

OSNOVE VEZIJ

zapiski predavanj

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Sodelavci Blaž Potočnik, Aljoša Praznik

UREJANJE DOKUMENTA

VERZIJA 01 REVIZIJA 02
DATUM 24. 2. 2009

ZADNJI POPRAVLJAL /
PREGLEDAL Blaž Potočnik, Aljoša Praznik

OPOMBE

POPRAVKI

24.02.2009 - Dodana manjkajoča 9 stran.

www.stromar.si

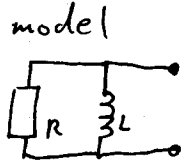
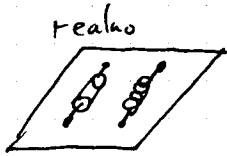
zbirka študijske literature na spletu

razmnoževanje dovoljeno le ob predhodnem dogovoru
v dokumentu lahko obstajajo napake

1.) Električno vezje

1.1. Realno elekt. vezje in model

Model električnega vezja



- zahteve za model:
 1. Mat. opisljiv
 2. Se da analizirati
 3. Rezultat analize mora biti blizu realnega

Zakaj model

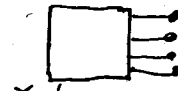
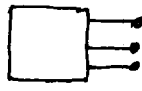
- drugače ne gre
- se da - obstajajo dobri modeli

Klasifikacija po priključkih:

- po priključnih sponkah

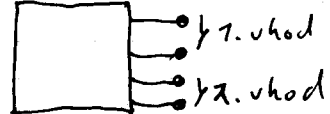
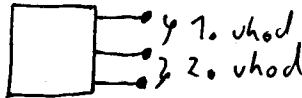
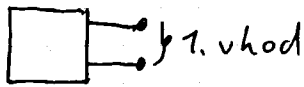


dvopol



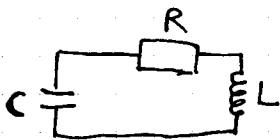
četveropol

- po vseh: vhod \equiv par priključnih sponk

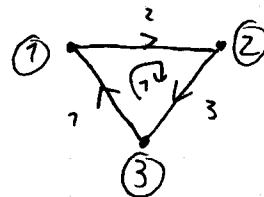


vhod \neq vzbujanje, odziv (signala)

Simbolična predstavitev vezja



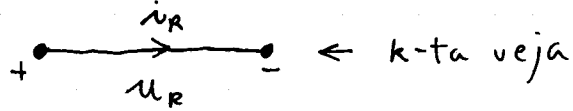
graf vezja:



- vozlišča
- veje
- okna

▣ Osnovne spremenljivke vezja

- vejni tokovi



$$i_1, \dots, i_B$$

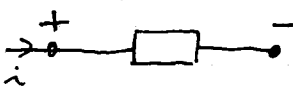
- vejne napetosti

$$u_1, \dots, u_B$$

▣ Referenčne in vezane polaritete

- referenčne pol.: + tok \rightarrow usmetitev

+ napetost \rightarrow polariteta

- vezane polar.: 
 $u = +R \cdot i$

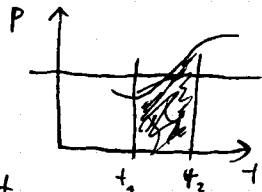
▣ Trenutna moč in energija

- trenutna moč:



$$p(t) = u(t) \cdot i(t)$$

- energija



$$W = P \cdot t$$

$$W(t_1, t_2) = \int_{t_1}^{t_2} p(t) \cdot dt$$

▣ Sistem oznak

- čas, signali: $u(t), i(t), x(t), u_g(t), \dots$

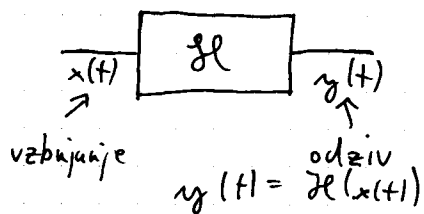
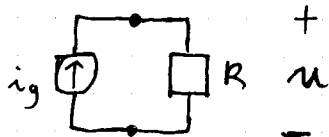
- konstante: K, U_0, I_{g0}, \dots

$$x(t) = X_0 \cos(\omega t + \varphi)$$

1.2. Električni sistem

Simbolični sistemski operator

- električni sistem \equiv vezje + vzbujanje + odziv



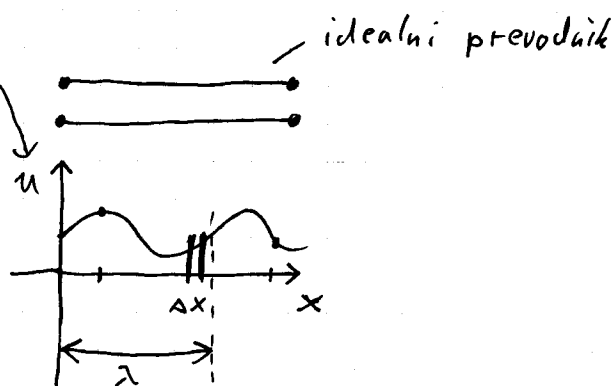
$$u = \mathcal{R} \cdot (i_g) \\ = R \cdot i_g$$

1.3. Lastnosti vezja in omejitve

Strnjeno vezje

- strnjeno vezje: $w(x, y, z, t) = w(t)$

- primer nestrnjene vezja



- vezje je strnjeno, če je

$$d_{\text{maks}} \ll \lambda$$

$$v = c = \lambda \cdot f \Rightarrow \lambda = \frac{c}{f}$$

- primeri: • $f = 0$ Hz ni omejitev

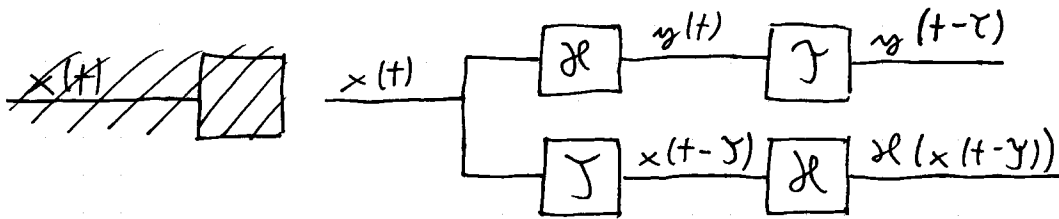
• $f = 50$ Hz; $\lambda = \frac{3 \cdot 10^8}{50} = \frac{3}{3} \cdot 10^7$ m

• $f = 1$ GHz; $\lambda = \frac{3 \cdot 10^8}{10^9} = 0,3$ m 3 mm velika vezje

• $f = 3$ GHz;

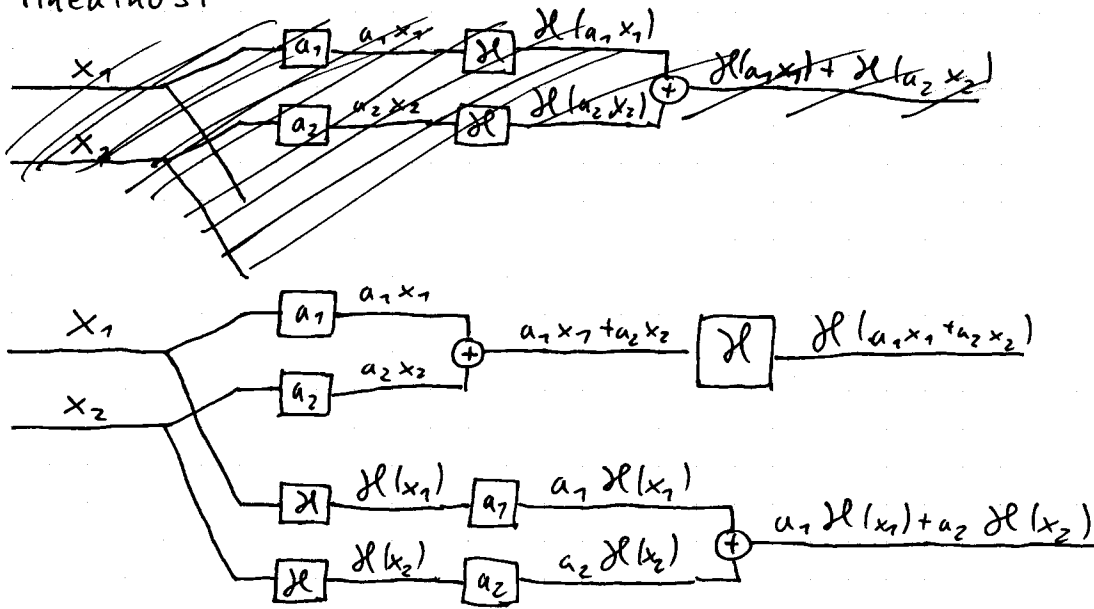
Linearnost in časovna invariantnost

- časovna invariantnost



Za vse $x(t)$, za vse τ ; $y(t-\tau) = \mathcal{H}(x(t-\tau))$

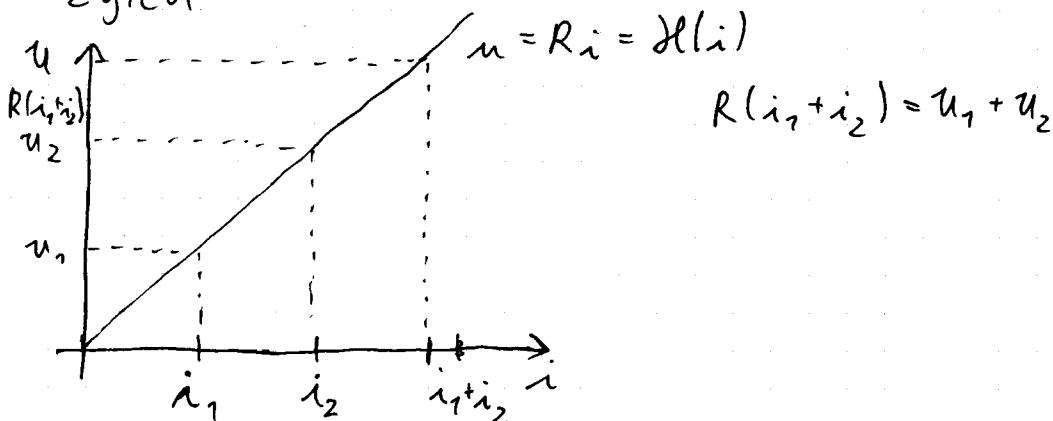
- linearnost



Za vse x_1, x_2 in a_1, a_2 velja:

$$\mathcal{H}(a_1 x_1 + a_2 x_2) = a_1 \mathcal{H}(x_1) + a_2 \mathcal{H}(x_2)$$

- zgled



- vezje je linearno $\Leftrightarrow \mathcal{H}$ je linearen
simbolični operator, ki predstavlja

- \Leftrightarrow ko ga predstavljajo linearne enačbe
- \Leftrightarrow ko ga sestavljajo linearni elementi

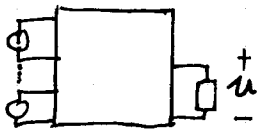
■ Sistemska enačba



$$\underline{a}_n \frac{d^n}{dt^n} y(t) + \dots + \underline{a}_1 \frac{d}{dt} y(t) + \underline{a}_0 y(t) = \underline{b}_m \frac{d^m}{dt^m} x(t) + \dots + \underline{b}_1 \frac{d}{dt} x(t) + \underline{b}_0 x(t)$$

konstante

■ Teorem o superpoziciji

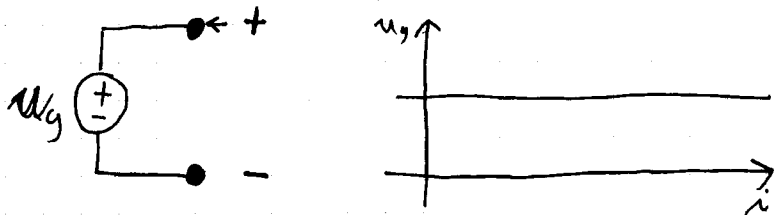


$$u' + u'' + \dots = u$$

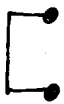
T: V linearnem vezju, ki ga sočasno na poljubnih mestih vzbujamo, je odziv v poljubnem trenutku na poljubnem mestu enak vsoti prispevkov posameznih vzbujanj.

■ Model vzbujanja vezja

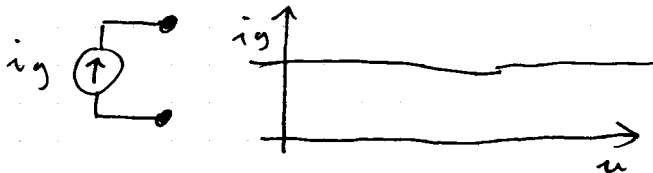
- neodvisni nap. vir



- realizacija $u_g(t) = 0 \text{ V}$ - nadomestilo s kratkim stikom



- neodvisni tokovni vir



- real. $i_g(t) = 0$



- nadomestilo z odprtimi spojkami

1.4. Kirchoffova zakona

▣ Tokov zakon

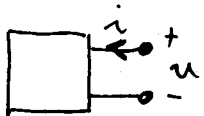
$$\sum_{k=1}^n i_k(t) = 0 \text{ A}$$

▣ Napetostni zakon

$$\sum_{k=1}^m u_k(t) = 0$$

1.5. Modeli elementov vezja

▣ Pasivni, reaktivni, aktivni dvopoli/elementi upor baterija

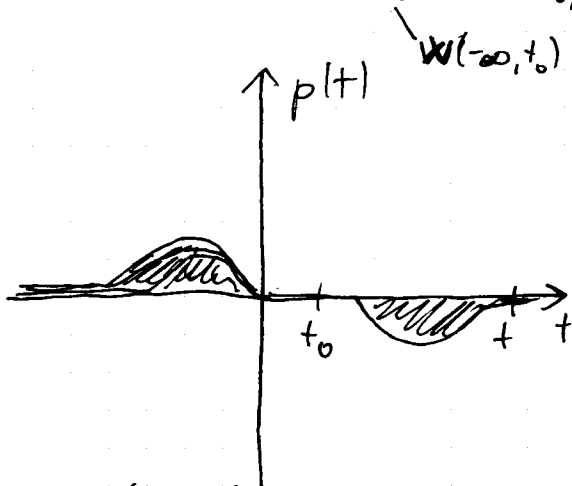


$W(t_1, t_2)$ "v dvopol"

- element/dvopol je pasiven:

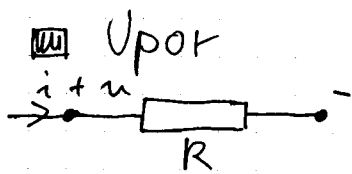
$$W(t_0) + W(t_0, t) \geq 0$$

vsek
 $\forall t_0 < t$
 vzbujanja



- element je aktiven, če ni pasiven

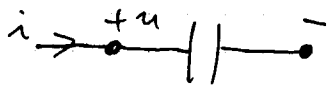
- reaktiven je primer pasivnega, ki shranjuje energijo



- karakter.: $u(t) = R i(t)$
 $i(t) = G u(t)$

- pasiven, ni reaktiven

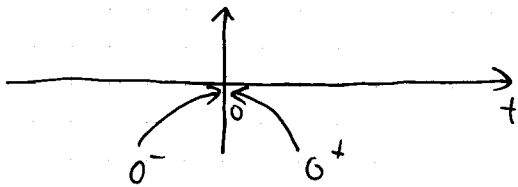
☐ Kondenzator



- karakter.: $u(t) = \frac{1}{C} \int_{-\infty}^t i(t') dt'$
 $i(t) = C \frac{du(t)}{dt}$

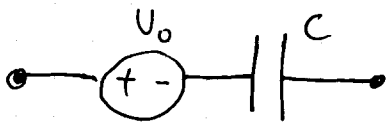
- nap na C je zveznai

$$u(0^-) = u(0^+)$$

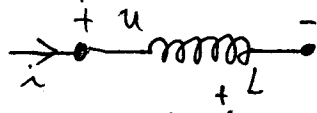


- model naelektrenega kondenzatorja:

$$u(t) = \underbrace{\frac{1}{C} \int_{-\infty}^{0^-} i(t') dt'}_{U_0} + \frac{1}{C} \int_{0^-}^t i(t') dt'$$



Tuljava



$$i(t) = \frac{1}{L} \int_{-\infty}^t u(t') dt'$$

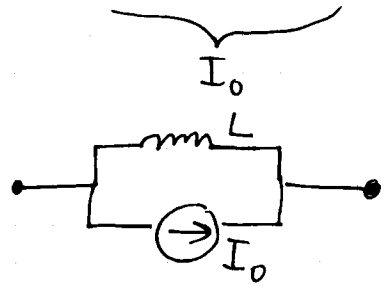
$$u(t) = L \frac{di(t)}{dt}$$

- tok tuljave je zvezen: -ker je rezultat integr. operacije isto za hord/mupe.

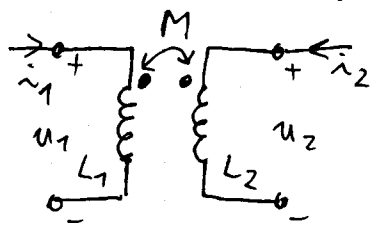
$$i(0^-) = i(0^+)$$

- model vzbujane tuljave

$$i(t) = \frac{1}{L} \int_{-\infty}^t u(t') dt' + \frac{1}{L} \int_{0^-}^t u(t') dt'$$



Sklop tuljav



$$u_1(t) = L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt}$$

$$u_2(t) = L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt}$$

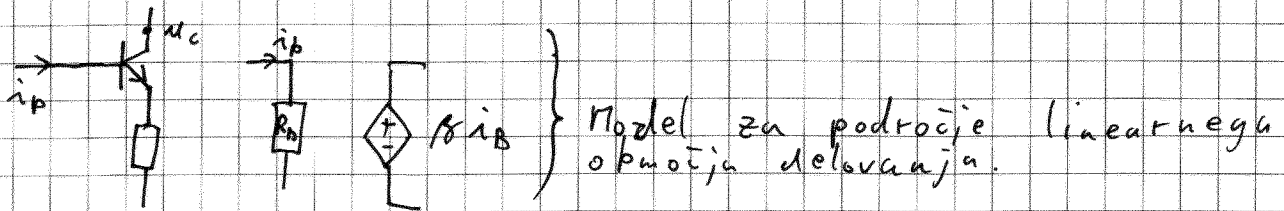
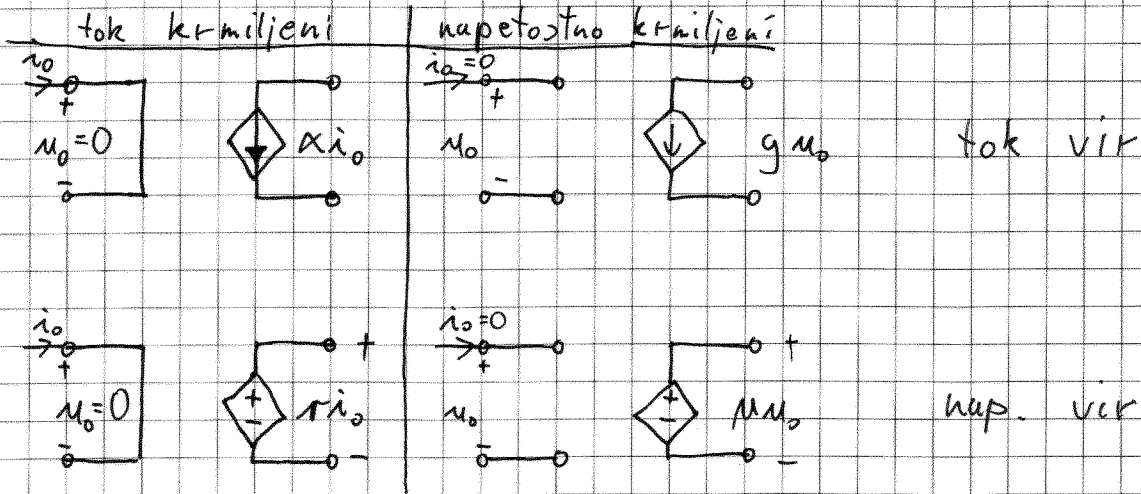
M... medsebojna induktivnost

$$M = k \sqrt{L_1 L_2}$$

... sklopni faktor

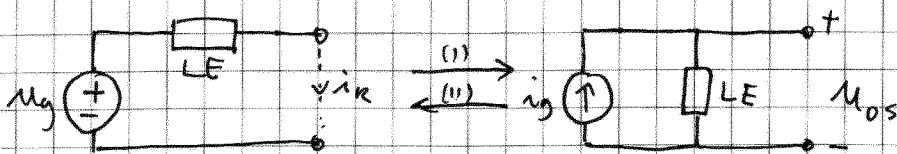
Krmiljeni viri

$$\frac{mm \leftarrow M}{L}$$

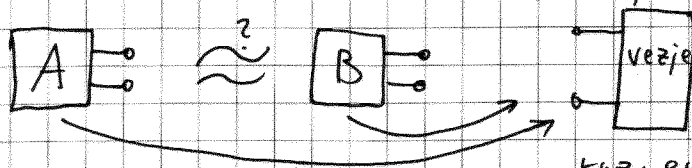


Moč, ki se porablja za krmiljenje je 0W.

Teorem o transformaciji virov

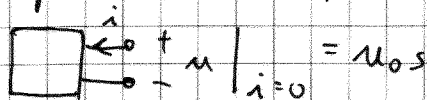


- ekvivalenca s staljšca priključnih spolk!

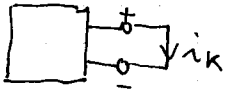


kazmerja v vezju so enaka, ne glede na vir = ekv. virov

- nap. odprtih spolk u_{os}



- kratkostični tok i_k :



- teoremi:

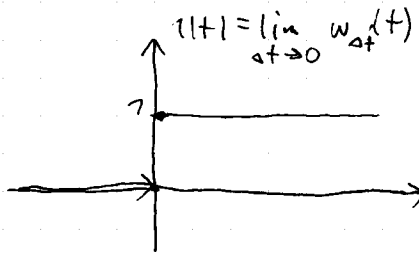
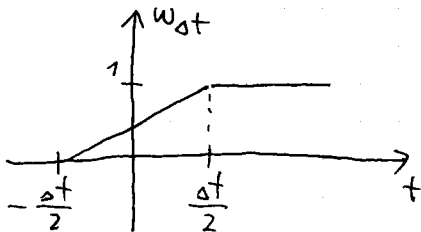
(I): - enak lin. el. LE

(II): - enak lin. el. LE
 $-i_g = i_k$
 $-u_g = u_{os}$

1.6. Temeljni el. signali in transf.

Temeljni el. signali

- signal je časovna funkcija s končno energijo
 - enofazna stopnica: $\tau(t)$



model: stikalo

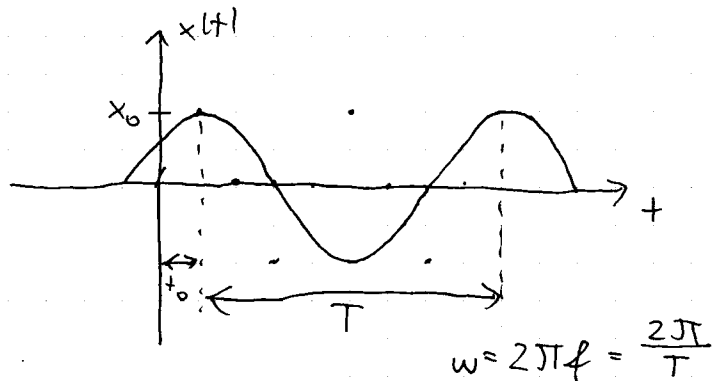
- harmonični signal

$$x(t) = x_0 \cos(\omega t + \varphi) =$$

amplitude
\omega \downarrow \text{frekvenca}
\varphi \downarrow \text{fazni zamik}

$$= x_0 \cos(\omega(t - t_0))$$

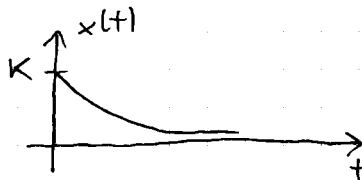
$$\varphi = -\omega t_0$$



med drugim predstavlja harm. stacionarno stanje

- eksponentni signal

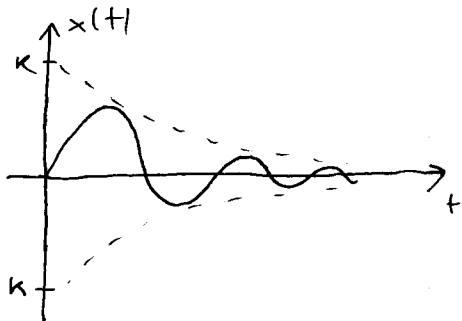
$$x(t) = k e^{\delta t}; \delta \leq 0$$



priznjenje kod.

