}}{R_2 C} = \frac{-v_g}{R_1 C}$$

$v_g(t) = 2u(t)$

$R_1 = 1\ M\Omega$

$R_2 = 10\ M\Omega$

$C = 1\ \mu F \qquad v_c(0) = 0$

$$v_{izh} = \underset{\text{prehodni}}{v_0} + \underset{\text{ustaljeno}}{v_s}$$

$t \to \infty \qquad \frac{dv_{izh}(t\to\infty)}{dt} = 0 \qquad v_{izh}(t\to\infty) = -\frac{R_2}{R_1}v_g$

$= -10\cdot 2$

$= \underline{-20\ V}$

$$\frac{dv_0}{dt} + \frac{v_0}{R_2C} = 0 \qquad v_0 = Ae^{st}$$

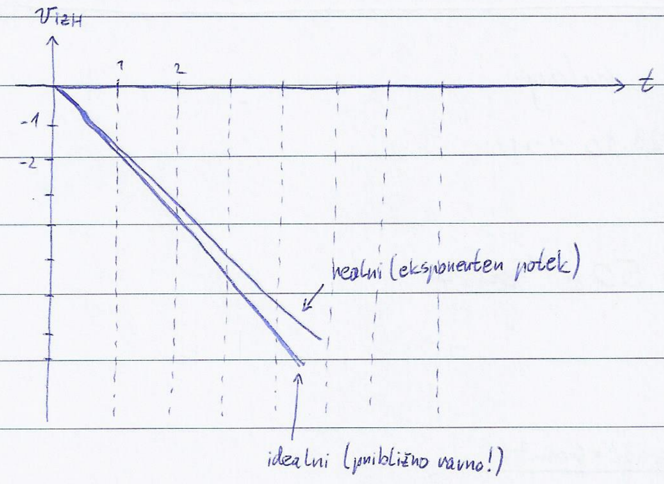
$$\left(s + \frac{1}{R_2C}\right)Ae^{st} = 0$$

$$s = -\frac{1}{R_2C} \qquad v_{izh}(0) = v_0(0) + v_s(0) =$$

$$= A - \frac{R_2}{R_1}v_g(0) = 0 \qquad A = \frac{R_2}{R_1}v_g(0) = 20V$$

rešitev

$$v_{izh} = 20\left(e^{\frac{t}{R_2C}} - 1\right) = -20\left(1 - e^{\frac{-t}{R_2C}}\right)\cdot u(t)$$



Primer Elementi vezja na sliki imajo vrednosti: Napajalna napetost operacijskih je ± 6 V. Pri $t=0$ skoči napetost iz 0 na 250 mV; kondenzatorji so prazni.

$R_a = 100\,k\Omega$

$R_1 = 500\,k\Omega$

$C = 0{,}1\,\mu F$

$R_b = 25\,k\Omega$

$R_2 = 100\,k\Omega$

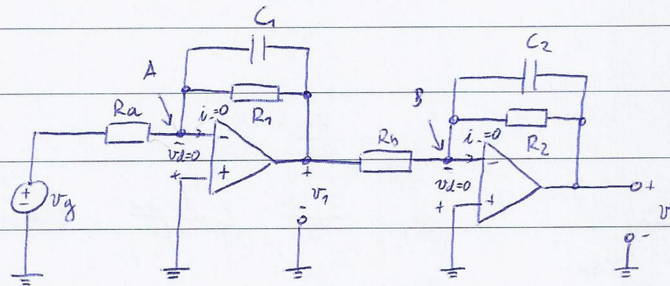
$C_2 = 1\,\mu F$

a) zapišite dif enačbo za v_{izh}

b) poiščite $v_{izh}(t)$ za $t \geq 0$

c) zapišite dif en z V_1

d) poiščite $V_1(t)$ za $t \geq 0$



tudi nizkopasovno sito,

kaskadni integrator

A:

$$\frac{v_g - 0}{R_a} + \frac{v_1 - 0}{R_1} + C_1 \frac{d(v_1 - 0)}{dt} = 0$$

$$\hookrightarrow \frac{dv_1}{dt} + \frac{1}{R_1 C_1} v_1 = -\frac{v_g}{R_a C_1} \quad \Big| \; \tau_1 = R_1 C_1$$

$$\frac{dv_1}{dt} = -\frac{v_1}{\tau_1} - \frac{v_g}{R_a C_1}$$

B:

$$\frac{v_1 - 0}{R_b} + \frac{v_{izh} - 0}{R_2} + C_2 \frac{d(v_{izh} - 0)}{dt} = 0$$

$$\hookrightarrow \frac{dv_{izh}}{dt} + \frac{1}{R_2 C_2} v_{izh} = -\frac{v_1}{R_b C_2} \quad \Big| \; \tau_2 = R_2 C_2$$

$$\hookrightarrow \frac{d^2 v_{izh}}{dt^2} + \frac{1}{\tau_2} \frac{dv_{izh}}{dt} = -\frac{1}{R_b C_2} \frac{dv_1}{dt}$$

iz te enačbe dobimo $\frac{dv_{izh}}{dt} = 0$!!

$$v_1 = -R_b C_2 \frac{dv_{izh}}{dt} - \frac{R_b C_2}{\tau_2} v_{izh}$$

dobimo:

$$\frac{d^2 v_{izh}}{dt^2} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \frac{dv_{izh}}{dt} + \frac{1}{\tau_1 \tau_2} v_{izh} = \frac{v_g}{R_a C_1 R_b C_2}$$

$$s^2 + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) s + \frac{1}{\tau_1 \tau_2} = 0$$

$$s_{1,2} = -\frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)^2 - \frac{1}{\tau_1 \tau_2}}$$ baje...

$$s_{1,2} = -\frac{1}{2}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \pm \frac{1}{2}\left(\frac{1}{\tau_1} - \frac{1}{\tau_2}\right)$$

$$s_1 = -\frac{1}{\tau_1}$$

$$s_2 = -\frac{1}{\tau_2}$$ ker sta korena realna, zeta > 1, baje

a) $\tau_1 = R_1 C_1 = 0{,}05\ s$

$\tau_2 = R_2 C_2 = 0{,}1\ s$

$\frac{v_g}{R_a C_1 R_b C_2} = 1000\ \frac{V}{s^2}$

$$\frac{d^2 v_{izh}}{dt^2} + 30 \frac{d v_{izh}}{dt} + 200\, v_{izh} = 1000$$

b) $s_1 = -10\ \frac{rad}{s}$ $\quad s_2 = -20\ \frac{rad}{s}$

$v_{izh} = v_0 + v_s$

$v_s = v_{izh}(t \to \infty) = \underbrace{(250 \cdot 10^{-3})}_{\text{vhod}} \underbrace{\left(-\frac{500}{100}\right)}_{\frac{R_1}{R_a}} \underbrace{\left(-\frac{100}{25}\right)}_{\frac{R_2}{R_b}} = \underline{5V}$

$v_{izh} = A_1 e^{s_1 t} + A_2 e^{s_2 t} + \cancel{v_s}\ 5V$

$A_1 = ?$, $A_2 = ?$

$v_{izh}(0) = 0$ $\qquad A_1 + A_2 = -5$

$\frac{d v_{izh}}{dt}(0) = 0$ $\qquad -10 A_1 - 20 A_2 = 0$ $\Rightarrow$ $A_1 = -10V$, $A_2 = 5V$

rešitev

$$\underline{v_{izh} = (-10 e^{-10t} + 5 e^{-20t} + 5)\ V \qquad t \geq 0}$$

c) $\frac{dv_1}{dt} + 20 v_1 = -25$

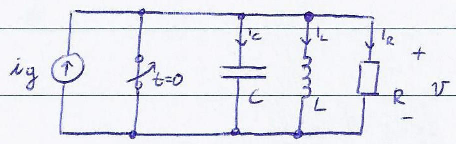
d) $v_1(t \to \infty) = 0{,}25\left(-\frac{500}{100}\right) = \underline{-1{,}25\ V}$

$$\underline{v_1 = -1{,}25\ V(1 - e^{-20t})\ V, \quad t \geq 0}$$

Primer   Vzporedno RLC vezje z idealnim tokovnim virom

$i_g = I\, u(t)$

$\underline{I = i_g}$



$+v(0)=0$

$i_C + i_L + i_R = i_g$

$$C\frac{dv(t)}{dt} + \frac{1}{L}\int_0^t v(\tau)\,d\tau + \frac{v}{R} = i_g$$

baje problem $\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v = \frac{1}{C}\frac{di_g(t)}{dt}$

$v = L\frac{di_L}{dt}$

$$Cv' + i_L + \frac{v}{R} = i_g$$

$$LC\, i_L'' + \frac{L}{R} i_L' + i_L = I = i_g \quad , \quad i_L(0)=0 \quad , \quad \frac{di_L(0)}{dt} = \frac{v(0)}{L} = 0$$

$i_L = i_o + i_s \qquad \underline{i_s = i_g}\ , \ t \ge 0$

$i_L = A_1 e^{s_1 t} + A_2 e^{s_2 t} + I \qquad s_1 \ne s_2$

$i_L = (B_1 + B_2 t)\, e^{st} + I \qquad s_1 = s_2 = s \quad (\text{ko } \xi = 1)$

$i_L(0) = 0 = A_1 + A_2 + I$

$\frac{di_L(0)}{dt} = s_1 A_1 + s_2 A_2 + 0 = 0$

$$\begin{bmatrix} 1 & 1 \\ s_1 & s_2 \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \frac{1}{s_2 - s_1}\begin{bmatrix} s_2 & -1 \\ -s_1 & 1 \end{bmatrix}\begin{bmatrix} I \\ 0 \end{bmatrix}$$

$A_1 = \frac{s_2}{s_1 - s_2} I$

$A_2 = \frac{s_1}{s_1 - s_2} I$

rešitev (za realne različne korene)

$$i_L(t) = I\left[\frac{1}{s_1 + s_2}\left(s_2 e^{s_1 t} - s_1 e^{s_2 t}\right) + 1\right]u(t)$$

za enake korene:

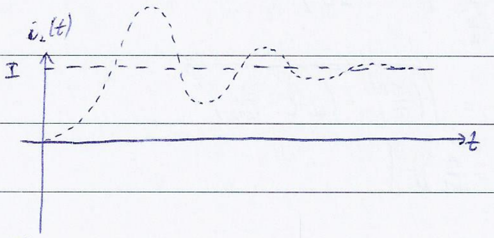
$$i_L(t) = I\left[(st - 1)\, e^{s_0 t} + 1\right]u(t)$$

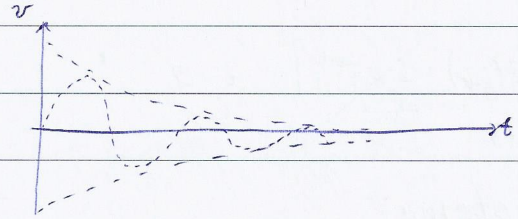
za kompleksne korene

za $\sigma < w_n$ $\quad s_{1,2} = -\sigma \pm \underbrace{\sqrt{\sigma^2 - w_n^2}}_{w_D}$

$$s_{1,2} = -\sigma \pm j w_D$$

$$i_L(t) = I\left[1 - \left(\cos w_D + \frac{\sigma}{w_D} \sin w_D t\right) e^{-\sigma t}\right]$$





Primer določimo odziv na sinusni vhodni signal za prejšnjo nalogo

zapišimo diferencialno enačbo za napetost

$$i_g = [I_m \cos(wt + \phi_i)]\, u(t) \qquad i_L(0) = 0$$

$$= \mathrm{Re}[I_m e^{jwt}]\, u(t) \qquad v(0) = 0$$

$$I = I_m e^{j\phi_i}$$

zapišimo

$$i_C + i_R + i_L = i_g$$

$$C\frac{dv(t)}{dt} + \frac{v}{R} + \frac{1}{L}\int_0^t v(\tau)\,d\tau = i_g$$

$$\frac{d^2v(t)}{dt^2} + \frac{1}{RC}\frac{dv(t)}{dt} + \frac{1}{LC}v(t) = \frac{1}{C}\frac{di_g(t)}{dt} \quad , t \geq 0 \quad , v(0) = 0$$

$$\frac{dv(0)}{dt} = \frac{i_g(0)}{C} \text{ (vidimo iz enačbe)}$$

$$= \frac{I_m \cos\phi_i}{C}$$

prehodni, ustaljeno

$$v = v_o + v_s$$

$$\frac{d^2v_o}{dt} + \frac{1}{RC}\frac{dv_o}{dt} + \frac{1}{LC}v_o = 0 \quad , t \geq 0 \longrightarrow v_o(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\left(s^2 + \frac{1}{RC}s + \frac{1}{CL} = 0\right)$$

$$\vdots$$

$$\frac{d^2v_s}{dt} + \frac{1}{RC}\frac{dv_s}{dt} + \frac{1}{LC}v_s = \frac{1}{C}\frac{di_g(t)}{dt}$$

rešimo naje tole ...

neka impedanca

$$v_s(t) = \mathrm{Re}\Big[\underbrace{\frac{P(jw)}{Q(jw)} I}_{\text{napetost}} e^{jwt}\Big] = \mathrm{Re}\left[V e^{jw}\right] = V_m \cos(wt + \phi_v)$$

$$\boxed{\frac{V}{I} = \frac{P(jw)}{Q(jw)} = Z(jw) = \frac{1}{Y(jw)}}$$

desna stran

$$v_s(t) = Re\left[\frac{\frac{1}{C}j\omega}{(j\omega)^2+\frac{1}{RC}j\omega+\frac{1}{LC}}\right] I e^{j\omega t} = \quad (s=j\omega)$$

$$= Re\left[Z(j\omega)\, I e^{j\omega t}\right] \qquad Z(j\omega) = \frac{1}{\frac{1}{R}+j(\omega C-\frac{1}{\omega L})}$$

za fazni kot arctan $\frac{Im}{Re}$

$$v_s(t) = Re\left[\frac{I_m e^{j\phi_i} e^{j\omega t}}{|Y|\, e^{j\arctan\left(\frac{\omega C-\frac{1}{\omega L}}{\frac{1}{R}}\right)}}\right], \quad |Y| = \sqrt{\left(\frac{1}{R}\right)^2+\left(\omega C-\frac{1}{\omega L}\right)^2}$$

$$v_s(t) = \frac{I_m}{|Y|}\cos\left[\omega t+\phi_i-\arctan\left(R(\omega C-\frac{1}{\omega L})\right)\right]$$

$$v(t) = v_0+v_s = A_1e^{s_1t}+A_2e^{s_2t}+Re\left[Z(j\omega)\, I e^{j\omega t}\right], \quad t\geq 0$$

Odziv LTI sistema

na enotin impulz

superpozicija...

ničelno začetno stanje

$$x(t) = \delta(t) \rightarrow y(t) = h(t)$$

prvega reda

$$a_1\frac{dy}{dt}+a_0y = b_0x$$

trenutek pred signalom

$$x(t) = \delta(t), \quad y(0) = 0$$

impulz doda začetno stanje !

$t=0^-$

$t=0$

$t=0^+$

zanima nas za $t\geq 0^+$

$$a_1\frac{dh}{dt}+a_0h = 0 \xrightarrow[\text{lahko}]{\text{sedaj}} a_1\frac{dh}{dt}+a_0h = 0$$

$$h(0^+) = ?$$

$$h(t) = Ae^{st}$$

za $t=0$

$$(a_1s+a_0)Ae^{st} = 0$$

$$a_1\frac{dh}{dt}+a_0h = b_0\delta \quad / \int_{0^-}^{0^+} dt$$

$$s = -\frac{a_0}{a_1} \qquad h(0^+) = A = \frac{b_0}{a_1}$$

$$\int_{0^-}^{0^+} a_1\,dh + \int_{0^-}^{0^+} a_0h\,dt = \int_{0^-}^{0^+} b_0\delta\,dt$$

$$\underline{h(t) = \frac{b_0}{a_1}e^{-\frac{a_0}{a_1}t}\cdot u(t)}, \quad t>0^+$$

$$a_1[\underbrace{h(0^+)}_{?}-\underbrace{h(0^-)}_{\text{pred signalom}=0}] + 0 = b_0$$

$$h(0^+) = \frac{b_0}{a_1}$$

Primer $\frac{dy}{dt} + 4y = x \quad , \quad y(0^-) = 0$

$x(t) = \delta(t)$

$y(t) = h(t)$

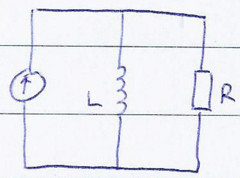
$h(0^+) = \frac{b_0}{a_1} = 1$

rešujemo $\frac{dh}{dt} + 4h = 0 \quad , \quad t > 0^+$

$s + 4 = 0$

$s = -4 \qquad h(t) = e^{-4t} \cdot u(t)$

Primer



$i_g = \delta(t) \quad , \quad i_L(0^-) = 0$

$\frac{v}{R} + i_L = i_g \qquad v = L\frac{di_L}{dt}$

$\frac{L}{R}\frac{di_L}{dt} + i_L = i_g$

$h(0^+) = \frac{b_0}{a_1} = \frac{R}{L} \qquad \frac{dh}{dt} + \frac{R}{L}h = 0$

$s + \frac{R}{L} = 0$

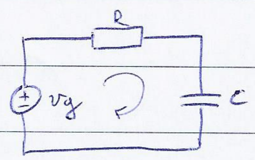
$s = -\frac{R}{L}$

rešitev: $h(t) = \frac{R}{L} e^{-\frac{R}{L}t} \cdot u(t)$

napetost na tuljavi $v_L = v = L\frac{di(t)}{dt} = \cancel{L}\frac{R}{\cancel{L}}\left(-\frac{R}{L}e^{-\frac{R}{L}t}u(t) + \underbrace{e^{-\frac{R}{L}t}\delta(t)}_{1}\right)$

$v_L = R\left(\delta(t) - h(t)\right)$

Primer



$v_C(0^-) = 0 \qquad v_R + v_C = v_g \qquad v_C = v_g - v_R$

$v_g = \delta(t) \qquad \cancel{iR + \frac{1}{C}\int_0^t i\,d\tau = v_g} \qquad v_C = v_g - Ri$

$\cancel{i'R + \frac{1}{C}i = v_g} \qquad v_C = v_g - R\frac{dv_C}{dt}C$

$\frac{dv_C}{dt} + \frac{1}{RC}v_C = v_g\frac{1}{RC}$

LTI, n-tega reda

$$a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt} + a_0 y = b_0 x \quad / \int_{0^-}^{0^+} dt$$

$$y(0^-)=0, \quad y'(0^-)=0, \dots \quad y^{n-1}(0^-)=0$$

ne more biti zvezna, ~~da pri odvodu~~ dobimo δ, baje ...

$$x(t) = \delta(t), \quad y(t) = h(t), \quad t > 0^+$$

$$\int_{0^-}^{0^+} f(t)\,dt = 0$$

omejena fja

$$a_n \cdot [h^{n-1}(0^+) - \underbrace{h^{n-1}(0^-)}_{\substack{0 \\ \text{menda}}}] + 0 = b_0$$

$$\frac{d^{n-1} h(0^+)}{dt^{n-1}} = \frac{b_0}{a_n}$$

$$h(0^+)=0, \quad h'(0^+)=0, \dots \quad h^{n-2}(0^+)=0 \qquad a_n \frac{d^n h}{dt^n} + \dots + a_1 \frac{dh}{dt} + a_0 h = 0, \quad t > 0^+$$

Iz de enačbe sistema izhaja, da morajo biti odziv $h(t)$ in njegovih $n-1$ odvodov povsod zvezni, sicer bi se na levi strani enačbe pojavil vsaj en odvod δ funkcije, kar pa ni dopustno, saj na desni strani nastopa le δ fja brez odvodov.

Primer: določite odziv na pulz

mora biti nezvezen!

$$\frac{d^2 y}{dt^2} + 4\frac{dy}{dt} + 3y = x \quad / \int_{0^-}^{0^+} dt$$

homogena:

$$\frac{d^2 h}{dt^2} + 4\frac{dh}{dt} + 3h = 0, \quad t > 0^+$$

$$x(t) = \delta(t), \quad y(t) = h(t)$$

$$y(0^-) = h(0^-) = 0 \qquad h(0^+)=0 \quad \frac{dh(0^+)}{dt} = \frac{b_0}{a_2} = 1$$

$$\hookrightarrow \quad h(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \quad t > 0^+$$

$$s^2 + 4s + 3 = 0$$

$$s_1 = -1, \quad s_2 = -3$$

$$h(t) = (A_1 e^{-t} + A_2 e^{-3t}) \cdot u(t)$$

$$\overset{?}{=} \qquad \overset{?}{=}$$

$$h(0^+) = 0 = A_1 + A_2 \rightarrow A_2 = -A_1 \qquad \Rightarrow \qquad A_1 = +\frac{1}{2}$$

$$\frac{dh(0^+)}{dt} = 1 = (-A_1 - 3A_2) \qquad A_2 = -\frac{1}{2}$$

$$\underline{h(t) = \left(\frac{1}{2} e^{-t} - \frac{1}{2} e^{-3t}\right) u(t)}$$

<u>Primer</u> $\quad \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 13y = x \quad / \int_{-0}^{+0} dt$

$$y(0^-) = 0$$

$$x(t) = \delta(t) \quad , \quad y(t) = h(t) \qquad \frac{d^2h}{dt^2} + 4\frac{dh}{dt} + 13h = 0$$

$$h(0^+) = 0 \qquad s^2 + 4s + 13 = 0$$

$$h'(0^+) = 1 = \frac{b_0}{a_2} = 1 \qquad s_{1,2} = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 13}}{2}$$

$$= -2 \pm \sqrt{2^2 - 13} = \underline{-2 \pm j3}$$

$$= -\sigma \pm j\omega_d$$

$$h(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$h(t) = e^{-\sigma t}(A \cos \omega_0 t + jB \sin \omega_d t) \cdot u(t)$$

$$h(0^+) = 0 = A$$

$$h'(0^+) = 1 = -\sigma A + \omega_0 B$$

$$A_2 = B = \frac{1}{\omega_0}$$

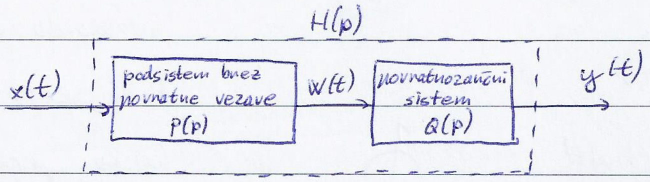
$$\underline{h(t) = \left(\frac{1}{\omega_0} e^{-\sigma t} \sin \omega_0 t\right) u(t)}$$

$$a_n \frac{d^n y}{dt^n} + \ldots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + \ldots + b_1 \frac{dx}{dt} + b_0 x$$ 30.10.2022

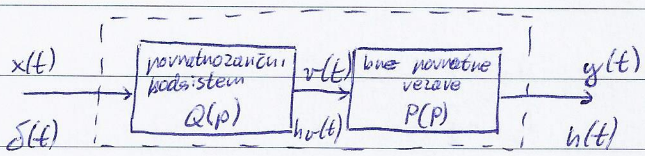
$$Q(p)\, y(t) = P(p)\, x(t)$$

$$w(t) = P(p)\, x(t) \quad \leftarrow \text{podsistem bez povratne vezave}$$

$$Q(p)\, y(t) = w(t)$$



LTI ... podsistema lahko zamenjamo



$$Q(p)\, v(t) = x(t)$$

$$P(p) \cdot v(t) = y(t)$$

rešujemo impulzni odziv za posamezni podsistem

Impulzni odziv sistema n-tega reda, ki ga opisuje DE $(m>0)$ določimo v dveh korakih

1) določimo impulzni odziv povratnega podsistema, ki ga opisuje enačba $Q(p)\cdot v(t) = x(t)$, pri tem uporabimo že opisani postopek določanja impulznega odziva sistema, ki ga opisuje DE brez odvodov vhodnega signala

2) uporabimo predpis $P(p)$ v enačbi $y(t) = P(p)\, v(t)$ in dobimo impulzni odziv celotnega sistema

$$h(t) = P(p)\, h_v(t)$$

Primer $\quad a_1 \frac{dy}{dt} + a_0 y = b_1 \frac{dx}{dt} + b_0 x \qquad$ x je pulz

1) $\quad Q(p)\, h_v(t) = \delta(t) \qquad b_0 = 1$

$t > 0^+$

$$h_v(t) = \frac{b_0}{a_1} e^{-\frac{a_0}{a_1}t} \cdot u(t)$$ vmesni signal

2) $\quad h(t) = P(p)\, h_v(t)$

$$h(t) = b_1 \frac{d h_v(t)}{dt} + b_0 h_v(t)$$

$$h(t) = +b_1 \frac{b_0}{a_1}\left(\frac{-a_0}{a_1} e^{-\frac{a_0}{a_1}t} u(t) + \overbrace{e^{-\frac{a_0}{a_1}t}}^{1} \delta(t)\right) + \frac{b_0}{a_1} e^{-\frac{a_0}{a_1}t} u(t)$$

uredimo $\quad h(t) = \frac{b_1}{a_1}\delta(t) + \left(\frac{b_0}{a_1} - \frac{b_1 a_0}{a_1^2}\right) e^{-\frac{a_0}{a_1}t}\, u(t)$

pazimo, da niso koeficienti 0, ker lahko pride do singularnosti

$a_1 b_0 - b_1 a_0 \neq 0$

neka oblika $h(p)$?

Primer:

$$y(t) = \frac{b_1 p + b_0}{a_1 p + a_0} x(t) = \quad p = \frac{d}{dt}$$

$$= \begin{cases} \left(\frac{b_1}{a_0} p + \frac{b_0}{a_0}\right) x(t), & \text{če } a_1 = 0 \\ \left(\frac{b_1}{a_1} + \frac{a_1 b_0 - a_0 b_1}{a_1^2 (p + \frac{a_0}{a_1})}\right) x(t), & \text{če } a_1 \neq 0 \end{cases}$$

~~Primer~~:

prenosna f $H(p)$	Opis	Odziv na en. impulz $h(t)$
$\frac{b_1}{a_0} p$	diferenciator	$\frac{b_1}{a_0} \delta'(t)$
$\frac{b_0}{a_1 p}$	integrator	$\frac{b_0}{a_1} u(t)$
$\frac{b_1 p + b_0}{a_0}$	diferenciator s puščanjem	$\frac{b_0}{a_0} \delta(t) + \frac{b_1}{a_0} \delta'(t)$
$\frac{b_0}{a_1 p + a_0}$	integrator s puščanjem (nizko-pasovni filter)	$\frac{b_0}{a_1} e^{-\frac{a_0}{a_1} t} u(t)$?
$\frac{b_1 p}{a_1 p + a_0}$	visoko-pasovno sito	$\frac{b_1}{a_1} \delta(t) - \frac{b_1 a_0}{a_1^2} e^{-\frac{a_0}{a_1} t} u(t)$
$\frac{b_1 (p - \frac{a_0}{a_1})}{a_1 (p + \frac{a_0}{a_1})}$	vse-prepustni filter o_O	$\frac{b_1}{a_1} \left(\delta(t) - 2 \frac{a_0}{a_1} e^{-\frac{a_0}{a_1} t} u(t)\right)$

Akta®

DE drugega reda

$$a_2\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_0y=b_2\frac{d^2x}{dt}+b_1\frac{dx}{dt}+b_0x \qquad \delta(t)$$

$$Q(p)\ h_v(t)=\delta(t)\ \Big|\int_{0^-}^{0^+}dt \qquad b_0=1$$

$$Q(p)\ h_v(t)=0\ ,\ t>0^+$$

možne rešitve

$$\frac{dh(0^+)}{dt}=\frac{1}{a_2}\ ,\ h(0^+)=0$$

$$h_v(t)=\frac{1}{a_2}\frac{1}{s_1-s_2}\left(e^{s_1t}-e^{s_2t}\right)u(t) \qquad s_1\neq s_2$$

$$h_v(t)=\frac{1}{a_2}e^{-\sigma t}\frac{1}{\omega_0}\sin\omega_0t\cdot u(t) \qquad s_{1,2}=-\sigma\pm j\omega_0$$

množenje s t

$$h_v(t)=\frac{t}{a_2}e^{st}\cdot u(t) \qquad s_1=s_2=s$$

rešimo še $h(t)=P(p)\ h_v(t)$

6.11.2012

$$y(t)=\frac{P(p)}{Q(p)}x(t)=\underbrace{\frac{b_2p^2+b_1p+b_0}{a_2p^2+a_1p+a_0}}_{H(p)}x(t)$$

<u>Primer</u> $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=\frac{dx}{dt}+2x$

1) $Q(p)\cdot h_v(t)=\delta(t)$

$$\frac{d^2h_v}{dt^2}+4\frac{dh_v}{dt}+3h_v=0\ ,\ t>0^+$$

$$h_v(0^+)=0$$

$$\frac{dh_v}{dt}(0^+)=\frac{b_0}{a_2}=\frac{1}{a_2}$$

$h_v=Ae^{st}$

$s^2+4s+3=0$

$s_1=-1$

$s_2=-3$

$h_v(t)=(A_1e^{-t}+A_2e^{-3t})\cdot u(t) \qquad h_v(0)=0=A_1+A_2 \qquad A_1=-A_2$

h

$$h_v'(0) = 1 = s_1 A_1 + s_2 A_2 = -A_1 - 3A_2$$

$$A_2 = -\frac{1}{2}$$

$$A_1 = +\frac{1}{2}$$

rešitev

$$h_v(t) = \left(\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}\right) \cdot u(t)$$

lahko bi uporabili

$$h_v(t) = \frac{1}{a_2}\frac{1}{s_1 - s_2}\left(e^{+s_1 t} - e^{s_2 t}\right) \cdot u(t)$$ „splošna oblika rešitve"

2) $P(p)\, h_v(t) = h(t)$

$$h(t) = \frac{dh_v}{dt} + 2h_v =$$

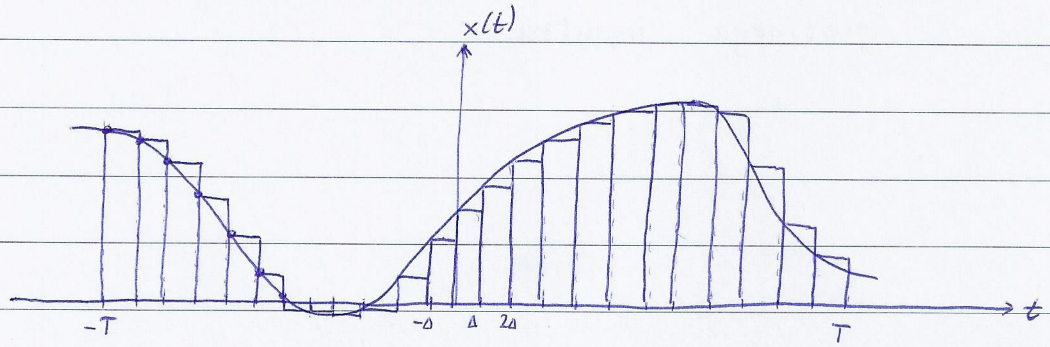
$$= \left(-\frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t}\right)u(t) + \underbrace{\left(\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}\right)\delta(t)}_{\phi} + 2\left(\frac{1}{2}e^{-t} - \frac{1}{2}e^{-3t}\right)u(t) =$$

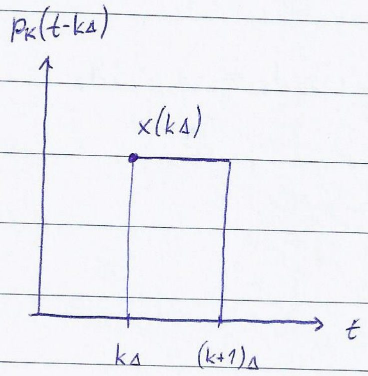
$$= \left(\frac{1}{2}e^{-t} + \frac{1}{2}e^{-3t}\right) \cdot u(t) = h(t)$$

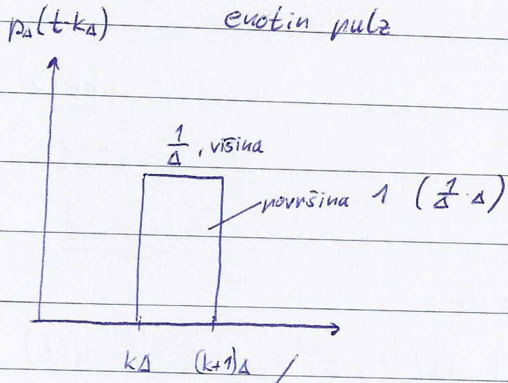
Konvolucija

Operacija, ki omogoča določitev odziva LTI na poljuben odziv, če poznamo odziv na enotin impulz.

Predstavitev časovno zveznega signala z neprekinjenim zaporedjem pulzov







$$p_K(t-k\Delta) = \frac{x(k\Delta)\cdot\Delta}{\frac{1}{\Delta}\Delta}\, \overset{(t-k\Delta)}{p_\Delta} = x(k\Delta)\cdot p_\Delta(t-k\Delta)\cdot\Delta$$

aproksimacija signala

$$x(t) \approx \sum_{k=-\frac{T}{\Delta}}^{\frac{T}{\Delta}} x(k\Delta)\, p_\Delta(t-k\Delta)\cdot\Delta$$

Lim

$\Delta \to 0$

$k\Delta \to \tau$

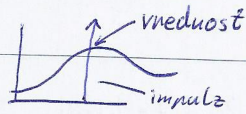
$\Delta \to d\tau$

$p_\Delta(t-k\Delta) \to \delta(t-\tau)$

$\sum \to \int$

potem $$x(t) = \int_{-T}^{T} x(\tau)\,\delta(t-\tau)\,d\tau$$

splošno $$\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}$$



Lastnost sita / tipalna lastnost enotinega impulza

Konvolucijski integral

Imamo LTI:

1) na vhod pošljemo $\underset{\text{enotin impulz}}{p_\Delta(t)} \rightarrow \underset{\text{odziv}}{h_\Delta(t)}$

2) če zakasnimo vhod, bo zakasnjen tudi izhod $p_\Delta(t-k\Delta) \rightarrow h_\Delta(t-k\Delta)$

3) impulz pomnožimo, tudi izhod je pomnožen $\underbrace{x(k\Delta)}_{\text{vrednost}}\Delta \cdot p_\Delta(t-k\Delta) \rightarrow x(k\Delta)\Delta\, h_\Delta(t-k\Delta)$

4) seštevanje prispevkov $y \approx \sum_{k=-\frac{T}{\Delta}}^{\frac{T}{\Delta}} x(k\Delta)\, h_\Delta(t-k\Delta)\,\Delta$, $-T \leq t \leq T$

$\Delta \rightarrow 0$

$$y = \int_{-T}^{+T} x(\tau)\, h(t-\tau)\, d\tau$$

$$y = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\boxed{\text{konvolucijski integral} \quad y = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau}$$

konvolucijski teorem:

Odziv sistema $y(t)$ na poljuben odziv lahko izračunamo s konvolucijo vzbujanja $x(t)$ in odzivom na enotin impulz $h(t)$.

