OSNOVE ROBOTIKE

3.10.20

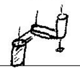
10. oktober, navodila za lab vaje / diplomska soba
zapiski tudi na mero.si

Bajd ... OSNOVE ROBOTIKE

22.10 razdelo vaj

Bajd + Springer + robotics [www]

kot v robotiki ϑ ϑ ϑ ϑ ϑ ϑ
razdalja d

antropomorfni - človeku podoben
scara robot 

inteligentno gibanje robotskih mehanizmov

10.10.20

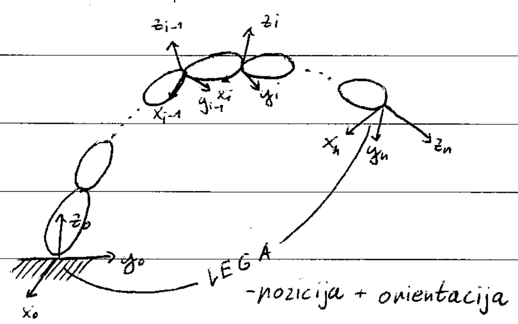
Robotika

- zasnovani od človeka
  - mobilni
  - v zraku
  - v vodi
- biološko zasnovani
  - človek
    - industrija { • robotske roke
    - • robotska zapestja
    - humanoidi { • prsti
    - • noge
  - živali
    - • kače
    - • ribe
    - • 4,6,8 - nožni roboti

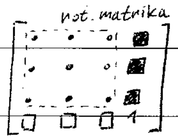
SERIJSKA VERIGA SEGMENTOV IN SKLEPOV

več kot 6 prost. stopenj → redundanten
mnogo več → hiperredundanten

PREMIK - translacija (d) + rotacija (ϑ)



Lega in premik → homogena transformacijska matrika



LEGA • orientacija ■ pozicija

PREMIK • rotacija ■ translacija

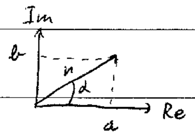
□ perspektiva

ROTACIJA	ORIENTACIJA
- okrog poljubne osi	- rot. matrika
- okrog x, y, z	- Eulerjevi koti
	- Roll Pitch Yaw koti
	- kvaternioni

razširitev kompleksnih števil

$z = a + ib$

$z = re^{i\alpha}$



Hamilton

$$\boxed{q = q_0 + q_1 i + q_2 j + q_3 k}$$

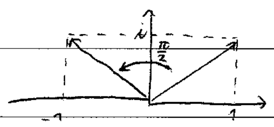
kvaternion

množenje je rotacija v ravnini

$z_1 \cdot z_2 = r_1 \cdot r_2 \, e^{i(\alpha+\beta)}$

$z_1 = 1 + i =$

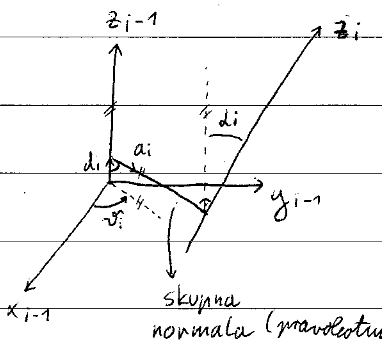
$z_2 = i = e^{i\frac{\pi}{2}}$



$z_1 z_2 = (1+i)i = -1 + i$

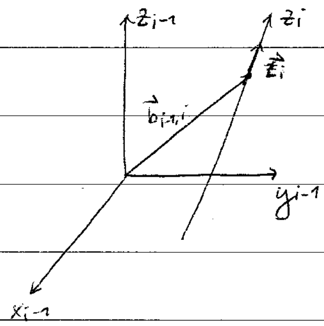
geometrijski model robota

A) Skalarni Denavit-Hartenberg



4 parametri

B) Vektorski parametri



6 parametrov

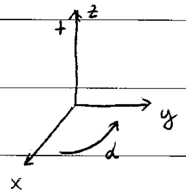
Inverzni model robot

Izpit:

1. premik (20%)
2. DH (30%)
3. vektorski model (20%)
4. kvaternioni (30 %)

bajd+ springer+ robotics www ... 3. poglavje

Orientacija



pozitivna smer obratna uri iz smeri z

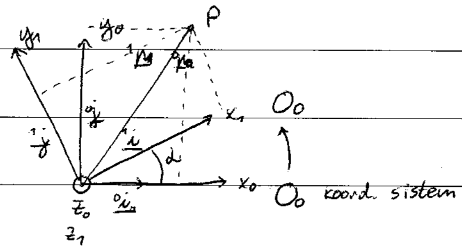
ali pravilo desne roke ali vijak

$\vec{a}$

$\underline{a}$ ← vektor

$\underline{\underline{a}}$ ← matrika

DAFQ !?



O_1:

$$ {}^1\underline{p} = {}^1p_x\,{}^1\underline{i} + {}^1p_y\,{}^1\underline{j} + {}^1p_z\,{}^1\underline{k} $$

O_0:

$$ {}^0\underline{p} = {}^0p_x\,{}^0\underline{i} + {}^0p_y\,{}^0\underline{j} + {}^0p_z\,{}^0\underline{k} \quad \Big/ \cdot {}^0\underline{i} \;\; \text{množimo} \quad \Big/ \cdot {}^0\underline{j} $$

$$ {}^0\underline{p}\,{}^0\underline{i} = {}^0p_x = {}^1p_x\,{}^1\underline{i}\,{}^0\underline{i} + {}^1p_y\,{}^1\underline{j}\,{}^0\underline{i} + {}^1p_z\,{}^1\underline{k}\,{}^0\underline{i} $$

$$ {}^0\underline{p}\,{}^0\underline{j} = {}^0p_y = -''-\,{}^0\underline{j} + -''-\,{}^0\underline{j} + -''-\,{}^0\underline{j} $$

$$ {}^0\underline{p}\,{}^0\underline{k} = {}^0p_z = -''-\,{}^0\underline{k} + -''-\,{}^0\underline{k} + -''-\,{}^0\underline{k} $$

$$ \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix} = {}^0\underline{\underline{R}}_1 \begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix} $$

$$ {}^0\underline{\underline{R}}_1 = \begin{array}{c} \overbrace{\quad x_1 \quad y_1 \quad z_1 \quad}^{O_1} \\ \begin{bmatrix} {}^1\underline{i}\,{}^0\underline{i} & {}^1\underline{j}\,{}^0\underline{i} & {}^1\underline{k}\,{}^0\underline{i} \\ {}^1\underline{i}\,{}^0\underline{j} & {}^1\underline{j}\,{}^0\underline{j} & {}^1\underline{k}\,{}^0\underline{j} \\ {}^1\underline{i}\,{}^0\underline{k} & {}^1\underline{j}\,{}^0\underline{k} & {}^1\underline{k}\,{}^0\underline{k} \end{bmatrix} \end{array} \begin{array}{c} x_0 \\ y_0 \\ z_0 \end{array} \Bigg\} O_0 $$

OBRATNA, ampak skalarni produkt komutativen

$$ {}^0\underline{\underline{R}}_1 = \begin{bmatrix} \cos\vartheta_{ii} & & \\ & \ddots & \\ & & \cos\vartheta_{kk} \end{bmatrix} $$

vsi koti ...

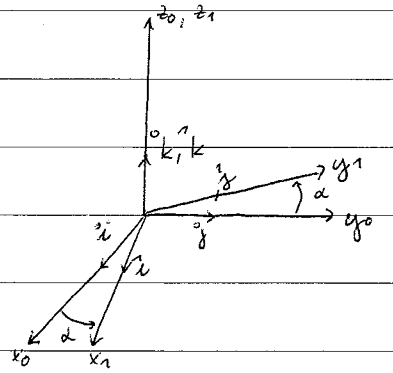
$$ {}^1\vec{p} = {}^1p_x\,{}^1\vec{i} + p_y\,{}^1\vec{j} + p_z\,{}^1\vec{k} $$

$$ \begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix} = {}^1\underline{\underline{R}}_0 \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix} $$

$$ {}^1\underline{\underline{R}}_0 = \begin{array}{c} \begin{array}{ccc} x_0 & y_0 & z_0 \end{array} \\ \begin{bmatrix} {}^0\underline{i}\,{}^1\underline{i} & {}^0\underline{j}\,{}^1\underline{i} & {}^0\underline{k}\,{}^1\underline{i} \\ & {}^0\underline{j}\,{}^1\underline{j} & {}^0\underline{k}\,{}^1\underline{j} \\ & & {}^0\underline{k}\,{}^1\underline{k} \end{bmatrix} \end{array} \begin{array}{c} x_1 \\ y_1 \\ z_1 \end{array} $$

$$ \Rightarrow \boxed{{}^1\underline{\underline{R}}_0 = \left({}^0\underline{\underline{R}}_1\right)^{-1} = \left({}^0\underline{\underline{R}}_1\right)^{T}} \;,\quad \det \underline{\underline{R}} = 1 $$

ortogonalna matrika



$$\underline{\underline{R}}_{z,\alpha} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotacija oko z

$${}^0k\,{}^1k = 1$$
$${}^0i\,{}^1i = \cos\alpha$$
$${}^0j\,{}^1j = \cos\alpha$$
$${}^0i\,{}^1j = \cos(90+\alpha) = -\sin\alpha$$
$${}^0j\,{}^1i = \cos(90-\alpha) = \sin\alpha$$

$$\underline{\underline{R}}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \qquad \underline{\underline{R}}_{y,\alpha} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

rotacija oko x　　　　rotacija oko y

$O_0 \quad O_1 \quad O_2$　　zajedničko ishodište

P

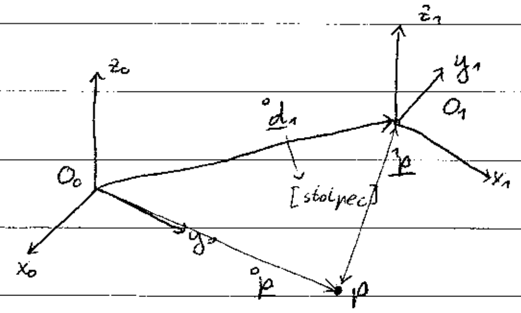
$${}^0p \quad {}^1p \quad {}^2p$$
$${}^0p = {}^0\underline{\underline{R}}_1\,{}^1p$$
$${}^1p = {}^1\underline{\underline{R}}_2\,{}^2p$$
$${}^0p = {}^0\underline{\underline{R}}_1\,{}^1\underline{\underline{R}}_2\,{}^2p$$

$$\boxed{{}^0\underline{\underline{R}}_n = {}^0\underline{\underline{R}}_1\,{}^1\underline{\underline{R}}_2 \cdots {}^{n-1}\underline{\underline{R}}_n}$$

