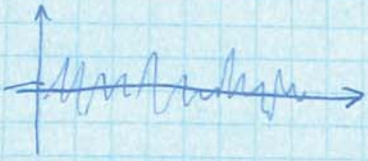
A1 5.3.2012

login: as24
pass: urna7401

$f(t)$
↳ realna spremenljivka



$f(t) \cong \tilde{f}(t)$

$$\boxed{\tilde{f}(t) = C_0 \varphi_0(t) + C_1 \varphi_1(t) + \ldots + C_n \varphi_n(t)}$$

sres
napaka aproksimacija

$$\overline{\varepsilon^2} = \int_{t_0}^{t_0+T} (f(t) - \tilde{f}(t))(f(t) - \tilde{f}(t))\,dt$$

srednja kvadratna napaka

$$\boxed{\int_{t_0}^{t_0+T} \varphi_i(t) \cdot \varphi_j(t) = \begin{cases} K_i\,;\ i=j \\ 0\,;\ i \neq j \end{cases}}$$

ortogonalnost

$$C_i = \frac{1}{K_i} \int_{t_0}^{t_0+T} f(t)\,\varphi_i(t)\,dt$$

koeficienti

$$\overline{\varepsilon^2} = \frac{1}{\underset{(t_2-t_1)}{T}} \left(\int_{t_0}^{t_0+T} f(t) f(t)\,dt - \sum_{i=0}^{n} K_i C_i C_i \right)$$

srednja kvadratna napaka

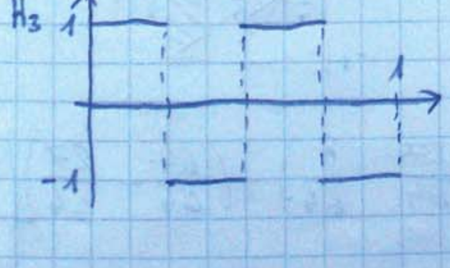
Walsh:
$K_i = 1$
definicijsko območje $[0, 1]$

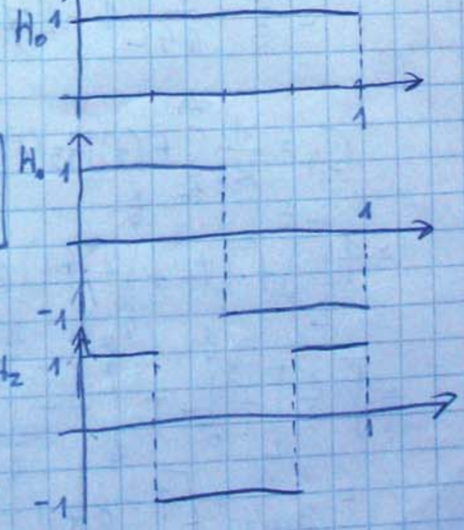
$H(0) = 1$

$$H(m) = \begin{bmatrix} H(m-1) & H(m-1) \\ H(m-1) & -H(m-1) \end{bmatrix}$$

$$H(1) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H(2) = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \\ H_0 \quad H_3 \quad H_1 \quad H_2 \end{array}$$





Naloga: Signal $x(t)=t^2$ izrazite z približkom $\tilde{x}(t)$, ki ga tvorijo prve 4 Walsheve temeljne funkcije. Določite srednjo kvadratno napako in skicirajte približek.

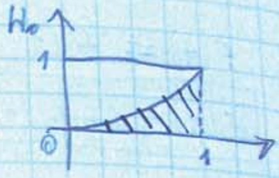
$x(t) = t^2$

$\tilde{x}(t) = ?$

$\varepsilon^2 = ?$

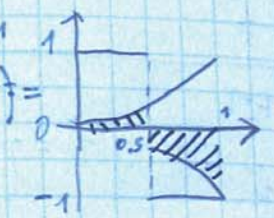
$$\tilde{x}(t) = C_0 f_0(t) + C_1 f_1(t) + C_2 f_2(t) + C_3 f_3(t)$$

$K_0 = K_1 = K_2 = K_3 = 1$



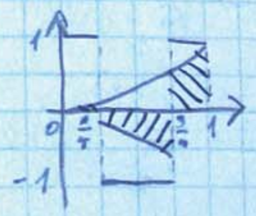
$$C_0 = \frac{1}{K_0}\int_{t_0}^{t_0+T} f(t)\, H_0(t)\, dt = \frac{1}{1}\int_0^1 t^2 \cdot 1\, dt =$$

$$= \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}\left[1-0\right] = \underline{\underline{\frac{1}{3}}} \qquad C_0 = \frac{1}{3}$$



$$C_1 = \frac{1}{K_1}\int_{t_0}^{t_0+T} f(t)\, H_1(t)\, dt = \frac{1}{1}\left(\int_0^{\frac{1}{2}} t^2 \cdot 1\, dt + \int_{\frac{1}{2}}^{1} t^2 \cdot (-1)\, dt\right) =$$

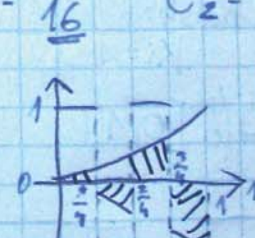
$$= \frac{t^3}{3}\Big|_0^{\frac{1}{2}} - \frac{t^3}{3}\Big|_{\frac{1}{2}}^{1} = \frac{1}{3}\left[\frac{1}{8} - \left(1 - \frac{1}{8}\right)\right] = \frac{1}{3}\left(\frac{1}{8} - \frac{7}{8}\right) = \frac{1}{\cancel{3}}\left(-\frac{\cancel{6}\,1}{\cancel{8}\,4}\right) = -\frac{1}{4}$$



$$C_2 = \frac{1}{K_2}\int_{t_0}^{t_0+T} f(t)\, H_2(t)\, dt = \frac{1}{1}\left(\int_0^{\frac{1}{4}} t^2 \cdot 1\, dt + \int_{\frac{1}{4}}^{\frac{3}{4}} t^2 \cdot (-1)\, dt + \int_{\frac{3}{4}}^{1} t^2 \cdot 1\, dt\right) =$$

$$= \frac{t^3}{3}\Big|_0^{\frac{1}{4}} - \frac{t^3}{3}\Big|_{\frac{1}{4}}^{\frac{3}{4}} + \frac{t^3}{3}\Big|_{\frac{3}{4}}^{1} = \frac{1}{3}\left[\frac{1}{64} - \left(\frac{27}{64} - \frac{1}{64}\right) + \left(\frac{64}{64} - \frac{27}{64}\right)\right] =$$

$$= \frac{1}{3}\cdot\frac{1}{64}\left[1 - 27 + 1 + 64 - 27\right] = \frac{1}{\cancel{3}}\cdot\frac{1}{\cancel{64}\,16}\cdot\cancel{12}\,\cancel{4}\,1 = \underline{\frac{1}{16}} \qquad C_2 = \frac{1}{16}$$



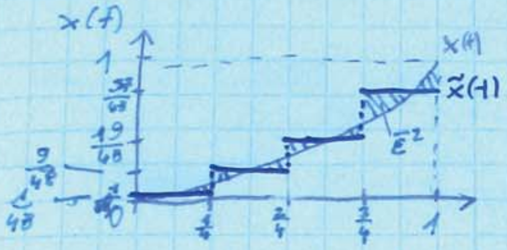
$$C_3 = \frac{1}{K_3}\int_{t_0}^{t_0+T} f(t)\, H_3(t)\, dt =$$

$$= \frac{1}{1}\left[\int_0^{\frac{1}{4}} t^2 \cdot 1\, dt + \int_{\frac{1}{4}}^{\frac{2}{4}} t^2 \cdot (-1)\, dt + \int_{\frac{2}{4}}^{\frac{3}{4}} t^2 \cdot 1\, dt + \int_{\frac{3}{4}}^{1} t^2 \cdot (-1)\, dt\right] =$$

$$= \frac{1}{3}\left[t^3\Big|_0^{\frac{1}{4}} - t^3\Big|_{\frac{1}{4}}^{\frac{2}{4}} + t^3\Big|_{\frac{2}{4}}^{\frac{3}{4}} - t^3\Big|_{\frac{3}{4}}^{1}\right] = \frac{1}{3}\left[\frac{1}{64} - \left(\frac{8}{64} - \frac{1}{64}\right) + \left(\frac{27}{64} - \frac{8}{64}\right) - \left(\frac{64}{64} - \frac{27}{64}\right)\right] =$$

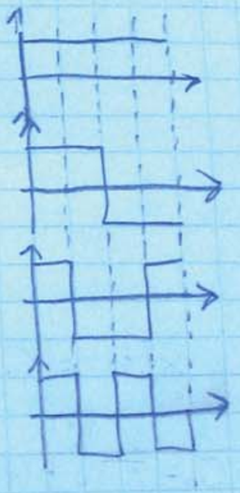
$$= \frac{1}{3}\,\frac{1}{64} = \left[1 - 8 + 1 + 27 - 8 - 64 + 27\right] = -\frac{1}{8} \qquad C_3 = -\frac{1}{8}$$

$$\tilde{x}(t) = \frac{1}{3}H_0(t) - \frac{1}{4}H_1(t) + \frac{1}{16}H_2(t) - \frac{1}{8}H_3(t) \qquad \left(\frac{1}{4}\right)^3 = \frac{1}{64} \quad t^3$$



$$\overline{\varepsilon^2} = \frac{1}{1}\int_0^1 t^2 \cdot t^2\, dt - \sum_{i=0}^{n=3} K_i C_i C_i = 1\int_0^1 t^4\, dt - \Big(\overset{C_0^2}{\frac{1}{3}\cdot\frac{1}{3}} + \overset{C_1^2}{\frac{1}{4}\cdot\frac{1}{4}} + \overset{C_2^2}{\frac{1}{16}\cdot\frac{1}{16}} + \overset{C_3^2}{\frac{1}{8}\cdot\frac{1}{8}}\Big) =$$

$$= \frac{t^5}{5}\Big|_0^1 - \left(\frac{1}{9} + \frac{1}{16} + \frac{1}{256} + \frac{1}{64}\right) = \frac{1}{5} - \frac{1}{9} - \frac{1}{16} - \frac{1}{256} - \frac{1}{64} = 0.00686$$



$0 \leq t \leq \frac{1}{4}$ $\quad \frac{1}{3} - \frac{1}{4} + \frac{1}{16} - \frac{1}{8} = \frac{1}{48}$ $\quad \leftarrow \frac{1}{3}H_0(t) - \frac{1}{4}H_1(t) + \ldots$

$\frac{1}{4} \leq t \leq \frac{2}{4}$ $\quad \frac{1}{3} - \frac{1}{4} - \frac{1}{16} + \frac{1}{8} = \frac{9}{48}$

$\frac{2}{4} \leq t \leq \frac{3}{4}$ $\quad \frac{1}{3} + \frac{1}{4} - \frac{1}{16} - \frac{1}{8} = \frac{19}{48}$

$\frac{3}{4} \leq t \leq 1$ $\quad \frac{1}{3} + \frac{1}{4} + \frac{1}{16} + \frac{1}{8} = \frac{37}{48}$

Haar:

$K_0 = K_1 = 1$

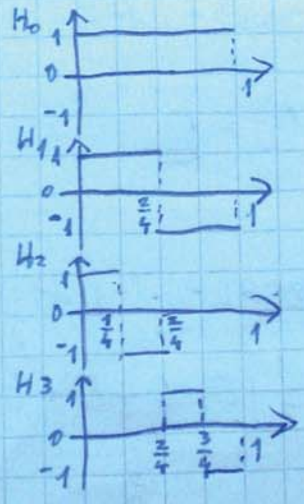
$K_2 = K_3 = \frac{1}{2}$

$K_4 = K_5 = K_6 = K_7 = \frac{1}{4}$

$K_i = \frac{1}{2^m}$

$2^m;\ m=1$

$m=2$



Naloga: Signal $x(t) = -\cos \pi t$ na intervalu med -1 in 1 aproksimiraj z prvimi 4imi Haarovimi funkcijami. Določi še srednjo kvadratno napako aproksimacije.

