Akustika - vaje

14.10.2013

1) $m = 10g$

$F = 1N$

$x_0 = 1cm$

$x(t) = ?$

$v_0 = 0$

$x(t) = A \cdot \cos(\omega_0 t + \varphi)$ [m] $\omega_0 = 2\pi f_0$

$$A = \sqrt{x_0^2 + \left(\frac{v_0^2}{\omega_0^2}\right)} = x_0$$

$$\varphi = \text{arctg}\left(\frac{-v_0}{\omega_0 x_0}\right) = 0 \Rightarrow \varphi = 0°$$

$F = -k \cdot x$

$F = k \cdot x$

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{F}{x \cdot m}} = \sqrt{\frac{1}{0.01 \cdot 10^{-2}}} = 100\ s^{-1}$$

2) a)

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{2M}}$$

b) $$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{2k}{M}}$$

$F = m \cdot g = k_{eff} \cdot x$

$\frac{F}{2} = k_1 x = k_2 x \rightarrow F = 2kx$

$\Big\} = k_{eff} \cdot x = 2kx$

$k_{eff} = 2k$

izpit* c) $F = -k_1 \cdot x_1 = -k_2 x_2$

$x_1 = x_2 \frac{k_2}{k_1}$

$$\omega_0 = \sqrt{\frac{k}{2M}}$$

$$F = k_{eff} \cdot (x_1 + x_2) = -k_{eff} \cdot x_2\left(1 + \frac{k_2}{k_1}\right) = -k_2 \cdot x_2 \Rightarrow k_{eff} = \frac{k_2}{\left(1 + \frac{k_2}{k_1}\right)} = \frac{k}{2}$$

d) $F = -k_1 x_1 - k_2 x_2 = -k_1 x - k_2 x = x(\underbrace{-k_1 - k_2}_{k_{eff}})$

$k_{eff} = 2k$

$\omega_0 = \sqrt{\frac{2k}{M}}$

3) $v_{max}(t = T/2) = V$

$v(t) = \dot{x}(t)$

$x(t) = ?$

$\dot{x}(t) = A \cdot \cos(\omega_0 t + \varphi)$

$v(t) = -A \cdot \omega_0 \sin(\omega_0 t + \varphi)$

$v(t) = V \sin(\omega_0 t + \varphi) \rightarrow A = -\frac{V}{\omega_0}$

$v(T/2) = V \Rightarrow \sin(\omega_0 \frac{T}{2} + \varphi) = 1$

$\frac{\omega_0 T}{2} + \varphi = \frac{\pi}{2} \Rightarrow \omega_0 = 2\pi f = \frac{2\pi}{T}$

$\frac{2\pi}{T} \cdot \frac{T}{2} + \varphi = \frac{\pi}{2} = \varphi = \frac{\pi}{2} - \pi = -\frac{\pi}{2} = \varphi$

$x = A \cdot \cos(\omega_0 t + \varphi) = \frac{-V}{\omega_0} \cdot \cos(\omega_0 t - \frac{\pi}{2}) = -\frac{V}{\omega_0} \cdot \sin(\omega_0 t)$

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4) $x(t) = 10 \cdot \sin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right)$

$T = ?$

$v_{max} = ?$ $T = 6\,s$

$a_{max} = ?$

$t(x_{max}) = ?$

$$v(t) = \dot{x}(t) = 10 \cdot \frac{\pi}{3} \cos\left(\frac{\pi}{3}t + \frac{\pi}{3}\right)$$

$$v_{max} = \frac{10\pi}{3} = \frac{314}{3} = 10{,}5\ \frac{m}{s}$$

$$a(t) = -10 \cdot \left(\frac{\pi}{3}\right)^2 \sin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right)$$

$$a_{max} = \frac{10 \cdot \pi^2}{9} = 11\ \frac{m}{s^2}$$

$$x(t) = 10 \cdot \sin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right) = 10\,m \Rightarrow \arcsin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right) = \frac{\pi}{2} = 0.5\,s$$

5) $m = 10g$

$x = 10 \cdot \sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)$

$F(t) = ?$

$A = ?$ $x_{max}, v_{max}, a_{max}, f_{max}$

$t\left(W_k = \frac{1}{2}\hat{W}_k\right)$

$x_{max} = 10\,m$

$v_{max} = 10.5\,\frac{m}{s}$

$a_{max} = +11\,\frac{m}{s^2}$

$f_{max} = 0.11\,N$

$F(t) - k \cdot x = m \cdot a$

$\dot{x}(t) = \frac{10\pi}{3}\cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = v(t)$

$\ddot{x}(t) = -\frac{10\pi^2}{9}\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = a(t)$

$F(t) = \frac{0.01 \cdot (-10)\pi^2}{9}\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = -0.11 \cdot \sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)\,\frac{m\,kg}{s^2}$

$W_k = m\frac{v^2}{2} \Rightarrow \hat{W}_k = m \cdot \frac{\hat{v}^2}{2} = 0.01 \cdot \frac{10.5^2}{2} =$

$W_k = \frac{1}{2}\hat{W}_k = \frac{1}{2}mv^2 = \frac{1}{2}\frac{1}{2}m\hat{v}^2 \quad v^2 = \frac{1}{2}\hat{v}^2$

$\hat{v}^2 \cdot \cos^2\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = \frac{1}{2}\hat{v}^2 \quad /\sqrt{\ }$ $\quad \cos\left(\frac{\pi}{3}t + \frac{\pi}{6}\right) = \frac{1}{\sqrt{2}}$

$\frac{\pi}{3}t + \frac{\pi}{6} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\pi}{4} = \frac{\pi}{3}t + \frac{\pi}{6} \rightarrow t = 0.25\,s$

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6) DN a) $= 30\,m$ b) $= 0.15\,\frac{m}{s}$ c) $-0.75\,\frac{m}{s^2}$

7) $W_p(t) = \frac{1}{2} k x^2(t)$

$W_k(t) = \frac{1}{2} m v^2(t)$ ali velja $\hat{W}_p = \hat{W}_k$

$\hat{W}_p(t) = \frac{1}{2} k \hat{x}^2(t) = \frac{1}{2} m \hat{v}^2(t) = \hat{W}_k(t)$

$\dot{x}(t) = -A\omega_0 \sin(\omega_0 t + \varphi)$

$k\hat{x}^2 = m A^2 \omega_0^2$

$k A^2 = m A^2 \omega_0^2$

$k = m \cdot \omega_0^2 \Rightarrow \omega_0 = \sqrt{\frac{k}{m}}$

AKUSTIKA – vaje (2)

21.10.2013

1) $t_1 = 4s$, $x_1 = 5cm$
$t_2 = 8s$, $x_2 = 3cm$
$t_3 = ?$, $x_3 = 1cm$

$$x(t) = A \cdot e^{-\beta t} \cdot \cos(\omega_1 t + \varphi)$$

$$\left.\begin{array}{l} x_1(t_1) = A \cdot e^{-\beta t_1} \\ x_2(t_2) = A \cdot e^{-\beta t_2} \end{array}\right\} \%$$

$$\frac{x_1}{x_2} = \frac{e^{-\beta t_1}}{e^{-\beta t_2}} = e^{-\beta(t_1+t_2)} = e^{-\beta(t_1-t_2)} = \frac{x_1}{x_2} \quad // \ln$$

$$-\beta(t_1 - t_2) = \ln\frac{x_1}{x_2} \Rightarrow \beta = \frac{\ln\frac{x_1}{x_2}}{-(t_1-t_2)} = \frac{\ln\frac{5}{3}}{4s} = 0.128\,s^{-1}$$

$$\ln\frac{x_1}{x_3} = -\beta(t_1 - t_3) \Rightarrow t_3 = \frac{\ln\frac{x_1}{x_3}}{\beta} + t_1 = 16{,}6\,s$$

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2) $m_1 = 0.5kg$
$m_2 = 0.2kg$
$x_2 = 40mm + x_1$
$x(1s) = \frac{X_0}{e}$
$\Downarrow$
$\tau = 1s$ relaksacijski čas

$R_m = ?$ $\omega_d = ?$ $A = ?$ $\phi = ?$

$$x(t) = A \cdot e^{-\beta t} \cdot \cos(\omega_d t + \phi)$$

$$\boxed{\tau = \frac{2m}{R_m}} \qquad R_m = \frac{2m}{\tau} = \frac{2 \cdot 0.5kg}{1s} = 1\frac{kg}{s}$$

$$\omega_d = \sqrt{\omega_0^2 - \beta^2} \; ; \; \beta = \frac{1}{\tau} = 1s^{-1}$$

$$\omega_d = \frac{k}{m} \; ; \; k = \frac{F}{x} = \frac{m_2 g}{x_2} = \frac{0.2 \cdot 10}{40mm} = 49 N/m = k$$

$\omega_0 = \frac{k}{m_1} = 9.9\ s^{-1}$ $\qquad$ $\omega_d = 9.85\ s^{-1}$

$x(t=0) = 40\ mm = A \cdot e^0 \cdot \cos(\phi) = A \cdot \cos(\phi)$

$v = \dot{x} = -A \cdot e^{-\beta t}(\beta \cdot \cos(\omega_d t + \phi) + \omega_d \sin(\omega_d t + \phi))$

$v(t=0) = 0$!

$v = -A(\beta \cdot \cos(\phi) + \omega_d \sin(\phi)) \quad /:-A$

$0 = \beta \cos\phi + \omega_d \sin\phi$

$\beta \cos\phi = -\omega_d \sin\phi$

$\frac{\beta}{\omega_d} = -tg\phi \Rightarrow \phi = -arctg\frac{\beta}{\omega_d} = -5.8°$

