

FIZIKA - VAJE

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literatura:

ODVOD

$f(x)$	$\frac{df}{dx} = f'(x)$
C	$\emptyset$
kx^n	knx^{n-1}
e^x	e^x
$\sin x$	$\cos x$
$\cos x$	$-\sin x$

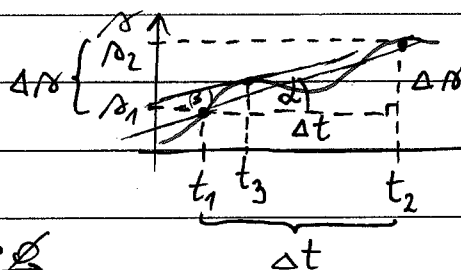
vaji iz fizike 1, zbirka vaj

physics fe.uni-lj.si → STUDENTS

KINEMATIKA

$$\bar{v} = \frac{\Delta x}{\Delta t} \quad \Delta t \rightarrow 0$$

$$v = \frac{ds}{dt}$$



$$\tan \alpha = \frac{\Delta x}{\Delta t} = \bar{v}$$

$$\tan \beta = \frac{ds}{dt} = v$$

$$\bar{a} = \frac{\Delta v}{\Delta t} \quad \Delta t \rightarrow 0$$

$$a = \frac{dv}{dt}$$

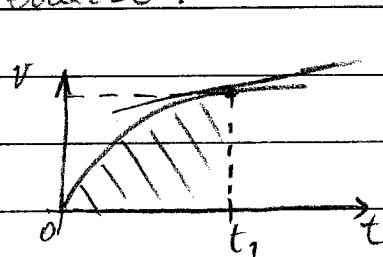
INTEGRAL

$f(x)$	$\int f(x) dx = F(x) + C$
k	$kx + C$
kx^n	$k \frac{x^{n+1}}{n+1} + C$
e^x	e^x
x^{-1}	$\ln x + C$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$

Hitrost avta je podana z enačbo:

$v = kt^{1/2}$

$$k = 5 \frac{m}{s^{3/2}}$$



$$\left[\frac{m}{s^{3/2}} \cdot s^{1/2} \right] = \left[\frac{m}{s^{3/2-1/2}} \right] = \left[\frac{m}{s} \right]$$

$$v(t=0) = v_0 = \emptyset$$

Kolikšen je pospešek v 30s in koliko poti opravi?

$$t_1 = 30s$$

$$a(t_1) =$$

$$s(t_1) =$$

$$a = \frac{dv}{dt} = \frac{d(kt^{1/2})}{dt} = \frac{k}{2} \cdot t^{-1/2} = \frac{k}{2\sqrt{t}}$$

$$a(t_1) = \frac{5 \cdot \frac{m}{s^{3/2}}}{2 \cdot \sqrt{30s}} = \frac{5 \cdot \frac{m}{s^{3/2}}}{2 \cdot \sqrt{30} \cdot s^{1/2}} \approx 0,46 \frac{m}{s^2}$$

$$ds = v dt$$

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v dt$$

$$\int_{s(t_1)}^{s(t_2)} ds = \int_0^{t_1} v dt = \int_0^{t_1} kt^{1/2} dt$$

$$\sum \Delta s_i = \sum \vec{v}_i \cdot \Delta t_i$$

$$\lim_{t \rightarrow 0} a(t) = \infty$$

$$s(t_1) - \emptyset = \frac{2k^0}{3} \cdot t_1^{3/2} - \emptyset \Rightarrow s(t_1) = \frac{2 \cdot 5 m/s^{3/2}}{3} \cdot (30s)^{3/2} \Rightarrow$$

$$\Rightarrow 547,7m$$

5. nal., str. 5

Vozilu, ki se giblje s hitrostjo v_0 se ustavi motor; kolikšno pot opravi v t_1 , če je pojemek a ; kolikšna je v_1 ?

$$v_0 = 4 \frac{\text{m}}{\text{s}}$$

$$a = -g v^2$$

$$g = 0,8 \frac{1}{\text{s}}$$

$$t_1 = 15 \text{ s}$$

$$s(t_1) = ?$$

$$v(t_1) = ?$$

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$-g v^2 = \frac{dv}{dt} \quad / \cdot dt$$

$$-g v^2 dt = dv \quad / \int$$

$$-g \int_0^{t_1} dt = \int_{v_0}^{v_1} \frac{dv}{v^2}$$

$$-g \int_0^{t_1} dt = \int_{v_0}^{v_1} dv v^{-2}$$

$$+g \left[t \right]_0^{t_1} = + \left[\frac{1}{v} \right]_{v_0}^{v_1}$$

$$(0,8 \frac{1}{\text{s}} \cdot 15 \text{ s}) - (0) = \frac{1}{v_1} - \left(\frac{1}{4 \text{ m/s}} \right)$$

$$12 \frac{\text{s}}{\text{s}} + \frac{1}{4} \frac{\text{s}}{\text{m}} = \frac{1}{v_1} \implies \underline{\underline{v_1 \approx 0,08 \frac{\text{m}}{\text{s}}}}$$

$$v = \frac{ds}{dt} \implies v dt = ds \quad | \int_{t_1}^{t_2}$$
$$\int_0^{t_1} v dt = \int_0^{s_1} ds$$

$$\boxed{\int_0^{t_1} \frac{1}{g t + \frac{1}{v_0}} dt = \int ds}$$

$$\boxed{R: s_1 = \frac{1}{g} \cdot \ln(1 + g v_0 t_1)}$$
$$s_1 = 4,9 \text{ m}$$

12. naloga
4,6

1.

$$v = v_0 + At + Bt^2$$

$$v_0 = 1 \text{ m/s}$$

$$A = 0,2 \text{ m/s}^2$$

$$B = 0,03 \text{ m/s}^3$$

$$t_1 = 10 \text{ s}$$

$$t_0 = 0$$

$$v(t_1)$$

$$s(t_1)$$

$$a = \frac{dv}{dt} = \frac{v_0 + At_1 + Bt_1^2}{dt} = 0 + A + 2Bt_1 =$$

$$a = 0,2 \text{ m/s}^2 + 2 \cdot 0,03 \text{ m/s}^3 \cdot 10 \text{ s} =$$

$$a = 0,8 \text{ m/s}^2$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt$$

$$v dt = ds \Rightarrow \int v dt = \int ds$$

$$\int_0^{t_1} v dt = \int_0^{t_1} ds$$

$$s = v_0 t_1 + \frac{At_1^2}{2} + \frac{Bt_1^3}{3} = 1 \text{ m/s} \cdot 10 \text{ s} + \frac{0,2 \text{ m/s}^2 \cdot (10 \text{ s})^2}{2} + \frac{0,03 \text{ m/s}^3 \cdot (10 \text{ s})^3}{3}$$

$$s = 1 \text{ m/s} \cdot 10 \text{ s} + \frac{0,2 \text{ m/s}^2 \cdot 100 \text{ s}^2}{2} + \frac{0,03 \text{ m/s}^3 \cdot 1000 \text{ s}^3}{3}$$

$$s = 10 \text{ m} + 10 \text{ m} + 10 \text{ m} = 30 \text{ m}$$

$$s = 10 \text{ m} + 10 \text{ m} + 10 \text{ m} = 30 \text{ m}$$

2.

$$a = kt^{1/2}$$

$$k = (1 \text{ m} \times \text{s}^{-5/2})$$

$$t_0 = 0$$

$$t_1 = 10 \text{ s}$$

$$v = ?$$

$$s = ?$$

$$a = \frac{dv}{dt} \Rightarrow a dt = dv \Rightarrow \int a dt = \int dv \Rightarrow v = \int a dt$$

$$\int a dt = \int dv \Rightarrow v = \int a dt$$

$$v = \int_0^{t_1} kt^{1/2} dt = k \int_0^{t_1} t^{1/2} dt = k \cdot \frac{2}{3} t^{3/2} \Big|_0^{t_1}$$

$$v = \frac{2}{3} kt^{3/2} - 0 = \frac{2}{3} (1 \text{ m} \times \text{s}^{-5/2}) \cdot 10^{3/2} = 21,08 \frac{\text{m}}{\text{s}}$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt$$

$$\int v dt = \int ds \Rightarrow s = \int v dt$$

$$s = \int_0^{t_1} \frac{2}{3} kt^{3/2} dt = \frac{2}{3} k \int_0^{t_1} t^{3/2} dt = \frac{2}{3} k \cdot \frac{2}{5} t^{5/2} \Big|_0^{t_1}$$

$$s = \frac{4}{15} kt^{5/2} - 0 = \frac{4}{15} (1 \text{ m} \times \text{s}^{-5/2}) \cdot 10^{5/2} = 84,3 \text{ m}$$

4. $N_0 = 3 \text{ m/s}$

$$k = 2 \text{ s}^{-1}$$

$$t = 0,5 \text{ s}$$

$$v = ?$$

$$a = -kv$$

$$\frac{dv}{dt} = -kv$$

$$\int_{v_0}^v \frac{dv}{v} = - \int_0^t k dt$$

$$\ln v \Big|_{v_0}^v = -kt$$

$$\ln v - \ln v_0 = -kt$$

$$\ln \frac{v}{v_0} = -kt$$

$$\ln v = -v_0 kt$$

$$v = v_0 e^{-kt}$$

$$v = \frac{ds}{dt} \Rightarrow v dt = ds \Rightarrow \int v dt = \int ds$$

$$s = \int_0^t v dt = v_0 \int_0^t e^{-kt} dt = -\frac{v_0}{k} e^{-kt} \Big|_0^t = -\frac{v_0}{k} e^{-kt} + \frac{v_0}{k}$$

$$s = \frac{v_0}{k} (1 - e^{-kt})$$

6. $v_0 = 3 \text{ m/s}$

$$a = \frac{dv}{dt} \Rightarrow -kv^2 = \frac{dv}{dt} \Rightarrow -kv^2 dt = dv$$

$$a = -kv^2$$

$$k = 0,2 \frac{1}{\text{m}}$$

$$t = 0,8 \text{ s}$$

s, v, a

$$dt = -\frac{dv}{kv^2}$$

$$\int_0^t dt = \int_{v_0}^v -\frac{dv}{kv^2} \Rightarrow \int_0^t dt = -\frac{1}{k} \int_{v_0}^v \frac{dv}{v^2}$$

$$\int v^{-2} dv = -v^{-1} + C$$

$$t = \frac{1}{kv} \Big|_{v_0}^v = \frac{1}{kv} - \frac{1}{kv_0} \Rightarrow \frac{1}{k} \left(\frac{1}{v} - \frac{1}{v_0} \right)$$

$$v(t) = \frac{v_0}{1 + kv_0 t} = \frac{3 \text{ m/s}}{1 + 0,2 \frac{1}{\text{m}} \cdot 3 \frac{\text{m}}{\text{s}} \cdot 0,8 \text{ s}} = 2,02 \text{ m/s}$$

$$a = -0,2 \cdot (2,02 \text{ m/s})^2 = -0,82 \frac{\text{m}}{\text{s}^2}$$

$$s = \int_0^t v(t) dt = \int_0^t \frac{v_0 dt}{1 + kv_0 t} = \int_0^t \frac{v_0 \frac{dm}{kv_0}}{\int m k v_0} = \frac{1}{k} \ln |m| \Big|_0^t = \frac{1}{k} \ln |1 + kv_0 t|$$

$$1 + kv_0 t = m$$

$$kv_0 dt = dm$$

$$dt = \frac{dm}{kv_0}$$

$$s = \frac{1}{0,2} \text{ m} \cdot \ln |1 + 0,2 \frac{1}{\text{m}} \cdot 3 \frac{\text{m}}{\text{s}} \cdot 0,8 \text{ s}| = 1,96 \text{ m}$$

(5.)

$$v_0 = 4 \frac{m}{s}$$

$$t_1 = 15 s$$

$$a = -g v^2$$

$$g = 9,8 m/s^2$$

$$v(t_1) = 0,08 \frac{m}{s}$$

$$v(t) = \frac{1}{2t + \frac{1}{v_0}}$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt$$

$$\int_{s(t_1)}^{s(t_1)} ds = \int_0^{t_1} v dt$$

$$s(t_1) = \int_0^{t_1} \frac{dt}{2t + \frac{1}{v_0}} = \int_0^{t_1} \frac{v_0 dt}{2tv_0 + 1} = \int_1^m \frac{v_0 dm}{2v_0 m} = \frac{1}{2} \int_1^m \frac{dm}{m} =$$

$$\left. \begin{aligned} 2tv_0 + 1 &= m \\ 2v_0 dt &= dm \\ dt &= \frac{dm}{2v_0} \end{aligned} \right\} = \frac{1}{2} \cdot \ln(m) \Big|_1^m = \frac{1}{2} \ln(2tv_0 + 1) \Big|_0^{t_1} =$$

$$= \frac{1}{2} \ln(2t_1 v_0 + 1) = \underline{4,86 m}$$

ENAKOMERNO POSPEŠENO GIBANJE

$$\vec{a} = \text{konst.}$$

$$d\vec{v} = \vec{a} dt / \int \Rightarrow \int d\vec{v} = \int \vec{a} dt$$

$$\vec{v}(t=0) = v_0$$

$$\int d\vec{v} = \vec{a} \int dt$$

$$\vec{v} \Big|_{v_0}^{v_1} = \vec{v}_1 - \vec{v}_0 = \vec{a} t \Big|_0^{t_1}$$

$$ds = \vec{v} \cdot dt$$

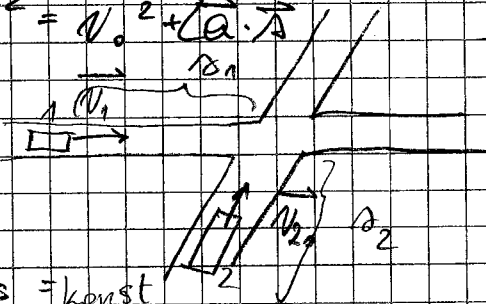
$$\vec{v}_1 - \vec{v}_0 = \vec{a} t_1$$

$$\vec{v}_1 = \vec{v}_0 + \vec{a} t_1$$

$$\vec{s}(t) = \vec{v}_0 t + \frac{\vec{a} t^2}{2}$$

$$v^2 = v_0^2 + 2\vec{a} \cdot \vec{s}$$

16. / str. 10



$$s_1 = v_1 \cdot t \Rightarrow t = \frac{s_1}{v_1} = \frac{30 m}{5 m/s} = 6 s$$

$$v_1 = 5 m/s = \text{konst.}$$

$$a_1 = 0$$

$$s_1 = 30 m$$

$$s_2 = 70 m$$

$$v_2 = 5 m/s$$

$$a_2 = \text{konst.}$$

$$a_2$$

$$s_2 = v_2 t + \frac{a_2 t^2}{2}$$

$$2(s_2 - v_2 t) = a_2 t^2 = 2 \cdot (70 m - 5 m/s \cdot 6 s) = 36 m^2$$

$$a_2 = \frac{(140 - 60) m}{36 s^2} = \frac{80 m}{36 s^2} = 2,22 m/s^2$$

13/str. 11

$$\begin{aligned} R_1 &= 50 \text{ m} \\ t_1 &= 5 \text{ s} \\ R_2 &= 50 \text{ m} \\ t_2 &= 7 \text{ s} \\ a &= \text{konst.} \\ a &= ? \end{aligned}$$

VLAK

 $\vec{v}_0$ $\vec{v}_1 = ?$

$$R_1 = v_0 t_1 + \frac{a t_1^2}{2}$$

$$R_2 = v_0 t_2 + \frac{a t_2^2}{2}$$

$$v_0 = v_0 + a t_1$$

$$v_0 = v_0 - a t_1$$

$$R_1 + R_2 = v_0 (t_1 + t_2) + \frac{a (t_1 + t_2)^2}{2}$$

$$\frac{R_1 + R_2}{t_1 + t_2} = \frac{R_1}{t_1} - \frac{a t_1}{2} + \frac{a (t_1 + t_2)}{2}$$

$$a = \frac{2}{t_2} \left(\frac{R_1 + R_2}{t_1 + t_2} - \frac{R_1}{t_1} \right) =$$

$$a = \frac{2}{7 \text{ s}} \left(\frac{100 \text{ m}}{12 \text{ s}} - \frac{50 \text{ m}}{5 \text{ s}} \right) =$$

$$a = \frac{2}{7 \text{ s}} \left(\frac{100 \text{ m}}{12 \text{ s}} - 10 \text{ m/s} \right) = -0,48 \frac{\text{m}}{\text{s}^2}$$

$$\frac{R_1 - a t_1^2}{2 t_1}$$

Kolikšna je razdalja med kamnoma 3 s potem, ko vržemo drugi kamen?

$$h = 200 \text{ m}$$

$$v_0 = 3 \frac{\text{m}}{\text{s}}$$

$$t_0 = 0$$

$$t_1 = 1 \text{ s}$$

$$v_0 = 30 \frac{\text{m}}{\text{s}}$$

$$t_2 = 4 \text{ s}$$

$$d(t_2) = ?$$

vržemo prvi kamen

vržemo drugi kamen

$$d = h - R_1 - R_2$$

$$R_1 = \frac{g t_1^2}{2} + v_0 t_1$$

$$R_2 = v_0 t_2 - \frac{g t_2^2}{2}$$

$$d = 200 \text{ m} - v_0 t_1 - \frac{g t_1^2}{2} - v_0 t_2 + \frac{g t_2^2}{2}$$

$$d = 200 \text{ m} - 3 \frac{\text{m}}{\text{s}} \cdot 4 \text{ s} - \frac{10 \frac{\text{m}}{\text{s}^2} \cdot 16 \text{ s}^2}{2} - 30 \frac{\text{m}}{\text{s}} \cdot 3 \text{ s} + \frac{10 \frac{\text{m}}{\text{s}^2} \cdot 9 \text{ s}^2}{2} =$$

$$d = 200 \text{ m} - 12 \text{ m} - 80 \text{ m} - 90 \text{ m} + 45 \text{ m} = 63 \text{ m}$$

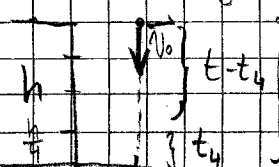
14./str. 9

NAVTIČNI MET

$$v_0 = 20 \frac{\text{m}}{\text{s}}$$

$$t_0 = 1 \text{ s}$$

$$t = ?$$

V zadnji sekundi meta naredi $\frac{1}{4}$ poti

$$h = v_0 t + \frac{g t^2}{2}$$

$$\frac{3}{4} h = v_0 (t - t_0) + \frac{g (t - t_0)^2}{2}$$

$$\frac{3}{4} \left(v_0 t + \frac{g t^2}{2} \right) = v_0 (t - t_0) + \frac{g (t - t_0)^2}{2}$$

$$6v_0 t + 3gt^2 = 8v_0 t - 8v_0 t_4 + 4g(t^2 - 2tt_4 + t_4^2)$$

$$6v_0 t + 3gt^2 = 8v_0 t - 8v_0 t_4 + 4gt^2 - 8gt_4 t + 4gt_4^2$$

$$-2v_0 t - gt^2 + 8gt_4 t - 4gt_4^2 + 8v_0 t_4$$

$$-gt^2 + (-2v_0 + 8gt_4)t + 8v_0 t_4 - 4gt_4^2 = 0$$

$$Ax^2 + Bx + C = 0$$

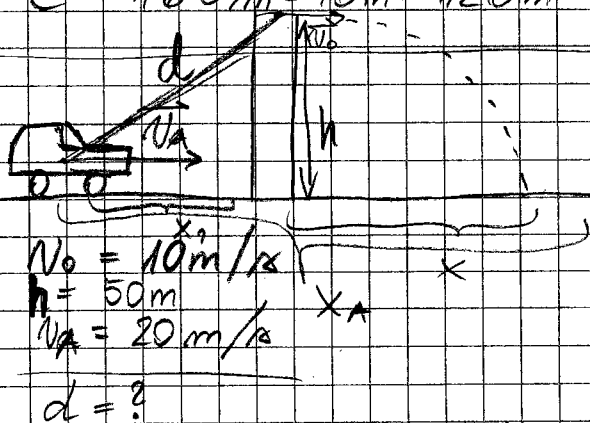
$$A = -10 \frac{\text{m}}{\text{s}^2}$$

$$B = -40 \frac{\text{m}}{\text{s}} + 80 \frac{\text{m}}{\text{s}} = 40 \frac{\text{m}}{\text{s}}$$

$$C = 160 \text{ m} - 40 \text{ m} = 120 \text{ m}$$

$$t_{1,2} = \frac{-40 \frac{\text{m}}{\text{s}} \pm \sqrt{1600 \frac{\text{m}^2}{\text{s}^2} + 4800 \frac{\text{m}^2}{\text{s}^2}}}{-20 \frac{\text{m}}{\text{s}^2}}$$

$$t_{1,2} = \frac{-40 \frac{\text{m}}{\text{s}} \pm 80 \frac{\text{m}}{\text{s}}}{-20 \frac{\text{m}}{\text{s}^2}}$$



$$t_1 = 2 \text{ s}$$

$$t_2 = 6 \text{ s}$$

KAMEN:

$$h = \frac{gt^2}{2}$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{100 \text{ m}}{10 \frac{\text{m}}{\text{s}^2}}} = \sqrt{10} \text{ s}$$

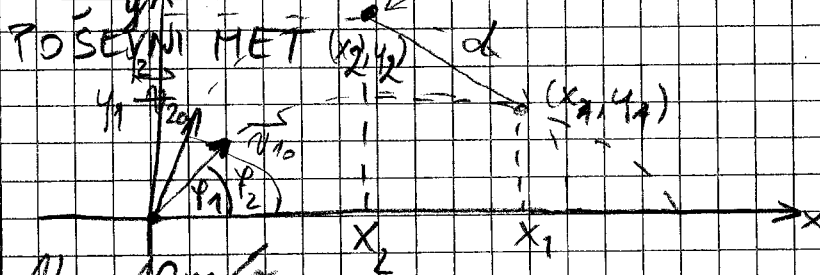
$$X = v_0 t = 10 \text{ m/s} \cdot \sqrt{10} \text{ s} = 31,6 \text{ m}$$

AVTO:

$$X_A = v_A t = 20 \text{ m/s} \cdot \sqrt{10} \text{ s} = 63,2 \text{ m}$$

$$X_0 = X_A - X = 63,2 \text{ m} - 31,6 \text{ m} = 31,6 \text{ m}$$

$$d = \sqrt{X_0^2 + h^2} = \sqrt{1000 \text{ m}^2 + 2500 \text{ m}^2} = \sqrt{3500 \text{ m}^2} = 59,16 \text{ m}$$



$$v_0 = 10 \text{ m/s}$$

$$\phi_1 = 60^\circ$$

$$\phi_2 = 70^\circ$$

$$v_{2y} = v_0 \sin \phi_2 - gt = 0 \Rightarrow$$

$$t = \frac{v_0 \sin \phi_2}{g} = \frac{10 \text{ m/s} \cdot \sin 70^\circ}{10 \frac{\text{m}}{\text{s}^2}} =$$

$$d = ?$$

Ko je drugo kraglo najvišje.

$$t = 0,94 \text{ s}$$

$$X_1 = v_0 \cos \phi_1 t = 10 \text{ m/s} \cdot \frac{1}{2} \cdot 0,94 \text{ s} = 4,7 \text{ m}$$

$$X_2 = v_0 \cos \phi_2 t = 10 \text{ m/s} \cdot 0,34 \cdot 0,94 \text{ s} = 3,2 \text{ m}$$

$$y_1 = v_0 \sin \phi_1 t - \frac{gt^2}{2} = 3,7 \text{ m}$$

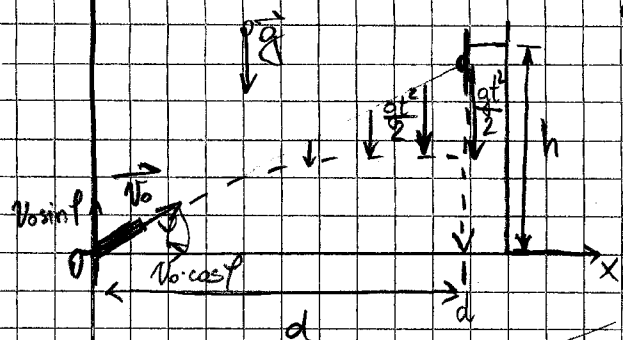
$$y_2 = v_0 \sin \phi_2 t - \frac{gt^2}{2} = 4,4 \text{ m}$$

$$d = \sqrt{(X_2 - X_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{1,25 + 0,49} = 1,66 \text{ m}$$

EXPERIMENT 2 OPICA:

opica in izstrellek zaradi gravitacije padata.



