

# FIZIKA II

1 UNI

## Zapiski predavanj

Šolsko leto 2010/2011  
Izvajalec Aleš Iglič

Avtor dokumenta Damjan Sirnik  
Skeniranje Damjan Sirnik



### UREJANJE DOKUMENTA

VERZIJA	01.01
DATUM	23.5.2012

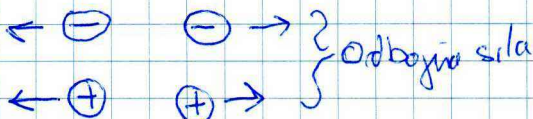
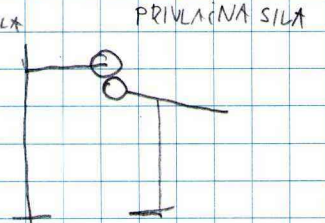
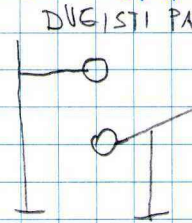
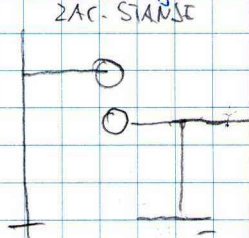
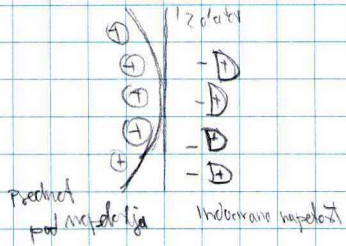
### OPOMBE

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ELEKTRON → JAVNA

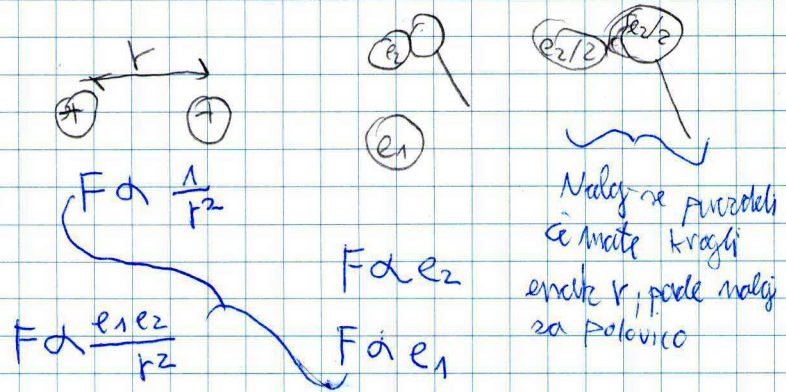
Papir je izolator a se lahko tem nalipi na magnetno polje zaradi influence.

Razdelujemo T mi - el. naloge. Problemni električni. Problem 2000 leti od električne



$\epsilon_0 = 1,6 \cdot 10^{-19} \text{ As}$

Sila el. polja se kvadratno razreda.



COULOMBOV ZAKON

$$F = k \cdot \frac{e_1 e_2}{r^2}$$

$k = \text{Coulombova konstanta} = 9 \cdot 10^9 \text{ N} \cdot \text{m}^2 / (\text{As})^2$

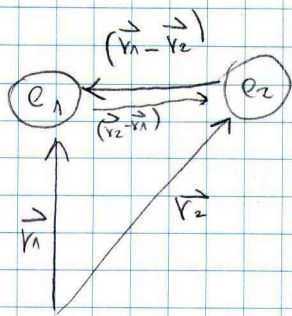
$k = \frac{1}{4\pi\epsilon_0}$

$\epsilon_0 = \text{indukcijska konstanta} = 8,85 \cdot 10^{-12} \text{ As/Vm}$

$$F = \frac{e_1 e_2}{4\pi\epsilon_0 r^2}$$

Nima veze z dielektričnostjo; momenta se na kvadrat prostora.

$\epsilon_0 = \frac{1}{4\pi k} \approx 3 \cdot 10^9 \text{ Vs}$

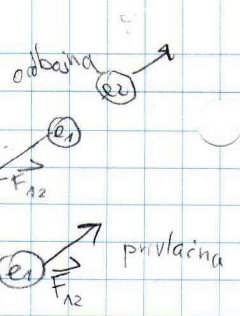


$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2} \left( \frac{\vec{r}_{12}}{r_{12}} \right)$$

enotni vektor

$$\left| \frac{\vec{r}_{12}}{r_{12}} \right| = 1$$

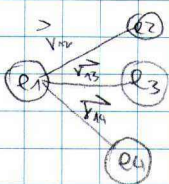
$q_1 > 0, q_2 > 0 : q_1 q_2 > 0$   
 $q_1 < 0, q_2 < 0 : q_1 q_2 > 0$   
 $q_1 > 0, q_2 < 0 : q_1 q_2 < 0$   
 $q_1 < 0, q_2 > 0 : q_1 q_2 < 0$



$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{21}^2} \left( \frac{\vec{r}_{21}}{r_{21}} \right)$$

$$\vec{F}_{21} = \vec{F}_{12} \Rightarrow \vec{F}_{12} + \vec{F}_{21} = 0$$

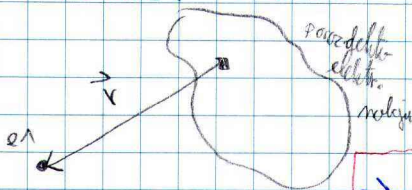
### POSLOŠITEV:



$$\vec{F}_1 = \sum_{j=2}^n \frac{q_1 q_j}{4\pi\epsilon_0 r_{1j}^2} \left( \frac{\vec{r}_{1j}}{r_{1j}} \right)$$

št. nabojev v okolici

### POSLOŠITEV V OKVIRU KONTINUUMSKEGA NABOJA



$$\sum_j \rightarrow \int_{r_j} \rightarrow r$$

$$\sum_{q_j} \rightarrow \int_{q_j} \rightarrow dq$$

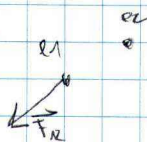
$\rho = \frac{dq}{dV} \Rightarrow dq = \rho dV$   
 $\sigma = \frac{dq}{dA} \Rightarrow dq = \sigma dA$   
 $\lambda = \frac{dq}{dL} \Rightarrow dq = \lambda dL$

$$\vec{F}_1 = \int \frac{q_1 dq}{4\pi\epsilon_0 r^2} \left( \frac{\vec{r}}{r} \right)$$

## ELEKTRIČNO POLJE (JAKOST EL. POLJA $\vec{E}$ )

$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2} \left( \frac{\vec{r}_{12}}{r_{12}} \right)$$

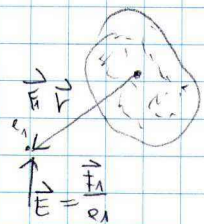
$$\vec{E}_2 = \frac{\vec{F}_{12}}{q_2} = \frac{q_1}{4\pi\epsilon_0 r_{12}^2} \left( \frac{\vec{r}_{12}}{r_{12}} \right)$$



### POSLOŠITEV:

$$\vec{F}_1 = \int \frac{q_1 dq}{4\pi\epsilon_0 r^2} \left( \frac{\vec{r}}{r} \right)$$

$$\vec{E} = \frac{\vec{F}_1}{q_1} = \int \frac{dq}{4\pi\epsilon_0 r^2} \left( \frac{\vec{r}}{r} \right)$$



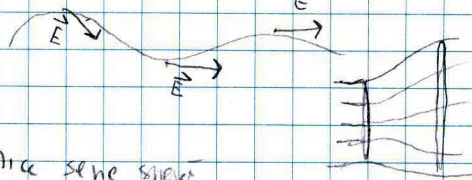
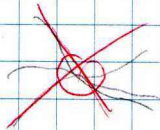
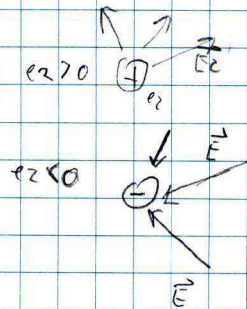
$q_1 =$  testni naboj

Testni naboj mora biti šim manjši, da ne zmoti porazdelitve. Velikost test naboja ni pomembna, saj  $\epsilon_0$  ampake manjšini, kot  $\epsilon_0$  ne more biti

Kvadrati - detekci 2 nalezum maxymim od  $\epsilon_0$  ( $1/3, 2/3$  e ipd.)  $\epsilon_2 > 0$

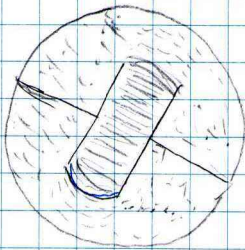
$$\vec{F} = e\vec{E}$$

→ Seta ma el. nalezuj w el. polju  
zaburzeni  $\vec{E}$

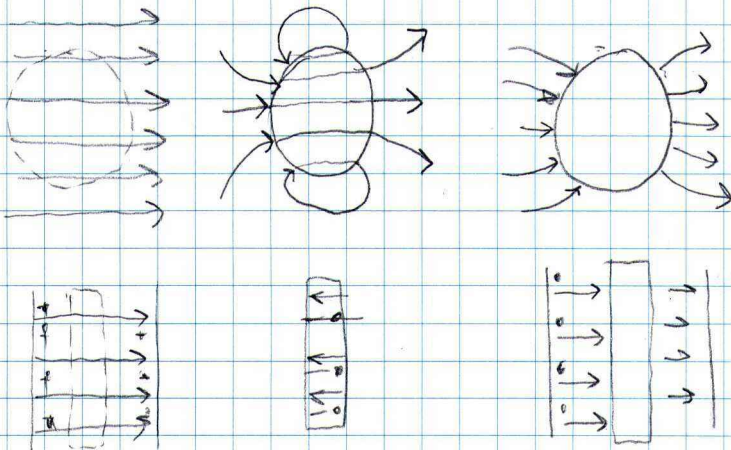


Silnice sene sniegi  
oznaczuju rektu kierunki polju  
potem miedzy dwi osmi

### VAN DER GRAAFS GENERATOR

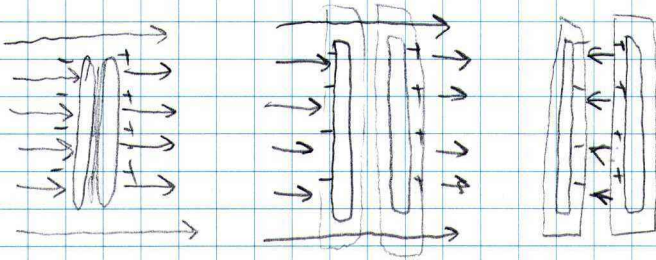


# INFLUENCA

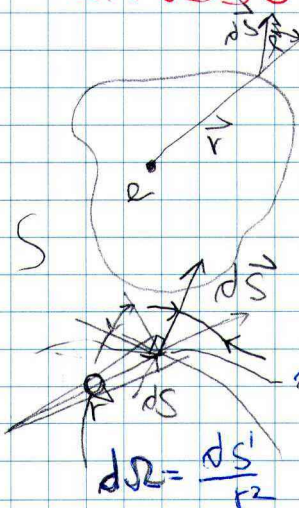


Površina preobčutka postane konpotencialna ploskev

## NEPOSREDNA MERITEV GOSTOTE EE. POLJA



## GAUSSOV ZAKON O ELEKTRIČNEM PRETOKU



$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \frac{\vec{r}}{r}$$

$$\oint_A \vec{E} \cdot d\vec{S} = \oint \frac{q \vec{r} \cdot d\vec{S}}{4\pi\epsilon_0 r^3} = \oint \frac{q r' dS \cos\alpha}{4\pi\epsilon_0 r^3} = \frac{q}{4\pi\epsilon_0 r^2} \oint dS'$$

$$= \frac{q}{4\pi\epsilon_0} \oint d\Omega = \frac{q}{4\pi\epsilon_0} 4\pi = \frac{q}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\Omega = \frac{4\pi r^2}{r^2} = 4\pi$$

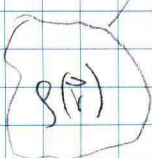
$$d\Omega = \frac{dS}{r^2}$$

$$dS \cos\alpha = dS'$$

$$d\Omega = \frac{dS'}{r^2}$$

POSREDOŠITEV:

$$\oint \vec{E} \cdot d\vec{S} = \frac{\sum e_i}{\epsilon_0}$$



$$\sum e_i \rightarrow \int \rho(\vec{r}) dV$$

↑  
Vol. gostota na obseju

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = \int \rho(\vec{r}) dV$$

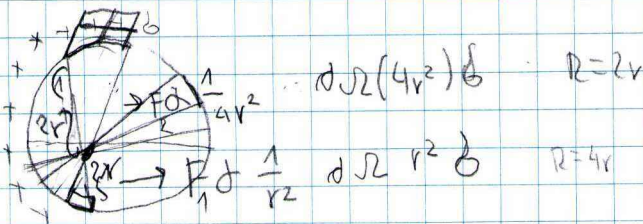
$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \rho(\vec{r}) - \text{v diferencialni obliki}$$

Druge oblike Coulombovega zakona, r parameter dnoy obliki

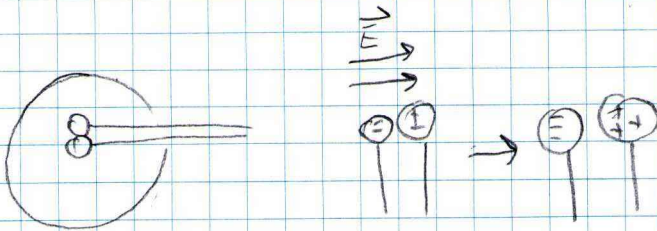
GAUSSOV ZAKON O EL. PRETOKU V INTEGRALNI OBLIKI

$$\vec{\nabla} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\vec{E} = (E_x, E_y, E_z)$$



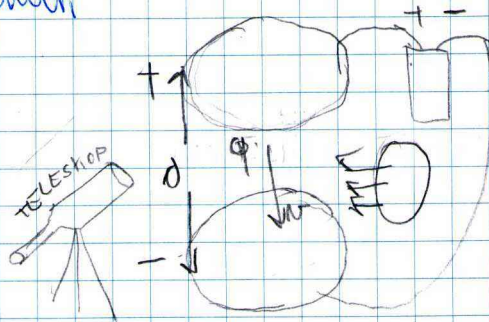
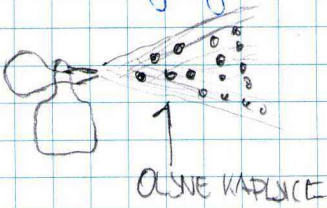
Kjer velji Faradajeva kletka (čisto terno)  
potem velji Coulombov zakon



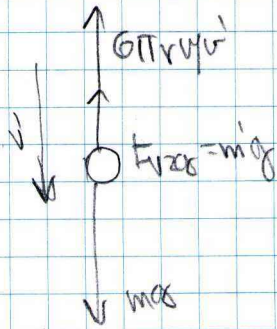
### MILLIKANOV POSKUS

28.02.11

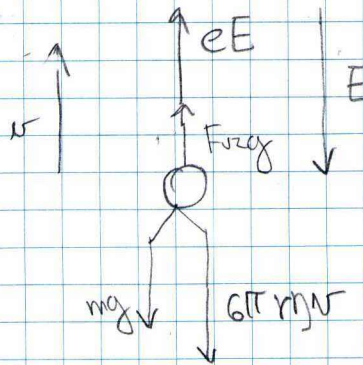
-el malej je kumulativen



$E=0$



$E \neq 0$



ZA STACIONARNO STANJE:  $E=0$

ZA  $E \neq 0$ :

$$mg = mg + 6\pi r \eta v'$$

$$eE + mg = mg + 6\pi r \eta v$$

max

$$\Rightarrow 6\pi r \eta v' + \frac{4}{3} \rho \pi r^3 g = \frac{4}{3} \rho \pi r^3 g$$

$$\Rightarrow 6\pi r \eta v' = 0$$

$$v' = 0$$

$$\frac{6\pi r \eta}{2g(\rho_s - \rho)} = r$$

$$\frac{6\pi r \eta}{2g(\rho_s - \rho)} = r$$

$$eE - 6\pi r \eta v' = 6\pi r \eta v$$

$$\Rightarrow eE = 6\pi r \eta (v + v')$$

$$\Rightarrow e = \frac{6\pi r \eta (v + v')}{E}$$

$$m = \frac{4}{3} \rho \pi r^3$$

$$m' = \frac{4}{3} \rho \pi r^3$$

$$e = ne_0, \text{ kjer } 1, 2, 3$$

$$e_0 = 1.6 \cdot 10^{-19} \text{ As}$$

→ el. nabij mi zvečen, ampak

	(C)	(kg)
Elektron	$-1,60217672 \times 10^{-19}$	$9,10938 \times 10^{-31}$
Proton	$+1,60217672 \times 10^{-19}$	$1,67262 \times 10^{-27}$
Neutron	0	$1,67492 \times 10^{-27}$

## GIBANJE ELEKTRIČNIH NABOJEV V SNОВI (V EL. POLJU)

VAKUUM:  
 $E \neq 0$

$$m \frac{d\vec{v}}{dt} = e\vec{E} \Rightarrow \vec{a} = \frac{e\vec{E}}{m} \quad (\text{VELIKA ZA } \frac{v}{c} \gtrsim 0,1)$$

SVELO. HITROST

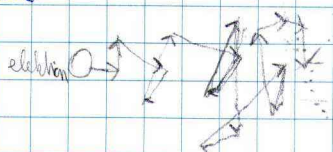
POSPEŠEVANJE  
LINEARNO



HITROST NE MORE BIT VEČJA OD  
SVELO. OBNOG HITROSTI.

### GIBANJE NABOJA V KOVINI:

V KOVINI PREDVODNI ELEKTRONI, KI TEKAJO PIP. HITROST JE ZELO MALA  
V ATOME  $m = \frac{d\vec{p}}{dt} = e\vec{E}$



$$\vec{F}_{av} = -k_e \vec{v}$$

Newton:

$$m \frac{d\vec{v}}{dt} = e\vec{E} - k_e \vec{v}$$

$$\frac{d\vec{v}}{dt} = \frac{e}{m} \vec{E} - \frac{k_e}{m} \vec{v}$$

$$\frac{d\vec{v}}{dt} + \frac{1}{\tau} \vec{v} = \frac{e}{m} \vec{E}$$

$\tau = \frac{m}{k}$  RELAKSACIJSKI ČAS

$$\vec{E} = 0: \int \frac{d\vec{v}}{dt} + \frac{1}{\tau} \vec{v} = 0$$

$$\int \frac{d\vec{v}}{dt} = - \int \frac{1}{\tau} \vec{v}$$

$$\ln \vec{v} = - \frac{t}{\tau} \Rightarrow \vec{v} = \vec{v}_0 \cdot e^{-t/\tau}$$

$E = \text{konst}$

$$\left\langle \frac{d\vec{v}}{dt} \right\rangle + \frac{\langle \vec{v} \rangle}{\tau} = \frac{e}{m} \vec{E}$$

$\left\langle \frac{d\vec{v}}{dt} \right\rangle = 0$

$$\Rightarrow \frac{\langle \vec{v} \rangle}{\tau} = \frac{e}{m} \vec{E}$$

### RELAKSACIJSKI ČAS

Snov	$\tau$ [s]
kovina	$\sim 10^{-14}$
varčelektivna v plinu	$\sim 10^{-9}$
Sončna kovina	$\sim 10^2$
netvezadni plini	$\sim 10^5$

GIBANJE:

$$\Rightarrow \beta = \frac{e\tau}{am}$$

$$\vec{v} = \frac{e\tau}{m} \vec{E}$$

$$I = \frac{dq}{dt}$$

$$j = \frac{I}{S} = \frac{1}{S} \frac{dq}{dt}$$

$$= \frac{1}{S} \frac{d(e n v l)}{dt} = \frac{1}{S} e n v \frac{dl}{dt}$$

$$\langle \vec{v} \rangle = \frac{e\tau}{m} \vec{E}$$

Ohmov zakon

$$\langle \vec{v} \rangle = -\beta \vec{E}$$

$$= \frac{1}{S} e n v \frac{dl}{dt} = e n v$$

$$j = neov$$

$$\vec{j} = neo \langle \vec{v} \rangle$$

$$n = \frac{N}{V} \text{ [m}^{-3}\text{]}$$

$$e = e n v = e \cdot \left(\frac{N}{V}\right) v$$

Ohmovi ZAKON

$$\vec{j} = ne_0 \langle \vec{v} \rangle \quad \vec{v} = \frac{e_0 \tau}{m} \vec{E}$$

$$\vec{j} = ne_0 \frac{e_0 \tau}{m} \vec{E}$$

SPECIFIČNA PŘEVODNOST / VĚRNOST

$$\sigma = \frac{ne_0^2 \tau}{m}$$

$$\vec{j} = \frac{ne_0^2 \tau}{m} \vec{E}$$

SPLAČNA OBLIKA

$$\frac{I}{S} = \sigma \frac{U}{l}$$

$$\left(\frac{l}{S}\right) I = U$$

$$\Rightarrow R = \frac{l}{\sigma S}$$

$$RI = U$$

$$\sigma_0 = \sigma_0 [1 + \alpha(T - T_0)]$$

Vpovprečje je odvisna od temperature

## ELEKTROSTATSKA POTENCIALNA ENERGIJA

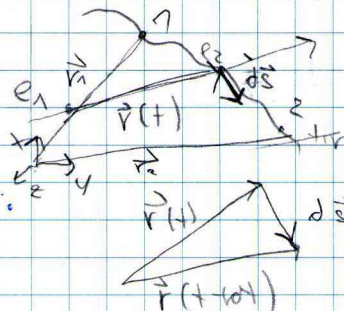
3.3.11

$$\vec{A} = \Delta W_k + W_{ost}$$

Delo vseh ostalih sil ni eno gravitacijske  
Delo vseh ostalih sil ni eno gravitacijske in električne.

$$\vec{A}_{ost} + \int \vec{F}_e \cdot d\vec{s} = \Delta W_k + \Delta W_{ost}$$

$$\Delta W_{ost} = \Delta W_k + \Delta W_{ost} - \int \vec{F}_e \cdot d\vec{s} \quad \rightarrow \text{Nagledano delo el. sile}$$



$$\vec{F}_e = \frac{e_1 e_2}{4\pi \epsilon_0 r^2} \left(\frac{\vec{r}}{r}\right)$$