(IZPIT)

$x(t) = -\cos \pi t$
$[-1, 1]$

$\tilde{x}(t) = ?$

$\overline{\varepsilon^2} = ?$

$u(t) = at + b \quad [-1, 1] \to [0, 1]$

$$\left.\begin{aligned} 0 &= a \cdot (-1) + b \\ 1 &= a \cdot 1 + b \end{aligned}\right\} +$$

$1 = 2b \Rightarrow b = \frac{1}{2} \Rightarrow a = \frac{1}{2} \qquad u(t) = \frac{1}{2}t + \frac{1}{2}$

$\int \cos(at) = \frac{\sin at}{a} + C$

$\hat{H}_0(t) = H_0(u(t)) \qquad \hat{H}_2 = H_2(u(t))$

$\hat{H}_1(t) = H_1(u(t)) \qquad \hat{H}_3 = H_3(u(t))$

$$\boxed{\hat{K}_i = \frac{K_i}{a}}$$

$\hat{K}_i = \frac{K_i}{a} = 2K_i$

$\hat{K}_0 = \hat{K}_1 = \underline{\underline{2}}$

$\hat{K}_2 = \hat{K}_3 = \underline{\underline{1}}$

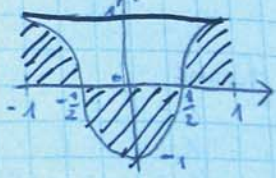
$$\tilde{x}(t) = C_0 \hat{H}_0(t) + C_1 \hat{H}_1(t) + C_2 \hat{H}_2(t) + C_3 \hat{H}_3(t)$$

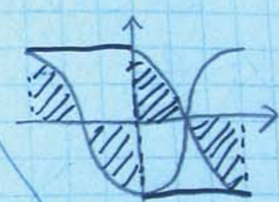
$$C_0 = \frac{1}{\hat{K}_0} \int_{t_0}^{t_0+T} f(t) \hat{H}_0(t)\, dt = \frac{1}{2} \int_{-1}^{1} (-\cos \pi t)\, dt =$$

$$= \frac{1}{2} \left(-\frac{\sin \pi t}{\pi} \Big|_{-1}^{1} \right) = -\frac{1}{2}(0 - 0) = \underline{\underline{0}}$$

lahko vidimo že iz slike

$C_0 = 0$

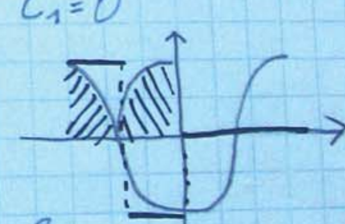




$$C_1 = \frac{1}{2} \left[\int_{-1}^{0} (-\cos \pi t) \cdot 1 \cdot dt + \int_{0}^{1} (-\cos \pi t) \cdot (-1)\, dt \right] =$$

$$= \frac{1}{2} \left[-\frac{\sin \pi t}{\pi} \Big|_{-1}^{0} + \frac{\sin \pi t}{\pi} \Big|_{0}^{1} \right] = \frac{1}{2} [0 - 0 + 0 + 0] = \underline{\underline{0}}$$

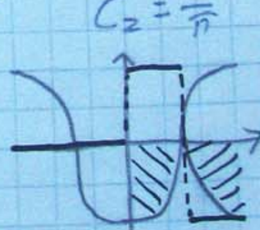
$C_1 = 0$



$$C_2 = \frac{1}{\hat{K}_2} \int_{t_0}^{t_0+T} f(t) \hat{H}_2(t)\, dt = \frac{1}{1} \left[-\int_{-1}^{-\frac{1}{2}} \cos \pi t\, dt + \int_{-\frac{1}{2}}^{0} \cos \pi t\, dt \right] =$$

$$= \frac{1}{1} \left[-\frac{\sin \pi t}{\pi} \Big|_{-1}^{-\frac{1}{2}} + \frac{\sin \pi t}{\pi} \Big|_{-\frac{1}{2}}^{0} \right] = \frac{1}{\pi} \left(-(-1 - 0) + (0 + 1) \right) = \frac{2}{\pi}$$

$C_2 = \frac{2}{\pi}$



$$C_3 = \frac{1}{\hat{K}_3} \int_{t_0}^{t_0+T} f(t) \hat{H}_3(t)\, dt = \frac{1}{1} \left[-\int_{0}^{\frac{1}{2}} \cos \pi t\, dt + \int_{\frac{1}{2}}^{1} \cos \pi t\, dt \right] =$$

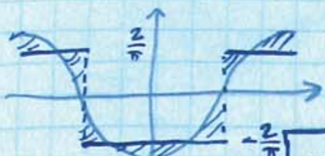
$$= -\frac{\sin \pi t}{\pi} \Big|_{0}^{\frac{1}{2}} + \frac{\sin \pi t}{\pi} \Big|_{\frac{1}{2}}^{1} = \frac{1}{\pi} \left(-(1 - 0) + (0 - 1) \right) = -\frac{2}{\pi}$$

$C_3 = -\frac{2}{\pi}$

lahko razvidiš iz sl... naspro... prejšnje

$$\overline{\varepsilon^2} = 0.0947153$$

Kolikšna je razlika v napaki aproksimacije, če aproksimiramo z 3 namesto 4 Haarovimi osnovnimi funkcijami?



$$\Delta \overline{\varepsilon^2} = \overline{\varepsilon_3^2} - \overline{\varepsilon_4^2} =$$

$$= \left(\frac{1}{t_2-t_1}\int f(t)f(t) + \sum_{i=0}^{2} K_i C_i C_i\right) + \left(\frac{1}{t_2-t_1}\int f(t)f(t) + \sum_{i=0}^{3} K_i C_i C_i\right) =$$

$$\boxed{\Delta\overline{\varepsilon^2} = \frac{1}{t_2-t_1}\hat{K}_3 C_3 C_3} = \frac{1}{1-(-1)}\cdot 1\cdot\left(-\frac{2}{\pi}\right)\left(-\frac{2}{\pi}\right) = \frac{4}{\pi^2}$$

IZPIT

$\boxed{L1}$ 12.3.2012

P

$$P_f = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} f(t)\cdot\overline{f(t)}\,dt \qquad E_f = \lim_{T\to\infty}\int_{-T}^{T} f(t)\overline{f(t)}\,dt$$

$$P_f = \frac{1}{T}\int_{t_1}^{t_1+T} f(t)\cdot\overline{f(t)}\,dt \qquad E_f < \infty$$

perioda

$P_f < \infty \quad P_f \neq 0$

Naloga: Dan je signal $x(t) = \cos t$, ker je signal periodičen določi njegovo moč P_f na dva načina:

a) po splošnem načinu
b) po zvezi za periodične signale

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$P_f = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\cos^2 t\,dt =$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left[\frac{1}{2}+\frac{\cos 2t}{2}\right]dt =$$

$$\lim_{x\to\infty}\frac{\sin x}{x} = 0$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\frac{1}{2}t\Big|_{-T}^{T} + \frac{\sin 2t}{4}\Big|_{-T}^{T}\right] =$$

$$= \lim_{T\to\infty}\frac{1}{2T}\left[\frac{1}{2}(T-(-T)) + \frac{1}{4}\left[\sin 2T - \sin(-2T)\right]\right] =$$

$$= \lim_{T\to\infty}\frac{1}{\cancel{2T}}\left[\frac{\cancel{2T}}{2} + \underbrace{\frac{\sin 2T}{2}}_{0}\right] = \frac{1}{2}$$

$$\omega = \frac{2\pi}{T} = 1 \Rightarrow T = 2\pi$$

$$PF = \frac{1}{2\pi}\int_0^{2\pi} \cos^2(t)\, dt = \frac{1}{2\pi}\left[\frac{1}{2}t\Big|_0^{2\pi} + \frac{\sin 2t}{4}\Big|_0^{2\pi}\right] =$$

$$= \frac{1}{\cancel{2\pi}}\left[\frac{1}{2}[\cancel{2\pi}] + 0\right] = \frac{1}{2}$$

$$\int e^{at} = \frac{e^{at}}{a}$$

$$\lim_{T\to\infty} e^{-at} = 0$$

$PF = \lim$

A2 19.3.2012

Fourierova vrsta $\qquad f(t) = f(t+T) \qquad T$ perioda

REALNA:
$$\boxed{\tilde{f}(t) = \underbrace{\frac{a_0}{2}}_{c_0} + \sum_{n=1}^{\infty} \underbrace{a_n \cos(n\omega t)}_{c_1} + \underbrace{b_n \sin(n\omega t)}_{c_2}}$$

KOMPLEKSNA:
$$\boxed{\tilde{f}(t) = \sum_{n=-\infty}^{\infty} F(n)\, e^{jn\omega t}}$$

$$e^{jn\omega t} = \cos n\omega t + j\sin \omega t$$

$$\omega = \frac{2\pi}{T}$$

Dirichletovi pogoji:

1.) $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$

2.) maximumi in minimumi (končno število)

3.) nezveznosti (končno število

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(n\omega t)\, dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(n\omega t)\, dt$$

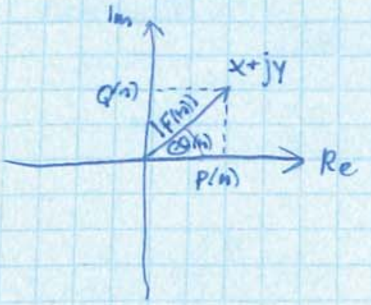
kompleksni spekter → $F(n) = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\, e^{-jn\omega t}\, dt$

$$F(n) = \frac{a_n - b_n}{j}$$

$$a_n = F(n) + \overline{F(n)}$$

$$b_n = j(F(n) - \overline{F(n)})$$

$$F(n) = \underbrace{P(n)}_{\text{realni spekter}} + j\underbrace{Q(n)}_{\text{imaginarni spekter}} = \underbrace{|F(n)|}_{\text{amplitudni spekter}}\, e^{j\overbrace{\Theta(n)}^{\text{fazni spekter}}}$$



$$\overline{x+jy} = x - jy$$

$$|F(n)| = \sqrt{F(n)\overline{F(n)}} = \sqrt{P^2(n) + Q^2(n)} = \frac{1}{2}\sqrt{a_n^2 + b_n^2}$$

$$\Theta(n) = \begin{cases} -\operatorname{arctg}\frac{b_n}{a_n}; & a_n > 0 \\ -\operatorname{arctg}\frac{b_n}{a_n} \pm \pi; & a_n < 0 \end{cases} = \begin{cases} \operatorname{arctg}\frac{Q(n)}{P(n)}; & P(n)> \\ \operatorname{arctg}\frac{Q(n)}{P(n)} \pm \pi; & P(n) \end{cases}$$