$40\ mm = A \cdot \cos\phi \Rightarrow A = 40{,}21\ mm$

AKUSTIKA - vaje

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3) $x_0 = 1cm$

$v(t=0) = 0!$

$A, \phi = ?$

$$x = A \cdot e^{-\beta t} \cdot \cos(\omega_d t + \phi)$$

$$\omega_d = \sqrt{\omega_0^2 - \beta^2}$$

$$\frac{\beta}{\omega_0} = 0.1 \Rightarrow \beta = 0.1 \cdot \omega_0$$

a) $A = 1{,}005\ cm$

$\phi =$ ~~-11,5~~ $-5{,}77°$

b) $A = 1{,}021\ cm$

$\phi = -11{,}5°$

c) $A = 1.048\ cm$

$\phi = -17{,}5°$

4)

$$x = \frac{F}{\omega|Z_m|} \sin(\omega t - \Theta) \quad Z_m = \sqrt{R_m^2 + \left(\omega \cdot m - \frac{k}{\omega}\right)^2} \quad f(t) = F \cdot \cos(\omega t)$$

$$; \beta = \frac{R_m}{2m} ; k = \omega_0^2 \cdot m$$

$\omega_1 = 320\ s^{-1}$

$\omega_0 = 300\ s^{-1}$, $x_1 = 6cm$

$\omega_2 = 340\ s^{-1}$, $x_2 = 5cm$

$\beta = ?$

$$\frac{x_1}{x_2} = \frac{\cancel{F}\,\omega_2|Z_{m2}|}{\omega_1|Z_{m1}| \cdot \cancel{F}}$$

$$|Z_m| = \sqrt{\beta^2 4m^2 + \left(\omega m - \left(\frac{\omega_0^2 m}{\omega}\right)\right)^2} = m\sqrt{4\beta^2 + \left(\omega - \frac{\omega_0^2}{\omega}\right)^2} = m\sqrt{4\beta^2 + \left(\omega\left(1 - \frac{\omega_0^2}{\omega^2}\right)\right)^2}$$

$$\frac{x_1}{x_2} = \frac{\omega_2 \sqrt{4\beta^2 + \omega_1^2\left(1 - \frac{\omega_0^2}{\omega_1^2}\right)^2}}{\omega_1 \sqrt{4\beta^2 + \omega_2^2\left(1 - \omega_0^2/\omega_2^2\right)^2}} \Rightarrow \overbrace{\frac{x_1 \omega_1}{x_2 \omega_2}}^{\xi} = \frac{\sqrt{4\beta^2 + \alpha}}{\sqrt{4\beta^2 + \gamma}}$$

$$\xi = \frac{\sqrt{4\beta^2 + \alpha}}{\sqrt{4\beta^2 + \gamma}} \quad /^2 \quad \Rightarrow \boxed{\beta = \frac{\gamma - \xi \cdot \alpha}{4 \cdot (\xi - 1)}} \qquad \beta = \underline{57.8}\ s^{-1}$$

$$\left.\begin{aligned} \xi &= 1.28 \\ \alpha &= 1501\ \mathrm{rad}/s^2 \\ \gamma &= 5668\ \mathrm{rad}/s^2 \end{aligned}\right\} \text{Zračunamo}$$

5) $f(t) = F \cdot \cos(\omega t)$

$$v(t) = \frac{F}{|Z_m|}\cos(\omega t - \theta)$$

$$a(t) = -\frac{F\omega}{|Z_m|}\sin(\omega t - \theta) \Rightarrow \alpha$$

$$\frac{da(\omega)}{d\omega} = 0 \Rightarrow \text{extremi} \qquad |Z_m| = \sqrt{R_m^2 + \left(\omega m - \frac{k}{\omega}\right)^2}$$

$k = \omega_0^2 \cdot m = \text{konst.}$

$R_m = 2m \cdot \beta = \text{konst.}$

③

AKUSTIKA – vaje

21. 10. 2013

$$\frac{d}{d\omega}\left(\frac{\omega}{|Z_m|}\right)$$

$$\left(\frac{u}{v}\right)' = \frac{v \cdot u' - v' \cdot u}{v^2}$$

$$y(\omega)^n = n \cdot y^{n-1} \cdot y'$$

⟹ ČUNGA LUNGA izpeljava

$$R_m^2 + \left(\omega m - \frac{k}{\omega}\right)^2 - \omega^2 m^2 + \frac{k^2}{\omega^2} = 0$$

$$R_m^2 + \frac{2k^2}{\omega^2} - 2mk = 0 \quad / \cdot \omega^2$$

$$2k^2 \cdot \omega^2 R_m^2 - 2mk\omega^2 = 0$$

$$\omega^2 (R_m^2 - 2mk) + 2k^2 = 0 \qquad k = \omega_0^2 m$$

$$\omega = \frac{\omega_0}{\left(1 - \frac{R_m^2}{2\omega_0^2 m^2}\right)^{1/2}}$$


ČLANSKA IZKAZNICA
ZASLUŽKI
NAPOTNICE
MOJA DELA
MOJE IZKUŠNJE
UGODNOSTI
POSLOVALNICE

AKUSTIKA - vaje

21.10.2013

6)

$$\Theta = \mathrm{arctg}\left(\frac{\omega m - k/\omega}{R_m}\right) = 0$$

$$\frac{\omega m - k/\omega}{R_m} = 0$$

$$\omega m = \frac{k}{\omega} \Rightarrow \omega = \sqrt{\frac{k}{m}} = \omega_0$$

AKUSTIKA - vaje

28.10.2013

1) $\phi_1 = 0°$
$\phi_2 = 30°$
$\phi_3 = 45°$

$$x(t) = A' \cdot \cos(\omega t + \phi)$$

$$A' = \sqrt{\sum (A_m \cos\phi_m)^2 + (A_m \cdot \sin\phi_m)^2}$$

$$tg\,\phi = \frac{\sum A_m \sin\phi_m}{\sum A_m \cos\phi_m}$$

n	$\sin\phi_n$	$\cos\phi_n$
1	0	1
2	1/2	$\sqrt{3}/2$
3	$\sqrt{2}/2$	$\sqrt{2}/2$

$$A^2 = \left(A \cdot 1 + A \cdot \frac{\sqrt{3}}{2} + A \cdot \frac{\sqrt{2}}{2}\right)^2 + A^2\left(0 + \frac{1}{2} + \frac{\sqrt{2}}{2}\right)^2 = A^2\left[\left(1 + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\right)^2 + \left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right)^2\right] = 8{,}12 A^2$$

$$A' = 2{,}85 A$$

$$tg\,\phi = \frac{A \cdot \left(0 + \frac{1}{2} + \frac{\sqrt{2}}{2}\right)}{A \cdot \left(1 + \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\right)} \Rightarrow = 0{,}47 \Rightarrow \phi = \underline{\underline{0{,}44\ rad}}$$

2. a) $\omega_1 = 3 s^{-1}$ $\omega_2 = 5 s^{-1}$

Pogoj: $\boxed{\frac{\omega_2}{\omega_1} \in \mathbb{Q}}$; $\omega_2 > \omega_1$

za periodičnost

$$\frac{\omega_2}{\omega_1} = \frac{5}{3} \in \mathbb{Q} \ ✓$$

$$D = 1 \Rightarrow \omega = \underline{\underline{1 \frac{rad}{s}}}$$

↑ največji skupni delitelj

b) $\omega_1 = 6\,s^{-1}$ $\Rightarrow \frac{6}{4} \in \mathbb{Q} \Rightarrow D = 2 \Rightarrow \underline{\underline{\omega = 2\,s^{-1}}}$

$\omega_2 = 4\,s^{-1}$

c) $\omega_1 = 1\,s^{-1}$ $\Rightarrow$ <u>ni periodično</u> $\frac{\sqrt{2}}{1} \notin \mathbb{Q}$

$\omega_2 = \sqrt{2}\,s^{-1}$

d) $\omega_1 = 0.71\,s^{-1}$ $\Rightarrow \frac{1.41\,s^{-1}}{0.71\,s^{-1}} \Rightarrow \frac{\frac{141}{100}}{\frac{71}{100}} = \underline{\underline{\frac{141}{71} \in \mathbb{Q}}}$ $\underline{\underline{D = 0.01}} \Rightarrow \underline{\underline{\omega = 0.01\,s^{-1}}}$

$\omega_2 = 1.41\,s^{-1}$

AKUSTIKA – vaje // nihanje strune 28.10.2013

Nihanje strune: (PROSTA)

$$y(t,x) = A \cdot \cos(\omega t - kx)$$

↑ porazdelitev nihanja vzdolž strune

1) $\rho_L = 0.1\,g/cm$

$$Y = 4 \cdot \cos(3t - 2x)$$

$\varphi, f, A, \lambda, k = ?$

$A = 4\,cm$, $k = 2\,\frac{rad}{cm}$, $\omega = 3\,rad/s \Rightarrow f = 0.48\,Hz$

$k = \frac{2\pi}{\lambda} \Rightarrow c = \frac{\omega}{k} = 1.5\,cm/s$ $\lambda = \frac{c}{f} = \frac{2\pi}{k} = 3{,}14\,cm$

b) $v = ?$ $v(t=0, x=0)$ $v = \frac{\partial y}{\partial t} = -A \cdot \omega \sin(\omega t - kx)$

$$v(0,0) = -A \cdot \omega \cdot \sin(0) = 0\,m/s$$

2) Vpeta na eni strani:

$$Y(t,x) = \frac{F}{k \cdot T} \frac{\sin(k(L-x))}{\cos(kL)} \cdot e^{j\omega t}$$

$T = 5N$

$\rho_L = 0.01 kg/m$

$L = 0.44m$

$\frac{\lambda}{2} = 0.1m$

$$f = \frac{c}{\lambda} = \frac{1}{\lambda} \cdot \sqrt{\frac{T}{\rho_L}} \quad ; c = \sqrt{\frac{T}{\rho_L}}$$

$$f = 112 Hz$$

b) $A_{max} = 0.02m$

$f = 112 Hz$

$y(0) = ?$

$$y(x) = K \frac{\sin(k(L-x))}{\cos(kL)}$$

$$A_{max} \cdot \sin(k(L-x))$$

$$y(0) = A_{max} \cdot \sin(kL) \Rightarrow A_{max} \cdot \sin\left(\frac{2\pi}{\lambda} \cdot L\right) = 0.02m \cdot \sin\left(\frac{2\pi}{0.2m} \cdot 0.44m\right)$$

$$y(0) = A_{max} \cdot 0.95 = 1.9cm$$

3) $L = 1m$

$\lambda_v = 0.3m$

d) $x_v = L - \frac{n\pi}{k} = L - \frac{n\lambda}{2}$; $n = 0, 1, 2 \ldots \frac{2L}{\lambda}$

n	x_v [m]	x_{av} [m]
0	1m	
		0.925
1	0.85	
		0.775
2	0.7	
		0.625
3	0.55	
		0.475
4	0.4	
		0.326
6	0.1	
		0.025

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b) $y(t,x) = A_{max} \frac{\sin(k(L-x))}{\cos(kL)} \cdot e^{j\omega t}$

$A_r = \frac{A_{vh}}{A_{max}} = \frac{\sin(k \cdot L)}{1} = 0.87$; $A_{vh} = 0.87 \cdot A_{max}$

c) $c = 100 \frac{m}{s}$ $f_r = \frac{(2n-1)}{4} \cdot \frac{c}{L}$; $n=1$: min

$f_r = \frac{2-1}{4} \cdot \frac{100\ m}{1\ s \cdot m} = 25 Hz$

$f_{ar} = \left(\frac{n}{2}\right)\left(\frac{c}{L}\right)$; $n = 1, \ldots$

$f_{ar\ min} = \frac{c}{2L} = \frac{100\ m}{2\ s \cdot m} = 50 Hz$

d) $Z_{m0} = -j \rho_L \cdot c \cdot ctg(kL)$ $Z_m = \frac{F}{v}$

$Z_{m0}(f=f_r) = -j \cdot \rho_L \cdot c \cdot \frac{\cos(kL)}{\sin(kL)} = -j \rho_L c \frac{\cos\left(\frac{2\pi f}{c} L\right)}{\sin\left(\frac{2\pi f}{c} \cdot L\right)} = -j \rho_L \cdot c \cdot \frac{\cos\left(\frac{2\pi 25 \cdot 1}{100}\right)}{\sin\left(\frac{2\pi 25}{100}\right)} = 0$