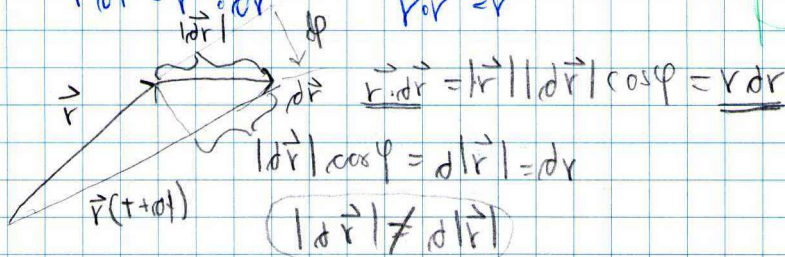
$$d\vec{r} = \vec{r}(t+dt) - \vec{r}(t)$$

$$-\int \vec{F}_e \cdot d\vec{s} = -\int \vec{F}_e \cdot d\vec{r} = -\int \frac{e_1 e_2}{4\pi \epsilon_0 r^2} \frac{\vec{r} \cdot d\vec{r}}{r} = \int_{r_1}^{r_2} \frac{e_1 e_2}{4\pi \epsilon_0 r^2} \frac{r dr}{r} = \frac{e_1 e_2}{4\pi \epsilon_0} \frac{1}{r} \Big|_{r_1}^{r_2} = \frac{e_1 e_2}{4\pi \epsilon_0 r_2} - \frac{e_1 e_2}{4\pi \epsilon_0 r_1}$$

$$d(\vec{r} \cdot \vec{r}) = \vec{r} \cdot d\vec{r} + d\vec{r} \cdot \vec{r} = 2\vec{r} \cdot d\vec{r}$$

$$d(r^2) = 2r dr = 2\vec{r} \cdot d\vec{r} \quad r = |\vec{r}|$$

$$r dr = \vec{r} \cdot d\vec{r} \quad \vec{r} \cdot \vec{r} = r^2$$



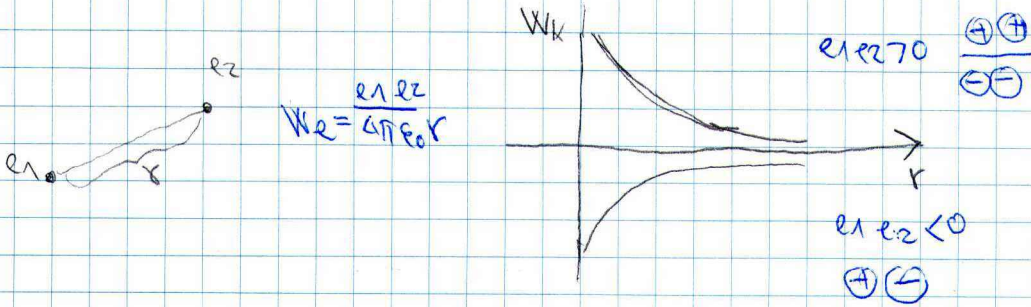
$$|\vec{r} \cdot d\vec{r}| = r |d\vec{r}| \cos \varphi = r dr$$

⇒ Ni odvisni po velikini  
poti groma, ampake samo  
od  $r_1$  in  $r_2$

$$W_{e2} - W_{e1} = \Delta W_e$$

$$W_e = \frac{e_1 e_2}{4\pi \epsilon_0 r}$$

Com Sol  $\rightarrow$  Program



$$A_{\text{ost}} = \Delta W_k + \Delta W_{\text{p.p.}} - \int \vec{f} \cdot d\vec{s}$$

$\frac{dW_e}{dWe}$

POSPLOŠENO

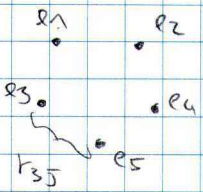
$$A_{\text{ost}} = \Delta W_k + \Delta W_{\text{g.p.}} + \Delta W_e$$

$$A_{\text{ost}} = 0 \Rightarrow \Delta W_k + \Delta W_{\text{g.p.}} + \Delta W_e = 0$$

$$\Delta (W_k + W_{\text{g.p.}} + W_e) = 0$$

$$W_k + W_{\text{g.p.}} + W_e = \text{konst}$$

POSPLOŠITVEN

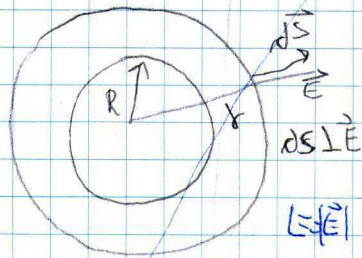
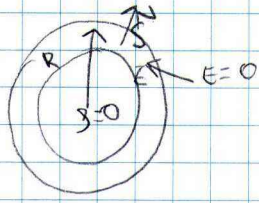
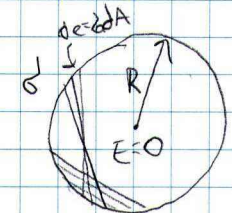


$$W_e = \sum_{i,j} \frac{q_i q_j}{4\pi \epsilon_0 r_{ij}} \cdot \frac{1}{2}$$

$$W_e = \int_V \frac{1}{2} \epsilon_0 E^2 dV = \int_0^R \int_0^{2\pi} \int_0^\pi \frac{1}{2} \epsilon_0 \frac{Q^2}{4\pi^2 r^4} r^2 \sin\theta dr d\theta d\phi$$

$$= \int_0^R \frac{2\pi R^2 \pi}{\epsilon_0} \frac{1}{r^2} dr$$

PRIMER



GAUSSOV ZAKON

$$\epsilon_0 \oint \vec{E} \cdot d\vec{s} = \rho \int dV$$

$$\epsilon_0 E \int dS = Q$$

$$\epsilon_0 E(r) \int dS = 2 \cdot 4\pi R^2$$

$$\epsilon_0 E(r) 4\pi r^2 = 2 \cdot 4\pi R^2$$

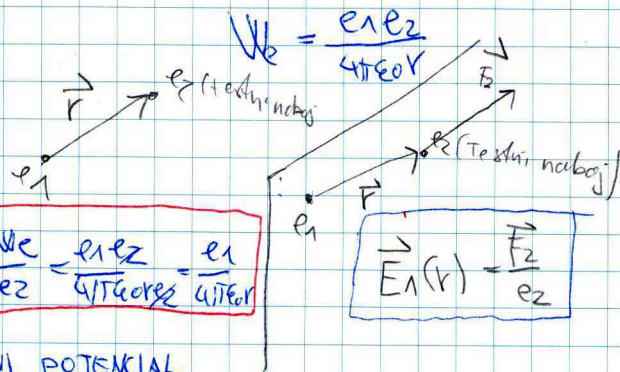
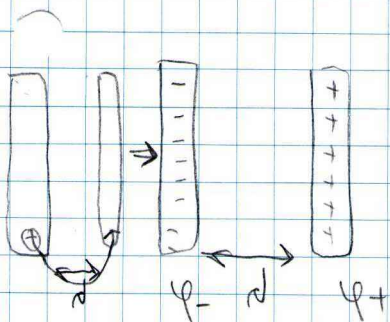
$$\Rightarrow E(r) = \frac{2R^2}{\epsilon_0 r^2}$$

$$e = 2 \cdot 4\pi R^2$$

$$e^2 = 16\pi^2 R^4$$

$$W_e = \int \frac{1}{2} \epsilon_0 E^2 dV = \frac{2 \cdot 2^2 R^4 \pi}{\epsilon_0} \int_0^R \frac{1}{r^2} r^2 dr = \frac{2 \cdot 2^2 R^4 \pi}{\epsilon_0} R = \frac{e^2}{8\pi \epsilon_0 R}$$

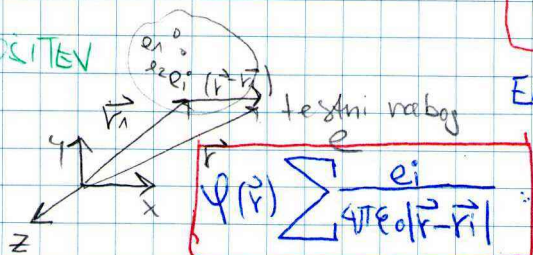
# ELEKTRIČNI POTENCIAL



$$\varphi_1(r) = \frac{W_e}{e_2} = \frac{e_1 e_2}{4\pi\epsilon_0 r e_2} = \frac{e_1}{4\pi\epsilon_0 r}$$

$$\vec{E}_1(r) = \frac{\vec{F}_2}{e_2}$$

POSPLOČITEV



$$\varphi(r) = \sum \frac{e_i}{4\pi\epsilon_0 |r - r_i|}$$

ELEKTRIČNI POTENCIAL

## ZVEZA MED $\varphi$ IN $\vec{E}$

$$\varphi(\vec{r}) = \varphi(x, y, z)$$

$$\vec{r} = (x, y, z)$$

$$y(x)$$

$$dy = \left(\frac{dy}{dx}\right) dx$$

$$d\varphi = \frac{\partial\varphi}{\partial x} dx + \frac{\partial\varphi}{\partial y} dy + \frac{\partial\varphi}{\partial z} dz$$

$$= \left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right) \cdot (dx, dy, dz)$$

$$d\varphi = \nabla\varphi \cdot d\vec{r} = \text{grad } \varphi \cdot d\vec{r}$$

∇ - Nabla operator

$$dW_e = - \int \vec{F}_e \cdot d\vec{r} = - \int \vec{E} \cdot d\vec{r}$$

$$W_e = e_2 \varphi(r)$$

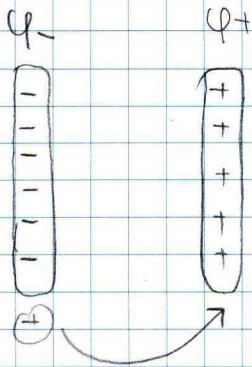
↓

$$d(\varphi) = -e \vec{E} \cdot d\vec{r}$$

$$d\varphi = -\vec{E} \cdot d\vec{r}$$

$$\left. \begin{aligned} d\varphi &= d\varphi \\ -\vec{E} \cdot d\vec{r} &= \text{grad } \varphi \cdot d\vec{r} \end{aligned} \right\}$$

$$\vec{E} = -\text{grad } \varphi$$

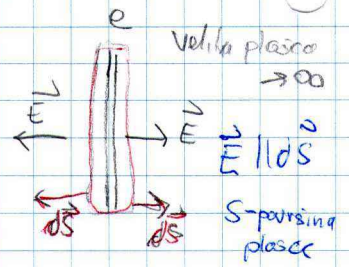


$$A = \Delta W_k + \Delta W_{gp} + \Delta W_e$$

$$dA = de\phi_+ - de\phi_- = de(\phi_+ - \phi_-) = Ude$$

$$dA = Ude$$

$$dA = \int Ude$$

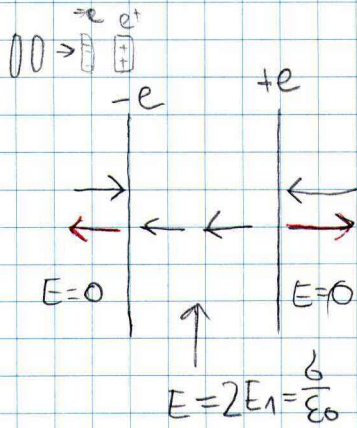


$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = \epsilon_0 \int E dS$$

$$= \epsilon_0 E \int dS = \epsilon_0 E 2\pi R l$$

$$= \int \rho dV$$

$$\epsilon_0 E 2\pi R l = \int \rho dV \Rightarrow E = \frac{\rho R l}{2\epsilon_0}$$



$$E = -\frac{d\phi}{dx}$$

$$\int d\phi = -\int E dx = El$$

$$U = El \Rightarrow E = \frac{U}{l}$$

$$U = \frac{q}{\epsilon_0 l} \Rightarrow C = \epsilon_0 \frac{S}{l}$$

$$U = \frac{q}{\epsilon_0 S}$$

$$q = \epsilon_0 \frac{S}{l} U$$

$$q = CU \Rightarrow U = \frac{q}{C}$$

$$A = dA = \int Ude = \int \frac{q}{C} de = \frac{q^2}{2C} = \frac{C^2 U^2}{2C} = \frac{1}{2} C U^2$$

ENERGIJA  
NAELEKTRENEGA KONDENZATORJA

$$W = \frac{1}{2} \sum_{i,j} \frac{e_i e_j}{4\pi \epsilon_0 r_{ij}} = \dots = \int \frac{1}{2} \epsilon_0 E^2 dV$$

$$W = \frac{1}{2} \epsilon_0 E^2$$

Gostota energije  
el. polja kondenzatorja

$$\epsilon=0 \quad \left| \quad \epsilon \neq 0 \quad \right| \quad \epsilon=0 \quad W_C = \frac{1}{2} C U^2 = \frac{1}{2} C \epsilon^2 l^2 = \frac{1}{2} \epsilon_0 \frac{S}{l} \epsilon^2 l^2 = \frac{1}{2} \epsilon_0 \epsilon^2 S l = \frac{1}{2} \epsilon \epsilon^2 V$$

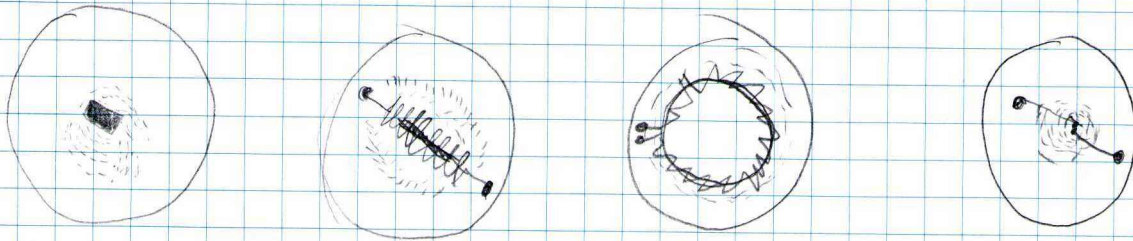
$V = Sl$

TUDI SPLOŠNA IZPELJAVA, TO IZPELJAVA  
NA POSEBNEM PRIMERU

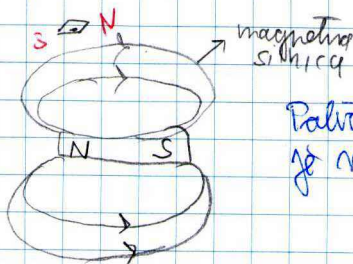
$$\Rightarrow W_E = \frac{1}{2} \epsilon_0 E^2$$

# MAGNETNO POLJE

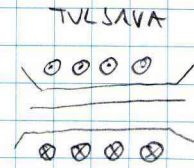
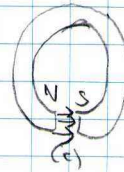
7.3.2011



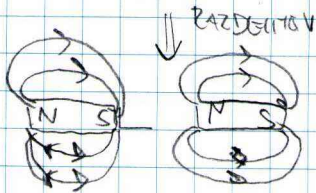
TOROID



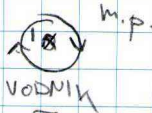
Polevni magnet je magnetni dipol



TULSAVA

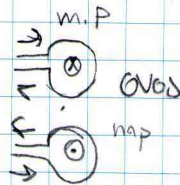


RAZDELOV



m.p.

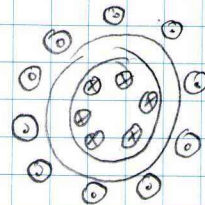
VODNIK



m.p.

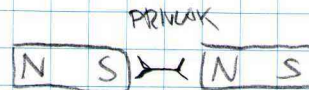
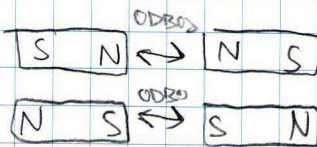
OVOD

nap



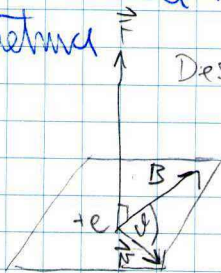
TOROID

N → Severni pol  
S → južni pol



V vsaki točki mag. polja je smer gibanja  $\vec{v}$  obkoležena s smerjo  $\vec{v}$  halata hokeja v ravnostnem krogu središčni (N) pol vrhunske silnice magnetne

Desno sredi vijk



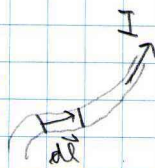
ENOTA za B:  $B \downarrow \frac{F}{e v} = \frac{N S}{A s m} = \frac{N}{A m} = \frac{N m}{A m^2} = \frac{J}{A m^2} = \frac{V A s}{A m^2} \frac{V s}{m^2} = 1 T$

DEF B:  $\vec{F} = e \vec{v} \times \vec{B}$

$\frac{V s}{m^2} = 1 T$  (tesla)

V žici

$d\vec{F} = de \vec{v} \times \vec{B}$   
 $d\vec{F} = I d\vec{l} \times \vec{B}$



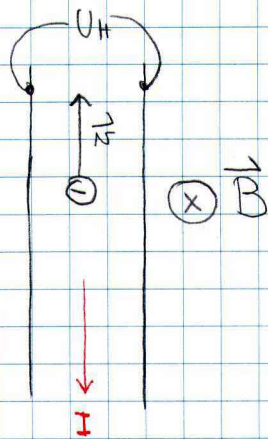
$\vec{F}_e = I \vec{l} \times \vec{B}$

RAVEN VODNIK

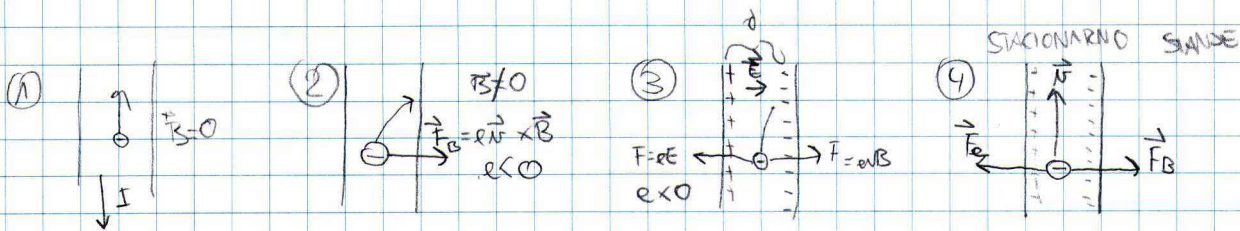
$d\vec{F} = I d\vec{l} \times \vec{B} \Rightarrow \vec{F}_e = \int I d\vec{l} \times \vec{B}$

SILA NA VODNIK TO KATEREM TEČE TOKI

# HALL-ON POŠAV



$U_H \equiv$  HALLOVA NAPETOST



## V STACIONARNEM STANJU

$$F_E = F_B$$

$$eE = e v_d B$$

$$\frac{U_H}{d} = v_d B$$

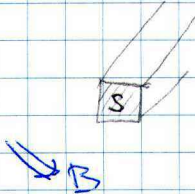
Prop. hitrost



$$E = \frac{U_H}{d} \text{ - Hallova napetost}$$

$$j = ne \langle v \rangle \Rightarrow \langle v \rangle = \frac{j}{ne} = \frac{I}{neS}$$

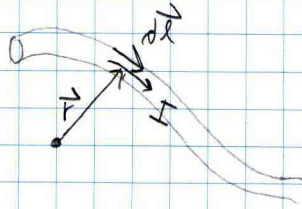
$$\frac{U_H}{d} = \frac{IB}{neS}$$



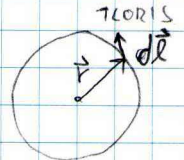
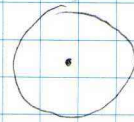
# BIOT - SAVARTOV ZAKON

10.3.11

$$B = \frac{\mu_0 I}{4\pi} \int \frac{\vec{r} \times d\vec{l}}{r^3}$$



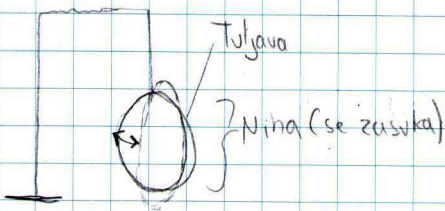
1. PRIMER - OKROGLA TOKOVNA ZANKA



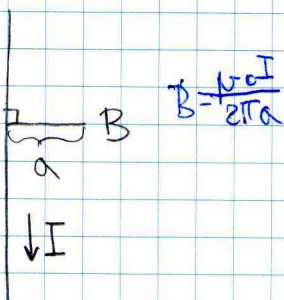
$\vec{F} \perp d\vec{l}$       $|\vec{r}| = r = \text{konst}$

$$\begin{aligned} \Rightarrow B &= \frac{\mu_0 I}{4\pi} \int \frac{I dl}{r^2} = \frac{\mu_0 I}{4\pi r^2} \int dl \\ &= \frac{\mu_0 I 2\pi r}{4\pi r^2} = \frac{\mu_0 I}{2r} \end{aligned}$$

Magnetna polje deluje z navorom



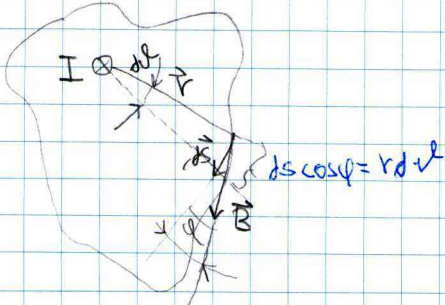
2. PRIMER - DOLG RAVEN VODNIK



# AMPEROV ZAKON

Integriramo mag. polje po zaključeni liniji

PRIMER:



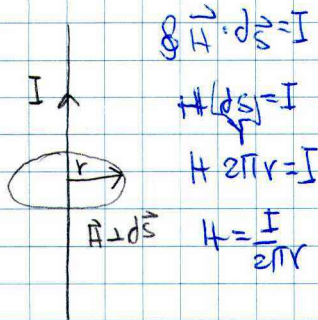
$$\oint \vec{B} \cdot d\vec{s} = \oint B ds \cos\theta = \oint \frac{\mu_0 I r dl}{2\pi r} = \frac{\mu_0 I}{2\pi} \int_0^{2\pi} dl = \mu_0 I$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{s} = I \mu_0 \int \vec{j} \cdot d\vec{s}$$

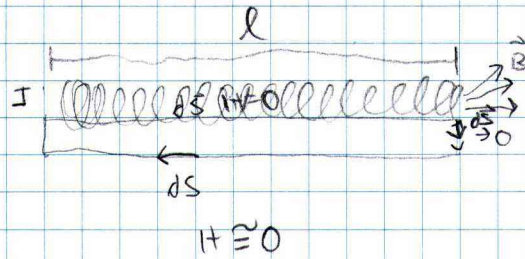
$$\vec{B} = \mu_0 \vec{H}$$

$$\oint \vec{H} \cdot d\vec{s} = \int \vec{j} \cdot d\vec{s}$$

1. PRIMER:



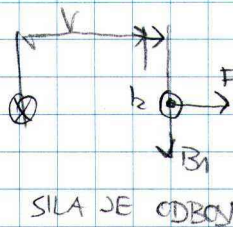
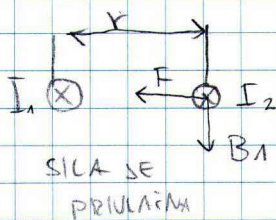
2. PRIMER - DOLGA TULSAVA



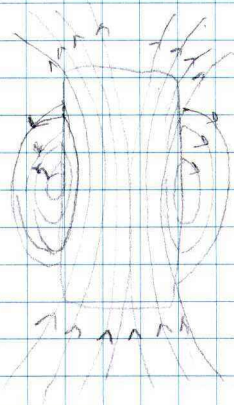
$\oint \vec{H} \cdot d\vec{s} = Hl = NI \Rightarrow H = \frac{NI}{l}$   
 $\uparrow$   
ST. CROSEV

$B = \mu_0 \frac{NI}{l}$

SILA MED VZPREDNIMA VODNIKOMA

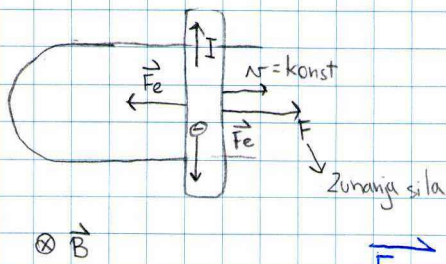


Če ločita ločena in sta smeri različni poslednje odboj, če pa sta nasprotno smer pa je poslednje privlačno-odbojna sila  
 - privlačna sila  
 Vektorski produkt mag. polja pravega vodnika z tokom



