PRIMER:

Dan je sistem 3 linearnih algebrajskih enačb s 3 neznankami.

① $10 = x_1 + 2x_2 - 3x_3 + 1$

② $5 = 2x_1 - x_2 - x_3 + 2$

③ $10 = 3x_1 - 2x_2 - 5x_3 + 3$

① + ③ $20 = 4x_1 - 8x_3 + 4$

$\boxed{4 = x_1 - 2x_3}$

① + 2·② $20 = 5x_1 - 5x_3 + 5$

$\boxed{3 = x_1 - x_3}$

$1 = -x_3$

$\boxed{\begin{matrix} x_3 = -1 \\ x_1 = 2 \\ x_2 = 2 \end{matrix}}$

$\Rightarrow$ x_1, x_2, x_3 = signali

$\Rightarrow$ cifre: 1, 2, 3 $\rightarrow$ dodana stvar - vzbujanje

$\rightarrow$ kar dobimo ven je izhod

$$\left.\begin{aligned} y_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1u_1 \\ y_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2u_2 \\ y_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_3u_3 \end{aligned}\right\} \text{red sistema je 3}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}_{3\times3} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_{3\times1}$$

ENAKI za množenje !!!

• vektor ima eno dimenzijo enako 1 (pri nas bodo stolpični)

$\boxed{y = A\cdot x + B\cdot u}$ vektor

matrika

$$y - Bu = A\cdot x$$

$$\underline{A^{-1}(y - Bu)} = A^{-1}\cdot A x = \underline{I\cdot x = x}$$

$$A^{-1} = \begin{bmatrix} 1^{+} & 2^{-} & -3^{+} \\ 2^{-} & -1^{+} & -1^{-} \\ 3^{+} & -2^{-} & -5^{+} \end{bmatrix}^{-1} = \frac{1}{3+14+4} \begin{bmatrix} 3 & 7 & -1 \\ 16 & 4 & 8 \\ -5 & -5 & -5 \end{bmatrix}^{T} = \frac{1}{20} \begin{bmatrix} 3 & 16 & -5 \\ 7 & 4 & -5 \\ -1 & 8 & -5 \end{bmatrix}$$

determinanta

kofaktorji → poddeterminanta, ki ostane

$$x = \frac{1}{20} \begin{bmatrix} 3 & 16 & -5 \\ 7 & 4 & -5 \\ -1 & 8 & -5 \end{bmatrix} \left(\begin{bmatrix} 10 \\ 5 \\ 10 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right) =$$

$$= \frac{1}{20} \begin{bmatrix} 3 & 16 & -5 \\ 7 & 4 & -5 \\ -1 & 8 & -5 \end{bmatrix} \left(\begin{bmatrix} 10 \\ 5 \\ 10 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right) = \frac{1}{20} \begin{bmatrix} 3 & 16 & -5 \\ 7 & 4 & -5 \\ -1 & 8 & -5 \end{bmatrix} \begin{bmatrix} 9 \\ 3 \\ 7 \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 40 \\ 40 \\ -20 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ ⇒ VSE BO ODVISNO OD ČASA !!!

$$\left.\begin{array}{l} x_1 = x_1(t) \\ x_2 = x_2(t) \\ x_3 = x_3(t) \end{array}\right\} x = x(t) \longrightarrow \text{SPREMENLJIVKE STANJ}$$

⇒ pridemo do <u>LINEARNIH DIFERENCIALNIH ENAČB</u>

$$\dot{x}(t) = Ax(t) + Bu(t)$$

↑ odvod bo naš odziv (ne več y)

$$\boxed{\dot{x} = Ax + Bu}$$

$$\dot{X}(t) = \underline{A}x(t) + Bu(t) \qquad (\circledast)$$

osnovna matrika sistema — vektor spremenljivk stanj — matrika vzbujanja — vektor vzbujanja

* vzbujanje pride iz zunanjega sistema; pomembna je matrika A, ki govori o obnašanju sistema
* reaktivni element: kondenzator, tuljava
* RL / RC RLC - vezja

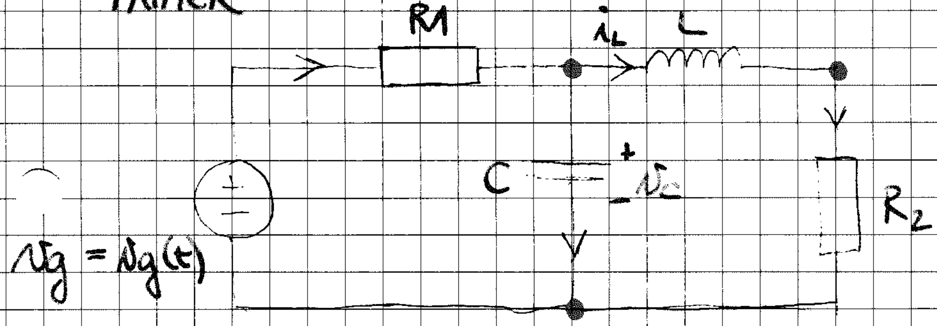
$$X = \begin{bmatrix} i_L \\ v_C \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} \dot{i}_L \\ \dot{v}_C \end{bmatrix} \qquad u = \begin{bmatrix} i_g \\ v_g \end{bmatrix}$$

spremenljivke stanj

* $v_L = L\dot{i}_L$

  $i_C = C\dot{v}_C$

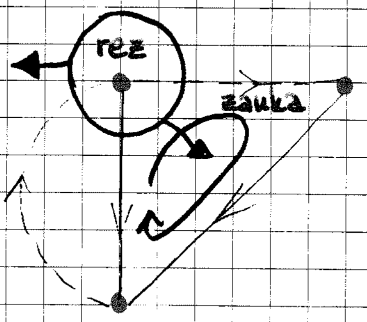
PRIMER



* MODELIRANJE SISTEMOV: zapisat dan sistem v naravi - opišemo z enačbo ($\circledast$)
* za vezja obstaja nek postopek
* DOGOVOR: tok teče s + na -

- narišemo "poseben" graf vezja (poenostavljeno vezje)
- določimo vozlišča, v graf vstavimo veje, kjer so kondenzatorji (veje so z isto smerjo)
- povezat je treba vsa vozlišča, vendar ne smemo sklenit zanke (pomagamo si z vejami, kjer so upornosti - ne nujno vse veje; NIKOLI pa

si ne pomagamo z vejami, kjer so tuljave)



- kar ostane noter v vezju, damo noter v graf kot KITE (vedno je to TULJAVA)

- napetostni generator: KRATEK STIK } pri risanju grafa jih zanemarimo
  tokovni generator: ODPRTE SPONKE

- OSNOVNI REZ (kjer je kondenzator): prereže vejo s kondenzatorjem in poljubno število kit ⇒ krožnica; smer reza je smer toka v veji s kondenzatorjem

- vsaki tuljavi pripada ena OSNOVNA ZANKA: zajamemo le kito, kjer je tuljava in poljubno število vej → smer je enaka smeri toka skozi tuljavo, ki ji pripada

→ osnovni rez: TOKOVNA ENAČBA → predznak glede na smer reza: če se smeri ujemata je pozitiven predznak, če se pa ne pa negativen

$$i_C + i_L + i_{R1} = 0$$ → ni vedno enako vozliščni enačbi

→ osnovna zanka: NAPETOSTNA ENAČBA

$$v_L + v_{R2} - v_C = 0$$

$$X = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$ spremenljivke stanj ↗ vrstni red elementov ni važen (le pazimo ko pišemo matrike)

$u = v_g$ - skalar (ni vektor)!

$$C\dot{v}_C = -i_L + i_{R1}$$
$$L\dot{i}_L = v_C - v_{R2}$$
→ zapisat moramo drugače: izraženo s spremenljivkami stanj in vzbujanjem

$$\boxed{\dot{X} = AX + Bu}$$ MODEL

→ $i_{R1} = \frac{v_{R1}}{R_1} = \frac{v_g - v_C}{R_1}$

→ $v_{R2} = R_2 \cdot i_{R2} = R_2 \cdot i_L$

$$C\dot{u}_C = -i_L - \frac{1}{R_1}u_C + \frac{1}{R_1}u_g \quad \Big| : C$$
$$L\dot{i}_L = u_C - R_2 i_L \quad \Big| : L$$

$$\dot{x} = \begin{bmatrix} \dot{i}_L \\ \dot{u}_C \end{bmatrix}$$

$$\dot{i}_L = -\frac{R_2}{L} i_L + \frac{1}{L} u_C$$

$$\dot{u}_C = -\frac{1}{C} i_L - \frac{1}{R_1 C} u_C + \frac{1}{R_1 C} u_g$$

$$\dot{x} = \begin{bmatrix} -\frac{R_2}{L} & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{R_1 C} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{R_1 C} \end{bmatrix} u$$

- na spremembo toka v vezju vplivajo vsi elementi ⇒ vsak element se mora v matriki vsaj enkrat pojavit

$$\boxed{y = Cx + Du}$$

C: izhodna matrika

D: vhodno-izhodna matrika sistema

y: vektor izhodov (sistema) ⇒ niso ODVODI! (lahko vas zanima karkoli drugega)

$$y = \begin{bmatrix} u_{R1} & u_{R2} & u_L & u_C \end{bmatrix}^T = \begin{bmatrix} u_{R1} \\ u_{R2} \\ u_L \\ u_C \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$y = y(t)$

* $y = Cx + Du$

$u_{R1} = u_g - u_C$

$u_{R2} = R_L \cdot i_L$

$u_L = \cancel{L\dot{i}_L} = u_C - u_{R2} = u_C - R_2 i_L$

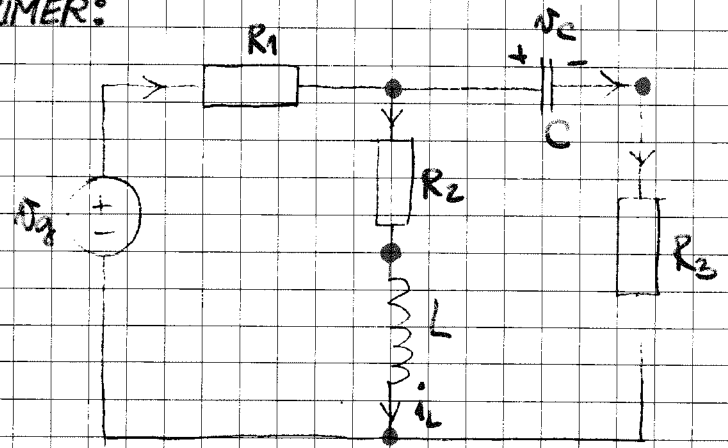
($L\dot{i}_L$:) ne koristi za to matrično enačbo

! $\boxed{u_C = u_C}$ ⇒ je že sama po sebi spremenljivka stanj

$$y = \begin{bmatrix} 0 & -1 \\ R_2 & 0 \\ -R_2 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

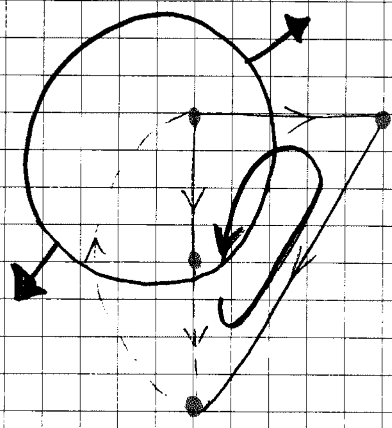
dva stolpca = dve spremenljivki stanj

PRIMER:



$$X = \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$u = v_g$$



- po dveh različnih poteh pridemo do istega rezultata ⇒ OK
- iskanje toka ali napetosti na upornosti je isti problem!!! ⇒ vsakič iščemo dve količini naenkrat
- zanko lahko delamo čez kondenzator, čez tuljavo pa ne!!!