ODZIV JE NESKONČEN PRI ω_r, OBRATNO PRI ω_{ar} ko je ∞

za 30Hz : $j \cdot \rho_L \cdot 32{,}6 \frac{m}{s}$

4) $f = 82.41\,Hz$

$L = 77\ cm$ $\quad \lambda = 2L$

$c = f \cdot \lambda = f \cdot 2L = \underline{\underline{127\ m/s}}$

5) $v = 425\ m/s$ $\quad f = ~~287\,Hz~~\ \underline{\underline{278\,Hz}}$

$L = 76.5\,cm$

n	λ
0	$L = \lambda/2$
1	$L = 2 \cdot (\lambda/2)$
2	$L = 3 \cdot (\lambda/2)$
3	$L = 4 \cdot (\lambda/2)$

$f = \frac{c}{L}$

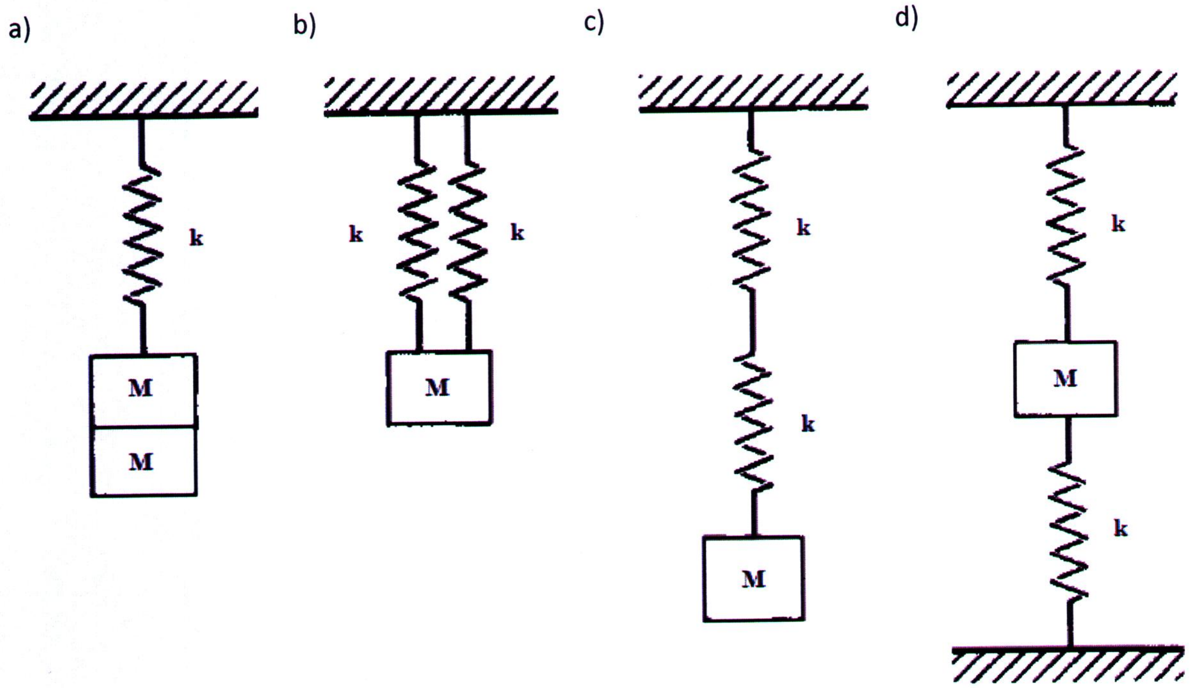
6) $T_2 = 2 \cdot T_1$ $\quad c = \sqrt{\frac{T}{S_L}}$ $\quad c = \lambda \cdot f \Rightarrow f = \frac{c}{\lambda}$ $\quad f = \frac{1}{\lambda}\sqrt{\frac{T}{S_L}}$

$$\frac{f_2}{f_1} = \frac{\sqrt{\frac{T_2}{S_L}}}{\sqrt{\frac{T_1}{S_L}}} = \frac{\sqrt{2 \cdot T_1}}{\sqrt{T_1}} = \underline{\underline{\sqrt{2}}}$$

Nedušeno nihanje

1. Visečo vijačno vzmet z maso 10g potegnemo s silo 1 N, pri čemer se vzmet raztegne za 1 cm. Vzmet nato hipoma izpustimo. Določite enačbo nihanja vzmeti.

2. Določite naravne frekvence sistemov na spodnjih štirih slikah.



3. Nihalo na vijačno vzmet, ki niha s frekvenco ω_0, doseže največjo hitrost nihanja V ob času t=T/2. Določite x(t).

4. Nihanje nekega telesa opišemo z enačbo $x(t) = 10\sin\left(\frac{\pi}{3}t + \frac{\pi}{3}\right)[m]$. Določite nihajni čas, največjo hitrost in največji pospešek nihanja. Po kolikšnem času telo prvič doseže skrajno lego pri nihanju?

5. Nihanje telesa z maso 10g opisuje enačba $x(t) = 10\sin\left(\frac{\pi}{3}t + \frac{\pi}{6}\right)[m]$. Določite silo, ki povzroča nihanje. Kolikšne so amplitude nihanja, hitrosti nihanja, pospeška nihanja in sile, ki deluje na telo? Po kolikšnem času je kinetična energija telesa enaka polovici amplitude kinetične energije?

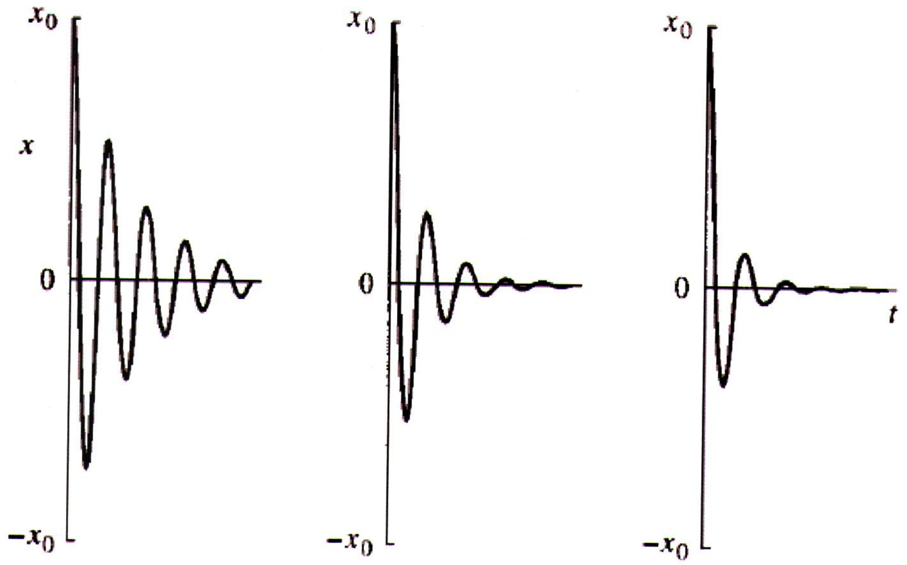
6. Nihalo na vijačno vzmet z naravno frekvenco nihanja 5 rad/s odmaknemo 30 mm iz ravnovesne lege in spustimo. Poiščite a) amplitudo nastalega gibanja, b) največjo doseženo hitrost in c) začetni pospešek.

7. Pokažite, da sta pri vsakem nedušenem nihanju vijačne vzmeti največji kinetična in potencialna energija enaki.

Dušeno nihanje

8. Masa 0.5 kg je obešena na vijačno vzmet. Ko na vzmet pripnemo dodatno maso 0.2 kg, se vzmet raztegne za 40 mm. Ko novo dodano maso v trenutku odstranimo z vzmeti, vzmet s prvotno maso 0.5 kg zaniha. Amplituda nihanja se v eni sekundi zmanjša na 1/e svoje prvotne vrednosti. Izračunajte R_m, ω_d, A in kot ϕ.

9. Vijačno vzmet odmaknemo 1cm iz prvotne lege, jo zadržimo in hipoma izpustimo. Določite amplitudo A in fazo ϕ, če velja
   a) β/ω_0=0.1
   b) β/ω_0=0.2
   c) β/ω_0=0.3



(a) (b) (c)

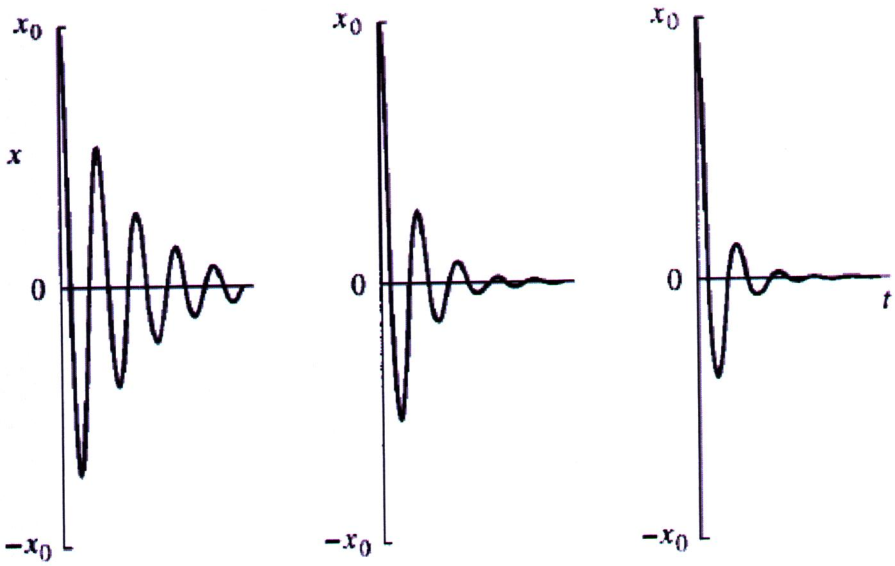
http://demonstrations.wolfram.com/ForcedOscillatorWithDamping/

Dušeno nihanje

1. Po 4 s nihanja je amplituda nihanja vzmeti 5 cm, po 8 s pa 3 cm. Po kolikšnem času bo amplituda nihanja padla na 1 cm?

2. Masa 0.5 kg je obešena na vijačno vzmet. Ko na vzmet pripnemo dodatno maso 0.2 kg, se vzmet raztegne za 40 mm. Ko novo dodano maso v trenutku odstranimo z vzmeti, vzmet s prvotno maso 0.5 kg zaniha. Amplituda nihanja se v eni sekundi zmanjša na 1/e svoje prvotne vrednosti. Izračunajte R_m, ω_d, A in kot ϕ.

3. Vijačno vzmet odmaknemo 1cm iz prvotne lege, jo zadržimo in hipoma izpustimo. Določite amplitudo A in fazo ϕ, če velja
   a) $\beta/\omega_0=0.1$
   b) $\beta/\omega_0=0.2$
   c) $\beta/\omega_0=0.3$



(*a*) (*b*) (*c*)

Vsiljeno nihanje

Izpit →

4. Nihalo z naravno krožno frekvenco 300 rad/s vsiljeno niha s krožno frekvenco 320 rad/s in amplitudo 6 cm. Ko krožno frekvenco vsiljenega nihanja povečamo na 340 rad/s, začne nihalo nihati z amplitudo 5 cm. Izračunajte koeficient β.

