

pri recepciji:

- MEHANIKA in TERMODINAMIKA - Igljč
- VISOKOŠOLSKA FIZIKA I. del - Klodnik
- FIZIKA 1. del - Strnad
- ZBIRKA FIZIKALNIH PROBLEM OV Z REŠITVAMI 1. del - Klodnik
- VAJE IZ FIZIKE - Igljč
- PHYSICS with modern physics - R.A. Serway

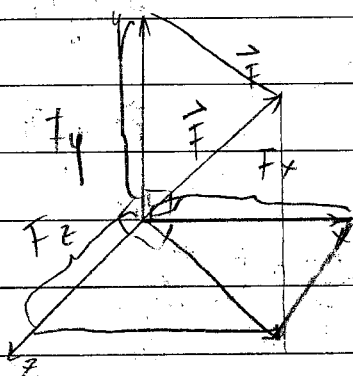
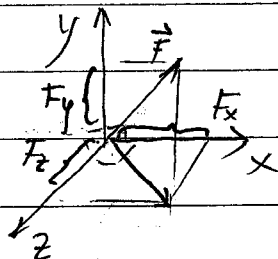
Vektorski račun

skalor - 1 podatek ; m (1 kg)

vektor - 2 podatka - smer in velikost, $\vec{F}$ (SN) $F = |\vec{F}|$

K.S.

- kartezični k.s.



$$\vec{F} = (F_x, F_y, F_z)$$
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

- vektorski produkt: $\vec{a} \times \vec{b}$

- skalarni produkt: $\vec{a} \cdot \vec{b}$

- seštevanje

- odštevanje



physics.fe.uni-lj.si/students

matematične osnove

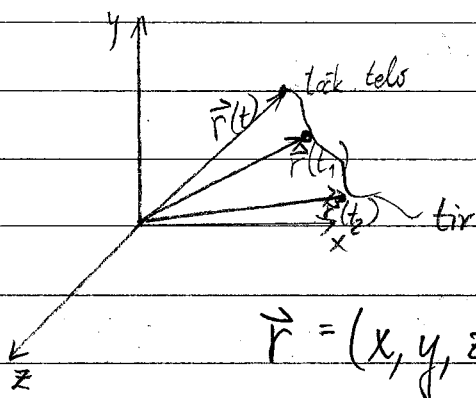
FIZIKA - MEHANIKA

- kinematika - ^{samo.}opišemo gibanje

točkasto telo

kartezijni k. s.

tir neodvisen od časa $\rightarrow$ telo miruje



$$\vec{r} = (x, y, z); x(t), y(t), z(t)$$

hitrost:

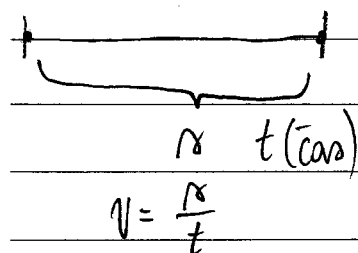
- sprememba lege telesa
poseben primer: premo gibanje

enakomerno

$v = \text{konst.}$

enakomerno

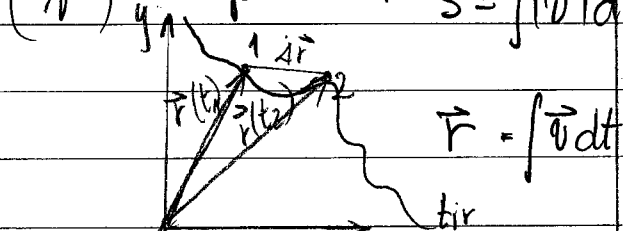
pospešeno



$$v = \frac{x}{t}$$

splošna definicija hitrosti:

($\vec{v}$) $s = \int |\vec{v}| dt$



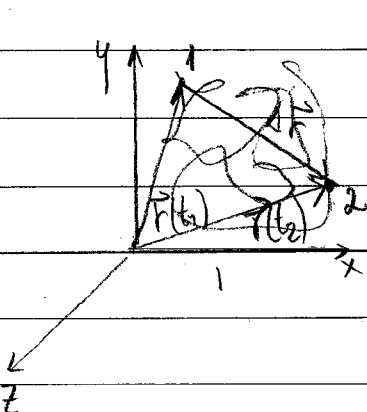
$$\Delta t = t_2 - t_1$$

$$\Delta \vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

porpčna hitrost:

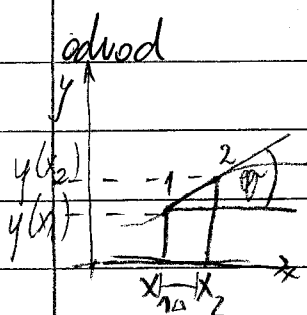
$$\vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

$$\vec{v} \propto \Delta \vec{r}$$



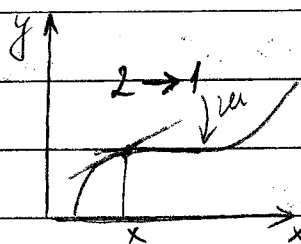
$$v = \lim_{\Delta t \rightarrow 0} \Delta \vec{r} / \Delta t = \frac{d\vec{r}}{dt} \quad \text{odkud vektorja}$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$



$$tg l = \frac{\Delta y}{\Delta x} \quad \Delta y = y(x_2) - y(x_1)$$

$$\Delta x = x_2 - x_1$$



$$\lim_{\Delta x \rightarrow 0} tg l = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right) = \vec{r} = (x, y, z) \quad \text{vrijednost vektor}$$

$$= (v_x, v_y, v_z)$$

primer:

$$y = x^2$$

$$\lim_{\Delta \rightarrow 0} \frac{y(x_2) - y(x_1)}{x_2 - x_1}$$

$$x_2 = x_1 + \Delta$$

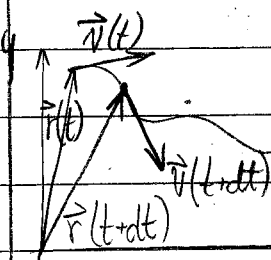
$$\lim_{\Delta \rightarrow 0} \frac{(x_1 + \Delta)^2 - x_1^2}{\Delta}$$

$$\lim_{\Delta \rightarrow 0} \frac{x_1^2 + 2x_1\Delta + \Delta^2 - x_1^2}{\Delta}$$

$$= \lim_{\Delta \rightarrow 0} \frac{2x_1\Delta}{\Delta} = 2x_1$$

$$(x^2)' = 2x$$

POSPEŠEK (t. tela)



$$\vec{v}(t) \quad \vec{v}(t+dt) \quad d\vec{v} = \vec{v}(t+dt) - \vec{v}(t)$$

$$\vec{v} = (v_x, v_y, v_z)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\frac{dv_x}{dt}, \frac{dv_y}{dt}, \frac{dv_z}{dt} \right) = a_x, a_y, a_z$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

poseben primer: enakomerno gibanje (t. telesa)

$$\vec{a} = 0$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 0 \Rightarrow \vec{v} = \vec{v}_0 = \text{konst.}$$

$$\vec{v}_0 = \frac{d\vec{r}}{dt} \Rightarrow \vec{r}(t=0) = \vec{r}_0$$

$$\int_0^t \vec{v}_0 dt = \int_{\vec{r}_0}^{\vec{r}(t)} d\vec{r} \Rightarrow \vec{v}_0 t = \vec{r} - \vec{r}_0 \Rightarrow \boxed{\vec{r} = \vec{r}_0 + \vec{v}_0 t}$$

poseben primer: enakomerno pospešeno gibanje (t. telesa)

$$\vec{a} = \vec{a}_0 = \text{konst.}$$

$$\vec{a}_0 = \frac{d\vec{v}}{dt} \Rightarrow \int_0^t \vec{a}_0 dt = \int_{\vec{v}_0}^{\vec{v}(t)} d\vec{v} \Rightarrow \vec{a}_0 t = \vec{v} - \vec{v}_0 \Rightarrow \boxed{\vec{v} = \vec{v}_0 + \vec{a}_0 t}$$

$$\vec{v} = \vec{v}_0 + \vec{a}_0 t \Rightarrow \frac{d\vec{r}}{dt} = \vec{v}_0 + \vec{a}_0 t$$

$$\int_{\vec{r}_0}^{\vec{r}(t)} d\vec{r} = \int_0^t \vec{v}_0 dt + \int_0^t \vec{a}_0 t dt \Rightarrow \boxed{\vec{r}(t) - \vec{r}_0 = \vec{v}_0 t + \frac{\vec{a}_0 t^2}{2}}$$

$$\vec{r}_0 = 0, \vec{v}_0 = (v_{0x}, v_{0y}, v_{0z})$$

$$\vec{a}_0 = (a_{0x}, a_{0y}, a_{0z})$$

zapiš po komponentah:

$$\boxed{\begin{aligned} x(t) &= v_{0x} t + \frac{1}{2} a_{0x} t^2 \\ y(t) &= v_{0y} t + \frac{1}{2} a_{0y} t^2 \\ z(t) &= v_{0z} t + \frac{1}{2} a_{0z} t^2 \end{aligned}}$$

FIZIKA

poseben primer: prosti pad (t. telesa)

$$\vec{g} = (0, -g, 0)$$

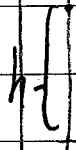
$$g \approx 9.81 \frac{m}{s^2}$$

v vakuumu

NOTE

$$v = \frac{m}{s}$$

$$a = \frac{m}{s^2}$$



$$a_y = \frac{dv_y}{dt} = -g$$

ob času $t=0$

$$v_y(t=0) = 0$$

$$y(t=0) = h$$

$$\int dv_y = \int -g dt$$

$$v_y = -gt$$

$$v_y = \frac{dy}{dt} = -gt$$

$$\int_h^y dy = -g \int_0^t dt$$

$$y - h = -\frac{1}{2}gt^2$$

$$\Rightarrow y = h - \frac{1}{2}gt^2$$

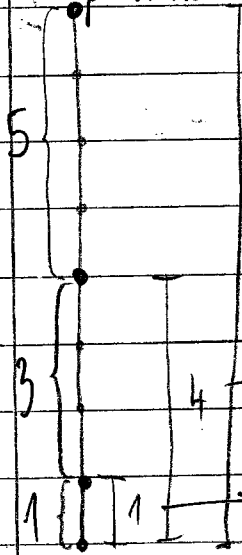
kdaj pride telo na tla:

$$0 = h - \frac{1}{2}gt_p^2 \Rightarrow t_p = \sqrt{\frac{2h}{g}}$$

hitrost ob padcu na tla:

$$v_p = -gt_p = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

eksperiment:



$$g \rightarrow t_{p3} \propto \sqrt{9} = 3$$

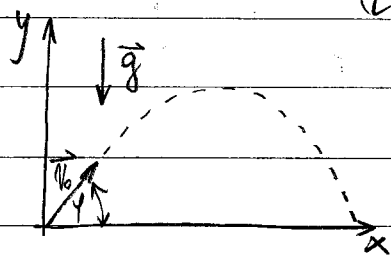
$$4 \rightarrow t_{p2} \propto \sqrt{4} = 2$$

$$1 \rightarrow t_{p1} \propto \sqrt{1} = 1$$

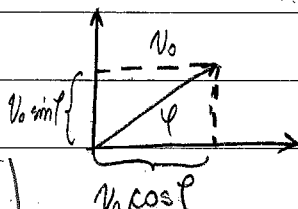
$$t_p \propto \sqrt{h}$$

poseben primer: poševni met (vakuum)
(t. telesa)

prosti pad
$t = 0,00452 \text{ s}$
$r = 1 \text{ m}$
$g = ?$

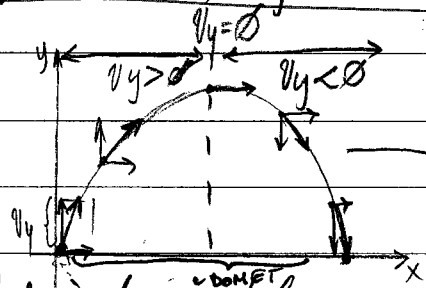


$$\vec{v}_0 = (v_0 \cos \varphi, v_0 \sin \varphi)$$



$$v_{xp} = v_0 \cos \varphi; v_x = v_0 \cos \varphi$$

$$v_{yp} = v_0 \sin \varphi; v_y = v_0 \sin \varphi - gt$$



kdaj doseže telo max. višino

$$v_y = 0 = v_0 \sin \varphi - g t_m = 0$$

$$\Rightarrow t_m = \frac{v_0 \sin \varphi}{g}$$

$$\vec{r} = (x(t), y(t))$$

$$x = \int_0^t v_x dt = \int_0^t v_0 \cos \varphi dt = v_0 \cos \varphi t \quad \rightarrow \quad t = \frac{x}{v_0 \cos \varphi}$$

$$y = \int_0^t v_y dt = \int_0^t (v_0 \sin \varphi - gt) dt = v_0 \sin \varphi t - \frac{1}{2} g t^2$$

$$y = v_0 \sin \varphi \frac{x}{v_0 \cos \varphi} - \frac{1}{2} g \frac{x^2}{v_0^2 \cos^2 \varphi}$$

$$y = \tan \varphi x - \frac{g x^2}{2 v_0^2 \cos^2 \varphi}$$

DOMET:

$$0 = \frac{\sin \varphi}{\cos \varphi} D - \frac{g D^2}{2 v_0^2 \cos^2 \varphi}$$

R: I. $D = 0$ - ni rešitev

$$\frac{\sin \varphi}{\cos \varphi} = \frac{g D}{2 v_0^2 \cos^2 \varphi} \quad \swarrow \times$$

$$0 = D \left(\frac{\sin \varphi}{\cos \varphi} - \frac{g D}{2 v_0^2 \cos^2 \varphi} \right) \quad \text{II.} \quad \left(\frac{\sin \varphi}{\cos \varphi} - \frac{g D}{2 v_0^2 \cos^2 \varphi} \right) = 0$$

$$D = \frac{2 v_0^2 \sin \varphi \cos \varphi}{g \cos \varphi}$$

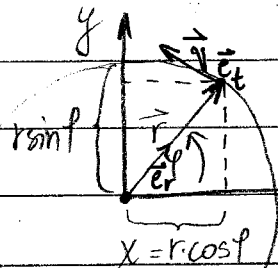
$$D = \frac{v_0^2 2 \sin \varphi \cos \varphi}{g} = \frac{v_0^2 \sin 2 \varphi}{g}$$

MAX. DOMET

$$2\varphi = 90^\circ$$

$$\varphi = 45^\circ$$

KROŽENJE (točkastega telesa)



$$r = |\vec{r}|, \vec{r} = (x, y, z)$$

$$\vec{r} = (r \cos \varphi, r \sin \varphi, \phi)$$

$\vec{e}_r$

$\vec{r}$ je enak po obhodu,
pot je različna od ϕ

$$\downarrow$$

$$2\pi r, 4\pi r, \dots$$

mirovanje: $\varphi = \text{konst.}$

kroženje: $\varphi = \varphi(t), r = |\vec{r}| = \text{konst.}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(-r \sin \varphi \frac{d\varphi}{dt}, r \cos \varphi \frac{d\varphi}{dt}, \phi \right)$$

$$w = \frac{d\varphi}{dt} = \dot{\varphi} \quad \vec{v} = (-r \dot{\varphi} \sin \varphi, r \dot{\varphi} \cos \varphi, \phi)$$

$$\vec{v} = (-r w \sin \varphi, r w \cos \varphi, \phi) = r w (-\sin \varphi, \cos \varphi, \phi)$$

$$\vec{v} = r \dot{\varphi} (-\sin \varphi, \cos \varphi, \phi) = r w \vec{e}_t$$

$\vec{e}_t \rightarrow$ enotni vektor - vrednost $\dot{\varphi} = 1$

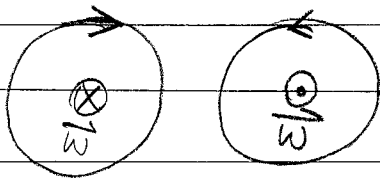
$$|\vec{e}_t| = \sqrt{\sin^2 \varphi + \cos^2 \varphi + \phi} = 1 \rightarrow \text{v smeri tangente na tir}$$

$$|\vec{e}_r| = \sqrt{\cos^2 \varphi + \sin^2 \varphi + \phi} = 1 \rightarrow \text{v smeri radija}$$

$$\vec{e}_z = (0, 0, 1), \quad \vec{e}_t = \vec{e}_z \times \vec{e}_r$$

$$\vec{v} = r w [\vec{e}_z \times \vec{e}_r] = \underbrace{w \vec{e}_z}_{\vec{\omega}} \times \underbrace{r \vec{e}_r}_{\vec{r}} = \vec{\omega} \times \vec{r}$$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}} \quad \boxed{\vec{\omega} = (0, 0, \omega)}$$



Pospešek pri kroženju

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} (\vec{\omega} \times \vec{r}) = \underbrace{\frac{d\vec{\omega}}{dt}}_{\vec{L}} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} =$$

$$\boxed{\vec{a} = \vec{L} \times \vec{r}}$$

Vektor kotnega pospeška kaže v isto smer kot vektor kotne hitrosti $\Rightarrow$ pravokotno na ravnini kroženja.

$$\vec{L} = \frac{d\vec{\omega}}{dt} = \frac{d}{dt} (0, 0, \omega) = (0, 0, \frac{d\omega}{dt})$$

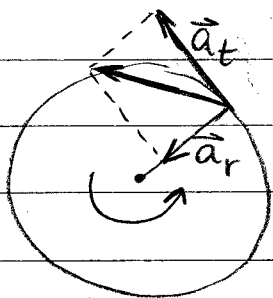
$$\vec{a} = \vec{L} \times \vec{r} = \vec{\omega} \times \vec{v} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \underbrace{(\vec{\omega} \cdot \vec{r})}_{0} \vec{\omega} - \underbrace{\vec{r}(\vec{\omega} \cdot \vec{\omega})}_{\omega^2} =$$

$$\boxed{\vec{a} = \vec{L} \times \vec{r} - \vec{r}\omega^2}$$

$$\vec{L} \times \vec{r} = L \vec{e}_z \times r \vec{e}_r = Lr (\vec{e}_z \times \vec{e}_r) = Lr \vec{e}_t$$

$$\vec{a} = Lr \vec{e}_t - \vec{r}\omega^2 = \underbrace{Lr \vec{e}_t}_{\vec{a}_t} - \underbrace{r\omega^2 \vec{e}_r}_{\vec{a}_r} \quad \vec{r}$$

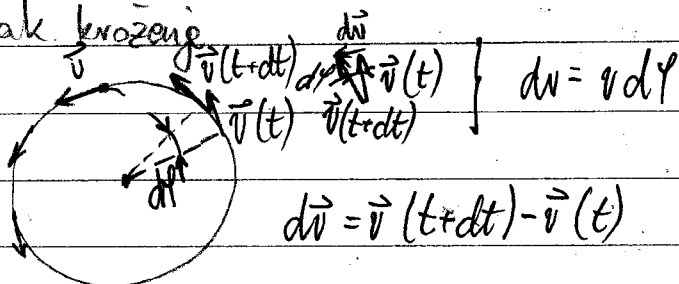
$$\boxed{L = 0 = \frac{d\omega}{dt} = \text{konst}}$$



$$\begin{aligned} \vec{a} &= \vec{a}_t + \vec{a}_r \\ \vec{a}_t &= Lr \vec{e}_t \\ \vec{a}_r &= -\vec{r}\omega^2 \end{aligned}$$

tang. pospešek
radialni pospešek

Enak kroženje



$$d\vec{v} = \vec{v}(t+dt) - \vec{v}(t)$$

$$a_r = \frac{dv}{dt} = \frac{v d\phi}{dt} = v \frac{d\phi}{dt} = v\omega = \omega^2 r$$

FIZIKA

$$\begin{aligned} a_t &= \alpha_r \\ a_r &= r\omega^2 \\ a &= \sqrt{a_t^2 + a_r^2} \\ \omega &= \frac{d\varphi}{dt} \\ \alpha &= \frac{d\omega}{dt} \end{aligned}$$

poseben primer: enakomerno kroženje

$$\alpha = 0 \Rightarrow \alpha = \frac{d\omega}{dt} = 0 \Rightarrow \omega = \omega_0 = \text{konst}$$

$$\omega = \frac{d\varphi}{dt} = \omega_0 \Rightarrow \int_{\varphi_0}^{\varphi} d\varphi = \int_0^t \omega_0 dt$$

$$\varphi_0 = \varphi(t=0) \quad \varphi - \varphi_0 = \omega_0 t \Rightarrow \varphi = \omega_0 t$$

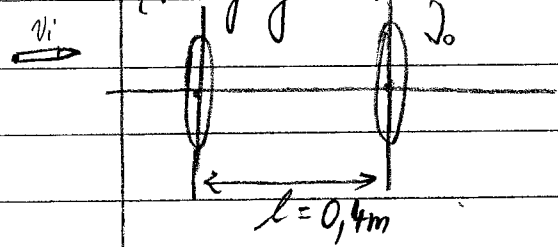
$$\vec{r} = (x, y) : \begin{cases} x = r \cdot \cos(\omega_0 t) \\ y = r \cdot \sin(\omega_0 t) \end{cases}$$

$$\text{obhodni čas: } t_0 = \frac{2\pi r}{v_0} = \frac{2\pi r}{\omega_0 r} = \frac{2\pi}{\omega_0}$$

$$\omega_0 = 2\pi / t_0 = 2\pi f_0$$

frekvenca $[Hz = s^{-1}]$

eksperiment: izstrelka
merjenje hitrosti



$$\begin{aligned} v_i &= \frac{l}{t} = \frac{2\pi r \omega_0}{\varphi} \\ \varphi &= 2\pi \gamma t \Rightarrow t = \frac{\varphi}{2\pi \gamma} \end{aligned}$$

$$\gamma_0 = 1800/60$$

$$\varphi = 35^\circ = \frac{7\pi}{36}$$

$$v_i = \frac{0,4m \cdot 2\pi \cdot \frac{1800}{60s}}{35 \cdot 2\pi / 360} = \frac{0,4m \cdot 1800 \cdot 360}{35 \cdot 60s} = 123,4 \frac{m}{s}$$

poseben primer:
enakomerno pospešeno gibanje

$$\alpha = \alpha_0 = \text{konst}, \omega_0 = \omega(t=0)$$

$$\alpha_0 = \frac{d\omega}{dt} \Rightarrow \int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha_0 dt$$

$$\omega = \omega_0 + \alpha_0 t$$

$$\alpha_0 t = \omega - \omega_0$$

$$w = \frac{dp}{dt} \Rightarrow \int_0^t (w_0 + \alpha_0 t) dt = \int_0^p dp$$

$$w_0 t + \frac{1}{2} \alpha_0 t^2 = p$$

$$w = w_0 + \alpha_0 t \Rightarrow t = \frac{w - w_0}{\alpha_0} \quad w_0 \frac{(w - w_0)}{\alpha_0} + \frac{\alpha_0}{2} \frac{(w - w_0)^2}{\alpha_0^2} = p / 2 \alpha_0$$

$$= 2 \alpha_0 w_0 (w - w_0) + (w - w_0)^2 = 2 \alpha_0 p$$

$$(2 w_0 w - 2 w_0^2 + w^2 - 2 w w_0 + w_0^2) = 2 \alpha_0 p$$

$$w^2 = w_0^2 + 2 \alpha_0 p \quad - \text{ zveza med kotno hitrostjo in zasukom}$$

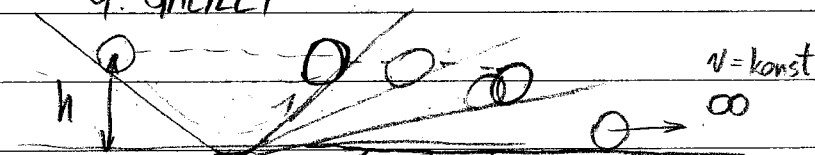
DINAMIKA

Vzrok za silo je vedno v telesu, ki pritiska na drugo telo.

ARISTOTEL

$$v \propto F \Rightarrow F \neq 0 \Rightarrow v \neq 0$$

G. GALILEI



NEW. ZAKONI ZA TOČKASTOTELO:

I. Newtonov zakon:

Telo se giblje po ravnici s stalno hitrostjo ali stoj na mestu, če nanj ne deluje nobena sila.

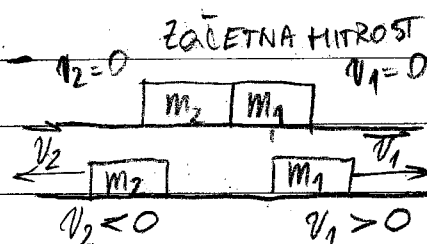
II. Newtonov zakon:

$$\vec{F} = m \cdot \vec{a}$$

III. Newtonov zakon:

zakon akcije in reakcije

$$\vec{F}_{21} + \vec{F}_{12} = 0$$



FIZ.