OPICA:

$$x_{op} = d$$

$$y_{op} = h - \frac{gt^2}{2}$$

IZSTRELEK:

$$x_{iz} = v_0 \cos \varphi \cdot t$$

$$y_{iz} = v_0 \sin \varphi \cdot t - \frac{gt^2}{2}$$

ZADETEK:

Kot mora biti vedno enak.

$$x_{op} = x_{iz} \Rightarrow d = v_0 \cos \varphi \cdot t \Rightarrow t = \frac{d}{v_0 \cos \varphi}$$

$$y_{op} = y_{iz} \Rightarrow h - \frac{gt^2}{2} = v_0 \sin \varphi \cdot t - \frac{gt^2}{2} \Rightarrow t = \frac{h}{v_0 \sin \varphi}$$

$$\frac{d}{v_0 \cos \varphi} = \frac{h}{v_0 \sin \varphi} \Rightarrow h = \frac{\sin \varphi}{\cos \varphi} \cdot d = \tan \varphi \cdot d$$

$$\boxed{\frac{h}{d} = \tan \varphi}$$

Merimo direktno v opico.

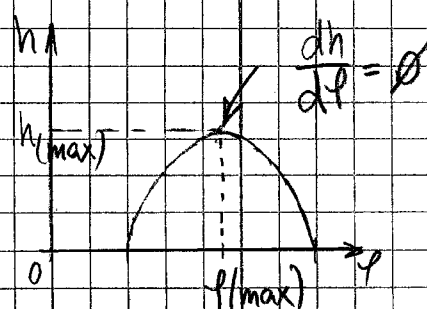
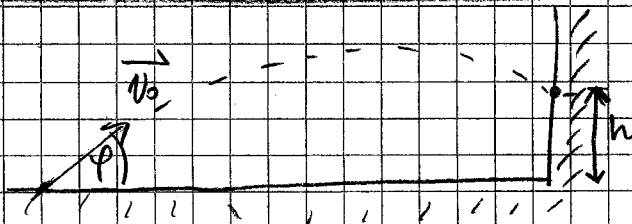
30. naloga, str. 21

$$v_0 = 16 \text{ m/s}$$

$$d = 7 \text{ m}$$

$$\varphi(h = \max)?$$

$$h(\varphi)$$



$$d = v_0 \cos \varphi \cdot t \Rightarrow t = \frac{d}{v_0 \cos \varphi}$$

$$h = v_0 \sin \varphi \cdot t - \frac{gt^2}{2} = v_0 \sin \varphi \cdot \frac{d}{v_0 \cos \varphi} - \frac{g}{2} \left(\frac{d}{v_0 \cos \varphi} \right)^2$$

$$h(\varphi) = \tan \varphi \cdot d - \frac{gd^2}{2v_0^2 \cos^2 \varphi}$$

$$h(\varphi) = d \cdot \tan \varphi - \frac{gd^2}{2v_0^2} \cdot \frac{1}{\cos^2 \varphi}$$

$$h'(\varphi) = 0$$

$$\frac{d}{\cos^2 \varphi} - \frac{gd^2}{2v_0^2} \cdot \frac{2 \sin \varphi}{\cos^3 \varphi} = 0$$

$$\frac{d}{\cos^2 \varphi} \left(1 - \frac{gd}{v_0^2} \frac{\sin \varphi}{\cos \varphi} \right) = 0 \Rightarrow \left(1 - \frac{gd}{v_0^2} \tan \varphi \right) = 0$$

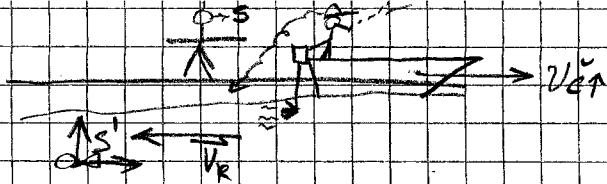
$$\tan \varphi = \frac{v_0^2}{gd}$$

$$\Rightarrow \varphi = \arctan \frac{v_0^2}{gd}$$

$$\frac{(16 \frac{\text{m}}{\text{s}})^2}{(9.8 \frac{\text{m}}{\text{s}^2}) \cdot 7 \text{ m}} = \arctan \frac{256 \frac{\text{m}^2}{\text{s}^2}}{68.6 \frac{\text{m}^2}{\text{s}^2}} = \arctan 3.73 \Rightarrow \varphi = 74.9^\circ$$

21. naloga / str. 11

$$\begin{aligned} v_R &= -5,5 \text{ km/h} \\ v_{\text{čt}} &= 9 \text{ km/h} \end{aligned} \quad \left. \begin{array}{l} \text{glede na} \\ \text{breg} \end{array} \right\}$$



kapitonu pade kapa v vodo

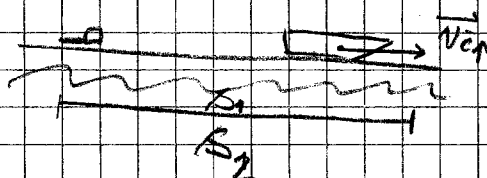
$$t_1 = 27 \text{ min}$$

$$v_{\text{čt}}' = -v_{\text{čt}} \rightarrow \text{glede na vodo je hitrost enaka}$$

$$t_2 = ?$$

GLEDE NA VODO (S'):

- kapa miruje



$$S_1 + S_2 = 0$$

$$v_{\text{čt}}' t_1 + v_{\text{čt}}' t_2 = 0$$

$$v_{\text{čt}} t_1 - v_{\text{čt}} t_2 = 0$$

$$t_1 = t_2 = 27 \text{ min}$$

GLEDE NA BREG (S):

$$v_{\text{čt}}' = ?$$

$$v_{\text{čt}}' = -v_{\text{čt}}$$

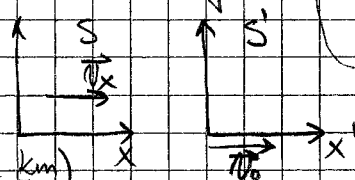
$$v_{\text{čt}} - v_R = - (v_{\text{čt}} - v_R)$$

$$v_{\text{čt}} = -v_{\text{čt}} + 2v_R$$

$$v_{\text{čt}} = -9 \text{ km/h} + 2(-5,5 \frac{\text{km}}{\text{h}})$$

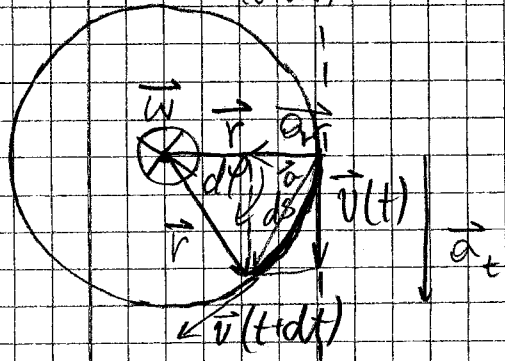
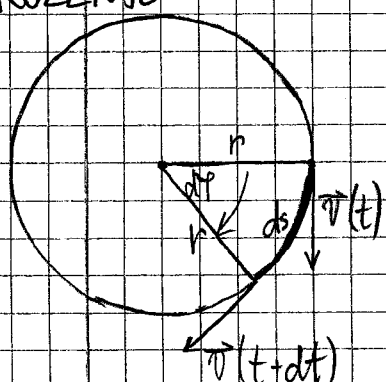
$$v_{\text{čt}} = -20 \text{ km/h}$$

GALILEI-jeva (klasična) transformacija:



$$v_x' = v_x - v_0$$

KROŽENJE



$$\varphi = \frac{s}{r} [\text{rad}]$$

$$v = \frac{ds}{dt} = \frac{r \cdot d\varphi}{dt} = r\omega$$

KOTNA HITROST:

$$\omega = \frac{d\varphi}{dt}$$

KOTNI POSPEŠEK:

$$\alpha = \frac{d\omega}{dt}$$

KROŽILNA HITROST:

$$v = r \cdot \omega$$

$$\vec{v} = \vec{r} \times \vec{\omega}$$

37. nal / str. 27

VENTILATOR

$$\nu_0 = 50 \text{ Hz}$$

$$\omega_0 = 2\pi\nu_0 = 100\pi \frac{\text{rad}}{\text{s}}$$

$$\alpha = \text{konst.}$$

$$t_1 = 100 \text{ s} \Leftarrow \text{se ustavi}$$

$$\omega(t_1) = \varnothing$$

$$N = ?$$

$$N = \frac{\varphi}{2\pi} = ?$$

$$N = \frac{5000\pi \text{ rad}}{2\pi} = \underline{\underline{2500}}$$

$$a = \text{konst}$$

$$\alpha = \text{konst}$$

$$\begin{aligned} v &= v_0 + at & v &\rightarrow w \\ \alpha &= \nu_0 t + \frac{at^2}{2} & \alpha &\rightarrow \varphi \\ & & a &\rightarrow \alpha \end{aligned}$$

$$\omega(t_1) = \omega_0 + \alpha t_1$$

$$\varnothing = \omega_0 + \alpha t_1$$

$$\alpha = -\frac{\omega_0}{t_1} = \frac{100\pi \frac{\text{rad}}{\text{s}}}{100 \text{ s}} = \pi \frac{\text{rad}}{\text{s}^2}$$

$$\varphi = \omega_0 t_1 + \left(-\frac{\omega_0}{t_1}\right) \cdot \frac{t_1^2}{2}$$

$$\varphi = \omega_0 t_1 + \frac{\omega_0 t_1}{2} = \frac{\omega_0 t_1}{2} = \frac{100\pi \frac{\text{rad}}{\text{s}} \cdot 100 \text{ s}}{2} = 5000\pi \text{ rad}$$

TANGE KOMPONENTE KROŽNEGA POSPEŠKA

- tangentska komponenta:
tangentska

$$a_t = \frac{dv}{dt} = \frac{r \cdot d\omega}{dt} = r \cdot \alpha$$

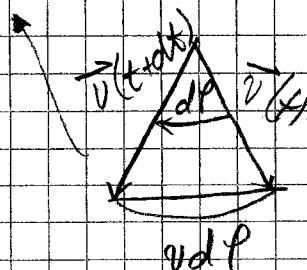
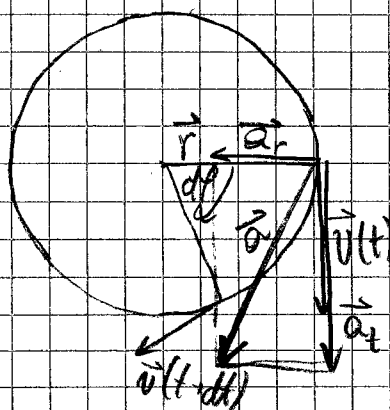
- radialna komponenta:

$$a_r = \left| \frac{d\vec{v}}{dt} \right| = \frac{v \cdot d\varphi}{dt} = v \cdot \omega$$

$v = \text{konst}$

$$a_r = v \cdot \omega = \omega^2 \cdot r = \frac{v^2}{r}$$

$$\vec{a}_r = \vec{\omega} \times \vec{v}$$



$$\omega_0 = 0,02 \frac{\text{rad}}{\text{s}}$$

$$\omega(t) = \omega_0 + kt^{\frac{1}{2}}$$

$$k = 0,5 \text{ s}^{-\frac{3}{2}}$$

$$r = 5 \text{ m}$$

$$t = 10 \text{ s}$$

$$v(t_1) = ?$$

$$a_t(t_1) = ?$$

$$a_r(t_1) = ?$$

$$v(t_1) = r\omega(t_1) = 5 \text{ m} \cdot \left(0,02 \frac{\text{rad}}{\text{s}} + 0,05 \text{ s}^{-\frac{3}{2}} \cdot \sqrt{10 \text{ s}}\right)$$

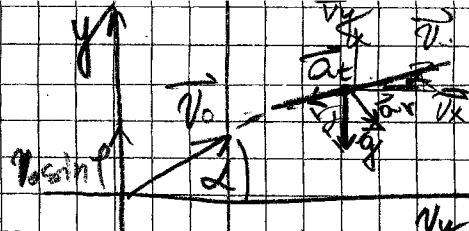
$$v(t_1) = 8,0 \frac{\text{m}}{\text{s}}$$

$$a_r(t_1) = \frac{v^2}{r} = \frac{64 \frac{\text{m}^2}{\text{s}^2}}{5 \text{ m}} = 12,8 \frac{\text{m}}{\text{s}^2}$$

$$a_t(t_1) = r \cdot \alpha$$

$$a_t = \frac{r \cdot k}{2\sqrt{t}} = \frac{5 \text{ m} \cdot 0,5 \text{ s}^{-\frac{3}{2}}}{2 \cdot \sqrt{10 \text{ s}}} = 0,4 \frac{\text{m}}{\text{s}^2}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d(\omega_0 + kt^{\frac{1}{2}})}{dt} = 0 + \frac{1}{2} kt^{-\frac{1}{2}} = \frac{k}{2\sqrt{t}}$$



$$\begin{aligned}\varphi &= 50^\circ \\ v_0 &= 30 \text{ m/s} \\ t_1 &= 2 \text{ s}\end{aligned}$$

$$\sin \alpha = \frac{v_y}{v}$$

$$a_{\text{v smeni gibanja}} = ? = a_t$$

$$a_t = -g \frac{v_y}{v}$$

$$a_t = -g \sin \alpha$$

$$\begin{aligned}v_y &= v_0 \sin \varphi - gt \\ v &= \sqrt{v_x^2 + v_y^2}\end{aligned}$$

$$= \sqrt{(v_0 \cos \varphi)^2 + (v_0 \sin \varphi - gt)^2}$$

$$v_x = v_0 \cos \varphi \quad \rightarrow \quad \sqrt{v_0^2 \cos^2 \varphi + v_0^2 \sin^2 \varphi - 2v_0 \sin \varphi gt + g^2 t^2}$$

$$a_t \approx -1,69 \frac{\text{m}}{\text{s}^2}$$

$$\sqrt{371,86 + 11,43} = 19,58 \frac{\text{m}}{\text{s}}$$

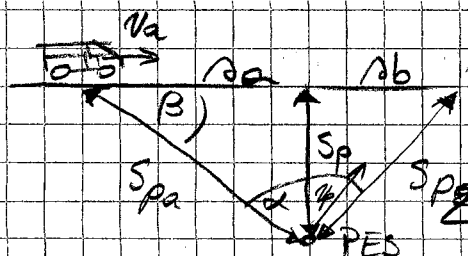
$$a_t = \frac{dv}{dt} = \frac{d(\sqrt{(v_0 \cos \varphi)^2 + (v_0 \sin \varphi - gt)^2})}{dt}$$

DN!

Sila 20. nal. / str. 11

$$\begin{aligned}
 S_p &= 50 \text{ m} \\
 v_a &= 10 \text{ m/s} \\
 v_p &= 3 \text{ m/s} \\
 r_{pa} &= 200 \text{ m}
 \end{aligned}$$

$$\angle = ?$$

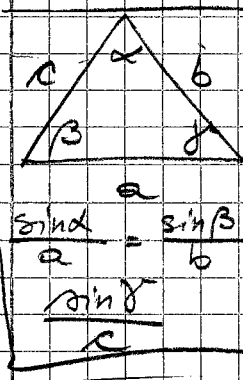


$$\begin{aligned}
 S_a + S_b &= v_a \cdot t \\
 S_{p2} &= v_p \cdot t
 \end{aligned}$$

$$\frac{\sin \alpha}{S_a + S_b} = \frac{\sin \beta}{S_{p2}}$$

$$\sin \beta = \frac{r_{pa}}{r_{pa}}$$

$$\begin{aligned}
 \sin \alpha &= \frac{S_p (S_a + S_b)}{r_{pa} \cdot r_{pa}} = \frac{S_p \cdot v_a \cdot t}{v_p \cdot t \cdot S_{p2}} = \frac{S_p \cdot v_a}{v_p \cdot S_{p2}} \\
 \sin \alpha &= \frac{50 \text{ m} \cdot 10 \text{ m/s}}{3 \text{ m/s} \cdot 200 \text{ m}} = \frac{500}{600} \Rightarrow \alpha = 56,4^\circ
 \end{aligned}$$



Newton - ov zakon:

$$\vec{F} = m \cdot \vec{a}$$

$$\sum_i \vec{F}_i + \vec{F}_s = m \cdot \vec{a}$$

vse zunanje sile

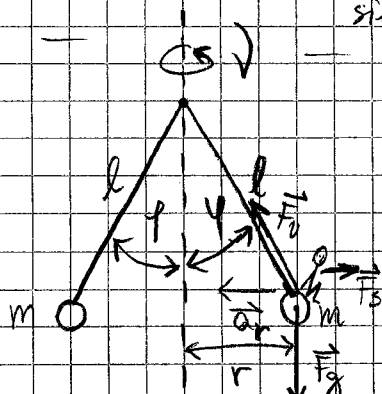
sistemska sila

$$\vec{F}_s = -m \vec{a}_s$$

pospešek opazovalnega sistema

primer:

$$\begin{aligned}
 l &= 0,8 \text{ m} \\
 \varphi &= 60^\circ \\
 \omega &= ?
 \end{aligned}$$

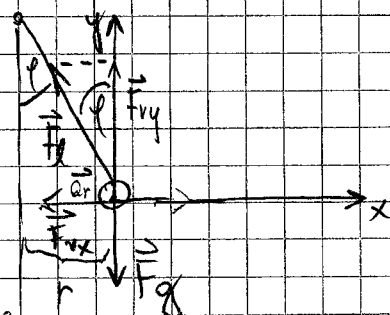


Na 2h 0,8m dolgih vrvičah sta pritrjeni kroglički.

Skolikšino v krožni kroglji okrog navpičnice, če sta od nje oddaljeni za 60°?

1. NEPOSREŠEN SISTEM - LABORATORIJSKI SISTEM

kroženje je pospešeno gibanje.



$$r = l \cdot \sin \varphi$$

$$m \cdot \omega^2 \cdot l \cdot \cos \varphi = m \cdot g$$

$$\omega^2 = \frac{g}{l \cdot \cos \varphi}$$

$$\omega = \sqrt{\frac{g}{l \cdot \cos \varphi}} = \sqrt{\frac{9,8 \frac{\text{m}}{\text{s}^2}}{0,8 \text{ m} \cdot \frac{1}{2}}} = \sqrt{\frac{9,8}{0,4 \text{ s}^2}} = 4,95 \frac{1}{\text{s}}$$

$$\omega = 2\pi \nu \quad \nu = \frac{\omega}{2\pi} = \frac{4,95}{2\pi \text{ s}} = 0,79 \text{ Hz}$$

$$y: -F_g + F_{ra} \cdot \cos \varphi = m \cdot a_y$$

$$F_r \cdot \cos \varphi = F_g$$

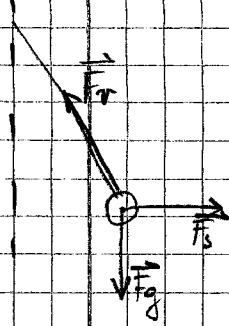
$$x: -F_r \cdot \sin \varphi = m \cdot a_x = ar$$

$$-F_r \cdot \sin \varphi = m(\omega^2 \cdot r)$$

$$F_r \cdot \sin \varphi = m \cdot \omega^2 \cdot l \cdot \sin \varphi$$

$$F_r = m \omega^2 \cdot l$$

(2.) SYSTEM KROGLICE



$$\vec{F}_s = -m\vec{a}_s \Rightarrow a_r$$

$$F_s = -m(-\omega^2 r) = m\omega^2 r$$

$$\underline{X}: -F_r \sin \varphi + m\omega^2 r = 0$$

Sile na klanec

$$\begin{aligned} \alpha &= 10^\circ \\ v_0 &= 14 \text{ m/s} \\ k_t &= 0,1 \\ s_1 &= 20 \text{ m} \\ v_2(s) &=? \end{aligned}$$

$$\underline{y}: -F_s + F_n = 0$$

$$\underline{X}: -F_d - F_t = m \cdot a$$

$$-mg \cdot \sin \alpha - mg \cdot \cos \alpha \cdot k_t = m \cdot a$$

$$a = -g(\sin \alpha + k_t \cdot \cos \alpha)$$

$$v^2 = v_0^2 + 2as$$

$$v_2^2 = v_0^2 + 2 \cdot (-g(\sin \alpha + k_t \cdot \cos \alpha)) \cdot s_1$$

$$= (14 \text{ m/s})^2 - 2 \cdot 9,8 \frac{\text{m}}{\text{s}^2} (\sin 10^\circ + 0,1 \cdot \cos 10^\circ) \cdot 20 \text{ m}$$

$$= \sqrt{196 \frac{\text{m}^2}{\text{s}^2} - 106,67 \frac{\text{m}^2}{\text{s}^2}} = 9,45 \frac{\text{m}}{\text{s}} \quad v=0$$

4. nal. / str. 29

$$\begin{aligned} \alpha &= 10^\circ \\ t_2 &= 2t_1 \\ k_t &=? \end{aligned}$$

$$\begin{aligned} A_1 + A_2 &= 0 \\ \text{GOR:} \\ a_1 &= -g(\sin \alpha + k_t \cos \alpha) \end{aligned}$$

bol:

$$a_2 = ?$$

$$-F_d + F_t = m \cdot a_2$$

$$-m \cdot g \cdot \sin \alpha + k_t \cdot mg \cdot \cos \alpha = m \cdot a_2$$

$$a_2 = -g(\sin \alpha - k_t \cos \alpha)$$

$$s_1 = v_0 t_1 + \frac{a_1 t_1^2}{2}$$

$$s_1 + s_2 = 0$$

$$v = v_0 + \frac{a_1 t_1}{1}$$

$$v_0 = \frac{a_1 t_1}{2}$$

$$s_1 = -a_1 t_1^2 + \frac{a_1 t_1^2}{2} = -\frac{a_1 t_1^2}{2}$$

$$s_2 = \frac{a_2 t_2^2}{2}$$

$$-\frac{a_1 t_1^2}{2} + \frac{a_2 t_2^2}{2} = 0$$

$$a_2 4 t_1^2 = + a_1 t_1^2$$

$$4a_2 = a_1$$

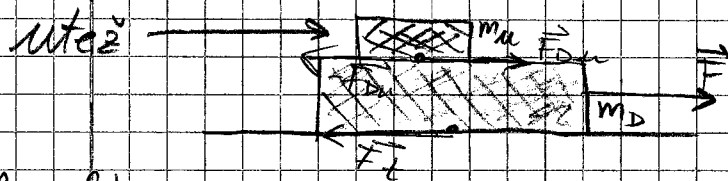
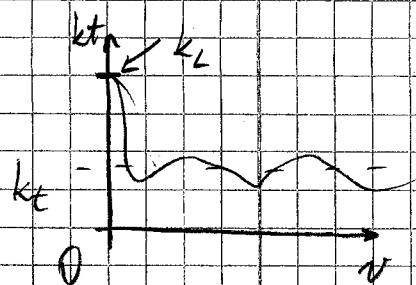
$$4g(\sin \alpha - k_t \cos \alpha) = 2g(\sin \alpha + k_t \cos \alpha)$$

$$4\sin \alpha - 4k_t \cos \alpha = \sin \alpha + k_t \cos \alpha$$

$$3\sin \alpha = 5k_t \cos \alpha$$

$$k_t = \frac{3\sin \alpha}{5\cos \alpha} = \frac{3}{5} \tan \alpha$$

$$k_t \approx 0,1$$



$$\begin{aligned} m_D &= 2 \text{ kg} \\ m_u &= 1 \text{ kg} \\ k_t &= 0,4 \\ k_f &= 0,3 \end{aligned}$$

$$F_{Du} \leq k_l m_u g \leq$$

meja (zdrs):

$$F_{Du} = k_l m_u g$$

$$F = (m_D + m_u) k_t + k_t (m_D + m_u) g$$

$$k_l m_u g = m_u a$$

$$a = k_l g$$

$$F = g(m_D + m_u) \cdot (k_t + k_l) \approx 3 \text{ kg} \cdot 10 \frac{\text{m}}{\text{s}^2} \cdot 0,7 = 21 \text{ N}$$

Na vodoravni podlagi je deska, na njej je utež. k_t med desko in podlago je 0,4; k_f med utežjo in desko je 0,3. S kolikšno silo moramo potegniti, da utež zdrsne?