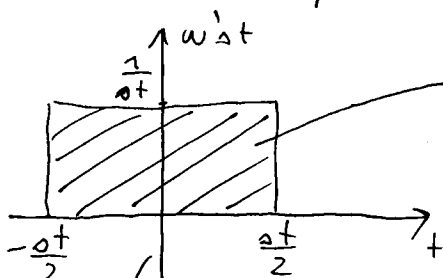
- dušeno nihanje

$$x(t) = K e^{\delta t} \cos(\omega t + \varphi); \delta \leq 0$$



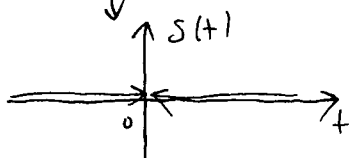
realni nihajni krog

- Dirackov impulz



plošč. = 1 \neq f(ω); neodv. od frekvence

$\delta t \rightarrow 0$

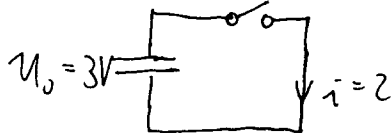


$$\delta(t) = \frac{d}{dt} 1(t)$$

seleksijska lastnost (za δ):

$$\int_{-\infty}^{\infty} f(t) \delta(t - t_0) dt = f(t_0)$$

Zgled



$$i(t) = C \frac{d u(t)}{dt} = C \frac{d}{dt} (U_0 \cdot 1(t)) = C U_0 \delta(t)$$

- matričnu oblika

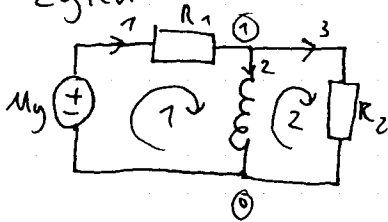
$$\underline{u} = \underline{Z}(D)\underline{i} + \underline{u}_g - \underline{Z}(D)\underline{i}_g$$

MATRIKA TOKOVNIH
VEJNIH OPERATORJEV

$$\underline{i} = \underline{Y}(D)\underline{u} + \underline{i}_g - \underline{Y}(D)\underline{u}_g$$

MATRIKA NAPETOSTNIH
VEJNIH OPERATORJEV

- Zgled



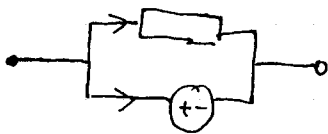
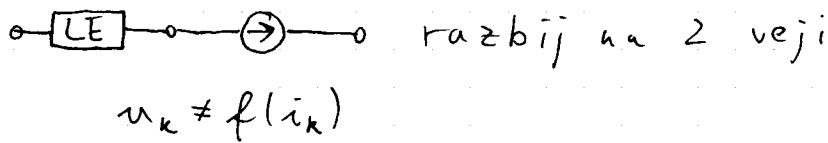
$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix}}_{\underline{Z}(D)} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} -u_g \\ 0 \\ 0 \end{bmatrix} - \underline{Z}(D) \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

\uparrow \underline{u} -vejni nap. $\underline{Z}(D)$ \uparrow \underline{i} -vejni tokovi

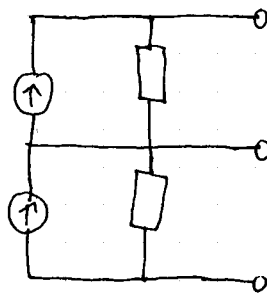
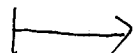
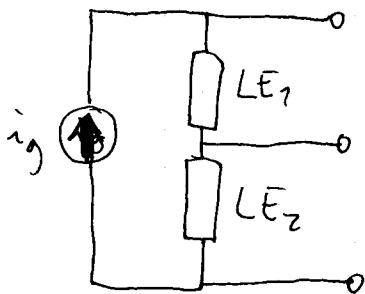
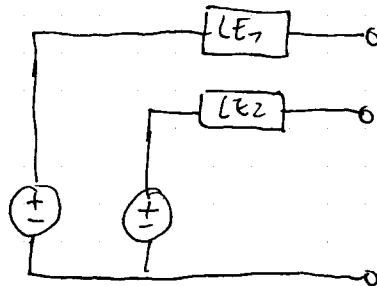
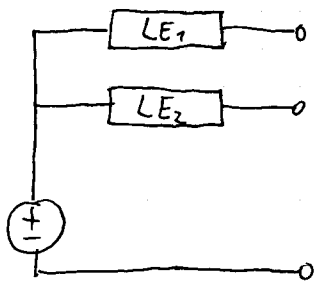
▣ Nekaterne lastnosti $\underline{Z}(D)$ in $\underline{Y}(D)$

- RLC vezje (\equiv brez krmiljenih virov) $\Rightarrow \underline{Z}(D), \underline{Y}(D)$ sta diag. matriki
- $\underline{Y}(D) = \underline{Z}(D)^{-1}$

▣ Kaj je ni veža - pomožna transformacija



- pomožna transformacija



2.3. Temeljne enačbe

Matrlična oblika ^(Tehove kit. enačbe) TKE

$$A \underline{i} = \underline{0} \quad \forall N-1$$

$\underline{i} = \vec{i} \Rightarrow$ vektor

Matrlična oblika ^{napetostnih} NKE:

$$M \cdot \underline{u} = \underline{0} \quad \left. \vphantom{M \cdot \underline{u} = \underline{0}} \right\} C \cdot \text{vrstic}$$

Ekonomika enačb

- št. spremenljivk:

$$\underline{i} = \begin{bmatrix} i_1 \\ \vdots \\ i_B \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_B \end{bmatrix}; \quad 2B$$

- št. enačb:

$$\left. \begin{array}{l} \text{TKE: } N-1 \\ \text{NKE: } C \\ \text{kurak. vej: } B \end{array} \right\} \begin{array}{l} \text{Enl.} \\ B \end{array} \left. \vphantom{\begin{array}{l} \text{TKE: } N-1 \\ \text{NKE: } C \\ \text{kurak. vej: } B \end{array}} \right\} 2B$$

2.4. Postavitve enačb sistema
- tri metode

Vejna metoda

- spremljivke metode: $\underline{i} = \begin{bmatrix} i_1 \\ \vdots \\ i_B \end{bmatrix}$

- zapis enačb:

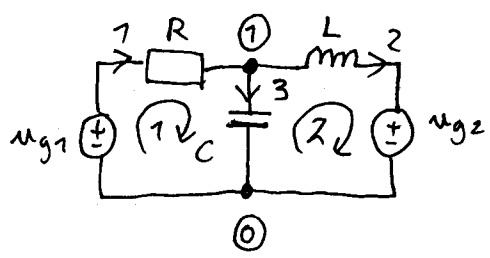
$$\text{TKE} \Rightarrow A \underline{i} = \underline{0}$$

$$\text{NKE} : M \underline{u} = \underline{0} : M(Z(D) \underline{i} + \underline{u}_g - Z(D) \underline{i}_g) = \underline{0}$$

$$M Z(D) \underline{i} = M Z(D) \underline{i}_g - M \underline{u}_g$$

$$\underbrace{\begin{bmatrix} A \\ \vdots \\ M Z(D) \end{bmatrix}}_{B \times B} \underline{i} = \begin{bmatrix} \underline{0} \\ \vdots \\ M Z(D) \underline{i}_g - M \underline{u}_g \end{bmatrix}$$

- zglede

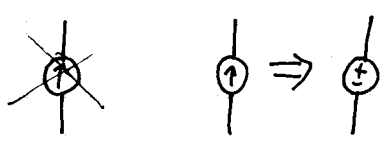


$$\begin{aligned} N &= 2 \\ B &= 3 \\ C &= 2 \end{aligned}$$

TKE: ①: $-i_1 + i_2 + i_3 = 0 \quad \left. \vphantom{-i_1 + i_2 + i_3 = 0} \right\} N-1=1$

NKE: ②: $-u_{g1} + R \cdot i_1 + \frac{1}{CD} i_3 = 0$
 ③: $-\frac{1}{CD} i_3 + LD i_2 + u_{g2} = 0 \quad \left. \vphantom{-\frac{1}{CD} i_3 + LD i_2 + u_{g2} = 0} \right\} C$

- omejitve:



Zančna metoda

- spremljivke: zančni tokovi $[i_{z1}, \dots, i_{zc}]^T = \underline{i}_z$

- zapis enačb

$$\underline{i}_z = M^T \underline{i}_z$$

NKE: $M \underline{u} = \underline{0}$

$MZ(D) \underline{i} = MZ(D) \underline{i}_g - M \underline{u}_g$ (glej vežo metodo)

$$\boxed{MZ(D) M^T \underline{i}_z = MZ(D) \underline{i}_g - M \underline{u}_g} \rightarrow \boxed{Z_z(D) \underline{i}_z = \underline{u}_{zg}}$$

- zglede (veže kot prejšnji zglede)

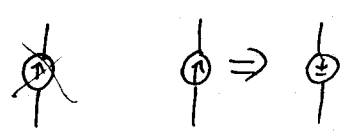
i_{z1}, i_{z2} :

①: $-u_{g1} + R \cdot i_{z1} + \frac{1}{CD} \cdot (i_{z1} - i_{z2}) = 0$
 ②: $\frac{1}{CD} (i_{z2} - i_{z1}) + LD i_{z2} + u_{g2} = 0 \quad \left. \vphantom{\frac{1}{CD} (i_{z2} - i_{z1}) + LD i_{z2} + u_{g2} = 0} \right\} C$

- matrična oblika

$Z_z(D) \equiv$ zančna matrika tokovnih operatorjev
 $\underline{u}_{zg} \equiv$ stolpec zančnih nap. virov

- omejitve:

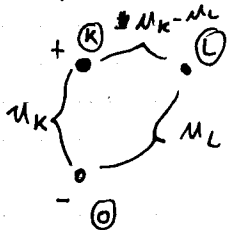


Teorema transformaciji virov

$$\underline{i}_z = Z_z(D)^{-1} \underline{u}_{zg}$$

Vozliščna metoda

- spremaaljičke: vozliščne napetosti $\underline{u}_v = [u_{v1}, \dots, u_{v(N-1)}]$



$$\underline{u}_v = A^T \underline{u}_v$$

- zapis enačb

TKE: $A \cdot \underline{i} = 0$

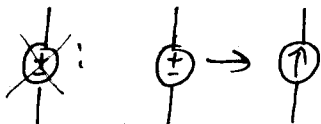
$$A (Y(D) \underline{u} + \underline{i}_g - Y(D) \underline{u}_g) = 0$$

$$\underbrace{A Y(D) A^T}_{Y_N(D)} \underline{u}_v = \underbrace{A Y(D) \underline{u}_g - A \underline{i}_g}_{\underline{i}_{vg}}$$

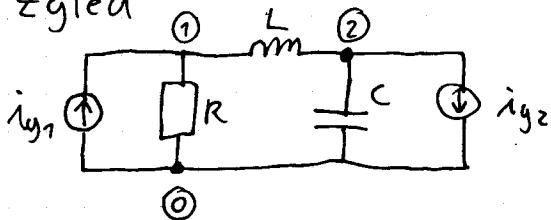
$$\boxed{Y_N(D) \underline{u}_v = \underline{i}_{vg}}$$

$Y_N(D) \equiv$ vozliščna matrika nap. operatorjev
 $\underline{i}_{vg} \equiv$ stolpec vozliščnih tokovnih virov

- omejitve



- zgled



- u_{v1}, u_{v2}



TKE:

$$\textcircled{1} +i_{g1} + \frac{u_{v1}}{R} + \frac{1}{LD} (u_{v1} - u_{v2}) = 0$$

$$\textcircled{2} \frac{1}{LD} (u_{v2} - u_{v1}) + CD u_{v2} + i_{g2} = 0$$

- matrična rešitev

$$\boxed{\underline{u}_v = Y_N(D)^{-1} \underline{i}_{vg}}$$

$$\begin{bmatrix} \frac{1}{R} + \frac{1}{LD}, & -\frac{1}{LD} \\ -\frac{1}{LD}, & \frac{1}{LD} + CD \end{bmatrix} \begin{bmatrix} u_{v1} \\ u_{v2} \end{bmatrix} = \begin{bmatrix} i_{g1} \\ i_{g2} \end{bmatrix}$$

- neposreden zapis enačb $Z_z(D)$ in $Y_v(D)$

$$Z_z(D) = [z_{ik}]_{ik}$$

$$Y_v(D) = [y_{ik}]_{ik}$$

$$z_{ik} = \sum t \cdot N \cdot \sigma$$

(1)
 izk

$$y_{ik} = \sum h \cdot N \cdot \sigma$$

(1)
 izk

vsota vseh tokovih vejnih operatorjev } vsota nap. vejnih operatorjev, preko katerih
 preko katerih k-ti zračni tok } k-ta vozliščna nap. črpa tok iz i-tega
 prispeva napetost v i-to zanko } vozlišča

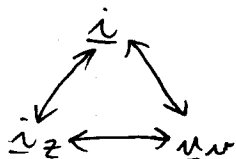
$$y_{21} = -\frac{1}{LD}$$

izbira in primerjava metod

Metoda	št. enačb
vejna	B
zračna	C
vozliščna	N-1

← najslabša
 } B

- splošna je le vozliščna metoda
- vejna in zračna le za ravninska vezja



↔ poljubna količina

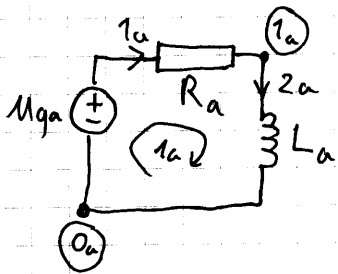
2.5. Dualnost

Opis dualnosti

- opis:

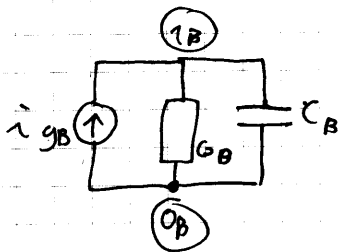
Vezji sta si dualni, če ju opisujeta matematično enaka sistema enačb za napetosti in tokove.

- zgled



$$\textcircled{1a} : -U_{ga} + R_a i_{z1} + L_a D i_{z1} = 0$$

$$-3V + 6x_1 + 10^{-3} D x_1 = 0$$



$$-3A + 6x_1 + 10^{-3} D x_1 = 0$$

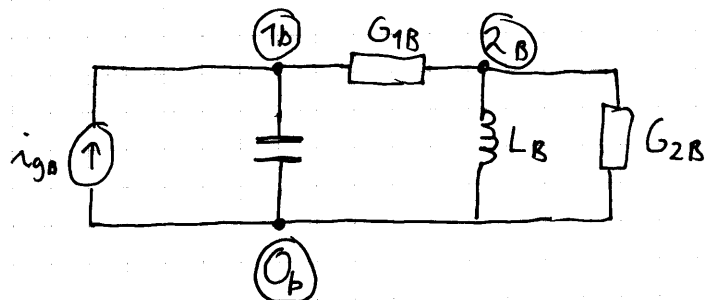
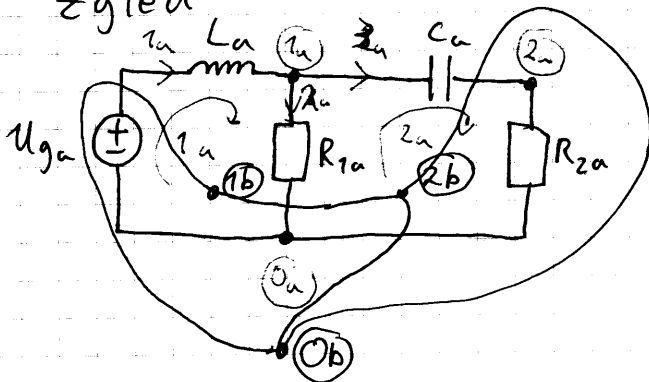
$$-i_{gB} + G_B u_{v1} + C_B D u_{v1} = 0$$

Dualni pojmi

nap. \leftrightarrow tok
zap. vezava \leftrightarrow vzporedna vezava
vozlisce \leftrightarrow okno
 $R \leftrightarrow G$
 $L \leftrightarrow C$

Postopek določanja dualnega vezja

- zgled



- dualni elementi \neq

- koraki

1. označite orig. vezje

2. izdelajte graf dualnega vezja

- vozlišča: ref. vozlišče + ostala
- veje

3. izdelajte dualno vezje

* dualni elementi

$$C_{Bh} = L_{Ah} \quad (1 \mu F = 1 \mu H)$$

↳ kotirano

$$i_{gBh} = u_{gAh} \quad (1 A = 1 V)$$

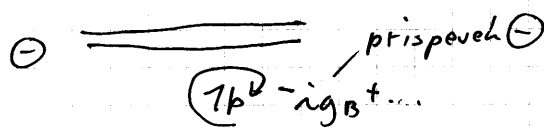
$$G_{1Bh} = R_{1Ah} \quad (1 S = 1 S)$$

$$L_{1Ah} = L_{Bh}$$

$$G_{2Bh} = R_{2Ah}$$

* svet tok. vira

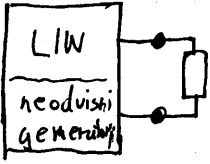
↳ : -uga + ...



- le za ravinska vezja

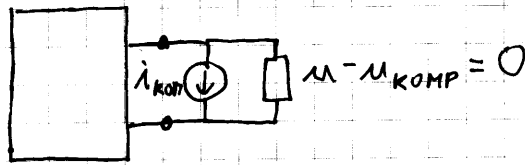
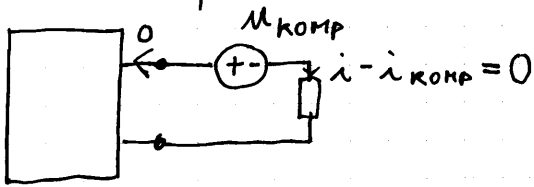
2.6. Theveninov in Nortonov teorem

▣ Problem, ekvivalenca

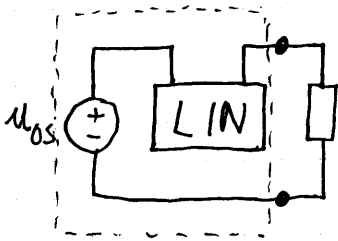


- ekvivalentno s statišča priključnih sponk

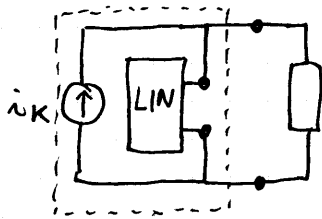
▣ Miselni poskus



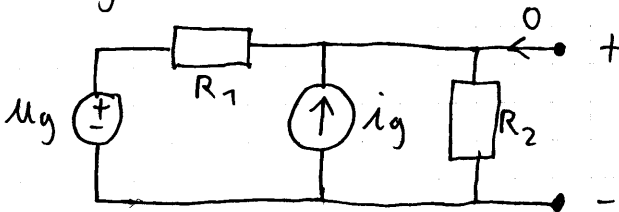
▣ Theveninov ekvivalentni dvopol



▣ Nortonov ekvivalentni dvopol

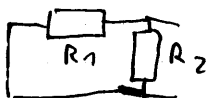


+ zgled

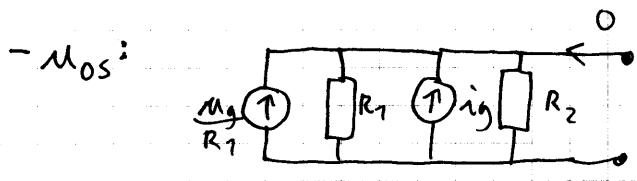


• Theveninov dvopol:

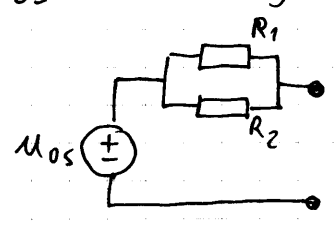
- lin:



zvezka

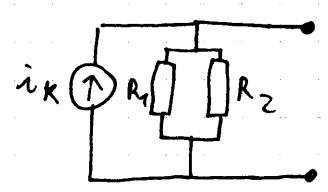


$$u_{os} = \left(\frac{u_g}{R_1} + i_g \right) \frac{R_1 \cdot R_2}{R_1 + R_2}$$

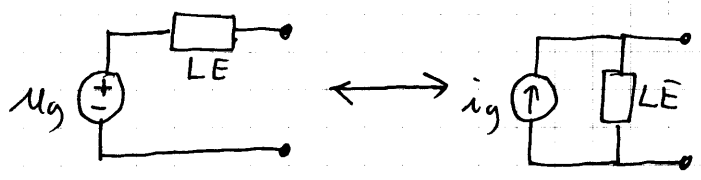


• Nortonov duopol:

$$i_k = i_g + \frac{u_g}{R_1}$$



+ opomba



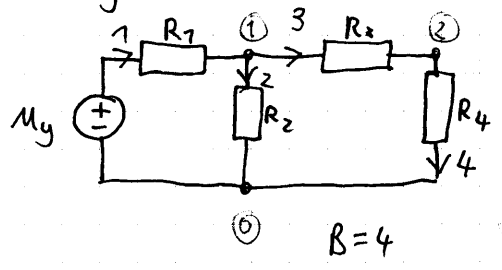
2.7. Tellegenov teorem

~~Teorem~~ Teorem

$$\sum_{k=1}^B u_k(t) \cdot i_k(t) = 0$$

$p_k(t)$

- zgled



$$\begin{aligned} \sum_{k=1}^4 u_k i_k &= u_1 i_1 + u_2 i_2 + u_3 i_3 + u_4 i_4 = \\ &= -u_{v1} i_1 + u_{v1} i_2 + (u_{v1} - u_{v2}) i_3 + u_{v2} i_4 = \\ &= u_{v1} \underbrace{(-i_1 + i_2 + i_3)}_0 + u_{v2} \underbrace{(-i_3 + i_4)}_0 = 0 \end{aligned}$$

Alternativna oblika teorema

$$\sum_{k=1}^B m_k(t_1) \cdot i_k(t_2) = 0$$

$$\sum_{k=1}^B \overbrace{m_k(t_1)}^{(1)} \cdot \overbrace{i_k(t_2)}^{(2)} = 0$$

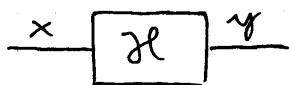
VEJME NAP. VEJMI TJA.
V VEZJU 1 V VEZJU 2

pogoji: vezji 1 in 2 imata skupen graf

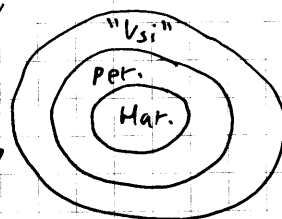
Klasična analiza

3. Klasična analiza

3.1. Sistemsko enačba in njena rešitev

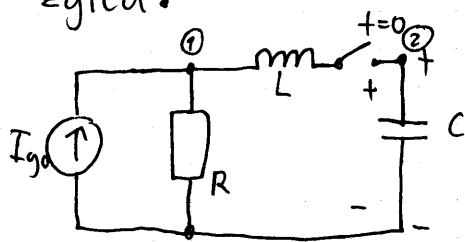


$$(a_n D^n + \dots + a_1 D + a_0) y = F(t)$$



- za vse signale

- zgled:



$$u_c(0^-) = u_0 = 1V, I_{g0} = 1A$$

$$R = 6\Omega, L = 1H, C = \frac{1}{5}F$$

1. DE $t \geq 0$

$$- y = y_h + y_p$$

HOMOGENA PARTIKULARNA

- n... red DE \leq št. reakt. elem.