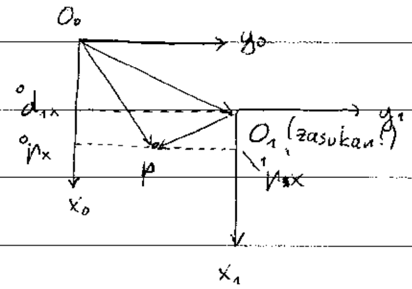
17.10.2012

Lega

orientacija + pozicija



Torisno gledano:



od prej:

$$^1\underline{p}' = {}^0\underline{\underline{R}}_1 \, {}^1\underline{p}$$

rotacija

$$^0\underline{p} = {}^1\underline{p}_1' + {}^0\underline{d}_1$$

$$\boxed{^0\underline{p} = {}^0\underline{\underline{R}}_1 \, {}^1\underline{p} + {}^0\underline{d}_1}$$

$$^0p_x = {}^0d_{1x} + {}^1p_x'$$

$$^0p_y = {}^0d_{1y} + {}^1p_y'$$

$$^0p_z = {}^0d_{1z} + {}^1p_z'$$

Zaporedne lege

tri koordinatni sistemi

$O_0 \quad O_1 \quad O_2$

P

$^0\underline{p} \quad {}^1\underline{p} \quad {}^2\underline{p}$

$$^0\underline{p} = {}^0\underline{\underline{R}}_1 \, {}^1\underline{p} + {}^0\underline{d}_1$$

$$^1\underline{p} = {}^1\underline{\underline{R}}_2 \, {}^2\underline{p} + {}^1\underline{d}_2$$

$$^0\underline{p} = \underbrace{{}^0\underline{\underline{R}}_1 \, {}^1\underline{\underline{R}}_2}_{\boxed{^0\underline{\underline{R}}_2 = {}^0\underline{\underline{R}}_1 \cdot {}^1\underline{\underline{R}}_2}} \, {}^2\underline{p} + \underbrace{{}^0\underline{\underline{R}}_1 \, {}^1\underline{d}_2 + {}^0\underline{d}_1}_{^0\underline{d}_2}$$

$$\boxed{^0\underline{d}_2 = {}^0\underline{\underline{R}}_1 \, {}^1\underline{d}_2 + {}^0\underline{d}_1}$$

zaporedne ~~pozicije~~ lege

$$\hookrightarrow \; {}^0\underline{p} = {}^0\underline{\underline{R}}_2 \, {}^2\underline{p} + {}^0\underline{d}_2$$

zaporedne rotacije

$${}^{0}\underline{p} = R\,{}^{1}\underline{p} + \underline{d}$$

homogena matrika (orient. + pozicija)

$$\begin{bmatrix} {}^{0}\underline{p} \\ 1 \end{bmatrix} = \underline{\underline{H}} \begin{bmatrix} {}^{1}\underline{p} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^{0}\underline{p} \\ 1 \end{bmatrix} = \begin{bmatrix} \underline{\underline{R}} & \underline{d} \\ \underline{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{1}\underline{p} \\ 1 \end{bmatrix}$$

~~tri ničle~~
(perspektiva)

$$\begin{bmatrix} {}^{0}R_1 & {}^{0}d_1 \\ \underline{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{1}R_2 & {}^{1}d_2 \\ \underline{0} & 1 \end{bmatrix} = \begin{bmatrix} {}^{0}R_1\,{}^{1}R_2 & {}^{0}R_1\,{}^{1}d_2 + {}^{0}d_1 \\ \underline{0} & 1 \end{bmatrix}$$

$$\Rightarrow \quad {}^{0}\underline{\underline{H}}_1\,{}^{1}\underline{\underline{H}}_2 = {}^{0}\underline{\underline{H}}_2$$

$$\boxed{{}^{0}\underline{\underline{H}}_n = {}^{0}\underline{\underline{H}}_1\,{}^{1}\underline{\underline{H}}_2 \cdots {}^{n-1}\underline{\underline{H}}_n}$$

inverzna homogena matrika

ortogonalna matrika (inverzna enaka transponirani (?))

~~množimo iz desne~~

$${}^{0}\underline{p} = \underline{\underline{R}}\,{}^{1}\underline{p} + \underline{d} \quad / \ \underline{\underline{R}}^T$$

$$\underline{\underline{R}}^T\,{}^{0}\underline{p} = \underline{\underline{R}}^T \underline{\underline{R}}\,{}^{1}\underline{p} + \underline{\underline{R}}^T \underline{d}$$

$${}^{1}\underline{p} = \underline{\underline{R}}^T\,{}^{0}\underline{p} - \underline{\underline{R}}^T \underline{d}$$

$$\boxed{\underline{\underline{H}}^{-1} = \begin{bmatrix} \underline{\underline{R}}^T & -\underline{\underline{R}}^T \underline{d} \\ \underline{0} & 1 \end{bmatrix}}$$

Lega

$\sin\vartheta = s\vartheta$

$\cos\vartheta = c\vartheta$

rotacija okoli x:
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\vartheta & -s\vartheta & 0 \\ 0 & s\vartheta & c\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

okoli y:
$$\begin{bmatrix} c\vartheta & 0 & s\vartheta & 0 \\ 0 & 1 & 0 & 0 \\ -s\vartheta & 0 & c\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

okoli z:
$$\begin{bmatrix} c\vartheta & -s\vartheta & 0 & 0 \\ s\vartheta & c\vartheta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer $\underline{\underline{H}} = trans(4,-3,7)\ Rot(y,90)\ Rot(z,90) =$

enotska

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} =$$

enica v drugi vrstici, prepišeno drugi stolpec v prvi

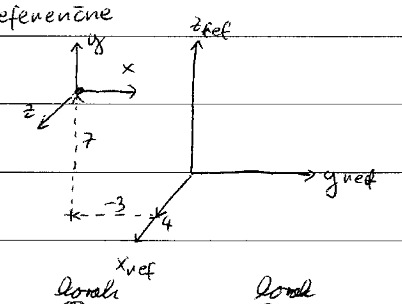
$$= \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

y os v smeri z ref

$$= \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ x_{ref} \\ y_{ref} \\ z_{ref} \\ \end{array}$$

drugi stolpec prve leve matrike

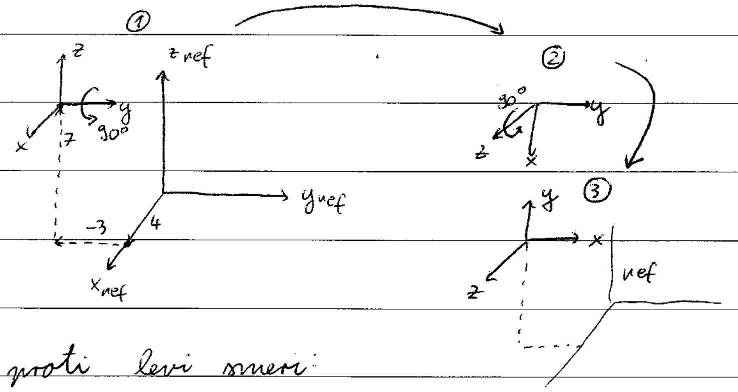
x os v smer y referenčne



korak ① korak ② korak ③

trans (4,-3,7) rot (y,90) rot (z,90)

premik referenčnega koordinatnega glede na relativnega koor. sistem



branje proti levi smeri:

premik referenčnega koordinatnega glede na referenčnega

① korak rot (z, 90)

② korak rot (y, 90)

③ trans (4,-3,7)

Premik koordinatnega sistema

$$\begin{bmatrix} \bullet & \bullet & \bullet & \times \\ \bullet & \bullet & \bullet & \times \\ \bullet & \bullet & \bullet & \times \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

lega < • orientacija, × pozicija

premik < • rotacija, × translacija

premultiplikacija:
$\begin{bmatrix} \text{premik} \\ \underline{\underline{P}} \end{bmatrix} \begin{bmatrix} \text{lega} \\ \underline{\underline{L}} \end{bmatrix}$ premik glede na referenčni koordinatni sistem

postmultiplikacija:
$\begin{bmatrix} \text{lega} \\ \underline{\underline{L}} \end{bmatrix} \begin{bmatrix} \text{premik} \\ \underline{\underline{P}} \end{bmatrix}$ premik glede na relativni koordinatni sistem

---

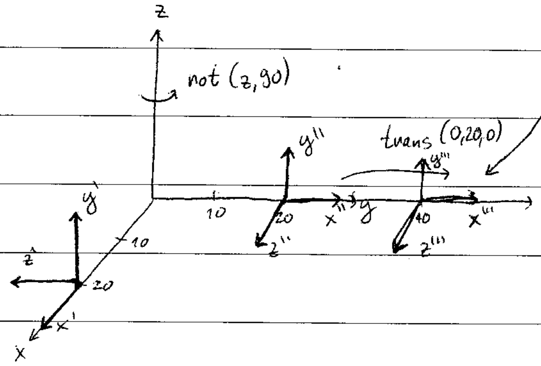
$$H = L = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{lega}$$

$$T = P = \text{trans}(0,20,0)\ \text{rot}(z,90) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

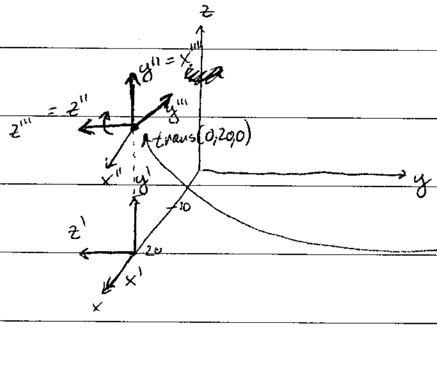
$\longleftarrow$

$\rightarrow$ premulti. $\underline{\underline{P}} \cdot \underline{\underline{L}} = \begin{bmatrix} 0 & 0 & 1 & \cancel{40} \\ 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

je pravilno!