$\vec{F}_e = I \int d\vec{l} \times \vec{B}$

# INDUKCIJA



$$\vec{F} = (-e_0) \vec{v} \times \vec{B}$$

$$\vec{F}_e = I \vec{l} \times \vec{B}$$

$$\vec{F} + \vec{F}_e = 0 \iff v = \text{konst}$$

$$\vec{F}_e = I \int d\vec{l} \times \vec{B} \quad \oint \vec{E} \cdot d\vec{s} = 0$$

$$\vec{E} = \text{grad } \varphi$$

$$V = \int \vec{v} \cdot \vec{E} \cdot d\vec{s}$$

$$\vec{F}_{\text{zun}} = -\vec{F}_e = -I(\vec{l} \times \vec{B})$$

$$-I(\vec{l} \times \vec{B}) \vec{v} = UI$$

DELO:  $dA = \vec{F} \cdot \vec{v} dt = UI dt$

$\underbrace{\vec{F} \cdot \vec{v}}_{d\vec{s}} \quad \underbrace{dt}_{P}$

$$U_i dt = \vec{v} \cdot (\vec{B} \times \vec{l}) dt = -\vec{v} dt (\vec{l} \times \vec{B})$$

$$= -d\vec{s} (\vec{l} \times \vec{B}) = -B(d\vec{s} \times \vec{l})$$

$$= \vec{B} \cdot d\vec{s} \Rightarrow U_i = \frac{d\Phi_m}{dt} = -\frac{d\Phi_m}{dt}$$

$$(\vec{B} \times \vec{l}) \vec{v} = U_i$$

$$U_i = \vec{v} (\vec{B} \times \vec{l})$$

INDUKCIJSKI  
ZAKON (POSEBNA OBLIKA)

$$U_i dt = -\vec{B} \cdot d\vec{s}$$

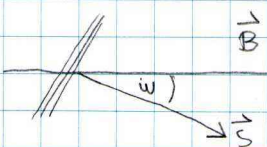
$$U_i = -\frac{d\Phi_m}{dt} = -\frac{d(\vec{B} \cdot \vec{s})}{dt} = -\frac{d\Phi_m}{dt}$$

POSPLOŠITEV:

$$U_i = -\frac{d\Phi_m}{dt}$$

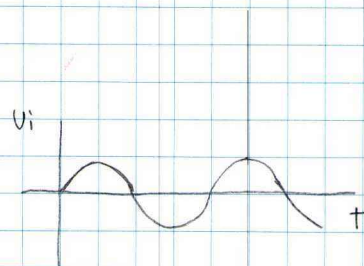
$$\Phi_m = \int \vec{B} \cdot d\vec{s}$$

PRIMER:



$$\Phi_m = \vec{B} \cdot \vec{s} = BS \cos(\omega t)$$

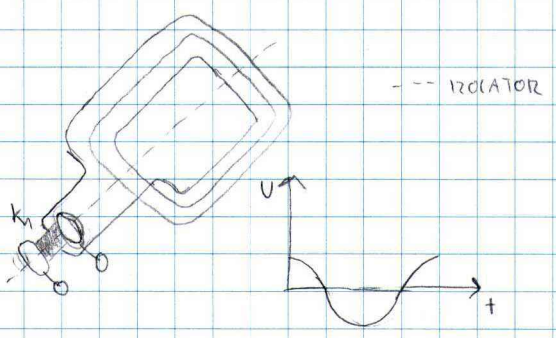
$$|U_i| = \left| \frac{d\Phi_m}{dt} \right| = BS \omega \sin(\omega t)$$



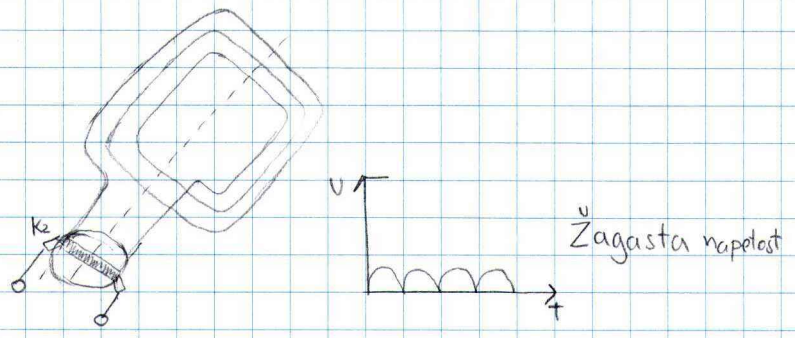
## LENZOVO PRAVILO

Inducirana napetost sproči ne tok, ki se upira spremembi, ki jo  
inducirana napetost spročila

# GENERATORJI ELEKTRIČNEGA TOKA



DINAMOSTATOR S KOLEKTORJEM ZA RMENIČNO NAPETOST



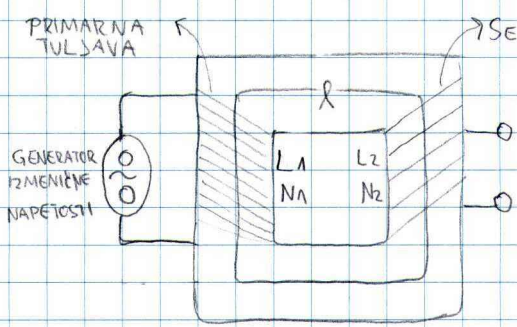
DINAMOSTATOR S KOMUTATORJEM ZA ENOSMERNO NAPETOST

$$U_i = - \frac{d\Phi_m}{dt} \Rightarrow \int U_i dt = - \int d\Phi_m = - \Phi_m \Big|_{\Phi_{m\text{zač}}}^{\Phi_{m\text{kone}}} = \Phi_{m\text{zač}} - \Phi_{m\text{kone}} = \Delta \Phi_m$$

SUNEN NAPETOSTI

kmeničen tok je spremenjen zaradi prenosa energije na velike razdalje

## TRANSFORMATOR



l - SREDNJI OBSEG SEDRA

Z istim presežamo magnetni pretok in dosežemo, da je magnetni pretok skozi tuljavo sklenjen

$$\left. \begin{aligned} U_1 &= \frac{d\Phi_1}{dt} = -N_1 \frac{d(BS)}{dt} \\ U_2 &= \frac{d\Phi_2}{dt} = -N_2 \frac{d(BS)}{dt} \end{aligned} \right\} 0/0 \Rightarrow \frac{U_{1,0}}{U_{2,0}} = \frac{N_1}{N_2}$$

$$\Phi_1 = N_1 BS \quad \Phi_2 = N_2 BS$$

- IDEALNI TRANSFORMATOR:
- 1.) Magnetna pretoka skozi tuljavi sta enaka
  - 2.) Upor obeh tuljav za eno samo napetost je zanemarljiv

Zaradi spremembe magnetnega pretoka se inducira napetost na primarni in sekundarni tuljavi:

$$\frac{(U_1)_0}{(U_2)_0} = \frac{N_1}{N_2}$$

Tokotok na sekundarni tuljavi sklenemo skozi Ohmski upor:

$$\frac{(I_1)_0}{(I_2)_0} = \frac{N_2}{N_1}$$

EFEKTIVNA MOČ:

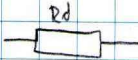
$$\bar{P} = I_g U_g$$

$$P_1 = P_2$$

$$I_1 U_{g2} = I_2 U_{g1} \Rightarrow \frac{I_1}{I_2} = \frac{U_{g2}}{U_{g1}} = \frac{U_{o2}}{U_{o1}} = \frac{N_2}{N_1}$$

## DALJNOVOD

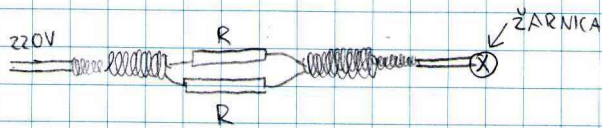
$$P = IU$$



IZGUBA NA DALJNOVODU:

$$P_d = I^2 R_d$$

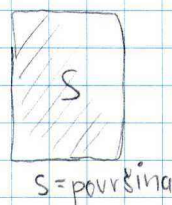
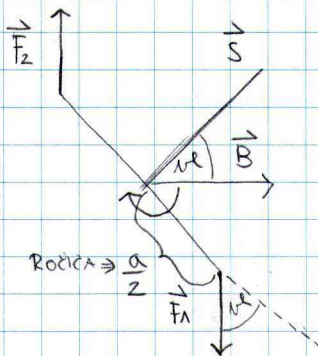
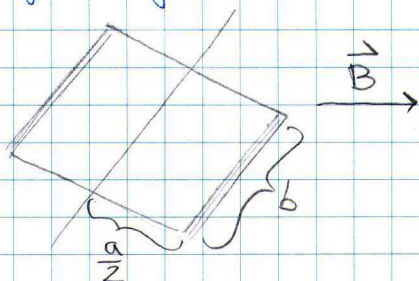
MODEL:



Na principu transformatorja deluje tudi navilni aparat

# NAVOR NA TOKOVNO ZANKO

Na tokovno zanko, ki ima magnetni dipolni moment  $\vec{p}_m$  v magnetnem polju deluje navor



Ker po žici teče tok dolimo na obeh straneh sile  
SILA NA RAVNI VODNIK V  $\vec{B}$

$$\vec{F} = I \vec{l} \times \vec{B}$$

$$M = \frac{a}{2} I b B \sin \alpha + \frac{a}{2} I b B \sin \alpha$$

$$= I a b B \sin \alpha = I S B \sin \alpha$$

$$M = I S B \sin \alpha$$

$p_m = IS \rightarrow$  magnetni dipolni moment tokovne zanke

$$p_m = IS$$

Če ima zanka več vrst:

$$p_m = NIS$$

$$\vec{M} = \vec{p}_m \times \vec{B}$$

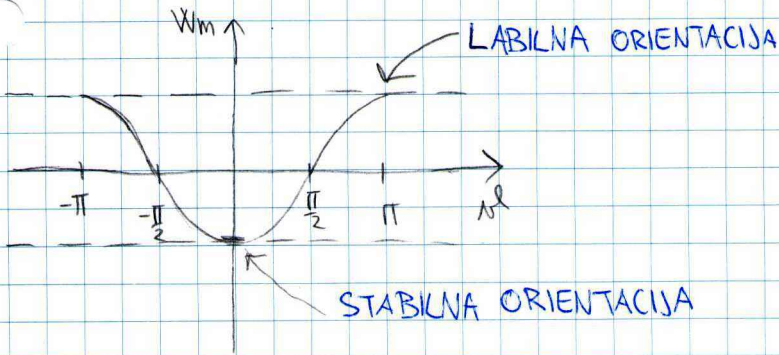
NAVOR NA TOKOVNO ZANKO  
ALI MAGNETNI DIPOLNI MOMENT  
V VEKTORSKI OBLIKI

$$A = \int M d\alpha = \int p_m B \sin \alpha d\alpha = -p_m B \cos \alpha$$

$$W_m = -p_m B \cos \alpha$$

ENERGIJA MAGNETNEGA DIPOLA  
 $\vec{p}_m$  V ZUNANJEM MAGNETNEM  
POLJU  $B$

EVA START:



$\alpha = 0: W_m = -p_m B(\min) \quad \vec{p}_m \uparrow \uparrow \vec{B}$

$\alpha = \pi: W_m = +p_m B(\max) \quad \vec{p}_m \downarrow \uparrow \vec{B}$

### NIHAJNI ČAS MAGNETNICE

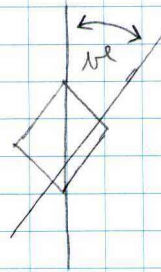
$M = J \dot{\alpha}$

$\sigma = \frac{a^2 \ddot{\alpha}}{d^2} \equiv \ddot{\alpha}$

$-p_m B \sin \alpha < J \ddot{\alpha}$

$\sin \alpha = \alpha$

$-p_m B \alpha = J \ddot{\alpha}$

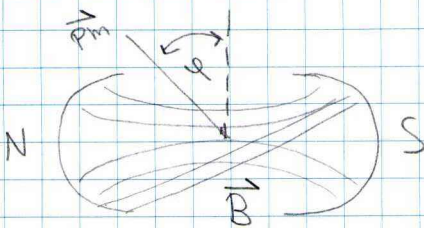


REŠITEV:  $\alpha = \alpha_0 \sin\left(\left[\frac{2\pi}{t_0}\right]t + \delta\right)$

$\ddot{\alpha} = -\alpha_0 \left(\frac{2\pi}{t_0}\right)^2 \sin\left(\left[\frac{2\pi}{t_0}\right]t + \delta\right)$

$\left(\frac{p_m B}{J}\right) \alpha = \left(\frac{2\pi}{t_0}\right)^2 \alpha \Rightarrow t_0 = 2\pi \sqrt{\frac{J}{p_m B}}$

### MAGNETNI MOMENT TULJAVE



$\vec{p}_m$  je mehako vektor pravokoten na  $\vec{B}$

$M = p_m B \sin \alpha = p_m B$

### RAVNOVESNO STANJE

$M_B = M_D$

$p_m B = D \psi$

$N I S B = D \psi$

$I = \frac{D}{N S B} \psi$

$I \propto \psi$

konstanta preobnosne vzmeti

Tok JE SORAZMERN ZASUKU

Iglič: Voda je najbolj pomembna tekočina poleg VINA!

## MERJENJE TOKOVNEGA SUNKA

$$\int I dt$$

$$\int M dt = \delta F = J \omega_0$$

$$\int_{pm} B dt \Rightarrow \int I N S B dt = J \omega$$

$$\int I dt = \frac{J \omega_0}{N S B}$$

$$\varphi = U_0 \sin\left(\frac{2\pi}{T_0} t + \varphi\right)$$

$$\omega = \frac{d\varphi}{dt} = \frac{2\pi}{T_0} U_0 \cos\left(\frac{2\pi}{T_0} t + \varphi\right)$$

EVA END

## MAXWELLOVE ENAČBE

(OSNOVNI ZAKONI ELEKTROTEHNIKE)

$$\oint \vec{D} \cdot d\vec{S} = \int \rho_e dV$$

ZAKON O ELEKTRIČNEM PRETOKU (GAUSSOV ZAKON)

$$\oint \vec{B} \cdot d\vec{S} = 0$$

ZAKON O MAGNETNEM PRETOKU

$$\oint \vec{H} \cdot d\vec{S} = \int \left( \vec{j}_e + \frac{\partial \vec{D}}{\partial t} \right) d\vec{S}$$

ZAKON O NEGATIVNI NAPETOSTI (AMPEROV ZAKON)

$$\oint \vec{E} \cdot d\vec{S} = - \int \left( \frac{\partial \rho_e}{\partial t} \right) dV$$

INDUKCIJSKI ZAKON (FARADAYEV ZAKON)

Dodatno mora veljati še zakon o ohranitvi energije

$$\oint \vec{j}_e \cdot d\vec{S} = - \int \frac{\partial \rho_e}{\partial t} dV$$

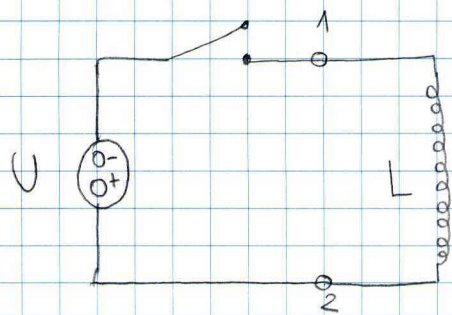
ko poznamo prostorske odčitnosti  $\vec{E}$  in  $\vec{B}$ :

$$\vec{F} = e\vec{E} + e\vec{v} \times \vec{B}$$

LORENTZOVA SILA

Voda ima dielektričnost okoli 80

# LASTNA INDUKCIJA



$$U + U_L = 0$$

Generator s konstantno pozitivno napetostjo priključimo (izključimo) na idealno tuljavo:

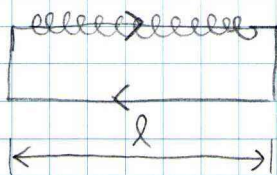
a) Ob času  $t \approx 0$  ~~po~~ priključitvi tere po tuljavi opremenljiva **NARAŠČAJOČE** el. tok. Ker je ~~dI/dt~~  $dI/dt > 0$  se v tuljavi inducira napetost, ki li po LENZOVEM pravilu el. tok v nasprotni smeri  $\Rightarrow$  **TULJAVA DELUJE KRATEK ČAS KOT UPOR**

$$\frac{dI}{dt} > 0 \Rightarrow U_L = -L \frac{dI}{dt} < 0 \Rightarrow U > 0$$

b) Ob času  $t \approx 0$  **PO IZKLSUČITVI** tere po tuljavi opremenljiva **PADAJOČE** el. tok. Ker je  $dI/dt < 0$  se v tuljavi inducira napetost, ki li po LENZOVEM pravilu el. tok v smeri začrtane toka  $\Rightarrow$  **TULJAVA DELUJE KRATEK ČAS KOT GENERATOR**

$$\frac{dI}{dt} < 0 \Rightarrow U_L = -L \frac{dI}{dt} > 0 \Rightarrow U < 0$$

## IZPELJAVA:



$$\oint \vec{B} \cdot d\vec{S} = \mu_0 I N$$

$$B_L = NI \mu_0 \Rightarrow B = \frac{NI \mu_0}{l}$$

$$\phi_m = NBS = N \frac{NI \mu_0 S}{l} = \left( \frac{\mu_0 N^2 S}{l} \right) I; \quad L = \mu_0 \frac{N^2 S}{l}$$

$$U_i = \frac{d\phi_m}{dt} \quad \leftarrow \quad \phi_m = LI$$

$$U_i = \frac{dLI}{dt} \Rightarrow U_i = -L \frac{dI}{dt}$$

# I ZMENIČNI TOK PO TULJAVI

Granilna napetost  $U_0 \sin(\omega t)$

$$U_0 \sin(\omega t) - U_L = 0$$

$$U_0 \sin(\omega t) - L \frac{dI}{dt} = 0$$

$$U_0 \sin(\omega t) = L \frac{dI}{dt}$$

$$dI = \frac{U_0}{L} \sin(\omega t) d(\omega t)$$

$$I = \frac{U_0}{L} \int \sin(\omega t) d(\omega t) = \frac{U_0}{\omega L} \cos(\omega t) = \frac{U_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$$

$$I(t) = I_0 \sin(\omega t - \frac{\pi}{2}) ; I_0 = \frac{U_0}{\omega L}$$

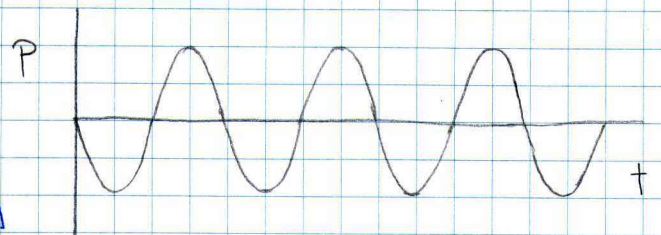
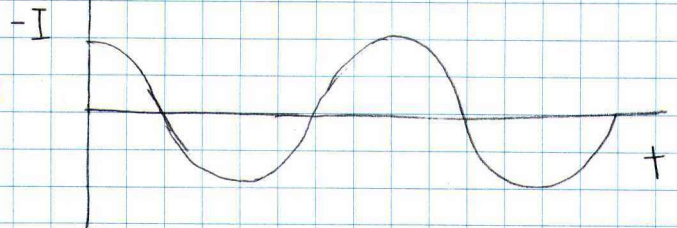
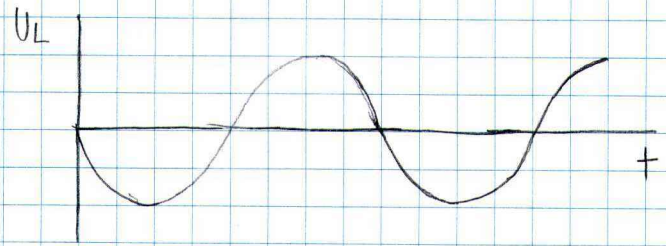
$$U_L = -U_0 \sin(\omega t) = U_0 \sin(\omega t - \pi)$$

$$\left. \begin{array}{l} I = I_0 \cos(\omega t) \\ U = U_0 \cos(\omega t - \delta) \end{array} \right\} P = IU = I_0 U_0 \cos(\omega t - \delta)$$

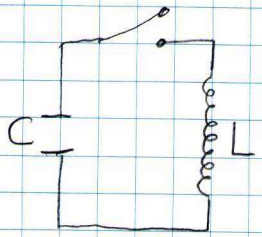
$$[\cos \omega t \cos \delta - \sin \omega t \sin \delta]$$

$$P = I_0 U_0 (\cos^2 \omega t \cos \delta - \sin^2 \omega t \sin \delta \cos \omega t)$$

$$P = I_0 U_0 \frac{1}{2} \cos \delta = \frac{1}{2} I_0 U_0 \cos \delta$$



# IDEALNI NIHAJNI KROG



$$U_L + U_C = 0 \quad e = CU$$

$$U_C(t=0) = U_0 \quad -L \frac{dI}{dt} - \frac{e}{C} = 0 \quad / \frac{d}{dt}$$

$$-L \frac{d^2 I}{dt^2} - \frac{(de)}{C} = 0$$

Rešitev:

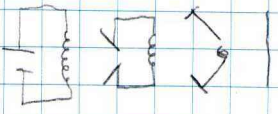
$$I = I_0 \sin(\omega_0 t)$$

$$\frac{d^2 I}{dt^2} = -I_0 \omega_0^2 \sin(\omega_0 t) = -\omega_0^2 I$$

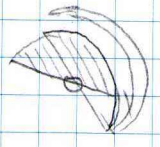
$$I(t) = -L \frac{d^2 I}{dt^2} - \frac{I}{C} = 0 \Rightarrow \frac{d^2 I}{dt^2} = -\frac{1}{LC} I$$

$$\omega_0^2 I = -\frac{1}{LC} I$$

$$\omega_0^2 = \frac{1}{LC} \quad \text{LASTNA FREKVENCA IDEALNEGA NIHAJNEGA KROGA}$$



DIPOLNA ANTENA  
(EM Valovanje)



VRTILSKI KONDENZATOR  
(Z ZASUJOM SPREMENJAMO KAPACITETO)

## NAPETOST NA TULJAVI

$$U_L = -L \frac{dI}{dt} = -LI_0 \omega_0 \cos(\omega_0 t) \rightarrow \text{Največja napetost na tuljavi}$$

## NAPETOST NA KONDENZATORJU

$$U_L + U_C = 0 \Rightarrow U_C = -U_L = LI_0 \omega_0 \cos(\omega_0 t) \quad \text{Največja napetost na kondenzatorju}$$

$$W_C = \frac{1}{2} C U_0^2$$

$$U_0 = LI_0 \omega_0$$

$$\int U_L I dt = \int L \frac{dI}{dt} I dt = \frac{1}{2} LI^2$$

ENERGIJA TULJAVE

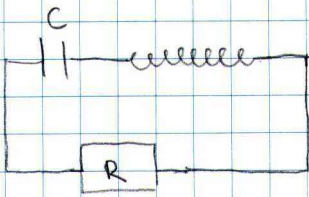
Energija delovanja nih kroga:

$$W = \frac{1}{2} C U_0^2 = \frac{1}{2} L I_0^2$$

V splošnem:

$$W = \frac{1}{2} C U^2 + \frac{1}{2} L I^2 = \frac{1}{2} C [I_0 \omega_0^2 C \cos^2(\omega_0 t)] + \frac{1}{2} L I_0^2 \sin^2(\omega_0 t) = \frac{1}{2} L I_0^2 [\cos^2(\omega_0 t) + \sin^2(\omega_0 t)] = \frac{1}{2} L I_0^2 = \frac{1}{2} C U_0^2$$

# DUŠENI ELEKTRIČNI NIHAJNI KROG



$$U_C + U_L + U_R = 0$$

$$-\frac{e}{C} = -L \frac{dI}{dt} - RI = 0 \quad / \frac{d}{dt}$$

$$-\frac{I}{C} - L \frac{d^2 I}{dt^2} = -R \frac{dI}{dt} = 0$$

$$-\frac{I}{LC} - \frac{d^2 I}{dt^2} + \frac{R}{L} \frac{dI}{dt} = 0$$

$$\frac{d^2}{dt^2} + \frac{R}{L} \frac{dI}{dt} + I\omega_0^2 = 0 \Rightarrow I(t)$$