$$\textcircled{1} -I_{g0} + \frac{u_{v1}}{R} + \frac{1}{LD} (u_{v1} - u_{v2}) = 0$$

$$\textcircled{2} \frac{1}{LD} (u_{v2} - u_{v1}) + CD \Delta u_{v2} = 0$$

$$u = u_{v2}$$

$$\rightarrow \frac{u_{v1}}{R} + CD u_{v2} = I_{g0}$$

$$u_{v1} = R (I_{g0} - CD u_{v2})$$

$$\rightarrow \frac{1}{LD} (u_{v2} - R (I_{g0} - CD u_{v2})) + CD u_{v2} = 0$$

$$-R I_{g0} + RCD u_{v2} + u_{v2} + LCD^2 u_{v2} = 0$$

$$\boxed{(LCD^2 + RCD + 1) u_{v2} = R I_{g0}} \quad n=2$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow \\ a_2 & a_1 & a_0 & F(t) \end{matrix}$$

Homogena rešitev:

+ karak. polinom: $P(p) = a_n p^n + \dots + a_1 p + a_0$

+ ničle: $P(p) = 0$: p_1, \dots, p_n ; $\text{Re}(p_i) \leq 0$

+ glede na red ničel: 1. red: $y_k(t) = K_1 e^{p_1 t} + \dots + K_n e^{p_n t}$
• p_k reda n : $(K_1 + K_2 t + \dots + K_n t^{n-1}) e^{p_k t}$

2. Hom:

$$P(p) = L(p^2 + R Cp + 1) = 0$$

$$p_{1,2} = \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC} \begin{matrix} \rightarrow -1 \text{ s}^{-1} \\ \rightarrow -5 \text{ s}^{-1} \end{matrix}$$

+ glede na tip. ničel: • $p \in \mathbb{R}$: $K e^{\delta t}$; $\delta = \text{Re}(p) = p$ $e^{j\varphi} = \cos \varphi + j \sin \varphi$
• $p \in \mathbb{C}$: p, p^* : $K_1 e^{p t} + K_1^* e^{p^* t} = K e^{\delta t} \cos(\omega t + \varphi)$; $\delta = \text{Re}(p)$, $\omega = |\text{Im}(p)|$

+ Homogena rešitev ima n prostih konstant: K_1, \dots, K_n

$$u_H(t) = K_1 e^{-t} + K_2 e^{-5t}$$

Partikularna rešitev

+ z nastavkom
+ oblik. nastavka

$F(t)$	nast. $y_p(t)$
K	A
$K e^{\delta t}$	$A e^{\delta t}$
$\cos(\omega t), \sin(\omega t)$	$A_1 \cos(\omega t) + A_2 \sin(\omega t)$
\vdots	\vdots

+ vstavimo v dif. enačbo \rightarrow sistem enačb za konstante nastavka

3. Par. reš.

$$F(t) = \text{konstanta} \Rightarrow u_p(t) = A$$

$$L C D^2 A + R C D A + A = R I_g$$

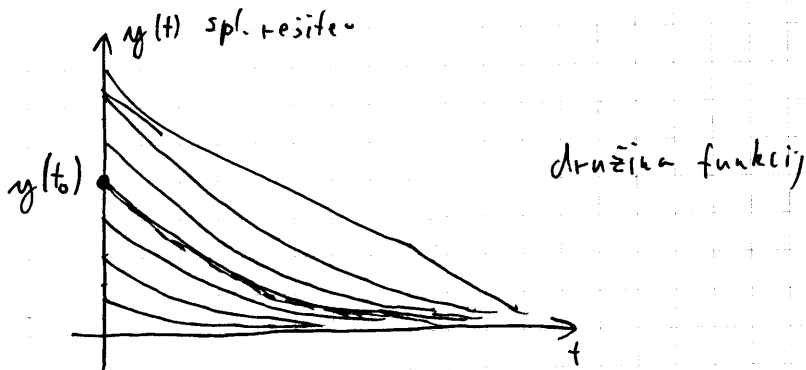
$$u_p(t) = R \cdot I_g$$

iv) Splošna rešitev

$$y(t) = \underbrace{y_h(t)}_{k_1 \dots k_n} + \underbrace{y_p(t)}_{\text{nobene, prave konstante}}$$

4. splošna reš:

$$u(t) = k_1 e^{-t} + k_2 e^{-5t} + 6V$$



v) Posebna rešitev

+ začetni pogoj: $t=0^+$

$$y(0^+), D y(0^+), \dots, D^{n-1} y(0^+)$$

+ začetni pogoj vstavimo v splošno rešitev
→ sistem enačb za k_1, \dots, k_n

+ rešimo k_1, \dots, k_n

v sistem enačb vezja vstavimo $t=0^-$

- upoštevamo začetno energijo to je: $u_C(0^-), i_L(0^-)$
- upoštevamo zveznost $u_C(0^-) = u_C(0^+)$
 $i_L(0^-) = i_L(0^+)$

• s tem smo dobili sistem navadnih enačb za ZP.

5. Posebna rešitev

$$n=2; \text{ ZP} = u(0^+), D u(0^+)$$

- začetne energije: $u_C(0^-) = 1V$ $i_L(0^-) = 0A$

- zveznost $u_C(0^+) = 1V$ $i_L(0^+) = 0A$

- vstavimo $t=0^+$ v sistem enačb:

$$-1A + \frac{u_C(0^+)}{R} + i_L(0^+) = 0$$

$$-i_L(0^+) + C D u(0^+) = 0$$

$$D u(0^+) = 0 \frac{V}{s}$$

$$u(0^+) = u_C(0^+) = 1V$$

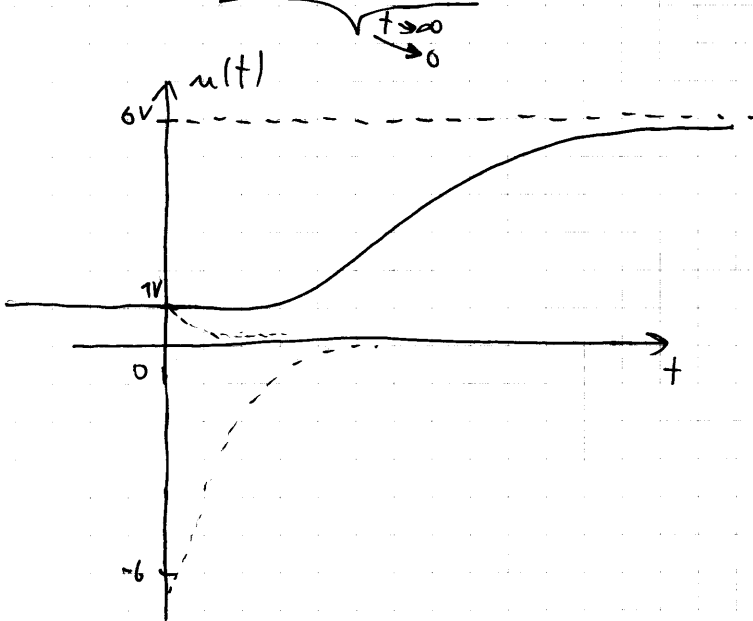
Začetne pogoje določimo tako, da v sistem enačb vezja (dobljenih z eno od treh metod) vstavimo vrednosti $u_C(0^+)$, $i_L(0^+)$ za vse L in C . S tem smo dobili sistem navadnih enačb za iskane začetne pogoje.

ŽP vstavimo v splošno rešitev.

$$D_m(t) = -k_1 e^{-t} - 5k_2 e^{-5t}$$

$$\begin{aligned} u(0^+) &= k_1 + k_2 + 6V = 7V \\ D_m(0^+) &= -k_1 - 5k_2 = 0 \end{aligned} \quad \left. \begin{array}{l} \\ + \end{array} \right\} \begin{aligned} -4k_2 &= -5 \Rightarrow k_2 = +\frac{5}{4} V \\ k_1 &= -5k_2 = -\frac{25}{4} V \end{aligned}$$

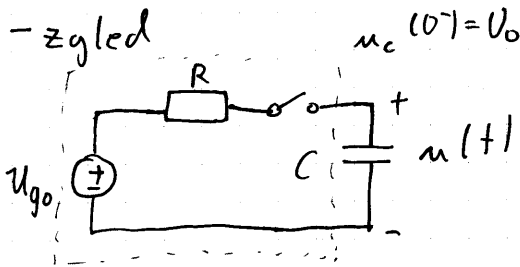
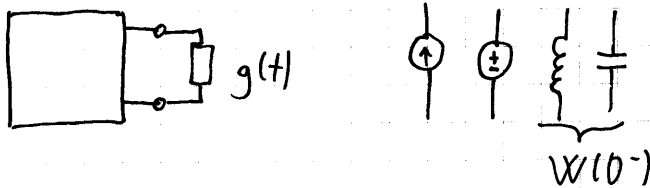
$$u(t) = -\frac{25}{4} e^{-t} + \frac{5}{4} e^{-5t} + 6V \quad \begin{array}{l} \text{Posebna rešitev.} \\ t \geq 0 \end{array}$$



3.2. Interpretacija rešitve

~~1111~~ ~~Lustni in vsiljen odziv~~

$$- y(t) = y_n(t) + y_p(t)$$



$$u(t) = \underbrace{(U_0 - U_{g0}) e^{-\frac{t}{RC}}}_{u_n(t)} + \underbrace{U_{g0}}_{u_p(t)}$$

$$W(0^-) = C \frac{U_0^2}{2}$$

~~1111~~ Lustni in vsiljen odziv

- $y_n(t)$: lustni odziv
- $y_p(t)$: vsiljen odziv

~~1111~~ Odziv na začetno stanje in odziv na vzbujanje

$$- y(t) = y_n(t) + y_p(t)$$

- zglej:

$$u(t) = U_0 e^{-\frac{t}{RC}} + U_{g0} (1 - e^{-\frac{t}{RC}})$$

- začetno stanje: stanja reaktivnih elementov ($u_c(0^-), i(0^-)$)

- začetna vrednost odziva $y(0^+)$

~~1111~~ Prehodni pojav in stacionarno stanje

$$- y(t) = y_0(t) + y_s(t)$$

- prehodni pojav: $y_0(t) \xrightarrow{t \rightarrow \infty} 0$

- $y_s(t) = y(t) - y_0(t)$: kar ostane

- zgled

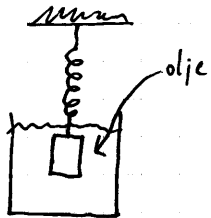
$$u_0(t) = (U_0 - U_{g0}) e^{-\frac{t}{\tau_c}} \xrightarrow{t \rightarrow \infty} 0$$

$$u_{gs}(t) = U_{g0}$$

- primeri stac. stanj:

- enosmerno stac. stanje: vsi u_i so konstantni
- harmonično stac. stanje: vsi u_i so harmonični

~~Tipi~~ Tipi lastnega odziva



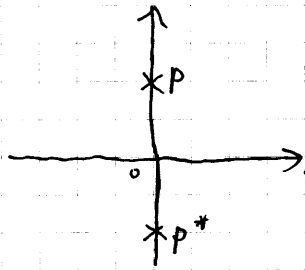
- zgled:

- 0 ničlah polinoma

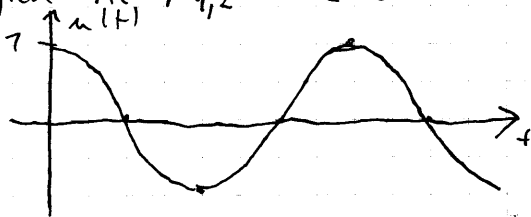
• nedušeno nihanje:

$$\operatorname{Re}(p_i) = 0; i = 1, \dots, n$$

$$\operatorname{Im}(p_i) \neq 0$$

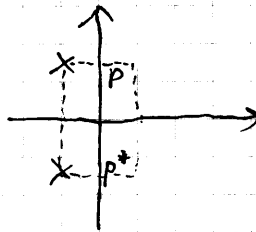


+ zgled: $\operatorname{Re}(p_{1,2}) = -\frac{R}{2} = 0 \Rightarrow R = 0 \Omega$



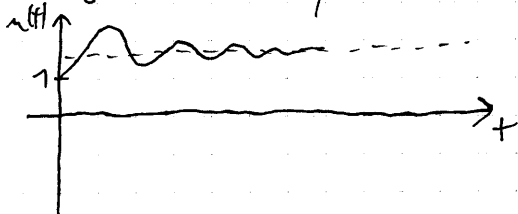
• dušeno nihanje

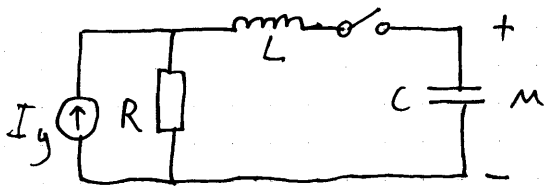
$$\operatorname{Re}(p_i) < 0; \operatorname{Im}(p_i) \neq 0$$



+ zgled

$$D_s = R^2 - 20 < 0; R < \sqrt{20} = 2\sqrt{5} \Omega$$





$$u_c(0^-) = U_0 = 1V$$

$$I_{g0} = 1A$$

$$L = 1H$$

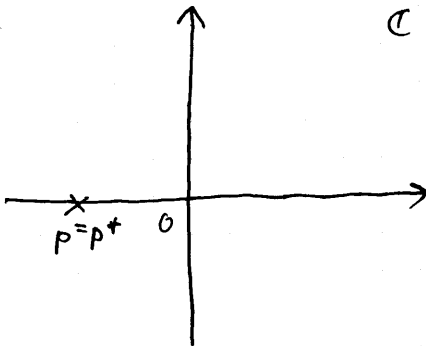
$$C = \frac{1}{5} F \quad R \dots \text{izbitam}$$

$$p_{1,2} = \frac{1}{2} (-R \pm \sqrt{R^2 - 20})$$

D_s

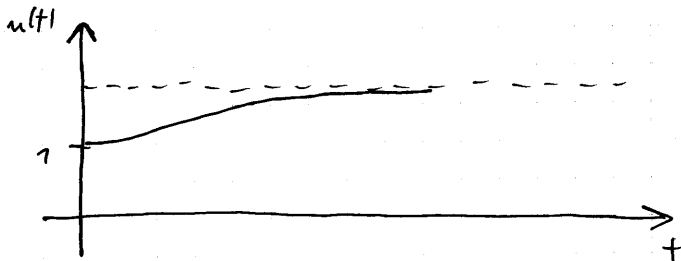
Kritično dušenje

- $st(p) > 1$, najmanjša tako dušenje
 \hookrightarrow stopnja



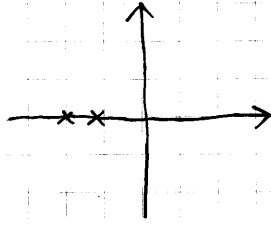
- zglede

$$D_s = 0: \Rightarrow R^2 - 20 = 0 \Rightarrow R = 2\sqrt{5} \Omega$$

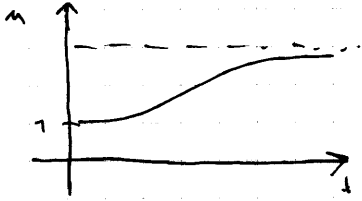


17) Nadkritično dušenje

$$\underbrace{\operatorname{Im}(p_i) = 0}_{n_i \in \mathbb{R}}, \operatorname{Re}(p_i) < 0$$



- zglede $D_s > 0$; $R^2 - 20 > 0$
 $R > 2\sqrt{5}\Omega$



- sklep: • nihanje le za $\operatorname{Im}(p) \neq 0$

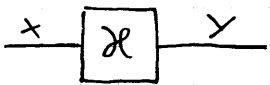
$$k_1 e^{p_1 t} + k_2 e^{p_2 t} \xrightarrow{\operatorname{Im}(p) = 0} k_1 e^{\delta_1 t} + k_2 e^{\delta_2 t}; \delta_1, \delta_2 < 0$$

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

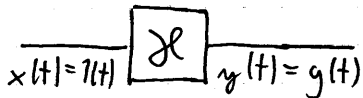
$$\xrightarrow{\operatorname{Im}(p) \neq 0} k_1 e^{(\delta + j\omega)t} + k_2 e^{(\delta - j\omega)t}$$

• nihanje le za $n \geq 2$
 \hookrightarrow št. reáln. el. \equiv št. L, C

4. Konvolucija



17) Prva oblika - aproksimacija $\approx 1(t)$:



$$g(t) = \mathcal{L}^{-1}(1(t))$$

Tedaj za poljubno vzbujanje $x(t)$ velja

$$y(t) = \mathcal{L}^{-1}(x(t)) = \int_{-\infty}^{\infty} x'(\tau) g(t - \tau) d\tau = (x' * g)(t)$$

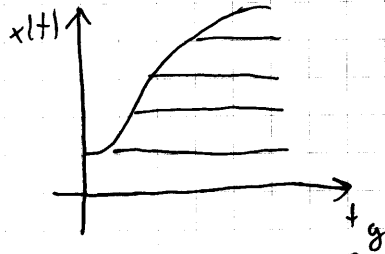
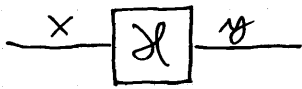
- velja \approx LTI sisteme
 \hookrightarrow linearne časovno invariantne

Iz nati:

- konv. sig. x_1 in x_2

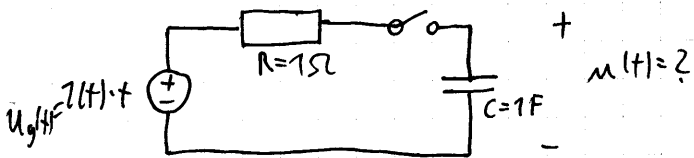
$$(x_1 * x_2)(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t - \tau) d\tau$$

- skica dokaza



$$\tilde{x}(t) = \sum_{n \rightarrow \infty}^h \underbrace{1(t)}_{x(t)} \longrightarrow \tilde{y}(t) = \sum_{n \rightarrow \infty}^h \underbrace{\mathcal{L}(1(t))}_{g(t)}$$

- zgljed



$$1. g(t) = \mathcal{L}(1(t))$$

- s klasičnu analizu:

$$u(t) = k e^{-t} + 1$$

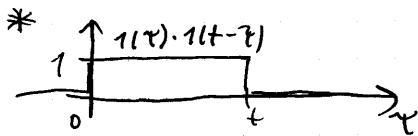
$$\text{ZP: } u(0) = 0; k = -1$$

$$u(t) = (1 - e^{-t}) 1(t) = g(t)$$

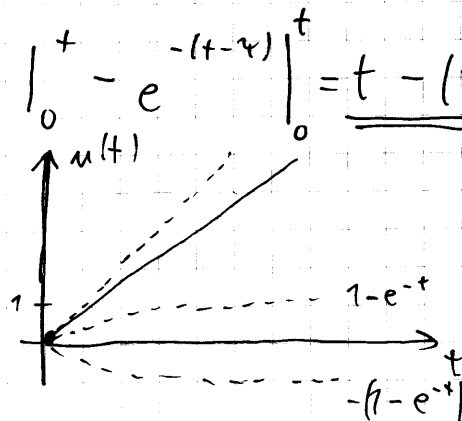
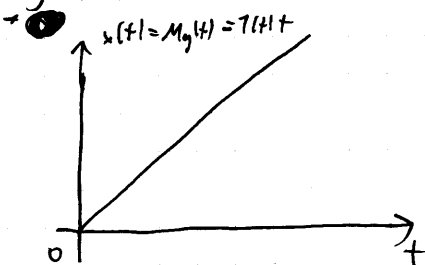
Odziv na $u_g(t) = 1(t) \cdot t$

$$u_g'(t) = 1 \stackrel{V}{=} 1(t) \text{ za } t > 0 \quad (u_g(t) = t \text{ za } t > 0)$$

$$u(t) = (u_g' * g)(t) = \int_{-\infty}^{\infty} 1 * g(t-\tau) d\tau = \int_{-\infty}^{\infty} \underbrace{(1 - e^{-(t-\tau)})}_{g(t-\tau)} \cdot \underbrace{1(\tau)}_{u_g'(\tau)} d\tau =$$



$$= \int_0^t (1 - e^{-(t-\tau)}) d\tau = \tau \Big|_0^t - e^{-(t-\tau)} \Big|_0^t = \underline{\underline{t - (1 - e^{-t})}}$$



Druga oblika teorema - aproksimacija z $\text{imp}(t)$

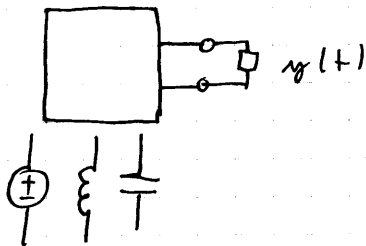
$$x = \delta(t) \quad \boxed{\mathcal{L}} \quad y = h(t) \quad h(t) = \mathcal{L}(\delta(t)) \rightarrow \text{Dirac. delta}$$

Tedaj za poljubno vzbujanje $x(t)$ velja

$$y(t) = (x * h)(t) = \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

Interpretacija odziva

$$y(t) = \underbrace{\int_{-\infty}^0 x(\tau) h(t - \tau) d\tau}_{y_z(t)} + \underbrace{\int_0^{\infty} x(\tau) h(t - \tau) d\tau}_{y_v(t)}$$

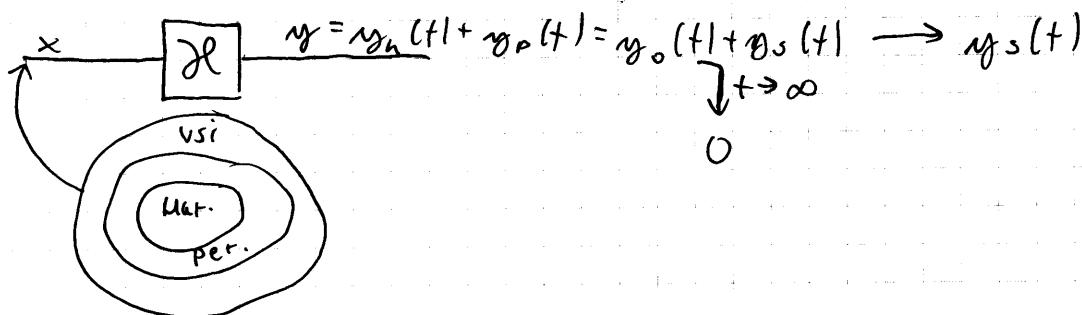


-primer $h(t) = \frac{1}{RC} \cdot e^{-\frac{t}{RC}} \mathcal{1}(t)$

5. Izmenična analiza

5.1. Uvod

Odziv vezja in izmenično stac. stanje



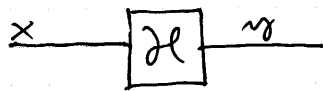
- če je $x(t) = X_0 \cos(\omega t + \varphi)$

• ker je $x(t)$ harmonična je $\frac{F(t)}{\text{desen stran. DE}}$ tudi harmonična

$$\Rightarrow y_s(t) = y_p(t) = A \cos(\omega t + \varphi)$$

če je vzbujanje harmonično, je stac. stanje tudi harmonično. in sicer

z isto kot ω frekvenca.



$$x(t) = X_0 \cos(\omega t + \varphi) \quad y(t) = Y_0 \cos(\omega t + \Psi)$$

$$(X_0, \omega, \varphi) \xrightarrow{?} (Y_0, \omega, \Psi)$$

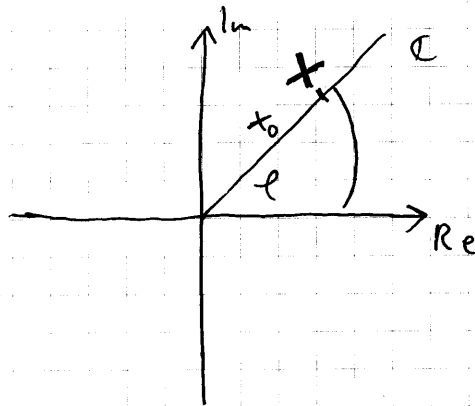
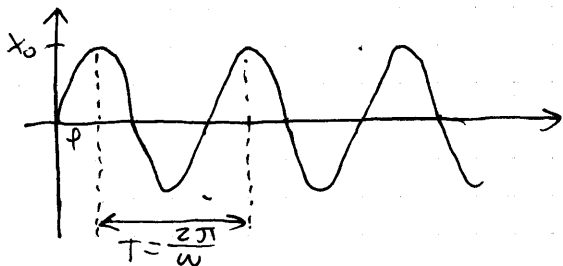
$$(X_0, \varphi) \xrightarrow{?} (Y_0, \Psi)$$

$$X = X_0 e^{j\varphi} \xrightarrow{H(j\omega)} Y_0 e^{j\Psi} = Y$$

- zato $x(t) \rightarrow X$... kazalec

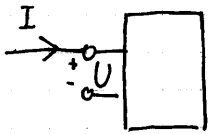
5.2. Domene kazalcev

Definicija kazalca



$$z = |z| e^{j\varphi}; \quad \varphi = \text{Arg}(z)$$

Sistemska funkcija in imitanca



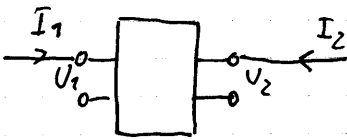
$$\left. \begin{aligned} I &= |I| \cdot e^{j\phi_i} \\ U &= |U| \cdot e^{j\phi_u} \end{aligned} \right\} X, Y \quad (Y = H(j\omega) \cdot X)$$

- sist. funkcija
 $H(j\omega) = \frac{Y}{X}$ - odziv / - vzbujanje

- posebni primeri sistem. funkcij

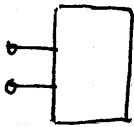
$$\left. \begin{aligned} \bullet \text{ impedanca: } Z(j\omega) &= \frac{U}{I} \\ \bullet \text{ admitanca: } Y(j\omega) &= \frac{I}{U} \end{aligned} \right\} \begin{array}{l} \text{reaktanca} \\ \text{imitanca} \\ \text{susceptanca} \end{array} = R(\omega) + jX(\omega) = G(\omega) + jB(\omega)$$

- za dvouhodno vezje



• prevajalni funkciji: $\frac{U_2}{I_1}, \frac{I_2}{U_1}$

Kompleksna moč



- delovna moč: $P = \frac{1}{2} U_0 I_0 \cos(\phi_u - \phi_i) \neq f(\omega)$ - ni odvisna od krožne frek.

