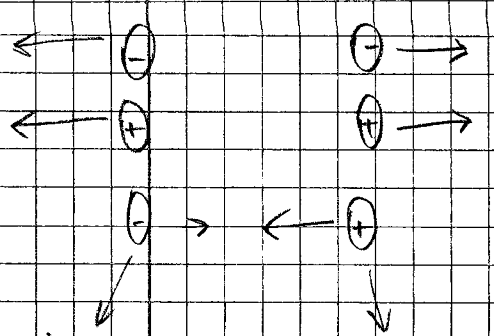
Coulombov zakon

21. 2. 2010

Izolator nima prevodnih elektronov.



} odbojna sila

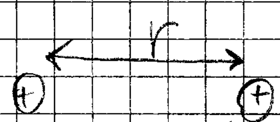
} privlačna sila

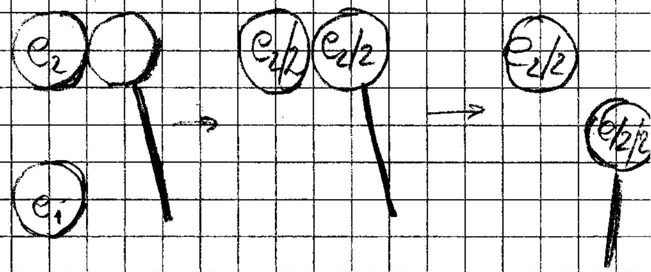
naboj elektrona

naboj protona

$e_0 = 1{,}6 \cdot 10^{-19}\,As$

El. sila je odvisna od razdalje.





$$\left.\begin{array}{l} F \propto \dfrac{1}{r^2} \\ F \propto e_1 \\ F \propto e_2 \end{array}\right\} F \propto \frac{e_1 e_2}{r^2}$$

$$\underset{\text{Coulombova konstanta}}{F = K}\,\frac{e_1 e_2}{r^2} = \boxed{\frac{e_1 e_2}{4\pi \varepsilon_0 r^2}}$$

influenčna konstanta

$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\,As/Vm$

Predpostavljamo, da snov ne vpliva na interakcijo med dvema nabojema.

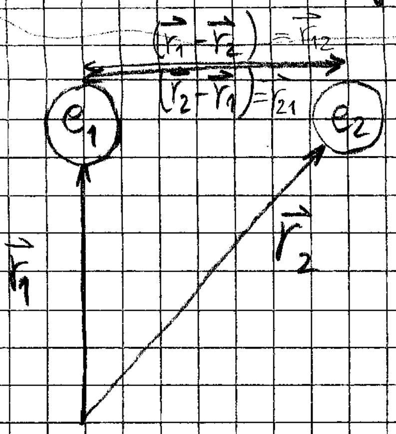
Coulombov zakon:

Sila med dvema nabojema v praznem prostoru.

ε_0 - se nanaša na lastnost praznega prostora, nima zveze s snovjo, dielektričnostjo.

$$\boxed{c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \cong 3 \cdot 10^8\,\frac{m}{s}}$$

Van der Graafov generator - deluje na principu influence

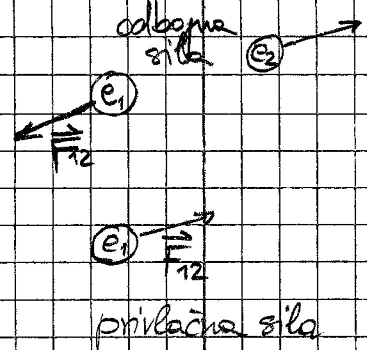


Sila deluje na daljavo

$$\vec{F}_{12} = \frac{e_1 e_2}{4\pi\varepsilon_0 r_{12}^2}\left(\frac{\vec{r}_{12}}{r_{12}}\right)$$

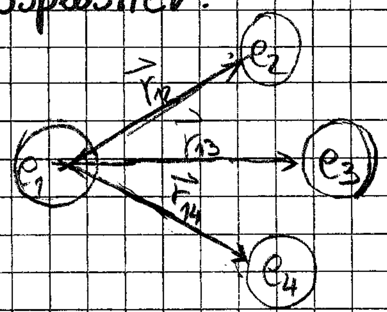
$$\left.\begin{array}{l} e_1 > 0,\ e_2 > 0\ ;\ e_1 e_2 > 0 \\ e_1 < 0,\ e_2 < 0\ ;\ e_1 e_2 > 0 \end{array}\right\}$$

$$\left.\begin{array}{l} e_1 < 0,\ e_2 > 0 \\ e_1 > 0\ \ e_2 < 0 \end{array}\right\} e_1 e_2 < 0$$



$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \underbrace{\vec{F}_{12} + \vec{F}_{21} = 0}_{\text{3. Newtonov zakon}}$$

posplošitev:

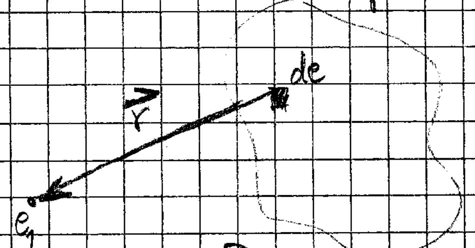


n ... št. nabojev v sistemu

$$\vec{F}_1 = \sum_{j=2}^{n} \frac{e_1 e_j}{4\pi\varepsilon_0 r_{1j}^2}\left(\frac{\vec{r}_{1j}}{r_{1j}}\right)$$

posplošitev:

porazdelitev el. naboja



$$\sum_j \rightarrow \int,\quad \vec{r}_{1j} \rightarrow \vec{r}$$

$$e_j \rightarrow de$$

$$\vec{F}_1 = \int \frac{e_1\, de}{4\pi\varepsilon_0 r^2}\left(\frac{\vec{r}}{r}\right)$$

$$\rho = \frac{de}{dV}$$

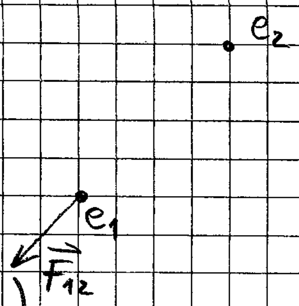
$$\sigma = \frac{de}{dA}$$

$$q = \frac{de}{dl}$$

$$\left.\begin{array}{l} de = \rho\, dV \\ de = \sigma\, dA \\ de = q\, dl \end{array}\right\}$$

Električno polje (jakost el. polja $\vec{E}$)

$$\vec{F}_{12} = \frac{e_1 e_2}{4\pi\varepsilon_0 r_{12}^2}\left(\frac{\vec{r}_{12}}{r_{12}}\right)$$



$e_1 \equiv$ testni naboj

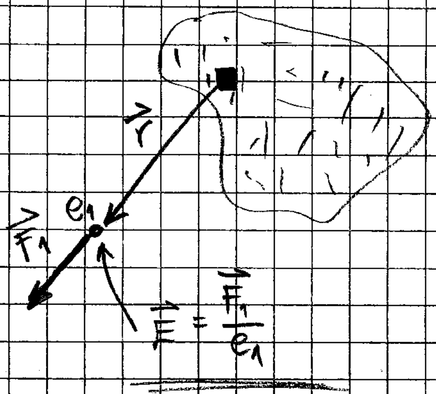
$$\vec{E}_2 = \frac{\vec{F}_{12}}{e_1} = \frac{e_2}{4\pi\varepsilon_0 r_{12}}\left(\frac{\vec{r}_{12}}{r_{12}}\right)$$

Jakost polja naboja e_2 na mestu kjer je naboj e_1.

posplošitev:

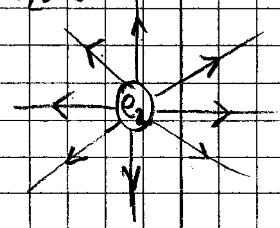
$$\vec{F}_1 = \int \frac{e_1\, de}{4\pi\varepsilon_0 r^2}\left(\frac{\vec{r}}{r}\right)$$

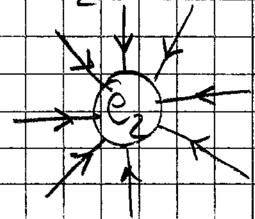
$$\vec{E} = \frac{\vec{F}_1}{e_1} = \int \frac{de}{4\pi\varepsilon_0 r^2}\left(\frac{\vec{r}}{r}\right)$$



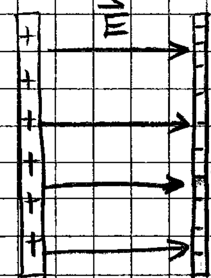
V principu bi moral biti testni naboj čim manjši da ne vpliva na porazdelitev in ne bi dobili točnih meritev. Manjši kot e_0 ne more biti.

$$\boxed{\vec{F}_1 = e_1\vec{E}}$$ → sila na testni naboj v polju.

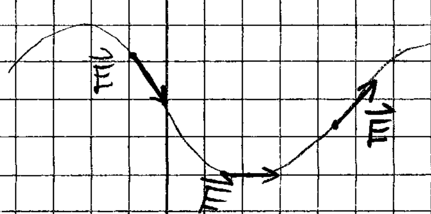


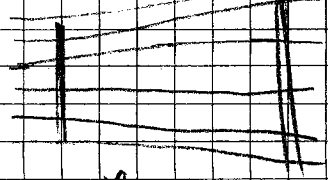


kondenzator:



smer polja:

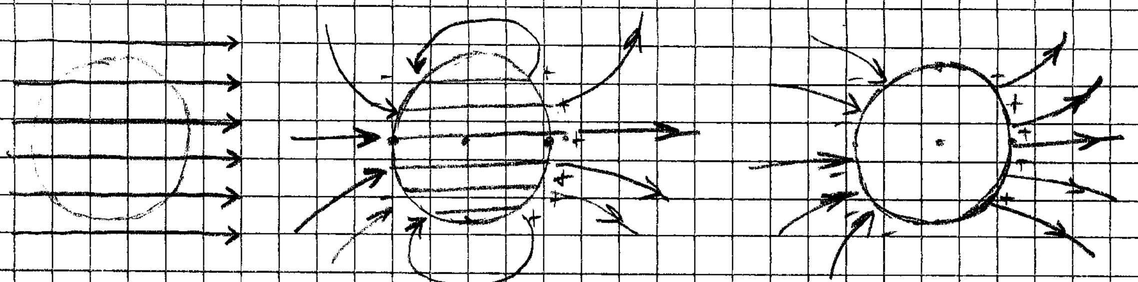




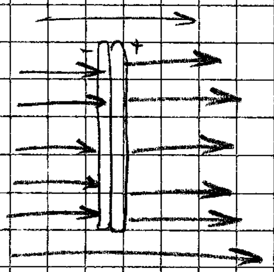
silnica

Silnice se ne smejo sekati, ker bi polje v tisti točki imelo 2 smeri in ne bi bilo definirano.

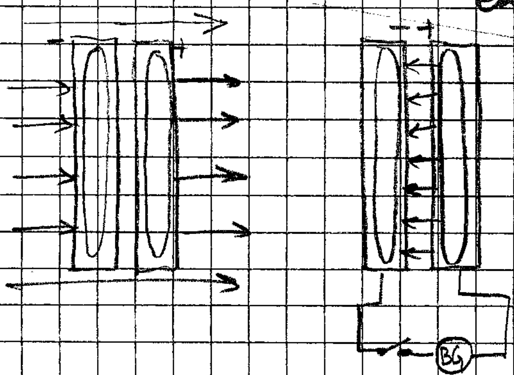
Influenca

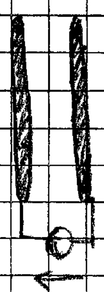


površina prevodnika postane ekvipotencialna ploskev



neposredna metoda merjenja influence



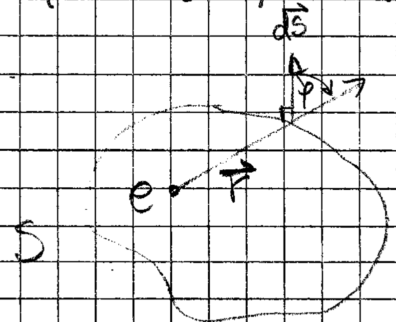


$$\boxed{D = \frac{e}{S}}$$

izmerimo influcirani naboj na ploščah

24. 2. 2011

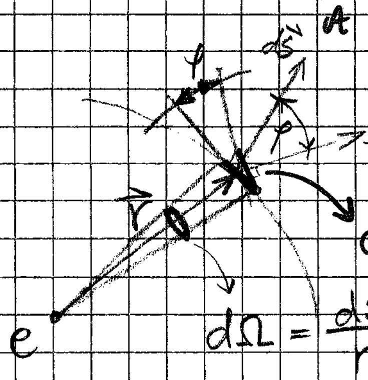
Gauss-ov stavek o električnem pretoku



$$\vec E = \frac{e}{4\pi\varepsilon_0 r^2}\cdot\frac{\vec r}{r}$$

$$\oint_A \vec E\cdot d\vec S = \oint \frac{e\,\overbrace{\vec r\cdot d\vec S}}{4\pi\varepsilon_0 r^3} = \oint \frac{e\,\cancel{r}\,dS\cos\varphi}{4\pi\varepsilon_0 r^{\cancel{3}\,2}} =$$

$$= \frac{e}{4\pi\varepsilon_0}\oint\frac{dS'}{r^2} = \frac{e}{4\pi\varepsilon_0}\oint d\Omega =$$



$$dS\cos\varphi = dS'$$

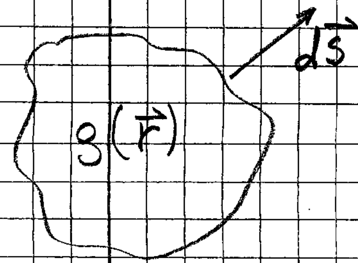
iz C-zakona

$$= \frac{e}{\cancel{4\pi}\varepsilon_0}\cdot\cancel{4\pi} = \frac{e_{not}}{\varepsilon_0}$$

$$d\Omega = \frac{dS'}{r^2}$$

$$d\Omega = 4\pi$$

$$\boxed{\oint_A \vec E\cdot d\vec S = \frac{e_{not}}{\varepsilon_0}}$$

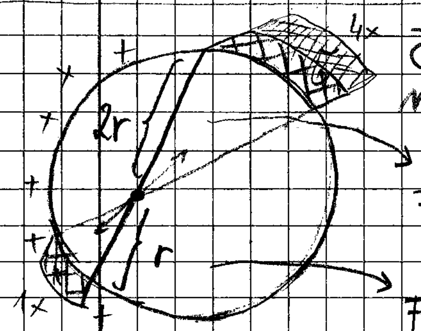


$\sum_i e_i \Rightarrow \int \rho(\vec{r})\,dV$

volumska gostota naboja

$$\boxed{\varepsilon_0 \oint_A \vec{E}\cdot d\vec{s} = \int_V \rho(\vec{r})\,dV}$$

Gauss-ov stavek o el. pretoku (v vakuumu)



Če je krogla naelektrena na površini, v notranjosti ni polja.

$F \propto \frac{1}{4r^2}$ $\quad d\Omega\, 4r^2\sigma \Rightarrow R = 2r$

$F \propto \frac{1}{r^2}$ $\quad d\Omega\, r^2\sigma \Rightarrow R = r$

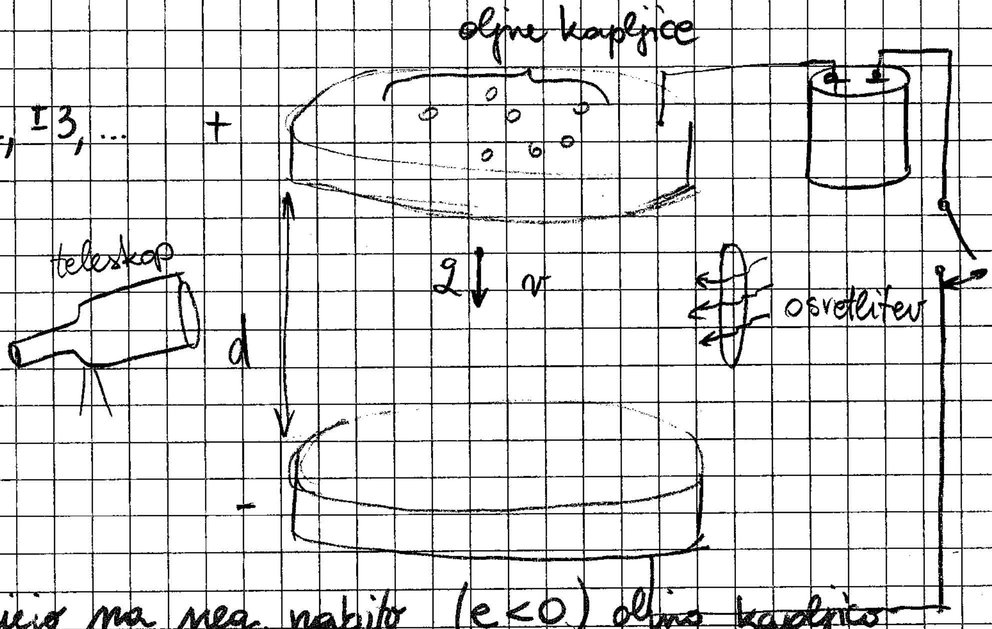
28.2.2011

Millikenov poskus — električni naboj je kvantiziran

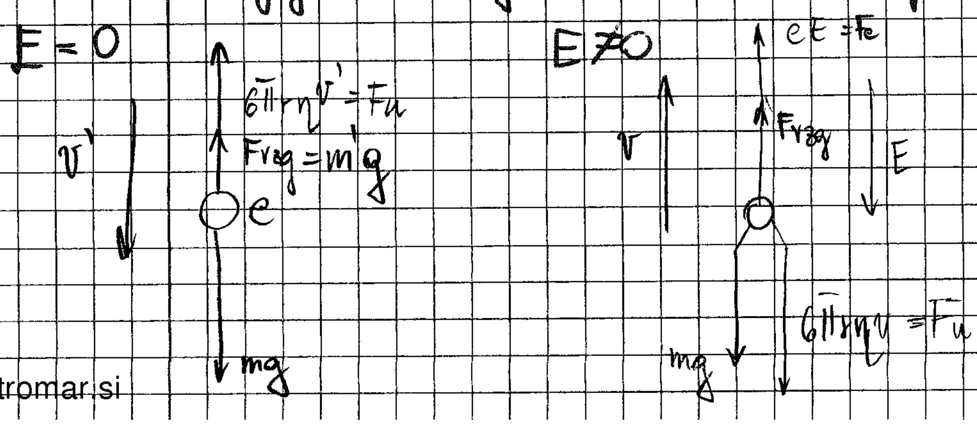
$e = ne_0$

$n = 0, \pm 1, \pm 2, \pm 3, \ldots$

$\boxed{e_0 = 1{,}6\cdot 10^{-19}\,\text{As}}$



Sile, ki delujejo na neg. nabito $(e<0)$ oljno kapljico



$\vec{E}=0$:

$$* \quad mg = m'g + 6\pi r \eta v'$$

$\vec{E}\neq 0$:

$$eE + m'g = mg + 6\pi r \eta v$$

neznanki: e, r

Cilj tega poskusa je določitev naboja posameznih kapljic.

znani parametri:

$E, \eta, \rho_k, \rho_z, v, v'$

$$eE - 6\pi r\eta v' = 6\pi r \eta v \Longrightarrow$$

$$eE = 6\pi r\eta(v+v') \Longrightarrow$$

$$\boxed{e = \frac{6\pi r\eta(v+v')}{E}}$$

Za isto kapljico izmerimo v in v' ter izračunamo naboj.

$$m_{kapl.} = \frac{4\pi r^3}{3}\cdot \rho_k$$

$$m' = \frac{4\pi r^3}{3}\rho_z \Rightarrow \text{masa izpodrinjenega zraka}$$

$$* \quad 6\cancel{\pi}\cancel{r}\eta v' + \frac{4\cancel{\pi} r^{\cancel{3}\,2}}{3}\rho_z g = \frac{4\cancel{\pi} r^{\cancel{3}\,2}}{3}\rho_k g$$

$$6\eta v' + \frac{4}{3}r^2\rho_z g = \frac{4}{3}r^2\rho_k g$$

$$6\eta v' = \frac{4}{3}g r^2(\rho_k - \rho_z)$$

$$r^2 = \frac{\frac{9}{2}\eta v'}{g(\rho_k-\rho_z)} = \frac{9}{2}\cdot\frac{\eta v'}{g(\rho_k-\rho_z)}$$

$$\boxed{r = \sqrt{\frac{9}{2}\frac{\eta v'}{g(\rho_k-\rho_z)}}}$$

Rezultat poskusa je pokazal, da so naboji kapljic večkratniki osnovnega naboja e_0. e_0 ni zvezen, je diskreten.

elektron (e)	$-1{,}602\cdot 10^{-19}$ As	$9{,}11\cdot 10^{-31}$ kg
proton (p)	$+1{,}602\cdot 10^{-19}$ As	$1{,}672\cdot 10^{-27}$ kg
nevtron	0	$1{,}674\cdot 10^{-27}$ kg

GIBANJE EL. NABOJEV V SNOVI V EL. POLJU

Vakuum:

$\vec{E} \neq 0$ $\quad \underbrace{m\frac{d\vec{v}}{dt}}_{\vec{a}} = e\vec{E} \Rightarrow \boxed{\vec{a} = \frac{e\vec{E}}{m}} \Longrightarrow$ za $\frac{v}{c_0} \lesssim 0{,}1$

c_0 … hitrost svetlobe

Povprečna hitrost elektronov v kovini je bistveno manjša kot v vakuumu.
Predpostavimo, da sta F_u in hitrost sorazmerna

$$\vec{F}_u = -k\vec{v}$$

Newton-ov zakon:

$$m\frac{d\vec{v}}{dt} = e\vec{E} - k\vec{v} \quad / : m$$

$$\boxed{\tau = \frac{m}{k}} \Longrightarrow \text{relaksacijski čas}$$

$$\frac{d\vec{v}}{dt} = \frac{e\vec{E}}{m} - \frac{k}{m}\vec{v}$$

$$\boxed{\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau} = \frac{e}{m}\vec{E}}$$

$\vec{E} = 0: \quad \frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau} = 0$

$$\int_{\vec{v}_0}^{\vec{v}} \frac{d\vec{v}}{\vec{v}} = -\int_0^t \frac{dt}{\tau}$$

$$\ln\vec{v}\Big|_{\vec{v}_0}^{\vec{v}} = -\frac{t}{\tau} \Longrightarrow \boxed{\vec{v} = \vec{v}_0 e^{-t/\tau}}$$

Relaksacijski časi elektronov (τ) v nekaterih snoveh:

kovine	$\sim 10^{-14}$
razelektritve v plinih	$\sim 10^{-8}$
sončni električni veter	$\sim 10^{2}$
medzvezdni plin	$\sim 10^{5}$

$\tau(s)$

$$\left\langle\frac{d\vec{v}}{dt}\right\rangle + \frac{\langle\vec{v}\rangle}{\tau} = \frac{e}{m}\vec{E} \quad ; \quad \vec{E} = \text{konst.}$$

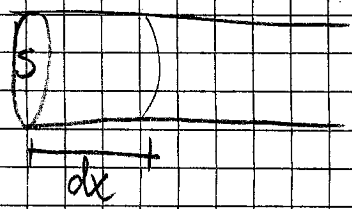
$$\left|\frac{e\tau}{m}\right| \text{ … gibljivost}$$

v ničnem povprečju: $\left\langle\frac{d\vec{v}}{dt}\right\rangle = 0$

$$\frac{\langle\vec{v}\rangle}{\tau} = \frac{e}{m}\vec{E} \Rightarrow \boxed{\langle\vec{v}\rangle = \frac{e\tau}{m}\vec{E}}$$

"Ohmov zakon"

$$\langle \vec{v} \rangle = \pm \beta \vec{E}$$



$$I = \frac{de}{dt}$$

$$j = \frac{I}{S} = \dots$$ gostota el. toka $$= \frac{1}{S}\frac{de}{dt} = \frac{1}{S}\frac{d(e_0 n V)}{dt} = \frac{1}{S} e_0 n \frac{dV}{dt}$$

$n = \frac{N}{V}$ $[m^{-3}]$ – prostorninska porazdelitev

$$e = e_0 n V = e_0 \left(\frac{N}{V}\right) V$$

$$dV = dx \cdot S$$

$$= \frac{1}{S} e_0 n \frac{dV}{dt} = \frac{1}{\cancel{S}} e_0 n \frac{dx\,\cancel{S}}{dt} = e_0 n v$$

$$\underbrace{\frac{dx}{dt}}_{v}$$

$$\boxed{j = n e_0 v} \Longrightarrow \boxed{\vec{j} = n e_0 \langle \vec{v} \rangle}$$

$$\langle \vec{v} \rangle = \frac{e\tau}{m}\vec{E}$$

$$\vec{j} = n e_0 \frac{e_0 \tau}{m}\vec{E} = \boxed{\frac{n e_0^2 \tau}{m}\vec{E}} =$$

$$\boxed{\sigma = \frac{n e_0^2 \tau}{m}}$$

$$\boxed{\vec{j} = \sigma \vec{E}}$$ "Ohmov zakon"

tenzor; prevodnost v vseh smereh ni nujno enaka

3.3.2011

$$\frac{I}{S} = \sigma \frac{U}{l}$$

$$\rho_\Omega = \frac{1}{\sigma}$$

$$\underbrace{\left(\frac{l}{\sigma S}\right)}_{R} I = U$$

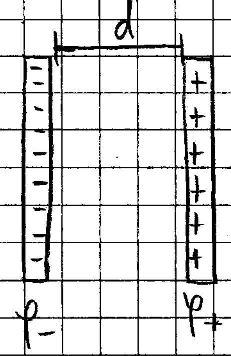
$$\boxed{RI = U}$$

$$\rho = \rho_0\left(1+\alpha\left(T-T_0\right)\right)$$

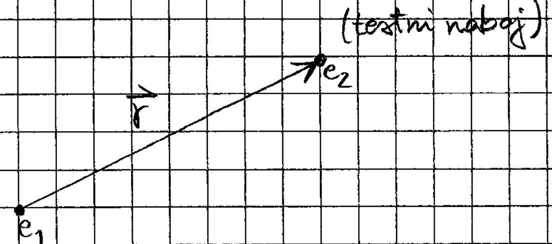
$$E(r)=\frac{\sigma R^2}{\varepsilon_0 r^2} \quad ; \quad \vec{E}(r)=\frac{\sigma R^2}{\varepsilon_0 r^2}\left(\frac{\vec{r}}{r}\right)$$

$$W_e=\int \frac{1}{2}\varepsilon_0 E^2 dV=\int_R^\infty \frac{1}{2}\varepsilon_0 \frac{\sigma^2 R^4}{\varepsilon_0^2 r^4}\cdot 4\pi r^2 dr = 2\int_R^\infty \frac{\sigma^2 R^4 \pi}{\varepsilon_0}\frac{1}{r^2}dr =$$

$$=2\frac{\sigma^2 R^4\pi}{\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_R^\infty = 2\frac{\sigma^2 R^4 \pi}{\varepsilon_0 R} = 2\frac{\sigma^2 R^3\pi}{\varepsilon_0}$$



električni potencial:

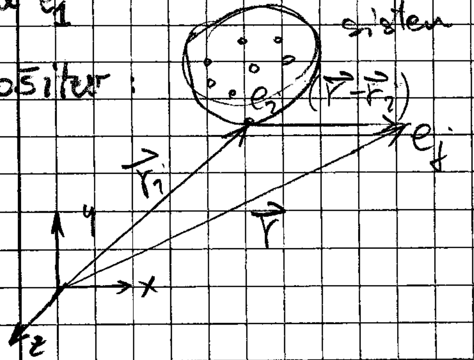


$$W_1=\frac{e_1 e_2}{4\pi\varepsilon_0 r}$$

$$\varphi_1(r)=\frac{W}{e_2}=\frac{e_1 e_2}{4\pi\varepsilon_0 r e_2}=\frac{e_1}{4\pi\varepsilon_0 r},$$

el. potencial e_1

posplošitev:



Zveza med φ in $\vec{E}$

$$\boxed{d\varphi=\nabla\varphi\, d\vec{r}=\operatorname{grad}\varphi\cdot d\vec{r}}$$

$$\varphi(\vec{r})=\varphi(x,y,z)$$

$$\vec{r}=(x,y,z)$$

$$dW_e=-\int \underbrace{\vec{F}_e}_{\vec{F}_e=e\cdot\vec{E}}\cdot d\vec{r}=\underline{-\int e\vec{E}\cdot d\vec{r}}$$

$$\varphi_1(r) = \frac{W_e}{e_2} = \frac{e_1 \cancel{e_2}}{4\pi\varepsilon_0 r \cancel{e_2}} = \frac{e_1}{4\pi\varepsilon_0 r} \Rightarrow \boxed{W_e = e_2 \varphi_1(r)}$$

$$\varphi(\vec{r}) = \sum_i \frac{e_i}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_i|}$$

$$d(e\varphi) = -e\vec{E} \cdot d\vec{r}$$

$$\boxed{d\varphi = -\vec{E} \cdot d\vec{r}}$$

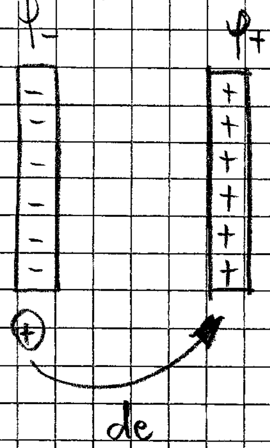
$$d\varphi = d\varphi$$

$$-\vec{E}\, d\vec{r} = +\,\mathrm{grad}\,\varphi \cdot d\vec{r}$$

$$\boxed{\vec{E} = -\,\mathrm{grad}\,\varphi(\vec{r})}$$

7.3.2011

KONDENZATOR

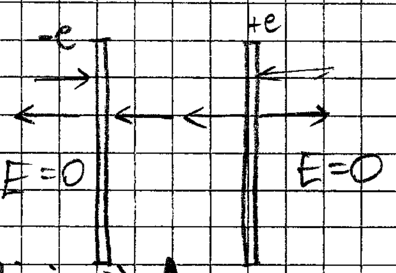


$$A = \Delta W_k + \Delta W_{g,p} + \Delta W_e$$

$$dA = de\,\varphi_+ - de\,\varphi_- = de\,\underbrace{(\varphi_+ - \varphi_-)}_{U} = U \cdot de$$

$$\underline{dA = U \cdot de}$$

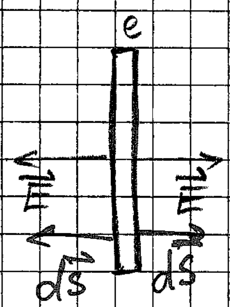
$$dA = \int U \cdot de$$



zunaj plošč se polje izniči, znotraj pa se sešteje.

$$\boxed{E = 2E_1 = \frac{\sigma}{\varepsilon_0}}$$

velika plošča $\to \infty$



$\vec{E} \parallel d\vec{S}$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \varepsilon_0 \int E \cdot dS =$$

$$\varepsilon_0 E \cdot \int dS = \underline{\varepsilon_0 E \cdot 2S}$$

S = površina plošče

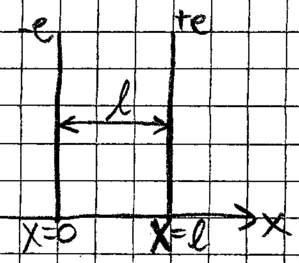
celoten objeti naboj $\Rightarrow$

$$\sigma \cdot \cancel{S} = \varepsilon_0 E\, 2 \cancel{S}$$

$$\boxed{E_1 = \frac{\sigma}{2\varepsilon_0}} \text{ za eno ploščo}$$

$$E = -\frac{d\varphi}{dx}$$

$$\int d\varphi = -\int E\,dx$$



$$U = \int d\varphi = -\int E\,dx = E \cdot l$$

$$\boxed{U = E \cdot l}$$

$$E = \frac{U}{l}$$

ELEKTROSTATSKA POTENCIALNA ENERGIJA

$$\tilde{A}_{ost} = \Delta W_k + \Delta W_p$$

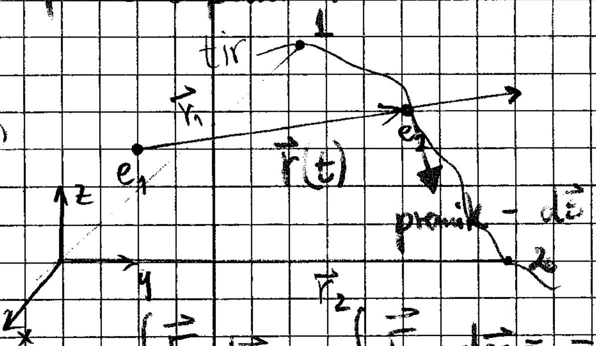
$$A_{ost} + \int \vec{F}_e \cdot d\vec{s} = \Delta W_k + \Delta W_{g,p}$$

delo vseh ostalih sil razen gravitacijske

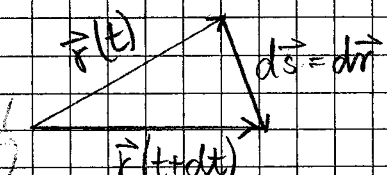
delo vseh ostalih sil razen gravitacijske in električne

$$A_{ost} = \Delta W_k + \Delta W_{g,p} - \int \vec{F}_e \cdot d\vec{s}$$ ⟹ negativno delo el. sile

poseben primer:



$$\vec{F}_e = \frac{e_1 e_2}{4\pi\varepsilon_0 r^2}\left(\frac{\vec r}{r}\right)$$



$$-\int \vec F_e \cdot d\vec s = -\int \vec F_e \cdot d\vec r = \underline{-\int \frac{e_1 e_2}{4\pi\varepsilon_0 r^2}\cdot\frac{\vec r\, d\vec r}{r}}\ *$$

$$d\vec r = \vec r(t+dt) - \vec r(t)$$

$$d(\underbrace{\vec r\cdot\vec r}_{r^2}) = \vec r\cdot d\vec r + d\vec r\cdot \vec r = 2\vec r\cdot d\vec r$$

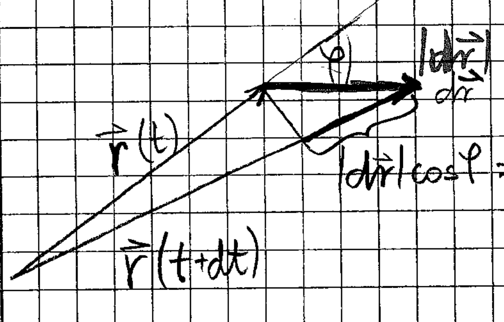
$$d(r^2) = 2r\,dr = 2\vec r\cdot d\vec r$$

$$d(r^2) = r\,dr = \vec r\, d\vec r$$

$$r = |\vec r|$$

$$\vec r\cdot\vec r = r^2$$

delo sile ni odvisno od poti, le od začetne in končne točke



$$|d\vec r|\cos\varphi = d\underbrace{|\vec r|} = \underline{dr}$$

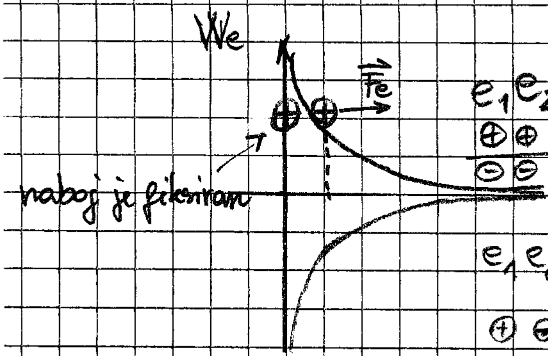
sprememba dolžine velikosti vektorja

$$\underline{\vec r\, d\vec r} = \underbrace{|\vec r||d\vec r|}_{r\,dr}\underbrace{\cos\varphi}_{dr} = \underline{r\,dr}$$

$$* \quad -\int \frac{e_1 e_2\, \overbrace{\vec r\cdot d\vec r}^{r\,dr}}{4\pi\varepsilon_0 r^2 r} = -\int \frac{e_1 e_2}{4\pi\varepsilon_0}\frac{r\,dr}{r^2 \cancel{r}} = -\frac{e_1 e_2}{4\pi\varepsilon_0}\int_{r_1}^{r_2}\frac{dr}{r^2} = \underline{\frac{e_1 e_2}{4\pi\varepsilon_0 r_2} - \frac{e_1 e_2}{4\pi\varepsilon_0 r_1}} =$$

$$= \underline{W_{e,2} - W_{e,1} = \Delta W_e}$$

$$\boxed{W_e = \frac{e_1 e_2}{4\pi\varepsilon_0 r}}$$



$e_1 e_2 > 0$ ⊕⊕ ⊖⊖

$e_1 e_2 < 0$ ⊕⊖

$$A_{ost} = \Delta W_k + \Delta W_{p,g} - \underbrace{\int \vec{F_e} \cdot d\vec{s}}_{\Delta W_e}$$

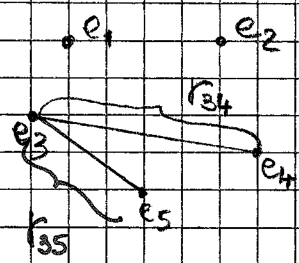
$$\boxed{A_{ost} = \Delta W_k + \Delta W_{p,g} + \Delta W_e}$$

$$A_{ost} = 0 \Rightarrow \Delta W_k + \Delta W_{g,p} + \Delta W_e = 0$$

$$\Delta(W_k + W_{g,p} + W_e) = 0$$

$$\boxed{W_k + W_{g,p} + W_e = \text{konst.}}$$

Posplošitev:

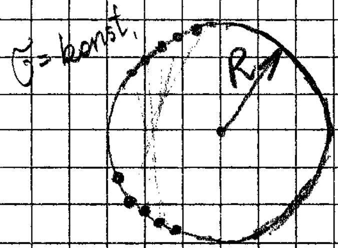


$$\boxed{W_e = \frac{1}{2} \sum_{i,j} \frac{e_i \cdot e_j}{4\pi\varepsilon_0 r_{ij}}}$$

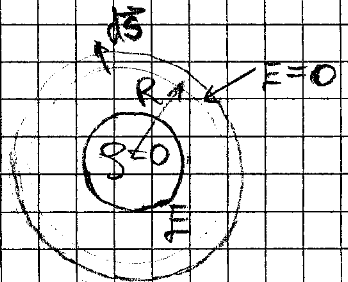
$$W_e = \int \underbrace{\frac{1}{2} \varepsilon_0 E^2}\, dV$$

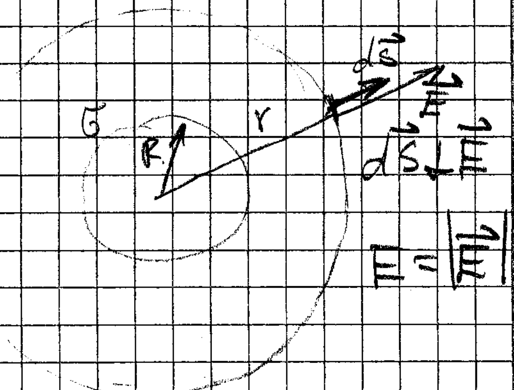
$$w_e = \frac{1}{2} \varepsilon_0 E^2$$

Primer:



$$W_e = \frac{1}{2} \sum_{i,j} \frac{e_i e_j}{4\pi\varepsilon_0 r_{ij}} \; ; \; W_1 = \int \frac{1}{2} \varepsilon_0 E^2 \, dV$$





$d\vec{S} \parallel \vec{E}$

$E = |\vec{E}|$

Gaussov stavek

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \sum e = \int \rho \, dV$$

$= E\,dS$

$$\varepsilon_0 E \int dS = \sigma 4\pi R^2$$

$$\varepsilon_0 E(r) \oint dS = \sigma 4\pi R^2$$

$$\varepsilon_0 E(r) \cancel{4\pi} r^2 = \sigma \cancel{4\pi} R^2$$

$$U = \frac{\sigma}{\varepsilon_0} l \quad , \quad \sigma = \frac{e}{S}$$

$$U = \frac{e l}{\varepsilon_0 S}$$

$$e = \underbrace{\varepsilon_0 \frac{S}{l}}_{C} U \qquad e = C \cdot U \qquad U = \frac{e}{C}$$

$$dA = \int U de = \int_0^e \frac{e}{C} de = \frac{1}{C} \cdot \frac{e^2}{2} = \underbrace{\frac{e^2}{2C} = \frac{C^2 U^2}{2C} = \frac{CU^2}{2}}$$

energija naelektrenega kondenzatorja.

sistem nabojev:

$$W = \frac{1}{2} \sum_{i,j} \frac{e_i e_j}{4\pi\varepsilon_0 r_{ij}} = \ldots\ldots = \underbrace{\frac{1}{2} \int E^2 \varepsilon_0 dV}$$

$$U = e \cdot l$$

$E = 0$ | $E \neq 0$ | $E = 0$

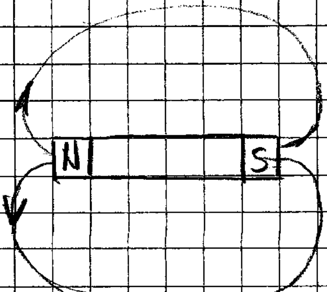
$$W_C = \frac{1}{2} C U^2 = \frac{1}{2} C E^2 l^2 = \frac{1}{2} \varepsilon_0 \frac{S}{l} E^2 l^2 =$$

$$= \frac{1}{2} \varepsilon_0 E^2 \underbrace{S \cdot l}_{V} = \underbrace{\frac{1}{2} \varepsilon_0 E^2 V}$$

$$\underbrace{w_e = \frac{1}{2} \varepsilon_0 E^2}$$

gostota energije $\vec{E}$

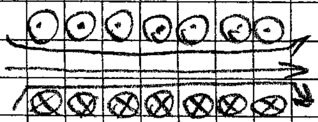
MAGNETNO POLJE



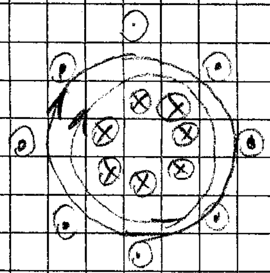
magnetna silnica

V vsaki m. polju je njegova smer določena s smerjo, v kateri kaže v ravnovesni legi severni (N) pol vsestransko ~~kaže~~ vsestransko vrtljive magnetnice.

tuljava:



toroid:



$$\vec{F}_B = e\vec{v} \times \vec{B}$$

gostota mag polja je definirana preko sile na naboj. gibajoči se

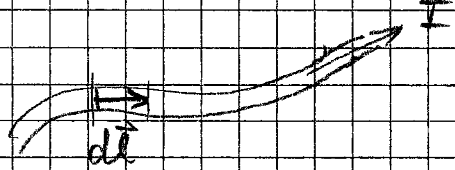
$$B \quad \left[T = \frac{Vs}{m^2}\right]$$

nevtronska zvezda	elektromagnet	površina Zemlje	medzvezdni prostor
$10^8\,T$	$10T \sim 1T$	$10^{-4}\,T$	$10^{-10}\,T$

V žici:

$$d\vec{F} = de\,\vec{v} \times \vec{B}$$

$$d\vec{F} = I\,\underbrace{dt\,\vec{v}}_{d\vec{l}} \times \vec{B}$$

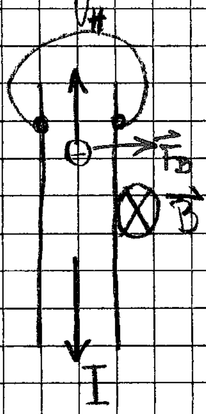


$$\boxed{d\vec{F} = I\,d\vec{l} \times \vec{B}} \Rightarrow \boxed{\vec{F}_e = \int (I\,d\vec{l} \times \vec{B})}$$ ... sila na vodnik po katerem teče tok

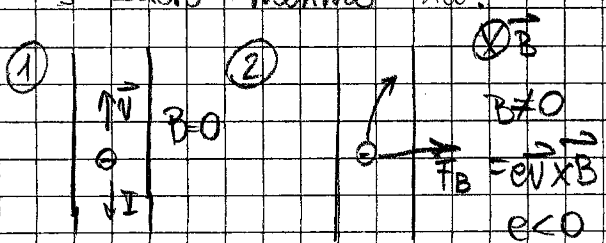
- raven vodnik:

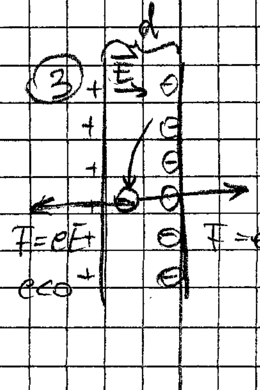
$$\boxed{\vec{F}_e = I\vec{l} \times \vec{B}}$$

Hall-ov pojav

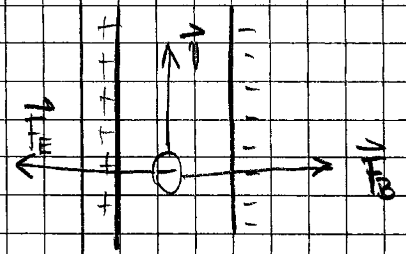


Hall-ova sonda meri napetost med dvema točkama, s katero merimo silo.





stacionarno stanje



sili sta enaki, ker se izenačijo naboji.

To izmerimo pri Hall-ovem poskusu, dobimo U_H

$$\vec{F}_E = \vec{F}_B$$

$$eE = evB$$

$$E = vB$$

$$\frac{U_H}{d} = \langle v \rangle B$$

↓

povprečna hitrost elektronov

$$E = \frac{U_H}{d}$$

$$n = \frac{N}{V}$$

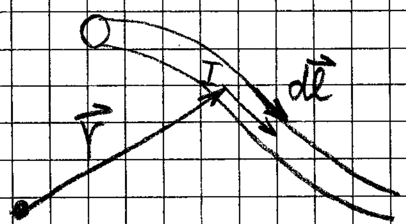
$$j = ne\langle v \rangle \Rightarrow \langle v \rangle = \frac{j}{ne} = \frac{I}{neS}$$

$$\frac{U_H}{d} = \frac{I \cdot B}{ne_0 S}$$ } iz tega določimo B

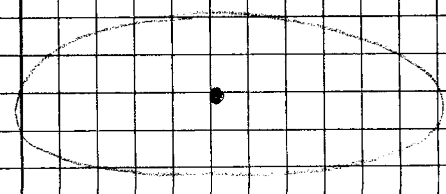
10.3.2011

Biot-Savart-ov zakon

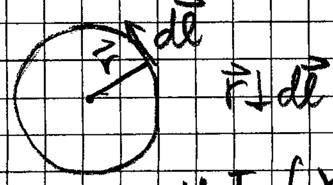
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{r} \times d\vec{l}}{r^3}$$



primer:
- okrogla tokovna zanka



tloris



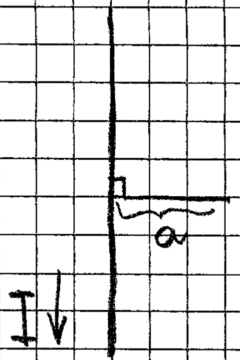
$$\vec{r} \perp d\vec{l}$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{\cancel{r}\, dl}{r^{\cancel{3}\,2}} =$$

$$= \frac{\mu_0 I}{4\pi r^2} \int \underbrace{dl}_{2\pi r} = \frac{\mu_0 I\, 2\cancel{\pi r}}{4\cancel{\pi} r^{\cancel{2}}} =$$

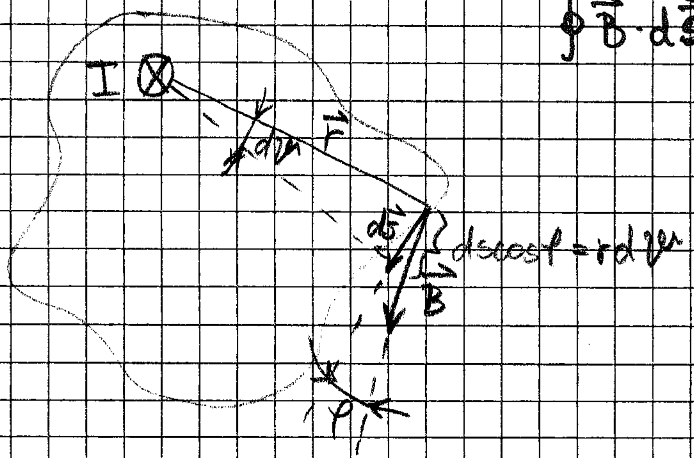
$$\boxed{B = \frac{\mu_0 I}{2r}}$$

- Dolg raven vodnik



$$B=\frac{\mu_0 I}{2\pi a}$$

Ampere-ov zakon



$$\oint \vec{B}\cdot d\vec{s} = \oint B\,\underbrace{ds\cos\varphi}_{r\,d\vartheta} =$$

$$= \oint \frac{\mu_0 I}{2\pi \cancel{r}}\,\cancel{r}\,d\vartheta =$$

$$= \frac{\mu_0 I}{2\pi}\oint d\vartheta =$$

$$= \frac{\mu_0 I}{\cancel{2\pi}}\cdot\cancel{2\pi} =$$

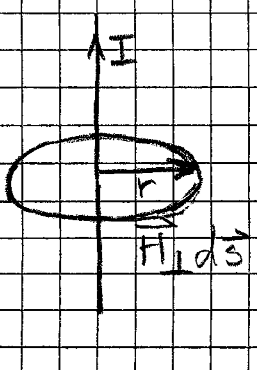
$$\boxed{B=\mu_0 I}$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I = \mu_0 \int \vec{J}_e\, d\vec{S}$$

$$\vec{B}=\mu_0\vec{H}$$

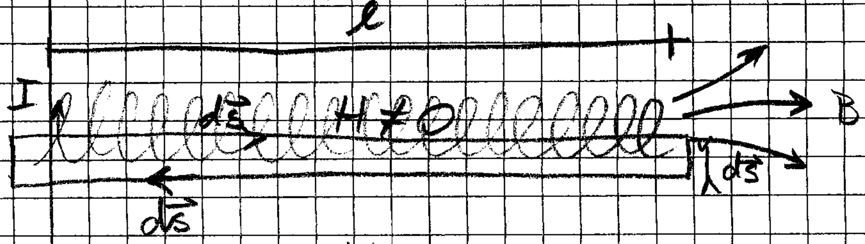
$$\boxed{\oint \vec{H}\,d\vec{s} = \int \vec{J}_e\cdot d\vec{S}}$$

1. primer:



$$\left.\begin{aligned} \oint \vec{H}\cdot d\vec{s} &= I \\ H\int ds &= I \\ H\,2\pi r &= I \end{aligned}\right\} \Rightarrow \underline{\underline{H=\frac{I}{2\pi r}}}$$

2. primer: dolga tuljava z gostimi navoji:



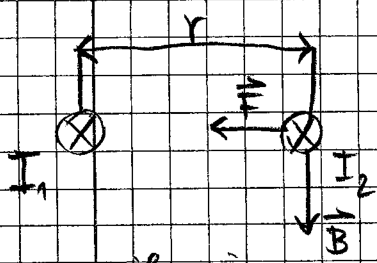
$$H \cong 0$$

$$\oint \vec{H} d\vec{s} = Hl = N \cdot I \Rightarrow H = \frac{NI}{l}$$

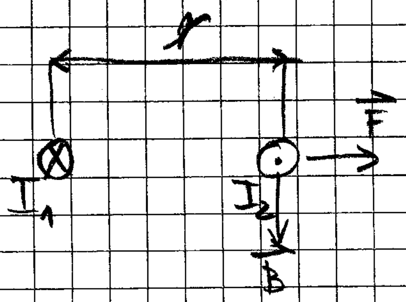
N ↓ št. ovojev

$$\boxed{B = \mu_0 \frac{NI}{l}}$$

SILA MED ~~2~~ VZPOREDNIMA DOLGIMA VODNIKOMA



- sila je privlačna
- toka tečeta v isto smer



- sila je odbojna
- toka tečeta v nasprotnih smereh

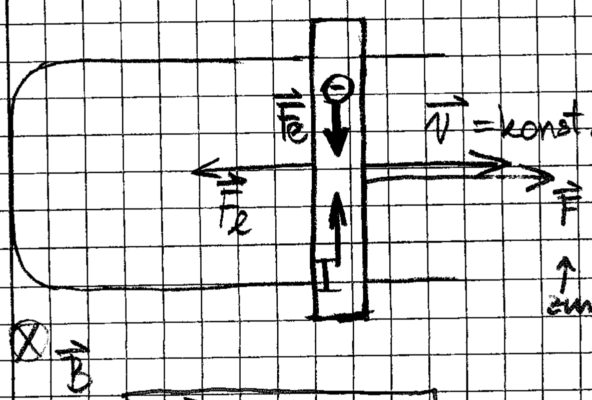
INDUKCIJA

$$\Phi_m = \int \vec{B} \cdot d\vec{S}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$\vec{F}_e = (e_0)\vec{v} \times \vec{B}$$

$$\vec{F}_l = I\vec{l} \times \vec{B}$$



$\vec{F}$ (sila, ki uravnovesi $\vec{F}_e$)
↑
zunanja sila

$$\boxed{\vec{F} + \vec{F}_l = 0} \Longleftrightarrow \vec{v} = \text{konst.}$$

$$\vec{F} = -\vec{F}_\ell = -I(\vec{\ell}\times\vec{B})$$

delo:

$$dA = \vec{F}\cdot\underbrace{\vec{v}\cdot dt}_{d\vec{s}} = U\cdot I\,\cancel{dt}$$

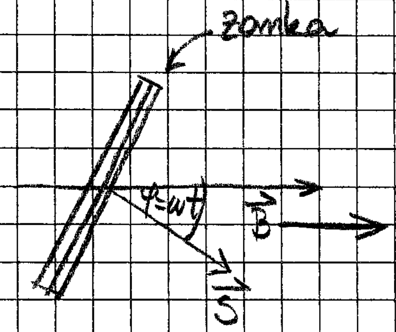
$$-\cancel{I}(\vec{\ell}\times\vec{B})\cdot\vec{v} = U\cdot\cancel{I}$$

$$\boxed{(\vec{B}\times\vec{\ell})\,\vec{v} = U_i}$$

INDUKCIJSKI ZAKON
(POSPLOŠENO)

$$U_i = -\frac{\vec{B}\cdot d\vec{S}}{dt} = -\frac{d\phi_m}{dt}$$

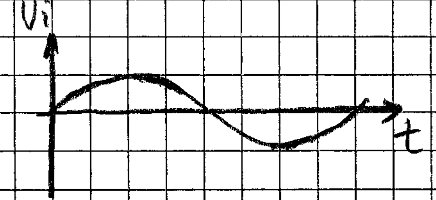
Primer:



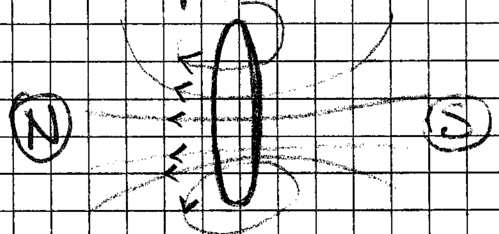
$$\phi_m = \vec{B}\cdot\vec{S} = B\cdot S\cdot\cos(\omega t)$$

$$|U_i| = \left|\frac{d\phi_m}{dt}\right| = BS\sin(\omega t)$$

ω = konst



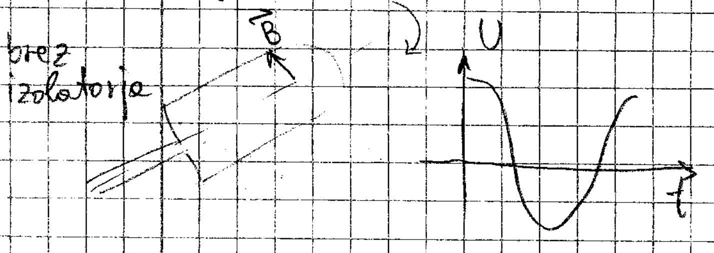
Lenz-ovo pravilo

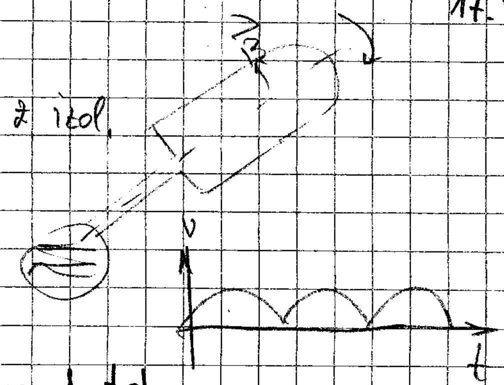


Ind. nap. požene tok, ki se upira spremembi, ki je inducirano napetost povzročila!

17.3.2011

Poskus: generator el. toka



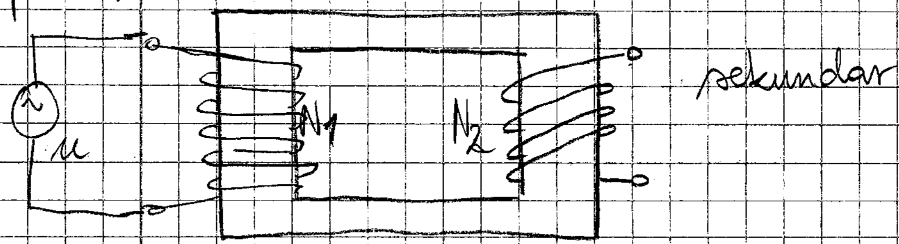


vsaka sprememba pretoka nam da sunek toka.

$$\boxed{U_i = -\frac{d\phi_m}{dt} \,/\, dt}$$

$$\underbrace{\int U_i dt}_{\text{sunek napetosti}} = -\int d\phi_m \Rightarrow -\phi_m \Big|_{\phi_{mz}}^{\phi_{mk}} = \boxed{\phi_{mz} - \phi_{mk} = \Delta\phi_m}$$

TRANSFORMATOR



$$\boxed{\phi_1 = N_1 \cdot B \cdot S}$$

Z jedrom povečamo ϕ_m in dosežemo, da je ϕ_m skozi obe tuljavi sklenjen.