$\rightarrow$ post multi $\underline{\underline{Y}} = \underline{\underline{L}} \cdot \underline{\underline{P}} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$



$$= \begin{bmatrix} 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

pravilno!

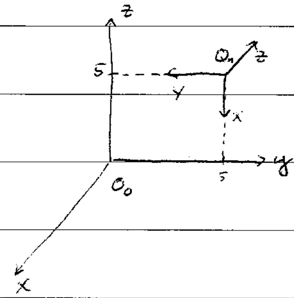
$P = \text{trans}(0,20,0)\ \text{rot}(z,90)$

$\longrightarrow$

za izpit 6 primerov za 1 nalogo (rama začetna + končna, premik ? ...)

1. NALOGA NA IZPITU

24.10.2012 Primer



$Rot(z, -\frac{\pi}{2})$
$Trans(0,-3,0)$
$Rot(x, \frac{\pi}{2})$

Zasuči O_1 za $\frac{\pi}{2}$ v smeri ur. kazalca okrog z sistema O_0

Translacija z 3 v neg. smeri O_0 v y

Rotacija $\frac{\pi}{2}$ pozitivno okrog x sistema O_0

$$\underline{\underline{H}}_1 = \begin{array}{c|cccc} & x & y & z & \text{relativnega} \\ \hline \text{referenčnega} & \\ \end{array} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{trenutna lega}$$

$\xleftarrow{\text{obratni vrstni red}}$

$$\underline{T} = Rot(x, \tfrac{\pi}{2}) \cdot Trans(0,-3,0) \cdot Rot(z, -\tfrac{\pi}{2})$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Rot(z, -\tfrac{\pi}{2}) \end{bmatrix} =$$

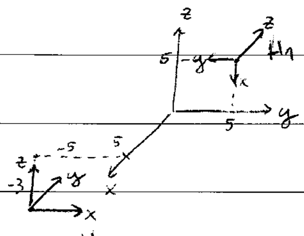
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

nova lega

$$\underline{\underline{H}}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

referenčni, najprej translacija

Primer glede na ref. sistem



H_2

H_1 in H_2 odčitamo iz risbe

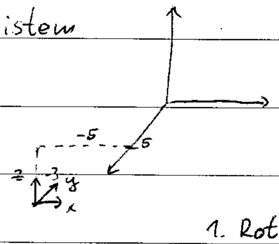
?

$$\underline{\underline{H}}_2 = \underline{\underline{T}} \cdot \underline{\underline{H}}_1$$

$$\underline{\underline{T}} = \underline{\underline{H}}_2 \cdot \underline{\underline{H}}_1^{-1} \qquad H^{-1} = \begin{bmatrix} \underline{\underline{R}}^T & -\underline{\underline{R}}^T \underline{d} \\ \underline{0} & 1 \end{bmatrix} \qquad -\underline{\underline{R}}^T \underline{d} = - \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} =$$

$$\underline{\underline{T}} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & +5 \\ 0 & -1 & 0 & +5 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad = - \begin{bmatrix} -5 \\ -5 \\ 0 \end{bmatrix}$$

Primer ref koordinatni sistem



1. $Rot(z, -\frac{\pi}{2})$

$\underline{\underline{T}}$ enako kot prva naloga!

2. $Trans(0,-3,0)$

3. $Rot(x, \frac{\pi}{2})$

$\underline{\underline{H_2}} = \underline{\underline{T}} \cdot \underline{\underline{H_1}}$

T^{-1}:

odčitamo iz risbe

$\underline{\underline{H_1}} = \underline{\underline{T}}^{-1} \underline{\underline{H_2}}$

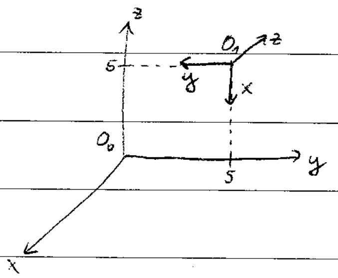
$$-\underline{\underline{R}}^T \underline{d} = -\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix} = -\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{\underline{T}}^{-1} = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{H_1}} = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer relativni koordinatni sistem



1. $Rot(z, -\frac{\pi}{2})$

2. $Trans(0,-3,0)$

3. $Rot(x, \frac{\pi}{2})$

$$\underline{\underline{T}}' = Rot(z, -\tfrac{\pi}{2})\, Trans(0,-3,0)\, Rot(x, \tfrac{\pi}{2})$$

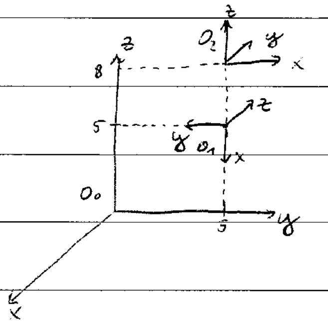
postmultiplikacija! $= \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\downarrow$

$\underline{\underline{H_2}} = \underline{\underline{H_1}}\, \underline{\underline{T}}$

$$\underline{\underline{H_2}} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer



H_1 in H_2 odčitamo

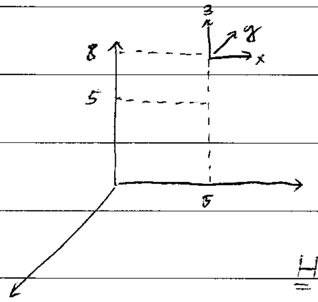
$\underline{H}_2 = \underline{H}_1 \cdot \underline{T}_1'$

$\boxed{\underline{T}_1' = \underline{H}_1^{-1} \underline{H}_2}$

premik $\underline{T}_1'$ je lega O_2 glede na O_1

$$\underline{T}_1' = \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer



$\underline{H}_2$ odčitamo

$\underline{T}$ izračunamo kot primer 4.

$\underline{H}_2 = \underline{H}_1 \underline{T}_1'$

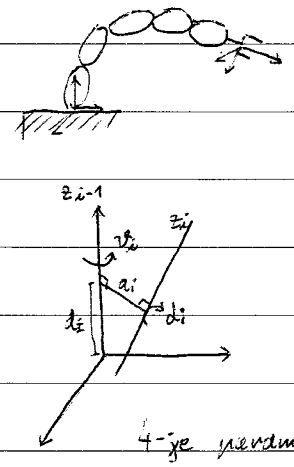
$\underline{H}_1 = \underline{H}_2 \underline{T}_1'^{-1}$

$$-\underline{R}^T \underline{d} = -\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

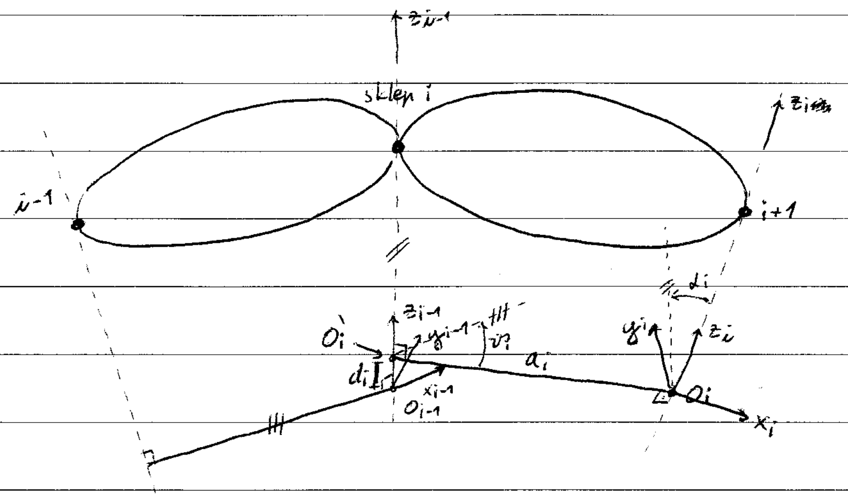
$$\underline{H}_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Geometrijski model robota z DH-skalarnimi parametri

7.11.2012



4-je parametri



DH pravila:

- izberi os z_i vzdolž osi sklepa $i+1$
- postavi koor. izhodišče O_i na presečišče normale in osi z_i
- postavi x_i vzdolž skupne normale, gleda od i proti $i+1$
- y_i po desnosučnem

DH parametri:

- a_i je razdalja med O_i in O_i' vzdolž x_i
- α_i je kot med z_{i-1} in z_i okrog x_i
- d_i je razdalja med O_{i-1} in O_i' vzdolž z_{i-1}
- ϑ_i je kot med x_{i-1} in x_i okrog z_{i-1}