REZ: $i_C + i_L - i_{R1} = 0$ *

ZANKA: $v_L - v_{R2} - v_C + v_{R3} = 0$

$i_{R1} =$

(R_2) $v_{R2} = R_2 \cdot i_{R2} = R_2 \cdot i_L$

(R_1, R_3) $v_{R1} + v_C + v_{R3} = v_g$

$i_{R1} - i_L - i_{R3} = 0 \quad / \cdot R_3$

$R_1 i_{R1} + v_C + v_{R3} = v_g$] +

$R_3 i_{R1} - R_3 i_L - v_{R3} = 0 \Longrightarrow v_{R3} = R_3 i_{R1} - R_3 i_L$

$$(R_1+R_3)\, i_{R_1} = v_g - v_c + R_3 i_L$$

$$i_{R_1} = \frac{R_3}{R_1+R_3} i_L - \frac{1}{R_1+R_3} v_C + \frac{1}{R_1+R_3} v_g$$

- preverimo, da je enačba prav $\Rightarrow$ tudi na drugi strani morajo deli predstavljat tok

$$\Rightarrow v_{R_3} = \frac{R_3^2}{R_1+R_3} i_L - \frac{R_3}{R_1+R_3} v_C + \frac{R_3}{R_1+R_3} v_g - R_3 i_L \qquad \frac{R_1R_3+R_3^2}{R_1+R_3}$$

$$v_{R_3} = -\frac{R_1R_3}{R_1+R_3} i_L - \frac{R_3}{R_1+R_3} v_C + \frac{R_3}{R_1+R_3} v_g$$

$$* \Rightarrow C\dot{v}_c = -i_L + i_{R_1} = -i_L + \frac{R_3}{R_1+R_3} i_L - \frac{1}{R_1+R_3} v_c + \frac{1}{R_1+R_3} v_g$$

$$L\dot{i}_L = v_C - v_{R_2} + v_{R_3} =$$

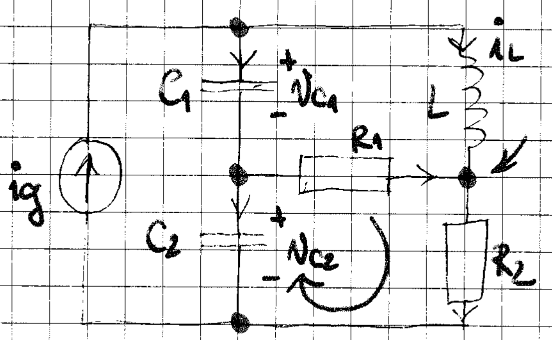
$$= v_C - R_2 i_L - \frac{R_1R_3}{R_1+R_3} i_L - \frac{R_3}{R_1+R_3} v_C + \frac{R_3}{R_1+R_3} v_g$$

$$C\dot{v}_c = -\frac{R_1}{R_1+R_3} i_L - \frac{1}{R_1+R_3} v_C + \frac{1}{R_1+R_3} v_g$$

$$L\dot{i}_L = \left(-R_2 - \frac{R_1R_3}{R_1+R_3}\right) i_L + \frac{R_1}{R_1+R_3} v_C + \frac{R_3}{R_1+R_3} v_g$$

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \dot{X} = \begin{bmatrix} -\frac{R_2}{L} - \frac{R_1R_3}{(R_1+R_3)L} & \frac{R_1}{(R_1+R_3)L} \\ -\frac{R_1}{(R_1+R_3)C} & -\frac{1}{(R_1+R_3)C} \end{bmatrix} X + \begin{bmatrix} \frac{R_3}{(R_1+R_3)L} \\ \frac{1}{(R_1+R_3)C} \end{bmatrix} u$$

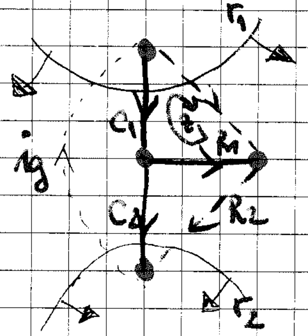
vaja 11 (izpit 5.1)



$X = [i_L \; u_{C1} \; u_{C2}]^T; \quad u = i_g$

a) $\boxed{\dot{X} = AX + Bu}$

$\Rightarrow$ 3 reaktivni elementi $\rightarrow$ tokovni vir vrišemo v graf kot kito!!!



r_1: $i_{C1} + i_L - i_g = 0$

r_2: $i_{C2} + i_{R2} - i_g = 0$

z: $u_L - u_{R1} - u_{C1} = 0$

$C_1 \dot{u}_{C1} = -i_L + i_g$

$C_2 \dot{u}_{C2} = -i_{R2} + i_g = -\frac{R_1}{R_1+R_2} i_L - \frac{1}{R_1+R_2} u_{C2} + i_g$

$L \dot{i}_L = -\frac{R_1 R_2}{R_1+R_2} i_L + u_{C1} + \frac{R_1}{R_1+R_2} u_{C2}$

$\rightarrow$ iščemo enačbo, kjer ne bomo pridelali novih odvodov spremenljivk stanj

$i_L + \overset{\frac{u_{R1}}{R_1}}{i_{R1}} - i_{R2} = 0 \quad / \cdot R_2$

$u_{R1} + u_{R2} - u_{C2} = 0$

$+$

$R_2 i_L - u_{C2} + \left(1 + \frac{R_2}{R_1}\right) u_{R1} = 0$

$\underline{u_{R1}} = -\frac{R_1 R_2}{R_1+R_2} i_L + \frac{R_1}{R_1+R_2} u_{C2}$

$\underline{i_{R2}} = \frac{u_{R1}}{R_1} + i_L = \frac{R_1}{R_1+R_2} i_L + \frac{1}{R_1+R_2} u_{C2}$

$$\dot{X} = [\dot{i}_L \quad \dot{u}_{C1} \quad \dot{u}_{C2}]^T$$

→ matrika A je vedno kvadratna

$$\dot{X} = \begin{bmatrix} -\frac{R_1R_2}{(R_1+R_2)L} & \frac{1}{L} & \frac{R_1}{(R_1+R_2)L} \\ -\frac{1}{C_1} & 0 & 0 \\ -\frac{R_1}{(R_1+R_2)C_2} & 0 & -\frac{1}{(R_1+R_2)C_2} \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix} u$$

b) Določite $\boxed{y = Cx + Du}$

$$y = \begin{bmatrix} i_{R1} \\ u_{C2} \end{bmatrix}$$

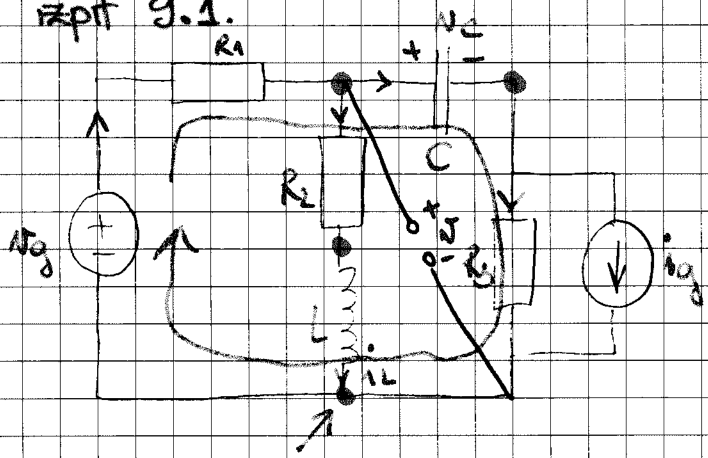
$$y_1 = i_{R1} = \frac{u_{R1}}{R_1} = -\frac{R_2}{R_1+R_2} i_L + \frac{1}{R_1+R_2} u_{C2}$$

$$y_2 = u_{C2} = u_{C2}$$

$$y = \begin{bmatrix} -\frac{R_2}{R_1+R_2} & 0 & \frac{1}{R_1+R_2} \\ 0 & 0 & 1 \end{bmatrix}_{2\times 3} X + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

2×3 (→ tri spremenljivke stanj)

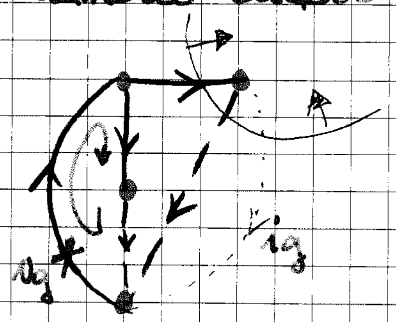
izpit 9.1.



$$X = [i_L \quad u_C]^T$$

$$u = [i_g \quad u_g]^T$$

a) določite matično enačbo $\dot{x} = Ax + Bu$



→ TOPOLOŠKI POSTOPKI

$$i_C - i_{R_3} - i_g = 0$$

$$u_L - u_g + u_{R_1} + u_{R_2} = 0$$

$$\underline{u_{R_2}} = R_2 \cdot i_{R_2} = \underline{R_2 \cdot i_L}$$

$$\rightarrow u_{R_1} + u_C + u_{R_3} - u_g = 0$$

$$i_{R_1} - i_L - i_{R_3} = i_g \quad / \cdot R_3 \quad +$$

$$\underline{u_{R_1}} = \frac{R_1 R_3}{R_1 + R_3} i_L - \frac{R_1}{R_1 + R_3} u_C + \frac{R_1 R_3}{R_1 + R_3} i_g + \frac{R_1}{R_1 + R_3} u_g$$

$$i_{R_3} = -i_{R_1} + i_L + i_g = -\frac{u_{R_1}}{R_1} + i_L + i_g$$

$$\underline{i_{R_3}} = -\frac{R_1}{R_1 + R_3} i_L - \frac{1}{R_1 + R_3} u_C - \frac{R_1}{R_1 + R_3} i_g + \frac{1}{R_1 + R_3} u_g$$

$$C \dot{u}_C = i_{R_3} + i_g = -\frac{R_1}{R_1 + R_3} i_L - \frac{1}{R_1 + R_3} u_C + \frac{R_3}{R_1 + R_3} i_g + \frac{1}{R_1 + R_3} u_g$$

$$L \dot{i}_L = -u_{R_1} - u_{R_2} + u_g = \left(-R_2 - \frac{R_1 R_3}{R_1 + R_3}\right) i_L + \frac{R_1}{R_1 + R_3} u_C - \frac{R_1 R_3}{R_1 + R_3} i_g + \frac{R_3}{R_1 + R_3} u_g$$

$$\dot{x} = \begin{bmatrix} \left(-R_2 - \frac{R_1 R_3}{R_1 + R_3}\right) \frac{1}{L} & \frac{R_1}{(R_1 + R_3) L} \\ -\frac{R_1}{(R_1 + R_3) C} & -\frac{1}{(R_1 + R_3) C} \end{bmatrix} x + \begin{bmatrix} -\frac{R_1 R_3}{(R_1 + R_3) L} & \frac{R_3}{L (R_1 + R_3)} \\ \frac{R_3}{(R_1 + R_3) C} & \frac{1}{(R_1 + R_3) C} \end{bmatrix} u$$

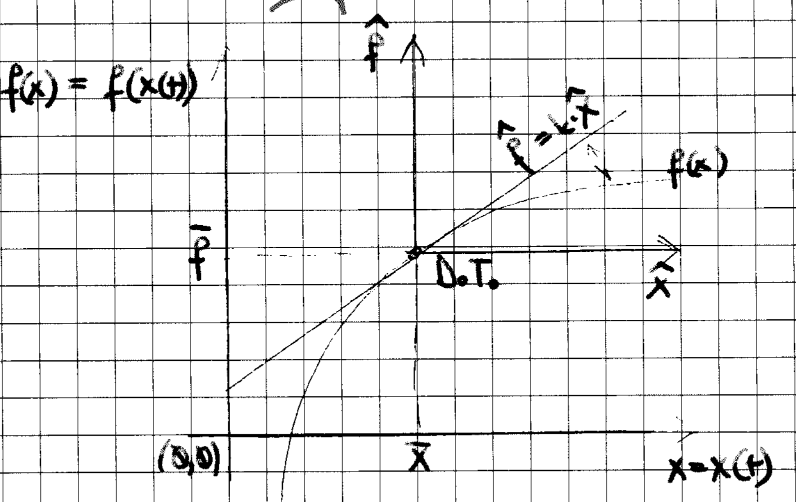
b) $\boxed{y = Cx + Du}$ $\qquad y = [u \;\; u_C]^T$

$$y_1 = u = u_C + u_{R_3} = -\frac{R_1 R_3}{R_1 + R_3} i_L + \frac{R_1}{R_1 + R_3} u_C - \frac{R_1 R_3}{R_1 + R_3} i_g + \frac{R_3}{R_1 + R_3} u_g$$

$$y_2 = I = i_C$$

$$y=\begin{bmatrix} -\frac{R_1R_3}{R_1+R_3} & \frac{R_1}{R_1+R_3} \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} -\frac{R_1R_3}{R_1+R_3} & \frac{R_3}{R_1+R_3} \\ 0 & 0 \end{bmatrix} u$$