Definicija mase: $\frac{v_1}{|v_2|} = \frac{m_2}{m_1} \Rightarrow m_1 \cdot v_1 = -m_2 v_2$

$$m_1 \Delta v_1 = -m_2 v_2$$

$$\Delta v_1 = v_1 - v_{1 \text{ zač}} = v_1$$

$$\Delta v_2 = v_2 - v_{2 \text{ zač}} = v_2$$

$$m_1 \frac{\Delta v_1}{\Delta t} = -m_2 \frac{\Delta v_2}{\Delta t}$$

limita: $\Delta t \rightarrow dt$

$$m_1 \frac{dv_1}{dt} = -m_2 \frac{dv_2}{dt} \Rightarrow m_1 a_1 = -m_2 a_2$$
$$\frac{d}{dt} (m_1 \vec{v}_1 + m_2 \vec{v}_2) = 0$$

Definicija sile:

$$m_1 a_1 = F_{21}, \quad m_2 a_2 = F_{12} \Rightarrow \boxed{\vec{F} = m \vec{a}} \quad \text{II. New. zakon}$$

$$F_{21} = -F_{12} \Rightarrow \boxed{F_{21} + F_{12} = 0} \Rightarrow \text{III. New. zakon}$$

vsota vseh sil je 0!

$$\vec{G}_1 = m_1 \vec{v}_1 \leftarrow \text{gibalna količina}$$

$$\vec{G}_2 = m_2 \vec{v}_2$$

$$\boxed{m_1 \vec{v}_1 + m_2 \vec{v}_2 = \vec{0}} \rightarrow \text{zakon o ohranitvi gibalne količine 2 točkasti telesi}$$

$$\boxed{\vec{F} = m \vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{G}}{dt}}$$

$$\vec{F} = m \frac{d\vec{v}}{dt} \cdot dt$$

$$\int \vec{F} dt = \int m d\vec{v} \Rightarrow \int \vec{F} dt = m \int d\vec{v} \Rightarrow \int \vec{F} dt = m \Delta \vec{v} = m \vec{v}_{\text{kon}} - m \vec{v}_{\text{zač}}$$

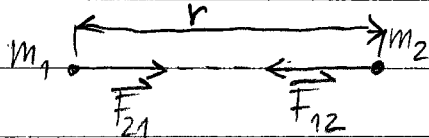
$$\boxed{\int \vec{F} dt = \Delta \vec{G}}$$

$\rightarrow$ surnek sile

$$\int \vec{F} dt = \Delta \vec{G} \Rightarrow \text{če je } \int \vec{F} dt = 0 \Rightarrow \Delta \vec{G} = 0 \Rightarrow \vec{G}_{\text{kon}} = \vec{G}_{\text{zač}} \Rightarrow m_1 \vec{v}_1 = m_2 \vec{v}_2$$

$$m_1 = m_2$$
$$|\vec{v}_1| = |\vec{v}_2|$$

CENTRALNE SILE:



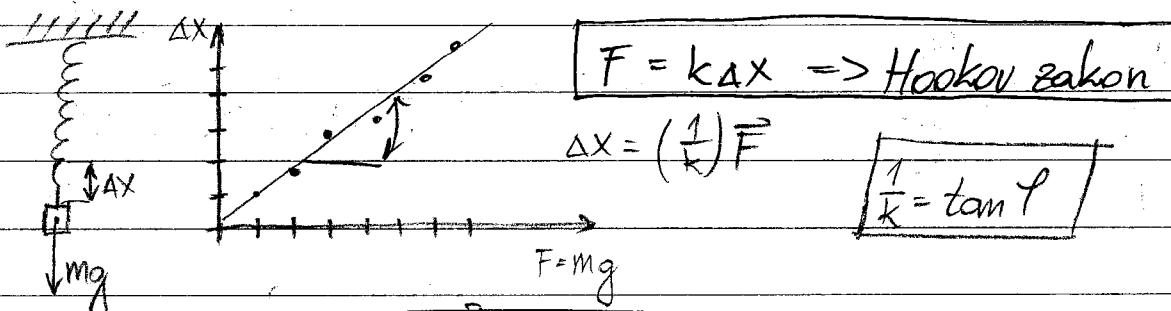
← enotski vektor

$$\left(\frac{\vec{r}}{r}\right) = 1$$

primeri:

- gravitacijska $\Rightarrow \vec{F} = -G \frac{m_1 m_2}{r^2} \left(\frac{\vec{r}}{r}\right)$
- Coulombova $\Rightarrow \vec{F}_C = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \left(\frac{\vec{r}}{r}\right)$

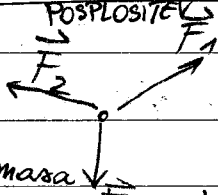
$\vec{F} \Rightarrow$ deformacija; sile lahko deformirajo telo



II. New. zakon

$$\vec{F} = m\vec{a}$$

↑ vztrajnostna masa



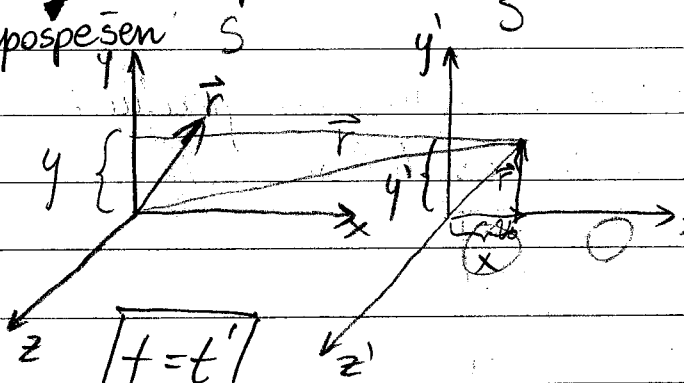
$$\sum_i \vec{F}_i = m\vec{a}$$

$$\vec{F} = \sum_i \vec{F}_i$$

če je $\vec{F} = 0 \Rightarrow \vec{a} = 0 \Rightarrow \vec{v} = \text{konst.}$

inercialni opazovalni sistem

↑
nepospešen S



$v_0 = \text{konst.}$

$$\vec{r} = (x, y, z)$$

$$\vec{r}' = (x', y', z')$$

$$t = t'$$

$$x = x'$$

$$y = y'$$

$$z = z'$$

v splošnem

$$x = x' + v_0 t$$

$$y = y'$$

$$z = z'$$

Galilei-eve transformacije

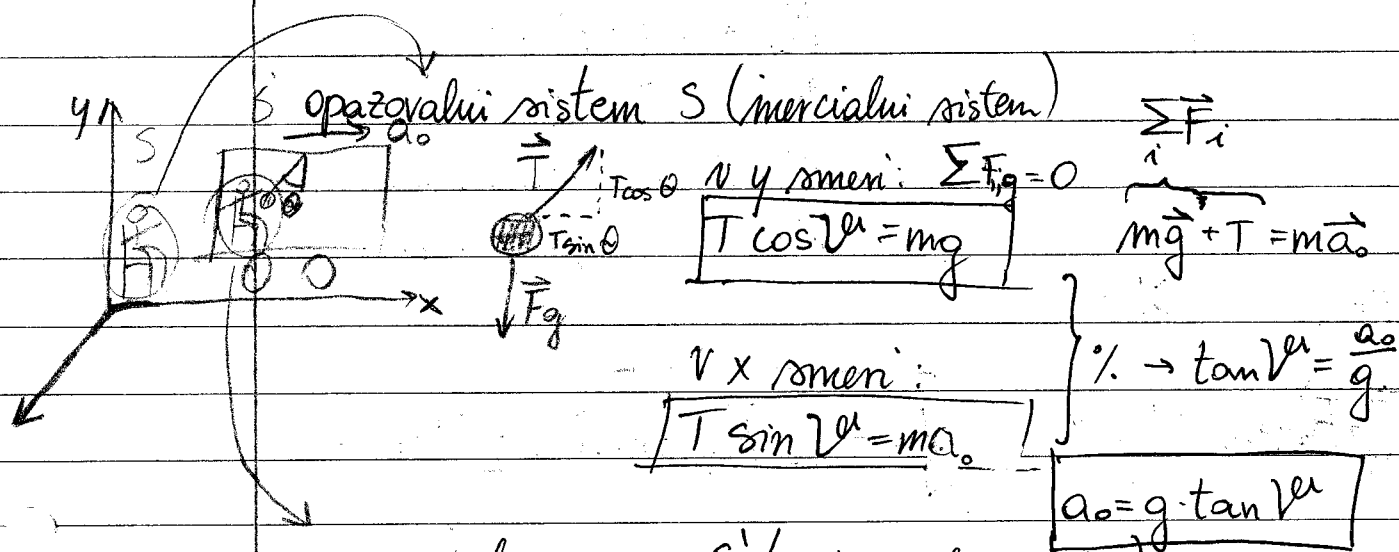
$$v_0 \approx \frac{c}{10}$$

pri večjih hitrostih transformacije ne

veljajo več.

$\theta \neq 0 \rightarrow$ neinercialni sistem

$\theta = 0 \rightarrow$ inercialni sistem



opazovalni sistem S' (neinercialni sistem)

$\vec{v} = \vec{0}, \vec{a} = 0$ v y' smeri: $T \cos \varphi = mg$

v x' smeri: $T \sin \varphi \neq m\vec{a}' = 0$

vpeljemo sistemsko silo

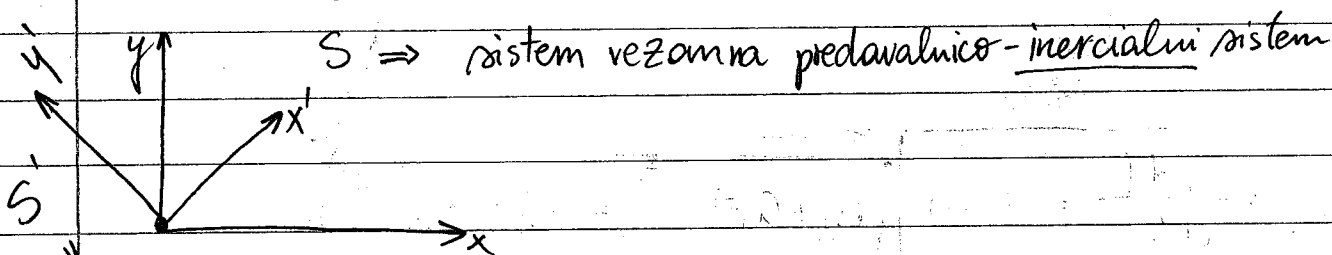
$T \sin \varphi + \vec{F}_s = 0$

$\vec{F}_s = -T \sin \varphi = m \cdot a_0$

$\vec{F}_s = -m\vec{a}_0$

sistemsko sila

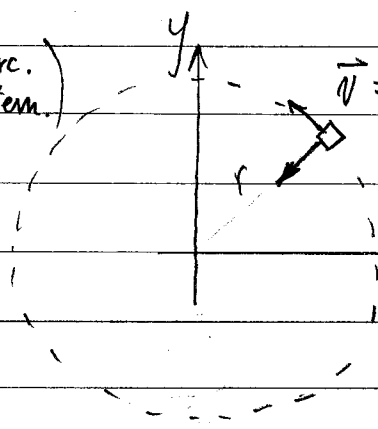
Centrifugalna sistemsko sila.



neinercialni opazovalni sistem - vrteči se op. sistem

kotna hitrost vrtenja = ω

S: (merc. sistem) $\vec{v} = \omega r \vec{e}_t$ New. zakon:

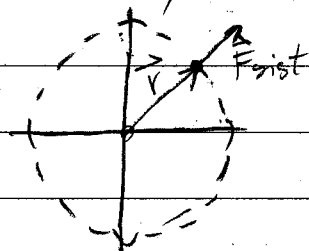


$$\vec{F}_{cp} = m(-\vec{F}\omega^2)$$

centripetalna sila radialni pospešek

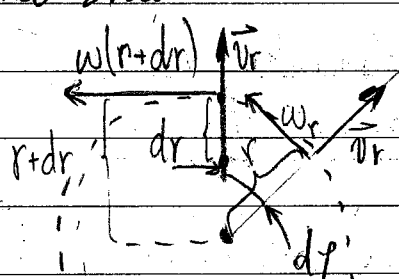
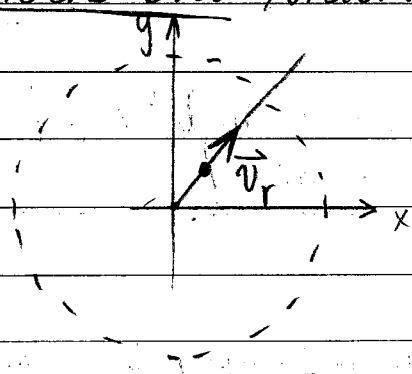
S': (neinerc. sistem) $\vec{a}' = \emptyset$

$$\vec{F}_{cp} + \vec{F}_{sist} = m \cdot \vec{a}' = \emptyset \Rightarrow \boxed{\vec{F}_{sist} = -\vec{F}_{cp} + m\vec{r}\omega^2 = \vec{F}_{ct}}$$

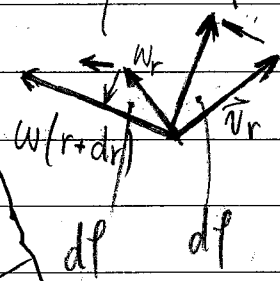


centrifugalna sistemsko sila

Coriolis-ova sistemsko sila



prirastek hitrosti - iz tega izhaja Cor. sila



$$a_1 = \frac{v_r d\varphi}{dt} = v_r \omega$$

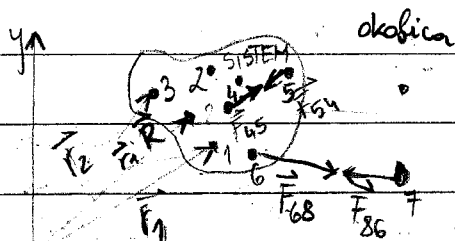
$$a_2 = \frac{\omega dr}{dt} = \omega v_r$$

$$a_3 = \underbrace{\omega r \omega}_{a_r} = r\omega^2$$

$$a_2 + a_1 \Rightarrow \vec{F}_{cor} = (-2\vec{\omega} \times \vec{v}_r) m$$

F12

New. zakon za sistem točkastih teles



zunanje sile - $\vec{F}_{ji}$
notranje sile - $\vec{F}_{ij}$

m_i = masa i-tega telesa

$$m_i \cdot \frac{d^2 \vec{r}_i}{dt^2} = \sum_j \vec{F}_{ji} + \vec{F}_i \quad m_1 \frac{d^2 \vec{r}_1}{dt^2} = \sum_j \vec{F}_{j1} + \vec{F}_1$$

$$\vec{F}_i = \sum_k \vec{F}_{ki}$$

→ vsota vseh notranjih sil na maso m_i v
vsota vseh zun. sil, ki v sistemu
delujejo na maso m_i v sistemu.

$$\sum_i m_i \frac{d^2 \vec{r}_i}{dt^2} = \sum_{i,j} \vec{F}_{ji} + \sum_i \vec{F}_i$$

$$\sum_i \vec{F}_i = \sum m_i \frac{d^2 \vec{r}_i}{dt^2} = \frac{d^2}{dt^2} \left(\sum_i m_i \vec{r}_i \right) = \left(\sum_i m_i \right) \left[\frac{\left(\sum_i m_i \vec{r}_i \right)}{\left(\sum_i m_i \right)} \right] \frac{d^2}{dt^2}$$

$$\sum_i m_i = m \leftarrow \text{celotna masa sistema}$$

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

$$\sum_i \vec{F}_i = m \frac{d^2 \vec{R}}{dt^2}$$

za točkasto telo vektor marnega
središča (težišča)

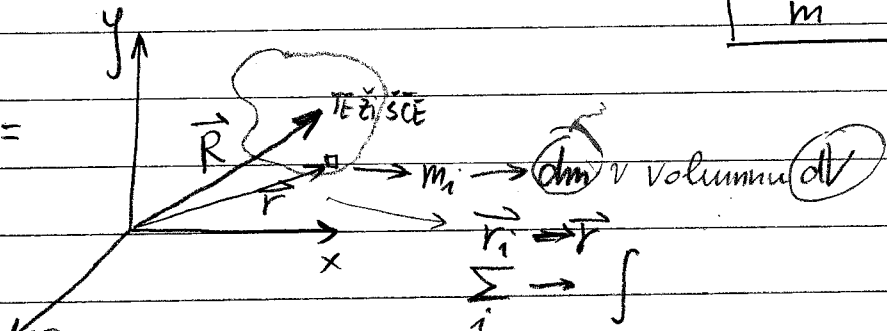
enaki sta
enaki

Razdalje med točkastimi telesi niso fiksne.

TOGO TELO:

$$\sum_i \vec{F}_i = m \frac{d^2 \vec{R}}{dt^2} =$$

$$\vec{R} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{\int \vec{r} dm}{m}$$



$$dm = \rho dV \quad \rho(\vec{r}) = \frac{dm}{dV} \leftarrow \text{gostota v mestu } \vec{r}$$

| ρ je $\rho(\vec{r}) = \text{konst}$ |

$$\vec{R} = \frac{\int \vec{r} dm}{m} = \frac{\int \vec{r} \rho dV}{\int \rho dV} = \frac{\rho \int \vec{r} dV}{\rho \int dV} = \frac{\int \vec{r} dV}{V}$$

$$\sum_i \vec{F}_i \equiv \vec{F} = m \frac{d^2 \vec{r}}{dt^2} = \frac{d^2(m\vec{r})}{dt^2} = m \cdot \vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}$$

← $\vec{v}$: rýchlosť ťažiska
 ← $\vec{a}$: zrýchlenie ťažiska
 ← $m\vec{v}$: hybnosť celého telesa
 ← $\vec{G}$: ťažisko

$$\int \vec{F} dt = \int dG = \Delta \vec{G} = \vec{G}_{\text{kon}} - \vec{G}_{\text{zac}}$$

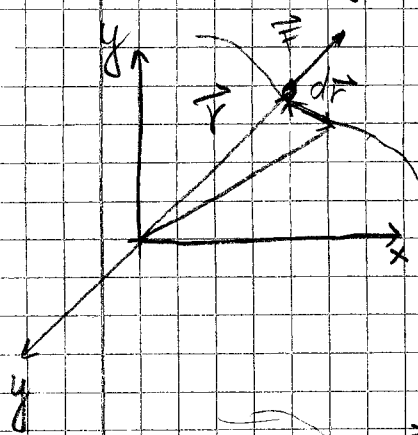
→ smere nile $\text{ke je } \int \vec{F} dt = 0 \Rightarrow \Delta \vec{G} = 0 \Rightarrow \vec{G}_{\text{kon}} = \vec{G}_{\text{zac}}$

ZAKON O OHRANIŤUJ GIB. KOLIČINE

KINETICKÁ ENERGIA

točkarstvo telo:

Kinetična energija (za točkasto telo) / skalarni produkt

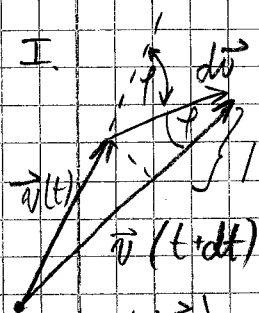


$$A = \int \vec{F} \cdot d\vec{r} = \int m \frac{d\vec{v}}{dt} \cdot \vec{v} dt = \int m \vec{v} \cdot d\vec{v}$$

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow d\vec{r} = \vec{v} dt$$

$$d\vec{v} = \vec{v}(t+dt) - \vec{v}(t)$$



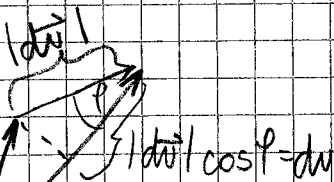
$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \varphi$$

$$|d\vec{v}| \neq d|\vec{v}|$$

$$\vec{v} \cdot d\vec{v} = |\vec{v}| |d\vec{v}| \cos \varphi$$

$$d\vec{v}$$

$$\boxed{\vec{v} \cdot d\vec{v} = v dv}$$



$$\text{II. } d(\vec{v} \cdot \vec{v}) \leftarrow \left| d(v^2) = dv^2 + dv^2 \right|$$

$$\vec{v} \cdot d\vec{v} + d\vec{v} \cdot \vec{v} = 2\vec{v} \cdot d\vec{v}$$

$$\Rightarrow d(v^2) = 2v dv = 2\vec{v} \cdot d\vec{v} \Rightarrow \boxed{\vec{v} \cdot d\vec{v} = v dv}$$

$$A = \int_{v_1}^{v_2} m v dv = \left. \frac{1}{2} m v^2 \right|_{v_1}^{v_2} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \frac{1}{2} m (v_2^2 - v_1^2)$$

$$W_k = \frac{1}{2} m v^2$$

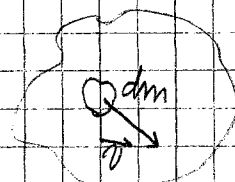
$$A = W_{k2} - W_{k1} = \Delta W_k$$

kinetična energija

$$\boxed{A = \int \vec{F} \cdot d\vec{r} = \Delta W_k}$$

POSPLOŠITEV ZA TOGO TELO:

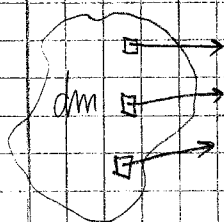
$$W_k = \int \frac{1}{2} dm \vec{v}$$



lahko razdeljimo na kinetično translacijsko in rotacijsko energijo.

poseben primer:

gibanje takega telesa po premici brez vrtenja.



$$W_k = \frac{1}{2} \int dm v^2 = \frac{1}{2} v^2 \int dm = \frac{1}{2} m v^2$$

$$A = \int \vec{F} \cdot d\vec{r} = \Delta W_k$$

$$; \text{ če je } \vec{F} = \emptyset \Rightarrow A = \emptyset \Rightarrow \Delta W_k = \emptyset$$

zakon o ohranitvi kinetične energije

$$W_{k1} = W_{k2}$$

uporablja se pri obravnavi trkov.

$$\int \vec{F} dt = \Delta \vec{G}$$

$$; \text{ če je } \int \vec{F} dt = \emptyset \Rightarrow \Delta \vec{G} = \emptyset \Rightarrow \vec{G}_1 = \vec{G}_2$$

zakon o ohranitvi gibalne količine

prožni (elastični) trk - del začetne

temperatura nam pove povprečno hitrost molekul v snovi.

energije se spremeni - kin. en. se ohranja
- gib. kol. se ohranja

$$\left. \begin{array}{l} m_1 \vec{v}_1 \\ m_2 \vec{v}_2 = \emptyset \end{array} \right\} \text{ pred trkom}$$

$$\left. \begin{array}{l} m_1 \vec{v}_1' \\ m_2 \vec{v}_2' \end{array} \right\} \text{ po trku}$$

$$\begin{array}{l} G_1 = G_2 : m_1 v_1 = m_1 v_1' + m_2 v_2' \\ W_{k1} = W_{k2} : \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 \end{array} \quad \begin{array}{l} / : m_1 \Rightarrow v_1 = v_1' + \frac{m_2}{m_1} v_2' \\ / : m_1 \Rightarrow v_1^2 = v_1'^2 + \frac{m_2}{m_1} v_2'^2 \end{array}$$

$$\frac{m_2}{m_1} = 2$$

neprožni trk - del zač. en. se spremeni v deformacijsko energijo.

$$\begin{array}{l} m_1 \vec{v}_1 + m_2 \vec{v}_2 = \emptyset \\ G_1 = G_2 \\ (m_1 + m_2) \vec{v}_2 \end{array}$$

$$m_1 v_1 = (m_1 + m_2) \tilde{v}_2$$

$$\tilde{v}_2 = \frac{m_1 v_1}{(m_1 + m_2)}$$

$$\begin{aligned} v_1' & \left\{ \begin{aligned} v_1' &= v_1 + \alpha v_2' \Rightarrow v_1' = v_1 - \alpha v_2' \\ v_2' &= v_2 + \alpha v_1' \Rightarrow v_2' = (v_1 - \alpha v_2')^2 + \alpha v_2'^2 \end{aligned} \right. \\ v_1^2 &= v_1'^2 - 2v_1\alpha v_2' + \alpha^2 v_2'^2 + \alpha v_2'^2 \end{aligned}$$

$$2v_1\alpha v_2' = \alpha^2 v_2'^2 + \alpha v_2'^2$$

$$2v_1\alpha v_2' = v_2'^2 \alpha (\alpha + 1) \Rightarrow v_2' = \frac{2v_1}{1+\alpha}$$

poseben primer: $m_1 = m_2$ ($\alpha = 1$)

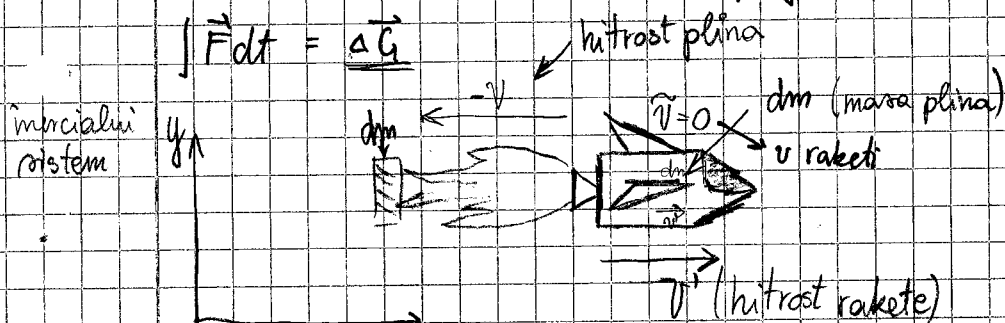
$$v_2' = v_1$$

$$v_1' = 0$$

$$v_1' = v_1 - \frac{2v_1}{1+\alpha}$$

EXPERIMENT: Vozilo na reaktivni pogon (vozilek na šibe)

$$\int \vec{F} dt = \Delta \vec{G}$$



$$\underbrace{dm(-v)}_{\text{končna gib. količina}} - \underbrace{dmv'}_{\text{začetna gib. količina}} = F_0 dt$$

$$-dm(v+v') = F_0 dt / dt$$

$$\boxed{\phi_m = \frac{dm}{dt} \Rightarrow \text{masni pretok}}$$

$$\underbrace{-\frac{dm}{dt}}_{\phi_m} \underbrace{(v+v')}_{v_0} = F_0$$

$$\boxed{F_0 = -\phi_m \cdot v_0} \leftarrow \text{silna plinov}$$

$$\boxed{F = -F_0 = \phi_m \cdot v_0} \leftarrow \text{silna na raketo}$$



$$\phi_m t_0 = m_0 \leftarrow \text{masa goriva}$$

ob času $t = t_0$

$$m = (m_1 + m_0) - \underbrace{\phi_m t_0}_{m_0} = m_1$$

$$m = \underbrace{(m_1 + m_2)}_{\tilde{m}} - \phi_m t$$

$$m(t) = \tilde{m} - \phi_m \cdot t$$

$$\sum \vec{F}_i = m(t) = \frac{d\vec{w}}{dt}$$

↑
masa
rakete

$$\phi_m v_0 = \underbrace{(\tilde{m} - \phi_m t)}_{m(t)} \frac{dv}{dt} \quad \text{— enačba gibanja}$$

diferencialna enačba

$$\phi_m \int_0^{t_0} \frac{dt}{(\tilde{m} - \phi_m t)} = \int_0^{v_{\max}} \frac{dv}{v_0}$$

$$\begin{aligned} (\tilde{m} - \phi_m t) &= x \\ -\phi_m dt &= dx \\ dt &= \frac{dx}{-\phi_m} \end{aligned}$$

$$\phi_m \int_0^{t_0} \frac{dx}{-\phi_m x} = \frac{1}{v_0} \int_0^{v_{\max}} dv$$

$$\frac{1}{v_0} \int_0^{v_{\max}} dv = \frac{v_{\max}}{v_0}$$

obrnemo meje, da
se znebimo "—" / m_1

$$\int_{m_1}^{m_0} \frac{dx}{x} = \ln x \Big|_{m_1}^{m_0} = \ln \tilde{m} - \ln m_1 = \ln \frac{\tilde{m}}{m_1}$$

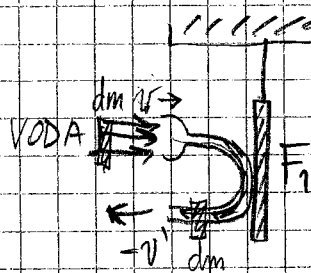
$$\ln \frac{\tilde{m}}{m_1} = \frac{v_{\max}}{v_0} \Rightarrow v_{\max} = v_0 \ln \left(\frac{m_1 + m_0}{m_1} \right)$$

Nismo upoštevali upora in gravitacije; predpostavljamo, da je raketa v vakuumu.

m_1 mora biti čim manjša
da dobimo večjo
hitrost.

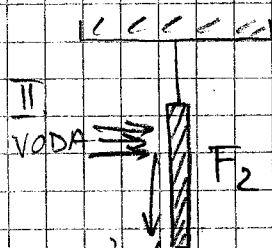
SILA CURKA

I.



$$F_1 > F_2$$

II.



F_{curka} je manjša

masni pretok: ρ gostota

$$\phi_m = \frac{dm}{dt} = \frac{d(\rho V)}{dt} = \rho \frac{dV}{dt} = \rho \phi_v$$

volumski pretok

$$\phi_v = \frac{dV}{dt}$$

$$I. \quad F_0 dt = \Delta G = dm(v') - dm \cdot v = -dm(v' + v) \approx F_0 dt \quad /dt$$

$$F_0 = - \frac{dm}{dt} (v' + v) = - \rho \phi_v (v' + v) \quad \checkmark \text{ sila odziva na curko}$$

silna curka na oviro

$$F_c = -F_0 = \rho \phi_v (v' + v)$$

II.

$$F_c = \rho \phi_v v$$

$$v' \approx \phi$$

FIZ

Sila trenja in lepenja

Sila lepenja moramo premagati, če želimo telo spraviti v gibanje, je večja od sile trenja.

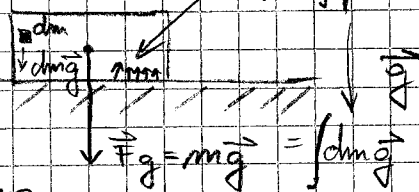
Sila trenja je sorazmerna masi telesa.

$$\vec{F}_P = \int p d\vec{S}$$

$$F_{tr} \propto mg \Rightarrow F_{tr} = k_{tr} \cdot F_P, n$$

$$F_e \propto mg \Rightarrow F_e = k_e \cdot F_P, n$$

v smeri
normalne

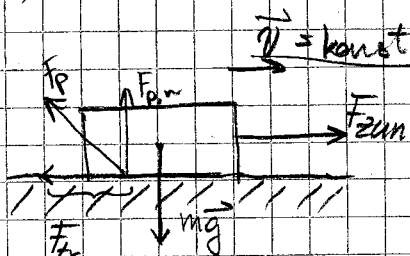


Sila podlage je ploskovno porazdeljena sila.

v ravnovesju (v vertikalni smeri)

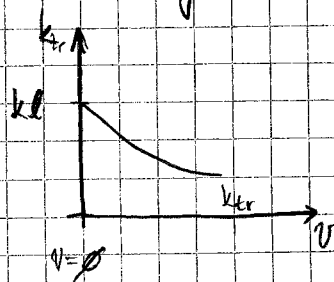
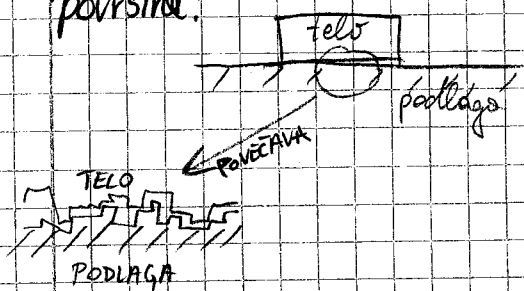
$$\vec{F}_P + \vec{F}_g = \vec{0}$$

$$\vec{F}_P = -\vec{F}_g = -m\vec{g}$$



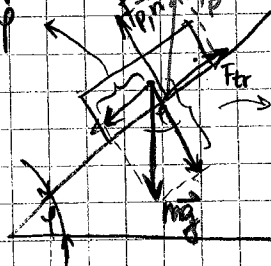
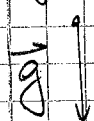
Sila trenja nastane zaradi interakcije med telesom in podlago, ker ni idealne površine.

če je $v = konst \Rightarrow F_{tr} = F_{zum}$



Drenje po klancu navzdol

$$F_d = mg \cdot \sin \varphi$$



$$F_s = F_{g,n} = mg \cdot \cos \varphi$$

F_d poganja telo naprej.

$F_d \geq F_{tr}$, drugače se telo ne premakne.