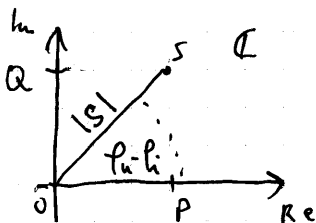
$$U = U_0 \cdot e^{j\phi_u} \quad I = I_0 \cdot e^{j\phi_i}$$

$$U \cdot I^* = U_0 I_0 e^{j(\phi_u - \phi_i)}$$

$$P = \frac{1}{2} \text{Re}(U \cdot I^*) = U_0 \cdot I_0 \cos(\phi_u - \phi_i)$$

- kompleksna moč: $S = \frac{1}{2} U \cdot I^* = P + jQ$
Re(S) Im(S)
delovna moč jalova moč

- močastni trikotnik



- navidezna moč: $|S|$

5.3, Zračna in vozliščna metoda:

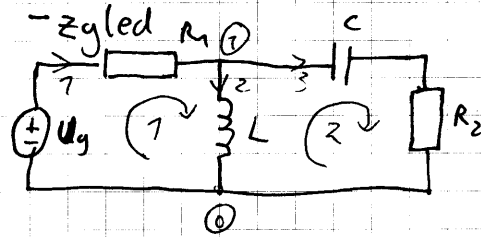
- Kir. zakoni veljajo za kazalce

Zračna metoda

- splemenljivke: $I_{z1} \dots I_{zc}$

- zapis enačb

NKE



$$\frac{1}{j\omega C} = -j \frac{1}{\omega C}$$

I_{z1}, I_{z2}

$$\begin{aligned} \textcircled{1} \quad & -U_g + R_1 I_{z1} + j\omega L (I_{z1} - I_{z2}) = 0 \\ \textcircled{2} \quad & j\omega L (I_{z2} - I_{z1}) + \frac{1}{j\omega C} I_{z2} + R_2 I_{z2} = 0 \end{aligned}$$

$$\begin{bmatrix} R_1 + j\omega L & -j\omega L \\ -j\omega L & j\omega L + R_2 + \frac{1}{j\omega C} \end{bmatrix} \begin{bmatrix} I_{z1} \\ I_{z2} \end{bmatrix} = \begin{bmatrix} U_g \\ 0 \end{bmatrix}$$

U_g - veh. zrač. nap. virov

$Z_z(j\omega)$ - zračna impe. matrika

- matrična oblika

$$\underline{Z}_z(j\omega) \underline{I}_z = \underline{U}_g$$

sistem navadnih enačb.

- nepostreden zapis $Z_z(j\omega) = [Z_{ik}]$

$Z_{ik} = \sum \text{impedanc...}$

\textcircled{i} I_{zk}

Vsota impedanc preko katerih k-ti zračni tok prispeva napetost v i-tem oknu

Vozliščna metoda

- spre: $U_{v1}, \dots, U_{v(n-1)}$

- enačbe: TKE

- matrična oblika

$$\underline{Y}_z(j\omega) \underline{U}_v = \underline{I}_g$$

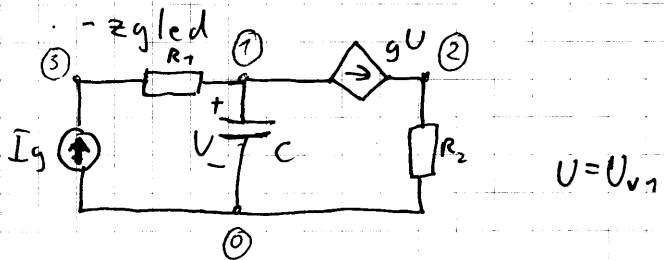
- nepostreden zapis:

$$\underline{Y}_v(j\omega) = [y_{ik}]$$

\textcircled{i} U_{vk}

$y_{ik} = \sum \text{admitanc...}$

Vsota admitanc preko katerih k-ta voz. napetost črpa tok iz i-tega vozlišča



U_{v1}, U_{v2}, U_{v3}

$$\begin{aligned} \textcircled{1} \quad & \frac{U_{v1} - U_{v3}}{R_1} + j\omega C U_{v1} + g U_{v1} = 0 \\ \textcircled{2} \quad & -g U_{v1} + \frac{U_{v2}}{R_2} = 0 \\ \textcircled{3} \quad & -I_g + \frac{U_{v3} - U_{v1}}{R_1} = 0 \end{aligned}$$

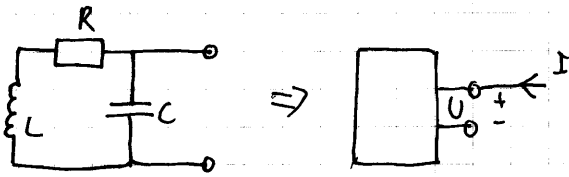
$$\begin{bmatrix} \frac{1}{R_1} + j\omega C + g & 0 & -\frac{1}{R_1} \\ -g & \frac{1}{R_2} & 0 \\ -\frac{1}{R_1} & 0 & \frac{1}{R_1} \end{bmatrix} \begin{bmatrix} U_{v1} \\ U_{v2} \\ U_{v3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_g \end{bmatrix}$$

Alketa $\underline{Y}_v(j\omega)$
vozliščna admitančna matrika

I_g - veh. vozl. tok. virov
35

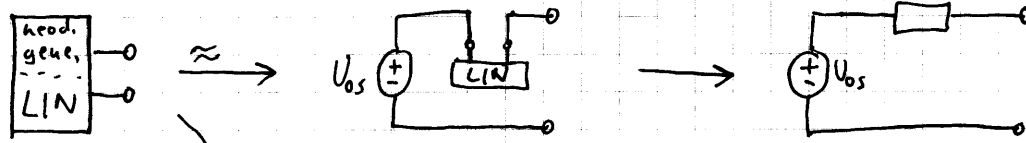
- opomba: za RLC vezju sta $Z_z(j\omega)$ in $Y_v(j\omega)$ simetrični.

5.4. Enovhodna vezja - dvopoli

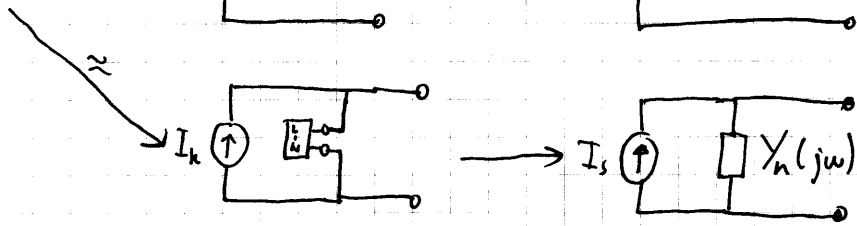


$$\begin{cases} -m, & x > 0 \\ +n, & x < 0 \end{cases}$$

Theveninov in Nortonov teorem dvopol



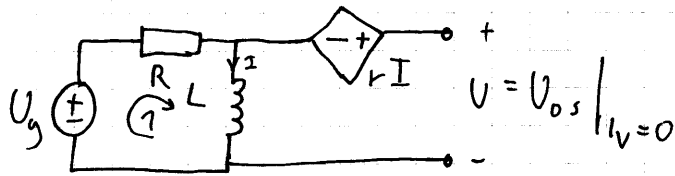
$$Z_n(j\omega) = R(\omega) + jX(\omega)$$



$$Y_n(j\omega) = G(\omega) + jB(\omega)$$

$$\begin{cases} +n & B > 0 \\ -m & B < 0 \end{cases}$$

- zgled: Thev. in Nort. dvopol



- opomba

- $Z_n(j\omega) = \frac{1}{Y_n(j\omega)}$
- $U_{os} = Z_n(j\omega) \cdot I_k$

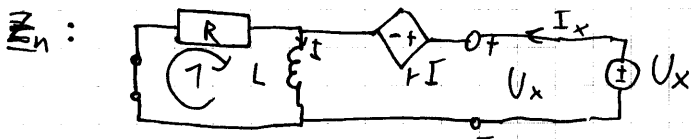
• Theveninov dvopol

$$U_{os} = rI + j\omega LI$$

$$\textcircled{1} -U_y + RI_{z1} + j\omega LI_{z1}; I_{z1} = I$$

$$I = I_{z1} = \frac{U_y}{R + j\omega L}$$

$$U_{os} = (r + j\omega L)I = \frac{r + j\omega L}{R + j\omega L} U_y = |U_{os}| e^{j\phi}$$



$$Z_n(j\omega) = \frac{U_x}{I_x} \quad U_x = rI + j\omega LI$$

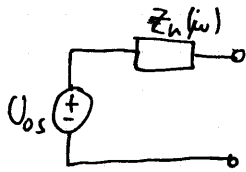
$$\textcircled{1} RI_{z1} + j\omega L(I_{z1} + I_x) = 0$$

" $U_x = aI_x + bI_x$ "

$$I_{z1} = \frac{-j\omega L}{R + j\omega L} I_x$$

$$U_x = (r + j\omega L) I = (r + j\omega L) (I_x - I_x \frac{j\omega L}{r + j\omega L}) / I_x$$

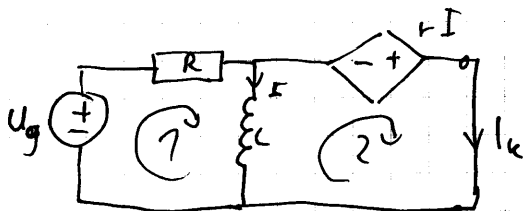
$$Z_n(j\omega) = \frac{(r + j\omega L) R}{R + j\omega L} = R(\omega) + jX(\omega)$$



• Nortonov dvopol

$$Y_n(j\omega) = \frac{1}{Z_n(j\omega)}$$

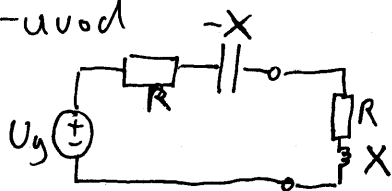
• I_k



$$I_k = \underbrace{\frac{(r + j\omega L)}{(R + j\omega L)} U_g}_{U_{0s}} \underbrace{\left(\frac{r + j\omega L}{R + j\omega L} R \right)^{-1}}_{Z_n(j\omega)^{-1}} = \frac{U_g}{R} ?$$

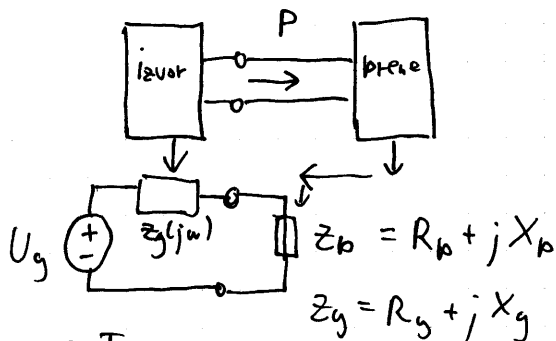
Teorem o maksim. prenosu moći

- uvod



$$P = \frac{1}{2} U_0 I_0 \cos(\varphi_n - \varphi_l)$$

- splotno



• Teorem

$$P = P(R_b, X_b)$$

$$\frac{\partial P}{\partial R_b} = 0, \quad \frac{\partial P}{\partial X_b} = 0$$

$$P = \text{Re}\left(\frac{1}{2} U_b I^*\right)$$

$$R_b = R_g \quad X_b = -X_g$$

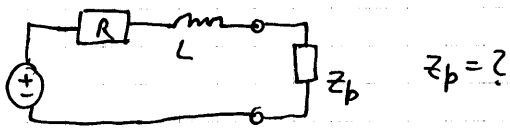
$$Z_b = -Z_g$$

- pri pogojih:

(i) R_p ni mogoče izbrati $X_p = -X_g$

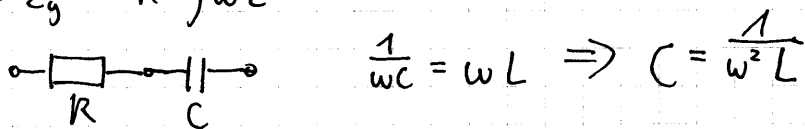
(ii) X_p ni mogoče izbrati $R_p = |Z_g| = \sqrt{R_g^2 + X_g^2}$

- zglej

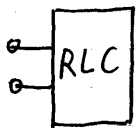


$$Z_g = R + j\omega L \quad X_p < 0$$

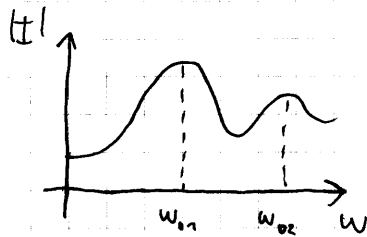
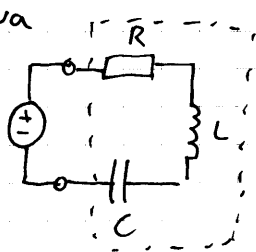
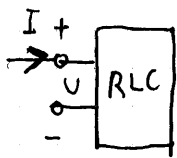
$$Z_p = Z_g^* = R - j\omega L$$



Resonanca



Res.: tokova



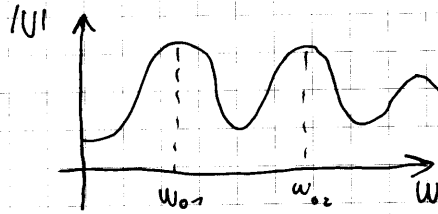
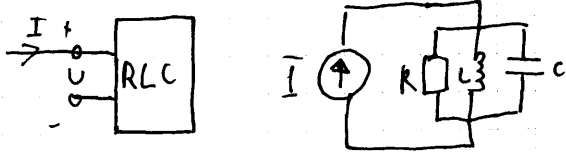
- razmere: U vsiljajen, da $|U| = \text{konst.}$, opazujem $|I|$

- izračun ω_0 : približne formule za ω_0 :

$$|I| = \frac{|U|}{|Z|} = \frac{|U|}{\sqrt{R^2(\omega) + X^2(\omega)}}$$

$$\text{Im}(Z(j\omega)) = X(\omega) = 0$$

Res.: napetostna



- razhete: I vsiljujem, |I| = konstanten, opazujem |U|

- prib. formula

$$|U| = \frac{|I|}{|Y|} = \frac{|I|}{\sqrt{G^2(\omega) + B^2(\omega)}}$$

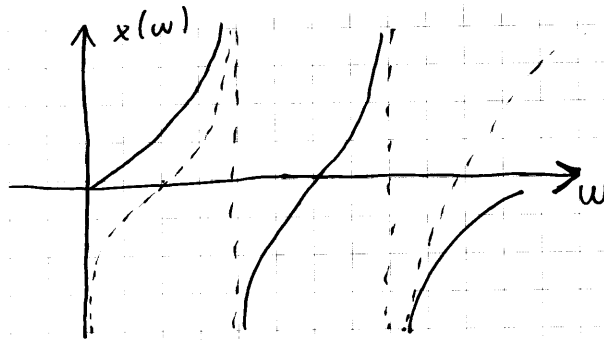
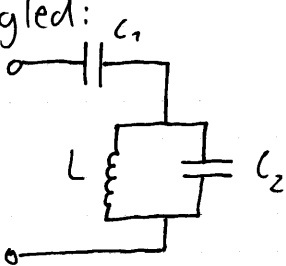
$$\text{Im } Y(j\omega) = B(\omega_0) = 0$$

- opomba: Približne formule kažejo, da sistem prvega reda nima resonančnih frekvenc.

Resonanca: LC vezja

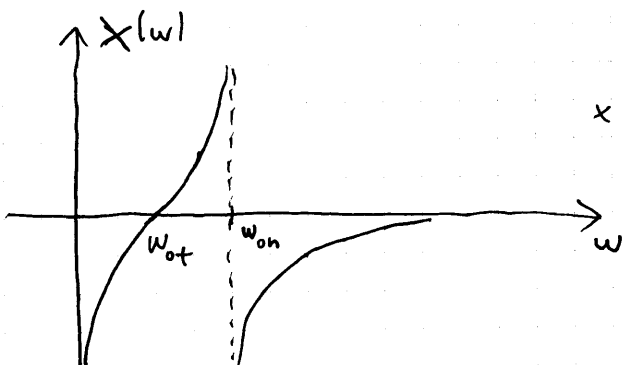
- velja: $\frac{dX(\omega)}{d\omega} > 0$ kjer obstaja

- zgled:



- tok. Res.:

$$Z = \frac{1}{j\omega C_1} + \frac{j\omega L \cdot \frac{1}{j\omega C_2}}{j\omega L + \frac{1}{j\omega C_2}} \cdot j\omega C_2 = j \underbrace{\left(\frac{\omega L}{1 - \omega^2 L C_2} - \frac{1}{\omega C_1} \right)}_{X(\omega)}$$



$$X(\omega) = \frac{\omega L}{1 - \omega^2 L C_2} - \frac{1}{\omega C_1} = 0$$

$$\omega_0^2 \cdot L C_1 = 1 - \omega_0^2 L C_2$$

$$\omega_{0f} = \frac{1}{\sqrt{L(C_1 + C_2)}}$$

- nap. res.:

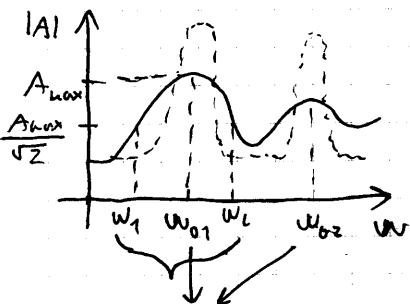
$$Y(j\omega) = \frac{1}{jX(\omega)}$$

$$B(\omega) = \frac{\omega C_1 (1 - \omega^2 LC_2)}{\omega^2 LC - (1 - \omega^2 LC_2)}$$

$$\omega_0 = 0$$

$$\omega_{0n} = \frac{1}{\sqrt{LC_2}}$$

Kvaliteta



A... opazovan odziv

$$20 \log \frac{1}{\sqrt{2}} = -3 \text{ dB}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

- pribl. formule za $Q(\omega_0)$

• energijska

$$Q(\omega_0) = \omega_0 \frac{\sum W_c(\omega_0)}{\sum P(\omega_0)}$$

maks. energije tuljiv
= $\omega_0 \frac{\sum W_c(\omega_0)}{\sum P(\omega_0)}$
- delovna moč upora

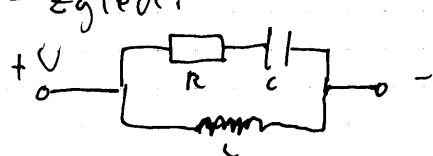
Samo v res.
freh. velju
enakost.

• imi tančne formule:

- impedančne: $Q(\omega_0) = \frac{X_c(\omega_0)}{R(\omega_0)} = - \frac{X_c(\omega_0)}{R(\omega_0)}$

- admittančne: $Q(\omega_0) = \frac{B_c(\omega_0)}{G(\omega_0)} = - \frac{B_c(\omega_0)}{G(\omega_0)}$

- zgled:



nap. res. in kval.

• nap. res.:

$$Y(j\omega) = \frac{1}{j\omega L} + \frac{1}{R + \frac{1}{j\omega C}} = \frac{1}{j\omega L} + \frac{j\omega C}{1 + j\omega RC} = \frac{\omega^2 RC^2}{1 + (\omega RC)^2} + j \left(\frac{\omega C}{1 + (\omega RC)^2} - \frac{1}{\omega L} \right)$$

$B(\omega)$

$$B(\omega) = \frac{\omega C}{1 + (\omega RC)^2} - \frac{1}{\omega L} = 0$$

$$\omega_0^2 LC = 1 + (\omega_0 RC)^2$$

$$\omega_0 = \frac{1}{\sqrt{LC - (RC)^2}}$$

okvaliteta

$$Q(\omega_0) = \omega_0 \frac{\sum W_L(\omega_0)}{\sum P(\omega_0)}$$

- $\sum P(\omega_0)$:

$$P(\omega_0) = \frac{1}{2} R \cdot |I_R|^2 = \frac{1}{2} R \left| \frac{U}{R + j\omega_0 C} \right|^2 = \left| \frac{j\omega_0 C U}{1 + j\omega_0 RC} \right|^2 = \frac{1}{2} R \frac{(\omega_0 C)^2 |U|^2}{1 + (\omega_0 RC)^2}$$

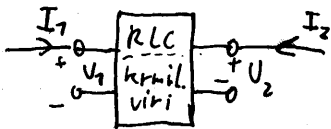
- $\sum W_L(\omega_0)$:

$$W_L(\omega_0) = \frac{1}{2} L |I_L|^2 = \frac{1}{2} L \left| \frac{U}{j\omega_0 L} \right|^2 = \frac{1}{2} L \frac{|U|^2}{(\omega_0 L)^2}$$

$$Q(\omega_0) = \omega_0 \frac{\frac{1}{2} L \frac{|U|^2}{(\omega_0 L)^2}}{\frac{1}{2} R \frac{(\omega_0 C)^2 |U|^2}{1 + (\omega_0 RC)^2}} = \omega_0 R \frac{1 + (\omega_0 RC)^2}{(\omega_0 L)^2 (\omega_0 C)^2}$$

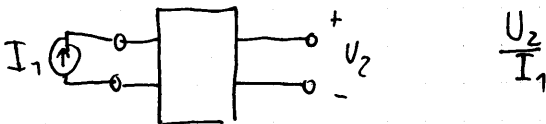
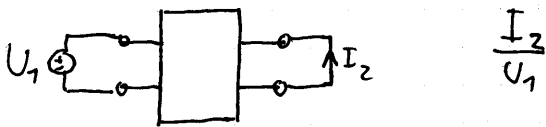
5.5. Dvovhodna vezja - četveropoli

Uvod



Teorem o recipročnosti

- recipročnost



- Teorem: RLC četveropoli so recipročni

Simetričnost

četrupol je simetričen, če se pri menjavi vhodov ohranijo vse sistemske funkcije.