$F(-n) = \overline{F(n)}$ soda funkcija 

$P(n) = P(-n)$ soda funkcija

$Q(n) = -Q(-n)$ liha funkcija 

$|F(n)| = |F(-n)|$ soda

$\Theta(-n) = -\Theta(n)$ liha

$f(-t) = f(t)$

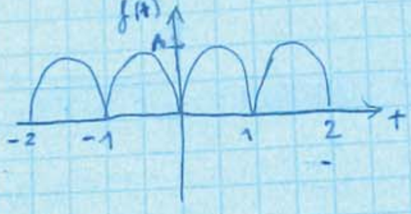
soda: $\boxed{\begin{array}{l} F(n) = P(n) = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\cos n\omega t\, dt \\ Q(n) = 0 \end{array}}$

$f(-t) = -f(t)$

liha: $\boxed{\begin{array}{l} P(n) = 0 \\ jQ(n) = F(n) = -j\frac{2}{T}\int_0^{\frac{T}{2}} f(t)\sin n\omega t\, dt \end{array}}$

Naloga: Izrazi periodično funkcijo $f(t)$ v fourierovo vrsto, določi koeficiente realne F.v. ter zapiši še realno F.v. Na koncu še določi in nariši tudi amplitudni ter fazni spekter

$$f(t) = \begin{cases} A\sin \pi t \,; & 0<t<1 \,, \quad A>0 \\ f(t) = f(t+T_0) \,; & \forall T \,; \; n \in \mathbb{Z} \end{cases}$$



$T = 1$

$\omega = \frac{2\pi}{T} = 2\pi$

1. sodost ali lihost

$Q(n) = 0$

$$P(n) = F(n) = \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\cos(n\omega t)\,dt = 2\int_0^{\frac{1}{2}} A\sin \pi t \cdot \cos(n 2\pi t)\,dt$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$P(n) = 2A\int_0^{\frac{1}{2}} \frac{1}{2}\left[\sin(\pi t + n2\pi t) + \sin(\pi t - n2\pi t)\right]dt =$$

$$= A\int_0^{\frac{1}{2}} \left(\sin((1+2n)\pi t) + \sin((1-2n)\pi t)\right)dt =$$

$$= A\left[-\frac{\cos((1+2n)\pi t)}{(1+2n)\pi} - \frac{\cos((1-2n)\pi t)}{(1-2n)\pi}\right]_0^{\frac{1}{2}} =$$

$$= A\left[-\left(0 - \frac{1}{(1+2n)\pi}\right) - \left(0 - \frac{1}{(1-2n)\pi}\right)\right] =$$

$$= A\left(+\frac{1}{(1+2n)\pi} + \frac{1}{(1-2n)\pi}\right) =$$

$$= \frac{A(1-2n) + A(1+2n)}{(1-4n^2)\pi} = \frac{2A}{(1-4n^2)\pi}$$

komp. F.v.

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2A}{(1-4n^2)\pi} e^{jn2\pi t} = \frac{2A}{\pi}\sum_{n=-\infty}^{\infty} \frac{e^{jn2\pi t}}{(1-4n^2)}$$

$$a_n = F(n) - \overline{F(n)} = \frac{4A}{(1-4n^2)\pi} \qquad a_0 = \frac{4A}{\pi}$$

$$b_n = j\left(F(n) - \overline{F(n)}\right) = 0$$

realna F.V.

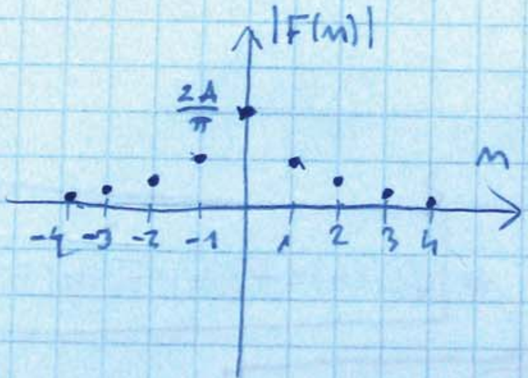
$$f(t) = \frac{4A}{2\pi} + \sum_{n=1}^{\infty} \frac{4A}{(1-4n^2)\pi} \cos(n 2\pi t) = \cancel{2A}$$

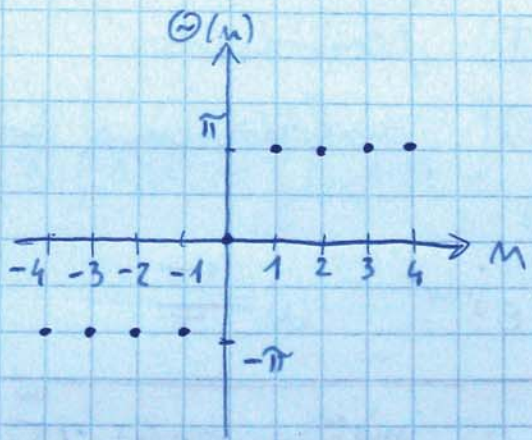
$$= \frac{2A}{\pi} + \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{(1-4n^2)} \cos(2n\pi t)$$

$$F(n) = \frac{2A}{(1-4n^2)\pi}$$

$$|F(n)| = \begin{cases} \frac{2A}{\pi} \;; & n=0 \\ \frac{2A}{(4n^2-1)\pi} & n \neq 0 \end{cases}$$

$$\Theta(n) = \begin{cases} \mathrm{arctg} \frac{\overset{=0}{Q(n)}}{P(n)} \;; & P(n) > 0 \\ \mathrm{arctg} \frac{Q(n)}{P(n)} \pm \pi & P(n) < 0 \end{cases} = \begin{cases} \pi \;; & n>0 \\ 0 \;; & n=0 \\ -\pi \;; & n<0 \end{cases}$$





A3 2.4.2012

Priprave na kolokvij: ČETRTEK ob 13:00

$f(t) = f(t+T)$ Dirichlet

$\rho_{ij}(\tau) = \frac{1}{T}\int_{t_0}^{t_0+T} \overline{f_i(t)}\, f_j(t+\tau)\, dt$

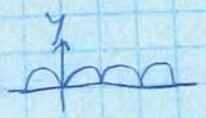
$i=j$ avtokorelacija

$i \neq j$ križna korelacija

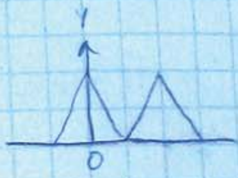
$\rho_{ii}(\tau) = \frac{1}{T}\int_{t_0}^{t_0+T} \overline{f_i(t)}\, f_i(t+\tau)\, dt$

Lastnosti:

1.) $\rho_{ii}(\tau) = \overline{\rho_{ii}(-\tau)}$ soda



2.) $\rho_{ii}(\tau) = \rho_{ii}(\tau+T)$ periodična



3.) $\rho_{ii}(0) \geq |\rho_{ii}(\tau)|$ maksimum v točki 0

$\rho_{ii}(0)$ $=$ moč signala!!

4.) $f(t) = \sum_{n=-\infty}^{\infty} F(n) e^{jn\omega t}$ $\omega = \frac{2\pi}{T}$

$$\boxed{\phi_{ii}(n) = F_i(n)\cdot\overline{F_i(n)} = |F_i(n)|^2}$$

Fourierova vrsta:

$$\boxed{\rho_{ii}(\tau) = \sum_{n=-\infty}^{\infty} \phi_{ii}(n) e^{jn\omega\tau} = \phi_{ii}(0) + 2\sum_{n=1}^{\infty} \phi_{ii}(n) \cos n\omega\tau}$$

močnostni spekter

če $f(t) \in \mathbb{R}$

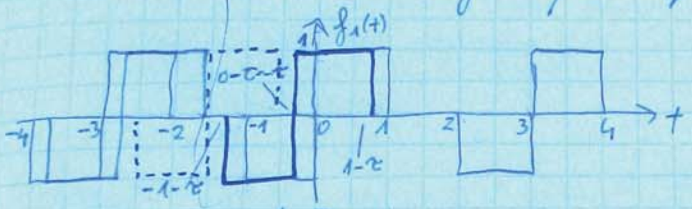
5.) $\rho_{ii}(\tau)$ zvezna, če je $f(t)$ odsekoma zvezna

6.) $f_2(t) = f_1(t-t_0)$

$\rho_{11}(\tau) = \rho_{22}(\tau)$

Za križno korelacijo veljajo vse točke razen tretje.

Naloga: Za periodičen signal $f_1(t)$ določi avtokorelacijo in skiciraj njen potek



$T = 3 \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

$\varphi_{ii}(\tau) = \varphi_{ii}(\tau + T)$

$0 \leq \tau \leq 1$ — $\quad 2 \leq \tau \leq 3$ —

$1 \leq \tau \leq 2$ ----

$$\varphi_{ii}(\tau) = \frac{1}{T}\int_{t_0}^{t_0+T} f_i(t)\, f_i(t+\tau)\,dt$$

↳ premik v levo

1. $$\varphi_{ii}(\tau) = \frac{1}{3}\Big(\cancel{\int_{-1-\tau}^{-1} 0\cdot(-1)\,dt} + \int_{-1}^{-\tau}(-1)(-1)\,dt + \int_{-\tau}^{0}(-1)\cdot(1)\,dt + \int_{0}^{1-\tau} 1\cdot 1\,dt + \cancel{\int_{1-\tau}^{1} 1\cdot 0\,dt}\Big) =$$

$$= \frac{1}{3}\Big(\int_{-1}^{\tau} 1\,dt + \int_{-\tau}^{0}(-1)\,dt + \int_{0}^{1-\tau} 1\,dt\Big) =$$

$$= \frac{1}{3}\Big(t\Big|_{-1}^{-\tau} - t\Big|_{-\tau}^{0} + t\Big|_{0}^{1-\tau}\Big) = \frac{1}{3}(-\tau + 1 - \tau + 1 - \tau) = \frac{1}{3}(2 - 3\tau) = -\tau + \frac{2}{3} =$$

$$= -\tau + \frac{2}{3}$$

2. $$\varphi_{ii}(\tau) = \frac{1}{3}\Big(\int_{-1-\tau}^{-2}(-1\cdot(-1))\,dt + \int_{-2}^{-\tau} 0\cdot(-1)\,dt + \int_{-\tau}^{-1} 0\cdot 1\,dt +$$

$$= \frac{1}{3}$$

$$= \frac{1}{3}\big(-(2+1+\tau) - (1-\tau+1)\big) = \frac{1}{3}(+2-1-\tau-1+\tau-1) = -\frac{1}{3}$$

3. $$\varphi_{ii}(\tau) = \frac{1}{3}\Big(\int_{-1-\tau}^{-3}(-1)(-1)\,dt + \int_{-3}^{-\tau} 1\cdot(-1)\,dt + \int_{-\tau}^{-2} 1\cdot 1\,dt + \int_{-2}^{1-\tau} 0\cdot 1\,dt\Big)$$

$$= \frac{1}{3}\Big(\int_{-1-\tau}^{-3} 1\,dt + \int_{-3}^{-\tau}(-1)\,dt + \int_{-\tau}^{-2} 1\,dt\Big) = \frac{1}{3}\Big(t\Big|_{-1-\tau}^{-3} - t\Big|_{-3}^{-\tau} + t\Big|_{-\tau}^{-2}\Big) =$$