$y(t) = x(t) * h(t)$ oblika zapisa

Lastnosti:

1) komutativnost $y(t) = x(t) * h(t) = h(t) * x(t)$

$\xi = t - \tau$

$$y(t) = \int_{-\infty}^{\infty} x(t-\xi)\, h(\xi)\, d\xi$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

2) ~~distributivnost~~ asociativnost $y(t) = x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t) = \ldots$

3) distributivnost $y(t) = x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$

4) konvolucijski integral in stabilnost

„BIBO"

$|x(t)| < B$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \right| \leq \int_{-\infty}^{\infty} |x(\tau)|\, |h(t-\tau)|\, d\tau \leq$$

$$\leq B \int_{-\infty}^{\infty} |h(\tau)|\, d\tau$$

sistem je stabilen, če je njegov odziv na enotin impulz absolutno integrabilen

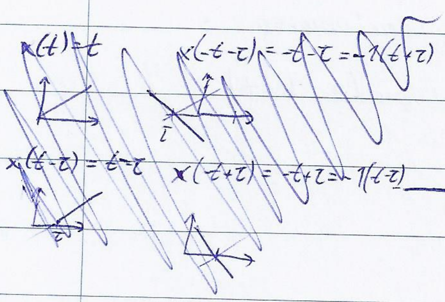
$$\int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$$

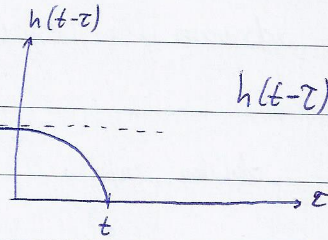
5) konvolucijski integral in kavzalnost

$t = 0$

$$y = \int_{0}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

$t < 0$, $x(t) = 0$





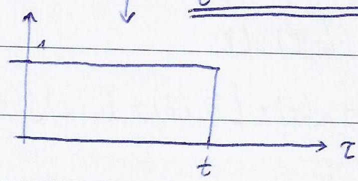
$h(t-\tau) = 0$ za $\tau > t$

7.11.2012 6) povezava odziva enotin impulz – enotina stopnica

odziv na stopnico $g(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau =$

$$= \int_{0}^{t} h(\tau)\, \underbrace{u(t-\tau)}_{1}\, d\tau = \underline{\underline{\int_{0}^{t} h(\tau)\, d\tau}}$$

$0 \leq \tau \leq t$



$\underline{\underline{h(t) = \frac{dg(t)}{dt}}}$ obratna pot

$$y(t) = \int_{-\infty}^{\infty} \overbrace{h(\tau)}^{v'} \, \overbrace{x(t-\tau)}^{u} \, d\tau = \qquad \int u(x) v'(x)\,dx = uv - \int u' v\,dx$$

$$= \underbrace{g(\tau)\, x(t-\tau) \Big|_{-\infty}^{\infty}}_{\emptyset} - \int_{-\infty}^{\infty} g(\tau) \frac{dx(t-\tau)}{d\tau}\, d\tau$$

$$y(t) = -\int_{-\infty}^{\infty} g(\tau) \frac{dx(t-\tau)}{d\tau}\, d\tau \qquad t-\tau = \xi$$

$$-d\tau = d\xi$$

$$\boxed{y(t) = \int_{-\infty}^{\infty} g(t-\tau) \frac{dx(\tau)}{d\tau}\, d\tau}$$

Primer sistem prvega reda ima odziv ... določimo odziv na stopnico

$$a_1 \frac{dy}{dt} + a_0 y = b_0 x$$

odziv na enotin impulz:

$$h(t) = y(t) = \frac{b_0}{a_1} e^{-\frac{a_0}{a_1}t} u(t)$$

Lahko spustimo, če v mejah

$$\hookrightarrow g(t) = \int_0^t h(\tau)\,d\tau = \int_0^t \frac{b_0}{a_1} e^{-\frac{a_0}{a_1}\tau} \, d\tau = \frac{b_0}{a_1}\left(-\frac{a_1}{a_0}\right) e^{-\frac{a_0}{a_1}\tau} \Big|_0^{\tau} =$$

$$= \frac{b_0}{a_1}\left(1 - e^{-\frac{a_0}{a_1}t}\right) \cdot u(t)$$

7) sinusno ustaljeno stanje

vhodni signal $x(t) = 2\cos\omega t = e^{j\omega t} + e^{-j\omega t} = x_1(t) + x_2(t)$

$$x_1(t) = e^{j\omega t} \qquad -\infty \le t \le \infty$$

$$y_{ust}(t) = \int_{-\infty}^{\infty} h(\tau)\, x_1(t-\tau)\, d\tau =$$

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{j\omega(t-\tau)} = e^{j\omega t} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau}\, d\tau}_{\text{fourier}} = e^{j\omega t} H(j\omega)$$

$$H(j\omega) = \int_{-\infty}^{\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

$$y_{2ust} = \int_{-\infty}^{\infty} h(\tau)\, x_2(t-\tau)\, d\tau = e^{-j\omega t} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{j\omega\tau}\, d\tau}_{H(-j\omega)} = e^{-j\omega t} H(-j\omega)$$

$$y_{ust} = y_{1ust}(t) + y_{2ust}(t) =$$
$$= e^{j\omega t} H(j\omega) + e^{-j\omega t} H(-j\omega)$$

faza

$$H(j\omega) = |H(j\omega)|\, e^{j\angle H(j\omega)}$$
$$H(-j\omega) = |H(j\omega)|\, e^{-j\angle H(j\omega)}$$

$$y_{ust} = e^{j\omega t} |H(j\omega)|\, e^{j\angle H(j\omega)} + e^{-j\omega t} |H(j\omega)|\, e^{-j\angle H(j\omega)} =$$
$$= |H(j\omega)| \left(e^{j(\omega t + \angle H(j\omega))} + e^{-j(\omega t + \angle H(j\omega))} \right) =$$
$$= 2|H(j\omega)| \cos(\omega t + \underline{\angle H(j\omega)})$$

odziv premakujen za fazo $\quad -\infty \le t \le \infty$

$$\boxed{\begin{array}{l} x(t) = A \cdot \cos(\omega t + \phi) \\ \text{odziv} \\ y_{ss}(t) = A|H(j\omega)| \cos(\omega t + \phi + \angle H(j\omega)) \end{array}}$$

enačba ustaljenega sinusnega stanja, LTI

Primer visokopasovni zvezni filter, ki duši signale nizkih frekvenc, ima odziv na enotin impulz

$$h(t) = \delta(t) - 10 e^{-10t} u(t)$$

vhodni signal $x(t) = 5 + 5 \cdot \cos 10t \quad -\infty \le t \le \infty$

$y_{ss}(t) = ?$

$$H(j\omega) = \int_0^{\infty} [\delta(t) - 10^{-10t}]\, e^{-j\omega\tau}\, d\tau = 1 - \frac{10}{10+j\omega} = \underline{\underline{\frac{j\omega}{10+j\omega}}}$$

$$H(j0) = 0$$

$$H(j10) = \frac{j10}{10+j10} = \underline{0{,}707\, e^{j\,0{,}785}}$$

$y_{ss}(t) = A\,|H(j\omega)|\cos(\omega t + \phi + \angle H(j\omega))$

$y_{ss}(t) = 3{,}53\cos(10t + 0{,}785)\,, \quad -\infty \le t \le \infty$

alternativna pot določanja frekvenčnega odziva

$$\sum_{k=0}^{n} a_k \frac{d^k y}{dt^k} = \sum_{k=0}^{m} b_k \frac{d^k x}{dt^k}$$

vzbujanje $x(t) = e^{j\omega t}$ $\quad \frac{d^k x}{dt^k} = (j\omega)^k e^{j\omega t}$

$$y(t) = e^{j\omega t} H(j\omega)\,, \quad \frac{d^k y}{dt^k} = (j\omega)^k H(j\omega)\, e^{j\omega t}$$

$$\sum_{k=0}^{n} a_k H(j\omega)(j\omega)^k \cancel{e^{j\omega t}} = \sum_{k=0}^{m} b_k (j\omega)^k \cancel{e^{j\omega t}}$$

$\cancel{H(j\omega)} =$

$$\boxed{H(j\omega) = \frac{\sum_{k=0}^{m} b_k (j\omega)^k}{\sum_{k=0}^{n} a_k (j\omega)^k}}$$

Primer pasovno zvezni filter

$$\ddot{y}(t) + 2\dot{y}(t) + 100\,y(t) = 100\cdot\dot{x}(t)$$

a) določimo frekvenčni odziv filtra $H(j\omega)$

b) določimo ustaljeni odziv na signal $x(t) = 10 + 10\cdot\cancel{\sin}\cos 10t + 10\cdot\cos 100t$

$s^2 + 2s + 100 = 0$ $\quad H(j\omega) = \frac{j\omega\,100}{(j\omega)^2 + 2j\omega + 100}$

$s_{1,2} = -1 \pm j\,9{,}95$

stabilen sistem

$H(j0) = 0$

$H(j10) = \frac{\cancel{j}1000}{\cancel{-100} + 2\cancel{j}10 + \cancel{100}} = 50$

$H(j100) = 1\,e^{-j\,1{,}55}$ ← ?

faza

Akta®

$$y_{ss}(t) = A|H(j\omega)| \cos(\omega t + \phi + \angle H(j\omega))$$

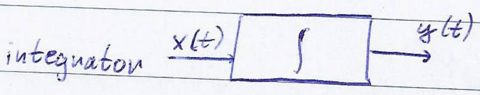
$$y_{ss}(t) = 500 \cos 10t + 10 \cos(100t - 1{,}55)$$

Metode določanja konvolucijskega integrala

1) analitično
2) grafično
3) numerično

impulzni odziv

1) $$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

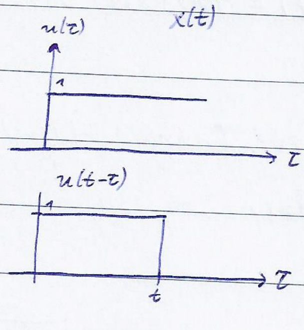


$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

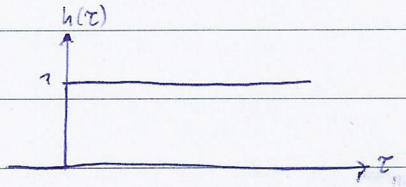
$$x(t) = t \cdot u(t)$$

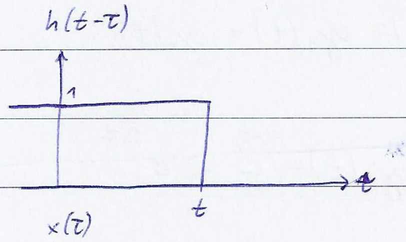
$$h(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau = u(t)$$

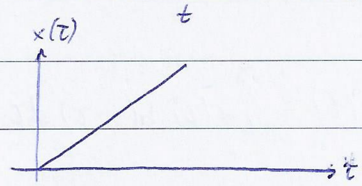
$$y(t) = \int_{-\infty}^{\infty} \underbrace{\tau(u(\tau))}_{\substack{\text{vhod} \\ x(t)}} u(t-\tau)\, d\tau = \int_{0}^{t} \tau\, d\tau = \frac{t^2}{2}$$

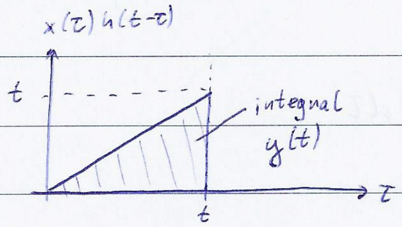


2) $$y(t) = \int_{-\infty}^{\infty} \underbrace{\tau u(\tau)}_{\substack{\text{ulaz} \\ x(t)}} \underbrace{u(t-\tau)}_{h(t-\tau)} \, d\tau$$









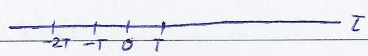
$$y(t) = \frac{t \cdot t}{2} = \frac{t^2}{2} \quad , \quad t > 0$$

3) $x(t) \rightarrow x(nT) \qquad t = nT$

$y(t) \rightarrow y(nT)$

T spustimo $x(n)$, $y(n)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$



$$y(t) = \ldots + \int_{-2T}^{-T} h(\tau)\, x(nT-\tau)\, d\tau + \int_{-T}^{0} h(\tau)\, x(nT-\tau)\, d\tau + \int_{0}^{T} h(\tau)\, x(nT-\tau)\, d\tau + \ldots$$

$$y(nT) \approx \ldots + T \cdot h(-2T)\, x(nT+2T) + T \cdot h(-T)\, x(nT+T) + T \cdot h(0)\, x(nT+0) + \ldots$$

$$y(nT) = T \sum_{m=-\infty}^{\infty} h(mT) \cdot x(nT - \overset{?}{mT}) = \underbrace{T \sum_{m=-\infty}^{\infty} h(m)\, x(n-m) = \underbrace{T \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)}_{T \cdot \text{conv}}}$$

T spustimo ... kao

Celotni odziv kavzalnega LTI sistema

$$y(t) = y_z(t) + y_v(t)$$

$$y_z(t) = \sum_{k=1}^{n} C_k e^{s_k t}$$

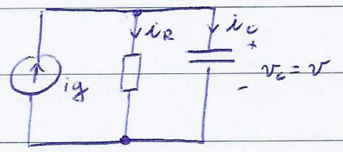
$$y_v(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = \sum_{k=1}^{n} C_k e^{s_k t} + \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$\downarrow \qquad \downarrow$$

$$y(t) = y_{pp}(t) + y_{ss}(t)$$

<u>Primer:</u> odloži odziv vezja



$$i_c + i_R = i_g$$

$$C\frac{dv}{dt} + \frac{v}{R} = i_g \quad , t>0 \quad , v(0) = V_0$$

$$i_g = I \cdot u(t)$$

$$v(0) = V_0$$

$$v(t) = v_z(t) + v_v(t)$$

$$v_z(t) = \Big| \; C\frac{dv_z}{dt} + \frac{v_z}{R} = 0 \quad , v(0) = V_0$$

$$v_z(t) = V_0 e^{-\frac{t}{RC}} \cdot u(t)$$

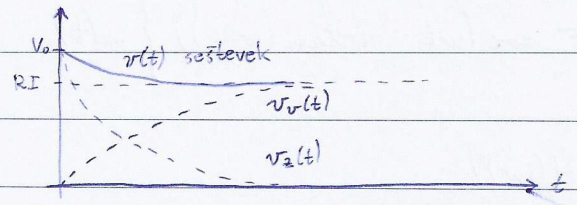
$$v_v(t) = \Big| \; C\frac{dv_v}{dt} + \frac{v_v}{R} = I \quad , t \geq 0, \; v_v(0) = 0$$

$$v_v(t) = RI\left(1 - e^{-\frac{t}{RC}}\right) \cdot u(t) \quad \leftarrow \text{postopek?}$$

$$v(t) = v_v(t) + v_z(t)$$

$$= \underbrace{V_0 e^{-\frac{t}{RC}}}_{\text{odziv začetno stanje}} + \underbrace{RI\left(1 - e^{-\frac{t}{RC}}\right)}_{\text{ustaljeno}} \quad , t \geq 0$$

$$v(t) = v_{pp} + v_{ss} = \underbrace{(V_0 - RI)e^{-\frac{t}{RC}}}_{} + \underbrace{RI}_{}$$



$$v_c(t) = v_c(\infty) + [v_c(t_0) - v_c(\infty)]\, e^{-\frac{t-t_0}{R_{th} C}}$$

sinusno vzbujanje $\quad i_g(t) = I \cdot \cos \omega t \cdot u(t)$

$$v(0) = V_0$$

$$v = v_z + v_s$$

$$C \frac{dv}{dt} + \frac{v}{R} = i_g \qquad v_z = V_0\, e^{-\frac{t}{RC}}\, u(t)$$

impulzni: $C \frac{dh}{dt} + \frac{h}{R} = \delta(t)$

$$h(t) = \frac{b_0}{a_1} e^{-\frac{a_0}{a_1} t}\, u(t) \quad \text{splošna oblika}$$

$$h(t) = \frac{1}{C} e^{-\frac{1}{RC} t} \cdot u(t)$$

$$x(t) = i_g(t) = i \cdot \mathrm{Re}[e^{j\omega t}]$$

$$v_v = \int_0^t x(\tau)\, h(t-\tau)\, d\tau = \int_0^t I \cdot \mathrm{Re}[e^{j\omega t}]\, \frac{1}{C} e^{-\frac{1}{RC}(t-\tau)} d\tau =$$

$$= \frac{I}{C} e^{-\frac{t}{RC}} \int_0^t \mathrm{Re}[e^{(j\omega + \frac{1}{RC})\tau}]\, d\tau$$

$$v_v(t) = \frac{I}{C} \mathrm{Re}\left[\frac{e^{j\omega t} - e^{-\frac{t}{RC}}}{j\omega + \frac{1}{RC}}\right] u(t) =$$

$$= I \cdot R \cdot \mathrm{Re}\left[\frac{e^{j\omega t} - e^{-\frac{t}{RC}}}{1 + j\omega RC}\right] u(t) = \dots$$

$$v(t) = \left(V_0 e^{-\frac{t}{RC}} + R \cdot I \cdot \mathrm{Re}\left[\frac{e^{j\omega t} - e^{-\frac{t}{RC}}}{1 + j\omega RC}\right]\right) u(t)$$

$$v(t) = v_{pp}(t) + v_{ss}(t) = \left\{ \underbrace{\left(V_0 - RI\, \mathrm{Re}\left[\frac{1}{1+j\omega RC}\right]\right) e^{-\frac{t}{RC}}}_{\text{prehodni pojav}} + \underbrace{RI\, \mathrm{Re}\left[\frac{e^{j\omega t}}{1+j\omega RC}\right]}_{\text{ustaljeno}} \right\} u(t)$$

$$v(t) = \left\{ \left[V_0 - \frac{R \cdot I}{1+(\omega RC)^2} \right] e^{-\frac{t}{RC}} + \frac{R \cdot I}{\sqrt{1+(\omega RC)^2}} \cos(\omega t - \arctan(\omega RC)) \right\} u(t)$$

$$y_{ss} = A\,|H(j\omega)| \cos(\omega t + \phi + \angle H(j\omega))$$

$$|H(j\omega)| = \frac{R}{\sqrt{1+(\omega RC)^2}}$$

$$\angle H(j\omega) = -\arctan(\omega RC)$$

kako drugače do $H(j\omega)$

$$H(j\omega) = \int_{-\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

Odziv na poljuben signal z metodo neposrednega integriranja

$$C \frac{dv}{dt} + \frac{v}{R} = i_g, \quad t>0, \quad v(0) = V_0$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{i_g}{C} \quad \Big/ \cdot e^{\frac{t}{RC}}$$

$$\underbrace{e^{\frac{t}{RC}} \frac{dv}{dt} + e^{\frac{t}{RC}} \frac{v}{RC}} = e^{\frac{t}{RC}} \frac{i_g}{C}$$

$$\frac{d}{dt} v \cdot e^{\frac{t}{RC}} = \frac{i_g}{C} e^{\frac{t}{RC}} \Big/ \int_0^t dt$$

$$v\, e^{\frac{t}{RC}} = \int_0^t \frac{i_g}{C} e^{\frac{\tau}{RC}} d\tau + v(0) \quad \Big/ \cdot e^{-\frac{t}{RC}}$$

$$v(t) = \underbrace{\underbrace{v(0)}_{V_0} e^{-\frac{t}{RC}}}_{\text{odziv na začetno stanje}} + \underbrace{\frac{1}{C}\int_0^t e^{-\frac{t-\tau}{RC}} i_g(\tau)\, d\tau}_{\text{odziv na vhodni signal}}$$

Posplošitev iskanja odziva

$$\frac{dy}{dt} + ay = bx \quad / e^{at} \qquad , \quad y(t_0) \text{ zač. stanje}$$

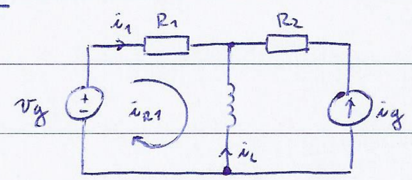
$$\frac{dy}{dt} e^{at} + ay\, e^{at} = bx\, e^{at}$$

$$\frac{d}{dt}\left(y e^{at}\right) = bx\, e^{at} \quad / \int_{t_0}^{t} d\tau$$

$$y(\tau) e^{a\tau} \Big|_{t_0}^{t} = b \int_{t_0}^{t} x(\tau) e^{a\tau} d\tau$$

$$y(t) = y(t_0)\, e^{-a(t-t_0)} + b \int_{t_0}^{t} e^{-a(t-\tau)} x(\tau)\, d\tau$$

Primer



$i_L(0) = 1\,A$

$R_1 = 1\,\Omega$

$R_2 = 2\,\Omega$

$L = 1\,H$

$v_g(t) = e^{-t} \cdot u(t)$

$i_g(t) = 2 \cdot u(t)$

$i_1 = (-3+t)e^{-t} \cdot u(t)$

$i_{R1} = i_1$

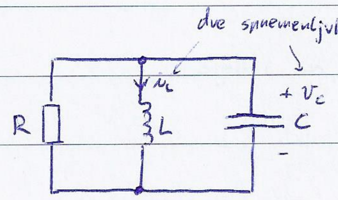
$i_1 = i_{1z} + i_{1vv} + i_{1vt}$

Reševanje sistemov v prostoru stanj

zapišemo več DE prvega reda, teh je toliko, kot je red sistema

13.11.2012

Odziv na začetno stanje



$$L \frac{di_L}{dt} = v_L = v_C$$

$$C \frac{dv_C}{dt} + \frac{v_C}{R} + i_L = 0$$

uvedimo

i_L, v_C

$$\left.\begin{aligned} \frac{dI_L}{dt} &= \frac{v_C}{L} \\ \frac{dv_C}{dt} &= -\frac{i_L}{C} - \frac{v_C}{RC} \end{aligned}\right\} \text{enačbi stanj}$$

i_L, v_C } spremenljivki stanj

$[i_L(0), v_C(0)]$ začetno stanje

$$\underline{x}(t) = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}$$ vektor spremenljivk stanj

dimenzij $n \times 1$

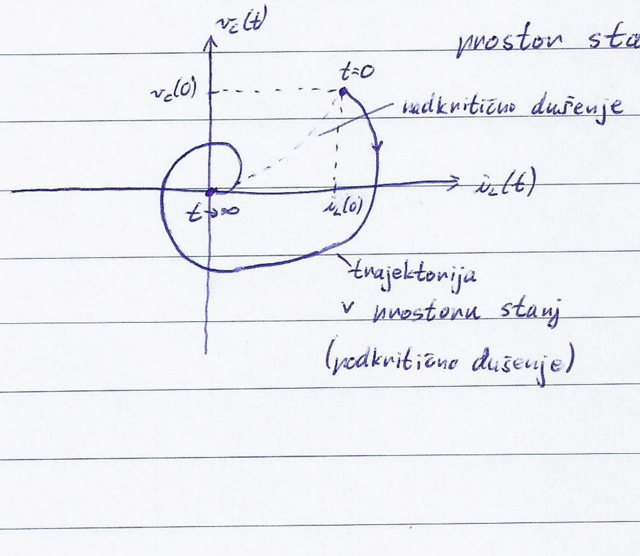
↑ št. dinamičnih elementov oz. spremenljivk stanj

$$\underline{x}(0) = \begin{bmatrix} i_L(0) \\ v_C(0) \end{bmatrix}$$ vektor začetnega stanja

$$\begin{bmatrix} \frac{di_L(t)}{dt} \\ \frac{dv_C(t)}{dt} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{\underline{A} \dots n \times n} \cdot \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix}$$

osn. matrika sistema

prostor stanj



$$\frac{d\underline{x}(t)}{dt} = \underline{\dot{x}}(t) = \underline{A} \cdot \underline{x}(t)$$

homogena enačba stanj

začetno stanje

$$\underline{x}(0) = \underline{X}_0$$

$$\frac{dx}{dt} = ax \quad , \; x(0) \; , \; t \geq 0$$

$$\frac{dx}{x} = a\,dt$$

$x(t) = e^{at} x(0) \; , \; t \geq 0$ rešitev skalarne homogene DE

$$\underline{x}(t) = e^{\underline{A}t} \underline{x}(0) \; , \; t \geq 0$$

matrika

$e^{\underline{A}t} = \underline{\phi}(t)$ ... matrika prehajanja stanj

kot preslikava iz $\underline{x}(t_0)$ v $\underline{x}(t)$

če $\underline{X}_0 = \underline{x}(t_0)$, posplošeno

$$\underline{x}(t) = e^{\underline{A}(t-t_0)} \underline{x}(t_0) \quad , \quad 0 \leq t_0 \leq t$$

1) Osnovne lastnosti matrike prehajanja stanj

1) $\phi(t_0 - t_0) = \phi(0) = I$

2) $\phi(t_1 - t_0) = [\phi(t_0 - t_1)]^{-1}$, $[\phi(t)]^{-1} = \phi(-t)$

3) $\phi(t_1 + t_2) = \phi(t_1)\,\phi(t_2) = \phi(t_2)\,\phi(t_1)$

4) $\phi(t_2 - t_1)\,\phi(t_1 - t_0) = \phi(t_2 - t_0)$ za vsak t_0, t_1, t_2

Približno računanje poti v prostoru stanj

$\frac{dx(t)}{dt} = \underline{A}\,\underline{x}(t)$ zapis predstavlja "hitrost" $x(t)$ v času t

Δt, $t=0$

$$\frac{d\underline{x}(0)}{dt} = \underline{A}\,\underline{x}(0) = \underline{A}\cdot\underline{x}_0$$

$$x(\Delta t) \doteq \underline{x}_0 + \frac{d\underline{x}(0)}{dt}\Delta t = x_0 + Ax_0\Delta t$$

$$x(2\Delta t) \doteq x(\Delta t) + x(\Delta t)\cdot\Delta t$$

$$\vdots$$

$$\underline{x}[(k+1)\Delta t] = \underline{x}(k\Delta t) + \underline{A}\,\underline{x}(k\Delta t)\Delta t = \underline{\underline{(I + \underline{A}\Delta t)\,\underline{x}(k\Delta t)}} \qquad k=0,1,2\ldots N$$

Primer: RLC vezje

$R = 1\Omega$ $\qquad i_L(0) = 1A$

$L = 1H$ $\qquad v_C(0) = 1V$

$C = 1F$

$$\underline{x} = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

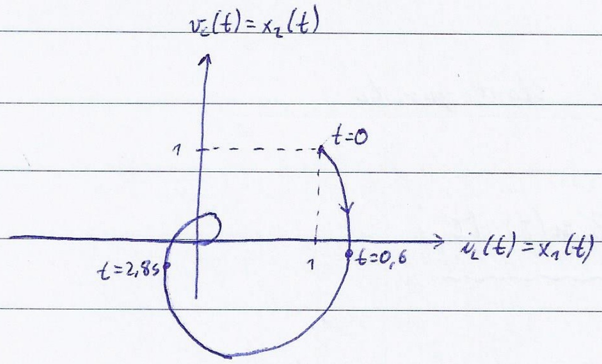
$$\underline{\dot{x}}(t) = \underline{A}\cdot\underline{x}(t)$$

korak $\Delta t = 0{,}2\,s$

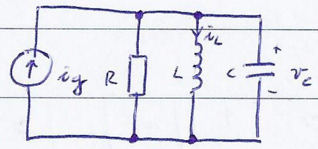
1) $$\begin{bmatrix} x_1(0{,}2) \\ x_2(0{,}2) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{x(0)} + \underbrace{0{,}2}_{\Delta t}\underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{x(0)} = \begin{bmatrix} 1.2 \\ 0.6 \end{bmatrix}$$

2) $$\begin{bmatrix} x_1(0.4) \\ x_2(0.4) \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.6 \end{bmatrix} + 0.2\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1.2 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 1.32 \\ 0.24 \end{bmatrix}$$

posplošeno: $$\underline{x}[(k+1)\Delta t] = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 0.2\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}\right)\underline{x}(k\Delta t) = \begin{bmatrix} 1 & 0.2 \\ -0.2 & 0.8 \end{bmatrix}\underline{x}(k\Delta t)$$



Celotni odziv v prostoru stanj



za spremenljivke stanj (ponavadi) izberemo veličine dinamičnih elementov

$$\frac{di_L}{dt} = \frac{v_C}{L}$$

$$i_g = C\frac{dv_C}{dt} + \frac{v_C}{R} + i_L$$

$$\frac{dv_C}{dt} = -\frac{i_L}{C} - \frac{v_C}{RC} + \underbrace{\frac{i_g}{C}}_{\text{vzbujanje}}$$

ostane enaka!

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} i_g$$

$$\underline{x} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

vektor vzbujanja!

$$\hookrightarrow \quad \dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

vhodna matrika

matrika prehajanja stanj

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} \quad / \, e^{-At} \qquad , \quad \underline{x}(0) = \underline{X}_0$$

$$e^{-At}\dot{\underline{x}} = e^{-At}\,\underline{x} + e^{-At}\,\underline{B}\,\underline{u}$$

14.11.2012

$$\frac{d}{dt}\left[e^{-At}\underline{x}(t)\right] = e^{-At}\,\underline{B}\,\underline{u}$$

$$e^{-At}\underline{x}(t) = \int_0^t e^{-A\tau}\,B\,u(\tau)\,d\tau + \underbrace{K}_{\underline{x}(0)} \quad / \cdot e^{At}$$

$$\boxed{\underline{x}(t) = \underbrace{e^{At}\,\underline{x}(0)}_{\substack{\text{odziv} \\ \text{na zač.} \\ \text{stanje}}} + \underbrace{\int_0^t e^{A(t-\tau)}\,\underline{B}\,\underline{u}(\tau)\,d\tau}_{\substack{\text{odziv na} \\ \text{vzbujanje}}}}$$

že $\underline{x}_0 = \underline{x}(t_0)$ $\quad 0 \le t_0 \le t$ $\quad$ štart pri t_0

$$x(t) = e^{A(t-t_0)} x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)} B\, u(\tau)\, d\tau$$

Primer 1) poiščemo matriko prehajanja stanj

$R = \frac{2}{3}\Omega$ $\quad i_L(0) = 1\,A$

$L = 1\,H$ $\quad v_C(0) = 1\,V$

$C = \frac{1}{2}F$ $\quad i_g(t) = u(t)$ enotina stopnica

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} i_g = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 2 \end{bmatrix}}_{B} i_g$$

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

$$x(t) = e^{At}\, \underline{x}(0) + \int_0^t e^{A(t-\tau)}\, \underline{B} \cdot \underline{u}(\tau)\, d\tau$$

enotska matrika

$$\phi(t) = e^{At} = I + At + \frac{1}{2!}A^2t^2 + \frac{1}{3!}A^3t^3 + \dots$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 6 & 7 \\ -14 & -15 \end{bmatrix}$$

$$\phi(t) = e^{At} = \begin{bmatrix} 1 - t^2 + t^3 + \dots & t - \frac{3}{2}t^2 + \frac{7}{6}t^3 + \dots \\ -2t + 3t^2 - \frac{7}{3}t^3 + \dots & 1 - 3t + \frac{7}{2}t^2 - \frac{5}{2}t^3 + \dots \end{bmatrix}$$

"hitro" se da videti!

$$\phi(t) = e^{At} \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = e^{\underline{A}t} X(0) + \int_0^t e^{\underline{A}(t-\tau)} \cdot B \cdot u(\tau)\, d\tau =$$

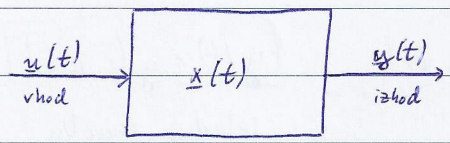
$$= e^{\underline{A}t} \underline{x}(0) + \int_0^t \begin{bmatrix} 2e^{-(t-\tau)} - 2e^{-2(t-\tau)} \\ -2e^{-(t-\tau)} + 4e^{-2(t-\tau)} \end{bmatrix} d\tau$$

$$\underline{x}(t) = \underbrace{\begin{bmatrix} 3e^{-t} - 2e^{-2t} \\ -3e^{-t} + 4e^{-2t} \end{bmatrix}}_{\text{odziv na začetno stanje}} + \underbrace{\begin{bmatrix} 1 - 2e^{-t} + e^{-2t} \\ 2e^{-t} - 2e^{-2t} \end{bmatrix}}_{\text{odziv na vhodni signal}}, \quad t \geq 0$$

Zasnova enačb v prostoru stanj

Pri obravnavi sistemov v prostoru stanj imamo opravka s tremi vrstami spremenljivk:

- vhodne spremenljivke $\underline{u}(t)$, dim $r \times 1$ (št vhodov)
- izhodne spremenljivke $\underline{y}(t)$, dim $m \times 1$ (št izhodov)
- spremenljivke stanj $\underline{x}(t)$, dim $n \times 1$



Možni so različni zapisi istega sistema v pr. stanj, vendar je št spremenljivk v pr. stanj pri vseh enako. Ker dinamični sistem vsebuje spominske elemente, so izhodi odvisni od zgodovine vhoda. Pri zveznih din. vezjih so spominski elementi integratorji. Ker integratorji določajo obnašanje sistema, izberemo njihove izhode kot spremenljivke stanj. Pri analizi vezij sta spremenljivki i_L in v_C. Za popoln opis vedenja din. sistema potrebujemo n spremenljivk, ki predstavljajo vektor spremenljivk stanj x. Vektor spremenljivk stanje, je vektor, ki določa stanje sistema $\underline{x}(t)$ za

vsak trenutek $t \geq t_0$, potem ko je znano začetno stanje $\underline{x}(t_0) = \underline{x}_0$ in vhodni signal $\underline{u}(t)$ na celotnem časovnem intervalu $[t_0, t)$.
V splošnem so izhodne spremenljivke lahko različne od spremenljivk stanj, vendar jih lahko izrazimo s spremenljivkami stanj $x_1, x_2 \ldots x_n$ in z vhodnimi spremenljivkami $u_1, u_2, \ldots u_r$.

r vhodov
m izhodov
n spr. stanj

$$\left.\begin{aligned} \dot{x}_1(t) &= f_1(x_1, x_2 \ldots x_n; u_1, u_2 \ldots u_r; t) \\ &\vdots \\ \dot{x}_n(t) &= f_n(x_1, x_2 \ldots x_n; u_1, u_2 \ldots u_r; t) \end{aligned}\right\} f(x,u,t)$$

splošen zapis

$$y_1(t) = g_1(x_1, x_2 \ldots x_n; u_1, u_2 \ldots u_r; t)$$
$$\vdots$$
$$y_m(t) = g_m(x_1, x_2 \ldots x_n; u_1, u_2 \ldots u_r; t)$$

$$\underline{x}(t) = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad \underline{y}(t) = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \qquad \underline{u}(t) = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_r \end{bmatrix}$$

$$\underline{f}(\underline{x}, \underline{u}, t) = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} \qquad \underline{g}(\underline{x}, \underline{u}, t) = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix}$$

$$\boxed{\dot{\underline{x}}(t) = \underline{f}(\underline{x}, \underline{u}, t)} \qquad \boxed{\underline{y}(t) = \underline{g}(\underline{x}, \underline{u}, t)}$$

enačba stanj — izhodna enačba

Če je sistem linearen ali če obravnavamo lineariziran sistem, se enačbe poenostavijo.