Izjeme:

1) z_i in z_{i-1} sta vzporedni $\rightarrow d_i = 0$

2) z_i in z_{i-1} se sekata $\rightarrow x_i \perp z_{i-1}$ in z_i

O_i je v presečišču

3) bazni koordinatni sistem $\rightarrow z_0$ vzdolž prvega sklepa

O_0 v središču prvega sklepa

4) koordinatni sistem na vrhu $\rightarrow x_n \perp z_{n-1}$

5) translacijski sklep $\rightarrow$ izhodišče O_i na začetek translacijskega gibanja

matrika ${}^{i-1}A_i$ (med i in $i-1$):

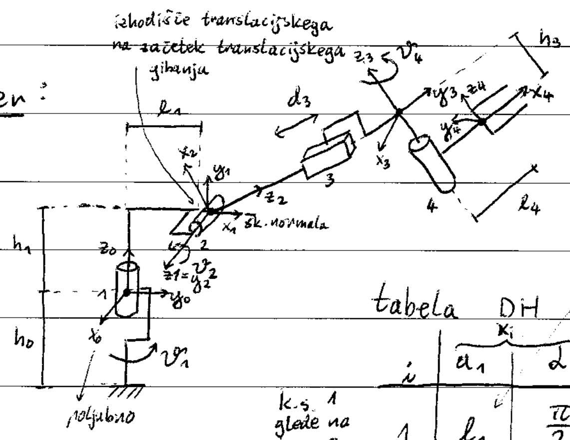
$${}^{i-1}A_i = \mathrm{Trans}(0,0,d_i)\ \mathrm{Rot}(z_{i-1}, \vartheta_i)\ \mathrm{Trans}(a_i,0,0)\ \mathrm{Rot}(x_i, \alpha_i) =$$

$\sin\vartheta_i = s\vartheta_i = s_i$

$\cos\vartheta_i = c\vartheta_i = c_i$

$$= \begin{bmatrix} c\vartheta_i & -s\vartheta_i & 0 & 0 \\ s\vartheta_i & c\vartheta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\vartheta_i & -s\vartheta_i c\alpha_i & s\vartheta_i s\alpha_i & a_i c\vartheta_i \\ s\vartheta_i & c\vartheta_i c\alpha_i & -c\vartheta_i s\alpha_i & a_i s\vartheta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer:



translacijski sklep od 0 do d_3

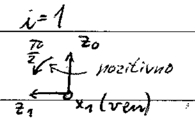
vzdolž x_1 za l_1 vzdolž z_{i-1} za h_0

tabela DH parametrov:

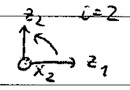
	i	a_i (x_i)	α_i (x_i)	d_i (z_{i-1})	ϑ_i (z_{i-1})	
k.s. 1 glede na k.s. 0	1	l_1	$\frac{\pi}{2}$	h_1	ϑ_1	
-"-	2	0	$\frac{\pi}{2}$	0	ϑ_2	
:	3	0	$\frac{\pi}{2}$	d_3	$\frac{\pi}{2}$	!
:	4	l_4	0	$-h_3$	ϑ_4	

translacijski nima premika.

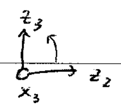
$i=1$



$i=2$



$i=3$



$$ {}^{0}A_1 = \begin{bmatrix} c1 & 0 & s1 & l_1 c_1 \\ s1 & 0 & -c1 & l_1 s1 \\ 0 & 1 & 0 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

geometrijski model

$$ \underline{\underline{T}} = \underline{\underline{{}^{0}A_1}}\ \underline{\underline{{}^{1}A_2}}\ \underline{\underline{{}^{2}A_3}}\ \underline{\underline{{}^{3}A_4}} = \text{fun!} $$

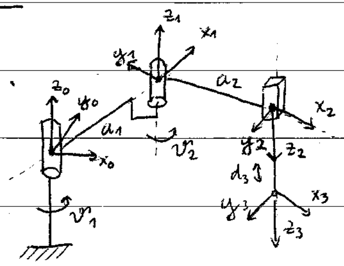
$$ {}^{1}A_2 = \begin{bmatrix} c2 & 0 & -s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

kot smo delali prej ...

$$ {}^{2}A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}A_4 = \begin{bmatrix} c4 & -s4 & 0 & l_4 c_4 \\ s4 & c4 & 0 & l_4 s4 \\ 0 & 0 & 1 & -h3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

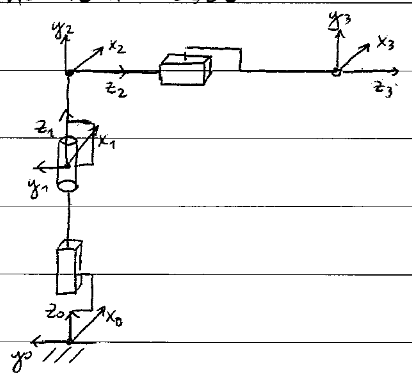
Primer:



	x_1		z_{i-1}	
i	a_i	α_i	d_i	ϑ_i
1	a_1	0	0	ϑ_1
2	a_2	π	0	ϑ_2
3	0	0	d_3	0

21.11.2012

Cilindrični robot



$$
\begin{array}{c|cc|cc}
 & \multicolumn{2}{c|}{\overbrace{\qquad\qquad}^{x_i}} & \multicolumn{2}{c}{\overbrace{\qquad\qquad}^{z_{i-1}}} \\
i & a_i & \alpha_i & d_i & \vartheta_i \\
\hline
1 & 0 & 0 & d_1 & 0 \\
2 & 0 & -\frac{\pi}{2} & d_2 & \vartheta_2 \\
3 & 0 & 0 & d_3 & 0
\end{array}
$$

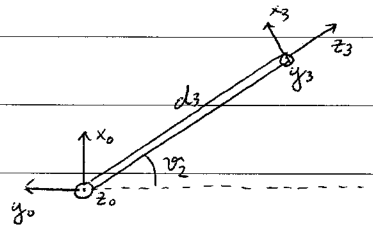
spremenljivke

$$
{}^{0}\underline{\underline{A}}_1 = \begin{bmatrix} c\vartheta_1 & -s\vartheta_1 c\alpha_1 & s\vartheta_1 s\alpha_1 & a_1 c\vartheta_1 \\ s\vartheta_1 & c\vartheta_1 c\alpha_1 & -c\vartheta_1 s\alpha_1 & a_1 s\vartheta_1 \\ 0 & s\alpha_1 & c\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\underline{\underline{T}} = {}^{0}\underline{\underline{A}}_1 \cdot {}^{1}\underline{\underline{A}}_2 \cdot {}^{2}\underline{\underline{A}}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} d_2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} =
$$

$$
= \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_1+l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 & 0 & s_2 & d_3 s_2 \\ s_2 & 0 & -c_2 & -d_3 c_2 \\ 0 & 1 & 0 & d_1+l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

to se baje naresti tudi na pamet:

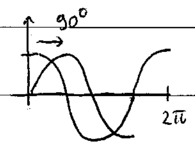


rotacijska matrika

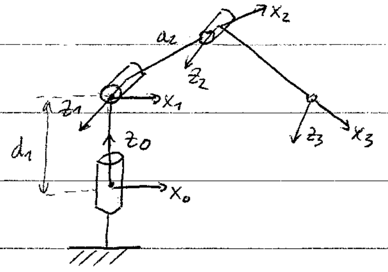
$$
\begin{array}{cc}
 & \begin{array}{ccc} x_3 & y_3 & z_3 \end{array} \\
 & \begin{bmatrix} c\vartheta_2 & c90 & c(90-\vartheta_2) \\ c(90-\vartheta_2) & c90 & c(180-\vartheta_2) \\ c90 & c0 & c90 \end{bmatrix} \begin{array}{c} x_0 \\ y_0 \\ z_0 \end{array}
\end{array}
$$

$$\cos(90^\circ - x) = \sin x$$

$$+\cos(x - 90^\circ) = \sin x$$

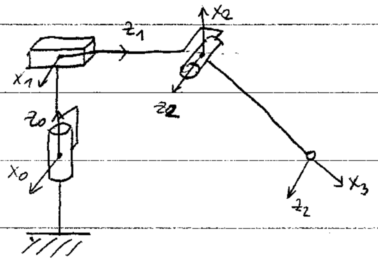


Antropomorfni model

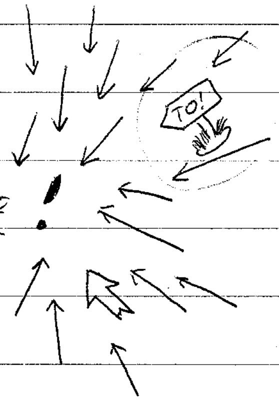


i	a_i	α_i	d_i	ϑ_i
	x_1		z_{i-1}	
1	0	$\frac{\pi}{2}$	d_1	ϑ_1
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

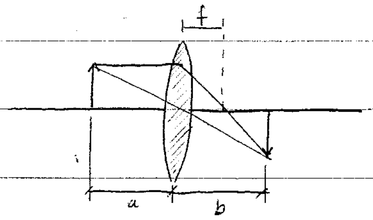
Še en primer



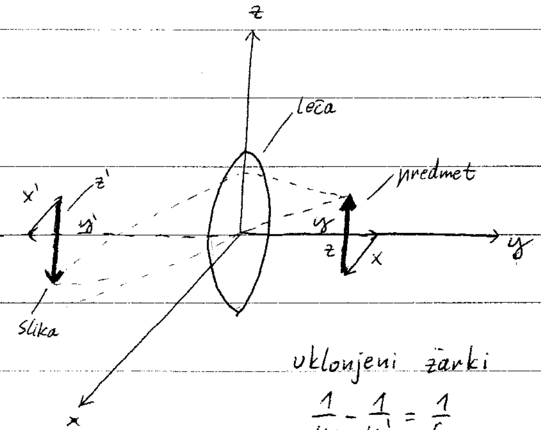
i	a_i	α_i	d_i	ϑ_i
1	0	$-\frac{\pi}{2}$	d_1	ϑ_1
2	0	$-\frac{\pi}{2}$	d_2	$-\frac{\pi}{2}$
3	a_3	0	0	ϑ_3