LINEARIZACIJA NELINEARNIH SISTEMOV



- DELOVNA TOČKA = točka opazovanja $(\bar{f}, \bar{x})$ ⇒ za majhen odmik od delovne točke lahko krivuljo opazujemo kot linearno s postavimo tangento v delovni točki
- izhodišče (0,0) prestavimo v delovno točko

$$x = \hat{x} + \bar{x}$$
$$f = \hat{f} + \bar{f}$$

→ nominalne vrednosti

→ inkrementalne vrednosti

- za majhne odmike od koordinatnega sistema je približek podoben realni vrednosti

⇒ kako priskati tangento? TAYLORJEVA VRSTA

$$\boxed{f(x) = f(\bar{x}) + \left.\frac{df(x)}{dx}\right|_{x=\bar{x}}(x-\bar{x}) + \frac{1}{2!}\cdot\left.\frac{d^2f(x)}{dx^2}\right|_{x=\bar{x}}(x-\bar{x})^2+\dots}$$

- neskončna vrsta
- za linearni približek upoštevamo prva dva LINEARNA člena

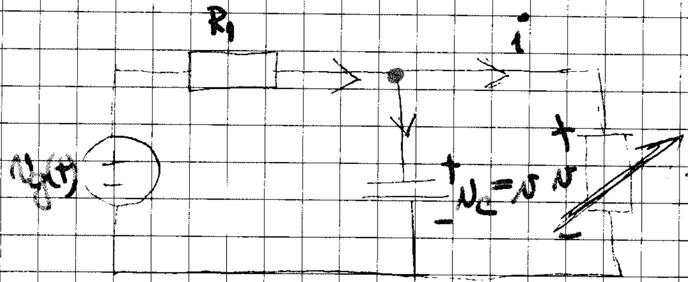
$$\boxed{f \approx \bar{f} + \left.\frac{df}{dx}\right|_{\bar{x}}(x-\bar{x})}$$

$$f-\bar f = \overbrace{\frac{df}{dx}\Big|_{\bar x}}^{k}(x-\bar x)$$

$$\boxed{\hat f = k\hat x}$$

- začetne vrednosti niso enake v obeh sistemih (za konstantno spremenjene vrednosti → izhodišče opazovanja se spremeni)
- delovno točko ponavadi dobimo z vzbujanjem

PRIMER:



R_2 → nelinearen element

$R_1 = 1$ $\quad i = 2v^2$

$C = \frac{1}{2}$ $\quad v_g(t) = 18 + A\cos\omega t$

– linearizirajte v okolici delovne točke

① najprej modeliramo sisteme

$$i_{R_1} - i_C - i = 0$$

$$\frac{v_g - v}{R_1} - C\dot v - 2v^3 = 0$$

identificiramo nelinearni člen in aproksimiramo s prvima dvema členoma Taylorjeve vrste

$$\boxed{\frac{1}{2}\dot v + v + 2v^3 = 18 + A\cos\omega t}$$ NELINEARNI MODEL

nelinearni člen → ne moremo take enačbe zapisat v matrični obliki

② določitev D.T.

Delovna točka podana kot stanje, kjer ni več sprememb v sistemu pri čemer za vzbujanje vzamemo povprečno vrednost vzbujanja.

$$\frac{1}{2}\dot v + v + 2v^3 = 18 + A\cos\omega t$$

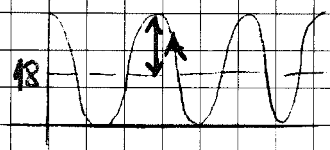
→ ni sprememb v sistemu = odvodi spremenljivk so enaki nič

$$\frac{1}{2}\cdot 0 + \bar v + 2\bar v^3 = 18$$

$$\bar v + 2\bar v^3 = 18$$

$\bar{v}(1+2\bar{v}^2) = 18$

$\underline{\bar{v} = 2} \Rightarrow \bar{i} = 2\bar{v}^3 = 16$



$$\boxed{DT = (\bar{v}, \bar{i}) = (2, 16)}$$

③ $i = i(v) = 2v^3$

$i \approx \bar{i} + \frac{di}{dv}\Big|_{\bar{v}} (v - \bar{v}) =$

$= 16 + 6v^2\big|_{\bar{v}}\, \hat{v} =$

$= 16 + 6\bar{v}^2 \hat{v}$

$$\boxed{i = 16 + 24\hat{v} \approx 2v^3}$$

④ PREOBLIKOVANJE ([illegible] model)

$\frac{1}{2}\dot{v} + v + 2v^3 = 18 + A\cos\omega t$

– [illegible] in [illegible]:

$\frac{1}{2}(\dot{\bar{v}} + \dot{\hat{v}}) + (\bar{v} + \hat{v}) + (16 + 24\hat{v}) = 18 + A\cos\omega t$

mora se odštet sicer je narobe $\bar{v}$ = konst.

$$\boxed{\frac{1}{2}\dot{\hat{v}} + 25\hat{v} = A\cos\omega t}$$ LINEAREN MODEL

$v(0) = 3$ → začetni pogoj

$\hat{v}(0) = v(0) - \bar{v}$; $v = \bar{v} + \hat{v}$

$\underline{\hat{v}(0) = 1}$

PRIMER: $\dot{x}_1 = x_2$

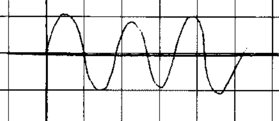
$\dot{x}_2 = -|x_1|\cdot x_1 - 2x_1 - 2x_2^3 - 3 + \overbrace{A\cdot\sin t}^{u(t)}$

nelinearni členi

D.T.

$0 = \bar{x}_2$

$0 = -|\bar{x}_1|\cdot \bar{x}_1 - 2\bar{x}_1 - \cancel{2\bar{x}_2^3} - 3 + 0$



$|\bar{x}_1|\cdot \bar{x}_1 + 2\bar{x}_1 + 3 = 0$

$\bar{x}_1 \ge 0 \qquad \bar{x}_1^2 + 2\cdot\bar{x}_1 + 3 = 0 \Rightarrow$ Im

$\bar{x}_1 < 0 \qquad -\bar{x}_1^2 + 2\bar{x}_1 + 3 = 0$

$\bar{x}_1^2 - 2\bar{x}_1 - 3 = 0$

PAZI POGOJE!

$(\bar{x}_1 + 1)(\bar{x}_1 - 3) = 0$

$\boxed{\bar{x}_1 = -1}$ ✓

$\bar{x}_2 = 3 > 0$ ✗

$$\boxed{DT \ (\bar{x}_1, \bar{x}_2) = (-1, 0)}$$

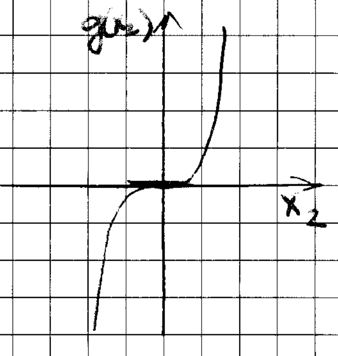
$$f(x_1) = |x_1|\,x_1 \approx f(\bar{x}_1) + \frac{df(x_1)}{dx_1}\Big|_{\bar{x}_1}(x_1 - \bar{x}_1) =$$

$$= |\bar{x}_1|\cdot\bar{x}_1 + 2|\bar{x}_1|\cdot\hat{x}_1 = -1 + 2\hat{x}_1$$

$$\frac{d(|x_1|x_1)}{dx_1} = \frac{x_1}{|x_1|}\cdot x_1 + |x_1|\cdot 1 = \frac{x_1^2}{|x_1|} + \frac{|x_1||x_1|}{|x_1|} = \frac{2x_1^2}{|x_1|} = 2|x_1|$$

$$\boxed{\frac{d|x_1|}{x_1} = \frac{x_1}{|x_1|}}$$

$$g(x_2) = x_2^3 \approx g(\bar{x}_2) + \frac{dg(x_2)}{dx_2}\Big|_{\bar{x}_2}(x_2 - \bar{x}_2) = \bar{x}_2^3 + 3\bar{x}_2^2\cdot\hat{x}_2 = 0$$

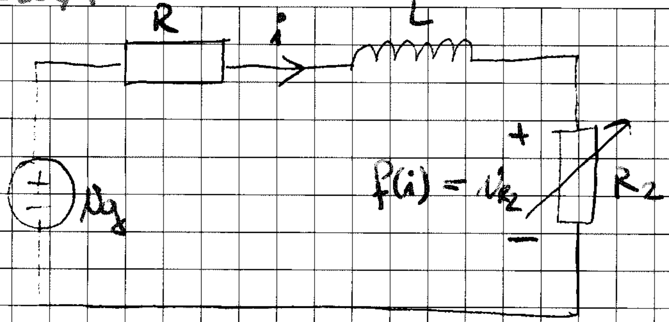


$$\dot{\hat{x}}_1 = \bar{x}_2 + \hat{x}_2 = \hat{x}_2$$

$$\dot{\hat{x}}_2 = 1 - 2\hat{x}_1 - 2(\overset{-1}{\bar{x}_1} + \hat{x}_1) - 2\cdot 0 - 3 + A\cdot\sin t$$

$$\boxed{\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 \\ \dot{\hat{x}}_2 &= -4\hat{x}_1 + A\sin t \end{aligned}}$$

IZPITNA NALOGA



a) Modelirajte sistem z DE

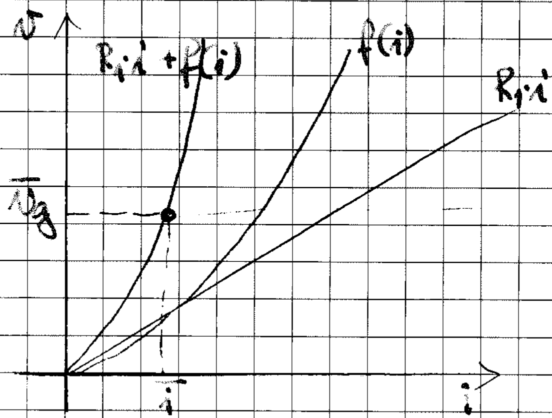
$v_{R_1} + v_L + v_{R_2} = v_g$

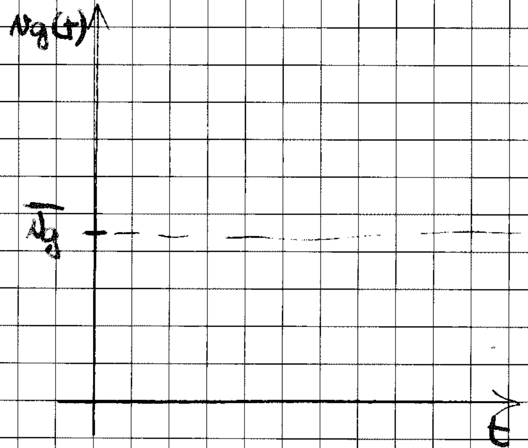
$\boxed{R_1 i + L\dot{i} + f(i) = v_g}$ NELINEAREN MODEL

b) D.T.; $v_g(t) = \bar{v}_g = konst.$

$R_1 \bar{i} + L\cdot 0 + f(\bar{i}) = \bar{v}_g$

$\boxed{R_1 \bar{i} + f(\bar{i}) = \bar{v}_g}$





c) linearizirajte model okoli delovne točke

⇒ poiščemo [illegible] → TAYLOR

$f(i) = f(\bar{i}) + \left.\frac{df(i)}{di}\right|_{\bar{i}} (i - \bar{i})$

neka konstanta K

$\boxed{f(i) \approx f(\bar{i}) + K\hat{i}}$

$R_1(\bar{i} + \hat{i}) + L\dot{\hat{i}} + f(\bar{i}) + K\cdot\hat{i} = v_g = \bar{v}_g + \hat{v}_g$

$$\boxed{R_1 \cdot i + L \cdot \dot{\hat{i}} + k \cdot \hat{i} = \hat{U}_g}$$ LINEAREN MODEL

d) zapišite dani model v matrični obliki

$$\dot{x} = Ax + Bu$$

$$\dot{\hat{i}} = -\frac{R_1 + k}{L} \cdot \hat{i} + \frac{1}{L} \cdot \hat{U}_g \qquad x = \hat{i} \quad u = \hat{U}_g$$

$$\dot{x} = \left[-\frac{R_1 + k}{L}\right] x + \left[\frac{1}{L}\right] u$$

ZVEZNI SISTEMI: čas zvezen
DISKRETNI SISTEMI: diskretni časovni trenutki (vrednosti)