Zaradi spremenljivega ϕ_m se napetost na primarju in sekundarju

$$\frac{U_1}{U_2} = \frac{N_1}{N_2}$$

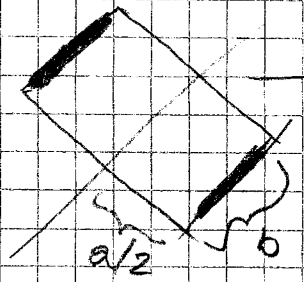
IDEALNI TRANS.

- ϕ_m skozi tuljavi je isti
- za enosmerno napetost je upor obeh tuljav zanemarljiv

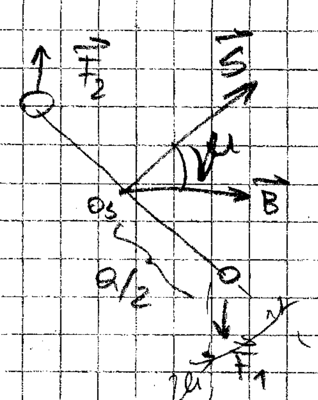
$$\left.\begin{aligned} U_1 &= -\frac{d\phi_1}{dt} = -N_1 \frac{d(BS)}{dt} \\ U_2 &= -\frac{d\phi_2}{dt} = -N_2 \frac{d(BS)}{dt} \end{aligned}\right\} \Rightarrow \boxed{\frac{U_1}{U_2} = \frac{N_1}{N_2}}$$

moč: $\boxed{\overline{P} = \overline{I} \cdot U_{ef}}$ $\boxed{\overline{P_1} = \overline{P_2}}$

NAVOR NA TOK. ZANKO



s strani



$\vec{S}$ – površina zanke

sili se seštevata, nasprotno sta enaki, zanko vrtita

$F = I\,\vec{e} \times \vec{B}$

↑ smer toka

$$\boxed{\vec{M} = \vec{r} \times \vec{F} = |r| \cdot |F| \cdot \sin \varphi}$$

$$M = \frac{a}{2} \cdot IbB \cdot \sin\vartheta + \frac{a}{2} IbB \sin\vartheta = I\underbrace{ab}_{S}B \cdot \sin\vartheta =$$

$$\boxed{M = I \cdot S \cdot B \cdot \sin\vartheta}$$

p_m ... dipolni moment zanke

$p_m = I \cdot \vec{S}$ smer je določena s prsti so v smeri toka kamor kaže palec

$$\boxed{p_m = N \cdot I \cdot \vec{S}} \rightarrow \text{če ima več ovojev}$$

$$\boxed{\vec{M} = \vec{p}_m \times \vec{B}}$$

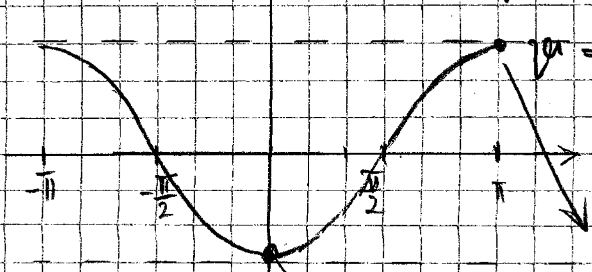
$\vec{M} = p_m B \cdot \sin\vartheta$

$A = \int M\, d\vartheta = \int p_m B \sin\vartheta\, d\vartheta = -p_m B \cos\vartheta \Big|_{\vartheta_z}^{\vartheta_k} \Rightarrow$

$\Rightarrow \boxed{W_m = -p_m \cdot B \cos\vartheta}$ — energija mag. dipola $\vec{p}_m$ v zun. mag. polju $\vec{B}$.

$\vartheta = 0: W_m = -p_m B$ (min) $\vec{p}_m \uparrow \uparrow \vec{B}$

$\vartheta = \pi: W_m = +p_m B$ (max) $\vec{p}_m \downarrow \uparrow \vec{B}$

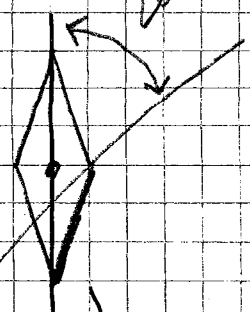


labilna lega

stabilna lega

Nihajni čas magnetnice:

$$M = J\cdot\alpha\ ;\quad \alpha = \frac{d^2\vartheta}{dt} \equiv \ddot{\vartheta}$$



$$-p_m B \sin\vartheta = J\cdot\ddot{\vartheta}\ ;\quad \sin\vartheta \simeq \vartheta$$

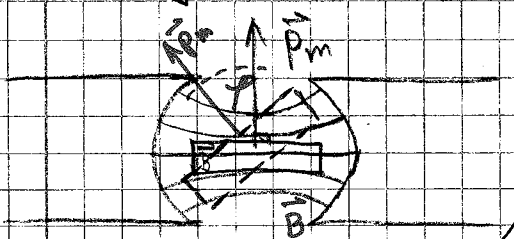
↓ vztrajnostni moment igle

$$-\left[\frac{p_m\cdot B}{J}\right]\vartheta = \ddot{\vartheta}$$

... rešitev $\vartheta = \vartheta_0 \sin\left(\left[\frac{2\pi}{t_0}\right]\cdot t + \delta\right)$

$$\ddot{\vartheta} = -\vartheta_0\left[\frac{2\pi}{t_0}\right]^2 \sin\left(\left[\frac{2\pi}{t_0}\right]t + \delta\right) = \left[\frac{2\pi}{t_0}\right]^2\vartheta$$

$$-\left[\frac{p_m\cdot B}{J}\right]\cancel{\vartheta} = \left[\frac{2\pi}{t_0}\right]^2\cancel{\vartheta} \implies \boxed{t_0 = 2\pi\sqrt{\frac{J}{p_m B}}}$$



$\vec{p}_m = NI\vec{S}$

$\vec{p}_m \perp \vec{B}$

$M = p_m B \underset{\approx 90^\circ}{\sin\vartheta} = p_m B$

ravnovesno stanje

$M_B = M_D$

$\boxed{p_m B = D\varphi}$

$NISB = D\varphi$

$I = \left[\frac{D}{NSB}\right]\varphi$

$I \propto \varphi$

instrument lahko uporabljamo tudi za merjenje tokovnih sunkov.

merimo $\int I\cdot dt$

$$\vec{M} = \frac{d\vec{\Gamma}}{dt} \implies \vec{M}dt = d\vec{\Gamma}$$ ← vrtilna količina

$$\boxed{\int \vec{M}dt = \Delta\vec{\Gamma}} = J\cdot\omega_0$$

$\varphi = \varphi_0 \sin(\omega t)$, $\omega = \frac{2\pi}{t_0}$

$$\omega = \frac{d\varphi}{dt} = \underbrace{\varphi_0\left(\frac{2\pi}{t_0}\right)}_{\omega_0}\cos\left(\frac{2\pi}{t_0}t\right)$$

$\int p_m B\, dt$

$\int INSB\, dt = J\omega_0$

$$\boxed{\int I\, dt = \frac{J\omega_0}{NSB}} = \frac{J}{NSB}\frac{2\pi}{t_0}\varphi_0$$

MAXWELLOVE ENAČBE

$\oint \vec{D}\cdot d\vec{S} = \int \rho_e \, dV$ zakon o el. pretoku - Gaussov stavek

$\oint \vec{B}\cdot d\vec{S} = 0$ zakon o mag. pretoku

$\oint \vec{H} d\vec{s} = \int \left(\vec{J}_e + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{S}$ Ampere-ov zakon

$\oint \vec{E} d\vec{s} = \int \left(\frac{\partial \vec{B}}{\partial t}\right) d\vec{S}$ Faraday-ev zakon

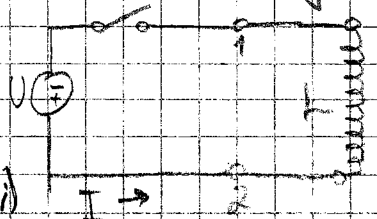
$\oint \vec{J}_e \cdot d\vec{S} = -\int \frac{\partial \rho_e}{\partial t} dV$ zakon o ohranitvi naboja

$D = \varepsilon_0 \varepsilon_r E$

$B = \mu_0 \mu_r H$

LASTNA INDUKCIJA (L)

$U_i = -\frac{d\Phi_m}{dt}$



$U + U_L = 0$

1. $t \approx 0$ (po priključitvi); po tuljavi steče spremenljiv naraščujoč tok

$\frac{dI}{dt} > 0 \Rightarrow U_L = -L\frac{dI}{dt} < 0 \Rightarrow U > 0$

tuljava za kratek čas deluje kot upor

2. $t \approx 0$ (po izključitvi); steče padajoč el. tok.

$\frac{dI}{dt} < 0 \Rightarrow U_L = L\frac{dI}{dt} > 0 \Rightarrow U < 0$

tuljava za kratek čas deluje kot generator

Lenzovo pravilo;

$\int \vec{B}\cdot d\vec{s} = \mu_0 I N$

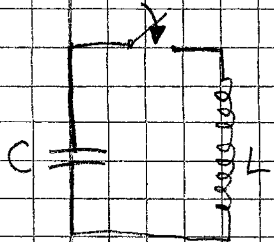
$Bl = NI\mu_0 \Rightarrow B = \frac{NI\mu_0}{l}$; $\Phi_m = NBS = \frac{NI\mu_0}{l} S \cdot N = \underbrace{\left[\frac{\mu_0 N^2 S}{l}\right]}_{L} \cdot I$

$\boxed{L = \frac{\mu_0 N^2 S}{l}}$

$\boxed{U_i = -\frac{d\Phi_m}{dt} = -L\frac{dI}{dt}}$

$\boxed{\Phi_m = L \cdot I}$ $\boxed{\overline{P} = \frac{1}{2} I_0 U_0 \cos\varphi}$

Idealni nihajni krog



Nedušeni el. nihajni krog; $R=0$

$U_L + U_C = 0$

$U_C(t=0) = U_0$

napetost naelektrenega kondenzatorja

$$-L\frac{dI}{dt} - \frac{e}{C} = 0 \Big/ \frac{d}{dt}$$ ← diferencialna enačba

Id. nihanje obstaja samo na atomski ravni.

$$-L\frac{d^2I}{dt^2} - \frac{\left(\frac{de}{dt}\right)}{C} = 0 \qquad \frac{de}{dt} \rightarrow I$$

$$-L\frac{d^2I}{dt^2} - \frac{I}{C} = 0$$

enačba nihanja ⟹ $$\frac{d^2I}{dt^2} = -\frac{1}{LC}I$$ diferencialna enačba za el. tok

rešitev:

$I = I_0 \sin(\omega_0 t)$

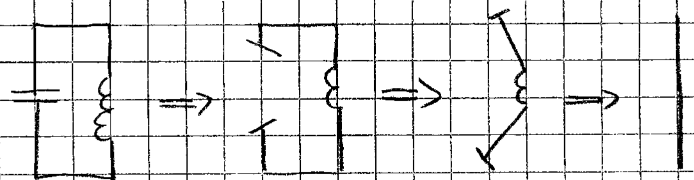
$$\frac{d^2I}{dt^2} = -I_0\,\omega_0^2 \sin(\omega_0 t) = -\omega_0^2 I$$

$$-\omega_0^2 \cancel{I} = -\frac{1}{LC}\cancel{I}$$

$$\boxed{\omega_0^2 = \frac{1}{LC}}$$

lastna krožna frekvenca idealnega nihajnega kroga

El. polje je samo v kondenzatorju, v tuljavi pa je samo magnetno polje.



⟹ dipolna antena - oddaja EM valovanje

$U_L = -L\frac{dI}{dt} = -L I_0 \omega_0 \cos(\omega_0 t)$ → največja možna napetost na tuljavi

$U_L + U_C = 0 \Rightarrow U_C = -U_L = L I_0 \omega_0 \cos(\omega_0 t)$ → –II– na kondenzatorju

$$\boxed{U_0 = L I_0 \omega_0}$$

Energija se ohranja, ker ni upora; ni izgub. En. se pretaka med kondenzatorjem in tuljavo.

$$W_C = \frac{1}{2}CU_C^2$$

$$\int U_L I\,dt = \int_0^{I_0} L\frac{dI}{\cancel{dt}} I\,\cancel{dt} = \frac{1}{2}LI^2 = W_L$$

Energija id. nih. kroga : $W=\frac{1}{2}CU_0^2=\frac{1}{2}LI_0^2$

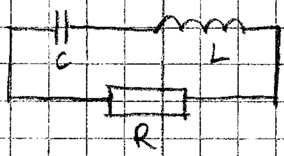
V splošnem : $W=\frac{1}{2}CU_C^2+\frac{1}{2}LI^2=\frac{1}{2}\underbrace{CL^2}I_0\,\omega_0^2\cos^2(\omega_0 t)+\frac{1}{2}LI_0^2\sin^2(\omega_0 t)\Rightarrow$

$\Rightarrow \frac{1}{2}LI_0^2\left[\underbrace{\cos^2(\omega_0 t)+\sin^2(\omega_0 t)}_{1}\right]=\boxed{\frac{1}{2}LI_0^2=\frac{1}{2}CU_0^2}$

$\cancel{C}L^{\cancel{2}}=\frac{1}{\cancel{LC}}=L \qquad I_0=\frac{U_0}{L\omega_0}$

Dušeni el. nih. krog:

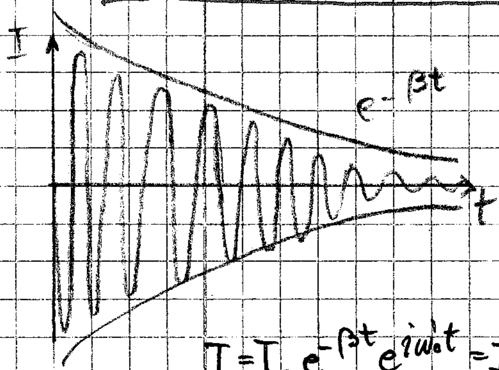
Dodamo še ohmsko upornost



$\underline{U_C+U_R+U_L=0}\Longrightarrow -\frac{e}{C}-L\frac{dI}{dt}-RI=0\;/\frac{d}{dt}\Rightarrow$

$\Rightarrow -\frac{I}{C}-L\frac{d^2I}{dt^2}-R\frac{dI}{dt}=0\;/:L\Rightarrow -\frac{I}{LC}-\frac{d^2I}{dt^2}-\frac{R}{L}\frac{dI}{dt}\Rightarrow$

$\Rightarrow\boxed{\frac{d^2I}{dt^2}+\frac{R}{L}\frac{dI}{dt}+I\omega_0^2=0}\Longrightarrow I(t)$



nastavek za rešitev:

$\beta\rightarrow$ koeficient dušenja

$I=I_0e^{-\beta t}\cos(\omega_0' t)$

$e^{i\omega_0' t}=\cos(\omega_0' t)+i\sin(\omega_0' t)$

$I=I_0e^{-\beta t}e^{i\omega_0' t}=I_0e^{(i\omega_0'-\beta)t}$

$\frac{dI}{dt}=I_0(i\omega_0'-\beta)e^{(i\omega_0'-\beta)t}$

$\frac{d^2I}{dt^2}=I_0(i\omega_0'-\beta)e^{(i\omega_0'-\beta)t}=I_0(-\omega_0'^2-2\omega_0'\beta i+\beta^2)\,e^{(i\omega_0'-\beta)t}$

$\underbrace{I_0e^{(i\omega_0'^2-\beta)t}}\left[\underbrace{-\omega_0'^2-2\omega_0'\beta i+\beta^2+\frac{R}{L}(i\omega_0')-\frac{R}{L}\beta+\omega_0^2}_{0}\right]=0$

V splošnem ta člen ne more biti 0.

$\boxed{\begin{aligned}a+bi&=0\\ a&=0\\ b&=0\end{aligned}}$

$-\omega_0'^2-2\omega_0'\beta i+\frac{R}{L}(i\omega_0')-\frac{R}{L}\beta+\omega_0^2+\beta^2=0$

$(\underbrace{-\omega_0'^2-\frac{R}{L}\beta+\beta^2+\omega_0^2}_{0})+i(\underbrace{\frac{R}{L}\omega_0'-2\omega_0'\beta}_{0})=0$

$\frac{R}{L}=2\beta\Rightarrow\boxed{\beta=\frac{1}{2}\frac{R}{L}}$

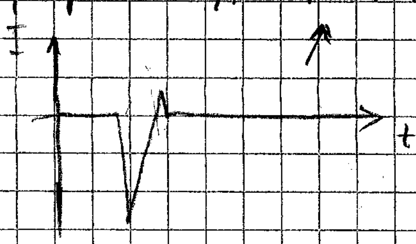
$-\omega_0'^2-2\beta^2+\beta^2+\omega_0^2\Rightarrow$

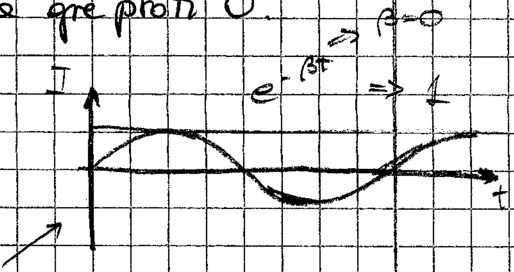
$\Rightarrow\boxed{\omega_0'=\sqrt{\omega_0^2-\beta^2}}$

$\beta<\omega_0$: $I=I_0e^{-\beta t}(\cos(\omega_0' t+\delta))$

Če je $R=0$; $\begin{cases}\beta=0\\ \omega_0'=\omega_0\\ I=I_0\cos(\omega_0 t)\end{cases}$

če je β prevelik, ni več nihanja; amplituda gre proti 0.

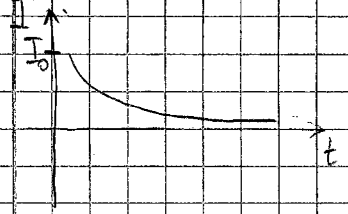




če je $\beta = 0$; funkcija preide v sinusno funkcijo

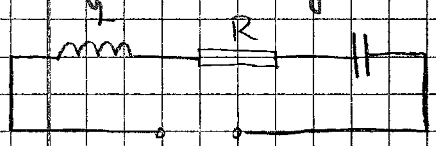
$\beta > \omega_0$: $\omega_0' = \sqrt{\omega_0^2 - \beta^2} = i\sqrt{\beta^2 - \omega_0^2} = i\alpha$

$$I = I_0 e^{-\beta t} e^{i\omega_0' t} = I_0 e^{-\beta t} e^{i i \sqrt{\beta^2 - \omega_0^2}\, t} = I_0 e^{-\beta t} e^{-\sqrt{\beta^2 - \omega_0^2}\, t} = I_0 e^{-\left(\beta + \sqrt{\beta^2 - \omega_0^2}\right) t}$$



pri $\beta > \omega_0$; tok eksponentno pada

Vsiljeno nihanje električnega nih. kroga:



$U_g = U_0 \cos(\omega t)$

$$U_g + U_L + U_R + U_C = 0$$

$$U_g - L\frac{dI}{dt} - IR - \frac{e}{C} = 0 \Big/ \frac{d}{dt}$$

$$\frac{dU_g}{dt} - L\frac{d^2I}{dt^2} - R\frac{dI}{dt} - \frac{I}{C} = 0$$

$I_0 \in \mathbb{R}e$ $\quad U_g = U_0 e^{i\omega t}$

$U_0 \in \mathbb{C}$ $\quad I = I_0 e^{i\omega t}$

$i^2 = -1$

$$e^{i\omega t}\left[\underbrace{U_0 i\omega + L I_0 \omega^2 - I_0 R i\omega - \frac{I_0}{C}}_{0}\right] = 0$$

$$\frac{dI}{dt} = I_0 i\omega e^{-i\omega t}$$

$$\frac{d^2I}{dt^2} = -I_0 \omega^2 e^{i\omega t}$$

impedanca

$$i\omega U_0 + L I \omega^2 - I_0 R i\omega - \frac{I_0}{C} = 0 \,/\, i\omega$$

$$U_0 = -\frac{L I_0 \omega^2 i}{i i\omega} + I_0 R + \frac{I_0 i}{i\omega C i}$$

$$U_0 = I_0 \overbrace{\left(\overbrace{L\omega i}^{Z_L} + \overbrace{R}^{Z_R} - \overbrace{\frac{i}{\omega C}}^{Z_C}\right)}^{Z} = I_0\left[R + i\left(L\omega - \frac{1}{\omega C}\right)\right]$$

$$U_0 = \underbrace{I_0 L\omega i}_{U_L} - \underbrace{I_0 \frac{i}{\omega C}}_{U_C} + \underbrace{I_0 R}_{U_R}$$

$$\boxed{U_0 = I_0 Z}$$

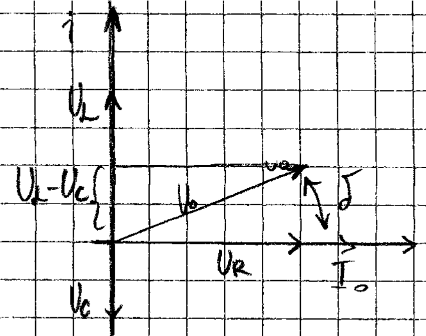
impedance se seštevajo

$Z_R = R$

$Z_L = L\omega i$

$Z_C = \frac{1}{\omega C}$

$Z = Z_L + Z_R + Z_C$



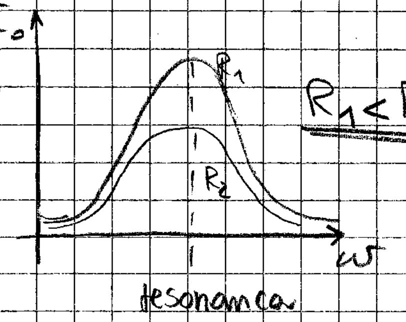
$$U_0^2 = U_0 U_0^* = \left[R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2\right] I_0^2 \Rightarrow$$

$$\Rightarrow I_0 = \frac{|U_0|}{\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}}$$

tok in napetost sta zamaknjena za nek kot δ.

$$\boxed{\tan\delta = \frac{L\omega - \frac{1}{\omega C}}{R}}$$

$$I_0 = \frac{|U_0|}{\sqrt{R^2 + \left(L\omega - \frac{1}{\omega C}\right)^2}}$$



$R_1 < R_2$

max: $L\omega_{RES} - \frac{1}{\omega C_{RES}} = 0 \Rightarrow \underbrace{\omega_{RES}^2 = \frac{1}{\omega C} = \omega_0^2}$

resonančna frekvenca je enaka nedušeni frekvenci nihajnega kroga.

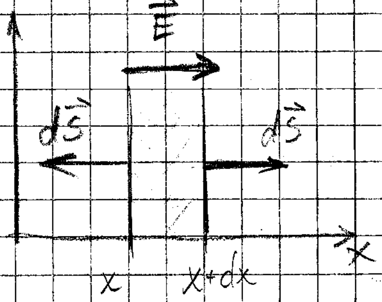
Povprečna moč:

$$\overline{P} = \frac{1}{2} I_0 \underbrace{|U_0| \cos\delta}_{U_R = I_0 R} = \frac{1}{2} I_0^2 R$$

24. 3. 2011

POISSONOVA ENAČBA

1-D



$\rho(x) \equiv$ volumska porazdelitev naboja

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{S} = \int \rho \, dV$$

$$\varepsilon_0 \left[E(x+dx)\cancel{S} - E(x)\cancel{S}\right] = \rho(x)\,dx\,\cancel{S}$$

$$\varepsilon_0 \left[\underbrace{E(x+dx) - E(x)}_{dE}\right] = \rho(x)\,dx$$

$$\varepsilon_0 \frac{dE}{dx} = \rho(x) \qquad E = -\frac{d\varphi}{dx}$$

el. potencial

$$\boxed{\frac{d^2\varphi}{dx^2} = -\frac{\rho(x)}{\varepsilon_0}}$$

Poissonova enačba v 1-D

V splošnem

$$\boxed{\vec{E} = -\mathrm{grad}\,\varphi}$$

GAUSSOV ZAKON V INTEGRALNI OBLIKI

$$\underbrace{\varepsilon_0 \oint_S \vec{E}\cdot d\vec{S} = \int_V \rho\, dV}$$

$$\varepsilon_0\, \mathrm{div}\vec{E} = \rho(\vec{r}) \qquad \varepsilon_0\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right] = \rho(\vec{r})$$

GAUSSOV ZAKON V DIFERENCIALNI OBLIKI

$$\boxed{\varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho(\vec{r})}$$

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{E} = (E_x, E_y, E_z)$$

$$\varepsilon_0 \vec{\nabla}\cdot\vec{E} = \rho(\vec{r})$$

$$\varepsilon_0 \vec{\nabla}\cdot(-\vec{\nabla}\varphi) = \rho(\vec{r})$$

$$-\varepsilon_0 \nabla^2 \varphi = \rho(\vec{r})$$

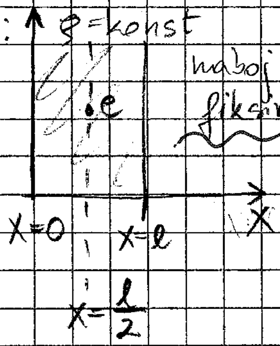
$$\underline{\Delta \equiv \nabla^2 = \text{Laplace}}$$

$$\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon_0}} \quad \text{POISSONOVA ENAČBA V 3-D}$$

$$\frac{\partial^2 \varphi(\vec{r})}{\partial x^2} + \frac{\partial^2 \varphi(\vec{r})}{\partial y^2} + \frac{\partial^2 \varphi(\vec{r})}{\partial z^2} = \underline{-\frac{\rho(\vec{r})}{\varepsilon_0}}$$

POISSONOVA ENAČBA

1. primer:



naboj je fiksiran

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho_0}{\varepsilon_0}, \quad \rho = \text{konst.} \quad / \int$$

$$\frac{d\varphi}{dx} = -\frac{\rho_0}{\varepsilon_0}x + B \quad / \int$$

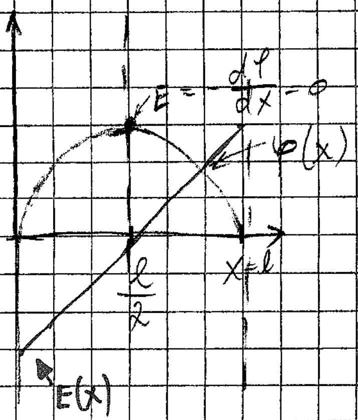
$$\varphi(x) = -\frac{\rho_0}{\varepsilon_0}\frac{x^2}{2} + Bx + C = 0$$

$$F\left(x = \frac{l}{2}\right) = 0 \Rightarrow \underbrace{E\left(x = \frac{l}{2}\right)} = 0$$

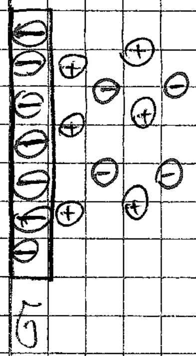
$$E(x) = \frac{d\varphi}{dx} = \frac{\rho_0}{\varepsilon_0}x - B$$

$$E\left(x = \frac{l}{2}\right) = \frac{\rho_0 l}{2\varepsilon_0} - B = 0 \Rightarrow B = \boxed{\frac{\rho_0 l}{2\varepsilon_0}}$$

$$\boxed{\varphi(x) = -\frac{\rho_0 x^2}{\varepsilon_0 2} + \frac{\rho_0 l}{2\varepsilon_0}x}$$