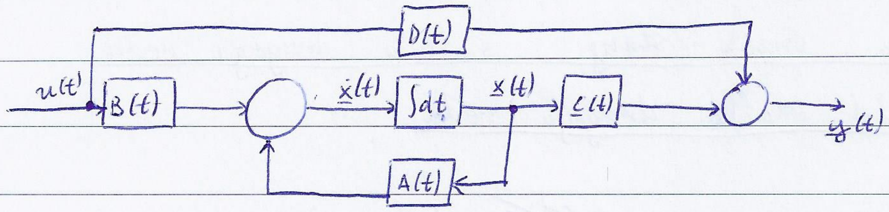
$$\dot{\underline{x}}(t) = \underline{A}(t)\underline{x}(t) + \underline{B}(t)\underline{u}(t)$$
$$\underline{y}(t) = \underline{C}(t)\underline{x}(t) + \underline{D}(t)\underline{u}(t)$$

$\underline{A}$ ... matrika stanj

$\underline{B}$ ... vhodna matrika

$\underline{C}$ ... izhodna matrika

$\underline{D}$ ... vhodno-izhodna matrika

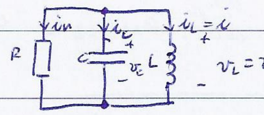


Če imamo LTI sistem velja zapis $\boxed{\underline{\dot{x}}(t) = \underline{A}\,\underline{x}(t) + \underline{B}\,\underline{u}(t)}$ enačba stanj v normalni obliki

$$\underline{y}(t) = \underline{C}\,\underline{x}(t) + \underline{D}\,\underline{u}(t)$$

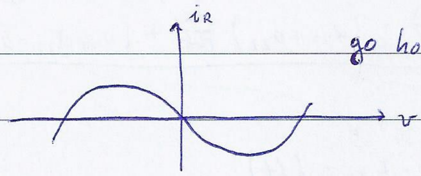
Če je $\underline{y} = \underline{x}$, velja $\underline{y}(t) = \underline{C}\,\underline{x} = \underline{I}\,x(t) = x(t)$

Primer



$$i_R = g(v) = -\alpha v + \beta v^3$$

$\alpha, \beta \ldots$ konst



go home resistor, you're drunk!

$x_1 = i_L = i$

$x_2 = v_C = v$

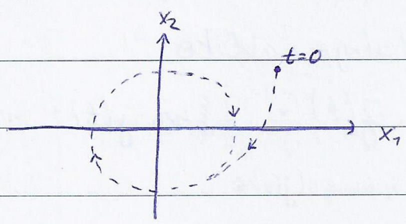
$$L\frac{di_L}{dt} = v \qquad \frac{di_L}{dt} = \frac{v}{L}$$

$$i_C = -i_L - i_R$$

$$i_C = C\frac{dv_C}{dt}$$

$$\frac{dv}{dt} = -\frac{i_L}{C} - \frac{g(v)}{C}$$

$$\underline{\dot{x}} = \begin{bmatrix} \frac{di}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} \frac{v}{L} \\ -\frac{i}{C} - \frac{g(v)}{C} \end{bmatrix} = f(\underline{x})$$



Pretvorba enačb stanj sistema drugega reda v dif. enačbo drugega reda

$$\frac{dx_1}{dt} = a_{11} x_1 + a_{12} x_2 + u_1(t) \rightarrow \left(\frac{d}{dt} - a_{11}\right)x_1 - a_{12} x_2 = u_1(t) \;\Big|\; \left(\frac{d}{dt} - a_{22}\right)$$

$$\frac{dx_2}{dt} = a_{21} x_1 + a_{22} x_2 + u_2(t) \rightarrow -a_{21} x_1 + \left(\frac{d}{dt} - a_{22}\right)x_2 = u_2(t) \;\Big|\; a_{12}$$

$$\Downarrow$$

$\dot{x} = Ax + B \cdot u$

$\downarrow$

2×2

$$\left(\frac{d}{dt} - a_{22}\right)\left(\frac{d}{dt} - a_{11}\right)x_1 - \left(\frac{d}{dt} - a_{22}\right)a_{12} x_2 = u_1(t)\left(\frac{d}{dt} - a_{22}\right)$$

1) $\frac{d^2x_1}{dt} - (a_{11}+a_{22})\frac{dx_1}{dt} + a_{11}a_{22}x_1 - \cancel{a_{12}\frac{dx_2}{dt}} + \cancel{a_{12}a_{22}x_2} = \frac{du_1}{dt} - a_{22}u_1$

+

2) $-a_{12}a_{21}x_1 + \cancel{a_{12}\frac{dx_2}{dt}} - \cancel{a_{12}a_{22}x_2} = a_{12}\, u_2(t)$

$$\Downarrow$$

$$\boxed{\frac{d^2x_1}{dt^2} - (a_{11}+a_{22})\frac{dx_1}{dt} + (a_{11}a_{22} - a_{12}a_{21})x_1 = \frac{du_1}{dt} - a_{22}u_1 + a_{12}u_2}$$

za x_2 upoštevamo simetrija, zamenjamo indekse

$$\frac{d^2x_2}{dt^2} - (a_{11}+a_{22})\frac{dx_2}{dt} + (a_{11}a_{22} - a_{12}a_{21})x_2 = \frac{du_2}{dt} - a_{11}u_2 + a_{21}u_1$$

splošno

$$\frac{d^2x}{dt} + 2\sigma\frac{dx}{dt} + \omega_n^2 x = f(t)$$

$\sigma = -\frac{1}{2}(a_{11}+a_{22}) = \xi\omega_n$

$\omega_n = \sqrt{a_{11}a_{22} - a_{12}a_{21}}$

$f(t)$:

x_1: $f_1(t) =$

Mnoge fizikalne sisteme (RLC vezje ali nek k) lahko modeliramo z DE drugega reda naslednje oblike

$$\ddot{y}(t) + 2\xi\omega_n\dot{y}(t) + \omega_n^2 y(t) = K\cdot u(t)$$

uvedemo spremenljivke

$x_1 = y$ $\qquad \dot{x}_2 = -2\xi\omega_n x_2 - \omega_n^2 x_1 + K\cdot u(t)$

$\dot{x}_1 = \dot{y}$ $\nearrow$ $\qquad \dot{x}_1 = x_2$

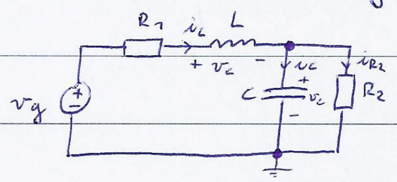
$x_2 = \dot{y} = \dot{x}_1$

$\dot{x}_2 = \ddot{y}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K \end{bmatrix} u(t)$$

$$\underline{\dot{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

<u>Primer:</u> zapišimo enačbe stanj za vezje $\dot{x} = Ax + Bu$



ni spremenljivka stanj!

$$\text{spremenljivki}\quad \underline{X} = \begin{bmatrix} v_c(t) \\ i_L(t) \end{bmatrix}$$

$$\frac{di_L}{dt} = \frac{v_L}{L} = \frac{v_g - i_L R_1 - v_c}{L} = \frac{v_g}{L} - \frac{i_L R_1}{L} - \frac{v_c}{L}$$

$$\frac{dv_c}{dt} = \frac{i_c}{C} = \frac{i_L - i_{R2}}{C} = \frac{i_L}{C} - \frac{v_c}{R_2 C}$$

$\cancel{v_c(t) = x_1}$ $\quad i_L(t) = x_1$

$\cancel{v_L(t) = x_2}$ $\quad v_c(t) = x_2$

$$\cancel{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{L} & -\frac{R_1}{L} \\ & \end{bmatrix}}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_2 C} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} +$$

če rečemo

$$\begin{bmatrix} \frac{dv_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2 C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_1}{L} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_g$$

Zapišite enačbo drugega reda za v_c

$$\frac{d^2 v_c}{dt} + \Big(\underset{a_{11}}{\frac{1}{R_2 C}} + \underset{a_{22}}{\frac{R_1}{L}}\Big)\frac{dv_c}{dt} + \Big(\frac{R_1}{R_2 C L} + \frac{1}{LC}\Big) v_c = \frac{1}{LC} v_g$$

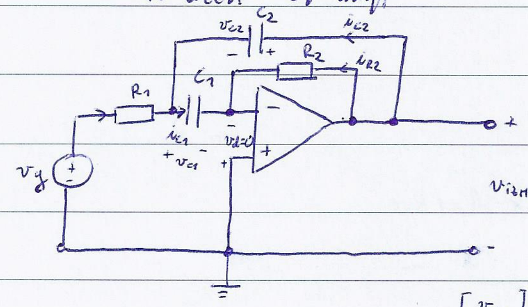
$$u_1 = 0$$

$$u_2 = \frac{1}{L} v_g$$

Akta®

Primer v_{C1}, v_{C2}, v_{izH}

idealen op-amp



spremenljivki $\underline{X} = \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix}$, $v_{izH} = ?$

$v_{izH} = v_{C1} + v_{C2}$

$$\frac{dv_{C1}}{dt} = \frac{i_{C1}}{C_1} = \frac{-1}{R_2 C_1}(v_{C1} + v_{C2}) \qquad v_{C1} - v_{R2} + v_{C2} = 0$$

$$\frac{dv_{C2}}{dt} = \frac{i_{C2}}{C_2}$$

$$i_{C1} = -i_{R2}, \quad v_{R2} = R_2 i_{R2} = -R_2 i_{C1} \rightarrow i_{C1} = -\frac{v_{R2}}{R_2}$$

$$i_{C1} = -\frac{1}{R_2}(v_{C1} + v_{C2})$$

$$+ i_{C2} - i_{C1} + \frac{v_g - v_{C1}}{R_1} = 0$$

$$i_{C2} = -\frac{1}{R_2}(v_{C1} + v_{C2}) - \frac{1}{R_1}(v_g - v_{C1})$$

$$i_{C2} = v_{C1}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) - v_{C2}\left(\frac{1}{R_2}\right) - \frac{v_g}{R_1}$$

$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2 C_1} & -\frac{1}{R_2 C_1} \\ \frac{1}{R_1 C_2} - \frac{1}{R_2 C_2} & -\frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{R_1 C_2} \end{bmatrix} v_g$$

$$\frac{d^2 v_{C1}}{dt^2} + \left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right)\frac{dv_{C1}}{dt} + \frac{1}{R_1 R_2 C_1 C_2} v_{C1} = \frac{1}{R_1 R_2 C_1 C_2} v_g$$

$u_1 = 0$

$u_2 = -\frac{1}{R_1 C_2} \cdot v_g$

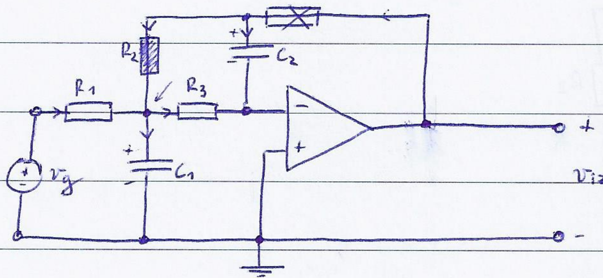
$$\frac{d^2 v_{C2}}{dt^2} + (\;-||-\;)\frac{dv_{C2}}{dt} + (-||-)\, v_{C2} = -\frac{1}{R_1 C_2} - \frac{1}{R_1 R_2 C_1 C_2} v_g$$

seštejemo

$$\frac{d^2}{dt^2}\underbrace{(v_{C1} + v_{C2})}_{v_{izH}} + \underbrace{\left(\frac{1}{R_2 C_1} + \frac{1}{R_2 C_2}\right)}_{2\delta = 2\xi\omega_n}\frac{d}{dt}\underbrace{(v_{C1} + v_{C2})}_{v_{izH}} + \frac{\overbrace{v_{C1} + v_{C2}}^{v_{izH}}}{R_1 R_2 C_1 C_2} = -\frac{1}{R_1 C_2}\frac{dv_g}{dt}$$

Lastna frekvenca ω_n^2

Primer v_{izh}



nizkopasovno sito

spremenljivke $\underline{X} = \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix}$

$$\frac{dv_{C1}}{dt} = \frac{i_{C1}}{C_1}$$

$$\frac{dv_{C2}}{dt} = \frac{i_{C2}}{C_2}$$

vozlišče $i_{R1} + i_{R2} = i_{R3} + i_{C1}$

$i_{C1} = -i_{R3} + i_{R1} + i_{R2}$

• $i_{C1} = -\frac{v_{C1}}{R_3} + \frac{v_g - v_{C1}}{R_1} + \frac{v_{C2} - v_{C1}}{R_2}$

$i_{C2} = -i_{R3}$

$i_{R3} = \frac{v_{C1}}{R_3}$

• $i_{C2} = -\frac{v_{C1}}{R_3}$

$i_{C1} = -v_{C1}\left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2}\right) + v_{C2}\left(+\frac{1}{R_2}\right) + \frac{v_g}{R_1}$

$$\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C_1} - \frac{1}{R_2C_1} - \frac{1}{R_3C_1} & +\frac{1}{R_2C_1} \\ -\frac{1}{R_3C_2} & 0 \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C_1} \\ 0 \end{bmatrix} v_g$$

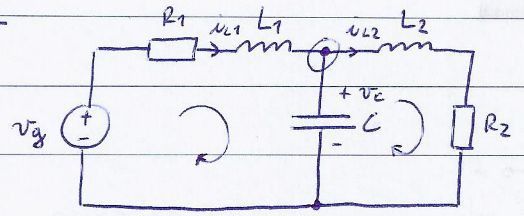
$v_{izh} =$ ~~[illegible]~~ v_{C2}

$$\frac{d^2 v_{izh}}{dt^2} + \frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\frac{dv_{izh}}{dt} + \frac{1}{R_2R_3C_1C_2} v_{izh} = -\frac{1}{R_2R_3C_1C_2} v_g$$

$u_1 = \frac{1}{R_1C_1} v_g$

$u_2 = 0$

20.11.2017 Primer



$$\underline{x} = \begin{bmatrix} i_{L1} \\ i_{L2} \\ u_c \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\underline{y} = \begin{bmatrix} v_{R1} \\ v_{R2} \end{bmatrix}$$

$$v_g(t) = R_1 x_1 + L_1 \frac{dx_1}{dt} + x_3$$

$$x_1 = x_2 + C\frac{dx_3}{dt}$$

$$x_3 = L_2 \frac{dx_2}{dt} + R_2 x_2$$

$$\frac{dx_1}{dt} = -\frac{R_1}{L_1}x_1 - \frac{x_3}{L_1} + \frac{v_g}{L_1}$$

$$\frac{dx_3}{dt} = \frac{x_1}{C} - \frac{x_2}{C}$$

$$\frac{dx_2}{dt} = \frac{x_3}{L_2} - \frac{R_2}{L_2}x_2$$

$$A = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix}$$

$$\underline{y} = \underline{C}\underline{x} + \underline{D}\underline{u}$$

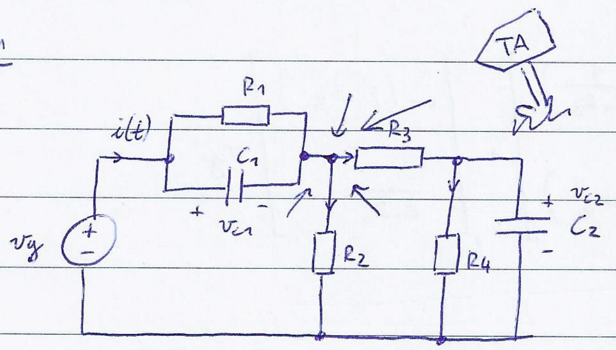
$$B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix}$$

$$v_{R1} = R_1 x_1$$

$$v_{R2} = R_2 x_2$$

$$\underline{y} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} [\overset{v_g}{u}]$$

Primer



za analizo

$v_g = 10 \cdot u(t)$

$R_1 = R_2 = 1$

$R_3 = 2$

$R_4 = 0.5$

$C_1 = 2$

$C_2 = 4$

$$\underline{x} = \begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix}$$

$y = i(t)$ $\qquad \underline{\dot{x}} = A x + B u$

$$v_g = v_{C1} + R_3 \overbrace{\left(\frac{v_{C2}}{R_4} + C_2 \frac{dv_{C2}}{dt}\right)}^{i_{R3}} + v_{C2}$$

$$i(t) = \frac{v_{C1}}{R_1} + C_1 \frac{dv_{C1}}{dt} \overset{?}{=} \frac{R_3}{R_2}\left(\frac{v_{C2}}{R_4} + C_2 \frac{dv_{C2}}{dt}\right) + \frac{v_{C2}}{R_2} + \frac{v_{C2}}{R_4} + C_2 \frac{dv_{C2}}{dt}$$

? $\hookrightarrow$ $$v_g = v_{C1} + \left(\frac{R_3}{R_4} + 1\right) v_{C2} + C_2 R_3 \frac{dv_{C2}}{dt}$$

? $\hookrightarrow$ $$\frac{v_{C1}}{R_1} + C_1 \frac{dv_{C1}}{dt} = \left(\frac{R_3}{R_2 R_4} + \frac{1}{R_2} + \frac{1}{R_4}\right) v_{C2} + \left(1 + \frac{R_3}{R_2}\right) C_2 \frac{dv_{C2}}{dt}$$

(2) $$\frac{dv_{C2}}{dt} = -\frac{1}{R_3 C_2} v_{C1} - \left(\frac{1}{R_4 C_2} + \frac{1}{R_3 C_2}\right) v_{C2} + v_g \frac{1}{R_3 C_2}$$

$$C_1 \frac{dv_{C1}}{dt} = -\frac{v_{C1}}{R_1} + \left(\frac{R_3}{R_2 R_4} + \frac{1}{R_2} + \frac{1}{R_4}\right) v_{C2} + \left(1 + \frac{R_3}{R_2}\right)\Big[-\frac{1}{R_3 C_2} v_{C1} -$$
$$- \left(\frac{1}{R_4 C_2} + \frac{1}{R_3 C_2}\right) v_{C2}\Big] C_2 + \left(1 + \frac{R_3}{R_2}\right) C_2 v_g \frac{1}{R_3 C_2}$$

$$\frac{dv_{C1}}{dt} = -\left[\frac{1}{R_1 C_1} + \left(1 + \frac{R_3}{R_2}\right)\frac{1}{R_3 C_1}\right] v_{C1} + \left[\left(\frac{R_3}{R_2 R_4} + \frac{1}{R_2} + \frac{1}{R_4}\right)\frac{1}{C_1} - \left(\frac{1}{R_4 C_1} + \frac{1}{R_3 C_1}\right) \cdot \left(1 + \frac{R_3}{R_2}\right)\right] v_{C2} + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)\frac{1}{C_1} v_g$$

(1) $$\frac{dv_{C1}}{dt} = -\left[\frac{1}{C_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\right] v_{C1} - \frac{1}{R_3 C_1} v_{C2} + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)\frac{1}{C_1} v_g$$

$$\begin{bmatrix} \frac{dv_{c1}}{dt} \\ \frac{dv_{c2}}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) & -\frac{1}{C_1 R_3} \\ -\frac{1}{C_2 R_3} & -\frac{1}{C_2}\left(\frac{1}{R_3}+\frac{1}{R_4}\right) \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1}\left(\frac{1}{R_2}+\frac{1}{R_3}\right) \\ \frac{1}{C_2 R_3} \end{bmatrix} v_g$$

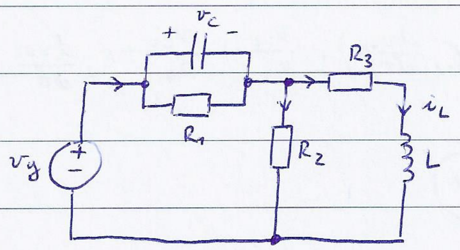
$y(t) = i(t)$

$$i(t) = \underline{C}\underline{x} + \underline{D}\underline{u}$$

$$i(t) = \frac{v_{c1}}{R_1} + C_1 \frac{dv_{c1}}{dt} = \dots\dots = -\frac{3}{2} v_{c1} - \frac{1}{2} v_{c2} + 1.5\, v_g$$

$$C = \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \qquad D = \begin{bmatrix} \frac{3}{2} \end{bmatrix}$$

D.N.



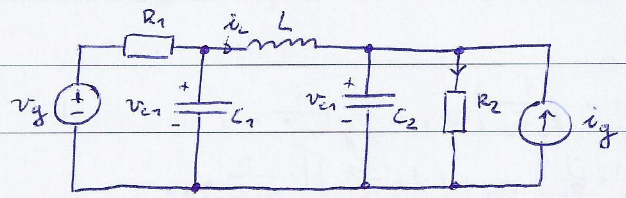
$R_1 = R_2 = 1\ \Omega$

$R_3 = \frac{1}{2}\ \Omega$

$L = \frac{1}{2}\ H$

$C = 2\ F$

$$\underline{x} = \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$



$R_1 = \frac{1}{4}\ \Omega$

$R_2 = 2\ \Omega$

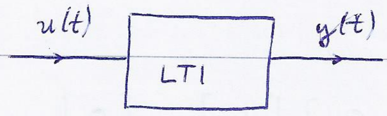
$L = 2H$

$C_1 = C_2 = \frac{1}{2}\ F$

$$\underline{x} = \begin{bmatrix} i_L \\ v_{c1} \\ v_{c2} \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} i_g \\ v_g \end{bmatrix}$$

Pretvorba DE višjega reda v prostor stanj

21.11.2012



SISO

$$y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 \dot{y} + a_0 y = b_0 u$$

za spremenljivke izberemo y in odvode

dva primera - na desni odvodi in brez odvodov

$$y(0) \;,\; y^{(n-1)}(0) \;,\; u(t) \;,\; t \geq 0$$

to izberemo za spremenljivke stanj

$$x_1 = y \qquad \dot{x}_1 = x_2$$

$$x_2 = \dot{y} \qquad \dot{x}_2 = x_3$$

$$\vdots \qquad \vdots$$

$$x_n = y^{(n-1)} \qquad \dot{x}_{n-1} = x_n$$

$$\dot{x}_n = y^{(n)} = b_0 \cdot u - a_{n-1} y^{(n-1)} - \dots - a_1 \dot{y} - a_0 y$$

$$= b_0 u - a_0 x_1 - a_1 x_2 - \dots - a_{n-1} x_n$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & & & & & & \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & & -a_{n-2} & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ b_0 \end{bmatrix} u$$

Kadar spremenljivke prostora stanj lahko zapišemo v tej obliki, jih imenujemo fazne spremenljivke, prostor pa fazni prostor.

$$y = x_1 \qquad y = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dots & 0 \end{bmatrix} u$$

Primer

$$\dddot{y} + 3\ddot{y} + 4\dot{y} + y = 7u$$

1) izberemo spremenljivke

$x_1 = y$

$x_2 = \dot{y}$

$x_3 = \ddot{y}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Drugič, če ima DE na desni strani odvode:

$$y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 \dot{y} + a_0 y = b_m u^{(m)} + \dots + b_1 \dot{u} + b_0 u$$

recimo, da $m \leq n$

Izberemo druge spremenljivke (prostor stanj ne dopušča odvodov na desni).

$\boxed{y_1 = x_1 + \beta_0 u}$ n-krat odvajamo $\longrightarrow$ $\dot{y} = \dot{x}_1 + \beta_0 \dot{u} = x_2 + \beta_1 u + \beta_0 \dot{u}$

$\dot{x}_1 = x_2 + \beta_1 u$ $\qquad$ $\ddot{y} = \dot{x}_2 + \beta_1 \dot{u} + \beta_0 \ddot{u} =$

$= x_3 + \beta_2 u + \beta_1 \dot{u} + \beta_0 \ddot{u}$

$\dot{x}_2 = x_3 + \beta_2 u$ $\qquad$ $\dddot{y} = \dot{x}_3 + \beta_2 \dot{u} \dots =$

....

$= x_4 + \beta_3 u + \beta_2 \dot{u} + \beta_1 \ddot{u} + \beta_0 \dddot{u}$

$\dot{x}_n = -a_0 x_1 - a_1 x_2 - \dots - a_{n-1} x_n + \beta_n u$

$\vdots$

$y^{(n-1)} = x_n + \beta_{n-1} u + \beta_{n-2} \dot{u} + \dots + \beta_1 u^{n-2} + \beta_0 u^{(n-1)}$

$y^{(n)} = \dot{x}_n + \beta_{n-1} \dot{u} + \beta_{n-2} \ddot{u} + \dots \beta_1 u^{(n-1)} + \beta_0 u^{(n)}$

Po vstavitvi odvodov v izhodiščno enačbo opazimo, da izpadejo členi $a_0 x_1 \dots a_{n-1} x_n$ in, da se leva stan enačba reducira na linearno kombinacijo u-ja in njegovih odvodov, koeficienti so konst a_i in β_i. Na desni imamo podobno kombinacijo, tu so koeficienti β_i.
Po identifikaciji dobimo...

$$u^{(n)}: \quad b_n = \beta_0$$
$$u^{(n-1)}: \quad b_{n-1} = \beta_1 + a_{n-1}\beta_0$$
$$u^{(n-2)}: \quad b_{n-2} = \beta_2 + a_{n-2}\beta_0 + a_{n-1}\beta_1$$
$$\vdots$$
$$u: \quad b_0 = \beta_n + a_0\beta_0 + a_1\beta_1 + \dots + a_{n-1}\beta_{n-1}$$

Kako določimo β_i

$$\beta_0 = b_n$$
$$\beta_1 = b_{n-1} - a_{n-1}\beta_0$$
$$\beta_2 = b_{n-2} - a_{n-2}\beta_0 - a_{n-1}\beta_1$$
$$\dots$$
$$\beta_n = b_0 - a_0\beta_0 - a_1\beta_1 - a_2\beta_2 - \dots - a_{n-1}\beta_{n-1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & & \\ & & & & & \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-2} & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \beta_0 u$$

Začetni pogoj $\underline{x}(0)$

$$x_1(0) = y(0) - B_0 u(0)$$

$$x_2(0) = \dot{y}(0) - B_0 \dot{u}(0) - B_1 u(0)$$

$$\vdots$$

$$x_n(0) = y^{(n-1)}(0) - B_0 u^{(n-1)} - B_1 \overset{?}{u^{(n-2)}} - \ldots - B_{n-2}\dot{u} - B_{n-1} u$$

Primer

$$\dddot{y} + 3\ddot{y} + 4\dot{y} + y = 2\ddot{u} + 5\dot{u} + 7u$$

$$B_0 = \overset{n=3}{b_n} = 0$$

$$B_1 = b_{n-1} - a_{n-1} B_0 = 2$$

$$B_2 = b_{n-2} - a_{n-2} B_0 - a_{n-1} B_1 = -1$$

$$B_3 = b_{n-1} - a_{n-3} B_0 - a_{n-2} B_1 - a_{n-1} B_2 = 2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \overset{B_0}{[0]}\, u$$

matlab: $[A, B, C, D] = tf2ss\underbrace{(\overset{\text{števec}}{num}, \overset{\text{imenovalec}}{den})}_{H(p)}$ → v obratni smeri $[num, den] = ss2tf(a, b, c, d, iu)$

$$H(p) = \frac{2p^3 + 9p^2 + 14p + 30}{p^3 + 4p^2 + 6p + 12} = \frac{P(p)}{Q(p)}$$

$$\dddot{y} + 4\ddot{y} + 6\dot{y} + 12y = 2\dddot{u} + 9\ddot{u} + 14\dot{u} + 30u$$

$B_0 = 2$ $\quad B_3 = 8$

$B_1 = 1$

$B_2 = -2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -6 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 8 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [2]\, u$$

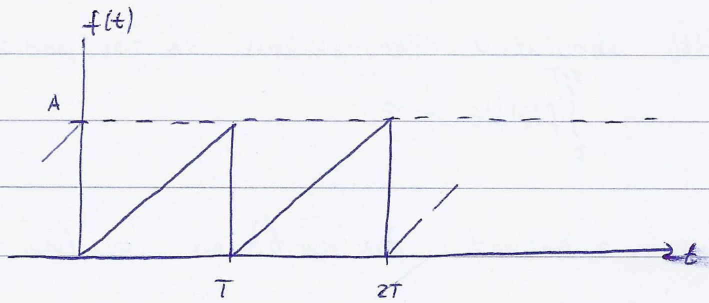
Spektralna analiza

Naredimo vsoto lažje rešljivih problemov.

(kot konvolucija)

Spektralna analiza temelji na principu superpozicije, ki velja za LTI. Omogoča, da zapletene čas. fje izrazimo z vsoto enostavnih, osnovnih funkcij. Pri spektralni analizi so to harmonične fje. Množico harm. signalov, ki v vsoti predstavljajo poljuben signal, imenujemo spekter, posamezne komponente pa harmonske komponente.

Primer



$$f(t) = \frac{A}{T} \cdot t \qquad 0 \le t \le T$$

$$f(t \pm nT) = f(t) \qquad n = 1, 2 \ldots$$

to je baje problem, lahko pa uporabimo fourierjevo vrsto

Periodične lahko zapišemo v obliki vsote periodičnih fj.

1) fourierjeva trigon. vrsta
   -II- kosinusna vrsta
2) fourierjeva eksponentna vrsta

} za periodične signale

3) fourierjev integral } za vse signale (tudi aperiodične)

Fourierova trigonometrična vrsta

(1) $$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n w_0 t) + b_n \sin(n w_0 t)\right]$$

$$w_0 = \frac{2\pi}{T}$$

T ... osnovna perioda

w_0 ... osnovna frekvenca signala

Zadostni pogoji → Dirichletovi pogoji

1) $f(t)$ sme imeti na intervalu ene periode končno število nezveznosti

2) $f(t)$ sme imeti na intervalu ene periode končno število maximumov in minimumov

3) $f(t)$ absolutno integrabilna na eni periodi

$$\int_{\tau}^{\tau+T} |f(t)|\,dt < \infty$$

Če enačbo (1) pomnožimo z enkrat $\cos(n w_0 t)$ in $\sin(n w_0 t)$ in integriramo $\int_0^T$ se členi pokrajšajo

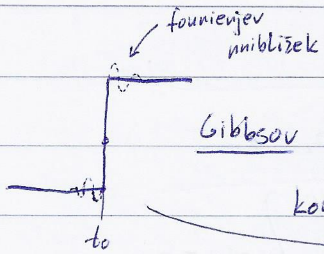
$$\boxed{\begin{aligned} a_0 &= \frac{1}{T}\int_{\tau}^{\tau+T} f(t)\,dt \qquad \text{DC komponenta} \\ a_n &= \frac{2}{T}\int_{\tau}^{\tau+T} f(t)\cos(n w_0 t)\,dt \\ b_n &= \frac{2}{T}\int_{\tau}^{\tau+T} f(t)\sin(n w_0 t)\,dt \end{aligned}}$$

Končno členov →

$$f(t) \doteq f_k(t) = a_0 + \sum_{n=1}^{k}\left[a_n \cos(n w_0 t) + b_n \sin(n w_0 t)\right]$$

$$\overline{e^2(t)} = \frac{1}{T}\int_0^T \left(f(t) - f_k(t)\right)^2 dt$$

$\overline{e^2(t)}$ ↓ najmanjši srednji kvadratični pogrešek približka



<u>Gibbsov</u> fenomen (konica 9 %)

konica zelo kratka, $W \to 0$ $\longrightarrow$ $f(t_0) = \frac{1}{2}\left(f(t_{0+}) - f(t_{0-})\right)$

desna limita ($f(t_{0+})$), leva limita ($f(t_{0-})$)

$|a_n| \leq \frac{M}{n^{k+1}}$, $|b_n| \leq \frac{M}{n^{k+1}}$

$k = 0, 1, 2 \ldots$

k … število odvodov, kjer odvod $f(t)$ postane nezvezen

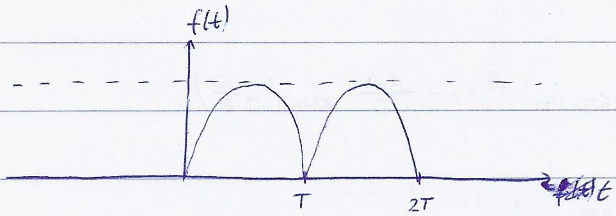
Najpočasneje upada kvadratni / žagasti signal $|a_n| \leq \frac{1}{n}$
(če signal je nezvezen)

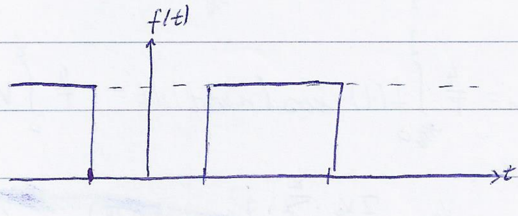
<u>Primer</u> Dobro je pogledati tip funkcije (sodost, lihost ...), ker so nekateri členi 0!

① $f(-t) = f(t)$ soda $\rightarrow b_n = 0$

$$a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\,dt$$

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(n w_0 t)\,dt$$

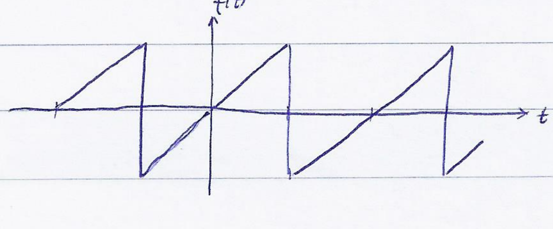




② $f(-t) = -f(t)$ liha $a_0 = 0$

$$a_n = 0$$

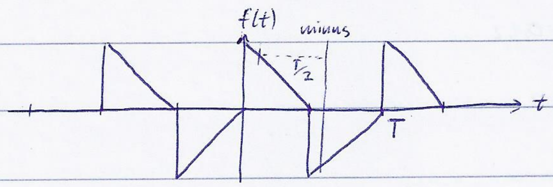
$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(n w_0 t)\,dt$$



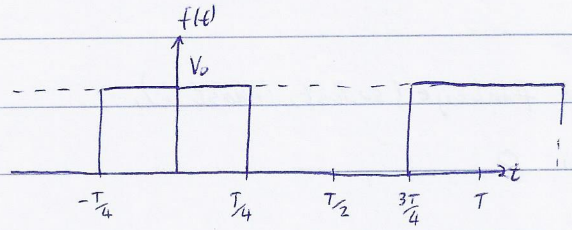
$f(t)$ ... polvalna simetrija

$$f\left(t \pm \frac{T}{2}\right) = -f(t) \qquad a_0 = a_n = b_n = 0\ , \text{ za } n = 0, 2, 4, 6 \ldots$$

$$\left.\begin{aligned} a_n &= \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cos(n\omega_0 t)\,dt \\ b_n &= \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\sin(n\omega_0 t)\,dt \end{aligned}\right\} n = 1, 3, 5, 7 \ldots$$



Primer Pravokotni pulzi ...



soda fja!