* $F_{P,n} = mg \cdot \cos \varphi \Rightarrow$ ravnovesje nil v smeri normalne na površino

* v smeri klanca (gibanje)

$$ma = mg \cdot \sin \varphi - F_{tr}$$

$$ma = mg \cdot \sin \varphi - k_{tr} \cdot mg \cdot \cos \varphi$$

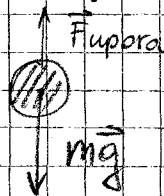
$$a = g(\sin \varphi - k_{tr} \cdot \cos \varphi)$$

experiment: (določanje k_{tr})

$a = 0 \Rightarrow$ tik preden se telo začne gibati

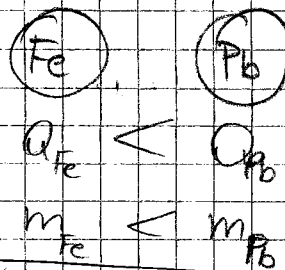
$$0 = g(\sin \varphi - k_{tr} \cdot \cos \varphi) \Rightarrow k_{tr} = \frac{\sin \varphi}{\cos \varphi} = \tan \varphi$$

komentar: padanje telesa v zraku, če so hitrosti visoke $F_n \propto v^2$



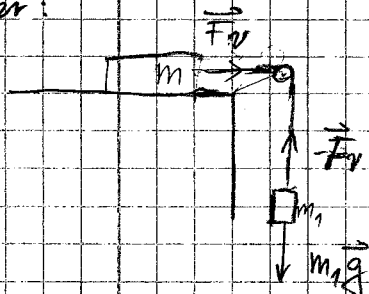
$$ma = \frac{mg - F_{upora}}{m}$$

$$a = g - \frac{F_{upora}}{m}$$



Gostejše telesa padajo hitreje.

primer:



pospešek je konstanten, vendar ni enak g .

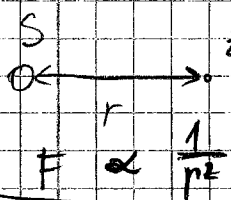
$$F_v = m \cdot a$$

$$m_1 g - F_v = m_1 a$$

$$a < g$$

$$m_1 g = a(m_1 + m) \Rightarrow a = \frac{m_1 g}{(m_1 + m)}$$

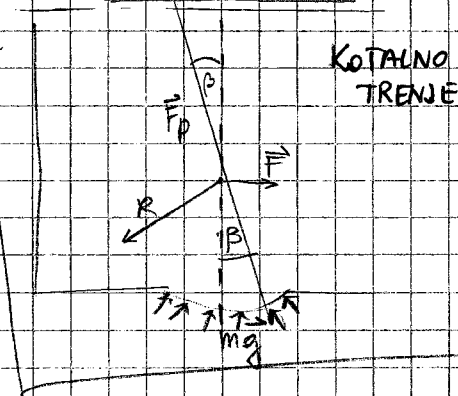
GRAVITACIJA



Oba planeta obratujeta kot točkasti telesi

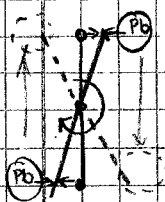
$$F = G \frac{m_1 m_2}{r^2} \Rightarrow \text{gravitacijski zakon}$$

gravitacijska konstanta



CAVENDISHOVA TEHNIKA

- priprava za izmeritev gravitacijske konstante $\Rightarrow G = 6,67 \cdot 10^{-11} \text{ Nm}^2 / \text{kg}^2$



pred se zavrti, vendar žica preprečuje vrtenje zaradi torzije.

S premikom se spremeni kot, izračunamo navor, nato mlo, mase poznamo, iz vseh podatkov pa izračunamo G .

Sila med dvema točkastima telesoma $(a+x)$

$F = G \frac{mM}{r^2}$
 $dF = G \frac{dM \cdot m}{(a+x)^2}$
 $dM = \rho dV = \rho dx S$
 $dF = G \frac{\rho S \cdot dx \cdot m}{(a+x)^2}$

$$\int dF = \int_0^b G \frac{\rho S dx}{(a+x)^2} = G \frac{mM}{a(a+b)}$$
 limita: $b \rightarrow 0 \Rightarrow F = G \frac{mM}{a^2}$

② gravitacijska sila med lupino in točk. telesom (maso):

$dF = G \frac{dM \cdot m}{S^2}$
 $F = \int dF = G \frac{mM}{r^2}$

③ gravitacijska sila med polno kroglo in točkasto maso m:

$dF = G \frac{dM m}{r^2}$ (masa ene lupine)
 celotna sila $F = \int dF = \int G \frac{dM m}{r^2} = G \frac{m}{r^2} \int dM = G \frac{Mm}{r^2}$

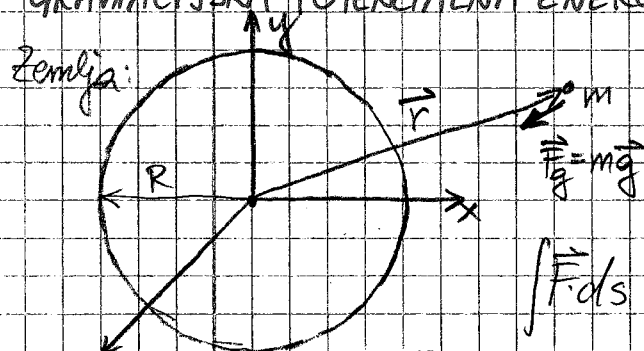
Pogoji, da pride do medsebojnih sil, je homogenost teles (palice, točke, lupine).

zemlja polmer zemlje ($\approx 6400 \text{ km}$)
 $r = R + h$ nadmorska višina
 $F_g = G \frac{mM}{r^2}$ masa zemlje
 $F_g = G \frac{Mm}{(R+h)^2} \rightarrow g$
 $F_g = m \cdot g$

$$g = G \frac{M}{(R+h)^2} \cdot \frac{R^2}{R^2} = G \frac{M}{R^2} \frac{R^2}{(R+h)^2} = g_0 \frac{R^2}{(R+h)^2}$$

$$g(h=0) = g_0 \leftarrow \approx 9,8 \frac{\text{m}}{\text{s}^2}$$

GRAVITACIJSKA POTENCIALNA ENERGIJA



$$\vec{F}_g = -G \frac{mM}{r^2} \left(\frac{\vec{r}}{r} \right); \quad \left| \frac{\vec{r}}{r} \right| = 1$$

$$\int \vec{F} \cdot d\vec{s} = \Delta W_k$$

$$\vec{F} = \vec{F}_{\text{ost}} + \vec{F}_g \quad (\text{delo vseh ostalih sil, razen } F_g)$$

$$\int \vec{F} \cdot d\vec{r} = \int (\vec{F}_{\text{ost}} + \vec{F}_g) \cdot d\vec{r} = \Delta W_k$$

$$A_{\text{ost}} = \int \vec{F}_{\text{ost}} \cdot d\vec{r} = \Delta W_k - \int \vec{F}_g \cdot d\vec{r}$$

$$\int \vec{F}_g \cdot d\vec{r} = \int -G \frac{mM}{r^2} \cdot \frac{\vec{r} \cdot d\vec{r}}{r} = G \int \frac{mM}{r^3} \vec{r} \cdot d\vec{r}$$

$$\vec{r} \cdot d\vec{r} = \underbrace{|\vec{r}|}_{r} \underbrace{|d\vec{r}| \cos \varphi}_{dr} = r \cdot dr$$

$$d(\vec{r} \cdot \vec{r}) = \vec{r} \cdot d\vec{r} + d\vec{r} \cdot \vec{r} = 2\vec{r} \cdot d\vec{r} \Rightarrow \vec{r} \cdot d\vec{r} = \frac{1}{2} d(r^2)$$

$$= G \int \frac{mM}{r^3} r dr = G \int \frac{mM}{r^2} dr = G \int \frac{mM}{r^2} dr = -G \frac{mM}{r} \Big|_{r_1}^{r_2} = -G \frac{mM}{r_2} - \left(-G \frac{mM}{r_1} \right) = \Delta W_p$$

$$= -G \frac{mM}{r_2} - \left(-G \frac{mM}{r_1} \right) = \Delta W_p$$

$$W_p = -G \frac{mM}{r}$$

gravitacijska potencialna energija

$$A_{\text{ost}} = \int \vec{F}_{\text{ost}} \cdot d\vec{r} = \Delta W_k - \underbrace{\int \vec{F}_g \cdot d\vec{r}}_{W_p} = \Delta W_k + W_p$$

za točkasto telo:

$$A_{\text{ost}} = \Delta W_k + W_p$$

$$W_p = -G \frac{mM}{(R+h)} = -G \frac{mM}{r}$$

$$W_p = mgh; h \ll R$$

V srednji šoli mislo upoštevati dejstvo, da se g_0 spreminja z višino.

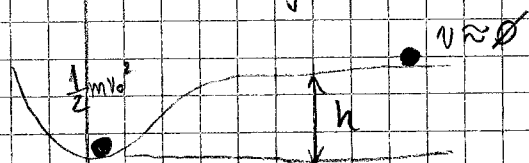
$$W_p = -G \frac{mM}{(R+h)} = -G \frac{mM}{R(1+\frac{h}{R})} \approx -G \frac{mM}{R} \left(1 - \frac{h}{R} + \frac{h^2}{R^2} - \dots\right) \approx -G \frac{mM}{R} \left(1 - \frac{h}{R}\right) =$$

$$= -G \frac{mM}{R} + G \frac{mM}{R} \frac{h}{R} \approx \text{konst.} + mg_0 h$$

ob predpostavki $\frac{h}{R} \ll 1$

$$W_p \approx mg_0 h ; \text{ če je } \frac{h}{R} \ll 1$$

primer: računanje ubežne hitrosti:



$$A_{\text{ost}} = \Delta W_k + \Delta W_p$$

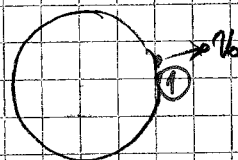
$$A_{\text{ost}} = 0 \Rightarrow \Delta W_k + \Delta W_p = 0$$

$$\Delta (W_k + W_p) = 0$$

$$W_k + W_p = \text{konst}$$

$$W_{k2} + W_{p2} = W_{k1} + W_{p1}$$

Zakon ohranitvi kinetične in potencialne energije.



$$\frac{1}{2} m v_0^2 - G \frac{mM}{R} = 0 + 0 = 0$$

v_0 = ubežna hitrost

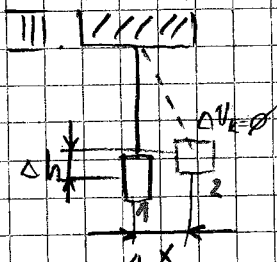
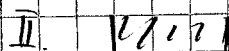
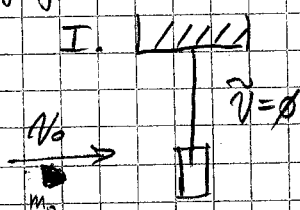
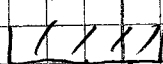
$$v_0 = \left(\frac{2GM}{R} \right)^{1/2} = \left(2 \left(\frac{GM}{R^2} \right) R \right)^{1/2} = (2g_0 R)^{1/2} \approx 11,2 \frac{\text{km}}{\text{s}}$$

EXPERIMENT: Merjenje hitrosti razstrelka s pomočjo balističnega nihala:

$$L = 590 \text{ cm}$$

$$m = 310 \text{ g}$$

$$m_0 = 0,5 \text{ g}$$



pri trku $\Delta G = 0$

$$m_0 v_0 = (m_0 + m) v \Rightarrow v_0 = \frac{(m_0 + m) v}{m_0}$$

$$W_k + W_p = \text{konst}$$

$$\frac{1}{2} (m_0 + m) v^2 = (m_0 + m) g_0 \Delta h$$

$$v_1 = \sqrt{2g_0 \Delta h}$$

$$v_0 = \frac{m_0 + m}{m_0} \sqrt{2g_0 \Delta h}$$

$$\Delta x = 19 \text{ cm}$$

$$\sin \varphi = \frac{\Delta x}{l} \Rightarrow \varphi \approx \frac{\Delta x}{l} = \frac{19 \text{ cm}}{590 \text{ cm}}$$

$$\Delta h = l - l \cos \varphi = l(1 - \cos \varphi) = l(1 - \cos(\frac{\Delta x}{l}))$$

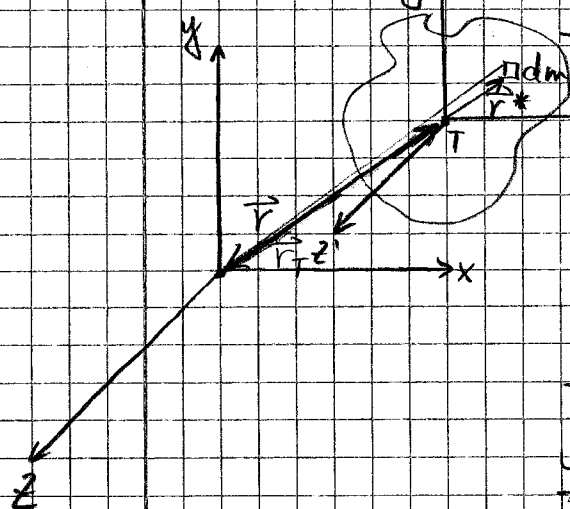
$$V_0 = \frac{m_0 + m}{m} \sqrt{2g \Delta h}$$

KINETIČNA ENERGIJA TOGEGA TELESA

5.11.2010

točk. telo: $W_k = \frac{1}{2} m v^2$

$$W_k = \int \frac{1}{2} dm v^2 = \int \frac{1}{2} dm (\vec{v} \cdot \vec{v})$$



$\vec{v} =$ hitrost
mase dm

$$\vec{r} = \vec{r}_T + \vec{r}^* \quad \frac{d\vec{r}}{dt} = \frac{d\vec{r}_T}{dt} + \frac{d\vec{r}^*}{dt}$$

$$\vec{v} = \vec{v}_T + \vec{v}^*$$

$$W_k = \int \frac{1}{2} dm (\vec{v}_T + \vec{v}^*) \cdot (\vec{v}_T + \vec{v}^*) = \int \frac{1}{2} dm v_T^2 + 2\vec{v}_T \cdot \int \frac{1}{2} dm \vec{v}^* + \int \frac{1}{2} dm v^{*2} =$$

$$= \frac{1}{2} \int dm v_T^2 + \cancel{\vec{v}_T \cdot \int dm \vec{v}^*} + \frac{1}{2} \int dm v^{*2} =$$

$$\vec{r}_T = \int \frac{\vec{r} dm}{dm} \quad (\text{težišče v laboratorijskem sistemu})$$

$$\vec{v}^* = \frac{d\vec{r}^*}{dt}; \quad \vec{r}_T^* = \frac{\int \vec{r}^* dm}{\int dm} = \vec{0} \quad \int dm \vec{v}^* = \frac{\int \vec{v}^* dm}{\int dm} = \vec{0}$$

$$W_k = \underbrace{\frac{1}{2} v_T^2 \int dm}_{\left[\frac{1}{2} m v_T^2 \right]} + \frac{1}{2} \int dm v^{*2}$$

TRANSLACIJSKA
KINETIČNA
ENERGIJA

$$\uparrow R\omega$$

$$\frac{1}{2} \int R^2 \omega^2 dm = \frac{1}{2} \omega^2 \int R^2 dm$$

$$J = \int R^2 \cdot dm$$

VZTRAJNOSTNI MOMENT OKOLI
TEŽIŠČNE OSI

→ rotacijska W_k

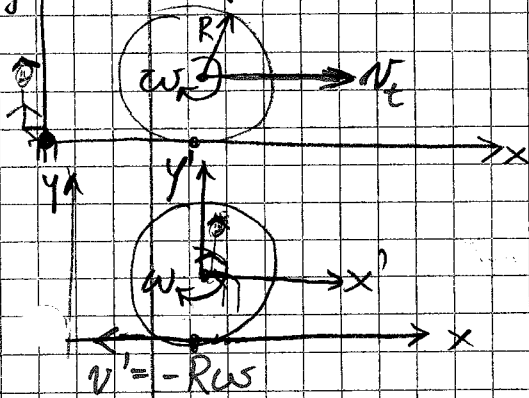
$$W_k = \frac{1}{2} m v_T^2 + \frac{1}{2} \int^* \omega^2$$

→ kinetična energija tega telesa

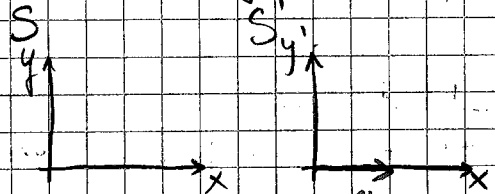
Zaradi gibanja težišča in rotacije okrog trenutne osi je vsa masa zbrana v središču.

PRIMER:

y W_k valja, ki se kotili po klancu navzdol (brez postrsanja)



Galilei - pva transformacija za hitrost



$$v = v' + v_T$$

$$0 = -R\omega + v_T$$

$$v_T = R \cdot \omega$$

$$W_k = \frac{1}{2} m v_T^2 + \frac{1}{2} \int^* \omega^2$$

$$\int^* = \int r^2 dm = \int r^2 \rho dV =$$

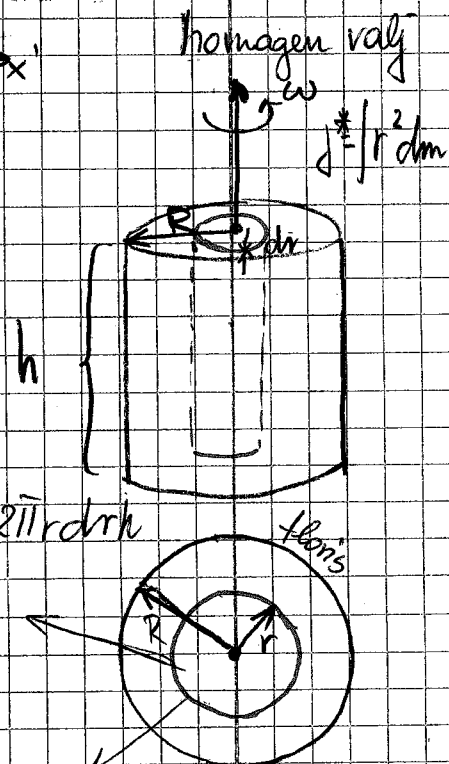
$$\int^* = \int r^2 \rho h 2\pi r dr = \int 2\pi \rho h r^3 dr =$$

$$\int^* = 2\pi \rho h \left[\frac{r^4}{4} \right]_0^R = 2\pi \rho h \frac{R^4}{4} = 2\pi \rho h \frac{R^4}{4} = \frac{1}{2} \pi \rho^2 h R^2 = \frac{1}{2} m R^2$$

$$W_k = \frac{1}{2} m v_T^2 + \frac{1}{4} m v_T^2 = \frac{3}{4} m v_T^2 \rightarrow (\text{kotaleči se valj})$$

kraka
S

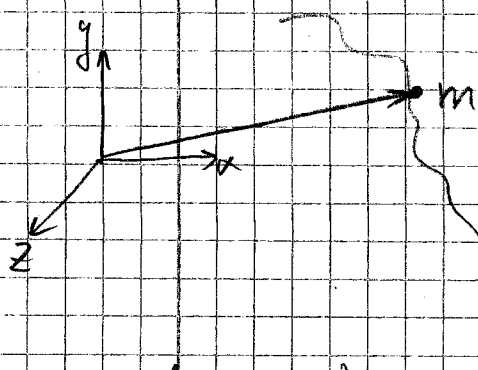
$$\int^* = \frac{2}{5} m R^2$$



$$dS = 2\pi r dr$$

IZREK O VRTILNI KOLIČINI

točkasto telo:



$$\vec{r} = \vec{r}(t)$$

$$\vec{a} = \frac{d^2 \vec{r}}{dt^2}$$

Newton-ov zakon:

$$m\vec{a} = \vec{F} / x \vec{r}$$

$$\vec{r} \times m\vec{a} = \vec{r} \times \vec{F}$$

$$\frac{d}{dt} (\vec{r} \times m\vec{v}) = \underbrace{\frac{d\vec{r}}{dt} \times m\vec{v}}_{(\vec{v} \times m\vec{v}) = \emptyset} + \vec{r} \times m \underbrace{\frac{d\vec{v}}{dt}}_{\vec{a}} = \vec{r} \times m\vec{a}$$

$$\frac{d(\vec{r} \times m\vec{v})}{dt} = \vec{M}$$

$\vec{M} = \vec{r} \times \vec{F}$
$\vec{L} = \vec{r} \times m\vec{v}$

NAVOR

VRTILNA KOLIČINA

$$\frac{d\vec{L}}{dt} = \vec{M}$$

Newtonov zakon za vrtenje

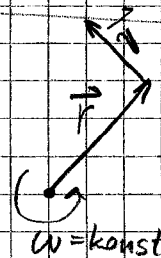
$$M = |\vec{M}| = |\vec{r} \times \vec{F}| = rF \sin \varphi$$

POGOJA ZA RAVNOVESJE

$$\sum_i \vec{F}_i = \vec{0}$$

$$\sum_i \vec{M}_i = \vec{0}$$

primer: VRTILNA KOLIČINA VREĆE TOČKASTE MASE:



$$v = r\omega$$

$$\vec{v} \perp \vec{r} \quad \varphi = 90^\circ$$

$$L = |\vec{r} \times m\vec{v}| = rmv \sin 90^\circ$$

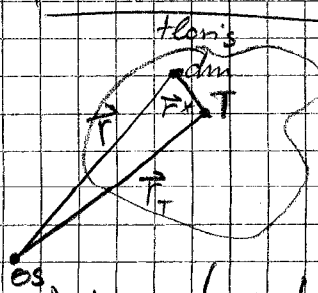
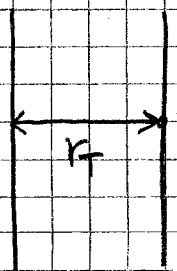
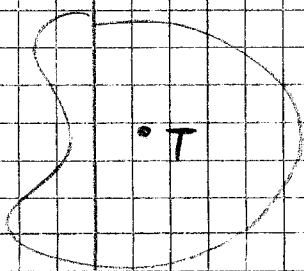
$$L = rmv = rmr\omega =$$

$$J\omega = L = mr^2 \omega$$

VRTILNA KOLIČINA

~~OSTATNA TOČKASTA MASA~~

Steiner-jovo pravilo



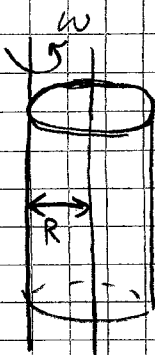
$$\vec{r} = \vec{r}_T + \vec{r}^*$$

$$J = \int r^2 dm = \int (\vec{r}_T + \vec{r}^*) \cdot (\vec{r}_T + \vec{r}^*) dm = \int r_T^2 dm + 2\vec{r}_T \cdot \int \vec{r}^* dm + \int r^{*2} dm =$$

$$r_T^2 \int dm + \int r^{*2} dm =$$

$$J = J^* + mr_T^2$$

primer:

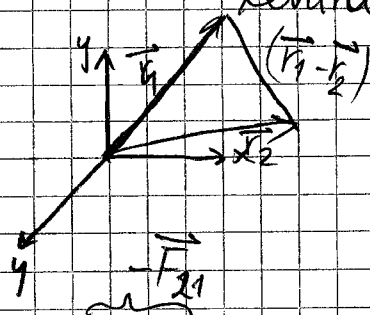
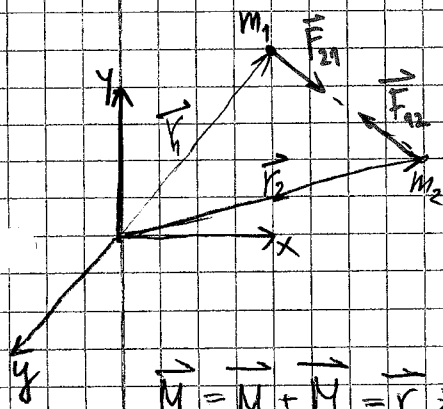


$$J = J^* + mR^2 = \frac{1}{2} mR^2 + mR^2 = \frac{3}{2} mR^2$$

NAVORI NOTRANJNH SIL

(primer: 2 točkasti telesni)

Sili ki povezuje točkasti telesni sta centralni sili



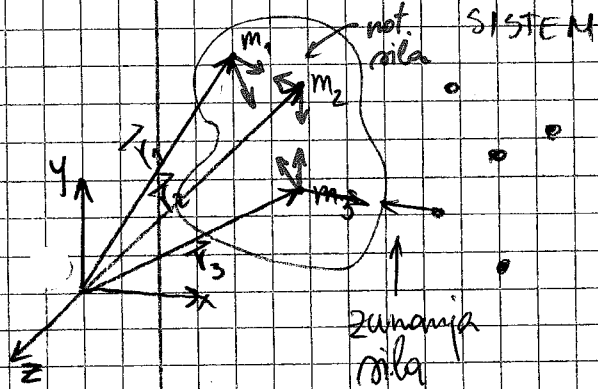
III. N.2.

$$\vec{F}_{12} + \vec{F}_{21} = \vec{0}$$

$$\vec{M} = \vec{M}_1 + \vec{M}_2 = \vec{r}_1 \times \vec{F}_{21} + \vec{r}_2 \times \vec{F}_{12} = \vec{r}_1 \times \vec{F}_{21} - \vec{r}_2 \times \vec{F}_{21} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{21} = \vec{0}$$

Vsota momentov notranjih sil n-točkastih teles je enaka 0

IZREK O VRTILNI KOLIČINI ZA SISTEM TOČKASTIH TELES



$$m_i \vec{a}_i = \sum_j \vec{F}_{ji} + \vec{F}_i$$

$$\vec{F}_i = \vec{r}_i \times m_i \vec{a}_i$$

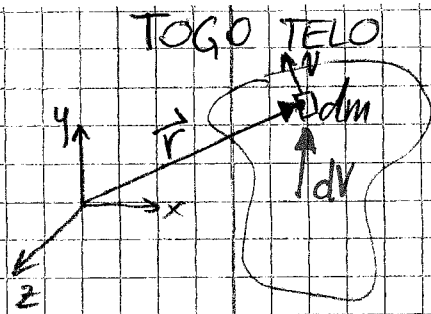
$$\vec{F} = \sum_i \vec{F}_i = \sum_i \vec{r}_i \times m_i \vec{a}_i$$

vrtilna količina sistema točkastih mas

$$\frac{d\vec{F}}{dt} = \frac{d}{dt} \left(\sum_i \vec{r}_i \times m_i \vec{v}_i \right) = \sum_i \left[\left(\frac{d\vec{r}_i}{dt} \times m_i \vec{v}_i \right) + \left(\vec{r}_i \times m_i \frac{d\vec{v}_i}{dt} \right) \right]$$

$$\begin{aligned} \frac{d\vec{F}}{dt} &= \vec{r}_i \times m_i \frac{d\vec{v}_i}{dt} = \vec{r}_i \times m_i \vec{a}_i = \sum_i (\vec{F}_i \times \sum_j \vec{F}_{ji} + \vec{F}_i) = \\ &= \sum_i (\vec{r}_i \times \vec{F}_{ji}) + \sum_i (\vec{r}_i \times \vec{F}_i) = \sum_i \vec{M}_i \end{aligned}$$

navar vseh notranjih sil



gostota
 $\rho = \rho(\vec{r})$
 $m_i = dm$
 $\vec{r}_i = \vec{r}$
 $\frac{m_i}{V_i} = \rho$

$$\vec{P} = \sum_i \vec{F}_i \times m_i \vec{v}_i$$

↑ ↑ ↑
 $\vec{r}$ dm $\vec{v}$

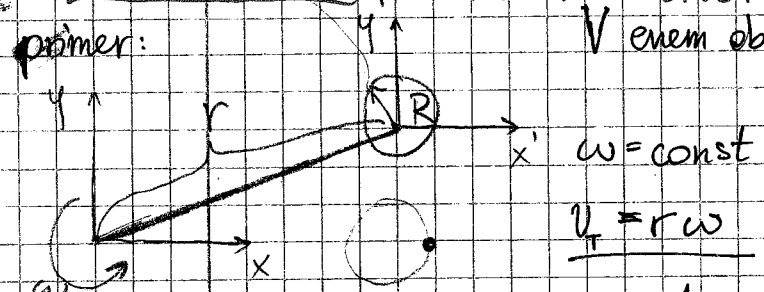
kontinuumski opis
 $\sum_i = \int$ $\vec{P} = \int (\vec{r} \times dm \vec{v})$
 $dm = \rho(\vec{r}) dV$
 vrtilna količina telesa

$$\frac{d\vec{P}}{dt} = \sum_i \vec{M}_i = \vec{M} / dt$$

$$d\vec{P} = \vec{M} dt$$

$$\Delta \vec{P} = \int \vec{M} dt$$

KINETIČNA ENERGIJA KROŽEČE KROGLE 8. 11. 10
 V enem obhodu se krogla enkrat zavrti.