NASTAVEK ZA REŠITEV:  $I = I_0 e^{-\beta t} \cos(\omega_0 t)$

$\beta$  - koeficient dušenja

$$e^{i\omega t} = \cos(\omega t) + i \sin(\omega t)$$

$$I = I_0 e^{-\beta t} e^{i\omega_0 t} = I_0 e^{(i\omega_0 - \beta)t}$$

$$\frac{dI}{dt} = I_0 (i\omega_0 - \beta) e^{(i\omega_0 - \beta)t} = I_0 (-\omega_0^2 - 2\omega_0 \beta i + \beta^2) e^{(i\omega_0 - \beta)t}$$

$$I_0 e^{(i\omega_0 - \beta)t} \left[ -\omega_0^2 - 2\omega_0 \beta i + \beta^2 + \frac{R}{L} (i\omega_0) - \frac{R}{L} \beta + \omega_0^2 \right] = 0$$

$$\underbrace{\left( -\omega_0^2 + \beta^2 - \frac{R}{L} \beta + \omega_0^2 \right)}_{=0} + i \underbrace{\left( \frac{R}{L} - 2\omega_0 \beta \right)}_{=0} = 0$$

$$-\omega_0^2 + \beta^2 - \frac{R}{L} \beta + \omega_0^2 = 0$$

$$-\omega_0^2 + \beta^2 - \omega_0^2 = 0$$

$$\frac{R}{L} = 2\beta \Rightarrow \beta = \frac{1}{2} \frac{R}{L}$$

$$\Rightarrow \omega_0' = \sqrt{\omega_0^2 - \beta^2}$$

$\Rightarrow$  Če  $R=0$ :

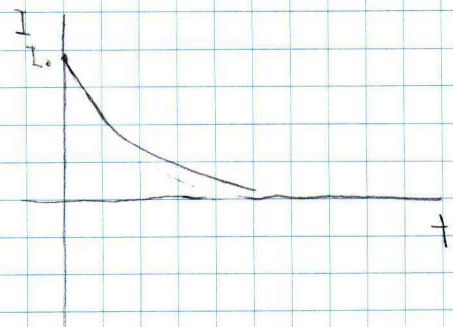
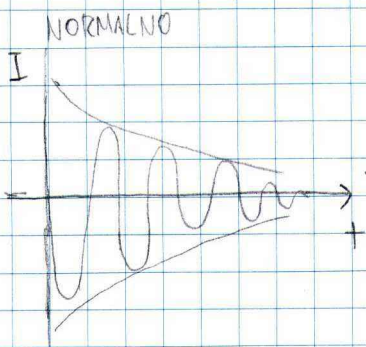
$$\begin{aligned} \beta &= 0 \\ \omega_0' &= \omega_0 \\ I &= I_0 \cos(\omega t) \end{aligned}$$

$$\beta > \omega_0: \omega_0' = \sqrt{\omega_0^2 - \beta^2} = i \sqrt{\beta^2 - \omega_0^2} = i\delta$$

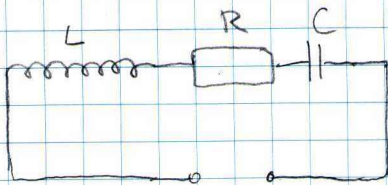
$$I = I_0 e^{-\beta t} e^{i\omega_0 t} = I_0 e^{-\beta t} e^{-\delta t}$$

$$\beta > \omega_0: I = I_0 e^{-(\beta + \sqrt{\beta^2 - \omega_0^2}) t}$$

$$\beta < \omega_0: I = I_0 e^{-\beta t} \cos(\omega_0 t + \delta)$$



# V SILSNO NIHANJE EL. NIHANEGA KROGA



$$U_g = U_0 \cos(\omega t)$$

$$I = I_0 e^{i\omega t}$$

$$U_g = U_0 e^{i\omega t}$$

$$I_0 \in \mathbb{R}_e$$

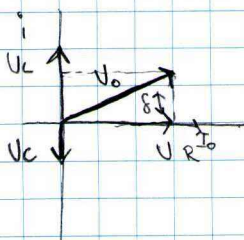
$$U_0 \in \mathbb{C}$$

$$U_g + U_L + U_R + U_C = 0$$

$$U_g - L \frac{dI}{dt} - IR - \frac{e}{C} = 0 \quad / \frac{d}{dt}$$

$$\frac{dU_g}{dt} - L \frac{d^2 I}{dt^2} - R \frac{dI}{dt} - \frac{I}{C} = 0$$

KAZALČNI DIAGRAM



$$e^{i\omega t} \left[ \underbrace{U_0 i\omega + LI\omega^2 - IRi\omega - \frac{I}{C}}_{=0} \right] = 0$$

$$i\omega U_0 + LI\omega^2 - IRi\omega - \frac{I}{C} = 0 \quad / i\omega$$

$$U_0 = -\frac{LI\omega^2}{i\omega} + IR + \frac{I}{i\omega C}$$

$Z = \frac{U_0}{I} \rightarrow$  IMPEDANCA

$$U_0 = I_0 \left[ \underbrace{+L\omega i}_{z_L} + \underbrace{R}_{z_R} - \underbrace{\frac{i}{\omega C}}_{z_C} \right] = I_0 \left[ R + i \left( L\omega - \frac{1}{\omega C} \right) \right]$$

$$U_0 = \underbrace{I_0 L\omega i}_{U_L} - \underbrace{I_0 \frac{i}{\omega C}}_{U_C} + \underbrace{I_0 R}_{U_R}$$

$$z_R = R$$

$$z_L = L\omega i$$

$$z_C = \frac{i}{\omega C}$$

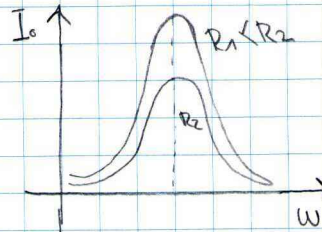
$$U_0 = I_0 Z$$

$$Z = z_R + z_L + z_C$$

$$U_0^2 = U_0 U_0^* = \left[ R^2 + \left( L\omega - \frac{1}{\omega C} \right)^2 \right] I_0^2 \Rightarrow I_0 = \frac{|U_0|}{\sqrt{R^2 + \left( L\omega - \frac{1}{\omega C} \right)^2}}$$

FAZNI ZAMIK:

$$\tan \delta = \frac{L\omega - \frac{1}{\omega C}}{R} \quad ; \quad I_0 = \frac{|U_0|}{\sqrt{R^2 + \left( L\omega - \frac{1}{\omega C} \right)^2}}$$

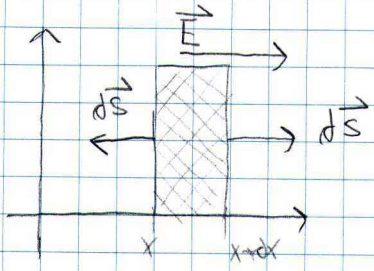


$$\text{MAX: } L\omega_{\text{RES}} - \frac{1}{\omega_{\text{RES}} C} = 0 \Rightarrow \omega_{\text{RES}}^2 = \frac{1}{LC} = \omega_0^2$$

$$\bar{P} = \frac{1}{2} I_0 |U_0| \cos \delta$$

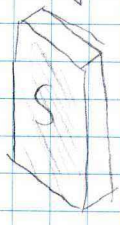
$$U_R = I_0 R$$

# POISSONOVA ENAČBA



$\rho(x)$  = volumetska porazdelitev naboja

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = \int \rho dV$$



$$\epsilon_0 [E(x+dx)S - E(x)S] = \rho(x) dx S$$

$$\epsilon_0 \frac{dE}{dx} = \rho(x)$$

$$\epsilon_0 \frac{dE}{dx} = \rho(x)$$

$$E = -\frac{d\phi}{dx}$$

$$\epsilon_0 \frac{d}{dx} \left( \frac{d\phi}{dx} \right) = \rho(x)$$

$$\frac{d^2 \phi}{dx^2} = -\frac{\rho(x)}{\epsilon_0}$$

POISSONOVA ENAČBA V 1-DIMENZIJI

## ISKLOM

$\vec{E} = -\text{grad} \phi$

$$\epsilon_0 \oint \vec{E} \cdot d\vec{S} = \int \rho dV \Rightarrow \epsilon_0 \text{div} \vec{E} = \rho(\vec{r})$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \rho(\vec{r})$$

$$\vec{\nabla} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$$\epsilon_0 \left[ \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = \rho(\vec{r})$$

V-3D:

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \rho(\vec{r})$$

$$\epsilon_0 \vec{\nabla} \cdot (-\nabla \phi) = \rho(\vec{r})$$

$$-\epsilon_0 \nabla^2 \phi = \rho(\vec{r}) \Rightarrow \nabla^2 \phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

$$\Delta \phi = \frac{\rho}{\epsilon_0}$$

POISSONOVA ENAČBA V 3-DIMENZIJI

$\Delta = \nabla^2$   $\Delta \equiv$  Laplace

$$\Delta \phi(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}, \quad \Delta = \vec{\nabla} \cdot \vec{\nabla} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

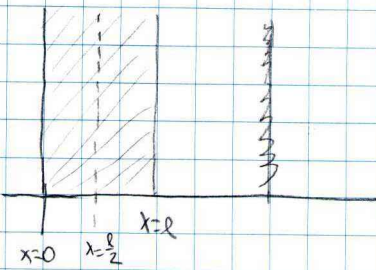
$$\frac{\partial^2 \phi(\vec{r})}{\partial x^2} + \frac{\partial^2 \phi(\vec{r})}{\partial y^2} + \frac{\partial^2 \phi(\vec{r})}{\partial z^2} = -\frac{\rho(\vec{r})}{\epsilon_0}$$

DIELEKTRIČNA "KONSTANTA"  $\epsilon_r(\vec{r})$

$$\vec{\nabla} \cdot [\epsilon_0 \epsilon_r(\vec{r}) \vec{E}] = \rho(\vec{r})$$

Restoracija FEM - Comsol

PP - FUNKSIAN NABOS



$$\frac{d^2\varphi}{dx^2} = \frac{\rho_0}{\epsilon_0}, \quad \rho_0 = \text{konstant}$$

$$\frac{d\varphi}{dx} = -\frac{\rho_0}{\epsilon_0}x + B$$

$$\varphi(x) = -\frac{\rho_0 x^2}{2\epsilon_0} + Bx + C$$

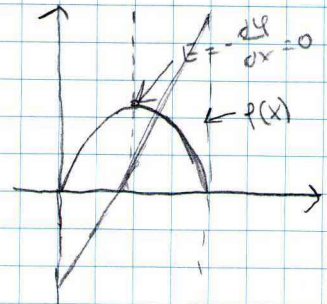
$$\varphi(x=\frac{l}{2})=0 \Rightarrow E(x=\frac{l}{2})=0$$

$$E(x) = -\frac{d\varphi}{dx} = \frac{\rho_0}{\epsilon_0}x - B$$

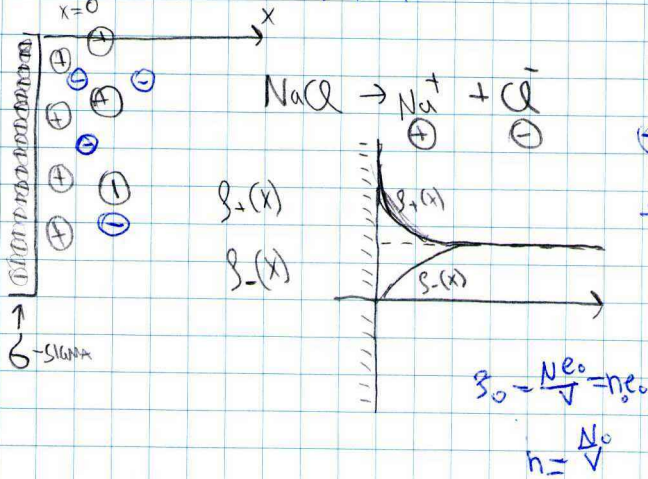
$$E(x=\frac{l}{2}) = \frac{\rho_0 l}{2\epsilon_0} - B = 0 \Rightarrow B = \frac{\rho_0 l}{2\epsilon_0}$$

$$\varphi(x) = -\frac{\rho_0 x^2}{2\epsilon_0} + \frac{\rho_0 l}{2\epsilon_0}x$$

$$\varphi(x=0), \varphi(x=l) = 0$$



PP2: - EL. DVO SNA PLAST



IDEALNI PLIN MAXWELLONA PORAZENJEV

$$e^{-\frac{W_{pot}}{kT}} \quad \text{BOUZMA NAVI FAKTOR}$$

$$\begin{aligned} \oplus W_{pot} = e_0 \varphi(x) &\Rightarrow \rho_+(x) = \rho_0 e^{-\frac{W_{pot}}{kT}} = \rho_0 e^{-\frac{e_0 \varphi}{kT}} \\ \ominus W_{pot} = -e_0 \varphi(x) &\Rightarrow \rho_-(x) = \rho_0 e^{-\frac{W_{pot}}{kT}} = \rho_0 e^{-\frac{-e_0 \varphi}{kT}} \end{aligned}$$

$$\Rightarrow \rho(x) = \rho_+(x) + \rho_-(x) = \rho_0 e^{-\frac{e_0 \varphi}{kT}} + \rho_0 e^{\frac{e_0 \varphi}{kT}}$$

$$\frac{d^2\varphi}{dx^2} = \frac{\rho_0}{\epsilon_0} \left( e^{-\frac{e_0 \varphi}{kT}} + e^{\frac{e_0 \varphi}{kT}} \right) \approx \frac{2\rho_0}{\epsilon_0} \text{sh}\left(\frac{e_0 \varphi}{kT}\right)$$

$$\frac{d^2\varphi}{dx^2} = \frac{2\rho_0}{\epsilon_0} \text{sh}\left(\frac{e_0 \varphi}{kT}\right) = \frac{2n_0 e_0}{\epsilon_0} \text{sh}\left(\frac{e_0 \varphi}{kT}\right)$$

$$\epsilon_0 \rightarrow \epsilon_r \epsilon_0$$

$$\epsilon_r \text{ H}_2\text{O} \approx 78,5$$

$$\frac{d^2\varphi}{dx^2} = \frac{2n_0 e_0}{\epsilon_r \epsilon_0} \text{sh}\left(\frac{e_0 \varphi}{kT}\right)$$

$$\frac{d^2\varphi}{dx^2} = \frac{zn_0 e_0}{\epsilon_r \epsilon_0} \left( \frac{e_0 \varphi}{kT} \right) + \frac{e_0 \varphi}{kT}$$

RESITEV ZA MAJHNE  $\frac{e_0 \varphi}{kT} < 1$

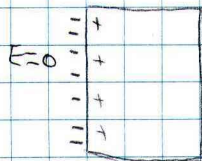
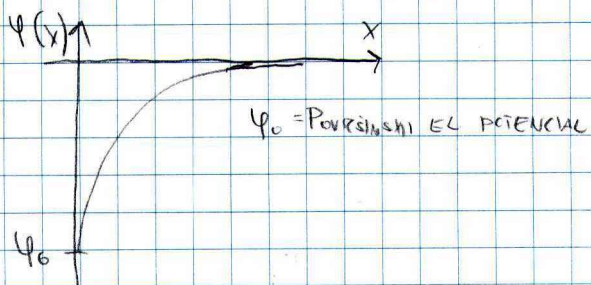
$$kT = \frac{1}{40} e_0 V$$

$$\frac{d^2\varphi}{dx^2} = \frac{zn_0 e_0}{\epsilon_r \epsilon_0} \frac{e_0 \varphi}{kT}$$

$$\frac{d^2\varphi}{dx^2} = \frac{zn_0 e_0^2}{\epsilon_r \epsilon_0 kT} \varphi \rightarrow K^2$$

$$\Rightarrow \lambda_D = \frac{2n_0 e_0}{\epsilon_r \epsilon_0 kT}$$

$\frac{d^2\varphi}{dx^2} = K^2 \varphi$  LINEARIZIRANA  
PB ENACBA

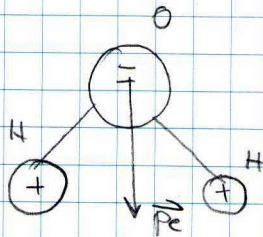


# DIELEKTRIČNE LASTNOSTI SNOVI

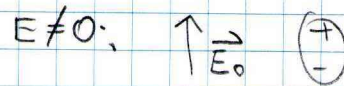
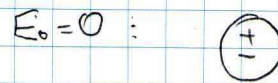
**POLARNE** - Molekule imajo permanentni dipolni moment

**NEPOLARNE** - imajo veliki dipolni moment enak 0. Če jih damo v el. polje pa dolže inducirani dipolni moment - nastane permanentni.

POLARNA MOLEKULA

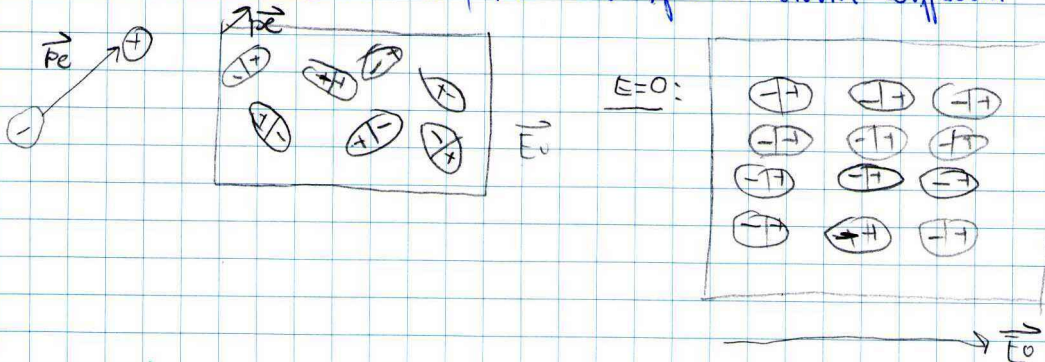


NEPOLARNA MOLEKULA

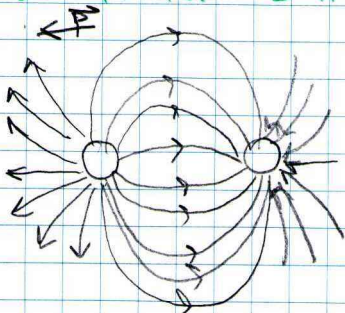


INDUCIRANI DIPOLNI MOMENT

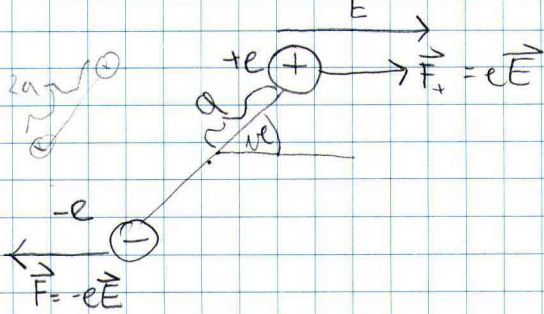
⇒ Voda obtehljavnost okoli 80, ker ima permanentni dipolni moment.



ELEKTRIČNI DIPOL



# SILA IN NAVOR V ZUNANJEM POLJU DIPOLA



$$M = M_+ + M_- = aeE + (-a)(-e)E = 2aeE \sin \alpha$$

$$M = edE \sin \alpha = p_e E \sin \alpha$$

$$\vec{M} = \vec{p}_e \times \vec{E}$$

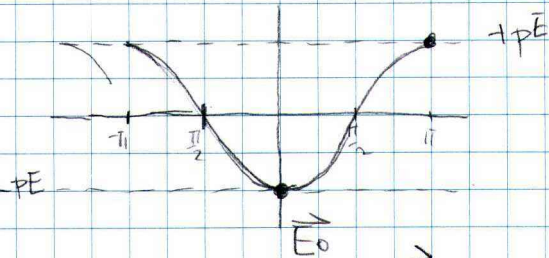
$$W_e = \int M d\alpha = \int p_e E \sin \alpha d\alpha = -p_e E \cos \alpha \Big|_{\alpha=0}^{\alpha=\pi} = -\vec{p}_e \cdot \vec{E}$$

$$W_e = -\vec{p}_e \cdot \vec{E}$$

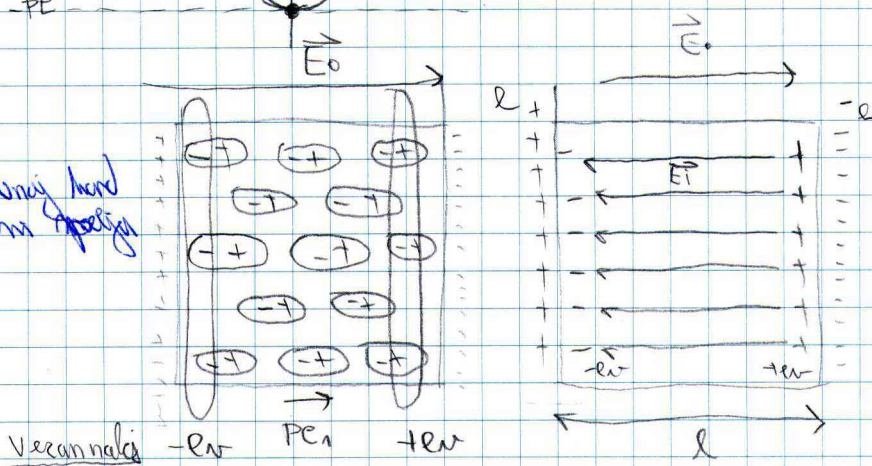
$$\begin{aligned} \alpha=0 & \uparrow p_e \uparrow E : W_e = -p_e E \\ \alpha=\pi & \uparrow p_e \uparrow E : W_e = +p_e E \end{aligned}$$

- Najmanjša energija  
- Največja energija

Večji timp manj stabilnosti; manjša delj razenitve



Zunaj hard  
in splošno



$$\begin{aligned} \langle \rho(x) \rangle &= \rho(x) - \frac{dP}{dx} \\ \langle \rho(x) \rangle &= \rho(x) - \nabla \cdot \vec{P} \\ E_0 \cdot \vec{E} &= \langle \rho(x) \rangle \\ \nabla \cdot (E_0 \vec{P}) &= \langle \rho(x) \rangle \end{aligned}$$

Polarizacija

CELOTNO POLJE:

$$\begin{aligned} E &= E_0 - E_i \\ E &= E_0 - \frac{P}{\epsilon_0} \end{aligned}$$

POLARIZACIJA:

$$\vec{P} = n \langle \vec{p}_e \rangle, n = \frac{N}{V}$$

DIPOLNI MOMENT VER. NAJBA:  $eVl = P$   
 $N \langle \vec{p}_e \rangle = eVl$

OD ENÉ MOL:

$$Pv = n \langle \vec{p}_e \rangle V = N \langle \vec{p}_e \rangle = N \langle \vec{p} \rangle$$

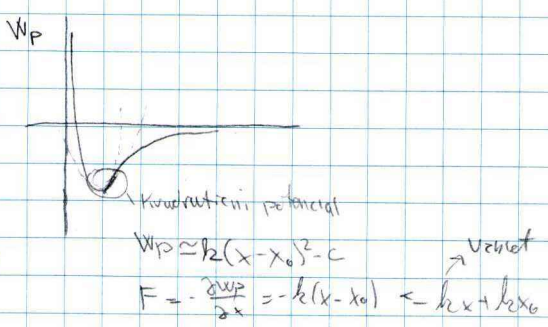
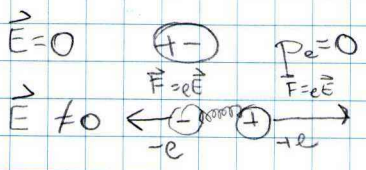
$N \langle \vec{p}_e \rangle = \chi \epsilon_0 E$   
 $PS \chi = \epsilon_0 \chi E \Rightarrow \chi E = PS$