2. primer: električna dvojna plast

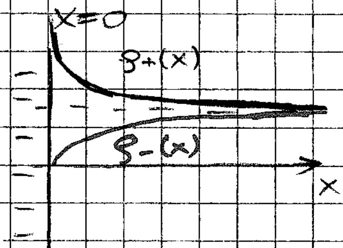


$NaCl \rightarrow Na^+ + Cl^-$

$\oplus \quad \ominus$

$\rho_+(x)$

$\rho_-(x)$



$$\frac{d^2\varphi}{dx^2} = -\frac{\rho(x)}{\varepsilon_r\varepsilon_0}$$

pozitivnih delcev je več kot negativnih

$$\oplus: W_+ = +e_0\varphi(x) \Rightarrow \rho_+(x) = \rho_0 e^{-\frac{W_+}{kT}} = \rho_0 e^{-\frac{e_0\varphi}{kT}}$$

$$\ominus: W_- = -e_0\varphi(x) \Rightarrow \rho_-(x) = \rho_0 e^{-\frac{W_-}{kT}} = \rho_0 e^{+\frac{e_0\varphi}{kT}}$$

$$\rho_0 = \frac{Ne}{V} = n_0 e_0$$

$$n_0 = \frac{N_0}{V}$$

$$\rho(x) = \rho_+(x) + \rho_-(x) = \rho_0 e^{-\frac{e_0\varphi}{kT}} - \rho_0 e^{+\frac{e_0\varphi}{kT}}$$

$$\frac{d^2\varphi}{dx^2} = \frac{\rho_0}{\varepsilon_0}\frac{2}{2}\left(e^{\frac{e_0\varphi}{kT}} - e^{-\frac{e_0\varphi}{kT}}\right) \Longrightarrow sh\,x$$

$$\boxed{\frac{d^2\varphi}{dx^2} = \frac{2\rho_0}{\varepsilon_0}\, sh\left(\frac{e_0\varphi}{kT}\right) = \frac{2n_0e_0}{\varepsilon_0}\, sh\left(\frac{e_0\varphi}{kT}\right)}$$

rešitev za majhne $\left(\frac{e_0\varphi}{kT}\right) \ll 1$

$kT = \frac{1}{40}\, e_0V$

$$sh(x) = x + \frac{x^2}{6} + \ldots$$

$$\frac{d^2\varphi}{dx^2} = \frac{2n_0e_0}{\varepsilon_r\varepsilon_0}\cdot\frac{e_0\varphi}{kT}$$

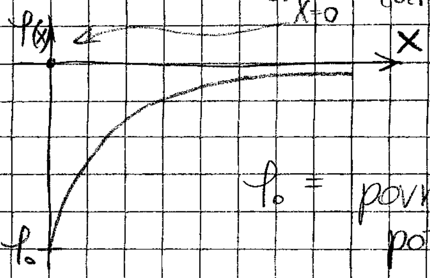
$$\frac{d^2\varphi}{dx^2} = \underbrace{\frac{2n_0e_0^2}{\varepsilon_r\varepsilon_0 kT}}_{K^2}\varphi$$

$$\boxed{\frac{d^2\varphi}{dx^2} = K^2\varphi}$$

linearizirana PB enačba

$$\varphi = \varphi_0 e^{-Kx}$$

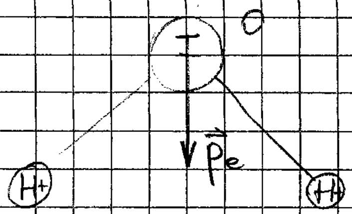
$$\left.\frac{d\varphi}{dx}\right|_{x=0} = -\frac{\sigma}{\varepsilon_0\varepsilon_r} \qquad E\,0$$

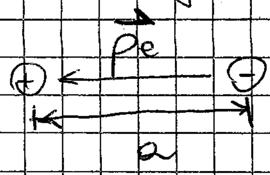


φ_0 = površinski dielektrični potencial

DIELEKTRIČNE LASTNOSTI SNOVI

POLARNE: molekule nimajo permanentnega dipolnega momenta. (Voda)

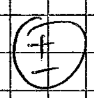




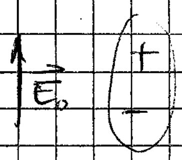
$\varepsilon_{r_{vode}} \cong 80$

NEPOLARNE:

$E_0 = 0$:

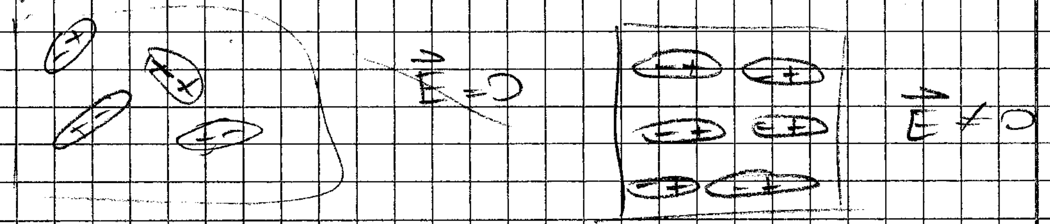


$E_0 \neq 0$:

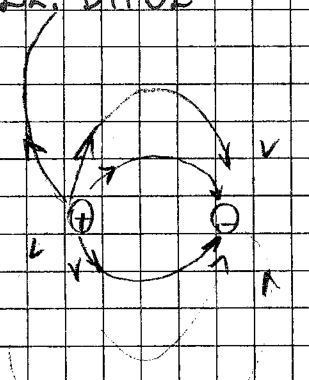


inducirani dipolni moment

SNOV V $\vec{E}$



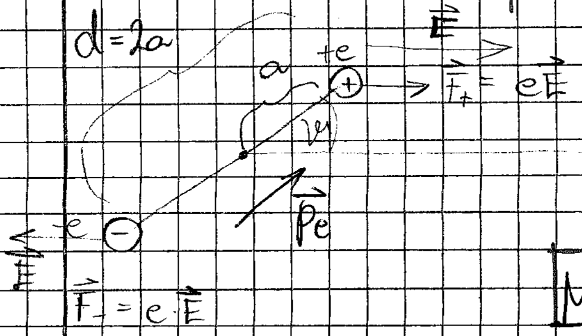
EL. DIPOL



SILA IN NAVOR na dipol

28. 3. 2011

$d = 2a$



$$M = M_+ + M_- = \left(a\cdot eE + (-a)(-e)E\right)\cdot \sin\varphi$$

$$= 2aeE \sin\varphi = dEe\cdot\sin\varphi$$

$$\boxed{M = \underbrace{e\cdot d}_{p_e}\cdot E\cdot \sin\varphi}$$

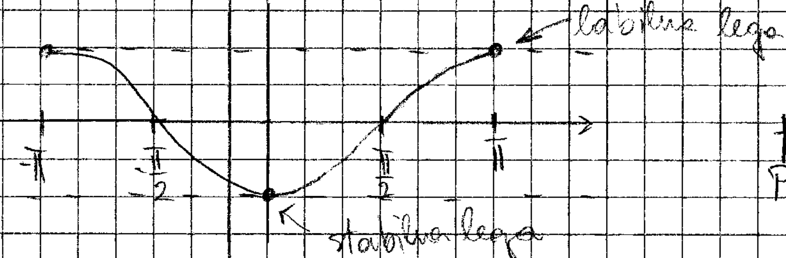
$$\boxed{\vec{M} = \vec{p}_e \times \vec{E}}$$

$$W_e = \int M\, d\vartheta = \int_{\vartheta_{zač}}^{\vartheta_{konč}} p_e \cdot E \sin\vartheta = -p_e E \cos\vartheta \Big|_{\vartheta_z}^{\vartheta_k} = -\vec{p_e} \cdot \vec{E}$$

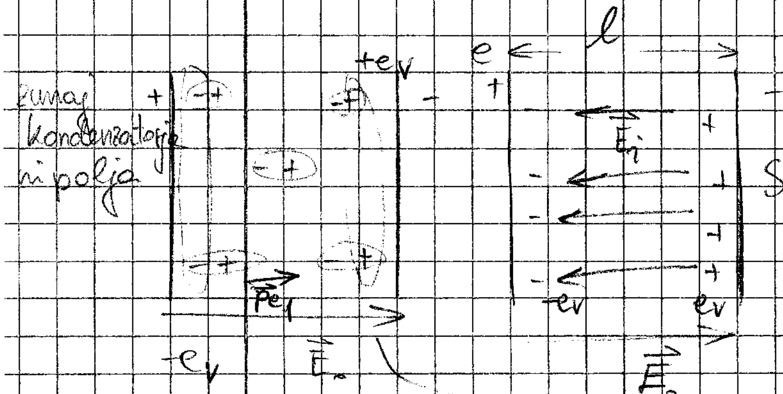
$$\boxed{W_e = -\vec{p_e} \cdot \vec{E}}$$

$\vartheta = 0$ $\quad \uparrow \vec{p_e} \quad \uparrow \vec{E} \quad ; \quad W_e = -p_e \cdot E$

$\vartheta = \pi$ $\quad \downarrow \vec{p_e} \quad \uparrow \vec{E} \quad ; \quad W_e = +p_e \cdot E$ — največja možna energija; ko je moment usmerjen v nasprotni smeri zunanjega polja.



To velja za polarne molekule.
Pri nepolarnih je moment 0.



$\vec{p_{e_1}} \equiv$ dipolni moment ene molekule

$e_v \equiv$ vezani naboj

Volumska gostota v makroskopskem smislu je 0.

$$\left(-\nabla \cdot \langle \varepsilon_0 \nabla \varphi \rangle = \langle \rho(x) \rangle\right)$$

$E = E_0 - E_i$ → polje vezanega naboja

$E = E_0 - \frac{e_v}{S\varepsilon_0}$ Dejansko polje se zaradi prispevka vezanih nabojev zmanjša.

polarizacija

$$\vec{P} = n \langle \vec{p_e} \rangle \quad ; \quad n = \frac{N}{V}$$

$$P \cdot V = n \langle \vec{p_e} \rangle V = \frac{N}{\cancel{V}} \langle \vec{p_e} \rangle \cancel{V} = N \langle \vec{p_e} \rangle$$

$\underbrace{N \langle \vec{p_e} \rangle}$ $= e_v l$

$P \cdot V = e_v l$

$P \cdot S \cdot \cancel{l} = e_v \cancel{l}$ $\quad \boxed{e_v = P \cdot S}$

$$E = E_0 - \frac{P \cdot S}{S\varepsilon_0} \Big/ \varepsilon_0$$

$$\boxed{\varepsilon_0 E_0 = \varepsilon_0 E - P}$$ – novo električno polje

$$\boxed{D = \varepsilon_0 E - P}$$ – gostota el. polja

v splošnem: $\chi = \chi(E)$

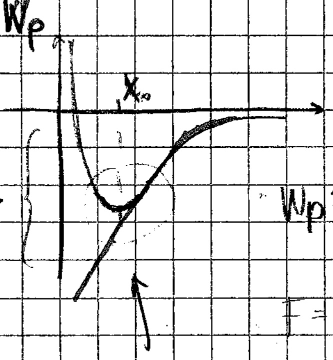
$P = \varepsilon_0 \chi E$
susceptibilnost

$$D = \varepsilon_0 E - \varepsilon_0 \chi E = \varepsilon_0 \overbrace{(1+\chi)}^{\varepsilon_r} E = \varepsilon E$$
dielektričnost

- nepolarne molekule

$\vec{E} = 0$ (+ −) $\vec{p}_e = 0$

$\vec{E} \neq 0$ $\vec{F} = e\vec{E}$ $\vec{F} = e\vec{E}$
−e +e



$W_p \cong k(x - x_0)^2 - C$

$F = -\frac{\partial W_p}{\partial x} = -k(x - x_0) = -kx + kx_0$

v okolici minimuma je funkcija podobna paraboli

$eE = kd$

$d = \frac{eE}{k} \Rightarrow p_e = de =$

$$\boxed{p_e = \frac{e^2 E}{k}}$$

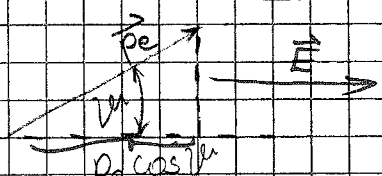
induciran dipolni moment

$p_e \propto E$

$$P = n p_e = \underbrace{\frac{n e^2}{k}}_{\varepsilon_0 \chi} E = E \varepsilon_0 \chi$$

$$\boxed{\chi = \frac{n e^2}{\varepsilon_0 k}}$$

- polarne molekule



$$P = n\langle \vec{p}_e \rangle = n\langle p_e \cos\vartheta \rangle = n p_e \underbrace{\langle \cos\vartheta \rangle}_{0}$$

$$\underline{\underline{P = 0}}$$

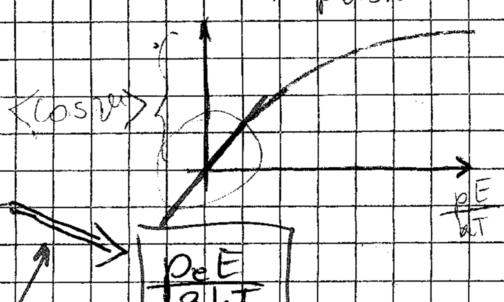
$W_d = -p_e E \cos\vartheta$
v splošnem

$E = 0$

$$\langle \cos\vartheta \rangle = \frac{\int \cos\vartheta \, d\Omega}{\int d\Omega}$$

$E \neq 0$

$$\langle \cos\vartheta \rangle = \frac{\int \cos\vartheta \, e^{-\frac{W_d}{kT}} d\Omega}{\int d\Omega \, e^{-\frac{W_d}{kT}}}$$



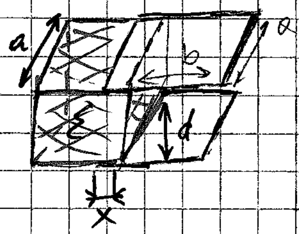
$$\boxed{\frac{p_e E}{3kT}}$$

za $\frac{p_e E}{kT} \ll 1$

$$\varepsilon_0 \chi = \frac{n p_e^2}{3kT} \Rightarrow \chi = \frac{n p_e^2}{3\varepsilon_0 kT} \qquad \varepsilon = 1+\chi = 1+\frac{n p_e^2}{3\varepsilon_0 kT}$$

31. 3. 2011

SILA NA DIELEKTRIK



Ko vstavimo dielektrik se naboj ne spremeni.

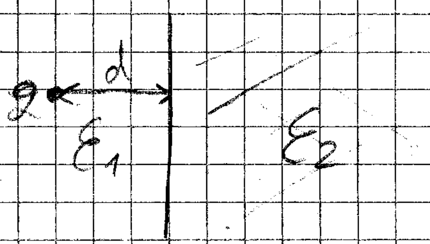
sistem obravnavamo kot vzp. vezane kondenzatorja.

$$e = CU_c \Rightarrow U_c = \frac{e}{C} \qquad W_c = \frac{1}{2}CU_c^2 = \frac{1}{2}\cdot\frac{e^2}{C}$$

$$W_x = \frac{1}{2}\,\frac{C_0^2 U_0^2}{\varepsilon_0 \frac{ax}{d}(\varepsilon-1)+\varepsilon_0\frac{ab}{d}} \qquad W(x) < W(x=0)$$

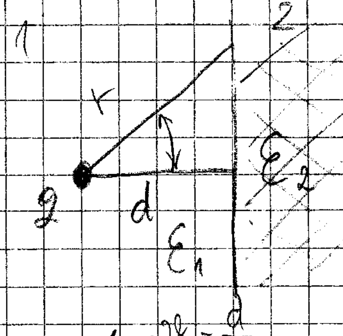
Sila na dielektrik ≡ sila med nabojem na ploščah in dipoli v dielektriku.

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_1}\,\frac{(\varepsilon_1-\varepsilon_2)}{(\varepsilon_1+\varepsilon_2)}\,\frac{q^2}{4d^2}$$

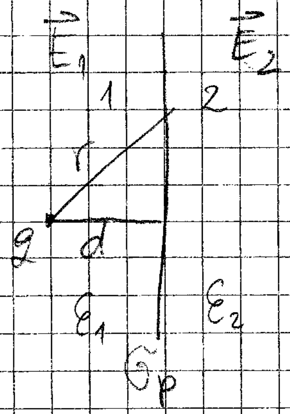


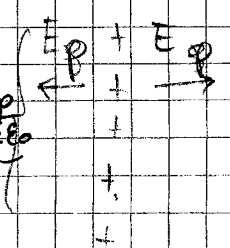
1. če $\varepsilon_1 > \varepsilon_2 \Rightarrow F > 0$
2. če $\varepsilon_1 < \varepsilon_2 \Rightarrow F < 0$

Sila na točk. naboj na meji



$\cos\vartheta = \frac{d}{r}$



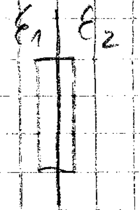


$$F_p = \pm\frac{\sigma_p}{2\varepsilon\varepsilon_0}$$

$$E_{1n} = \frac{q}{4\pi\varepsilon_0\varepsilon_1}\cdot\frac{1}{r^2}\cdot\frac{d}{r} - \frac{\sigma_p}{2\varepsilon_1\varepsilon_0}$$

$$E_{2n} = \frac{q}{4\pi\varepsilon_0\varepsilon_1}\cdot\frac{1}{r^2}\cdot\frac{d}{r} + \frac{\sigma_p}{2\varepsilon_0\varepsilon_1}$$

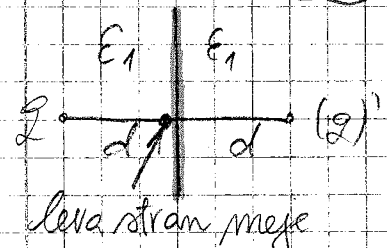
mejni pogoj



$$-\varepsilon_1 E_{1n}\, dA + \varepsilon_2 E_{2n}\, dA = 0$$

$$E_p = \frac{-\sigma_p}{2\varepsilon_0 \varepsilon_1} = -\left(\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\right) \frac{1}{4\pi \varepsilon_0 \varepsilon_1} \frac{q}{r^2} \frac{d}{r} \rightarrow q'$$

prispevek na levi strani meje zaradi $\varepsilon_1 \neq \varepsilon_2$ izgleda kot polje točk. naboja



SNOV V MAGNETNEM POLJU

$$\boxed{W_p = -\vec{p}_m \cdot \vec{B}} \rightarrow \quad \vartheta = 0: W_p = -p_m \cdot B \quad \vec{p}_m \uparrow\uparrow \vec{B}$$

$$\vartheta = \pi: W_p = p_m B \quad \vec{p}_m \downarrow\uparrow \vec{B}$$

$\vec{p}_m \uparrow\uparrow \vec{B}$

Pri večji temperaturi se molekule bolj intenzivno gibljejo okoli svoje ravnovesne lege.

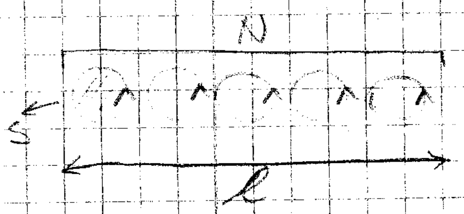
$B = \underbrace{B_0}_{\text{zun. mag. polje}} + B_m$

paramagnetne so snovi, katere dipoli se obrnejo v smeri zunanjega mag. polja.

Primer:
1-D prostor

$B = B_0 + B_n \leftarrow$ notr. mag. polje

magnetizacija $\vec{M} = n \langle \vec{p}_m \rangle, \; n = \frac{N}{V}$



$$B_m = \mu_0 \frac{N \cdot I}{l} \cdot \frac{S}{S} = \mu_0 \frac{N \cdot p_m}{V} = \mu_0 n \cdot p_m = \mu_0 M$$

$$\boxed{\vec{B}_m = \mu_0 \vec{M}}$$

$M = \chi \cdot H$
$\uparrow$
susceptibilnost

$$B = B_0 + B_m = B_0 + \mu_0 M \qquad (1+\chi) = \mu \leftarrow \text{permeabilnost}$$

$$B = \mu_0 H + \mu_0 \chi H = \mu_0 (1+\chi) H$$

$$\boxed{\vec{B} = \mu_0 \mu \vec{H}}$$

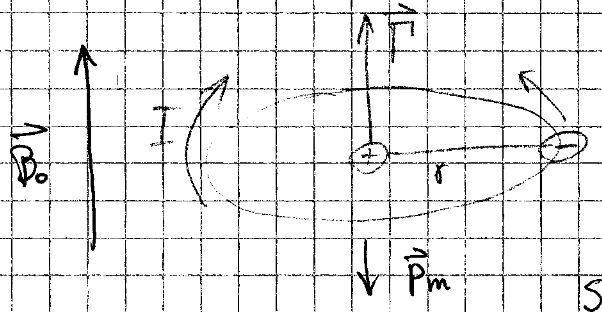
$\chi > 0 \Rightarrow \mu > 1$: param. snovi

$\chi < 0 \Rightarrow \mu < 1$: diam. snovi

χ zelo velik $\Rightarrow \mu(H) \gg 1$ feram. snovi

(B in H) nista sorazmerna)

KLASIČNI MODEL DIAMAGNETIZMA



$\vec{\Gamma} \equiv$ vrtilna količina

$\vec{\Gamma} = m\vec{r} \times \vec{v}$

$\vec{p}_m = I \cdot S = \frac{e_0 v \pi r^2 m_e}{2\pi r \, m_e} = -\frac{e_0}{2m_e} \underbrace{m_e r v}_{\Gamma}$

$S = \pi r^2$

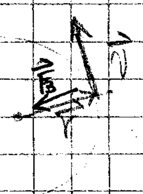
$t_0 =$ obhodni čas

$t_0 = \frac{2\pi r}{v}$

$I = \frac{e_0}{t_0} = \frac{e_0 v}{2\pi r}$

$$\boxed{\vec{p}_m = -\frac{e_0}{2m_e} \Gamma}$$

$\varphi(\vec{v}, \vec{B}) = 90°$



$\vec{F}_B = (-e_0) \, \vec{v} \times \vec{B}$

N - zakon (za enak. kroženje)

$m_e a_r = F_B$

$m_e \frac{v^2}{r} = e_0 v B \quad / r$

$m v = e_0 B r$

$\Gamma = m_e r v = e_0 r^2 B_0$

$p_m = -\frac{e_0}{2m_e} \cdot e_0 r^2 \mu_0 H$

$M = n \cdot p_m = -\frac{\mu_0 n e_0^2 \langle r^2 \rangle}{2m_e} \cdot H$

$Z \equiv$ št. elektronov v atomu

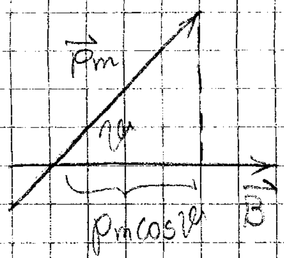
$M = \underbrace{-\frac{\mu_0 n e_0^2 \langle r^2 \rangle Z}{2m_e}}_{\chi} \cdot H \Longrightarrow \chi = -\frac{\mu_0 n Z e_0^2 \langle r^2 \rangle}{2m_e}$

Paramagneti

4.4.2011

$\vec{p}_m \uparrow$ $T \to 0$ $\vec{B} \uparrow$ $\Big\}$ $\vartheta = 0$, $W_m = -p_m B$

$$W_m = -\vec{p}_m \cdot \vec{B}$$



termična energija $\sim kT$

magnetizacija: $\vec{M} = n \langle \vec{p}_m \rangle = n p_m \langle \cos\vartheta \rangle$

$$n = \frac{N}{V} \qquad \langle \cos\vartheta \rangle = \frac{\int \cos\vartheta \, e^{-\frac{W_m}{kT}} \, d\Omega}{\int e^{-\frac{W_m}{kT}} \, d\Omega} \cong$$

$d\Omega$ ← prostorski kot

$$\cong \boxed{\frac{p_m B}{3kT}}$$

$\frac{p_m B}{kT} < 1$

Vsak paramagnet je tudi diamagnet.

$\mathcal{L}(x)$ = Langevinova funkcija

$$M = n p_m \langle \cos\vartheta \rangle = \frac{n p_m^2 B_0}{3kT} \qquad B_0 = \mu_0 H \qquad \cong \boxed{\underbrace{\frac{n \mu_0 p_m^2}{3kT}}_{\chi} \cdot H}$$

χ – susceptibilnost

$$\boxed{M = \chi \cdot H}$$

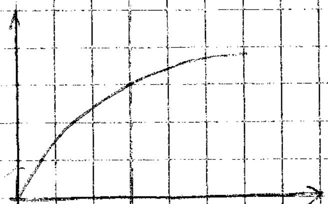
če je $\frac{p_m B_0}{kT} < 1$

$$\boxed{M = C \cdot \frac{B_0}{T}}$$

C → Curie-jeva konstanta

Curie-jev zakon

$$\chi = \frac{n \cdot \mu_0 p_m^2}{3kT}$$



$$\mu = 1 + \chi$$

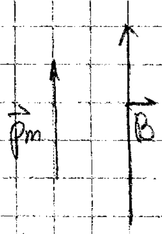
Eksperiment:

Dve kroglici na pečkah, ena je param. drugo pa dia., vmes damo magnet.

Paramagnetno potegne med pola magneta, teži k ~~polju~~ močnejšemu polju.

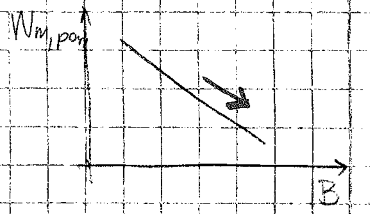
Diamagnetno odvrne od polov magneta, v smeri manjšega mag. polja.

paramagneti

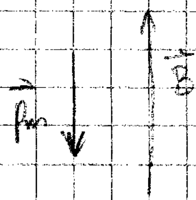


$W_m = -\vec{p}_m \cdot \vec{B} = -p_m \cdot B \cdot \cos\vartheta$

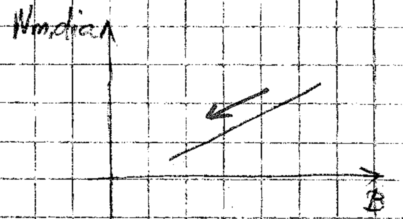
$\begin{cases} W_{m,par} = -p_m B \\ \vartheta = 0 \end{cases}$



diamagneti



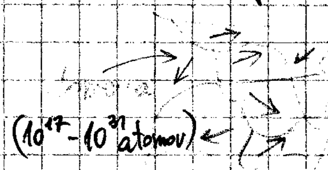
$\begin{cases} W_{m,dia} = +p_m \cdot B \\ \vartheta = \pi \end{cases}$



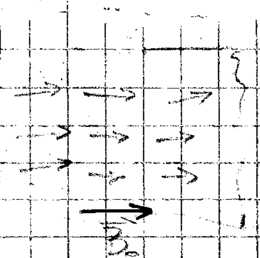
Feromagneti

V nemagnetnem stanju snovi so domene usmerjene naključno.

$\vec{B}_0 = 0$



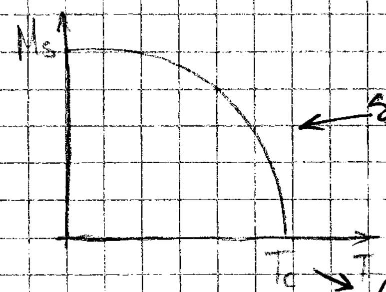
Če temp. pade po Curie-jevo temperaturo je vsak feromagnet je paramagnet



ureditev domen v zunanjem mag. polju $\vec{B}_0$; μ ni konstantna

$$\boxed{\mu(H) = \frac{1}{\mu_0} \cdot \frac{dB}{dH}}$$

$\bar{\mu} \equiv$ povprečna permeabilnost



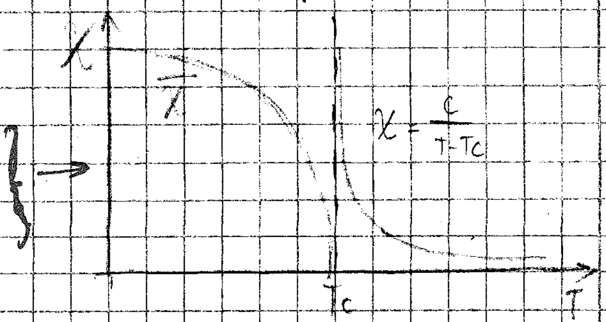
spontana magnetizacija znotraj domen (tudi brez $\vec{B}_0$)

$T_c \rightarrow$ Curie-jeva temperatura

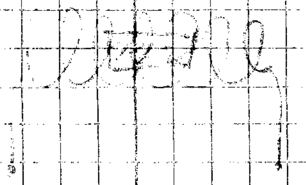
Če je $T \geq T_c$ ferom. snovi izgubijo spontano magnetizacijo in postanejo paramagneti

velja Curie-Weissov zakon:

$$\boxed{\chi = \frac{C}{T - T_c}}$$



Barkhausen-ov poskus



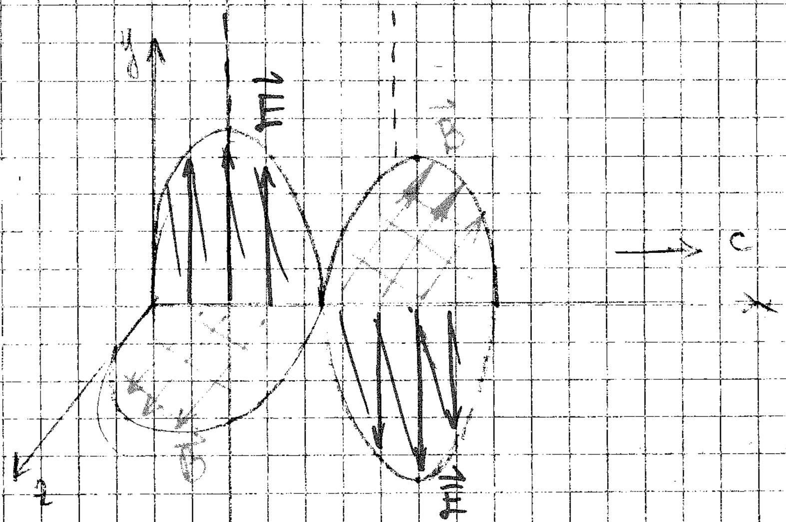
domene ~~dipoli~~ v tuljavi se premikajo, inducira se napetost.