Perspektivična transformacija



$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$



uklonjeni žarki

$$\frac{1}{y} - \frac{1}{y'} = \frac{1}{f}$$

neuklonjeni žarki

$$\frac{x}{y} = \frac{x'}{y'}$$

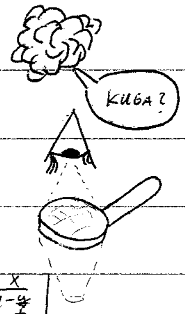
$$\frac{1}{y'} = \frac{1}{y} - \frac{1}{f}$$

$$y' = \frac{yf}{f-y}$$

$$y' = \frac{x'y}{x}$$

$$\frac{x'y}{x} = \frac{yf}{f-y}$$

$$\boxed{x' = \frac{x}{1-\frac{y}{f}} \qquad z' = \frac{z}{1-\frac{y}{f}} \qquad y' = \frac{y}{1-\frac{y}{f}}}$$



$$\underline{\underline{P}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{f} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1-\frac{y}{f} \end{bmatrix} = \begin{bmatrix} \frac{x}{1-\frac{y}{f}} \\ \frac{y}{1-\frac{y}{f}} \\ \frac{z}{1-\frac{y}{f}} \\ 1 \end{bmatrix}$$

Primer

Leža v $[x, z]$

$f=2$

$T'[-1,-3,-2]$

$$y' = \frac{y}{1-\frac{y}{f}}$$

$$y'(1-\frac{y}{f}) = y$$

$$y' - y'\frac{y}{f} = y$$

$$y\left(+\frac{y'}{f}+1\right) = +y'$$

$$y = \frac{y'}{\frac{y'}{f}+1} = \underline{\underline{6}}$$

$$x = \cancel{-\tfrac{1}{2}}\ \underline{\underline{2}}$$

$$z = \underline{\underline{4}}$$

$$\underline{T}' = \underline{P} \cdot \underline{T}$$

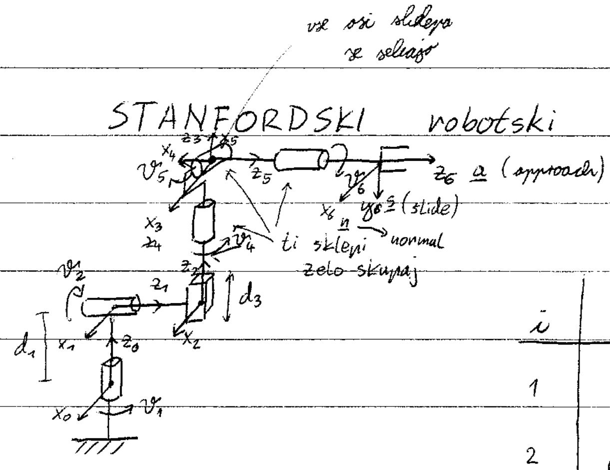
$$\underline{T} = \underline{P}^{-1}\, \underline{T}'$$

$$P \cdot P^{-1} = I \qquad P^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{f} & 0 & 1 \end{bmatrix}$$

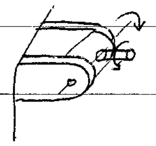
v tisti osi, kjer je gorišču razdalja

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -2 \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \\ 1 \end{bmatrix}$$

STANFORDSKI robotski manipulator 28.11.2012



i	a_i	α_i	d_i	ϑ_i
	x_i		z_{i-1}	
1	0	$-\frac{\pi}{2}$	d_1	ϑ_1
2	0	$\frac{\pi}{2}$	d_2	ϑ_2
3	0	0	d_3	0
4	0	$-\frac{\pi}{2}$	0	ϑ_4
5	0	$\frac{\pi}{2}$	0	ϑ_5
6	0	0	d_6	ϑ_6



$$ {}^{0}\underline{\underline{A}}_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{4}\underline{\underline{A}}_5 = \begin{bmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}\underline{\underline{A}}_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{5}\underline{\underline{A}}_6 = \begin{bmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}\underline{\underline{A}}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

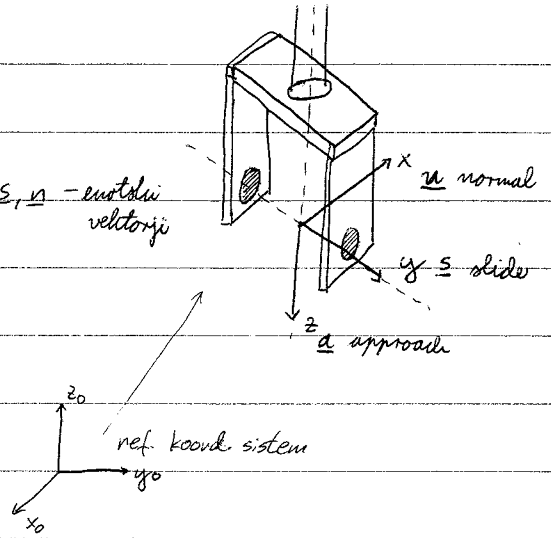
$$ {}^{3}\underline{\underline{A}}_4 = \begin{bmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

matrika zapestja

$$ {}^{3}\underline{\underline{A}}_6 = {}^{3}\underline{\underline{A}}_4 \cdot {}^{4}\underline{\underline{A}}_5 \cdot {}^{5}\underline{\underline{A}}_6 = $$

$$ = \begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & c4s5 & c4s5d6 \\ s4c5c6 + c4s6 & -s4c5s6 + c4c6 & s4s5 & s4s5d6 \\ -s5c6 & s5s6 & c5 & c5d6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Orientacija



$\underline{a}, \underline{s}, \underline{n}$ – enotski vektorji

②

$$\underline{R} = \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix}$$

$\underline{n} = \underline{s} \times \underline{a}$

$\underline{s} \cdot \underline{s} = 1$

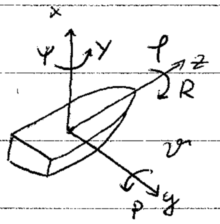
$\underline{a} \cdot \underline{a} = 1$

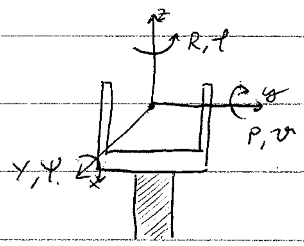
$\underline{s} \cdot \underline{a} = 0$

③

$\boxed{\text{RPY način}}$ roll, pitch, yaw

$\downarrow$ z $\quad\downarrow$ y $\quad\downarrow$ x

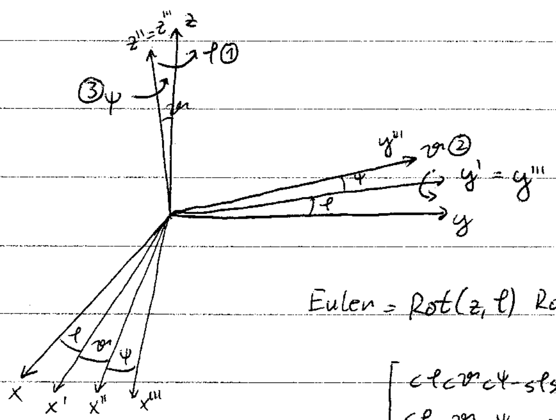




$$\underline{RPY} = Rot(z,\varphi)\, Rot(y,\vartheta)\, Rot(x,\psi) =$$

$$= \begin{bmatrix} c\varphi c\vartheta & c\varphi s\vartheta s\psi - s\varphi c\psi & c\varphi s\vartheta c\psi + s\varphi s\psi \\ c\vartheta s\varphi & s\varphi s\vartheta s\psi + c\varphi c\psi & s\varphi s\vartheta c\psi - c\varphi s\psi \\ -s\vartheta & c\vartheta s\psi & c\vartheta c\psi \end{bmatrix}$$

$\boxed{\text{Eulerjevi koti}}$



možnih je več vrstnih redov (z, y, z) naša

XYZ

XZY

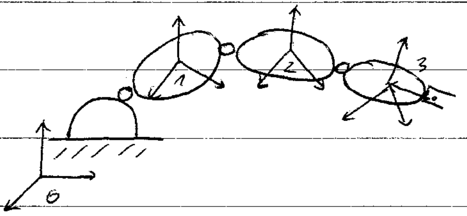
XYX

⋮

$$\text{Euler} = Rot(z,\varphi)\, Rot(y',\vartheta)\, Rot(z'',\psi) =$$

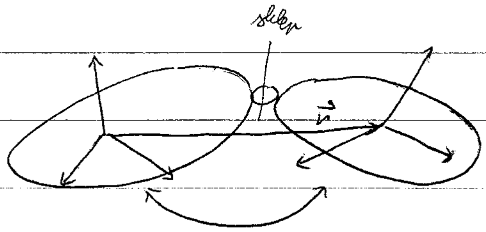
$$= \begin{bmatrix} c\varphi c\vartheta c\psi - s\varphi s\psi & -c\varphi c\vartheta s\psi - s\varphi c\varphi & c\varphi s\vartheta \\ s\varphi c\vartheta c\psi + c\varphi s\psi & -s\varphi c\vartheta s\psi + c\varphi c\psi & s\varphi s\vartheta \\ -s\vartheta c\psi & s\vartheta s\psi & c\vartheta \end{bmatrix}$$

5.12.2012



$$^0H_3 = {}^0H_1\,{}^1H_2\,{}^2H_3$$

odvajanje
- hitrost
- pospeški
- ...