$b_n = 0$

$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt = \frac{1}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} V_0\,dt = \frac{V_0}{2}$$

$$a_n = \frac{4}{T}\int_{0}^{\frac{T}{2}} f(t)\cdot\cos(n\omega_0 t)\,dt = \frac{4}{T}\int_0^{\frac{T}{4}} V_0\cos\left(n\frac{2\pi}{T}t\right) = \frac{2V_0}{n\pi}\sin\left(n\frac{\pi}{2}\right)$$

$$f_K = \frac{V_0}{2} + \frac{2V_0}{\pi}\sum_{n=1}^{K}\frac{1}{n}\sin\left(n\frac{\pi}{2}\right)\cos\left(n\frac{2\pi}{T}\cdot t\right)$$

zanimivo!

$$a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \overset{\downarrow}{=} c_n \cos(n\omega_0 t + \varphi_n)$$

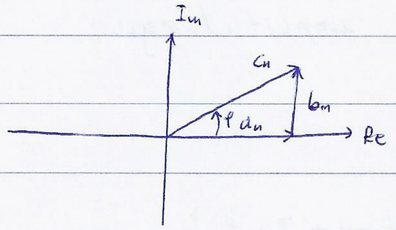
$$a_n - j b_n = c_n e^{j\varphi_n}$$

$$c_n = \sqrt{a_n^2 + b_n^2}$$

↑
amplituda posameznega harmonika

$$\varphi_n = -\arctan \frac{b_n}{a_n} \quad , a_n > 0$$

$$\varphi_n = -\arctan\left(\frac{b_n}{a_n}\right) \pm \pi \,, a_n < 0 \,, +\pi \,, b_n < 0$$



Trigonometrična kosinusna vrsta

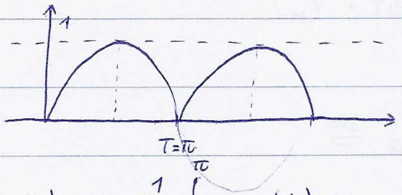
$$f(t) = c_0 + \sum_{n=1}^{\infty} c_n \cdot \cos(n\omega_0 t + \varphi_n)$$

$c_0 = a_0$

c_n ... amplitudni spekter
φ_n ... fazni spekter } črtasta spektra

Primer amplitudni spekter

$$f(t) = |\sin(\omega_0 t)| \qquad \omega_0 = \frac{2\pi}{\pi} = 2\,\frac{rad}{s}$$

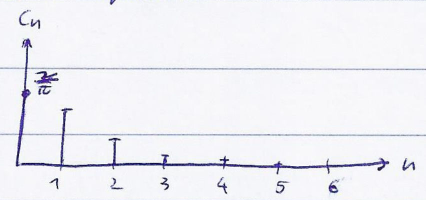


$$a_0 = \frac{1}{T}\int_0^T \sin(\omega_0 t)\,dt = \frac{1}{\pi}\int_0^{\pi} \sin(t)\,dt = -\frac{1}{\pi}\cos(t)\Big|_0^{\pi} = \frac{2}{\pi}$$

$$a_n = \frac{4}{T}\int_0^{T/2} \sin(\omega_0 t)\cos(n\omega_0 t)\,dt = \frac{4}{\pi} \dots = -\frac{4}{\pi}\Big[\cos \dots = \frac{4}{\pi(1-4n^2)}$$

$$c_n = \sqrt{a_n^2 + b_n^2} = |a_n| = \left| \frac{4}{\pi(1-4n^2)} \right|$$

amplitudni spekter:



$n\omega_0 = \omega_n$

Časovna zakasnitev signala se odraža samo na faznem spektru, ne vpliva pa na amplitudnega.

$t \to t - \tau$

$$f(t-\tau) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t \underline{- n\omega_0 \tau} + \varphi_n)$$

Signala sta si podobna, če je eden glede na drugega le zakasnjen in(ali) amplitudno skaliran.

Časovno skaliranje signala ne spremeni njegovih spekt. amplitud c_n, skalirajo se le pripadajoče harmonske frekvence.

$t \to a \cdot t \quad (a > 0)$

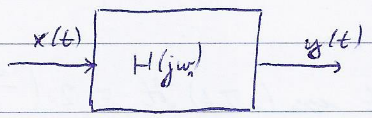
$$f(at) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 at + \varphi_n)$$

$\omega_n' = n \cdot \omega_0'$

$\omega_0' = a\,\omega_0 = a\,\frac{2\pi}{T}$

Odziv sistema na spekter vzbujanja

$|H(jw)|$ $\quad\quad$ $\angle H(jw)$



vzbujanje

$\downarrow$

$$x(t) = x_0 + \sum_{n=1}^{\infty} x_n \cos(n w_0 t + \varphi_{xn})$$

izhod:

$$y(t) = y_0 + \sum_{n=1}^{\infty} y_n \cos(n w_0 t + \varphi_{yn})$$

$$y_n = |H_n|\, x_n$$

$$\varphi_{yn} = \varphi_{xn} + \Theta_n$$

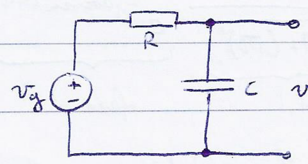
$$\downarrow$$

$$\angle H(jw)$$

fourierjev transform impulznega odziva

$$\boxed{y(t) = H_0 x_0 + \sum_{n=1}^{\infty} |H_n|\, x_n \cos(w_n t + \varphi_{xn} + \Theta)}$$

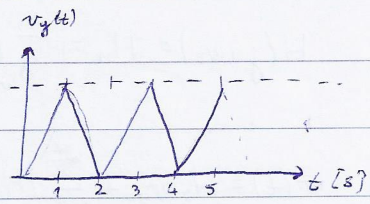
Primer izračunajte odziv!



$R = 1\,\Omega$

$C = 1\,F$

$T = 2\,s$



$$w_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \ \frac{rad}{s}$$

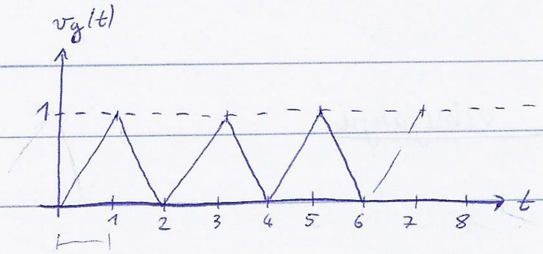
$$a_0 = v_{g0} = \frac{1}{2}\,V$$

$$b_n = 0$$

$$a_n = \frac{4}{T}\int_0^{T/2} v_g(t) \sin(n w_0 t)\,dt = 2\int_0^1 t \cdot \cos(n\pi t)\,dt = 2\left(\frac{\cos n\pi t}{(n\pi)^2} + \frac{t \sin(n\pi t)}{n\pi}\right)\Bigg|_{t=0}^{t=1} =$$

$$= 2\left(\frac{\cos(n\pi)}{(n\pi)^2} + 0 - \frac{1}{(n\pi)^2} - 0\right) = \begin{cases} 0 & n \text{ sod} \\ -\frac{4}{(n\pi)^2} & n \text{ lih} \end{cases}$$

27.11.2012



$T = 2s$ $\qquad\qquad\qquad\qquad \omega_0 = \frac{2\pi}{T} = \pi$

$a_0 = \frac{1}{2}$ $\qquad b_n = 0$

$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} v_g(t)\cos(n\omega_0 t)\,dt = 2\int_0^1 t\cdot\cos(n\pi t)\,dt = 2\cdot\left(\frac{\cos(n\pi)}{(n\pi)^2} + 0 - \frac{1}{(n\pi)^2} - 0\right) =$

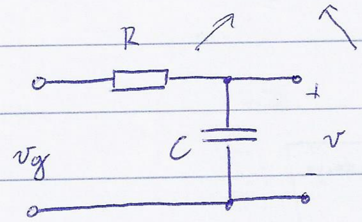
$u = t$

$v = \frac{\sin(n\pi t)}{n\pi}$

$= \begin{cases} 0 & , n \text{ sod} \\ -\frac{4}{(n\pi)^2} & , n \text{ lih} \end{cases}$

$a_0 = \frac{1}{2}\,V$

$|a_n| = v_{gn} = \frac{4}{(n\pi)^2}\,V$, $\varphi_{v_{gn}} = \pi$



$$H(j\omega) = \frac{v}{v_g} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + Rj\omega C} = \frac{1}{1 + j\omega RC}$$

$$H(jn\omega_0) = H_n = \left.\frac{V}{V_g}\right|_{\omega = n\omega_0 = n\pi} = \frac{1}{1 + jn\pi} = \underbrace{\frac{1}{\sqrt{1 + (n\pi)^2}}}_{|H_n|}\,\underbrace{e^{-j\arctan(n\pi)}}_{\text{faza}} = |H_n|e^{j\Theta_n}$$

- $v(t) = H_0 v_{g0} + \sum_{n=1}^{\infty} |H_n| v_{gn}\cos(n\pi t + \varphi_{v_{gn}} + \Theta_n) =$

$$= \frac{1}{2} + \sum_{n=1,3,5}^{\infty} \frac{1}{\sqrt{1 + (n\pi)^2}}\,\frac{4}{(n\pi)^2}\cdot\cos(n\pi t + \pi - \mathrm{arctg}(n\pi)) \quad V$$

$v_0 = 0{,}5V$

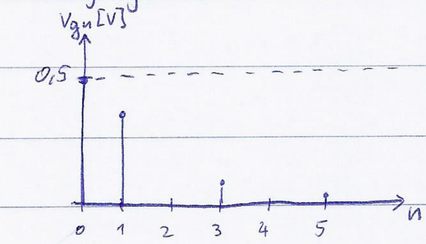
$v_1 = \ldots = H_1 v_{g1} = \frac{1}{\sqrt{1+\pi^2}}\,\frac{4}{\pi^2} = 0{,}123\ V$

$v_2 = 0$

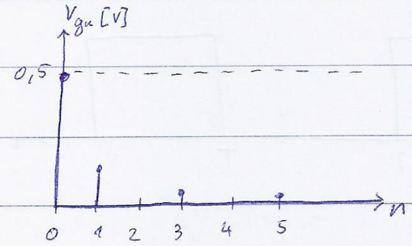
$v_3 = 0{,}005\ V$

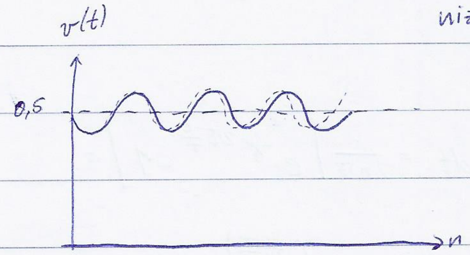
$v_4 = 0$ $\qquad v_5 = 0{,}001\ V$

Vzbujanje



odziv





nizkoprepustni

Fourierova eksponentna vrsta

$$f(t) = \sum_{n=-\infty}^{\infty} A_n e^{jn\omega_0 t}$$

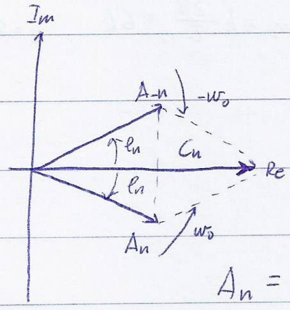
$$A_n = \frac{1}{T}\int_{\tau}^{\tau+T} f(t)\, e^{-jn\omega_0 t}\, dt$$

največkrat vzamemo $\tau = 0$ ali $-\frac{T}{2}$

neka izpeljava:

$$c_n \cos(n\omega_0 t + \varphi_n) = \overset{A_n}{\frac{c_n}{2}} \cos(n\omega_0 t + \varphi_n) + \overset{A_{-n}}{\frac{c_n}{2}} \cos(-n\omega_0 t - \varphi_n)$$

$$= Re[A_n e^{jn\omega_0 t}] + Re[A_{-n} e^{-jn\omega_0 t}] = A_n e^{jn\omega_0 t} + A_{-n} e^{-jn\omega_0 t}$$



$$A_n = \frac{c_n}{2} e^{j\varphi_n}$$

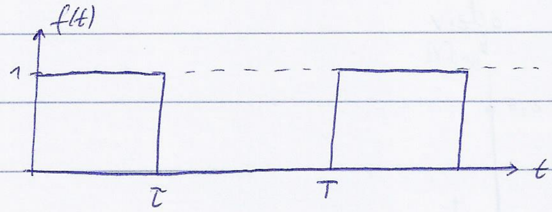
$$A_{-n} = \frac{c_n}{2} e^{-j\varphi_n}$$

$$A_n = |A_n| e^{j\phi_n} = \frac{a_n - jb_n}{2}$$

$$|A_{\pm n}| = \frac{c_n}{2}$$

$$\phi_{\pm n} = \pm \varphi_n \quad n \in N$$

Primer Poiščimo fourier. eks. vrsto za signal



$\tau = \frac{T}{6}$

$T = 2\pi\, s$

$$A_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt = \frac{1}{T}\int_0^{\tau} e^{-jn\frac{2\pi}{T}t}\, dt = \frac{j}{2n\pi}\left[e^{-jn2\pi\frac{\tau}{T}} - 1\right] =$$

$$= \frac{\tau}{T}\,\frac{\sin\left(n\pi\frac{\tau}{T}\right)}{n\pi\frac{\tau}{T}}\, e^{-jn\pi\frac{\tau}{T}}$$

$$= \frac{\tau}{T}\cdot \operatorname{sinc}\left(n\pi\frac{\tau}{T}\right) e^{-jn\pi\frac{\tau}{T}}$$

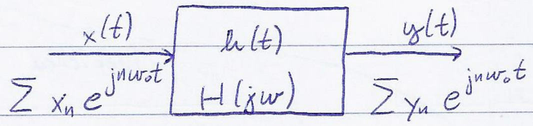
$$f(t) = \frac{\tau}{T}\sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(n\pi\frac{\tau}{T}\right) e^{-jn\pi\frac{\tau}{T}}\, e^{jn2\pi\frac{t}{T}}$$

$$|A_n| = \underbrace{\frac{1}{6}}_{\frac{\tau}{T}}\left|\operatorname{sinc}\left(n\frac{\pi}{6}\right)\right|$$

$$\phi_n = \longrightarrow = \begin{cases} -n\frac{\pi}{6}, & \operatorname{sinc}\left(\frac{n\pi}{6}\right) > 0 \\ -n\frac{\pi}{6} + \pi, & \operatorname{sinc}\left(\frac{n\pi}{6}\right) < 0 \end{cases}$$

Ničle: $n\pi\frac{\tau}{T} = k\pi \rightarrow n\omega_0 = k\frac{2\pi}{\tau} = 6k$, $k = 1, 2, 3 \ldots$

Odziv vezja na spekter vzbujanja



$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jnw_0 t} \qquad y(t) = \sum_{n=-\infty}^{\infty} Y_n e^{jnw_0 t}$$

pri − in + sta imaginarna, vendar sta skupaj realna

$$Y_n = H_n X_n$$

mnogokratnik w_0

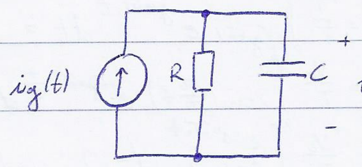
$$y(t) = \sum_{n=-\infty}^{\infty} H_n x_n e^{jnw_0 t}$$

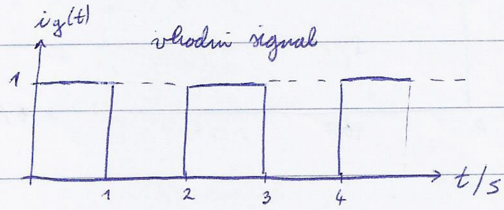
$$|Y_n| = |H_n||X_n| \quad \text{... amplitudni spekter}$$

$$\phi_{yn} = \phi_{xn} + \Theta_n \quad \text{... fazni spekter}$$

Θ_n: $\angle H(jnw_0)$

Primer: four. eks. vrsta odziv v(t), amplitudni in fazni spekter





$R = 1\Omega$

$C = 1F$

$T = 2s$

$w_0 = \frac{2\pi}{T} = \pi \ \frac{rad}{s}$

$$I_{g0} = \frac{1}{T}\int_0^T i_g(t)\,dt = \frac{1}{2}\int_0^1 1\,dt = \frac{t}{2}\Big|_0^1 = \underline{\frac{1}{2}\,A}$$

$$I_{gn} = \frac{1}{T}\int_0^T i_g(t) e^{-jnw_0 t}\,dt = \frac{1}{2}\int_0^1 e^{-jn\pi t}\,dt = \frac{1}{-2jn\pi}\left(e^{-jn\pi t} - 1\right) =$$

$$= \begin{cases} 0, & n \text{ sod} \\ \frac{1}{jn\pi}, & n \text{ lih} \end{cases}$$

$$H(jw) = \frac{V(jw)}{i_g(jw)} = \frac{R}{1 + jnw_0 RC} =$$

$$= \frac{1}{\sqrt{1+(n\pi)^2}}\, e^{-j\arctan(n\pi)}\ \Omega$$

$$\to Y_n = \frac{1}{R} + jn\omega_0 C = \frac{1}{H_n} = \frac{1}{Z_n}$$

$$v(t) = \sum_{n=-\infty}^{\infty} V_n e^{j\omega_n t} = \sum_{n=-\infty}^{\infty} \underbrace{\overset{H_n}{Z_n} I_{gn}}_{V_n} e^{jn\omega_0 t} = \frac{1}{2} + \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{|n|\sqrt{1+(n\pi)^2}} \, e^{j[\underbrace{n\pi t}_{\text{frekvenca}} \underbrace{- \operatorname{arctg}(n\pi) - \overbrace{\tfrac{n}{|n|}\tfrac{\pi}{2}}^{\text{liha; kot pri } i_g}}_{\text{faza}}]} \ V$$

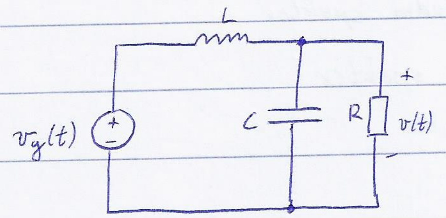
$$V_0 = \frac{1}{2}$$

$$|V_n| = \begin{cases} 0, & n \text{ sod} \\ \dfrac{1}{|n|\pi\sqrt{1+(n\pi)^2}}, & n \text{ lih} \end{cases}$$

faza:

$$\phi_n = -\arctan(n\pi) - \frac{n}{|n|}\frac{\pi}{2} \ \text{rad}$$

<u>Primer</u> valovitovitost napetosti na izhodu gladilnika polnousmerjene AC omrežne napetosti $v_g(t)$



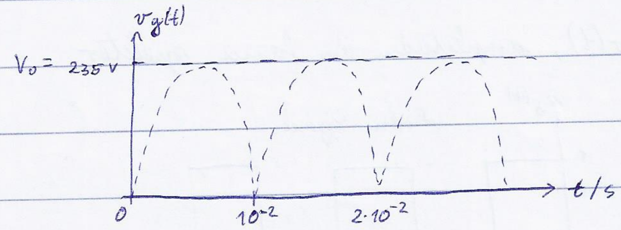
$L = 10\ H$

$C = 20\ \mu F$

$R = 1\ k\Omega$

$$v_g(t) = V_0 \, |\sin(100\pi t)|$$

$T = 10\ ms$



$$V_{gn} = \frac{1}{T}\int_0^T v_g(t)\, e^{-jn\omega_0 t} dt =$$

$$= \frac{V_0}{T}\int_0^T \sin\left(\frac{\pi}{T}t\right) e^{-jn2\pi\frac{t}{T}} dt =$$

$$= \frac{V_0}{\pi}\left[\frac{e^{-jn2\pi\frac{t}{T}}}{1-4n^2}\left(-j2n\sin\left(\frac{\pi t}{T}\right) - \cos\left(\frac{\pi t}{T}\right)\right)\right]_0^T =$$

$$= \frac{650}{\pi(1-4n^2)} V$$

$$H(j\omega) = \frac{v(j\omega)}{v_g(j\omega)} = \frac{\dfrac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}}{j\omega L + \dfrac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}}$$

$$= \frac{j\omega RC\,(1 + j\omega RC)}{j\omega L + \dfrac{j\omega RC}{1 + j\omega RC}} \overset{\omega = n\omega_0}{\underset{\text{magic!}}{=}} \frac{R}{R(1 - n^2\omega_0^2 LC) + jn\omega_0 L}$$

$$V_n = H_n V_{gn}$$

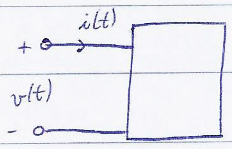
n	$\lvert V_n \rvert$
0	207 V
1	0,882 V
2	0,044 V
3	0,008 V

v spektru nastopajo polovične vrednosti

$$\left|\underbrace{\frac{2V_1}{V_0}}\right| = 0{,}0085$$

ocena valovitosti 0,85 %

Moč in efektivna vrednost



$$p(t) = v(t)\, i(t)$$

$$v(t) = \sum_{m=-\infty}^{\infty} V_m e^{jm\omega_0 t}$$

$$i(t) = \sum_{n=-\infty}^{\infty} I_n e^{jn\omega_0 t}$$

neka ortogonalnost

moč prek ene periode: $P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{T}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\int_0^T V_m I_n e^{j(m+n)\omega_0 t}\,dt$

$m \neq n$ vrednost integrala 0

$m = -n$ integral obstaja

$$P = \sum_{n=-\infty}^{\infty} V_{-n} I_n = \sum_{m=-\infty}^{\infty} V_n^* I_n$$

$$P = \sum_{n=-\infty}^{\infty} |I_n|^2 R_n \qquad Z_n = R$$

$$\boxed{\frac{1}{T}\int_0^T i^2(t)\,dt = \sum_{n=-\infty}^{\infty} |I_n^2|}$$

f prostor

t prostor

Parsevallov teorem

Numerično računanje koeficientov

periodo $f(t)$ razdelimo na M delov

$$\Delta t = \frac{T}{M} \leftarrow \text{perioda}$$

$$a_0 \doteq \underset{\substack{\uparrow \\ \text{diskretiziran}}}{a_{D0}} = \frac{1}{M}\sum_{m=0}^{M-1} f(m\Delta t)$$

$$a_n \doteq a_{DN} = \frac{2}{M}\sum_{m=0}^{M-1} f(m\Delta t)\cos\left(mn\frac{2\pi}{M}\right)$$

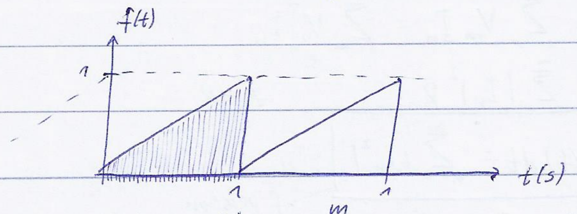
$$b_n \doteq b_{DN} = \frac{2}{M}\sum_{m=0}^{M-1} f(m\Delta t)\sin\left(mn\frac{2\pi}{M}\right)$$

za eksponentno vrsto:

$$A_n \doteq A_{DN} = \frac{1}{M}\sum_{m=0}^{M-1} f(m\Delta t)\, e^{-jmn\frac{2\pi}{M}}$$

uporabne vrednosti po numeričnih do $n \leq \frac{M}{2}$
(spekter postane periodičen s periodo M)
iz tega izhaja Nyquistov teorem

Primer: Na osnovi trigonometrične vrste numerično izračunajmo spekter za žagasti signal



$$f(m\Delta t) = t\Big|_{t=\frac{m}{32}} = \frac{m}{32} \qquad m \text{ od } 0 \text{ do } 31$$

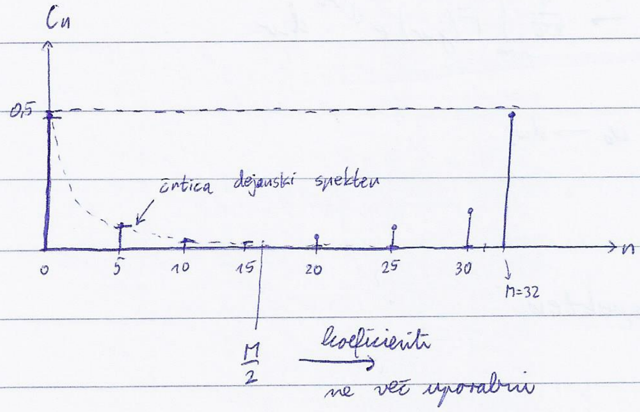
$$a_0 = \frac{1}{32}\sum_{m=0}^{31}\frac{m}{32} = 0.484 \quad (\text{blizu } 0.5)$$

$$a_n = \frac{1}{16}\sum_{m=0}^{31}\frac{m}{32}\cos\left(m\cdot n\cdot\frac{\pi}{16}\right)$$

$$b_n = \frac{1}{16}\sum_{m=0}^{31}\frac{m}{32}\sin\left(m\cdot n\cdot\frac{\pi}{16}\right)$$

eks.

$\hookrightarrow \quad C_n = \sqrt{a_n^2 + b_n^2}$



Fourierjev transform

aperiodični signali

Limitiranje four. eks. vrste $(T \to \infty)$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt = \mathcal{F}[f(t)]$$

frekvenčni spekter zvezen!

$\omega \ldots [-\infty, \infty]$

Zadostni pogoj je absolutna integrabilnost $f(t)$

$$\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$$

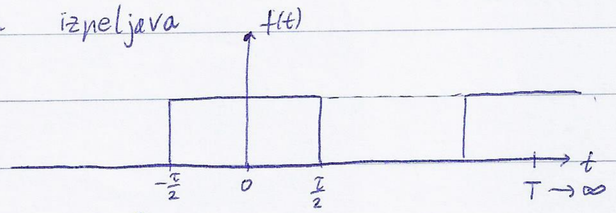
mnogo funkcij tem pogoju ne ustrezajo!

Pomagamo si lahko z $\delta(\omega)$.

Inverzni transform:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, d\omega = \mathcal{F}^{-1}[F(j\omega)]$$

Neka izpeljava



$$A(jn\omega_0) = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$\underbrace{T \cdot A(jn\omega_0)}_{F(j\omega)} = \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-jn\omega_0 t}\, dt \quad , \quad T \to \infty \quad , \quad n\omega_0 \longrightarrow \omega$$

$\Delta\omega = \omega_0 = \frac{2\pi}{T}$

dimenzija $f(t)$ krat čas!

„amplitudna gostota"

$$f(t) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \omega_0 F(j\omega_0 n) e^{jn\omega_0 t} \rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

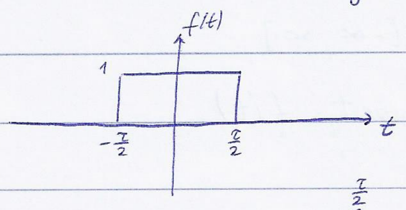
$$\frac{\omega_0}{2\pi} = \frac{2\pi}{2\pi T} = T^{-1} \qquad T \rightarrow \infty, \quad \omega_0 \rightarrow d\omega$$

$F(j\omega)$, iščemo amplitudni spekter

Spekter amplitudne gostote in fazni spekter

$$F(j\omega) = \underbrace{|F(j\omega)|}_{\substack{\text{spekter amplitudne gostote} \\ \text{(soda fja)}}} e^{j\underbrace{\phi(\omega)}_{\substack{\text{faza} \\ \text{(liha fja)}}}}$$

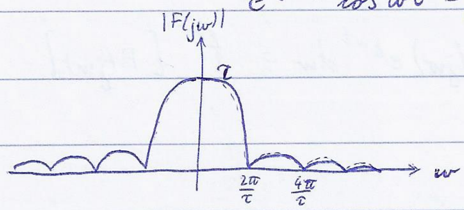
Primer spekter pravokotnega pulza



ker je signal sod lahko pišemo tako

$$F(j\omega) = 2\int_0^{\frac{\tau}{2}} \cos(\omega t) dt = \frac{2}{\omega} \sin\left(\frac{\omega\tau}{2}\right) = \tau \cdot \mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

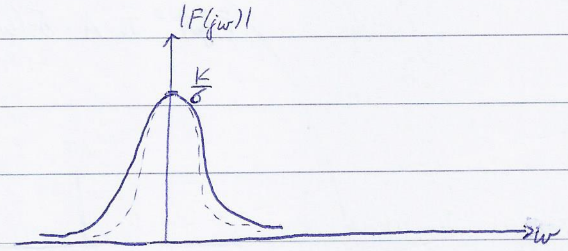
$$e^{-j\omega t} = \underbrace{\cos\omega t}_{} - j\sin\omega t$$



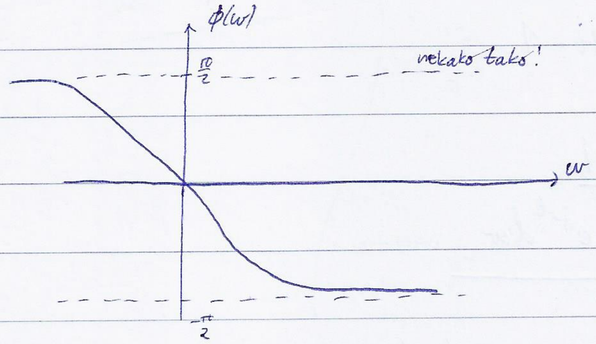
Primer spekter signala $f(t) = K e^{-\sigma t} \cdot u(t)$

$$F(j\omega) = K \int_{-\infty}^{\infty} e^{-\sigma t} u(t) \cdot e^{-j\omega t}\, dt = K \int_{0}^{\infty} e^{-(\sigma + j\omega)t}\, dt = \frac{-K}{\sigma + j\omega}\, e^{-(\sigma+j\omega)t} \Big|_0^{\infty} =$$

$$= \frac{K}{\sigma + j\omega}$$



$$|F(j\omega)| = \frac{K}{\sqrt{\sigma^2 + \omega^2}}$$



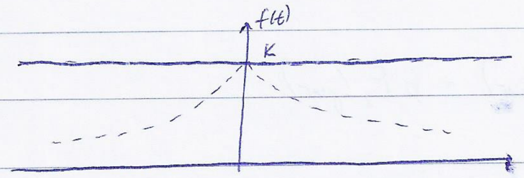
$$\phi = -\arctan \frac{\omega}{\sigma}$$

Primer transform konstante K

$f(t) = K$

↳ $f'(t) = K \cdot e^{-a|t|}$

$$F(j\omega) = \int_{-\infty}^{0} K e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} K e^{-at} e^{-j\omega t}\, dt =$$



$$= \frac{K}{a - j\omega} + \frac{K}{a + j\omega} = \frac{2aK}{a^2 + \omega^2}$$

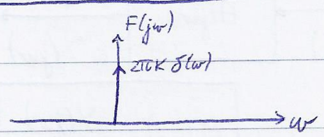
$a \to 0$ (f' se približa f)

pri $\omega = 0$ vrednost resultata nedoločena

$$\int_{-\infty}^{\infty} \frac{a}{a^2 + \omega^2} = \operatorname{arctg} \frac{\omega}{a} \Big|_{-\infty}^{\infty} = \pi$$

$$\int_{-\infty}^{\infty} \frac{2aK}{a^2 + \omega^2}\, d\omega = 2\pi \cdot K$$

$$F[K] = 2\pi \cdot K \cdot \delta(\omega)$$

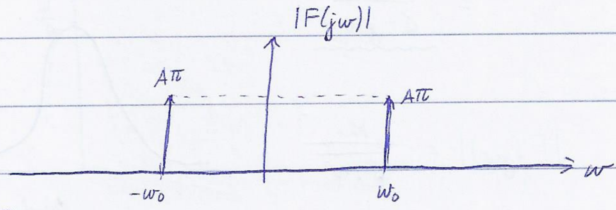


Primer fourierjev transform $\underline{f(t) = A \cdot \cos(\omega_0 t)}$

$$F(j\omega) = \int_{-\infty}^{\infty} A \cdot \cos(\omega_0 t)\, e^{-j\omega t}\, dt = \frac{A}{2} \int_{-\infty}^{\infty} e^{-j(\omega-\omega_0)t}\, dt + \frac{A}{2} \int_{-\infty}^{\infty} e^{-j(\omega+\omega_0)t}\, dt =$$

$$=$$

$$F(j\omega) = \pi \cdot A \cdot \delta(\omega - \omega_0) + \pi \cdot A\, \delta(\omega + \omega_0)$$



$$\boxed{\int_{-\infty}^{\infty} f(t)\, \delta(t-\tau)\, dt = f(\tau)}$$

$f(t) = e^{-j\omega t}$ $\quad \tau = 0$

$$\int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt = 1$$

$$\underline{\mathcal{F}[\delta(t)] = 1}$$

$$\underline{\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t}\, d\omega}$$

↳ zamenjava spremenljivk t, ω

ker soda fja!?

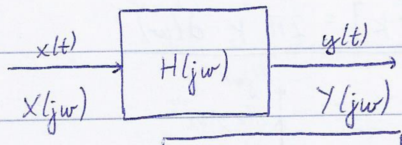
$$2\pi\, \delta(\omega) = \int_{-\infty}^{\infty} e^{\pm j\omega t}\, dt$$

Lastnosti fourierovih transformov

$$\mathcal{F}[a f_1(t) + b f_2(t)] = a F_1(j\omega) + b F_2(j\omega)$$

$$\mathcal{F}[f(t-\tau)] = \int_{-\infty}^{\infty} f(t-\tau)\, e^{-j\omega(t-\tau)}\, dt = F(j\omega) e^{-j\omega\tau}$$

$$\mathcal{F}\left[\frac{d^n f(t)}{dt^n}\right] = (j\omega)^n F(j\omega)$$



$$\boxed{H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}} \qquad \boxed{H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt}$$

$$\boxed{\begin{aligned} x(t) &= \delta(t) \\ Y(j\omega) &= H(j\omega) \end{aligned}}$$

Laplasova transformacija

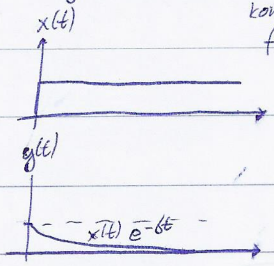
4.12.2012

- stabilnost
- lega korenov nam določa obnašanje
- def. prevajalne fje

fourierova: $X(j\omega) = \int_{-\infty}^{+\infty} x(t)\, e^{-j\omega t}\, dt$, $\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$

inverz: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$ $(\delta(\omega))$

neka fja $y(t) = x(t)\, \underbrace{e^{-\sigma t}}_{\text{konvergenčni faktor}}$ (sedaj je $y(t)$ absolutno integrabilna)



$$Y(j\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-\sigma t} e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-(\sigma + j\omega)t} dt =$$

$$= X(\sigma + j\omega)$$

inverz: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\sigma + j\omega)\, e^{(\sigma + j\omega)t}\, d\omega$

uvedemo $s = \sigma + j\omega$ (kompleksna frekvenca)

$\hookrightarrow ds = j\, d\omega$

$\hookrightarrow d\omega = \frac{1}{j} ds$

$$\boxed{\begin{aligned} X(s) &= \int_{-\infty}^{\infty} x(t) e^{-st} dt \\ x(t) &= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds \end{aligned}}$$

dvostranski Laplaceov transform

σ v s mora biti taksen, da zaduši funkcijo x (konvergenca, absolutna integr.)