→ Rešujemo eno diferencialno enačbo n-tega reda za odziv:

PRIMER:

$$\frac{d^3 y(t)}{dt^3} + \frac{3}{2} \frac{d^2 y(t)}{dt^2} + 2 \frac{dy(t)}{dt} + y(t) = u(t)$$

$u(t)$... prisilno vzbujanje
$y(t)$... odziv sistema

$$\dddot{y} + \frac{3}{2}\ddot{y} + 2\dot{y} + y = u \Rightarrow \left[\text{ena DE reda 3}\right]$$

$$\left.\begin{array}{l} y\text{... odziv} \\ u\text{... vzbujanje} \end{array}\right\} \text{sistema}$$

→ uvajamo nove spremenljivke stanj za zmanjšanje reda (več enačb nižjega reda)

$$\left.\begin{array}{l}
x_1 = y \Rightarrow \dot{x}_1 = \dot{y} = x_2 \\
x_2 = \dot{y} \Rightarrow \dot{x}_2 = \ddot{y} = x_3 \\
x_3 = \ddot{y} \Rightarrow \dot{x}_3 = \dddot{y} = -\frac{3}{2}\ddot{y} - 2\dot{y} - y + u \\
\qquad\qquad\qquad = -\frac{3}{2}x_3 - 2x_2 - x_1 + u
\end{array}\right\} \text{iščemo } \boxed{\dot{x} = Ax + Bu}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -\frac{3}{2} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \quad \left[\text{tri DE reda 1}\right] !!!$$

→ upoštevat moramo še drugo enačbo: odvisnost izhoda od vzbujanja in spremenljivk stanj

$$y = Cx + Du$$

$$y = [1\ 0\ 0]\,x \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ x: spremenljivke stanj (podajajo notranjo ureditev stanja sistema → ni nujno da nas vse te zanimajo).

$$\boxed{x(t) = \underbrace{e^{At}x(0)}_{x_Z(t)} + \underbrace{\int_0^t e^{A(t-\tau)} B\cdot u(\tau)\,d\tau}_{x_V(t)}}$$

$x_Z(t)$ – odziv na začetne vrednosti
$x_V(t)$ – odziv na vzbujanje

→ to je vektorska oblika rešitve: matrična DE → soodvisnost spremenljivk

$t=0$ ... začetek opazovanja

$e^{At} = \Phi(t)$ → matrika prehajanja stanj (kako x-i med sabo prehajajo)

B ... matrika vzbujanja

POSTOPKI:

① določanje e^{At} z Laplaceovo transformacijo

$$\dot{x} = Ax + Bu \;/\; \mathcal{L}$$

$$s\cdot X(s) - x(0) = A\,X(s) + B\cdot U(s)$$

s je skalar!

$$(sI - A)X(s) = x(0) + B\cdot U(s) \quad \Rightarrow \text{množimo z leve}$$

$$I = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix}$$

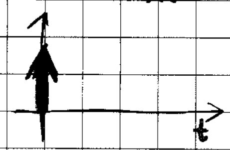
$$\boxed{X(s) = (sI-A)^{-1}\cdot x(0) + (sI-A)^{-1}\cdot B\cdot U(s)}$$

$$\boxed{\mathcal{L}\{e^{At}\} = (sI-A)^{-1}}$$

Vaja

$$\dot{x} = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} \delta(t)$$

$\delta(t) \rightarrow$ enotin impulz (širina 0, neskončna amplituda)



- Koliko je red danega sistema? 2 (dve enačbi za 1. red)

$\boxed{x(0) = 0}$ – začetno stanje; če ni podano pa predpostavljamo!

$x(t) = ?$

$$(sI-A)^{-1} = \left(s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} \right)^{-1} = \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} \right)^{-1} =$$

$$= \begin{bmatrix} s+2 & 2 \\ 1 & s+3 \end{bmatrix}^{-1} = \frac{1}{(s+2)(s+3) - 1\cdot 2} \begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+4} \begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix} = \frac{1}{(s+1)(s+4)} \begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix}$$

- INVERZNA MATRIKA (2x2): diagonalna člena zamenjata položaj $\rightarrow$ izvendiagonalna zamenjata predznak

$$X = (sI-A)^{-1} x(0) + (sI-A)^{-1}\cdot B\cdot U \; ; \quad x(0) = 0 \; ; \quad \mathcal{L}\{\delta(t)\} = 1$$

$$X = (sI-A)^{-1}\cdot B\cdot U$$

$$X = \frac{1}{(s+1)(s+4)} \begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot 1$$

$$X = \frac{1}{(s+1)(s+4)} \cdot \begin{bmatrix} 2s+4 \\ s \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$\mathcal{L}^{-1}\{X\} = x$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$X = \begin{bmatrix} \frac{2s+4}{(s+1)(s+4)} \\ \frac{s}{(s+1)(s+4)} \end{bmatrix} = \begin{bmatrix} \frac{\frac{2}{3}}{s+1} + \frac{\frac{4}{3}}{s+4} \\ \frac{-\frac{1}{3}}{s+1} + \frac{\frac{4}{3}}{s+4} \end{bmatrix}$$

→ Velja za različne korene in ko je stopnja imenovalca večja od stopnje števca ⇒ PARCIALNI ULOMKI

$$x(t) = \mathcal{L}^{-1}\{X\} = \begin{bmatrix} \frac{2}{3}e^{-t} + \frac{4}{3}e^{-4t} \\ -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2e^{-t} + 4e^{-4t} \\ -e^{-t} + 4e^{-4t} \end{bmatrix}$$

→ Rešitev velja samo za čas, ki nam ga je dovoljeno opazovat $\boxed{t \geq 0}$

VAJA

$x_1(t) = i_{L1}(t)$

$x_2(t) = i_{L2}(t)$

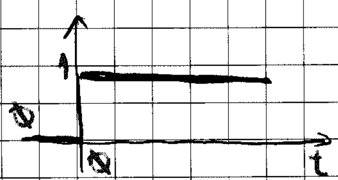
$$A = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix}$$

$$\boxed{x_2(t) = -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t}}$$

PRIMER 3

$$\dot{x} = \begin{bmatrix} -1 & -1 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 4 \\ 0 \end{bmatrix} u(t)$$

$u(t)$ → enotina stopnica



$x(0) = 0$

$x(t) = ?$

$$X = (sI - A)^{-1} \cdot B \cdot U \qquad (x(0) = 0)$$

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & 1 \\ -1 & s+2 \end{bmatrix}^{-1} = \frac{1}{s^2+3s+3}\begin{bmatrix} s+2 & -1 \\ 1 & s+1 \end{bmatrix}$$

$$\mathcal{X} = \frac{1}{s^2+3s+3}\begin{bmatrix} s+2 & -1 \\ 1 & s+1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \end{bmatrix} \cdot \frac{1}{s} \longleftarrow \mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$= \frac{1}{s(s^2+3s+3)}\begin{bmatrix} 4s+8 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{\frac{8}{3}}{s} + \frac{-4\left(\frac{2}{3}s+1\right)}{s^2+3s+3} \\ \frac{\frac{4}{3}}{s} + \frac{-4\left(\frac{1}{3}s+1\right)}{s^2+3s+3} \end{bmatrix}$$

$$\boxed{\begin{aligned} \mathcal{L}\{e^{bt}\sin at\} &= \frac{a}{(s-b)^2+a^2} \\ \mathcal{L}\{e^{bt}\cos at\} &= \frac{s-b}{(s-b)^2+a^2} \end{aligned}}$$

$$\mathcal{X} = \begin{bmatrix} \frac{\frac{8}{3}}{s} + \frac{-4\left(\frac{2}{3}\left(s+\frac{3}{2}\right)\right)}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} \\ \frac{\frac{4}{3}}{s} + \frac{-4\left(\frac{1}{3}\left(s+\frac{3}{2}\right)+\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}}\right)}{\left(s+\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} \end{bmatrix}$$

$$x = \mathcal{L}^{-1}\{\mathcal{X}\} = \begin{bmatrix} \frac{8}{3} - \frac{8}{3}e^{-\frac{3}{2}t}\cdot\cos\frac{\sqrt{3}}{2}t \\ \frac{4}{3} - \frac{4}{3}e^{-\frac{3}{2}t}\cdot\cos\frac{\sqrt{3}}{2}t - \frac{4}{\sqrt{3}}e^{-\frac{3}{2}t}\cdot\sin\frac{\sqrt{3}}{2}t \end{bmatrix}; \quad t \geq 0$$

enotina stopnica: $u(t)$ ali 1; $t > 0$

VAJA

$$\ddot{y}_1 + 2\dot{y}_1 = 2u_1 + u_2 + 2u_3$$

$$\ddot{y}_2 + \dot{y}_2 = u_1 + u_3 + u_2$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad \boxed{n=4}$$

→ red sistema je 4

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$x_1 = y_1 \rightarrow \dot{x}_1 = \dot{y}_1 = x_2$$

$$x_2 = \dot{y}_1 \Rightarrow \dot{x}_2 = \ddot{y}_1 = -2\dot{y}_1 + 2u_1 + u_2 + 2u_3 = -2x_2 + 2u_1 + u_2 + 2u_3$$

$$x_3 = y_2 \Rightarrow \dot{x}_3 = \dot{y}_2 = x_4$$

$$x_4 = \dot{y}_2 \Rightarrow \dot{x}_4 = \ddot{y}_2 = -\dot{y}_2 + u_1 + u_2 + u_3 = -x_4 + u_1 + u_2 + u_3$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} u \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x + \cancel{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} u$$

identificiramo kateri y spada k kateremu x-u

DOLOČANJE $\Phi(t)$ S POMOČJO CAYLEY – HAMILTONOVEGA TEOREMA

$\Phi(t) = e^{At}$ – matrika prehajanja stanj

- temelji na lastnih vrednostih matrike A

$\boxed{g(\lambda) = |A - \lambda I|}$ → determinanta ... ZNAČILNA ENAČBA (karakteristični polinom)

$\boxed{g(\lambda) = 0}$ → λ – lastne vrednosti matrike A (toliko kot je dimenzija kvadratne matrike)

$$f(\lambda) = \sum_{k=0}^{n-1} \alpha_k \lambda^k \Rightarrow \alpha_k \Rightarrow f(A) = \sum_{k=0}^{n-1} \alpha_k A^k$$

funkcijsko odvisnost zapišemo kot končno vsoto (n – red sistema)

- če velja funkcijska odvisnost za lastne vrednosti, velja ista funkcijska odvisnost za matriko, ki pripada lastnim vrednostim.

$$\boxed{e^{\lambda t} = \sum_{k=0}^{n-1} \alpha_k \lambda^k \Rightarrow \alpha_k \Rightarrow e^{At} = \sum_{k=0}^{n-1} \alpha_k A^k}$$