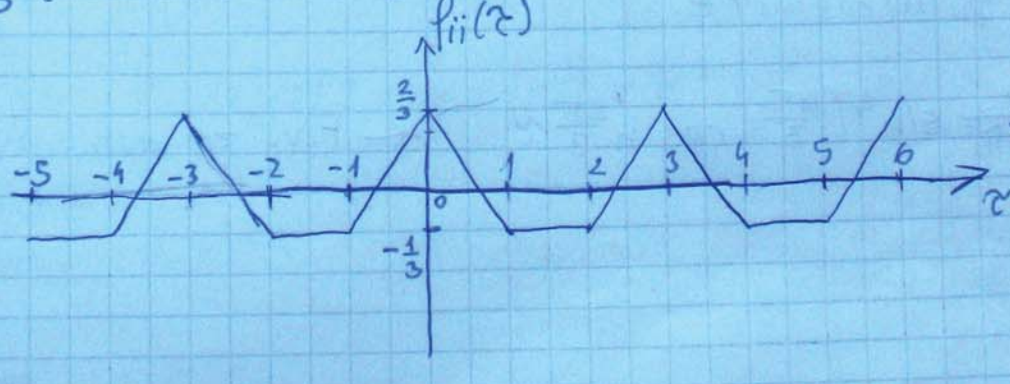
$$= \frac{1}{3}\big(-3+1+\tau - (-\tau+3) + (-2+\tau)\big) = \frac{1}{3}(-2+\tau+\tau-3-2+\tau) =$$

$$= \frac{1}{3}(3\tau - 7) = \tau - \frac{7}{3}$$

$0 \leq \tau \leq 1: \varphi_{ii}(\tau) = -\tau$

$1 \leq \tau \leq 2: \varphi_{ii}(\tau) = -\frac{1}{3}$

$2 \leq \tau \leq 3: \varphi_{ii}(\tau) = \tau - \frac{7}{3}$



Za signal $f(t)$ na sliki določi fouriegovo vrsto avtokorelacije

1.) $\varphi_{ii}(\tau) \rightarrow$ F.v. 2 načina za določanje F.v.

2.) preko močn. spektra

$$\Phi_{ii}(n) = F(n)\overline{F(n)} = |F_i(n)|^2$$

$$F(n) = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)e^{-jn\omega t}\,dt =$$

$$= \frac{1}{3}\left(\int_{-1}^{0}(-1)e^{-jn\omega t}dt + \int_{0}^{1} 1\, e^{-jn\omega t}dt\right) =$$

$$= \frac{1}{3}\left(-\int_{-1}^{0} e^{-jn\frac{2\pi}{3}t}dt + \int_{0}^{1} e^{-jn\frac{2\pi}{3}t}dt\right) =$$

$$n \neq 0 \quad = \frac{1}{\cancel{3}}\left(-\frac{\cancel{3}e^{-jn\frac{2\pi}{3}\cdot t}}{-jn2\pi}\Big|_{-1}^{0} + \frac{\cancel{3}e^{-jn\frac{2\pi}{3}t}}{-jn2\pi}\Big|_{0}^{1}\right) = -\frac{1}{jn2\pi}\left(-e^{-jn\frac{2\pi}{3}t}\Big|_{-1}^{0} + e^{-jn\frac{2\pi}{3}t}\Big|_{0}^{1}\right) =$$

$$= -\frac{1}{jn2\pi}\left(-\left(1 - e^{jn\frac{2\pi}{3}}\right) + \left(e^{-jn\frac{2\pi}{3}} - 1\right)\right) =$$

$$= -\frac{1}{jn2\pi}\left(-1 + e^{jn\frac{2\pi}{3}} + e^{-jn\frac{2\pi}{3}} - 1\right) =$$

$e^{jx} = \cos x + j\sin$

$\sin^2\alpha = \frac{1-\cos 2}{2}$

$$= -\frac{1}{jn2\pi}\left(e^{jn\frac{2\pi}{3}} + e^{-jn\frac{2\pi}{3}} - 2\right) =$$

$$= -\frac{1}{jn2\pi}\left(\cos\left(n\frac{2\pi}{3}\right) + \cancel{j\sin\left(n\frac{2\pi}{3}\right)} + \cos\left(n\frac{2\pi}{3}\right) - \cancel{j\sin\left(n\frac{2\pi}{3}\right)} - 2\right) =$$

$$= -\frac{1}{jn\cancel{2}\pi}\left(\cancel{2}\cos\left(n\frac{2\pi}{3}\right) - \cancel{2}\right) = \frac{1}{jn\pi}\left(1 - \cos\left(\frac{2n\pi}{3}\right)\right) =$$

$$= \frac{1\cdot j}{jn\pi\cdot j}\, 2\sin^2\left(\frac{n\pi}{3}\right) = -\frac{2j}{n\pi}\sin^2\left(\frac{n\pi}{3}\right) = F(n)$$

$$\Phi_{ii}(n) = F(n)\overline{F(n)} = \frac{4}{n^2\pi^2}\sin^4\left(\frac{n\pi}{3}\right)$$

$$\varphi_{ii}(\tau) = \varphi_{ii}(0) + 2\sum_{n=1}^{\infty}\Phi_{ii}(n)\cos n\omega\tau =$$

$$= 0 + 2\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\sin^4\left(\frac{n\pi}{3}\right)\cos n\frac{2\pi}{3}\tau$$ ← F.v. za avtokorelacijo

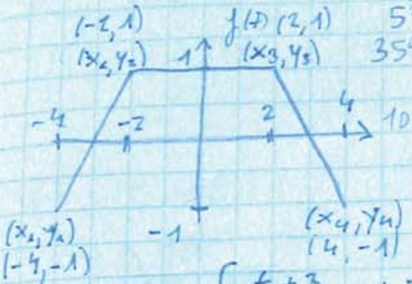
sin 0 = 0

5.4.2012

Priprave na kolokvij

- izražava s haarovimi ali walshovimi funkcijami
- konvolucija

Naloga 1: Analizirajmo signal $f(t)$ na sliki.



5% **a)** zapiši signal $f(t)$ z matematično formulo

35% **b)** aproksimiraj signal na intervalu od -4 do 4 s prvimi štirimi haarovimi funkcijami

10% **c)** kakšna je razlika v napaki, če signal aproksimiramo samo s prvimi tremi haarovimi funkcijami

a) $f(t) = \begin{cases} t+3 & ; -4 \le t \le -2 \\ 1 & ; -2 \le t \le 2 \\ -t+3 & ; 2 \le t \le 4 \\ 0 & ; \text{sicer} \end{cases}$

1. $k = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1+1}{-2+4} = \dfrac{2}{2} = \underline{1}$

$y = kx + n$

$-1 = 1 \cdot (-4) + n$

$\underline{\underline{3 = n}}$

$y = x + 3$

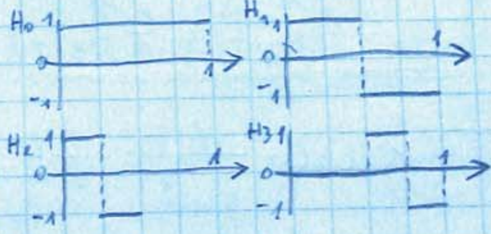
2. $k = \dfrac{-1-1}{4-2} = \dfrac{-2}{2} = -1$

$y = kx + n$

$-1 = -1 \cdot 4 + n$

$\underline{n = 3}$

$y = -x + 3$



$K_0 = K_1 = 1$

$K_2 = K_3 = \frac{1}{2}$

b) $u(t) = at + b$

$[-4, 4] \rightarrow [0, 1]$

$0 = -4a + b$

$1 = 4a + b$

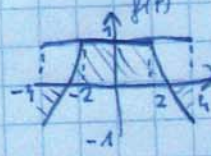
$1 = 8a \Rightarrow a = \frac{1}{8}$

$\hat{K}_i = \dfrac{K_i}{a} = 8K_i$

$\hat{K}_0 = \hat{K}_1 = 8$

$\hat{K}_2 = \hat{K}_3 = 4$

$$C_0 = \frac{1}{K_0} \int_{t_1}^{t_2} f(t)\, \hat{H}_0(t)\, dt = \frac{1}{8}\left(\int_{-4}^{-2} (t+3)\, dt + \int_{-2}^{2} 1\, dt + \int_{2}^{4} (-t+3)\, dt \right) =$$

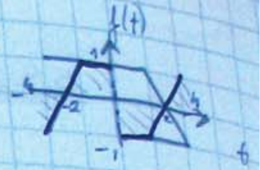


$$= \frac{1}{8}\left(\left(\frac{t^2}{2} + 3t\right)\Big|_{-4}^{-2} + t\Big|_{-2}^{2} + \left(-\frac{t^2}{2} + 3t\right)\Big|_{2}^{4} dt \right) =$$

$2 - 8 \quad -8 + 12 = 0 \qquad 4 \qquad -8 + 12 + 2 - 6 = 0$

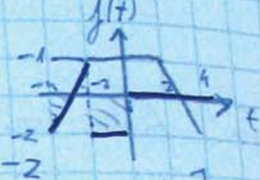
$$= \frac{1}{8}\left[\left(\frac{4}{2} - 6 - \left(\frac{16}{2} - 12\right)\right) + (2+2) + \left(-\frac{16}{2} + 12 - \left(-\frac{4}{2} + 6\right)\right) \right] =$$

$$= \frac{1}{8}[4] = \frac{1}{2}$$



$$C_1 = \frac{1}{\hat{K}_1}\int_{t_1}^{t_2} f(t)\,\hat{H}_1(t)\,dt =$$

$$= \frac{1}{8}\left[\int_{-4}^{-2}(t+3)\,dt + \int_{-2}^{0} 1\,dt + \int_{0}^{2}(-1)\,dt + \int_{2}^{4}(-t+3)\cdot(-1)\,dt\right] = 0$$

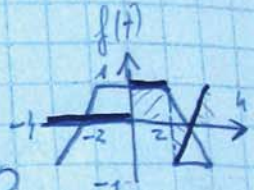


$$C_2 = \frac{1}{\hat{K}_2}\int_{t_1}^{t_2} f(t)\,\hat{H}_2(t)\,dt = \frac{1}{4}\left[\int_{-4}^{-2}(t+3)\,dt + \int_{-2}^{0}(-1)\,dt\right] =$$

$$2-6-8+12=0$$

$$= \frac{1}{4}\left[\left(\frac{t^2}{2}+3t\right)\Big|_{-4}^{-2} - (t)\Big|_{-2}^{0}\right] = \frac{1}{4}\left[\frac{4}{2}-6-\left(\frac{16}{2}-12\right)-\left(0-(-2)\right)\right] =$$

$$= \frac{1}{4}[-2] = \underline{\underline{-\frac{1}{2}}}$$



$$C_3 = \frac{1}{\hat{K}_3}\int_{t_1}^{t_2} f(t)\,\hat{H}_3(t)\,dt = \frac{1}{4}\left[\int_{0}^{2} 1\,dt + \int_{2}^{4}(-t+3)\cdot(-1)\,dt\right] =$$

$$8-12-2+6=0$$

$$= \frac{1}{4}\left[(t)\Big|_{0}^{2} + \left(\frac{t^2}{2}-3t\right)\Big|_{2}^{4}\right] = \frac{1}{4}\left[(2-0)+\left(\frac{16}{2}-12-\left(\frac{4}{2}-6\right)\right)\right] =$$

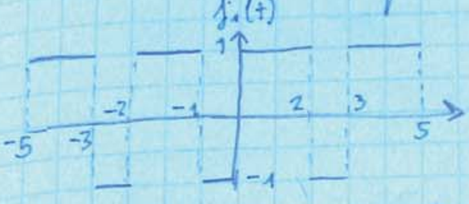
$$= \frac{1}{4}[2] = \underline{\underline{\frac{1}{2}}} \qquad C_0 = \frac{1}{2} \quad C_1 = 0 \quad C_2 = -\frac{1}{2} \quad C_3 = \frac{1}{2}$$

$$\underline{\underline{\tilde{f}(t) = \frac{1}{2}\hat{H}_0(t) - \frac{1}{2}\hat{H}_2(t) + \frac{1}{2}\hat{H}_3(t)}}$$

c) $$\Delta\overline{\varepsilon^2} = \hat{K}_3 C_3 C_3 \frac{1}{\underbrace{t_2-t_1}_{4-(-4)}} = \frac{1}{8}\,\underbrace{4}_{\hat{K}_3}\cdot\underbrace{\frac{1}{2}}_{C_3}\cdot\underbrace{\frac{1}{2}}_{C_3} = \frac{1}{8}$$

$$-4 \le t \le -2 \qquad \frac{1}{2} + 0 - \frac{1}{2} = 0$$

Naloga 2: Izračunaj avtokorelacijo periodičnega signala podanega na sliki.