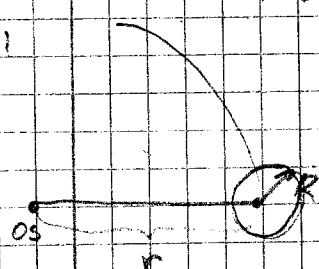


1. način: $W_k = \frac{1}{2} m v_t^2 + \frac{1}{2} J^* \omega^2$
 $= \frac{1}{2} m v_t^2 + \frac{1}{2} \cdot \frac{2}{5} m R^2 \cdot \omega^2$

$J^* = \frac{2}{5} m R^2$ → vztrajnostni moment krogle
 $= \frac{1}{2} m r^2 \omega^2 + \frac{1}{2} \cdot \frac{2}{5} m R^2 \omega^2$
 $= \frac{1}{2} m \omega^2 (r^2 + \frac{2}{5} R^2)$

limita ($R \ll r$); $W_k \approx \frac{1}{2} m r^2 \omega^2 = \frac{1}{2} m v_t^2$

2. način:



$$W_k = \frac{1}{2} J \omega^2 = \frac{1}{2} \left(\frac{2}{5} m R^2 + m r^2 \right) \omega^2$$

Steiner:
 $J = J^* + m r^2 = \frac{2}{5} m R^2 + m r^2$

- Luping

mejni primer

New. zakon:

$$F_g = mg = m a_r$$

$$mg = m r \frac{v^{*2}}{r}$$

izguba W_p se spremeni v W_k

$$mgh = \frac{1}{2} m v^{*2} \Rightarrow v^{*2} = 2gh$$

$$v^* = R\omega$$

$$g = \frac{2gh}{r} \Rightarrow h = \frac{r}{2} \Rightarrow \text{enačba ni dovolj natančna}$$

upoštevamo še $\frac{1}{2} J \omega^2$:

$$mgh = \frac{1}{2} m v^{*2} + \frac{1}{2} J \omega^2 =$$

$$= \frac{1}{2} m v^{*2} + \frac{1}{2} \cdot \frac{2}{5} m R^2 \cdot \frac{v^{*2}}{R^2} = m \left(\frac{1}{2} + \frac{1}{5} \right) v^{*2} =$$

$$= \left[m \cdot \frac{7}{10} v^{*2} = mgh \right] \Rightarrow v^{*2} = \frac{10}{7} gh$$

$$g = \frac{v^{*2}}{r} = \frac{10}{7} \frac{gh}{r} \Rightarrow \left[h = \frac{7}{10} r \right] \Rightarrow \text{z upoštevanjem rotacijske WK doseže kroglica večjo pot.}$$

Togo telo

$$\vec{\Gamma} = \int \vec{r} \times \vec{v} dm$$

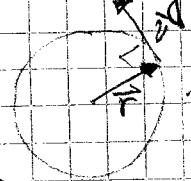
točkasto telo:

$$\Gamma = J \omega$$

$$\uparrow$$

$$mr^2$$

kroženji točk. tel.



$$\vec{\Gamma} = \vec{r} \times m \vec{v}$$

$$\Gamma = r m v \cdot \sin 90^\circ =$$

$$= r m v \omega = \underbrace{mr^2}_{\text{inertanca}} \omega = J \omega$$

togo telo (v splošnem)

$$\vec{\Gamma}, \vec{\omega}$$

tenzor (matrika)

$$\vec{\Gamma} = J \vec{\omega}$$

$$\begin{bmatrix} J_{xx} & D_{xy} & D_{xz} \\ D_{yx} & J_{yy} & D_{yz} \\ D_{zx} & D_{zy} & J_{zz} \end{bmatrix} \text{ (simetrija) } = J$$

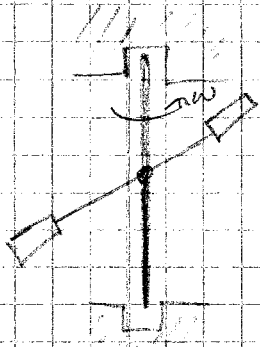
$$\vec{\Gamma} = \begin{bmatrix} J_{xx} & D_{xy} & D_{xz} \\ D_{yx} & J_{yy} & D_{yz} \\ D_{zx} & D_{zy} & J_{zz} \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} J_{xx}\omega_x + D_{xy}\omega_y + D_{xz}\omega_z \\ J_{yy}\omega_y + D_{yx}\omega_x + D_{yz}\omega_z \\ J_{zz}\omega_z + D_{zx}\omega_x + D_{zy}\omega_y \end{bmatrix}$$

množenje tenzorja z vektorjem

$D_{nn} \rightarrow$ deviacijski momenti

EKSPERIMENT

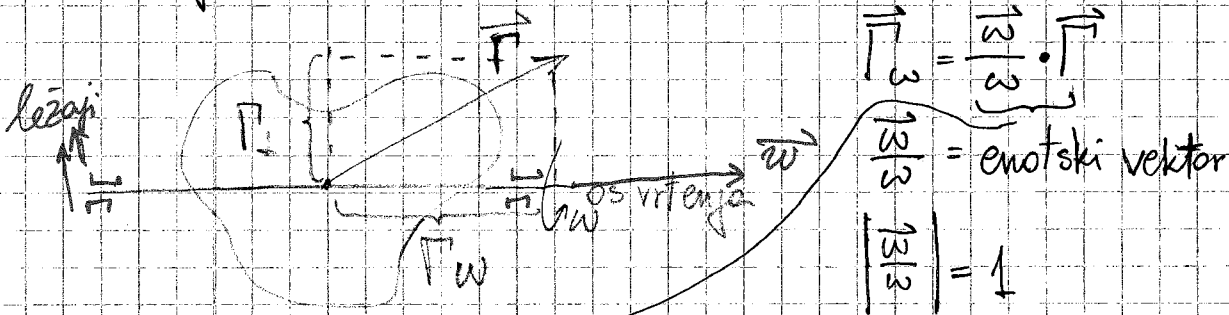
$$\vec{\Gamma} = J \vec{\omega} \Rightarrow \vec{\Gamma} \parallel \vec{\omega}$$



točka se vrti v treh smereh, namo svetli laser. Ko je točka prosta se vrti okoli ene izmed glavnih os, ponašajo, kakor ti, ste, ki ima najmanjšo WK. Vektor vrtilne količine je usmerjen navzgor.

Če os vpišemo & točka premika po vertikalni osi.

- os vrtenja je vsiljena:



$$\frac{\omega}{\omega} \Gamma \cdot \cos \varphi$$

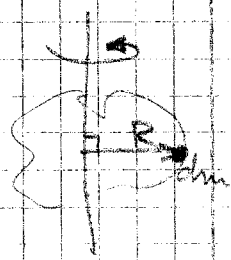
$$\Rightarrow \Gamma_{\omega} = \frac{\omega}{\omega} \int \vec{r} \times \vec{v} \, dm = \frac{\omega}{\omega} \cdot \left(\vec{r} \times (\vec{r} \times \vec{\omega}) \right) dm =$$

$$= \frac{\omega}{\omega} \cdot \left[r^2 \vec{\omega} - \vec{r} (\vec{r} \cdot \vec{\omega}) \right] dm = \int \left[r^2 \vec{\omega} - \frac{r^2 \omega^2 \cos^2 \varphi}{\omega} \right] dm$$



$$\vec{r} \cdot \vec{\omega} = r \omega \cos \varphi$$

$$\int \omega r^2 [1 - \cos^2 \varphi] dm = \omega (r \sin \varphi)^2 dm = \omega \int R^2 dm$$



$$\Gamma_{\omega} = J_{\omega} \omega$$

vrtilna količina

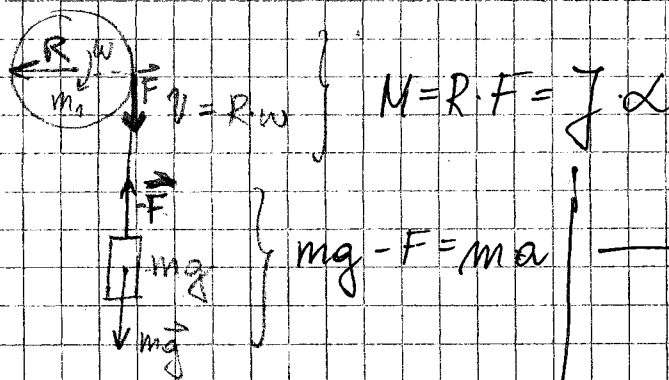
na vsiljeni osi vrtenja

$$M_{\omega} = \frac{d\Gamma_{\omega}}{dt} = \frac{d(J_{\omega} \omega)}{dt} = J_{\omega} \frac{d\omega}{dt} = J_{\omega} \alpha$$

"Newton-ov" zakon vrtenja
Če deli fiksnе osi
toplega telesa

primer:

8.11.10



$$v = R\omega / \frac{d}{dt}$$

$$\frac{dv}{dt} = R \frac{d\omega}{dt}$$

$$a = R \cdot \alpha$$

$$\alpha = \frac{a}{R}$$

$$R \cdot F = \frac{1}{2} m_1 \cdot R^2 \cdot \alpha \Rightarrow R \cdot F = \frac{1}{2} m_1 \cdot R \cdot a$$

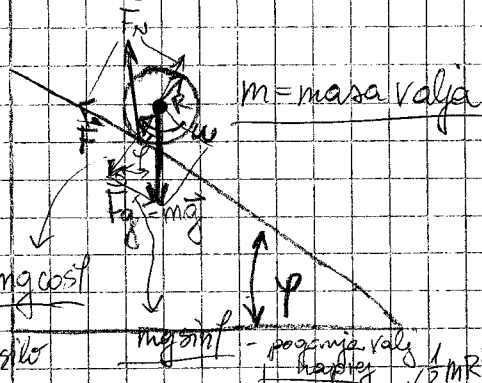
$$\frac{a}{R} = \frac{1}{2} m_1 \cdot a$$

$$F = \frac{1}{2} m_1 \cdot a \Rightarrow mg - \frac{1}{2} m_1 \cdot a = ma$$

$$a = \frac{mg}{\frac{1}{2} m_1 + m}$$

$$mg = a \left(\frac{1}{2} m_1 + m \right)$$

KOTAJENJE VALJA PO KLANKU NAVZDOL BREZ PODSAVANJA 12.11.10



Uravnovesilna sila
podlage
navor sile F:1

$$2. \quad mg \sin \phi - F = m \cdot a$$

$$1. \quad R \cdot F = \frac{1}{2} m R^2 \cdot \alpha$$

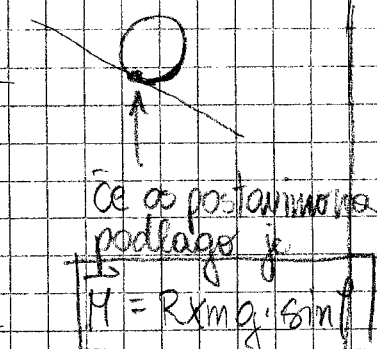
$$F = \frac{1}{2} m \cdot a$$

$$2. \quad mg \sin \phi - \frac{1}{2} m a = m a$$

$$mg \sin \phi = \frac{3}{2} m a$$

$$g \sin \phi = \frac{3}{2} a$$

$$a = \frac{2}{3} g \sin \phi$$



$$M = R \cdot m g \cdot \sin \phi$$

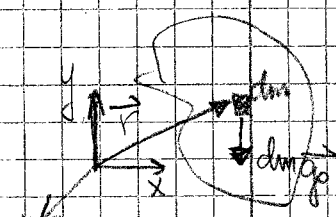
$$v = R \omega / \frac{d}{dt}$$

$$\frac{dv}{dt} = R \cdot \frac{d\omega}{dt}$$

$$a = R \cdot \alpha$$

$$\alpha = \frac{a}{R}$$

Navor sile teže:



$$d\vec{M} = \vec{r} \times d\vec{m} \vec{g}$$

$$\vec{M} = \int d\vec{M} = \int \vec{r} \times d\vec{m} \vec{g} =$$

$$= \left(\int \vec{r} dm \times \vec{g} \right) = \left(\int \vec{r} dm \right) \times \vec{g} =$$

$$\vec{r}_1 \times \vec{g}_0 + \vec{r}_2 \times \vec{g}_0 + \vec{r}_3 \times \vec{g}_0 + \dots =$$

$$= (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \dots) \times \vec{g}_0$$

$$= \left(\int \vec{r} dm \right) \times \vec{g}_0 = m \vec{r}_F \times \vec{g}_0 =$$

$$\vec{M} = \vec{r}_F \times m \vec{g}_0$$

pospešek homogenega valja pri kotajenju brez podsaivanja

$$F = \frac{1}{2} m a = \frac{1}{2} \cdot m \cdot \frac{2}{3} g \sin \varphi = \boxed{\frac{1}{3} m g \sin \varphi} \quad \text{komponenta sile podlage}$$

prazen valj:

$$\boxed{J = \int r^2 dm}$$

$$r^2 \in R^2 \Rightarrow R^2 / dm = \boxed{R^2 \cdot m}$$



$$M = F R = m R^2 \cdot \left(\frac{a}{R}\right) \Rightarrow \boxed{m a R = F}$$

$$m g \sin \varphi - F = m a$$

$$g \sin \varphi - a = a$$

$$g \sin \varphi = 2a$$

$$\boxed{a = \frac{1}{2} g \sin \varphi}$$

$$\boxed{F = m a = m \frac{1}{2} g \sin \varphi}$$

HOMOGEN VALJ

IMA VEČJI POSPEŠEK

ZARADI VEŠTAJNOSTNEGA

MOMENTA

Valj bi začel podravati, ko bi se bila F_n izenacila s silo F_{lep} .

$$F_{lep} = k_{lep} m g \cos \varphi$$

$$F_{tr} = k_f m g \cos \varphi$$

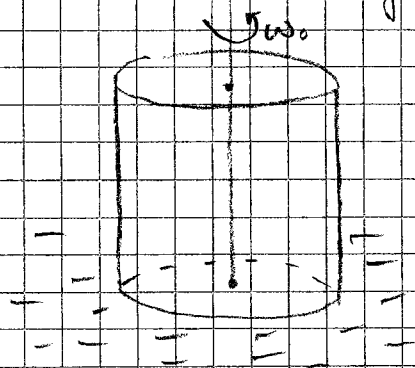
ni podravanja: $F = \frac{1}{3} m g \sin \varphi < \overbrace{k_{lep} m g \cos \varphi}^{F_{lep}}$

$$\boxed{\tan \varphi < 3 k_{lep}}$$

$$\boxed{M = J \cdot \alpha}$$

primer:

- N - zakon za vrtenje doli fiksne osi,



$$M_z = -k w$$

↑
kotna
hitrost
vrtenja

$$M_z = J \cdot \alpha = -k \cdot w$$

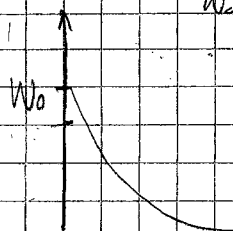
$$-k w = J \frac{dw}{dt}$$

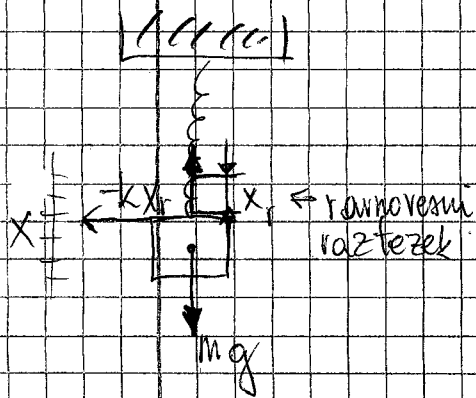
$$\int_{w_0}^w \frac{dw}{w} = - \int_0^t \frac{k}{J} dt, \quad w_0 = w(t=0)$$

$$= \ln w \Big|_{w_0}^w = - \frac{k}{J} t =$$

$$= \ln w - \ln w_0 = \ln \left(\frac{w}{w_0} \right) = - \frac{k}{J} t$$

$$\boxed{w = w_0 e^{(-\frac{k}{J} t)}}$$





↑ ravnovesju: $mg - kx_r = 0$

$$\hat{X} = X_r + x$$

↑ celotni raztezek vzmeti

N - zakon za gibanje:

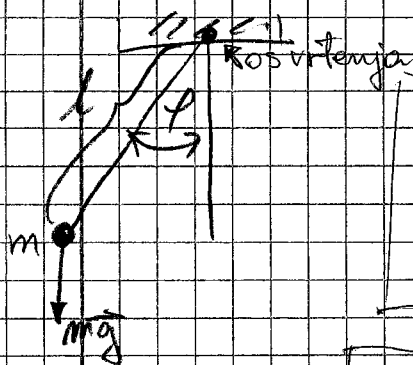
$$m \ddot{\hat{X}} = mg - k \hat{X} = mg - kx_r - kx$$

$$m \ddot{X} = -kX / m$$

$$\ddot{X} = -\frac{k}{m} X \Rightarrow X = X_0 \sin(\omega_0 t + \varphi)$$

$$\boxed{\omega_0 = \frac{2\pi}{T_0} = \sqrt{\frac{k}{m}}}$$

Matematično krmhalo:



$$M = J \cdot \alpha = -X \cdot mg \cdot \sin \varphi = ml^2 \cdot \ddot{\varphi}$$

$$\omega = \frac{d\varphi}{dt} = \dot{\varphi}$$

$$\alpha = \frac{d^2\varphi}{dt^2} = \ddot{\varphi}$$

$$-g \sin \varphi = l \ddot{\varphi} \quad \varphi \ll 1 \Rightarrow \sin \varphi \approx \varphi$$

↓
v radianih

$$\varphi \ll 1 \Rightarrow -g \varphi \approx l \cdot \ddot{\varphi}$$

$$\boxed{\ddot{\varphi} = -\frac{g}{l} \varphi}$$

postavke za rešitev:

$$\varphi = \varphi_0 \sin(\omega_0 t + \varphi)$$

$$\omega = \frac{d\varphi}{dt} = \dot{\varphi} = \left(\frac{2\pi}{T_0}\right) \varphi_0 \cos\left(\frac{2\pi}{T_0} t + \varphi\right)$$

$$\boxed{\omega_0 = \frac{2\pi}{T_0} \varphi_0}$$

$$\alpha = \frac{d^2\varphi}{dt^2} = \ddot{\varphi} = -\left(\frac{2\pi}{T_0}\right)^2 \varphi_0 \sin\left(\frac{2\pi}{T_0} t + \varphi\right)$$

$$\boxed{\ddot{\varphi} = -\left(\frac{2\pi}{T_0}\right)^2 \varphi}$$

$$\left(-\frac{2\pi}{T_0}\right)^2 \varphi_0 \sin\left(\frac{2\pi}{T_0} t + \varphi\right) = -\frac{g}{l} \varphi_0 \sin\left(\frac{2\pi}{T_0} t + \varphi\right)$$

$$\left(\frac{2\pi}{T_0}\right)^2 = \frac{g}{l}$$

$$\frac{2\pi}{T_0} = \sqrt{\frac{g}{l}} \Rightarrow$$

$$\boxed{T_0 = 2\pi \sqrt{\frac{l}{g}}}$$

Fyzické nikholo:

$$M = J \cdot \ddot{\varphi}, \quad \vec{F}_g = m\vec{g}$$

$$-r_T \cdot F_g \cdot \sin \varphi = J \cdot \ddot{\varphi}$$

$$-r_T mg \varphi = J \ddot{\varphi}$$

$$\ddot{\varphi} = - \left[\frac{r_T}{J} mg \right] \varphi$$

$$\left(\frac{2\pi}{t_0} \right)^2 = \left(\frac{r_T}{J} mg \right)^{-1}$$

$$t_0 = 2\pi \sqrt{\frac{J}{r_T mg}}$$

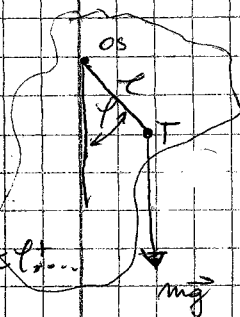
rešitev:

$$\varphi = \varphi_0 \cdot \sin\left(\frac{2\pi}{t_0} t + \varphi\right)$$

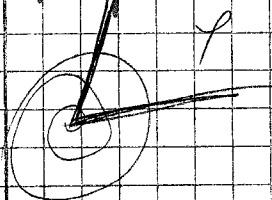
limit: mat. nikholo

$$J = ml^2$$

$$t_0 = 2\pi \sqrt{\frac{J}{r_T mg}} = 2\pi \sqrt{\frac{ml^2}{l \cdot mg}} = 2\pi \sqrt{\frac{l}{g}}$$



Nikholo na polzasto vemet:



$$M = -D\varphi$$

$$-D\varphi = J \ddot{\varphi}$$

$$\ddot{\varphi} = - \frac{D}{J} \varphi$$

$$\left(\frac{2\pi}{t_0} \right)^2 = \frac{D}{J}$$

$$\left(\frac{2\pi}{t_0} \right)^2 = \frac{D}{J}$$

$$t_0 = 2\pi \sqrt{\frac{J}{D}}$$

$$\varphi = \varphi_0 \sin\left(\frac{2\pi}{t_0} t + \varphi\right)$$

Dušené nihanje:

$$F_d = -2\beta m \dot{x}$$

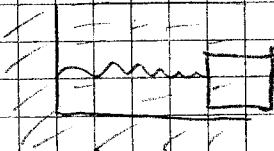
koeficient trenja

← Stokes-ov zákon



$$F_d = 6\pi r \eta v$$

viskozita
hltrost



$$F_{TR} = k_{TR} m \dot{x}$$

N - zákon:

$$m\ddot{x} = -kx - 2\beta m \dot{x}$$

$$\ddot{x} + 2\beta \dot{x} + \frac{k}{m} x = 0$$

$$\omega_0^2 = \frac{k}{m}$$

ze nedušené nihanje

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = 0$$

Izrek o ohranitvi vrtilne količine (za vrtenje telesa okoli fiksne osi)

$$\vec{M} = \vec{J} \vec{\alpha}, \text{ vrtilna količina } \vec{L} = \vec{J} \vec{\omega}, \quad J = \int r^2 dm$$

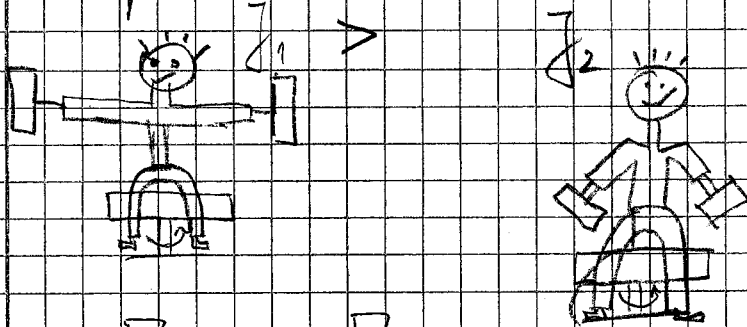
$$\vec{M} = J \frac{d\vec{\omega}}{dt} = \frac{d(J\vec{\omega})}{dt} = \frac{d\vec{L}}{dt}$$

$$\int \vec{M} dt = \int d\vec{L} \Rightarrow \vec{L}_{kon} - \vec{L}_{zac} \quad \boxed{\int \vec{M} dt = \Delta \vec{L}}$$

Če je $\int \vec{M} dt = 0 \Rightarrow \Delta \vec{L} = 0 \Rightarrow \vec{L}_{kon} = \vec{L}_{zac}$ ohranitev vrtilne količine

izrek o vrtilni kol. (o sumku momentov)

primer uporabe izreka o ohranitvi vrtilne količine:



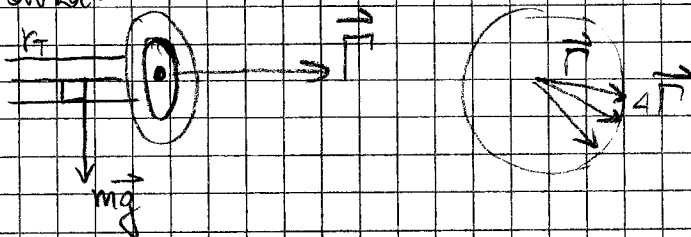
$$J_1 = J_2$$

$$J_1 \omega_1 = J_2 \omega_2$$

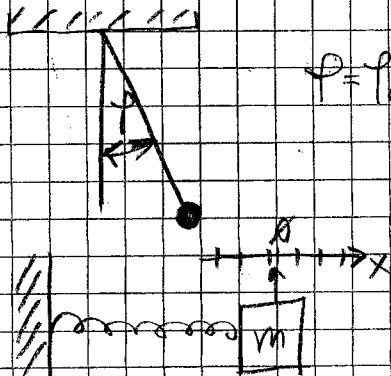
$$\omega_1 = \frac{J_2}{J_1} \omega_2 \Rightarrow$$

$$\boxed{\omega_1 < \omega_2 \text{ , ker } \frac{J_2}{J_1} < 1}$$

vrtenka:

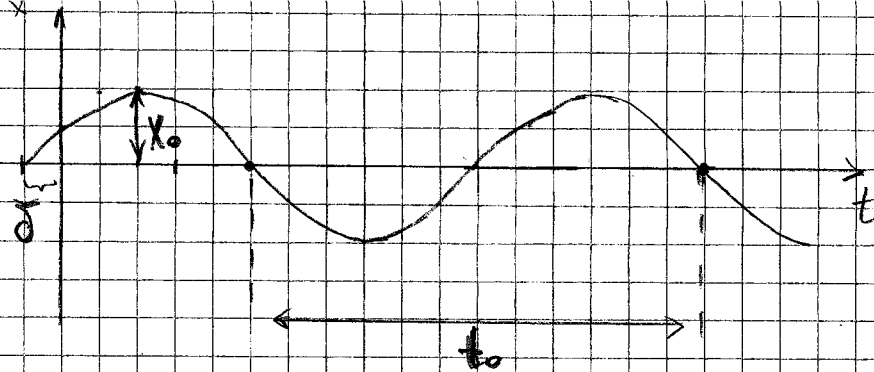


HARMONIČNO NIHANJE



$$\varphi = \varphi_0 \sin\left(\frac{2\pi}{T_0} t + \varphi\right) \Rightarrow \text{sučino sinusno (harmonično) gibanje}$$

$$x = x_0 \sin\left(\frac{2\pi}{T_0} t + \varphi\right) \Rightarrow \text{linearno sinusno (harmonično) gibanje}$$



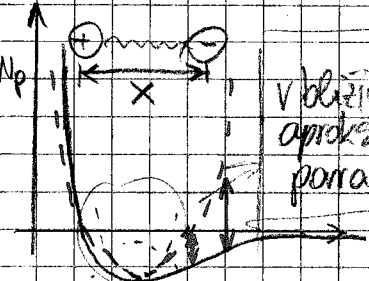
$$\omega_0 t_0 = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{t_0}$$

$$\nu_0 = \frac{1}{t_0}$$

$t_0 \equiv$ lastni nihajni čas ω_0

$$\omega_0 = 2\pi \nu_0$$

$\Rightarrow$ kristalna mreža



v bližini minimuma opredeljeno poraboli

$$W_p = \frac{1}{2} K x^2$$

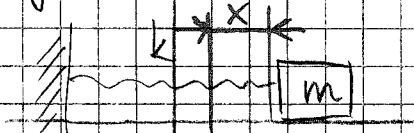
$$F = - \frac{dW_p}{dx} \Rightarrow \text{dub} = -F dx$$

$$F = -K \cdot x \quad \text{-- Hookov zakon}$$

Nedušeno harmonično nihanje.