⇒ $A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$; $e^{At} = \Phi(t) = ?$

$$g(\lambda) = |A - \lambda I| = \left| \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = \begin{vmatrix} -2-\lambda & 0 \\ 1 & -1-\lambda \end{vmatrix} = (-2-\lambda)(-1-\lambda)$$

$$g(\lambda) = 0 \Rightarrow \boxed{\begin{matrix} \lambda_1 = -2 \\ \lambda_2 = -1 \end{matrix}}$$

$$\boxed{e^{\lambda t} = \sum_{k=0}^{n-1} \alpha_k \lambda^k \overset{n=2}{=} \alpha_0 \lambda^0 + \alpha_1 \cdot \lambda^1 = \alpha_0 + \alpha_1 \cdot \lambda}$$

$$e^{\lambda_1 t} = \alpha_0 + \alpha_1 \lambda_1$$
$$e^{\lambda_2 t} = \alpha_0 + \alpha_1 \lambda_2$$

$$\left.\begin{matrix} e^{-2t} = \alpha_0 - 2\alpha_1 \\ e^{-t} = \alpha_0 - \alpha_1 \end{matrix}\right\} \Rightarrow \begin{matrix} \alpha_1 = e^{-t} - e^{-2t} \\ \alpha_0 = 2e^{-t} - e^{-2t} \end{matrix}$$

vedno najprej α_1 računamo!!!
POZOR PRI VSTAVLJANJU!

$$e^{At} = \sum_{k=0}^{n-1} \alpha_k A^k \overset{n=2}{=} \alpha_0 I + \alpha_1 A$$

$$e^{At} = (2e^{-t} - e^{-2t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^{-t} - e^{-2t}) \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$\boxed{e^{At} = \begin{bmatrix} e^{-2t} & 0 \\ e^{-t} - e^{-2t} & e^{-t} \end{bmatrix}}$$

KONČNA REŠITEV

⇒ $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$; $e^{At} = ?$

$$g(\lambda)=\begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^3$$

$g(\lambda)=0 \Rightarrow \boxed{\lambda_{1,2,3}=2}$ ena 3-kratna lastna vrednost

$$e^{\lambda t}=\sum_{k=0}^{n-1}\alpha_k\lambda^k=\alpha_0+\alpha_1\lambda+\alpha_2\lambda^2 \Rightarrow e^{2t}=\alpha_0+2\alpha_1+4\alpha_2$$

odvajamo po λ!!!

z odvajanjem naredimo stvari linearno neodvisne

$$te^{\lambda t}=\alpha_1+2\alpha_2\lambda \Rightarrow te^{2t}=\alpha_1+4\alpha_2$$

$$t^2e^{\lambda t}=2\alpha_2 \Rightarrow t^2e^{2t}=2\alpha_2$$

$$\boxed{\alpha_2=\frac{t^2e^{2t}}{2}} \quad \boxed{\alpha_1=te^{2t}-2t^2e^{2t}} \quad \boxed{\alpha_0=e^{2t}-2te^{2t}+2t^2e^{2t}}$$

$$e^{At}=\sum_{k=0}^{n-1}\alpha_k A^k \overset{n=3}{=} \alpha_0 I+\alpha_1 A+\alpha_2 A^2$$

$$e^{At}=(e^{2t}-2te^{2t}+2t^2e^{2t})\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+(te^{2t}-2t^2e^{2t})\begin{bmatrix}2&1&0\\0&2&1\\0&0&2\end{bmatrix}+\frac{1}{2}t^2e^{2t}\begin{bmatrix}4&4&1\\0&4&4\\0&0&4\end{bmatrix}$$

$$A^2=\begin{bmatrix}2&1&0\\0&2&1\\0&0&2\end{bmatrix}\begin{bmatrix}2&1&0\\0&2&1\\0&0&2\end{bmatrix}=\begin{bmatrix}4&4&1\\0&4&4\\0&0&4\end{bmatrix}$$

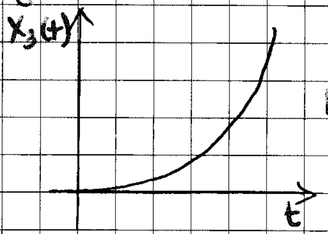
$$e^{At}=\begin{bmatrix}e^{2t} & te^{2t} & \frac{1}{2}t^2e^{2t}\\ 0 & e^{2t} & te^{2t}\\ 0 & 0 & e^{2t}\end{bmatrix}$$

⟹ Ali bi sistem lahko obstajal v resničnem svetu?

$$X(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad u = 0$$

$$X(t) = e^{At} \cdot X(0) + \int_0^t e^{At} \cdot B \cdot u(\tau)\, d\tau \overset{u=0}{=} e^{At} \cdot X(0) = \begin{bmatrix} e^{2t} + te^{2t} + \frac{1}{2}t^2 e^{2t} \\ e^{2t} + te^{2t} \\ e^{2t} \end{bmatrix}$$

$$X_3(t) = e^{2t}$$



Ne obstaja!!!

$$\Longrightarrow \quad \dot{X}(t) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

a) Določite $\Phi(t)$ (C.-H.) [10%]

$$g(\lambda) = \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (2-\lambda)^2$$

$$g(\lambda) = 0 \Rightarrow \boxed{\lambda_{1,2} = 2}$$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda \qquad \boxed{\alpha_0 = e^{2t} - 2te^{2t}}$$

$$te^{\lambda t} = \alpha_1 \Rightarrow \boxed{\alpha_1 = te^{2t}}$$

$$e^{At} = \alpha_0 I + \alpha_1 A = (e^{2t} - 2te^{2t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + te^{2t} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\boxed{e^{At} = \begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t} \end{bmatrix}}$$

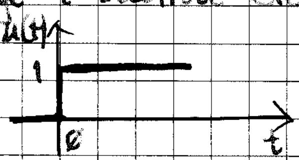
Ker je A diagonalna je tudi matrika e^{At} diagonalna

b) Določite splošno rešitev sistema na vzbujanje z enotino stopnico [10%]

$u = u(t)$ .... enotina stopnica

$X(0) = [1 \; -1]^T$

$\underline{X(t)} =$



$$X(t) = e^{At} \cdot X(0) + \int_0^t e^{A(t-\tau)} B\, u(\tau)\, d\tau$$

$$x(t)=\begin{bmatrix} e^{2t} & 0 \\ 0 & e^{2t}\end{bmatrix}\cdot\begin{bmatrix}2\\-2\end{bmatrix}+\int_0^t\begin{bmatrix} e^{2(t-\tau)} & 0 \\ 0 & e^{2(t-\tau)}\end{bmatrix}\cdot\begin{bmatrix}1\\1\end{bmatrix}1\cdot d\tau$$

$$x(t)=\begin{bmatrix}2e^{2t}\\-2e^{2t}\end{bmatrix}+\int_0^t\begin{bmatrix}e^{2(t-\tau)}\\e^{2(t-\tau)}\end{bmatrix}d\tau=\begin{bmatrix}2e^{2t}\\-2e^{2t}\end{bmatrix}+\begin{bmatrix}-\frac{1}{2}e^{2(t-\tau)}\\-\frac{1}{2}e^{2(t-\tau)}\end{bmatrix}\Bigg|_0^t$$

$$e^{2(t-\tau)}=e^{2t}\cdot e^{-2\tau}\Rightarrow \underline{\underline{-\tfrac{1}{2}e^{2(t-\tau)}}}\ \text{integral}$$

$$\begin{bmatrix}2e^{2t}\\-2e^{2t}\end{bmatrix}+\begin{bmatrix}-\frac{1}{2}+\frac{1}{2}e^{2t}\\-\frac{1}{2}+\frac{1}{2}e^{2t}\end{bmatrix}$$

$$\boxed{x(t)=\begin{bmatrix}-\frac{1}{2}+\frac{5}{2}e^{2t}\\-\frac{1}{2}-\frac{5}{2}e^{2t}\end{bmatrix}}\ ;\ \boxed{t>0}$$

→ začnemo opazovat pri začetnih pogojih; pozitivnem času

c) Kdaj doseže $x_1(t)$ vrednost nič? [5%]

$$x_1(t)=-\frac{1}{2}+\frac{5}{2}e^{2t}$$

$$0=-\frac{1}{2}+\frac{5}{2}e^{2t}$$

$$\frac{1}{5}=e^{2t}\ /\ln\ \Rightarrow\ \boxed{\frac{1}{2}\cdot\ln\frac{1}{5}=t_0}=-\frac{1}{2}\ln 5<0$$

$x_1(t)$ nič v <u>času opazovanja</u> NE DOSEŽE!!!

$$\Rightarrow\quad A=\begin{bmatrix}3/2 & 0 & 1\\-1/2 & 1/2 & -1/2\\-1/2 & 0 & 0\end{bmatrix};\ e^{At}=?\ \text{([illegible])}$$

$$\Delta(\lambda)=\begin{vmatrix}3/2-\lambda & 0 & 1\\-1/2 & \frac{1}{2}-\lambda & -1/2\\-1/2 & 0 & -\lambda\end{vmatrix}=\left(\frac{1}{2}-\lambda\right)\left(\lambda^2-\frac{3}{2}\lambda+\frac{1}{2}\right)=\left(\frac{1}{2}-\lambda\right)(\lambda-1)\left(\lambda-\frac{1}{2}\right)$$

• imaginarnih vrednosti ne [illegible] !

$$g(\lambda) = 0 \Rightarrow \boxed{\begin{aligned} \lambda_{1,2} &= \tfrac{1}{2} \\ \lambda_3 &= 1 \end{aligned}}$$

$$e^{\lambda t} = \alpha_0 + \alpha_1\lambda + \alpha_2\lambda^2 \Rightarrow \left.\begin{aligned} e^{t} &= \alpha_0 + \alpha_1 + \alpha_2 \\ e^{\frac{1}{2}t} &= \alpha_0 + \tfrac{1}{2}\alpha_1 + \tfrac{1}{4}\alpha_2 \end{aligned}\right\} -$$

$$t e^{\lambda t} = \alpha_1 + 2\alpha_2\lambda \Rightarrow t e^{\frac{1}{2}t} = \alpha_1 + \alpha_2 \quad / \cdot \tfrac{1}{2}$$

$$\left.\begin{aligned} e^{t} - e^{t/2} &= \tfrac{1}{2}\alpha_1 + \tfrac{3}{4}\alpha_2 \\ \tfrac{1}{2} t e^{\frac{1}{2}t} &= \tfrac{1}{2}\alpha_1 + \tfrac{1}{2}\alpha_2 \end{aligned}\right\} -$$

$$\boxed{\alpha_2 = 4e^{t} - 4e^{t/2} - 2t\cdot e^{t/2}}$$

$$\alpha_1 = 2e^{t} - 2e^{t/2} - \tfrac{6}{4}\alpha_2$$

$$\boxed{\alpha_1 = -4e^{t} + 4e^{t/2} + 3te^{t/2}}$$

$$\alpha_0 = e^{t} - \alpha_1 - \alpha_2$$

$$\boxed{\alpha_0 = e^{t} - te^{t/2}}$$

$$e^{At} = \alpha_0 I + \alpha_1 A + \alpha_2 A^2 =$$

$$e^{At} = (e^{t} - te^{t/2})\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + (-4e^{t} + 4e^{t/2} + 3te^{t/2})\begin{bmatrix} 3/2 & 0 & 1 \\ -1/2 & 1/2 & -1/2 \\ -1/2 & 0 & 0 \end{bmatrix} +$$

$$+ (4e^{t} - 4e^{t/2} - 2te^{t/2})\begin{bmatrix} 7/4 & 0 & 3/2 \\ -3/4 & 1/4 & -3/4 \\ -3/4 & 0 & -1/2 \end{bmatrix}$$

$$e^{At} = \begin{bmatrix} 2e^{t} - e^{t/2} & \emptyset & 2e^{t} - 2e^{t/2} \\ -e^{t} + e^{t/2} & e^{t/2} & -e^{t} + e^{t/2} \\ -e^{t} + e^{t/2} & \emptyset & -e^{t} + 2e^{t/2} \end{bmatrix}$$ DN