5% a) Zapiši signal $f_1(t)$ z matematično formulo in določi osnovno periodo T

30% b) Izračunaj avtokorelacijo $\varphi_{11}(\tau)$ signala $f_1(t)$

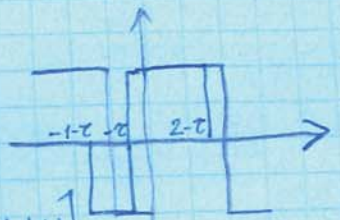
15% c) Skiciraj izračunano avtokorelacijo.

a)

$$f(t)=\begin{cases} 1; & 0\leq t\leq 2 \\ -1; & 2\leq t\leq 3 \\ f(t+T); & \forall t \end{cases}$$

b) $0\leq\tau\leq 1$

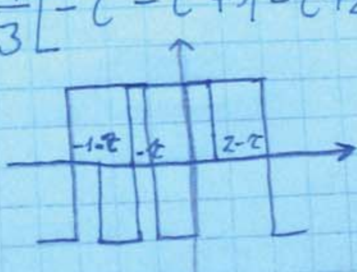
$$\varphi_{11}(\tau)=\frac{1}{T}\int_{t_0}^{t_0+T} f_1(t)\cdot f_1(t+\tau)\,dt=$$



$$=\frac{1}{3}\left[\int_{-1-\tau}^{-1}(-1)\cdot(1)\,dt+\int_{-1}^{-\tau}(-1)\cdot(-1)\,dt+\int_{-\tau}^{0}(-1)\cdot 1\,dt+\int_{0}^{2-\tau}1\cdot 1\,dt\right]=$$

$$=\frac{1}{3}\left[\int_{-1-\tau}^{-1}(-1)\,dt+\int_{-1}^{-\tau}1\,dt+\int_{-\tau}^{0}(-1)\,dt+\int_{0}^{2-\tau}1\,dt\right]=\frac{1}{3}\left[-t\Big|_{-1-\tau}^{-1}+t\Big|_{-1}^{-\tau}-t\Big|_{-\tau}^{0}+t\Big|_{0}^{2-\tau}\right]=$$

$$=\frac{1}{3}\left[-(\cancel{-1}+\cancel{1}+\tau)+(-\tau+1)-(0+\tau)+(2-\tau)\right]=\frac{1}{3}\left[-\tau-\tau+1-\tau+2-\tau\right]=$$



$$=-\frac{4}{3}\tau+3$$

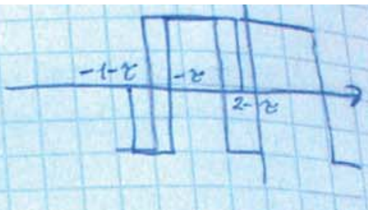
$1\leq\tau\leq 2$

$$\varphi_{11}(\tau)=\frac{1}{T}\int_{t_0}^{t_0+T} f_1(t)\cdot f_1(t+\tau)\,dt=$$

$$=\frac{1}{3}\left[\int_{-1-\tau}^{-\tau}-1\,dt+\int_{-\tau}^{-1}1\,dt+\int_{-1}^{0}-1\,dt+\int_{0}^{2-\tau}1\,dt\right]=\frac{1}{3}\left[-t\Big|_{-1-\tau}^{-\tau}+t\Big|_{-\tau}^{-1}-t\Big|_{-1}^{0}+t\Big|_{0}^{2-\tau}\right]=$$

$$=\frac{1}{3}\left[-(\cancel{-\tau}+1+\cancel{\tau})+(-1+\tau)-(0+1)+(2-\tau)\right]=$$

$$=\frac{1}{3}\left[-1\cancel{-1}+\cancel{\tau}\cancel{-1}\cancel{+2}-\cancel{\tau}\right]=\frac{1}{3}\cdot 1=\frac{1}{3}$$



$2 \le \tau \le 3$

$\rho_{11}(\tau) =$

$$= \frac{1}{3}\left[\int_{-1-\tau}^{-3} 1\,dt + \int_{-3}^{-\tau}(-1)\,dt + \int_{-\tau}^{-1} 1\,dt + \int_{-1}^{2-\tau}(-1)\,dt\right] =$$

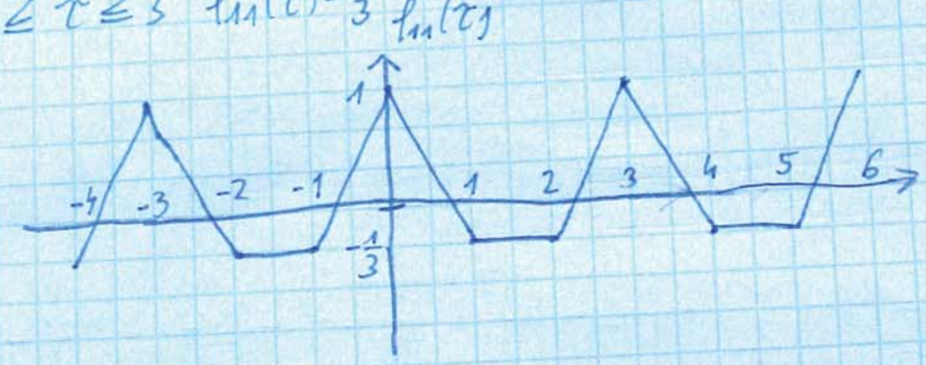
$$= \frac{1}{3}\left[t\Big|_{-1-\tau}^{-3} - t\Big|_{-3}^{-\tau} + t\Big|_{-\tau}^{-1} - t\Big|_{-1}^{2-\tau}\right] = \frac{1}{3}\left[-3+1+\tau - (-\tau+3) + (-1+\tau) - (2-\tau+1)\right]$$

$$= \frac{1}{3}\left[-2+\tau+\tau-3-1+\tau-2+\tau-1\right] = \frac{1}{3}\left[4\tau-9\right] = \frac{4\tau}{3}-3$$

$0 \le \tau \le 1 \quad \rho_{11}(\tau) = -\frac{4}{3}\tau+1$

$0 \le \tau \le 2 \quad \rho_{11}(\tau) = -\frac{1}{3}$

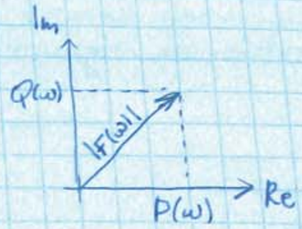
$2 \le \tau \le 3 \quad \rho_{11}(\tau) = \frac{4\tau}{3}-3$



$\boxed{A4}$ **23.4.2012**

$f(t)$

1. $\int_{-\infty}^{\infty} |f(t)|\, dt < \infty$
2. max, min
3. nezveznosti



= FT: $$\boxed{F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt}$$

kompleks. spekter

IFT: $$\boxed{f(t) = \mathcal{F}^{-1}(F(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, d\omega}$$

inverzna Fourierova transformacija

$f(t) \Leftrightarrow F(\omega)$

$$F(\omega) = P(\omega) + jQ(\omega) = |F(\omega)|\, e^{j\Theta(\omega)}$$

$|F(\omega)|$: spekter amplitudne gostote; $\Theta(\omega)$: spekter fazne gostote

$$|F(\omega)| = \sqrt{F(\omega)\overline{F(\omega)}} = \sqrt{P(\omega)^2 + Q(\omega)^2}$$

$$\Theta(\omega) \begin{cases} \operatorname{arctg} \frac{Q(\omega)}{P(\omega)} & P(\omega) > 0 \\ \operatorname{arctg} \frac{Q(\omega)}{P(\omega)} \pm \pi; & P(\omega) < 0 \end{cases}$$

$|F(\omega)|^2$ spekter močnostne gostote

Lastnosti:

$P(\omega) = P(-\omega)$

$Q(\omega) = -Q(-\omega)$

$F(\omega) = \overline{F(-\omega)}$

$\Theta(\omega) = -\Theta(-\omega)$

$|F(\omega)| = |F(-\omega)|$

Sodi $f(t)$

$$F(\omega) = 2\int_{0}^{\infty} f(t) \cos(\omega t)\, dt$$

Liha $f(t)$

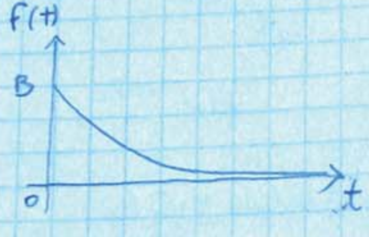
$$F(\omega) = -2j\int_{0}^{\infty} f(t) \sin(\omega t)\, dt$$

$y = kx + n$
$y = kt + n$
$1t + 2$

Naloga:

Izračunaj kompleksni spekter, spektra fazne in amplitudne gostote za signale:

$$f(t) = \begin{cases} Be^{-at} ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}, \quad B, a > 0$$



$$\int e^{at} dt = \frac{e^{at}}{a}$$

$$\lim_{t \to \infty} e^{-t} = 0$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt =$$

$$= \int_{-\infty}^{\infty} B e^{-at} e^{-j\omega t} dt = B \int_{0}^{\infty} e^{(-a-j\omega)t} dt =$$

0 - lahko zamenjamo ker je pred t=0 signal enak nič!