$-p_m \cdot B$

$$U_i = -\frac{d\phi_m}{dt}$$

Priklopjeno je na zvočnik, sliši se hreščanje zaradi premikanje magneta v tuljavi.

ELEKTROMAGNETNO VALOVANJE



raven elektromagnetski val

Maxwell-ove enačbe:

$$\oint \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{S}$$

$$\oint \vec{H}\cdot d\vec{s} = \frac{\partial}{\partial t}\int_S \vec{D}\cdot d\vec{S}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\cdot\frac{\partial^2 E}{\partial x^2}$$

$$\frac{\partial^2 H}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\cdot\frac{\partial^2 H}{\partial x^2}$$

valovni enačbi za E in H (1-D)

$E = E_0 \cdot \cos(\omega t - kx)$ ← rešitev: potujoče valovanje

$k = \frac{\omega}{c}$ hitrost širjenja motnje

$$\frac{\partial^2 E}{\partial x^2} = -E_0 k^2 \cos(\omega t - kx)$$

$$\frac{\partial^2 E}{\partial t^2} = -E_0 \omega^2 \cos(\omega t - kx)$$

$$\frac{\frac{\partial^2 E}{\partial x^2}}{\frac{\partial^2 E}{\partial t^2}} = \frac{k^2}{\omega^2} = \frac{\cancel{\omega^2}}{\cancel{\omega^2} c_0^2} = \frac{1}{c_0^2}$$

$$\Longrightarrow \frac{\partial^2 E}{\partial t^2} = c_0^2\frac{\partial^2 E}{\partial x^2}$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E}{\partial x^2}$$

$$c_0^2 = \frac{1}{\varepsilon_0\mu_0}$$

$$\boxed{c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}}$$

→ hitrost svetlobe je elektromagnetno valovanje

$$c_0 \cong 3\cdot 10^8\ \frac{m}{s}$$

$\varepsilon \neq 1$ $\quad \varepsilon_0 \rightarrow \varepsilon_r \varepsilon_0$

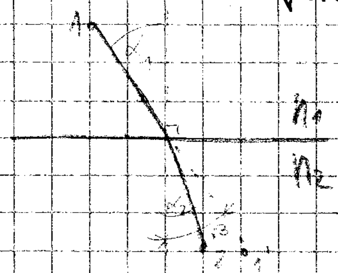
$\mu \neq 1$ $\quad \mu_0 \rightarrow \mu_r \mu_0$

$$C = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0}}$$

lomni količnik

Za prozorne snovi:

$$\mu = 1 \rightarrow c = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_0}} = \frac{c_0}{\sqrt{\varepsilon}} = \frac{c_0}{n}, \quad n = \sqrt{\varepsilon}$$



Spekter EM valovanja

- vidna svetloba

rdeča $\doteq$ 700 nm
vijolična $\doteq$ 400 nm $\Big\} \doteq 10^{15}$ Hz

- infrardeča $\doteq 10^{-5}$ m $\Rightarrow 10^{13}$ Hz
- UV $\doteq 10^{-8}$ m $\Rightarrow 10^{16}$ Hz

7.4.2011

Gostota energijskega toka v EM valovanju

$$\overline{j} = \overline{w} \cdot c_0, \quad c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}, \quad \vec{E} = \vec{B} \times \vec{c_0}$$

$$w = w_E + w_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 \Rightarrow \quad B = \mu_0 H$$

$$= \frac{1}{2}\varepsilon_0 \overline{E^2} + \frac{1}{2}\cancel{\mu_0}\frac{B^2}{\mu_0^{\cancel{2}}} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} \quad B = \frac{E}{c_0}$$

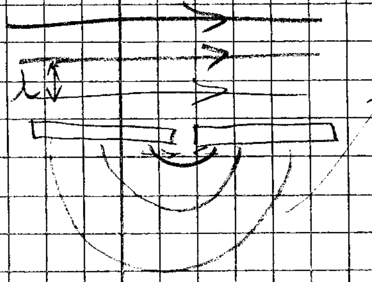
$$\overline{w} = \frac{1}{2}\varepsilon_0 \overline{E^2} + \frac{1}{2}\frac{\overline{E^2}}{\mu_0 c_0^2} = \frac{1}{2}\varepsilon_0 \overline{E^2} + \frac{1}{2}\frac{\overline{E^2}\varepsilon_0 \cancel{\mu_0}}{\cancel{\mu_0}} = \overline{E^2}\varepsilon_0 = \varepsilon_0 E_0^2 \cdot \underbrace{\left(\overline{\cos^2(\omega t - kx)}\right)}_{1/2}$$

$$E = E_0 \cos(\omega t - kx) \qquad \boxed{w = \frac{1}{2}\varepsilon_0 E_0^2}$$

$$P = \int j \, dS = \frac{p_0^2 \omega^4}{12\pi \varepsilon_0 c_0^3} \qquad j = c\overline{w} = \frac{p_0 \omega^4}{32\pi^2 \varepsilon_0 c_0^3} \cdot \frac{\sin^2 \vartheta}{r^2}$$

$$j = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

$c_0 = \frac{18,2\,m}{60 \cdot 10^{-9}\,s} = 0,3 \cdot 10^9 \frac{m}{s} = \underline{\underline{3 \cdot 10^8 \frac{m}{s}}}$ } dolžina predavalnice + čas potovanja žarka

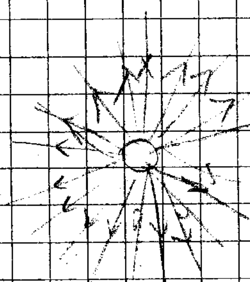


uklon: valovanje se širi tudi za ovire - valovna dolžina je reda velikosti ovire; radijski valovi

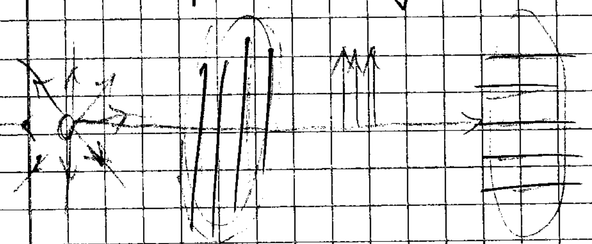
televizijski in mobilni valovi so kratkovalovni in potrebujejo direkten stik, zato so bolj pogosti oddajniki

Polarizacija sončne svetlobe:

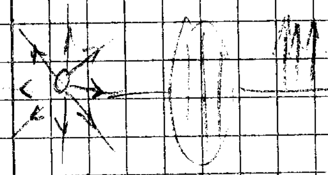
- nepolarizirana svetloba (EM. val.)



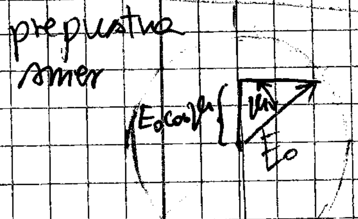
linearno polarizacijo lahko naredimo s polarizatorji;



ne pride nič; $\underline{\cos 90° = 0}$



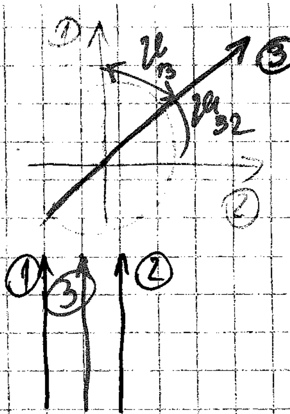
$\overline{\cos^2 \varphi} = \frac{1}{2}$



$\overline{j} = \overline{w} \cdot c_0 = \frac{1}{2} \varepsilon_0 E_0^2 c_0$

$\left.\begin{array}{l} \overline{j}_{vp} = \frac{1}{2} \varepsilon_0 E_0^2 c_0 \\ \overline{j}_{prep} = \frac{1}{2} \varepsilon_0 (E_0 \cos\varphi)^2 c_0 \end{array}\right\}$ prepustnost

$\frac{\overline{j}_{prep}}{\overline{j}_{vp}} = \frac{\cancel{\frac{1}{2}\varepsilon_0 E_0^2} \cos^2\varphi \cdot \cancel{c_0}}{\cancel{\frac{1}{2}\varepsilon_0 E_0^2 c_0}} = \cos^2\varphi$



{ prepustne smeri

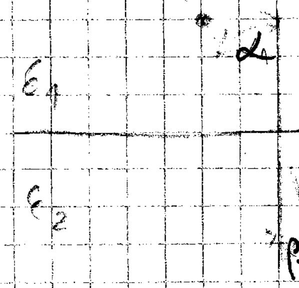
Če nimamo ③ polarizatorja je $\cos 90° = 0$ in skozi ne pride nič

Ravno nasprotno kot pri absorbciji, kjer se ob dodajanju vmesnih stvari zadeva zmanjša.

$$j_{sip} \propto \omega^4 \propto \frac{1}{\lambda^4}$$

LOMNI ZAKON

Fermatov princip - svetloba potuje med točko 1 in točko 2 tako, da porabi najmanj časa in energije.



$$c_1 = \frac{c_0}{\sqrt{\varepsilon_1}} = \frac{c_0}{n_1}, \quad n_1 = \sqrt{\varepsilon_1}$$

$$c_2 = \frac{c_0}{\sqrt{\varepsilon_2}} = \frac{c_0}{n_2}; \quad n = \sqrt{\varepsilon_2}$$

$$\boxed{\frac{\sin\alpha}{\sin\beta} = \frac{c_1}{c_2}}$$

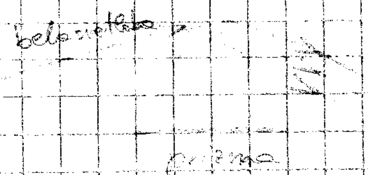
LOMNI ZAKON

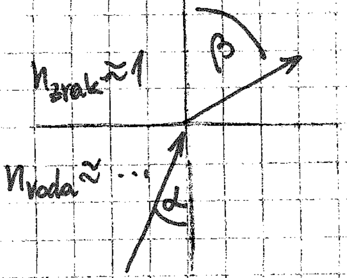
$$\boxed{\frac{\sin\alpha}{\sin\beta} = \frac{n_2}{n_1}}$$

TOTALNI ODBOJ

Če z belo svetlobo posvetimo na prizmo se le-ta razkloni v spekter barv.

lomni količnik je odvisen od val. dolžine, zato pride do razklona.





$$\frac{\sin\alpha}{\sin\beta} = \frac{n_v}{1}$$

alfa kritični

$\alpha = \alpha_c$

$\beta = \frac{\pi}{2}$

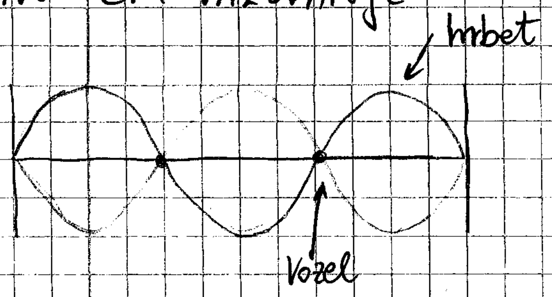
$$\Downarrow$$

$$\frac{\sin\alpha_c}{1} = n_v$$

$$\underline{\alpha_c = \arcsin(n_v)}$$

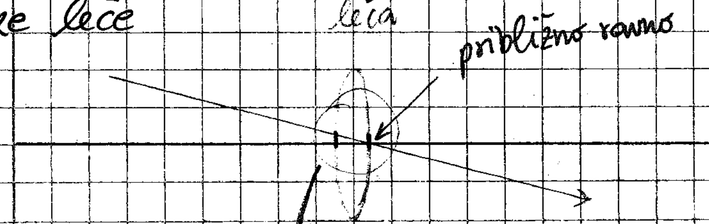
$\alpha_{c_{stekla}} \doteq 42°$

STOJNO EM VALOVANJE

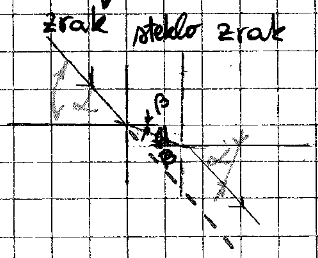


Optični aparati

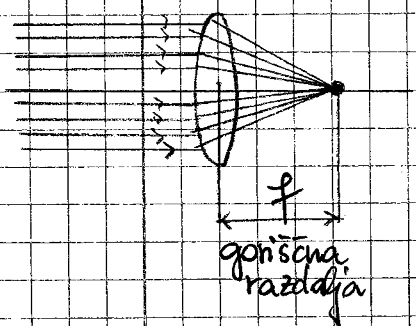
– tanke leče



pri tanki leči je vzporedni premik zelo majhen, zato rišemo, kot da gre direktno skozi.



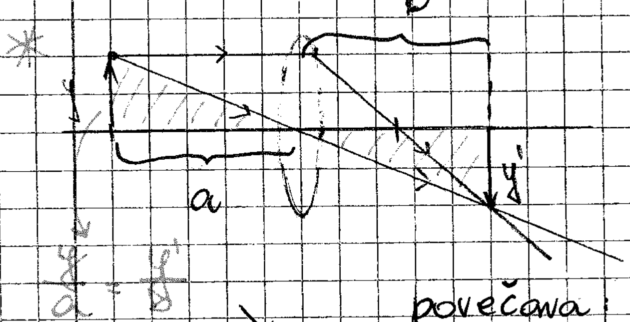
Pri tankih lečah vedno podamo gorišče.



Bikonkavna leča – razpršilna leča, ima navidezno gorišče in sicer na vhodni strani leče.

Bikonveksna leča – zbiralna leča

– bikonveksna leča



enačba tanke leče

$$\boxed{\frac{1}{a}+\frac{1}{b}=\frac{1}{f}}$$

povečava:

$$M=\frac{y'}{y}=\frac{b}{a}=\frac{\cancel{a}f}{\cancel{a}(a-f)}=\frac{f}{(a-f)}$$

$$\frac{1}{b}=\frac{1}{f}-\frac{1}{a}$$

$$\frac{1}{b}=\frac{a-f}{af}$$

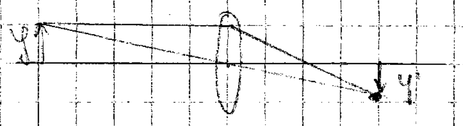
$$\boxed{b=\frac{af}{a-f}}$$

$f>0$

$a>0$

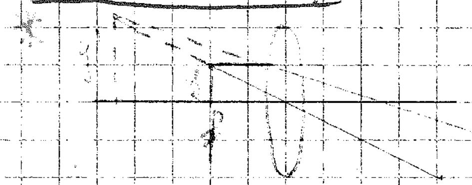
$b>0$

če je a velik, dobimo majhno povečavo



$a > 0$
$b < 0$ } dobimo navidezno sliko; podaljški žarkov se sekajo zadaj

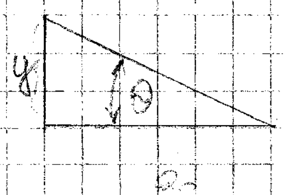
- bikonkavna leča



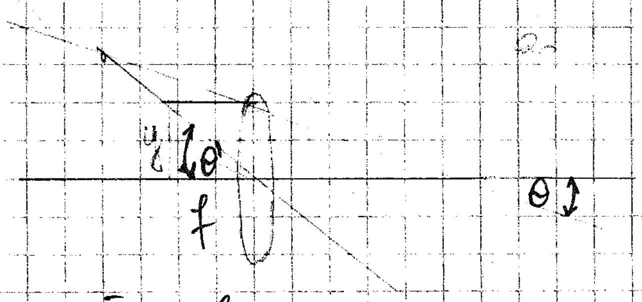
} princip lupe; poveča zorni kot: če damo npr.: mravljo v gorišče, prihajajo žarki v oko pod večjim kotom, žarki se sekajo v podaljških, slika nastane v neskončnosti

$a_0 \equiv$ normalna zorna razdalja

$$\boxed{M = \frac{\operatorname{tg}\theta'}{\operatorname{tg}\theta}}$$



$\Rightarrow \operatorname{tg}\theta = \frac{y}{a_0}$



$$\operatorname{tg}\theta' = \frac{y}{f}$$

$f < 0$
$a > 0$
$b < 0$

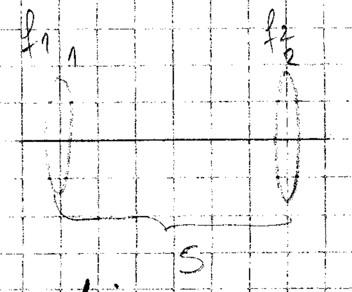
povečava lupe:

$$M = \frac{\operatorname{tg}\theta'}{\operatorname{tg}\theta} = \frac{y \cdot a_0}{f\, y} = \frac{a_0}{f}$$

Sistem dveh leč

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{s}{f_1 f_2}}$$

$s \equiv$ razdalja med lečama



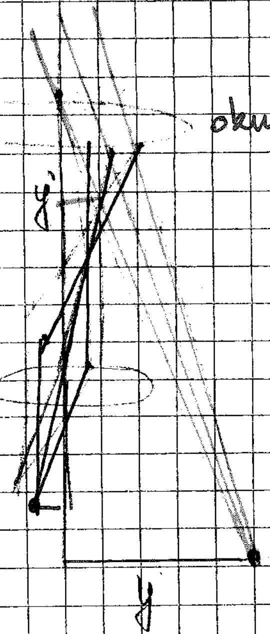
Pri debelih lečah enačbe za tanke leče ne veljajo

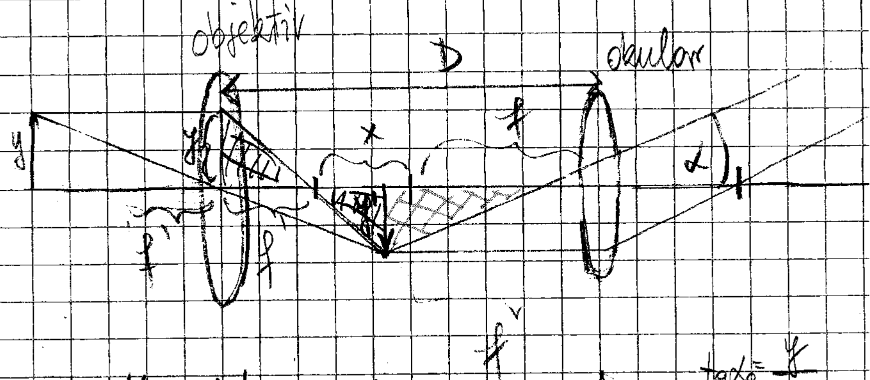
radiji:

- bikonveksna; $r > 0$; $r' > 0$ $\Rightarrow \frac{1}{f} = (n-1)\left(\frac{1}{r} + \frac{1}{r'}\right)$
- bikonkavna; $r < 0$; $r' < 0$ $\Rightarrow \frac{1}{f} = -(n-1)\left(\frac{1}{|r|} + \frac{1}{|r'|}\right)$

Mikroskop:

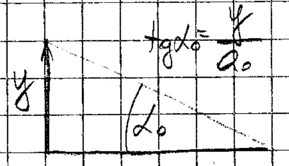
2 leči – okular in objektiv





$$\frac{y}{f'} = \frac{y'}{x} \Rightarrow \frac{y'}{y} = \frac{x}{f'}$$

$$\operatorname{tg}\alpha = \frac{y'}{f}$$



$a_0 \approx 25\,cm$

normalna zorna razdalja

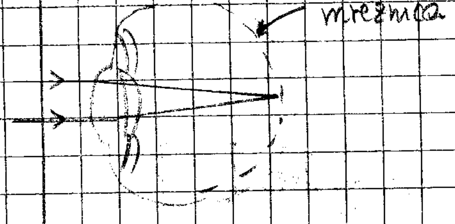
povečava:

$$\boxed{M = \frac{\operatorname{tg}\alpha}{\operatorname{tg}\alpha_0} = \frac{y'}{f}\frac{a_0}{y} = \frac{x\,a_0}{ff'} = \frac{(D-f-f')a_0}{ff'}}$$

Pri mikroskopu ni pomembna samo povečava, ampak tudi ločljivost, ki je odvisna od valovne dolžine

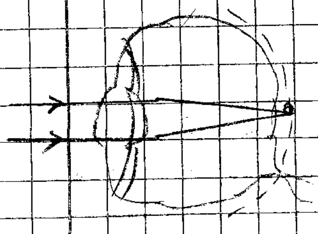
Oko:

kratkovidnost:
Slika nastane pred mrežnico, leče nima dovolj velike goriščne razdalje



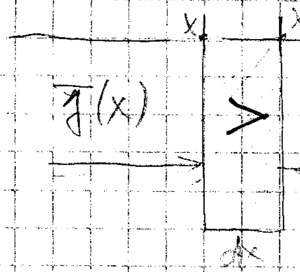
$D = \frac{1}{f'}$; $a_0 > 0$

daljnovidnost:
Slika nastane za mrežnico



$D = \frac{1}{f'}$; $a_0 < 0$

Absorpcija – pojemanje gostote en. toka svetlobe (EMV), ko energija potuje skozi snov.
Pojema z debelino plasti prehoda.



$j(x+dx) = j + dj$

$dj \propto -j\,dx$

za majhne debeline velja sorazmernost

$$\boxed{dj = -\mu j\,dx}$$

Vpadna gostota en. toka EMV

$$dj = -\mu j\,dx \quad /:j$$

$$\int_{j_0}^{j(x=d)} \frac{dj}{j} = -\mu \int_0^d dx$$

$$\ln j \Big|_{j_0}^{j(x=d)} = -\mu d$$

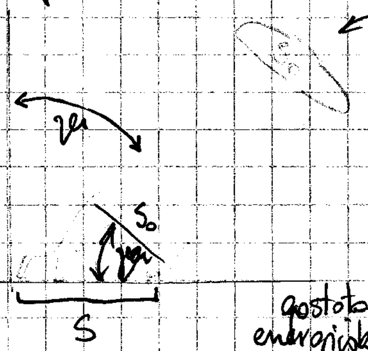
$$\ln \frac{j(x=d)}{j_0} = -\mu d$$

$$\boxed{j(x=d) = j_0\, e^{-(\mu d)}}$$ → eksponentni zakon absorpcije

Več kot je polarizatorjev, manjša je gostota en. toka, zaradi absorpcije.

Fotometrija: 14. 4. 2011

– Osvetljenost:



← površina curka

$$\cos\vartheta = \frac{S_0}{S}$$

$$E = \frac{P}{S}$$ ← energijski tok [W]

E – gostota energijskega toka (osvetljenost)

* V smeri normale na S je osvetljenost maksimalna. Če svetimo pod kotom se površina poveča, en. tok je isti – osvetljenost je manjša.

$$S = \frac{S_0}{\cos\vartheta} \qquad \boxed{E = \frac{P}{S_0}\cdot\cos\vartheta} \qquad \boxed{E = j\cdot\cos\vartheta}$$

⇓

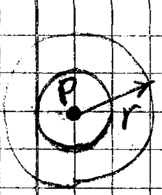
j – gostota en. toka v curku

– Svetilnost :

a) Točkasto telo :

$I \equiv$ svetilnost

$4\pi \equiv$ poln prostorski kot

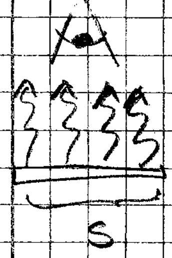


$$\boxed{j = \frac{P}{4\pi r^2} = \frac{I}{r^2}}$$

$$I = \frac{P}{4\pi}$$

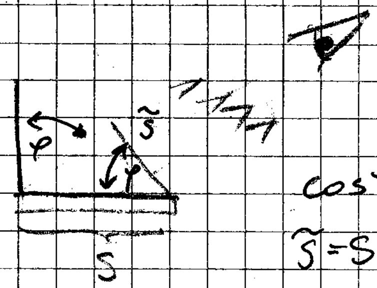
b) ploskovno svetilo :

* v tem primeru je enako.



$$I = B_0 \cdot S$$

↑ svetlost



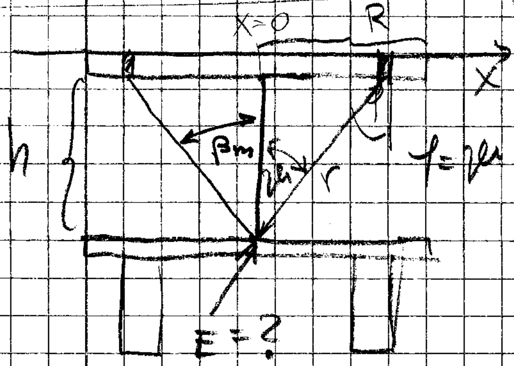
$$\cos\varphi = \frac{\tilde{S}}{S}$$

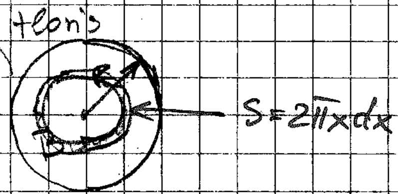
$$\tilde{S} = S \cdot \cos\varphi$$

$$I = B\tilde{S} = B \cdot S \cdot \cos\varphi$$

$$B = B(\varphi)$$

Lambert : $B(\varphi) = B_0$





$$dE = \frac{dI}{r^2} \cos\vartheta$$

$$\varphi = \vartheta$$

$$dI = B \cdot 2\pi x dx \cos\varphi$$

$$dE = \frac{B 2\pi x dx \cos^2\vartheta}{R^2}$$

D.N.

$$E = \int \frac{B 2\pi x dx \cos^2\vartheta}{r^2} = \dots = \frac{\pi B}{2}(1 - \cos 2\beta_m)$$

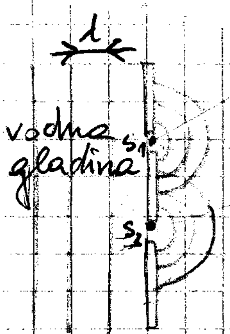
limita : neskončno svetilo :

$$R \to \infty \Rightarrow \beta_m = \frac{\pi}{2} ; \quad \cos(\underbrace{\tfrac{2\pi}{2}}_{\beta_m}) = 1$$

$$\boxed{E_\infty = \pi B}$$

INTEFERENČNI POJAVI PRI EMV

– inteferenca



Fazna razlika je konstantna, ni odvisna od ~~časa~~, drugače ne bi bilo bili vzorca (vrh na vrh – ojačitev)..!
Sta na isti valovni fronti, nihata koherentno

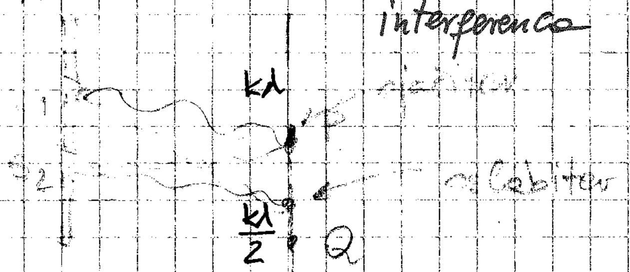
Na ojačitvah so svetla področja, na oslabitvah pa temna območja

konstruktivna interferenca

amplitude se seštevajo

destruktivna interferenca

Huygens-ovo načelo



en izvor svetlobe/valovanja je vedno koherenten.

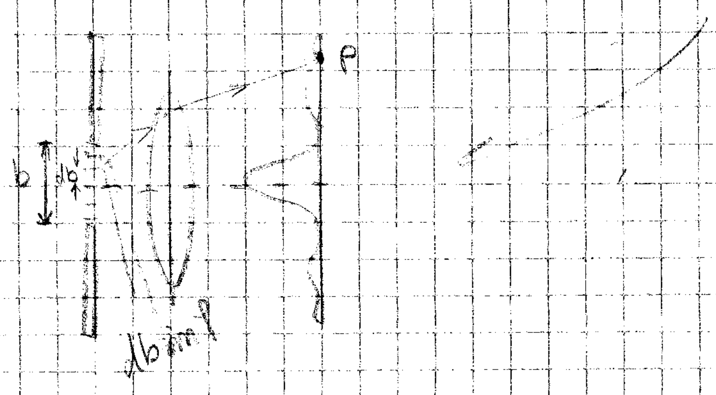
uklon – valovanje se širi tudi za mrežo, velikost reže je manjša od valovne dolžine.

Vsako točko med ~~režami~~ ~~obravnadm~~ obravnavamo kot izvor točkastega svetila.