DIAGONALIZACIJA MATRIKE A

$\dot{x} = \underline{Ax} + Bu \qquad \Phi(t) = e^{At}$

- DIAGONALNA MATRIKA: izven diagonale so ničle
- preslikamo matriko v drug prostor, da bo tam ta diagonalna
  $\Rightarrow$ linearna preslikava imenovana PODOBNOSTNA TRANSFORMACIJA

$A = \Theta \cdot \Lambda \cdot \Theta^{-1}$

$\underline{\Lambda = \Theta^{-1} \cdot A \cdot \Theta}$

$\Lambda = \begin{bmatrix} \lambda_1 & & \emptyset \\ & \lambda_2 & \\ & & \ddots \\ \emptyset & & & \lambda_n \end{bmatrix} \rightarrow$ lastne vrednosti matrike A

$\boxed{e^{At} = \Theta \cdot e^{\Lambda t} \cdot \Theta^{-1}}$

$\Rightarrow$ večkratne lastne vrednosti: matrika ni več diagonalna

$\boxed{\lambda_1 \neq \lambda_2 \neq \ldots \neq \lambda_n}$

$\Theta = [\, z_1 \,\vdots\, z_2 \,\vdots\, \ldots \,\vdots\, z_n \,] \rightarrow$ vsak stolpec je lastni vektor

$\lambda_1 \leftrightarrow z_1$

$\vdots$

$\lambda_n \leftrightarrow z_n$

$\rightarrow$ ohranjamo vrstni red lastnih vrednosti in lastnih vektorjev

$$e^{At} = \begin{bmatrix} e^{\lambda_1 t} & & & \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ & & & e^{\lambda_n t} \end{bmatrix}$$ $\rightarrow$ če ne bi bila diagonalna, tega ne bi smeli narediti

$\left.\begin{matrix}\Lambda \\ C\end{matrix}\right\} \rightarrow$ jordanova kanonična forma

PRIMER $A=\begin{bmatrix}-5 & 2\\ -1 & -2\end{bmatrix}$; $e^{At}=?$

značilni polinom: $g(\lambda)=|A-\lambda\cdot I|=\begin{vmatrix}-5-\lambda & 2\\ -1 & -2-\lambda\end{vmatrix}=\lambda^2+7\lambda+12=(\lambda+3)(\lambda+4)$

$g(\lambda)=0 \Rightarrow \underline{\lambda_1=-3}\quad \underline{\lambda_2=-4}$

funkcija dveh parametrov: $\boxed{f(\lambda,\mu)=\frac{g(\lambda)-g(\mu)}{\lambda-\mu}}=$ → značilni polinom

$$=\frac{\lambda^2+7\lambda+12-(\mu^2+7\mu+12)}{\lambda-\mu}=\frac{(\lambda-\mu)(\lambda+\mu)+7(\lambda-\mu)}{\lambda-\mu}=$$

$f(\lambda,\mu)=\underline{\lambda+\mu+7}$ || $f(\lambda,\mu)\leftrightarrow C(\lambda)$; $\lambda\to\lambda I$; $\mu\to A$

preslikava v matrični prostor

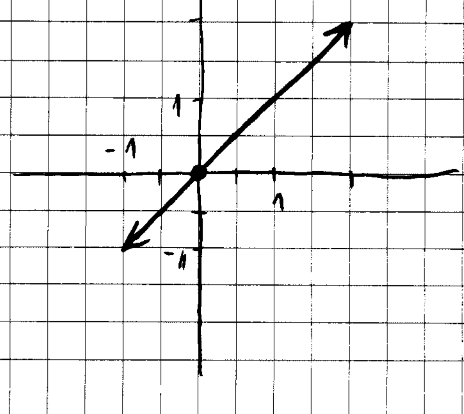
$C(\lambda)=\lambda\cdot I+A+7\cdot I$

$$C(\lambda)=\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}+\begin{bmatrix}-5 & 2\\ -1 & -2\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$

$$C(\lambda)=\begin{bmatrix}\lambda+2 & 2\\ -1 & \lambda+5\end{bmatrix}$$

vsak stolpec predstavlja lastni vektor, ki pripada tej lastni vrednosti

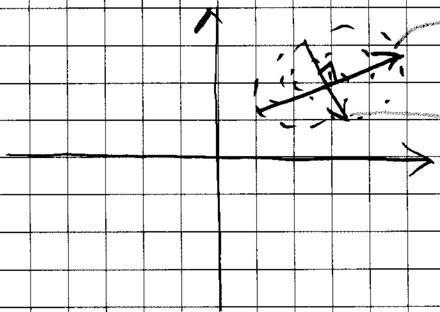
$C(\lambda_1=-3)=\begin{bmatrix}-1 & 2\\ -1 & 2\end{bmatrix}\Rightarrow z_1=\begin{bmatrix}1\\ 1\end{bmatrix}$ → do konstante natančno!!!



⇒ lastne vektorje upoštevamo do konstante natančno (potek v prostoru)

$$C(\lambda_2=-4)=\begin{bmatrix}-2 & 2\\ -1 & 1\end{bmatrix} \Rightarrow g_2=\begin{bmatrix}2\\1\end{bmatrix}$$

$\Longrightarrow$ ANALIZA OSNOVNIH KOMPONENT (PCA): kompresija, zgoščevanje podatkov



prva lastna vrednost: v smeri največjega zgoščevanja podatkov

pravokotna na prvo (druga lastna vrednost)

$$\Lambda=\begin{bmatrix}-3 & 0\\ 0 & -4\end{bmatrix} \Longrightarrow e^{\Lambda t}=\begin{bmatrix}e^{-3t} & 0\\ 0 & e^{-4t}\end{bmatrix}$$

$$\Theta=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$$

$$e^{At}=\Theta\cdot e^{\Lambda t}\cdot\Theta^{-1}=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}\begin{bmatrix}e^{-3t} & 0\\ 0 & e^{-4t}\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}^{-1}=$$

$$=\begin{bmatrix}e^{-3t} & 2e^{-4t}\\ e^{-3t} & e^{-4t}\end{bmatrix}\cdot\frac{1}{-1}\begin{bmatrix}1 & -2\\ -1 & 1\end{bmatrix}=\begin{bmatrix}-e^{-3t}+2e^{-4t} & 2e^{-3t}-2e^{-4t}\\ -e^{-3t}+e^{-4t} & 2e^{-3t}-e^{-4t}\end{bmatrix}$$

$\boxed{\text{DN}}$ $$A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix};\quad e^{At}=?$$

$\Longrightarrow$

$$\dot{x}=\begin{bmatrix}0 & 1\\ -\frac{1}{6} & \frac{5}{6}\end{bmatrix}x+\begin{bmatrix}0\\1\end{bmatrix}u=Ax+Bu$$

$$y=\begin{bmatrix}-1 & 2\end{bmatrix}x=Cx$$

a) Novo spremenljivko stanj w izberemo tako, da postane osnovna matrika sistema diagonalna, pri čemer jih dobimo z neko transformacijo z dosedanjimi spremenljivkami stanj

$$w = T \cdot x \; ; \; T = ?$$

$$\Downarrow$$

$$x = T^{-1} w = \Theta \cdot w$$

$$\dot{x} = Ax + Bu = \Theta \cdot \Lambda \cdot \Theta^{-1} x + B \cdot u \,\Big|\cdot \Theta^{-1}$$

$$\Theta^{-1}\dot{x} = \Lambda \cdot \Theta^{-1} x + \Theta^{-1} B \cdot u$$

$$\dot{w} = \underline{\underline{\Lambda}} \cdot w + \Theta^{-1} \cdot B \cdot u$$

$$w = \Theta^{-1} \cdot x$$

$$\boxed{T = \Theta^{-1}}$$

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -\lambda & 1 \\ -\frac{1}{6} & \frac{5}{6} - \lambda \end{vmatrix} = \lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = \left(\lambda - \frac{1}{2}\right)\left(\lambda - \frac{1}{3}\right)$$

$$g(\lambda) = 0 \Rightarrow \underline{\underline{\lambda_1 = \frac{1}{3}}} \;,\; \underline{\underline{\lambda_2 = \frac{1}{2}}}$$

$$\boxed{A q = \lambda \cdot q}$$

$$(A - \lambda I) \cdot q = 0$$

$$\Rightarrow A q_1 = \lambda_1 q_1$$

$$\begin{bmatrix} 0 & 1 \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} q_{11} \\ q_{21} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} q_{11} \\ q_{21} \end{bmatrix}$$

$$q_{21} = \frac{1}{3} q_{11}$$

$$-\frac{1}{6} q_{11} + \frac{5}{6} q_{21} = \frac{1}{3} q_{21}$$

$$-\frac{1}{6} q_{11} = -\frac{1}{2} q_{21}$$

$$\boxed{\frac{1}{3} q_{11} = q_{21}}$$

$$\Rightarrow q_{1} = \begin{bmatrix} 1 \\ \frac{1}{3} \end{bmatrix}$$

$$\Rightarrow A q_2 = \lambda_2 \cdot q_2$$

$$\begin{bmatrix} 0 & 1 \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix} \begin{bmatrix} q_{12} \\ q_{22} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} q_{12} \\ q_{22} \end{bmatrix}$$

$$q_{22} = \frac{1}{2} q_{12}$$

$$q_2 = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 1 & 1 \\ \frac{1}{3} & \frac{1}{2} \end{bmatrix} \Rightarrow T = \Theta^{-1} = \frac{1}{\frac{1}{6}} \begin{bmatrix} \frac{1}{2} & -1 \\ -\frac{1}{3} & 1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -2 & 6 \end{bmatrix}$$

→ rešitev je več: odvisno kako izberemo lastne vektorje

b) Zapišite matrično enačbo: $\dot{\omega} = A'\omega + B'u$

A' = diagonalna matrika
B' = kanonični

$$B' = \Theta^{-1} \cdot B = \begin{bmatrix} 3 & -6 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \end{bmatrix}$$

$$A' = \Lambda = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$\dot{\omega} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \omega + \begin{bmatrix} -6 \\ 6 \end{bmatrix} u$$

→ isti sistem, le spremenljivke stanj so druge

c) Zapišite $y = C'\omega + \cancel{D'u}$ → 0 ⇒ ni D-ja!

$$y = Cx = C\Theta \cdot \omega = \begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ \frac{1}{3} & \frac{1}{2} \end{bmatrix} \omega = \underline{\underline{\begin{bmatrix} -\frac{1}{3} & 0 \end{bmatrix} \omega = y}}$$

⇒ z diagonalizacijo

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}; \; e^{At} = ?$$

$$g(\lambda) = \begin{vmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2 = (\lambda+1)(\lambda+2)$$

$$g(\lambda) = 0 \Rightarrow \underline{\lambda_1 = -1}, \; \underline{\lambda_2 = -2}$$

$$f(\lambda, \mu) = \frac{g(\lambda) - g(\mu)}{\lambda - \mu} = \frac{\lambda^2 + 3\lambda + 2 - (\mu^2 + 3\mu + 2)}{\lambda - \mu} = \frac{(\lambda-\mu)(\lambda+\mu) + 3(\lambda-\mu)}{\lambda - \mu}$$

$$f(\lambda, \mu) = \lambda + \mu + 3 = \underline{\underline{\lambda + 3 + \mu}}$$

$$C(\lambda) = (\lambda+3)I + A = \begin{bmatrix} \lambda+3 & 1 \\ -2 & \lambda \end{bmatrix}$$

$$C(\lambda_1=-1) = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \Rightarrow q_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