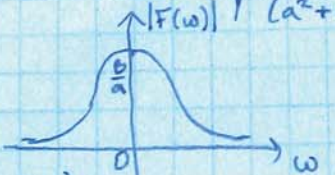
$$= -\frac{B}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty} =$$

$$= -\frac{B}{a+j\omega}\left[0 - 1\right] = \frac{B(a-j\omega)}{a+j\omega(a-j\omega)} =$$

$$= \underline{\underline{\frac{Ba - Bj\omega}{a^2+\omega^2}}} = \overbrace{\frac{Ba}{a^2+\omega^2}}^{P(\omega)} - j \overbrace{\frac{B\omega}{a^2+\omega^2}}^{Q(\omega)}$$

$$|F(\omega)| = \sqrt{P(\omega)^2 + Q(\omega)^2} = \sqrt{\frac{B^2a^2}{(a^2+\omega^2)^2} + \frac{B^2\omega^2}{(a^2+\omega^2)^2}} = \sqrt{\frac{B^2\cancel{(a^2+\omega^2)}}{(a^2+\omega^2)^2}} =$$

$$= \underline{\underline{\frac{B}{\sqrt{a^2+\omega^2}}}}$$



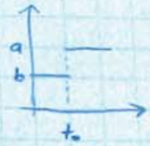
$$\Theta(\omega) = \begin{cases} \mathrm{arctg} \frac{Q(\omega)}{P(\omega)} ; & P(\omega) > 0 \\ \mathrm{arctg} \frac{Q(\omega)}{P(\omega)} \pm \pi ; & P(\omega) < 0 \end{cases}$$

$$= \mathrm{arctg} \frac{Q(\omega)}{P(\omega)} = \mathrm{arctg} \frac{-\frac{B\omega}{\cancel{a^2+\omega^2}}}{\frac{Ba}{\cancel{a^2+\omega^2}}} = \mathrm{arctg} -\frac{\omega}{a} = -\mathrm{arctg}\frac{\omega}{a}$$

Lastnosti Fourierjeve transformacije

$$\mathcal{F}\left(\frac{d^n f(t)}{dt^n}\right) = (j\omega)^n F(\omega)$$

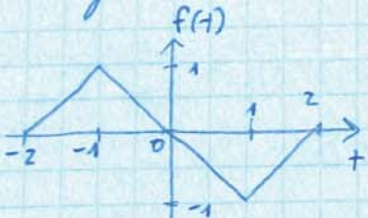
$$\left.\frac{df(t)}{dt}\right|_{t=t_0} = \delta(t-t_0)\left[\overset{a}{f^+(t_0)} - \overset{b}{f^-(t_0)}\right]$$



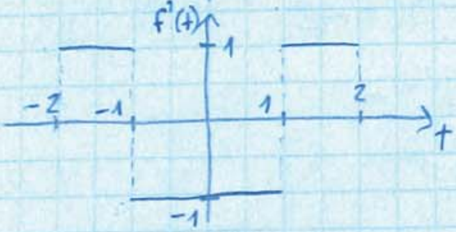
$$\mathcal{F}(\delta(t)) = 1$$

$$\mathcal{F}(\delta(t-t_0)) = e^{-j\omega t_0}$$

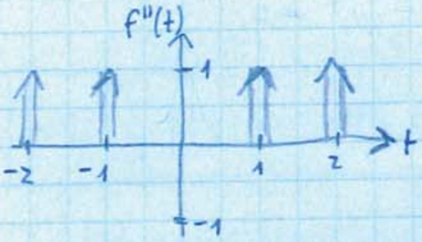
Naloga: Izračunaj kompleksni spekter signala $f(t)$ na sliki; izračunaj še spektra amplitudne in močnostne gostote.



$$f(t) = \begin{cases} t+2 \ ; & -2 \leq t \leq -1 \\ -t \ ; & -1 \leq t \leq 1 \\ t-2 \ ; & 1 \leq t \leq 2 \\ 0 \ ; & \text{sicer} \end{cases}$$



$$f'(t) = \begin{cases} 1 \ ; & -2 \leq t \leq -1 \\ -1 \ ; & -1 \leq t \leq 1 \\ 1 \ ; & 1 \leq t \leq 2 \\ 0 \ ; & \text{sicer} \end{cases}$$



$$\frac{d^2 f}{dt^2} = \delta(t+2)[1-0] + \delta(\overset{t-(-1)}{t+1})[-1-1] + \delta(t-1)[1+1] + \delta(t-2)[0-1] =$$

$t-t_0$

$$= \delta(t+2) - 2\delta(t+1) + 2\delta(t-1) - \delta(t-2) \quad \Big| \mathcal{F}$$

$$\boxed{\begin{aligned} 2\cos(z) &= e^{jz} + e^{-jz} \\ 2j\sin(z) &= e^{jz} - e^{-jz} \end{aligned}}$$

$$(j\omega)^2 F(\omega) = e^{j\omega 2} - 2e^{j\omega} + 2e^{-j\omega} - e^{-2j\omega}$$

$$-\omega^2 F(\omega) = e^{j2\omega} - e^{-j2\omega} - 2(e^{j\omega} - e^{-j\omega})$$

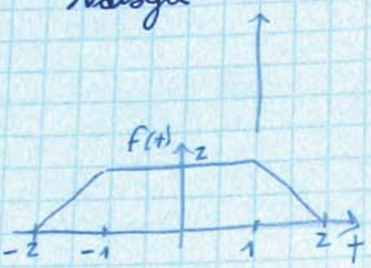
$$-\omega^2 F(\omega) = 2j\sin(2\omega) - 2(2j\sin\omega) \quad \Big/ -\omega^2 \neq 0$$

$$F(\omega) = -\frac{2j}{\omega^2}\left[\sin 2\omega - 2\sin\omega\right]$$

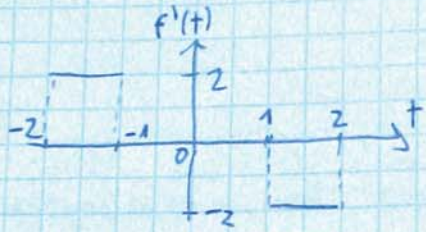
$$|F(\omega)| = \frac{2}{\omega^2}\left|\sin 2\omega - 2\sin\omega\right|$$

$$|F(\omega)|^2 = \frac{4}{\omega^4}(\sin 2\omega - 2\sin\omega)^2$$

Naloga:



$$f(t)=\begin{cases} 2t+2; & -2 \le t \le -1 \\ 2; & -1 \le t \le 1 \\ -2t+2; & 1 \le t \le 2 \\ 0; & \text{sicer} \end{cases}$$



$$f'(t)=\begin{cases} 2 \\ 0 \\ -2 \\ 0 \end{cases}$$

$$\frac{d^2f}{dt^2}=\delta(t+2)[2]+\delta(t+1)[0-2]+$$

$$+\delta(t-1)[-2]+\delta(t-2)[2]$$

$$\frac{d^2f}{dt^2}=2\delta(t+2)-2\delta(t+1)-2\delta(t-1)+2\delta(t-2) \quad \Big| \mathcal{F}$$

$$(j\omega)^2F(\omega)=2e^{j2\omega}-2e^{j\omega}-2e^{-j\omega}+2e^{-j2\omega}=$$

$$-\omega^2F(\omega)=2(e^{j2\omega}+e^{-j2\omega})-2(e^{j\omega}+e^{-j\omega})$$

$$-\omega^2F(\omega)=2(2\cos(2\omega))-2(2\cos\omega) \quad \Big| :-\omega^2 \neq 0$$

$$F(\omega)=-\frac{4}{\omega^2}\left[\cos(2\omega)-\cos(\omega)\right]$$

$$|F(\omega)|=\frac{4}{\omega^2}\left|\cos(2\omega)-\cos(\omega)\right|$$

$$|F(\omega)|^2=\frac{16}{\omega^4}\left(\cos(2\omega)-\cos(\omega)\right)^2$$

1. odpreš m datoteko
2. function $y = $ funkcija3a(t)

⋮

3. shraniš z imenom funkcija3a

$\boxed{A5}$ 14.5.2012

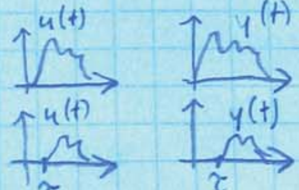
$u(t) \rightarrow \boxed{LSS} \rightarrow y(t)$ LSS - linearen stacionaren sistem

- Linearnost:

$$a u_1(t) + b u_2(t) = a y_1(t) + b y_2(t)$$

- Stacionarnost

$$u(t) \rightarrow y(t)$$
$$u(t-\tau) \rightarrow y(t-\tau)$$



(ne generira se signal pred vhodom)

•

1.) $\delta(t)$

odziv na enotin impulz

$$\delta(t-\tau) \rightarrow h(t-\tau)$$

2.) $$\boxed{y(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau) u(t-\tau)\, d\tau}$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau) h(t-\tau)\, d\tau \quad / \mathcal{F}$$

$$Y(\omega) = U(\omega) \cdot H(\omega)$$

$$\Rightarrow H(\omega) = \frac{Y(\omega)}{U(\omega)} \quad / \mathcal{F}^{-1} \Rightarrow h(t)$$

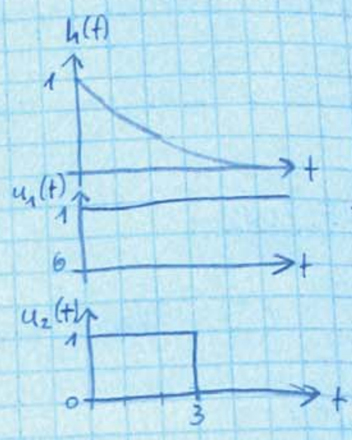
prevajalna funkcija LSS

Naloga: Določi in skiciraj odziv LSS, ki ima odziv na enotin impulz $h(t)$:

$$h(t) = \begin{cases} 0; & t \leq 0 \\ e^{-t}; & t \geq 0 \end{cases}$$

a) $u_1(t) = \begin{cases} 0; & t < 0 \\ 1; & t \geq 0 \end{cases}$

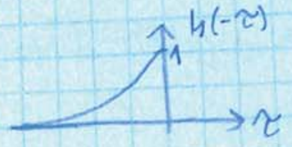
b) $u_2(t) = \begin{cases} 0; & t < 0 \\ 1; & 0 \leq t \leq 3 \\ 0; & t > 3 \end{cases}$



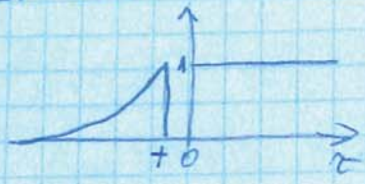
a) $y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$

$u_1(t)$

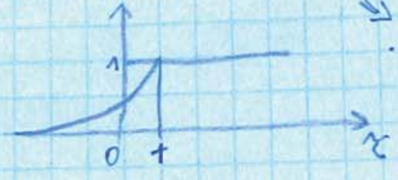
↳ premik zrcaljene funkcije v desno



I) $t < 0$

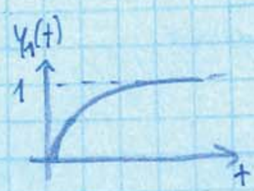


II) $\infty > t > 0$



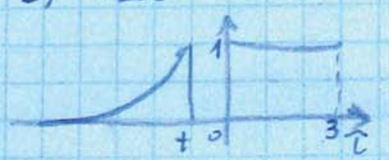
$$\Rightarrow y(t) = \int_0^t 1\, e^{-(t-\tau)}\, d\tau = \int_0^t e^{-t} e^{\tau}\, d\tau = e^{-t} e^{\tau}\Big|_0^t = e^{-t}(e^t - 1) = \underline{\underline{1 - e^{-t}}}$$