Enostranska Laplaceova transformacija

$$\boxed{\begin{aligned} X(s) &= \int_{0^-}^{\infty} x(t)\, e^{-st}\, dt \\ x(t) &= \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\omega} X(s)\, e^{st}\, ds \end{aligned}}$$

transform obstaja, če $\lim\limits_{t\to\infty} e^{-\sigma t} x(t) = 0$

$\sigma > 0$ lahko $x(t) = A e^{Bt}$

problem nastane $x(t) = A \cdot e^{t^2}$

Lastnosti Laplaceove trans.

$$X(s) = \mathcal{L}[x(t)]$$

$$x(t) = \mathcal{L}^{-1}[X(s)]$$

1) Linearnost $\mathcal{L}[\alpha x_1(t) + \beta x_2(t)] = \alpha X_1(s) + \beta X_2(s)$

2) Čas. premaknitev $\mathcal{L}[x(t-\tau)\, u(t-\tau)] = e^{-\tau s} X(s)$

3) Množ. z eksponentom $\mathcal{L}[e^{at} x(t)] = X(s-a)$

4) Čas. skaliranje $\mathcal{L}[x(at)] = \frac{1}{a} X\left(\frac{s}{a}\right)$, $a > 0$

5) Odvajanje $\mathcal{L}\left[\frac{dx}{dt}\right] = sX(s) - x(0^-)$

$$\mathcal{L}\left[\frac{d^n x}{dt^n}\right] = s^n X(s) - s^{n-1} x(0^-) - s^{n-2} \frac{dx(0^-)}{dt} - \dots - \frac{d^{n-1} x(0^-)}{dt^{n-1}}$$

6) Integriranje $\mathcal{L}\left[\int^{\infty} x(\tau)\, d\tau\right] = \frac{X(s)}{s}$

$$\mathcal{L}\left[\int_{-\infty}^{t} x(\tau)\, d\tau\right] = \frac{X(s)}{s} + \frac{1}{s} \underbrace{\int_{-\infty}^{0^-} x(\tau)\, d\tau}_{x(0^-)}$$

7) Množenje s časom $\mathcal{L}[t \cdot x(t)] = -\frac{dX(s)}{ds}$

$$\mathcal{L}[t^n \cdot x(t)] = (-1)^n \frac{d^n X(s)}{ds^n}$$

kot ds

8) Deljenje s časom $\mathcal{L}\left[\frac{x(t)}{t}\right] = \int_{s}^{\infty} X(u)\, du$

9) Časovna konvolucija $X_1(s) \cdot X_2(s) = \mathcal{L}\left[\int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau\right]$

Primer $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 8y = x(t) \quad / \mathcal{L}$

$$y(0^-) = 10$$
$$\frac{dy(0^-)}{dt} = 2$$

$$s^2 Y(s) - s y(0^-) - \frac{dy(0)}{dt} + 4(sY(s) - y(0^-)) + 8Y(s) = X(s)$$

$$(s^2+4s+8)Y(s) = 10s + 42 + X(s)$$

$$Y(s) = \frac{10s+42+X(s)}{s^2+4s+8}$$

Primer $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$$\frac{dy}{dt} = x \qquad sY(s) - y(0^-) = X(s)$$
$$Y(s) = \frac{X(s)}{s} - \frac{1}{s}y(0^-)$$

Primer $\boxed{y(t) = \int_0^t h(\tau)\,x(t-\tau)\,dt \iff Y(s) = H(s)\cdot X(s)}$

$H(s)$ ... prevajalna / prenosna fja. sistema

Primer odziv RLC vezja na enotin impulz je podan z izrazom 5.12.2012

$$h(t) = (e^{-t} - e^{-2t})\,u(t)$$

signal $x(t) = u(t)$

določite odziv $y(t)$, preverite izraz s konv. integralom

$$Y(s) = H(s)\cdot X(s) \qquad H(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{1}{(s+1)(s+2)} \qquad Re[s] > -1$$

$$X(s) = \frac{1}{s} \qquad Re[s] > 0$$

$$Y(s) = \frac{1}{(s+1)(s+2)}\cdot\frac{1}{s} = \frac{0{,}5}{s} - \frac{1}{s+1} + \frac{0{,}5}{s+2} \qquad Re[s] > 0$$

$$y(t) = (0{,}5\,e^{-t} + 0{,}5\,e^{-2t})\,u(t)$$

$$y(t) = \int_0^t h(\tau)\underbrace{x(t-\tau)}_{u(t-\tau)}\,d\tau = \int_0^t h(\tau)\,d\tau = \int_0^t (e^{-\tau} - e^{-2\tau})\,d\tau = [-e^{-t}+1] + 0{,}5[e^{-2t}-1] =$$

$$= 0{,}5\,e^{-t} + 0{,}5\,e^{-2t}, \quad t \geq 0$$

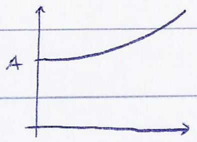
Nadaljevanje lastnosti

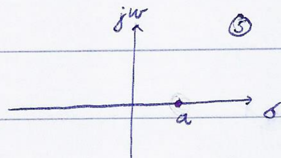
10) $\mathcal{L}\left[x_1(t)x_2(t)\right] = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X_1(u)\,X_2(s-u)\,du$

Frekvenčna konvolucija

Pomembnejši Laplaceovi trans.

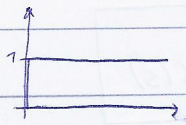
1) $Ae^{at}u(t) \Rightarrow \frac{A}{s-a}$

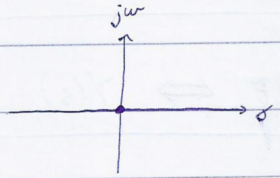




$Re[s] > Re[a]$

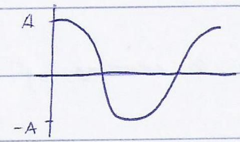
2) $u(t) \longrightarrow \frac{1}{s}$ (kot prejšnji primer $A=1$, $a=0$)

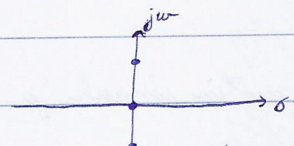




3) $A\cdot\cos(\omega t)\,u(t) \Rightarrow \mathcal{L}\left[A\,\frac{e^{j\omega t}+e^{-j\omega t}}{2}\right] = \frac{A\cdot 0{,}5}{s-j\omega} + \frac{A\cdot 0{,}5}{s+j\omega} =$

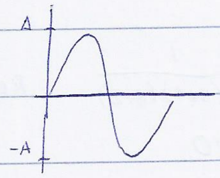
$= \frac{A\cdot s}{s^2+\omega^2}$

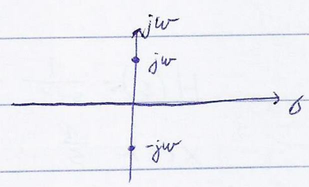




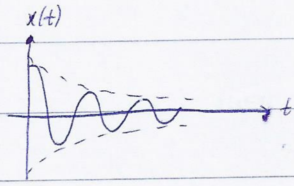
Kompleksna frekvenca

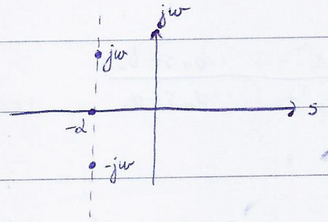
4) $A\cdot\sin(\omega t)\,u(t) \Rightarrow \mathcal{L}\left[A\,\frac{e^{j\omega t}-e^{-j\omega t}}{2j}\right] = \frac{0{,}5\,\frac{A}{j}}{s-j\omega} - \frac{0{,}5\,\frac{A}{j}}{s+j\omega} = \frac{A\cdot\omega}{s^2+\omega^2}$



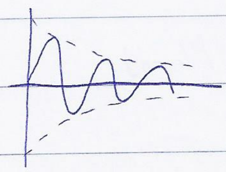


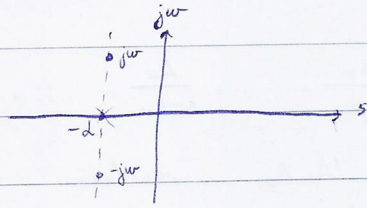
5) $A\cdot e^{-\alpha t}\cos(\omega t)\,u(t) \Rightarrow \dfrac{A(s+\alpha)}{(s+\alpha)^2+\omega^2}$





6) $A e^{-\alpha t}\sin(\omega t)\,u(t) \Rightarrow \dfrac{A\omega}{(s+\alpha)^2+\omega^2}$



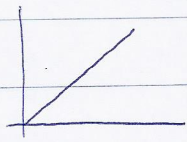


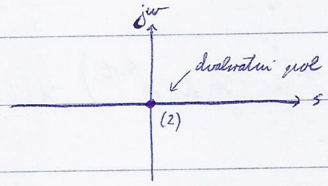
7) $r(t) \Rightarrow \dfrac{1}{s^2}$

$u(\tau) \longrightarrow \dfrac{1}{s}$

$$\mathcal{L}\left[\int_0^t x(\tau)\,d\tau\right] = \frac{X(s)}{s} = \frac{1}{s^2}$$

$$\mathcal{L}\left[t\cdot x(t)\right] = -\frac{dX(s)}{ds} = \frac{1}{s^2}$$





8) $\delta(t) \Rightarrow 1$

$$\mathcal{L}\left[\delta(t-\tau)\right]$$

$\delta(t-\tau) \Rightarrow e^{-s\tau}$

9) $A t e^{\alpha t} \Rightarrow -\cancel{A\frac{d}{dt}}\ \dfrac{A}{(s-\alpha)^2}$

Iskanje inverza z metodo parcialnih ulomkov

$$Y(s) = \frac{P(s)}{Q(s)} = \frac{b_m s^m + \dots + b_1 s + b_0}{a_n s^n + \dots + a_1 s + a_0} \quad \text{recimo } n > m$$

iščemo korene $Q(s) = a_n (s-s_1)(s-s_2) \dots (s-s_n)$

recimo vsi koreni različni med sabo

$$Y(s) = \frac{A_1}{s-s_1} + \frac{A_2}{s-s_2} + \dots + \frac{A_k}{s-s_k} + \dots + \frac{A_n}{s-s_n} \quad / (s-s_k)$$

$A_1, A_2, \dots$ residuumi

$$(s-s_k) Y(s) = \frac{A_1(s-s_k)}{s-s_1} + \dots + A_k + \dots \frac{A_n(s-s_k)}{s-s_n} \Bigg|_{s=s_k}$$

$$\boxed{A_k = \Big[(s-s_k) Y(s) \Big]_{s=s_k}} \quad k = 1, 2, \dots n$$

za vse korene različne!

potem

$$y(t) = \left(A_1 e^{s_1 t} + A_2 e^{s_2 t} + \dots + A_n e^{s_n t}\right) u(t)$$

Primer

$$Y(s) = \frac{7s^3 + 60s^2 + 117s + 104}{s^4 + 6s^3 + 21s^2 + 26s} \; \dots\dots = \frac{4}{s} - \frac{3}{s+2} + \frac{3+j4}{s+2+j3} + \frac{3-j4}{s+2-j3}$$

poiščemo korene

$$\Longrightarrow y(t) = \left(4 - 3e^{-2t} + (3+j4) e^{-(2+j3)t} + (3-j4) e^{-(2-j3)t}\right) u(t)$$

$$Q(s) = s(s+2)(s+2+j3)(s+2-j3)$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+2+j3} + \frac{D}{s+2-j3}$$

oziroma

$$y(t) = \left(4 - 3e^{-2t} + 6e^{-2t} \cos 3t + 8e^{-2t} \sin 3t\right) u(t)$$

$$A = (s - \overset{0}{s_A}) Y(s) \Big|_{s=s_A=0} = \frac{7s^3 + 60s^2 + 117s + 104}{(s+2)(s+2+j3)(s+2-j3)} \Bigg|_{s=0} = \underline{\underline{4}}$$

$$B = (s+2) Y(s) \Big|_{s=-2} = \frac{-11-}{(s+2+j3)(s+2-j3)s} \Bigg|_{s=-2} = \underline{\underline{-3}}$$

$$C = (s+2+j3) Y(s) \Big|_{s=-2-j3} = \frac{-11-}{s(s+2)(s+2-j3)} \Bigg|_{s=-2-j3} = \underline{\underline{3+j4}}$$

$$D = \underline{\underline{3-j4}}$$

večkratni koreni

$$Y(s) = \frac{A_1}{s-s_1} + \dots + \frac{A_{i,1}}{(s-s_i)} + \frac{A_{i,2}}{(s-s_i)^2} + \dots + \frac{A_{i,n}}{(s-s_i)^n} + \dots + \frac{A_n}{s-s_n}$$

s_i n-kraten

$$A_{i,n} = \left[(s-s_i)^n Y(s)\right]_{s=s_i}$$

$$A_{i,n-1} = \frac{d}{ds}\left[(s-s_i)^n Y(s)\right]_{s=s_i}$$

$$\vdots$$

$$A_{i,n-k} = \frac{1}{k!}\frac{d^k}{ds^k}\left[(s-s_i)^n Y(s)\right]_{s=s_i}$$

$$\vdots$$

$$A_{i,1} = \frac{1}{(n-1)!}\frac{d^{n-1}}{ds^{n-1}}\left[(s-s_i)^n Y(s)\right]_{s=s_i}$$

$$\mathcal{L}^{-1}\left[\frac{A}{(s+a)^k}\right] = \frac{A}{(k-1)!}\, t^{k-1} e^{-at}$$

Primer

$$Y(s) = \frac{10(s+2)}{(s+1)^2(s+3)}$$

$$Y(s) = \frac{A_{1,1}}{(s+1)} + \frac{A_{1,2}}{(s+1)^2} + \frac{A_2}{s+3}$$

$$A_{1,2} = \left[(s+1)^2 Y(s)\right]_{s=-1} = \left.\frac{10(s+2)}{s+3}\right|_{s=-1} = 5$$

$$A_{1,1} = \frac{d}{ds}\left[(s+1)^2 Y(s)\right]_{s=-1} = \frac{d}{ds}\left[\frac{10(s+2)}{s+3}\right]_{s=-1} = \left.\frac{10(s+3) - 10(s+2)}{(s+3)^2}\right|_{s=-1} = 2{,}5$$

$$A_2 = \left[(s+3)\, Y(s)\right]_{s=-3} = -2{,}5$$

drug način

$$\left.\frac{10(s+2)}{(s+1)^2(s+3)}\right|_{s=0} = \frac{A_{1,1}}{1} + \frac{\overset{A_{1,2}}{5}}{1} - \frac{\overset{A_2}{2{,}5}}{3}$$

$$A_{1,1} = 2.5$$

rešitev

potem $Y(s) = \frac{2.5}{s+1} + \frac{5}{(s+1)^2} - \frac{2.5}{s+3}$

$\hookrightarrow y(t) = \left(2{,}5\, e^{-t} + 5te^{-t} - 2{,}5\, e^{-3t}\right)$

neki

$$X(0^+) = \lim_{s\to\infty} sX(s)$$

velja

$$\int_0^\infty \frac{dx}{dt} e^{-st} dt = sX(s) - x(0^-)$$

$$x(0^+) = X(0^-)$$

Teorem o končni vrednosti

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$\lim_{s\to 0} \int_0^\infty \frac{dx}{dt} e^{-st} dt = \lim_{s\to 0} \left[sX(s) - x(0^-)\right]$$

$$\lim_{t\to\infty} x(t) - \cancel{x(0^-)} = \lim_{s\to 0} sX(s) - \cancel{x(0^-)}$$

Laplaceova transformacija periodičnih kavzalnih funkcij

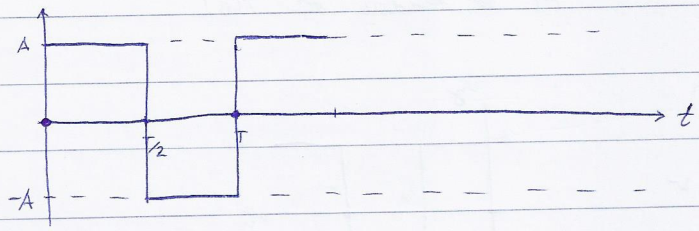
periodični signal $x(t)$

prva perioda $x_1(t)$

$$X_1(s) = \int_0^T x(t)\, e^{-st}\, dt$$

celotni $X(s) = X_1(s)\left(1 + e^{-sT} + e^{-s2T} + e^{-s3T} \dots\right) = \dfrac{X_1(s)}{1-e^{-sT}}$

$$\sum_{n=0}^{\infty} e^{-snt} = \frac{1}{1-e^{-st}} \qquad |e^{-sT}| < 1$$



$$x_1(t) = A u(t) - 2A\, u\left(t - \tfrac{T}{2}\right) + A u(t - T)$$

$$X_1(s) = \frac{A}{s}\left(1 - 2e^{-s\frac{T}{2}} + e^{-sT}\right) = \frac{A}{s}\left(1 - e^{-s\frac{T}{2}}\right)^2$$

potem:

$$X(s) = \frac{X_1(s)}{1-e^{-sT}} \overset{\text{še nekaj korakov}}{=} \frac{A\left(1-e^{-s\frac{T}{2}}\right)}{s\left(1+e^{-s\frac{T}{2}}\right)}$$

Odziv na kavzalno periodično vzbujanje

$$Y(s) = H(s)\, X(s) = \frac{H(s)\, X_1(s)}{1-e^{-sT}}$$

$0 = 1 - e^{-sT} \rightarrow s = j\, 2\pi \frac{n}{T}$

(neskončno korenov)

harmoniki signala

$$Y(s) = H(s)X_1(s)\left(1 + e^{-sT} + e^{-s2T} + e^{-s3T} + \dots\right)$$

zanima nas

$$(n-1)T < t < nT$$

$$y_1(t) = \mathcal{L}^{-1}[H(s)X_1(s)]$$

$$y(t) = y_1(t)u(t) + y_1(t)u(t-T) + \dots + y_1(t-nT+T)u(t-nT+T)$$

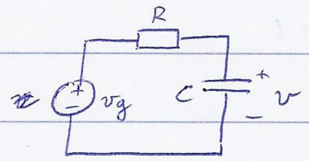
$$y_{ss}(t) = \mathcal{L}^{-1}[H(s)X_1(s)] - \sum_{i=1}^{n} A_i e^{s_i t}$$

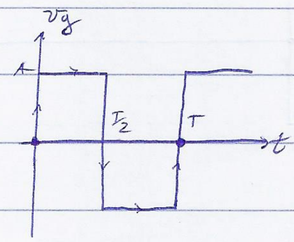
laplaceov transform prve periode

$s_i = 1, 2, \dots n$ poli od $H(s)$

A_i so residui od $Y(s)$

<u>Primer</u>





$R = 5\,M\Omega$ $A = 20\,V$

$C = 1\,\mu F$ $T = 2\,s$

$v(0) = 0$

$$RC\frac{dv}{dt} + v = v_g \quad /\mathcal{L}$$

$$(RCs + 1)V(s) = V_g(s) + \cancel{RCv(0)}$$

$$V(s) = \frac{V_g(s)}{RCs+1} = \frac{\frac{1}{RC}V_g}{s + \frac{1}{RC}}$$

$$v_{g1}(t) = Au(t) - 2Au(t - \tfrac{T}{2}) + Au(t-T)$$

$$V_{g1}(s) = \frac{A}{s}\left(1 - 2e^{-s\frac{T}{2}} + e^{-sT}\right)$$

$$V_g(s) = \frac{V_{g1}(s)}{1 - e^{-sT}}$$

množimo

$$H(s) = \frac{V(s)}{V_g(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} = \frac{2}{s+2}$$

odziv na prvo periodo

$$v_1(t) = \mathcal{L}^{-1}[H(s)V_{g1}(s)] = \mathcal{L}^{-1}\left[\frac{40(1 - 2e^{-s} + e^{-2s})}{s(s+2)}\right]$$

$$\frac{40}{s(s+2)} = \frac{A}{s} + \frac{B}{(s+2)} \qquad A = \left.\frac{40}{s+2}\right|_{s=0} = 20$$

$B = -20$

$$\cancel{v_1(t) = \mathcal{L}^{-1}\left[\frac{20}{s} - \frac{20}{s+2}\right] =} \quad ?$$

$$v_1(t) = 20\left[u(t) - 2u(t-1) + u(t-2)\right] - 20\left[e^{-2t}u(t) - 2e^{-2(t-1)}u(t-1) + e^{-2(t-2)}u(t-2)\right] =$$

$$= \underbrace{20(1-e^{-2t})u(t) - 40(1-e^{-2(t-1)})u(t-1)}_{} + 20(1-e^{-2(t-2)})u(t-2)$$

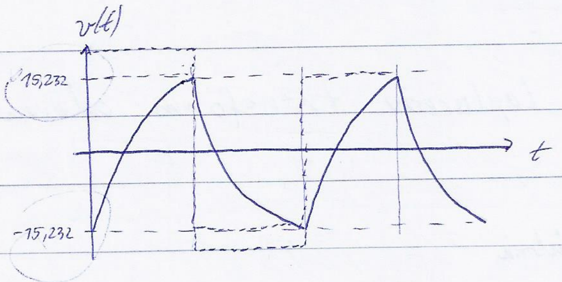
ta člen lako kao spustimo (velja za naslednjo periodo)

en residuum

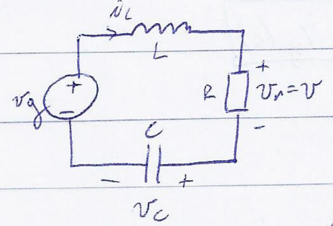
$$v_{ss}(t) = v_1(t) - A' e^{-2t} = \_\_\_ - 15,232\, e^{-2t} u(t)$$

$$V(s) = \frac{40(1-2e^{-s}+e^{-2s})}{s(s+2)(1-e^{-2s})}$$

$$A' = \left.\frac{40(1-2e^{-s}+e^{-2s})}{s(1-e^{-2s})}\right|_{s=-2} = 15,232\ V$$



Primer: 12.12.2012



$i_L(0^-) = 0$

$v_c(0^-) = 0$

$i = \frac{v}{R}$

$v_L = L\frac{di}{dt}$

$\mathcal{L}[v(t)] = V(s)$

$v_c = v_c(0^-) + \frac{1}{C}\int_0^t i\, d\tau$

$$\frac{v}{\cancel{R}} + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\, d\tau = v_g$$

$$\frac{L}{R}\frac{dv}{dt} + v + \frac{1}{RC}\int_0^t v\, d\tau = v_g \qquad v_g = \delta(t)$$

$$v = h(t)$$

$$\frac{L}{R}\frac{dh}{dt} + h + \frac{1}{RC}\int_0^t h\, d\tau = \delta(t) \quad /\mathcal{L}$$

$$\left(\frac{L}{R}s + 1 + \frac{1}{RCs}\right)H(s) = 1$$

$$H(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{R}{L}\frac{s}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

$$A = \left[(s-s_1)H(s)\right]_{s=s_1} = \frac{R}{L}\frac{s}{s-s_2}\Big|_{s=s_1} = \frac{R}{L}\frac{s_1}{s_1-s_2}$$

$$B = \text{-}\text{''}\text{-} = \frac{R}{L}\frac{s_2}{s_2-s_1} = -\frac{R}{L}\frac{s_2}{s-s_2}$$

$$H(s) = \frac{R}{L}\left(\frac{s_1}{(s-s_1)(s_1-s_2)} - \frac{s_2}{(s-s_2)(s_1-s_2)}\right)$$

$$\mathcal{L}^{-1}[H(s)] = h(t) =$$

$$= \frac{R}{L}\,\frac{s_1 e^{s_1 t} - s_2 e^{s_2 t}}{s_1 - s}\cdot u(t)$$

Prevajalna funkcija vezja je Laplaceov transform odziva na impulz.

⋮

$$\left(\frac{L}{R}s + 1 + \frac{1}{RCs}\right)V(s) = V_g(s)$$

poljubna

$$\frac{V(s)}{V_g(s)} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = H(s)$$

$$\boxed{V(s) = H(s)\,V_g(s)}$$

Prevajalna fja vezja je kvocient Laplace. transforma izhodnega signala in laplace. transforma vhodnega signala (pri 0. začetnem)

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}[H(s)X(s)] = \mathcal{L}^{-1}\left[\frac{P(s)}{Q(s)}X(s)\right]$$

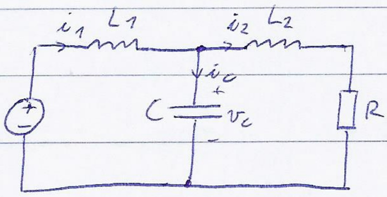
Prenosne/prevajalne fje so pomembne. Prenosno fjo LTI sistema dobimo iz diferencialke ali iz mat. postopkov ali modeliranja.

$$\sum_{k=1}^{n} a_k \frac{d^k y}{dt^k} = \sum_{k=1}^{m} b_k \frac{d^k x}{dt^k} \quad / \mathcal{L}$$

$$\overbrace{\left[a_n s^n + \dots + a_1 s + a_0\right]}^{Q(p)(s)} Y(s) = \overbrace{\left[b_m s^m + \dots + b_1 s + b_0\right]}^{P(p)(s)} X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{P(s)}{Q(s)} = \frac{\sum_{k=0}^{m} b_k s^k}{\sum_{k=0}^{n} a_k s^k}$$

Primer



$L_1 = 1H$
$L_2 = 1H$
$C = 1F$
$R = 1\Omega$

več tuljav → zančna metoda

$$H_1(s) = \frac{V_g(s)}{I_1(s)} \qquad H_2(s) = \frac{I_2(s)}{V_g(s)}$$

$$H_3(s) = \frac{I_2(s)}{I_1(s)}$$

$$L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + R i_2 = v_g \xRightarrow{\mathcal{L}} L_1 s I_1 + L_2 s I_2 + R I_2 = V_g$$

$$L_1 \frac{di_1}{dt} + \frac{1}{C}\int i_c \, d\tau = v_g \Rightarrow L_1 s I_1 + \frac{1}{Cs} I_c = V_g$$

$$i_1 = i_2 + i_c \Rightarrow I_1 = I_c + I_2$$

$$I_c = C s V_g - L_1 C s^2 I_1$$

$$I_2 = I_1 - I_c$$

$$I_2 = I_1 - C s V_g + L_1 C s^2 I_1$$

$$L_1 s I_1 + (L_2 s + R)\left[(1 + L_1 C s^2) I_1 - C s V_g\right] = V_g$$

lahko izračunamo $H_1(s)$, damo še vrednosti elementov

$$V_g\left[1 + Cs(L_2 s + R)\right] = \left(L_1 s + L_2 s + R + L_1 L_2 C s^3 + R L_1 C s^2\right) I_1$$

$$H_1(s) = \frac{V_g(s)}{I_1(s)} = \frac{s^3 + s^2 + 2s + 1}{s^2 + s + 1}$$

pač balinaš naprej

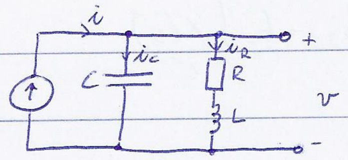
$$H_2(s) = \frac{I_2(s)}{V_g(s)} = \frac{1}{s^3 + s^2 + 2s + 1}$$

$$H_3(s) = \frac{I_2(s)}{I_1(s)} = \frac{1}{s^2 + s + 1}$$

Štromar.si

12.12.2012 P12 16 h 8.1.2012

Primer:



$H(s) = ?$

$i(t) = ?$

$$H(s) = \frac{V(s)}{I(s)}$$

narišite lego polov in ničel v s-ravnini

$R = 1\Omega$, $C = 1F$, $L = 1H$

$h(t) = ?$

$i = i_C + i_R$ $\quad v = i_R R + L\frac{di_R}{dt}$ $\quad i_C = C\frac{dV}{dt}$ $\quad \mathcal{L}$

$I = I_C + I_R$ $\quad V = I_R R + L s I_R$ $\quad I_C = C_s V$

$$I_R = \frac{V}{R+Ls}$$

a)

$$I = CsV + \frac{V}{R+Ls}$$

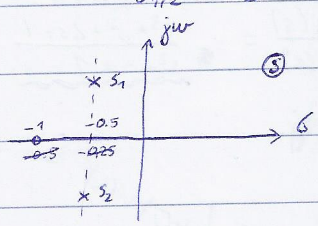
$$H(s) = \frac{V(s)}{I(s)} = \frac{R+Ls}{s^2LC+sRC+1}$$ ← lahko bi dobili tudi z impedancami

$$Y = Cs + \frac{1}{R+Ls}$$

$$Z_{VH} = \frac{R+Ls}{s^2LC+sRC+1}$$

b) $H(s) = \frac{V(s)}{I(s)} = \frac{s+1}{s^2+s+1}$ $\quad n = -1$

$$s_{1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4}-1} = -0.5 \pm j0.5\sqrt{3}$$



dušeno nihanje

$$h(t) = \mathcal{L}^{-1}(H(s)) = \mathcal{L}^{-1}\left[\frac{s+1}{\cancel{s}(s+0.5+j0.5\sqrt{3})(s+0.5-j0.5\sqrt{3})}\right] =$$

$$= \mathcal{L}^{-1}\left[\frac{A}{s+0.5-j0.5\sqrt{3}} + \frac{B}{s+0.5+j0.5\sqrt{3}}\right] = \mathcal{L}^{-1}\left[\frac{0.5-j\frac{0.5}{\sqrt{3}}}{s+0.5-j0.5\sqrt{3}} + \frac{0.5+j\frac{0.5}{\sqrt{3}}}{s+0.5+j0.5\sqrt{3}}\right] =$$

$A, B = ?$

$$A = \left[\frac{s+1}{s+0.5+j0.5\sqrt{3}}\right] = \ldots = 0.5 - j\frac{0.5}{\sqrt{3}}$$

$s_1 = \cancel{-0.5+0.5j}$

$s_1 = -0.5 + j0.5\sqrt{3}$

konj. kompleksna!

$$B = 0.5 + j\frac{0.5}{\sqrt{3}}$$

$$h(t) = \left[\left(0.5 - j\frac{0.5}{\sqrt{3}}\right)e^{(-0.5+j0.5\sqrt{3})t} + \left(0.5+j\frac{0.5}{\sqrt{3}}\right)e^{(-0.5-j0.5\sqrt{3})t}\right]u(t)$$

c)

$$= 0.5\,e^{-0.5t}\left(e^{j0.5\sqrt{3}t} + e^{-j0.5\sqrt{3}t}\right) + \frac{0.5}{\sqrt{3}j}\,e^{-0.5}\left(e^{j0.5\sqrt{3}t} - e^{-j0.5\sqrt{3}t}\right) =$$

$$h(t) = e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)\cdot u(t)$$

Če imamo obliko:

$$\frac{M+jN}{s+\alpha+j\omega} + \frac{M-jN}{s+\alpha-j\omega} \Rightarrow$$

$$\Rightarrow e^{-\alpha t}\left(2M\cos\omega t + 2N\sin\omega t\right) =$$

$$= 2|K|\,e^{-\alpha t}\cos(\omega t + \Theta)\;u(t)$$

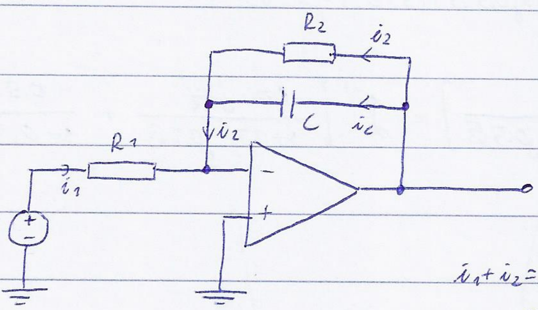
$$|K| = \sqrt{M^2+N^2}$$

$$\Theta = -\arctan\frac{N}{M}$$

!

$$H(s) = \frac{V_{izH}}{V_g}$$

Primer:



$$i_1 + i_2 = 0$$

$$i_1 = \frac{v_g}{R_1}$$

$$i_2 = \frac{v_{izH}}{R_2} + C\frac{dv_{izH}}{dt}$$

$$\Downarrow \mathcal{L}$$

$$I_1 + I_2 = 0$$

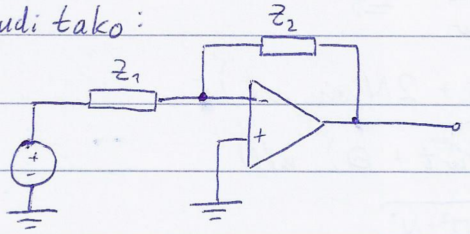
$$I_1 = \frac{V_g}{R_1}$$

$$I_2 = V_{izH}\left(\frac{1}{R_2} + Cs\right)$$

$$\frac{V_g}{R_1} = -V_{izH}\left(\frac{1+R_2Cs}{R_2}\right)$$

$$H(s) = \frac{V_{izH}}{V_g} = \underline{-\frac{R_2}{R_1}\left(\frac{1}{1+R_2Cs}\right)}$$

Lahko tudi tako:



$$\frac{V_{izH}}{V_g} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{\frac{1}{\frac{1}{R_2}+Cs}}{R_1} = -\frac{R_2}{R_1}\frac{1}{1+R_2Cs}$$

Celotni odziv z Laplaceovo transformacijo

dif. enačba:

$$\sum_{k=0}^{n} a_k \frac{d^k y}{dt^k} = \sum_{k=0}^{m} b_k \frac{d^k x}{dt^k}$$

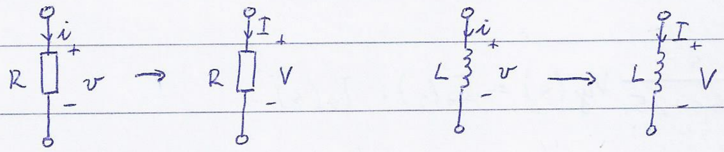
zač. stanje:

$$y(0),\ y'(0) \dots y^{(n-1)}(0),\ x(0),\ x'(0) \dots x^{(m-1)}(0)$$

$$\Downarrow \mathcal{L}$$

$$Q(s)\,Y(s) = \underbrace{P_z(s)}_{\text{začetno stanje}} + P(s)\,X(s)$$

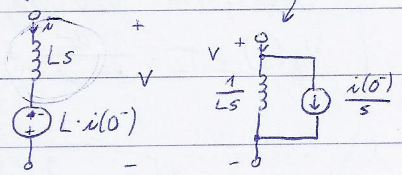
$$Y(s) = \frac{P_z(s)}{Q(s)} + \underbrace{\frac{P(s)}{Q(s)}}_{H(s)} X(s)$$



$$V(s) = L \cdot s \cdot I(s) - L\,i(0^-)$$

$$I(s) = \frac{V(s)}{Ls} + \frac{i(0^-)}{s}$$

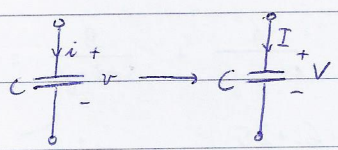
$$v = L\frac{di}{dt}, \quad i(0^-)$$



prevedimo enačbi za tuljavo v čas. prostor:

$$v(t) = L\frac{di}{dt} - \underbrace{L\,i(0^-)\,\delta(t)}_{\substack{\text{v času } t=0 \\ \text{se v tuljavo} \\ \text{"vpiše" začetna} \\ \text{vrednost}}}$$

$$i(t) = i(0^-) + \frac{1}{L}\int_0^t v(\tau)\,d\tau$$



$$i_C = C\frac{dv}{dt}, \quad v(0^-)$$

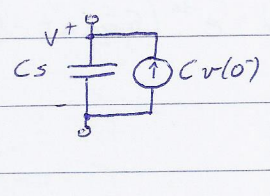
$$I(s)\cancel{V(s)} = CsV - Cv(0^-)$$

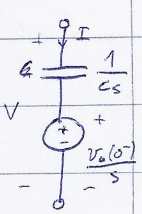
$$V(s) = \frac{I(s)}{Cs} + \frac{v(0^-)}{s}$$

še za kondenzator:

$$i(t) = C\frac{dv}{dt} - C\,v(0^-)\,\delta(t)$$

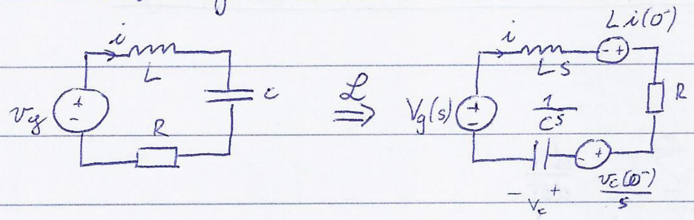
$$v(t) = v(0^-) + \frac{1}{C}\int_0^t i(\tau)\,d\tau$$





Primer:

celotni odziv vezja



$v_g = 12 \sin 5t \cdot u(t)$

$i_L(0^-) = 5A \qquad v_C(0^-) = 1V$

$L = 1H$

$C = 0.04F$

$R = 6\Omega$

$$LsI - Li(0^-) + RI + \frac{1}{Cs}I + \frac{v_c(0)}{s} = V_g$$

$$\frac{L}{s}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)I = \underbrace{Li(0^-) - \frac{v_c(0^-)}{s}}_{\text{zač. stanje}} + V$$

$$I = \frac{i(0^-)s - \frac{v_c(0^-)}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} + \frac{\frac{s}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} V_g$$

$$I = I_z(s) + I_v(s)$$

$$I(s) = \frac{5s-1}{s^2+6s+25} + \frac{s}{s^2+6s+25} V_g(s) = I_z(s) + I_v(s)$$

transform vzbujanja:

$$V_g(s) = 12 \cdot \frac{\omega}{s^2+\omega^2} = 12 \cdot \frac{5}{s+25} = \frac{60}{s^2+25}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \mathcal{L}^{-1}[I_z(s)] + \mathcal{L}^{-1}[I_v(s)]$$

$$I_z(s) = \frac{A}{s+3-j4} + \frac{B}{s+3+j4} = \frac{2.5+j2}{s+3-j4} + \frac{2.5-j2}{s+3+j4}$$

uporabimo isto bližnjico...