UTEŽ:

$$F_{Du} = m_u a$$

UTEŽ + DESKA:

$$F - F_t = (m_D + m_u) a$$

$$F_t = k_t (m_D + m_u) g$$

Na vod. podlagi je zaboj z maso M , pritrjena je vrh, prek lahkega skripca speljana do telesa z maso m . Kolikšna je sila trenja, ko se začnejo gibanjati?



$$a_1 = \frac{2\Delta s_1}{t_1^2} = \frac{2 \text{ m}}{1,21 \text{ s}^2} = 1,65 \frac{\text{m}}{\text{s}^2}$$

$$F_g - F_t = (m + M) a$$

$$F_t = -(m + M) a + F_g$$

$$F_t = mg - (m + M) a$$

$$F_t = 0,4 \cdot 10 \frac{\text{m}}{\text{s}^2} - 1,6 \text{ kg} \cdot 1,65 \frac{\text{m}}{\text{s}^2}$$

$$F_t = 4 - 2,64 = 1,36 \text{ N}$$

$$\begin{aligned} M &= 1,2 \text{ kg} \\ m &= 0,4 \text{ kg} \\ \Delta s_1 &= 1 \text{ m} \\ t_1 &= 1,1 \text{ s} \end{aligned}$$

$$F_t = ?$$

ZABOJ:

$$F_v - F_t = Ma$$

UTEŽ:

$$F_v = ma$$

MEGA VESOLJSKA LADJA

$$R = 1 \text{ km}$$

$$\underline{\vec{F}_s = \vec{F}_c} \Rightarrow -\cancel{m} \cdot \vec{a}_s = m \cdot \vec{a}_p$$

$$\gamma = ?$$

$$\vec{a}_r = \vec{\omega} \times (\vec{\omega} \times \vec{R})$$

$$|\vec{Q}_H| = \omega^2 \cdot R$$

$$\omega^2 R = g$$

$$w = \sqrt{\frac{10 \frac{m}{s^2}}{1000 \frac{m}{s^2}}} = 91 \frac{1}{s}$$

$$J = \frac{W}{2\pi} = \frac{0,1 \frac{g}{s}}{2\pi} = \frac{1}{62,8 \frac{s}{min}} \approx \frac{1}{min}$$

$$V_t = 2v \cdot r$$

$$\frac{dr_e}{dt} = \omega \frac{dr}{dt} = \omega \cdot r$$

$\vec{v}_T = 5 \text{ m/s}$
 $m = 75 \text{ kg}$

$$F_c = -ma_r = m\omega^2 \cdot r$$

$$F_{\text{COR}} = I_m \cdot \omega \cdot v_r$$

$$= 2.75 \text{ kg} \cdot \frac{1}{10 \cancel{\text{s}}} \cdot 5 \frac{\text{m}}{\cancel{\text{s}}} =$$

$$= \boxed{175 \text{ N}}$$

VRTENJE $\vec{v}_r$:

A vector diagram on a grid background. A vector labeled $\vec{v}_r(t+dt)$ points from the origin to the top-right. A horizontal vector labeled $\vec{v}_r(t)$ points from the origin to the right. A vertical vector labeled v_{rdy} points from the tip of $\vec{v}_r(t)$ to the tip of $\vec{v}_r(t+dt)$. A small angle $d\theta$ is indicated between the two vectors.

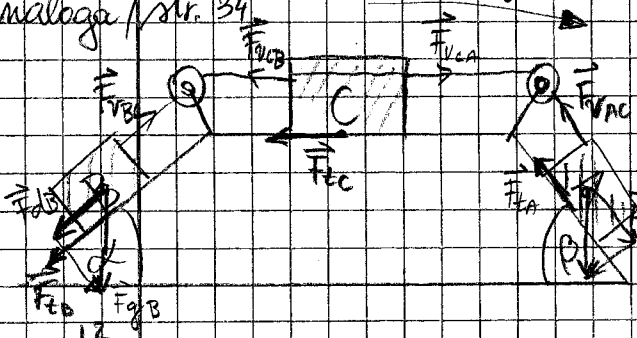
$$\frac{v_r \cdot dr}{dt} = v_r \cdot \omega$$

$$Q_{\text{exp}} = 2 \text{ W } \checkmark$$

$$\vec{a}_{\text{COR}} = -2\vec{\omega} \times \vec{r}$$

14. malaga / str. 34

11. 11. 2010



$$\begin{aligned} m_A &= 5 \text{ kg} \\ m_B &= 3 \text{ kg} \\ m_C &= 2 \text{ kg} \\ \alpha &= 30^\circ \\ \beta &= 60^\circ \\ \mu_k &= 0,1 \end{aligned}$$

$$f = 3$$

$$A(t_n) = ?$$

$$F_{dA} > F_{dB}$$

$$A = \frac{at^2}{2}$$

$$\left. \begin{array}{l} A: F_{dA} - F_{tAC} - F_{tA} = m_A \cdot a \\ B: F_{tBC} - F_{dB} - F_{tB} = m_B \cdot a \\ C: F_{tCA} - F_{tCB} - F_{tC} = m_C \cdot a \end{array} \right\} \Rightarrow F_{dA} - F_{tA} - F_{dB} - F_{tB} - F_{tC} = a(m_A + m_B + m_C)$$

$$a = \frac{F_{dA} - F_{cA} - F_{dB} - F_{cB} - F_{cC}}{m_A + m_B + m_C} = \frac{m_A \cdot g \cdot \sin \beta - k \cdot m_A \cdot g \cdot \cos \beta - m_B \cdot g \cdot \sin \alpha - k \cdot m_B \cdot g \cdot \cos \alpha - k \cdot m_C}{m_A + m_B + m_C}$$

$$Q = \frac{(43,3 - 25 - 15 - 2,6 - 2) \cdot 10^{-3} \text{ m/s}^2}{10^{-3}} = 2,12 \frac{\text{m}}{\text{s}^2}$$

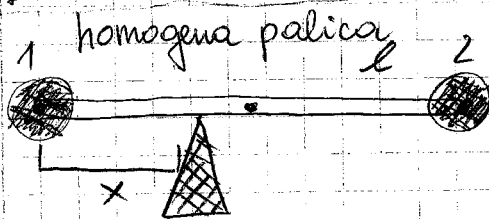
$$S = \frac{(2,12 \frac{\text{m}}{\text{s}})^2 \cdot (3\text{s})^2}{2} = 9,54 \text{ m}$$

RAČUNANJE TEŽIŠČA SISTEMA TELES

$$\vec{r}^* = \frac{\sum_i m_i \cdot \vec{r}_i}{\sum_i m_i}$$

$$\vec{r}^* = \frac{\int \vec{r} \, dm}{\int dm}$$

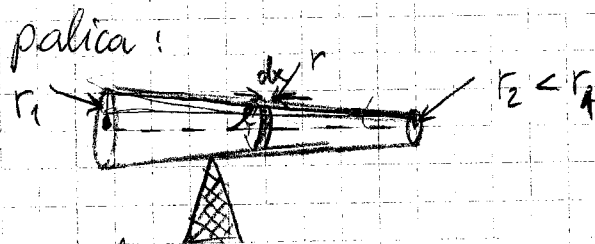
$l = 1m$
 $m_l = 0,5kg$
 $m_1 = 1kg$
 $m_2 = 2kg$
 $x^* = ?$



$$x^* = \frac{m_1 \cdot 0 + m_2 \cdot l + m_l \cdot \frac{l}{2}}{m_l + m_1 + m_2}$$

$$x^* = \frac{2kg \cdot 1m + 0,25kg \cdot 0,5m}{3,5kg} =$$

$$x^* = \frac{2,25m}{3,5} = 0,64m = 64cm$$



$\rho = 700 kg/m^3$
 $l = 1m$
 $r_1 = 10cm$
 $r_2 = 2cm$

$$k = \frac{\Delta r}{\Delta x} = \text{const}$$

$x^* = ?$ (od levega težišča)

$$x^* = \frac{\int x \, dm}{\int dm}$$

$$DM: x^* = \frac{\int_0^l x \rho \pi r^2 dx}{\int_0^l \rho 2\pi r^2 dx}$$

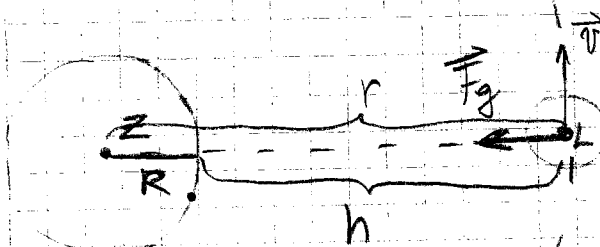
$$dm = \rho \cdot dV = \rho \cdot \pi r^2 \cdot dx$$

$R: 39cm$

$r = \text{linearna funkcija } x!$

$$r(x) = \left(\frac{2}{25}x\right)$$

GRAVITACIJSKI ZAKON:



$$\vec{F}_g = m\vec{g}$$

$$F_g = G \frac{m_2 \cdot m}{r^2}$$

$$g = G \frac{m_2}{r^2}$$

$$g_0 = G \frac{m_2}{R^2} \quad (R=r)$$

$$g = g_0 \frac{R^2}{r^2} =$$

$$g = g_0 \frac{R^2}{(R+h)^2}$$

$g_0 = 9,81 \frac{m}{s^2}$
 $R = 6400 km$

$$h = \left(g = \frac{g_0}{4}\right) = ?$$

$$g = \frac{g_0}{4} = g_0 \frac{R^2}{(R+h)^2}$$

$$4R^2 = (R+h)^2$$

$$4R^2 = R^2 + 2Rh + h^2$$

$$h^2 + 2Rh - 3R^2 = 0$$

$$h_{1,2} = \frac{-2R \pm \sqrt{4R^2 + 12R^2}}{2} = \frac{-2R \pm 4R}{2}$$

$$h_1 = \frac{2R}{2} = R$$

$$h_2 = \frac{-6R}{2} = -3R$$

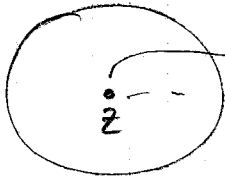
masa lune

$$\frac{g_{02}}{g_{0L}}$$

$$\frac{m_2}{R_2^2} = \frac{m_L}{R_L^2}$$

$$\frac{81m_L}{(3,7R_L)^2} = \frac{m_L}{R_L^2}$$

$$\frac{81}{13,69} = \underline{\underline{5,9 \approx 6}}$$



$$M_2 = 81m_L$$

$$r = 385000 \text{ km}$$

$$x = ?$$

$$|\vec{F}_{g2}| = |\vec{F}_{gL}|$$

$$M \cdot g_2 = M \cdot g_L$$

$$\frac{m_2}{(r-x)^2} = \frac{m_L}{x^2}$$

$$\frac{81m_L}{r^2 - 2rx + x^2} = \frac{m_L}{x^2}$$

$$r^2 - 2rx + x^2 = 81x^2$$

$$r^2 - 2rx - 80x^2 = 0$$

$$x_1 = -\frac{r}{80}$$

$$x_2 = \frac{r}{10}$$

$$x_2 = 38.500 \text{ km}$$

na kateri višini kroži satelit?

$$v = 6 \text{ km/s}$$

$$g_0 = 9,81 \text{ m/s}^2$$

$$R = 6400 \text{ km}$$

$$a_r = \frac{v^2}{r} = \omega^2 \cdot r$$

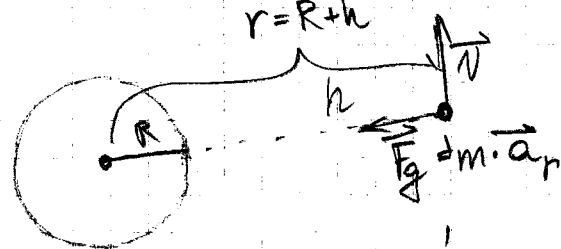
$$F_g = mg = ma_r$$

$$g = g_0 \frac{R^2}{(R+h)^2} = \frac{v^2}{(R+h)}$$

$$\frac{g_0 R^2}{v^2} = R+h$$

$$h = \frac{g_0 R^2}{v^2} - R = \frac{(6400 \cdot 10^3 \text{ m})^2 \cdot 9,81 \frac{\text{m}}{\text{s}^2}}{(6 \cdot 10^3 \text{ m/s})^2} - 6400 \cdot 10^3 \text{ m} =$$

$$= \underline{\underline{4761,6 \text{ km} \approx 4800 \text{ km}}}$$



GEOSTACIONARNI SATELIT:

$$t_0 = 24 \text{ h}$$

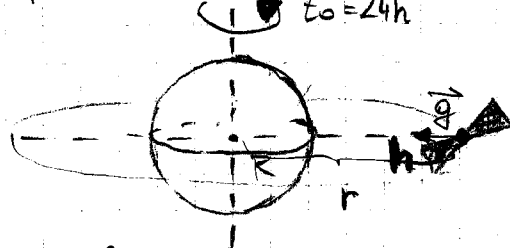
$$R = 6400 \text{ km}$$

$$g_0 = 9,81 \text{ m/s}^2$$

$$h = ?$$

$$\omega = \frac{2\pi}{t_0} = \frac{2\pi}{24 \text{ h}} = \frac{1 \cdot \pi}{12 \text{ h}}$$

$$a_r = \omega^2 \cdot r$$



$$g_0 \frac{R^2}{(R+h)^2} = \left(\frac{2\pi}{t_0}\right)^2 \cdot (R+h)$$

$$g = a_r \Rightarrow \frac{g_0 R^2 t_0^2}{4\pi^2} = (R+h)^3 \Rightarrow \sqrt[3]{\frac{g_0 R^2 t_0^2}{4\pi^2}} - R = h$$

$$h = 35954 \text{ km} \approx 36 \cdot 10^3 \text{ km}$$

GIBALNA KOLIČINA:

IZREK O GIB. KOL.

$$\vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} \int$$

$$\int_{t_1}^{t_2} \vec{F} \cdot dt = \int_{\vec{v}_1}^{\vec{v}_2} m d\vec{v} = m \cdot \vec{v}_2 - m \cdot \vec{v}_1 = \Delta \vec{G}$$

$$\int \vec{F} dt = \Delta \vec{G}$$

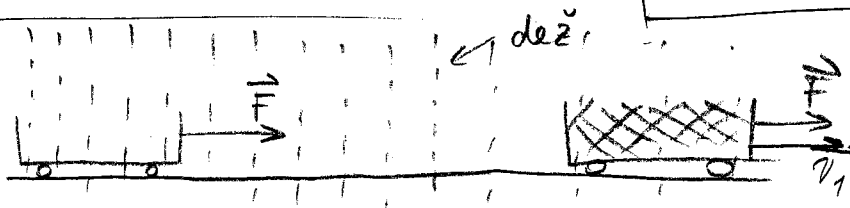
SUNEK ZUNANJIH SIL

$$\vec{G} = m \cdot \vec{v}$$

7. naloga / str. 71

$$m_0 = 1 \text{ kg}$$

$$F = 0,8 \text{ N}$$



$$\phi_m = \frac{dm}{dt} = \frac{95 \text{ g}}{1 \text{ s}} = \frac{95 \text{ g}}{\text{s}}$$

$$v_0 = 0$$

$$v(t_1 = 14 \text{ s}) = ?$$

$$m(t) = m_0 + \phi_m \cdot t$$

$$F = m \cdot a$$

$$a = \frac{F}{m} = \frac{F}{m_0 + \phi_m \cdot t}$$

$$a = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v_1} dv = \int_0^{t_1} a dt$$

$$v_1 = \int_0^{t_1} \frac{F dt}{m_0 + \phi_m \cdot t}$$

$$\frac{dv}{dt}$$

$$m = m_0 + \phi_m \cdot t$$

$$dm = \phi_m \cdot dt$$

$$dt = \frac{dm}{\phi_m}$$

metoda ni prava!

$$v_1 = \frac{F}{\phi_m} \int_{m_0}^{m_1} \frac{dm}{m} = \frac{F}{\phi_m} \cdot \ln|m| \Big|_{m_0}^{m_1} = \frac{F}{\phi_m} \ln \frac{m_1}{m_0}$$

$$v_1 = \frac{F}{\phi_m} \ln \frac{m_0 + \phi_m \cdot t_1}{m_0} = \frac{F}{\phi_m} \ln \left(1 + \frac{\phi_m \cdot t_1}{m_0} \right)$$

$$v_1 = 7,1 \frac{\text{m}}{\text{s}}$$

$$\lim_{t \rightarrow \infty} v = \infty$$

(2) VODA, KI PADE NA VOZIČEK V t_1 , + VOZIČEK:

$$\int_0^{t_1} F dt = \Delta G$$

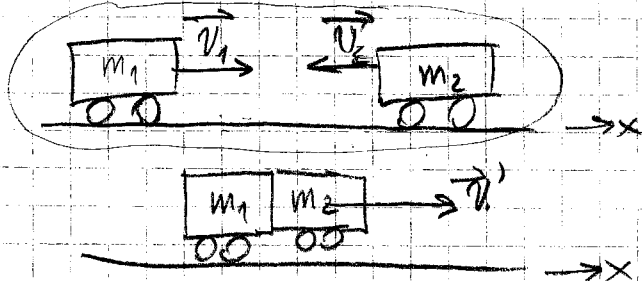
$$F \cdot t_1 = m_1 v_1$$

$$F \cdot t_1 = (m_0 + \phi_m t_1) \cdot v_1$$

$$v_1 = \frac{F \cdot t_1}{(m_0 + \phi_m \cdot t_1)} = 4,8 \frac{\text{m}}{\text{s}}$$

$$\lim_{t \rightarrow \infty} v = \frac{F}{\phi_m} = \frac{F}{\phi_m}$$

DO TE VREDNOSTI HITROST NARAŠČA, NATO PA VOZIČEK NE POSPEŠUJE VEČ, VES SUNEK SILE SE PORABI ZA POSPEŠEVANJE KAPLJIC DO HITROSTI VOZIČKA.



$$m_1 = 2 \text{ kg} \quad m_2 = 5 \text{ kg}$$

$$v_1 = 3 \text{ m/s} \quad v_2 = -1,5 \text{ m/s}$$

$$v' = ?$$

$$\int \vec{F} dt = \Delta \vec{G} \Rightarrow \vec{G}_z = \vec{G}_k$$

$$m_1 \cdot v_1 + m_2 \cdot v_2 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 \cdot v_1 + m_2 \cdot v_2}{(m_1 + m_2)}$$

$$v' = \frac{6 \frac{\text{kg} \cdot \text{m}}{\text{s}} - 7,5 \frac{\text{kg} \cdot \text{m}}{\text{s}}}{7 \text{ kg}} = -0,21 \frac{\text{m}}{\text{s}}$$

18.11.2010

1.) točkasto telo na dani razdalji:

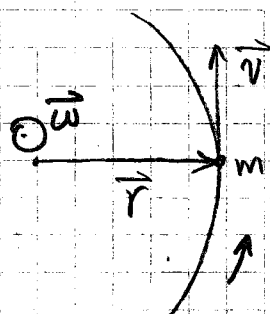
$$\Delta \vec{L} = \vec{M} \Delta t$$

$$\vec{L} = \vec{r} \times \vec{G} = \vec{r} \times m\vec{v}$$

$$\vec{L} = m \vec{r} \times \vec{v} = m \vec{r} \times (\vec{\omega} \times \vec{r})$$

$$\vec{L} = m \vec{r} \cdot \vec{r} \vec{\omega} - m (\vec{\omega} \cdot \vec{r}) \vec{r}$$

$$\vec{L} = m \vec{r}^2 \vec{\omega} - m (\vec{\omega} \cdot \vec{r}) \vec{r}$$

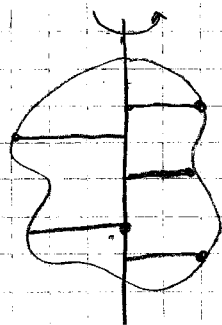


vztrajnostni moment

$$\vec{L} = m \vec{r}^2 \cdot \vec{\omega} = J \cdot \vec{\omega}$$

2.) togo telo:

$$J = \int r^2 dm$$



1. TOČK. TELO m

$$J = m \cdot r^2$$

2. TANEK OBRČ r

$$J = m \cdot r$$

3. HOMOGEN VAL r_i

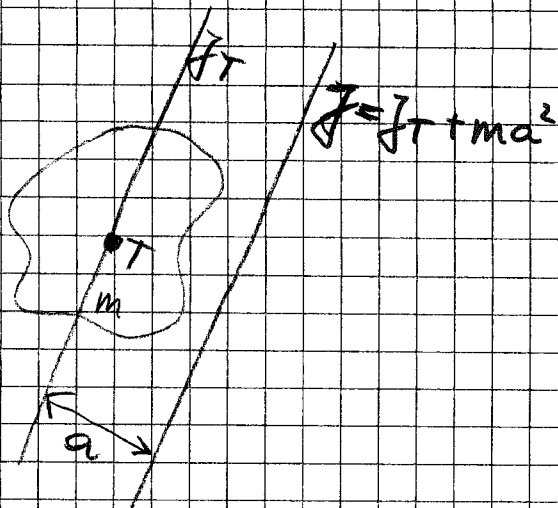
$$J = \frac{m r^2}{2}$$

4. KROGLA $J = \frac{2}{5} m r^2$

5. $J = \frac{m l^2}{3}$

6. $J = \frac{m l^2}{12}$

Steinerjeva formula



$$d\vec{P} = \vec{M} dt$$

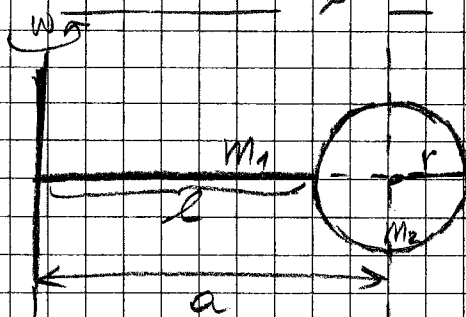
$$\vec{M} = \frac{d\vec{P}}{dt} = \frac{d}{dt} (J \vec{\omega}) = \frac{dJ}{dt} \vec{\omega} + J \frac{d\vec{\omega}}{dt} \quad \boxed{\vec{M} = J \cdot \vec{\alpha}} \Rightarrow \text{N-zakon za vrtenje}$$

$$m_1 = 0,4 \text{ kg}$$

$$l = 0,3 \text{ m}$$

$$m_2 = 0,6 \text{ kg}$$

$$r = 0,12 \text{ m}$$



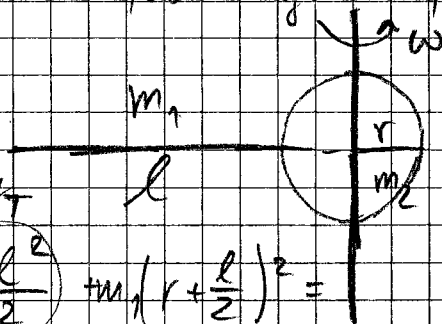
$$J = \frac{m_1 l^2}{3}$$

$$J = \frac{2}{5} m_2 r^2$$

$$\textcircled{1} \quad J = \frac{m_1 l^2}{3} + \frac{2}{5} m_2 r^2 + m_2 (l+r)^2 = \frac{0,4 \text{ kg} \cdot (0,3 \text{ m})^2}{3} + \frac{2}{5} \cdot 0,6 \text{ kg} \cdot (0,12 \text{ m})^2 + 0,6 \text{ kg} \cdot (0,42 \text{ m})^2$$

$$J = 0,012 \text{ kg m}^2 + 0,00346 \text{ kg m}^2 + 0,111 \text{ kg m}^2 = 0,12 \text{ kg m}^2$$