$e^{-t} \cdot e^{t} = e^{t-t} = e^0 = 1$

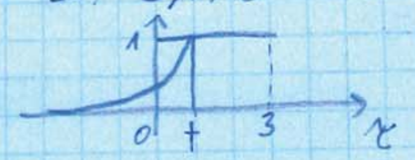


b) $u_2(t)$

I) $t < 0$

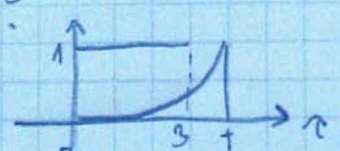


II) $3 > t > 0 \Rightarrow y(t) = 1 - e^{-t}$ (enako kot prej)

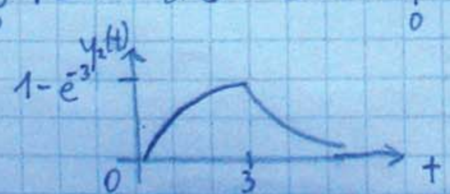


III) $t > 3$

⇓

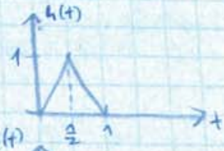


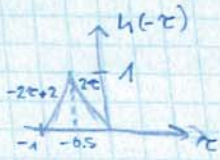
$$y(t) = \int_0^3 1 e^{-t+\tau}\, d\tau = e^{-t} e^{\tau}\Big|_0^3 = e^{-t}(e^3 - e^0) = 19.08\, e^{-t}$$



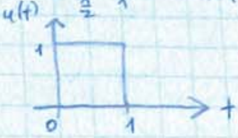
Naloga: Določi in skiciraj odziv LSS, ki ima odziv na enotin impulz $h(t)$:

$$h(t) = \begin{cases} 2t \, ; & 0 \le t \le \frac{1}{2} \\ -2t+2 \, ; & \frac{1}{2} \le t \le 1 \\ 0 \, ; & \text{sicer} \end{cases}$$



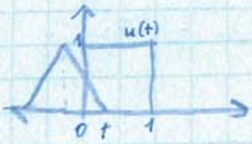


$$u(t) = \begin{cases} 1 \, ; & 0 \le t \le 1 \\ 0 \, ; & \text{sicer} \end{cases}$$



I) $t < 0 \, ; \quad y(t) = 0$

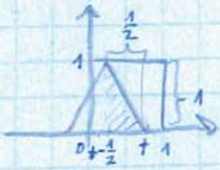
II) $0 \le t \le \frac{1}{2}$



$$y(t) = \int_0^t 1\,(2(t-\tau))\,d\tau = 2\int_0^t (t-\tau)\,d\tau =$$

$$= \left(2t\tau - \cancel{2}\frac{\tau^2}{\cancel{2}}\right)\Big|_0^t = 2t^2 - t^2 = \underline{\underline{t^2}}$$

III) $\frac{1}{2} \le t \le 1$



$$y(t) = \int_0^{t-\frac{1}{2}} (-2(t-\tau)+2)\,d\tau + \frac{1}{4} =$$

$$= \int_0^{t-\frac{1}{2}} (-2t + 2\tau + 2)\,d\tau + \frac{1}{4} =$$

$$= \left(-2t\tau + \tau^2 + 2\tau\right)\Big|_0^{t-\frac{1}{2}} + \frac{1}{4} =$$

$$P = \frac{\frac{1}{2}\cdot 1}{2} = \frac{1}{4}$$

$$= \left(-2t\left(t-\tfrac{1}{2}\right) + \left(t-\tfrac{1}{2}\right)^2 + 2\left(t-\tfrac{1}{2}\right)\right) + \frac{1}{4} =$$

$$= -2t^2 + \cancel{t} + t^2 - \cancel{t} + \frac{1}{4} + 2t - 1 + \frac{1}{4} = \qquad +\cancel{1}\left(t^2 - t + \frac{1}{4}\right)$$

$$= \underline{\underline{-t^2 + 2t - \frac{1}{2}}}$$

IV) $1 \le t \le \frac{3}{2}$

$$y(t) = \frac{1}{4} + \int_{t-\frac{1}{2}}^{1} 2(t-\tau)\,d\tau = \frac{1}{4} + 2\int_{t-\frac{1}{2}}^{1} (t-\tau)\,d\tau =$$

$$= 2\left(t\tau - \frac{\tau^2}{2}\right)\Big|_{t-\frac{1}{2}}^{1} \quad 2\left(\left(t - t^2 + \tfrac{1}{2}t\right) - \left(\tfrac{1}{2} - \frac{\left(t-\frac{1}{2}\right)^2}{2}\right)\right) =$$

$$P = \frac{1}{4}$$

$$= 2\left(t - t^2 + \cancel{\tfrac{1}{2}}t - \frac{1}{2} + \frac{t^2}{2} - \frac{t}{\cancel{2}} + \frac{1}{8}\right) =$$

$$= 2\left(-\frac{t^2}{2} + t - \right.$$

V. $\frac{3}{2} \le t \le 2$ $\quad y(t) = \int_{t-1}^{1} (-2(t-\tau)+2)\,d\tau = \int_{t-1}^{1} (-2t+2\tau+2)\,d\tau =$

$$= (-2t\tau + \tau^2 + 2\tau)\Big|_{t-1}^{1} =$$

$$= -2t(1-t+1) + (1^2 - (t-1)^2) + 2(1-t+1)$$

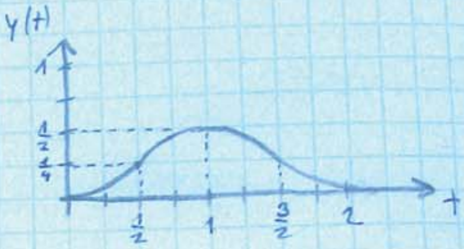
t^2-4t+4

$0 \le t \le \frac{1}{2}$; t^2

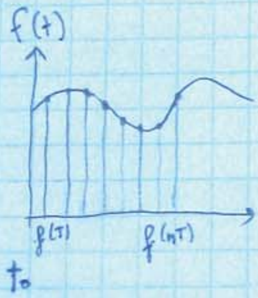
$\frac{1}{2} \le t \le 1$; $-t^2+2t-\frac{1}{2}$

$1 \le t \le \frac{3}{2}$; $-t^2+2t-\frac{1}{2}$

$\frac{3}{2} \le t \le 2$; t^2-4t+4



$\boxed{A6}$ 28.5.2012



največja frekvenca signala

$T = \frac{1}{2F} \qquad f = 2F = \frac{1}{T}$

$[0 \quad t_1]$

$N = \frac{t_1}{T} = 2t_1 F = t_1 f$

$f(t) \Rightarrow \mathcal{F}(f(t)) = F(\omega)$

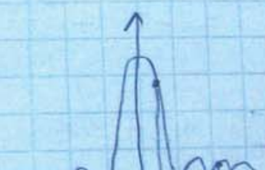
$\{f(nT)\} = \{f(0), f(T), f(2T), \dots, f((N-1)T)\}$

$n = 0, 1, 2, \dots, N$

DFT (diskretna fourierova transformacija)

$$\boxed{F_D(k\Omega) = \sum_{n=0}^{N-1} f(nT)\, e^{-jnTk\Omega}} \qquad \Omega = \frac{2\pi}{NT}$$

$k = 0, 1, 2, \dots$



$$\boxed{F(\omega)\Big|_{\omega = k\Omega} \approx T F_D(k\Omega)}$$

IDFT (inverzna diskretna fourierova transformacija)

$$\boxed{f(nT) = \mathcal{F}_D^{-1}\{F_D(k\Omega)\} = \frac{1}{N} \sum_{k=0}^{N-1} F_D(k\Omega)\, e^{jnTk\Omega}}$$

Naloga: DFT vhodnega signala $u(t)$ LSS je $\{U_D(k\Omega)\} = \{2, 1+j, 1, 1-j\}$. DFT odziva sistema $y(t)$ na vhodni signal je enak $\{Y_D(k\Omega)\} = \{2, -2j, -1, 2j\}$. Pri določitvi obeh DFT smo signala vzorčili s časovnim presledkom $T=1$.

a) Določi približno vrednost amplitudnega in faznega spektra prevajalne funkcije pri $\omega = \frac{\pi}{2}$

b) Določi približen odziv sistema na enotin impulz pri $t=1$.

$\{U_D(k\Omega)\} = \{2, 1+j, 1, 1-j\} \Rightarrow N=4$

$\{Y_D(k\Omega)\} = \{2, -2j, -1, 2j\}$

$T=1$

$\omega = \frac{\pi}{2}$

$t=1$

prevajalna funkcija

a) $\Omega = \frac{2\pi}{NT} = \frac{\cancel{2}\pi}{\cancel{4}_2 \cdot 1} = \frac{\pi}{2}$

$$H_D(k\Omega) = \frac{Y_D(k\Omega)}{U_D(k\Omega)} = \left\{\frac{2}{2}, \frac{-2j}{1+j}, \frac{-1}{1}, \frac{2j}{1-j}\right\} =$$

$$= \{\overset{k=0}{1}, \overset{k=1}{-1-j}, \overset{k=2}{-1}, \overset{k=3}{-1+j}\}$$

$\omega = k\Omega$

$\frac{\pi}{2} = k\frac{\pi}{2} \Rightarrow k=1$

$$H(\omega)\Big|_{\omega=\frac{\pi}{2}} = T H_D(k\Omega) \overset{k=1}{=} -1-j$$

$$\frac{-2j(1-j)}{(1+j)(1-j)} = \frac{-\cancel{2}j - \cancel{2}}{\cancel{1+1}} = -j-1$$

$$\frac{2j(1+j)}{(1-j)(1+j)} = \frac{2j-2}{2} = j-1$$

$$|H(\omega)|\Big|_{\omega=\frac{\pi}{2}} = \sqrt{(-1)^2 + (-1)^2} = \underline{\underline{\sqrt{2}}}$$

$$\Theta_H(\omega)\Big|_{\omega=\frac{\pi}{2}} = \pm\pi + \text{arctg}\frac{-1}{-1} = \underline{\underline{-\frac{3\pi}{4}}}$$

b)

$t = nT \quad ; n = 0, 1, \dots, N-1$

$nT = 1$

$\Omega = \frac{\pi}{2}$

$1 = n1 \Rightarrow n=1$

$$\boxed{e^{jx} = \cos x + j\sin x}$$

$H_D(k\Omega)$

$\downarrow$ IDFT

$h(nT)$

$$h(1) = \frac{1}{N}\sum_{k=0}^{N-1} H_D(k\Omega)\, e^{jk\Omega nT} = \frac{1}{N}\left[1e^{j0} + (-1-j)e^{j1\frac{\pi}{2}1} + (-1)e^{j2\frac{\pi}{2}1} + (-1+j)e^{j3\frac{\pi}{2}1}\right]$$

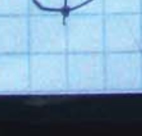
$$= \frac{1}{N}\left[1 + (-1-j)j - 1(-1) + (-1+j)\cdot(-j)\right] =$$

$$= \frac{1}{N}\left[1 \cancel{-j} \underbrace{-j^2}_{+1} + 1 \cancel{+j} \underbrace{-j^2}_{+1}\right] = \frac{1}{4}[4] = \underline{\underline{1}}$$

$e^{j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) =$
$= 0 + 1j = j$

$e^{j\pi} = \cos(\pi) + j\sin(\pi) =$
$= -1$

$e^{j\frac{3\pi}{2}} = \cos\left(\frac{3\pi}{2}\right) + j\sin\left(\frac{3\pi}{2}\right) =$
$= -j$