15.11.2010

Nihalo ne plžasto vzmet
to je odvisen od k vzmeti.



predpostavim, da mi traja je samo
sila vzmeti

N - zakon:

$$m \ddot{x} = -kx / m$$

$$\ddot{x} \equiv \frac{d^2 x}{dt^2} \quad (\text{pospešek})$$

$$\dot{x} = \frac{dx}{dt} \quad (\text{hitrost})$$

$$\ddot{x} = -\frac{k}{m} x$$

$$F = -kx$$

navstavek za rešitev:

$$x = x_0 \sin(\omega_0 t + \delta) \quad \text{fazni zamik}$$

amplituda

$$v = \dot{x} = \cos(\omega_0 t + \delta) \cdot \overset{x_0}{\omega_0} \quad |v_0 = x_0 \omega_0|$$

$$a = \ddot{x} = -x_0 \omega_0^2 (\cos \omega_0 t + \delta)$$

$$F = x_0 \omega_0^2 \cdot \sin(\omega_0 t + \delta) = \frac{k}{m} x_0 \sin(\omega_0 t + \delta)$$

$$\omega_0^2 = \frac{k}{m}$$

$$\omega_0 = \frac{2\pi}{t_0} = \sqrt{\frac{k}{m}}$$

$$t_0 = 2\pi \sqrt{\frac{m}{k}}$$

El. nihajni krog:

$$x(t) = y(t)e^{-\beta t}$$

$$\dot{x} = \dot{y}e^{-\beta t} - \beta ye^{-\beta t}$$

$$\ddot{x} = \ddot{y}e^{-\beta t} - \underbrace{2\beta \dot{y}e^{-\beta t}}_{-2\beta \dot{y}e^{-\beta t}} + \beta ye^{-\beta t}$$

$$\cancel{\ddot{y}e^{-\beta t}} - \cancel{2\beta \dot{y}e^{-\beta t}} + \beta^2 ye^{-\beta t} + \cancel{2\beta \dot{y}e^{-\beta t}} - \cancel{2\beta^2 ye^{-\beta t}} + \omega_0^2 ye^{-\beta t} = 0$$

$$e^{-\beta t} [\ddot{y} - \beta^2 y + \omega_0^2 y] = 0$$

$$\ddot{y} = \frac{1}{2}(\omega^2 - \beta^2)y$$

$$\omega_0'^2 = \omega_0^2 - \beta^2$$

$$y = X_0 \sin(\omega_0' t + \delta)$$

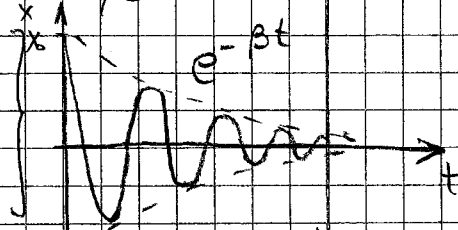
$$\ddot{y} = -\omega_0'^2 y$$

$$\omega_0 = \sqrt{\omega_0'^2 + \beta^2}$$

$$x = y(t)e^{-\beta t} = X_0 \sin(\omega_0' t + \delta) e^{-\beta t}$$

$$x(t) = X_0 e^{-\beta t} \sin(\omega_0' t + \delta)$$

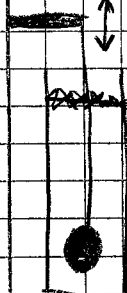
$$\omega_0' = \sqrt{\omega^2 - \beta^2}$$



poseben primer:

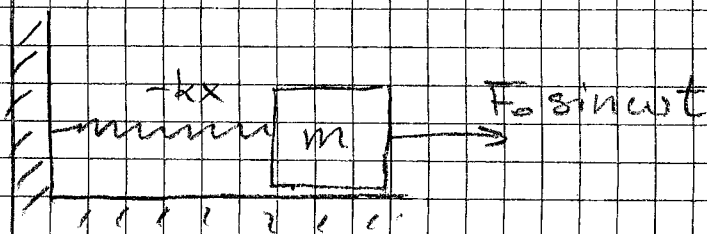
$|\beta = 0| \Rightarrow \omega_0' = \omega_0 \Leftarrow$ neodvisno nihanje ($x = X_0 \sin(\omega_0 t + \delta)$)

VSILJENO NIHANJE



opazi se pojav resonance (vsiljena frekvenca je blizu lastne frekvence).

primer:



$$F(t) = F_0 \sin \omega t$$

dušenje:

$$F_n = -2\beta m v$$

↑ koeficient
dušenja

$$a = \frac{d^2 x}{dt^2} = \ddot{x}$$

$$v = \frac{dx}{dt} = \dot{x}$$

N-zakon:

$$m a = -kx - 2\beta m v + F_0 \sin(\omega t)$$

sila
vzmeti

sila vzbujanja

$$m \ddot{x} + 2\beta m \dot{x} + kx = F_0 \sin(\omega t)$$

$$m \ddot{x} + 2\beta m \dot{x} + kx = 0$$

↑
homogen del
resitve

$$\text{resitev: } x = x_0 e^{-\beta t} \sin(\omega_0 t + \delta)$$

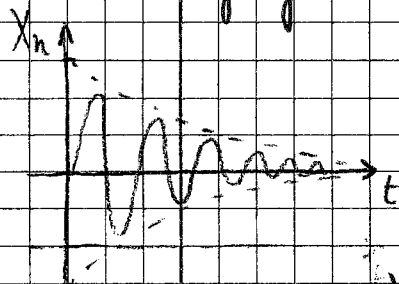
$$\omega_0 = \sqrt{\omega^2 - \beta^2}$$

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$x(t) = x_h(t) + x_p(t)$$

 $x_p(t)$ = partikularna
resitev nehomogen
dif. enačbe

Brez vzbujanja



$$x_h \rightarrow 0$$

$$x_p(t) = x_0 \sin(\omega t + \delta), \quad \omega = \text{krožna frekvenca vsiljenega nihanja}$$

$$x_p(t) = x_0 \sin(\omega t + \delta) = x_0 \sin \omega t \cos \delta + x_0 \cos \omega t \sin \delta$$

$$x_0 \cos \delta = A_1$$

$$x_0 \sin \delta = A_2$$

$$x_p(t) = A_1 \sin \omega t + A_2 \cos \omega t$$

$$\begin{aligned} \dot{x}_p &= A_1 \omega \cos \omega t - A_2 \omega \sin \omega t \\ \ddot{x}_p &= A_1 \omega^2 \sin \omega t - A_2 \omega^2 \cos \omega t \end{aligned} \quad \left. \vphantom{\begin{aligned} \dot{x}_p &= A_1 \omega \cos \omega t - A_2 \omega \sin \omega t \\ \ddot{x}_p &= A_1 \omega^2 \sin \omega t - A_2 \omega^2 \cos \omega t \end{aligned}} \right\} \text{vstavimo v dif. enačbo}$$

$$\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = \frac{F_0}{m} \sin \omega t$$

$$\cos \omega t [-A_2 \omega^2 + 2\beta A_1 \omega + \omega_0^2 A_2] = 0$$

$$\sin \omega t [A_1 \omega^2 - 2\beta A_2 \omega + \omega_0^2 A_1] = \frac{F_0}{m} \sin \omega t$$

$$\sin \omega t [-A_1 \omega^2 - 2\beta A_2 \omega + \omega_0^2 A_1] = \frac{F_0}{m} \sin \omega t$$

$$(\omega_0^2 - \omega^2) A_1 - 2\beta \omega A_2 = F_0/m$$

$$(\omega_0^2 - \omega^2) A_2 + 2\beta \omega A_1 = 0$$



$$\frac{A_2}{A_1} = \frac{x_0 \sin \delta}{x_0 \cos \delta} = \tan \delta = \frac{2\beta \omega}{\omega_0^2 - \omega^2}$$

$$A_1^2 + A_2^2 = x_0^2 \cos^2 \delta + x_0^2 \sin^2 \delta = x_0^2 \Rightarrow x_0 = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2 \omega^2}}$$

Če je: $\omega \ll \omega_0$

$$\frac{x_0}{\frac{F_0}{m \omega_0^2}} \approx 1 \Rightarrow x_0 \approx \frac{F_0}{m \omega_0^2}$$

$$\cancel{\frac{m \ddot{x}}{m}} + 2\beta \cancel{\frac{m \dot{x}}{m}} + \frac{kx}{m} = \frac{F_0}{m} \Rightarrow x = \frac{F_0}{\omega_0^2 m} \sin \omega t$$

$$\omega \approx \omega_0 \Rightarrow \frac{dx_0}{d\omega} = 0 \text{ je pri } \omega = \omega_{\max} = \sqrt{\omega_0^2 - 2\beta^2}$$

Za $\beta \rightarrow 0$: $\omega_{\max} \rightarrow \omega_0$ in $x_0 \rightarrow \infty$

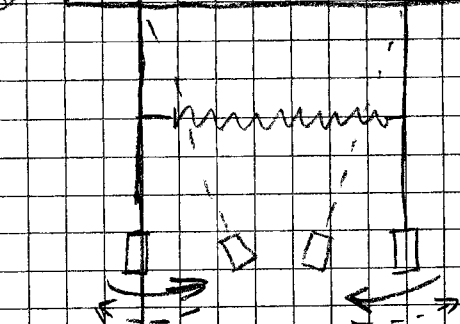
Če je $\omega \gg \omega_0 \Rightarrow x_0 \rightarrow 0$

- Fazni zamik: (odvisno od medija) voda / glicerina
- pri majhnih ω je nihanje usklajeno
- ob povečanju frekvence pride do zamika za pol nihanja
- ob zelo visokih frekvencah ~~amplituda~~ amplituda pade na 0

Sklopjeno nihanje

- Dva nihali v interakciji

2



$$M = J \cdot \ddot{\varphi} \quad \alpha \equiv \ddot{\varphi}$$

$$\begin{aligned} J \ddot{\varphi}_1 &= -mgl \sin \varphi_1 + kx b \cos \varphi_1 \\ J \ddot{\varphi}_2 &= -mgl \sin \varphi_2 - kx b \cos \varphi_2 \end{aligned}$$

raztezek vmeti
sila vmeti

če je $\varphi_1 \ll 1$ in $\varphi_2 \ll 1$

$$\begin{aligned} \sin \varphi_1 &\approx \varphi_1 & \cos \varphi_1 &\approx 1 \\ \sin \varphi_2 &\approx \varphi_2 & \cos \varphi_2 &\approx 1 \end{aligned}$$

$$\Delta x = b \sin \varphi_2 - b \sin \varphi_1 \approx b (\varphi_2 - \varphi_1)$$

$$\begin{aligned} \sin \varphi &\approx \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \dots \\ \cos \varphi &\approx 1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \dots \end{aligned}$$

ročka vmeti

$$J \ddot{\varphi}_1 = -mgl \varphi_1 + kb^2 (\varphi_2 - \varphi_1) / J; \quad J \ddot{\varphi}_2 = -mgl \varphi_2 - kb^2 (\varphi_2 - \varphi_1) / J$$

$$\omega_0^2 = \frac{mgl}{J}$$

$$D = \frac{kb^2}{J} \Rightarrow \text{sklopitvena konstanta}$$

$$\ddot{\varphi}_1 = -\omega_0^2 \varphi_1 + D (\varphi_2 - \varphi_1); \quad \ddot{\varphi}_2 = -\omega_0^2 \varphi_2 - D (\varphi_2 - \varphi_1)$$

$$\omega_1 = \omega_0; \quad \omega_2 = \sqrt{\omega_0^2 + 2D}$$

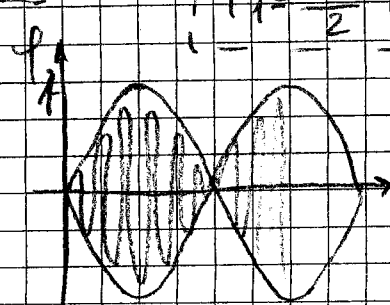
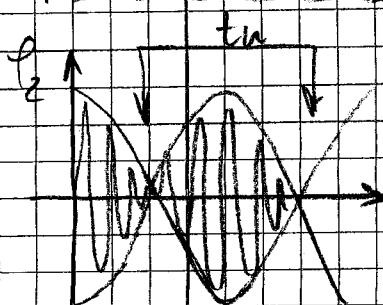
$$\varphi_1 = \varphi_0; \quad \varphi_2 = \varphi_0$$

$$\varphi_2 + \varphi_1 = \varphi_0 \cos \omega_1 t$$

$$\varphi_2 - \varphi_1 = \varphi_0 \cos \omega_2 t$$

$$\varphi_2 = \frac{\varphi_0}{2} (\cos \omega_1 t + \cos \omega_2 t)$$

$$\varphi_1 = \frac{\varphi_0}{2} (\cos \omega_1 t - \cos \omega_2 t)$$



UTRIPANJE

$$t_n \equiv \text{čas utripanja}$$

$$\frac{\omega_2 - \omega_1}{2} t_1 = \pi$$

$$x = x_0 \sin(\omega_1 t)$$

$$y = y_0 \sin(\omega_2 t - \delta)$$

$$\delta = 0$$

$$\delta = \frac{\pi}{4}$$

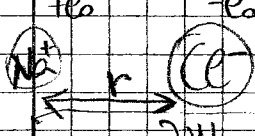
$$\delta = \frac{\pi}{2}$$

$$\delta = \frac{3\pi}{4}$$

$$\delta = \pi$$

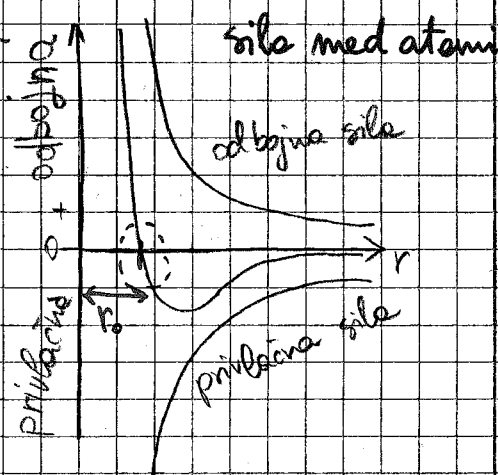
MEHANIKA TRDNIH TELES (DEFORMACIJA TRDNIH TELES)

$+e_0$ $-e_0$ - natezna deformacija

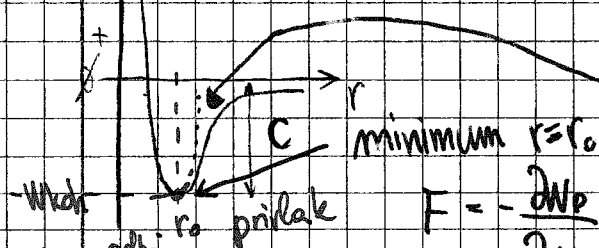


$$F = \frac{\partial W_p}{\partial r} = \frac{(e_0)(-e_0)}{4\pi\epsilon_0 r^2}$$

$$W_p = -\frac{e_0^2}{4\pi\epsilon_0 r}$$



potencialna energija



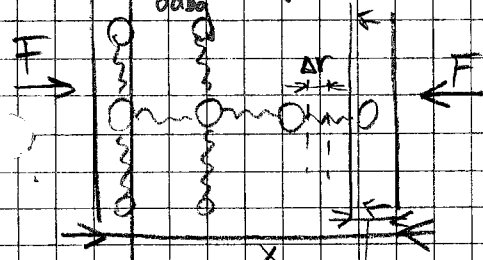
V okolici minimuma:

$$W_p = \frac{1}{2}k(r-r_0)^2 - C$$

$$\tilde{F} = -\frac{dW_p}{dr} = 2 \cdot \frac{1}{2}k(r-r_0) = k(r-r_0)$$

$$\tilde{F} = -k\Delta r$$

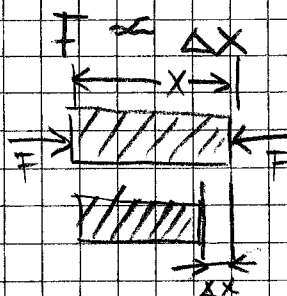
$$F = -\frac{\partial W_p}{\partial r} = 0$$



prožnostni modul (Youngov modul)

$$\frac{F}{S} = E \frac{\Delta x}{x}$$

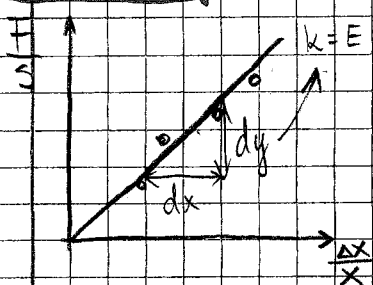
HOOKEV ZAKON



$$F \propto \Delta x$$

$$\frac{F}{S} \propto \left(\frac{\Delta x}{x}\right)$$

relativni raztezek



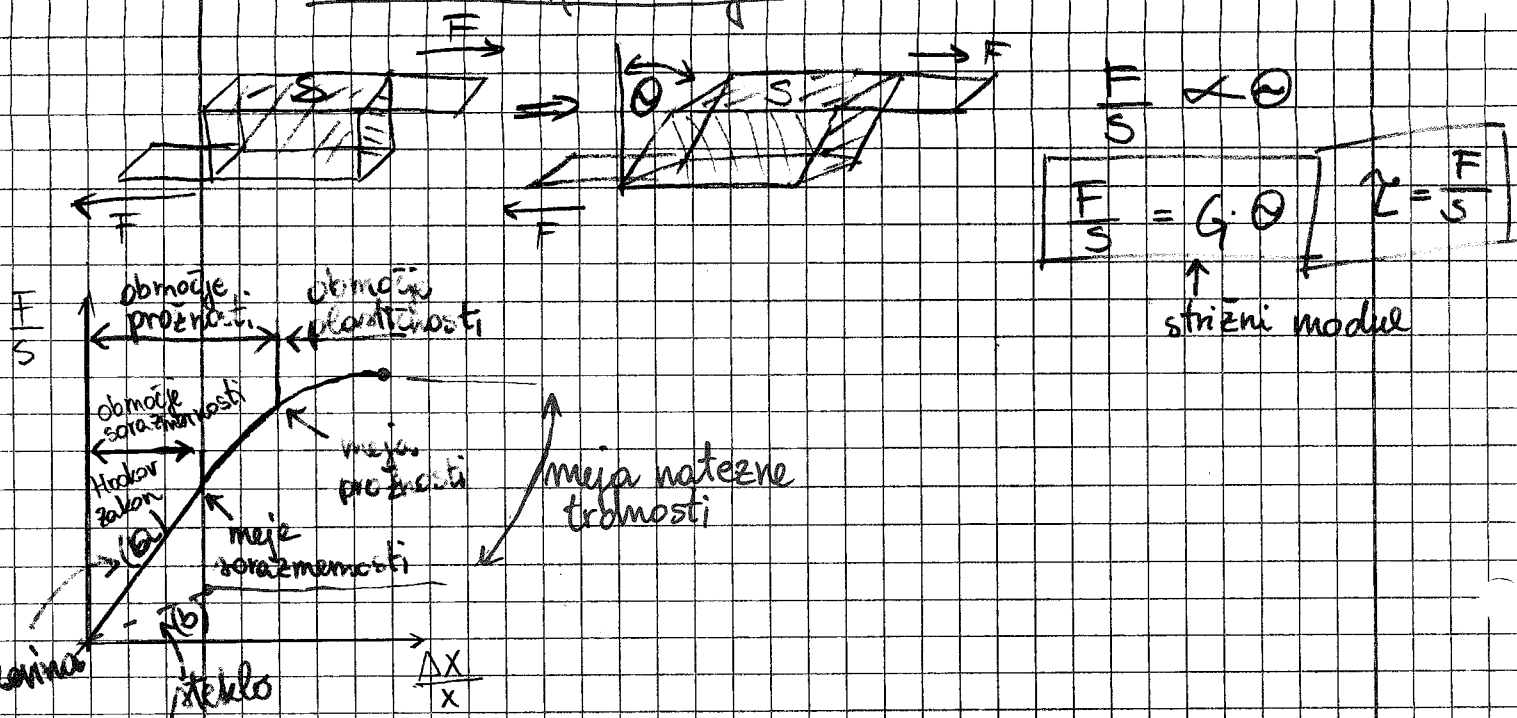
metoda najmanjših kvadratov; prilagoditev premice glede na točke

$$\sum \left(\underbrace{\left(\frac{\Delta x}{x}\right)}_{\text{izmerjeno}} - \underbrace{E \frac{\Delta x}{x}}_{\text{izračunano}} \right)^2$$

$$\sigma = \frac{F}{S} = \text{napetost (stress)}$$

$$\frac{\Delta x}{x} = \text{rel. raztezek (skrček, strain)}$$

tlak je ~~le~~ v primeru pravokotnosti na sile na podlagi.
- strižna deformacija



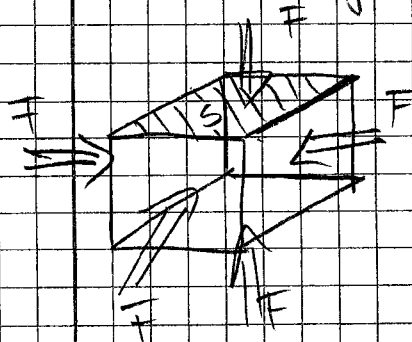
$$\frac{F}{S} = G \cdot \theta \quad \left[\tau = \frac{F}{S} \right]$$

↑
strižni modul

- ~~Mechanika~~

Mechanika kontinuumov

Vsestransko stiskanje



$$p = \frac{F}{S}$$

sprememba volumna

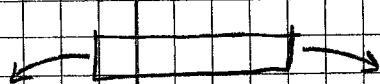
$$\frac{\Delta V}{V} \propto p$$

stisljivost

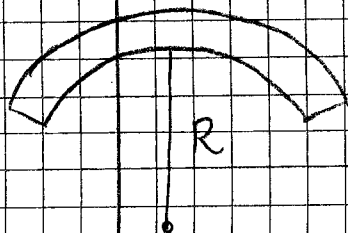
$$\frac{\Delta V}{V} = -\chi p$$

$$\frac{1}{\chi} \rightarrow \text{stisljivostni modul}$$

upogib:



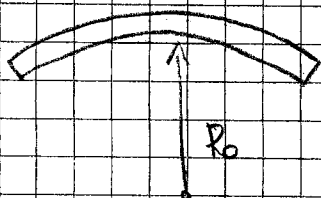
neutralna ravnina (ne se raztegne)



$$C = \frac{1}{R} \leftarrow \text{ukrivljenost}$$

upogibna energija $\propto \left(\frac{1}{R}\right)^2$

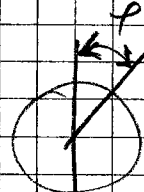
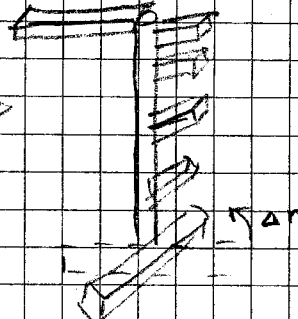
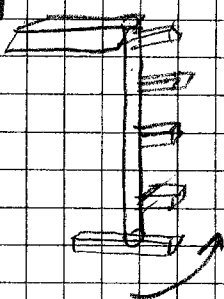
Če ima palica spontano ukrivljenost $C_0 = \frac{1}{R_0}$



upogibna energija $\propto \left(\frac{1}{R} - \frac{1}{R_0}\right)^2$

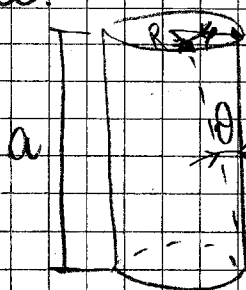
- torzija

torzija žice:



$$M = D \phi$$

torzija palice:



strižni modul

$$M = \left[\frac{\pi G R^4}{2a} \right] \phi$$

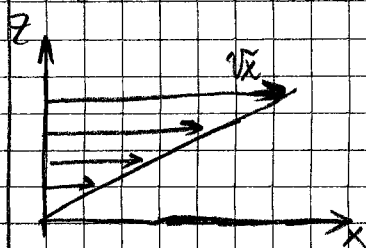
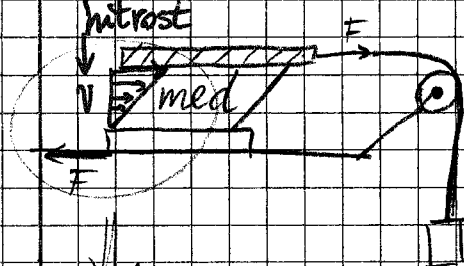
$D \rightarrow$ računa konstanta

KAPLJEVINE

Snovi, ki tvorijo kaplje (voda)

Tekočine so snovi, ki tečejo (tudi plin, etc.)

Mehanika tekočin

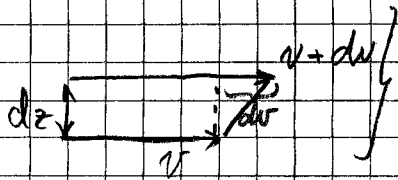


$$\frac{F}{S} \propto \frac{v_x}{z}$$

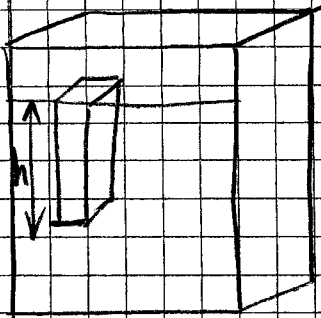
$$\frac{F}{S} = \eta \frac{v_x}{z}$$

Zakon o viskoznosti

viskoznost

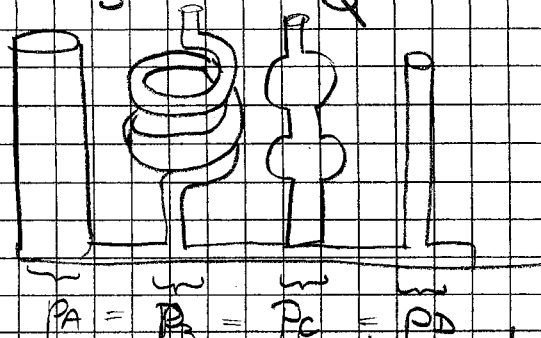


$$\frac{F}{S} = \eta \frac{dv}{dz}$$

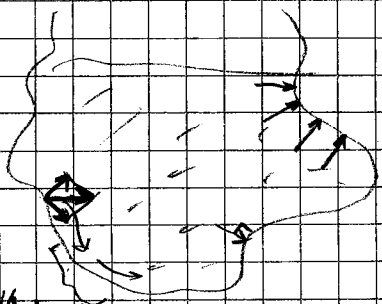


gostota ρ

$$p = \frac{mg}{S} = \frac{S \cdot h \cdot \rho \cdot g}{S} = \rho g h$$



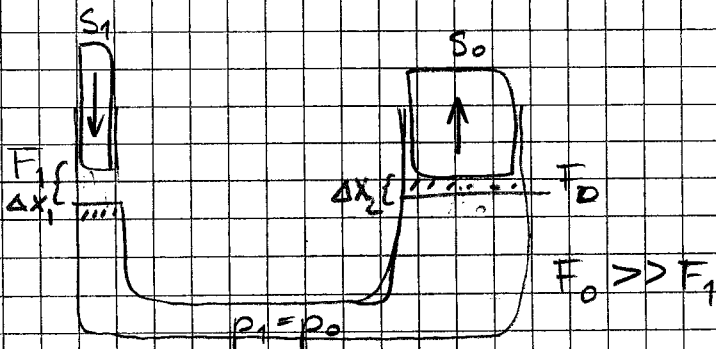
Tlak je na določeni globini v vseh smereh enak, je neodvisen od smeri.
Smeri ima vedno pravokotno na steno posode.



$$\frac{F}{S} = \rho \frac{dv}{dz}$$

Če sila ni 1 tokovi

v primeru stržne sile: ker tokovi ni tudi stržne sile ni, zato je sila na steno vedno $\propto$ smeri normale in narašča linearno v smeri globine



Pascalovo načelo

$$S_1 \Delta x_1 = S_2 \Delta x_2 \quad \left. \begin{array}{l} \text{zaradi} \\ \text{nestisljivosti} \\ \text{vode} \end{array} \right\}$$

Ker je voda skoraj nestisljiva. $p_1 = p_2$

$$\frac{F_1}{S_1} = \frac{F_2}{S_2} \quad \left. \begin{array}{l} \text{zaradi enakosti} \\ \text{tlakov} \end{array} \right\}$$