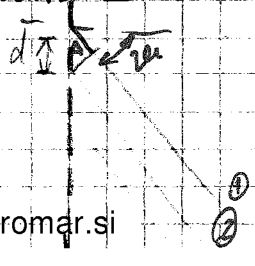
uklon in interferenca na reži s širino b

$$j = j_0 \frac{\sin^2(\pi b \sin\vartheta/\lambda)}{(\pi b \sin\vartheta/\lambda)^2}$$

$\Delta x \propto \frac{\lambda}{b}$ } širina reže



Uklonska mrežica



ojačitev

$$\Delta = d\sin\vartheta = N\lambda$$

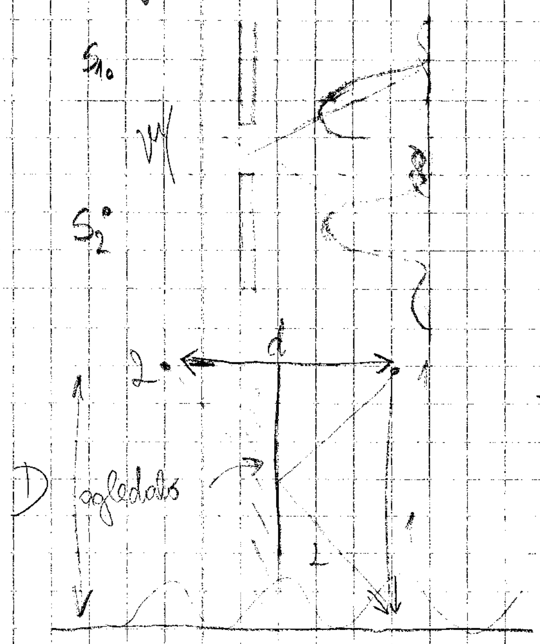
$$\boxed{\sin\vartheta_{max} = N\frac{\lambda}{d}}$$

oslabitev

$$\Delta = d\sin\vartheta = \frac{N}{2}\lambda$$

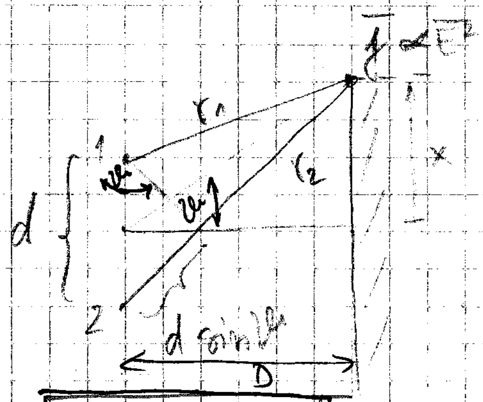
Ločljivost zaradi uklona

Pri mikroskopu: če hočemo povečati ločljivost, moramo zmanjšati val. dolžino → elektronski mikroskop, ker uporablja curek elektronov in ne svetlobe kot opt. mikroskop.



$\bar{j} \propto \overline{E^2}$

18.4.2011



$$\boxed{r_1 - r_2 = d\sin\vartheta}$$

$$E = E_0 \cos(\vec{k}\vec{r} - \omega t)$$
$$E_1 = E_0 e^{i(kr_1 - \omega t)}$$
$$E_2 = E_0 e^{i(kr_2 - \omega t)}$$

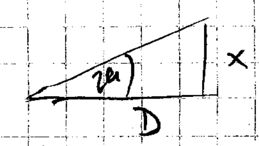
$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$
$$e^{-i\alpha} = \cos\alpha - \sin\alpha$$

Gostota energijskega toka na zaslonu je sorazmerna E^2

$$\bar{j} \propto \overline{E^2} = \overline{(E_1+E_2)^2} = (E_1+E_2)(E_1+E_2)^* = (E_1+E_2)(E_1^*+E_2^*) =$$
$$= E_1E_1^* + E_2E_1^* + E_1E_2^* + E_2E_2^* = 2E_0^2 + E_0^2 e^{ik(r_2-r_1)} + E_0^2 e^{ik(r_1-r_2)} =$$
$$= 2E_0^2\Big(1 + \underbrace{e^{ik(r_2-r_1)} + e^{-ik(r_2-r_1)}}_{2\cos[k(r_2-r_1)]}\Big) =$$
$$= 2E_0^2\left(1+\cos k(r_2-r_1)\right) =$$
$$= 2E_0^2 \cdot 2\cos^2\left[\frac{k(r_2-r_1)}{2}\right]$$
$$\bar{j} \propto \cos^2\left[\frac{k(r_2-r_1)}{2}\right] = \cos^2\left[\frac{k\,d\sin\vartheta}{2}\right] =$$
$$= \cos^2\left[\frac{2\pi d\sin\vartheta}{\lambda\, 2}\right] = \cos^2\left[\frac{\pi d\sin\vartheta}{\lambda}\right] =$$
$$= \cos^2\left[\frac{\pi d x}{\lambda D}\right]$$
$$\boxed{\bar{j} \propto \cos^2\left[\frac{\pi \cdot d \cdot x}{\lambda \cdot D}\right]}$$

točkasti izvor

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\cos 2x = \cos^2 x - (1-\cos^2 x)$$
$$\cos 2x = 2\cos^2 x - 1$$
$$1 + \cos 2x = 2\cos^2 x$$
$$2x = k(r_2 - r_1)$$
$$x = \frac{k}{2}(r_2 - r_1)$$
$$k = \frac{2\pi}{\lambda}$$



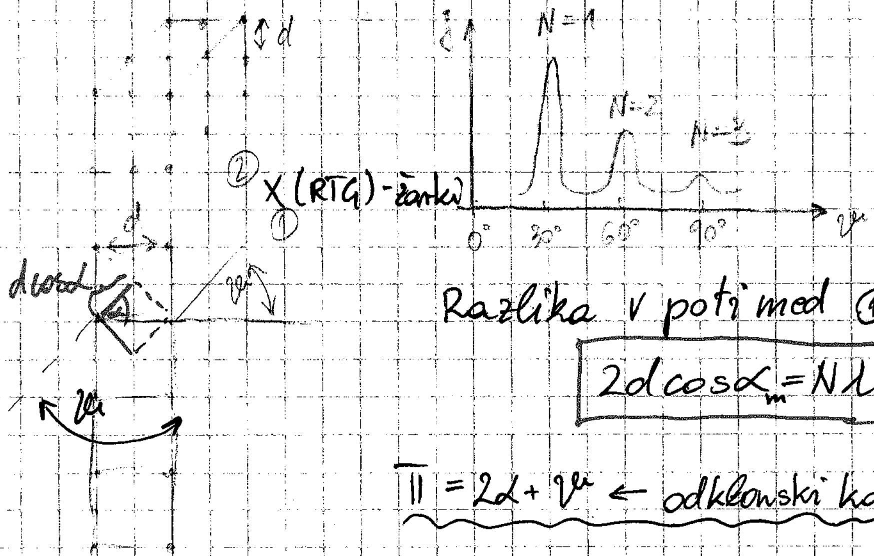
$$\mathrm{tg}\,\vartheta = \frac{x}{D}$$

$\vartheta \ll 1 \Rightarrow \cos\vartheta = 1$ ker je D velik

$$\boxed{\sin\vartheta = \mathrm{tg}\,\vartheta = \frac{x}{D}}$$

Bragg-ov uklon

NaCl - ionska kristalna mreža ; zanima nas razdalja med ~~molekulami~~ atomi.



Razlika v poti med ① in ②

$$\boxed{2d\cos\alpha_m = N\lambda}$$; ojačanje, N=1, 2, 3, ...

$\pi = 2\alpha + \vartheta \leftarrow$ odklonski kot

$\frac{\pi}{2} = \alpha + \frac{\vartheta}{2} \Rightarrow \alpha = \frac{\pi}{2} - \frac{\vartheta}{2}$

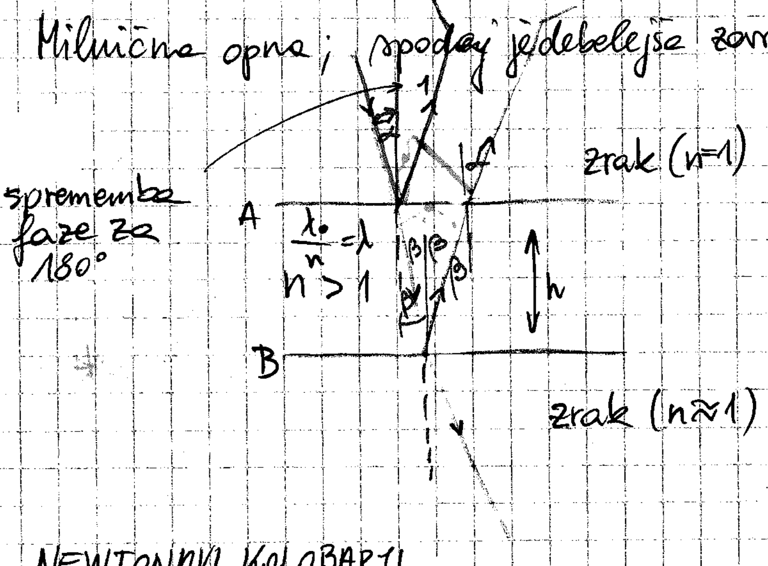
$\cos\alpha = \cos\left(\frac{\pi}{2} - \frac{\vartheta}{2}\right) = \sin\left(\frac{\vartheta}{2}\right)$

$2d\sin\left(\frac{\vartheta_m}{2}\right) = N\lambda$

$$\boxed{d = \frac{N\lambda}{2\sin\left(\frac{\vartheta}{2}\right)}}$$

Val. dolžina mora biti enaka razdalji med atomi, zato uporabimo rentgensko svetlobo, z vidno svetlobo ne moremo v tem primeru narediti nič.
Višje frekvence → večja energija

Milnična opna ; spodaj je debelejše zaradi gravitacije.



Pri odboju v optično ~~gostejem~~ gostejši snovi se faza obrne, pridobimo še en dodatno fazno razliko → $\frac{\lambda}{2}$

NEWTONOVI KOLOBARJI

milnica med tankimi lističi, razlike v kotu → svetle, temne proge.
metoda za merjenje debeline lističev

$\ast$ $pot_2 - pot_1 = 2 \cdot \cancel{h} \cos\beta$

dejanska razlika

Razlika po opt. poti je večja, ker se faza obrne in moramo prišteti še $\frac{\lambda}{2}$:

$$\boxed{2h\cos\beta + \frac{\lambda}{2} = N\lambda}$$ ← pogoj za ojačanje

$$2h\cos\beta = \frac{\lambda}{2}(2N-1)$$

$$2h\cos\beta = \frac{\lambda_0}{2n}(2N-1)$$

$$\frac{\sin\alpha}{\sin\beta} = n \quad / (\ )^2$$

lomni zakon na meji ~~zrak~~ in tanka opna

$$\frac{\sin^2\alpha_n}{\sin^2\beta} = n^2$$

$$\frac{\sin^2\alpha}{1-\cos^2\beta} = n^2$$

$$\sin^2\alpha_n = n^2 - n^2\cos^2\beta \Rightarrow n^2\cos^2\beta = n^2 - \sin^2\alpha_n \Rightarrow \cos^2\beta = 1$$

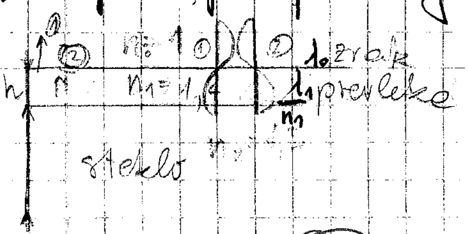
$$\boxed{\cos\beta = \sqrt{1 - \frac{\sin^2\alpha_n}{n^2}}}$$

$$\boxed{2h\sqrt{1 - \frac{\sin^2\alpha_n}{n^2}} = \frac{\lambda_0}{2n}(2N-1)}$$

INTERFERENČNI FILTER

deluje na principu absorbcije;

$\boxed{\alpha = 0}$



Da prepusti čim več, se mora najmanjkrat odbiti. → t.j., če bosta žarka ① in ② interferirala destruktivno.

$$J_{prep.} = J_{vpad.} - J_{odb}$$

$2h_N = N\frac{\lambda}{2}$, $N = 1, 3, 5$

$2h_N = N\frac{\lambda_0}{n_1 2}$; $N = 1, 3, 5$

$2h_N = N\frac{\lambda_0}{2 \cdot n_1}$; $N = 1, 3, 5 \ldots$

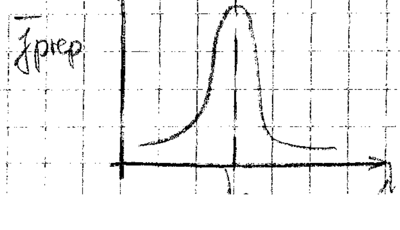
Želimo tanko plast, da imamo nizko absorpcijo. Izberemo $N = 1$ →

$N = 1$, $\boxed{h_1 = \frac{\lambda_0}{4n_1}}$

Uporablja se pri optiki

Če gledamo filter pod kotom, se spreminja svetloba.

Filter je v odvisnosti od val. dolžine in kota



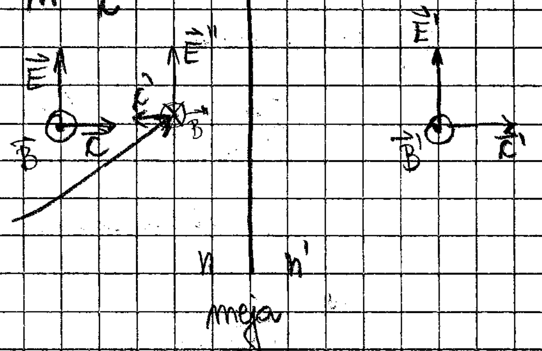
SPREMEMBA FAZE EMV PRI ODBOJU

21.4.2011

$\vec{E} = \vec{B} \times \vec{c}$ $\qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\varepsilon\varepsilon_0 \mu\mu_0}}$

$E = Bc$

$n = \frac{c_0}{c}$



odbiti del

$\boxed{E + E'' = E'}$

$H - H'' = H'$

$B = \mu\mu_0 H \implies H = \frac{B}{\mu\mu_0}$

$\mu = \mu'$

$H' = \frac{B'}{\mu\mu_0}$

$H'' = \frac{B''}{\mu\mu_0}$

$\boxed{B - B'' = B'}$

$\frac{E}{c} - \frac{E''}{c} = \frac{E'}{c'} \quad / \cdot c$

$\boxed{E - E'' = E' \frac{c}{c'}} = E' \frac{n'}{n}$

$+$

$2E = E' + E' \frac{n'}{n} = \underline{\underline{E'\left(1 + \frac{n'}{n}\right)}}$

$E' = \frac{2E}{1 + \frac{n'}{n}}$

$E - E'' = \frac{2E}{1+\frac{n'}{n}} \frac{n'}{n} = \frac{2E}{\frac{n}{n'}+1} = \frac{2n'}{n+n'} E$

$\boxed{E'' = E\left(1 - \frac{2n'}{n+n'}\right) = \left(\frac{n-n'}{n+n'}\right) E}$

$\underline{n > n'}$ (odboj na optično redkejši snovi)

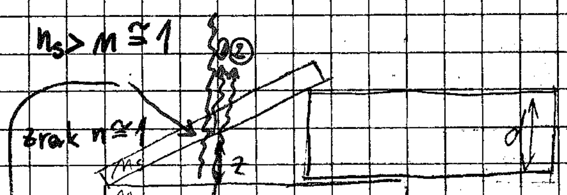
'$\vec{E}$ se odbije z isto fazo'

$\underline{n < n'}$ (odboj na opt. gostejši snovi)

'$\vec{E}$ se odbije z nasprotno fazo'

INTEFERENČNI FILTER (pt. I)

$n_s > n \cong 1$



odboj na opt. redkejši snovi

odboj na opt. gostejši snovi

$\delta = 2z + \frac{\lambda_0}{2}$ $\qquad N = 1, 2, 3, \ldots$

ojačitev:

$2z + \frac{\lambda_0}{2} = N\lambda_0$

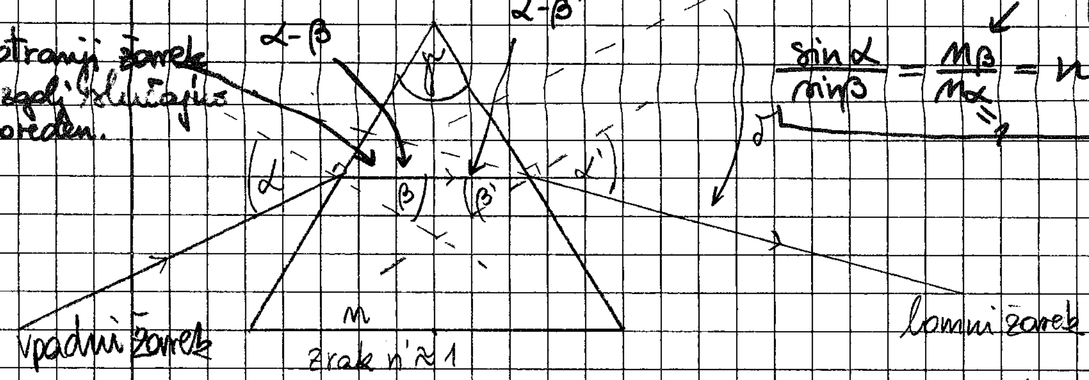
$2z = N\lambda_0 - \frac{\lambda_0}{2}$

$z = \frac{1}{4}(2N-1)\lambda_0$

$\underline{\underline{\Delta z \approx \frac{\lambda_0}{2} \approx 200\,\text{nm}}}$

LOM SVETLOBE NA PRIZMI

notranji žarek je zgolj [illegible] vzporeden.



lomni količnik prizme

$$\frac{\sin\alpha}{\sin\beta}=\frac{n_\beta}{n_\alpha}=n \qquad n_\alpha = 1$$

$$\underbrace{\frac{\sin\beta'}{\sin\alpha'}=\frac{1}{n}}$$

$$\underbrace{\delta=(\alpha-\beta)+(\alpha'-\beta')}=(n-1)\beta+(n-1)\beta'=$$

$$=(n-1)\,(\underbrace{\beta+\beta'}_{\gamma})$$

$$\boxed{\delta=(n-1)\cdot\gamma}$$

majhni koti:

$$\frac{\alpha}{\beta}=n$$

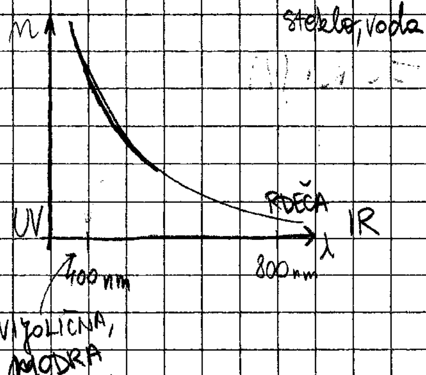
$$\frac{\beta'}{\alpha'}=\frac{1}{n}\Rightarrow\alpha'=n\beta'$$

disperzija:

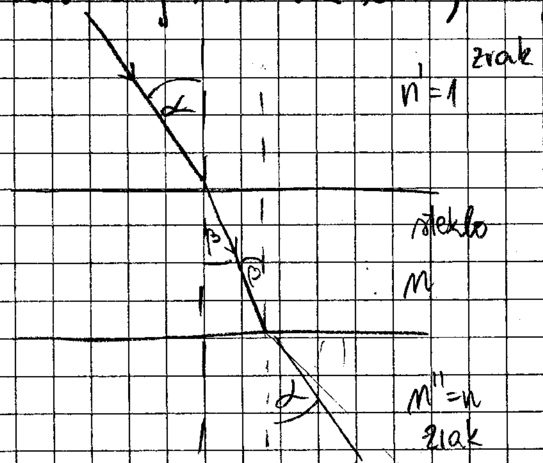
$$c=c(\lambda)$$

$$n=n(\lambda)$$

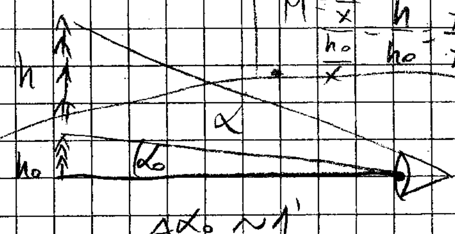
$$\delta(\lambda)=(n(\lambda)-1)\gamma$$



Večji kot je kot bolj smo na levi, večji lomni količnik

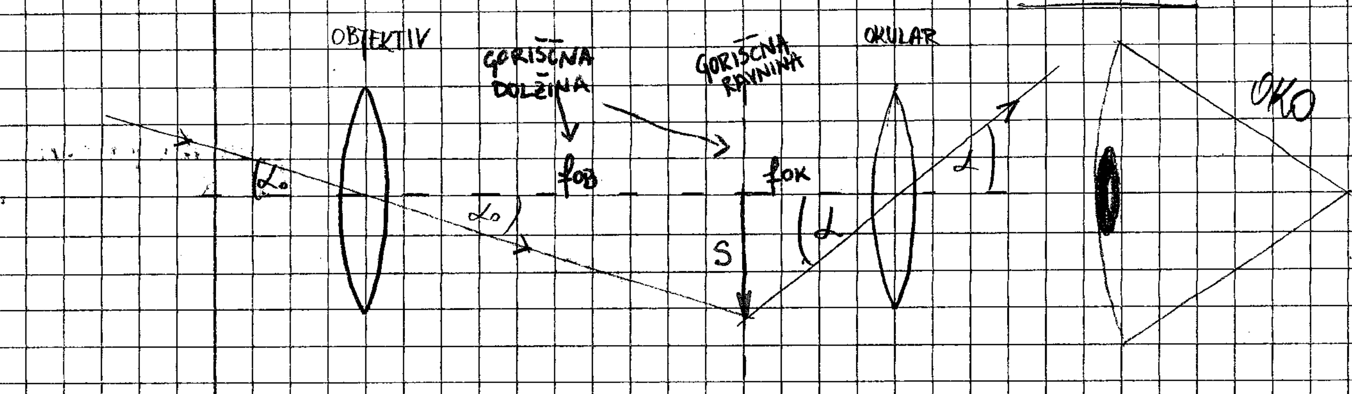


$$M=\frac{\frac{h}{x}}{\frac{h_0}{x}}=\frac{h}{h_0}=\frac{tg\,\alpha}{tg\,\alpha_0}$$



$$\Delta\alpha_0\sim 1'$$

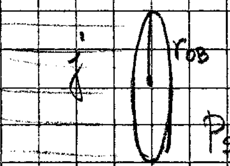
DALJNOGLED



POVEČAVA:

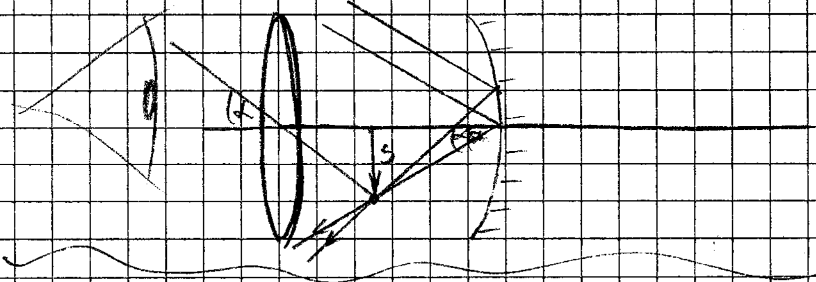
$$M = \frac{tg\alpha}{tg\alpha_0} = \frac{\frac{s}{f_{OK}}}{\frac{s}{f_{OB}}} = \frac{f_{OB}}{f_{OK}}$$

Na goriščni ravnini je slika najbolj ostra.



$$P_{SL} = P_{OB} = \frac{P}{4\pi r^2} \pi r_{OB}^2 \quad \frac{P}{4\pi r^2} = j$$

} osvetljenost slike $\quad j' \propto \frac{r_{OB}^2}{r^2}$

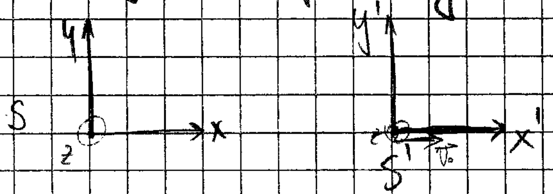


9.5.2011

POSEBNA TEORIJA

1. Galilejeva transformacija:

S' se giblje s hitrostjo v_0 v smeri x osi



inercialni - nepospešeni - opazovalni sistemi.

$X = x' + v_0 t \qquad x' = x - v_0 t \; / \frac{d}{dt}$

$y = y' \qquad y' = y$

$z = z' \qquad z' = z$

$\boxed{t = t'}$ absolutni čas

privzeli smo, da čas v obeh sistemih teče enako.

2. Maxwell $\Rightarrow$ EMV:

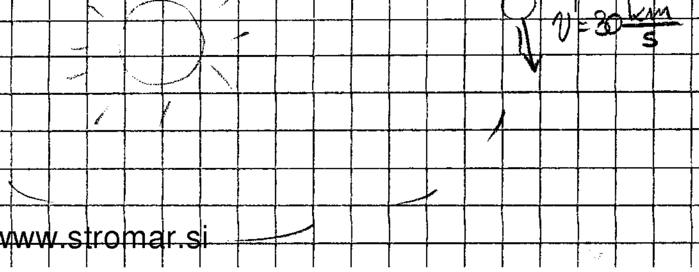
$$c_0 = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = \text{konst.}$$

$$\frac{dx'}{dt} = \frac{dx}{dt} - v_0 \frac{dt}{dt}$$

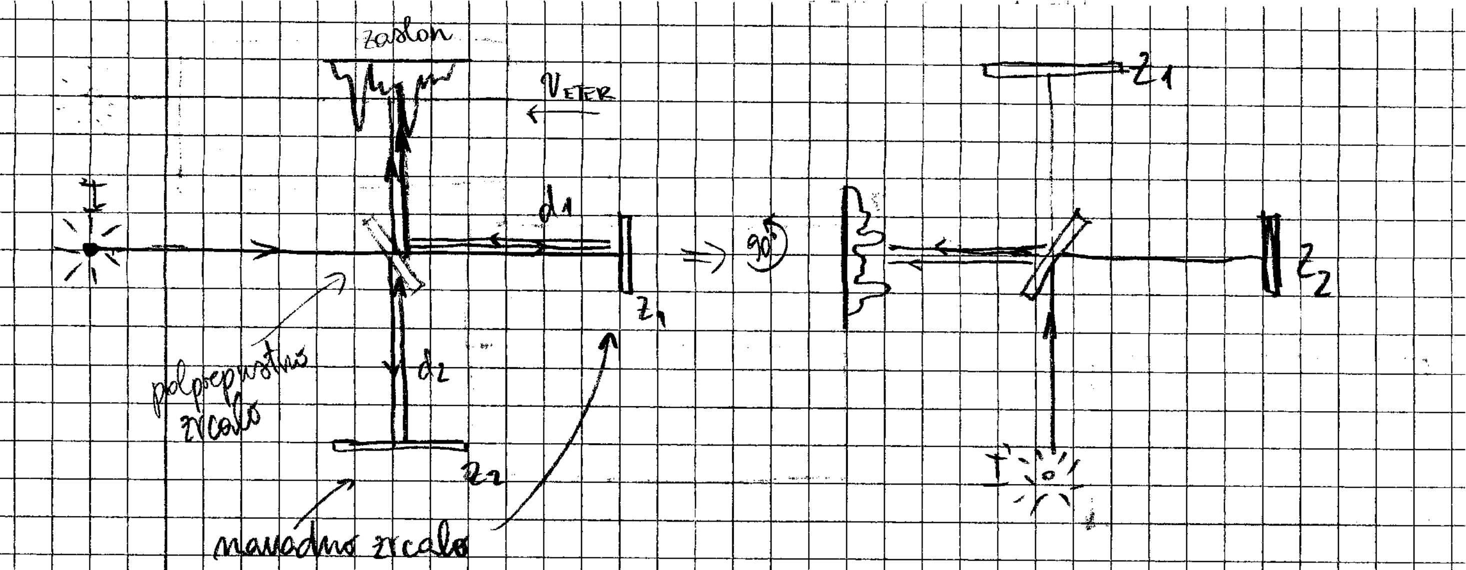
$$\boxed{v_x' = v_x - v_0} \quad ?$$

"ETER" - v njem veljata obe enačbi..

EKSPERIMENT: (Michelson in Morley)



Interferometer: vir svetlobe, ki oddaja EMV, polprepustno zrcalo (del fotonov gre skozi, del se odbije), dve navadni zrcali), na zaslonu opazujemo interferenčno sliko, različne poti, interferenca.



- Einstein (1905)

dve glavni načeli:

① c_0 = konst.