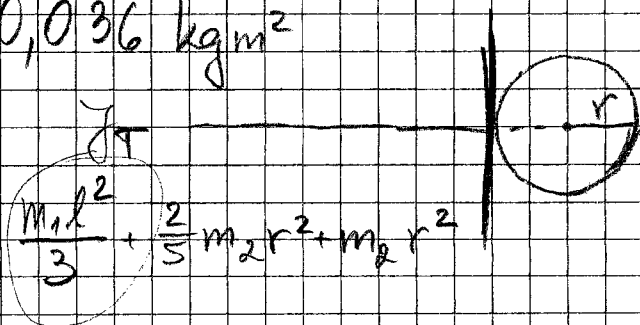
②



$$J = \frac{2}{5} m_2 r^2 + \frac{m_1 l^2}{12} + m_1 \left(r + \frac{l}{2} \right)^2 =$$

$$J = 0,036 \text{ kg m}^2$$

③



$$J = \frac{m_1 l^2}{3} + \frac{2}{5} m_2 r^2 + m_2 r^2$$

N - zakon za kotaljenje

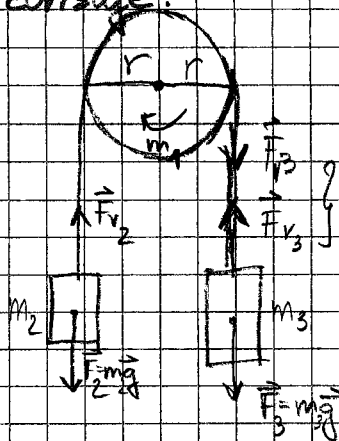
Skripco ima hom. valj z maso m_1 , vrtečiv je doli svoje vodoravne geo. osi. 2 uteži povežemo z zelo lahko vrvico in napeljeno preko skripca. S kolikšnim a se gibljeta uteži, ko ju izpustimo in kolikšna nla deluje na vrvici? Vrvica ne zdrsne.

$$\begin{aligned} m_1 &= 3 \text{ kg} \\ m_2 &= 2 \text{ kg} \\ m_3 &= 5 \text{ kg} \end{aligned}$$

$$a, F_2, F_3$$

$$\begin{aligned} M &= J \alpha \\ M &= r \times F \end{aligned}$$

$$a = r \alpha$$



$$m_3 g - F_3 = m_3 \cdot a$$

$$F_2 - m_2 g = m_2 \cdot a$$

$$F_3 r - F_2 r = \frac{m_1 r^2}{2} \cdot \frac{a}{r}$$

$$F_3 - F_2 = \frac{m_1 \cdot a}{2}$$

$$m_3 g - F_3 + F_2 - m_2 g + F_3 - F_2 = m_3 a + m_2 a + \frac{m_1 a}{2}$$

$$2g(m_3 - m_2) = a(2m_3 + 2m_2 + m_1)$$

$$a = \frac{2g(m_3 - m_2)}{(2m_3 + 2m_2 + m_1)} = \frac{2 \cdot 9,8 \frac{\text{m}}{\text{s}^2} (3 \text{ kg})}{(10 + 4 + 3) \text{ kg}} = \frac{6 \cdot 9,8 \text{ m/s}^2}{17}$$

$$a = 3,46 \text{ m/s}^2$$

$$F_2 = m_2(a + g)$$

$$F_2 = m_2 \left(\frac{2g(m_3 - m_2)}{2m_3 + 2m_2 + m_1} + g \right) = m_2 \cdot \left(g \frac{2m_3 - 2m_2 + 2m_2 + 2m_3 + m_1}{2m_3 + 2m_2 + m_1} \right)$$

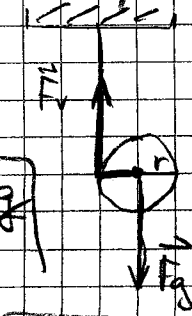
$$F_2 = m_2 g \left(\frac{4m_3 + m_1}{2m_3 + 2m_2 + m_1} \right) = 2 \cdot 9,8 \frac{\text{m}}{\text{s}^2} \left(\frac{20 - 3}{10 + 4 + 3} \right) = \left(\frac{17}{17} \right) \cdot 2 \cdot 9,8 \frac{\text{m}}{\text{s}^2} = 19,6 \text{ N}$$

$$F_3 = m_3 g \frac{4m_2 + m_1}{2m_2 + 2m_3 + m_1}$$

Na hom. valj z m_1 navijemo zelo lahko vrvico, katere drugi konec pritrdimo na strop. Valj začne padati in se vrteč okoli svoje vod. osi. Skolikšnim a pada težišče valja, a kolikšna F deluje na vrvici?

$$m = 2 \text{ kg}$$

$$F = \frac{2mg}{3 \cdot 2} = \frac{mg}{3}$$



$$mg - F = m \cdot a$$

$$M = J \alpha = F \cdot r = \frac{m r^2}{2} \cdot \frac{a}{r}$$

$$F = \frac{ma}{2}$$

$$mg - \frac{ma}{2} = ma$$

$$mg = ma + \frac{ma}{2}$$

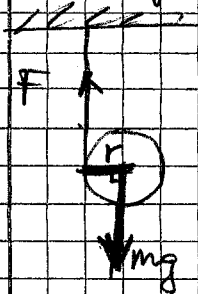
$$mg = a \left(m + \frac{m}{2} \right)$$

$$a = \frac{mg}{m + \frac{m}{2}}$$

$$a = \frac{2 \cdot 9,8 \frac{\text{m}}{\text{s}^2}}{3} = 6,53$$

$$a = 6,53$$

2) Steinerjev izrek



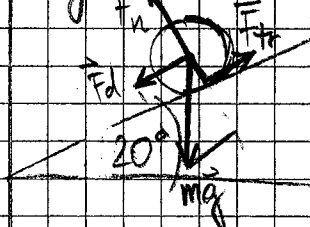
$$mgr = \left(\frac{mr^2}{2} + mr^2 \right) \frac{a}{r}$$

$$g = \frac{3a}{2}$$

$$a = \frac{2g}{3}$$

Hom. krogle je kotli mavedel po klancu z naklonom 20° . Kolikšen je koeficient in pospešek in koeficient trenja? Brez podrisavanja!

$\varphi = 20^\circ$
 a, k



$$ma = F_d - F_{tr} \Rightarrow ma = mg \sin \varphi - kmg \cos \varphi$$

$$M = F_{tr} \cdot r = J \frac{a}{r} \Rightarrow kmg \cos \varphi = \frac{2}{5} m r^2 \frac{a}{r^2}$$

$$\frac{g \sin \varphi - g \cos \varphi}{k \cos \varphi} = \frac{2}{5} a$$

$$a = g (\sin \varphi - k \cos \varphi)$$

$$k g \cos \varphi = \frac{2}{5} a$$

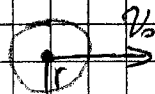
$$k \cos \varphi = \frac{2}{5} (\sin \varphi - k \cos \varphi)$$

$$k = \frac{\frac{2}{5} \sin \varphi}{\cos \varphi + \frac{2}{5} \cos \varphi} = \frac{2 \sin \varphi}{7 \cos \varphi} = \frac{2}{7} \tan \varphi = 0,1$$

$$a = g \left(\sin \varphi - \frac{2}{7} \frac{\sin \varphi}{\cos \varphi} \cos \varphi \right) = g \cdot \frac{5}{7} \sin \varphi = 2,5 \text{ m/s}^2$$

Homogen valj drži s konst. $v = 5 \text{ m/s}$ brez trenja po vodoravni podlagi. Nenadoma pade na hrupano podlago s koef. $k = 0,15$. Po kolikšnem času valj preneha drseti in se začne kotiti?

$v_0 = 5 \text{ m/s}$
 $k = 0,15$
 $t = ?$



$$F_t = m \cdot a$$

$$mgk = m \cdot a$$

zaviranje
težišča

$$a = gk$$

$$v = v_0 - at$$

$$v = v_0 - gkt$$

$$mgk \cdot x = \frac{mr^2}{2} \cdot \alpha$$

$$\alpha = \frac{2gk}{r}$$

$$\omega = \alpha \cdot t = \frac{2gk}{r} \cdot t$$

$$v = \omega r$$

$$v_0 - gkt = \frac{2gk}{r} t \cdot r$$

$$v_0 = 3gkt$$

$$t = \frac{v_0}{3gk} = \frac{5}{0,15 \cdot 3 \cdot 9,8 \frac{\text{m}}{\text{s}^2}} = 1,13 \text{ s}$$

$$t = 1,13 \text{ s}$$

DN (Homogena krogla)

- kinematika ($\vec{r}, \vec{v}, \vec{a}, \vec{\omega}, \dots$)
- sile (N-zakoni) ($\vec{F} = m\vec{a}, \vec{F}_s, \vec{F}_g, \dots$)
- manovr, kotanje
- gibalna količina, vrtilna količina

Izrek o gibalni količini:

$$\int \vec{F} \cdot dt = \Delta \vec{G}$$

$$\vec{G} = m \cdot \vec{v}$$

Izrek o vrtilni količini:

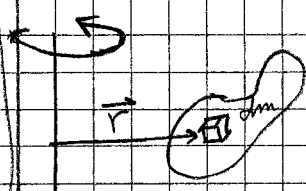
$$\int \vec{M} \cdot dt = \Delta \vec{\Gamma}$$

sume
zmanjših
manovrov

$$I = \int r^2 dm$$

$$\vec{F} = \int \vec{w}$$

$$\vec{\Gamma} = \int dm \vec{r} \times \vec{v}$$



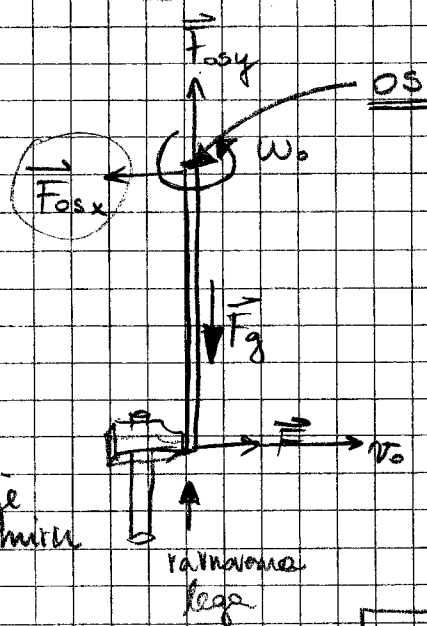
10/str. 73/

$$l = 1m$$

$$m = 1kg$$

$$\int F dt = 1Ns$$

$$v_0 = ?$$



$$\int M dt = \Delta \vec{\Gamma}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$M = r \cdot F \sin \varphi$$

$$\int l \cdot F \cdot dt = \int \vec{w}$$

$$= \vec{\Gamma}' - \vec{\Gamma}$$

$$l \cdot \int F dt = \int w_0 - 0$$

$$l \cdot \int F dt = \frac{ml^2}{3} \cdot \omega_0 / l$$

$$v_0 = \frac{3 \int F dt}{m} = \frac{3 Ns}{1kg} = \frac{3 \frac{kg \cdot m}{s}}{1kg} = \underline{3m/s}$$

$$I = \frac{ml^2}{3}$$

$$v_0 = \omega_0 \cdot l$$

$$\omega_0$$

17. / 18. 76

valjasta plošča na ledcu $\rightarrow$ trenja ni,

$$R = 0,7 \text{ m}$$

$$m_v = 45 \text{ kg}$$

$$m = 75 \text{ kg}$$

$$v = 1 \text{ m/s}$$

$$v = ? , v^* = ?$$

hitrost
težišča
plošče

$$J = \frac{mR^2}{2}$$

$$2m_v + 2m$$

$$\int \vec{F} dt = \Delta \vec{G} \Rightarrow \vec{G}_z = \vec{G}_k$$

$$x: 0 = m_v v_x^* - m \cdot v$$

$$v_x^* = \frac{m \cdot v}{m_v} = \frac{75 \text{ kg}}{45 \text{ kg}} \cdot 1 \text{ m/s} = 1,6 \frac{\text{m}}{\text{s}}$$

$$y: 0 = m_v v_y^* - m \cdot v$$

$$v_y^* = v_x^* = \frac{m}{m_v} \cdot v = 1,6 \frac{\text{m}}{\text{s}}$$

$$v^* = \sqrt{v_x^{*2} + v_y^{*2}} = \sqrt{2} v_x^* \approx 2,4 \frac{\text{m}}{\text{s}}$$

$$\int \vec{M} dt = \Delta \vec{\Gamma}$$

$$\vec{\Gamma}_z = \vec{\Gamma}_k$$

$$\sin 90^\circ = 1$$

$$\omega = 2\pi v$$

$$0 = J \cdot \omega - (m \cdot R \cdot v)$$

$$\omega = \frac{m R v}{J} = \frac{2}{m R} \cdot R \cdot v = \frac{2v}{m R} = 2 \text{ m/s}$$

$$m R \omega = \frac{m_v R^2}{2} \cdot \omega$$

$$\omega = \frac{2 m v}{m_v R}$$

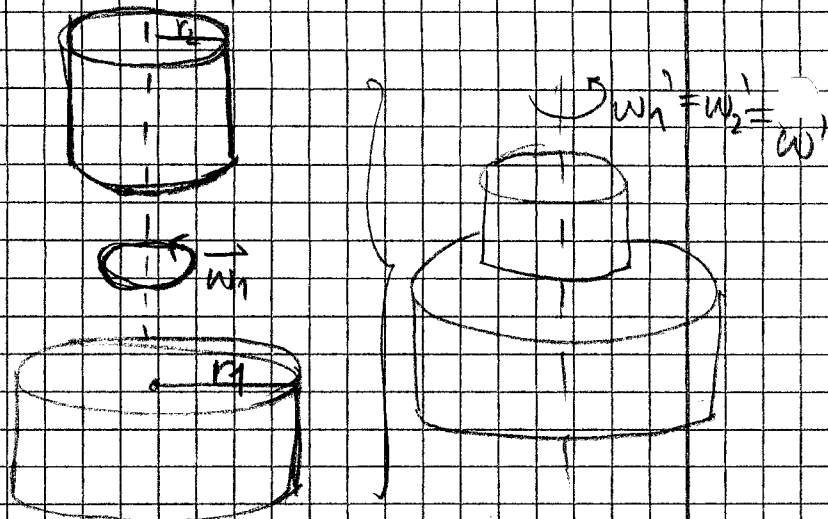
$$v = \frac{8 m v}{\pi m_v R} = \frac{75 \text{ kg} \cdot 1 \text{ m/s}}{\pi \cdot 45 \text{ kg} \cdot 0,7 \text{ m}} = 0,76 \text{ m/s}$$

15/str. 76

primer b): $\omega_2 = 0$

$$\left. \begin{array}{l} m_1 = 3 \text{ kg} \\ r_1 = 12 \text{ cm} \\ \omega_1 = 100 \frac{\text{rad}}{\text{s}} \end{array} \right\} \text{1. valj}$$

$$\left. \begin{array}{l} m_2 = 2 \text{ kg} \\ r_2 = 8 \text{ cm} \\ \omega_2 = 0 \end{array} \right\} \text{2. valj}$$



$$\int M dt = \Delta \Gamma \Rightarrow \Gamma = \Gamma'$$

ni zunanjih
momentov

$$\Gamma = J_1 \omega_1 + J_2 \omega_2$$

$$\Gamma' = J_1 \omega' + J_2 \omega' = \omega' (J_1 + J_2)$$

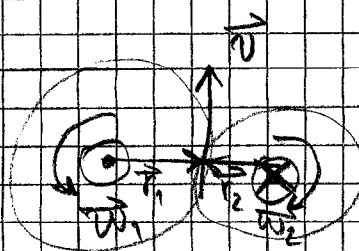
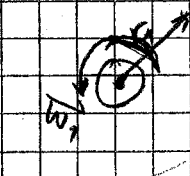
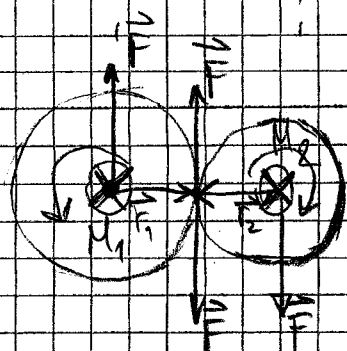
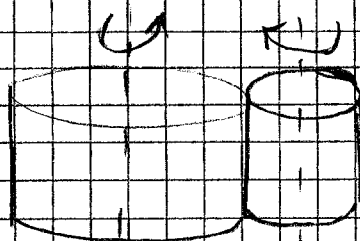
$$J_1 \omega_1 = (J_1 + J_2) \omega'$$

$$\omega' = \frac{J_1 \omega_1}{J_1 + J_2} = \frac{J_1 \omega_1}{1 + \frac{J_2}{J_1}}$$

$$\omega' = \frac{100 \text{ rad/s}}{1 + \frac{m_2 r_2^2}{m_1 r_1^2}} = \frac{100 \text{ rad/s}}{1 + \frac{2 \text{ kg} \cdot 64 \text{ cm}^2}{3 \text{ kg} \cdot 144 \text{ cm}^2}} = 77 \text{ rad/s}$$

primer a):

podatki so
enaki



$$\omega_1' = \omega_2'$$

$$\omega_1' r_1 = -\omega_2' r_2$$

$$\omega_2' = -\frac{r_1}{r_2} \omega_1'$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

DN: sistem
(VALJ 1 + VALJ 2)
steinerjovsko pravilo

1. valj:

$$\int \vec{M}_1 dt = \vec{L}_1' - \vec{L}_1$$

$$\vec{M} = \vec{r} \times \vec{F}$$

$$-\int r_1 F dt = \int_1 \omega_1' - \int_1 \omega_1$$

2. valj:

$$\int \vec{M}_2 dt = \vec{L}_2' - \vec{L}_2$$

$$-\int r_2 F dt = \int_2 \omega_2' - \int_2 \omega_2$$

$$\frac{-\int r_1 F dt}{-\int r_2 F dt} = \frac{\int_1 \omega_1' - \int_1 \omega_1}{\int_2 \omega_2'}$$

$$= \frac{r_1 \int F dt}{r_2 \int F dt} = \frac{\int_1 \omega_1' - \int_1 \omega_1}{\int_2 \omega_2'} \Rightarrow \frac{r_1}{r_2} \int_2 \omega_2' = \int_1 \omega_1' - \int_1 \omega_1$$

$$= \frac{r_1}{r_2} \int_2 \left(-\frac{r_1}{r_2} \omega_1' \right) = \int_1 \omega_1' - \int_1 \omega_1$$

$$= -\frac{r_1^2}{r_2^2} \cdot \frac{\int_2}{\int_1} \omega_1' = \omega_1' - \omega_1$$

$$\omega_1 = \omega_1' \left(1 + \frac{r_1^2 \int_2}{r_2^2 \int_1} \right)$$

$$\omega_1' = \frac{\omega_1}{1 + \frac{m_2 R_2^2}{m_1 R_1^2}}$$

$$= \frac{100 \text{ rad/s}}{1 + \frac{2}{3}}$$

$$\frac{m_2 R_2^2}{2}$$

$$\frac{m_1 R_1^2}{2}$$

$$= \frac{100 \text{ rad/s}}{\frac{5}{3}} = 60 \text{ rad/s}$$

$$60 \text{ rad/s}$$

$$\omega_2' = -\frac{r_1}{r_2} \omega_1' = -\frac{12 \text{ cm}}{8 \text{ cm}} \cdot 60 \text{ rad/s} = -90 \text{ rad/s}$$

$$v^2 = v_0^2 - 2gh$$

$$v_0^2 = 2gh$$

$$h = \frac{v_0^2}{2g}$$

2.12.2009 - 1. Kolokvij

(1.)

$$v_0 = 15 \text{ m/s}$$

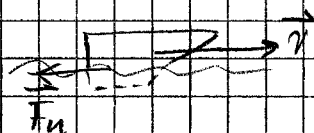
$$t_1 = 15 \text{ s}$$

$$v_1 = 5 \text{ m/s}$$

$$F_u = -k \cdot v$$

$$m = 70 \text{ kg}$$

k



$$F_u = m \cdot a$$

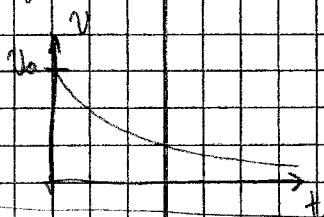
$$-k \cdot v = m \cdot \frac{dv}{dt}$$

$$\int_0^{t_1} \frac{-k}{m} dt = \int_{v_0}^{v_1} \frac{dv}{v}$$

$$\frac{-k}{m} \int_0^{t_1} dt = \int_{v_0}^{v_1} \frac{dv}{v}$$

$$\frac{-k}{m} \cdot t_1 = \ln v \Big|_{v_0}^{v_1} = \ln v_1 - \ln v_0$$

$$v(t) = v_0 e^{-\frac{k}{m}t}$$



$$\frac{-k}{m} t_1 = \ln \frac{v_1}{v_0}$$

$$k = m \frac{\ln \frac{v_1}{v_0}}{t_1} = \frac{70 \text{ kg}}{15 \text{ s}} \ln \frac{5}{15} = 5,12$$

(D)

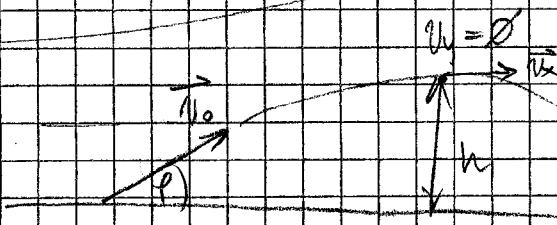
(2)

$$g_L = 1,6 \frac{\text{m}}{\text{s}^2}$$

$$v_0 = 5 \text{ m/s}$$

$$\varphi = 40^\circ$$

a) h



$$v_y = v_0 \sin \varphi - gt$$

$$v_0 \sin \varphi - gt = 0$$

$$t = \frac{v_0 \sin \varphi}{g} = 2 \text{ s}$$

$$h = v_0 \sin \varphi \cdot t - \frac{gt^2}{2}$$

$$h = \frac{v_0^2 \sin^2 \varphi}{g} - \frac{v_0^2 \sin^2 \varphi}{2g} = \frac{v_0^2 \sin^2 \varphi}{2g} = 3,2 \text{ m} \quad (B)$$

b)

$$v_x = v_0 \cdot \cos \varphi = 5 \text{ m/s} \cdot \cos 40^\circ = 3,83 \frac{\text{m}}{\text{s}} \quad (B)$$

c)

$$t = 2 \cdot \frac{v_0 \sin \varphi}{g} = 2 \cdot \frac{5 \cdot \sin 40^\circ}{1,6} = 4,02 \text{ s} \quad (A)$$

(3)

$$v = 8 \frac{\text{m}}{\text{s}}$$

$$R = 50 \text{ m}$$

$$a) k_L = ?$$

$$F_v = m \cdot a_{\frac{v^2}{R}}$$

$$k_L \cdot m \cdot g = m \cdot \frac{v^2}{R}$$

$$k_L = \frac{v^2}{gR} = \frac{64 \frac{\text{m}^2}{\text{s}^2}}{9,8 \frac{\text{m}}{\text{s}^2} \cdot 50 \text{ m}} = 0,13$$

(B)

$$b) a_t = ?$$

$$k_t = 0,8$$

$$F = m \cdot a$$

$$k_t m g = m (\sqrt{a_r^2 + a_t^2})$$

$$\sqrt{k_t^2 g^2 - a_r^2} = a_t$$

$$a_t = \sqrt{k_t^2 g^2 - \frac{v^4}{R^2}} = \sqrt{0,8^2 \cdot (9,8 \frac{\text{m}}{\text{s}^2})^2 - \frac{(8 \frac{\text{m}}{\text{s}})^4}{(50 \text{ m})^2}} = \sqrt{6,15 - 1,64} = 7,17 \frac{\text{m}}{\text{s}^2}$$