Delo sile:

$$F_1 : A_1 = F_1 \cdot \Delta x_1$$

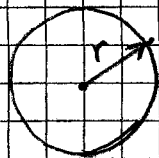
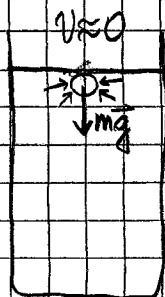
$$F_1 = \frac{S_1}{S_2} F_2$$

$$A = \frac{S_1}{S_2} \cdot F_2 \cdot \frac{1}{S_1} \cdot \Delta x_1 \quad \Delta x_1 = \frac{S_2}{S_1} \Delta x_2$$

$$A = F_2 \cdot \Delta x_2$$

Delo, ki ga vlečimo tudi odloži

Ar



Stokesov zakon

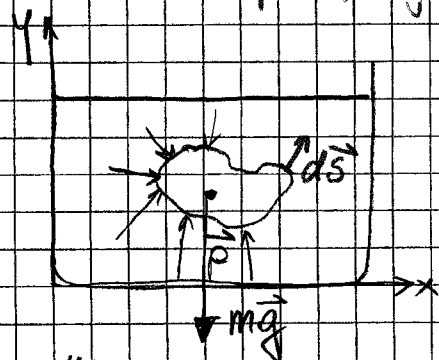
viskoznost

$$F_{\text{upora}} = 6\pi r \eta v$$

polmer kroglje

hitrost kroglje

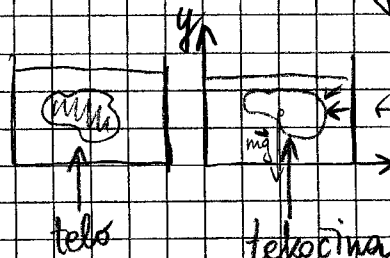
Sila tekočine na potopljeno telo: sila vzgona



$$\int_S p \cdot d\vec{S} = ?$$

$$p = \rho g h$$

vektor površine; vedno kaže v omenjeni normalki



$$F_1 = F_2$$

$$\int p d\vec{S} - m\vec{g} = \vec{0}$$

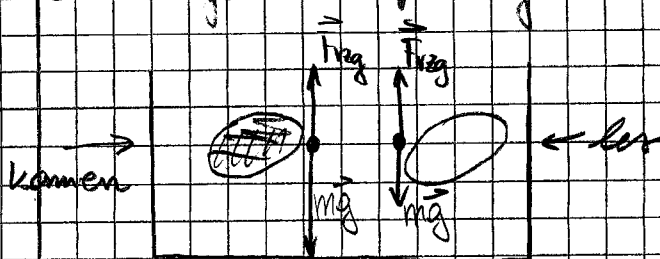
$$\int p d\vec{S} = m\vec{g}$$

maha tekočine

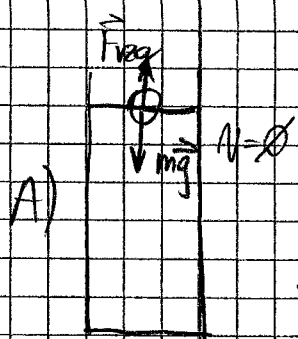
sila vzgona

$$\int p d\vec{S} = \rho_{\text{tek.}} V_{\text{tela}} \vec{g}$$

enaka je masi izpodrinjene tekočine



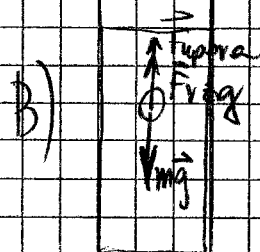
$\rho_{\text{kamena}} > \rho_{\text{vode}} \rightarrow$ telo potone
 $\rho_{\text{kora}} < \rho_{\text{vode}} \rightarrow$ telo plavne



ob času $t=0$

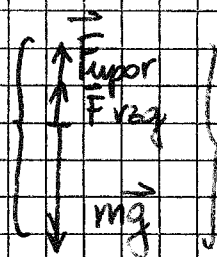
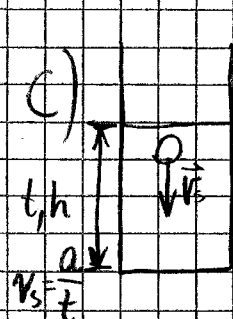
$$\left. \begin{aligned} \vec{m}\vec{g} - \vec{F}_{vzg} &> \vec{0} \\ \vec{m}\vec{g} - \vec{F}_{vzg} &= m\vec{a} \end{aligned} \right\} \begin{aligned} \vec{a} &> \vec{0} \\ \vec{v} &= \int \vec{a} dt \end{aligned}$$

ob času $t > 0$



dokler je $\vec{m}\vec{g} - \vec{F}_{vzg} - \vec{F}_{vzgora} > \vec{0}$

je $\vec{a} > \vec{0}$ in hitrost narašča

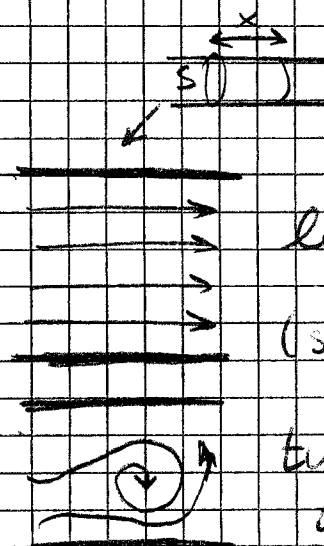
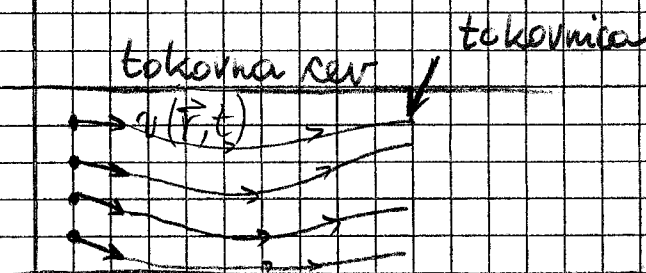


$$m\vec{g} - \vec{F}_{vzg} - \vec{F}_{vzgora} = \vec{0}$$

$$\Rightarrow \vec{a} = \vec{0} \Rightarrow \vec{v} = \vec{v}_s = \text{konst.}$$

$$m\vec{g} - \vec{F}_{vzg} - 6\pi\eta r\vec{v} = m \frac{dv}{dt} \Rightarrow \underline{v = v(t)}$$

hidrodinamika

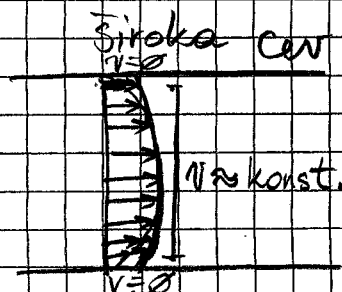
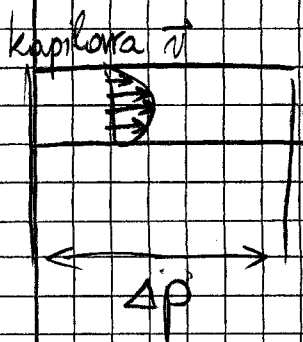


$$\phi_v = \frac{dV}{dt}$$

$$\phi_v = \frac{d(x \cdot \vec{v})}{dt} = \int \frac{d\vec{x}}{dt} \cdot \vec{v}$$

$$\boxed{\phi_v = \int \vec{v} \cdot d\vec{S}}$$

prizor, vseh
presekih hitrosti
je / pretekla celota



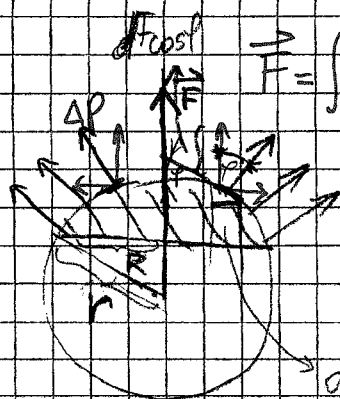
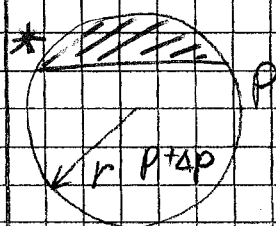
Če imamo neke realne
tekočine, kot je voda, ki
teče v kapilarni cevi, se
obnaša drugače, kot v široki cevi.

efekt kot pri rakah:

za plavanje prsti teku se premaknejo k
sredini cevi, kjer je hitrost največja.

V nasprotnem primeru se premaknejo na
steno cevi, kjer je hitrost najmanjša.

Bernoullijeva enačba



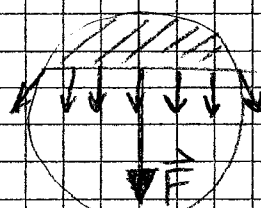
$$\vec{F} = \int d\vec{F} = \int \Delta p d\vec{S}$$

x komponenti se vedno odštejeta

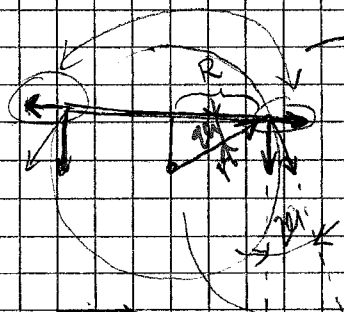
$$F = \int \Delta p dS \cos \theta$$

seštevanje vertikalne komponente

$$= \int \Delta p dS' = \boxed{\Delta p \pi R^2} *$$

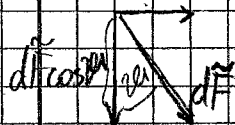


sila zaradi površinske napetosti

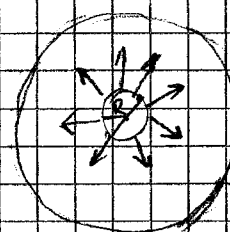


x komponente se odštejeta.

$$\cos \theta = \frac{R}{r}$$



TLOČIS



$$62 \pi R \cos \theta$$

Vravnovesje:

$$\Delta p \pi R^2 = 62 \pi R \cos \theta$$

$$2\sigma \cos \theta = \Delta p R$$

$$2\sigma \frac{R}{r} = \Delta p R$$

$$\Delta p = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

$$\Delta p = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

Laplace-ov zakon

krivinski radij

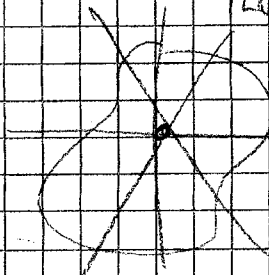
ravnina

ploskev

razlika tlakov je obratno sorazmerna z radijem.

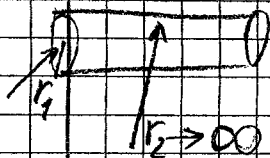
r... krivinski radij

TLOČIS



vertikalna ravnina

valjasta površina:



$C_1 = \frac{1}{r_1}$ → glavni ukrivljenosti
 $C_2 = \frac{1}{r_2}$ → v tem primeru je 0

kroglasta površina:

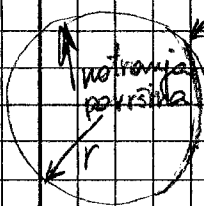
$r_1 = r_2 = r$
 $C_1 = C_2 = \frac{1}{r}$

$H = \frac{1}{2} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) = \frac{1}{2} (C_1 + C_2)$

↑
medija ukrivljenost

$\Delta p = 2\sigma H$

MEHURČEK

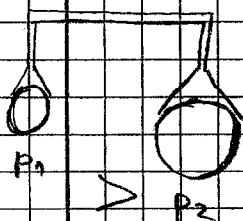


↑
zunanja površina

$\Delta p = \frac{4\sigma}{r}$

x2, ker sta 2 površini

POSKUS

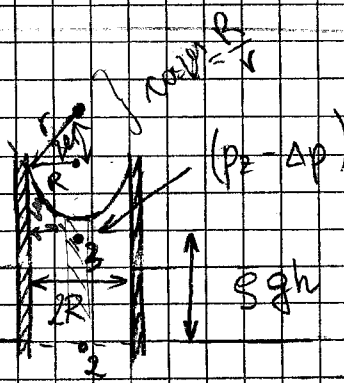


manjši mehurček se manjša, večji pa večja

KAPILARNI DVIG



↑
zunanji
stačni
tlak



v točki 3
 $p_z - \Delta p + \rho g h = p_z$

v točki 2

$\Delta p = \rho g h$

$\Delta p = \frac{2\sigma}{r}$

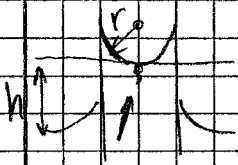
$$\rho g h = \frac{2\sigma}{r}$$

$$r = \frac{R}{\cos \varphi}$$

$$h = \frac{2\sigma}{\rho g R}$$

$$h = \frac{2\sigma \cdot \cos \varphi}{\rho g \cdot R}$$

enakba opisuje razmere na medlini kapilare



$$\cos \varphi > 0$$

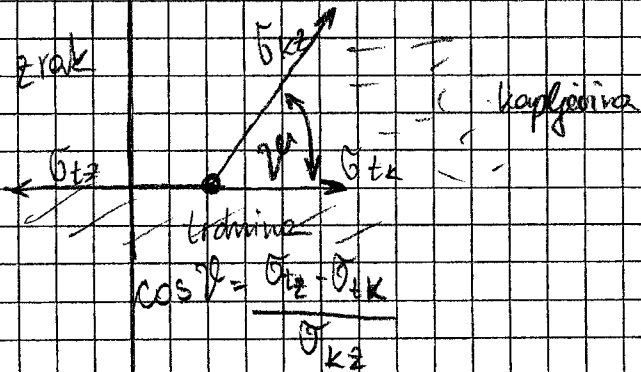
moči

moči



$$\cos \varphi < 0 \Rightarrow h < 0$$

ne moči



$$\cos \varphi = \frac{\sigma_{kz1} - \sigma_{kz2}}{\sigma_{kz}}$$

- moči: $\varphi < 90^\circ$; $\cos \varphi > 0$ ($\sigma_{kz2} > \sigma_{kz1}$)

- ne moči: $\varphi > 90^\circ$; $\cos \varphi < 0$ ($\sigma_{kz1} > \sigma_{kz2}$)

$$\sigma_{kz} = \sigma_{kz1} + \sigma_{kz2} \cos \varphi$$

TERMODINAMIKA

termodinamske spremenljivke:

T ... temperatura
 V ... volumen
 p ... pritisk

$$T \sim \langle v^2 \rangle$$

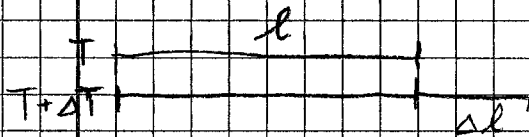
v plinu:

$$p = p(T, V)$$

enacba stanja - povezuje TD spremenljivke.

TEMPERATURNO RAZTEŽANJE SNovi

1. Linearno (če spustimo tok skozi žico, se raztegne)

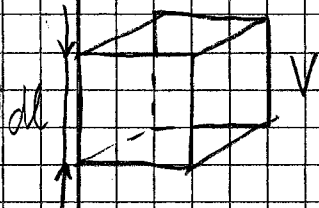


$$\frac{\Delta l}{l} \propto \Delta T$$

koefficient linearnega raztežka

$$\frac{\Delta l}{l} = \alpha \Delta T$$

$$\frac{dl}{l} = \alpha dt$$



$$\frac{\Delta V}{V} = \beta \Delta T$$

$$\frac{dV}{V} = \beta dT$$

$$dV = (l + dl)^3 - l^3$$

10. 12. 2010

$$\frac{dV}{V} = \frac{(l + dl)^3 - l^3}{l^3} = \frac{l^3 + 3l^2 dl + 3l dl^2 + dl^3 - l^3}{l^3} = \frac{3l^2 dl}{l^3} = \frac{3 dl}{l}$$

$$\frac{dV}{V} = \frac{3 dl}{l} = \frac{3 \alpha \Delta T}{1} \Rightarrow \beta = 3\alpha$$

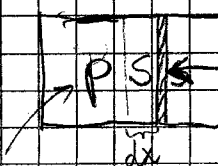
1. zakon termodinamike

$$\Delta W_n = A + Q$$

spenecba notranje energije

delo toplota

W_n = vsota vseh molekul, ki sestavljajo sistem



$$F = pS$$

$$dA = F dx = p \cdot \frac{dV}{S} \cdot S = p dV$$

$$A = - \int p dV$$

opravlja delo ← raztezanje: $dV > 0 \Rightarrow A < 0$

prejema delo ← stiskanje: $dV < 0 \Rightarrow A > 0$

$$Q [J]$$

predznak je zaradi sistema (prejemanje / opravljanje)

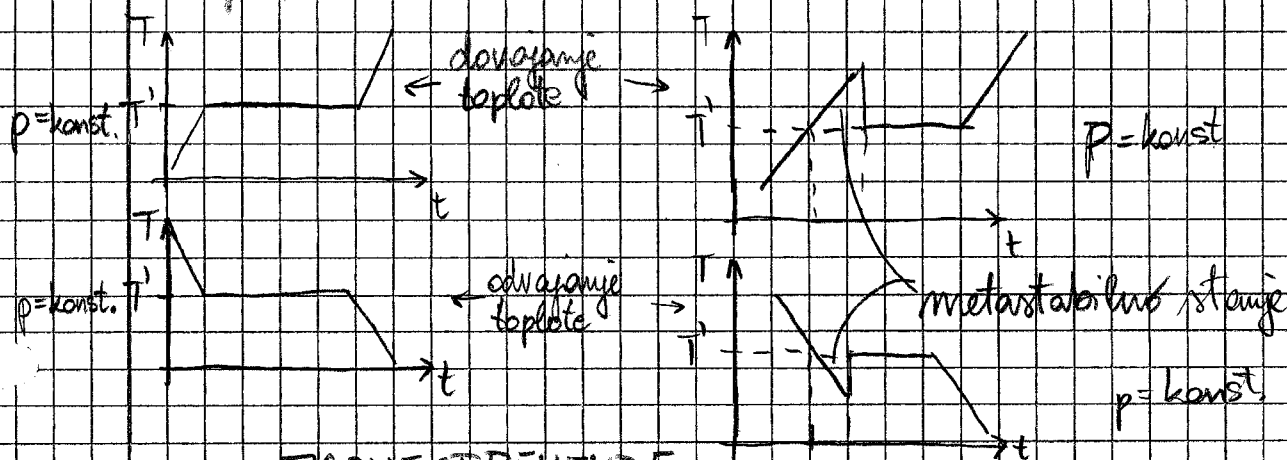
fazno stanje je odvisno od P, V in T .

izoterma - krivulja s stalno T
 izohora - p
 izobara - V

Fazni prehod - iz plinastega v tekoče stanje

Teočine so odvisne od stanja zato pri manjšanju volumna zelo naraste tlak.

Če odvajamo temp. pri stalnem tlaku; led se segreje do $0^\circ C$, led se tali, ko se stopi vsa led ~~se~~ se vedno dovajamo toplote in temp. naraste.



FAZNE SPREMEMBE

Specifična talilna toplota
 izparilna

Q_i

$$Q_t = q_t \cdot m$$

$$Q_i = q_i \cdot m$$

SEGREVANJE / OHLAJANJE

$$V = \text{konst.} \quad dW_n = dQ - p dV$$

$\Downarrow$

$dQ \Rightarrow$ merimo, ker ni f(x)

$$dV = 0$$

$$dW_n = dQ = c_m \cdot m \cdot dT$$

specifična toplota pri konst. V

$$p = \text{konst.}$$

$$H = W_n + pV$$

entalpija $dH = dW_n + dpV + p dV$

$$dp = 0$$

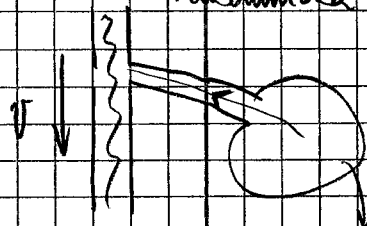
$$dH = dA = p dV + dQ + dpV + p dV$$

$$dH = dQ + V dp$$

$$dH = dQ = c_p m dT$$

specifična
toplota pri
konst. p.

Vakuumsko črpalno



manjšaj
podtlak

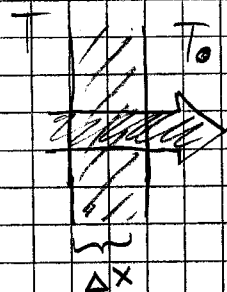
$$\rho + \frac{1}{2} \rho v^2 + \rho g h = \text{konst.}$$

če je večja hitrost je manjši tlak

tlak se je znižal, voda je zavrela pri 65°C .

PREVADANJE TOPLOTE

$$j = -\lambda \left[\frac{T_0 - T}{\Delta x} \right]$$



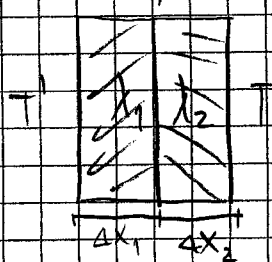
$$T_0 < T$$

toplota teče v področja z nižjo ~~temperaturo~~
visjo T na področje z nižjo T.

Toplotni tok: $P = \frac{dQ}{dt}$

gostota topl. toka: $j = \frac{dP}{dS}$

dve steni



$$R = R_1 + R_2$$

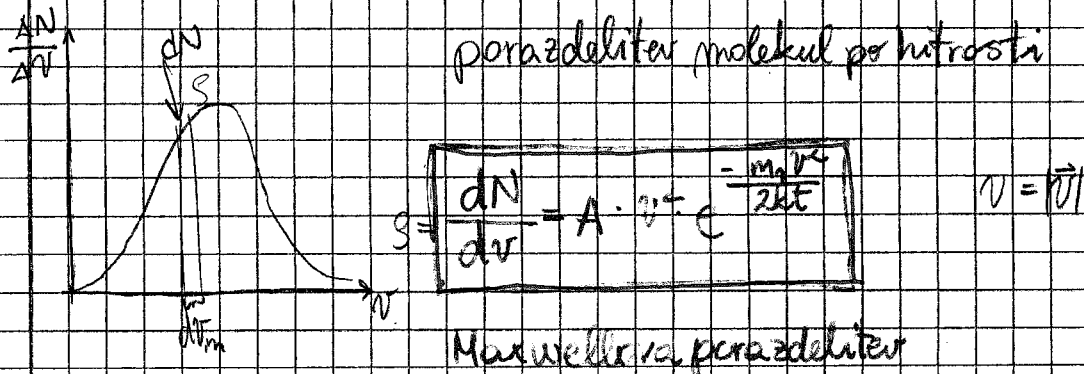
KINETIČNA TEORIJA PLINOV

- statistična fizika

poskusi:

Model plina ypesodi: Kroglice tresemo, skozi rezo padajo do različno oddaljenih reš.

Ponazorimo gibanje molekul v plinu; hitrost ni enaka, petela po neki funkciji



m_1 ... masa ene molekule

$\frac{m_1 v^2}{2}$... kinetična energija ene molekule

k ... Boltzmannova konstanta $\Rightarrow k = 1,38 \cdot 10^{-23} \text{ J/K}$

13.12.2010

$$\frac{dg}{dv} = 2Ave^{-\frac{m_1 v^2}{2kT}} - 2Av_m^2 \frac{m_1 v_m}{2kT} e^{-\frac{m_1 v_m^2}{2kT}} = 0$$

$$2Ave^{-\frac{m_1 v^2}{2kT}} \left(1 - v_m^2 \frac{m_1}{2kT} \right) = 0$$

maximalna hitrost

$$v_m = \left(\frac{2kT}{m_1} \right)^{1/2}$$

$$\langle v^2 \rangle = \int_0^{\infty} v^2 g dv$$

$$v_m^2 \sim kT$$

$$\int_0^{\infty} g dv = 1$$

$$A = \left(\frac{m_1}{2kT} \right)^{3/2} \frac{4}{\pi^{1/2}}$$

$$\langle v^2 \rangle = \frac{3kT}{m_1}$$

porpčni kvadrat velikosti hitrosti

poz. hitrost molekul v plinu = 0

$$\langle \vec{v} \rangle = 0$$

$$\vec{v} = (v_x, v_y, v_z)$$

$$v^2 = (v_x^2 + v_y^2 + v_z^2) / < >$$

$$<v^2> = <v_x^2> + <v_y^2> + <v_z^2> \quad \frac{1}{2} m_1$$

$$\frac{1}{2} m_1 <v^2> = \frac{1}{2} m_1 <v_x^2> + \frac{1}{2} m_1 <v_y^2> + \frac{1}{2} m_1 <v_z^2> = \frac{3}{2} kT$$

povprečna
translacijska
kinetična
energija

$$\frac{1}{2} kT$$

$$\frac{1}{2} kT$$

$$\frac{1}{2} kT$$

$$W_k = \underbrace{\frac{1}{2} m v^2}_{\text{trans. } W_k} + W_{\text{rot}}$$

Notranja energija idealnega plina:

plin, ki je razredčen do take mere, da so molekule na takem velikem razdalji da zanemarimo interakcijske energije med njimi

$$\rightarrow <W_k> = \frac{1}{2} m_1 <v^2> + W_{\text{rot}} = \left(\frac{1}{2} m_1 <v_x^2> + \left(\frac{1}{2} m_1 <v_y^2> + \left(\frac{1}{2} m_1 <v_z^2> \right) \right) = \frac{3}{2} kT$$

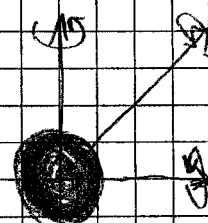
1. enoatomni molekuli plina:

$$<W_{\text{rot}}> = 0$$

$$J = \int r^2 dm \rightarrow 0$$

večkratnostni
moment

pri enoatomnih plinih
upostevamo trans. W_k .



Avogadrova
število

$$k \cdot N_A = R = 8314 \frac{\text{J}}{\text{kmol}}$$

Notranja energija:

$$W_N = N <W_k> = N \left(\frac{3}{2} kT \right) = \underbrace{\frac{3}{2}}_{\substack{\text{st. molekul} \\ m}} \cdot \underbrace{N}_{\substack{\text{masa plina} \\ M}} \cdot \underbrace{k}_{\substack{\text{Avogadrova} \\ \text{število}}} \cdot \underbrace{N_A}_{\substack{\text{mol} \\ M}} \cdot T = \frac{3}{2} \frac{mRT}{M}$$

splošnem

specifična gostota pri $V = \text{konst.}$

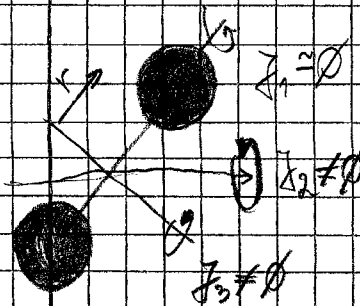
$$V = \text{konst.} : \Delta W_N = Q = C_v \cdot m \cdot \Delta T$$

$$C_v = \frac{2}{3} \frac{R}{M}$$

$$\Delta W_N = m C_v \Delta T$$

enoatomni plin, W_N
vedno velja po formuli

2. dvubatomne molekule plina:



$$\langle W_k \rangle = \langle W_{\text{trans}} \rangle + \langle W_{\text{rot}} \rangle = \underbrace{\left\langle \frac{1}{2} m_1 v_x^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} m_1 v_y^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} m_1 v_z^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} I_2 \omega_2^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} I_3 \omega_3^2 \right\rangle}_{\frac{1}{2} kT} = \frac{5}{2} kT$$

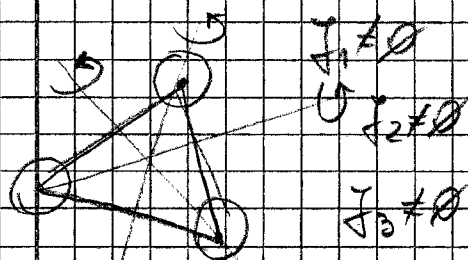
$$W_n = N \langle W_k \rangle = N \frac{5}{2} kT = \frac{5 N m_1 k N_A T}{2 m_1 N_A} = \frac{5}{2} n R T = m C_v T$$

$$C_v = \frac{5}{2} \frac{R}{M}$$

$$\Delta W_n = m C_v \Delta T$$

2 atomni plin

3. triatomni idealni plin:



$$\langle W_k \rangle = \underbrace{\left\langle \frac{1}{2} m_1 v_x^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} m_1 v_y^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} m_1 v_z^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} I_1 \omega_1^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} I_2 \omega_2^2 \right\rangle}_{\frac{1}{2} kT} + \underbrace{\left\langle \frac{1}{2} I_3 \omega_3^2 \right\rangle}_{\frac{1}{2} kT}$$

$$\langle W_k \rangle = \frac{6}{2} kT$$

$$W_n = N \langle W_k \rangle = N \frac{6}{2} kT = m \frac{6}{2} \frac{R}{M} T ; C_v = \frac{6}{2} \frac{R}{M}$$

	St. atomov	C_v	C_p	$\gamma = \frac{C_p}{C_v}$
1.)	1	$\frac{3}{2} \frac{R}{M}$	$\frac{5}{2} \frac{R}{M}$	1,66
2.)	2	$\frac{5}{2} \frac{R}{M}$	$\frac{7}{2} \frac{R}{M}$	1,4
3.)	3	$\frac{6}{2} \frac{R}{M}$	$\frac{8}{2} \frac{R}{M}$	1,33

zveza med C_v in C_p :

za id. plin $p, V_1, T_1 \rightarrow p, V_2, T_2$ ($p = \text{konst.}$) $\Rightarrow$ izobarna sprememba

$$\Delta W_M = A + Q \quad \leftarrow \quad - \frac{m}{M} R (T_2 - T_1)$$
$$C_v \cdot m (T_2 - T_1) = m \cdot C_p (T_2 - T_1) - p(V_2 - V_1) =$$

$$C_v \cancel{m(T_2 - T_1)} = m \cdot C_p \cancel{(T_2 - T_1)} - \cancel{\frac{m}{M} R (T_2 - T_1)}$$

$$C_v = C_p - \frac{R}{M} \quad \Rightarrow \quad \boxed{C_p = C_v + \frac{R}{M}}$$

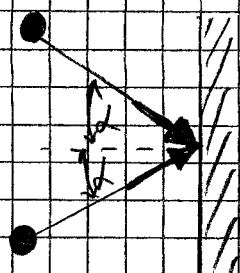
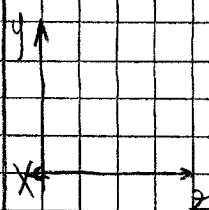
$$1.) \quad \frac{3}{2} \frac{R}{M} + \frac{R}{M} = \frac{5}{2} \frac{R}{M}$$

$$2.) \quad \frac{5}{2} \frac{R}{M} + \frac{R}{M} = \frac{7}{2} \frac{R}{M}$$

$$3.) \quad \frac{6}{2} \frac{R}{M} + \frac{R}{M} = \frac{8}{2} \frac{R}{M}$$

Kinetična energija teorija plinov (idealni plin)

- lastnosti molekul $\Rightarrow$ lastnosti plinov
- prožentrik molekule so steno:



stena je "gladka"

$$\vec{v} = (v_x, v_y, v_z)$$

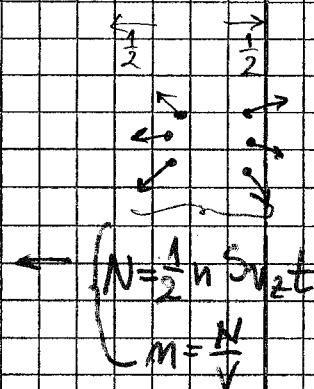
m_1 = masa molekule

$$\int F_z dt = -m_1 v_z - m_1 v_z = -2m_1 v_z$$

skupni umek
sil stene na
molekule

$$N \cdot \int F_z dt = -2m_1 v_z \cdot N$$

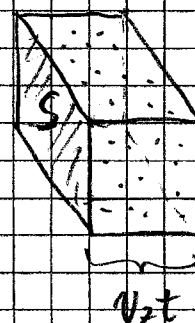
$$p = \frac{-N \int F_z dt}{S \cdot t} = 2m_1 v_z \cdot \frac{N}{S \cdot t}$$



$$p = n m_1 \langle v_z^2 \rangle$$

predpostavka:

hitrosti vseh
molekul
so enake.



$$\frac{1}{2} m_1 \langle v_z^2 \rangle = \frac{1}{2} kT \Rightarrow \langle v_z^2 \rangle = \frac{kT}{m_1}$$

$$p = n m_1 \cdot \frac{kT}{m_1} = n \cdot kT, \quad n = \frac{N}{V}$$

$$p = \frac{N}{V} kT \leftarrow \text{plinska enačba, ki velja za idealni plin.}$$

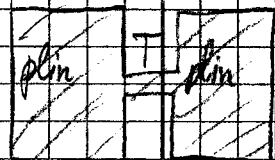
$$p = \frac{N m_1 k N_A T}{V m_1 N_A}$$

$$p = \frac{m}{\mu} \frac{R}{T}$$



$$\Delta W_h = A + Q = 0$$

$$A = 0$$



$$Q = 0 \quad (\text{Adiabatna sprememba})$$

$$\Delta W_h = C_v m \Delta T = 0 \Rightarrow \Delta T = 0$$

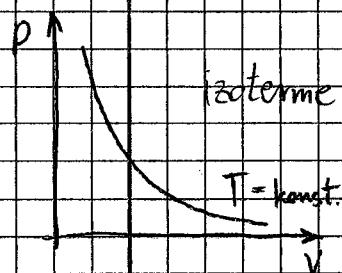
sprememba je prehitra, tako da se ne izmenja nič toplote - ADIABATNA sprememba.
Enako se zgodi, če je sistem zelo dobro izoliran.