$Q(s) = s^2 + 6s + 25$

$s_{1,2} = -3 \pm j4$

$$A = \left.\frac{5s-1}{s+3+j4}\right|_{s=-3+j4} = 2.5 + j2$$

$\downarrow *$

$B = 2.5 - j2$

$$i_z(t) = e^{-3t}(5\cos 4t - 4\sin 4t)\,u(t)$$

$\cancel{\frac{1}{i} = \frac{1}{\sqrt{-1}} \cdot \frac{\sqrt{-1}}{\sqrt{-1}} = \sqrt{-1}}$

$\frac{1}{i}\frac{(-i)}{(-i)} = \frac{-i}{-i^2} = \frac{-i}{+1} = -i$

$$I_v(s) = \frac{A}{s+3-j4} + \frac{B}{s+3+j4} + \frac{C}{s-j5} + \frac{D}{s+j5}$$

$$A = \frac{60s}{(s+3+j4)(s^2+25)}\Bigg|_{s=-3+j4} = j1.25$$

$B = -j1.25$

$C = -j$

$D = j$

$$I_v(s) = \frac{j1.25}{s+3-j4} + \frac{-j1.25}{s+3+j4} + \frac{-j}{s-j5} + \frac{j}{s+j5} \quad / \mathcal{L}^{-1}$$

$$i_v(t) = 1.25\, e^{-3t}\, 2\, \frac{e^{j(4t+90°)} + e^{-j(4t+90°)}}{2} + 2\, \frac{e^{j(5t-90°)} + e^{-j(5t-90°)}}{2}$$

$$= 2.5\, e^{-3t} \cos(4t+90°) + 2\cos(5t-90°) = 0$$

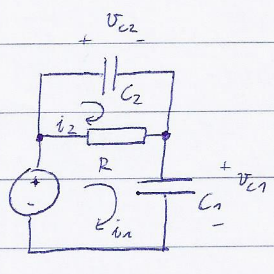
$$i_v(t) = [\underbrace{-2.5\, e^{-3t} \sin(4t)}_{\text{prelazni pojav}} + \underbrace{2\sin(5t)}_{\text{ustaljeno}}]\, u(t)$$

$$i(t) = i_z(t) + i_v(t) = i_0(t) + i_{ss}(t) =$$

$$= \cancel{5} \cdot \Big[\underbrace{e^{-3t}(5\cos 4t - 6.5 \sin 4t)}_{\text{prelazni pojav}} + \underbrace{2\sin 5t}_{\text{ustaljeno}}\Big]\, u(t)$$

$$\Big[\; y_{ss}(t) = \frac{Q(p)}{P(p)}\, e^{st}\Big|_{s=p}$$

Primer:



$C_1 = C_2 = 1F$ $\quad$ $v_g = 5 \cdot u(t)$ V

$R = 1\Omega$ $\quad$ $v_{c1}(0^-) = 1V$

$v_{c2}(0^-) = 2V$

kr durekt

$$R(I_1 - I_2) + \frac{1}{C_1 s} I_1 + \frac{v_{c1}(0^-)}{s} = V_g = \frac{5}{s}$$

$$\frac{1}{C_2 s} I_2 + \frac{v_{c2}(0^-)}{s} + R(I_2 - I_1) = 0$$

$$\frac{1}{i} = -i$$

dajmo v matriko

$$\begin{bmatrix} R + \frac{1}{C_1 s} & -R \\ -R & R + \frac{1}{C_2 s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_g - \frac{v_{C1}(0^-)}{s} \\ 0 - \frac{v_{C2}(0^-)}{s} \end{bmatrix}$$

vrednosti elementov

$$\begin{bmatrix} 1 + \frac{1}{s} & -1 \\ -1 & 1 + \frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{4}{s} \\ -\frac{2}{s} \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \overbrace{\frac{1}{(1+\frac{1}{s})^2 - 1} \begin{bmatrix} 1 + \frac{1}{s} & 1 \\ 1 & 1 + \frac{1}{s} \end{bmatrix}}^{\text{inv}} \begin{bmatrix} \frac{4}{s} \\ -\frac{2}{s} \end{bmatrix}$$

delimo števec z imenovalcem

$$I_1 = \frac{D_1}{D} = \frac{\frac{4}{s}(1+\frac{1}{s}) - \frac{2}{s}}{(1+\frac{1}{s})^2 - 1} = \frac{\frac{4}{s} + \frac{4}{s^2} - \frac{2}{s}}{\frac{1}{s^2} + \frac{2}{s}} = \frac{2s+4}{2s+1} = 1 + \frac{\frac{3}{2}}{s+\frac{1}{2}}$$

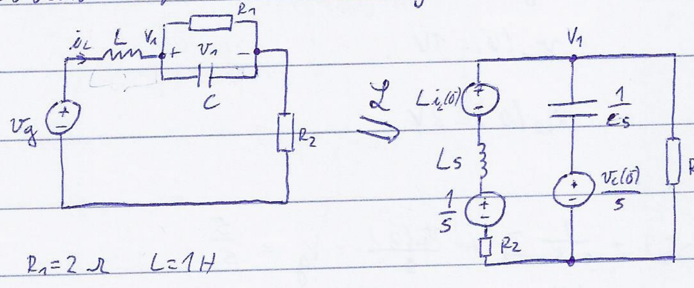
$$I_2 = \frac{D_2}{D} = \ldots = \frac{2s-2}{2s+1} = 1 + \frac{\frac{3}{2}}{s+\frac{1}{2}}$$

$$i_1(t) = \delta(t) + \frac{3}{2} e^{-\frac{1}{2}t} u(t)$$

$$i_2(t) = \delta(t) - \frac{3}{2} e^{-\frac{1}{2}t} u(t)$$

Primer:

določimo nap. na kondenzatorju



$R_1 = 2\,\Omega \quad L = 1\,H$

$R_2 = 3\,\Omega \quad C = 0.25\,F$

$v_g = u(t)\,, \quad i_L(0) = 2A$

$v_C(0^-) = 1V$

$$\frac{V_1}{R_1} + \left(V_1 - \frac{v_c(0)}{s}\right) Cs + \left(V_1 - Li(0) - \frac{1}{s}\right)\frac{1}{R_2 + Ls} = 0$$ tokovno vozlišče

$$V_1\left(\frac{1}{R_1} + Cs + \frac{1}{R_2+Ls}\right) = \frac{Li(0) + \frac{1}{s}}{R_2 + Ls} + Cs\frac{v_c(0)}{s}$$

vstavimo vrednosti

$$V_1\left(\frac{1}{2} + \frac{1}{4}s + \frac{1}{3+s}\right) = \frac{2+\frac{1}{s}}{3+s} + \frac{1}{4} \quad / \cdot 4$$

$$V_1 \frac{(2+s)(3+s)+4}{\cancel{3+s}} = \frac{8+\frac{4}{s}+3+s}{\cancel{3+s}}$$

$$V_1 = \frac{s^2+11s+4}{s(s^2+5s+10)} = \frac{A}{s} + \frac{B}{s+\frac{5}{2}-j\frac{\sqrt{15}}{2}} + \frac{C}{s+\frac{5}{2}+j\frac{\sqrt{15}}{2}}$$

$$\downarrow$$

$$s_{1,2} = -\frac{5}{2} \pm j\frac{\sqrt{15}}{2}$$

$$A = \frac{s^2+11s+4}{s^2+5s+10}\bigg|_{s=0} = \frac{2}{5}$$

$$B = \frac{s^2+11s+4}{s\left(s+\frac{5}{2}+j\frac{\sqrt{15}}{2}\right)}\bigg|_{s=-\frac{5}{2}+j\frac{\sqrt{15}}{2}} = \frac{3}{10} - j\frac{\sqrt{15}}{2}$$

$$C = B^* = \frac{3}{10} + j\frac{\sqrt{15}}{2}$$

$$V_1 = \frac{2}{5}\frac{1}{s} + \left(\frac{3}{10} - j\frac{\sqrt{15}}{2}\right)\frac{1}{s+\frac{5}{2}-j\frac{\sqrt{15}}{2}} + \left(\frac{3}{10}+j\frac{\sqrt{15}}{2}\right)\frac{1}{s+\frac{5}{2}+j\frac{\sqrt{15}}{2}}$$

$$v_1(t) = \left[\frac{2}{5} + e^{-\frac{5}{2}t}\left(\frac{3}{2}\cos\frac{\sqrt{15}}{2}t + \sqrt{15}\sin\frac{\sqrt{15}}{2}t\right)\right]\cdot u(t)$$

<u>Uporaba Laplaceove transformacije pri reševanju v prostoru stanj</u> LTI !

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u}$$

$$\underline{y} = \underline{C}\underline{x} + \underline{D}\underline{u}$$

za začetek recimo, da $\underline{u} = 0$, $\underline{x}(0^-) \cancel{\neq 0}$

$$\dot{x} = Ax \quad \Big\downarrow \mathcal{L}$$

$$sX(s) - x(0^-) = AX(s)$$

$$(sI - A)X(s) = x(0^-)$$

$$\underline{X}(s) = (s\underline{I} - \underline{A})^{-1}\,\underline{x}(0^-)$$

$$x(t) = \mathcal{L}^{-1}[X(s)] = \mathcal{L}^{-1}[(sI-A)^{-1} x(0^-)]$$

izhod: $Y(s) = C X(s)$

$$= C(sI-A)^{-1} x(0^-)$$

$$y(t) = \mathcal{L}^{-1}[Y(s)] = \mathcal{L}^{-1}[C(sI-A)^{-1} x(0^-)]$$

$$x(t) = e^{At} x(0^-) = \underbrace{\phi(t)}_{\text{matrika prehajanja stanj}} x(0^-)$$

$$\boxed{e^{At} = \mathcal{L}^{-1}[(sI-A)^{-1}] = \phi(t)}$$

sedaj recimo, da $u \neq 0$

$$sX(s) - x(0^-) = AX(s) + BU(s)$$

$$(sI-A)X(s) = x(0^-) + BU(s)$$

$$X(s) = \underbrace{(sI-A)^{-1}x(0^-)}_{\substack{\text{začetno}\\ \text{stanje}\\ \text{(odziv)}}} + \underbrace{(sI-A)^{-1} BU(s)}_{\substack{\text{vzbujanje}\\ \text{(odziv)}}}$$

$$x(t) = \mathcal{L}^{-1}[X(s)] = \mathcal{L}^{-1}[(sI-A)^{-1}x(0^-)] + \mathcal{L}^{-1}[(sI-A)^{-1}BU(s)]$$

$$Y(s) = CX(s) + DU(s)$$

$$Y(s) = C(sI-A)^{-1}x(0^-) + C(sI-A)^{-1}BU(s) + DU(s)$$

$$= C(sI-A)^{-1}x(0^-) + \underbrace{[C(sI-A)^{-1}B + D]}_{H(s)} U(s)$$

prenosna fja $\boxed{H(s) = C(sI-A)^{-1}B + D}$

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

Primer

$$\dot{x}(t) = \begin{bmatrix}-3 & -1\\ 2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix} u(t)$$

enotina stopnica

$$\dot{x} = Ax + Bu$$

$$\Phi(t) = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right] \qquad sI-A = \begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}-3 & -1\\ 2 & 0\end{bmatrix} = \begin{bmatrix}s+3 & 1\\ -2 & s\end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^2+3s+2}\begin{bmatrix}s\cancel{+3} & -1\\ 2 & s+3\end{bmatrix} = \frac{1}{(s+1)(s+2)}\begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix}$$

$s_{1,2} = -1, -2$

$$\frac{s}{(s+1)(s+2)} = \frac{A_{11}}{s+1} + \frac{B_{11}}{s+2} = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$\frac{-1}{(s+1)(s+2)} = A_{12} \ldots \quad A_{12} = -1, \quad B_{12} = 1$$

$$A_{21} = 2, \quad B_{21} = -2$$

$$A_{22} = 2, \quad B_{22} = -1$$

zač!

$$x_z(t) = \mathcal{L}^{-1}\left[(sI-A)^{-1}x(0^-)\right] = \begin{bmatrix}-e^{-t}+2e^{-2t} & -e^{-t}+e^{-2t}\\ 2e^{-t}-2e^{-2t} & 2e^{-t}-e^{-2t}\end{bmatrix} x(0^-)$$

$$x_v(t) = \mathcal{L}^{-1}\left[(sI-A)^{-1}B\,U(s)\right] = \mathcal{L}^{-1}\left[\frac{1}{(s+1)(s+2)}\begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}\frac{1}{s}\right] =$$

$$= \mathcal{L}^{-1}\begin{bmatrix}\frac{1}{(s+1)(s+2)}\\ \frac{2}{s(s+1)(s+2)}\end{bmatrix}$$

$$\frac{1}{(s+1)(s+2)} = \frac{A_1}{s+1} + \frac{B_1}{s+2}$$

$A_1 = 1$

$B_1 = -1$

$$\frac{2}{s(s+1)(s+2)} = \frac{A_2}{s} + \frac{B_2}{s+1} + \frac{C_2}{s+2}$$

$A_2 = 1$

$B_2 = -2$

$C_2 = 1$

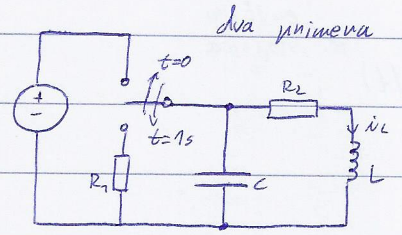
$$x_v(t) = \begin{bmatrix}e^{-t} - e^{-2t}\\ 1-2e^{-t}+e^{-2t}\end{bmatrix} \quad t \geq 0$$

$$x(t) = x_z(t) + x_v(t)$$

Primer tok skozi tuljavo

dva primera $0 < t < 1s$

$1s < t < \infty$



$v_g = 2u(t)$ ni začetnega stanja

18.12.2012 $R_1 = 1\Omega$

$C = 1F$

$$v_g(t) = v_c(t) = Ri_L + L\frac{di_L}{dt}$$

$L = 0.25H$

$$\frac{di_L}{dt} = -4i_L + 4v_g$$

$R_2 = 1\Omega$

$$A = -4 \qquad B = 4$$

a)

$$(sI - A) = s + 4$$

$$(sI - A)^{-1} = \frac{1}{s+4}$$

$$i_L(t) = \mathcal{L}^{-1}\left[(sI-A)^{-1} B\, U(s)\right] =$$

$$= 8\mathcal{L}^{-1}\left[\frac{1}{s(s+4)}\right] =$$

$$= 8\mathcal{L}^{-1}\left[\frac{A}{s} + \frac{B}{s+4}\right] = 8\mathcal{L}^{-1}\left[\frac{1}{4s} - \frac{1}{4(s+4)}\right] =$$

$$\frac{1}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

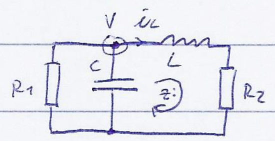
$$= 2u(t) - 2e^{-4t}u(t) \qquad 0 \le t \le 1$$

$$A = \left(\frac{1}{s+4}\right)\Big|_{s=0} = \frac{1}{4}$$

$$= \underline{\underline{2(1 - e^{-4t})u(t)}}$$

$$B = \frac{1}{s}\Big|_{s=-4} = -\frac{1}{4}$$

b) $1s < t < \infty$



z: $v_c = Ri_L + L\frac{di_L}{dt}$

V: $\frac{v_c}{R} + C\frac{dv_c}{dt} + i_L = 0$

$$\dot{x} = Ax + BU$$

$$A = \begin{bmatrix} -4 & 4 \\ -1 & -1 \end{bmatrix}$$

$$\frac{di_L}{dt} = -4i_L + 4v_c$$

$$\dot{x} = \begin{bmatrix} -4 & 4 \\ -1 & -1 \end{bmatrix} x + \cancel{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} 0$$

$$\frac{dv_c}{dt} = -i_L - v_c$$

$$\underline{x} = \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s+4 & -4 \\ 1 & s+1 \end{bmatrix}$$

$$\underline{x}(1) = \begin{bmatrix} 2-2e^{-4} \\ 2 \end{bmatrix} = \begin{bmatrix} 1.963 \\ 2 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{1}{s^2+5s+4+4}\begin{bmatrix} s+1 & 4 \\ -1 & s+4 \end{bmatrix}$$

$$e^{At} = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right]$$

$$(sI-A)^{-1} = \frac{1}{s^2+5s+8}\begin{bmatrix} s+1 & 4 \\ -1 & s+4 \end{bmatrix} = \cancel{(s+}$$

$$= \begin{bmatrix} \dfrac{s+2.5-\frac{1.5}{\sqrt{1.75}}\sqrt{1.75}}{(s+2.5)^2+1.75} & \dfrac{4\cdot\frac{\sqrt{1.75}}{\sqrt{1.75}}}{(s+2.5)^2+1.75} \\ \dfrac{-\frac{\sqrt{1.75}}{\sqrt{1.75}}}{(s+2.5)^2+1.75} & \dfrac{s+2.5+1.5\frac{\sqrt{1.75}}{\sqrt{1.75}}}{(s+2.5)^2+1.75} \end{bmatrix} = e^{-2.5t}\begin{bmatrix} \cos\sqrt{1.75}\,t - \frac{1.5}{\sqrt{1.75}}\sin\sqrt{1.75}\,t & \frac{4}{\sqrt{1.75}}\sin\sqrt{1.75}\,t \\ -\frac{1}{\sqrt{1.75}}\sin\sqrt{1.75}\,t & \cos\sqrt{1.75}\,t + \frac{1.5}{\sqrt{1.75}}\sin\sqrt{1.75}\,t \end{bmatrix}$$

<u>odziv</u>:

$$x(t) = e^{At}x(1) = e^{At}\begin{bmatrix} 1.963 \\ 2 \end{bmatrix} =$$

$$= e^{-2.5t}\begin{bmatrix} 1.963\cos wt + 3.32\sin wt \\ 2\cos wt + 0.784\sin wt \end{bmatrix} \qquad t \geq 1$$

$$w = \sqrt{1.75}\ s^{-1}$$

<u>Frekvenčna karakteristika sistemov</u>

Je lastnost sistema, ki pove, kako se sistem v ustaljenem stanju odziva na sinusni vhodni signal.

Ustaljeno stanje pri sinusnem vzbujanju

$$Y(s) = \overset{n>m}{H(s)}\,X(s) = \qquad x(t) = A\cos(wt+\varphi)$$

$$X(s) = \left(\frac{0.5Ae^{j\varphi}}{s-jw} + \frac{0.5Ae^{-j\varphi}}{s+jw}\right) = \cancel{\frac{c_1}{s-s_1}\ \frac{c_2}{s-s_2}}$$

$$\downarrow$$

$$= \frac{c_1}{s-s_1} + \frac{c_2}{s-s_2} + \ldots + \frac{c_n}{s-s_n} + \frac{c_w}{s-jw} + \frac{c_w^*}{s+jw}$$

$$y(t) = \left(C_1 e^{s_1 t} + C_2 e^{s_2 t} + \ldots + C_n e^{s_n t} + C_\omega e^{j\omega t} + C_\omega^* e^{-j\omega t}\right) u(t)$$

ustaljeno stanje $t \to \infty$

$y(t) \to y_{ss}(t)$

večkratni koreni

LTI sistem $\quad (s - s_j)^{r_j} \to t^{k_j} e^{s_j t}, \; k_j = 0, 1, 2, \ldots, r_j - 1$

gredo tudi proti nič

~~y(ss)~~

$$y_{ss}(t) = C_\omega e^{j\omega t} + C_\omega^* e^{-j\omega t}$$

$$C_\omega = Y(s)(s - j\omega)\Big|_{s=j\omega} = H(s)\left(\frac{0.5 A e^{j\varphi} \cancel{(s - j\omega)}}{\cancel{(s - j\omega)}} + \frac{0.5 A e^{-j\varphi}(s - j\omega)}{(s + j\omega)}\right)_{s=j\omega}$$

$$C_\omega = H(j\omega) 0.5 A e^{j\varphi} = 0.5 A |H(j\omega)| e^{j(\angle H(j\omega) + \varphi)}$$

$$C_\omega^* = H(-j\omega) 0.5 A e^{j\varphi} = 0.5 A |H(j\omega)| e^{-j(\angle H(j\omega) + \varphi)}$$

$$H(-j\omega) = \underset{\text{soda}}{|H(j\omega)|}\, e^{\underset{\text{liha}}{j\angle H(j\omega)}} = |H(j\omega)| e^{-j\angle H(j\omega)}$$

$$y_{ss}(t) = 0.5 A |H(j\omega)| \left(e^{j(\omega t + \angle H(j\omega) + \varphi)} + e^{-j(\omega t + H(j\omega) + \varphi)}\right)$$

$$\boxed{y_{ss}(t) = A |H(j\omega)| \cos(\omega t + \angle H(j\omega) + \varphi)}$$

$H(j\omega)$ je ful važna

Določitev frekvenčne karakteristike analitično

1) $H(j\omega) = H(s)\Big|_{s=j\omega}$

2) $H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} \, dt$

3) $\sum_{k=0}^{n} a_k \frac{d^k y}{dt^k} = \sum_{k=0}^{m} b_k \frac{d^k x}{dt^k} \quad \rightarrow H(j\omega) = \frac{\sum_{k=0}^{m} b_k (j\omega)^k}{\sum_{k=0}^{n} a_k (j\omega)^k}$

$$H(j\omega) = \mathrm{Re}[H(j\omega)] + j\,\mathrm{Im}[H(j\omega)] = \alpha + j\beta = |H(j\omega)|e^{j\angle H(j\omega)} = Me^{j\phi}$$ 19. 12. 2012

$$M = \sqrt{\alpha^2+\beta^2}$$

$$\phi(\omega) = \arctan\frac{\beta}{\alpha}$$

Predstavitev frekvenčne karakteristike

s pomočjo korenov in polov

členi samo realni

$$H(s) = k\,\frac{(s-n_1)(s-n_2)\ldots}{s^l(s-p_1)(s-p_2)\ldots} = k\,\frac{(s+\frac{1}{\tau_{n1}})(s+\frac{1}{\tau_{n2}})\ldots}{s^l(s+\frac{1}{\tau_{p1}})(s+\frac{1}{\tau_{p2}})\ldots}$$

$$H(j\omega) = K\,\frac{(1+j\omega\tau_{n1})(1+j\omega\tau_{n2})\ldots}{(j\omega)^l(1+j\omega\tau_{p1})(1+j\omega\tau_{p2})\ldots} =$$

$$= K\,\frac{(1+\frac{j\omega}{\omega_{n1}})(1+\frac{j\omega}{\omega_{n2}})\ldots}{(j\omega)^l(1+\frac{j\omega}{\omega_{p1}})(1+\frac{j\omega}{\omega_{p2}})\ldots} \qquad K = k\,\tau_{p1}\tau_{p2}\ldots/\tau_{n1}\tau_{n2}\ldots$$

Bodejev diagram

ampl. $M(\omega) = 20\cdot\log|H(j\omega)| = 20\cdot\log|K| + 20\log|1+\frac{j\omega}{\omega_{n1}}| + \ldots$

$$- 20\log|(j\omega)^l| - 20\log|1+\frac{j\omega}{\omega_{p1}}| + \ldots$$

faza $\phi(\omega) = \angle H(j\omega) = \arctan\frac{\mathrm{Im}[H(j\omega)]}{\mathrm{Re}[H(j\omega)]} =$

$$= (0 \text{ ali } \pm\pi) + \arg(1+\frac{j\omega}{\omega_{n1}}) + \ldots + \arg\frac{1}{(1+\frac{j\omega}{\omega_{p1}})}$$

Produkti znotraj faktoriziranega zapisa $|H(j\omega)|$ se spremenijo v vsoto. Vsak tak sumant se nato predstavi z asimptotskim potekom, ki določa

Tudi za fazni potek naredimo podobno

Bodejevi diagrami osnovnih členov

1) konstanta K

2) integrirni ali dif. člen $(j\omega)^{\pm 1}$

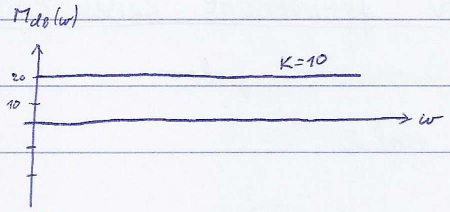
3) člen 1. reda $(1+j\omega\tau)^{\pm 1}$

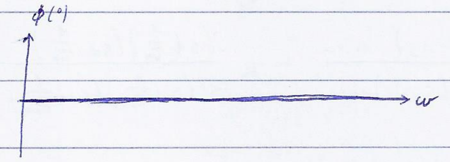
4) člen 2. reda $\left(1+2\xi\frac{j\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2\right)^{\pm 1}$

① $M_{dB} = 20 \cdot \log |K|$

$\phi = 0°$ $K>0$

$\phi = \pm\pi$ $K<0$





② $H(j\omega) = \frac{1}{j\omega}$

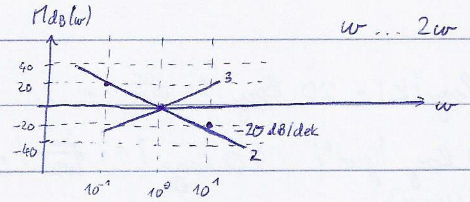
$M_{dB} = 20 \cdot \log \left|\frac{1}{j\omega}\right| = -20 \cdot \log \omega$ | $\omega = 1\,s^{-1}$ $M_{dB} = 0$ dB presečišče

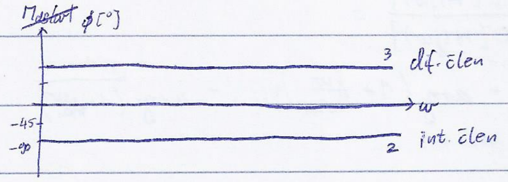
$\phi = \arctan \frac{-\frac{1}{\omega}}{0} = -90°$ zakasnilni kvaliter

strmina -20 dB na dekado

$\omega \ldots 10\omega$ dekada

$\omega \ldots 2\omega$ oktava





③ $H(j\omega) = j\omega$ — strmina 20 dB na dekado

$M_{dB} = 20 \cdot \log \omega$ — presečišče pri $\omega = 1\ s^{-1}$

$\phi = \text{arctg}\ \frac{\omega}{0} = 90^\circ$

višjega reda:

$H(j\omega) = \frac{1}{(j\omega)^n}$ — $H(j\omega) = (j\omega)^n$

$M_{dB} = -20 \cdot n \cdot \log \omega$ — $M_{dB} = 20 \cdot n \cdot \log \omega$

$\phi = -n\,90^\circ$ — $\phi = n\,90^\circ$

večje strmine!

④ $H(j\omega) = \frac{1}{1 + j\omega\tau}$

$M_{dB}(\omega) = -20 \log \sqrt{1 + (\omega\tau)^2}$

nf: $\omega \ll \frac{1}{\tau}$ — $M_{dB} = -20 \log 1 = 0\ dB$

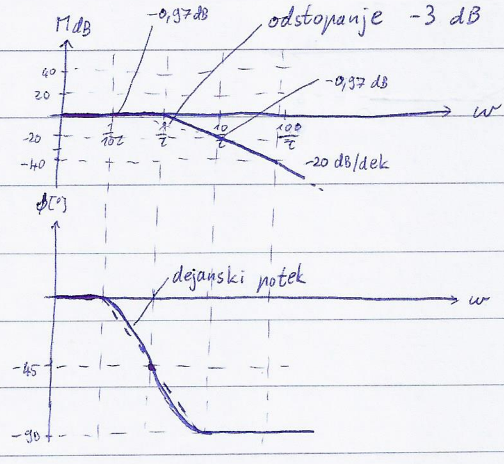
vf: $\omega \gg \frac{1}{\tau}$ — $M_{dB} = -20 \log \omega\tau$

$\omega = \frac{1}{\tau}$ — $M_{dB} = -20 \log 1 = 0\ dB$ — $\omega = \frac{1}{\tau}$ lomna frekvenca

$\omega = \frac{10}{\tau}$ — $M_{dB} = -20 \log 10 = -20\ dB$

$\phi = -\arctan \omega\tau$

ω	$\frac{1}{10\tau}$	$\frac{1}{2\tau}$	$\frac{1}{\tau}$	$\frac{2}{\tau}$	$\frac{10}{\tau}$
ϕ	$-5{,}7^\circ$	-27°	-45°	-63°	-84°

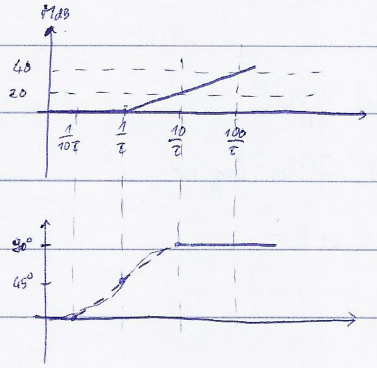


⑤ $H(j\omega) = 1 + j\omega\tau$

$M_{dB} = 20\log\sqrt{1+(\omega\tau)^2}$

$\omega \ll \frac{1}{\tau} \qquad M_{dB} = 0\ \text{dB}$

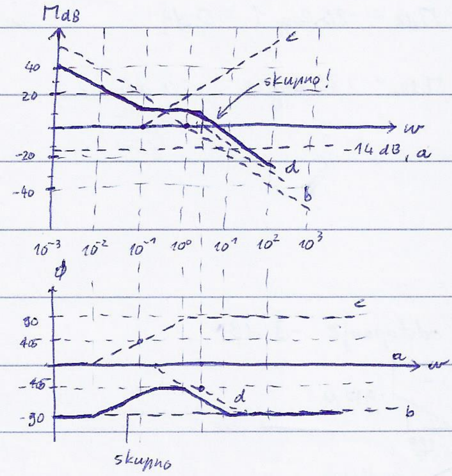
$\omega \gg \tau \qquad M_{dB} = 20\cdot\log\omega\tau$



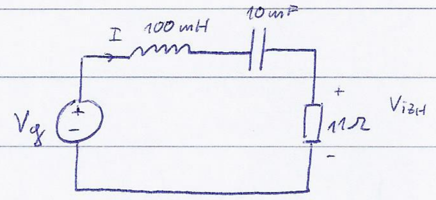
Primer

$$H(s) = \frac{4(s+0.1)}{s(s+2)}$$

$$H(j\omega) = \frac{4(j\omega+0.1)}{j\omega(j\omega+2)} = \frac{4\cdot 0.1\left(1+\frac{j\omega}{0.1}\right)}{j\omega\cdot 2\left(1+\frac{j\omega}{2}\right)} = \frac{\overset{a}{0.2}\ \overset{c}{\left(1+\frac{j\omega}{0.1}\right)}}{\underset{b}{j\omega}\ \underset{d}{\left(1+\frac{j\omega}{2}\right)}}$$



Primer



$$H = \frac{R}{Z_C + Z_L + R} = \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

$$= \frac{\frac{R}{L} \cdot s}{\frac{R}{L} + \frac{1}{sLC} + s \cdot s} = \frac{\frac{R}{L} s}{\frac{R}{L} s + \frac{1}{LC} + s^2}$$

$$H(s) = ?$$

$$i = \frac{v_{izh}}{R}$$

$$V_g = L \frac{di}{dt} + \frac{1}{C}\int_0^t i\,dt + \cancel{iR}\; v_{izh}$$

$$V_g = \frac{L}{R} \frac{dv_{izh}}{dt}$$

$$V_g = \frac{L}{R} s\, v_{izh} + \frac{1}{RCs} v_{izh} + v_{izh}$$

$$H(s) = \frac{v_{izh}}{v_g} = \frac{\frac{R}{L} s}{s^2 + \frac{R}{L} s + \frac{1}{LC}} = \frac{110\, s}{s^2 + 110 s + 1000} =$$

$$= \frac{110\, s}{(s+10)(s+100)}$$

$$H(j\omega) = \frac{110\, j\omega}{(j\omega + 10)(j\omega + 100)} =$$

$$= \frac{110\, j\omega}{10\left(1 + \frac{j\omega}{10}\right) 100 \left(1 + \frac{j\omega}{100}\right)} =$$

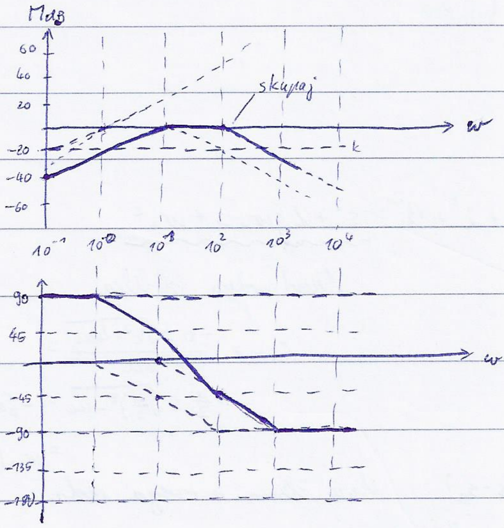
$$= \frac{0.11\; j\omega}{\left(1 + \frac{j\omega}{10}\right)\left(1 + \frac{j\omega}{100}\right)}$$

$$s^2 + 2\xi\omega_n + \omega_n^2 = 0$$

$\omega_n^2 = 1000 \qquad \omega_n = 31,62$

$2\xi\omega_n = 110$

$\xi = \frac{110}{2\omega_n} = 1.74$ nadkritično dušenje ...