② načelo relativnosti: "vsi nepospešeni inercialni sistemi so enakovredni"
(če se postavimo v en sistem in zapišemo zakone zanj, morajo veljati tudi v drugem sistemu)

Lorentzove transformacije:

(enaka sistema kot pri Galilejevih sistemih)

$* \quad \underset{S}{x} = \gamma\,(x' + v_0 t')$ $\qquad$ $\underset{S'}{x'} = \gamma\,(x - v_0 t)$

predpostavimo, da je prostor homogen in izotropen (zakoni veljajo v vseh smereh), tudi čas je homogen.

t': čas v sistemu S'

(sedaj) funkcija hitrosti: $\gamma = \gamma(v)$?

$\frac{dx'}{dt'}$

čas ni več absoluten, je relativen.

$* \quad \frac{x}{\gamma} = x' + v_0 t' \rightarrow t' = \frac{\frac{x}{\gamma} - x'}{v_0} = \frac{1}{v_0}\left(\frac{x}{\gamma} - \gamma x + v_0 \gamma t\right) =$

$= \cancel{\frac{1}{v_0}}\,\gamma\left(t - \frac{x}{v_0}\left(1 - \frac{1}{\gamma^2}\right)\right)$

$dt' = \gamma\left(dt - \frac{dx}{v_0}\left(1 - \frac{1}{\gamma^2}\right)\right)$

$$v_x' = \frac{dx'}{dt'} = \frac{\gamma\,(dx - v_0\,dt)}{\gamma\left(dt - \frac{dx}{v_0}\left(1 - \frac{1}{\gamma^2}\right)\right)} =$$

$$= \frac{\frac{dx}{dt} - v_0}{1 - \frac{dx}{dt}\,\frac{1}{v_0}\left(1 - \frac{1}{\gamma^2}\right)} =$$

$$= \frac{v_x - v_0}{1 - \frac{v_x}{v_0}\left(1 - \frac{1}{\gamma^2}\right)}$$

$\boxed{\begin{aligned} v_x &= c_0 \\ v_x' &= c_0 \end{aligned}}$

$$c_0 = \frac{c_0 - v_0}{1 - \frac{c_0}{v_0}\left(1 - \frac{1}{\gamma^2}\right)}$$

$$\cancel{1} - \frac{c_0}{v_0}\left(1-\frac{1}{\gamma^2}\right) = \cancel{1} - \frac{v_0}{c_0}$$

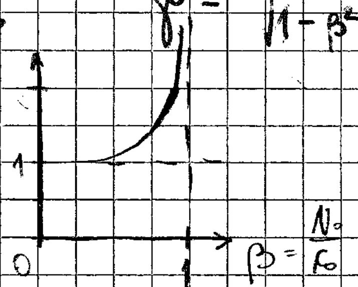
$$1-\frac{1}{\gamma^2} = \frac{v_0^2}{c_0^2}$$

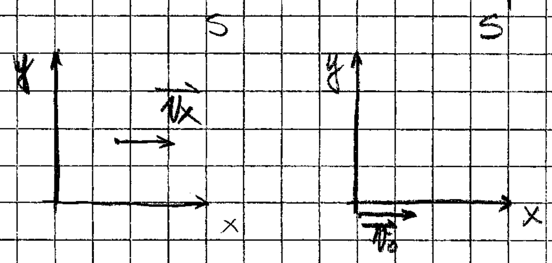
$$\frac{1}{\gamma^2} = 1-\frac{v_0^2}{c_0^2} \longrightarrow \boxed{\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c_0^2}}}} \qquad \boxed{v_x = \frac{v_x'+v_0}{1+\frac{v_x' v_0}{c_0^2}}}$$

$$\boxed{t' = \gamma\left(t - \frac{v_0}{c_0^2}x\right)}$$

$$\boxed{v_x' = \frac{v_x - v_0}{1-\frac{v_x v_0}{c_0^2}}} \qquad \boxed{t = \gamma\left(t' + \frac{v_0}{c_0^2}x'\right)}$$

$$\beta = \frac{v_0}{c_0} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$





$$\boxed{v_y} = \frac{dy}{dt} = \frac{dy'}{\gamma\left(dt' - \frac{v_0}{c_0^2}dx'\right)} = \frac{\overbrace{\frac{dy'}{dt'}}^{v_y'}}{\gamma\left(1-\frac{v_0 \overbrace{\frac{dx'}{dt'}}^{v_x'}}{c_0^2}\right)} = \boxed{\frac{v_y'}{\gamma\left(1-\frac{v_0 v_x'}{c_0}\right)}}$$

$$\boxed{v_z = \frac{dz}{dt} = \frac{v_z'}{\gamma\left(1-\frac{v_0 v_x'}{c_0^2}\right)}}$$

$\underbrace{\text{krajevna in časovna "koordinata" nista več neodvisni...}}_{\text{dogodki}}$

Sočasnost je relativna:

	S		S'
1. dogodek:	t_1	x_1	$t_1' = \gamma\left(t_1 - \frac{v_0}{c_0^2}x_1\right)$
2. dogodek:	$t_1 = t_2$	$x_2 \neq x_1$	$t_2' = \gamma\left(t_1 - \frac{v_0}{c_0^2}x_2\right)$

$\Delta t = t_2 - t_1 = 0$ ← dogodka sta sočasna

dogodka nista sočasna → $\Delta t' = \gamma\frac{v}{c_0^2}(x_2 - x_1) \neq 0$

PODALJŠANJE (DILATACIJA) ČASA: (v sistemu S ura miruje)

	S		S'
1. dogodek:	t_1	x_1	$t_1' = \gamma\left(t_1 - \frac{v_0}{c_0^2}x_1\right)$
2. dogodek:	t_2	$x_2 = x_1$	$t_2' = \gamma\left(t_2 - \frac{v_0}{c_0^2}x_2\right)$

$\Delta t = t_2 - t_1 = \tau$ ← lastni ča...

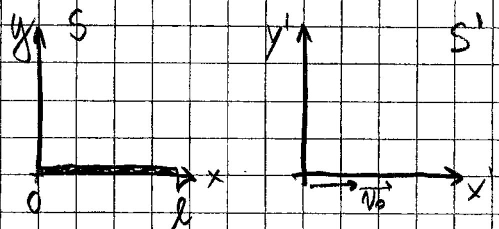
$$\Delta t' = t_2' - t_1' \qquad \Delta t' = t_2' - t_1' = \gamma t_2 - \gamma t_1 = \gamma \underbrace{(t_2 - t_1)}_{\Delta t}$$

najkrajši čas je v lastnem sistemu

$$\boxed{\Delta t' = \gamma \cdot \Delta t = \gamma \tau}$$

SKRČENJE DOLŽIN

	S	S'
1. Dogodek:	$x_1 = 0$	$t_1' = t$
2. Dogodek:	$x_2 = l$	$t_2' = t$



$t_1 = \dots \quad x_1' = \dots$

$t_2 = \dots \quad x_2' = \dots$

z dvema urama začnemo meriti, ko sta krajišči palice v izhodišču.

$$\Delta x = x_2 - x_1 = l$$

$$\Delta x = \gamma(\Delta x' + v_0 \underbrace{\Delta t'}_{t_2' - t_1' = 0}) = \gamma \Delta x'$$

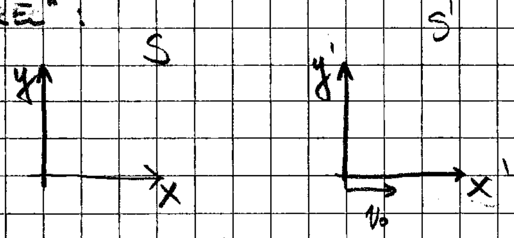
$$l = \gamma \cdot l' \qquad \boxed{l' = \frac{l}{\gamma}}$$

V sistemu kjer se palica giblje je dolžina krajša kot v drugih sistemih.

ŠTIRIRAZSEŽNI PROSTOR ≡ PROSTOR-ČAS

DOGODEK POPIŠE "SVETOVNI ČETVEREC":

$$^4x = (\underbrace{c_0 t}_{\text{časovni del}}, \underbrace{x, y, z}_{\text{krajevni del}}) = (c_0 t, \vec{r})$$



$$\beta = \frac{v_0}{c_0}$$

$$^4x \equiv x^\mu$$

$$c_0 t = \gamma (c_0 t' - \beta x')$$

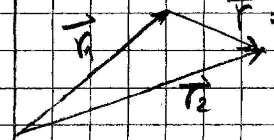
$$x = \gamma (x' - \beta c_0 t')$$

$$y = y'$$

$$z = z'$$

$$\boxed{\vec{r} \cdot \vec{r} = x^2 + y^2 + z^2}$$

– KLASIČNI 3-D prostor:



$$|\vec{r}| = \sqrt{\vec{r}\,\vec{r}} = \text{invarianta}$$

dolžina $|\vec{r}|$ je neodvisna; enaka je v vseh sistemih

- 4 - RAZSEŽNI PROSTOR:

$|\vec{r}|$ = skalarni produkt " svetovnega četverca "

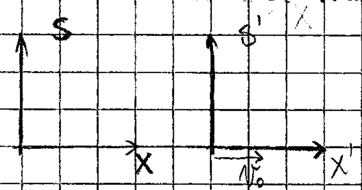
$$\underbrace{{}^4x \cdot {}^4x = \ominus c_0^2 t^2 + x^2 + y^2 + z^2}_{} \quad (+c_0^2t^2 - x^2 - y^2 - z^2)$$

dve signaturi ← odvisno od dogovora.

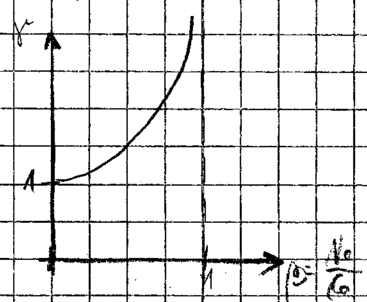
12.5.2011

$${}^4u = (\underbrace{u_0}_{\text{časovni del}}, \underbrace{u_1, u_2, u_3}_{\text{krajevni del}})$$

LORENTZOVE TRANSFORMACIJE



S, S' ... nepospešena (inercialna) sistema



$u_0' = \gamma(u_0 - \beta u_1)$ $\qquad \beta = \frac{v_0}{c_0} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$

$u_1' = \gamma(u_1 - \beta u_0)$ $\qquad {}^4u \cdot {}^4u$ = invarianta

$u_2' = u_2$

$u_3' = u_3$

- ČETVEREC HITROSTI:

$${}^4v = \frac{d({}^4x)}{d\tau} = \frac{d({}^4x)}{dt} \cdot \gamma$$

τ ... lastni čas (sistem v katerem ura miruje)

- gibajoče ure zaostajajo.

$${}^4v = \gamma\left(c_0\frac{dt}{dt}, \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right) =$$

$$\boxed{t = \gamma \cdot \tau} \Rightarrow dt = \gamma \cdot d\tau$$

$${}^4v = \gamma\,(c_0, v_x, v_y, v_z)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}}$$

$$\boxed{{}^4p = \left(\frac{W}{c_0}, m_0\, {}^4\vec{v}\right)}$$

- ČETVEREC GIBALNE KOLIČINE:

$${}^4p = m_0\,{}^4v = (m_0\gamma c_0; \underbrace{m_0\gamma\vec{v}}_{\vec{p}}) =$$ ← krajevni del

m_0 ... mirovna masa

$\vec{p} = m_0\gamma\vec{v}$ ... relativistična gibalna količina

$m_r = m_0\gamma$ ... relativistična masa

$E \equiv W$

$$= \left(\frac{E}{c_0}, \vec{p}\right)$$

$\frac{E}{c_0}$ ← energija $= m_0\gamma c_0 \Longrightarrow E = m_0\gamma c_0^2$ ... polna energija

$\boxed{E_0 = m_0 c_0^2}$ ... mirovna energija

$$W_k = T$$

RELATIVISTIČNA KIN. ENERGIJA

$$W_k = E - E_0 = m_0\gamma c_0^2 - m_0 c_0^2 = \underbrace{m_0 c_0^2(\gamma - 1)}$$

$$W_k = m_0 c_0^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}} - 1\right) = m_0 c_0^2\left(1 + \frac{1}{2}\frac{v^2}{c_0^2} + \sigma\left(\frac{v^4}{c_0^4}\right) - 1\right) =$$

nek člen

$$= \frac{1}{2} m_0 v^2 + \sigma\left(\frac{v^4}{c_0^4}\right)$$

$$\boxed{v \ll c_0 : W_k \cong \frac{1}{2} m v^2}$$

$$^4P = \left(\frac{E}{c_0}, \vec{P}\right) \qquad \vec{P} = m_0\gamma\vec{v}$$

V lastnem sistemu delec miruje

LASTNI SISTEM $(v=0)$

$$^4P \cdot {}^4P = -\frac{E^2}{c_0^2} + P^2 = \frac{-E_0^2}{c_0^2} \Longrightarrow \boxed{E^2 = E_0^2 + c_0^2 \cdot P^2}$$

$$\vec{P} = m_0\gamma\vec{v}$$
$$E = m_0\gamma c_0^2$$
$$\frac{E}{P} = \frac{\cancel{m_0\gamma} c_0^2}{\cancel{m_0\gamma} v} = \frac{c_0^2}{v}$$

RELATIVNA KIN. ENERGIJA NAM DA V LIMITI KLASIČNO KIN. ENERGIJO

DELEC IMA HITROST $v = c_0$:

$$\frac{E}{P} = \frac{c_0^2}{c_0} = c_0 \Longrightarrow E = c_0 \cdot P \longrightarrow E_0 = 0 \Rightarrow m_0 = 0$$

Delec mora imeti mirovno maso 0, v kolikor se "želi" gibati s svetlobno hitrostjo.

NEWTONOV ZAKON V 4 RAZSEŽNI OBLIKI

KLASIČNO: $\vec{F} = \frac{d\vec{G}}{dt}$; $\vec{G} = m\vec{v}$

RELATIVISTIČNO: $^4P = \left(\frac{E}{c_0}, \vec{P}\right)$ $\quad \vec{P} = m_0\gamma\vec{v}$ $\quad$ POLNA ENERGIJA

$$^4F = \frac{d(^4P)}{d\tau} = \gamma\frac{d(^4P)}{dt} = \gamma\left(\frac{1}{c_0}\frac{dE}{dt}, \frac{d\vec{P}}{dt}\right)$$

ČETVEREC SILE NA NAELEKTREN DELEC:

JAKOST EL. POLJA

$$^4F = \left(\frac{e\gamma\,\vec{E}\cdot\vec{v}}{c_0}, e\gamma\left(\vec{E} + \vec{v}\times\vec{B}\right)\right)$$

- KRAJEVNI DEL ČETVERCA SILE:

$$\boxed{\frac{d\vec{P}}{dt} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)} \qquad \boxed{\vec{P} = m_0\gamma\vec{v}}$$

$$U = -\int \vec{E}\,d\vec{s} = V_2 - V_1$$
$$V \equiv \varphi$$

- ČASOVNI DEL ČETVERCA SILE:

$$\frac{\cancel{\gamma}}{\cancel{c_0}}\frac{dE}{dt} = \frac{e\cancel{\gamma}\,\vec{E}\cdot\vec{v}}{\cancel{c_0}} \Longrightarrow \Delta E = e\int \vec{E}\cdot\vec{v}\,dt \Longrightarrow$$

($\vec{v}\,dt = d\vec{s}$)

$$\Longrightarrow \Delta E = -e\Delta\varphi \qquad \Delta\left(E + e\varphi\right) = 0$$

$$\boxed{E + e\varphi = \text{konst.}}$$

OHRANITVENE ENAČBE:

$$ {}^4F = \frac{d({}^4p)}{d\tau} \Rightarrow \Delta {}^4p = \int {}^4F\, d\tau \quad \Leftarrow \text{sunek sile} $$

$$ \int {}^4F\, d\tau = 0 \Rightarrow \Delta {}^4p \Rightarrow \begin{matrix} \Delta E = 0 \\ \Delta \vec{p} = 0 \end{matrix} \quad \begin{matrix} \Leftarrow \text{polna energija je konstantna} \\ \Leftarrow \text{gib. količina} \quad -\parallel- \end{matrix} $$

MAXWELLOVE ENAČBE:

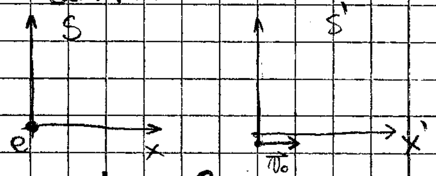
- VEKTOR ČETVEREC POTENCIALA

$$ \vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t} \qquad {}^4A = \left(\frac{\varphi}{c_0}, \vec{A}\right) \qquad \varphi \equiv V $$

$$ \vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \qquad {}^4j = (\rho \cdot c_0, \vec{j}) $$

gostota naboja — gostota el. toka

PRIMER: TOČKAST NABOJ MIRUJE V IZHODIŠČU SISTEMA S



$$ \underline{\underline{S}}: \ \varphi = \frac{e}{4\pi\varepsilon_0 r} \qquad \vec{A} = 0 $$

$$ \underline{\underline{S'}}: \ \frac{\varphi'}{c_0} = \gamma\left(\frac{\varphi}{c_0} - \beta \underset{0}{A_x}\right) = \gamma \frac{\varphi}{c_0} = \gamma \frac{e}{4\pi\varepsilon_0 r \cdot c_0}; \quad \varphi' = \gamma \frac{e}{4\pi\varepsilon_0 r} $$

$$ \underline{A_x'} = \gamma\left(\underset{0}{A_x} - \beta \frac{\varphi}{c_0}\right) = \gamma\beta\frac{\varphi}{c_0} = \underline{\gamma \frac{v_0}{c_0^2} \frac{e}{4\pi\varepsilon_0 r} \neq 0} $$

V sistemu kjer se naboj premika imamo el. tok.

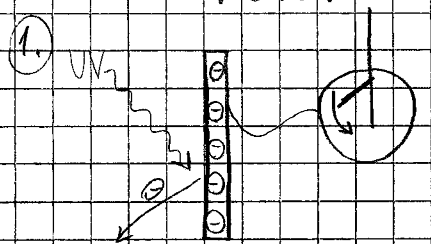
MAXWELLOVE ENAČBE S ČETVERCI:

$$ \Box\, {}^4A = \mu_0 ({}^4j) $$

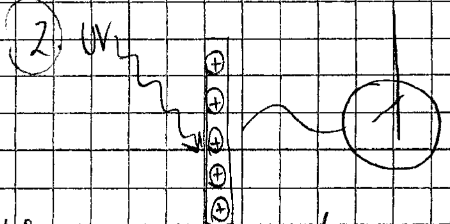
$$ \Box = \frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \qquad \text{D'ALAMBERTOV OPERATOR} $$

16.5.2011

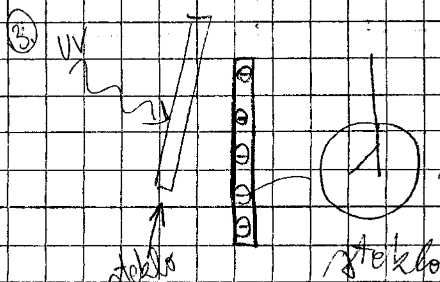
FOTOEFEKT



← pri negativno naelektreni ploščici se plošča razelektri



elektron se izbije, vendar ga k pozitivnemu naboju ponovno pritegne



če med UV svetilko in instrument postavimo steklo, se plošča ne razelektri.

steklo prepusti vidno svetlobo, UV načeloma ne prepušča

Gostota energijskega toka:

$$\overline{j} = c_0 \overline{w} \propto E_0^2$$

$$E_{abs} \propto \overline{j} \cdot S \cdot t \cdot a \propto t$$

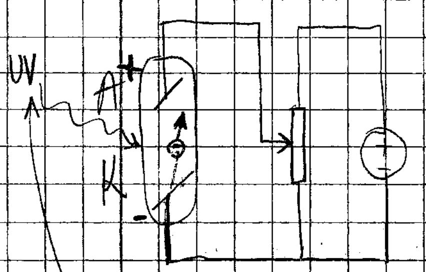
S – površina plosče

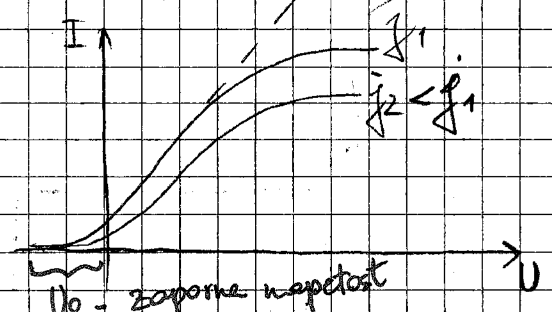
a – faktor odbitega en. toka – albedo

Če hočemo e^- izbiti, moramo dovesti energijo, da presežemo vezalno energijo v mreži, da se odtrgajo.

fotocelica

Zaprta steklena posoda, vakuum, katoda, anoda $I = \frac{U}{R}$, $I \propto U$





U_0 – zaporna napetost

ν – frekvenca svetlobe

j – gostota energijskega toka

Odvisnost UI ni v skladu z ohmovim zakonom

Da tok spravimo na 0, moramo dovesti negativno napetost, ki je odvisna od materiala katode.

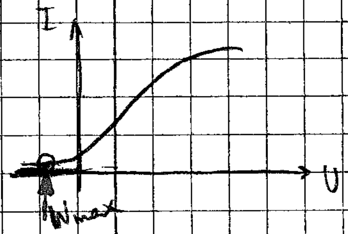
Z katode se izbijajo elektroni. Če ne obsevamo, ni toka v sistemu.

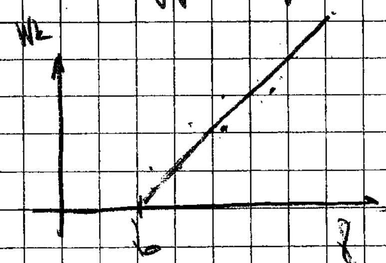
Število e^- je odvisno od gostote energijskega toka, št. elektronov je omejeno.

Elektroni imajo kinetično energijo, zato so zmožni samo priti do ...

$$W_{k,max} = \frac{m_e \cdot v_{max}^2}{2} = e_0 \cdot U_0$$

Vsa energija se porabi v polju.





$$W_{k,max} = h(\nu - \nu_0) = h\nu - h\nu_0$$

$h \equiv$ Planckova konstanta

$h = 6{,}62 \cdot 10^{-34}$ [Js] ; $h\nu$ [J]

$$\hbar = \frac{h}{2\pi}$$

$= h\nu - W_{izst}$ ← izstopno delo

$h\nu$ – absorbirana energija

$$\boxed{W_{k,max} = h\nu - W_{izst}}$$

– vezavne energije elektrona, energija, ki je potrebna da iztrgamo elektron iz kovinske mreže.

$$\boxed{h\nu \geq W_{izst}}$$

Samo v idealnem primeru dosežemo maks. energijo.

- klasično: $j \propto E_0^2$; $e_{abs} \propto c_0 E_0^2 t$

Energija se absorbira le v paketih - kvantih, absorbirana energija ni zvezna.

fotoni nimajo mase, imajo gib. količino.

$$E = W_0 + T \quad ; \quad W_0 = m_0 c_0^2$$

polna energija, lastna energija, kinetična energija

$$\boxed{W^2 = W_0^2 + c_0^2 P^2}$$ relativistična gibalna količina

fotoni:

$W_0 = 0$; $\boxed{W^2 = c_0^2 p^2} \rightarrow W = c_0 P_\nu$

polna energija:

$W = h\nu$; $\boxed{P_\nu = \frac{W}{c_0} = \frac{h\nu}{c_0} = \frac{h}{\lambda}}$ → gibalna količina fotona

$c_0 = \nu\lambda$

$$N_\nu = \frac{P \cdot t}{h\nu} = \frac{\bar{j} S t}{h\nu} \qquad \nu_0 \lambda_0 = c_0 \Rightarrow \nu_0 = \frac{c_0}{\lambda_0}$$

kovina	λ_0 [nm]
platina	230
volfram	280
srebro	290
Al, Zn	340
kalij	560

primer: anihilacija - izničenje

pozitron ⊕
elektron ⊖

$W_0 = m_0 c_0^2 = 0{,}51\,MeV$

T (kin. energija)

⊕ → ⊖ $v_e = 0$

Če nimamo zunanjih sil, se ohranjata gib. količina in polna energija.
Pri anihilaciji nastaneta 2 fotona, masa izgine.

ν' ν

1. ohranitev polne energije: → skupna polna en. pred trkom je enaka sk. polni en. po trku

$$\boxed{W_0 + T_p + W_0 = h\nu + h\nu'}$$

2. ohranitev gib. količine: skupna gib. količina -//- ...

$$\boxed{P_p = \frac{h\nu}{c_0} - \frac{h\nu'}{c_0}} \quad ; \quad \underbrace{c_0 P}_{m_0 \gamma v} = h(\nu - \nu')$$

$$W^2 = W_0^2 + c_0^2 P_p^2$$

$$(W_0 + T_p)^2 = W_0^2 + c_0^2 P_p^2$$

$$\cancel{W_0^2} + 2W_0 T_p + T_p^2 = \cancel{W_0^2} + c_0^2 P_p^2 \,/\sqrt{\ }$$

$$c_0 P_p = \sqrt{2W_0 T_p + T_p^2}$$

de Broglie:

delci: $p = \frac{h}{\lambda}$

$\boxed{\lambda_B = \frac{h}{p} = \frac{h}{mv}}$ de Broglie

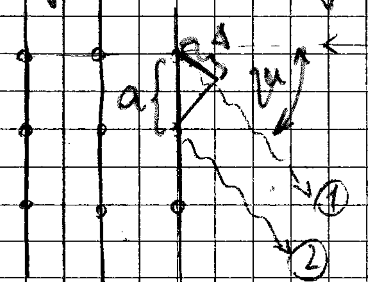
$m_e = 9{,}1\cdot 10^{-31}\,kg$

19. 5. 2011

$\frac{m_e v^2}{2} = e_0 U_0 \Longrightarrow v = \sqrt{\frac{2e_0U_0}{m_e}} \Longrightarrow \lambda_B = \frac{h}{m_e v} = \frac{h}{m_e\sqrt{\frac{2e_0U_0}{m_e}}} =$

$v = \frac{h}{\sqrt{2m_e e_0 U_0}}$

a) majhne U_0; majhne W_k, velik λ_B

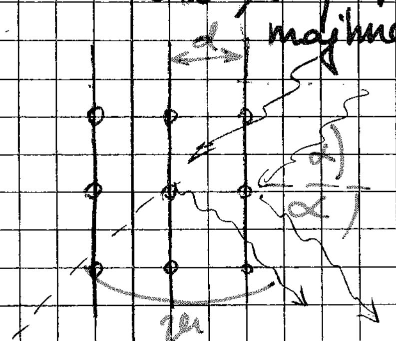


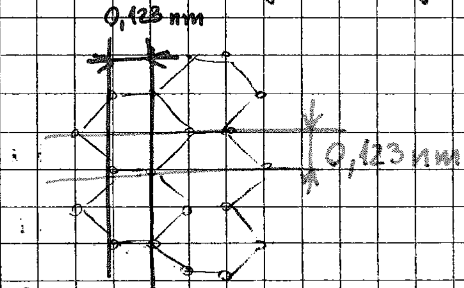
$\Delta = a\sin\vartheta = N\lambda_B$

$U_0 = 100\,V$; $\lambda_B = 0{,}123\,m$

Odbijejo se od prve plasti, ker imajo premajhno kinetično energije

b) velike U_0; večje W_k, majhne λ_B





Braggovo sipanje

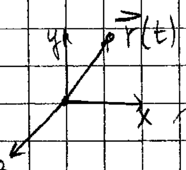
$U_0 = 1500\,V$; $\lambda_B \approx 0{,}03\,nm$

Newton-ova mehanika:

$\vec r = \vec r(t)$

$\vec v$

$\vec a$



$\sum \vec F_i = m\frac{d\vec v}{dt}$

$\lambda_B = \frac{h}{mv}$ Schrödingerjeva enačba

$x: \Delta p_x \Delta x \geq \hbar$

relativistična fizika:

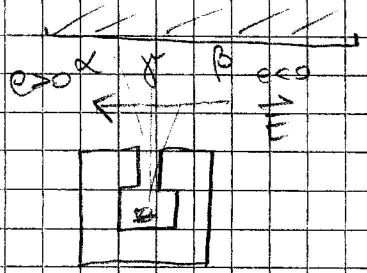
$\vec p = m_0\gamma\vec v \xrightarrow{\frac{v}{c_0}\to 0} m_0\vec v$

$T = m_0c_0^2(\gamma - 1) \xrightarrow{\frac{v}{c_0}\to 0} \frac{m_0v^2}{2}$

Kjer se delec giblje se voda kondenzira in vidimo sled (reaktivno detalo,...)

Visoko energijski fotoni so gama žarki

Beta žarki so elektroni, alfa žarki pa so helijeva jedra.