- plinska enačba za idealni plin:

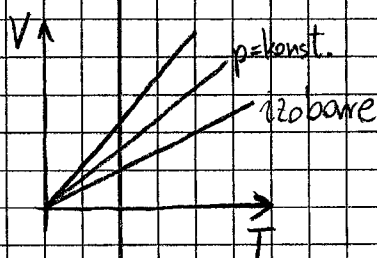
$$pV = \frac{m}{M} RT$$

- posebni primeri:

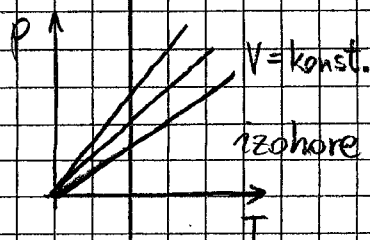
$$1) T = \text{konst.} \Rightarrow pV = \text{konst.} \Rightarrow p \propto \frac{1}{V}$$



$$2) p = \text{konst.} \Rightarrow V \propto T$$



$$3) V = \text{konst.} \Rightarrow p \propto T$$

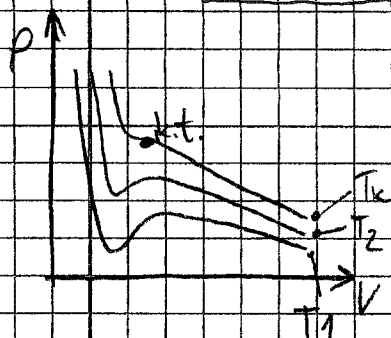


realni plini $\rightarrow$ van der Waalsova enačba
(kapilarnost)

$$\left(p + \frac{A}{V^2}\right)(V-B) = \left(\frac{m}{M}\right)RT$$

$B \propto$ volumen molekul

$A \sim$ privlačne intermolekularne sile
(silijske molekule skupaj podobno kot stene posode)

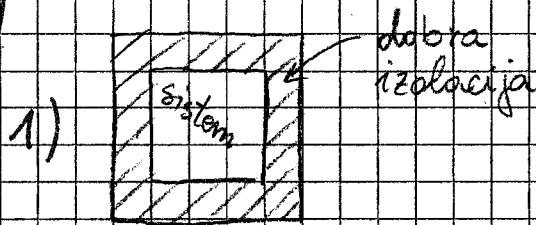


$$A = a \left(\frac{m}{M}\right)^2$$

$$B = \frac{bm}{M}$$

Adiabatske spremembe idealnega plina:

$$[Q=0]$$



2) Sprememba je hitra

$$p, V, T \rightarrow p_1, V_1, T_1$$

$$pV = \frac{m}{M}RT \Rightarrow \frac{pV}{T} = \frac{p_1 V_1}{T_1}$$

$$C_p - C_v = \frac{R}{M}$$

1. Zakon termodinamike:

$$dW_n = \underbrace{dQ}_0 + dA$$

$$dW_n = dA \quad p = \frac{m}{M} R \frac{T}{V}$$

$$C_v m dT = -p dV$$

$$C_v m dT = -\frac{m}{M} R \frac{dT}{V} T$$

$$C_v \int \frac{dT}{T} = -\left(\frac{R}{M}\right) \int \frac{dV}{V}$$

$$\int \frac{dT}{T} = \frac{1}{C_v (C_v - C_p)} \int \frac{dW}{V}$$

$$\int \frac{dT}{T} = \left(1 - \frac{C_p}{C_v}\right) \int \frac{dV}{V}$$

$K = \text{kapac}$

$$\int \frac{dT}{T} = (1 - K) \int \frac{dV}{V}$$

$$\ln T_2 = -(K-1) \ln V_2$$

$$\ln T_2 - \ln T = -(\kappa - 1) [\ln V_2 - \ln V]$$

$$\ln\left(\frac{T_2}{T}\right) = -(\kappa - 1) \ln\left(\frac{V_2}{V}\right)$$

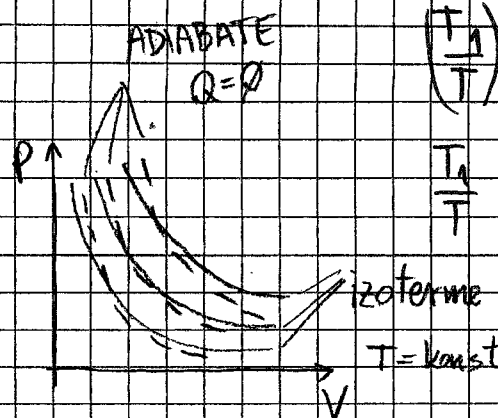
$$\ln\left(\frac{T_2}{T}\right) = \ln\left(\frac{V_2}{V}\right)^{-(\kappa-1)}$$

$$\left(\frac{T_2}{T}\right) = \left(\frac{V_2}{V}\right)^{-(\kappa-1)}$$

$$\frac{T_2}{T} = \left(\frac{V}{V_2}\right)^{\kappa-1}$$

$$T_1 V_1^{\kappa-1} = T V^{\kappa-1}$$

$$T V^{\kappa-1} = \text{konst.}$$



$$p_1 V_1^\kappa = p V^\kappa$$

$$\frac{T_1^\kappa}{p_1^{\kappa-1}} = \frac{T^\kappa}{p^{\kappa-1}}$$

izoterme

$$pV = \text{konst.}$$

$$d(pV) = d(\text{konst.}) = 0$$

$$dpV + p dV = 0$$

$$\left(\frac{dp}{dV}\right) = -\frac{p}{V}$$

$$T = \text{konst.}$$

$$\chi_T = -\frac{1}{V} \left(\frac{dV}{dp}\right) = \frac{1}{p}$$

$$T = \text{konst.}$$

izotermna stisljivost

adiabate:

$$pV^\kappa = \text{konst.}$$

$$d(pV^\kappa) = d(\text{konst.}) = 0$$

$$dpV^\kappa + p\kappa V^{\kappa-1} dV = 0$$

$$\left(\frac{dp}{dV}\right) = \frac{-\kappa p V^{\kappa-1}}{V^\kappa} = -\kappa \frac{p}{V}; \kappa = \frac{C_p}{C_v} > 1$$

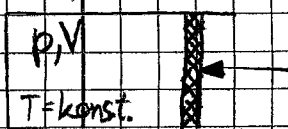
$$Q=0$$

$$\chi_Q = -\frac{1}{V} \left(\frac{dV}{dp}\right) = \frac{1}{p\kappa}$$

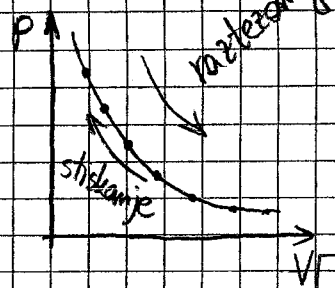
adiabatna stisljivost

$$\chi_Q < \chi_T$$

Delo pri izotermni spremembi:



okoliš



$$pV = \frac{m}{M} RT = \text{konst.}; dT=0$$

id. plin

$$dW_h = dQ + dA$$

$$C_v m dT = dQ + dA = 0$$

izotermna spr.:

- počasna sprememba, stena mora dobro prevajati, toplota v okolici se ne spremeni

$$A = - \int_{V_1}^{V_2} p dV = - \int_{V_1}^{V_2} \frac{1}{V} \frac{m}{M} RT dV = \frac{m}{M} RT \int_{V_2}^{V_1} \frac{dV}{V} = \frac{m}{M} RT \ln\left(\frac{V_1}{V_2}\right)$$

$p = \frac{1}{V} \frac{m}{M} RT$

raščiranje

$$V_2 > V_1$$

$$A < 0$$

$$Q > 0$$

toplota
dovajamo

stiskanje

$$V_2 < V_1$$

$$A > 0$$

$$Q < 0$$

toplota
odvajamo

Delo pri adiabatnih spremembah (za id. plin)

$$Q = 0$$

$$\Delta W_n = A + Q$$

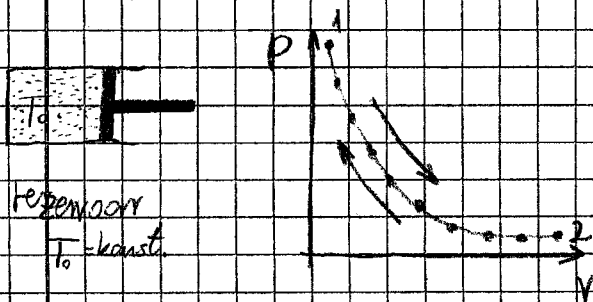
$$c_v m \Delta T = A$$

$$A = c_v m (T_2 - T_1)$$

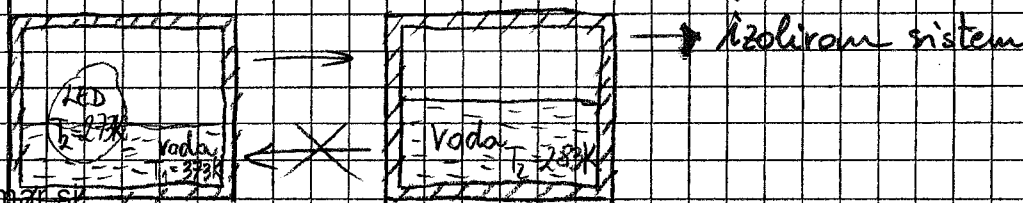
20. 12. 2010

- Reversibilna sprememba:

možna je obrnjena sprememba preko enakih vmesnih stanj kot pri prvotna sprememba
(če pri obeh spremembah ni nobenih trajnih sprememb v bližnji in daljni okolici)



- Irreversibilna sprememba: - ni možna obrnjena sprememba



$$Q = 0$$

ker je sistem izoliran

$\left. \begin{matrix} m_L, T_0 \\ m_V, T_1 \end{matrix} \right\}$ na začetku

$(m_V + m_L), T_2$ na koncu
 zmesna temp.

$$m_L q_{tal} + m_L c_p (T_2 - T_0) + m_V c_p (T_2 - T_1) = 0$$

talilna toplota segrevanje vode, ki
 nastala iz ledu

$c_p = 4188 \frac{J}{kg \cdot K}$
 $q_{tal} = 336 \cdot 10^3 \frac{J}{kg}$

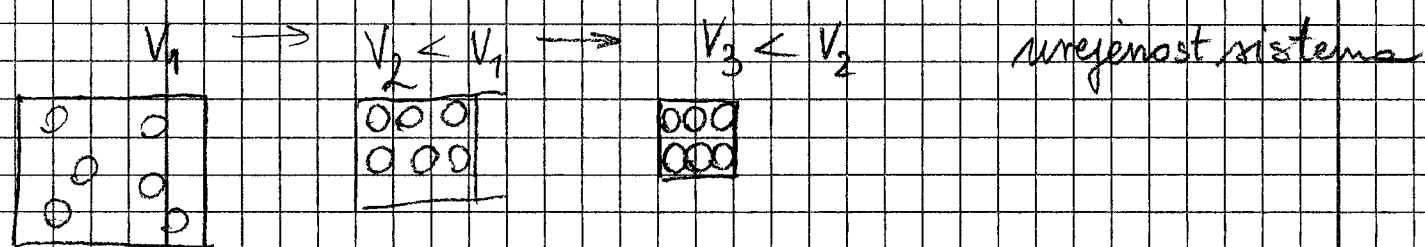
$$m_L q_{tal} + m_L c_p T_2 - m_L c_p T_0 + m_V c_p T_2 - m_V c_p T_1 = 0$$

$$T_2 (m_L c_p + m_V c_p) = c_p (m_L T_0 + m_V T_1) - m_L q_{tal} \Rightarrow T_2 = \frac{c_p (m_L T_0 + m_V T_1) - m_L q_{tal}}{c_p (m_L + m_V)}$$

primer:
 $T_1 = 273K$
 $T_2 = 373K$
 $m_1 = 1kg$
 $m_2 = 1kg$
 $T_3 = ?$

Entropijski zakon (II. zakon TD)

- ni možna krožna sprememba, pri kateri bi sistem prejel toploto iz topl. rezervoarja in oddal enako veliko delo.
- ni možna krožna sprememba, pri kateri bi se prenesla toplota s hladnejšega telesa na toplejše telo brez vloženega dela.



• primer: idealni plin
 izotermno referenčno razpenjanje

$T = konst. \Rightarrow dW = c_m dt = dA + dQ = -p dV + dQ = 0$
 $dQ = p dV$

$$p = \frac{1}{V} \frac{m}{M} RT$$

$$dQ = \frac{1}{V} \frac{m}{M} RT dV$$

$$\frac{dQ}{T} = \frac{dV}{V} = \frac{m}{M} R$$

$$\frac{dW}{V} < \frac{dQ}{T}$$

za reverzibilne spremembe:

$$dS \equiv \frac{dQ}{T}$$

S = entropija

za ireverzibilne spremembe:

$$dS > \frac{dQ}{T}$$

$$\Delta S \geq \int \frac{dQ}{T} \left[\frac{T}{K} \right] \quad \text{(entropijski zakon)}$$

2. zakon TD

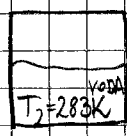
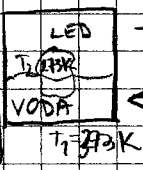
$$S = k \cdot \ln W$$

Boltzmann-ova formula

število načinov konfiguracije molekul

$$\int \frac{dQ}{T} = 0$$

sistem je izoliran



NAD. REVER. SPREMEMBE

$$\Delta S_0 = \frac{m_L \cdot L_{tal}}{T_0} \quad \text{rev. taljenje ledu}$$

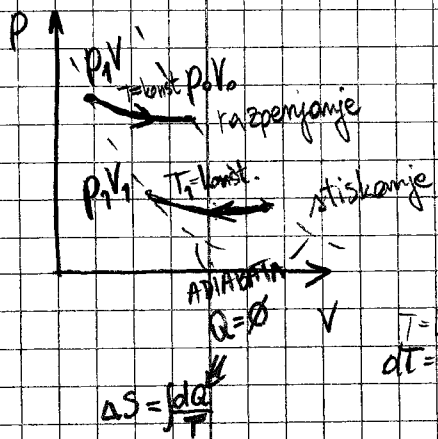
$$\Delta S_1 = \int_{T_0}^{T_2} \frac{m_L \cdot c_p dt}{T} = m_L c_p \ln \frac{T_2}{T_0} \quad \text{rev. segrevanje vode ki nastane iz ledu (m_L)}$$

$$\Delta S_2 = \int_{T_1}^{T_2} \frac{m_V \cdot c_p dt}{T} = m_V c_p \ln \frac{T_2}{T_1} \quad \text{rev. ohlajanje prvotne vode (m_V)}$$

$$\Delta S_0 + \Delta S_1 + \Delta S_2 = \Delta S$$

$$\Delta S > \int \frac{dQ}{T} = 0$$

ireverzibilna sprememba



Carnot-ova krožna sprememba

$$dW_n = c_v m dt = dQ - p dV = 0 = \underline{dQ = p dV}$$

$$\Delta S = \int \frac{dQ}{T} = \int \frac{p dV}{T}$$

$$\frac{p}{T} = \frac{m}{M} R \frac{1}{V}$$

RAZPENJANJE:

$$\Delta S = \frac{m}{M} R \ln \frac{V_0}{V}$$

STISKANJE:

$$\Delta S_1 = \frac{m}{M} R \ln \frac{V_1}{V_2}$$

ker smo na adiabatih:

$$\left. \begin{aligned} T V^{k-1} &= T_1 V_1^{k-1} \\ T V_0^{k-1} &= T_1 V_2^{k-1} \end{aligned} \right\} \frac{V_0}{V} = \frac{V_2}{V_1} \quad T = \text{konst}$$

$$\Delta S = \frac{m}{M} R \ln \frac{V}{V_0} = \left[-\frac{m}{M} R \ln \frac{V_0}{V} \right] = -\Delta S_1$$

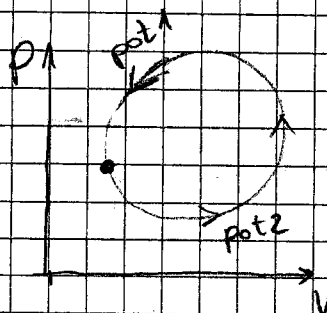
$$\Delta S + \Delta S_1 = 0$$

ENTROPIJA je ENOLICNA funkcija stanja:

$$\oint dS = \oint \frac{dQ}{T} = 0$$

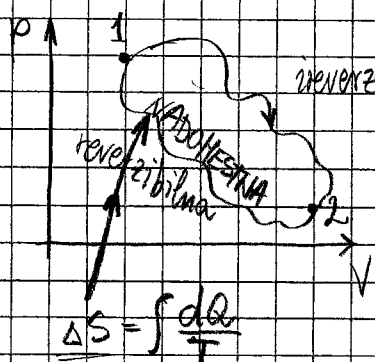
velja za poljubno reverzibilno krožno spremembo.

$$\oint \frac{dQ}{T} = \int_{p_1, T_1}^{p_2, T_2} \frac{dQ}{T} + \int_{p_2, T_2}^{p_1, T_1} \frac{dQ}{T} = 0$$



$$S - S' = \int_{p_1, T_1}^{p_2, T_2} \frac{dQ}{T} = \int_{p_1, T_1}^{p_2, T_2} \frac{dQ}{T}$$

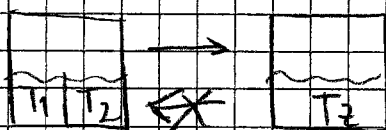
Ce želimo izračunati spremembo entr. pri kakšni spremembi (tudi reverz.) moramo prištati poljubno reverzibilno pot iz istega začetnega v isto končno stanje, po tej poti izračunati integral $\int \frac{dQ}{T}$.



$$\Delta S > \int \frac{dQ}{T}$$

$$\Delta S = S_2 - S_1$$

3.1.2014



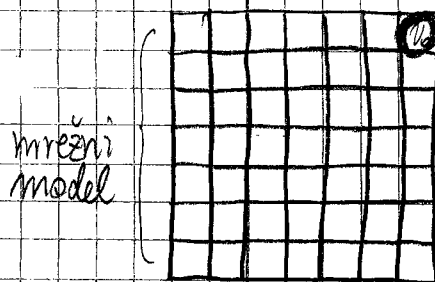
$$\int \frac{dQ}{T} = 0$$

$T_1 = 273K$
 $T_2 = 373K$
 $m_1 = 1kg$
 $m_2 = 1kg$

$T_2 = 323K$

$$\Delta S = m_1 c_p \ln \frac{T_2}{T_1} + m_2 c_p \ln \frac{T_2}{T_2} = 102 J/K > 0 \Rightarrow \text{neodmestna sprememba}$$

Sprememba entropije id. plina pri $T = \text{konst.}$, kjer je $V_1 \rightarrow V_2$



$$\mu = \frac{V}{N_0}$$

$$V = M \cdot v_0$$

↑
št. mrežnih mest

N = število molekul

$$M(M-1)(M-2) \dots (M-(N-1))$$

št. načinov porazdelitve molekul v mreži

$$S = k \ln W$$

$$W = \frac{M(M-1)(M-2) \dots (M-(N-1)) (M-N) \dots 1}{(M-N)!} = \boxed{\frac{M!}{N!(M-N)!}}$$

$$\ln W = \ln \left[\frac{M!}{N!(M-N)!} \right] \approx \ln M! - \ln N! - \ln (M-N)! \approx \left\{ \begin{array}{l} \text{predpostavimo, da ločimo} \\ \text{molekule med seboj} \end{array} \right.$$

Stirling-ova aproksimacija $\approx M \ln M - M - N \ln N + N - (M-N) \ln (M-N) + (M-N)$

$$\boxed{\ln n! \approx n \ln n - n}$$

n zelo velik

$$- (M-N) \ln \left[M \cdot \left(1 - \frac{N}{M} \right) \right] =$$

$$= - (M-N) \ln M - \ln (M-N) \ln \left(1 - \frac{N}{M} \right) =$$

$$= - M \ln M + N \ln M - (M-N) \ln \left(1 - \frac{N}{M} \right)$$

$$= \cancel{M \ln M - M} - \cancel{N \ln N + N} + \cancel{M \ln M} + \cancel{N \ln M} - (M-N) \ln \left(1 - \frac{N}{M} \right) + \cancel{M-N} =$$

$$= - N \ln N + N \ln M - (M-N) \ln \left(1 - \frac{N}{M} \right) =$$

$$= - N \ln \frac{N}{M} - (M-N) \ln \left(1 - \frac{N}{M} \right)$$

$$S = k \ln W = -kN \ln \left(\frac{N}{M} \right) - k(M-N) \ln \left(1 - \frac{N}{M} \right)$$

$$-(M-N) \underbrace{\ln\left(1 - \frac{N}{M}\right)}_{-\left(\frac{N}{M}\right)} \approx (M-N) \left(\frac{N}{M}\right) = N - \frac{N^2}{M} \approx N \quad \begin{matrix} \text{id. plin} \\ N \ll M \end{matrix}$$

$$S = -kN \ln\left(\frac{N}{M}\right) + kN$$

$$S = -kN \left(\ln\left(\frac{N}{M}\right) + 1 \right)$$

$$S = -k \left(N \ln\left(\frac{N}{M}\right) - N \right)$$

$$\begin{cases} x \ll 1 \\ \ln(1-x) \approx -x + \dots \end{cases}$$

$$T = \text{konst.} \rightarrow V_1 \rightarrow V_2$$

$$V = M \cdot v_0$$

volumen 1 molekule

$$V_1 = M_1 v_0$$

$$V_2 = M_2 v_0$$

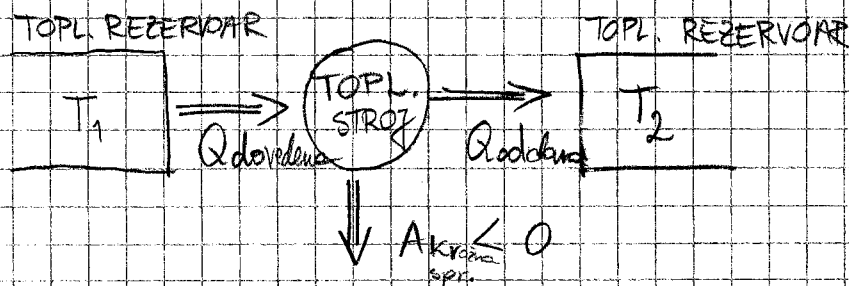
$$\Delta S = S_2 - S_1 = -k \left[N \ln \frac{N}{M_2} - N \right] + k \left[N \ln \frac{N}{M_1} - N \right] =$$

$$= -kN \ln \frac{N}{M_2} + kN \ln \frac{N}{M_1} = kN \left[\ln \frac{N}{M_1} - \ln \frac{N}{M_2} \right] = kN \ln \frac{M_2}{M_1}$$

$$\Delta S = \frac{R \left(\frac{K N_A N m_1}{N_A m_1} \right)}{M} \ln \frac{M_2 v_0}{M_1 v_0} = \frac{mR}{M} \ln \left(\frac{V_2}{V_1} \right)$$

IDEALNI TOPLOTNI STROJ

- krožna sprememba je reverzibilna
- stroj ponavlja Carnot-ovo krožno spremembo



dostajamo delo

$$T_1 > T_2$$

- prejema toploto in oddaja delo in pri tem ponavlja krožno spremembo.



$$\Delta W_n = A_{KR} + Q_{do} - |Q_{od}| = 0 \quad (1)$$

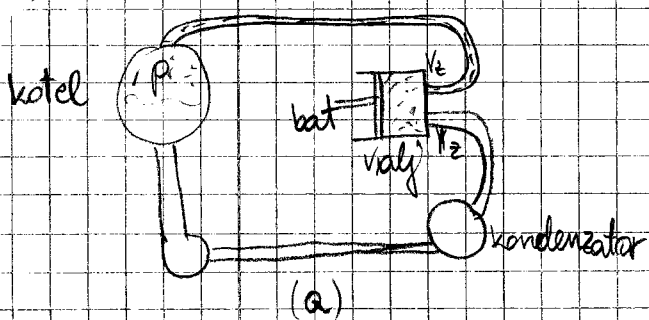
$$\Delta S = \frac{Q_{do}}{T_1} - \frac{|Q_{od}|}{T_2} = 0 \quad (2)$$

$$(1) \Rightarrow |A_{KR}| = -A_{KR} = Q_{do} - |Q_{od}|$$

$$(2) \Rightarrow \frac{|Q_{od}|}{Q_{do}} = \frac{T_2}{T_1}$$

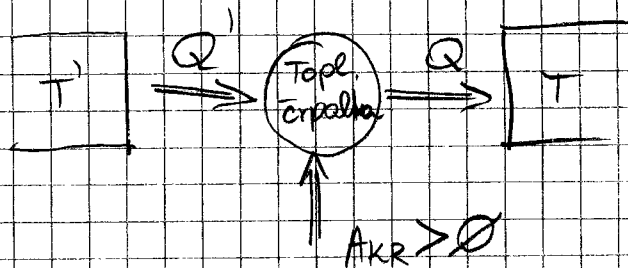
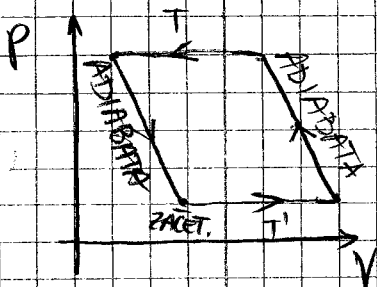
$$\text{izkoristek} \equiv \eta = \frac{|A_{KR}|}{Q_{do}} = \frac{Q_{do} - |Q_{od}|}{Q_{do}} = 1 - \frac{|Q_{od}|}{Q_{do}} = 1 - \frac{T_2}{T_1}$$

PARNI STROJ



$T_2 = T_1 \Rightarrow \eta = 0 \quad |A_{KR}| = 0$
izotermnega topl. stroja NI.