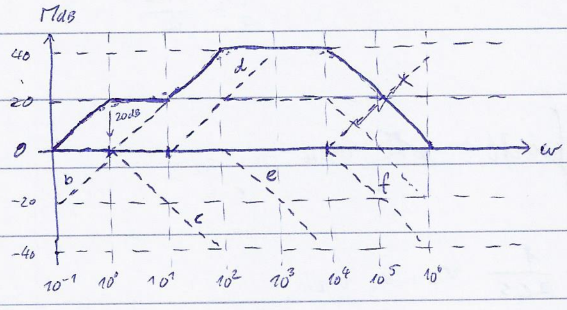


Primer najdi $H(j\omega)$ in $H(s)$

iz danega bodejevega diagrama

sistem z minimalno fazo

↳ (vse ničle stojijo na levi strani s-ravnine)



nf: $M_{dB} = K_{dB} + \overbrace{(b-a)}^{-2} 20 \cdot \log|j\omega|$

$\omega = 10^{-1}\, s^{-1}$

$0 = K_{dB} + 20 \cdot \log|0.1|$

$K = 20\, dB$

$$H(j\omega) = \frac{10\, j\omega \left(1 + \frac{j\omega}{10}\right) \xi}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{100}\right)\left(1+\frac{j\omega}{10\,000}\right)}$$

$$H(s) = \frac{10 s \left(1+\frac{s}{10}\right)}{\ldots\ldots}$$

------------------ Kompleksni koreni

$$H(s) = \frac{K}{(s+\alpha-j\beta)(s+\alpha+j\beta)}$$

$(a+b)(a-b)$

$a^2 + \cancel{ab} - \cancel{ba} - b^2$

$a^2 - b^2$

$$((s+\alpha)-j\beta)((s+\alpha)+j\beta) = (s+\alpha)^2 + \beta^2 = s^2 + 2\alpha s + \alpha^2 + \beta^2 = s^2 + 2\xi\omega_n s + \omega_n^2$$

standardna oblika

$\omega_n^2 = \alpha^2 + \beta^2$

$\xi\omega_n = \alpha$

$s_{1,2} = \omega_n\left(-\xi \pm \sqrt{\xi^2 - 1}\right)$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$= -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac} = -\xi\omega_n \pm \sqrt{(\xi\omega_n)^2 - \omega_n^2}$$

$$= \omega_n\left(-\beta \pm \sqrt{\xi^2 - 1}\right)$$

če $\xi \geq 1$ korena realna $(s-s_1)(s-s_2)$ dva člena prvega reda

če $\xi < 1$ $H(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{K}{\omega_n^2} \frac{1}{\left(1 + 2\xi\frac{s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2\right)}$

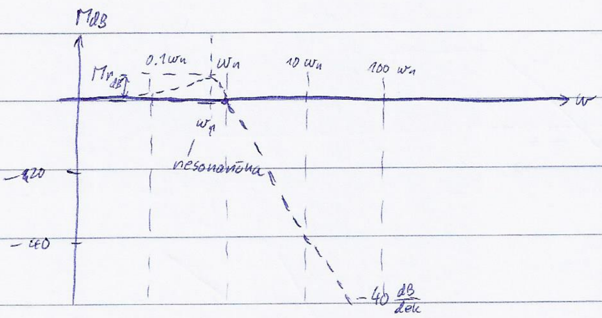
$$H(j\omega) = \frac{1}{1 + 2\xi\frac{j\omega}{\omega_n} + \left(\frac{j\omega}{\omega_n}\right)^2}$$

kako izgleda Bode?

$$M_{dB} = -20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

$\omega \ll \omega_n \qquad M_{dB} \doteq -20\log 1 = 0\ dB$

$\omega \gg \omega_n \qquad M_{dB} = -20\log\frac{\omega^2}{\omega_n^2} = -40\log\frac{\omega}{\omega_n}$



recimo $\xi = 0.3$

nižji ξ, višji vrh, bližje je ω_r ω_n

$M_{rdB} = ?$

$\omega_r = ? \qquad F(\omega) = \left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2 \qquad 0 \le \xi \le 0.707$

$\frac{dF(\omega)}{d\omega} = 0 \rightarrow \underline{\underline{\omega_r = \omega_n\sqrt{1-2\xi^2}}}$

tukaj izgine resonančni vrh

frekvenca dušenega nihanja

$\underline{\omega_d = \omega_n\sqrt{1-\xi^2}}$

$$M_r = H(j\omega)_{max} = H(j\omega_r) = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

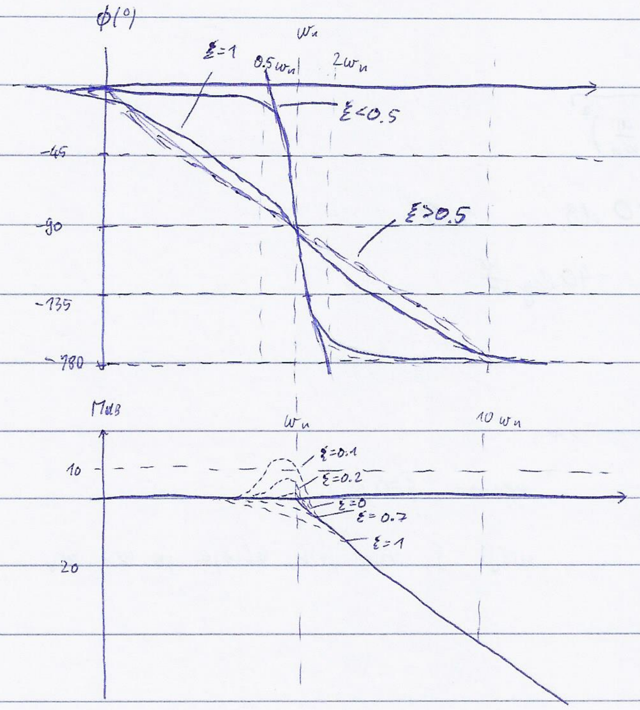
$$\underline{\underline{M_{rdB} = 20\cdot\log\frac{1}{2\xi\sqrt{1-\xi^2}}}}$$

Še faza: $\phi = -\mathrm{arctg}\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$

$\omega \ll \omega_n \qquad \phi(\omega) = 0°$

$\omega \gg \omega_n \qquad \phi(\omega) = -180°$

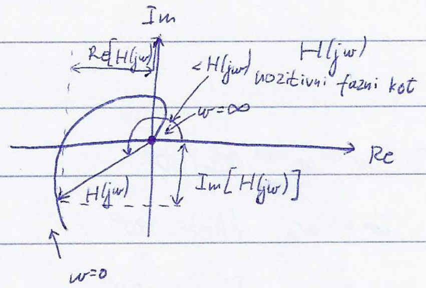
$\omega = \omega_n \qquad \phi(\omega_n) = -90°$



2.1.2013

Polarni diagram

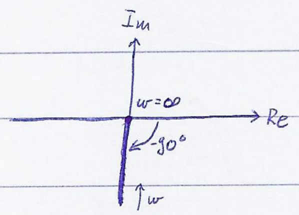
Predstavlja frek. karakteristiko $H(j\omega)$ v kompleksni ravnini.
Vsaka točka je podana s polarnim kompleksorjem $H(j\omega) = |H(j\omega)| e^{j\angle H(j\omega)}$
Če se frekvenca spreminja od 0 do ∞, zariše ta kompleksor polarni diagram.



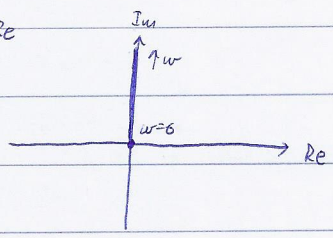
Polarni diagrami osnovnih členov

1) integrirni in diferen. člen

a) $H(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega} e^{j(-90°)}$

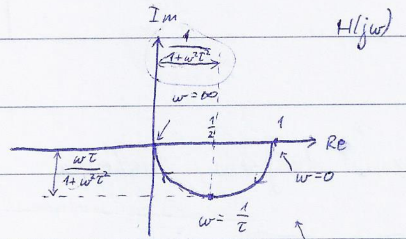


b) $H(j\omega) = j\omega = \omega e^{j90°}$



2) člen prvega reda

$$H(j\omega) = \frac{1}{1+j\omega\tau} = \frac{1}{\sqrt{1+\omega^2\tau^2}}\, e^{j(-\arctan \omega\tau)}$$



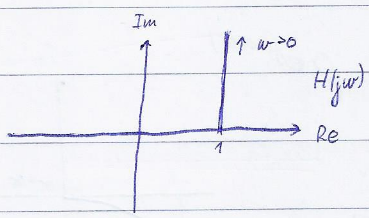
$\omega=0 \qquad \lim_{\omega\to 0} H(j\omega) = 1\, e^{j0^\circ}$

$\omega=\frac{1}{\tau} \qquad H(j\tfrac{1}{\tau}) = \frac{1}{\sqrt{2}}\, e^{j(-45^\circ)}$

$\omega\Rightarrow\infty \qquad H(j\omega) = 0\, e^{j(-90^\circ)}$

$$\frac{1}{1+j\omega\tau} = \frac{1-j\omega\tau}{1+(\omega\tau)^2}$$

$$H(j\omega) = 1+j\omega\tau$$

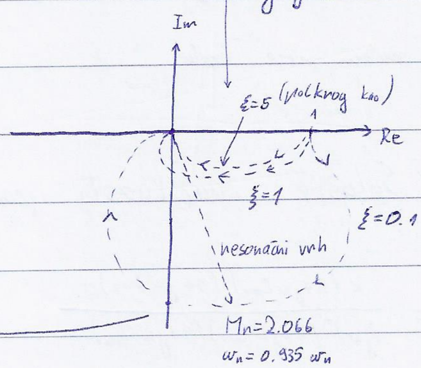


3) člen drugega reda

podobno

$$H(j\omega) = \frac{1}{1+2\xi\frac{j\omega}{\omega_n}+\left(\frac{j\omega}{\omega_n}\right)^2}$$

glej tudi list



$$\lim_{\omega\to 0} H(j\omega) = 1\, e^{j0^\circ}$$

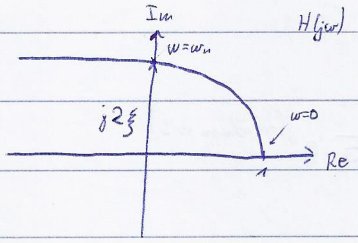
$$\lim_{\omega\to\infty} H(j\omega) = 0\, e^{j(-180^\circ)}$$

$$H(j\omega)\Big|_{\omega=\omega_n} = \frac{1}{j2\xi} = \frac{1}{2\xi}\, e^{j(-90^\circ)}$$

$$H(j\omega) = 1 + 2\xi\left(\frac{j\omega}{\omega_n}\right) + \left(\frac{j\omega}{\omega_n}\right)^2$$

$$\lim_{\omega \to 0} H(j\omega) = 1\, e^{j0^\circ}$$

$$\lim_{\omega \to \infty} H(j\omega) = \infty\, e^{j180^\circ}$$



Polarni diagram sistema prve vrste

$$H(j\omega) = \boxed{\frac{1}{j\omega(1+j\omega\tau)}} = -\frac{\tau}{1+(\omega\tau)^2} - j\frac{1}{1+(\omega\tau)^2}$$

$$\frac{1}{j\omega(1+j\omega\tau)} = \frac{1}{j\omega - \omega^2\tau} =$$

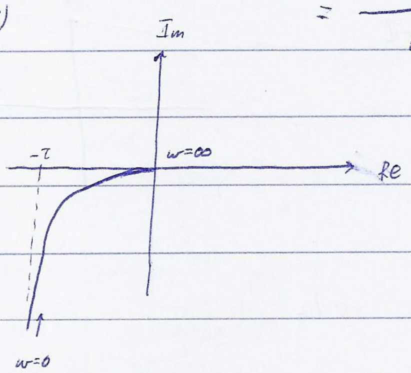
$$= \frac{-\omega^2\tau - j\omega}{-\omega^2\tau + j\omega \;/(-\omega^2\tau - j\omega)} =$$

$$= \frac{-\omega^2\tau - j\omega}{\omega^4\tau^2 + \omega^2} =$$

$$= \frac{-\tau}{1+\omega^2\tau^2} - \frac{j}{\omega + \omega^3\tau^2}$$

$$\lim_{\omega \to 0} H(j\omega) = -\tau - j\infty = \infty\, e^{j(-90^\circ)}$$

$$\lim_{\omega \to \infty} H(j\omega) = -0 - j0 = 0\, e^{j(-180^\circ)}$$



$l = 0$ sistem ničte vrste

$l = 1$ sistem prve vrste

Splošne značilnosti polarnih diagramov

$$H(j\omega) = \frac{k(1+j\omega\tau_{n1})(1+j\omega\tau_{n2})\ldots}{(j\omega)^l(1+j\omega\tau_{p1})(1+j\omega\tau_{p2})\ldots}$$

$$= \frac{b_m(j\omega)^m + \ldots + b_1 j\omega + b_0}{a_n(j\omega)^n + \ldots + a_1 j\omega + a_0}$$

za primere, ko $\underline{n > m}$

1) $l=0$

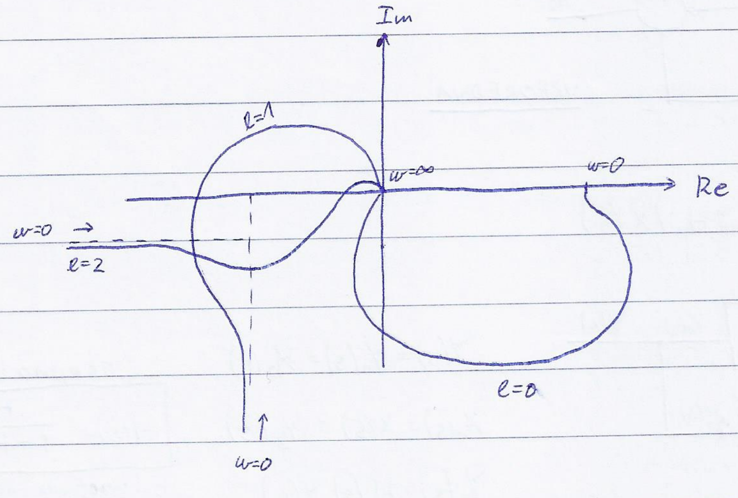
Začetna točka pri $w=0$ je na pozitivni realni osi, tangenta je pravokotna na realno os. Končna točka za $w=\infty$ je v koordinatnem izhodišču. Ena od koordinatnih osi predstavlja tangento na polarni diagram v izhodišču ($w=\infty$).

2) $l=1$

Člen jw v imenovalcu prispeva kot $-90°$ na celotnem frekvenčnem obm. Pri $w=0$ je abs vrednost ∞, fazni kot $-90°$ fazni diagram se približuje asimptoti, ki je pravokotna im. koord osi. Pri $w=\infty$ je abs. vr. 0, vstopa tangencialno na eno izmed osi.

3) $l>1$

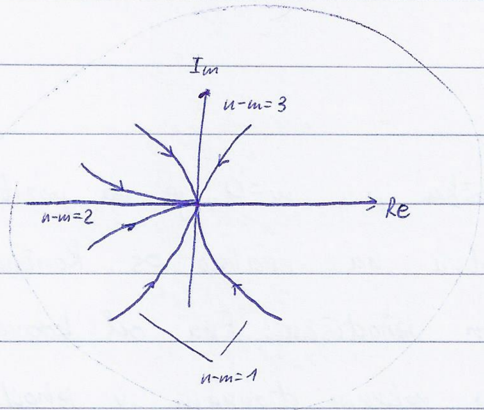
Člen $(jw)^l$ daje fazni kot $-l\cdot 90°$, pri $w=0$ je abs ∞, faza $-l\cdot 90°$. To je tudi kot, ki določa nizko frek. asimpt., ki je vzporedna z eno izmed koord. osi. Pri $w=\infty$ abs $=0$, krivulja v koord. izhodišče tangencialno na eno izmed osi.

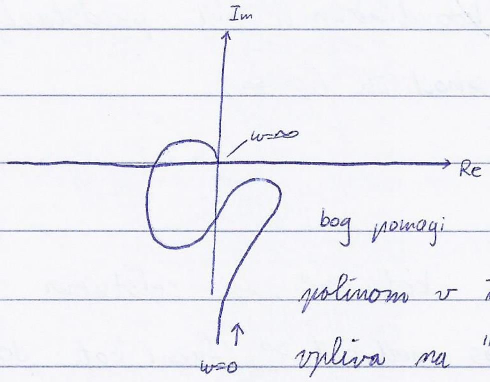


$n > m$

$\omega \to \infty$

$$H(j\omega) = \frac{b_m (j\omega)^m}{a_n (j\omega)^n} = \frac{b_m}{a_n} (j\omega)^{-(n-m)}$$



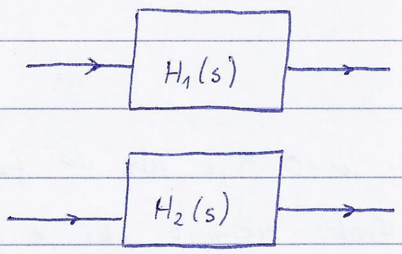


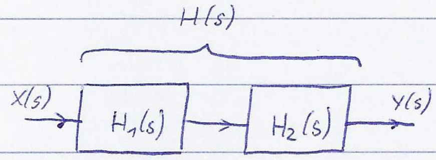
bog pomagi

polinom v števcu

vpliva na "kompleksnost"

Osnovne povezave med sistemi

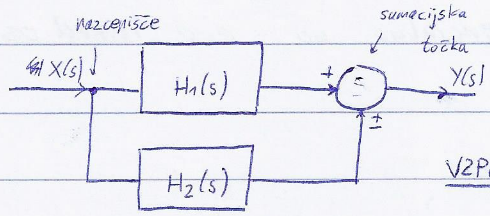




$$Y(s) = H_1(s)\, H_2(s)\, X(s)$$

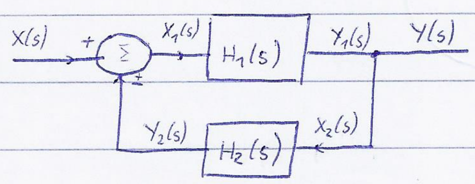
$$Y(s) = H(s)\, X(s)$$

KASKADA



VZPOREDNA

$$Y(s) = (H_1 \pm H_2)\, X(s)$$



$$Y(s) = Y_1(s) = \cancel{X_2}(s)$$

$$X_1(s) = X(s) \pm Y_2(s)$$

$$Y_2(s) = H_2(s)\, Y(s)$$

skupno:

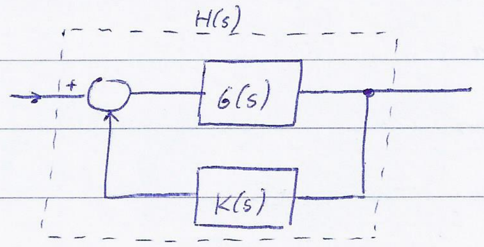
$$\boxed{H(s) = \frac{H_1(s)}{1 + H_1(s) H_2(s)}}$$

negativna povratna zanka

$\cancel{Y(s) = X(s)}$

$$Y(s) = H_1(s)\left[X(s) \pm H_2(s)\, Y(s)\right] \Rightarrow Y(s)\left[1 \mp H_1(s) H_2(s)\right] = H_1(s)\, X(s)$$

Določeno obliko povratne vezave vsebuje večina fizikalnih sistemov.
Pri povratnih sistemih je vhod neka funkcija izhoda...
Vhodni signal uravnavamo tako, da je razlika med dejanskim in želenim izhodnim čim manjša.
Sistem v direktni in povratni veji tvorita zaprto-zanko.
(zaprtozančni sistemi).



$$H(s)=\frac{G(s)}{1+K(s)G(s)}$$

poli

$$\underbrace{1+K(s)G(s)=0}_{\text{merilo povratne vezave}}$$

$W(s) = K(s)\,G(s)$

prevajalna fja. odprtozančnega sistema, ki ga dobimo, če zaprto zanko kjerkoli prekinemo

Stabilnost povratnih sistemov

Pri povratnih sist. so zahteve glede stabilnosti strožje, zato govorimo o ABS. in REL. stabilnosti.

Absolutna stabilnost

lega korenov $1+K(s)G(s)$ v s-ravnini

1) $1+K(s)G(s)=0$

2) Routhov stab. kriterij

   1) najprej zapišemo $1+K(s)G(s)=0$ v obliki $a_n s^n - a_{n-1} s^{n-1} + \dots + a_1 s + a_0 = 0$ HEH, FUNNY GUY

   2) koeficiente uredimo v tabelo:

$$
\begin{array}{c|ccccc}
s^n & a_n & a_{n-2} & a_{n-4} & a_{n-6} & \dots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & a_{n-7} & \dots \\
\hline
s^{n-2} & b_1 & b_2 & b_3 & b_4 & \dots \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 & \dots \\
\vdots & \vdots & & & & \\
s^3 & e_1 & e_2 & e_3 & & \\
s^2 & f_1 & f_2 & & & \\
s^1 & g_1 & & & & \\
s^0 & h_1 & & & &
\end{array}
$$

trikotna shema

zadosten pogoj $a_n, a_{n-1} \dots$ pozitivni

potreben pogoj .. menjava predznakov neki

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \qquad d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}} \qquad c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1} \qquad d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}$$

$$b_3 = \frac{a_{n-1}a_{n-6} - a_n a_{n-7}}{a_{n-1}} \qquad c_3 = \frac{b_1 a_{n-7} - a_{n-1} b_4}{b_1} \qquad d_3 = \frac{c_1 b_4 - b_1 c_4}{c_1}$$

Routhov stabilnostni krit. pravi, da je število korenov karakt. s pozitivnimi realnimi koreni enako številu menjav predznaka v prvem stolpcu, zato je potreben in zadosten pogoj, da so vsi koef. karakt. polinoma pozitivni in da so vsi elementi v prvem stolpcu pozitivni.

Primer

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

s^4	1	3	5
s^3	2	4	0
s^2	1	5	
s^1	-6	0	
s^0	5		

dve menjavi predznaka

Primer $s^3 + 3s^2 + 3s + 1 + K = 0$

s^3	1	3	0
s^2	3	$1+k$	0
s^1	$\frac{8-k}{3}$	0	
s^0	$1+k$		

$8-k>0$

$1+k>0$

$$\boxed{-1<k<8}$$

Pri povratnih sistemih želimo poleg informacije o abs. stabilnosti tudi inf. o tem tudi o tem, koliko je sistem oddaljen od meje stabilnosti in kako lahko to razdaljo spreminjamo: Rel. stabilnost.

Relativna stabilnost

Nyquistov stab. kriterij

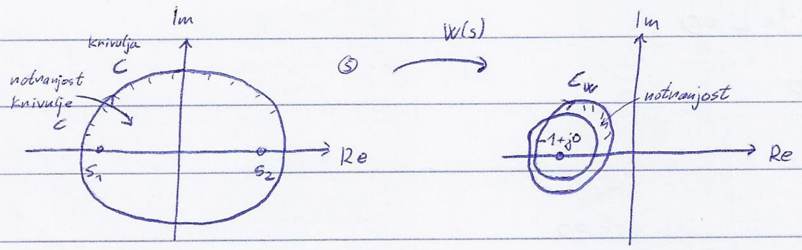
$$H(s) = \frac{G(s)}{1 + K(s)\,G(s)}$$

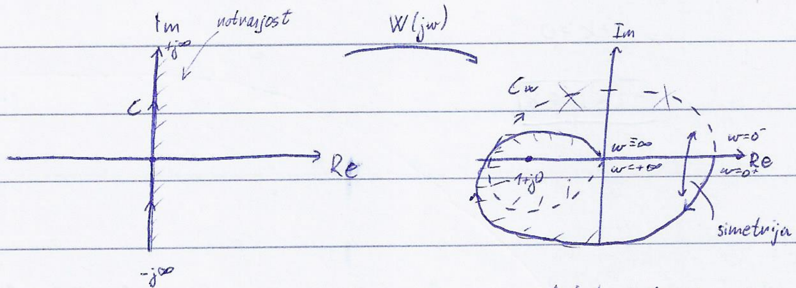
$$1 + \underbrace{K(s)\,G(s)}_{W(s)} = F(s) \quad \text{ničle } F(s)$$

$$F(s)\Big|_{s=s_1} = 0 = 1 + K(s)\,G(s)\Big|_{s=s_1}$$

$$s_1 \xrightarrow{\text{preslikamo}} W(s)$$

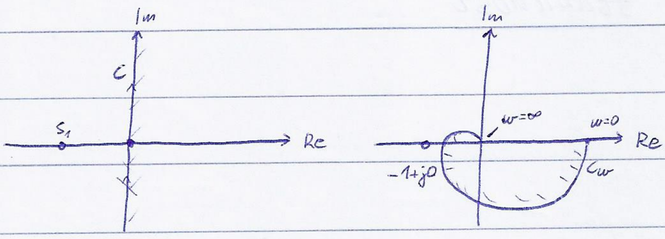
$$W(s_1) = \underline{\underline{-1 + j0}}$$

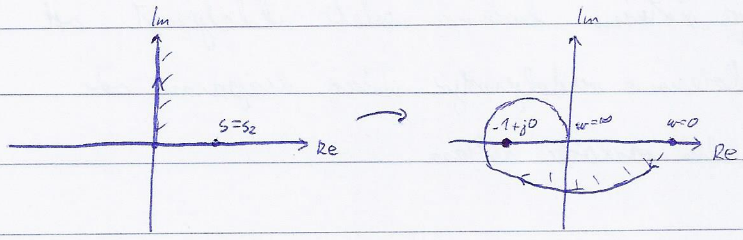


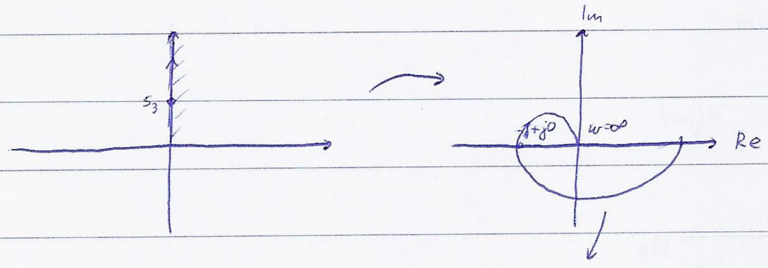


za stabilnost ta točka zunaj preslikave

$$1 + W(s) = 0\Big|_{s=s_1}$$







krivulja predstavlja

polarni diagram $W(j\omega)$

Za povratne sisteme, kjer sta oba sistema sama po sebi stabilna, se Nyquistov stavek glasi:
Povratni sistem je stabilen, če krivulja C_w v ravnini $W(j\omega)$ ki je slika imaginarne osi ravnine s ne obkroži točke v $W=-1+j0$. Ta točka se torej nahaja zunaj krivulje, ko preteče ω od 0 do ∞.

<u>Primer</u> $W(s) = G(s)K(s) = \dfrac{k}{s(s+1)(s+2)}$

za katere k bo povratni sistem stabilen?

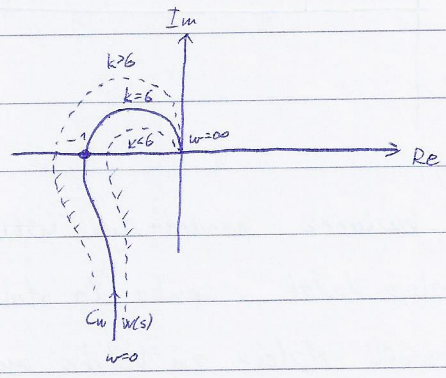
$$1+G(s)K(s)=0 = 1+\frac{k}{s(s+1)(s+2)}=0$$

$$s^3+3s^2+2s+k=0$$

s^3	1	2
s^2	3	k
s^1	$\frac{6-k}{3}$	0
s^0	k	

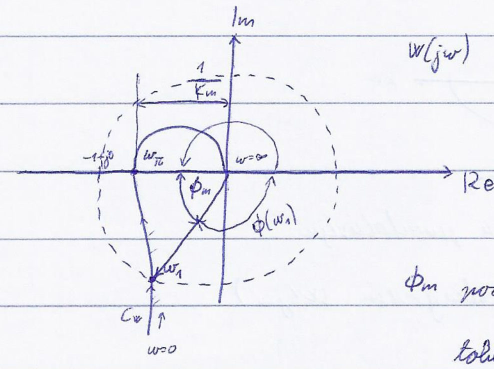
abs. stab.
$k<6$

$k>0$



Pri načrtovanju povratnega sistema hočemo vedeti oddaljenost od stabilnosti. Rel. stabilnost določena z oddaljenostjo polar. diagrama od točke $(-1+j0)$. Fazni in ojačevalni razloček izraža

STABILNI SISTEM



ϕ_m pozitivni fazni razloček

toliko kako obrnemo, da je sistem še stabilen

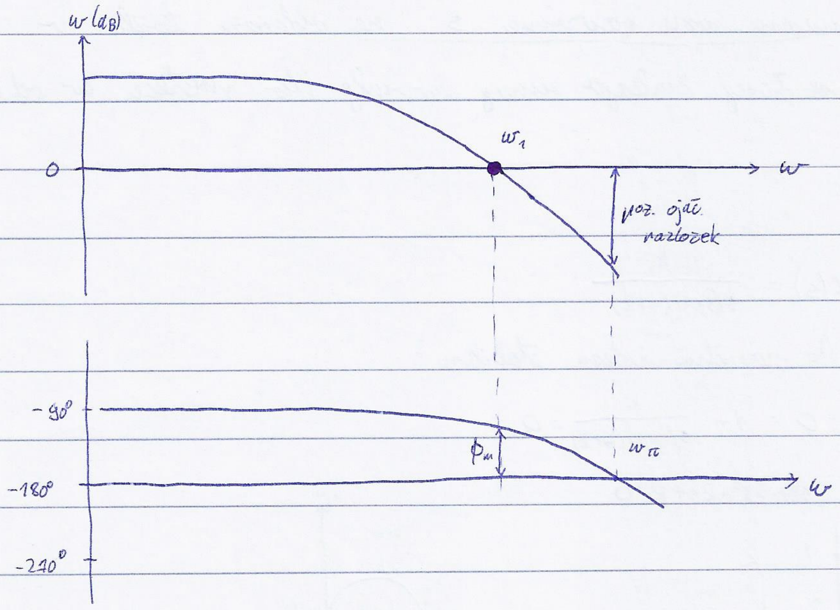
$\phi_m = 180° + \phi(\omega_1)$

$K_m > 1$ poz. ojačevalni razloček

$K_m = \frac{1}{|W(j\omega_\pi)|}$

8.1.2013

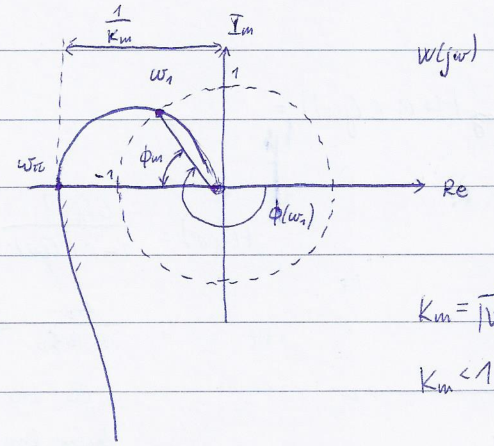
$K_{m_{dB}} = -20 \log |W(j\omega_\pi)|$



Poz. fazni razloček predstavlja tisti pozitivni kot (fazno zaostajanje), ki ga je potrebno dodati v zanko, da stabilni sistem postane mejno stabilen. Ojač. razloček določa, za koliko moramo spremeniti ojačanje odprtozančne prenos. fje $W(s)$, da zaprtozančni ~~zan~~sistem postane mejno stabilen.