5. Izpeljite splošni izraz za pospešek dušenega nihanja nihala na vijačno vzmet, ki mu nihanje vsiljuje sila $F\cos(\omega t)$. Pri katerih krožnih frekvencah je pospešek največji?

6. Pri katerih krožnih frekvencah je fazni kot mehanske impedance pri vsiljenem dušenem nihanju Θ enak 0?

http://demonstrations.wolfram.com/ForcedOscillatorWithDamping/

Linearne kombinacije harmoničnih vibracij

7. Neka točka tekočega medija niha pod vplivom treh harmoničnih vibracij iste amplitude in frekvence, ki pa se razlikujejo v fazi ($\phi_1=0$, $\phi_2=30°$, $\phi_3=45°$). Zapišite enačbo gibanja točke.

8. Nihanje je sestavljeno iz dveh harmonskih komponent. Kolikšna je krožna frekvenca nihanja, če imata harmonski komponenti krožni frekvenci
   a) ω_1 = 3 rad/s, ω_2 = 5 rad/s
   b) ω_1 = 6 rad/s, ω_2 = 4 rad/s
   c) ω_1 = 1 rad/s, ω_2 = $\sqrt{2}$ rad/s
   d) ω_1 = 0.71 rad/s, ω_2 = 1.41 rad/s

Linearne kombinacije harmoničnih vibracij

*grafično izpit

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http://www.wolframalpha.com

Nihanje strune

1. Po neskončno dolgi struni z linearno gostoto 0.1 g/cm se širi valovanje y=4 cos(3t-2x), pri čemer sta *x* in *y* dolžini, izraženi v *cm*, *t* pa je čas v sekundah.

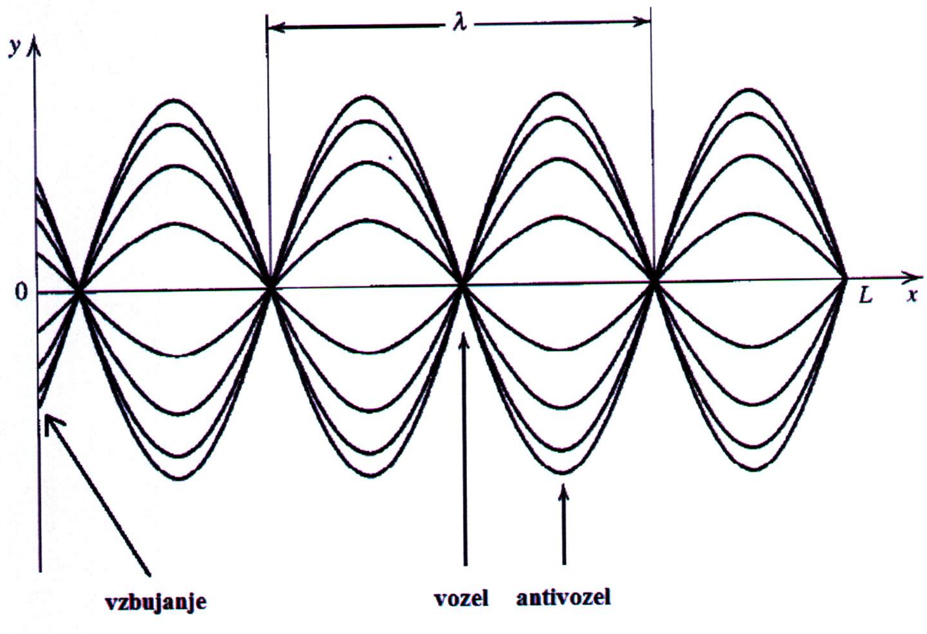
   a) Koliko znašajo amplituda, fazna hitrost, frekvenca, valovna dolžina in valovno število?

   b) Kolikšna je hitrost delcev elementa strune pri x=0 ob času t=0?

2. Struna z dolžino 0.44 m in linearno gostoto 0.01 kg/m je na eni strani togo vpeta, na drugi strani pa ji vsiljujemo prečne periodične (sinusne) vibracije. Struna je napeta s prožnostno silo 5 N.

   a) Izračunajte frekvenco, pri kateri je razdalja med vozli na struni 0.1 m.

   b) Kolikšna je amplituda sinusnega vzbujanja pri izračunani frekvenci, če je maksimalna amplituda nihanja na struni 0.02 m ?



3. Struna dolžine 1 m je na eni strani togo vpeta, na drugi strani pa jo vzbujamo s prečnim sinusnim valovanjem valovne dolžine 0.3 m.

   a) Določite mesta vozlov in antivozlov.

   b) Kolikšna je relativna amplituda nihanja na prostem koncu v primerjavi z največjo amplitudo na struni?

   c) Izračunajte najnižjo resonančno in antiresonančno frekvenco, če je hitrost fazna hitrost valovanja 100 m/s

   d) Izračunajte vhodno mehansko impedanco pri resonančni in antiresonančni frekvenci ter frekvenci 30 Hz.

4. Struna E na kitari niha s frekvenco 82.41 Hz. Dolžina strune je 77 cm. Kolikšna je hitrost valovanja strune?

5. Hitrost valovanja kitarske strune je 425 m/s. Določite osnovno frekvenco strune (prvi harmonik), če je dolžina strune 76.5 cm.

6. Koliko se spremeni frekvenca strune, če se njena napetost dvakrat poveča?

http://demonstrations.wolfram.com/TransverseStandingWaves/

Ravninski val

1. Ravninsko zvočno valovanje se širi po zraku s frekvenco 100 Hz in amplitudo zvočnega tlaka 2 Pa.

a) Izračunajte jakost in nivo jakosti (IL) zvočnega valovanja

b) Kakšna je amplituda hitrosti delcev?

c) Izračunajte efektivni zvočni tlak

d) Izračunajte nivo zvočnega tlaka (SPL)

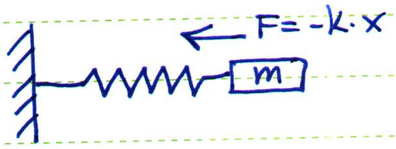
2. Pokažite, da ima ravninski val, ki potuje po zraku z efektivnim zvočnim tlakom 1μbar, nivo jakosti (IL) 74 dB *re* 10^{-12} W/m^2.

3. Kakšna je jakost ravninskega zvočnega vala, ki potuje po vodi z nivojem zvočnega tlaka SPL(1μbar) = 120 dB?

4. Kakšno je razmerje (v dB) med zvočnima tlakoma dveh ravninskih zvočnih valovanj, ki se razširjata v vodi in zraku, če imata obe valovanji enako jakost?

2.10.2013

AKUSTIKA



$$F = m \cdot a$$

$$-k \cdot x = m \cdot \ddot{x}$$

$$m \cdot \ddot{x} + k \cdot x = 0$$

$$m \cdot \frac{\partial^2 x}{\partial t^2} + \frac{k}{m} \cdot x = 0$$

$$\frac{\partial^2 x}{\partial t^2} + \omega_0^2 \cdot x = 0$$ enačba nihanja

rešimo z nastavkom: $x = A_1 \cdot \cos \gamma t$

$$\frac{\partial^2 x}{\partial t^2} = -A_1 \gamma^2 \cos \gamma t$$

$$-A_1 \gamma^2 \cos \gamma t + \omega_0^2 A_1 \cos \gamma t = 0$$

$$\gamma^2 = \omega_0^2 \Rightarrow \gamma = \omega_0$$

$$x_1 = A_1 \cos(\omega_0 t), \quad x_2 = A_1 \sin(\omega_0 t)$$

$$x = A_1 \cdot \cos(\omega_0 t) + A_2 \sin(\omega_0 t)$$

zač. pogoji: $t=0$, $x = x_0$, $v = v_0$

$$v = \dot{x} = A_2 \omega_0 \cos(\omega_0 t) - A_1 \omega_0 \sin(\omega_0 t)$$

$x_0 = A_1$

$v_0 = A_2 \cdot \omega_0$

$$x = x_0 \cos(\omega_0 t) + \frac{v_0}{\omega_0} \sin(\omega_0 t)$$

$A_1 = A \cdot \cos\phi \quad ; A_2 = -A \cdot \sin\phi$

$x = A \cdot \cos(\phi)\cos(\omega_0 t) - A\sin(\phi)\sin(\omega_0 t)$

$$x = A \cdot \cos(\omega_0 t + \phi)$$
$$v = -A \cdot \omega_0 \sin(\omega_0 t + \phi)$$
$$a = -A \cdot \omega_0^2 \cos(\omega_0 t + \phi)$$

zač. pogoji: $t=0$, $x = x_0$, $v = v_0$

$x_0 = A \cdot \cos\phi \rightarrow A = \frac{x_0}{\cos\phi}$ $\qquad \frac{x_0}{\cos\phi} = -\frac{v_0}{\omega_0 \sin\phi}$ $\qquad \frac{\sin\phi}{\cos\phi} = -\frac{v_0}{x_0\omega_0}$

$v_0 = -A\omega_0 \sin\phi \rightarrow A = -\frac{v_0}{\omega_0 \sin\phi}$

$$\phi = \operatorname{arc\,tg}\left(-\frac{v_0}{x_0\omega_0}\right)$$

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_0}\right)^2}$$

$E = E_{pr} + E_k \quad ; E_k = \frac{1}{2}mv^2 \quad ; E_{pr} = \int_x k \cdot x\,dx = k \cdot \frac{x^2}{2}$

$E = k \cdot \frac{x^2}{2} + \frac{1}{2}mv^2 = \frac{k}{2}A^2\cos^2(\omega_0 t + \phi) + \frac{k}{2}A^2\omega_0^2\sin^2(\omega_0 t + \phi)$

$\omega = \sqrt{k/m}$

$k = \omega_0^2 m \quad \Rightarrow \quad E = \frac{1}{2}m\omega_0^2 A^2$

(2)