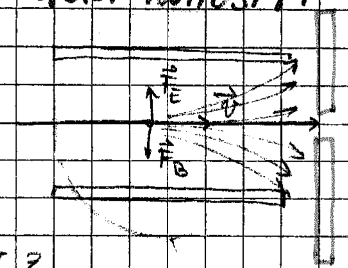
$\vec{F}_B = e\vec{v} \times \vec{B}$

$\vec{F}_E = e\vec{E}$

$\vec{F}_B = e\vec{v} \times \vec{B}$

$\vec{F}_E = e\vec{E}$

- selektor hitrosti:



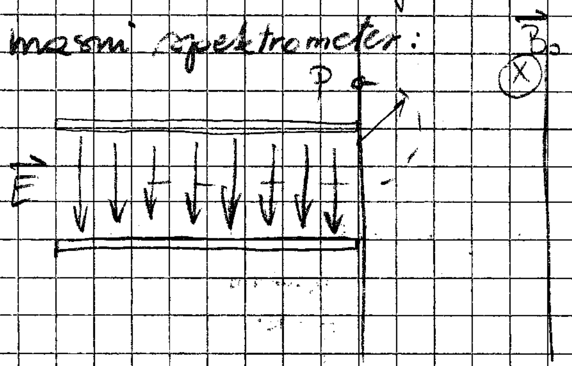
$\varphi = 90°$

$e \cdot vB = eE$

$\boxed{v = \frac{E}{B}}$

Izbira delcev z enako hitrostjo

- masni spektrometer:



II. zakon:

$ma_r = evB_0$

$m\frac{v^2}{r} = evB_0$

$\boxed{r = \frac{mv}{eB_0}}$ (2)

+ selektor hitrosti

$evB = eE$

$\boxed{v = \frac{E}{B}}$ (1)

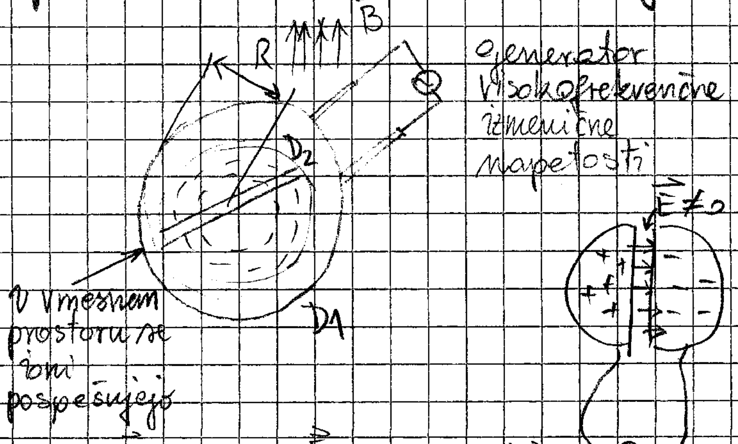
$\boxed{r = \frac{mE}{eB_0B} \rightarrow \frac{m}{e} = \frac{rB_0B}{E}}$

Thomsonov aparat za merjenje razmerja $\frac{e}{m}$ (specifični naboj)

$\frac{e}{m} \approx 0{,}77 \cdot 10^{11} \frac{As}{kg}$

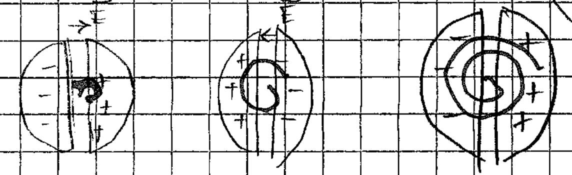
- Ciklotron:

2 votli elektrodi D_1, D_2 v evakuiranem prostoru. | Z mag. poljem ne moremo povečati kinetične energije delca, lahko samo spreminjamo pot. | Zato delamo v kombinaciji z el. poljem.



V elektrodah ni el. polja

$\boxed{r = \frac{mv}{eB}}$



Polariteto obrnemo ravno takrat ko delec pride v režo, frekvenca mora biti prava, drugače se delec zavre.

obhodni čas pri dani hitrosti v_0 $\quad t_0 = \frac{2\pi r}{v} \leftarrow \frac{mv}{eB} = \frac{2\pi}{\cancel{v}} \cdot \frac{m\cancel{v}}{eB} = \frac{2\pi m}{eB}$

Pridemo do zaključka, da frekvenca ni odvisna od hitrosti in jo lahko ohranjamo.

Za majhne hitrosti!!!

$$\boxed{\nu_c = \frac{1}{t_0} = \frac{eB}{2\pi m}}$$ ciklotronska frekvenca

Če hitrost postane prevelika ($\frac{c_0}{10}$), se frekvenca spremeni, ker Newtonova mehanika odpade, frekvenca postane odvisna od hitrosti, moramo jo stalno sinhronizirati.

Sinhrociklotron

$$\vec{p} = m_0 \gamma \vec{v}$$

$$\frac{d\vec{P}}{dt} = e\vec{v} \times \vec{B}$$

$$\frac{d(m_0 \gamma \vec{v})}{dt} = e\vec{v} \times \vec{B}$$

Ker je W_k konst. (mag. polje) $\Longrightarrow$

$|\vec{v}| = v = \text{konst.}$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}} = \text{konst.}$$

V mag. polju γ ni odvisna od časa!!

V splošnem: $\frac{d(m_0\gamma\vec{v})}{dt} = m_0 \left[\cancel{\frac{d\gamma}{dt}\vec{v}} + \gamma \frac{d\vec{v}}{dt} \right]$

$$\gamma m_0 \frac{d\vec{v}}{dt} = e\vec{v} \times \vec{B}$$

$$\boxed{\gamma m_0 \vec{a}_r = e\vec{v} \times \vec{B}}$$

$$\gamma m_0 a_r = evB$$

$$\gamma m_0 \frac{v^2}{r} = evB$$

$$\gamma m_0 \frac{v}{r} = eB \Rightarrow \boxed{r = \frac{\gamma m_0 v}{eB}}$$

$$\tilde{t}_0 = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{\gamma m_0 v}{eB} = \frac{2\pi}{eB}\gamma m_0$$

$$\boxed{\nu_{sc}} = \frac{1}{\tilde{t}_0} = \frac{eB}{2\pi\gamma m_0} = \boxed{\frac{1}{\gamma}\underbrace{\left[\frac{eB}{2\pi m_0}\right]}_{\nu_c}}$$

$$\frac{d\vec{P}}{dt} = e\vec{E} \qquad \vec{v} \parallel \vec{E} \qquad \vec{E} = \text{konst.}$$

$$\frac{d(m_0\gamma\vec{v})}{dt} = e\vec{E} \Rightarrow \frac{d(m_0\gamma v)}{dt} = eE \,/dt/\int$$

$$\int d(m_0\gamma v) = \int eE\,dt$$

$$\gamma = 1 - \frac{v^2}{c_0^2}$$

$$\boxed{m_0\gamma v = eEt}$$

$$\boxed{\frac{m_0 v}{\sqrt{1-\frac{v^2}{c_0^2}}} = eEt} \,/ m_0 c_0 \longrightarrow \frac{\frac{v}{c_0}}{\sqrt{1-\left(\frac{v}{c_0}\right)^2}} = \overbrace{\left(\frac{eE}{m_0 c_0}\right)}^{\alpha} t = \alpha t \,/ (\,)^2$$

$$\left(\frac{v}{c_0}\right)^2 = \alpha^2 t^2 \left[1-\left(\frac{v}{c_0}\right)^2\right] \Longrightarrow \boxed{v = c_0 \left[\frac{\alpha t}{\sqrt{1+\alpha^2 t^2}}\right]}$$ relativistično

Klasično:

$m\,dv = eE\,dt$

$v = \frac{eE}{m_0} t$

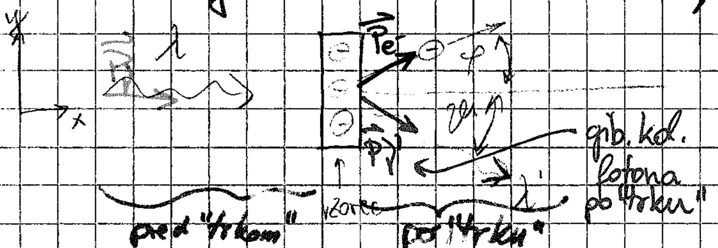
$t \to \infty \Rightarrow v \to \infty$

$$\lim_{t \to \infty} v = c_0 \frac{\alpha t}{\alpha t} = c_0$$

COMPTONSKO SIPANJE

23.5.2011

Žarek rentgenske svetlobe na kristal, sipane svetlobe nima iste λ.



$\boxed{\lambda \to \lambda'}$ (trk fotona s skoraj prostim elektronom)

V vzorcu so šibko vezani elektroni, fotoni trkajo vanje, fotonom se spremeni λ.

Pri trkanju delcev se ohranjata gib. količina in polna energija.

- Ohranitev polne energije:

$W_0 = 0{,}51\ \mathrm{MeV}$ ← lastna energija elektrona

$c_0 = \nu\lambda$

$W_\nu = h\nu = \frac{hc_0}{\lambda}$

$P_\nu = \frac{h\nu}{c_0}$

elektron pred trkom miruje.

$$\frac{hc_0}{\lambda} + W_0 = \frac{hc_0}{\lambda'} + W$$

{Vezalna energija je zanemarljiva}

$$\boxed{W^2 = W_0^2 + c_0^2 p^2}$$

gib. kol. elektrona po "trku"

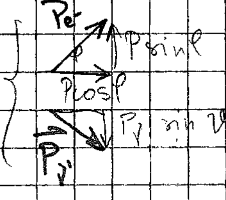
polna energija elektrona

$\boxed{W = W_0 + T}$ ← kin. en. el. po "trku"

- Ohranitev skupne gibalne količine:

x: $\boxed{\left(\frac{h}{\lambda}\right) = p_e \cos\varphi + \left(\frac{h}{\lambda'}\right)\cos\vartheta}$

y: $\boxed{p \sin\varphi = \left(\frac{h}{\lambda'}\right)\sin\vartheta}$



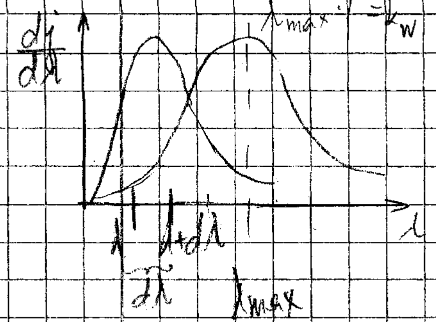
Comptonova enačba:

$$\boxed{\lambda' - \lambda = \frac{hc_0}{W_0}(1-\cos\vartheta)}\ ;\quad \lambda_c = \frac{hc_0}{W_0} = 0{,}0024\ \mathrm{nm}$$

Comptonova val. dolžina

SEVANJE ČRNEGA TELESA

j => gostota energijskega toka
λ => val dolžina



Spekter je odvisen od temperature

$k_W = 2{,}9 \cdot 10^{-3}\,mK$ ← Wienova konstanta

$\sigma = 5{,}67 \cdot 10^{-8}\,Wm/K$ ← Stefanova konstanta

$h = 6{,}6 \cdot 10^{-34}\,Js$ ← Planckova konstanta

$k = 1{,}38 \cdot 10^{-23}\,J/K$ ← Boltzmanova konstanta

$$\boxed{\frac{dj}{d\lambda} = \frac{2\pi h c^2}{\lambda^5} \Big/ \left[e^{\frac{hc}{\lambda kT}} - 1\right]}$$ ← Planckov zakon sevanja črnega telesa

$$\boxed{j} = \underbrace{\int_0^\infty \left(\frac{dj}{d\lambda}\right) d\lambda}_{} = \boxed{\sigma T^4}$$

"Če Plancka integriramo, dobimo Stefana"

Temperatura je v Kelvinih!

$$\boxed{\lambda_{max} \cdot T = k_W}$$ ← Wienov zakon

λ_{max} – kjer ima spekter črnega telesa maksimalno vrednost

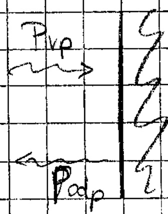
odvajamo Planckov zakon; maksimum

Črno telo : $j = \sigma T^4$

sivo telo (ne-črno)

stacionarno stanje: izsevani tok mora biti enak absorbiranemu toku, temperatura se ne spreminja.

odbojnost (albedo) : $a = \frac{P_{odb}}{P_{vp}}$



za sivo telo :

$P_{vp} = S\sigma T^4$

$P_{odb} = a P_{vp} = a S \sigma T^4$

$P_{abs.} = P_{vp} - P_{odb} = S\sigma T^4 - aS\sigma T^4 = \boxed{S(1-a)\sigma T^4} = P_{izs}$

$$\boxed{P_{izs} = j_{izs} \cdot S = S(\underbrace{1-a}_{e})\sigma T^4}$$

$$\boxed{j_{izs} = e\,\sigma T^4}$$

e – emisivnost; za črno telo je 1

primer: Segrevanje zemlje:

$P_{izs} = e\sigma T^4 S$

zaradi CO_2 se manjša.

e je odvisen od vsebnosti CO_2, če se CO_2 poveča, se poveča absorbcija IR žarkov!

P_{abs} ← v stacionarnem stanju zemlje

sončna svetloba

$P_{abs} = P_{izs}$

$\boxed{P_{abs}} = e\sigma T^4 S$

Če se e zmanjša za 5% → $e_n = 0{,}95e$

$\boxed{P_{abs}} = e_n \sigma T_n^4 S$

Obe absorbirana toka sta ista ⇒ tudi izsevana toka sta ista!

$e\cancel{\sigma}T^4\cancel{S} = e_n\cancel{\sigma}T_n^4\cancel{S}$

$\boxed{eT^4 = e_n T_n^4}$

$T\,[K]$

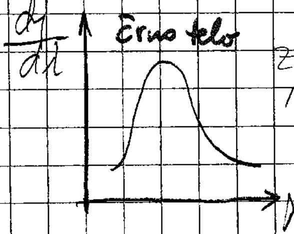
upr.: $eT^4 = 0{,}95e\,T_n^4$

$T_n = \frac{T}{\sqrt[4]{0{,}95}}$

T_n je večja ≈ 4K

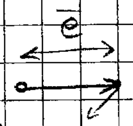
$T = 293K$

ČRTASTI SPEKTRI SEVANJA PLINOV

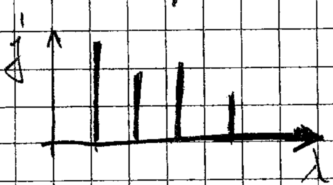


zvezen spekter

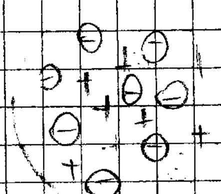
$\frac{mv^2}{2} = eU$



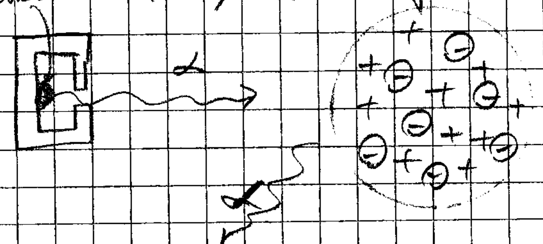
črtasti spekter



Thomsov model atoma:

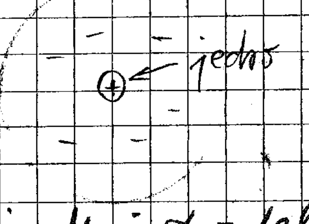


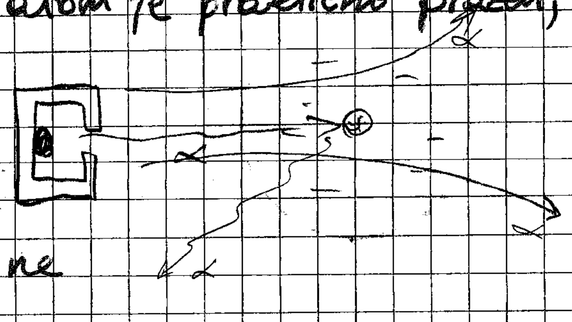
radioaktivna snov, ki seva alfa žarke (He jedra)



Rutherfordov model atoma:

Vsa masa je zbrana v jedru, atom je praktično prazen; primer osončje



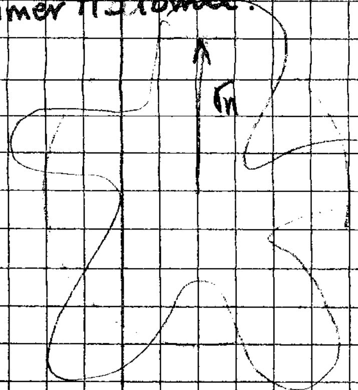


Pojasni odboj α-delcev; ne pojasni črtastega spektra

Bohrov model atoma

26.5.2011

-primer H-atoma:



fotoni: $h\nu = cP \Rightarrow P = \frac{h\nu}{c} = \frac{h}{\lambda}$

delci: $\boxed{\lambda_B = \frac{h}{mv}}$

$2\pi r_n = n\lambda_B$

$\boxed{\hbar = \frac{h}{2\pi}}$

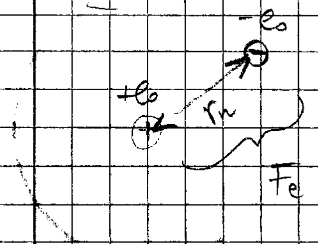
$2\pi r_n = n\frac{h}{m_e v} \Big/ m_e v$

$r_n = \frac{n\hbar}{m_e v_n}$

vrtilna količina: $\Gamma_n = m_e r_n v = n\left(\frac{h}{2\pi}\right) = n\hbar$

$\boxed{\Gamma_n = n\hbar}$

vrtilna količina je kvantizirana



$F_e = \frac{e_0^2}{4\pi\varepsilon_0 r_n^2}$

N-zakon:

$m_e\frac{v_n^2}{r_n} = \frac{e_0^2}{4\pi\varepsilon_0 r_n^2}$

$\boxed{v_n^2 \cdot r_n = \frac{e_0^2}{4\pi\varepsilon_0 m_e}}$

$v_n = \frac{e_0^2}{4\pi\varepsilon_0 n\hbar}$

$r_n = \frac{n^2\hbar\, 4\pi\varepsilon_0 \hbar}{m_e e_0^2} = \frac{n^2\hbar^2 4\pi\varepsilon_0}{m_e e_0^2}$

Energija:

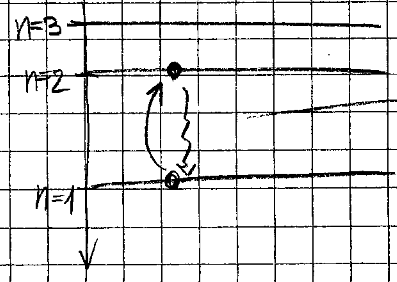
$W_n = \frac{1}{2}m_e v_n^2 - \frac{e_0^2}{4\pi\varepsilon_0 r_n} = \frac{m_e e_0^4}{2\cdot 16\pi^2\varepsilon_0^2\hbar^2\cdot n^2} - \frac{e_0^2 m_e e_0^2}{16\pi^2\varepsilon_0^2\hbar^2 n^2} =$

$W_n = -\frac{e_0^4 m_e}{32\pi^2\varepsilon_0^2\hbar^2 n^2}$

Tudi energija je kvantizirana

$-13{,}6\,eV = -13{,}6 \cdot 1{,}6 \cdot 10^{-19}\,As$

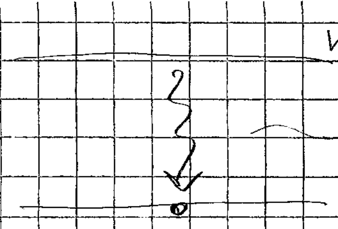
$\boxed{W_n = -\frac{13{,}6\,eV}{n^2}}$



$h\nu = -\frac{13{,}6\,eV}{4} - (-13{,}6\,eV)$

$h\frac{c_0}{\lambda} = 13{,}6\,eV - \frac{13{,}6}{4}\,eV$

$\frac{1}{\lambda} = \frac{1}{hc_0}\left[13{,}6 - \frac{13{,}6}{4}\right]eV$

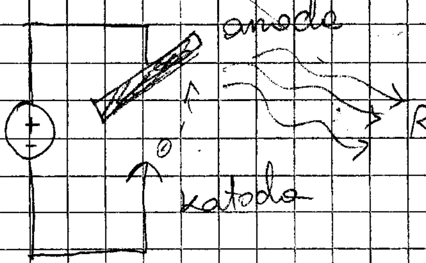


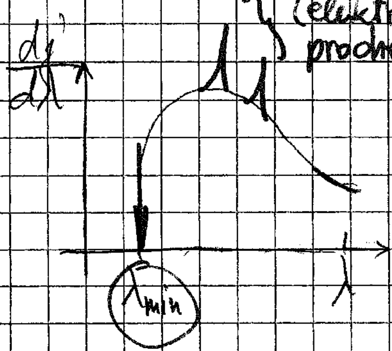
$$h\frac{c_0}{\lambda} = -\frac{13{,}6\,eV}{n^2} - \left(-\frac{13{,}6\,eV}{n'^2}\right)$$

$$\frac{1}{\lambda_{n',n}} = \underbrace{\frac{13{,}6\,eV}{hc_0}}_{R_y}\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)$$

$$\boxed{\frac{1}{\lambda_{n',n}} = R_y\left(\frac{1}{n'^2}-\frac{1}{n^2}\right)}$$

Zavorno sevanje (X-žarki, Rtg-žarki)





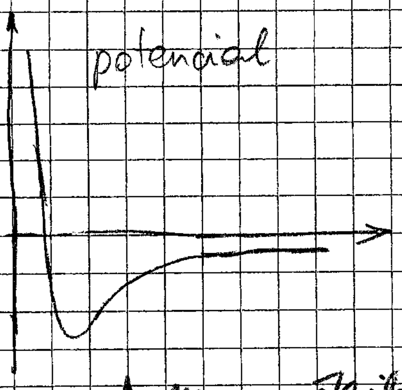
te črte so odvisne od materiala anode (elektroni lahko prodrejo globje, ...)

$$W_{kin,max} = \frac{1}{2}m_e v^2 = e_0U_0 = h\nu_{max} = h\frac{c}{\lambda_{min}}$$

$$e_0U_0 = h\frac{c}{\lambda_{min}} \Rightarrow \boxed{\lambda_{min} = \frac{hc}{e_0U_0}}$$

proton je sestavljen iz kvarkov uud in nosi naboj e_0.
nevtron -//- -//- udd in je brez naboja.

močna sila veže jedro.
EM sila veže atome.
težnost veže sončni sistem
šibka sila povzroči radioaktivni razpad.



$_Z X^A$ – A - masno število, Z - vrstno število

Jedrske reakcije

$$a + X \rightarrow V \rightarrow \begin{cases} a + X \\ a + X + \gamma \\ Y + b \end{cases}$$

a – delec, foton (γ žarek); X – jedro; V – masno jedro; γ – gama žarek (foton)

(masa se ne ohranja)

Polna energija in gib. količina se ohranjata.
Lastna energija se po reakciji poveča, zaradi α delca in njegove kin. energije.

Rad. razpad:

$N \rightarrow$ št. jeder

$dN = Ndt$, $dN - \lambda N dt$

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \quad , \quad N_0 = N(t=0)$$

$$\ln N - \ln N_0 = \ln\left(\frac{N}{N_0}\right) = -\lambda t \Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$

$$t_{1/2} = N(t = t_{1/2}) = \frac{N_0}{2} \Rightarrow$$

$$\Rightarrow \boxed{N = N_0 e^{-\frac{\ln 2 t}{t_{1/2}}}}$$

30.5.2011

- fotoefekt:

foton: $P_\gamma = \frac{h}{\lambda}$

energija: $W = h\nu$

- sevanje črnega telesa:

$$\frac{dj}{d\lambda} = \frac{2\pi hc^2}{\lambda^5} \Leftarrow \text{Planckov zakon}$$

- Stefanov zakon:

$$j = \sigma T^4$$

- emisijski spektri

$$E_n = \frac{-13{,}6\,eV}{n^2}$$

$n = 1,2,3,\ldots$

energijski nivoji vodikovega atoma so diskretni

elektron se vzbudi pri trku, preskoči v nižje stanje, izseva foton

$$Ry = \frac{13{,}6\,eV}{h c_0} \Leftarrow \text{Rydbergova konstanta}$$

-

- elektroni:

$$\underbrace{\lambda_B = \frac{h}{p_e} \rightarrow \frac{h}{m_e v}}$$

$$\underbrace{\underline{p} = \frac{2\pi}{2\pi}\frac{h}{\lambda} = \underline{k\hbar}, \quad \underline{W = \hbar\omega}}_{\text{fotoni}}$$

k ... valovni vektor

$k = \frac{2\pi}{\lambda}$

$\hbar = \frac{h}{2\pi}$

EMV:

$\overline{j} = \overline{w} \cdot c_0$

$\overline{j} \propto \overline{E^2}$

elektroni

$\boxed{p = \hbar k, \quad W = \hbar\omega}$, $\boxed{p = mv, \quad W = T(+V)}$

kin. energija

$k = \frac{mv}{\hbar} = \frac{p}{\hbar}$

$\omega = \frac{W}{\hbar} = \frac{T}{\hbar} = \frac{p^2}{2\hbar m}$

$T = \frac{1}{2}mv^2$

$p = mv$

$v = \frac{p}{m}$

$\underline{T = \frac{1}{2}\frac{p^2}{m}} = \underline{\frac{p^2}{2m}}$

- Valovna funkcija

$$\underline{\psi_p(x,t)} \propto e^{i(kx-\omega t)} = e^{i\left(\frac{p}{\hbar}x - \frac{W}{\hbar}t\right)} = \underline{e^{i(px - Wt)/\hbar}}$$

$\underline{W = W_{kin} + W_{pot}}$

kjer je na zaslonu bolj svetlo, je tam padlo več elektronov.

verjetnostna ... - verjetnost da tja pade elektron

$\rho \propto \psi^* \psi \quad , \quad \overline{j} \propto E^* E$

uklon curka na dveh mrežah: $\int \frac{dN}{N} = \int \rho\, dx \Rightarrow \frac{N}{N} = 1 \quad , \int \rho\, dx = 1$

KVANTNA MEHANIKA

- zakone in enačbe določimo z eksperimenti

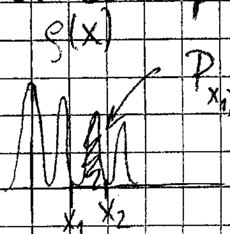
valovna funkcija:

$\psi = \psi_0\, e^{i(\vec{k}\vec{r} - \omega t)}$

$\vec{p} = \hbar\vec{k}$ gib. kol. elektrona

$\hbar\omega = W = W_k + W_p$ polna energija elektrona

Verjetnost, da bo delec padel na neko mesto je enaka ploščini pod krivuljo.



$P_{x_1 x_2} = \int_{x_1}^{x_2} \rho(x)\, dx$

$$\int_{-\infty}^{+\infty} \rho\, dx = 1$$

$$\int_V \psi^* \psi\, dV = 1 \quad , dV = (dx\,dy\,dz)$$

pričakovana vrednost:

$$\langle x \rangle = \int_{-\infty}^{\infty} x \rho \, dx = \int_{-\infty}^{\infty} x \, \psi^* \psi \, dx = \int_{-\infty}^{\infty} \psi^* x \psi \, dx$$

① $-i\hbar \frac{\partial}{\partial x} \psi = \cancel{\frac{i p_x}{\hbar}} \cancel{(-i\hbar)} \psi = p_x \psi$, $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$

② $i\hbar \frac{\partial}{\partial x} \psi \cancel{(i\hbar)} \left(+ \frac{iW}{\cancel{\hbar}} \right) \psi = W \psi$, $i\hbar \frac{\partial}{\partial t} \psi = W\psi$

operator polne energije $\hat{W} = i\hbar \frac{\partial}{\partial x}$

2.6.2011

verjetnostna gostota

$\rho = \psi^* \psi$

val. funkcije

kl. fizika:

$\vec{r}(t)$
$\vec{v}(t)$

$\psi = \psi_0 \, e^{i(p_x x - Wt)/\hbar}$

$\hat{T}\psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = \frac{p_x^2 \psi}{2m}$

$\hat{W}\psi = i\hbar \frac{\partial}{\partial t} \psi = W \psi$

delec ni v potencialu

če je delec prost - $V(x) = 0$

$W = T = \frac{p_x^2}{2m}$

$\hat{T} W = \hat{W} \psi$

$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi = i\hbar \frac{\partial}{\partial t} \psi$ Schrödingerjeva enačba za prosti delec (1-D)

zunanja sila:

$\vec{F} = -\mathrm{grad} V(\vec{r}, t) \longrightarrow H = \frac{p^2}{2m} + V(\vec{r}, t)$

$V \neq 0$

$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \psi = i\hbar \frac{\partial \psi}{\partial t}$ ← Schr. enačba s časovno odvisnostjo, rešitev je val. funkcija $\psi = \psi(\vec{r}, t)$

V kv. mehaniki lahko delec pride skozi steno tudi, če ima manjšo W_k kot je višina stene.