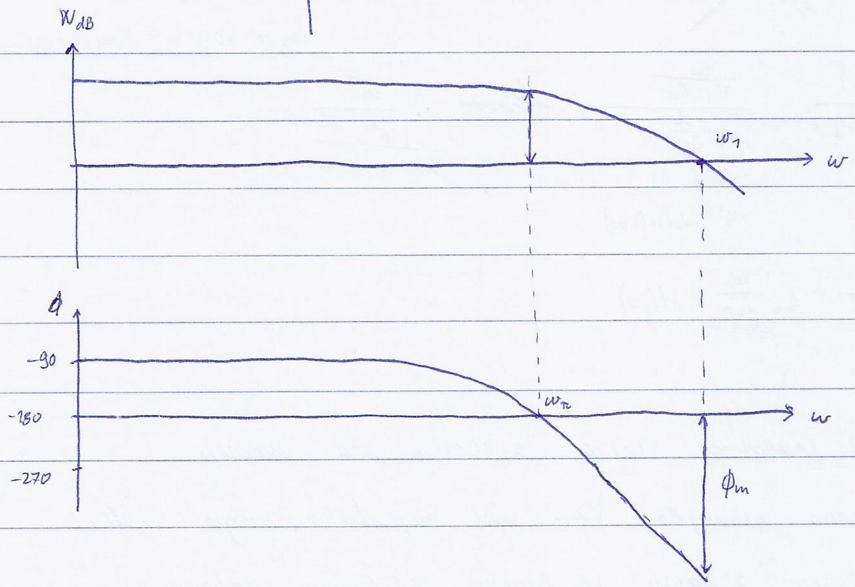
$\frac{17}{\sqrt{3}} \cdot \ln 2$

NESTABILEN SISTEM



$$K_m = \frac{1}{|W(j\omega_\pi)|}$$

$$K_m < 1$$



Povratni sistem z enim polom

$$G(s) = \frac{G_0}{1 + \frac{s}{\omega_p}}$$

$$H(s) = \frac{G(s)}{1 + \beta G(s)} = \frac{\frac{G_0}{1+\frac{s}{\omega_p}}}{1 + \beta \frac{G_0}{1+\frac{s}{\omega_p}}} = \frac{G_0}{1 + \frac{s}{\omega_p} + \beta G_0} = \frac{\frac{G_0}{1+\beta G_0}}{\frac{1}{1+\beta G_0} + \frac{s}{\omega_p(1+\beta G_0)} + \frac{\beta G_0}{1+\beta G_0}}$$

$$K(s) = \beta$$

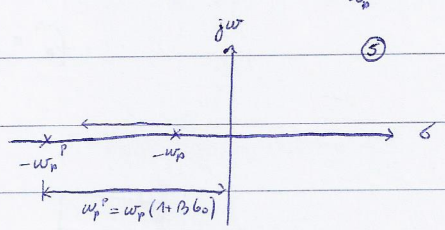
$$= \frac{\frac{G_0}{1+\beta G_0}}{1 + \frac{s}{\omega_p(1+\beta G_0)}} =$$

$$H_0 = \frac{G_0}{1+\beta G_0}$$

$$\omega_p^P = \omega_p (1 + \beta G_0)$$

$$= \frac{H_0}{1 + \frac{s}{\omega_p^P}}$$

$$\boxed{H_0 \omega_p^P = G_0 \omega_p}$$

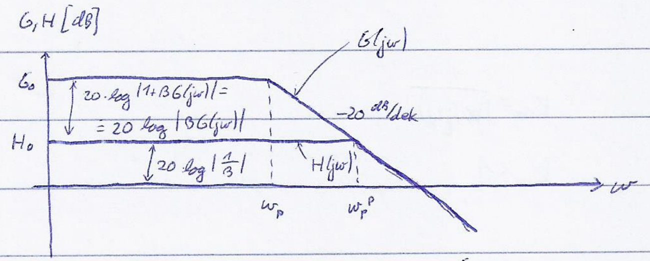


5

$$H(j\omega) = \frac{G(j\omega)}{1+\beta G(j\omega)}$$

$\underline{\beta G_0 \gg 1}$ (tudi ponavadi velja)

$$20 \cdot \log |H(j\omega)| = 20 \cdot \log |G(j\omega)| - 20 \cdot \log |1+\beta G(j\omega)| =$$
$$= 20 \log \left|\frac{1}{\beta}\right|$$



$\beta G_0 \cdot \frac{1}{\beta}$

$$H(j\omega) = \frac{G(j\omega)}{1+\beta G(j\omega)} = \frac{\frac{G_0}{1+\frac{j\omega}{\omega_p}}}{1+\beta\frac{G_0}{1+\frac{j\omega}{\omega_p}}} =$$

$$\text{nf:} \quad = \frac{G_0}{1+\beta G_0} = H_0$$

$$G_0 = H_0 (1+\beta G_0)$$
$$\log G_0 = \log H_0 + \log(1+\beta G_0)$$

$$\text{vf:} \quad H(j\omega) = \frac{G(j\omega)}{1+\beta G(j\omega)} = \frac{\frac{G_0}{1+\beta G_0}}{\cancel{1+}\frac{j\omega}{\omega_p^p}} = \cancel{G_0} \; G_0 \frac{\omega_p}{j\omega}$$

$$\omega_p^p = \omega_p (1+\beta G_0)$$

$$\text{vf:} \quad G(j\omega) = \frac{G_0}{\cancel{1+}\frac{j\omega}{\omega_p}} = G_0 \frac{\omega_p}{j\omega} = H(j\omega)$$

Povhatna vezava razširi pasovno širino sistema na račun ojačanja, ki se sorazmerno zmanjša. Ker pol zaprtozančnega sist. nikoli ne vstopi v desno s-ravn. je sistem z enim polom brezpogojno stabilen.

Primer $\quad f_p = 100\ Hz \quad G_0 = 10^5$, povratna zveza $\quad \beta = 0.01$

$f_p^p = ?$

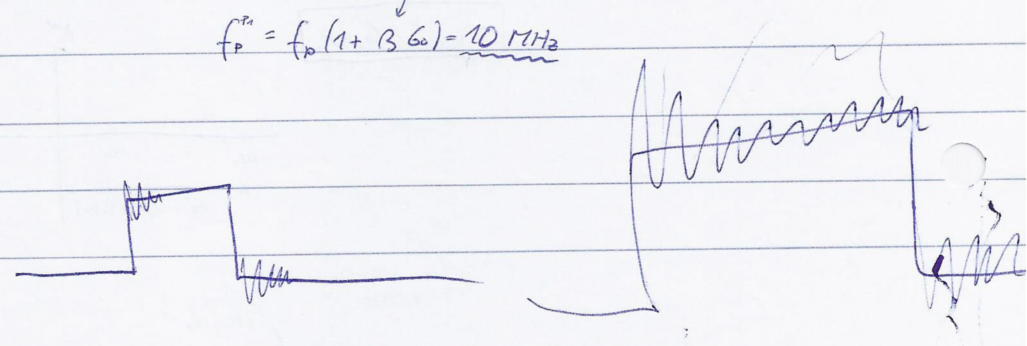
$$H_0 = \frac{G_0}{1+\beta G_0}$$

β spremenimo tako, da bo $H_0 = 1$, kam se premakne pol $\quad f_p^p = f_p(1+\beta G_0)$

a) $f_p(1+\beta G_0) =$

$= f_p(1+0.01\cdot 10^5) = \cancel{100\,k}\ Hz$ 100.1 kHz $= f_p^p$

$$1 = \frac{G_0}{1+\beta G_0}$$
$$\beta = \frac{G_0 - 1}{G_0} = 0.99999$$
$$f_p^{p_1} = f_p(1+\beta G_0) = \underline{10\ MHz}$$



$$\frac{\sqrt{4a}}{2} = 1$$

DVA POLA

poli $H(s)$: $\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right) + \beta G_0 = 0$

$$1 + \frac{s}{\omega_{P1}} + \frac{s}{\omega_{P2}} + \frac{s^2}{\omega_{P2}\omega_{P1}} + \beta G_0 = 0 \quad / \cdot \omega_{P1}\omega_{P2}$$

$$(1+\beta G_0)\omega_{P1}\omega_{P2} + s(\omega_{P1}+\omega_{P2}) + s^2 = 0$$

$$\underset{a}{1} s^2 + \underset{b}{(\omega_{P1}+\omega_{P2})} s + \underset{c}{(1+\beta G_0)\omega_{P1}\omega_{P2}} = 0$$

$$s_{1,2} = \cancel{\frac{-b \pm \sqrt{4ac}}{2a}} = \frac{-b}{2} \cancel{\pm \sqrt{ac}}$$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \underset{a=1}{=} \frac{-b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - c}$$

$$s_{1,2} = -\frac{\omega_{P1}+\omega_{P2}}{2} \pm \sqrt{\underbrace{\left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2 - (1+\beta G_0)\omega_{P1}\omega_{P2}}_{\text{~~to~~ menda rata kompleksno}}}$$

pola sovpadata, ko · enako 0 $(\xi = 1)$

$$\left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2 - (1+\beta G_0)\omega_{P1}\omega_{P2} = 0$$

$$(1+\beta G_0)\omega_{P1}\omega_{P2} = \left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2$$

$$\beta G_0 \omega_{P1}\omega_{P2} = \left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2 - \omega_{P1}\omega_{P2}$$

$$\beta = \frac{\left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2 - \omega_{P1}\omega_{P2}}{G_0\,\omega_{P1}\omega_{P2}} = \frac{\frac{1}{4}\frac{(\omega_{P1}+\omega_{P2})^2}{\omega_{P1}\omega_{P2}} - 1}{G_0}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \sqrt{(1+\beta G_0)\,\omega_{P1}\omega_{P2}}$$

$$\xi = \frac{\omega_{P1}+\omega_{P2}}{2\omega_n} =$$

$$\xi = \frac{\omega_{P1}+\omega_{P2}}{2\sqrt{(1+\beta G_0)\,\omega_{P1}\omega_{P2}}}$$

$$= \sqrt{\frac{(\omega_{P1}+\omega_{P2})^2}{4(1+\beta G_0)\omega_{P1}\omega_{P2}}} =$$

$$= \sqrt{\frac{\left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2}{(1+\beta G_0)\,\omega_{P1}\omega_{P2}}} = 1$$

$$\xi = \sqrt{\frac{\left(\frac{\omega_{P1}+\omega_{P2}}{2}\right)^2}{(1+\beta G_0)\,\omega_{P1}\omega_{P2}}}$$

$$\xi = \frac{\omega_{P1}+\omega_{P2}}{2\sqrt{(1+\beta G_0)\,\omega_{P1}\omega_{P2}}}$$

$$\frac{2\xi}{\omega_{P1}+\omega_{P2}} = \frac{1}{\sqrt{(1+\beta G_0)\omega_{P1}\omega_{P2}}} \quad /^2$$

$$(1+\beta G_0)\omega_{P1}\omega_{P2} = \left(\frac{\omega_{P1}+\omega_{P2}}{2\xi}\right)^2$$

$$1+\beta G_0 = \frac{\left(\frac{\omega_{P1}+\omega_{P2}}{2\xi}\right)^2}{\omega_{P1}\omega_{P2}}$$

$$\beta = \frac{\frac{(\omega_{P1}+\omega_{P2})^2}{4\,\omega_{P1}\omega_{P2}\,\xi^2} - 1}{G_0}$$

$B = 2\xi\omega_n$

$\xi = \dfrac{B}{2\omega_n}$

Povratni sistem z dvema poloma

9.1.2013

$$G(s) = \frac{G_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

$$H(s) = \frac{G(s)}{1+K(s)G(s)} = \frac{\frac{G_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}}{1+B\frac{G_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}} =$$

$$= \frac{G_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)+BG_0}$$

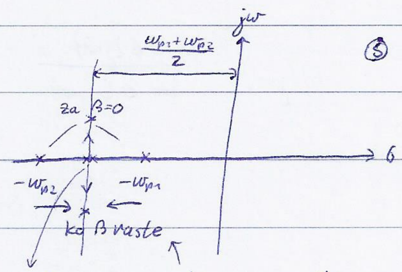
$1 + BG(s) = 0$

$$s^2 + (\omega_{p1}+\omega_{p2})s + \omega_{p1}\omega_{p2}(1+BG_0) = 0$$

WHAT?

$$s_{1,2} = -\frac{\omega_{p1}+\omega_{p2}}{2} \pm \sqrt{\left(\frac{\omega_{p1}+\omega_{p2}}{2}\right)^2 - \omega_{p1}\omega_{p2}(1+BG_0)}$$

še vedno brezpogojno stabilen

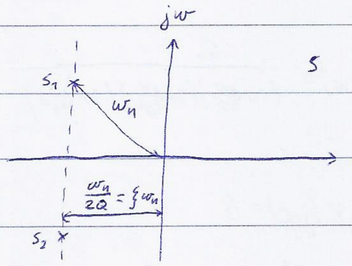


$$s^2 + \frac{\omega_n}{Q}s + \omega_n^2 = s^2 + \underbrace{2\xi\omega_n}\, s + \omega_n^2 = 0$$

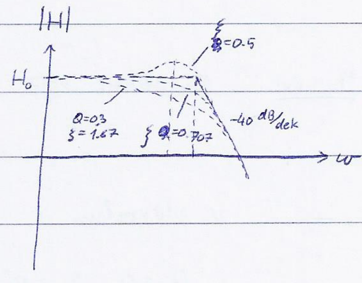
$$\omega_n = \sqrt{(1+BG_0)\,\omega_{p1}\omega_{p2}}$$

$$\frac{1}{2\xi} = Q = \frac{\sqrt{(1+BG_0)\,\omega_{p1}\omega_{p2}}}{\omega_{p1}+\omega_{p2}}$$

$$\xi = \frac{\omega_{p1}+\omega_{p2}}{2\sqrt{(1+BG_0)\,\omega_{p1}\omega_{p2}}}$$



$Q = \dfrac{1}{2\xi}$



$\xi = 0.707 = Q$

$Q = 0.5$

Primer $G_0 = 100$

$\omega_{p1} = 10^4\ \frac{rad}{s}$

$\omega_{p2} = 10^6\ \frac{rad}{s}$

ko $\xi = 1$, $Q = 0.5$

$$B = \frac{\frac{Q^2(\omega_{p1}+\omega_{p2})^2}{\omega_{p1}\omega_{p2}} - 1}{G_0} = \underline{0.245}$$ vpliv povratne vezave

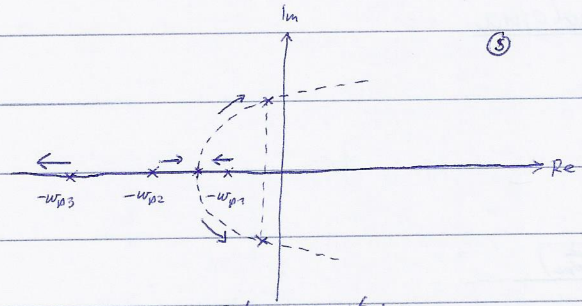
tudi: $Q = 0.707$

$$B = \text{-||-} = \underline{\underline{0.5}}$$

$$H_0(Q=0.707) = \frac{G_0}{1+BG_0} = \frac{100}{1+0.5\cdot 100} = \underline{\underline{1.96}}$$

pri kateri B pola zaprtozančnega sistema sovpadata? pri katerem B bo ξ 0.707 koliko je takrat enosmerno ojačanje?

Povratni sistem s tremi ali več poli



5

kaj, ko $B\uparrow$?

študij stabilnosti z $W(j\omega)$ (polarni) je zelo zamudno, $G(s)$ je znan, spreminjamo B

sistem postaja nestabilen!

- alternativno ugotavljanje stabilnosti povratnega sistema!

$$H(j\omega) = \frac{G(j\omega)}{1 + B\,G(j\omega)}$$

$$20\cdot\log|H(j\omega)| = 20\log|G(j\omega)| - 20\log|1 + B\,G(j\omega)|$$

$$\omega = 0 \rightarrow H(0) = \frac{G(0)}{1 + B\,G(0)}$$

NF

$$\text{če } B\,G(0) \gg 1 \rightarrow H(0) = \frac{1}{B}$$

$$20\log|H(j\omega)| = 20\log\left|\frac{1}{B}\right|$$

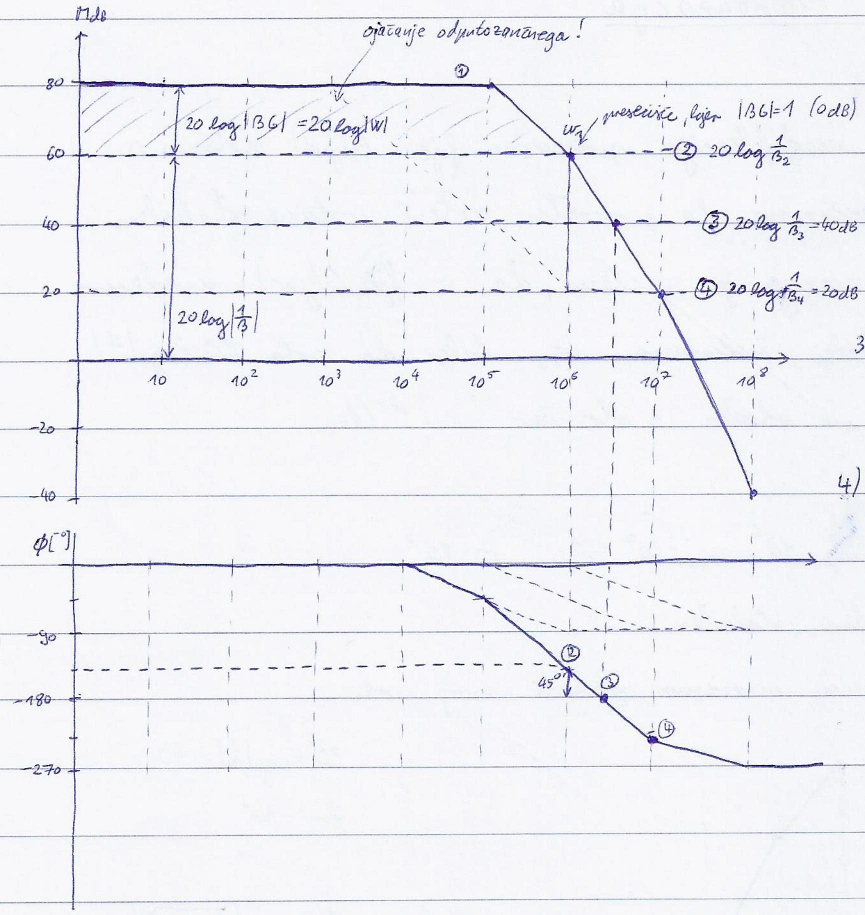
$$\underline{20\log|G(j\omega)|} - \underline{20\log\frac{1}{B}} = 20\log|B\,G(j\omega)|$$

gledamo njuno presečišče

<u>Primer</u> direktna veja $G(j\omega) = \dfrac{G_0}{\left(1 + \frac{j\omega}{10^5}\right)\left(1 + \frac{j\omega}{10^6}\right)\left(1 + \frac{j\omega}{10^7}\right)}$, $G_0 = 10^4$

v povratni veji $K(j\omega) = B$

$B_1 = 0$, $B_2 = 10^{-3}$, $B_3 = 10^{-2}$, $B_4 = 10^{-1}$ ugotovite stabilnost sistema



1) $\beta=0$ $H(j\omega)=G(j\omega)$

2) $\beta_2=10^{-3}$ $20\cdot\log\frac{1}{\beta_2}=60\,dB$; $\phi=-135°$

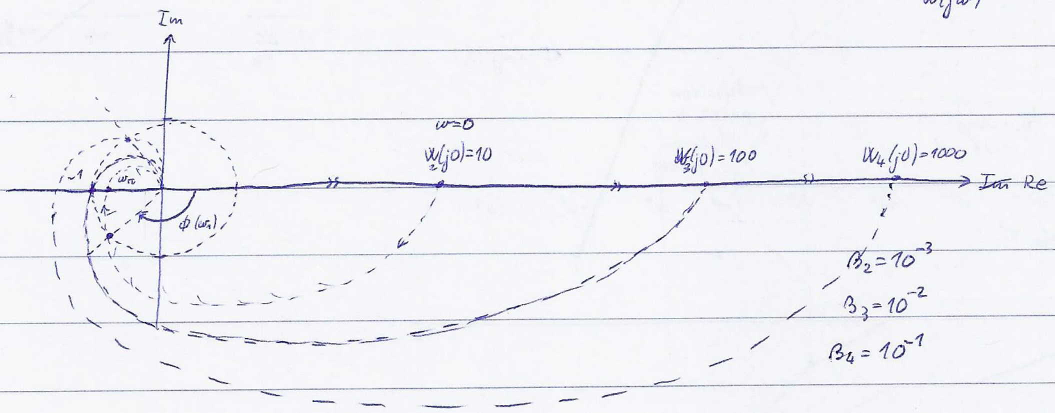
$20\log|\beta G|_2 = 20dB$ $\phi_m=45°$

3) $\beta_3=10^{-2}$ $20\cdot\log\frac{1}{\beta_3}=40\,dB$ $\phi=-180°$

$20\log|\beta G|_3 = 40dB$ $\phi_m=0°$

4) $\beta_4=10^{-1}$ $20\cdot\log\frac{1}{\beta_4}=20\,dB$ $\phi=-225°$

$20\log|\beta G|_4 = 60\,dB$ $\phi_m=-45°$ ni stabilen!



Ker pride do faznega premika 180° vedno na segmentu -40 dB/dek, je pravilo za stabilnost naslednje. Povratni sistem bo stabilen, če črta $20\cdot\log\frac{1}{\beta}$ seka krivuljo $20\log|G(j\omega)|$ na segmentu z naklonom -20 dB/dek, to pravilo zagotavlja, da bo fazni razloček najmanj 45°. Navedeno pravilo lahko posplošimo za primere, ko je β fja frekvence. $\beta=\beta(j\omega)$ Splošno pravilo pravi, da v presečišču krivulj $20\cdot\log\frac{1}{\beta(j\omega)}$ in $20\log|G(j\omega)|$ razlika strmin ne sme preseči 20 dB/dek.

Frekvenčna kompenzacija

Je metoda, pri kateri z modifikacijo prenosne fje $G(j\omega)$ sistema s stremi ali več poli dosežemo, da povratni sistem postane stabilen. Bistvo frekvenčne kompenzacije je v tem, da v fji $G(j\omega)$ uvedemo nov pol pri zadosti nizki frekvenci ω_{p0} tako, da črta $20\log|\frac{1}{\beta}|$, seka modificirano $G(j\omega)$ na odseku s strmino $-20\ dB/dek$

Primer: $G_0 = 10^5$, $\omega_{p1} = 10^5$, $\omega_{p2} = 10^6$, $\omega_{p3} = 10^7\ \frac{rad}{s}$, $\beta = 10^{-2}$

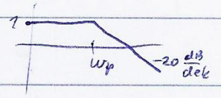
naredimo ta sistem stabilen

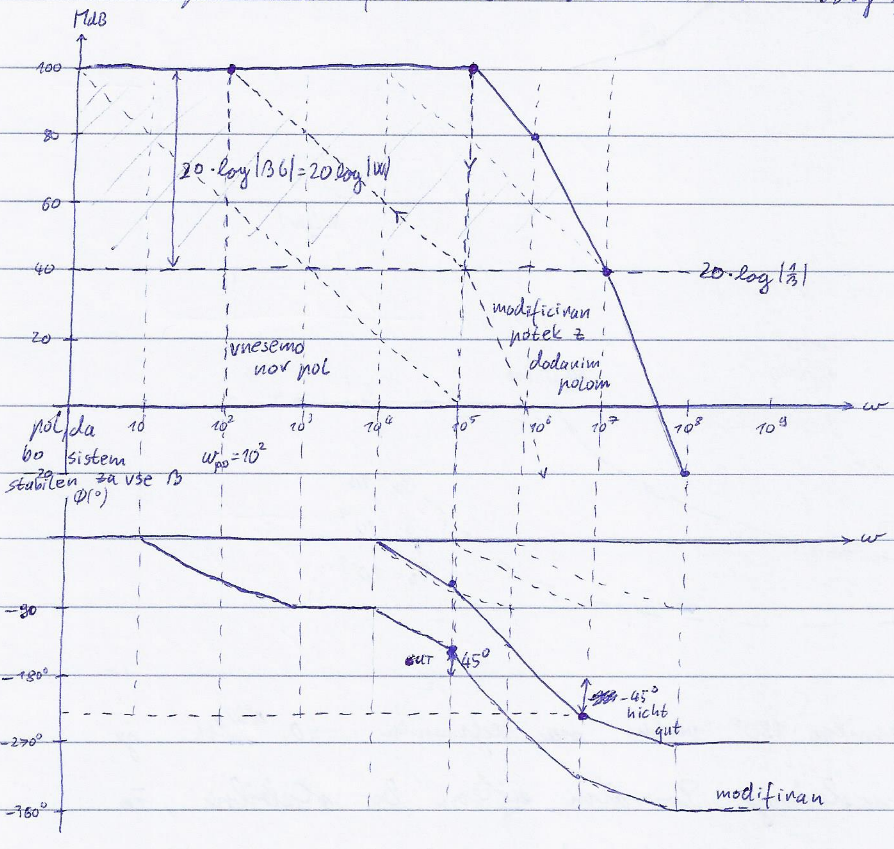
pri kateri frekvenci moramo dodati svoj pol

$20\cdot\log|\frac{1}{\beta}| = 40$

$\phi_m = -90°$

$\frac{1}{1+\frac{j\omega}{\omega_p}}$





Občutljivost sistema s povratno vezavo

$$\frac{\partial H}{\partial G} = \frac{1}{(1+\cancel{B}_{K}G)^2} \qquad \frac{\partial H}{\partial K} = \frac{-G^2}{(1+KG)^2}$$

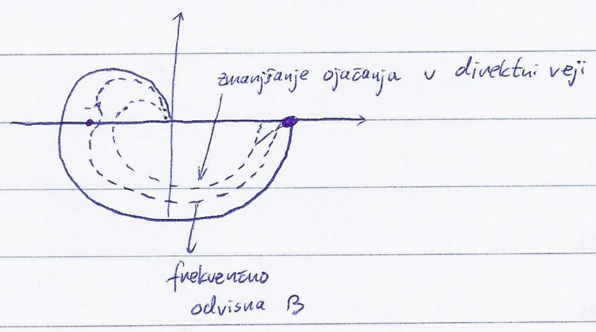
povratna veja

direktna veja

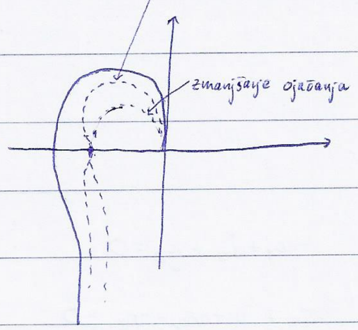
$$\boxed{\frac{\frac{\partial H}{\partial K}}{\frac{\partial H}{\partial G}} = -G^2}$$

v povratni veji uporabljamo pasivne elemente!

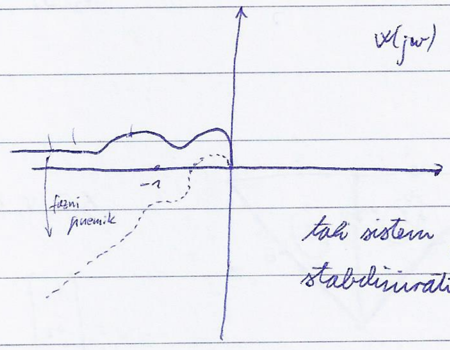
Povratni sistem 0. vrste



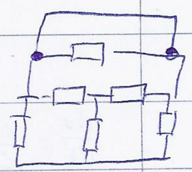
Povratni sistem 1. vrste



Povratni sistem 2. vrste



tak sistem je težko stabilizirati



TOPOLOGIJA VEZIJ

16.1.2013

SISTEMATIČEN PRISTOP K REŠEVANJU VEZIJ V PROSTORU STANJ

$\dot{x} = Ax + Bu$ $\quad v_c, i_L$

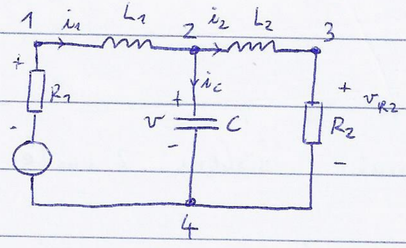
$y = Cx + Du$

$C\frac{dv_c}{dt} = i_c$ v veje drevesa

$L\frac{di_L}{dt} = u_L$ v lite drevesa

x, v_c, i_L — v vsako vejo kondenzatorja, vrišemo osnovni rez; v vsako lito tuljav, vrišemo osnovno zanko

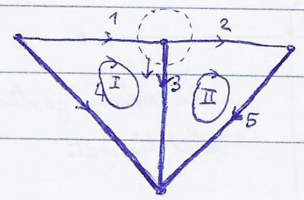
Primer



$v(0)$

$i_1(0)$

$i_2(0)$



$x = v, i_1, i_2$

$x = \begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix}$

Rez: $-i_1 + i_2 + \underline{i_c} = 0$

z I: $\underline{v_{L1}} + v - v_g - \underline{v_{R1}} = 0$

z II: $\underline{v_{L2}} + \underline{v_{R2}} - v = 0$

$i_c = C\frac{dV}{dt}$

$v_L = L\frac{di}{dt}$

$v_{R1} = -i_1 R_1$

$v_{R2} = i_2 R_2$

dobimo

$$\frac{dv}{dt} = \frac{i_1}{C} - \frac{i_2}{C}$$

$$\frac{di_1}{dt} = -\frac{R_1 i_1}{L_1} - \frac{v}{L_1} + \frac{v_g}{L_1}$$

$$\frac{di_2}{dt} = -\frac{R_2 i_2}{L_2} + \frac{v}{L_2}$$

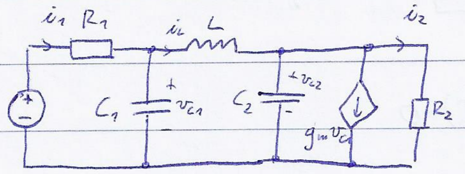
$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix} v_g$$

$$y = \begin{bmatrix} e_1 \\ v_{R2} \end{bmatrix} = \Big[-R_1 \quad 0 \quad \dots$$

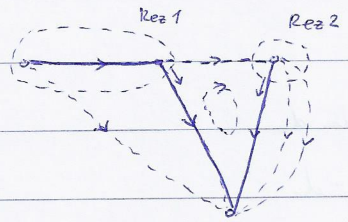
$v_{R2} = R_2 i_2$

$e_1 = v_g - R_1 i_1$

Primer



$$x = \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix}$$



Rez I: $C_1 \frac{dv_{c1}}{dt} = i_1 - i_L = \frac{v_g - v_{c1}}{R_1} - i_L$

Rez II: $C_2 \frac{dv_{c2}}{dt} = i_L - g_m v_{c1} - i_2 = i_L - g_m v_{c1} - \frac{v_{c2}}{R_2}$

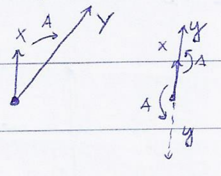
zanka: $L \frac{di_L}{dt} = v_{c1} - v_{c2}$

$$\begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 & -\frac{1}{C_1} \\ -\frac{g_m}{C_2} & -\frac{1}{R_2 C_2} & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C_1} \\ 0 \\ 0 \end{bmatrix} v_g$$

---

e^{At} matrika A $n \times n$, vektor x

linearna preslikava $Y = Ax$ lastne vrednosti, lastni vektorji



$y = \lambda x$ (če so soležne)

$Ax = \lambda x = \lambda I x$

$(A - \lambda I)x = 0$ λ... lastne vrednosti

Lastne vrednosti so tista števila, pri kateri ima netrivialne rešitve.

Homogen sistem ima netrivialne rešitve, če $\det(A - \lambda I) = 0$ sonda!

Metoda Cayley-Hamilton za določanje $e^{At}=\phi(t)$

če v polinomu:

$$Q(\lambda)=\lambda^n+a_{n-1}\lambda^{n-1}+\dots+a_1\lambda+a_0=0$$

nadomestimo λ za A, je polinom

$$Q(A)=A^n+a_{n-1}A^{n-1}+\dots+\cancel{A}\,a_1A+a_0$$

še vedno enak nič

$$\boxed{e^{At}=\alpha_0 I+\alpha_1 A+\alpha_2 A^2+\dots\alpha_{n-1}A^{n-1}}$$

$$\boxed{e^{\lambda t}=\alpha_0 I+\alpha_1 \lambda+\alpha_2 \lambda^2+\dots+\alpha_{n-1}\lambda^{n-1}}$$ iz tukaj dobimo alfe...

Primer

$$A=\begin{bmatrix}-3 & -1\\ 2 & 0\end{bmatrix} \qquad \det(A-\lambda I)=0$$

$$\lambda_1=-1$$

$n=2$

$$\lambda_2=-2$$

$e^{At}=\alpha_0 I+\alpha_1 A$, določimo α_0 in α_1

$e^{\lambda_1 t}\neq\alpha_0 I+\alpha_1\lambda_1 \qquad e^{-t}=\alpha_0-\alpha_1$

$e^{\lambda_2 t}=\alpha_0 I+\alpha_1\lambda_2 \qquad e^{-2t}=\alpha_0-2\alpha_1$

$$\alpha_0=2e^{-t}-e^{-2t}$$

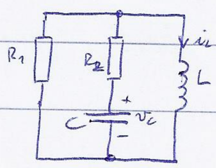
$$\alpha_1=e^{-t}-e^{-2t}$$

$$e^{At}=\alpha_0 I+\alpha_1 A=\alpha_0\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\alpha_1\begin{bmatrix}-3 & -1\\ 2 & 0\end{bmatrix}=\begin{bmatrix}\alpha_0-3\alpha_1 & -\alpha_1\\ 2\alpha_1 & \alpha_0\end{bmatrix}=$$

$$=\begin{bmatrix}-e^{-t}+2e^{-2t} & -e^{-t}+e^{-2t}\\ 2e^{-t}-2e^{-2t} & 2e^{-t}-e^{-2t}\end{bmatrix}$$

$2e$

Primer



$R_1=R_2=1\Omega$

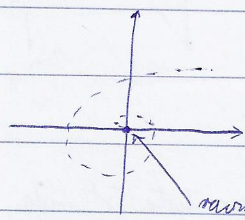
$C=1F$

$L=1H$

$x_1(0)=2$ V

$x_2(0)=2$ A

$$x=\begin{bmatrix}v_C\\ i_L\end{bmatrix} \qquad x(t)=2e^{-\frac{t}{2}}\begin{bmatrix}\cos\frac{t}{2}-\sin\frac{t}{2}\\ \cos\frac{t}{2}+\sin\frac{t}{2}\end{bmatrix}u(t)$$



ravnotežna točka

(žarišče)

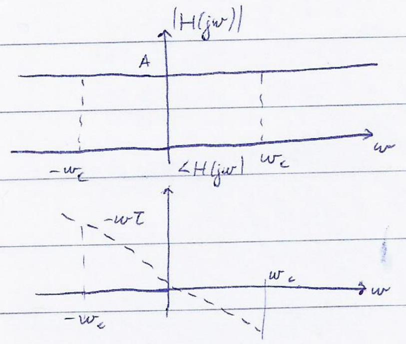
FILTRI

Sistem brez popačenj:

Ker vsak, tudi idealni, vnaša zakasnitve (vhod-izhod) imenujemo prenosni sistem brez popačenj, tisti sistem, ki prepušča signale brez spremembe, razen ojačanja in časovne zakasnitve

$$y(t) = A x(t-\tau)$$

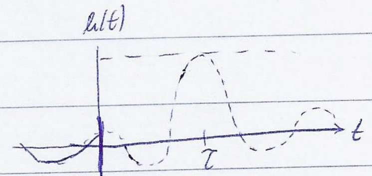
$$Y(j\omega) = A X(j\omega) e^{-j\omega\tau} \qquad H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = A e^{-j\omega\tau}$$



$$H(j\omega) = \begin{cases} A e^{-j\omega\tau} & -\omega_c \le \omega \le \omega_c \\ 0 & \end{cases}$$

$\downarrow$

$$h(t) = \mathcal{F}^{-1}[H(j\omega)] = A \frac{\omega_c}{\pi} \frac{\sin[\omega_c(t-\tau)]}{\omega_c(t-\tau)}$$



odziv pred signalom ... idealen filter nemogoč

Butterworth (amplitudna karakteristika maksimalno ravna v prepustnem in neprepustnem delu)

Chebyshev (hiter prehod)

$$|H(j\omega)|^2 = \frac{n(\omega^2)}{m(\omega^2)}$$ butterworth

$n(\omega^2) = n_0$ (baje najboljše)

$m(\omega^2) = a_0 + \underset{\overset{\shortparallel}{1}}{a_{2n}} \omega^{2n}$

<u>2n-1</u> odvodov enakih nič pri $\omega = 0$

$$|H(j\omega)|^2 = \frac{n_0}{a_0 + \omega^{2n}} = \frac{K^2}{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2n}} = \frac{K^2}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}}$$

$$\omega_c = \omega_p \varepsilon^{-\frac{1}{n}}$$

$\downarrow$

rob prepustnega pasu