(A)

$$J_0 = 130 \text{ Hz}$$

$$N = \frac{\varphi}{2\pi}$$

$$M = J \cdot \alpha$$

$$M = 15 \text{ Nm}$$

$$W = 2\pi$$

enak posp.
kroženje

$$J = 0,3 \text{ kg m}^2$$

$$N = ?$$

$$\alpha = \frac{M}{J} = \frac{15 \text{ Nm}}{0,3 \text{ kg m}^2} = 50 \frac{1}{\text{s}^2}$$

$$W^2 = W_0^2 + 2\alpha\varphi$$

ustanavlja se

$$\varphi = \frac{W_0^2}{2\alpha} = \frac{(4\pi)^2}{2 \cdot 50} = 66650,9^\circ$$

$$N = \frac{\varphi}{2\pi} = \frac{66650,9^\circ}{6,28} = 1061^\circ$$

(A)

(4)

$$m = 50 \text{ kg}$$

$$\varphi = 15^\circ$$

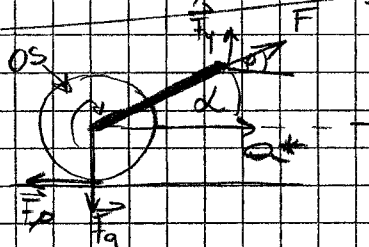
$$a) F = ?$$

$$F_y = F \cdot \sin \varphi$$

$$F \cdot \sin \varphi = m \cdot g$$

$$F = \frac{mg}{\sin \varphi} = \frac{50 \text{ kg} \cdot 9,8 \frac{\text{m}}{\text{s}^2}}{\sin 15^\circ} = 1893 \text{ N}$$

(C)



$$b) F = 250 \text{ N}$$

kotaljenje brez
podrsavanja

$$a^* = ?$$

$$F \cos \varphi - F_p = m \cdot a^*$$

$$N^* = R \cdot \rho$$

$$v^* = R \omega$$

$$a^* = R \alpha$$

$$M = J \cdot \alpha = R \cdot F_p$$

$$\frac{m R^2}{2} \frac{a^*}{R} = R \cdot F_p$$

$$F_p = \frac{1}{2} m a^*$$

$$F \cos \varphi = \frac{3}{2} m a^*$$

$$a^* =$$

$$\frac{2}{3} F \cos \varphi = 3,22 \frac{\text{m}}{\text{s}^2}$$

(A)

$$c) M_t = 6 \text{ Nm}$$

$$R = 20 \text{ cm} = 0.2 \text{ m}$$

$$a^* = ?$$

$$F \cos \varphi - F_p = m a^*$$

inoma 2 navora

$$R \cdot F_p - M_t = \frac{1}{2} m R^2 \cdot \frac{a^*}{R}$$

$$F_p - \frac{M_t}{R} = \frac{1}{2} m a^*$$



$$a^* = \frac{2}{3} \frac{(F \cos \varphi - \frac{M_t}{R})}{m} = 2.8 \text{ m/s}^2 \quad (c)$$

27.11.2008 - 1. kol

5.

$$v_0 = 45 \frac{\text{m}}{\text{s}}$$

$$a_0 = 0.5 \frac{\text{m}}{\text{s}^2}$$

$$a) v(t_1)$$

$$t_1 = 14 \text{ s}$$

$$a = \frac{dv}{dt}$$

$$v_1 = v_0 + a t_1 = 45 \frac{\text{m}}{\text{s}} + 0.5 \frac{\text{m}}{\text{s}^2} \cdot 14 \text{ s} = 52 \frac{\text{m}}{\text{s}} \quad (d)$$

enakomerno
pospešeno
gibanje

$$b) a_0(t) = a_0 e^{-t/\tau}$$

$$a = \frac{dv}{dt}$$

$$\tau = 10 \text{ s}$$

$$v_1 = ?$$

$$a dt = dv \quad | \int$$

$$\int_0^{t_1} a dt = \int_{v_0}^{v_1} dv$$

$$v_1 - v_0 = a_0 \int_0^{t_1} e^{-t/\tau} dt$$

$$v_1 - v_0 = a_0 \cdot \frac{e^{-t/\tau}}{(-\frac{1}{\tau})} \bigg|_0^{t_1}$$

$$v_1 = v_0 + (-a_0 \tau) (e^{-\frac{t_1}{\tau}} - e^{-0})$$

$$v_1 = v_0 + a_0 \tau (1 - e^{-\frac{t_1}{\tau}}) = 48.8 \frac{\text{m}}{\text{s}}$$

12.2003

$$\int e^{kx} dx = \frac{e^{kx}}{k}$$

27.11.2003

②

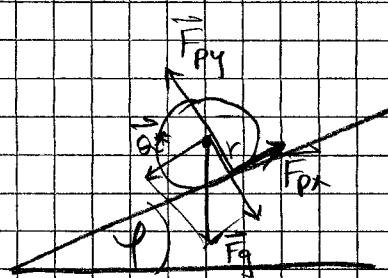
$$r = 7 \text{ cm}$$

$$m = 3 \text{ kg}$$

$$\varphi = 22^\circ$$

$$a^* = 1 \frac{\text{m}}{\text{s}^2}$$

$$J = ?$$



rotacijsko simetrično telo

$$M = J \cdot \alpha$$

$$r \cdot F_{fx} = J \cdot \frac{a^*}{r}$$

$$J = \frac{r^2 F_{fx}}{a^*}$$

$$F_g - F_{fx} = m a^*$$

$$F_{fx} = F_g - m a^* = m g \sin \varphi - m a^*$$

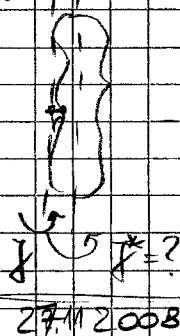
$$J = \frac{r^2 \cdot m (g \sin \varphi - a^*)}{a^*}$$

$$J = (0.07 \text{ m})^2 \cdot 3 \text{ kg} \left(\frac{9.8 \text{ m/s}^2}{1 \text{ m/s}^2} \cdot \sin 22^\circ - 1 \right)$$

$$J = \underline{0.039 \text{ kg m}^2} = \underline{39 \text{ g m}^2}$$

2. način

$$J^* = J + m r^2$$

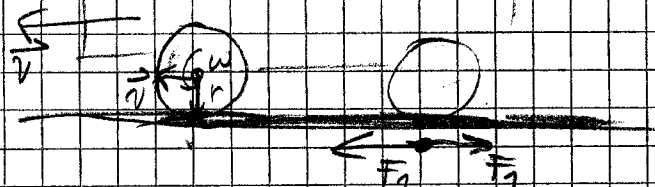


③

$$r = 21 \text{ cm}$$

$$a) \quad \omega = 9 \text{ Hz}$$

$$v = ?$$



$$v = \omega \cdot r = 2\pi \cdot 9 \cdot r = 18\pi \cdot 0.21 \text{ m/s} = 11.9 \frac{\text{m}}{\text{s}}$$

b)

$$M = 230 \text{ Nm}$$

$$m = 1500 \text{ kg}$$

$$a = ?$$

$$4 F_f = m a$$

$$M_1 = r \cdot F_f$$

$$4 \frac{M_1}{r} = m a$$

$$a = \frac{4M}{r m} = 2.9 \frac{\text{m}}{\text{s}^2}$$

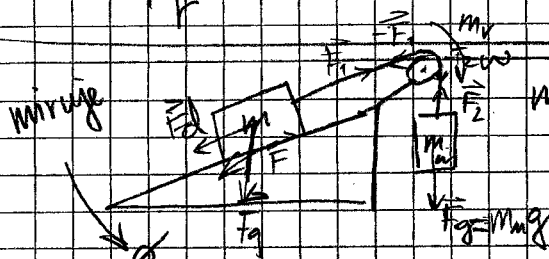
④

$$m = 14 \text{ kg}$$

$$\alpha = 30^\circ$$

$$a) \quad k = 0.1$$

$$m_{\text{max}} = ?$$



$$m a = m (\sin \alpha + k \cos \alpha) = 8.2 \text{ kg}$$

$$-F_f - F + m_2 g = 0$$

$$m_2 g = m g \sin \alpha + m g k \cos \alpha$$

b) $m_m = 30 \text{ kg}$
 $k_t = 0,08$
 $a = ?$
 $m_v < k m_u$
 $= 4,9 \frac{\text{m}}{\text{s}^2}$
 (B)

c) $m_u = 30 \text{ kg}$
 $k_t = 0,08$
 $m_v = 40 \text{ kg}$
 $a = ?$

KLADA m_i :

$$F_1 - F_d - F_t = m_i a$$

VALJ m_v :

$$R \cdot F_2 - R \cdot F_1 = \frac{1}{2} m_v a$$

$$M = 7 \text{ d}$$

UTEZ m_u :

$$F_g - F_A = m_u a$$

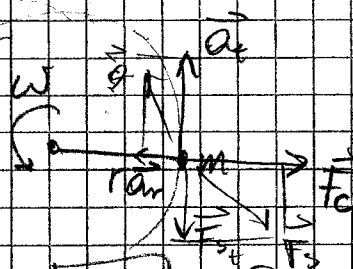
$$F_g - F_d - F_t = m_i a + m_u a + \frac{1}{2} m_v a$$

$$F_g - F_d - F_t = a (m_i + m_u + \frac{1}{2} m_v)$$

$$a = \frac{-m_i g \sin \alpha - k m_i g \cos \alpha + m_u g}{m_i + m_u + \frac{1}{2} m_v} = 3,5 \text{ m/s}^2 \quad (B)$$

2) $\omega = 0,5 \text{ rad/s}$
 $r = 2 \text{ m}$
 $m = 20 \text{ kg}$

F_c ?



$$|F_c| = m \omega r = m \omega^2 r = 10 \text{ N} \quad (B)$$

$$\omega = 0,5 \text{ rad/s}$$

$$\alpha = 0,25 \text{ rad/s}^2$$

$F_s = ?$

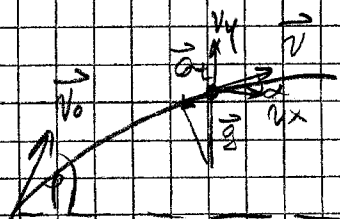
$$|F_s| = m \cdot |\vec{a}| = m \sqrt{a_r^2 + a_t^2} = m \sqrt{\omega^4 r^2 + \alpha^2 r^2}$$

$$= m r \sqrt{\omega^4 + \alpha^2} = 20 \text{ kg} \cdot 2 \text{ m} \sqrt{(0,5 \frac{\text{rad}}{\text{s}})^4 + (0,25 \frac{\text{rad}}{\text{s}^2})^2} = 14,1 \text{ N} \quad (C)$$

N_0, φ, t_1

$$v = |\vec{v}|$$

$$a_t = \frac{dv}{dt}$$



$$v = \sqrt{v_x^2 + v_y^2}$$

$$v_x = v_0 \cos \varphi$$

$$v_y = v_0 \sin \varphi - gt$$

$$a_t = \frac{d}{dt} \sqrt{v_0^2 \cos^2 \varphi + (v_0 \sin \varphi - gt)^2} = \frac{0 + 2(v_0 \sin \varphi - gt)(0 - g)}{2v} = -g \frac{v_0 \sin \varphi - gt}{v}$$

$$a_t = -\frac{g v_0 \sin \varphi}{v}$$

2007

7) $v = kt^{1/3}$
 $k = 3 \text{ m/s}^2$

$a = \frac{dv}{dt} = \frac{d(kt^{1/3})}{dt} = \frac{k}{3t^{2/3}}$

$t_1 = 19 \text{ s}$

$a(t_1) = \frac{k}{3t_1^{2/3}} = 0,14 \frac{\text{m}}{\text{s}^2} \quad (b)$

8) $s(t_1) = ?$

$v = \frac{ds}{dt}$
 $\int_0^{t_1} v dt = \int_0^{t_1} ds$

$s(t_1) = k \int_0^{t_1} t^{1/3} dt = k \frac{t^{4/3}}{4/3} \Big|_0^{t_1} = \frac{3k}{4} t_1^{4/3} = 114 \text{ m} \quad (b)$

4/71

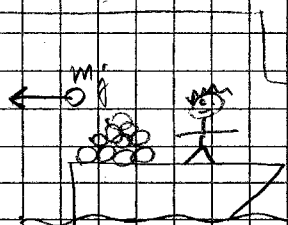
$s = 10 \text{ m}$
 $t = 5 \text{ s}$

$m_{sk} = 200 \text{ kg}$

$m_j = 0,2 \text{ kg}$

$v_j = 25 \text{ m/s}$

x_j



$G_B = G_K$

SISTEM COLNA

$0 = -m_j v_j + m_{sk} \Delta v$

$\Delta v = \frac{m_j v_j}{m_{sk}} = \frac{0,2 \text{ kg} \cdot 25 \text{ m/s}}{200 \text{ kg}} = 25 \cdot 10^{-3} \frac{\text{m}}{\text{s}}$

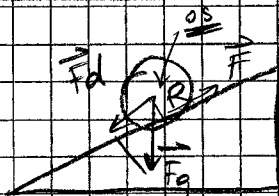
$\Delta = \Delta v t_0 + 2 \Delta v t_0 + 3 \Delta v \cdot (t - 2t_0)$

$\Delta = \frac{n+1}{2} \Delta v t_0$

$N(N+1) > \frac{2s}{\Delta v t_0} = \frac{20 \text{ m}}{25 \cdot 10^{-3} \cdot 5} \Rightarrow 160$
 $N = 13$

$\alpha = 45^\circ$

$k = ?$



KOTACIENJE BREZ PODSILAVANJA

$F \leq k_r mg \cos \theta$

$F_d - F = m \cdot a^*$

$mg \sin \theta - mg k \cos \theta = m \cdot a^*$

$M = F \cdot r = \int r \cdot dM$

$F \cdot R = \frac{1}{2} m R^2 \frac{a^*}{R}$

$F = \frac{m a^*}{2}$

$m a^* = 2F$

$F_d - F = 2F$

$F_d = 3F$

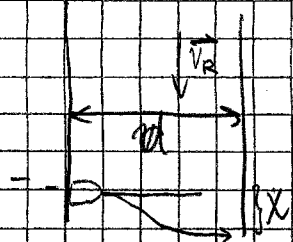
$F = \frac{F_d}{3}$

$k_r mg \cos \theta = \frac{mg \sin \theta}{3}$

$k_r = \frac{1}{3} \tan \theta = \frac{1}{3}$

$$v_R = 1 \frac{m}{s}$$

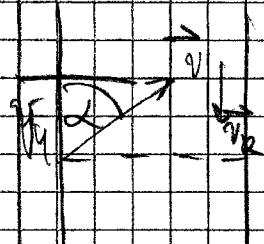
$$d = 3 \text{ m}$$



$$d = v \cdot t$$

$$t = \frac{d}{v}$$

$$x = v_R \cdot t = \frac{v_R}{v} \cdot d = 17 \text{ m}$$



$$v \cdot \cos \varphi = v_R$$

$$\cos \varphi = \frac{v_R}{v} = \frac{1}{2} \Rightarrow \varphi = 60^\circ$$

NIHANJE

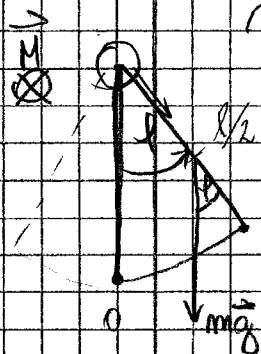
3.12.2010

$$l = 0.8 \text{ m}$$

palica je vrtljiva okoli rogoje vodoravne osi v krajnjem. Palico malo odmaknemo, s katerimi nihalnimi črtami začne to palica.

MALHNI ODMIKI ($\varphi \ll 1$)

$$\sin \varphi \approx \varphi$$



$$M = J \cdot \ddot{\varphi}$$

$$-mg \cdot \frac{l}{2} \sin \varphi = \frac{J}{3} \cdot \ddot{\varphi}$$

$$\ddot{\varphi} + \frac{3g}{2l} \sin \varphi = 0 \rightarrow \varphi(t)$$

$$\ddot{\varphi} + \frac{3g}{2l} \cdot \varphi = 0$$

$$\ddot{x} + \omega^2 x = 0$$

krožna frekvenca

NASTAVEK

amplituda odnaka

fazni zamik

$$x(t) = \hat{x}_0 \sin(\omega t + \delta)$$

$$v(t) = \dot{x}(t) = \omega \hat{x}_0 \cos(\omega t + \delta)$$

$$a(t) = \ddot{x}(t) = -\omega^2 \hat{x}_0 \sin(\omega t + \delta)$$

$$\omega t_0 = 2\pi$$

$$t_0 = \frac{2\pi}{\omega}$$

$$\omega = 2\pi \nu$$

$$= -\omega^2 \hat{x}_0 \sin(\omega t + \delta) + \hat{x}_0 \sin(\omega t + \delta) = 0$$

$$\ddot{x} + \omega^2 x = 0$$

konst. vredno mora biti pozitivno realno število

$$t_0 = 2\pi \sqrt{\frac{2l}{3g}} = 2\pi \sqrt{\frac{2 \cdot 0.8 \text{ m}}{3 \cdot 10 \frac{m}{s^2}}} \approx 1.45 \text{ s}$$

1. nal. str. 100

$$m = 0,3 \text{ kg}$$

$$k = 16 \frac{\text{N}}{\text{m}}$$

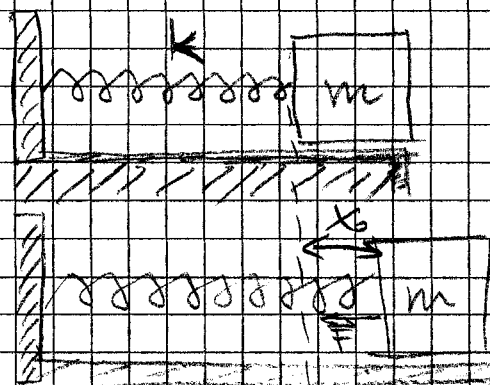
$$x_0 = 8 \text{ cm}$$

$$t_1 = 0,2 \text{ s}$$

ko gre utež prvič skozi
ravnovesno lego.

$$x(t_1) = ?$$

$$v(t_1) = ?$$



Hookeov zakon:

$$F = -k \cdot x$$

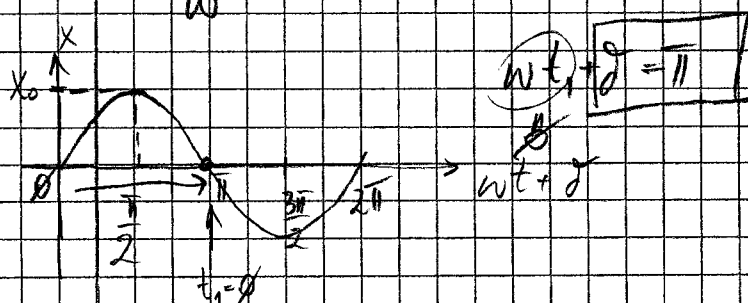
$$F = m \cdot a$$

$$-kx = m \cdot \ddot{x}$$

$$\ddot{x} + \left(\frac{k}{m}\right) x = 0$$

$$\omega^2$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{16 \frac{\text{N}}{\text{m}}}{0,3 \text{ s}^2 \cdot \text{kg} \cdot \text{m}^{-1}}} = 7,3 \frac{\text{rad}}{\text{s}}$$



$$x(t_1) = x_0 \cdot \sin(\omega t_1 + \phi) = 8 \text{ cm} \cdot \sin\left(7,3 \frac{\text{rad}}{\text{s}} \cdot 0,2 \text{ s} + \pi\right) = -0,0795 \text{ m}$$

$$v(t_1) = \omega \cdot x_0 \cos(\omega t_1 + \phi) = 7,3 \frac{\text{rad}}{\text{s}} \cdot 8 \text{ cm} \cdot \cos\left(\frac{4}{3}\right)$$

$$0,584 \cdot \cos\left(1,46 + \pi\right) = -6,46 \frac{\text{cm}}{\text{s}}$$

negativni
odmak

Max. sila, ki vrača harmonično nihajoče telo v ravnovesno lego je $3 \cdot 10^{-3} \text{ N}$.

$$W = 7 \cdot 10^{-5} \text{ J}$$

$$x_0 = 2$$

$$\omega^2 = \frac{k}{m}$$

$$F_0 = k \cdot x_0$$

$$W = \frac{1}{2} k x_0^2$$

$$W = \underbrace{\frac{1}{2} m v^2}_{W_k} + \underbrace{\frac{1}{2} k x^2}_{W_{pr}}$$

$$W = \frac{1}{2} m \omega^2 x_0^2 \cdot \cos^2(\omega t + \phi) + \frac{1}{2} k \cdot x_0^2 \sin^2(\omega t + \phi)$$

$$W = \frac{1}{2} m \frac{k}{m} x_0^2 \cos^2(\omega t + \phi) + \frac{1}{2} k x_0^2 \sin^2(\omega t + \phi)$$

$$W = \frac{1}{2} k x_0^2 (\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi))$$

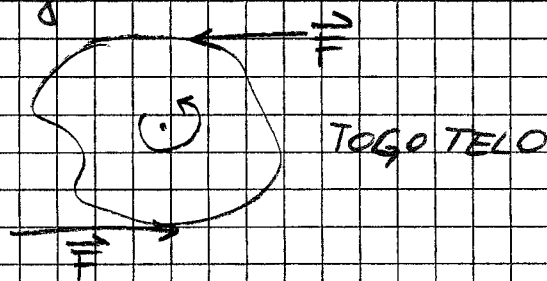
$$W = \frac{1}{2} k x_0^2 = \frac{1}{2} m v_0^2 = \text{konst}$$

$$\frac{W}{F} = \frac{\frac{1}{2} k x_0^2}{k x_0} = \frac{W}{F} = \frac{1}{2} x_0 \Rightarrow x_0 = \frac{2W}{F} = \frac{14 \cdot 10^{-3} \text{ Nm}}{3 \cdot 10^{-2} \text{ N}} = 4,67 \text{ cm}$$

$$A = \Delta W_k + \Delta W_p$$

$$W_k = \underbrace{\frac{1}{2} m v^2}_{\text{translacijska } W_k} + \underbrace{\frac{1}{2} I \omega^2}_{\text{rotacijska } W_k} \rightarrow \text{togo telo se lahko vrti}$$

$$A = \int \vec{F} \cdot d\vec{s} + \int \vec{M} \cdot d\vec{\varphi}$$



$$\Delta W_p = m \int g dh$$

$g = \text{konst.}$

$$\Delta W_p = mg \int dh = mgh$$

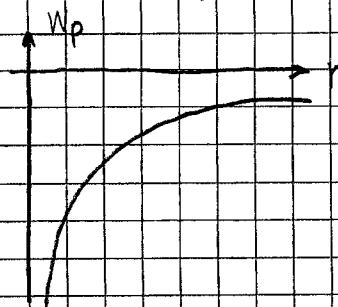
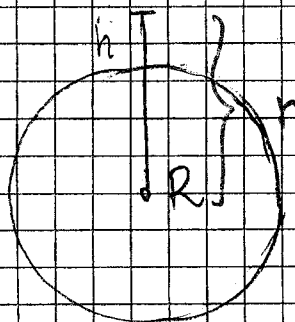
$g \neq \text{konst.}$

$$g = g_0 \frac{R^2}{(R+h)^2}$$

$$\Delta W_p = m g_0 \int \frac{R^2}{(R+h)^2} dh$$

$$\Delta W_p = m g_0 \int \frac{dr^2}{r^2} = -m g_0 \frac{R^2}{r} \Big|_{r_1}^{r_2} = -m g_0 \frac{R^2}{r}$$

$$W_p = -m g_0 \frac{R^2}{R+h}$$



40. mal. / str. 95

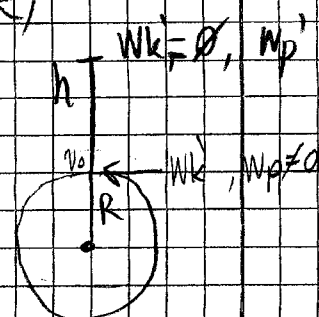
S kolikšno v_0 moramo izstreliti marmično kroglo, da bo dosegel višino 3000 km?

$$\begin{aligned} H &= 3000 \text{ km} \\ R &= 6400 \text{ km} \\ g_0 &= 9,81 \frac{\text{m}}{\text{s}^2} \\ v_0 &=? \end{aligned}$$

$$0 = \left(0 - \frac{1}{2} m v^2\right) + \left(-m g_0 \frac{R^2}{R+h} - (-m g_0 R)\right)$$

$$\frac{1}{2} m v^2 = m g_0 R - m g_0 \frac{R^2}{R+h}$$

$$v = \sqrt{2 g_0 R \left(1 - \frac{R}{R+h}\right)} = 6330,47 \frac{\text{m}}{\text{s}} = \underline{\underline{6,3 \frac{\text{km}}{\text{s}}}}$$



Količina mora biti hitrost, da nikoli več ne pade na Zemljo?