$\underline{R}$

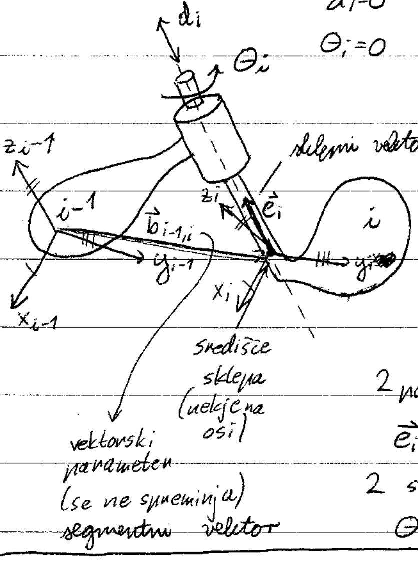
$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix} \qquad H = \left[\begin{array}{ccc|c} & R & & \vec{r} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$R_y = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}$$

$$R_z = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$d_i = 0$

$\Theta_i = 0$



$$^{i-1}H_i = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & b_{i-1,i} \\ 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

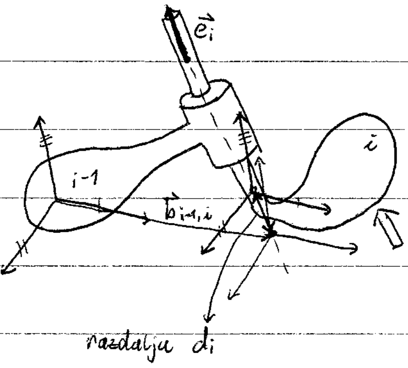
2 parametra

$\vec{e}_i,\ \vec{b}_{i-1,i}$

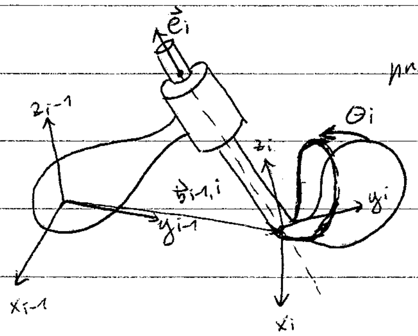
2 spremenljivki

$\Theta_i,\ d_i$

primer, če $d_i \neq 0$, $\Theta_i = 0$



$$^{i-1}H_i = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & b_{i-1,i} + d_i\vec{e}_i \\ 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$



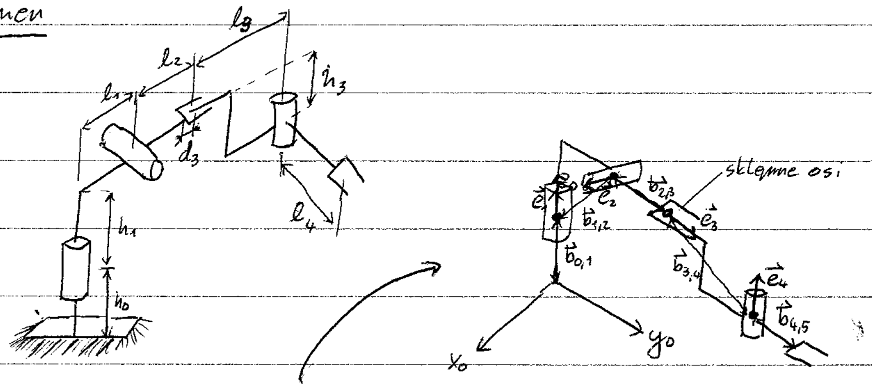
primer $d_i = 0,\ \Theta_i \neq 0$

$$^{i-1}H_i = \left[\begin{array}{ccc|c} & {}^{i-1}R_i & & \vec{b}_{i-1,i} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

primer $d_i \neq 0,\ \Theta_i \neq 0$

$$^{i-1}H_i = \left[\begin{array}{ccc|c} & {}^{i-1}R_i & & \vec{b}_{i-1,i} + d_i\vec{e}_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Primer



1) referenčna / začetna lega

$d_i = 0,\ \Theta_i = 0$

2) določiti središča sklepov

3) tabele !

	1	2	3	4
e_i	0	1	0	0
	0	0	1	0
	1	0	0	1

cool !

	1	2	3	4	5
$b_{i-1,i}$	0	0	0	0	0
	0	l_1	l_2	l_3	l_4
	h_0	h_1	0	$-h_3$	0

	1	2	3	4
d_i	0	0	d_3	0
Θ_i	Θ_1	Θ_2	0	Θ_4

$$ {}^{0}H_{1} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & h_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{1}H_{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & l_1 \\ 0 & s_2 & c_2 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}H_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 + d_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{3}H_{4} = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & l_3 \\ 0 & 0 & 1 & h_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{4}H_{5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

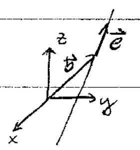
Vektorski model

12.12.2012

ponovitev

1. referenčna lega , ref. k.s.

   $d_i = 0$ (translacijski)



2. središča sklepov

3.

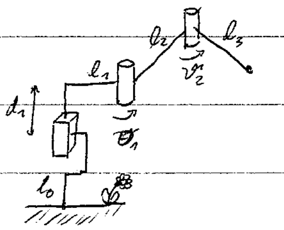
4.

5. matrike

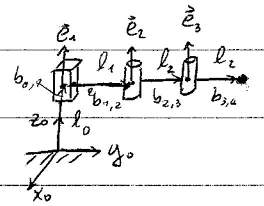
$$ {}^{i-1}H_{i} = \begin{bmatrix} R_x, R_y, R_z & \vec{b}_{i-1,i} \\ \underline{0} & 1 \end{bmatrix} $$

$$ {}^{i-1}H_{i} = \begin{bmatrix} \underline{\underline{I}} & d_i \vec{e}_i + b_{i-1,i} \\ \underline{0} & 1 \end{bmatrix} $$

Primer: SCARA robot



ref. lega



i	1	2	3
ϑ_i	0	ϑ_2	ϑ_3
d_i	d_1	0	0

$$\vec{e}_i: \begin{array}{ccc} 1 & 2 & 3 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{array}$$

$$\vec{b}_{i,i-1}: \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & l_1 & l_2 & l_3 \\ l_0 & 0 & 0 & 0 \end{array}$$

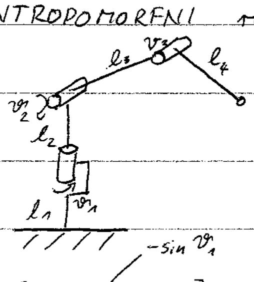
$$\underline{\underline{{}^0H_1}} = \begin{bmatrix} \underline{\underline{I}} & d_1\vec{e}_1 + b_{01} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

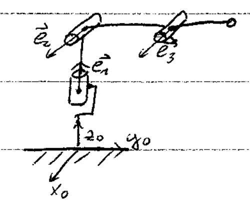
$$\underline{\underline{{}^1H_2}} = \begin{bmatrix} Rot\,z & b_{1,2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ s_2 & c_2 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{{}^2H_3}} = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ s_3 & c_3 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{{}^3H_4}} = \cancel{\begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$\underline{\underline{{}^3H_4}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer ANTROPOMORFNI robot





i	1	2	3
ϑ_i	ϑ_1	ϑ_2	ϑ_3
d_i	0	0	0

$$\vec{e}_i: \begin{array}{ccc} 1 & 2 & 3 \\ \hline 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}$$

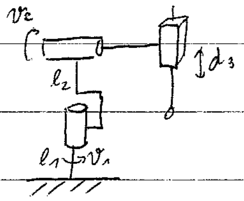
$$\vec{b}_{i,i-1}: \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & l_3 & l_4 \\ l_1 & l_2 & 0 & 0 \end{array}$$

$-\sin\vartheta_1$

$$\underline{\underline{{}^0H_1}} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{{}^1H_2}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{{}^2H_3}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & l_3 \\ 0 & s_3 & c_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \underline{\underline{{}^3H_4}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer STANFORDSKI robot



	1	2	3
ϑ_i	ϑ_1	ϑ_2	0
d_i	0	0	d_3

$$\vec{e}_i: \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline & 0 & 0 & 0 \\ & 0 & +1 & 0 \\ & 1 & 0 & 1 \end{array}$$

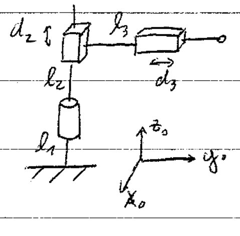
$$\vec{b}_{i,i-1}: \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline & 0 & 0 & 0 \\ & 0 & 0 & l_3 \\ & l_1 & l_2 & d_3 \end{array}$$

$b_{i,i-1}$ do središča naslednjega!

$$ {}^0\underline{\underline{H}}_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1\underline{\underline{H}}_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ 0 & 1 & 0 & 0 \\ -s_2 & 0 & c_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Primer CILINDRIČNI robot

$$ {}^0\underline{\underline{H}}_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^1\underline{\underline{H}}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^2\underline{\underline{H}}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3+d_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$



KVATERNIONI

posplošitev kompleksnih števil

$$\boxed{q = q_0 \cdot 1 + \underbrace{q_1 \underline{i} + q_2 \underline{j} + q_3 \underline{k}}_{\underline{q}}}$$

enotski vektorji, kompleksni!