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$$\frac{\partial^2 x}{\partial t^2} + \omega_0^2 \cdot x = 0 \quad ; \quad x = e^{\gamma t}$$

$$A \cdot \gamma^2 \cdot e^{\gamma t} + \omega_0^2 A e^{\gamma t} = 0$$

$$(A \cdot \gamma^2 + \omega_0^2 A) = 0$$

$$\gamma^2 + \omega_0^2 = 0$$

$$\gamma^2 = -\omega_0^2$$

$$\gamma = \pm j\omega_0$$

$$x = A_1 \cdot e^{j\omega_0 t} + A_2 \cdot e^{-j\omega_0 t}$$

$$x_0 = A_1 + A_2$$

$$A_1 = \frac{1}{2}\left(x_0 - j\frac{v_0}{\omega_0}\right)$$

$$A_2 = \frac{1}{2}\left(x_0 + j\frac{v_0}{\omega_0}\right)$$

$$x = x_0 \cos(\omega_0 t) + \frac{v_0}{\omega_0}\sin(\omega_0 t)$$

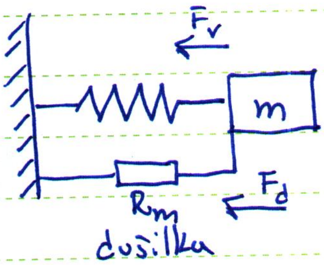
$$A = |A| \cdot e^{j\phi} \qquad |A| = \sqrt{a^2 + b^2}$$

$$\phi = \operatorname{arctg}\frac{b}{a}$$

$$x = |A| \cdot e^{j(\omega_0 t + \phi)}$$

AKUSTIKA 9.10.2013

Dušeno nihanje:



$$F_d = -R_m \cdot v = -R_m \cdot \frac{dx}{dt}$$

$$m \cdot a = -k \cdot x - R_m \cdot \frac{dx}{dt}$$

$$m \cdot \frac{d^2x}{dt^2} = -k \cdot x - R_m \cdot \frac{dx}{dt} \quad / :m$$

$$\frac{k}{m} = \omega_0^2$$

$$\frac{d^2x}{dt^2} + \omega_0^2 \cdot x + \frac{R_m}{m}\frac{dx}{dt} = 0$$

Rešitev v obliki: $x = A \cdot e^{\mu t}$

$$A \cdot \mu^2 e^{\mu t} + \frac{R_m}{m}\mu \cdot A e^{\mu t} + \omega_0^2 A e^{\mu t} = 0$$

$$A \cdot e^{\mu t}\left(\mu^2 + \frac{R_m}{m}\mu + \omega_0^2\right) = 0$$

$$\mu^2 + \frac{R_m}{m}\mu + \omega_0^2 = 0$$

$$\mu = \frac{-\frac{R_m}{m} \pm \sqrt{\frac{R_m^2}{m^2} - 4\omega_0^2}}{2}$$

$$\mu = -\frac{R_m}{2m} \pm \sqrt{\frac{R_m^2}{4m^2} - \omega_0^2}$$

$$\frac{R_m}{2m} = \beta \quad \text{... absorbcijski koef.}$$

$$\mu = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$$

$\beta < \omega_0$ pomeni da je rešitev Im

Poseben primer: $\beta = 0$

$\mu = \pm j\omega_0$; pridemo do idealnega nihanja

$$\omega_d < \omega_0$$

vpeljemo kotno freq. dušenega nihanja:

$$\omega_d = \sqrt{\omega_0^2 - \beta^2}$$

$$\boxed{\mu = -\beta \pm j\omega_d}$$

$$x(t) = A_1 \cdot e^{(-\beta + j\omega_d)t} + A_2 \cdot e^{(-\beta - j\omega_d)t}$$

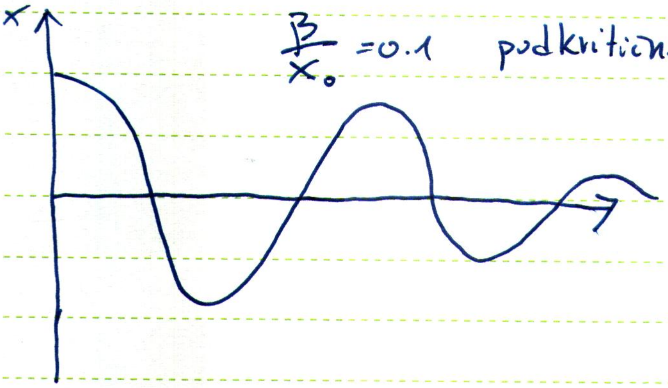
$$x(t) = e^{-\beta t}\left(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t}\right)$$

} splošna rešitev DE!

možna rešitev takega $Re[x(t)]$

$$x(t) = e^{-\beta t}\left(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t)\right)$$

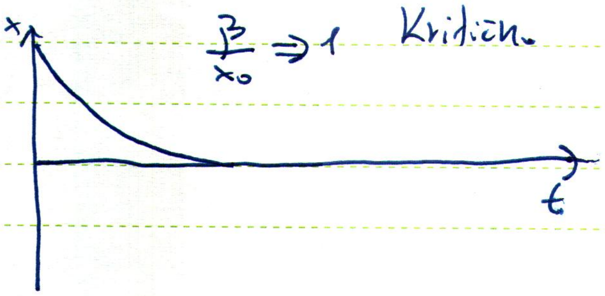
$\frac{\beta}{x_0} = 0.1$ podkritično



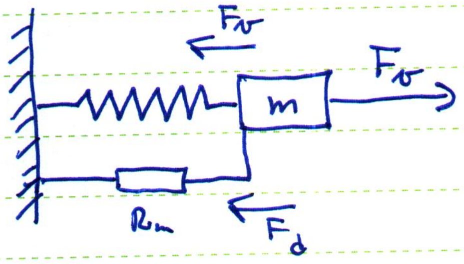
$\tau = \frac{1}{\beta}$ ... relaksacijski čas

ko pade amplituda pod $\frac{1}{e}$.

$\frac{\beta}{x_0} \Rightarrow 1$ Kritično



AKUSTIKA — vzbujanje — 9.10.2013



$$F_v = F\cdot e^{j\omega t}$$

$$F_v = F_1\cdot \cos(\omega t)$$ F ... konstanta $F = \frac{F_1}{m}$

$$m\frac{d^2x}{dt^2} = -k\cdot x - R_m\frac{dx}{dt} + F_1 e^{j\omega t}$$

$$m\frac{d^2x}{dt^2} + kx + R_m\frac{dx}{dt} = F\cdot e^{j\omega t}$$

Rešitev oblike: $x = A\cdot e^{j\omega t}$

$$-A\omega^2 m e^{j\omega t} + j R_m \omega A e^{j\omega t} + k A e^{j\omega t} = F\cdot e^{j\omega t}$$

~~$A\cdot e^{j\omega t}\left(-\omega^2 + j\frac{R_m}{m}\omega + \omega_0^2\right) = F\cdot e^{j\omega t}$~~

$$A\cdot e^{j\omega t}\left(-\omega^2 m + jR_m\omega + k\right) = F\cdot e^{j\omega t}$$

$$A = \frac{F}{-\omega^2 m + jR_m\omega + k} = \frac{F}{j\omega\left(j\omega m + R_m - j\frac{k}{\omega}\right)} = \frac{F}{j\omega\left(R_m + j\left(\omega m - \frac{k}{\omega}\right)\right)}$$

$$x(t) = \frac{F\, e^{j\omega t}}{j\omega\left(R_m + j\left(\omega m - \frac{k}{\omega}\right)\right)}$$

$$v(t) = \frac{F\cdot e^{j\omega t}}{R_m + j\left(\omega\cdot m - \frac{k}{\omega}\right)}$$

Impedanca (mehanska)

$$Z_m = R_m - jX_m$$

$$X_m = \omega \cdot m - \frac{k}{\omega}$$

$$x(t) = \frac{F \cdot e^{j\omega t}}{j\omega \cdot Z_m}$$

$$v(t) = \frac{F \cdot e^{j\omega t}}{Z_m}$$

$$Z_m = |Z_m| \cdot e^{j\Theta}$$

$$|Z_m| = \sqrt{R_m^2 + \left(\omega m - \frac{k}{\omega}\right)^2}$$

$$\Theta = \operatorname{arctg} \frac{\omega m - \frac{k}{\omega}}{R_m}$$

$$Z_m = \frac{F}{v}$$

$$v = j\omega X(t)$$

$$x(t) = \frac{v}{j\omega} = \frac{F_v}{j\omega Z_m}$$

$$x(t) = \frac{F \cdot e^{j\omega t}}{j\omega |Z_m| \cdot e^{j\Theta}} = \frac{F \cdot e^{j\omega t} \cdot e^{-j\Theta}}{j\omega |Z_m|} = \frac{F}{j\omega |Z_m|}(\cos\omega t + j\sin\omega t)(\cos\Theta - j\sin\Theta)$$

$$Re[x(t)] = \frac{F}{\omega |Z_m|}(-\cos(\omega t)\sin\Theta + \sin\omega t\cos\Theta) = \frac{F}{\omega |Z_m|} \cdot \sin(\omega t - \Theta)$$

$$v(t) = \frac{F}{|Z_m|}\cos(\omega t - \Theta)$$

AKUSTIKA 8.10.2013

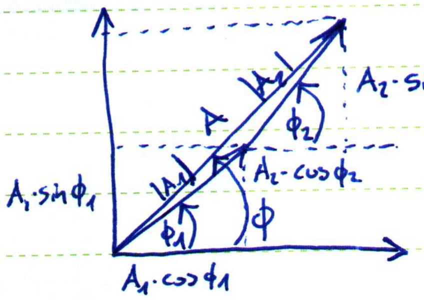
Več vzbujanj / linearne kombinacije različnih vzbujanj

· 2 vzbujanje pri isti frekvenci:

$x_1 = A_1 \cdot e^{j(\omega_1 t + \phi_1)}$; $x_2 = A_2 \cdot e^{j(\omega_1 t + \phi_2)} \Rightarrow x = x_1 + x_2$

$$x = A_1 e^{j(\omega_1 t + \phi_1)} + A_2 e^{j(\omega_1 t + \phi_2)} \Rightarrow A \cdot e^{j(\omega t + \phi)}$$

$$\boxed{\omega = \omega_1}$$



$$A = \left((A_1\cos\phi_1 + A_2\cos\phi_2)^2 + (A_1\sin\phi_1 + A_2\sin\phi_2)^2\right)^{\frac{1}{2}}$$

$$\phi = \operatorname{arctg} \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$$

$$\boxed{|A_1 - A_2| \leq A \leq |A_1 + A_2|}$$

splošna formula:

$$A = \sqrt{\left(\sum A_n \cos\phi_n\right)^2 + \left(\sum A_n \sin\phi_n\right)^2}$$

$$\phi = \operatorname{arctg} \frac{\sum A_n \sin\phi_n}{\sum A_n \cos\phi_n}$$

- Utripanja pri različnih frekvencah:

$$x_1 = A_1 e^{j(\omega_1 t + \phi_1)} \qquad x_2 = A_2 e^{j(\omega_2 t + \phi_2)}$$

- če je $\frac{\omega_1}{\omega_2}$ rac. število potem je gibanje periodično.