$\Rightarrow E = \epsilon_0 - \frac{PS}{\epsilon_0} / \epsilon_0 \Rightarrow \epsilon_0 E = \epsilon_0 E_0 - P \Rightarrow \epsilon_0 \vec{E}_0 = \epsilon_0 \vec{E} + P$   
 $\boxed{\epsilon_0 \vec{E}_0 = \epsilon_0 \vec{E} + P}$   
 $\boxed{D = \epsilon_0 \vec{E} + P}$

$P = \epsilon_0 \chi E$   
 $\leftarrow$  SUSCEPTIBILNOST

$D = \epsilon_0 E + \epsilon_0 \chi E$   
 $D = \epsilon_0 (1 + \chi) E$   
 $\epsilon = 1 + \chi$   $\epsilon$ -dielektričnost

NEPOLARNE MOLEKULE

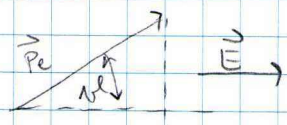


$eE = kx$  INDUCIRANI DIPOLNI MOMENT  
 $d = \frac{eE}{k} \Rightarrow p_e = de = \frac{e^2 E}{k}$

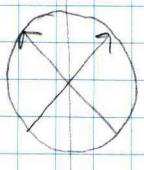
$P_e \propto E$   $\boxed{P_e = \frac{e^2 E}{k}}$   $P = n P_e = \frac{n e^2 E}{k}$   $\epsilon_0 \chi = \frac{n e^2}{k}$

POLARNE MOLEKULE

$P = \epsilon_0 \chi E$   $\chi = \frac{n e^2}{\epsilon_0 k}$   $\epsilon_0 = 1 + \chi = 1 + \frac{n e^2}{\epsilon_0 k} \lim_{E \rightarrow \infty} \epsilon \rightarrow 1$

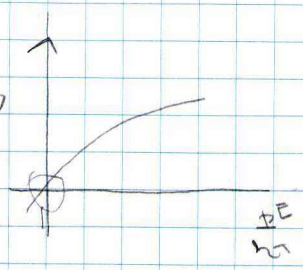


$p = n \langle \vec{p}_e \rangle = n \langle p_e \cos \alpha \rangle = n p_e \langle \cos \alpha \rangle$



$\langle \cos \alpha \rangle = \frac{\int \cos \alpha d\Omega}{\int d\Omega}$   $\text{za } E=0$

$W_d = -P_e E \cos \alpha$



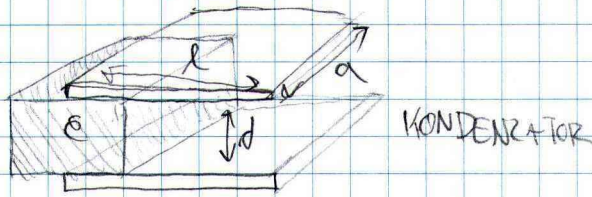
za  $\frac{P_e E}{kT} \ll 1$ :  $\langle \cos \alpha \rangle \approx \frac{P_e E}{3kT}$

$P = n P_e \langle \cos \alpha \rangle = \frac{n P_e^2 E}{3kT} = \epsilon_0 \chi E$

$\Rightarrow \epsilon_0 \chi = \frac{n P_e^2}{3kT} \Rightarrow \chi = \frac{n p_e^2}{3 \epsilon_0 kT}$

$\epsilon = 1 + \chi = 1 + \frac{n p_e^2}{3 \epsilon_0 kT}$

# SILA NA DIELEKTRIK



Ko nateramo dielektrik med plošči kondenzatorja (do globine  $x$ ) se moč na ploščah spremeni

$$e = C_0 U_0 = \epsilon_0 \frac{S}{d} U_0 \Rightarrow \text{konstanta razne } x$$

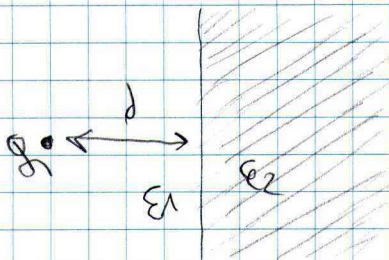
Imej  $C_0 = \epsilon_0 \frac{S}{d}$  = kapaciteta pri  $x=0$  ni  $U_0$  napetost med ploščama;

Pri  $x=x_0$ : Sistem obravnava kot dva neperiodno vezna kondenzatorja

$$W(x) = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} \frac{C_0^2 U_0^2}{\epsilon_0 \frac{ax}{d} + \epsilon_0 \frac{a(l-x)}{d}} = \frac{1}{2} \frac{C_0^2 U_0^2}{\epsilon_0 \frac{ax}{d} (\epsilon - 1) + \epsilon_0 \frac{al}{d}}$$

SILA NA DIELEKTRIK = sila med ploščama med ploščama ni dipol v dielektriku

$$F = - \frac{dW(x)}{dx} = \frac{1}{2} U_0^2 \frac{\epsilon_0 a l}{d} \frac{(\epsilon - 1)}{(\epsilon - 1)x + l}$$

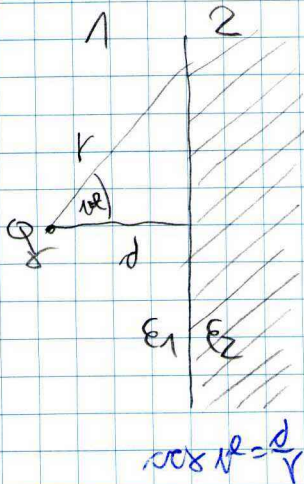


$\epsilon_1 > \epsilon_2 \Rightarrow F_{x0}$  odbojna sila stran od meje plošče

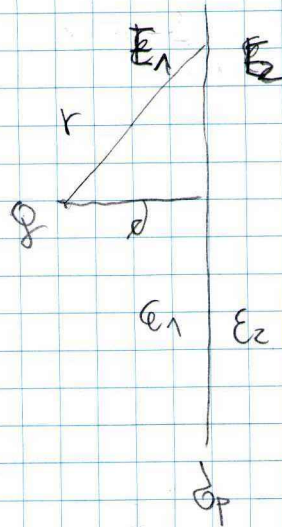
$\epsilon_1 < \epsilon_2 \Rightarrow F_{x0}$  privlačna sila proti meji plošči

$$F = \frac{1}{4\pi\epsilon_0 \epsilon_1} \frac{(\epsilon_1 - \epsilon_2)}{(\epsilon_1 + \epsilon_2)} \frac{q^2}{4d^2}$$

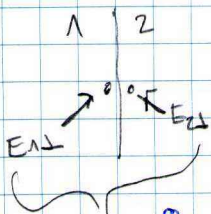
# SILA NA TOČKASTI NABOJ V BLIZINI MESE DVEH DIELEKTRIKOV



MODEL:  
 Predpostaviti, da  
 navedeni tudi  $\epsilon = \epsilon_0$   
 različno  $\epsilon$  in  
 pri upoštevanju  
 s površinsko  
 nabojem  $\sigma_p$   
 privedu



$\cos \alpha = \frac{d}{r}$



$E_p = \frac{\sigma_p}{2\epsilon_0\epsilon_0}$

$E_{1\perp} = \frac{q}{4\pi\epsilon_0\epsilon_1} \frac{1}{r^2} \frac{d}{r} - \frac{\sigma_p}{2\epsilon_0\epsilon_1}$   
 $E_{2\perp} = \frac{q}{4\pi\epsilon_0\epsilon_2} \frac{1}{r^2} \frac{d}{r} + \frac{\sigma_p}{2\epsilon_0\epsilon_2}$

Vr. polja skozi stični površini

ROBNI POGOJ

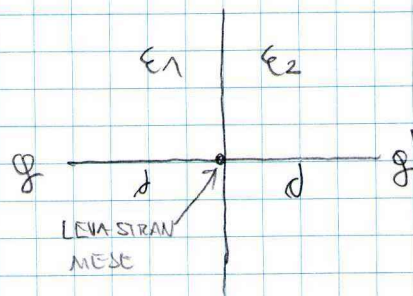
$\Rightarrow -\epsilon_1 E_{1\perp} + \epsilon_2 E_{2\perp} = 0 \Rightarrow \sigma_p \Rightarrow E_p = \frac{\sigma_p}{2\epsilon_0\epsilon_1} = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \frac{1}{4\pi\epsilon_0\epsilon_1} \frac{q}{r^2} \frac{d}{r}$

ROBNI POGOJ

$-\epsilon_1 E_{1\perp} dA + \epsilon_2 E_{2\perp} dA = 0$

PRISPEVEK in električnem polju na levi strani  
 med zračar  $\epsilon_1 \neq \epsilon_0$  napeta ima polje  
 kakovostni naboj  $q'$

$q' = \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} q$  v mediju  $\epsilon = \epsilon_1$



# SNOV V MAGNETNEM POLJU

$W_p = -\vec{p}_m \cdot \vec{B}$

$\rightarrow \begin{matrix} \mu^r = 0: & W_p = -\vec{p}_m \cdot \vec{B} & \vec{p}_m \uparrow \uparrow \vec{B} \\ \mu^r = -1: & W_p = +\vec{p}_m \cdot \vec{B} & \vec{p}_m \downarrow \uparrow \vec{B} \end{matrix}$

$\vec{p}_m \uparrow \uparrow \vec{B}$

Vzmenjajo mag. polje se preferenčni magnetni materialji in smer zmanjšuje polje, in pa n. sklobo z lencami proučimo

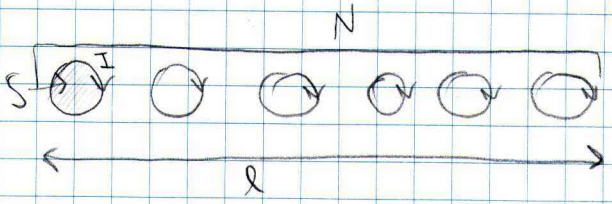
$B = B_0 + B_m$  → Notk. mag. polje

- Paramagneti - permanentni dipolni momenti
- Diamagneti - Nimajo permanentnega dipolnega momenta

Vsaki paramagnet je tudi diamagnet

## 1-DIMENZIONALNI PRIMER

MAGNETIZACIJA:  $\vec{M} = n \langle \vec{p}_m \rangle, n = \frac{N}{V}$



$B_m = \mu_0 \frac{NI}{l} \frac{S}{S} = \mu_0 \frac{N}{V} p_m$

$\frac{NI}{l} = M$

$V = lS$  VOLUME

$I S = p_m$  MAGNETNI MOMENT

$B_m = \mu_0 \vec{M}$

Magnetizacija je odvisna od magnetnega polja

$M = \chi H$

↑ SUSCEPTIBILNOST

$$B = B_0 + B_m = B_0 + \mu_0 M \quad \rightarrow M = \chi H =$$

$$B = \mu_0 H + \mu_0 \chi H = \mu_0 (1 + \chi) H$$

$1 + \chi = \mu \Rightarrow$  PERMEABILNOST

$$\vec{B} = \mu_0 \mu \vec{H}$$

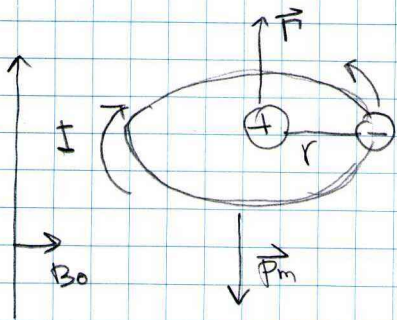
PARAMAGNETI  $\rightarrow$  Susceptibilnost  $> 0$  , Permeabilnost  $> 1$

DIAMAGNETI:  $\rightarrow$  Susceptibilnost  $< 0$  , Permeabilnost  $< 1$

$\chi$  zelo velika  $\Rightarrow \mu \gg 1$  } FEROMAGNETI  
 $B$  ni  $H$  medla odzivanja

### ISINGOV MODEL

### KLASIČNI MODEL DIAMAGNETIZMA

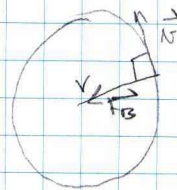


$\vec{\Gamma}$  VRTILNA KOLIČINA  
 $\vec{\Gamma} = m \vec{r} \times \vec{v}$

$$p_m = IS = \frac{e_0 \omega r^2}{2\pi R} \\ = \frac{e_0}{2me} \frac{m_e \omega r^2}{\Gamma}$$

$$S = \pi r^2 \\ I = \frac{e_0}{T_0} = \frac{e_0}{2\pi r}$$

$$T_0 = \frac{2\pi r}{v}$$



$$\vec{F}_B = (-e_0) \vec{v} \times \vec{B}$$

kvantni zakon:

$$m_e a r = F_B$$

$$m_e \frac{v^2}{r} = e_0 v B \quad / r$$

$$m_e v = e_0 r B$$

$$\Gamma = m_e v r = e_0 r^2 B$$

$$\vec{p}_m = - \frac{e_0}{2me} \vec{\Gamma}$$

$$p_m = \frac{e_0}{2me} e_0 r^2 \mu_0 H$$

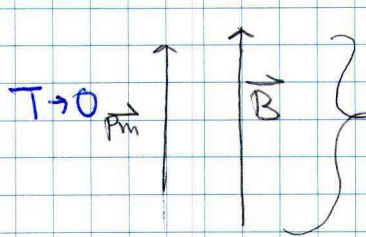
$$M = n p_m = - \frac{n e_0^2 r^2}{2me} H$$

$$M = - \frac{N_0 n e_0^2 \langle r^2 \rangle}{2me} H \Rightarrow \chi = - \frac{N_0 n e_0^2 \langle r^2 \rangle}{2me}$$

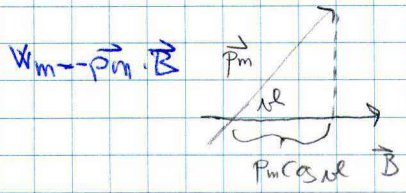
$\chi$  SUSCEPTIBILNOST

$$\chi \approx 10^{-5}$$

# PARAMAGNETI



$\alpha = 0$   
 $W_p = -pmB$



magnetizacija:  $\vec{M} = n \langle \vec{pm} \rangle$   
 $n = \frac{N}{V}$   $\langle \cos \alpha \rangle = \frac{\int \cos \alpha e^{-\frac{W_m}{kT}} d\Omega}{\int e^{-\frac{W_m}{kT}} d\Omega}$

Zemljeno energija  $\sim kT$

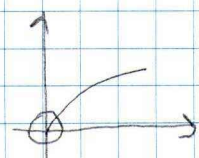
$\mathcal{L}(x) =$  LANGMUIROVA funkcija

$\Rightarrow \frac{pmB}{kT} \ll 1 : \approx \frac{pmB}{3kT}$

$M = n p m \langle \cos \alpha \rangle = \frac{n p m^2 B_0}{3kT} \Rightarrow B_0 = \mu_0 H = \frac{n \mu_0 p m^2}{3kT} H$

$M = \chi H$   
 $\chi =$  SUSCEPTIBILNOST

Če je  $\frac{pmB_0}{kT} \ll 1$ :  $\chi = \frac{n \mu_0 p m^2}{3kT}$   
 $\mu = 1 + \chi = 1 + \frac{n \mu_0 p m^2}{3kT}$

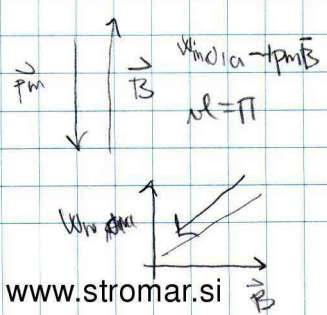
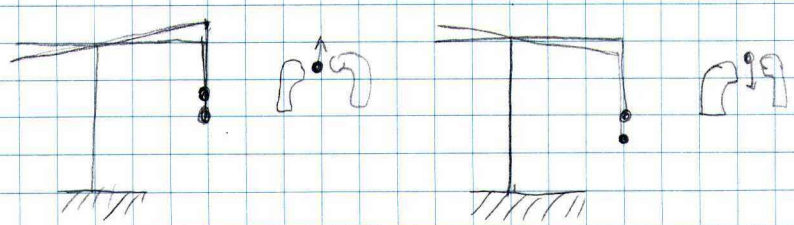


$M = C \frac{B_0}{T}$  Curieov zakon  
 $C = \frac{n p m^2}{3k}$

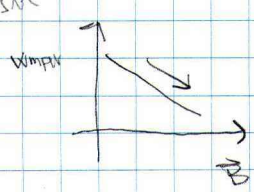
## POSKUS

DIAMAGNET

PARAMAGNET



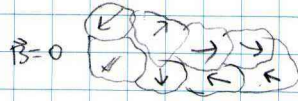
$W_m = \vec{pm} \cdot \vec{B} = -pmB \cos \alpha$   
 $W_{m,par} = -pmB$   
 $\alpha = 0$



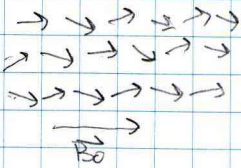
# FEROMAGNETI

## Isingov model

MAGNETNO STANJE SNOVI - s pomembno domeno revidije

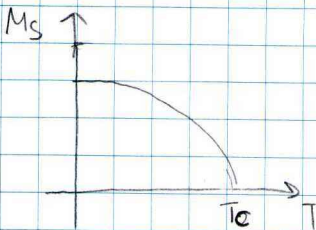


Vrste feromagnetni so tisti paramagnetni in temperaturo dovolj povišano zaradi fuzije prehodna



Vrednoti domene v znanjuim mag-polju  $\vec{B}_0$   
 $N$  ni konstanta  
 $N \Rightarrow$  povpr. pomanjšanost

$$\mu(H) = \frac{1}{\mu_0} \frac{\partial B}{\partial H}$$

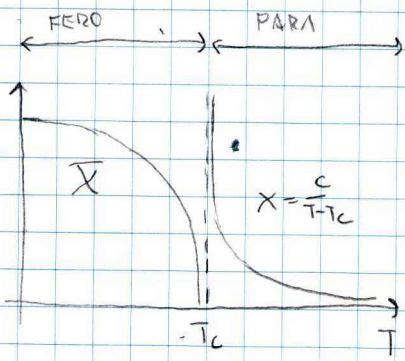


$T_c \rightarrow$  Curiejeva temp.  
 Spontana magnetizacija znižaj domene

če je  $T \geq T_c$  feromagnetni snovi izgubijo spontano magnetizacijo in postanejo paramagnetni.

za  $T < T_c$  za feromagnetne velja Curie-Weissov zakon

$$\chi = \frac{C}{T - T_c}$$

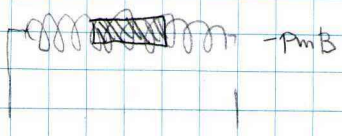


$\chi > 0 \Rightarrow \mu > 1$  : PARAMAGNETI  
 $\chi < 0 \Rightarrow \mu < 1$  : DIAMAGNETI

$\chi$  zelo velika  $\Rightarrow \mu(H) \gg 1$   
 $B$  in  $H$  nista sorazmerna : feromagnetne snovi

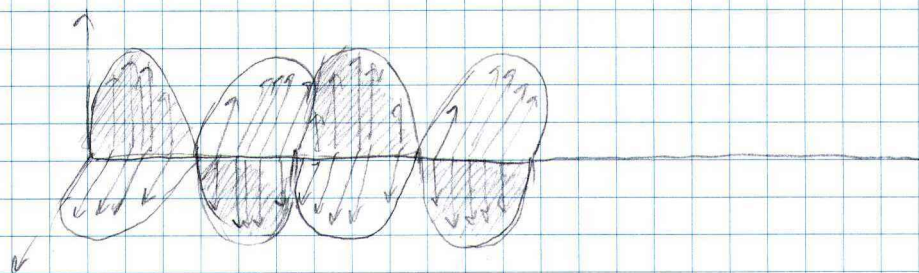
## BERKHANSEN-OV POKUS

Snov v tuljavo obkroži pol polja in v magnet



Magnetna revidija, Dipolni momenti se zaradi magnetnega polja obravnavajo kot mali magneti.  
 $U_i = -\frac{d\Phi_m}{dt}$   
 Snovi se znotraj obravnavajo kot domene

# ELEKTRO - MAGNETNO VALOVANJE



MAXWELLOVE ENAČBE:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{s}$$

$$\oint \vec{H} \cdot d\vec{s} = \frac{\partial}{\partial t} \int \vec{D} \cdot d\vec{s}$$

$$\left. \begin{aligned} \frac{\partial^2 E}{\partial x^2} &= \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 C}{\partial x^2} \\ \frac{\partial^2 H}{\partial x^2} &= \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 H}{\partial x^2} \end{aligned} \right\} \text{VALOVNI ENAČBI}$$

Rezultat: Potujejo valovanja:

$$E = E_0 \cos(\omega t - kx)$$

$$\left. \begin{aligned} \frac{\partial^2 E}{\partial x^2} &= -E_0 k^2 \cos(\omega t - kx) \\ \frac{\partial^2 E}{\partial t^2} &= -E_0 \omega^2 \cos(\omega t - kx) \end{aligned} \right\} \text{0/0}$$

$$\Rightarrow k = \frac{\omega}{c_0} \quad \text{H-tost sinjnih valov}$$

$$\frac{\frac{\partial^2 E}{\partial x^2}}{\frac{\partial^2 E}{\partial t^2}} = \frac{k^2}{\omega^2} = \frac{\omega^2}{\omega^2 c_0^2} = \frac{1}{c_0^2} \Rightarrow \frac{\partial^2 E}{\partial x^2} = c_0^2 \frac{\partial^2 E}{\partial t^2}$$

$$c_0^2 = \frac{1}{\epsilon_0 \mu_0}$$

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$c_0 \approx 3 \cdot 10^8 \text{ m/s}$$

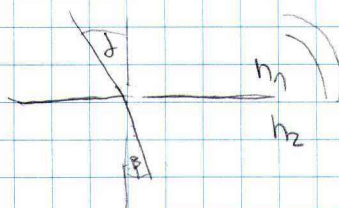
$$\epsilon \neq 1 \quad \epsilon_0 \rightarrow \epsilon \epsilon_0$$

$$\mu \neq 1 \quad \mu_0 \rightarrow \mu \mu_0$$

ZA PROZIRNE SNILI

$$n=1 \quad c = \frac{1}{\sqrt{\epsilon \epsilon_0 \mu \mu_0}} = \frac{c_0}{\sqrt{\epsilon \mu}}$$

$$n = \sqrt{\epsilon \mu} \Rightarrow \text{LOVNI KOLIČNIK}$$

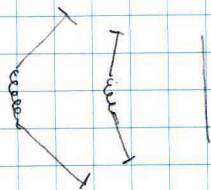




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$L \propto N^2$$

$$C \propto \frac{1}{d}$$



DIPOLNA ANTENA

## GOSTOTA ENERGIJSKEGA TOKA V EM VALOVANJU ZL.M

$$\vec{S} = \vec{W} \vec{C}_0$$

$$C_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

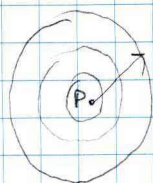
$$\vec{E} = \vec{B} \times \vec{C}_0 \Rightarrow E = B C_0$$

$$W = W_E + W_B = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 B^2 = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \mu_0 \frac{E^2}{\mu_0} = \frac{1}{2} E^2 \epsilon_0 + \frac{1}{2} \frac{B^2}{\mu_0}$$

$$= \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{E^2}{\mu_0 C_0^2} = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2} \frac{E^2 \epsilon_0 \mu_0}{\mu_0} = \epsilon_0 E^2 \quad \leftarrow E = E_0 \cos(\omega t - kx)$$

$$\epsilon_0 E_0^2 \cos^2(\omega t - kx) = \frac{1}{2} \epsilon_0 E_0^2$$

$$\vec{S} = \vec{W} \vec{C}_0 = \frac{1}{2} \epsilon_0 E_0^2$$



$$S = 4\pi r^2$$

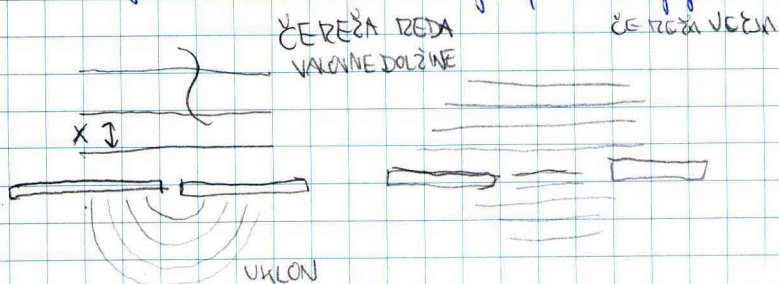
$$\vec{S} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