seštevanje:

$$p + q = (p_0 + q_0) + (p_1 + q_1)\underline{i} + (p_2 + q_2)\underline{j} + (p_3 + q_3)\underline{k}$$

množenje s skalarjem:

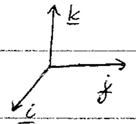
$$\dots \quad w q = \dots$$

konjugirani

$$q^* = q_0 - q_1 \underline{i} - q_2 \underline{j} - q_3 \underline{k}$$

$$\underline{i}^2 = \underline{j}^2 = \underline{k}^2 = -1 \overset{?}{=} (\underline{i}\underline{j}\underline{k})^2$$

množenje ni komutativno



$$\underline{k} = \underline{i}\underline{j}$$

$$\underline{j}\underline{i} = -\underline{k}$$

"imaginarni" del

$$q = q_0 + \underline{q}$$

$$pq = (p_0 + p_1 i + p_2 j + p_3 k)(q_0 + q_1 i + q_2 j + q_3 k) =$$

$$= p_0 q_0 + q_0 \overbrace{(p_1 i + p_2 j + p_3 k)}^{\underline{p}} + p_0 \overbrace{(q_1 i + q_2 j + q_3 k)}^{\underline{q}} +$$

$$+ p_1 q_1 \underline{i}^2 + p_2 q_1 \overset{-k}{(ji)} + p_3 q_1 k i +$$

$$+ p_1 q_2 \overset{k}{(ij)} + p_2 q_2 j^2 + p_3 q_2 k j +$$

$$+ p_1 q_3 ik + p_2 q_3 jk + p_3 q_3 \underline{k}^2 =$$

$$= p_0 q_0 + q_0 \underline{p} + p_0 \underline{q} - p_1 q_1 - p_2 q_2 - p_3 q_3 + (p_2 q_3 - p_3 q_2) i + (p_3 q_1 - p_1 q_3) j +$$

$$+ (p_1 q_2 - p_2 q_1) k$$

formula:

$$pq = \underbrace{p_0 q_0 - \overset{\text{skalarni}}{\underline{p}\underline{q}}}_{\text{SKALAR}} + \underbrace{p_0 \underline{q} + q_0 \underline{p} + \overset{\text{vektorski}}{\underline{p} \times \underline{q}}}_{\text{VEKTOR}}$$

r_0 produkt $\qquad\qquad r = p \cdot q$

$r_0 = p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3$

$r_1 = p_0 q_1 + q_0 p_1 + p_2 q_3 - p_3 q_2$

$r_2 = p_0 q_2 + q_0 p_2 + p_3 q_1 - p_1 q_3$ $\qquad\qquad r = r_0 + r_1 i + r_2 j + r_3 k$

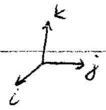
$r_3 = p_0 q_3 + q_0 p_3 + p_1 q_2 - p_2 q_1$

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Primer

$p = 3 + i - 2j + k$

$q = 2 - i + 2j + 3k$



$$(3+i-2j+k)(2-i+2j+3k) =$$

$$6 + 2i - 4j + 2k - 3i - i^2 + 2\overset{-k}{(ji)} - \overset{j}{(ki)} + 6j + 2\overset{k}{(ij)} - 4j^2 + 2\overset{-i}{(kj)} +$$

$$9k + 3\overset{-j}{(ik)} - 6\overset{i}{(jk)} + 3k^2 =$$

$$= 6 + 1 + 4 - 3 + (2 - 3 - 2 - 6)i + (-4 - 1 + 6 - 3)j +$$

$$+ (2 - 2 + 2 + 9)k$$

$$= \underline{\underline{8 - 9i - 2j + 11k}}$$

izračunajmo po drugi formuli $\tilde{p}$ $\qquad \tilde{q}$

$p = 3 + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \qquad (3 + \begin{bmatrix} +1 \\ -2 \\ 1 \end{bmatrix})(2 + \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}) =$

$q = 2 + \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} \qquad = 6 - (-1 - 4 + 3) + \begin{bmatrix} -3 \\ +6 \\ 9 \end{bmatrix} + \begin{bmatrix} +2 \\ -4 \\ 2 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 1 & -2 & 1 \\ -1 & 2 & 3 \end{bmatrix}$

$$= 6 - (-2) + \begin{bmatrix} -3 \\ 6 \\ 9 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix} + \begin{bmatrix} -8 \\ -4 \\ 0 \end{bmatrix} =$$

$$= \underline{\underline{8 + \begin{bmatrix} -9 \\ -2 \\ 11 \end{bmatrix}}}$$

še po tretji:

antisimetrična

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 3 & -1 & +2 & -1 \\ +1 & 3 & -1 & -2 \\ -2 & 1 & 3 & -1 \\ +1 & 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ -9 \\ -2 \\ 11 \end{bmatrix}$$

Rotacija + Lega s kvaternioni

$$\underline{r}_2 = \underline{R}\, r_1 \qquad q = q_0 + \underline{q}$$

$$\boxed{\underline{r}_2 = q\, r_1\, q^*} \qquad q^* = q_0 - \underline{q}$$

$$r_2 = 0 + \underline{r}_2$$

pa pojdimo

$$r_1 = 0 + \underline{r}_1$$

$$\underline{r}_2 = (q_0 + \underline{q})(0 + \underline{r}_1)q^* =$$

$$= (\underbrace{-\underline{q}\,\underline{r}_1}_{\text{ŠKALAR}} + \underbrace{q_0 \underline{r}_1 + \underline{q} \times \underline{r}_1}_{\text{VEKTOR}})(q_0 - \underline{q}) =$$

solata!

pravokotna, skalarni je 0

$$= -\cancel{\underline{q}\,\underline{r}_1 q_0} +$$

$$+ \cancel{q_0 \underline{r}_1 \underline{q}} + (\underline{q} \times \underline{r}_1)\underline{q} +$$

$$+ \underline{q}\,\underline{r}_1\underline{q} + q_0^2 \underline{r}_1 + q_0(\underline{q} \times \underline{r}_1) -$$

$$- q_0(\underline{r}_1 \times \underline{q}) - \underline{q} \times \underline{r}_1 \times \underline{q} =$$

$$+ q_0(\underline{q} \times \underline{r}_1) = \qquad -(\underline{q}\,\underline{q})\underline{r}_1 + (\underline{r}_1 \underline{q})\underline{q}$$

$$r_2 = q_0^2 \underline{r}_1 - (\underline{q} \cdot \underline{q})\underline{r}_1 + 2q_0(\underline{q} \times \underline{r}_1) + 2\underline{q}(\underline{q}\,\underline{r}_1)$$

$$\underline{q} \times \underline{r}_1 = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{1z} \end{bmatrix} \equiv \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ q_1 & q_2 & q_3 \\ r_{1x} & r_{1y} & r_{1z} \end{bmatrix}$$

$$2\underline{q}(\underline{q} \cdot \underline{r}_1) = 2\underline{q}\,\underline{q}^T r_1 = 2r_1 \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} [q_1\ q_2\ q_3]$$

$$r_2 = \left\{ (g_0^2 - g_1^2 - g_2^2 - g_3^2)\,\underline{\underline{I}} + 2g_0 \begin{bmatrix} 0 & -g_3 & g_2 \\ g_3 & 0 & -g_1 \\ -g_2 & g_1 & 0 \end{bmatrix} + 2 \begin{bmatrix} g_1^2 & g_1 g_2 & g_1 g_3 \\ g_1 g_2 & g_2^2 & g_2 g_3 \\ g_1 g_3 & g_2 g_3 & g_3^2 \end{bmatrix} \right\} \underline{r_1}$$

$$\underline{\underline{R}} = \begin{bmatrix} g_0^2 + g_1^2 - g_2^2 - g_3^2 & 2(g_1 g_2 - g_0 g_3) & 2(g_1 g_2 + g_0 g_2) \\ 2(g_1 g_2 + g_0 g_3) & g_0^2 - g_1^2 + g_2^2 - g_3^2 & 2(g_2 g_3 - g_0 g_1) \\ 2(g_1 g_3 - g_0 g_2) & 2(g_2 g_3 + g_0 g_1) & g_0^2 - g_1^2 - g_2^2 + g_3^2 \end{bmatrix}$$

$$\text{trace } \underline{\underline{R}} = r_{11} + r_{22} + r_{33} = 3g_0^2 - g_1^2 - g_2^2 - g_3^2$$ 2.1.2013

kvaternion pomeni rotacijo → enotski

$$\boxed{g_0^2 + g_1^2 + g_2^2 + g_3^2 = 1} \qquad \underline{g} = g_0 + g_1 \underline{i} + g_2 \underline{j} + g_3 \underline{k}$$

$$g_0 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}}$$
$$g_1 = \frac{1}{2}\sqrt{1 + r_{11} - r_{22} - r_{33}}$$
$$g_2 = \frac{1}{2}\sqrt{1 - r_{11} + r_{22} - r_{33}}$$
$$g_3 = \frac{1}{2}\sqrt{1 - r_{11} - r_{22} + r_{33}}$$

iz rotacijske matrike v kvaternione

$$\cancel{g_1 = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2} s_x}$$

$$\boxed{\underline{g} = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\, \underline{s}_{\text{enotski}}} =$$

vmesno:

$$\cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2} = \cos\vartheta$$
$$\cos^2\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2} = 1$$

vzeti

$$r_{11} = g_0^2 + g_1^2 - g_2^2 - g_3^2 = g_0^2 + 2g_1^2 - g_1^2 - g_2^2 - g_3^2 \;?$$

$$= 2g_0^2 + 2g_1^2 - 1 = 2\cos^2\frac{\vartheta}{2} + 2 s_x^2 \sin\frac{\vartheta}{2} - 1 =$$

vsota g_0, g_1, g_2, g_3

$$2\cos^2\frac{\vartheta}{2} = 1 + \cos\vartheta \qquad = c\vartheta + s^2 + v\vartheta$$

$$2\sin^2\frac{\vartheta}{2} = 1 - \cos\vartheta = \text{vers}\,\vartheta = v\vartheta$$

$$\underline{g} = \underbrace{\cos\frac{\vartheta}{2}}_{g_0} + \underbrace{\sin\frac{\vartheta}{2} s_x}_{g_1} + \underbrace{\sin\frac{\vartheta}{2} s_y}_{g_2} + \underbrace{\sin\frac{\vartheta}{2} s_z}_{g_3}$$