ω = največji skupni deljitelj od ω_1 in ω_2

zgled:

$\omega_2 = \omega_1 + \Delta\omega$

$$x = A_1 \cdot e^{j(\omega_1 t + \phi_1)} + A_2 e^{j((\omega_1 + \Delta\omega)t + \phi_2)} = \underline{e^{j\omega_1 t}\left(A_1 e^{j\phi_1} + A_2 e^{j(\Delta\omega t + \phi_2)}\right)}$$

rešitev oblike: $A \cdot e^{j(\omega_1 t + \phi)}$

$$A = A_1(\cos\phi_1 + j\sin\phi_1) + A_2(\cos(\Delta\omega t + \phi_2) + j\sin(\Delta\omega t + \phi_2))$$

$$= A_1\cos\phi_1 + A_2\cos(\Delta\omega t + \phi_2) + j(A_1\sin\phi_1 + A_2\sin(\Delta\omega t + \phi_2))$$

$$A = A_1^2\cos^2\phi_1 + A_2^2\cos^2(\Delta\omega t + \phi_2) + 2\cdot A_1 A_2 \cos\phi_1 \cos(\Delta\omega t + \phi_2) +$$
$$+ A_1^2\sin^2\phi_1 + A_2^2\sin^2(\Delta\omega t + \phi_2) + 2\cdot A_1 A_2 \sin\phi_1 \sin(\Delta\omega t + \phi_2)$$ //koren iz tega

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2 - \Delta\omega t)}$$

$$\Theta = \operatorname{arctg}\frac{A_1\sin\phi_1 + A_2\sin(\Delta\omega t + \phi_2)}{A_1\cos\phi_1 + A_2\cos(\Delta\omega t + \phi_2)}$$

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(4)

za: $A_1 = A_2 \qquad \phi_1 = \phi_2 = 0$

$$A = \sqrt{2A_1^2 + 2A_1^2 \cos(-\Delta\omega t)} = A_1 \sqrt{2 + 2\cos(-\Delta\omega t)}$$

$$\theta = \text{arctg} \frac{A_1 \sin(\Delta\omega t)}{A_1 + A_1 \cos(\Delta\omega t)} = \text{arctg} \frac{\sin(\Delta\omega t)}{1 + \cos(\Delta\omega t)}$$

$$\boxed{0 \leq A \leq 2A_1}$$ utripanje


ČLANSKA IZKAZNICA
ZASLUŽKI
NAPOTNICE
MOJA DELA
MOJE IZKUŠNJE
UGODNOSTI
POSLOVALNICE

16.10.2013

AKUSTIKA

Fourierjev transform:

pogoj: $\int_T |f(t)|\,dt < \infty$

$$f(t) = \frac{1}{\cancel{2}}A_0 + A_1\cos(\omega t) + \dots + B_1\sin(\omega t) + B_2\sin(2\omega t) + \dots$$

Če sta $f(t)$ in $f'(t)$ zvezna, potem vrsta konvergira.

Za SODE FUNKCIJE:

$$f(t) = f(-t) \Rightarrow B_n = 0$$

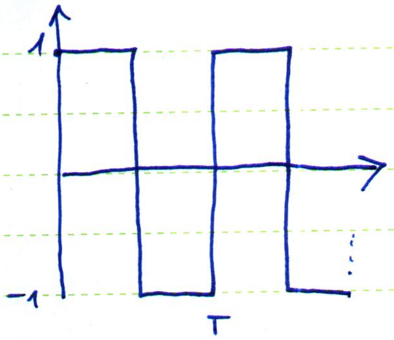
LIHE F:

$$f(t) = -f(-t) \Rightarrow A_n = 0$$

$$A_n = \frac{2}{T}\int_T f(t)\cdot\cos(n\omega t)\,dt$$

$$B_n = \frac{2}{T}\int_T f(t)\cdot\sin(n\omega t)\,dt$$

Primer:



$$f(t) = \begin{cases} 1 & 0 \le t \le T/2 \\ -1 & T/2 \le t \le T \end{cases}$$

$\int \sin(ax)dx = -\frac{\cos(ax)}{a}$

$$B_n = \frac{2}{T}\left[\int_0^{T/2} \sin(n\omega t)dt + \int_{T/2}^{T} (-1)\sin(n\omega t)dt\right] = \frac{2}{T}\left[\frac{-1}{n\omega}\cos(n\omega t)\Big|_0^{T/2} + \frac{1}{n\omega}\cos(n\omega t)\Big|_{T/2}^{T}\right]$$

$$= \frac{2}{Tn\cdot\omega}\left[\cos(n\omega T) - \cos(n\omega \tfrac{T}{2}) - \cos(n\omega \tfrac{T}{2}) + 1\right] =$$

$$= \frac{2}{Tn\omega}\left[\cos(n\omega T) - 2\cos(n\omega \tfrac{T}{2}) + 1\right] = \qquad \omega = \frac{2\pi}{T}$$

$$= \frac{2T}{T\cdot n\cdot 2\pi}\left[\cos(2\pi n) - 2\cdot\cos(n\pi) + 1\right] = \frac{1}{n\pi}\left[2 - 2\cdot\cos(n\pi)\right]$$

$n = 1, 3, 5, 7, 9, \ldots \qquad B_n = \frac{4}{n\pi}$

$$f(t) = \frac{4}{\pi}\left(\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t)\ldots\right)$$

Aperiodični signal:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}dt \qquad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\cdot e^{j\omega t}d\omega$$

SODA: $f(t) = f(-t) \Rightarrow Re$

LIHA: $f(t) = -f(-t) \Rightarrow Im$

$$X[\omega] = Re[X(\omega)] + j\,Im[X(\omega)]$$

$$X[\omega] = A\cdot x(\omega)\cdot e^{j\Theta x(\omega)}$$

$A(\omega) = |X(\omega)|$

$\Theta(\omega) = \arg(X(\omega))$

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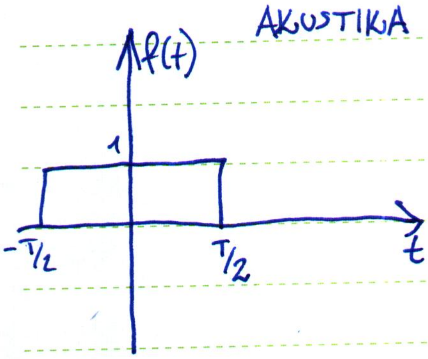
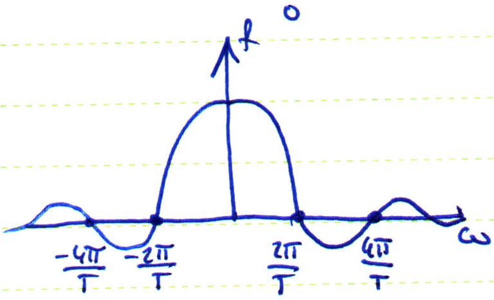


$$f(t) = \begin{cases} 1 & |t| \le T/2 \\ 0 & |t| > T/2 \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt = 2 \cdot \int_{0}^{T/2} \cos(\omega t)\, dt = \frac{2}{\omega} \sin(\omega t) \Big|_0^{T/2} =$$

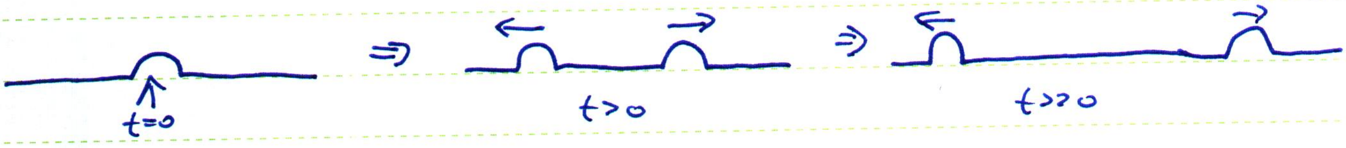
($\cos(\omega t)$ = $Re[e^{-j\omega t}]$)

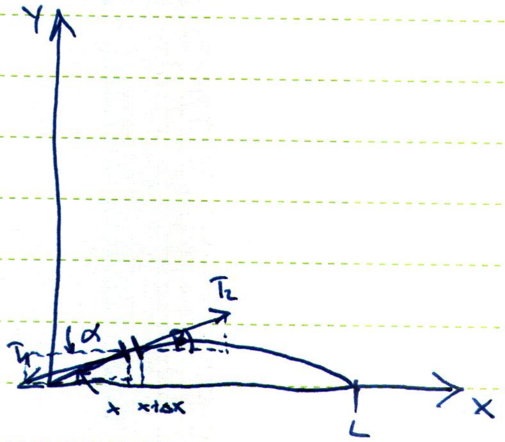
$$F(\omega) = \frac{2}{\omega} \sin\left(\omega \frac{T}{2}\right) = T \cdot \frac{\sin(\omega T/2)}{(\omega \cdot T/2)}$$



ničle: $\omega = \frac{2\pi n}{T}$

Nihanje strune:





T_1, T_2 - sila tenzije

- homogena struna: ρ_L linijska gostota kg/m
- $F_y = 0$
- transverzalno valovanje

$\cdot\, T_1 \cdot \cos\alpha = T_2 \cdot \cos\beta = T = konst$

$$T_2 \sin\beta - T_1 \sin\alpha = m \cdot a \qquad m = \rho_L \Delta x$$

$$T_2 \sin\beta - T_1 \sin\alpha = \rho_L \cdot \Delta x \cdot \frac{\partial^2 y}{\partial t^2} \quad /:T$$

$$\frac{T_2 \sin\beta}{T_2 \cos\beta} - \frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{\rho_L \Delta x}{T} \cdot \frac{\partial^2 y}{\partial t^2}$$

$$tg\beta - tg\alpha = \frac{\rho_L \Delta x}{T} \cdot \frac{\partial^2 y}{\partial t^2} \quad ; \quad tg\beta = \frac{f(x+\Delta x) - f_x}{\Delta x} = \frac{\partial f}{\partial x}$$

$$\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_x = \frac{\rho_L \Delta x}{T} \cdot \frac{\partial^2 y}{\partial t} \quad /:\Delta x$$

$$\frac{\partial^2 y}{\partial t} \cdot \frac{\rho_L}{T} = \frac{\partial^2 y}{\partial x} \qquad \frac{T}{\rho_L} = c^2$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t}}$$

1D valovna enačba

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splošna rešitev:

$$y(x,t) = y_1(ct-x) + y_2(ct+x)$$

$$ct_1 - x_1 = ct_2 - x_2$$

$$c(t_2 - t_1) = x_2 - x_1$$

$$c = \frac{x_2 - x_1}{t_2 - t_1}$$ fazna hitrost

$$v(x,t) = \frac{\partial y}{\partial t}$$ skupna hitrost

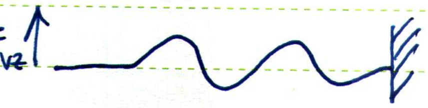
Napredujoči in odbiti val, ki se odštejeta na koncih, če je

struna vpeta:

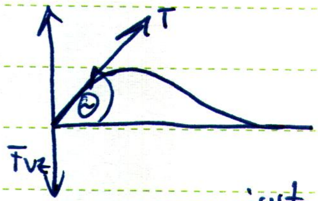
$$y(x,t) = y_1(ct-x) - y_1(ct+x)$$

Nevpeta struna: tu se val odbije nazaj z isto amp. ki se nato sešteje in dobimo višjo amp.