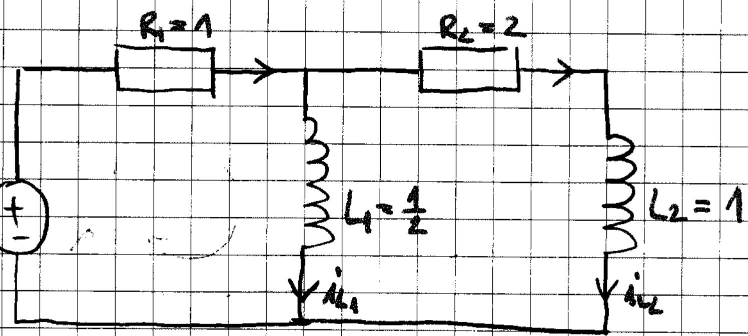
$$C(\lambda_2=-2) = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \Rightarrow q_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$$

$$e^{At} = \Theta \cdot e^{\lambda t} \Theta^{-1} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \cdot \frac{1}{-1} \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} e^{-t} & e^{-2t} \\ -e^{-t} & -2e^{-2t} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}$$

$\Rightarrow$



a) zapišite dano vezje v matrični obliki $\dot{x} = Ax + Bu$

$$x = [i_{L1} \; i_{L2}]^T \; ; \; u = v_g$$

$$v_{L1} + v_{R1} = v_g$$

$$v_{L2} + v_{R1} + v_{R2} = v_g$$

$$L_1 \dot{i}_{L1} + R_1(i_{L1} + i_{L2}) = v_g$$

$$L_2 \dot{i}_{L2} + R_1(i_{L1} + i_{L2}) + R_2 i_{L2} = v_g$$

$$\dot{x} = \begin{bmatrix} -\frac{R_1}{L_1} & -\frac{R_1}{L_1} \\ -\frac{R_1}{L_2} & -\frac{R_1+R_2}{L_2} \end{bmatrix} x + \begin{bmatrix} \frac{1}{L_1} \\ \frac{1}{L_2} \end{bmatrix} u$$

b) $\left.\begin{array}{l} x(0) = [0, 0]^T \\ i_1(t) \text{ in } i_2(t) = ? \Rightarrow x(t) = ? \end{array}\right\}$ s pomočjo Laplace-ove transformacije

$$\dot{x} = \begin{bmatrix} -2 & -2 \\ -1 & -3 \end{bmatrix} x + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$

$$X(s) = \cancelto{0}{(sI-A)^{-1}x(0)} + (sI-A)^{-1} \cdot B \cdot U(s)$$

$$X(s) = \begin{bmatrix} s+2 & 2 \\ 1 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \cdot 1 = \frac{1}{s^2+5s+4} \begin{bmatrix} s+3 & -2 \\ -1 & s+2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$$

$$= \frac{1}{(s+1)(s+4)} \cdot \begin{bmatrix} 2(s+2) \\ s \end{bmatrix}$$

$$X_1(s) = \frac{2(s+2)}{(s+1)(s+4)} = 2\left(\frac{\frac{1}{3}}{(s+1)} + \frac{\frac{2}{3}}{(s+4)}\right)$$

$$x_1(t) = \frac{2}{3}e^{-t} + \frac{4}{3}e^{-4t}$$

$$X_2(s) = \frac{s}{(s+1)(s+4)} = \frac{-\frac{1}{3}}{(s+1)} + \frac{\frac{4}{3}}{(s+4)}$$

$$x_2(t) = -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t}$$

$$x(t) = \begin{bmatrix} \frac{2}{3}\left(e^{-t} + 2e^{-4t}\right) \\ \frac{1}{3}\left(-e^{-t} + 4e^{-4t}\right) \end{bmatrix} ; \; t > 0$$ (za čase $t < 0$ Laplace-ova transformacija ni definirana)

c) določite RAVNOVESNA STANJA sistema (STACIONARNA) ... vrednosti, ki se ne spreminjajo več, se vzpostavijo ko prehodni pojav izzveni, $\bar{x}$ $\Rightarrow$ to so konstante ; $\bar{x} = x(t \to \infty)$)

$$\dot{x} = Ax + Bu$$

$$0 = A\dot{x} + Bu$$

$$\boxed{\bar{x} = -A^{-1} \cdot B \cdot u}$$

$$\bar{x} = \begin{bmatrix} \emptyset \\ \emptyset \end{bmatrix}$$

$\Longrightarrow$ Dan je linearen sistem:

$$\ddot{y}_1(t) + 2\dot{y}_1(t) - y_2(t) = 2u_1(t) + 2u_2(t)$$

$$\ddot{y}_2(t) + \dot{y}_2(t) - 4y_1(t) + 2y_2(t) = 2u_1(t) + u_2(t)$$

a) Uvedite nove spremenljivke stanj. Zapišite sistem v obliki $\dot{x} = Ax + Bu$, $u = [u_1, u_2]^T$

$$x_1 = y_1 \Longrightarrow \dot{x}_1 = \dot{y}_1 = x_2$$

$$x_2 = \dot{y}_1 \Longrightarrow \dot{x}_2 = \ddot{y}_1 = -2x_2 + x_3 + 2u_1 + 2u_2$$

$$x_3 = y_2 \Longrightarrow \dot{x}_3 = \dot{y}_2 = x_4$$

$$x_4 = \dot{y}_2 \Longrightarrow \dot{x}_4 = \ddot{y}_2 = -x_4 + 4x_1 - 2x_3 + 2u_1 + u_2$$

$$\dot{x} = \underbrace{\begin{bmatrix} \emptyset & 1 & \emptyset & \emptyset \\ \emptyset & -2 & 1 & \emptyset \\ \emptyset & \emptyset & \emptyset & 1 \\ 4 & \emptyset & -2 & -1 \end{bmatrix}}_{4\times 4} x + \begin{bmatrix} \emptyset & \emptyset \\ 2 & 2 \\ \emptyset & \emptyset \\ 2 & 1 \end{bmatrix} u$$

b) Določite enačbo $y = Cx + Du$; $y = [y_1, y_2]^T$, $D = \emptyset$

$$y = \begin{bmatrix} 1 & \emptyset & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 & \emptyset \end{bmatrix} x$$

c) Kakšen je red sistema? red sistema $\boxed{n=4}$; gledamo kot celoto in ne kot y_1 in y_2 posebej

$$e^{At} \Rightarrow x(t)$$

$\Longrightarrow$

$$A = \begin{bmatrix} \emptyset & 6 \\ -1 & 5 \end{bmatrix} ; \quad x(\emptyset) = \begin{bmatrix} \emptyset \\ 1 \end{bmatrix}$$

a) $\Phi(t) = e^{At} = ?$ (C.-H.)

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -\lambda & 6 \\ -1 & 5-\lambda \end{vmatrix} = \lambda^2 - 5\lambda + 6$$

$$g(\lambda) = \lambda^2 - 5\lambda + 6 = (\lambda - 2)(\lambda - 3) = 0$$

$$\boxed{\lambda_1 = +2} \quad \boxed{\lambda_2 = +3}$$

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda \iff \boxed{e^{At} = \alpha_0 I + \alpha_1 A} =$$

$$e^{2t} = \alpha_0 + 2\alpha_1$$
$$e^{3t} = \alpha_0 + 3\alpha_1$$

$$= (-2e^{3t} + 3e^{2t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + (e^{3t} - e^{2t}) \begin{bmatrix} 0 & 6 \\ -1 & 5 \end{bmatrix} =$$

$$\boxed{\begin{aligned} \alpha_1 &= e^{3t} - e^{2t} \\ \alpha_0 &= -2e^{3t} + 3e^{2t} \end{aligned}}$$

$$= \begin{bmatrix} -2e^{3t} + 3e^{2t} & 6e^{3t} - 6e^{2t} \\ -e^{3t} + e^{2t} & 3e^{3t} - 2e^{2t} \end{bmatrix} = \underline{\underline{e^{At}}}$$

b) sistem ni realen, ker gre odziv v neskončnost

STABILNOST: realni deli → lastne vrednosti za stabilnost ležijo na levi strani kompleksne ravnine

c) Določite matriko B tako, da bo sistem pri vzbujanju z $u(t) = \delta(t)$, da bo odziv sistema enak

$$y(t) = \begin{bmatrix} -6e^{2t} + 8e^{3t} \\ -2e^{2t} + 4e^{3t} \end{bmatrix} \qquad \boxed{y(t) = x(t)}$$

$$\dot{x} = Ax + Bu$$

→ enotin impulz ⇒ splača se delat z Laplace-om

$$X = Y = (sI - A)^{-1} x(0) + (sI - A)^{-1} \boxed{B}\, U(s) = (sI - A)^{-1} (x(0) + B \cdot U(s))$$

$$(sI - A) X = x(0) + B \cdot U$$

$$((sI - A) X - x(0)) U^{-1} = B$$

$$B = \left(\underset{2\times 2}{\begin{bmatrix} s & -6 \\ 1 & s-5 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} -\frac{6}{s-2} + \frac{8}{s-3} \\ -\frac{2}{s-2} + \frac{4}{s-3} \end{bmatrix}} - \underset{1\times 1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} \right) \cdot 1^{-1}$$

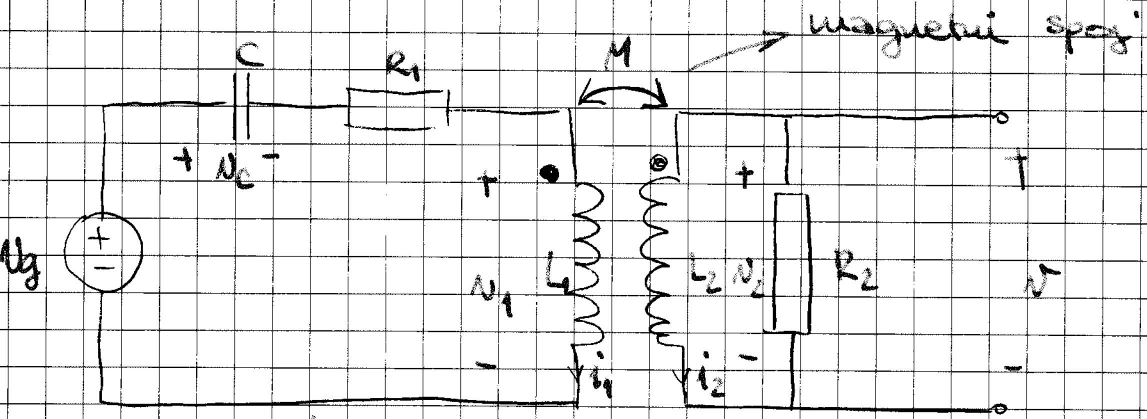
$$B = \begin{bmatrix} -\frac{6s}{s-2} + \frac{8s}{s-3} + \frac{12}{s-2} - \frac{24}{s-3} \\ -\frac{6}{s-2} + \frac{8}{s-3} - \frac{2s-10}{s-2} + \frac{4s-20}{s-3} \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{6\cancel{(s-2)}}{\cancel{s-2}} + \frac{8\cancel{(s-3)}}{\cancel{s-3}} \\ -\frac{2\cancel{(s-2)}}{\cancel{s-2}} + \frac{4\cancel{(s-3)}}{\cancel{s-3}} \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

- lahko računamo v časovnem, ker smo že imeli e^{At} →
<u>KONVOLUCIJSKI INTEGRAL</u>

$$\boxed{f(t) = \int_0^t f(t-\tau)\,\delta(\tau)\,d\tau}$$

⟹



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$X = \begin{bmatrix} v_C & i_1 & i_2 \end{bmatrix}^T$$

$$u = v_g$$

$(L_1 L_2 - M^2 > 0)$ → podatek determinanti

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} = \frac{1}{L_1 L_2 - M^2} \begin{bmatrix} L_2 & -M \\ -M & L_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\dot{i}_1 = \frac{L_2}{L_1 L_2 - M^2} v_1 - \frac{M}{L_1 L_2 - M^2} v_2$$

$$\dot{i}_2 = -\frac{M}{L_1 L_2 - M^2} v_1 + \frac{L_1}{L_1 L_2 - M^2} v_2$$

$i_C = i_1$

$\dot{v}_C = \frac{1}{C} i_1$

$v_2 = -R_2 i_2$

$v_1 + v_C + R_1 i_1 = v_g$

$v_1 = -v_C - R_1 i_1 + v_g$

$$\boxed{\dot{x} = Ax + Bu}$$

$$\dot{x} = \begin{bmatrix} 0 & \frac{1}{C} & 0 \\ -\frac{L_2}{L_1L_2-M^2} & -\frac{L_2R_1}{L_1L_2-M^2} & \frac{R_2M}{L_1L_2-M^2} \\ \frac{M}{L_1L_2-M^2} & \frac{R_1M}{L_1L_2-M^2} & -\frac{R_2L_1}{L_1L_2-M^2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{L_2}{L_1L_2-M^2} \\ -\frac{M}{L_1L_2-M^2} \end{bmatrix} u$$