- simet. \Rightarrow reci.

četrupolni parametri

$$\begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} P \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix}$$

$2 \times P$

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} = 6$$

- četrupolni parametri so enolično določeni, če obstajajo.

P. d. v.: impedančni parametri (param. dvovhod. vezij)

$$\begin{aligned} U_1 &= z_{11} I_1 + z_{12} I_2 \\ U_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \quad \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \underbrace{\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}}_Z \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$z_{11} = \left. \frac{U_1}{I_1} \right|_{I_2=0} \quad z_{12} = \left. \frac{U_2}{I_2} \right|_{I_1=0}$$

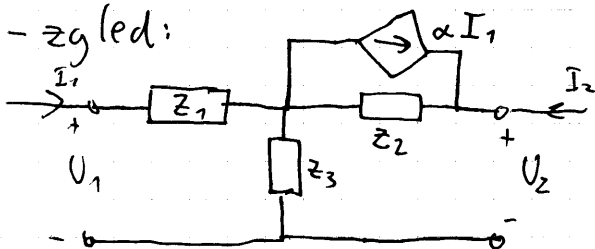
Param. z_{11} je impedanca 1. vhoda pri odprtju 2. vhoda.

$$z_{21} = \left. \frac{U_2}{I_1} \right|_{I_2=0} \quad z_{22} = \left. \frac{U_2}{I_2} \right|_{I_1=0}$$

- recipročni: $z_{12} = z_{21}$

- simetrični: $z_{12} = z_{21}$ & $z_{11} = z_{22}$

- zgled:



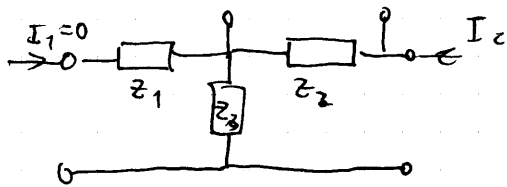
Či $I_2 = 0$:

$$z_{11} = \frac{U_1}{I_1} = z_1 + z_3$$

$$z_{21} = \left. \frac{U_2}{I_1} \right|_{I_2=0} \Rightarrow U_2 = \alpha I_1 z_2 + I_1 z_3 \quad /: I_1$$

$$\downarrow (U_2 = aI_1 + bI_2)$$

$$\underline{z_{21} = \alpha z_2 + z_3}$$

(ii) $I_1 = 0$:

$$z_{12} = \frac{U_1}{I_2} \Rightarrow U_1 = z_1 \cdot 0 + z_3 I_2 \quad /: I_2$$

$$\underline{z_{12} = z_3}$$

$$z_{22} = \frac{U_2}{I_2} = z_2 + z_3$$

$$z = \begin{bmatrix} z_1 + z_3 & , & z_3 \\ \alpha z_2 + z_3 & , & z_2 + z_3 \end{bmatrix}$$

• rec.: $z_{12} = z_{21}$

$$\alpha z_2 + z_3 = z_3$$

$$\underline{\alpha = 0}$$

• sim.: $\alpha = 0$ & $z_{11} = z_{22}$

$$z_1 + z_3 = z_2 + z_3$$

$$\underline{z_1 = z_2}$$

▮ P.d.v.: admitančni parametri

$$I_1 = y_{11} U_1 + y_{12} U_2$$

$$I_2 = y_{21} U_1 + y_{22} U_2$$

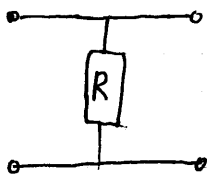
$$y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

- recipročnost: $y_{22} = y_{21}$

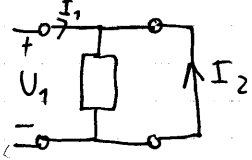
- simetričnost: recip. & $y_{11} = y_{22}$

- velja: $y = z^{-1}$, $z = y^{-1}$ ce obstaja

- zgled



(i) $U_2 = 0$:



$$y_{11} = \frac{I_1}{U_1} \Rightarrow \text{ne obstajajo}$$

imped. par. $Z = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$ ~~Z^{-1}~~ (ne obstaja Z^{-1})

▣ P. d. v.: verižni a in b

$$U_1 = a_{11} U_2 - a_{12} I_2$$

$$I_1 = a_{21} U_2 - a_{22} I_2$$

deter. $\Delta a = 1$

reci.

$$U_2 = b_{11} U_1 - b_{12} I_1$$

$$I_2 = b_{21} U_1 - b_{22} I_1$$

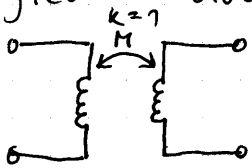
$$\Delta b = 1$$

reci. & $a_{11} = a_{22}$

simet.

reci. & $b_{11} = b_{22}$

- zglej: določiti a-par, sim., reci.



$$n = \frac{N_2}{N_1} = \frac{U_2}{U_1} = -\frac{I_1}{I_2}$$

(i) $I_2 = 0$;

$$a_{11} = \frac{U_2}{U_1} = \frac{1}{2}$$

$$a_{21} = \frac{I_1}{U_2} = \frac{0}{U_2} = 0 \text{ S}$$

(ii) $U_2 = 0$;

$$a_{12} = -\frac{U_1}{I_2} = -\frac{0}{I_2} = 0 \Omega$$

$$a_{22} = -\frac{I_1}{I_2} = n$$

$$\begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

• reci.:

$$\Delta a = a_{11} a_{22} - a_{12} a_{21} = \frac{1}{n} n - 0 = 1$$

• sim.:

rec. & $a_{11} = a_{22}$

$$\frac{1}{n} = n$$

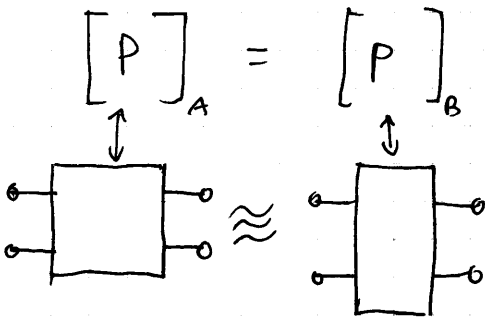
$$n = 1$$

▣ P.d.v.: hibridni h in g .

	recip.	sim.
$U_1 = h_{11} I_1 + h_{12} U_2$ $I_2 = h_{21} I_1 + h_{22} U_2$	$h_{12} = -h_{21}$	$\Delta h = 1$
$U_2 = g_{11} U_1 + g_{12} I_2$ $I_1 = g_{21} U_1 + g_{22} I_2$	$g_{12} = -g_{21}$	$\Delta g = 1$

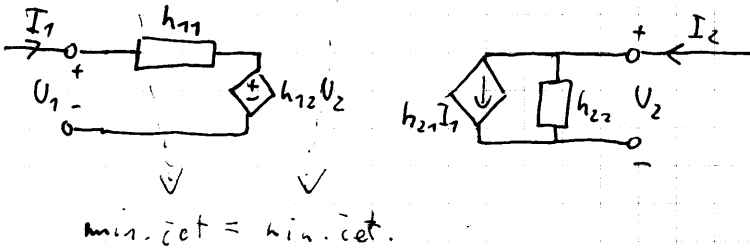
$h = g^{-1}$, $g = h^{-1}$ če \exists . ^{obstajajo}

▣ Ekvivalentna dvovhodna vezja

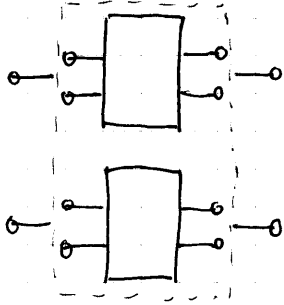


- minimalni četverpol za dane parametre

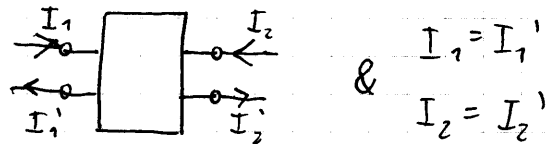
• za h par.



Združevanje dvovodnih vezij



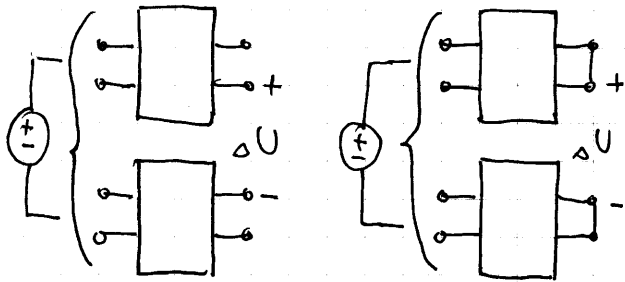
- tokovo ravnovesje



& $I_1 = I_1'$
 $I_2 = I_2'$

četrtopolni par. velja le v tokovem ravnovesju.

- Brunejev test



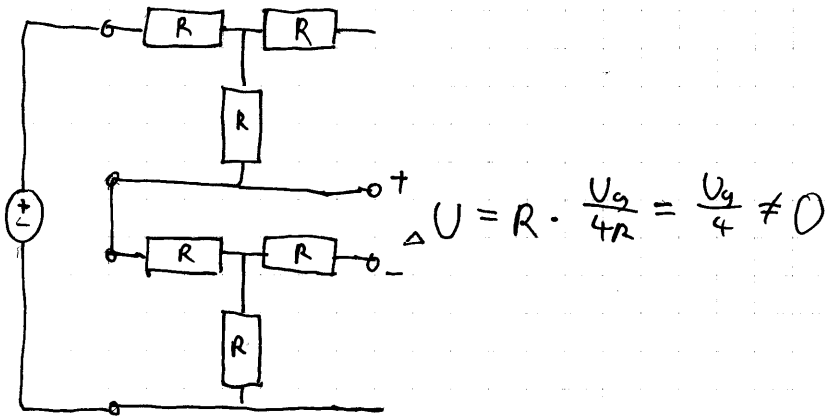
a.) zap.

b.) vzp.

test uspe, če je $\Delta U = 0$

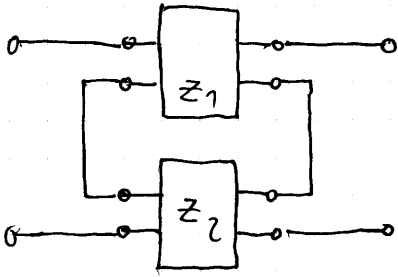
test uspe, če je $\Delta U = 0$

- zgled



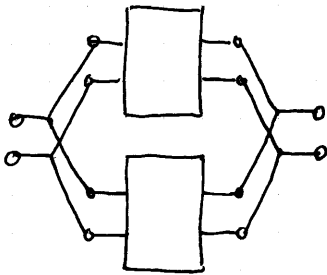
$$\Delta U = R \cdot \frac{U_0}{4R} = \frac{U_0}{4} \neq 0$$

- vezave:
- zaporedna



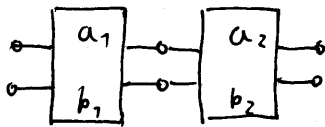
$z = z_1 + z_2$, če velja tokova ravnovesje
 \uparrow
 2×2

- veporedna



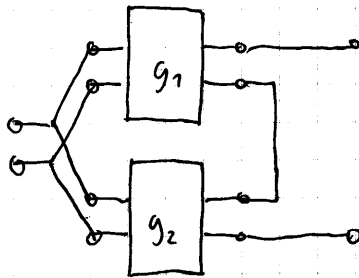
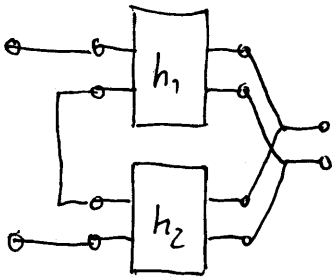
$y = y_1 + y_2$, če velja tok. ravn.

- verižna



$a = a_1 \cdot a_2$, $b = b_1 \cdot b_2$, če...

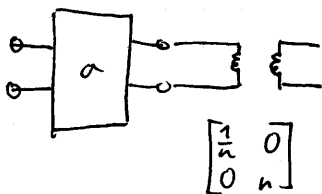
- hibridna h in g



$h = h_1 + h_2$, če...

$g = g_1 + g_2$, če...

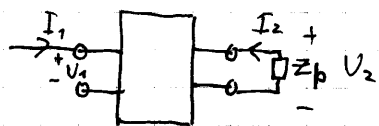
- zglede



Določi trans. toko, da ob dani vezavi ne bo spremenil delovanja četverpola

$a = a \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix} = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow a = 1$

III Vhodna impedanca in transformacija impedance premera



$$Z_V = \frac{U_1}{I_1} \text{ pri obrem. 2. vhodu.}$$

$$Z_V = Z_V(Z_p)$$

- izrazimo $Z_V = Z_V(Z_p)$ z imp. parametri

$$\left. \begin{aligned} U_1 &= z_{11} I_1 + z_{12} I_2 \\ U_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \right\} I_2 = \dots I_1$$

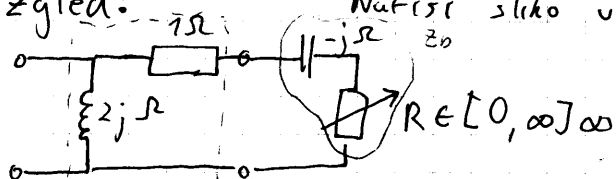
$$U_2 = -z_b I_2$$

$$Z_V = \frac{U_1}{I_1} = \frac{z_{11} I_1 + z_{12} I_2}{I_1} = z_{11} - z_{12} z_{21} \frac{1}{z_{22} + z_b}$$

$Z_V = Z_V(Z_b)$ je kompozitum

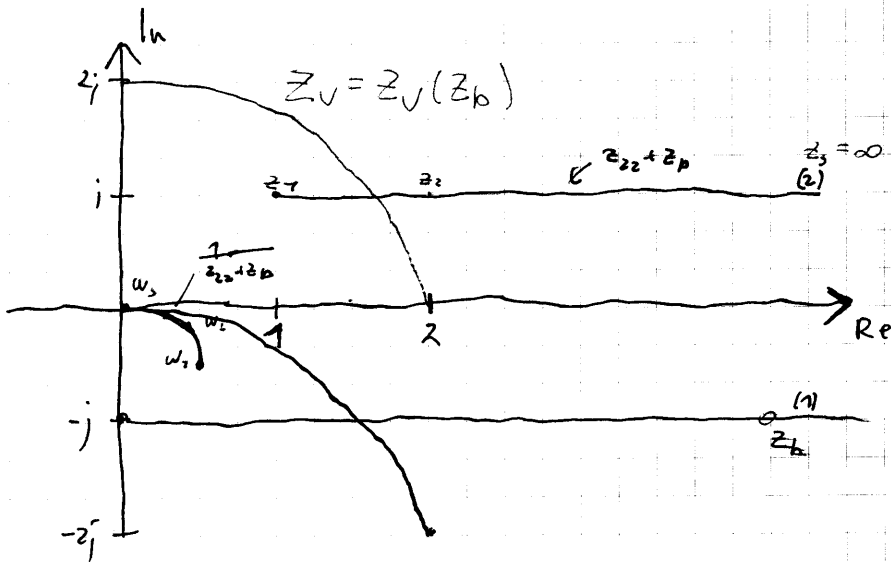
- 1.) Translacije
- 2.) Invertiranja
- 3.) Rotacije in skaliranja
- 4.) Translacije

- zglej: *Nariši sliko vhodne impedance.*



• imp. par.: $\begin{bmatrix} 2j & 2j \\ 2j & 1+2j \end{bmatrix}$

$$\bullet Z_V = Z_V(Z_b) = 2j - (2j)^2 \cdot \frac{1}{1+2j+Z_b} = 2j + 4 \frac{1}{1+2j+Z_b}$$



Potek:

- slika bremena (1)
- translucija (2)
- $z \rightarrow z_0 + z$
- invertiranje $\frac{1}{z} = \frac{1}{|z|} \cdot e^{-j\varphi}$

$$z_1 = 1+j \rightarrow w_1 = \frac{1}{1+j} = \frac{1-j}{2}$$

$$z_2 = 2+j \rightarrow w_2 = \frac{1}{2+j} = \frac{2-j}{5}$$

$$z_3 = \infty \rightarrow w_3 = 0$$

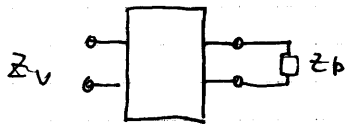
- rot. + skal.

$$z \rightarrow z_0 \cdot z = |z_0||z| \cdot e^{j(\varphi_0 + \varphi)}$$

$$4 = 4 \cdot e^{j0}$$

- translucija

Vhodna impedanca: karakteristična impedanca za simetrične četverpole



$$Z_V = Z_V(Z_b)$$

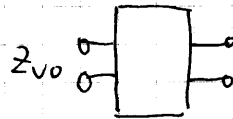
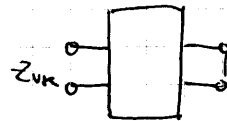
$$Z_K = Z_V \Big|_{Z_b = Z_K}$$

Impedanca bremea z lastnostjo, da se preslika vase na vhod.

- izračun

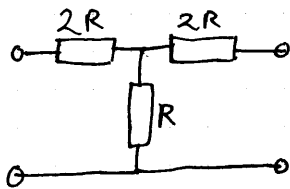
$$Z_K = Z_{11} - Z_{12} Z_{21} \frac{1}{Z_{22} + Z_K} \Big|_{\substack{Z_{12} = Z_{21} \\ Z_{11} = Z_{22}}}$$

$$Z_K = \pm \sqrt{\Delta Z} = \pm \sqrt{\frac{a_{12}}{a_{21}}} = \pm \sqrt{Z_{V0} \cdot Z_{VK}}$$



Transformacije

- zglede



• s četveropolnimi parametri

$$z = \begin{bmatrix} 3R & R \\ R & 3R \end{bmatrix}$$

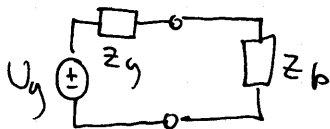
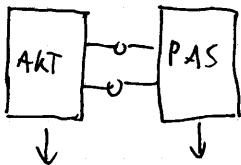
$$z_k = \pm \sqrt{\Delta} z = \pm \sqrt{8R^2} = 2\sqrt{2} R$$

$$\bullet \pm \sqrt{z_{VHK} \cdot z_{VHO}} = \pm \sqrt{\frac{8R}{3} \cdot 3R} = 2 \cdot \sqrt{2} R$$

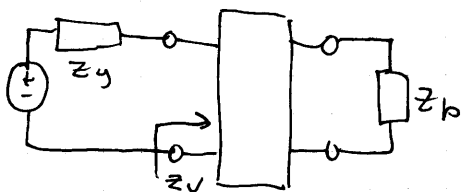
$$z_{VHO} = 3R$$

$$z_{VHK} = 2R + \frac{2R \cdot R}{2R + R} = \frac{8R}{3}$$

Vhodna impedanca: imped. prilagajanje



$$z_b \neq z_g^*$$



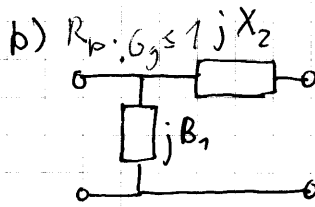
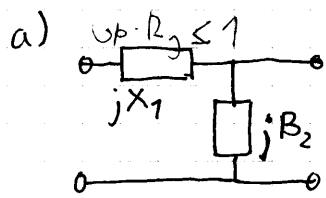
Od prilag. čet. pričakujemo:

- prilagodi breme:

$$z_v = z_g^*$$

- nič ne porabi zase: ~~R~~, LC

- kar se da preprost: 2. elementa



- izračun:

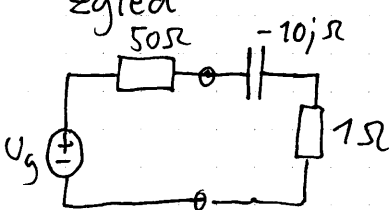
a) $B_2 = \pm \sqrt{\frac{G_p}{R_g} (1 - G_p R_g) - B_p}$

$X_1 = \pm \sqrt{\frac{R_g}{G_p} (1 - G_p R_g) - X_g}$

b.) $B_1 = \pm \sqrt{\frac{G_g}{R_b} (1 - R_b G_g) - B_g}$

$X_2 = \pm \sqrt{\frac{R_p}{G_g} (1 - R_p G_g) - X_p}$

- zglede



Prilagodi dano breme na dan generator.

$Z_g = 50 \Omega = R_g + jX_g$

$Z_p = 1 - 10j \Omega = R_p + jX_p \quad Z_p \neq Z_g^*$

• izbira: $R_g = 50$

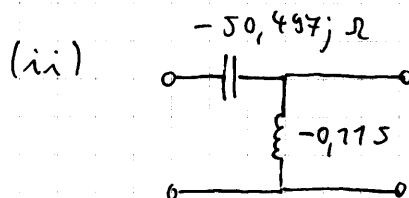
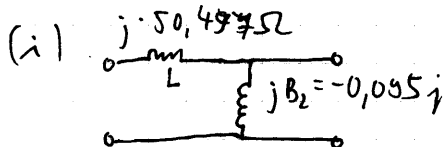
$G_p = \operatorname{Re} \left(\frac{1}{1 - 10j} \right) = \operatorname{Re} \left(\frac{1 + 10j}{101} \right) = \boxed{\frac{1}{101} \text{ S}}$

$G_p \cdot R_g = \frac{50}{101} \leq 1$

• določiti:

$B_2 = \begin{matrix} \text{(i)} \rightarrow -0,09 \text{ S} \\ \text{(ii)} \rightarrow -0,11 \text{ S} \end{matrix}$

$X_1 = \begin{matrix} \text{(i)} \rightarrow 50,497 \Omega \\ \text{(ii)} \rightarrow -50,497 \Omega \end{matrix}$



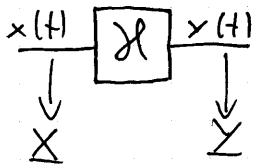
- opomba:

$$\omega L = 50,457$$

$$L = \frac{50,457}{\omega}$$

• prilaganje je frekvenčno odvisno!