Gostota energijskega toka pada s kvadratom razdalje

Poskus:

$$C_0 = \frac{18,2m}{60ns} = \frac{18,2 \cdot 10^{-9}}{60} \approx 3 \cdot 10^8 \text{ m/s}$$

Dolgi elm-valci  $\rightarrow$  so ravninski, ne uklanjajo na obeh

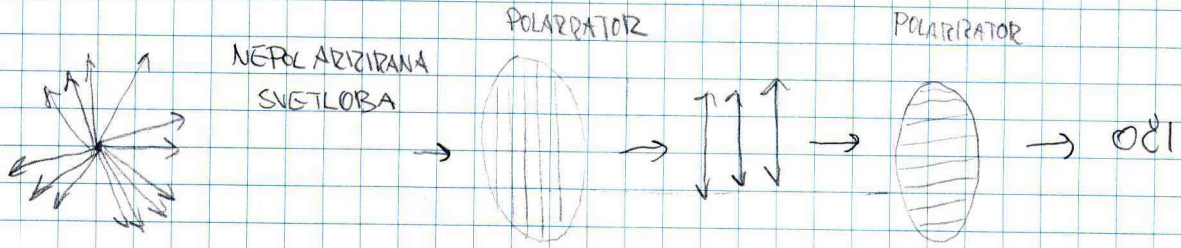


FM TV in mobilnih telefonih se žar čisto valci ne širijo

Doljšiki se širijo karadi vlnovca

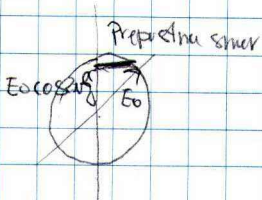
POLARIZACIJA

# POLARIZACIJA



Kalmo dva, rez lincej mi me. poride

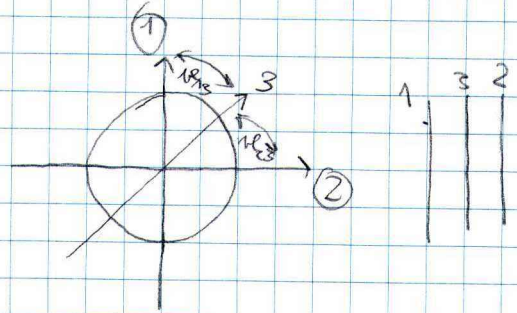
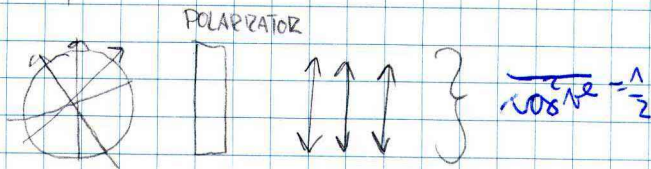
$$\bar{I} = \bar{I}_0 \cos^2 \alpha = \frac{1}{2} \epsilon_0 E_0^2 c$$



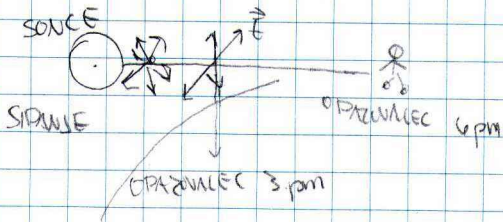
$$\bar{I}_{\text{podna}} = \frac{1}{2} \epsilon_0 E_0^2 c$$

$$\bar{I}_{\text{prep}} = \frac{1}{2} \epsilon_0 (E_0 \cos \alpha)^2 c$$

$$\text{Prepovednost} = \frac{\bar{I}_{\text{prep}}}{\bar{I}_{\text{podna}}} = \frac{\frac{1}{2} \epsilon_0 (E_0 \cos \alpha)^2 c}{\frac{1}{2} \epsilon_0 E_0^2 c} = \cos^2 \alpha$$

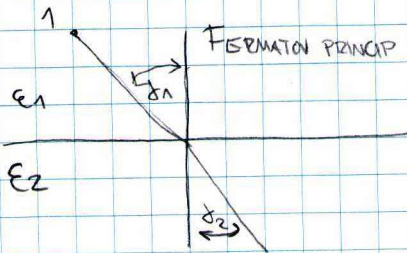


To je zmehnjajna zvezki polarizacija, to ni absorbcija



# LOMNI ZAKON

7.4.11



$$c_1 = \frac{c_0}{\sqrt{\epsilon_1}} = \frac{c_0}{n_1}, \quad n_1 = \sqrt{\epsilon_1}$$

$$c_2 = \frac{c_0}{\sqrt{\epsilon_2}} = \frac{c_0}{n_2}, \quad n_2 = \sqrt{\epsilon_2}$$

$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2}$$

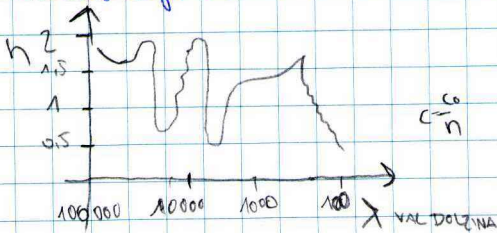
LOMNI ZAKON

$$\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1}$$

m

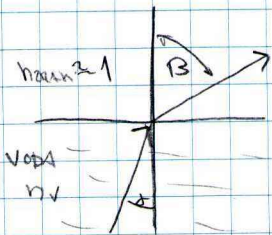
# TOTALNI ODBOJ

Disperzija - odvisnost lomnega koeficienta od valovne dolžine



Odušen zaradi tega, ker imajo molekule v meji med medijema lastne mikrajne žarke.

# TOTALNI ODBOJ



$$\alpha = \alpha_c$$

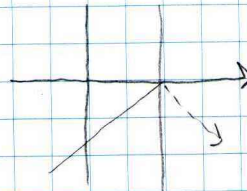
$$\beta = \frac{\pi}{2}$$

$$\frac{\sin \alpha}{\sin \beta} = \frac{n_2}{n_1}$$

$$\sin \alpha_c = \frac{n_1}{n_2} = \frac{c_2}{c_1} = \frac{v_2}{v_1} = \frac{h\nu_2}{h\nu_1} = \frac{\nu_2}{\nu_1}$$

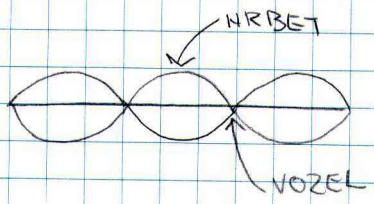
$$\alpha_c = \arcsin \left( \frac{n_1}{n_2} \right)$$

↳ KRITIČNI



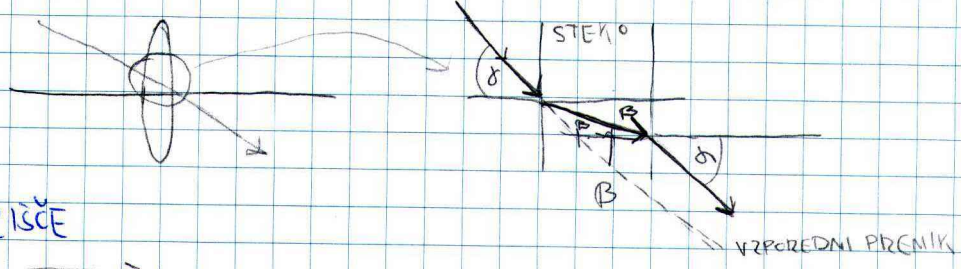
M.H.M

# STOJNO EM VALOVANJE

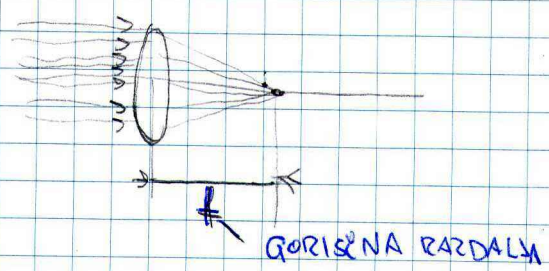


# OPTIČNI APARATI

## TANKE LEČE

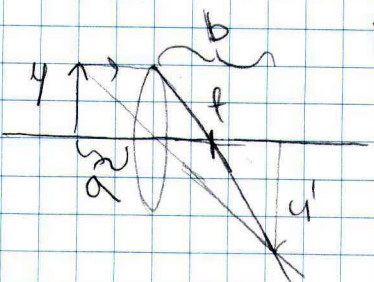


## GORIŠČE



Ločprijelna in zbiralna leča

## KONVEKSIJNA LEČA



### ENACBA

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$

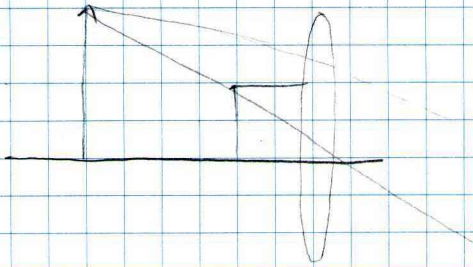
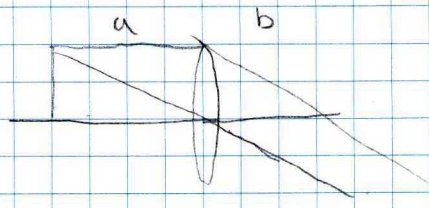
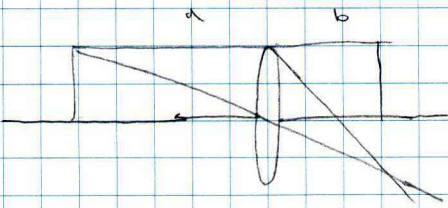
$$\frac{b}{y} = \frac{a'}{y'} = \frac{f}{a-f}$$

### POVEŠANA

$$M = \frac{y'}{y} \quad \frac{y}{a} = \frac{y'}{b} \Rightarrow \frac{y'}{y} = \frac{b}{a}$$

$$M = \frac{y'}{y} = \frac{b}{a} = \frac{f}{a-f}$$

## BIKONVEKSNA LEČA



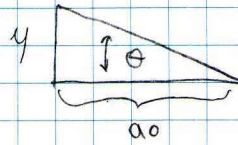
Ni NE VIDMO NA ZASLONU

## LUPA

$a_0 \equiv$  NORMALNA ZORNA RAZDALJA

$$M = \frac{\tan \theta'}{\tan \theta} = \frac{y a_0}{f a_0} = \frac{a_0}{f}$$

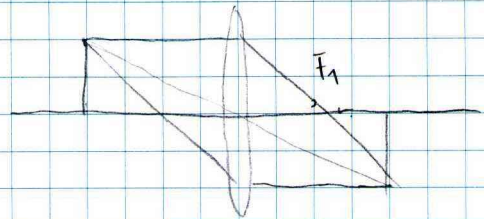
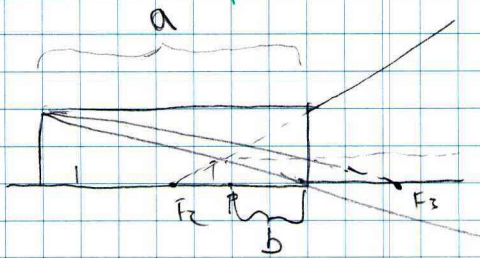
$$M = \frac{a_0}{f}$$



$$\tan \theta = \frac{y}{a_0}$$

## BIKONKAVNA LEČA

$f < 0$   
 $a > 0$   
 $b > 0$



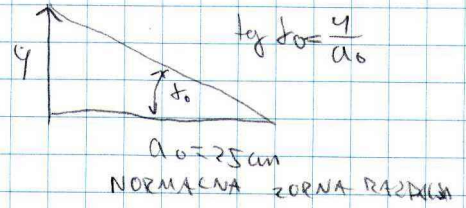
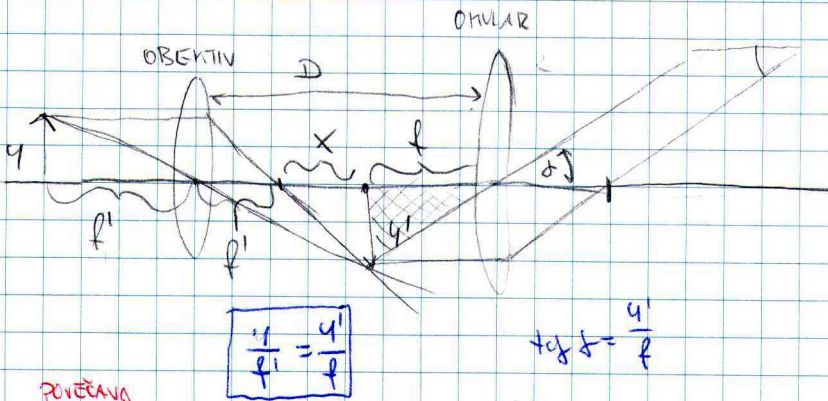
## SISTEM DVEH LEČ

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{s}{f_1 f_2}$$

NADOMESTNA GORIŠNA RAZDALJA

$s =$  razdalja med lečama

# MIKROSKOP



$$\frac{y'}{f'} = \frac{y}{f}$$

$$\tan \delta = \frac{y'}{f}$$

POVEČAVA

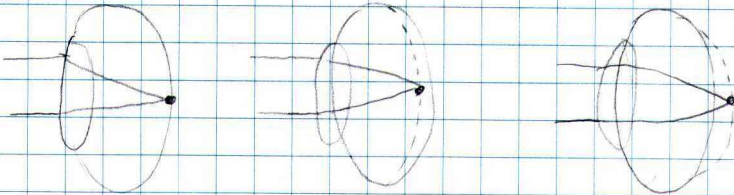
$$M = \frac{\tan \delta}{\tan \delta_0} = \frac{y' a_0}{f y} = \frac{x a_0}{f f'} = \frac{(D - f - f') a_0}{f f'}$$

↑  
LOČLJIVOST  
(valovna dolžina)

NORMALNO OČKO

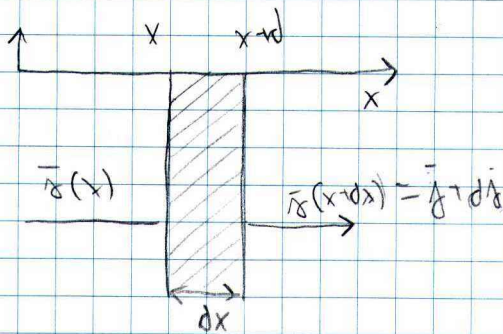
HRATOVIDNO OČKO

DALJNOVIDNO OČKO



DIOPTRIJ:  $\frac{1}{f}$

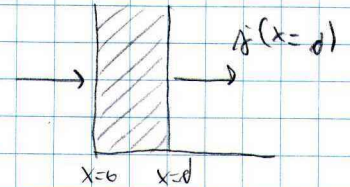
# ABSORPCIJA



$$dI = -I dx$$

$$dI = -\mu I dx$$

ABSORPCIJSKI KOEFICIENT



$$\ln \frac{I(x=d)}{I(x=0)} = \ln I(x=0) - \mu d$$

$$\ln \frac{I(x=d)}{I(x=0)} = -\mu d$$

$$I(x=d) = I(x=0) \cdot \exp(-\mu d)$$

EKSPONENTNI ABSORPCIJSKI ZAKON

$$dI = -\mu I dx$$

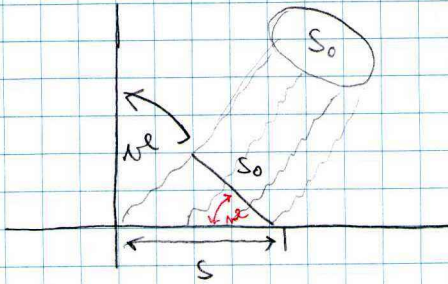
$$\int_{I(x=0)}^{I(x=d)} \frac{dI}{I} = -\mu \int_0^d dx$$

$$\ln I(x=d) = -\mu d$$

# FOTOMETRIJA

14.4.11

## OSVETLJENOST



$$\cos \alpha = \frac{S_0}{S}$$

$$S = \frac{S_0}{\cos \alpha}$$

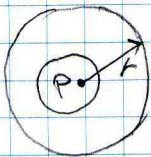
GOSTOTA ENERGIJSKEGA TOKA  $\Phi$

$$E = \frac{P}{S} \rightarrow \text{Energijski tok [W]}$$

$$= \frac{P}{S_0} \cos \alpha \rightarrow \text{Energijski tok na ravni}$$

## SVETILNOST

1. TOČKASTO Telo



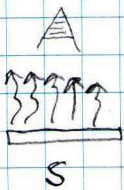
$$j = \frac{P}{4\pi r^2} = \frac{I}{r^2}$$

$$j = \frac{I}{r^2}$$

I - razsvetljava  
I =  $\frac{P}{4\pi}$   
4π - polni prostorski kot

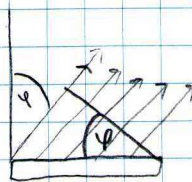
$$I = \frac{P}{4\pi}$$

2. PLOSKOVNO SVETILO



$$I = B_0 S$$

↑  
SVETLOST



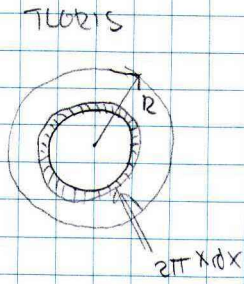
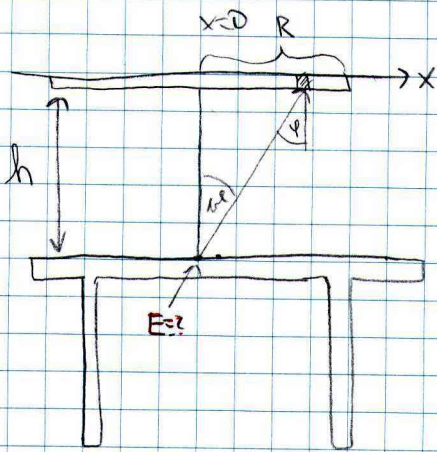
$$\cos \varphi = \frac{S'}{S}$$

$$S' = S \cos \varphi$$

B<sub>0</sub>-svetlost

$$I = BS' = BS \cos \varphi$$

Svetlost odhaja od točke. Če ni odhaja od točke poravnano točkasto svetilo  
LAMBERTOV SVETILO



$$dE = \frac{dI}{r^2} rR = \frac{B2\pi x dx \cos^2 \alpha}{r^2}$$

$$dI = B2\pi dx \cos^2 \alpha$$

$$r = \omega$$

$$E = \int \frac{2\pi B \cos^2 \alpha dx}{r^2}$$

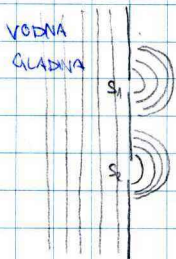
$$= \frac{\pi B}{2} (1 - \cos 2\alpha)$$

Limeta - zelo velika svetila ( $R \rightarrow \infty \rightarrow B_m \rightarrow \frac{1}{2}$ )  $\cdot \cos(2\frac{\pi}{2}) = 1$

$$E_{\infty} = \pi B$$

# INTERFERENČNI POSAVI PRI EMV

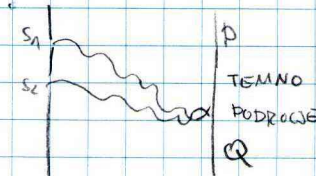
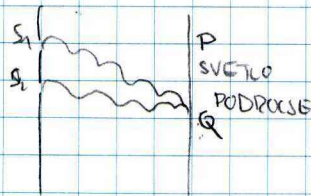
## INTERFERENCA



HEYGENSOV PRINCIP

IZVORA KOHERENTNA - svetloba v fazi. Slika iz različnih razdalj - različni fazni koeficienti

## YOUNGOV POSKUS



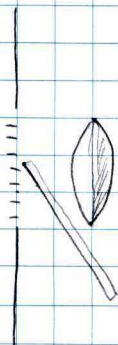
Travnica razlika med enim in obeh slitsih svetloba ima različni fazni koeficienti

Eni svetloba svetloba koherentna svetloba - vrh-nu-vrh

Huygensovo načelo vsako točko valovne fronte obravnava kot vir koherentne svetlobe

OPTIČNA MREŽICA → več 100 rež → 600 rež na mm

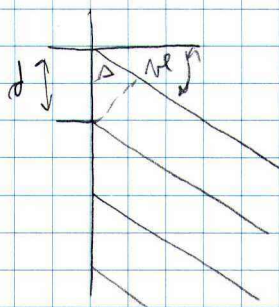
→ Dobimo več pik



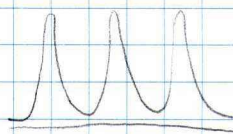
$$I = I_0 \frac{\sin^2(\pi b \sin \theta / \lambda)}{(\pi b \sin \theta / \lambda)^2}$$

$$I = I_0 \frac{\sin^2(\pi b \sin \theta / \lambda)}{(\pi b \sin \theta / \lambda)^2} \cdot \frac{\sin^2(N \pi d \sin \theta / \lambda)}{\sin^2(\pi d \sin \theta / \lambda)}$$

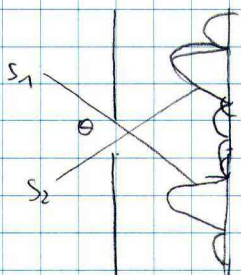
# UVLONSKA MREŽICA



Prostir za svetlobu:  
 $\Delta = d \sin \alpha = N \lambda$   
 $\sin \alpha \leq \sin \beta = N \frac{\lambda}{d}$  ;  $N = 1, 2, 3, 4$



# LOČLJIVOST svetlobnih valov



Pri optičnih mikroskopih svetloba lahko zmanjša valovno dolžino  
 $\Rightarrow$  Kalma elektroni mikroskop - pri črni svetlobi  
 pa lahko dolžina velike manjše valovne dolžine.