$$y(x,t) = y_1(ct-x) + y_1(ct+x)$$



$$y(x,t) = A\cdot e^{j(\omega t - kx)} + B\cdot e^{j(\omega t + kx)}$$



$$F_{vz} = -\sin\Theta\cdot T \quad \rightarrow \Theta \rightarrow 0$$

$$F\cdot e^{j\omega t} = -\left(\frac{\partial y}{\partial x}\right)_{x=0}\cdot T \qquad F\cdot e^{j\omega t} + T\left(\frac{\partial y}{\partial x}\right) = 0$$

$$\left(\frac{\partial y}{\partial x}\right)_{x=0} = -jk\cdot A\cdot e^{j(\omega t - kx)} + jk B\cdot e^{j(\omega t + kx)}\Big|_{x=0} = (-jk\cdot A + jkB)\,e^{j\omega t}$$

$$F\cdot \cancel{e^{j\omega t}} + T\cdot(-jkA + jkB)\cancel{e^{j\omega t}} = 0$$

$$F + T\cdot(-jkA + jkB)\cancel{e^{j\omega t}} = 0$$

$x=L$, $t=0$, $y=0$:

$$0 = A\cdot e^{-jkL} + B\cdot e^{jkL}$$

$$A = \frac{-B\cdot e^{jkL}}{e^{-jkL}}$$

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$$\frac{e^{jx}+e^{-jx}}{2} = \cos x$$

$$F + T(-jkA + jkB) = 0$$

$$F + Tjk\left(+\frac{B e^{jkL}}{e^{-jkL}} + B\right) = 0$$

$$F + jkT\cdot B\left(e^{jkL}/e^{-jkL} + 1\right) = 0$$

$$F e^{-jkL} + 2jkBT\cos(kL) = 0$$

$$B = -\frac{F\cdot e^{-jkL}}{2jkT\cos(kL)} \qquad A = \frac{F\cdot e^{+jkL}}{2jkT\cos(kL)}$$

$$y(x,t) = \frac{F}{2jkT\cos(kL)}\cdot\left[e^{j(\omega t-kx)+jkL} - e^{j(\omega t+kx)-jkL}\right]$$

$$Y(x,t) = \frac{F\cdot e^{j\omega t}}{2jkT\cos(kL)}\left[e^{jk(L-x)} - e^{-jk(L+x)}\right]$$

potujoča valova

$$y(x,t) = \frac{F\cdot e^{j\omega t}}{T\cdot k\cdot\cos(k\cdot L)}\cdot\sin(k(L-x))$$

stojni val (stojno valovanje)

- amp. je 0 → VOZLI

$$\sin(k(L-x)) = 0$$

$$k(L-x) = n\cdot\pi \quad ; n=0,1,2,3,\ldots$$

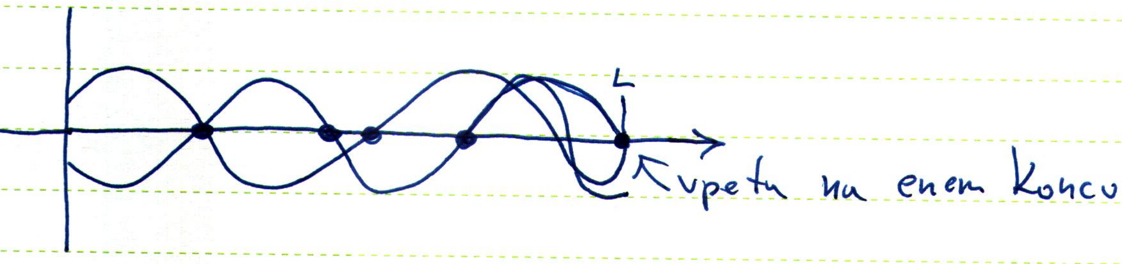
$$kL - kx = n\pi$$

$$kx = -n\pi + kL$$

$$x = L - \frac{n\pi}{k} \qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$x = L - \frac{\pi n\cdot\lambda}{2\pi} = L - \frac{n\lambda}{2} = x_v$$

$$L = \frac{n\lambda}{2} \quad n = \frac{2L}{\lambda} \qquad n = 0, 1, 2, \ldots \frac{2L}{\lambda}$$



na $\frac{\lambda}{4}$ dobimo max. amplituda

$$\cos(kL) = 0$$

$$kL = \frac{\pi}{2}(2n-1) \quad n=1,3,5,7 \quad \text{samo te so (lihi)}$$

$$\frac{2\pi f}{c}\cdot L = \frac{\pi}{2}(2n-1)$$

$$f_r = \frac{c}{L}\,\frac{(2n-1)}{4} \quad ; \text{resonančne frekvence}$$

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min Amplitudu:

$$\cos(kL) = 1$$

$$kL = n \cdot \pi$$

$$\frac{2\pi f}{c} \cdot L = n \cdot \pi$$

$$f_{ar} = \left(\frac{c}{L}\right) \cdot \left(\frac{n}{2}\right)$$ anti resonančna frekvenca

Impedanca:

$$Z_m = \frac{f}{v}$$ pri $x=0$; na vzbujanju

$$Z_m = \frac{F \cdot e^{j\omega t}}{\frac{\partial y}{\partial t}}$$

$$\frac{\partial y}{\partial t} = \frac{F \cdot e^{j\omega t} \cdot \sin(k(L-x))}{k \cdot T \cdot \cos(k \cdot L)} \cdot j\omega$$

$$Z_m = \frac{F \cdot e^{j\omega t} \cdot k \cdot T \cdot \cos(kL)}{F \cdot e^{j\omega t} \cdot \sin(k(L-x)) j\omega} = \frac{T \cdot k}{j\omega} \cdot ctg(kL)$$

$$T = c^2 \rho_L \quad , k = \frac{2\pi f}{c}$$

$$Z_m = \frac{\omega c^2 \rho_L}{c\, j \cdot \omega} \cdot ctg(kL) = -j \rho_L \cdot c \cdot ctg(k \cdot L)$$ čisto Im

$$Z_m = R_m + jX_m \quad ; R_m = 0$$

za $f = f_r \Rightarrow Z_m = 0$; $f = f_{ar} \Rightarrow Z_m = \infty$

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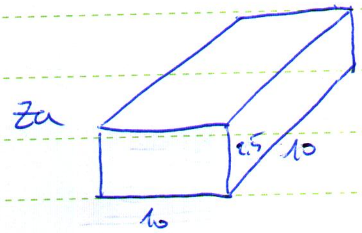
Akustika

-slajd 1/7

$$MFP = \frac{4V}{S}$$



$abc = 250\,m^3$ $\quad S = 2 \cdot 100 + 4 \cdot 25 = 300\,m^2$

$MFP = \frac{1000}{300} = 33.3\,m$ povprečna pot med odboji

-ovire na valovnih poteh slajd 10:

· <u>Akustična velikost:</u>

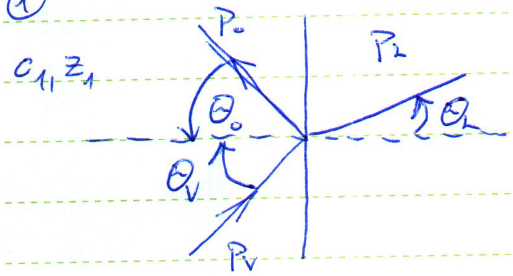
· <u>Akustična senca</u>

⇒ Ravninsko valovanje

⇒ Meja je popolna ravnina

⇒ Tekočine (oba medija) (kp in plin)

(1)



Iščemo $p_{2,1}$, I, W (c, z, θ_v)

$Z = \pm \rho_0 \cdot c$

$Z = \pm r + jX$ ← za ravninski val

$T = \frac{p_L}{p_V}$ koef. loma $\qquad T_I = \frac{I_L}{I_V}$

$R = \frac{p_o}{p_V}$ koef. odboja $\qquad R_I = \frac{I_o}{I_V}$

$I = \frac{p^2}{\rho_0 \cdot c} = \frac{p^2}{r}$

$W = A \cdot I$ ← moč

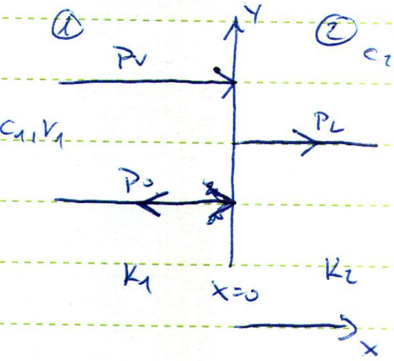
$T_I = \frac{p_L^2}{r_2} \cdot \frac{r_1}{p_V^2} = \frac{r_1}{r_2} \cdot |T|^2$

$T_W = \frac{W_L}{W_V} = \frac{A_L \cdot I_L}{A_V \cdot I_V} = \frac{A_L}{A_V} \cdot T_I$

$R_I = \frac{p_o^2}{r_1} \cdot \frac{r_1}{p_V^2} = |R|^2$

$R_W = \frac{W_o}{W_V} = R_I = |R|^2$

$\boxed{R_W + T_W = 1}$ zakon ohranitvi energije



$k = \frac{\omega}{c}$

$p = P \cdot e^{j(\omega t - \vec{k}\vec{r})}$

$p_V = P_V \cdot e^{j(\omega t - k_x \cdot x)}$

$p_o = P_o \cdot e^{j(\omega t + k_x x)}$

$p_L = P_L \cdot e^{j(\omega t - k_x \cdot x)}$

$p_V + p_o = p_L$

$P_V e^{j(\omega t - k_1 x)} + P_o e^{j(\omega t + k_1 x)} = P_L \cdot e^{j(\omega t - k_2 x)}$

$P_V + P_o = P_L \quad / : P_V$

$\boxed{1 + R = T}$

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$v_v + v_o = v_L$

$$\frac{p_v + p_o}{v_v + v_o} = \frac{p_L}{v_L} \quad \leftarrow \quad r = \pm\frac{p}{v}$$

$$\frac{p_v + p_o}{\frac{p_v}{r_1} - \frac{p_o}{r_1}} = \frac{p_L}{\frac{p_L}{r_2}}$$

$$\frac{p_v + p_o}{p_v - p_o} = \frac{r_2}{r_1} \Rightarrow p_v + p_o = \frac{r_2}{r_1}(p_v - p_o) \quad / : p_v$$

$$1 + R = \frac{r_2}{r_1}(1 - R)$$

$$1 + R = -\frac{r_2}{r_1}R + \frac{r_2}{r_1}$$

$$\boxed{R = \frac{r_2/r_1 - 1}{r_2/r_1 + 1}}$$

$$T = R + 1 = \frac{r_2/r_1 - 1 + r_2/r_1 + 1}{\frac{r_2}{r_1} + 1} = \boxed{\frac{2r_2}{r_2 + r_1} = T}$$

$$R_I = \frac{I_o}{I_v} = |R|^2 = \left(\frac{\frac{r_2}{r_1} - 1}{\frac{r_2}{r_1} + 1}\right)^2 \qquad I_L = \frac{r_1}{r_2}|T|^2 = \frac{r_1}{r_2} \cdot \frac{4r_2^2}{(r_1 + r_2)^2} = \boxed{\frac{4r_1r_2}{(r_2 + r_1)^2} = I_L}$$

ZGLED:

1) $r_1 < r_2$ (zrak < voda)