- UBEŽNA HITROST

$$v_u = \lim_{h \rightarrow \infty} \sqrt{2g_0 R \left(1 - \frac{R}{R+h}\right)} = \sqrt{2g_0 R}$$

$$v_u \approx 11 \frac{\text{km}}{\text{s}}$$

$W_k + W_p > 0 \Rightarrow$ mezezi sistem

izstrelek pobegne privlačni Zemlje

hiperkola

$W_k + W_p = 0$

parabola

elipsa

$W_k + W_p < 0$

vezani sistem

9. nal. / str. 80

$$\begin{aligned} m_v &= 100 \text{ kg} \\ 2R_v &= 2 \text{ m} \\ R &= 1 \text{ m} \end{aligned}$$

$$\begin{aligned} m &= 80 \text{ kg} \\ \vec{v} &= 0,25 \text{ Hz} \end{aligned}$$

$$\omega = 2\pi \vec{v}$$

$$A = ?$$

$$A = W_k' - W_k$$

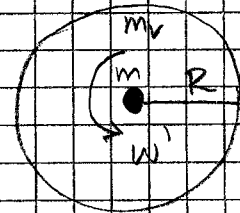
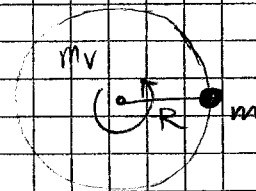
$$A = \frac{1}{2} \int_v \omega'^2 - \frac{1}{2} \left(\int_v + \int \right) \omega^2$$

$\frac{1}{2} m_v R^2 \quad m R^2$

$$A = \frac{1}{2} \left(\int_v + \int \right) \omega'^2 - \frac{1}{2} \left(\int_v + \int \right) \omega^2$$

$$A = \frac{1}{2} \left(\int_v + \int \right) \omega^2 \left(\frac{\left(\int_v + \int \right)}{\int_v} - 1 \right) = \frac{1}{2} \left(\int_v + \int \right) \omega^2 \left(\frac{\int}{\int_v} \right)$$

$$A = \frac{1}{2} \left(\frac{1}{2} m_v R^2 + m R^2 \right) 4\pi^2 (0,25)^2 \left(\frac{m R^2}{\frac{1}{2} m_v R^2} \right) = 256 \approx 0,06 \text{ kcal}$$



$$1 \text{ kcal} = \frac{17}{4200}$$

$$P_{\text{ivo}} (0,58) = 200 \text{ kcal}$$

$$P_{\text{ICA}} \sim 1000 \text{ kcal}$$

$$\int M d\vec{r} = \Delta \vec{r}$$

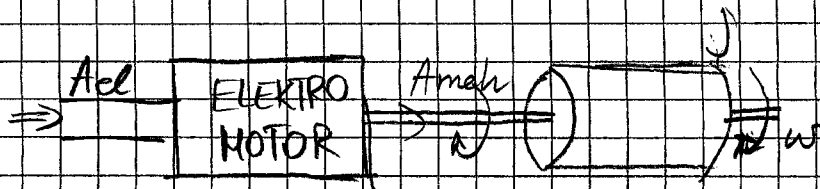
$$\vec{r} = \vec{r}'$$

$$(\int_v + \int) \omega = \int_v \omega' \Rightarrow$$

$$\omega' = \frac{(\int_v + \int) \omega}{\int_v}$$

MOC

$$D = \frac{dA}{dt}$$



$$J = 10 \text{ kg} \cdot \text{m}^2$$

$$t_1 = 35 \text{ s}$$

$$P = k \sqrt{t}$$

$$k = 10 \frac{\text{W}}{\sqrt{\text{s}}}$$

$$\mu = 0,25 \text{ (izkonistek)}$$

$$\omega_1$$

$$A_{\text{mech}} = \mu \cdot A_{\text{el}}$$

$$A_{\text{mech}} = \frac{1}{2} J \omega_1^2$$

$$\mu A_{\text{el}} = \frac{1}{2} J \omega_1^2$$

$$A_{\text{el}} = \int_0^{t_1} P dt$$

$$A_{\text{el}} = k \int_0^{t_1} t^{1/2} dt = \frac{2}{3} k t^{3/2} \Big|_0^{t_1} = \frac{2}{3} k t_1^{3/2}$$

$$\mu \cdot \frac{2}{3} k t_1^{3/2} = \frac{1}{2} J \omega_1^2$$

$$\omega_1 = \sqrt{\frac{2 \mu \frac{2}{3} k t_1^{3/2}}{\frac{1}{2} J}} = 2 \sqrt{\frac{\mu k t_1^{3/2}}{3 J}}$$

$$\omega_1 = 8,3 \frac{\text{rad}}{\text{s}}$$

$$\omega, R$$

$$\varphi = ?$$

① N-zakon

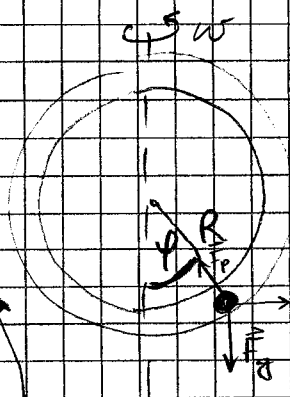
$$\cos \varphi = \frac{g W}{R}$$

② ravnotezna lega

$$W = W_k + W_p$$

$$W(\varphi) = ?$$

$$\frac{dW}{d\varphi} = 0$$



DN

SINUSNO NIHANJE

$$\ddot{x} = -\omega^2 x$$

$$\omega = 2\pi \nu = \frac{2\pi}{T}$$

fizično nihalo:

8. maloga, str. 101

$$l = 1 \text{ m}$$

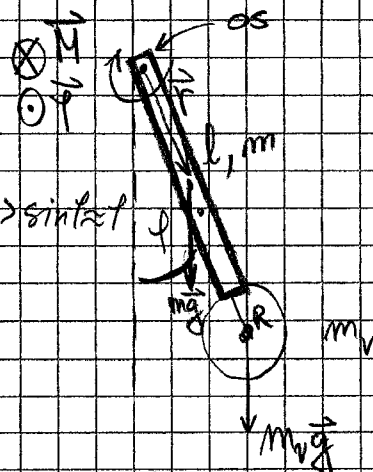
$$m = 4 \text{ kg}$$

$$m_v = 1 \text{ kg}$$

$$R = 0,2 \text{ m}$$

 $\varphi \ll 1$ (majhni odkliki) $\Rightarrow \sin \varphi \approx \varphi$

$$t_0 = ?$$



$$M = J \cdot \alpha$$

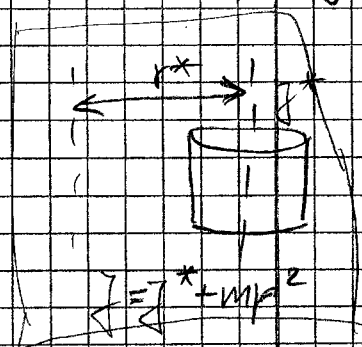
$$\vec{M} = \vec{r} \times \vec{F}$$

$$M = r \cdot F \cdot \sin \varphi$$

$$-\frac{l}{2} mg \sin \varphi - (l+R) m_v g \sin \varphi = J \cdot \alpha$$

$$= -\frac{l}{2} mg \sin \varphi - (l+R) m_v g \sin \varphi = J \cdot \ddot{\varphi}$$

$$\ddot{\varphi} = - \frac{\frac{l}{2} mg + (l+R) m_v g}{J} \sin \varphi$$



$$t_0 = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{J}{g \left[\frac{l}{2} m + (l+R) m_v \right]}} = 1,86 \text{ s}$$

$$J = J_l + J_v = \frac{1}{3} m l^2 + \frac{1}{2} m_v R^2 + m_v (l+R)^2$$

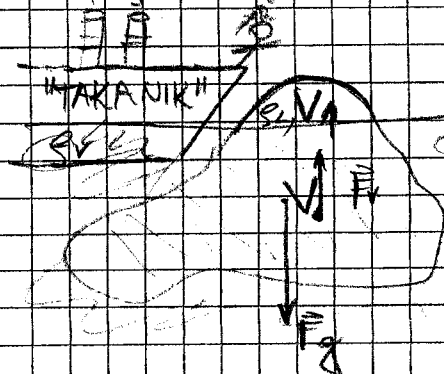
V klani vodi plava ledena gora z gostoto ledu $\rho = 900 \frac{\text{kg}}{\text{m}^3}$, $\rho_v = 1030 \frac{\text{kg}}{\text{m}^3}$, $V_1 = 195 \text{ m}^3$, kolikšen je celoten volumen ledena gore.

$$\rho_L = 900 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_v = 1030 \frac{\text{kg}}{\text{m}^3}$$

$$V_1 = 195 \text{ m}^3$$

$$V$$



ravnovesje:

$$F_g - F_v = 0$$

$$\rho_L \cdot V \cdot g - \rho_v (V - V_1) g = 0$$

$$\rho_L V = \rho_v (V - V_1)$$

$$\rho_L V - \rho_v V = -\rho_v V_1$$

$$V (\rho_v - \rho_L) = \rho_v V_1$$

$$V = \frac{\rho_v V_1}{\rho_v - \rho_L} = 1545 \text{ m}^3$$

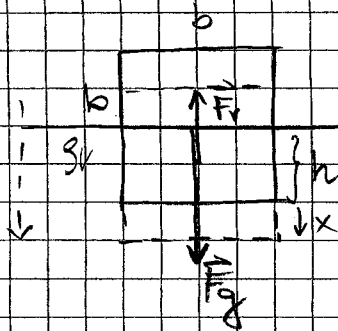
KOCKA

$$b = 10 \text{ cm}$$

$$\rho_k = 500 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_v = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$t_0 = ?$$



RAVNOVESJE:

$$F_g - F_v = 0$$

$$\rho_k \cdot b^3 \cdot g - \rho_v \cdot b^2 \cdot h \cdot g = 0$$

$$\rho_k \cdot b = \rho_v \cdot h$$

$$h = \frac{\rho_k}{\rho_v} \cdot b = \frac{5}{10} b = \frac{1}{2} \cdot b = \underline{\underline{5 \text{ cm}}}$$

potopljena je pol kocke

NIHANJE:

$$F_g - F_v = m_k \cdot a$$

$$\rho_k \cdot b^3 \cdot g - \rho_v \cdot b^2 (h+x) \cdot g = m_k \ddot{x}$$

$$\rho_k \cdot b^3 \cdot g - \rho_v \cdot b^2 (h \cdot g) - \rho_v \cdot b^2 \cdot x \cdot g = b^2 \rho_k \ddot{x}$$

$$-\rho_v \cdot b^2 \cdot x \cdot g = \rho_k \cdot m_k \ddot{x} \cdot b$$

$$-\rho_v \cdot x \cdot g = \rho_k \cdot m_k \ddot{x}$$

$$\ddot{x} = - \frac{\rho_v \cdot g}{\rho_k \cdot b} \cdot x$$

$$\omega^2$$

$$t_0 = \frac{2\pi}{\omega} = 2\pi \cdot \sqrt{\frac{\rho_k \cdot b}{\rho_v \cdot g}}$$

$$t_0 = 2\pi \cdot \sqrt{\frac{1}{2} \cdot \frac{500}{1000} \cdot \frac{1}{9.81}} = \frac{\sqrt{2} \pi}{10} \text{ s} \approx \underline{\underline{0,44 \text{ s}}}$$

+ UPOR V VODI

$$F_u = k \cdot v$$

$$F_g - F_v - F_u = m_k \cdot a$$

$$-\rho_v \cdot b^2 \cdot x \cdot g - k \cdot \dot{x} = m_k \cdot \ddot{x} \quad / : m_k$$

$$\ddot{x} + \frac{\rho_v \cdot b^2 \cdot g}{\rho_k \cdot b} \cdot x + \frac{k}{m_k} \cdot \dot{x} = 0$$

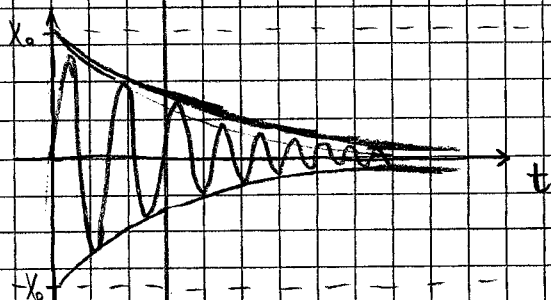
$$v = \dot{x}$$

REŠITEV:

$$x(t) = x_0 \cdot e^{-\beta t} \sin(\omega t + \delta)$$

$$\beta = \text{koeficient dušenja}$$

$$\omega' = \sqrt{\omega^2 - \beta^2}$$

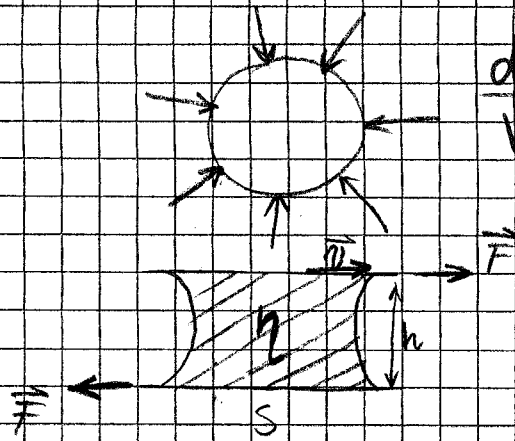


BERNOULLI -jeva enačba

$$(n=0, \chi=0)$$

viskoznost

stisljivost



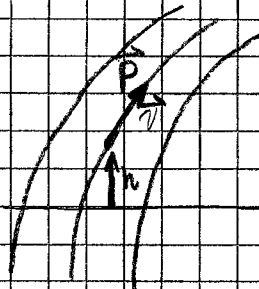
$$\frac{dW}{V} = -\chi \cdot dp$$

$$\frac{F}{S} = \eta \frac{v}{h}$$

$$\eta_{H_2O} \approx 10^{-3} \frac{kg}{m \cdot s}$$

$$\eta_{\text{smola}} \approx 10^7 \frac{kg}{m \cdot s}$$

$$\eta_{\text{zrak}} \approx 1,7 \cdot 10^{-4} \frac{kg}{m \cdot s}$$

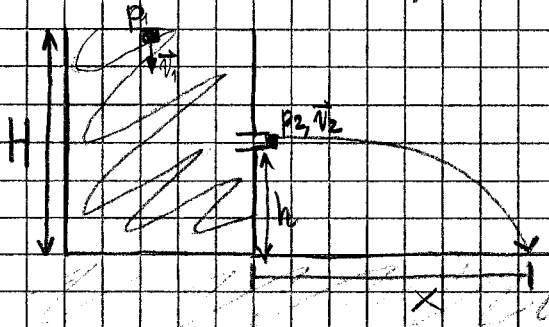


$$p = \rho g h + \frac{1}{2} \rho v^2 = \text{konst.}$$

$$H = 40 \text{ cm}$$

$$h = 30 \text{ cm}$$

domet curka
vode:



$$p_1 + \rho g H + \frac{1}{2} \rho v_1^2 = p_2 + \rho g h + \frac{1}{2} \rho v_2^2$$

p_0

$$\rho g H = \rho g h + \frac{1}{2} \rho v_2^2$$

$$v_2 = \sqrt{2g(H-h)}$$

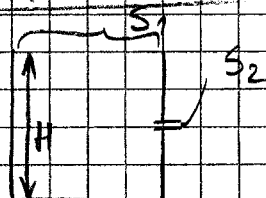
$$x = v_2 t = \sqrt{2g(H-h)} \cdot \sqrt{\frac{2h}{g}} = 2\sqrt{(H-h)h} = 0,35 \text{ m}$$

$$h = \frac{v_2^2}{2g} \Rightarrow t = \sqrt{\frac{2h}{g}}$$

Zanemarili smo
dr. upor, stisljivost,
viskoznost vode

$$H_0, h, S_1, S_2$$

$$x(t) = ?$$



KONTINUITETNA ENAČBA

$$\phi_{v1} = \phi_{v2}$$

$$S_1 \cdot v_1 = S_2 \cdot v_2$$

$$v_1 = \frac{S_2}{S_1} v_2$$

$$p_1 + \rho g H + \frac{1}{2} \rho v_1^2 = p_2 + \rho g h + \frac{1}{2} \rho v_2^2$$

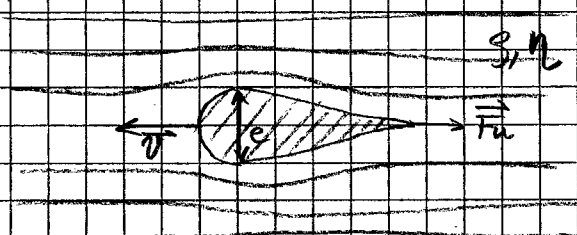
$$H(t) = H_0 - \int_0^t v_1 dt' = H_0 - \frac{S_2}{S_1} \int_0^t v_2 dt' \Rightarrow \rho g H_0 - \rho g \frac{S_2}{S_1} \int_0^t v_2 dt' + \frac{1}{2} \rho \left(\frac{S_2}{S_1} v_2 \right)^2 = \rho g h + \frac{1}{2} \rho v_2^2 \cdot \frac{d}{dt}$$

$$0 - \rho g \frac{S_2}{S_1} v_2 + \frac{1}{2} \rho \frac{S_2^2}{S_1^2} v_2 \frac{d}{dt} = 0 - \frac{1}{2} \rho v_2 \frac{d}{dt}$$

SILA UPORA:

$$Re = \frac{\rho \cdot S \cdot v}{\eta}$$

Reynolds-ov
številko



$$F_d = \begin{cases} Re < 0,5 : \text{linearni zakon upora} & F_d = 6\pi \cdot r \cdot \eta \cdot v \Rightarrow \text{krogla (Stokes-ova ena\u010dba)} \\ \updownarrow ??? \\ Re > 1000 : \text{kvadratni zakon upora} & F_d = \frac{1}{2} C_d \cdot \rho \cdot S \cdot v^2 \Rightarrow \text{krogla} \end{cases}$$

27. nal. / str. 131

OKROGEL KAMEN

KNADRATNI ZAKON UPORA:

$$F_d = \frac{1}{2} C_d \rho S v^2$$



$$r = 1 \text{ cm}$$

$$\rho_k = 2700 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_v = 1000 \frac{\text{kg}}{\text{m}^3}$$

RAVNOVESJE:

$$F_g - F_v - F_d = 0$$

$$\rho_k \cdot V \cdot g - \rho_v \cdot V \cdot g = \frac{1}{2} C_d \rho_v S v^2$$

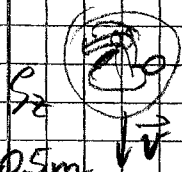
$$(\rho_k - \rho_v) \frac{4\pi}{3} r^3 g = \frac{1}{2} C_d \rho_v \pi r^2 v^2$$

$$v = \sqrt{\frac{8 (\rho_k - \rho_v) r \cdot g}{3 \rho_v C_d}} = \sqrt{\frac{8 (2700 - 1000) \frac{\text{kg}}{\text{m}^3} \cdot 1 \cdot 10^{-2} \text{ m} \cdot 9,8 \frac{\text{m}}{\text{s}^2}}{3 \cdot 1000 \frac{\text{kg}}{\text{m}^3} \cdot 0,4}} = 1,05 \frac{\text{m}}{\text{s}}$$

$$v = 1,05 \frac{\text{m}}{\text{s}}$$

$$Re \approx \frac{2r \cdot \rho \cdot v}{\eta} \approx \frac{2 \cdot 10^{-3} \text{ m} \cdot 1000 \frac{\text{kg}}{\text{m}^3} \cdot 1 \frac{\text{m}}{\text{s}}}{10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}}} = 20.000$$

SKOK IZ LETALA



"bombica"

RAVNOVESJE

$$F_g - F_u = 0$$

$$m \cdot g = \frac{1}{2} C_u \rho \cdot \pi r^2 \cdot v^2$$

$$r = 0,5 \text{ m}$$

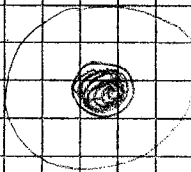
$$m = 70 \text{ kg}$$

$$v = ?$$

$$v = \sqrt{\frac{2mg}{C_u \rho \pi r^2}} = \sqrt{\frac{2 \cdot 70 \text{ kg} \cdot 9,8 \frac{\text{m}}{\text{s}^2}}{0,4 \cdot 1,3 \frac{\text{kg}}{\text{m}^3} \cdot \pi \cdot 0,25 \text{ m}^2}} = 58 \frac{\text{m}}{\text{s}}$$

$$v \approx 60 \text{ m/s} = 215 \frac{\text{km}}{\text{h}}$$

BIOLOŠKA CELICA



$$\rho_k = 1,05 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$$

$$\rho_v = 1,03 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$$

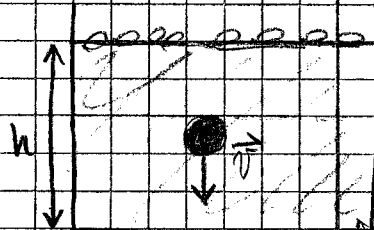
$$t = 10 \text{ h}$$

$$h = 10 \text{ cm}$$

$$\eta = 1,5 \cdot 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

$$r = ?$$

$$v = \frac{h}{t} = 1 \frac{\text{cm}}{\text{h}}$$



$$0,1$$

$$3600 \text{ s}$$

fiziološka raztopina

RAVNOVESJE

$$F_g - F_v - F_u = 0$$

$$\rho_k V g - \rho_v V g - C_u \pi r^2 \eta v = 0$$

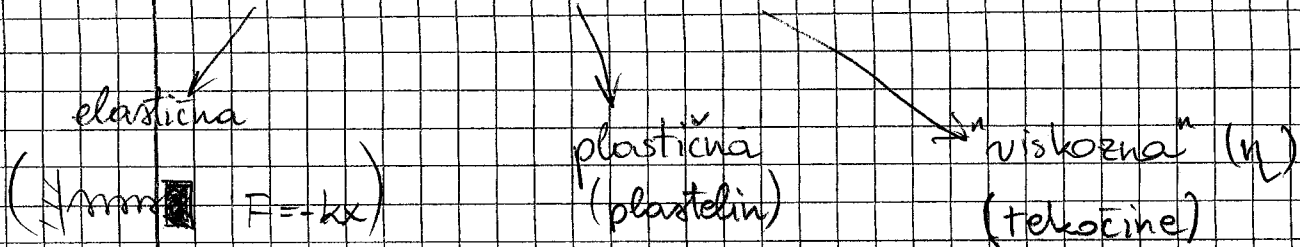
$$(\rho_k - \rho_v) \frac{4}{3} \pi r^3 g = C_u \pi r^2 \eta v$$

$$r = \sqrt{\frac{3 \eta v}{2g(\rho_k - \rho_v)}} = \sqrt{\frac{3 \cdot 1,5 \cdot 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}} \cdot 1 \cdot 27,7 \cdot 10^{-6} \frac{\text{m}}{\text{s}}}{2 \cdot 9,8 \cdot 10 \text{ m} (1,05 \cdot 10^3 - 1,03 \cdot 10^3) \frac{\text{kg}}{\text{m}^3}}} = 9,7 \mu\text{m}$$