Naloga: Podana sta signala: vhodni $u(nT)$ in prev. funk. $h(nT)$. Signala sta vzorčena z časovnim presledkom $T=\frac{1}{2}$. Izračunaj $Y(\omega)$.

$$h(nT)=\begin{cases} 0.5 & ; n=0 \\ 1 & ; n=1 \\ 0 & ; n=2 \\ -1 & ; n=3 \\ -0.5 & ; n=4 \\ 0 & ; n=5 \\ 0 & ; n=6 \\ 0 & ; n=7 \end{cases} \qquad u(nT)=\begin{cases} -0.5 & ; n=0 \\ -1 & ; n=1 \\ 0 & ; n=2 \\ 1 & ; n=3 \\ 0.5 & ; n=4 \\ 0 & ; n=5 \\ 0 & ; n=6 \\ 0 & ; n=7 \end{cases} \Rightarrow N=5$$

$Y(\omega)$? $\quad \omega=\frac{\pi}{2}$

$N=5 \quad T=\frac{1}{2}$

$$\Omega=\frac{2\pi}{NT}=\frac{2\pi}{5\frac{1}{2}}=\frac{4\pi}{5}$$

$$\omega=\frac{\pi}{2}=k\Omega$$

$$\frac{\pi}{2}=k\frac{4\pi}{5} \Rightarrow \cancel{k \neq \mathbb{Z}}$$

$$k=\frac{\frac{\cancel{\pi}}{\cancel{2}}}{\frac{\cancel{4}\cancel{\pi}2}{5}}=\frac{5}{2}$$

$$N=8 \quad \Omega=\frac{2\pi}{NT}=\frac{2\pi}{8\frac{1}{2}}=\frac{4\pi}{8}=\frac{\pi}{2} \qquad \omega=k\Omega$$

$$\frac{\pi}{2}=k\frac{\pi}{2} \Rightarrow k=1$$

$$H_D(k\Omega)=H_D(\Omega)=\sum_{n=0}^{7} h(nT)\,e^{-jn\frac{1}{2}\frac{\pi}{2}}=\sum_{n=0}^{7} h(nT)\,e^{-jn\frac{\pi}{4}}=$$

$$=0.5e^0+1e^{-j\frac{\pi}{4}}-1e^{-j\frac{3\pi}{4}}-0.5e^{-j\pi}=$$

$$=0.5+\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}-1\left(-\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}\right)-0.5\cdot(-1)=$$

$$=0.5+\frac{\sqrt{2}}{2}-\cancel{j\frac{\sqrt{2}}{2}}+\frac{\sqrt{2}}{2}+\cancel{j\frac{\sqrt{2}}{2}}+0.5=\underline{\underline{1+\sqrt{2}}}$$

$$e^{-j\frac{\pi}{4}}=\cos\frac{\pi}{4}-j\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}$$

$$e^{-j\frac{3\pi}{4}}=\cos\frac{3\pi}{4}-j\sin\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}$$

$$e^{-j\pi}=\cos\pi-j\sin\pi=-1$$

$$U_D(k\Omega)=\sum_{n=0}^{N-1} u(nT)\,e^{-jn\frac{\pi}{4}}=$$

$$=-0.5e^0-1e^{-j\frac{\pi}{4}}+1e^{-j\frac{3\pi}{4}}+0.5e^{-j\pi}=$$

$$=-0.5-\frac{\sqrt{2}}{2}+\cancel{j\frac{\sqrt{2}}{2}}-\frac{\sqrt{2}}{2}-\cancel{j\frac{\sqrt{2}}{2}}-0.5=$$

$$=\underline{\underline{-1-\sqrt{2}}}$$

$U_D(k\Omega)=-1-\sqrt{2}$

$$H(\omega)\Big|_{\omega=\frac{\pi}{2}}=TH(k\Omega)=\frac{1+\sqrt{2}}{2}$$

$$U(\omega)\Big|_{\omega=\frac{\pi}{2}}=TU(k\Omega)=\frac{-1-\sqrt{2}}{2}$$

$$Y(\omega)\Big|_{\omega=\frac{\pi}{2}}=H(\omega)\,U(\omega)=$$

$$=\frac{1}{4}(1+\sqrt{2})(-1-\sqrt{2})=$$

$$=\frac{1}{4}(-1-\sqrt{2}-\sqrt{2}-2)=$$

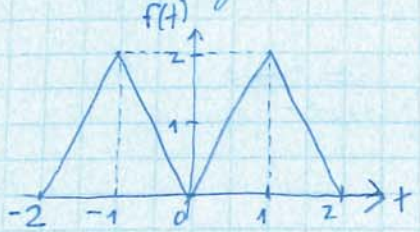
$$\left|Y(\omega)\right|\Big|_{\omega=\frac{\pi}{2}} = \sqrt{(1.457)^2} = 1.457$$

Priprave na kolokvij:

<u>4.6.2012</u>

Naloga: Podan je neperiodičen signal $f(t)$ na sliki.

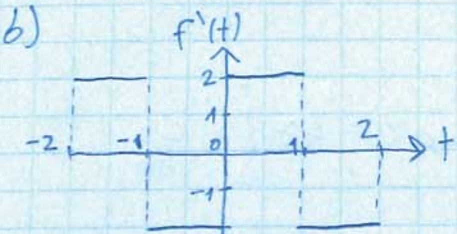
a) Zapiši signal $f(t)$ z matematično formulo
b) Izračunaj kompleksni spekter signala $f(t)$
c) Izračunaj še amplitudni in močnostni spekter signala $f(t)$.



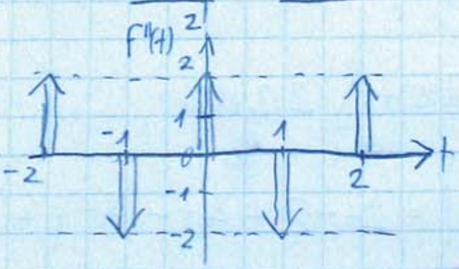
a)

$$f(t) = \begin{cases} 2t+4 & ; -2 \le t < -1 \\ -2t & ; -1 \le t < 0 \\ 2t & ; 0 \le t < 1 \\ -2t+4 & ; 1 \le t < 2 \\ 0 & ; \text{sicer} \end{cases}$$

b)



$$f'(t) = \begin{cases} 2 & ; -2 \le t < -1 \\ -2 & ; -1 \le t < 0 \\ 2 & ; 0 \le t < 1 \\ -2 & ; 1 \le t < 2 \\ 0 & ; \text{sicer} \end{cases}$$



$$\frac{d^2f}{dt^2} = \delta(t+2)\left[2-0\right] + \delta(t+1)\left[-2-2\right] + \delta t\left[2-(-2)\right]$$

$$+\delta(t-1)\left[-2-2\right] + \delta(t-2)\left[0-(-2)\right] =$$

$$= 2\delta(t+2) - 4\delta(t+1) + 4\delta t - 4\delta(t-1) + 2\delta(t-2) \;/\mathcal{F}$$

$$(j\omega)^2 F(\omega) = 2e^{2j\omega} - 4e^{j\omega} + 4 - 4e^{-j\omega} + 2e^{-2j\omega}$$

$$-\omega^2 F(\omega) = 4\cos(2\omega) - 8\cos(\omega) + 4 \quad /: -\omega^2$$

$$F(\omega) = -\frac{4}{\omega^2}\left(\cos(2\omega) - 2\cos(\omega) + 1\right)$$

c)

$$\left|F(\omega)\right| = \frac{4}{\omega^2}\left|\cos(2\omega) - 2\cos(\omega) + 1\right|$$

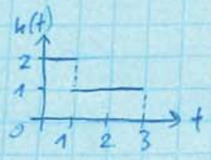
$$\left|F(\omega)\right|^2 = \frac{16}{\omega^4}\left(\cos(2\omega) - 2\cos(\omega) + 1\right)^2$$

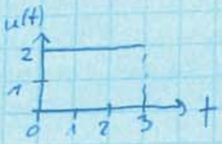
Naloga: LSS ima odziv na enotin impulz $h(t)$ podan na sliki.

a) Zapiši odziv na enotin impulz $h(t)$ in funkcijo vhodnega signala $u(t)$ z matematičnimi formulami $y(t)$

b) Dobi izhodni signal $y(t)$

c) Odziv sistema skiciraj



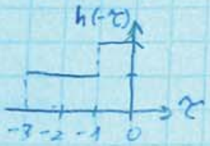


a)

$$h(t) = \begin{cases} 2 & ; \ 0 \leq t < 1 \\ 1 & ; \ 1 \leq t < 2 \\ 0 & ; \ \text{sicer} \end{cases}$$

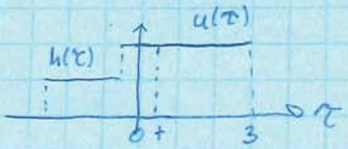
$$u(t) = \begin{cases} 2 & ; \ 0 \leq t < 3 \\ 0 & ; \ \text{sicer} \end{cases}$$

b) $y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$



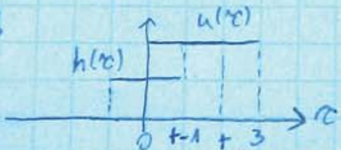
I) $t < 0 \quad y(t) = 0$

II) $0 \leq t < 1$



$$y(t) = \int_0^t 2 \cdot 2 \, d\tau = 4\tau \Big|_0^t = \underline{\underline{4t}}$$

III) $1 \leq t < 3$

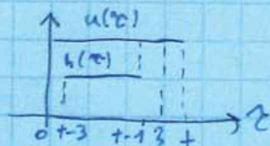


$$y(t) = \int_0^{t-1} 2 \cdot 1 \, d\tau + \int_{t-1}^{t} 2 \cdot 2 \, d\tau =$$

$$= 2\tau \Big|_0^{t-1} + 4\tau \Big|_{t-1}^{t} = 2t - 2 + \cancel{4t} - \cancel{4t} + 4 =$$

$$= \underline{\underline{2t + 2}}$$

IV) $3 \leq t < 4$

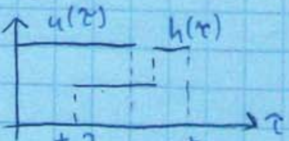


$$y(t) = \int_{t-3}^{t-1} 2 \cdot 1 \, d\tau + \int_{t-1}^{3} 2 \cdot 2 \, d\tau =$$

$$= 2\tau \Big|_{t-3}^{t-1} + 4\tau \Big|_{t-1}^{3} = \cancel{2t} - 2 - \cancel{2t} + 6 + 12 - 4t + 4$$

$$= \underline{\underline{20 - 4t}}$$

V $4 \leq t < 6$



$$y(t) = \int_{t-3}^{3} 2 \cdot 1 \, d\tau = 2\tau \Big|_{t-3}^{3} = 6 - 2t + 6 = 12 - 2$$

VI $6 < t \quad y(t) = 0$

$$y(t) = \begin{cases} 0 & ; \; t < 0 \\ 4t & ; \; 0 \leq t < 1 \\ 2t + 2 & ; \; 1 \leq t < 3 \\ 20 - 4t & ; \; 3 \leq t < 4 \\ 12 - 2t & ; \; 4 \leq t < 6 \\ 0 & ; \; 6 < t \end{cases}$$