IDEALNI HLADILNI STROJ



$$\Delta W_n = A_{KR} + Q' - |Q| = 0 \Rightarrow A_{KR} = |Q| - Q'$$

$$\Delta S = \frac{Q'}{T'} - \frac{|Q|}{T} = 0 \Rightarrow \frac{|Q|}{Q'} = \frac{T}{T'}$$

$$\frac{A_{KR}}{Q'} = \frac{|Q| - Q'}{Q'} = \frac{|Q|}{Q'} - 1 = \left(\frac{T}{T'} - 1 \right)$$

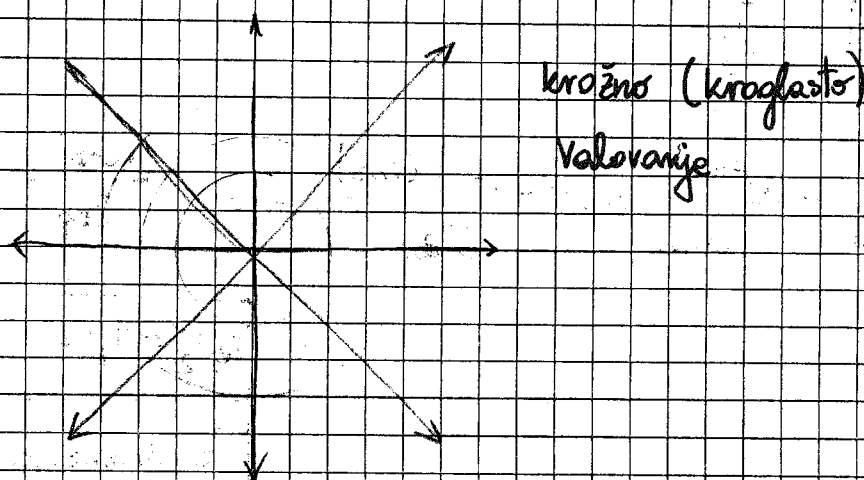
$$Q' = \frac{A_{KR}}{\left(\frac{T}{T'} - 1 \right)}$$

Potujoče valovanje:

7.1. 2011

valovna fronta: povezuje sosednja mesta z enakim odmikom

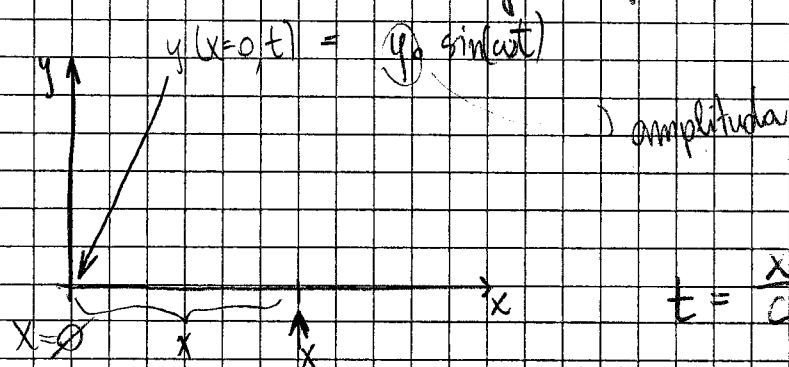
žarki: so pravokotni na val. fronte (kažejo smer valovanja)



Valovanje: širjenje energije (delci nihajo okrog ravnovesne lege)

• longitudinalno valovanje (vzdolžno)

• transversalno valovanje (prečno)



$$t = \frac{x}{c}$$

c = hitrost potovanja motnje

1-D primer:

Odmik na mestu x : $y(x, t) =$

$$= y_0 \sin\left(\omega\left(t - \frac{x}{c}\right)\right) =$$

$$= y_0 \sin\left(\omega t - \left(\frac{\omega}{c}\right)x\right) =$$

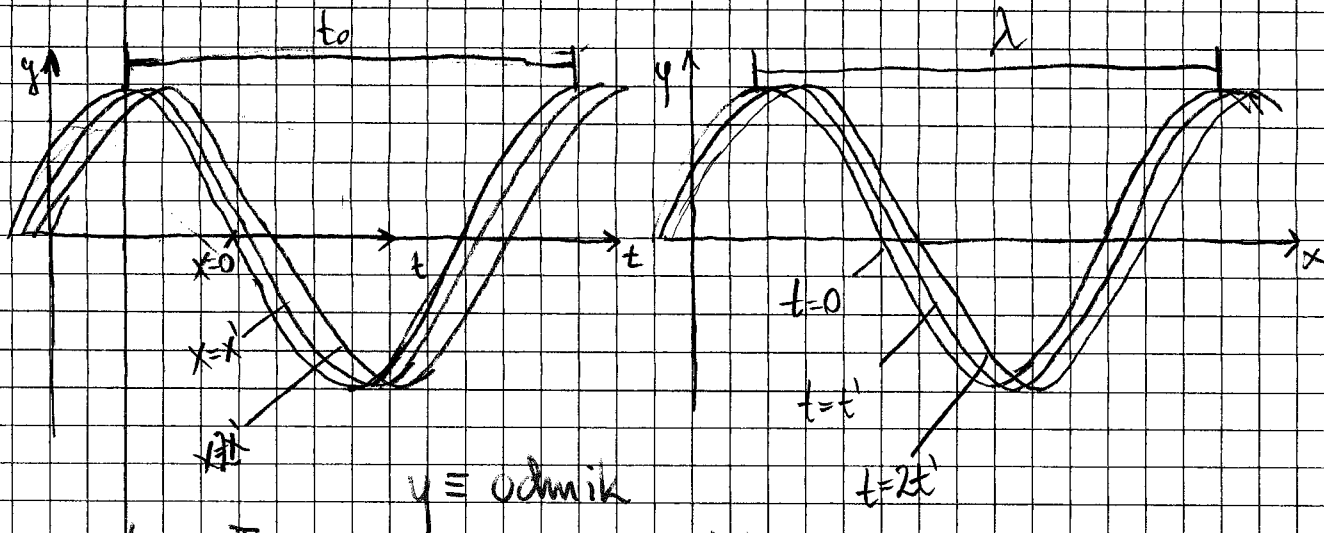
$$\left(\frac{\omega}{c}\right) = k$$

valovni vektor

$$y(x, t) = y_0 \sin(\omega t - kx)$$

3-D: $kx \rightarrow \vec{k} \cdot \vec{r}$

ω = krožna frekvenca ; $\omega = \frac{2\pi}{T} = 2\pi f$; $T = \frac{1}{f}$



$$\omega T_0 = 2\pi$$

↑
čas periode motnje

$$k = \frac{2\pi}{\lambda} \quad k = \frac{\omega}{c}$$

$$\frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2\pi f}{c}$$

$$\boxed{\frac{1}{\lambda} = \frac{f}{c}} \Rightarrow \boxed{c = \lambda f}$$

$$k\lambda = 2\pi$$

$$k = \frac{2\pi}{\lambda}$$

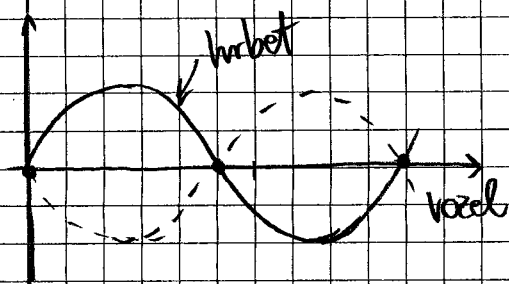
λ = valovna dolžina [m]

Stojere valovanje

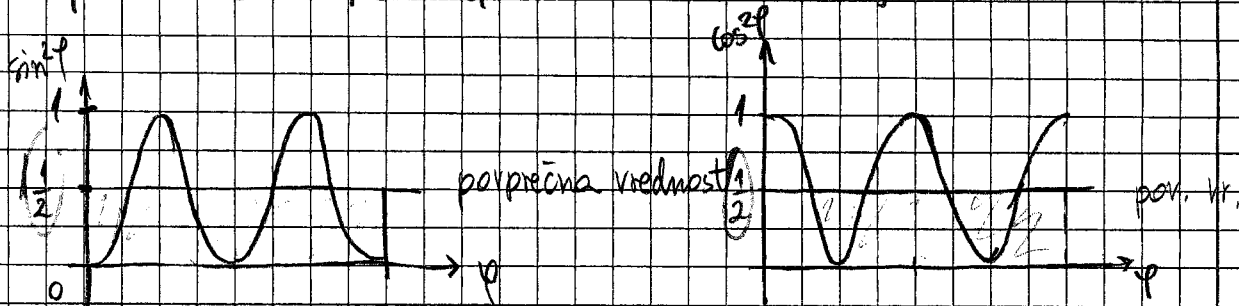
$$y = y_0 \sin(\omega t - kx) \pm y_0 \sin(\omega t + kx) = \begin{cases} 2y_0 \cos kx \cdot \sin \omega t, & (+) \\ 2y_0 \sin kx \cdot \cos \omega t, & (-) \end{cases}$$

Superpozicija (sestavljanje) dveh valovanj, ki potujeta v nasprotnih smereh (z enakima amplitudama in frekvencama)

- prosto krajšice → valovanje z oblijo z enako fazo (+)
- vprto krajšice → valovanje z oblijo z nasprotno fazo (-)



GOSTOTA ENERGIJSKEGA TOKA VALOVANJA



em tok:

$$P = \frac{dW}{dt}$$

energija, ki gre skozi dano ploščo S na enoto časa

gost. en. toku -

$$j = \frac{dP}{dS} \left[\frac{W}{m^2} \right]$$

gost. energije -

$$w = \frac{dW}{dV} \left[J/m^3 \right]$$

$$w = \frac{dW}{dV} = \frac{P dt}{S dt} = j \Rightarrow j = w \cdot c$$

$$y = y_0 \sin(\omega t - kx) \Rightarrow v = \frac{dy}{dt} = y_0 \omega \cos(\omega t - kx)$$

$$\bar{w} = \frac{1}{2} \rho v^2 = \frac{1}{2} \rho y_0^2 \omega^2 \cos^2(\omega t - kx) = \frac{1}{2} \rho y_0^2 \omega^2$$

$$\bar{j} = \bar{w} c = \frac{1}{2} \rho c y_0^2 \omega^2$$

$$w = \frac{1}{2} \rho v^2 = \text{deformacijska energija}$$

$$\text{def. en: } \frac{1}{2} k x^2$$

HITROST ŠIRJENJA MOTNJE (c)

$$1-D: y(x, t) = y_0 \sin(\omega t - kx), k = \frac{\omega}{c}$$

$$\left. \begin{aligned} \frac{\partial^2 y}{\partial x^2} &= -y_0 k^2 \sin(\omega t - kx) \\ \frac{\partial^2 y}{\partial t^2} &= -y_0 \omega^2 \sin(\omega t - kx) \end{aligned} \right\} \Rightarrow \frac{\frac{\partial^2 y}{\partial x^2}}{\frac{\partial^2 y}{\partial t^2}} = \frac{k^2}{\omega^2} = \frac{1}{c^2}$$

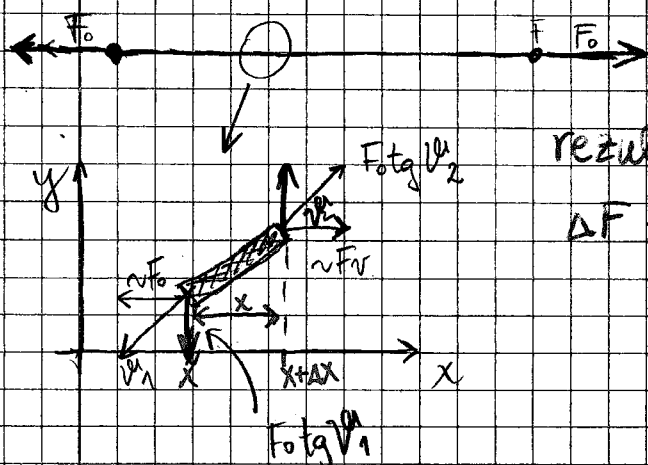
$$\boxed{\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}} \quad \text{valovna enačba 1-D}$$

↓

$$\text{rešitev: } y = y_0 \sin(\omega t - kx)$$

$$\omega^2 = \frac{1}{\epsilon_0 \mu_0} ; \quad \omega = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \approx 3 \cdot 10^8 \frac{\text{m}}{\text{s}}$$

primer: hitrost širjenja motnje po napeti struni (el. vrvi)



rezultantna sila:

$$\Delta F = F_0 \tan \varphi_2 - F_0 \tan \varphi_1 = F_0 (\tan \varphi_2 - \tan \varphi_1)$$

$$f(x + \Delta x) \approx f(x) + f'(x) \Delta x$$

$$f(x + \Delta x) - f(x) \approx f'(x) \Delta x$$

$$f(x_2) - f(x_1) \approx f'(x) \Delta x$$

$$\tan \varphi_2 - \tan \varphi_1 = \frac{\partial \tan \varphi}{\partial x} \Delta x$$

$$\Delta F = \frac{\partial (F_0 \tan \varphi)}{\partial x} \Delta x$$

$$\Delta F = \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} \right) \Delta x = \frac{\partial^2 y}{\partial x^2} \Delta x$$

Newtonov zakon:

$$\Delta F = \underset{\substack{\text{gostota} \\ \downarrow}}{\rho} \Delta x S \frac{\partial^2 y}{\partial t^2}$$

$$F_0 \Delta x \frac{\partial^2 y}{\partial x^2} = \rho S \Delta x \frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 y}{\partial t^2} = \left(\frac{F_0}{\rho S} \right) \frac{\partial^2 y}{\partial x^2}}$$

VALOVNA
ENACBA

rešitev: pot. val: $y = y_0 \sin(\omega t - kx)$;

$$c = \frac{F_0}{\rho S} \Rightarrow c = \sqrt{\frac{F_0}{\rho S}}$$

trdna snov:

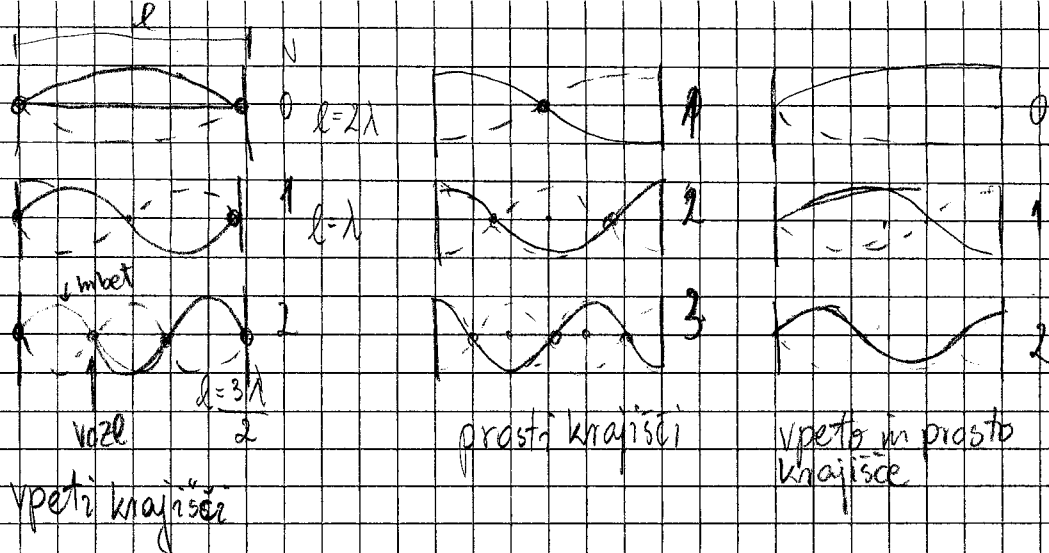
Youngov modul

$$E \Rightarrow c = \sqrt{\frac{E}{\rho}}$$

plin:

$$c = \sqrt{\frac{\gamma p}{\rho}}$$

↑
stlačljivost

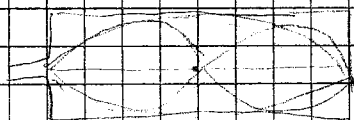


$$l = (N+1) \frac{\lambda_N}{2} \Rightarrow \lambda_N = \frac{2l}{(N+1)}$$

$$\nu_N = \frac{c}{\lambda_N} = \frac{c(N+1)}{2l}$$

piščal - longitudinalno valovanje (vzdolžno)

Kundtova piščal



$$\text{glasnost} = G = 10 \log \left(\frac{I}{I^*} \right)$$

$$I^* = 10^{-12} \text{ W/m}^2$$

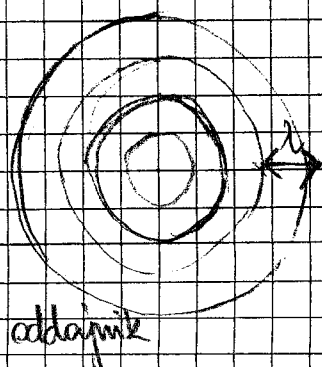
(najmanjša jakost zvoka, ki jo človek se sliši.)

DOPPLERJEV POJAV

1.) Sprejemnik se giblje: (spreminja frekvenco)

$$c = \lambda \nu \Rightarrow \lambda = \frac{c}{\nu}$$

λ - valovna dolžina ni odvisna



$$\lambda = \lambda'$$

sprejemnik

$$\lambda = \lambda'$$

$$\frac{c}{\nu} = \frac{c'}{\nu'}$$

$$c' = c + v$$

$$\frac{c}{\nu} = \frac{c+v}{\nu'}$$

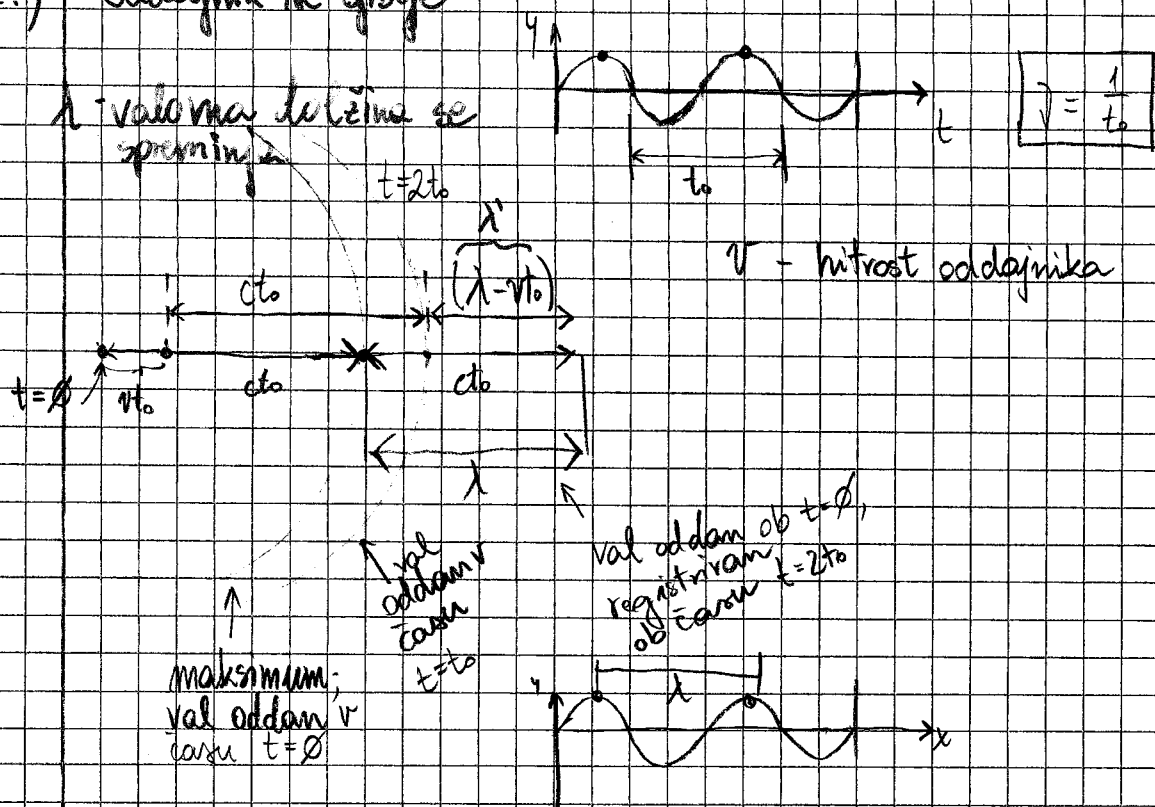
+ poslušalec se približuje
- poslušalec se oddaljuje

c - hitrost zvoka

frekvenco, ki jo sliši sprejemnik

$$\nu' = \nu \left(\frac{c+v}{c} \right) = \nu \left(1 \pm \frac{v}{c} \right)$$

2.) Oddajnik se giblje



oddajnik se giblje, poslušalec (sprejemnik) miruje

$$\lambda' = \lambda - vt_0$$

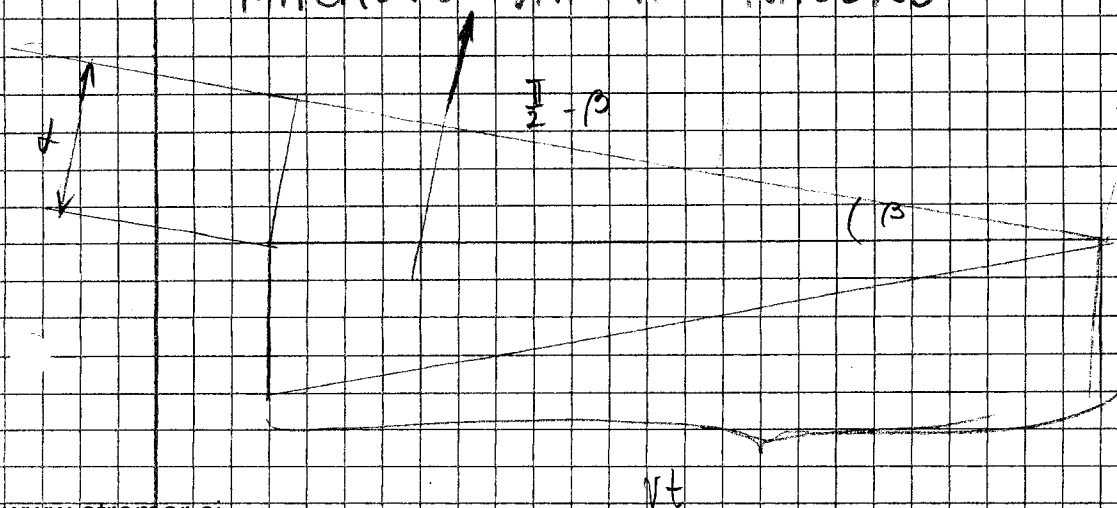
$$c = \lambda' f'$$

$$f' = \frac{c}{\lambda'} = \frac{c}{\lambda - vt_0} = \frac{c}{(\lambda - \frac{v}{f} \frac{1}{f})} = \frac{cf}{\lambda f - v} = \frac{f}{c - v} =$$

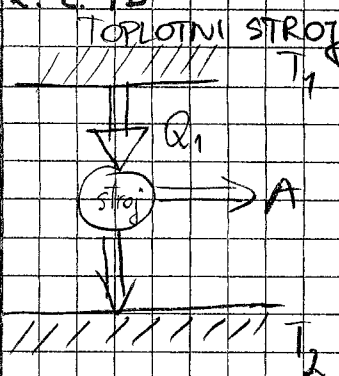
$$f' = \frac{f}{(1 \pm \frac{v}{c})}$$

- oddajnik se približuje sprejemniku
- oddajnik oddaljuje sprejemnika

MACHOVO VALOVNO NAČELO



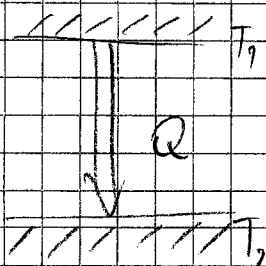
2. 2. TD



$$\left. \begin{aligned} -\frac{Q_1}{T_1} &= \Delta S_1 \\ \frac{Q_2}{T_2} &= \Delta S_2 \end{aligned} \right\} \Delta S = \Delta S_1 + \Delta S_2 = 0$$

$$\Delta S \geq 0$$

sprememba ni reverzibilna



$$-\frac{Q}{T_1}$$

$$\frac{Q}{T_2} - \frac{Q}{T_1} > 0$$

$$\frac{Q}{T_2}$$

Ce se entropija veča, se zmožnost sistema pretvorbe toplote v delo zmanjša

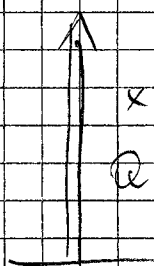
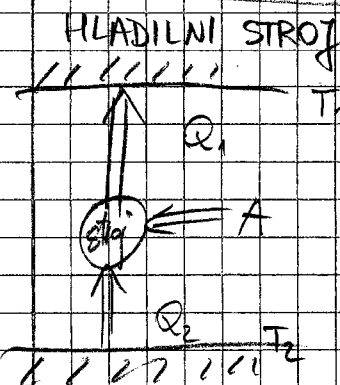
$$S = k \ln W$$

posamezne molekule

verjetnost, da bo sistem v nekem makroskopskem stanju
3. mikroskopskih stanj za makroskopsko stanje

Ko je sistem v ravnovesju, je energija pravilno porazdeljena

(hlak, ...)



$$\eta = \frac{Q_2}{A}$$

parameter za hladilnik

$$\eta = \frac{Q_2}{Q_1 - Q_2} \stackrel{\text{rev.}}{=} \frac{T_2}{T_1 - T_2}$$

$$Q_1 = Q_2 + A$$

idealni stroj: $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

velja za vse
reverzibilne
stroje

TOPLOTNA ČRPALKA

↓
parametar
za t.c.

$$\eta = \frac{Q_1}{A} = \frac{Q_1}{Q_1 - Q_2} \xrightarrow{\text{HOU.}} \frac{T_1}{T_1 - T_2}$$

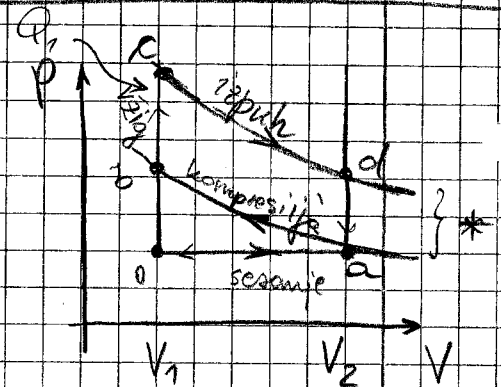
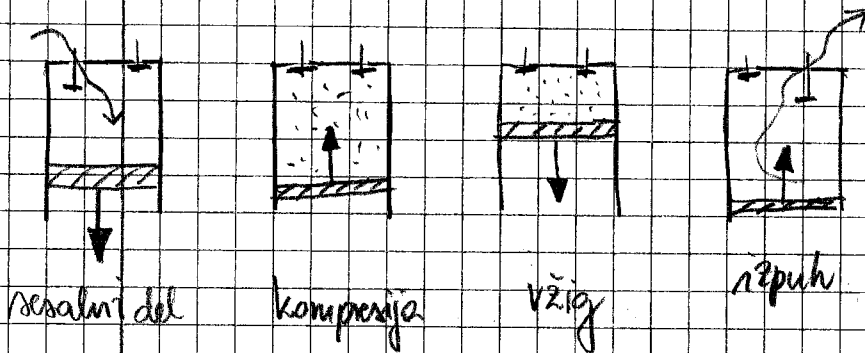
$$T_1 = 300 \text{ K } (27^\circ \text{C})$$

$$T_2 = 270 \text{ K } (-3^\circ \text{C})$$

$$\eta = \frac{300 \text{ K}}{30 \text{ K}} \approx 10 \frac{Q_1}{A}$$

$Q_1 = 10A \Rightarrow$ dobimo 10x več toplote kot vložimo dela

Motor z notranjim izgrevanjem



* adiabatni ; toplota se ne pretaka, ker je sprememba prehltra

$$\Delta V = 0 \Rightarrow A \equiv \int p dV = 0$$

$$\Delta W_n = Q$$

$b \rightarrow c$ $Q_1 = m C_v \Delta T = m C_v (T_c - T_b)$

$d \rightarrow a$ $Q_2 = m C_v (T_d - T_a)$

$c \rightarrow d$
 $b \rightarrow a$

$$\begin{aligned} T_c V_1^{\gamma-1} &= T_d V_2^{\gamma-1} \\ T_b V_1^{\gamma-1} &= T_a V_2^{\gamma-1} \end{aligned}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{T_d - T_a}{T_c - T_b}$$

$$\eta = 1 - \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$(T_c - T_b) V_1^{\gamma-1} = (T_d - T_a) V_2^{\gamma-1}$$

$$\frac{V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{(T_d - T_a)}{(T_c - T_b)}$$

$$\eta \approx 60\%$$

TOPL. RAZTEŽANJE SNOVI

β - temp. koeficient prostorninskega raztezeka:
relativna sprememba prostornine na enoto spremembe temperature

$$\beta = \frac{\Delta V/V}{\Delta T} \quad ; \quad \alpha = \frac{\Delta l/l}{\Delta T} \quad \beta = 3\alpha$$

$$\Delta V = \left(\frac{\Delta V}{\Delta T}\right)_p \Delta T + \left(\frac{\Delta V}{\Delta p}\right)_T \Delta p = 0$$

$\Downarrow$

$$V\beta$$

$\Downarrow$

$$-V\chi$$

$$\left(\frac{\Delta V}{V} = -\chi\right) \Delta p$$

$$V\beta \Delta T - V\chi \Delta p = 0$$

$$\Delta p = \frac{\beta \Delta T}{\chi}$$

$$\Delta l = \left(\frac{\Delta l}{\Delta T}\right)_0 \Delta T + \left(\frac{\Delta l}{\Delta \sigma}\right)_T \Delta \sigma = 0$$

$\Downarrow$

$$\alpha \Delta T + \frac{\chi}{E} \Delta \sigma = 0$$

$$\Delta \sigma = -\alpha E \Delta T$$

$$\left(\frac{F}{S}\right) = E \frac{\Delta l}{l}$$

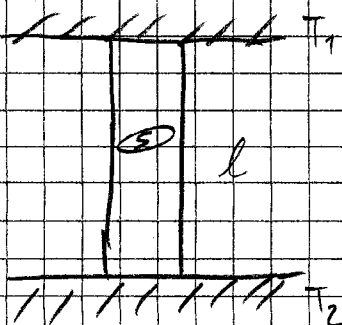
$$\frac{\Delta l}{\Delta \sigma} = \frac{l}{E}$$

sprememba

(natezne) napetosti

a) prevajanje - prenos toplote ^(energije) preko stika, preko trkov molekul

b) konvekcija



$$P = \frac{dQ}{dt}$$

$$P = \lambda S \frac{\Delta T}{l}$$

$\rightarrow$ toplotni tok

$$P = \frac{\Delta T}{R_T} \quad \left(1 = \frac{l}{\lambda S}\right)$$

$$R_T = \frac{l}{\lambda S} \rightarrow \text{toplotna upornost}$$