~~Prevalne~~ Prevalne lastnosti: prevajalna funkcija



$$H(j\omega) = \frac{Y}{X} \text{ - prevajalna funkcija}$$

$$Y = H(j\omega) \cdot X; \quad H(j\omega) = |H(j\omega)| \cdot e^{j\theta(\omega)}$$

$$|Y| = |H(j\omega)| \cdot |X|; \quad x = |X| \cdot e^{j\phi_x}; \quad y = |Y| \cdot e^{j\phi_y}$$

$$\phi_y = \theta(\omega) + \phi_x$$

- amplitudni odziv

- fazni odziv

- ojačanje:

$$A_{\text{Np}}(\omega) = \ln |H(j\omega)|$$

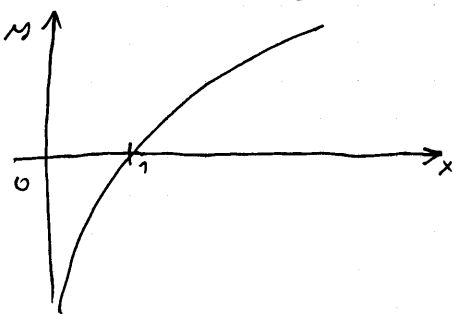
Neper

$$A_{\text{dB}}(\omega) = 20 \cdot \log_{10} |H(j\omega)|$$

- slabljenje:

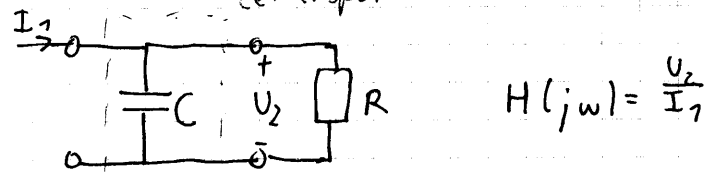
$$\alpha_{\text{Np}}(\omega) = \ln \frac{1}{|H(j\omega)|} = -A_{\text{Np}}(\omega) \quad \alpha_{\text{dB}}(\omega) = 20 \cdot \log_{10} \frac{1}{|H(j\omega)|} = -A_{\text{dB}}(\omega)$$

$$1 \text{ Np} = 20 \cdot \log_{10} e \text{ dB} = 8,686 \text{ dB}$$



- zgleđ: Določi amplitudni in fazni odziv.

četverpol



$$H(j\omega) = \frac{U_2}{I_1}$$

$$U_2 = I_1 \cdot \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = I_1 \cdot \frac{R}{1 + j\omega RC} \quad /: I_1$$

$$H(j\omega) = \frac{R}{1 + j\omega RC}$$

$$\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

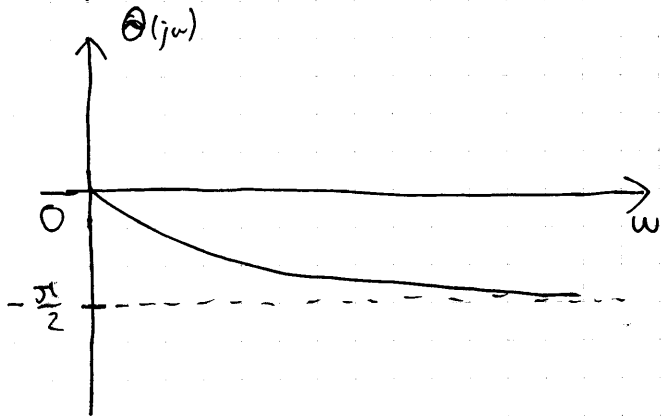
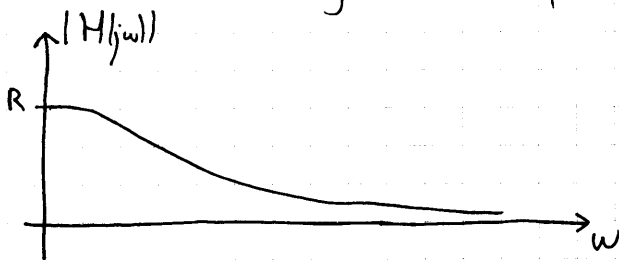
$$|H(j\omega)| = \frac{R}{\sqrt{1 + (\omega RC)^2}} \quad \text{- apl. odziv}$$

$$\Theta(\omega) = \text{Arg}(H(j\omega))$$

$$= 0 - \arctg(\omega RC) \quad \text{- faz. odziv}$$

$$\text{Arg}\left(\frac{z_1}{z_2}\right) = \text{Arg}(z_1) - \text{Arg}(z_2)$$

$$\frac{z_1}{z_2} = \frac{|z_1| \cdot e^{j\varphi_1}}{|z_2| \cdot e^{j\varphi_2}} = \frac{|z_1|}{|z_2|} e^{j(\varphi_1 - \varphi_2)}$$



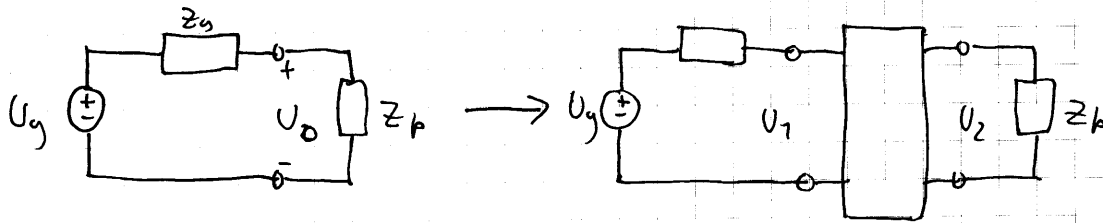
- opomba:

$$i_1(t) = x(t) = X_0 \cos(\omega t + \varphi_x)$$

$$u_2(t) = y(t) = X_0 \underbrace{\frac{R}{\sqrt{1 + (\omega RC)^2}}}_{|H(j\omega)|} \cdot \cos\left(\omega t + \varphi_x - \overbrace{\arctg(\omega RC)}^{\Theta(\omega)}\right)$$

$$\omega \left(t + \underbrace{\frac{\varphi_x}{\omega} - \frac{1}{\omega} \arctg(\omega RC)}_{t_0} \right)$$

Prevaljalne lastnosti: vstavitevna prevajalna funkcija



$$H_V(j\omega) = \frac{U_2}{U_o}$$

- vstavitevno ojačanje

$$A_{VNP} = (\ln |H_V(j\omega)|) \quad A_{VdB} = 20 \log |H_V(j\omega)|$$

- vstavitevno ojačanje je podano z razmerjem moči

$$\frac{P_2}{P_0} = \frac{\frac{|U_2|^2}{2R_b}}{\frac{|U_o|^2}{2R_b}} = \left| \frac{U_2}{U_o} \right|^2 = |H_V(j\omega)|^2$$

$$A_{VNP} = (\ln |H_V(j\omega)|) = \frac{1}{2} (\ln \frac{P_2}{P_0})$$

$$A_{VdB} = 10 \log \frac{P_2}{P_0} = 10 \log \frac{|U_2|^2}{|U_o|^2} = 20 \log \left| \frac{U_2}{U_o} \right| = 20 \log |H_V(j\omega)|$$

- vstavitvena $H_V(j\omega)$ je dana s četv. par.

$$U_o = U_g \frac{Z_b}{Z_g + Z_b}$$

$$U_2: \left. \begin{aligned} U_1 &= z_{11} I_1 + z_{12} I_2 \\ U_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \right\}$$

$$U_g = z_g I_1 + U_1$$

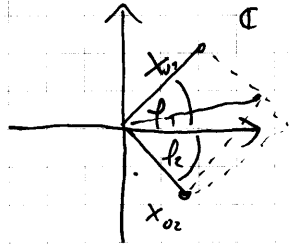
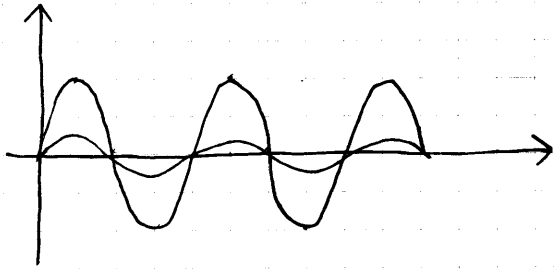
$$U_2 = -z_b I_2$$

$$U_2 \rightarrow \frac{U_2}{U_o} = H_V(j\omega)$$

5. Analiza s spektri

5.1. Fourierjeva trigonometrična vsota

Kaj je spekter?

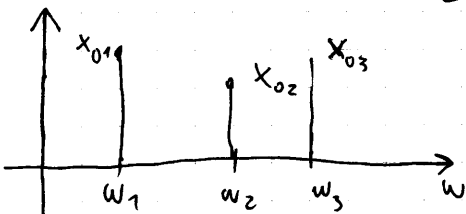


$$x_1(t) + x_2(t) = X_{01} \cdot \cos(\omega_1 t + \phi_1) + X_{02} \cdot \cos(\omega_2 t + \phi_2)$$

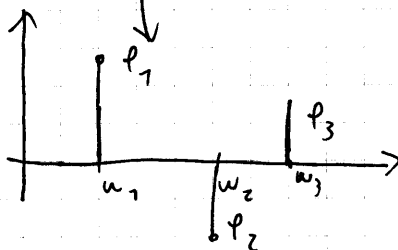
$\omega_1 = \omega_2$
↓
Je HARM. SIGNAL

$\omega_1 \neq \omega_2$
↓
NI HARM. SIGNAL

PREHOD SE IMENUJE FOURIERJEVA VASTA



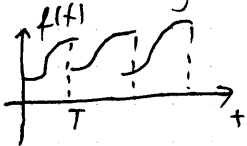
Amplitudni spekter



Fazni spekter

Opis, izračun koeficientov

- samo za periodične signale
- perioda: $T = \text{arg min} \{ f(T+t) = f(t) \}$
- izračun



- $T = \frac{2\pi}{\omega_1}$
- $\omega_1 = \frac{2\pi}{T}$

• cosmetna komponenta

$$a_0 = \frac{1}{T} \int_{\tau}^{\tau+T} f(t) dt$$

• za $n \geq 1$

$$a_n = \frac{2}{T} \int_{\tau}^{\tau+T} f(t) \cos(n\omega_1 t) dt$$

$$b_n = \frac{2}{T} \int_{\tau}^{\tau+T} f(t) \sin(n\omega_1 t) dt$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_1 t) + b_n \sin(n\omega_1 t))$$

3.12.08!

$$\Rightarrow a_0 = \frac{1}{T} \int_{\tau}^{\tau+T} f(t) dt$$

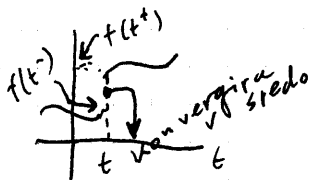
$$n \geq 1: a_n = \frac{2}{T} \int_{\tau}^{\tau+T} f(t) \cos(n\omega_1 t) dt \quad b_n = \frac{2}{T} \int_{\tau}^{\tau+T} f(t) \sin(n\omega_1 t) dt$$

$$\hat{f}(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\omega_1 t) + b_n \sin(n\omega_1 t))$$

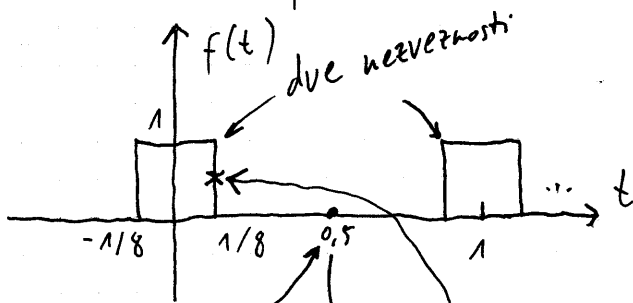
- konvergenca: Dirichletovi pogoji

- končno mnogo nezveznosti na periodo
- končno mnogo ekstremov na periodo
- $\int_{\tau}^{\tau+T} f(t) dt < \infty$

kadar so izpolnjeni, je $\hat{f}(t) = \frac{1}{2} (f(t^-) + f(t^+))$



- zgled: določi trigonometrično Furierovo vrsto in spekter danega signala.



- zgled: $\hat{f}(0,5) = 0$ $f(1/8) = \frac{1}{2}(1+0)$

• $T = 1s \rightarrow \omega_1 = \frac{2\pi}{T} = 2\pi \text{ rad/s}$; $\tau = -1/8$

• $a_0 = \frac{1}{T} \int_{\tau}^{\tau+T} f(t) dt = \int_{-1/8}^{1/8} 1 dt = \frac{1}{4}$

$$n \geq 1: a_n = \frac{2}{1} \int_{-1/8}^{1/8} 1 \cos(2\pi n t) dt =$$

$$= \left. \frac{\sin(2\pi n t)}{2\pi n} \right|_{-1/8}^{1/8} = \frac{\sin(n\pi/4) - \sin(-n\pi/4)}{2\pi n} =$$

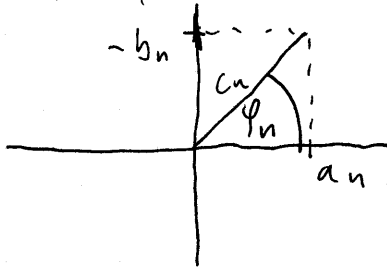
$$= \frac{2}{n\pi} \sin\left(n \frac{\pi}{4}\right)$$

$$b_n = \frac{2}{1} \int_{-1/8}^{1/8} 1 \sin(2\pi n t) dt = 2 \left. \frac{-\cos(2\pi n t)}{2\pi n} \right|_{-1/8}^{1/8} = 0$$

$$\hat{F}(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(n \frac{\pi}{4}\right) \cos(2\pi n t)$$

Amplitudni in fazni spekter

$$a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) = c_n \cos(n\omega_0 t + \varphi_n)$$



- amplitudni spekter: $\{c_n; n \geq 0\}$
 $c_0 = a_0; c_n = \sqrt{a_n^2 + b_n^2}$

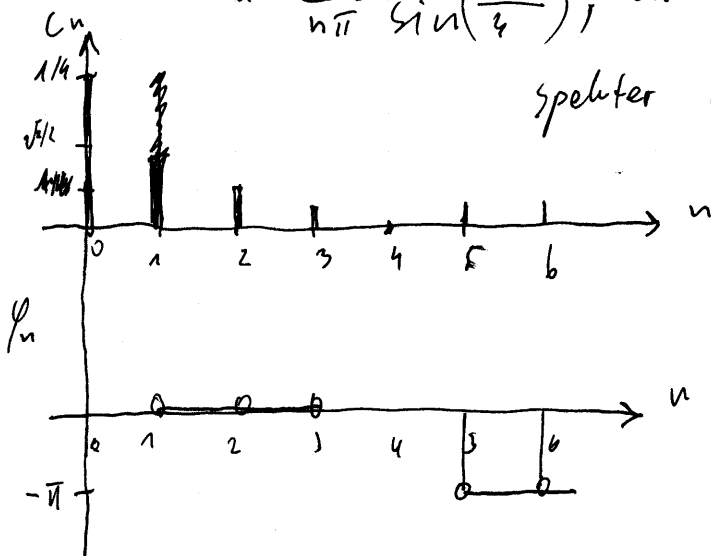
- fazni spekter: $\{\varphi_n; n > 0\}$ $\varphi_n = \text{Arg}[a_n - j b_n]$
 $\varphi_n \in [-\pi, \pi]$

- zgled: Določi fazni in amplitudni spekter za prejšnji zgled.

$$c_0 = a_0 = \frac{1}{4}$$

$$a_n = \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right); b_n = 0 \Rightarrow c_n = |a_n| = \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right)$$

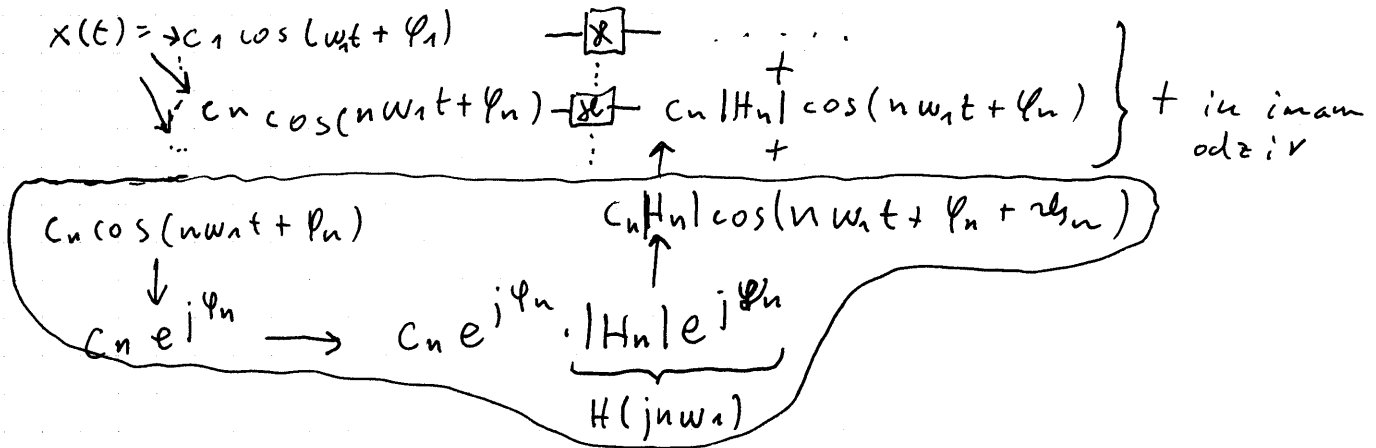
spekter upada (konvergenca)



Odziv vezja na spekter vzbujanja



1. Razvoj vzbujanja: $x(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_1 t + \varphi_n)$



2. Določi prevajalno funkcijo $H(j\omega)$

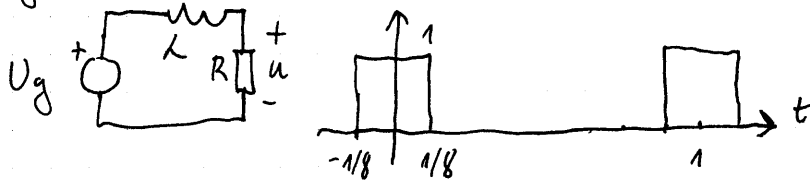
3. Odziv: $y(t) = c_0 H(0) + \sum_{n=1}^{\infty} c_n |H_n| \cos(n\omega_1 t + \varphi_n + \psi_n)$

- spekter odziva:

- amplitudi: $\{c_n |H_n|; n > 0\}$

- fazi: $\{\varphi_n + \psi_n; n > 0\}$

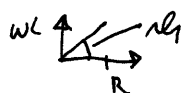
- zglej: določi spekter odziva



① $U_g(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos(2\pi n t)$

② Prev. fja.: $H(j\omega) = \frac{U}{U_g} = \frac{R}{R + j\omega L} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} \cdot e^{j \arctan \frac{\omega L}{R}}$

$U = \frac{R}{R + j\omega L} \cdot U_g$



Fazni odziv

ampl. odziv

$\psi_1(\omega)$

③ Vrsta odziva C_n od uzbujanja

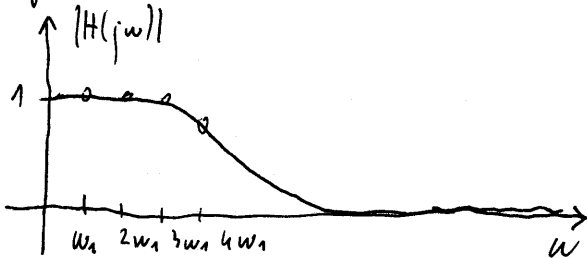
$$U_n(t) \approx \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cdot \frac{R}{\sqrt{R^2 + (2\pi nL)^2}} \cdot \cos(2\pi n t + \arctg\left(\frac{2\pi nL}{R}\right))$$

$H_n = H(jn\omega_n)$ $|H_n|$

- amplituda: $\left\{ \frac{2}{n\pi} \left(\sin\left(\frac{n\pi}{4}\right) \right) \frac{R}{\sqrt{R^2 + (2\pi nL)^2}} ; n > 0 \right\}$

- fazni spekter: $\left\{ \varphi_n + \arctg\left(\frac{2\pi nL}{R}\right) ; n > 0 \right\}$

- Opomba:



6.2. Fourierjeva eksponentna vrsta

Opis, izračun koef.

$$C_n \cos(n\omega_1 t + \varphi_n) =$$

$$= \underbrace{\frac{C_n}{2} e^{-j\varphi_n}}_{A_{-n}} e^{-jn\omega_1 t} + \underbrace{\frac{C_n}{2} e^{j\varphi_n}}_{A_{+n}} e^{jn\omega_1 t}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

eulerjeva inverzna formula

$$\hat{f}(t) = \sum_{n=-\infty}^{\infty} A_n e^{jn\omega_1 t}$$

$$A_0 = a_0 = \frac{1}{T} \int_{\tau}^{\tau+T} f(t) dt$$

$$A_n = \frac{1}{T} \int_{\tau}^{\tau+T} f(t) e^{-jn\omega_1 t} dt$$

- inter.: A_{-n}, A_n predstavljata kazalec harmonične komponente.

- konvergenca: enako, kot pri trigonometrični vrsti
- zveza med koef. trig. in eksp. vrste.

$$A_0 = a_0 = C_0$$

$$n > 0: A_n = \frac{a_n - j b_n}{2}$$

$$A_{-n} = A_n^*$$

- konvergenca:

Spekter eksponentne vrste:

- kompleksni spekter: $\{A_n; n \in \mathbb{Z}\}$

- amplitudni spekter:

$$\{|A_n|; n \in \mathbb{Z}\}$$

- fazni spekter

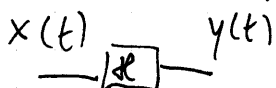
$$\{\text{Arg}(A_n) = \phi_n; n \in \mathbb{Z}\}$$

- simetrije: $|A_{-n}| = |A_n|$, soda funkcija

$\phi_{-n} = -\phi_n$, liha funkcija

Odziv vezja z eksponentno Fourierjevo vrsto

① Razvoj vzbujanja



$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega t}$$

② Prevojalna funkcija \checkmark izmenični analizi
 $H(j\omega)$

③ Vrsta odziva:

$$y(t) = \sum_{n=-\infty}^{\infty} X_n H(jn\omega_1) e^{jn\omega_1 t}$$

Glejte razlago k trigonometrični vrsti

- spekter odziva

- kompl: $\{X_n H_n; n \in \mathbb{Z}\}$

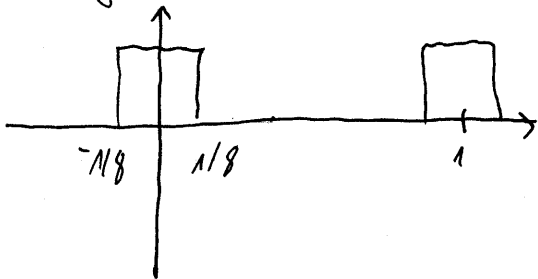
- ampl: $\{|X_n| |H_n|, n \in \mathbb{Z}\}$

- fazni: $\{\Phi_n + \varphi_n; n \in \mathbb{Z}\}$

kjer je

$$H(jn\omega_1) = |H_n| e^{j\varphi_n}$$

- zgled:



Poišči eksp. four. v. za dan signal.

- $T=1s$; $\omega_1 = \frac{2\pi}{T} = 2\pi \text{ rad/s}$

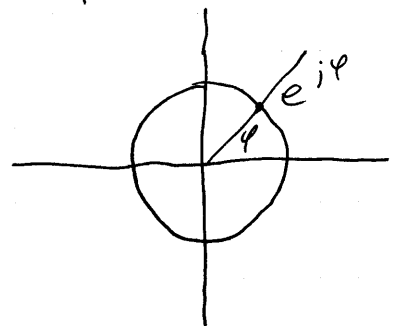
- $A_0 = a_0 = \frac{1}{4}$; $\tau = 1/8$

- $n \neq 0$; $A_n = \frac{1}{1} \int_{-1/8}^{1/8} 1 e^{jn2\pi t} dt =$

$$= \frac{e^{-jn2\pi t}}{-jn2\pi} \Big|_{-1/8}^{1/8} =$$

$$= \frac{e^{jn2\pi \cdot 1/8} - e^{-jn2\pi \cdot 1/8}}{-jn2\pi} \stackrel{\text{Euler}}{=} \frac{1}{n\pi} \sin\left(\frac{n\pi}{4}\right)$$

(a) $\int e^{at} dt = \frac{e^{at}}{a}$



- zglede 2: odziv vezja prejšnjega zgleda:

$$1) U_g(t) = \frac{1}{4} + \sum_{n \neq 0} \frac{1}{n\pi} \sin\left(\frac{n\pi}{4}\right) e^{jn2\pi t} \quad V$$

~~U_g(t)~~

$$2) H(j\omega) = \frac{R}{R + j\omega L} \quad (\text{glej prejšnji zglede})$$

$$3) U(t) = \frac{1}{4} + \sum_{n \neq 0} \frac{1}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cdot \frac{R}{R + j2\pi n h} \cdot e^{jn2\pi t}$$

frekvenco

Doloci vse tri spektre.