$R > 0 \Rightarrow$ torej se bo val odbil v fazi nazaj

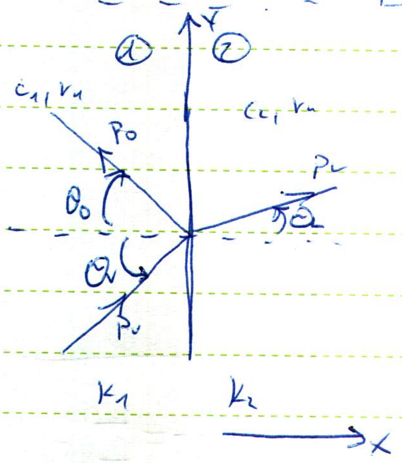
2) $r_1 > r_2$ (voda > zrak)

$R < 0 \Rightarrow$ odbije se nazaj v prot. fazi

3) $r_1 = r_2$

$R = 0 \Rightarrow$ ni odboja

(1–3:) $T > 0$ in realen



$$p = P \cdot e^{j(\omega t - \vec{k}\vec{r})}$$

$$\vec{k} = \vec{1}_x k_x + \vec{1}_y k_y$$

$$k_x = k \cdot \cos\theta_v$$

$$k_y = k \cdot \sin\theta_v$$

$$\boxed{p_v + p_o = p_L \quad @\ x = 0}$$

$$y \cdot \sin\theta_v = y \cdot \sin\theta_o$$

$$\sin\theta_v = \sin\theta_o \quad ; \ \theta_v \in \left[0, \tfrac{\pi}{2}\right]$$

$$\theta_v = \theta_o$$

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lom: $k_1 \cdot y \cdot \sin\theta_v = k_2 \cdot y \cdot \sin\theta_L$

$$\frac{\omega}{c_1} \sin\theta_v = \frac{\omega}{c_2} \cdot \sin\theta_L$$

$$\boxed{\frac{\sin\theta_v}{c_1} = \frac{\sin\theta_L}{c_2}}$$

$v_{v_x} + v_{o_x} = v_{2_x}$ $\qquad v = \pm \frac{p}{r}$ $\qquad$: za x smer

$$v_v \cdot \cos\theta_v + v_o \cdot \cos\theta_o = v_2 \cdot \cos\theta_L$$

$$\frac{p_v}{r_1}\cos\theta_v - \frac{p_o}{r_1}\cos\theta_o = \frac{p_L}{r_2}\cos\theta_L \qquad \theta_v = \theta_o$$

$$\frac{\cos\theta_v}{r_1}(p_v - p_o) = \frac{p_L}{r_2}\cos\theta_L \quad / : p_v$$

$$\frac{\cos\theta_v}{r_1}(1 - R) = T \frac{\cos\theta_L}{r_2}$$

$$(1-R) = T \cdot \frac{\cos\theta_L}{\cos\theta_v} \frac{r_1}{r_2} \qquad (+1 + R) = T$$

$$(1-R) = (1+R)\frac{\cos\theta_L}{\cos\theta_v}\frac{r_1}{r_2}$$

$$1 - \frac{r_1}{r_2}\frac{\cos\theta_L}{\cos\theta_v} = R\left(1 + \frac{r_1}{r_2}\frac{\cos\theta_L}{\cos\theta_v}\right) \Rightarrow \boxed{R = \frac{1 - \frac{r_1}{r_2}\frac{\cos\theta_L}{\cos\theta_v}}{1 + \frac{r_1}{r_2}\frac{\cos\theta_L}{\cos\theta_v}} = \frac{\frac{r_2}{r_1} - \frac{\cos\theta_L}{\cos\theta_v}}{\frac{r_2}{r_1} + \frac{\cos\theta_L}{\cos\theta_v}}}$$

$\cos\Theta_L = \sqrt{1-\sin^2\Theta_L}$ $\sin\Theta_L = \frac{c_2}{c_1}\cdot\sin\Theta_V$

$\cos\Theta_L = \sqrt{1-\left|\frac{c_2}{c_1}\right|^2\sin^2\Theta_L}$

Kdaj ne pride do odboja?
Torej gre vse skozi eg $\Rightarrow R=0$

$\frac{v_2}{v_1} = \frac{\cos\Theta_L}{\cos\Theta_V}$ pogoj

$\sin\Theta_V = \sqrt{\frac{\left(\frac{v_2}{v_1}\right)^2-1}{\left(\frac{v_2}{v_1}\right)^2-\left(\frac{c_2}{c_1}\right)^2}}$

Zgledi:

1) $c_1 > c_2 \Rightarrow \cos\Theta_L \in \mathbb{R}$ (voda – zrak)

$\sin\Theta_L = \frac{c_2}{c_1}\sin\Theta_V \quad \Theta_L < \Theta_V$

2) $c_1 < c_2 \Rightarrow \Theta_V < \Theta_K \quad \cos\Theta_L = 0 \Rightarrow \frac{c_1}{c_2}^2 = \sin^2\Theta_V$

$\sin\Theta_K = \frac{c_1}{c_2}$ $\Rightarrow$ kritični kot

$\Theta_L \in \mathbb{R} \Rightarrow \Theta_L > \Theta_V$

3) $c_1 < c_2 \quad \Theta_V > \Theta_K$

$\Rightarrow$ Vidimo usihajoče polje

$\cos\Theta_L = I_m$

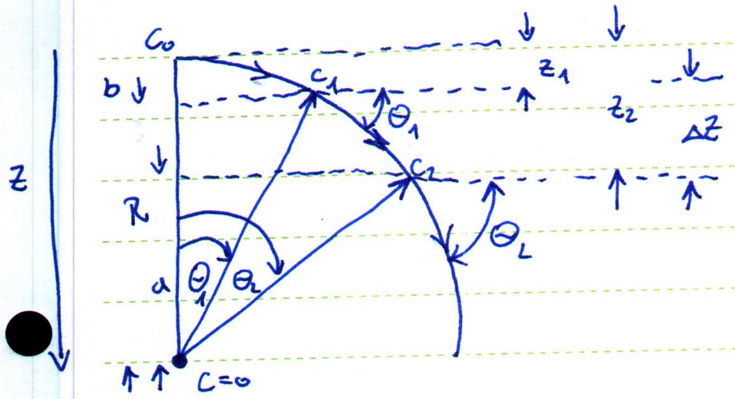
$T_W = ?$ $R_W = |R|^2$ $R_W + T_W = 1$

$$T_W = \frac{4\frac{v_2}{v_1}\cdot\frac{\cos\Theta_L}{\cos\Theta_V}}{\left(\frac{v_2}{v_1}+\frac{\cos\Theta_L}{\cos\Theta_V}\right)^2}$$

Za $c_1 > c_2 \parallel \Theta_V < \Theta_K$,
sicer je $T_W = 0$

AKUSTIKA → 3. ura   **Podvodna akustika**   20.11.2013

$g = \frac{c_2 - c_1}{\Delta z}$; gradient



$b = R \cdot \cos\Theta_1$

$a = R \cdot \cos\Theta_2$

$$g = \frac{c_2 - c_1}{\Delta z} = \frac{c_2 - c_1}{R(\cos\Theta_1 - \cos\Theta_2)}$$

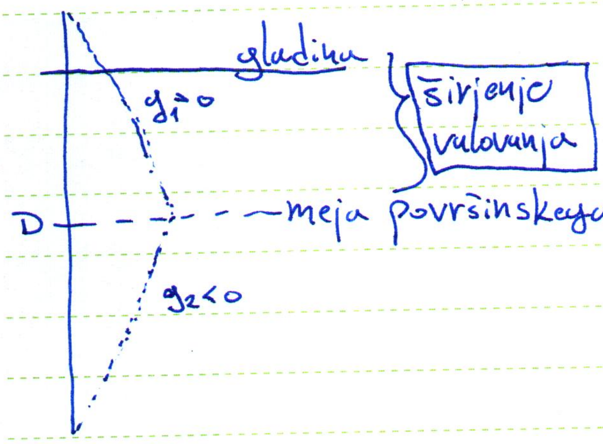
~~zanrceo~~:

(1) $c_1 = c_0$; $\Theta_1 = 0°$

(2) $c_2 = 0$; $\Theta_2 = 90° \mid \frac{\pi}{2}$

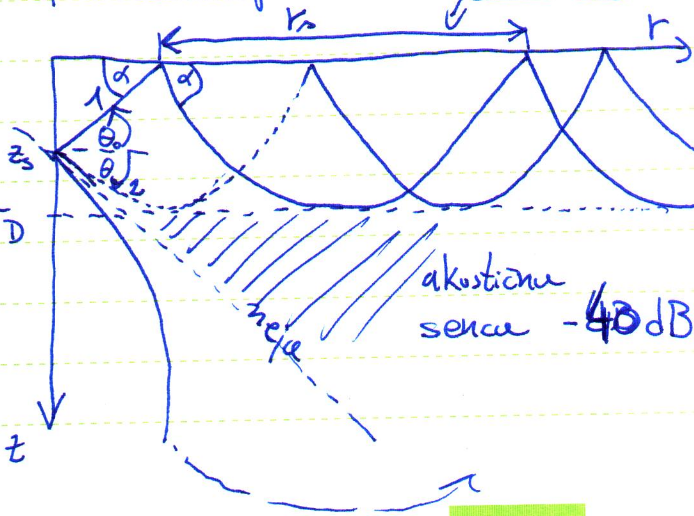
$$g = \frac{-c_0}{R} \Rightarrow R = \frac{-c_0}{g} = \boxed{\left|\frac{c_0}{g}\right| = R}$$ ← radij

Površje - gladina:

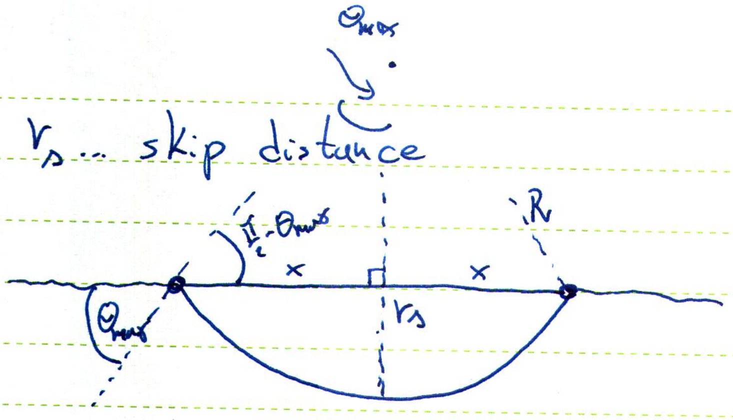


1,2 ekstremna primera

$r_t = \frac{D}{2 \cdot \Theta_0}$



r_s ... skip distance



$x = R \cdot \sin\theta_{max}$

$r_s = 2x = 2R\sin\theta_{max}$

$R = \frac{1500\ m/s}{0.015} \doteq 1\cdot 10^5 m$

Konvergenčne cone:

Kjer se izgubljeni žarki vrnejo iz globine in se prištejejo valovanju v coni širjenja. Dobimo ojačanje zvoka. V oceanu pride do tega po približno 15-70km od izvora

Področje globoke vode ~1000m globok zvočni kanal