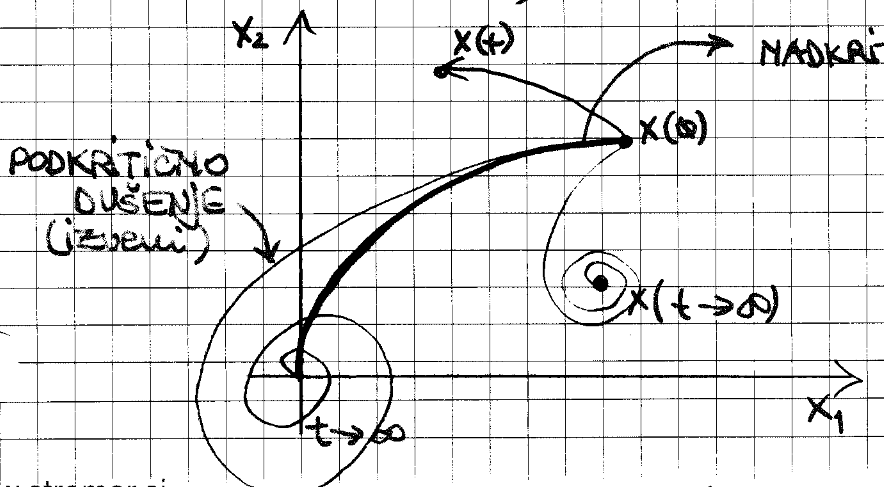
- Določite: $y = Cx + Du$, $y = v = v_{R_2}$
- Odziv $y(t)$ na enotino stopnico: $x(0) = [0 \;\; 0 \;\; 1]^T$

$R_1 = 4$, $R_2 = 2$, $C = \frac{1}{3}$, $L_1 = 1$, $L_2 = 1$, $M = 0$ → ni magnetnega sklopa (drugega dela veje ni ⇒ vse se dogaja v eni zanki)

LASTNOSTI SISTEMOV

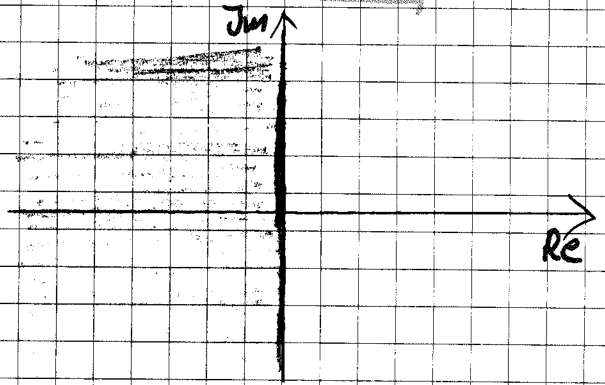
1 STABILNOST

Sistem je stabilen, če je njegov odziv pri poljubnem začetnem stanju izveni (gre proti 0) ali ostane omejen (gre proti konstantni vrednosti).



→ računska ugotovitev stabilnosti

$$\boxed{g(\lambda) = |A - \lambda I| \Rightarrow \lambda}$$



$\forall \, Re\{\lambda\} < 0 \Rightarrow$ stabilen sistem

→ če $\lambda = 0 \Rightarrow$ krozi (oscilira)
→ na meji stabilnosti

(2) VODLJIVOST (kontrolabilnost)

Sistem je vodljiv takrat, če ga je mogoče z izbranim vzbujanjem privesti iz poljubnega začetnega v poljubno končno stanje v končnem času.

n - red sistema

$$\boxed{M = \left[B \,\vdots\, AB \,\vdots\, A^2B \,\vdots\, \dots \,\vdots\, A^{n-1}B \right]_{n \times n}}$$

⇓

n - stolpcev

$\dot{x} = Ax + Bu$

→ M nesingularna ⇒ VODLJIV SISTEM: $\boxed{\det(M) \neq 0 \text{ in } \rho(M) = n}$
(rang: število neodvisnih stolpcev oz. vrstic)

(3) SPOZNAVNOST

Sistem je spoznaven, če je mogoče določiti njegovo stanje na osnovi končno dolgega opazovanja njegovega izhoda.

$$\boxed{N = \left[C^T \,\vdots\, A^T \cdot C^T \,\vdots\, (A^T)^2 C^T \,\vdots\, \dots \,\vdots\, (A^T)^{n-1} C^T \right]}$$

n - vrstic; stolpcev ni nujno n ; $y = Cx + Du$

→ N nesingularna ⇒ SPOZNAVEN SISTEM : $\boxed{\det(N) \neq 0 \text{ oz. } \rho(N) = n}$

PRIMER

$$\dot{x} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \quad ; \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Ali je sistem stabilen, vodljiv, spoznaven?

(1)

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -1-\lambda & 1 & 0 \\ 0 & -1-\lambda & 0 \\ 0 & 0 & -3-\lambda \end{vmatrix} = (-1-\lambda)^2(-3-\lambda)$$

$$g(\lambda) = 0 \Rightarrow \underline{\lambda_{1,2} = -1} \quad \underline{\lambda_3 = -3} \Rightarrow \text{STABILEN SISTEM !!!}$$

(2)

$$M = \begin{bmatrix} B & AB & \underset{\substack{\downarrow \\ A \cdot AB}}{A^2B} \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ 1 & -3 & 9 \end{bmatrix}, \quad \boxed{\det(M) \neq 0} \Rightarrow \text{sistem je vodljiv}$$

$$|M| = -1 \cdot 8 - 2 \cdot (-2) = \underline{-4}$$

(3)

$$N = \begin{bmatrix} C^T & \underset{\substack{\downarrow \\ (CA)^T}}{A^TC^T} & \underset{\substack{\downarrow \\ (C \cdot A \cdot A)^T}}{(A^T)^2C^T} \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{sistem ni spoznaven} \quad \boxed{\det(N) = 0}$$

PRIMER

$$\dot{x} = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u \quad ; \quad y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x + u$$

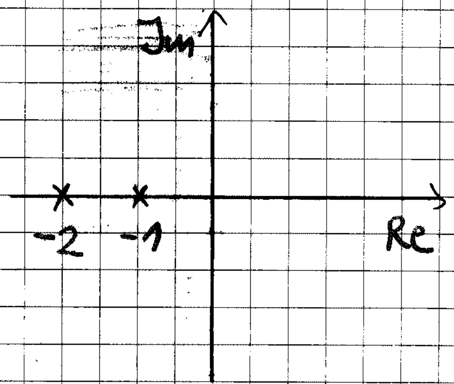
1

$$g(\lambda) = |A - \lambda I| = \begin{vmatrix} -3-\lambda & -1 & 0 \\ 2 & -\lambda & 0 \\ 0 & -1 & -1-\lambda \end{vmatrix} =$$

$$= (-1-\lambda)\left((-3-\lambda)(-\lambda) + 2\right) = (-1-\lambda)(\lambda+1)(\lambda+2) =$$

$$= -(\lambda+1)^2 (\lambda+2)$$

$$g(\lambda) = 0 \Rightarrow \boxed{\lambda_{1,2} = -1} \quad \boxed{\lambda_2 = 2}$$ STABILEN SISTEM!!



$$\underline{e^{-2t}, e^{-t}} \Rightarrow \underline{e^{\lambda t} \Rightarrow e^{At}}$$

2

$$M = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & -3 & 7 \\ 0 & 2 & -6 \\ 1 & -1 & -1 \end{bmatrix} \Longrightarrow$$ vodljiv sistem : $\det(M) \neq 0$

$$A \cdot B = \begin{bmatrix} -3 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}_{3\times 1} = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}_{3\times 1}$$

$$A^2 \cdot B = A \cdot A \cdot B = A \cdot (A \cdot B) = \begin{bmatrix} 7 \\ -6 \\ -1 \end{bmatrix}$$

$$|M| = 1(-2-6) + 1(18-14) = \underline{-4}$$

(3) $N = \begin{bmatrix} C^T & A^T C^T & (A^T)^2 \cdot C^T \end{bmatrix}$

$$N = \begin{bmatrix} 1 & -3 & 5 \\ \emptyset & -2 & 4 \\ 1 & -1 & 1 \end{bmatrix} \Big\} - \rho(N) = 2 \neq 3 = n$$

$\Rightarrow$ ni spoznaven sistem

$1(-2+4) + 1(-12+10) = \emptyset$

$$A^T C^T = (C \cdot A)^T =$$

$$C A = \begin{bmatrix} 1 & \emptyset & 1 \end{bmatrix}_{1\times3} \begin{bmatrix} -3 & -1 & \emptyset \\ 2 & \emptyset & \emptyset \\ \emptyset & -1 & 1 \end{bmatrix}_{3\times3} = \begin{bmatrix} -3 & -2 & -1 \end{bmatrix}$$

$$(A^T)^2 \cdot C^T = A^T \cdot A^T \cdot C^T = (C \cdot A \cdot A)^T = ((C \cdot A) \cdot A)^T$$

$$(C \cdot A) \cdot A = \begin{bmatrix} -3 & -2 & -1 \end{bmatrix} \begin{bmatrix} -3 & -1 & \emptyset \\ 2 & \emptyset & \emptyset \\ \emptyset & -1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 1 \end{bmatrix}$$

IZPITNA NALOGA Preprost model določanja položaja satelita

$$J \cdot \ddot{\Theta} = M$$

J.... vztrajnostni moment (konstanta)

M... vrtilni moment

Za določanje položaja sta odgovorna dva senzorja; prvi meri kot, drugi pa kotno hitrost.
Vrednost ε je napaka senzorja pri merjenju kotne hitrosti, ki je konstantna $\Rightarrow$ $\dot{\varepsilon} = \emptyset$

a) Zapišite model v obliki $\dot{x} = Ax + Bu$; $x = [\Theta \ \dot{\Theta} \ \varepsilon]^T$, $u = M$

$y_1 = \Theta$

$y_2 = \dot{\Theta} + \varepsilon$

$$x = \begin{bmatrix} \Theta \\ \dot{\Theta} \\ \varepsilon \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$x_1 = \Theta \Rightarrow \dot{x}_1 = \dot{\Theta} = x_2$$

$$x_2 = \dot{\Theta} \Rightarrow \dot{x}_2 = \ddot{\Theta} = \frac{M}{J} = \frac{1}{J}\cdot u$$

$$x_3 = \varepsilon \Rightarrow \dot{x}_3 = 0$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{J} \\ 0 \end{bmatrix} u$$

b) Zapišite odziv sistema: $y = Cx + Du$, $y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} x$$

c) Ali je sistem vodljiv oz. spoznaven?

$$M = \begin{bmatrix} B & AB & A^2B \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & \frac{1}{J} & 0 \\ \frac{1}{J} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$$ sistem ni vodljiv → vidimo po zadnjem stolpcu

$$N = \begin{bmatrix} C^T & \vdots & A^T C^T & \vdots & (A^T)^2 C^T \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & 0 & \vdots & 0 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 1 & 0 & \vdots & 0 & 0 \\ 0 & 1 & \vdots & 1 & 0 & \vdots & 0 & 0 \end{bmatrix}$$

determinante ne moremo definirat, če matrika ni kvadratna

$\rho(N) = 3 = 3 = n$

$\Rightarrow$ sistem je spoznaven!

$$((CA)\cdot A) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$