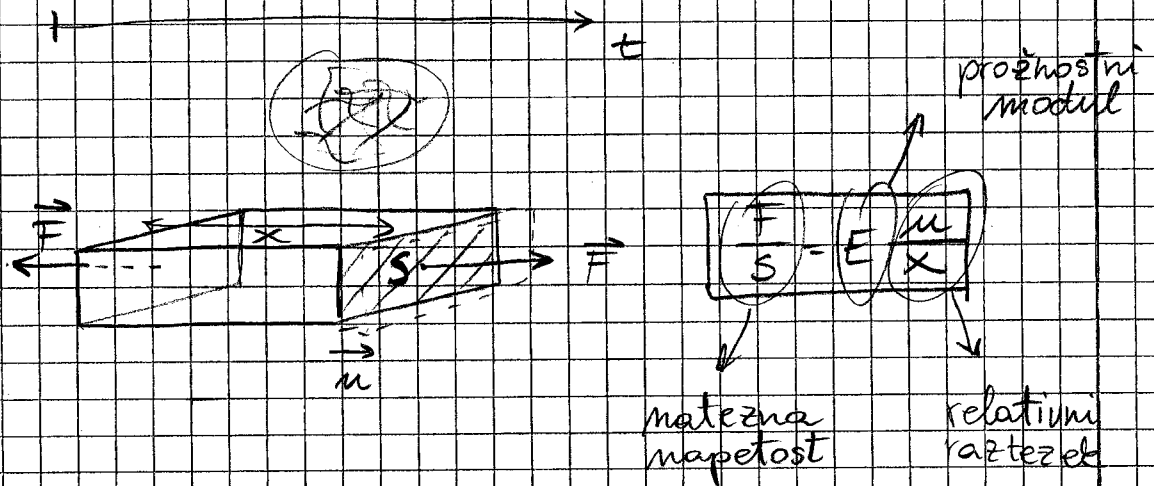
$$r = 9,7 \mu\text{m}$$

$$Re = \frac{2 \cdot 10^{-6} \text{ m} \cdot 10^3 \frac{\text{kg}}{\text{m}^3} \cdot 10^{-4} \frac{\text{m}}{\text{s}}}{1,5 \cdot 10^{-3} \frac{\text{kg}}{\text{m} \cdot \text{s}}} = 37 \cdot 10^{-6}$$

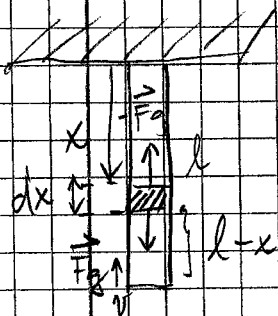
DEFORMACIJA ~~elastičnosti~~



Viskoelastična snov



svinčena palica; raztezek zaradi lastne teže, ko je obesena na strop



$$l = 0,75 \text{ m}$$

$$E = 2 \cdot 10^6 \frac{\text{N}}{\text{cm}^2}$$

$$\rho = 11,3 \frac{\text{g}}{\text{cm}^3}$$

$$\mu = ?$$

$$\frac{F_g}{S} = E \frac{du}{dx}$$

$$\frac{\rho \cdot S \cdot (l-x) \cdot g}{S} = E \frac{du}{dx}$$

$$F_g = mg = \rho V \cdot g =$$

$$= \rho \cdot S \cdot (l-x) \cdot g$$

$$u = l - x$$

$$du = -dx$$

$$\int du = \int \frac{\rho \cdot g}{E} (l-x) dx$$

$$\mu = \frac{\rho \cdot g}{E} \int_0^l (l-x) dx = -\frac{\rho \cdot g}{E} \int_l^0 u du = -\frac{\rho \cdot g}{E} \left[\frac{u^2}{2} \right]_l^0 =$$

$$\mu = \frac{\rho \cdot g}{E} \cdot \frac{l^2}{2} = \frac{11,3 \cdot 10^{-3} \text{ kg} \cdot 9,81 \text{ m/s}^2 \cdot (0,75 \text{ m})^2}{2 \cdot 2 \cdot 10^6 \text{ N/cm}^2} =$$

KDAJ SE OBEŠENA PALICA PRETRGA ZARADI LASTNE TEŽE?

materijalna trdnost zvinca

$$\sigma = 2 \frac{\text{KN}}{\text{cm}^2}$$

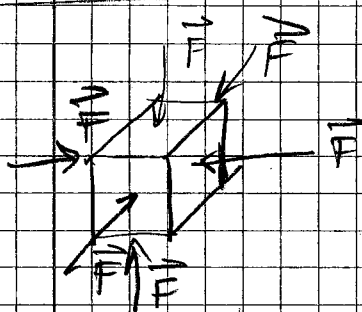
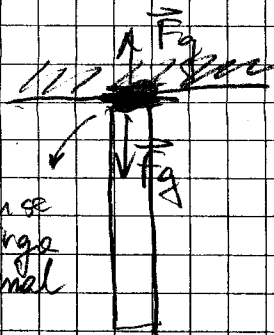
$$\sigma = \left(\frac{F}{S} \right)_{\text{max}}$$

$$l_0 = ?$$

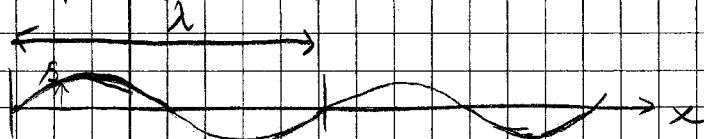
$$\sigma = \frac{mg}{S} = \frac{\rho S l_0 g}{S}$$

$$l_0 = \frac{\sigma}{\rho g} = \frac{2 \cdot 10^8 \text{ kg/m} \cdot \text{cm}^2}{9.8 \cdot 11.3 \cdot 10^{-3} \text{ kg/cm}^2} = 180 \text{ m}$$

tu se pretrga material



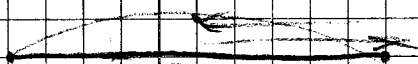
VALOVANJE



$$A = A_0 \cos(\omega t - kx)$$

$$k \cdot \lambda = 2\pi \Rightarrow k = \frac{2\pi}{\lambda}$$

struna



valovni vektor

ker se odbije

$$A = A_0 \cos(\omega t - kx) + A_0 \cos(\omega t + kx)$$

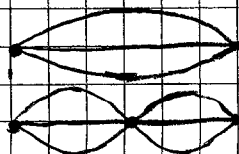
$$\cos \alpha + \cos \beta =$$

$$A = 2A_0 \cos kx \cos \omega t$$

stojede (stojno) valovanje

$$= 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos(-\alpha) = \cos \alpha$$



osnovni nihanjski način

1. vzbujeno stanje

$$C_u: 261,6 \text{ Hz}$$

$$C_5: 523,2 \text{ Hz}$$

struna:

$$m = 0,25 \text{ g}$$

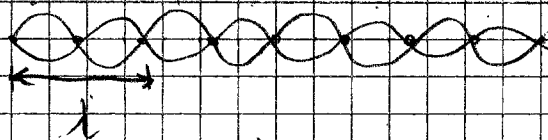
$$l = 1 \text{ m}$$

$$F = 4 \text{ N}$$

NIHA V 8
SEGMENTH:

$$\nu = ?$$

$$\lambda = \frac{l}{4}$$



$$4\lambda = l$$

$$c = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F \cdot l}{m}} = \sqrt{\frac{F \cdot l}{m}} = \sqrt{\frac{4 \text{ N} \cdot 1 \text{ m}}{0,25 \cdot 10^{-3} \text{ kg}}} = \sqrt{16 \cdot 10^3 \frac{\text{m}^2}{\text{s}^2}}$$

$$c = 126,5 \frac{\text{m}}{\text{s}}$$

$$\nu = \frac{c}{\lambda} = \frac{126,5 \frac{\text{m}}{\text{s}}}{0,25 \text{ m}} = 506 \text{ Hz}$$

HITROST ŠIRJENJA VALOVANJA NA STRUNI:

$$c = \sqrt{\frac{F}{\mu}}$$

$$c = \lambda \cdot \nu$$

$$m = \rho \cdot S \cdot l$$

$$c = \sqrt{\frac{F \cdot l}{m}} = \sqrt{\frac{F \cdot l}{\rho \cdot S \cdot l}} = \sqrt{\frac{F}{\rho \cdot S}}$$

1. maloga / str. 135

$$\nu = 284 \text{ Hz}$$

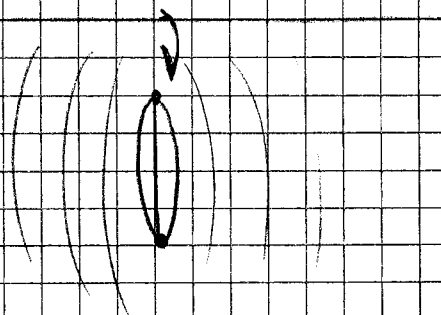
$$T = 25^\circ \text{C}$$

$$T_0 = 273 \text{ K}$$

$$c_0 = 331 \frac{\text{m}}{\text{s}}$$

$$\lambda = ?$$

$$\lambda = \frac{c(T)}{\nu} = \frac{345,8 \frac{\text{m}}{\text{s}}}{284 \frac{1}{\text{s}}} = 1,22 \text{ m}$$



$$c = \sqrt{\frac{K \cdot T_0}{M}}$$

$$c = \sqrt{\frac{K \cdot T}{M}}$$

$$\frac{c}{c_0} = \sqrt{\frac{T}{T_0}} \Rightarrow c = c_0 \sqrt{\frac{T}{T_0}}$$

$$c = 331 \frac{\text{m}}{\text{s}} \sqrt{\frac{298 \text{ K}}{273 \text{ K}}} = 345,8 \frac{\text{m}}{\text{s}}$$

$$\kappa = \frac{C_p}{C_v}$$

R = spl. pl. konst

M = kilomol. masa

T = temp.

PLIN

$$c = \sqrt{\frac{\kappa R T}{M}}$$

GLASNOST

gostota en. toka

$$I [\text{dB}] = 10 \cdot \log_{10} \left(\frac{I}{I_0} \right)$$

$$j = \frac{W}{S} = \frac{P}{S}$$

$$I_0 = 10^{-12} \frac{\text{W}}{\text{m}^2}$$

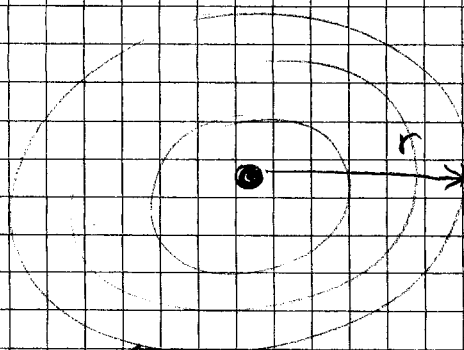
slušna meja pri frekvenci 1000 Hz

zvočnik

$$P = 500 \text{ W}$$

$$r = 10 \text{ m}$$

$$I [\text{dB}] = ?$$



krogla v prostoru

$$j(r) = \frac{P}{4\pi r^2} = \frac{500 \text{ W}}{4\pi \cdot 100 \text{ m}^2} = 0,398 \frac{\text{W}}{\text{m}^2} \approx \boxed{0,4 \frac{\text{W}}{\text{m}^2}}$$

površina
krogla

$$I [\text{dB}] = 10 \cdot \log_{10} \frac{0,4 \frac{\text{W}}{\text{m}^2}}{10^{-12} \frac{\text{W}}{\text{m}^2}} = 116 \text{ dB}$$

6.1.2011

"HELIKOPTER"

Dinamični vzgon ($\vec{F}_V$)

$$F_u = \frac{1}{2} C_u \rho \cdot S \cdot v^2$$

koeficient
upora

$$F_v = \frac{1}{2} C_v \cdot \rho \cdot S \cdot v^2$$

koeficient
dinamičnega
vzгона

$$\int F_y dt = \Delta G_y = m \cdot v_y$$

BERNOULLI-jeva enačba

$$p_0 + \frac{1}{2} \rho v_0^2 = p_1 + \frac{1}{2} \rho v_1^2$$

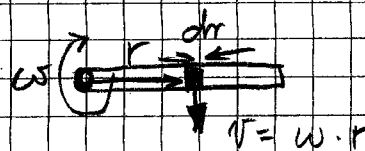
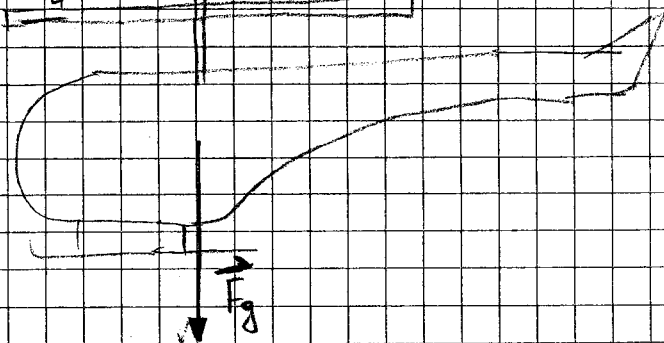
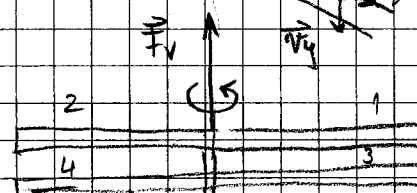
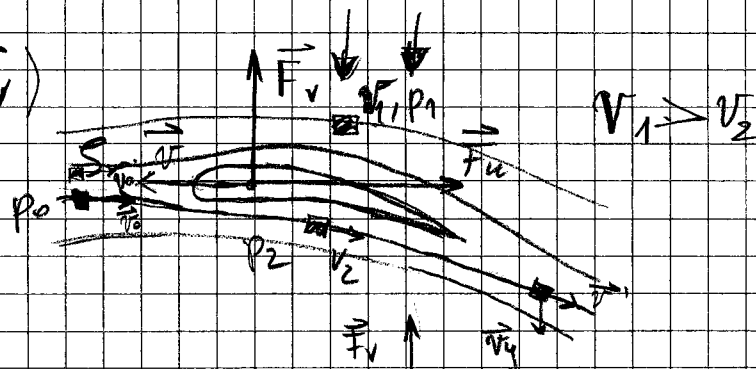
$$p_0 + \frac{1}{2} \rho v_0^2 = p_2 + \frac{1}{2} \rho v_2^2$$

$$p_2 - p_1 = \frac{1}{2} \rho (v_1^2 - v_2^2) > 0$$

en krak elise:

$$F_v = \int \frac{1}{2} C_v \rho \cdot v^2 \cdot dS$$

$\downarrow$
 $h \cdot dr$



$$F_v = \frac{1}{2} \rho \cdot c_v h \int_0^l v^2 dr = \frac{1}{2} \rho c_v h \cdot \omega^2 \int_0^l r^2 dr = \frac{1}{2} \rho c_v h \cdot \omega^2 \cdot \frac{l^3}{3}$$

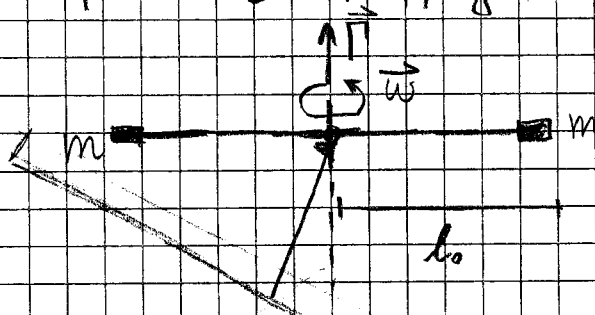
$$F_g = F_v$$

$$mg = 4 \cdot \frac{1}{2} \cdot c_v \cdot \rho \cdot h \cdot 4\pi^2 \omega^2 \frac{l^3}{3}$$

4 deli elise

$$\omega^2 = \frac{3mg}{8 c_v \pi^2 \cdot \rho \cdot l^3 \cdot h}$$

$$\omega = \sqrt{\frac{3 \cdot 0,1 \text{ kg} \cdot 10 \text{ m}}{4,8 \cdot 11 \cdot 5 \cdot 1 \cdot 1,4 \text{ kg} \cdot 10^{-3} \text{ m}^3 \cdot 0,5 \cdot 10^{-2} \text{ m}}} \approx 70 \text{ Hz (?)}$$



$$\vec{\tau} = \vec{J} \cdot \omega^2$$

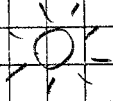
$$J = 2ml_0^2$$

$$\int \vec{M} dt = \Delta \vec{L} \Rightarrow \vec{L} = \text{konst}$$

GIROSKOP

2 rotorja me glavni gredi, da izniči navor ter se ne vrta okoli svoje osi

DOPPLERJEV POJAV



2. KOL. 2006 #12 1, UNI

$$\begin{aligned} \nu_0 &= 21000 \text{ Hz} \\ \nu &= 21450 \text{ Hz} \\ c_0 &= 1420 \frac{\text{m}}{\text{s}} \quad (\text{hitrost zvoka v vodi}) \end{aligned}$$

$$v = ?$$

$$c = \lambda \cdot \nu$$

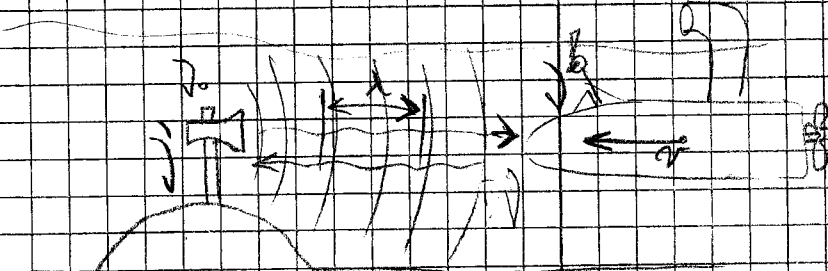
medbitni zvok

BAZA

POSLOVNICA

BAZA
MARJE
VIR
www.mar.si

$$\lambda_0 = \frac{\lambda}{c_0 + v}$$



$$\lambda = \lambda_0 \frac{c_0 + v}{c_0}$$

medbiti zvoke

PODMORNICA

BAZA

$$\lambda = \lambda' \Rightarrow \frac{c_0 + v}{\lambda} = \frac{c_0}{\lambda'} \Rightarrow \lambda' = \lambda \frac{c_0}{c_0 + v}$$

$$\lambda' = \lambda_0 \frac{c_0 + v}{c_0} \cdot \frac{c_0}{c_0 - v}$$

$$\lambda' = \lambda_0 \frac{c_0 + v}{c_0 - v}$$

$$\frac{\lambda'}{\lambda_0} (c_0 - v) = c_0 + v$$

$$\frac{\lambda'}{\lambda_0} c_0 - \frac{\lambda'}{\lambda_0} v = c_0 + v$$

$$\frac{\lambda'}{\lambda_0} c_0 - c_0 = v \left(1 + \frac{\lambda'}{\lambda_0} \right)$$

$$c_0 \left(\frac{\lambda'}{\lambda_0} - 1 \right) = v \left(1 + \frac{\lambda'}{\lambda_0} \right)$$

$$\lambda' = c_0 \frac{\frac{\lambda'}{\lambda_0} - 1}{\frac{\lambda'}{\lambda_0} + 1}$$

$$\lambda' = 1420 \frac{\text{m}}{\text{s}} \frac{\frac{2145}{2100} - 1}{\frac{2145}{2100} + 1} =$$

$$v = 15 \text{ m/s}$$

(13. nal. / str. 138)

Netopir

$$v = 0,025 \cdot c_0$$

$$\lambda_0 = 39000 \text{ Hz}$$

$$v_k = 80 \frac{\text{km}}{\text{h}} \approx 22 \frac{\text{m}}{\text{s}}$$

$$c_0 = 340 \frac{\text{m}}{\text{s}} \quad (\text{hitrost zvoke v zraku})$$

$$\lambda' = ?$$

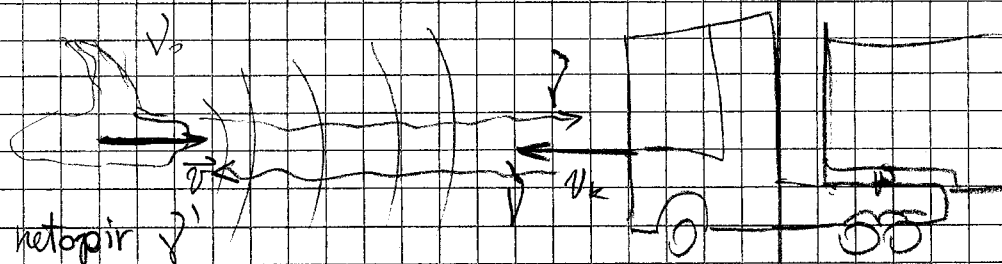
medbiti zvoke

netopir

kamion

$$\lambda_0 = \lambda$$

$$\frac{c_0 - v}{\lambda_0} = \frac{c_0 + v_k}{\lambda} \Rightarrow \lambda = \lambda_0 \frac{c_0 + v_k}{c_0 - v}$$



odbiti 80k

komion

metopir

$$\frac{c_0 - v_k}{v} = \frac{c_0 + v}{v'} \Rightarrow v' = v \frac{c_0 + v}{c_0 - v_k}$$

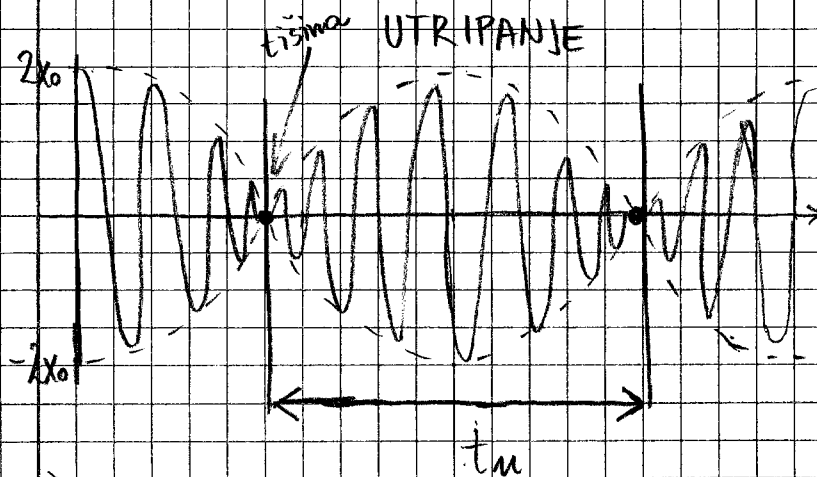
$$v' = v \cdot \frac{c_0 + v_k}{c_0 - v_k} \cdot \frac{c_0 + v}{c_0 - v} = v \cdot \frac{c_0 + v_k}{c_0 - v_k} \cdot \frac{1 + \frac{v}{c_0}}{1 - \frac{v}{c_0}} =$$

$$= 39000 \text{ Hz} \cdot \frac{362 \frac{\text{m}}{\text{s}}}{318 \frac{\text{m}}{\text{s}}} \cdot \frac{1,025}{0,975} \approx \underline{\underline{46670 \text{ Hz}}}$$

$$X = X_0 \cos \omega_1 t + X_0 \cos \omega_2 t$$

$$\omega_1 \approx \omega_2 \approx \bar{\omega}$$

$$X = 2X_0 \cos \left[\frac{(\omega_1 - \omega_2)t}{2} \right] \cdot \cos \left[\frac{(\omega_1 + \omega_2)t}{2} \right] \quad \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

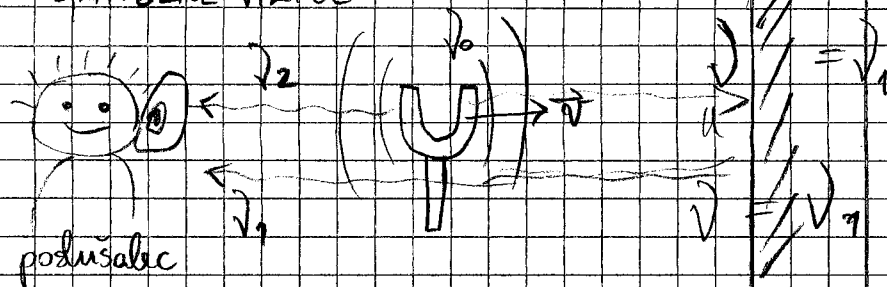


$$\frac{(\omega_1 - \omega_2)t_m}{2} = \pi$$

$$t_m = \frac{2\pi}{\omega_1 - \omega_2} = \frac{2\pi}{2\pi f_1 - 2\pi f_2}$$

$$v_m = \frac{1}{t_m} = |f_1 - f_2|$$

$$\begin{aligned}\lambda_0 &= 500 \text{ Hz} \\ v &= 2 \text{ m/s} \\ c_0 &= 332 \text{ m/s}\end{aligned}$$



$$\lambda_u = ?$$

(frekvenca utripanja zvoka pri poslušalcu)