- zglede: $u(t) = U_0 \cos(\omega t + \varphi)$

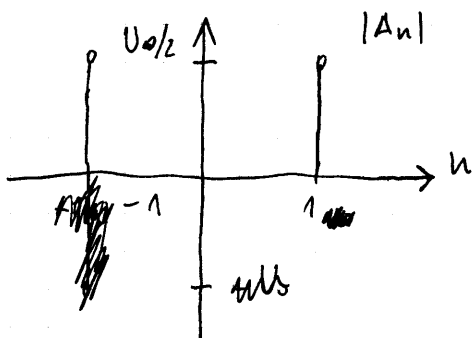
$$- T = \frac{2\pi}{\omega}, \quad \omega_1 = \frac{2\pi}{T} = \omega$$

$$- A_0 = 0V$$

$$n \neq 0 \quad A_n = \frac{1}{2\pi} \int_0^{2\pi} U_0 \cos(\omega t + \varphi) e^{-jn\omega t} dt =$$

$$= \begin{cases} \frac{U_0}{2} & ; n=1 \\ \frac{U_0}{2} & ; n=-1 \\ 0 & ; n \notin \{-1, 1\} \end{cases}$$

$$u(t) = A_{-1} e^{-j\omega t} + A_1 e^{j\omega t} = \frac{U_0}{2} e^{-j\omega t} + \frac{U_0}{2} e^{j\omega t}$$



17.12.08

- zgleđ:

$$u(t) = U_0 \cos(\omega_0 t + \varphi)$$

$$u(t) = \underbrace{\frac{U_0}{2} e^{-i\varphi}}_{A_{-1}} e^{-i\omega_0 t} + \underbrace{\frac{U_0}{2} e^{i\varphi}}_{A_1} e^{i\omega_0 t} = *$$

- z integrali: $T = \frac{2\pi}{\omega_0} \rightarrow \omega_1 = \frac{2\pi}{T} = \omega_0$

$A_0 = 0V$ povpr. vrednost signala

$A_n = 0$ za $\forall n \neq -1$ & $n \neq 1$

$$* = \sum_{n=-\infty}^{\infty} A_n e^{in\omega_1 t}$$

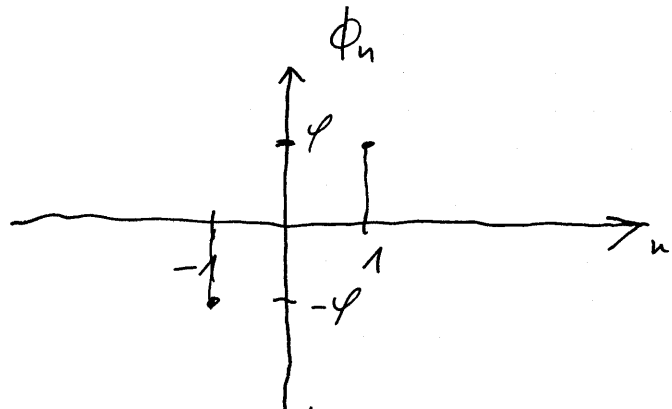
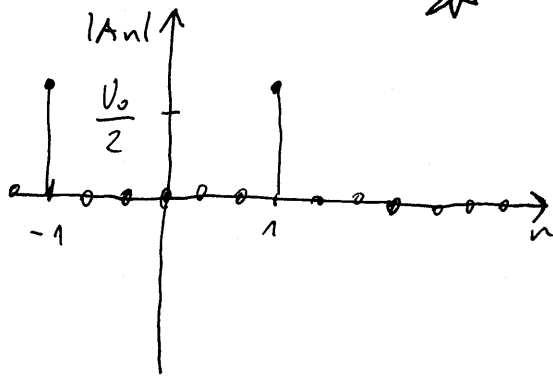
$\omega_0 = 1 \cdot \omega_1$
 $\omega_0 = \omega_1$

- spekter:

kompleksni spekter: $\left\{ \frac{U_0}{2} e^{-i\varphi}, \frac{U_0}{2} e^{i\varphi}, 0 \text{ sicer} \right\}$

amplitudni: $\left\{ \frac{U_0}{2}, \frac{U_0}{2}, 0 \text{ sicer} \right\}$

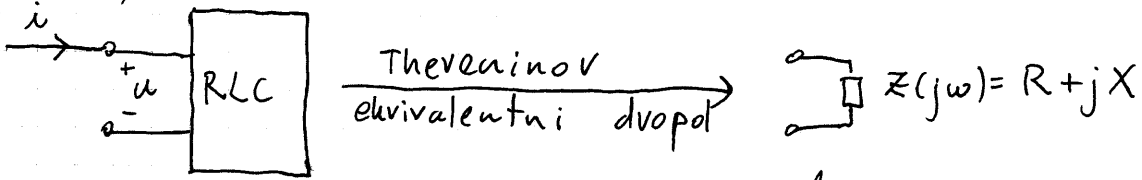
fazni: $\left\{ -\varphi, \varphi, 0 \text{ sicer} \right\}$



- kazalec signala: $U = U_0 e^{i\varphi}$

vsaka harmonična da en par (na vsaki strani en skok na diskretnem spektru)

Moc, efektivna vrednost in Parsevalov teorem

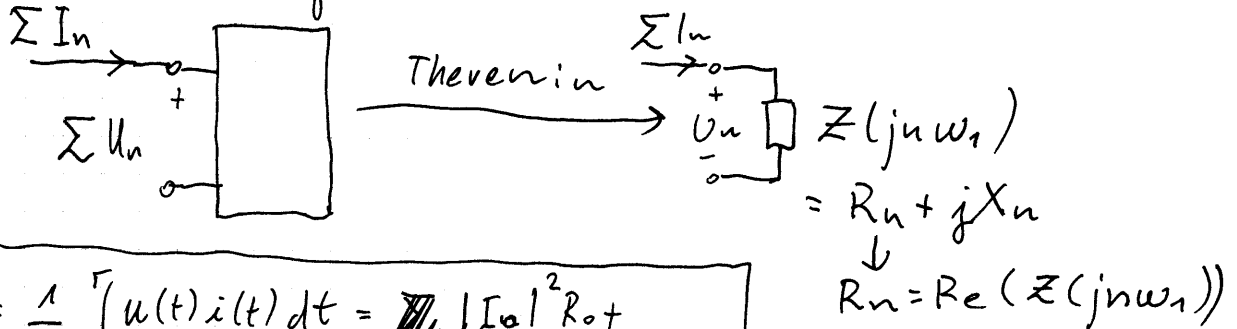


Delovna moc $P = \frac{1}{T} \int_0^T u(t) i(t) dt = \frac{1}{2} U_0 I_0 \cos(\varphi_u - \varphi_i)$

$P = \frac{1}{2} \operatorname{Re}(U I^*) = \frac{1}{2} |I|^2 R \rightarrow \frac{u^2}{R} = i^2 \cdot R$

To je vse res za harmonicne signale.

- periodicni signali:



$$P = \frac{1}{T} \int_0^T u(t) i(t) dt = \cancel{|I_0|^2 R_0} + \sum_{n \neq 0} \cancel{|I_n|^2 R_n}$$

$$= i^2 R_0 + \sum_{n=1}^{\infty} \frac{i_n^2}{2} R_n$$

- opazim: delovna moc je odvisna samo od amplitudnega spektra, nic pa od faznega.

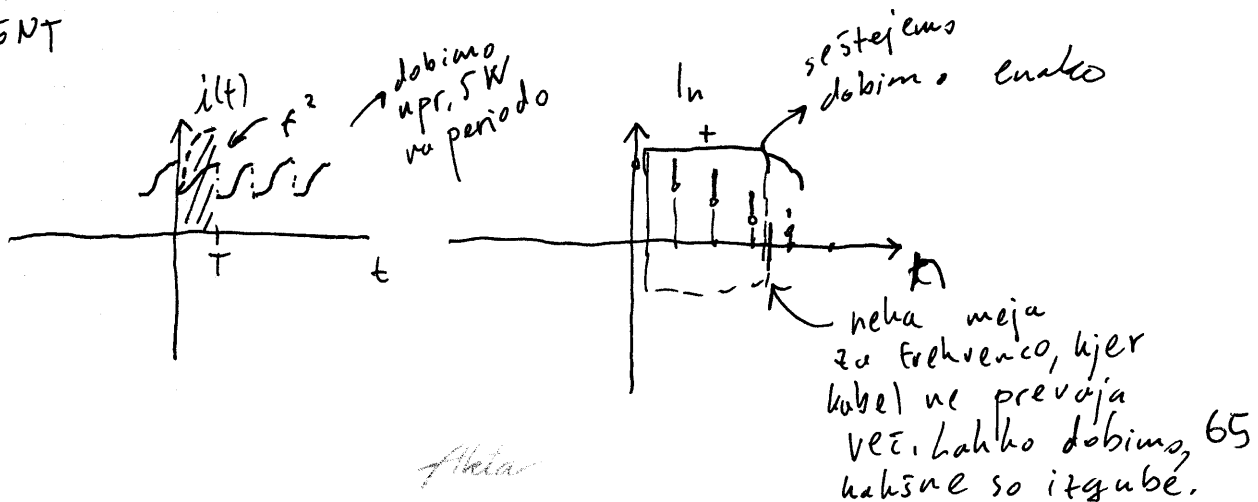
Za $R_n = 1$ in abstrakten signal $f(t)$

$$\frac{1}{2} \int_0^T f^2(t) dt = \sum_{n=-\infty}^{\infty} |F_n|^2 = f_0^2 + \sum_{n=1}^{\infty} \frac{f_n^2}{2}$$

Enako kot prej za $R=1$

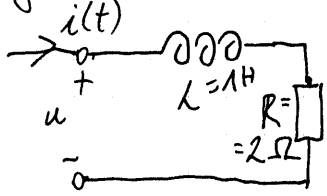
MOCI
SPEKTRALNIH
KOMPONENT

Opomba:



Alina

Zgled



$$i(t) = \underbrace{5}_{i_0} + \underbrace{7}_{i_1} \cos(\omega_1 t) + \underbrace{2}_{i_2} \cos(\omega_2 t) \quad \text{A}$$

$$Z(j\omega) = R + j\omega L$$

$$R_n = \text{Re}(Z(j\omega_n)) = R = 2 \Omega$$

$$P = 5^2 \cdot (2) + \frac{7^2}{2} \cdot (2) + \frac{2^2}{2} \cdot (2) = 103 \text{ W}$$

Parseval \downarrow R_0 R_1 R_2

$$\frac{1}{T} \int_0^T i^2(t) R dt$$

- efektivna vrednost:

- za harmoničen signal: $P = \frac{1}{2} U_0 I_0 \cos(\varphi_u - \varphi_i) : I_{ef} = \frac{I_0}{\sqrt{2}}$

- za periodičen signal:

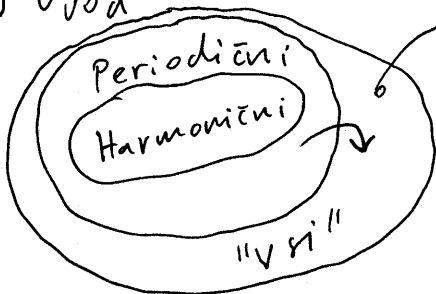
$$I_{ef} = \sqrt{\sum_{n=-\infty}^{\infty} |I_n|^2} = \sqrt{I_0^2 + \sum \frac{I_n^2}{2}}$$

- za zgled:

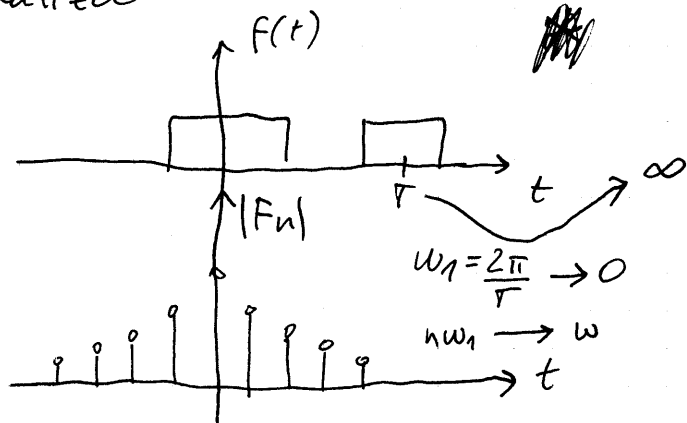
$$I_{ef} = \sqrt{5^2 + \frac{7^2}{2} + \frac{2^2}{2}} = 7,18 \text{ A}$$

6.3. Fourierjev transform

Uvod



klasična analiza



$$T \cdot A_n \rightarrow F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

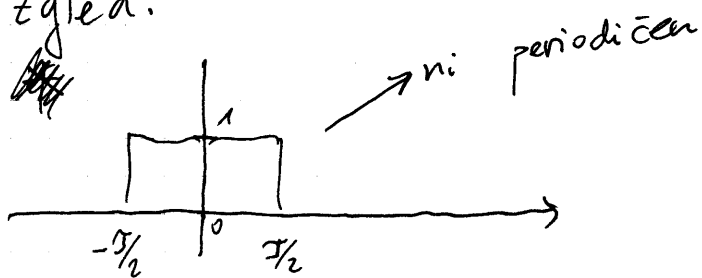
Opis, izračun in fazni prostor

$$- f(t) \leftrightarrow F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \mathcal{F}(f(t))$$

$$- \text{inverz: } \mathcal{F}^{-1}(F(j\omega)) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega = f(t)$$

- fazni prostor

- zgled:



$$F(j\omega) = \int_{-\pi/2}^{\pi/2} 1 \cdot e^{-j\omega t} dt = \left. \frac{e^{-j\omega t}}{-j\omega} \right|_{-\pi/2}^{\pi/2} =$$

$$= \frac{e^{j\omega\pi/2}}{j\omega} - \frac{e^{-j\omega\pi/2}}{j\omega} = \frac{2j \sin(\omega\pi/2)}{j\omega} =$$

$$- \text{enote: } f[V] \rightarrow F[V_s] \\ \alpha[A] \rightarrow [A_s]$$

$$- \text{konvergenca: } \int_{-\infty}^{\infty} |f(t)| dt < \infty !$$

Spekter: amplitudne gostote in fazni spekter

$$A_n = |A_n| e^{j\phi_n}$$

$$- F(j\omega) = \underbrace{|F(j\omega)|}_{\text{amplitudne gostote}} e^{j\phi(\omega)}_{\text{fazni spekter}}$$

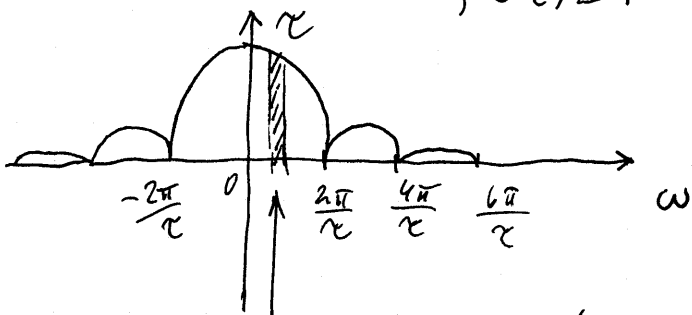
$$- \text{lastnosti: } \left. \begin{array}{l} |F(j\omega)| \dots \text{soda fja} \\ \phi(j\omega) \dots \text{liha fja} \end{array} \right\} F(-j\omega) = F(j\omega)^*$$

- zglede: $F(j\omega) = \frac{2 \sin(\omega\tau/2)}{\omega} = \tau \frac{\sin(\omega\tau/2)}{(\omega\tau/2)}$

- spekter amplitudne gostote

$$|F(j\omega)| = \tau \frac{|\sin(\omega\tau/2)|}{|\omega\tau/2|} *$$

$$\frac{\sin x}{x} \rightarrow 1$$



$$\frac{\omega\tau}{2} = k\pi, \text{ da ničle}$$

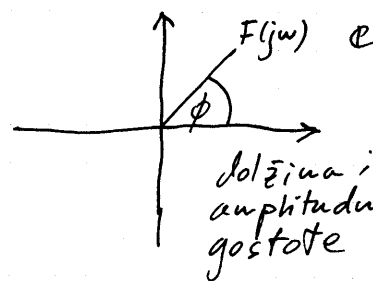
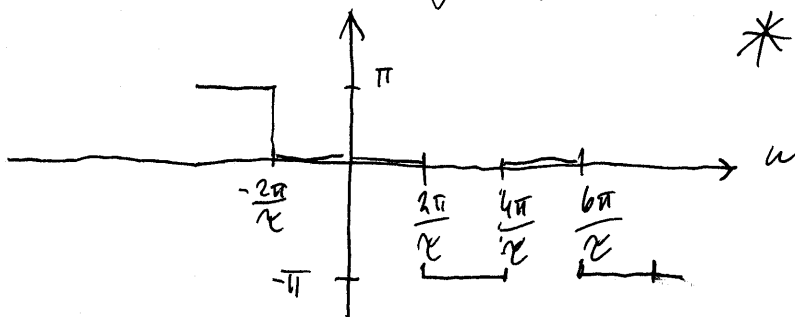
↓

$$\omega = \frac{2k\pi}{\tau}$$

energija signala na tej frekvenci

- fazni spekter

$$\phi(\omega) = \text{Arg}(F(j\omega)); \tau > 0$$



▣ Lastnosti Fourierjevega transformacije, tablica

- Fourierjev par: $f(t) \leftrightarrow F(j\omega) = \mathcal{F}(f(t))$ transformacija je injektivna!

- Lastnosti:

• Linearnost: $\mathcal{F}(a_1 f_1(t) + a_2 f_2(t)) = a_1 \mathcal{F}(f_1(t)) + a_2 \mathcal{F}(f_2(t))$

• Časovna zakasnitev: $\mathcal{F}(f(t-\tau)) = F(j\omega) e^{-j\omega\tau}$
↑
zakasnitev

• Časovno skaliranje: $\mathcal{F}(f(at)) = \frac{1}{|a|} F\left(\frac{j\omega}{a}\right)$

- odvod

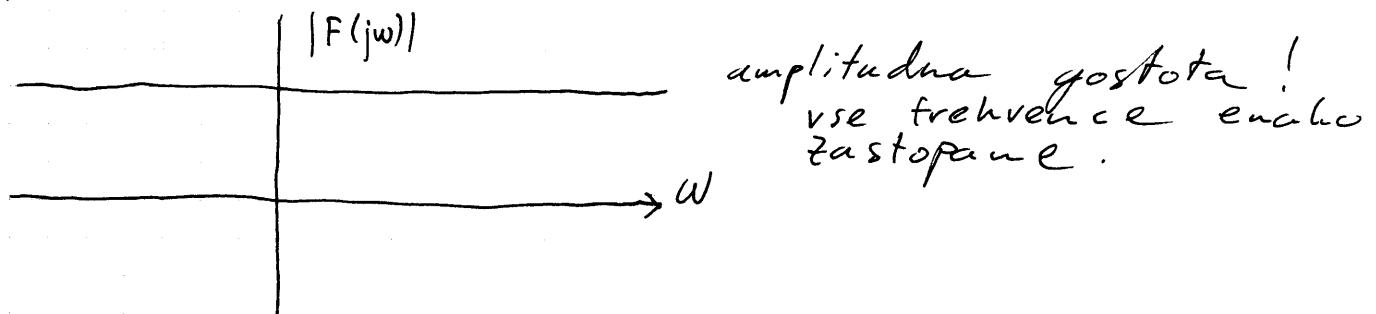
$$\mathcal{F}(D^n f(t)) = (j\omega)^n F(j\omega)$$

$$D \longrightarrow j\omega$$

- tablice

$f(t)$	$F(j\omega)$
$\delta(t)$	1
$1(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$ <small>dirakov δ</small>
$e^{-\alpha t} 1(t)$	$\frac{1}{\alpha + j\omega}$
\vdots	\vdots

- opomba: $\mathcal{F}(\delta(t)) = 1$ Pomembno.



Analiza vezij s Fourierjevo transformacijo



1. Transformiraj vzbujanje

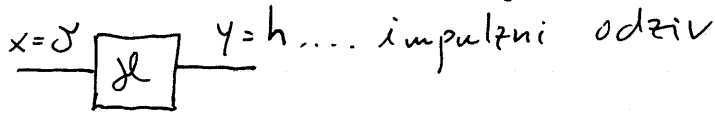
$$x(t) \leftrightarrow X(j\omega)$$

2. Izračunaj prevajalno funkcijo (Vizmenični analizi) $H(j\omega)$

$$3. Y(j\omega) = H(j\omega) \cdot X(j\omega)$$

4. Izračun inverza $y(t) \leftrightarrow Y(j\omega)$

- zglede 1: Določiti odziv na $\delta(t)$



1. Izračunaj transform od vzbujanja

$$\delta(t) \leftrightarrow 1$$

2. $H(j\omega) = \frac{Y}{X}$

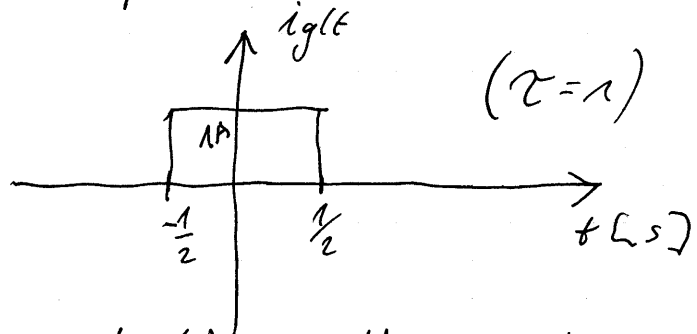
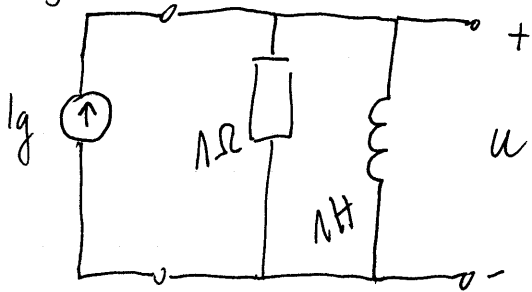
3. $Y(j\omega) = H(j\omega) \cdot 1 = H(j\omega)$

4. $h(t) \leftrightarrow H(j\omega)$

Impulzni odziv je ~~Fourierjev~~ Fourierjev par s prevajalno funkcijo.