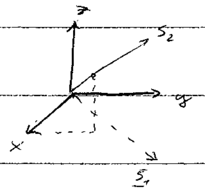
$$\underline{\underline{R}} = \begin{bmatrix} s_x^2 v\vartheta + c\vartheta & - & + \\ s_x s_y v\vartheta + s_z s\vartheta & s_y^2 v\vartheta + c\vartheta & - \\ s_x s_z v\vartheta - s_y s\vartheta & s_y s_z v\vartheta + s_x s\vartheta & s_z^2 v\vartheta + c\vartheta \end{bmatrix}$$

$\underline{s_x} = [1, 0\ 0]^T$

Primer: zapiši kvaternion za rotacijo $\frac{\pi}{2}$ okoli osi $x=y$, $z=0$

in rotacijo za $\frac{2\pi}{3}$ okoli osi $x=y$, $z=x$

s produktom obeh določi skupno

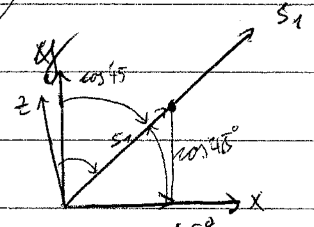


$$s_1 = [\cos 45°, \cos 45°, \cos 90°]^T = \frac{\sqrt{2}}{2}[1, 1, 0]^T$$

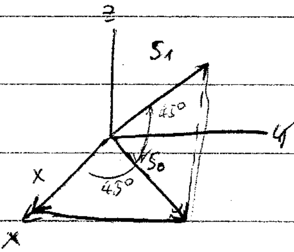
$$s_2 = \cancel{[\cos 45°, \cos 45°, \cos 45°]}$$

$$s = \frac{1}{\sqrt{3}} = s_{x2} = s_{y2} = s_{z2}$$

$$s_2 = \frac{1}{\sqrt{3}}[1, 1, 1]^T$$



$x = \cos 45° \cdot s$



$x = \cos 45 \cdot s_0$

$x = \frac{\sqrt{2}}{2} s_0$

$s_0 = \cos 45\ s_1$

$q = q_0 + q_1 i + q_2 j + q_3 k$

$i^2 = j^2 = k^2 = -1 = -1$

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\underline{s} \longrightarrow$$

$$s_1: \quad q = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} = \frac{\sqrt{2}}{2} + \frac{1}{2}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

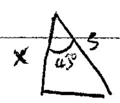
$$s_2: \quad p = \frac{1}{2} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1}{2} + \frac{1}{2}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$q \cdot p = \frac{1}{4}(\sqrt{2} + i + j)(1 + i + j + k) =$$

$$= \frac{1}{4}(\sqrt{2} + i + j + \sqrt{2}i + i^2 + ji + \sqrt{2}j + ij + j^2 + \sqrt{2}k + ik + jk) =$$

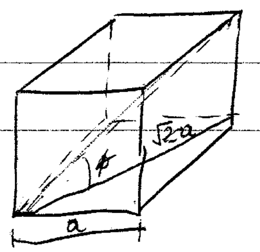
$$= \frac{1}{4}(\sqrt{2} + i + j + \sqrt{2}i - 1 - k + \sqrt{2}j + k - 1 + \sqrt{2}k - j + i) =$$

$$= \frac{1}{4}[(\sqrt{2} - 2) + (\sqrt{2} + 2)i + \sqrt{2}j + \sqrt{2}k]$$



$x = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} s_1$

$= \frac{1}{2} s_1$



$\phi = \arctan\frac{1}{\sqrt{2}}$

<u>Primer</u> $r_1 = [1,1,0]^T$ zarotiraj za $90°$ v pozitivni smeri okrog osi z!

vemo

$$r_2 = q\, r_1\, q^*$$

$$r_1 = 0 + \underline{r}_1$$

$$r_2 = 0 + \underline{r}_2$$

$$pq = p_0 q_0 - \underline{p}\cdot\underline{q} + p_0\underline{q} + q_0\underline{p} + \underline{p}\times\underline{q}$$

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\underline{s}$$

$$q = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$r_2 = \frac{\sqrt{2}}{2}\left(1+\begin{bmatrix}0\\0\\1\end{bmatrix}\right)\left(0+\begin{bmatrix}1\\1\\0\end{bmatrix}\right)\left(1-\begin{bmatrix}0\\0\\1\end{bmatrix}\right)\frac{\sqrt{2}}{2}$$

$$= \frac{1}{2}\left[-[0\,0\,1]\begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}i & j & k\\0 & 0 & 1\\1 & 1 & 0\end{bmatrix}\right]\left(1-\begin{bmatrix}0\\0\\1\end{bmatrix}\right) =$$

$$= \frac{1}{2}\left(-0 + \begin{bmatrix}1\\1\\0\end{bmatrix} + \begin{bmatrix}-1\\1\\0\end{bmatrix}\right)\left(1-\begin{bmatrix}0\\0\\1\end{bmatrix}\right) =$$

$$= \frac{1}{2}\left(0 + \begin{bmatrix}0\\2\\0\end{bmatrix}\right)\left(1-\begin{bmatrix}0\\0\\1\end{bmatrix}\right) =$$

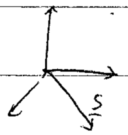
$$= \frac{1}{2}\left(-[0\,2\,0]\begin{bmatrix}0\\0\\1\end{bmatrix} + \begin{bmatrix}0\\2\\0\end{bmatrix} - \begin{bmatrix}i & j & k\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}\right)$$

$$= \frac{1}{2}\left(\begin{bmatrix}0\\2\\0\end{bmatrix} - \begin{bmatrix}2\\0\\0\end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix}-2\\2\\0\end{bmatrix} = \begin{bmatrix}-1\\1\\0\end{bmatrix}$$

simpl naloga bo na izpitu, verjetno!

<u>Primer</u> zarotiraj $\vec{j}$ za $180°$ okrog osi $x=y,\ z=0$

<u>rezultat je $\vec{i}$ sicer</u>



$$s = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right]^T$$

$$q = \cancel{\cos 90°} + \underbrace{\sin 90°}_{1}\frac{\sqrt{2}}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} = \frac{\sqrt{2}}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} = \frac{\sqrt{2}}{2}(\underline{i}+\underline{j})$$

$$q^* = \frac{\sqrt{2}}{2}(-\underline{i}-\underline{j}) \qquad \underline{r}_1 = \underline{j}$$

$$r_2 = q\, r_1\, q^*$$

$\underline{v}_2 = g\, v_1\, g^*$

$$v_2 = \frac{1}{2}(\underline{i}+j)(j)(-\underline{i}-j) = \frac{1}{2}(k-1)(-i-j) = \frac{1}{2}(\cancel{-j}+i+i\cancel{+j}) = \underline{\underline{i}}$$

20 %

30 %

OSNOVE ROBOTIKE

rotacije okrog osi:

$$\underline{\underline{R}}_{z,\alpha} = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \underline{\underline{R}}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\alpha & -s\alpha \\ 0 & s\alpha & c\alpha \end{bmatrix} \quad \underline{\underline{R}}_{y,\alpha} = \begin{bmatrix} c\alpha & 0 & s\alpha \\ 0 & 1 & 0 \\ -s\alpha & 0 & c\alpha \end{bmatrix}$$

$$p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} \qquad {}^0\underline{p} = {}^0\underline{\underline{R}}_1 \, {}^1\underline{p}$$

$${}^0\underline{p} = \underbrace{{}^0\underline{\underline{R}}_1 \, {}^1\underline{\underline{R}}_2}_{\text{množenje rot. matrik}} \, {}^2\underline{p}$$

$${}^1\underline{\underline{R}}_0 = \left({}^0\underline{\underline{R}}_1\right)^T$$

$${}^0\underline{p} = {}^0\underline{\underline{R}}_1 \, \underline{p}_1 + {}^0\underline{d}_1$$

$$\begin{bmatrix} {}^0\underline{p} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} \underline{\underline{R}} & \underline{d} \\ \underline{0} & 1 \end{bmatrix}}_{\underline{\underline{H}}} \cdot \begin{bmatrix} {}^1\underline{p} \\ 1 \end{bmatrix}$$

$$\underline{\underline{H}} \longrightarrow \underline{\underline{H}}^{-1} = \begin{bmatrix} \underline{\underline{R}}^T & -\underline{\underline{R}}^T\underline{d} \\ \underline{0} & 1 \end{bmatrix} \quad \text{LEGA!}$$

če premikamo glede na ref.

$\underline{\underline{T}} \cdot \underline{\underline{H}}$ $\quad \underset{\text{premik}}{\underline{\underline{P}}} \cdot \underset{\text{lega}}{\underline{\underline{L}}}$ premultiplikacija (branje z desne proti levi)

če premikamo glede na relativnega

$\underline{\underline{H}} \cdot \underline{\underline{T}}$ $\quad \underline{\underline{L}} \cdot \underline{\underline{P}}$ postmultiplikacija (branje z leve proti desni)