# FOBNARIC

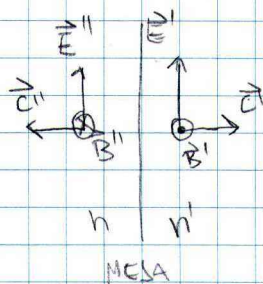
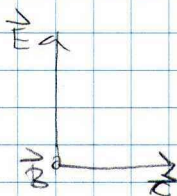
$$\vec{E} = \vec{B} \times \vec{c}$$

$$E = Bc$$

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$c = \frac{1}{\sqrt{\epsilon \mu}}$$

$$n = \frac{c_0}{c}$$



$$E + E'' = E'$$

$$B - B'' = B'$$

$$H - H'' = H'$$

$$B = \mu_0 H \Rightarrow H = \frac{B}{\mu_0}$$

$$\mu \approx \mu'$$

$$H' = \frac{B'}{\mu_0}$$

$$H'' = \frac{B''}{\mu_0}$$

$$\frac{E}{c} - \frac{E''}{c} = \frac{E'}{c} \quad / \quad c \Rightarrow E - E'' = E' \frac{c}{c'} = E' \frac{n'}{n}$$

$$2E = E' + E' \frac{n'}{n} = E' \left(1 + \frac{n'}{n}\right)$$

$$E' = \frac{2E}{1 + \frac{n'}{n}}$$

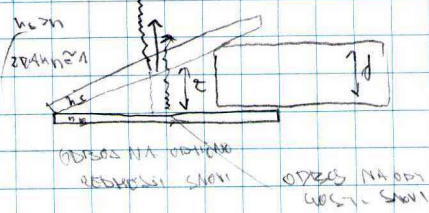
$$E - E' = \frac{2E}{1 + \frac{n'}{n}} \frac{n'}{n} = \frac{2E}{1 + \frac{n'}{n}} \frac{2n'}{n+n'}$$

$$E'' = E \left(1 - \frac{2n'}{n+n'}\right) = \frac{(n-n')}{n+n'} E \Rightarrow E'' = \frac{(n-n')}{(n+n')} E$$

$n > n'$  - Odboj na optično mešanici svetlobe...  $\vec{E}$  se odboji z isto fazo

$n < n'$  - Odboj na optično mešanici svetlobe...  $\vec{E}$  se odboji z nasprotno fazo

PP:



$$S = 2z + \frac{\lambda_0}{2}$$

ZA OSLAČITEV

$$2z \rightarrow \frac{\lambda_0}{2} = N\lambda_0$$

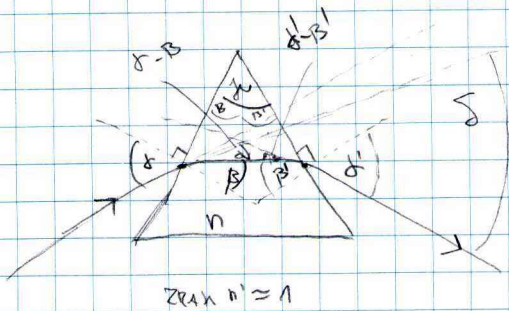
$N=1,2,3, \dots$

$$\Rightarrow 2z = N\lambda_0 - \frac{\lambda_0}{2}$$

$$z = \frac{1}{2}(2N-1)\lambda_0$$

$$\Delta z \approx \frac{\lambda_0}{2} \approx 200 \text{ nm}$$

## PRIZMA (LOM SVETLOB)



$$\frac{\sin \delta}{\sin \beta} = \frac{n \beta}{\beta'} = n$$

$$\frac{\sin \beta'}{\sin \beta} = \frac{1}{n}$$

$$\delta = (\delta - \beta) + (\delta' - \beta') = (n-1)\beta + (n-1)\beta' = (n-1)(\beta + \beta')$$

$$\Rightarrow \delta \approx (n-1)\delta$$

$$\text{Maj/min kot} = \frac{\delta}{\beta} = n \Rightarrow \delta = n\beta$$

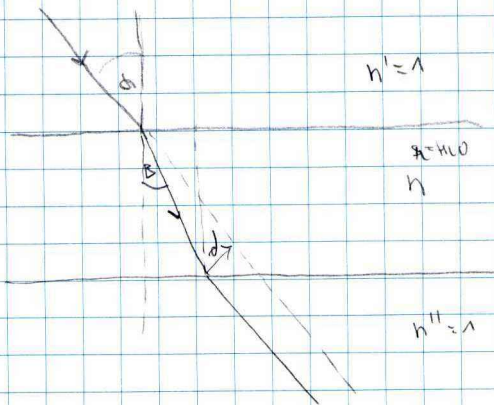
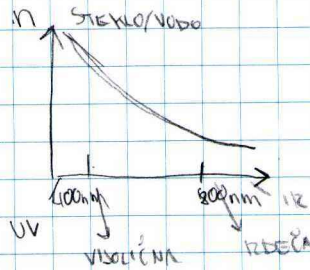
$$\frac{\beta'}{\beta} = \frac{1}{n} \Rightarrow \delta' = n\beta'$$

DISPERZNA:

$$c = c(\lambda)$$

$$n = n(\lambda)$$

$$\delta(\lambda) = (n(\lambda) - 1)\delta$$





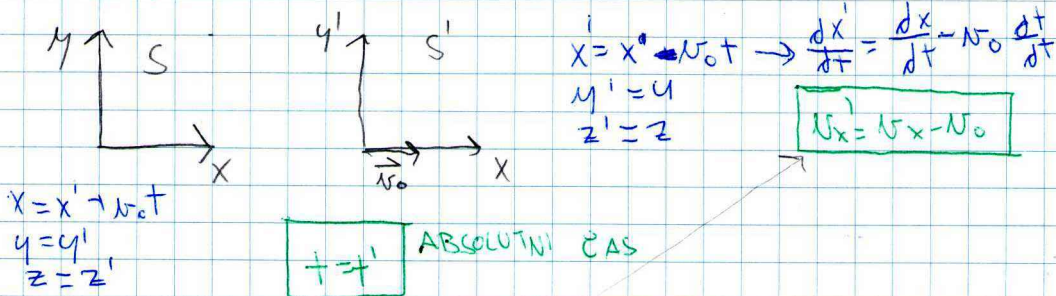
V sveto ni Fechnerja na vajah: No v sveto bo spet Sarka in se jej ni ne narobe če kdaj kaka zanka v roke pred tablo

# MODERNA FIZIKA

## POSEBNA TEORIJA RELATIVNOSTI (INERCIALNI - NEPOSPEŠENI OPAZOVALNI SIST.)

9.5.11

### 1. GALILEJEVA TRANSFORMACIJA



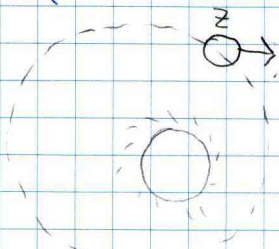
? Neki minimalni  $\Rightarrow$  uvedejo ETHER

### 2.) MAXWELL

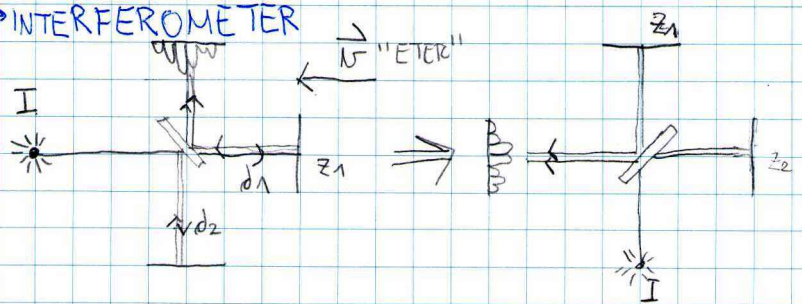
EMV:  $c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \text{konst}$

### EKSPERIMENT MICHELSONA IN MORLEY

"ETER"



INTERFEROMETER



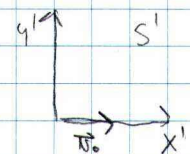
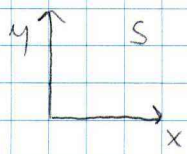
ALBERT EINSTEIN (1905):

DVE GLAVNI NAČELA:

1) Hitrost svetlobe  $c_0 = \text{konst}$

2) Načelo relativnosti - vsi nepospešeni opazovalni sistemi so enakovredni

# LORENTZOVE TRANSFORMACIJE



$$x = \gamma(x' - v_0 t')$$

$\downarrow$   
 $\gamma = \gamma(v) \rightarrow$  FUNKCIJA HITROSTI

$$x' = \gamma(x - v_0 t)$$

$$\frac{x}{\gamma} = x' - v_0 t' \Rightarrow t' = \frac{1}{v_0} \left( \frac{x}{\gamma} - x' \right) = \frac{1}{v_0} \left( \frac{x}{\gamma} - \gamma(x - v_0 t) \right)$$

$$= \gamma \left( t - \frac{x}{v_0} \left( 1 - \frac{1}{\gamma^2} \right) \right)$$

$$= t' = \gamma \left( t - \frac{v_0}{c^2} x \right)$$

$$\frac{dx'}{dt'} = \frac{\gamma(dx - v_0 dt)}{\gamma \left( dt - \frac{v_0}{c^2} dx \right)}$$

$$= \frac{\frac{dx}{dt} - v_0}{1 - \frac{v_0}{c^2} \frac{dx}{dt}} = \frac{v_x - v_0}{1 - \frac{v_x v_0}{c^2}}$$

- Homogenost prostora in časa
- izotropnost prostora

$$v_x = c_0 \Rightarrow v_x' = c_0$$

$$c_0 = \frac{c_0 - v_0}{1 - \frac{v_0}{c_0} \left( 1 - \frac{1}{\gamma^2} \right)}$$

$$\Rightarrow 1 - \frac{v_0}{c_0} \left( 1 - \frac{1}{\gamma^2} \right) = 1 - \frac{v_0}{c_0}$$

$$1 - \frac{1}{\gamma^2} = \frac{v_0^2}{c_0^2}$$

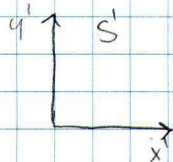
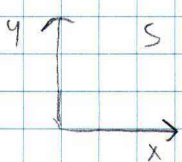
$$\frac{1}{\gamma^2} = 1 - \frac{v_0^2}{c_0^2}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_0^2}{c_0^2}}}$$

$$t' = \gamma \left( t - \frac{v_0}{c_0^2} x \right)$$

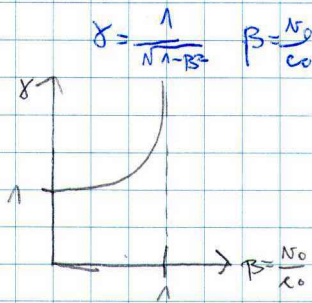
$$v_x' = \frac{v_x - v_0}{1 - \frac{v_x v_0}{c_0^2}}$$

DOKAZA SE ZADATA NA ISTEM  
NAZIVU ( $t_1 = t_2 - \Delta t$ )



$$\begin{aligned} x &= \gamma(x' + v_0 t') \\ y &= y' \\ z &= z' \\ t &= \gamma \left( t' + \frac{v_0}{c_0^2} x' \right) \end{aligned}$$

$$\begin{aligned} x' &= \gamma(x - v_0 t) \\ y' &= y \\ z' &= z \\ t' &= \gamma \left( t - \frac{v_0}{c_0^2} x \right) \end{aligned}$$



$$v_x = \frac{v_x' + v_0}{1 + \frac{v_x' v_0}{c_0^2}}$$

$$v_x' = \frac{v_x - v_0}{1 - \frac{v_x v_0}{c_0^2}}$$

$$v_y = \frac{dy}{dt} = \frac{dy'}{\gamma \left( dt' + \frac{v_0}{c_0^2} dx' \right)} = \frac{\frac{dy'}{dt'}}{\gamma \left( 1 + \frac{v_0}{c_0^2} \frac{dx'}{dt'} \right)} = \frac{v_y'}{\gamma \left( 1 + \frac{v_0 v_x'}{c_0^2} \right)}$$

$$v_z = \frac{dz}{dt} = \frac{v_z'}{\gamma \left( 1 - \frac{v_0 v_x'}{c_0^2} \right)}$$

Čas in prostorske koordinata  
nista neodvisni  $\rightarrow$  Določile

# SOČASNOST JE RELATIVNA

9.5.11

1. DOGODEK :  $t_1$   $x_1$        $t_1' = \gamma \left( t_1 - \frac{v_0}{c^2} x_1 \right)$

2. DOGODEK :  $t_2$   $x_2 \neq x_1$        $t_2' = \gamma \left( t_2 - \frac{v_0}{c^2} x_2 \right)$

$\Delta t = t_2 - t_1 = 0$   
SOČASNA

$\Delta t' = \gamma \frac{v_0}{c^2} (x_2 - x_1) \neq 0$   
NISTA SOČASNA

# PODALIŠANJE (DILATACIJA) ČASA

1. DOGODEK  $t_1$   $x_1$        $t_1' = \gamma \left( t_1 - \frac{v_0}{c^2} x_1 \right)$

2. DOGODEK  $t_2$   $x_2 = x_1$        $t_2' = \gamma \left( t_2 - \frac{v_0}{c^2} x_2 \right)$

$\Delta t = t_2 - t_1 = \gamma$   
LASTNI ČAS  
(VZA MIRUJE)

$\Delta t' = t_2' - t_1' = \gamma t_2 - \gamma t_1 = \gamma (t_2 - t_1)$

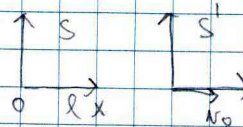
$\Delta t' = \gamma \cdot \Delta t = \gamma \gamma$

Čas med dogodkoma najmanj v  
lastnem sistemu. Drugod daljši

# SKRČENJE DOLŽINE

1. DOGODEK  $x_1 = 0$   $t_1 \dots t_1' = t$   $x_1' =$

2. DOGODEK  $x_2 = l$   $t_2 \dots t_2' = t$   $x_2' =$

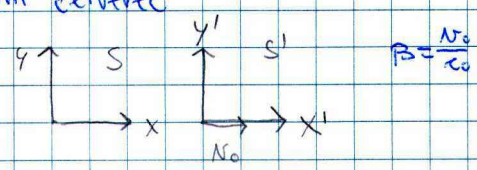


$\Delta x = x_2 - x_1 = l$   $\approx t_2' - t_1' = 0$   
 $\Delta x = \gamma (\Delta x' + v_0 \Delta t')$   
 $\Delta x = \gamma \Delta x'$   
 $l = \gamma \cdot l'$

$l' = \frac{l}{\gamma}$

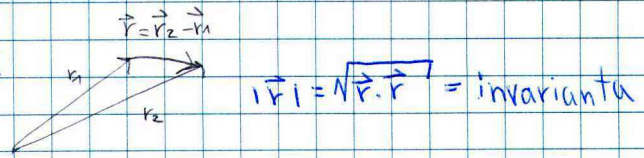
# ŠTIRIRAZSEŽNI PROSTOR $\equiv$ PROSTOR-ČAS (PROSTOR MIKOWSKEGA)

Drogolik prepis "Svetovni četverec"

$${}^4X = (\underbrace{ct}_{\text{ČASOVNI DEL}}, \underbrace{x, y, z}_{\text{PROSTORNI DEL}})$$


$$\begin{aligned} ct &= \gamma(ct' - \beta x') \\ x &= \gamma(x' - \beta ct') \\ y &= y' \quad z = z' \end{aligned}$$

"KLASIČNO" 3D:



Dobena oz. Analomni produkt tega vekt. je invarianta

"ŠTIRIRAZSEŽNI PROSTOR":

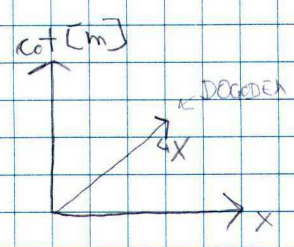
$${}^4X \cdot {}^4X = -c^2 t^2 + x^2 + y^2 + z^2$$

↓  
PREDZNAK DRUGIČEN

## SVETOVNI ČETVEREC

${}^4X \equiv X^\mu \rightarrow$  STANOVIK TJO NAPIŠE

$${}^4X = (ct, x, y, z) = (ct, \vec{r})$$



3D:  $|r| = \sqrt{r.r} = \text{invarianta}$

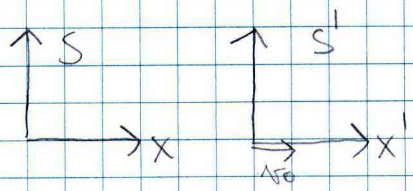
$$\left. \begin{aligned} {}^4X \cdot X &= -c^2 t^2 + x^2 + y^2 + z^2 \\ \text{ali} &= c_0^2 t^2 - x^2 - y^2 - z^2 \end{aligned} \right\} \text{DVE RAZLIČNI SIGNATURI}$$

STANOVIK

$${}^4V = (v_0, v_1, v_2, v_3)$$

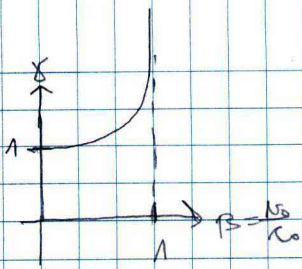
ČASOVNI DEL      PROSTORNI DEL

## LORENTZONE TRANSFORMACIJE



S, S' neposredna - INERCIALNA  
Če li kila prepisana li meli sistemske rale

$$v_0 = \gamma(v_0 - \beta v_1), \quad \beta = \frac{v_0}{c_0}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$



$$\begin{aligned} v'_0 &= \gamma(v_0 - \beta v_1) \\ v'_1 &= \gamma(v_1 - \beta v_0) \\ v'_2 &= v_2 \\ v'_3 &= v_3 \end{aligned}$$

## ČETVEREC HITROSTI

$${}^4v = \frac{dx}{d\tau} = \frac{dx}{dt} \cdot \gamma$$

$\gamma$ -LASTNI ČAS

"Mojing občas nam odur"

$${}^4v = \gamma \left( c_0 \frac{dt}{dt} + \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} \right) = \gamma (c_0, v_x, v_y, v_z)$$

$$\begin{aligned} \tau &= \gamma \cdot T \\ &= \gamma (c_0, v) \end{aligned}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c_0^2}}}$$

## ČETVEREC GIBALNE KOLIČINE

$\vec{G} = m\vec{v}$  v klasični fiz  $G, v$  - modern  $P$

$$\begin{aligned} {}^4p &= m_0 {}^4v = (m_0 \gamma c_0, m_0 \gamma \vec{v}) \\ &= \left( \frac{E}{c_0}, \vec{p} \right) \end{aligned}$$

$$\vec{P} = m_0 \gamma \vec{v}$$

RELATIVISTIČNA GIBALNA KOLIČINA

$m\gamma = m_0 \gamma$  RELATIVISTIČNA MASA

$$\frac{E}{c_0} = m_0 \gamma c_0 \Rightarrow E = m_0 \gamma c_0^2$$

POLNA ENERGIJA

$$E_0 = m_0 c_0^2$$

MIROVNA ENERGIJA

$$\begin{aligned} E &= W \\ W_K &= T \end{aligned}$$

$$W_K = E - E_0 = m_0 \gamma c_0^2 - m_0 c_0^2 = m_0 c_0^2 (\gamma - 1)$$

$$\Rightarrow W_K = m_0 c_0^2 \left( \frac{1}{\sqrt{1 - \frac{v^2}{c_0^2}}} - 1 \right) = m_0 c_0^2 \left( 1 + \frac{1}{2} \frac{v^2}{c_0^2} + \sigma \left( \frac{v^4}{c_0^4} \right) - 1 \right)$$

TAYLORJEVA VRSTA

$$W_K = \frac{1}{2} m_0 v^2 + \sigma \left( \frac{v^4}{c_0^4} \right)$$

Pri  $v \ll c_0$ :  $W_K \approx \frac{1}{2} m_0 v^2$

${}^4v, {}^4v =$  invarianta

LASNI SISTEM ( $v=0$ )

$${}^4p = \left( \frac{E}{c_0}, \vec{p} \right) \quad \vec{p} = m_0 \gamma \vec{v}$$

$$E^2 = E_0^2 + c_0^2 p^2$$

$${}^4p \cdot {}^4p = \frac{E^2}{c_0^2} - p^2 = -\frac{E_0^2}{c_0^2}$$

$$\vec{p} = m_0 \gamma \vec{v}$$

$$\vec{E} = m_0 \gamma c_0^2$$

$$\frac{E}{p} = \frac{m_0 \gamma c_0^2}{m_0 \gamma v} = \frac{c_0^2}{v}$$

DELEC  $v=c_0$ : (Skrajni primer)

$$\frac{E}{p} = \frac{c_0^2}{c_0} = c_0 \Rightarrow \boxed{E = c_0 p} \rightarrow E_0 = 0 \Rightarrow \boxed{m_0 = 0}^{\text{FOTON}}$$

Da se delec lahko uprži s svetlobno energijo mora imeti  $m_0 = 0$

## NEWTONOV ZAKON V ŠTIRIRAZSEŽNI OBLIKI

KLASIČNO

$$\vec{F} = \frac{d\vec{G}}{dt} \quad \vec{G} = m\vec{v}$$

RELATIVISTIČNO

$${}^4P = \left( \frac{E}{c_0}, \vec{p} \right) \quad \vec{p} = m_0 \gamma \vec{v}$$

POVA GIBENJA

$${}^4F = \frac{d({}^4P)}{d\tau} = \gamma \frac{d({}^4P)}{dt} = \gamma \left( \frac{1}{c_0} \frac{dE}{dt}, \frac{d\vec{p}}{dt} \right)$$

$$t = \gamma \tau, \quad dt = \gamma d\tau$$

Čeherec ali na maelektron delec:

$${}^4F = \left( \frac{e\gamma \vec{E} \cdot \vec{v}}{c_0}, e\gamma (\vec{E} + \vec{v} \times \vec{B}) \right)$$

SKOST EL. POLJA

KRAŠENI DELI:

$$\boxed{\frac{d\vec{p}}{dt} = e(\vec{E} + \vec{v} \times \vec{B})} \quad \boxed{\vec{p} = m_0 \gamma \vec{v}}$$

ČASOVNI DEL:

$$\frac{e\gamma}{c_0} \frac{dE}{dt} = \frac{e\gamma}{c_0} \vec{E} \cdot \vec{v} \Rightarrow \Delta E = e \int \vec{E} \cdot \vec{v} dt = -e\Delta\varphi$$

$$v = -\int \vec{E} \cdot d\vec{r} = v_2 - v_1$$

$$v = \varphi$$

$$\Delta(E + e\varphi) = 0$$

$$E + e\varphi = \text{konst}$$

## OHRANITVENI ZAKONI

$$\int \gamma F = \frac{d(\gamma P)}{d\gamma} \Rightarrow \Delta \gamma P = \int \gamma F d\gamma$$

$$\int \gamma F d\gamma = 0 \Rightarrow \Delta \gamma P \Rightarrow \boxed{\begin{matrix} \Delta E = 0 \\ \Delta P = 0 \end{matrix}}$$

## MAXWELLOVE ENAČBE

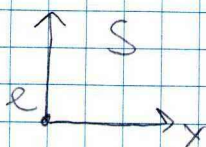
Vektor zbirave potenciala  $\gamma A = \left( \frac{\varphi}{c_0}, \vec{A} \right)$   $\varphi \equiv V$

(Jakost električnega polja)  $\vec{E} = -\vec{\nabla} \varphi - \frac{\partial \vec{A}}{\partial t}$

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

$\vec{J} = \left( \rho \cdot c_0, \vec{j} \right)$   
GOSTOTA NABEVA  
GOSTOTA ELEKTRO

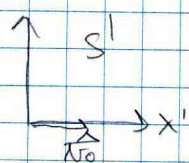
PRIMER: Točkovni naboje murgje v inercialnem sistemu S



S:  $\varphi = \frac{e}{4\pi\epsilon_0 r}$   $\vec{A} = 0$

$$\gamma_0 = \gamma (v_0 - \beta v_x)$$

$$\gamma v_x = \gamma (v_x - \beta v_0)$$



S':  $\frac{\varphi'}{c_0} = \gamma \left( \frac{\varphi}{c_0} - \beta A_x \right) = \gamma \frac{\varphi}{c_0} = \gamma \frac{e}{4\pi\epsilon_0 r c_0}$   
 $\varphi' = \gamma \frac{e}{4\pi\epsilon_0 r}$

$$A_x' = \gamma \left( A_x - \beta \frac{\varphi}{c_0} \right) = \gamma \beta \frac{\varphi}{c_0} = \gamma \frac{v_0}{c_0^2} \frac{e}{4\pi\epsilon_0 r} \neq 0$$