- zglede 2:

Določiti spektralni odziv



1. $i_g(t) \leftrightarrow I_g(j\omega) = \frac{\sin(\omega/2)}{\omega/2}$

$$i_g(t) = 1(t + 1/2) - 1(t - 1/2)$$

uporabimo \downarrow linearnost

$$\left(\frac{1}{j\omega} + \pi \delta(t) \right) e^{j\omega/2} - \left(\frac{1}{j\omega} + \pi \delta(t) \right) e^{-j\omega/2} = D.N. \dots$$

\uparrow zahasnitev \leftarrow

2. $H(j\omega) = \frac{U}{I_g} = \frac{R j\omega L}{R + j\omega L}$

$$U_g = I_g \frac{R j\omega L}{R + j\omega L} \quad /: I_g =$$

$$= \boxed{\frac{j\omega}{1 + j\omega}}$$

$$3. U(j\omega) = \frac{j\omega}{1+j\omega} \cdot \frac{\sin(\omega/2)}{\omega/2} = \frac{2j}{1+j\omega} \sin\left(\frac{\omega}{2}\right)$$

4. Opomba: prava $u(t)$ je Fourierjev par $U(j\omega)$

Spekter:

- amplitudna gostota $|U(j\omega)| = \frac{2}{\sqrt{1+\omega^2}} \left| \sin\left(\frac{\omega}{2}\right) \right|$

- fazi spekter:

$$\phi(\omega) = \text{Arg}(U(j\omega)) = \frac{\pi}{2} + \begin{cases} 0; \sin(\omega/2) \geq 0 \\ -\pi; \sin(\omega/2) < 0 \end{cases} - \text{atg}(\omega)$$

Arg

7.1.2009

Težave Fourierjeve transformacije

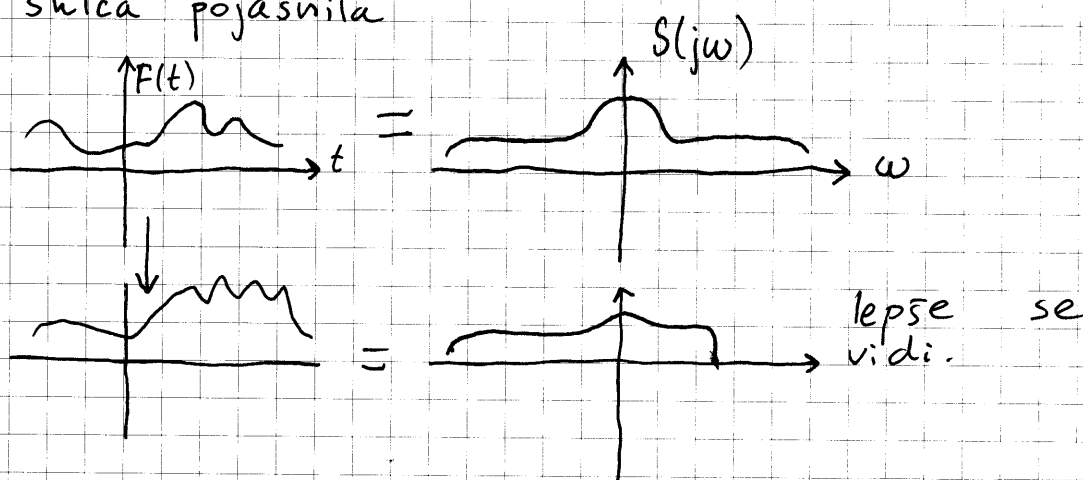
- $\mathcal{F}(f(t)) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = F(j\omega)$
- slabo konvergira: ~~\mathcal{F}~~ (konstantnega signala)!
- ne upošteva začetne energije!

Parsevalov teorem, spekter energijske gostote

- energija signala: $W = \int_{-\infty}^{\infty} f^2(t) dt$; $F(t) \leftrightarrow F(j\omega)$
 \downarrow
 $|F(j\omega)| e^{j\theta(s)}$
- Parseval: $W = \int_{-\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$

- spekter amplitudne gostote: $S(j\omega) = \frac{|F(j\omega)|}{2\pi}$.

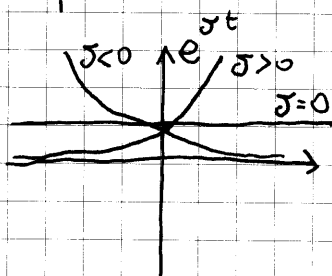
- shica pojasnila



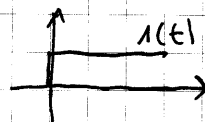
7. Laplaceova transformacija

7.1. Laplaceov transform

- vpeljava: $\mathcal{F}(f(t)) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$



$e^{-\sigma t}$, $\sigma > 0$, popravi konvergenco
 $\cdot 1(t)$ (oz. integrira se od 0)



Dobimo $\int_{-\infty}^{\infty} f(t) e^{-\overbrace{(G+j\omega)t}^s} dt =$

$$= \int_0^{\infty} f(t) e^{-st} dt =: \mathcal{L}(f(t))$$

s..... kompleksna frekvenca

$F(t) \leftrightarrow F(s)$ Laplaceov par

Npr.: $u_g(t) \leftrightarrow U_g(s)$

- Zgled: $\mathcal{L}(1(t)) = \int_0^{\infty} 1(t) e^{-st} dt = \frac{e^{-st}}{-s} \Big|_0^{\infty} = \frac{0-1}{-s} = \frac{1}{s}$

v območju int. vedno ena

$1(t) \leftrightarrow \frac{1}{s}$

$$e^{-st} = e^{-Gt} e^{-j\omega t} = e^{-Gt} \cdot e^{j\omega t}$$

$G > 0; \text{ vse } \rightarrow 0$

- domena s:

$s = G + j\omega; G > 0$ to pa je kompleksna polravnina

▣ Lastnosti, tablica

- linearnost: $\mathcal{L}(a_1 f_1(t) + a_2 f_2(t)) = a_1 F_1(s) + a_2 F_2(s)$

\uparrow \uparrow
 $f_1(t)$ $f_2(t)$

- časovna zakasnitev:

$$\mathcal{L}(f(t-\tau) 1(t-\tau)) = F(s) e^{-\tau s}$$

- frekvenčno skaliranje

$$\mathcal{L}(f(at)) = \frac{1}{a} F\left(\frac{s}{a}\right)$$

- odvod

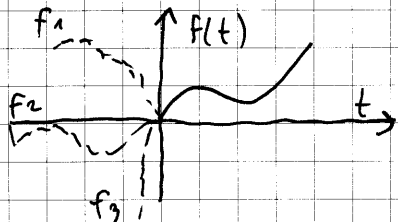
$$\mathcal{L}(DF(t)) = sF(s) - \underbrace{f(0^-)}_{\substack{\text{zapiš } \text{začetne} \\ \text{energije}}}$$

Tablice za transforme so na WWW.

- tablica

$F(t)$	$F(s)$
$1(t)$	$\frac{1}{s}$
$\delta(t)$	1
$e^{\delta t} 1(t)$	$\frac{1}{s-\delta}$
$t \cdot e^{\delta t} 1(t)$	$\frac{1}{(s-\delta)^2}$ ← dvojna realna ničla
$\cos(\omega_0 t) 1(t)$	$\frac{s}{s^2 + \omega_0^2}$ ← dvojna C ničla
$\sin(\omega_0 t) 1(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$

• $\mathcal{L}(\dots)$ ignorira $t < 0$! Zato povsod $1(t)$



$$F_1(s) = F_2(s) = F_3(s)$$

zato $\mathcal{L}^{-1}(F(s))(t) = 0$ za $t < 0$,
za $\forall F(s)$.

■ Računanje obratnega (inverznega) transformata

- vedno imamo $F(s) = \frac{Q(s)}{P(s)}$, kjer Q, P
polinoma in $st(Q) \leq st(P)$

- Potek izračuna

1. Če $st(Q) = st(P)$, polinoma delimo:

$$\frac{Q}{P} = a + \frac{\tilde{Q}}{P}; \quad st(\tilde{Q}) < st(P)$$

2. Razcep P :

$$P(s) = 0 \rightarrow s_1, \dots, s_n \quad \leftarrow n = st(P)$$

|||
red sistema

||
št. reaktivnih elementov

Glede na red (stopnjo) ničel:

(i) Vse ničle $st(s_i) = 1$

$$F(t) = (k_1 e^{s_1 t} + \dots + k_n e^{s_n t}) 1(t)$$

$$k_i = F(s) (s - s_i) \Big|_{\text{vstavi } s=s_i} \quad 1 \leq i \leq n$$

(ii) Vsaj kakšna ničla višjega reda: za vsako ničlo s_0 , ko $st(s_0) = m > 1$

$$(k_1 + t k_2 + \dots + t^{m-1} k_m) e^{s_0 t} 1(t)$$

$$k_i = \left(\frac{1}{(m-1)!} \frac{d^{m-i}}{ds^{m-i}} \left(F(s) (s - s_0)^m \right) \right) \Big|_{\text{vstavi } s=s_0}$$

- zglej: $F(s) = \frac{s-1}{(s+3)(s+5)}$ Poišči Laplaceov par tej zadevi.

$$st(p) = 2; \quad st(a) = 1$$

$$P(s) = (s+3)(s+5) = 0 \quad s_1 = -3 \quad s_2 = -5$$

$$f(t) = (k_1 e^{-3t} + k_2 e^{-5t}) 1(t)$$

$$k_1 = \frac{s-1}{(s+3)(s+5)} \underbrace{(s+3)}_{(s-i)} \Big|_{s=-3} = \frac{-3-1}{-3+5} = \frac{-4}{2} = -2$$

$$k_2 = \frac{s-1}{(s+3)(s+5)} \underbrace{(s+5)}_{(s-i)} \Big|_{s=-5} = \frac{-6}{-2} = 3$$

$$F(t) = (-2 e^{-3t} + 3 e^{-5t}) 1(t)$$

- zglej: $\frac{s}{s+1} = F(s)$

$st(p) = st(a)$; delimo

$$\frac{s}{s+1} = \underbrace{1}_{\delta(t)} - \underbrace{\frac{1}{s+1}}_{e^{-t} 1(t)} \quad \left. \begin{array}{l} \leftarrow \text{ } \rightarrow \\ \leftarrow \text{ } \rightarrow \end{array} \right\} f(t) = \delta(t) - e^{-t} 1(t)$$

Poglejmo v tablice.

- zglied:

$$F(s) = \frac{2s+7}{(s+4)(s^2+6s+25)}$$

$$P(s) = 0 : s_1 = -4 \frac{1}{s} \leftarrow \text{sekunde}$$

$$s_{2,3} = \frac{-6 \pm \sqrt{36-100}}{2} = \frac{-6 \pm 8j}{2} \begin{matrix} \nearrow s_2 = -3 + 4j \frac{1}{s} \\ \searrow s_3 = -3 - 4j \frac{1}{s} \end{matrix}$$

$$f(t) = (k_1 e^{-4t} + k_2 e^{(-3+4j)t} + k_3 e^{(-3-4j)t}) \cdot 1(t)$$

$$k_1 = \frac{2s+7}{(s+4)(s^2+6s+25)} \Big|_{s=-4} = \frac{2(-4)+7}{16-24+25} = -\frac{1}{17}$$

$$k_2 = \frac{2s+7}{(s+4)(s^2+6s+25)} \Big|_{s=-3+4j} = \frac{2s+7}{(s-s_2)(s-s_3)} \Big|_{s=-3+4j}$$

$$= \frac{2s+7}{(s+4)(s-(-3-4j))} \Big|_{s=-3+4j} = \frac{-8+8j+7}{(-3+4j+4)(-3+4j-(-3-4j))} =$$

$$= \frac{1+8j}{(1+4j) \cdot 8j} = 0,224 e^{j83,1^\circ} = \frac{k}{2} e^{j\varphi}$$

$$k_3 = \dots = k_2^*$$

$$f(t) = -\frac{1}{17} e^{-4t} + \frac{k}{2} e^{j\varphi} e^{(-3+4j)t} +$$

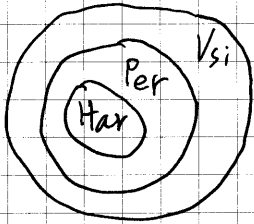
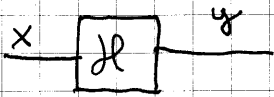
$$+ \frac{k}{2} e^{-j\varphi} e^{(-3-4j)t} \Big|_{1(t)} =$$

$$= \left(-\frac{1}{17} e^{-4t} + \underbrace{k}_{2 \cdot 0,224} e^{-3t} \cos(4t + 83,1^\circ) \right) \Big|_{1(t)}$$

~~Summe~~

7.2. Analiza vezij z Lapl. transform

■ Sistemska fja



$$(a_n D^n + \dots + a_1 D + a_0) y(t) = (b_m D^m + \dots + b_1 D + b_0) x(t) \quad / \mathcal{L}$$

$$\mathcal{L}(Df(t)) = sF(s) - F(0^-)$$

$$(a_n s^n + \dots + a_1 s + a_0) Y(s) - A(s) = (b_m s^m + \dots + b_1 s + b_0) X(s) - B(s)$$

$$\begin{aligned} \mathcal{L}(D^2 f(t)) &= s \mathcal{L}(Df(t)) - Df(0^-) \\ &= s(sF(s) - F(0^-)) - DF(0^-) \\ &= s^2 F(s) - sF(0^-) - DF(0^-) \end{aligned}$$

$$Y(s) = \frac{a(s)}{P(s)} X(s) + \frac{A(s) - B(s)}{P(s)}$$

\downarrow $H(s)$ sistemska fja $\quad Y_z(s)$
 $= H(s) X(s) = Y_v(s)$

Laplaceov transform odziva na začetno stanje.

Laplaceov transform odziva na vzbujanje Y_v .

Dobili smo

$$Y(s) = Y_v(s) + Y_z(s)$$

$$\updownarrow \quad \updownarrow \quad \updownarrow$$

$$y(t) = y_v(t) + y_z(t)$$

Opazimo: $H(s) = \frac{a(s)}{P(s)} = H(j\omega) \Big|_{j\omega = s}$

$$H(s) = \frac{Y_v(s)}{X(s)}$$

Kirchoffovi zakoni veljajo tudi za Laplacea!

Model vezja v Laplaceovem prostoru

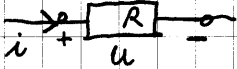
- topološki opis (graf) je enak
- modeli elementov
 - neodvisen vir:

$$x(t) \leftrightarrow X(s)$$

- krmiljeni viri:

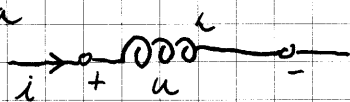
$$\begin{aligned} x_2(t) &= k \cdot x_1(t) \quad | \mathcal{L} \\ X_2(s) &= k X_1(s) \end{aligned}$$

(linearnost transformacije)

- upor: 

$$\begin{aligned} u(t) &= R i(t) \\ U(s) &= R I(s) \\ Z_R(s) &= R \dots \text{impedanca} \\ Y_R(s) &= 1/R = G \dots \text{admitanca} \end{aligned}$$

- tuljava

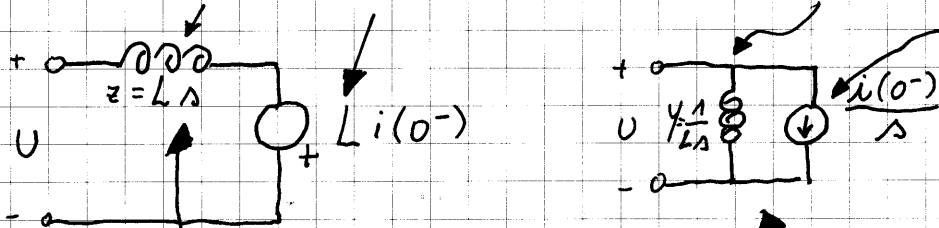


$$u(t) = L D i(t)$$

uporabimo $\mathcal{L}(Df(t)) = s F(s) - F(0^-)$

$$U(s) = L (\cancel{s I(s)} - i_L(0^-)) =$$

$$= L s I(s) - L i_L(0^-) \longrightarrow I(s) = \frac{1}{L s} U(s) + \frac{i(0^-)}{s}$$



nevzbujana tuljava ($w(0^-) = 0$)

teorem o transformaciji virov

$$Z_L(s) = \frac{U(s)}{I(s)} = L s [\Omega]$$

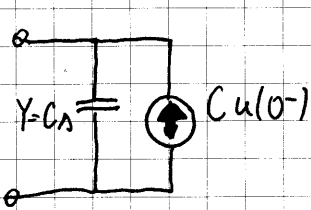
za nevzbujano tuljavo

$$Y_L(s) = \frac{I(s)}{U(s)} = \frac{1}{L s}$$

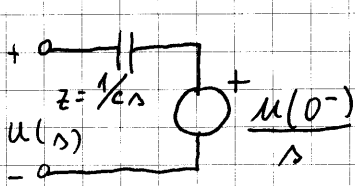
• kondenzator

$$\begin{array}{c} \xrightarrow{i} \\ \text{---} \parallel \text{---} \\ \text{---} \end{array} \quad i(t) = C \dot{u}(t)$$

$$I(s) = C s U(s) - C u(0^-)$$



$$U(s) = \frac{1}{C s} I(s) + \frac{u(0^-)}{s}$$



$$\left. \begin{array}{l} Y_c(s) = c s \text{ adm.} \\ Z_c(s) = 1/c s \text{ imp.} \end{array} \right\} \text{ imitance}$$

- 0 energiji

$$W_L(0^-) = \frac{1}{2} L i^2(0^-); \quad W_L(0^-) = 0 \text{ kadar } i_L(0^-) = 0$$

$$W_C(0^-) = \frac{1}{2} C u^2(0^-); \quad W_C(0^-) = 0 \text{ kadar } u_C(0^-) = 0$$

zacetne energije

Analiza vezja v Lapl. prostoru

Potek:

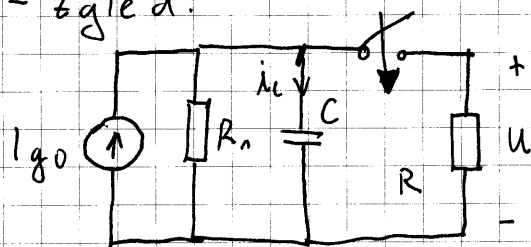
- ① Model vezja v Lapl.
 - transformiram generatorje
 - -||-
 - linearne elemente

② Opis vezja (tri-metode)

③ Izračun $Y(s)$

④ Obratni transform
 $y(t) \leftrightarrow Y(s)$

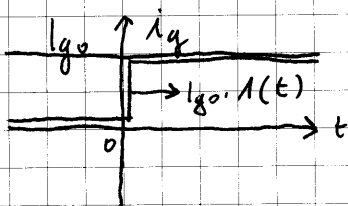
- zgle d:



Tablica:

$f(t)$	$F(s)$
$\delta(t)$	1
$1(t)$	$1/s$
$e^{\sigma t} \cdot 1(t)$	$1/(s - \sigma)$
$t e^{\sigma t} \cdot 1(t)$	$1/(s - \sigma)^2$
$\cos(\omega_0 t) \cdot 1(t)$	$s / (s^2 + \omega_0^2)$
$\sin(\omega_0 t) \cdot 1(t)$	$\omega_0 / (s^2 + \omega_0^2)$

① - transf. gen.

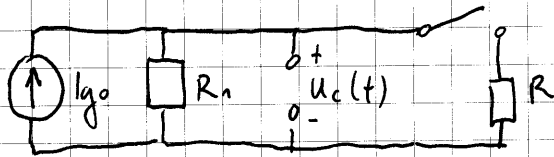


$$I_{g0} \cdot 1(t) \leftrightarrow \frac{I_{g0}}{s}$$

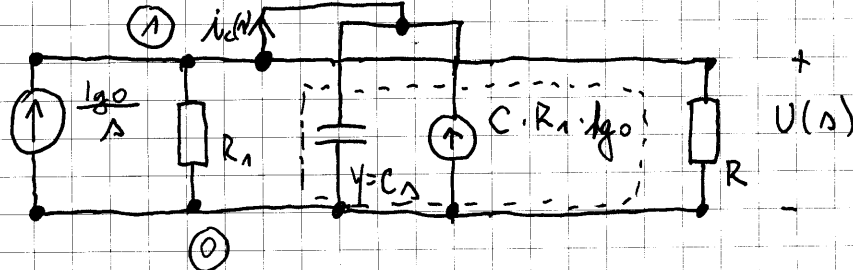
Vse generatorje lahko pri transformaciji v Laplacea pri negativnih časih postavim na nič. Tega pa ne napravimo pri izračunu začetnega stanja vezja.

- transf. elem.: $\forall i_L(0^-)$ in $u_C(0^-)$ poiščemo

Za $t < 0$ določimo stacionarno stanje (enosmerno)



$$u_C(t) = I_{g0} \cdot R_n \Rightarrow u_C(0^-) = I_{g0} \cdot R_n$$



② opis

$$\textcircled{1} - \frac{I_{g0}}{s} + \frac{U_{V1}}{R_n} + C s U_{V1} - C R_n I_{g0} + \frac{U_{V1}}{R} = 0 \quad [As]$$

$$U(s) = U_{V1}$$

③ Izračun $U(\Delta)$:

$$U_{v1} \left(\frac{1}{R_1} + C\Delta + \frac{1}{R_2} \right) = \frac{I_{g0}}{\Delta} + R_1 C I_{g0}$$

$$\bullet U(\Delta) = U_{v1} = \frac{R_1 R}{R_1 + R + R_1 R C \Delta} I_{g0} \frac{1 + R_1 C \Delta}{\Delta}$$

④ Obratni transform

$$u(t) \leftrightarrow U(\Delta)$$

$$\left(Y(\Delta) = \frac{Q(\Delta)}{P(\Delta)} \right)$$

ali: je $\Delta t.Q = \Delta t.P$
 $\uparrow \quad \uparrow$
 $1 \neq 2 \quad \checkmark$ ni treba deliti

$$-P(\Delta) = \Delta(R_1 R C \Delta + R_1 + R) = 0$$

$$\begin{array}{l} \Delta_1 = 0 \\ \uparrow \\ \Delta t. \Delta_1 = 1 \end{array} \quad \begin{array}{l} \Delta_2 = \frac{R_1 + R}{R_1 R C} \\ \uparrow \\ \Delta t. \Delta_2 = 1 \end{array}$$

$$u(t) = \left(k_1 e^{0 \cdot t} + k_2 e^{-\frac{R_1 + R}{R_1 R C} t} \right) 1(t)$$

$$k_1 = U(\Delta) (\Delta - \Delta_1) \Big|_{\Delta = \Delta_1} = \frac{R_1 R}{R_1 + R + R_1 R C \Delta} I_{g0} \frac{1 + R_1 C \Delta}{\Delta} \Big|_{\Delta = 0} =$$

$$= \frac{R_1 R}{R_1 + R} I_{g0}$$

$$k_2 = U(\Delta) (\Delta - \Delta_2) \Big|_{\Delta = \Delta_2} = \frac{R_1 R}{R_1 + R + R_1 R C \Delta} I_{g0} \frac{1 + R_1 C \Delta}{\Delta} \left(\Delta + \frac{R_1 R}{R_1 R C} \right) \Big|_{\Delta = \Delta_2} =$$

razcep



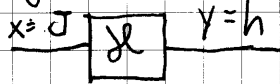
$$= \frac{R_1 R}{R_1 R C} I_{g0} \frac{1 + R_1 C \Delta}{\Delta} \Big|_{\Delta = -\frac{R_1 + R}{R_1 R C}} = \frac{I_{g0}}{\cancel{R_1 R C}} \cdot \frac{1 + R_1 C \frac{-(R_1 + R)}{R_1 R C}}{\frac{-(R_1 + R)}{R_1 R C}}$$

$$k_2 = I_{g0} \frac{R_1 R - R_1(R_1 + R)}{-(R_1 + R)} = R_1 I_{g0} \frac{R_1}{R_1 + R}$$

$$u(t) = I_0 \frac{R_1}{R_1 + R} \left(R + R_1 e^{-\frac{R_1 + R}{R_1 R C} t} \right) \cdot 1(t)$$

za $t \geq 0$, za $t \leq 0$ je $u(t) = R_1 I_0$ (stac. stanje.)

- zglej:



- za $t < 0$ je $x(t) = \delta(t) = 0$, zato je $W(0^-) = 0$, in $H(s) = H_v(s)$

$$(W(0^-) = 0 \rightarrow Y_v(s) = 0 \rightarrow Y(s) = Y_v(s) = H_v(s))$$

- vemo:

$$Y_v(s) = H(s) X(s)$$

$$\parallel H_v(s) = \parallel H(s) \cdot \parallel \mathcal{L}(\delta) = 1$$

\downarrow

\downarrow

$$h(t) \leftrightarrow H(s) = H(j\omega) \Big|_{j\omega = s}$$

Impulzni odziv $h(t)$ je laplaceov par sistemske funkcije v laplaceovi domeni.

Če v sistemske fjo v izmenični analizi namesto $j\omega$ vstavimo s , dobimo $H(s)$. Njen inverzni laplaceov transform je enak inputnemu odzivu $h(t)$.

- Konvolucija

$$y(t) = (h * x)(t)$$

in imamo odziv za vsak signal.