## MAXWELLOVE ENAČBE S ČETVEPCI

$$\square \gamma A = \mu_0 \gamma j$$

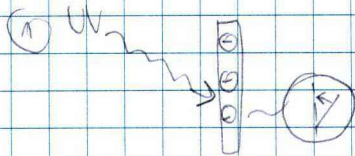
$$\square = \frac{1}{c_0^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

# FOTOEFEKT

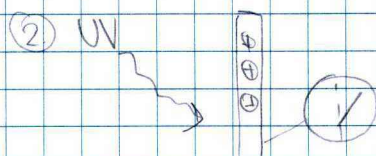
18.5.11

## Poskus

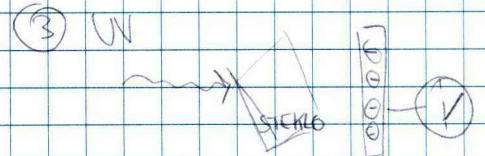
- Z UV svetlobo na žilpico. Če navelikujemo moč svetlobe, malo je & raven porabe
- Če pa povečujemo pa ne
- Če pa imajo določeno dolžino valovne dolžine



UV žarek izloča elektrone iz ploščice



Pravilni napetost potegne izloče elektrone manj



V steklu se UV del svetlobe absorbira

Za izločanje  $Z^-$  mora biti dovolj velik  $\nu$ . Tega se ne da uveljaviti s klasično fiziko

$$\bar{j} = \epsilon_0 \bar{w} \sin^2 \theta \quad \leftarrow \text{kaprica valovne mot EMW}$$

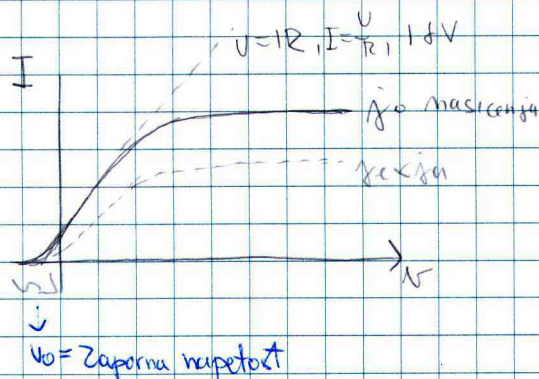
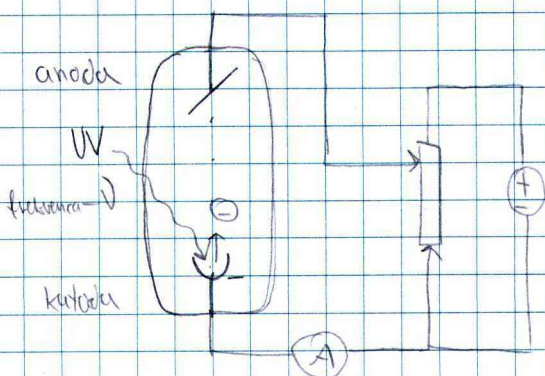
$$E_{\text{abs}} = \bar{j} S t \quad \text{površina plošče}$$

$$E_{\text{abs}} = \alpha j S t \quad \alpha \text{-koefficient}$$

$E_{\text{abs}} \propto t$  bi morali dati opazovati, pa to imajo dovolj  $W$  za izločanje Verama  $W$  Ampereh to me gre

Emulsijski je ista leta objavljen 3 različne stvari

## FOTOCELKA



$$\bar{j} = \text{gostota energijskega toka}$$

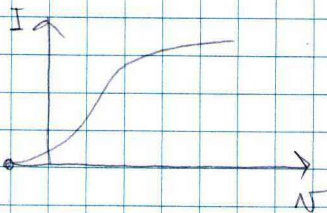
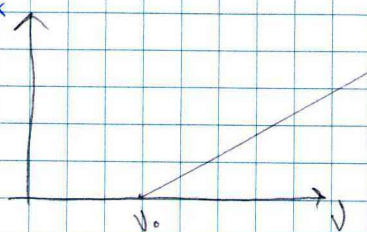
Plata nasičenja vsi  $e^-$  uveljavijo. Zaporna napetost je potencialna, ker mora biti dovolj velik  $\nu$ . Polekna da vsi  $e^-$  uveljavijo

$$\frac{m_e v_{max}^2}{2} = e_0 V_0 = A e \quad W_k \rightarrow W_{pe}$$

||

$W_{kmax}$  je pomenoma  $V_0$ , lahko maximo

$W_{kmax}(V) \rightarrow W_{kmax}$

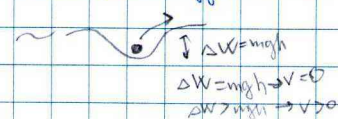


$$W_k = h(\nu - \nu_0) = h\nu - h\nu_0 = h\nu - W_{izst}$$

$$W_{kmax} = h\nu - W_{izstop}$$

absorbirana energija

vezavna energija elektronov



$h$  - Planckova konstanta -  $h$   $h = \frac{h}{2\pi}$

### [h\*nu] [eV]

Nmogo mi enake energije, ker so različne smeri se razdelijo, nekateri trčijo v steno

Konje menčična:  $W = mgh$

Klasična je  $E = \frac{1}{2} m v^2$ ,  $E_{pot}$  in  $E_{kin}$

Energija fotonov je paralelna  $\rightarrow$  Energija fotonov

Fotoni imajo cikelno količino, more pa nimajo  
Pri trkih se oblikuje polna energija

$$W_{polna} = W_0 + T_{kinetika}$$

$$W_0 = m_0 c^2$$

$$W^2 = W_0^2 + c_0^2 p^2 \quad - \text{relativnost}$$

FOTON:  $W_0 = 0$  - lastna energija

$$W^2 = c_0^2 p^2 \quad - \text{nel. cikelna količina}$$

$$W = h\nu \quad - \text{Polna energija}$$

$\downarrow$

$$c_0 p \rightarrow \text{Gibalna kol. fotona}$$

$$p = \frac{W_0}{c_0} = \frac{h\nu}{c_0}$$

$$c_0 = 2\lambda$$

$$p = \frac{h}{\lambda}$$

nel. cikelna kol. fotona

St. absorpcijski foton:

$$N_0 = \frac{P_t}{h\nu} = \frac{A \cdot \Delta t}{h\nu} = \frac{A \cdot \Delta t}{h\nu}$$

Material	$\lambda$ (nm)
platina	230
voltrom	280
stebro	290
Al, cink	340
kavčij	560

redigono delo

$$W_i = h\nu_0 = h \frac{c_0}{\lambda}$$

Primer: Anihilacija (izničevanje) ( $\neq$  kreacija)

POZITRON  $\oplus$   
ELEKTRON  $\ominus$   $W_0 = m_0 c_0^2 = 0,511 \text{ MeV}$



V relativni fiziki se bosta območja nekega gibanja in polna energija, če mi svinčeno zmanjšamo silo



Obrneta se E in P

1.) Obrneta polne energije

$$W_0 + T_P + W_0 = h\nu - h\nu'$$

2.) Obrneta gibalne količine:

$$P = \frac{h\nu}{c_0} - \frac{h\nu'}{c_0} \Rightarrow c_0 P = h(\nu - \nu') = \sqrt{2W_0 T_P + T_P^2}$$

gib. kol. pozitrona

$\Rightarrow \nu, \nu'$

$$c_0 P = W$$

$$P_0 = \frac{h\nu}{c_0}$$

$$c_0 P = m_0 c_0 P$$

$$W^2 = W_0^2 + c_0^2 P^2$$

$$(W_0 + T_P)^2 = W_0^2 + c_0^2 P^2$$

$$\sqrt{2W_0 T_P + T_P^2} = c_0 P$$

interferenčni invar. z računom  $e^-$  svetlobe na kristal grafita

Prejemo samo sferični. Svetloba lahko episkopoma kot naključje ali kot delce  
kot z delci lahko kot z naključnim delcem

de Broglie: Propozicija delcem nalene lastnosti - kar ni ta eniče

$P = \frac{h}{\lambda}$ , pripisujemo delcem nalene delnice!

$$\lambda_B = \frac{h}{P} = \frac{h}{mv} \quad \lambda_B = \frac{h}{mv}$$

$$W = cP$$

$$P = \frac{W}{t} = \frac{h\nu}{t} = \frac{h}{\lambda}$$

GIBANJA KUL  
FOTONOV

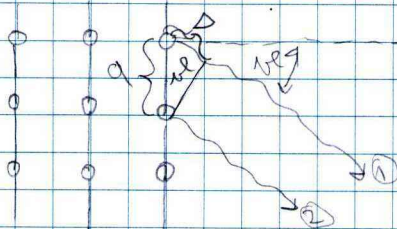
$$\lambda_B = \frac{h}{mv}$$

de Broglie  
 $m_e = 9,1 \cdot 10^{-31}$  kg

$$\frac{m_e v^2}{2} = e_0 U_0 \Rightarrow v = \sqrt{\frac{2e_0 U_0}{m_e}} \Rightarrow \lambda_B = \frac{h}{m_e v} = \frac{h}{m_e \sqrt{\frac{2e_0 U_0}{m_e}}} = \frac{h}{\sqrt{2m_e e_0 U_0}}$$

Večji napetost  
manjša valovna  
dolžina

A.) Majhna  $U_0 \Rightarrow$  majhna  $W_k$ , velika  $\lambda_B$



$$\Delta = a \sin \theta = N \lambda_B$$

$$U_0 = 100V, \lambda_B = 0,123 \text{ nm}$$

B.) Velika napetost  $U_0 \Rightarrow$  večja  $W_k$ , manjša  $\lambda_B$

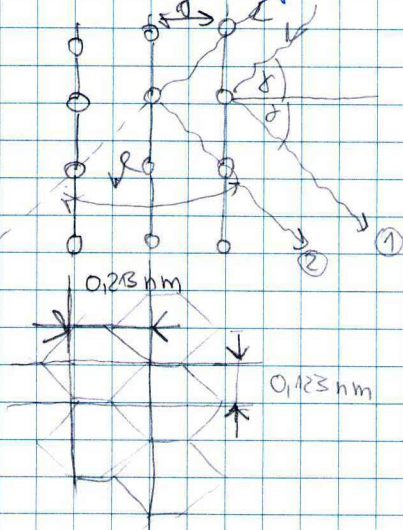
Braggova sipanje

$$U_0 = 1500V$$

$$\lambda_B \approx 0,052 \text{ nm}$$

$$\lambda_B = \frac{h}{mv}$$

de Broglie



Newtonova - mehanika

$$\vec{r} = \vec{r}(t)$$

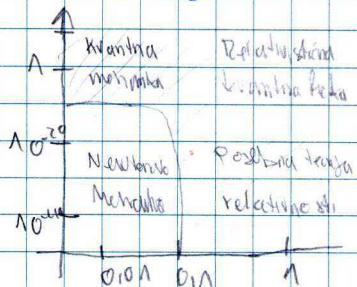
$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{r}(t)$$

$$\sum \vec{F}_i = m \frac{d\vec{v}}{dt}$$

Schrödinger E - mehaniki Newtonova mehanika



$$h = 6,6 \cdot 10^{-34} \text{ Js}$$

Ne da se hitrost obkroži leže ni hitrost  
Al eno al drugo

de Broglie prvi hvala k relativistični fiz

Rel. brzina:

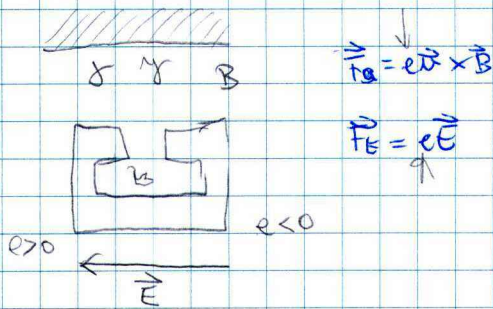
$$\vec{p} = m_0 \gamma_e \vec{v} \rightarrow m_0 \vec{v} \quad \text{Newtonova mehanika}$$

$$T = m_0 c \alpha^2 (\gamma_e - 1) \rightarrow \frac{m_0 c \alpha^2}{2} \quad \frac{v}{c} \rightarrow 0$$

Tipične veličnosti:   
 Radij  $5 \times 10^{-11} \text{ m} \sim 10^{-10} \text{ m} = \text{\AA}$    
 Radij  $1,2 \cdot 10^{-15} \text{ m} \sim 10^{-15} \text{ m}$    
 } Delci medu  $10^{-10} - 10^{-15} \text{ m}$

## MEGLIČNA CELICA

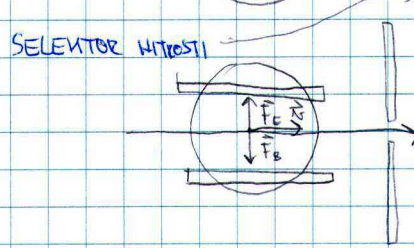
Zrak z nekega izvora svetlobe preučimo preučimo preostanoma, da se odloči   
 → z nekega izvora nastane zrak svetlobe   
 → svetloba svetloba se spreje na sferični površini svetlobe   
 kroglice (konvencionalna)   
 → Svetlobi nastane delci svetlobe svetlobe svetlobe



$$\vec{r}_a = e \vec{v} \times \vec{B}$$

$$\vec{F}_E = e \vec{E}$$

Lahko namrečena žarka z magnetno poljem   
 → klna delcev z enako hitrostjo



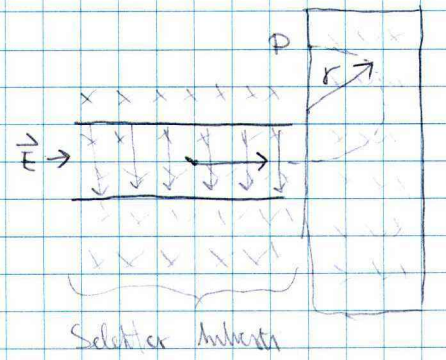
$$\vec{F}_B = e \vec{v} \times \vec{B}$$

$$\vec{F}_E = e \vec{E}$$

$$v B = E$$

$$v = \frac{E}{B}$$

## MASNI SPENTROMETER



$$m a_r = e v B_0$$

$$m \frac{v^2}{r} = e v B_0$$

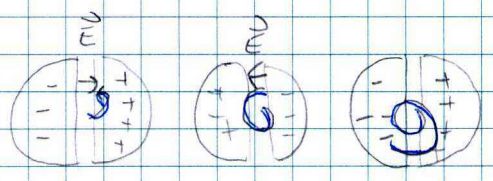
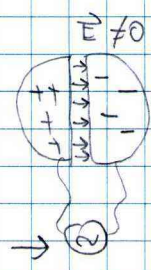
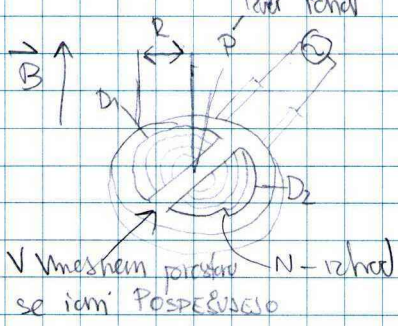
$$r = \frac{m v}{e B_0} \Rightarrow v = \frac{m E}{e B_0 B} \Rightarrow \frac{m}{e} = \frac{r B_0 B}{E}$$

1859-1860  
 Thomsonov aparat za merjenje  $e/m \rightarrow$  Specifični naboj  
 hitrosti  $\rightarrow$  elektroni  
 Thomson:  $\frac{e}{m} \sim 0,77 \cdot 10^{11} \text{ As/kg}$

**CIKLOTRON**

V električnem ni el. polja! (rotacijski produkt)

2 rotli elektroni  $\rightarrow D_1, D_2$   
 V električnem polju



$\vec{F}_B \perp d\vec{s}$   
 $A = \oint \vec{F} \cdot d\vec{s} = 0$

Obhodni čas delca:  
 pri dani hitrosti  $v$

$t_0 = \frac{2\pi r}{v} = \frac{2\pi m v r}{v e B r} = \frac{2\pi m}{e B}$

$\nu_c = \frac{1}{t_0} = \frac{e B}{2\pi m}$  **CIKLOTRONSKA FREKVENCA**

Frekvenca pri vsaki hitrosti  $v$  ostane enaka

**SINHROCIKLOTRON**

$\vec{p} = m \gamma \vec{v}$

$\frac{d\vec{p}}{dt} = e \vec{v} \times \vec{B}$

$\frac{d(m_0 \gamma \vec{v})}{dt} = e \vec{v} \times \vec{B}$

$\gamma m_0 \frac{d\vec{v}}{dt} = e \vec{v} \times \vec{B}$

$\gamma m_0 \frac{d\vec{v}}{dt} = e \vec{v} \times \vec{B}$

Ker je kinetična energija konstantna  $\rightarrow |\vec{v}| = \text{konst. } v$

V splošnem  $\frac{d(m_0 \gamma \vec{v})}{dt} = m_0 \left[ \frac{d\gamma}{dt} \vec{v} + \gamma \frac{d\vec{v}}{dt} \right]$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \text{konst.}$

$\gamma m_0 \frac{d\vec{v}}{dt} = e \vec{v} \times \vec{B}$   
 $\gamma m_0 \frac{d\vec{v}}{dt} = e \vec{v} \times \vec{B}$   
 $\gamma m_0 \frac{d\vec{v}}{dt} = e \vec{v} \times \vec{B} \rightarrow r = \frac{\gamma m_0 v}{e B}$

$\frac{t_0}{\gamma} = \frac{2\pi r}{v} = \frac{2\pi \gamma m_0 v}{v e B} = \frac{2\pi \gamma m_0}{e B}$

$\nu_c = \frac{1}{t_0} = \frac{e B}{2\pi \gamma m_0} = \frac{1}{\gamma} \left[ \frac{e B}{2\pi m_0} \right]$

$$\frac{d\vec{p}}{dt} = e\vec{E}$$

Σ me sommerer das  
wen plus ni beibehalten

$$\frac{d(m_0 \gamma v)}{dt} = eE$$

$$\frac{d(m_0 \gamma v)}{dt} = eE$$

/dt

$$\int d(m_0 \gamma v) = \int eE dt$$

E = konst  
v = l/E

$$m_0 \gamma v = eE \int dt = eEt$$

$$\rightarrow m_0 \gamma v = eEt$$

$$\frac{m_0 \gamma v}{\sqrt{1 - \frac{v^2}{c^2}}} = eEt$$

$$\frac{\frac{v}{c}}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \left(\frac{eE}{m_0 c^2}\right) t = \gamma t \quad (1)^2$$

$$\left(\frac{v}{c}\right)^2 = \gamma^2 [1 - \left(\frac{v}{c}\right)^2]$$

$$\left(\frac{v}{c}\right)^2 [1 + \gamma^2] = \gamma^2$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - \gamma^2}}}$$

$$\lim_{v \rightarrow c} \gamma = \lim_{v \rightarrow c} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \infty$$

•  $m_0 \gamma v = eEt$   
 $m_0 \gamma v = eEt$   
 $\gamma = \frac{eEt}{m_0 v}$   
 $v \rightarrow c$   
 $\Rightarrow \gamma \rightarrow \infty$

• FOTOEFEKT  
FOTONI

- FOTON:  $p = \frac{h}{\lambda}$

- ENERGIJA:  $w = h\nu$   
 $w = \hbar \omega$

• SEVANJE BT:

- Planckov zakon in Stefanov zakon

• COMPTONSKO SIFRANJE

- Comptonova valovna dolžina

• ŽRTOVI SPECTRI

- Energijski nivoji

ELEKTRON

$\lambda_B = \frac{h}{p} = \frac{h}{m_e v}$

$p = \frac{2\pi h}{\lambda} = \hbar k$

$\hbar \frac{2\pi}{\lambda} = \hbar k$

VALOVNI VEKTOR

EM Valovnjak:  $\vec{A} = \vec{A}_0 \cos(\vec{k} \cdot \vec{r} - \omega t)$   
 $\vec{A} \propto \vec{E}$

ELEKTRON:

$p = \hbar k$ ,  $p = m v$   
 $w = \hbar \omega$ ,  $w = T + V$

$T = \frac{1}{2} m v^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m}$

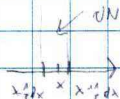
VALOVNA F. ZA DELECE

Postava:  $\Psi(x, t) \propto e^{i(\frac{p}{\hbar}x - \frac{w}{\hbar}t)} = e^{i(px - wt)/\hbar}$

$\Psi^* \Psi$  - Verjetnostna gostota elektronov na razdalji

WILSON CURVA ELEKTRONOV

$\frac{dN}{N} = p dx$





# OSNOVE KVANTNE MEHANIKE

## NAČELA KVANTNE MEHANIKE

### - NAČELO STATISTIČNA OPAŠI

- Izbira prostora za prostorsko merjenje delcev na makro e. golcevirje nepravilni
- V splošnem se mišne nepravilni za množico delcev
- Vpeljemo verjetnostno gostoto  $\rho = \psi^* \psi$   
za določeno lokacijo, navedeno o navedenju

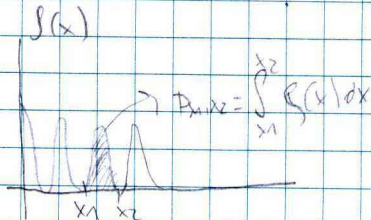
### VALOVNA FUNKCIJA

$$\psi = \psi_0 e^{i(kx - \omega t)}$$

$$\vec{p} = \hbar \vec{k}$$

- Gibalna kol

$$\hbar \omega = W = W_k + W_p \rightarrow \text{povna energija elektrona}$$



Integral verjetnostne gostote:  $\int_{-\infty}^{\infty} \rho(x) dx = 1$

Prilobitvina vrednost  $\langle x \rangle = \int_{-\infty}^{\infty} x \rho dx = \int_{-\infty}^{\infty} x \psi^* \psi dx = \int_{-\infty}^{\infty} \psi^* x \psi dx$

### - HEISENBERGOVO NAČELO NEODLOČNOSTI

$$\Delta x \cdot \Delta p_x \geq \hbar$$

$$\Delta y \cdot \Delta p_y \geq \hbar$$

$$\Delta z \cdot \Delta p_z \geq \hbar$$

valovna kol. Izbira se lahko sprejme med elektroni

### - NAČELO SUPERPOZICIJE

$$|\psi|^2 \geq |c_1 \psi_1|^2 + |c_2 \psi_2|^2$$

### OPERATORJI

Ne moremo prejeti glatke delnice in lege hkrati. Lahko razne ene stranici na enkrat

$$\psi = \psi_0 e^{i(\frac{p_x}{\hbar} x - \frac{W}{\hbar} t)}$$

$$i\hbar \frac{\partial \psi}{\partial x} = \frac{1}{\hbar} p_x (i\hbar) \psi = p_x \psi$$

$$\frac{\partial \psi}{\partial t} = \frac{-iW}{\hbar} \psi = -W \psi \quad ; \quad i\hbar \frac{\partial}{\partial t} \psi = W \psi \quad \text{OP. POLNE E. VREDNOSTE}$$

$$i\hbar \frac{\partial}{\partial x} \psi = p_x \psi \quad \text{OP. GIB. KOL}$$

OPERATOR KOMPONENTNE GIBANJE V KOLIČNE

$$\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

OPERATOR KIN ENERGIJE

$$T = \frac{p^2}{2m} = \frac{\hbar^2}{2m} \nabla^2$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

SCHRODINGERJEVA ENAČBA