NEODBITI ZVOK:

VILICE

POSLUŠALEC

$$\lambda_0 = \lambda_2$$

$$\frac{c_0 + v}{\lambda_0} = \frac{c_0}{\lambda_2} \Rightarrow \lambda_2 = \lambda_0 \frac{c_0}{c_0 + v} = 500 \text{ Hz} \cdot \frac{332}{334} = \underline{\underline{497 \text{ Hz}}}$$

ODBITI ZVOK:

VILICE

STENA

$$\lambda_0 = \lambda_1$$

$$\frac{c_0 - v}{\lambda_0} = \frac{c_0}{\lambda_1} \Rightarrow \lambda_1 = \lambda_0 \frac{c_0}{c_0 - v} = 500 \text{ Hz} \cdot \frac{332}{330} = \underline{\underline{503 \text{ Hz}}}$$

približno enaka frekvenca

$$\lambda_u = |\lambda_1 - \lambda_2| = \lambda_1 - \lambda_2 = 6 \text{ Hz}$$

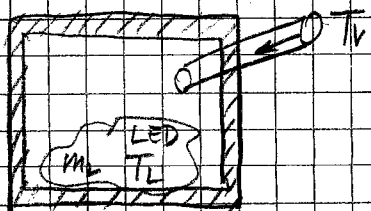
6 x na sekundo bo nastopala tišina

TERMODINAMIKA

12.1.2011

$$\begin{aligned}\Delta W_n &= A + Q \\ A &= - \int p dV\end{aligned}$$

Toplotno izolirana posoda



Koliko vode s T_v moramo pripeljati v posodo z dvema kilogramoma ledu, da se bo ves led stabiliziral?

$$\begin{aligned}T_v &= 50^\circ \text{C} \\ m_w &= 2 \text{ kg} \\ T_i &= 0^\circ \text{C} \\ c_p &= 4200 \frac{\text{J}}{\text{kg} \cdot \text{K}}\end{aligned}$$

$$Q_{\text{LAT}} = 3,3 \cdot 10^5 \text{ J/kg}$$

$$m_v = ?$$

$$T_k = 0^\circ\text{C}$$

$$m_L \cdot g_{\text{TAL}} + m_V \cdot c_p \cdot \Delta T = 0$$

$$m_V = \frac{-m_L \cdot g_{\text{TAL}}}{c_p \Delta T} = \frac{-m_L \cdot g_{\text{TAL}}}{c_p (T_k - T_V)} = \frac{-2 \text{ kg} \cdot 3,3 \cdot 10^5 \frac{\text{J}}{\text{kg}}}{4200 \frac{\text{J}}{\text{kg} \cdot \text{K}} (-50 \text{ K})} =$$

$$m_V \approx 3,1 \text{ kg}$$

42 / str. 163

$$m_V = \rho \cdot V = 1 \text{ kg}$$

$$W = m_V \cdot g_{\text{PRE}} - m_V \cdot c_p \Delta T$$

$$W = 1 \text{ kg} \cdot 3 \cdot 10^5 \frac{\text{J}}{\text{kg}} - 1 \text{ kg} \cdot 4200 \frac{\text{J}}{\text{kg} \cdot \text{K}} \cdot 30 \text{ K}$$

$$W \approx 2,87 \text{ MJ} = 685 \text{ kcal}$$

$$V = 1 \text{ l} = 1 \text{ dm}^3 =$$

$$T_0 = 8^\circ\text{C}$$

$$T_2 = 38^\circ\text{C}$$

$$g_{\text{pr}} = 3 \cdot 10^6 \frac{\text{J}}{\text{kg}} \quad (\text{ENERGIJSKA VREDNOST})$$

$$c_p = 4200 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

$$\rho = 1000 \frac{\text{kg}}{\text{m}^3} \approx 1 \frac{\text{kg}}{\text{dm}^3}$$

$$W = ?$$

$$1 \text{ kcal} = \frac{1 \text{ J}}{4200}$$

IDEALNI PLIN

$$pV = \frac{m}{M} RT$$

M ... kilomolska masa $[\frac{\text{kg}}{\text{kmol}}]$

R ... splošna plinska konstanta

$$8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}}$$

$$N_A \dots 6,02 \cdot 10^{23}$$

8. nal. / str. 146

$$V = 200 \text{ m}^3$$

$$T = 150^\circ\text{C}$$

$$p = 10^5 \text{ Pa}$$

$$T_0 = 18^\circ\text{C}$$

$$m_B = 30 \text{ kg}$$

$$m_t$$

$$M_{\text{ZR}} = 29 \text{ kg/kmol}$$

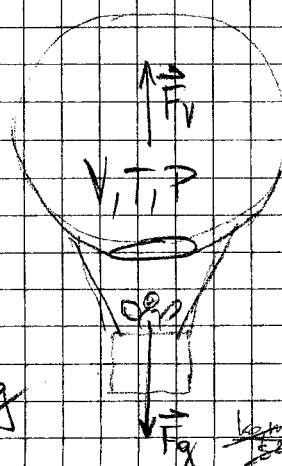
$$F_g = F_v$$

$$[m_B + m_t + m_{\text{ZR}}(T)] g = m_Z(T_0) g$$

$$m_t = m_Z(T_0) - m_Z(T) - m_B$$

$$m_t = \frac{pVM}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right) - m_B = \frac{pVM}{R} \left(\frac{T - T_0}{T_0 T} \right) - m_B = \frac{10^5 \text{ Pa} \cdot 200 \text{ m}^3 \cdot 29 \text{ kg/kmol}}{8314 \frac{\text{J}}{\text{kmol} \cdot \text{K}} \cdot 10^3 \cdot 10^3 \text{ K}} - 30 \text{ kg}$$

$$m_t \approx 4,5 \text{ kg}$$

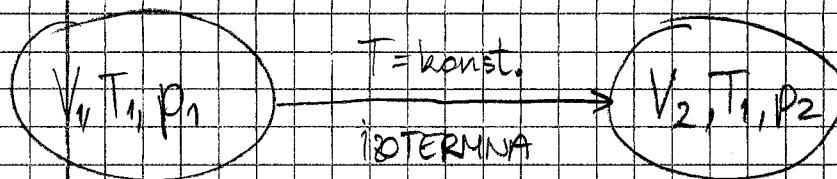


$$m = \frac{pVM}{RT}$$

$$\frac{pV}{T} = \frac{mR}{M} = \text{konst.}$$

$$\begin{aligned} T &= \text{konst.} \\ V &= \text{konst.} \\ p &= \text{konst.} \end{aligned}$$

$$\begin{aligned} pV &= \text{konst.} \\ \frac{p}{T} &= \text{konst.} \\ \frac{V}{T} &= \text{konst.} \end{aligned}$$



IDEALNI PLIN

$$\Delta W_n = m \cdot c_v \Delta T$$

$$C_p = c_v + \frac{R}{M}$$

$$\begin{aligned} V &= 1 \text{ m}^3 \\ T_1 &= 30^\circ\text{C} \\ p_1 &= 10^5 \text{ Pa} \\ p_2 &= 10^6 \text{ Pa} \end{aligned}$$

$$\Delta W_n = 0, \text{ temperatura se ne spreminja}$$

$$\begin{aligned} \Delta W_n &= A + Q \\ 0 &= A + Q \\ -A &= Q \end{aligned}$$

$$\begin{aligned} Q &= ? \\ \Delta W_n &= ? \\ \Delta S &= ? \end{aligned}$$

$$\begin{aligned} Q &= -A = \int_{V_1}^{V_2} p dV = \\ &= p_1 V_1 \int_{V_1}^{V_2} \frac{dV}{V} = \\ &= p_1 V_1 \ln \frac{V_2}{V_1} \end{aligned}$$

$$\begin{aligned} p_1 V_1 &= p_2 V_2 \Rightarrow V_2 = \frac{p_1 V_1}{p_2} \\ p_1 \cdot V_1 &= p \cdot V \Rightarrow p = \frac{p_1 V_1}{V} \end{aligned}$$

$$Q = -A = p_1 V_1 \ln \frac{p_1}{p_2}$$

$$Q = -A = 10^5 \text{ N/m}^2 \cdot 1 \text{ m}^3 \cdot \ln \frac{1}{10} = 10^5 \ln \frac{1}{10} \text{ Nm} \approx -230 \text{ kJ}$$

ADIABATNA (IZENTROPNA) SPREMEBA : $Q = 0$

$$pV^\gamma = \text{konst.}$$

$$\gamma = \frac{c_p}{c_v}$$

$$\begin{aligned} pV^\gamma &= \text{konst.} \\ \frac{pV}{T} &= \text{konst.} \end{aligned}$$

$$\frac{pV^\gamma}{pV} = \frac{T V^{\gamma-1}}{T} = \text{konst.}$$

$$\frac{T^\gamma}{p^{\gamma-1}} = \text{konst.}$$

2-atomni plin

$$V = 2\ell$$

DVOATOMNI IDEALNI PLIN

ADIABATNO STISK

$$Q = 0$$

$$P_1, V_1, T_1$$

$$P_2, V_2, T_2$$

$$V_1 = 2\ell = 2\text{dm}^3$$

$$T_1 = 20^\circ\text{C}$$

$$P_1 = 10^5\text{Pa}$$

$$V_2 = \frac{V_1}{2}$$

$$\Delta W_n = A + Q$$

$$Q = 0$$

$$\Delta W_n = A = m C_V \Delta T$$

$$C_p = C_v + \frac{R}{M} = \frac{5}{2} \frac{R}{M}$$

$$A = ?$$

$$\Delta W_n = ?$$

$$C_v = \frac{5}{2} \frac{R}{M}$$

$$m = \frac{pVM}{RT_1}$$

$$m C_V \Delta T = \frac{P_1 V_1 M}{RT_1} \cdot \frac{5}{2} \frac{R}{M} (T_2 - T_1)$$

$$T \cdot V^{\gamma-1} = T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$\Delta W_n = A = \frac{5}{2} \frac{P_1 V_1}{\gamma} \left(\left(\frac{V_1}{V_2} \right)^{\gamma-1} - 1 \right) =$$

$$= \frac{5}{2} P_1 V_1 \left(\left(\frac{V_1}{V_2} \right)^{\gamma-1} - 1 \right) = \frac{5}{2} 10^5 \frac{\text{N}}{\text{m}^2} \cdot 2 \cdot 10^{-3} \text{m}^3 \cdot \left(2^{\frac{5}{2}} - 1 \right) =$$

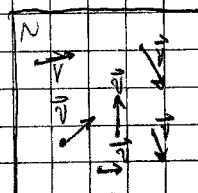
$$= 160 \text{ J}$$

$$pV = \frac{m}{M} RT = \frac{N m_0}{N_A M} RT$$

$$pV = N k_B T$$

BOLTZMANN-OVA KONSTANTA

$$k_B = \frac{R}{N_A} = 1,38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$$



$$\overline{W_{k, \text{transl.}}} = \frac{3}{2} k_B T$$

$$\frac{1}{2} m_1 \overline{v_x^2} = \frac{1}{2} k_B T$$

$$\frac{1}{2} m_1 \overline{v_x^2} + \frac{1}{2} m_1 \overline{v_y^2} + \frac{1}{2} m_1 \overline{v_z^2} = \frac{1}{2} m_1 \overline{v^2} \Rightarrow \overline{W_{k, \text{transl.}}}$$

2. KOL. FIZ. I
PETEK, 28.1.2011
OB 10⁰⁰
PRIJAVA NA IZPIT

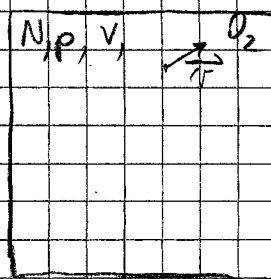
Kisik (O_2)

$$N = 2 \cdot 10^{20}$$

$$v_{eff} = \sqrt{v^2} = 600 \text{ m/s}$$

efektivna
hitrost

$$p = 1,28 \cdot 10^3 \text{ Pa}$$



$$M_{O_2} = 32 \frac{\text{kg}}{\text{kmol}}$$

$$V = ? \quad V = \frac{N k_B T}{p}$$

$$\frac{1}{2} m_1 \overline{v^2} = \frac{3}{2} k_B \cdot T$$

$$T = \frac{m_1 \overline{v^2}}{3 k_B}$$

$$V = \frac{N \cdot \cancel{k_B} \cdot m_1 \overline{v^2}}{3 \cancel{k_B} \cdot p}$$

$$M_{O_2} = N_A \cdot m_1$$

$$V = \frac{N \cdot M_{O_2} \cdot \overline{v^2}}{3 p N_A} = \frac{2 \cdot 10^{20} \cdot 32 \text{ kg} \cdot \left(600 \frac{\text{m}}{\text{s}}\right)^2}{6,02 \cdot 10^{26} \cdot 3 \cdot 1,28 \frac{\text{N}}{\text{m}^2} \cdot 10^3} = \frac{2,3 \cdot 10^{27} \cdot 10}{6,02 \cdot 10^{29}} = \underline{\underline{1 \text{ dm}^3}}$$

Kolikšno W_k imajo molekule kisika pri $T = 20^\circ\text{C}$?

$$\overline{W_{k1}} = ?$$

$$T = 20^\circ\text{C}$$

$$v_{eff} = \sqrt{v^2} = ?$$

$$\overline{W_k} = 3 \cdot \frac{1}{2} k_B T$$

$$\overline{W_{kr}} = 2 \cdot \frac{1}{2} k_B T$$

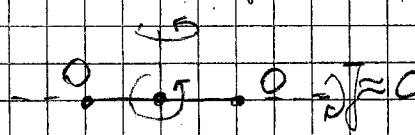
$$\overline{W_{k1}} = \overline{W_{k,trans}} + \overline{W_{k,rot}} = \frac{5}{2} k_B T = \frac{5}{2} \cdot 1,38 \cdot 10^{-23} \text{ J/K} \cdot 293 \text{ K} = \underline{\underline{10^{-20} \text{ J}}}$$

$$\overline{W_{k,trans}} = \frac{1}{2} m_1 \overline{v^2} = \frac{3}{2} k_B T$$

$$\overline{v^2} = \frac{3 k_B T}{m_1} = \frac{3 k_B T \cdot N_A}{M_{O_2}}$$

$$v_{eff} = \sqrt{v^2} = \sqrt{\frac{3 k_B T \cdot N_A}{M_{O_2}}} = \sqrt{\frac{3 \cdot 1,38 \cdot 10^{-23} \cdot 6,02 \cdot 10^{26} \cdot 293 \text{ K} \cdot \text{J/K}}{32 \text{ kg}}}$$

$$= \underline{\underline{478 \frac{\text{m}}{\text{s}}}}$$



Vsaka prostostna
stopnja prinese $\frac{1}{2} k_B T$

$$j = \frac{dP}{ds}$$

$$j = -\lambda \frac{dT}{dx}$$

$$P = \frac{dQ}{dt}$$

nal. 55/ntr. 170

$$\lambda = 1,8 \cdot 10^{-2} \frac{\text{J}}{\text{msK}}$$

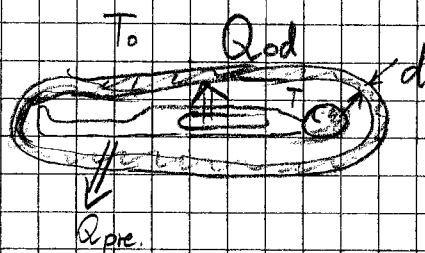
$$T = 37^\circ\text{C}$$

$$T_0 = 10^\circ\text{C}$$

$$S = 1,8 \text{ m}^2$$

$$P_{\text{od}} = 40 \text{ J/s}$$

d



$$Q_{\text{od}} = Q_{\text{pre}} \quad / \cdot \frac{d}{dt}$$

$$P_{\text{od}} = P_{\text{pre}}$$

$$j_{\text{pre}} = \frac{P_{\text{pre}}}{S} = -\lambda \frac{dT}{x}$$

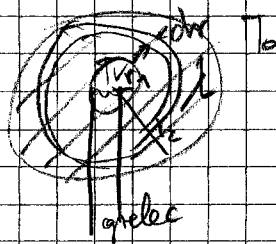
$$\frac{P_{\text{od}}}{S} = -\lambda \frac{T_0 - T}{x}$$

$$x = \frac{-\lambda (T_0 - T) S}{P} = \frac{\lambda S (T - T_0)}{P}$$

$$x = \frac{1,8 \cdot 10^{-2} \frac{\text{J}}{\text{msK}} \cdot 1,8 \text{ m}^2 (27^\circ\text{K})}{40 \text{ J/s}} \approx 2,2 \text{ cm}$$

Posoda ima doliko krogle z notr. radijem r_1 in zmr. radijem r_2 , toplotna prevodnost $\lambda = 0,2 \text{ W/mK}$, v posodi je voda s temp. 95°C , zmr. pa je $T_0 = 10^\circ\text{C}$. Kolikšno moč mora imeti grelec, da vzhreva temperaturo vode?

$$\begin{aligned} r_1 &= 10 \text{ cm} \\ r_2 &= 20 \text{ cm} \\ \lambda &= 0,2 \text{ W/mK} \\ T_v &= 95^\circ\text{C} \\ T_0 &= 10^\circ\text{C} \\ P_{\text{grelec}} &=? \end{aligned}$$



$$P_g = P_{\text{pre}} = j \cdot S = -\lambda S \frac{dT}{dr}$$

$$P_g = -4\pi r^2 \cdot \lambda \cdot \frac{dT}{dr}$$

$$\int_{r_1}^{r_2} \frac{P_g}{4\pi \lambda} \frac{dr}{r^2} = \int_{T_v}^{T_0} dT \Rightarrow -\frac{P_g}{4\pi \lambda} \left(-\frac{1}{r}\right) \Big|_{r_1}^{r_2} = T_0 - T_v \Rightarrow \frac{P_g}{4\pi \lambda} \left(\frac{1}{r_2} - \frac{1}{r_1}\right) = T_0 - T_v$$

$$P_g = \frac{4\pi \lambda (T_0 - T_v)}{\frac{1}{r_2} - \frac{1}{r_1}} = \frac{4\pi \cdot 0,2 \frac{\text{W}}{\text{mK}} (-85 \text{ K})}{-\frac{1}{5 \text{ m}}} = \underline{\underline{42,7 \text{ W}}}$$

$$\frac{dV}{V} = \beta \cdot dT$$

koeficient prostorninskega raztezka

$$\frac{dL}{L} = \alpha \cdot dT$$

koeficient linearnega raztezka

IZOTRUPNE
SNUGI

$$\beta = 3\alpha$$

V topi izolirani posodi je 1 kg vode pri temp. $T = 280 \text{ K}$. V posodo potopimo aluminijasto kroglo $T_1 = 350 \text{ K}$ in poičakamo, da se vzpostavi ravnovesje. Masa kroglice je $0,2 \text{ kg}$, $\alpha_{al} = 22,9 \cdot 10^{-6} \text{ K}^{-1}$, $c_{pal} = 880 \text{ J/kgK}$. Izmajmo zmesno temp. koeficient $\frac{\Delta V}{V}$.

$$m_v = 1 \text{ kg}$$

$$T_v = 280 \text{ K}$$

$$T_{al} = 350 \text{ K}$$

$$m_{al} = 0,2 \text{ kg}$$

$$\alpha_{al} = 22,9 \cdot 10^{-6} \text{ K}^{-1}$$

$$c_{pal} = 880 \text{ J/kgK}$$

$$c_{pv} = 4200 \text{ J/kgK}$$

$$\frac{\Delta V}{V} \approx \beta \Delta T$$

$$\frac{\Delta V}{V} \approx 3\alpha \Delta T$$

$$\boxed{\frac{dV}{V} = \beta dT}$$

$$m_{al} \cdot c_{pal} \cdot (T_k - T_{al}) + m_v \cdot c_{pv} \cdot (T_k - T_v) = 0$$

$$\frac{\Delta V}{V} = ?$$

$$T_k (m_{al} \cdot c_{pal} + m_v \cdot c_{pv}) = m_{al} \cdot c_{pal} \cdot T_{al} + m_v \cdot c_{pv} \cdot T_v$$

$$T_k = \frac{m_{al} \cdot c_{pal} \cdot T_{al} + m_v \cdot c_{pv} \cdot T_v}{m_{al} \cdot c_{pal} + m_v \cdot c_{pv}} = \frac{0,2 \text{ kg} \cdot 880 \text{ J/kgK} \cdot 350 \text{ K} + 1 \text{ kg} \cdot 4200 \text{ J/kgK} \cdot 280 \text{ K}}{0,2 \text{ kg} \cdot 880 \text{ J/kgK} + 1 \text{ kg} \cdot 4200 \text{ J/kgK}} =$$

$$T_k = 282,8 \text{ K}$$

$$\frac{\Delta V}{V} = 3\alpha_{al} (T_k - T_{al}) = 3 \cdot 22,9 \cdot 10^{-6} \text{ 1/K} (282,8 \text{ K} - 350 \text{ K}) =$$

$$= 68,7 \cdot (-67,2 \text{ K}) = \underline{\underline{-4,6 \cdot 10^{-3}}}$$

IDEALNI PARNI STROJ

Oceni kolikšna je razlika v delu ~~stroja~~ ki se dobi iz 1kg črnega premaga pri v id. parnem stroju, če ga uporabimo poleti ali v jeseni

$$g_{sej} = 80 \cdot 10^6 \text{ J/kg}$$

Carnotova krožna sprememba:

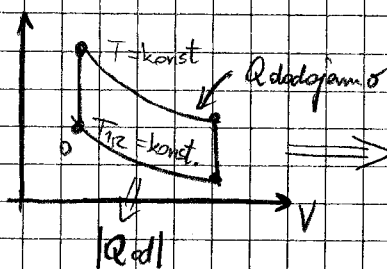
$$T_P = 25^\circ\text{C}$$

$$T_J = 7^\circ\text{C}$$

$$T_{KOTEL} = 44^\circ\text{C}$$

$$\Delta A = A_2 - A_1 = ?$$

$$\eta_c = \frac{|A|}{Q_{do}}$$



izkoristek

$$\eta_c = 1 - \frac{T_{1,2}}{T}$$

(eta)

$$|A_1| = Q_{do} \cdot \eta_c = m \cdot g_{sej} \left(1 - \frac{T_P}{T_K} \right)$$

$$|A_2| = Q_{do} \cdot \eta_c = m \cdot g_{sej} \left(1 - \frac{T_J}{T_K} \right)$$

$$\Delta A = m g_{sej} \left(1 - \frac{T_J}{T_K} - 1 + \frac{T_P}{T_K} \right) = m g_{sej} \left(\frac{T_P - T_J}{T_K} \right) = 80 \cdot 10^6 \text{ J} \cdot \frac{18\text{K}}{273\text{K}} = \approx 0,76 \text{ MJ}$$

2. KoloKVij 2010 FIZI

3. $W_p(r) = -G \frac{m_1 m_2}{r}$

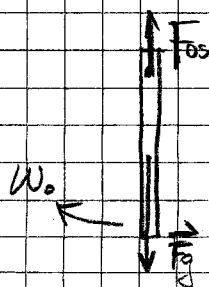
a) $+\frac{1}{r}$ b) $+\frac{1}{r^2}$ (c) $-\frac{1}{r}$ d) $-\frac{1}{r^2}$

5. Acetrip = ? = $\int \vec{F}_c \cdot d\vec{s} \Rightarrow \vec{F}_c \perp d\vec{s}$; skalarni produkt = 0

a) poz. (b) 0 c) ∞ d) neg.

6. $l = 1,5\text{m}$; kdaj je sila na nihalo v osi največja?

(a) $F=0$ b) $F=\max$ c) $F_{osi}=0$ d) $F_{osi}=\text{konst}$



$$F=0$$

$$F_{os} = F_g + m a_r = mg + m \omega_0^2 \cdot r$$

4. $X_0 = 2X_0$; $W_n = ?$

a) 4x manjša b) 2x večja c) 2xm (d) max

$$W = \frac{kx_0^2}{2}$$

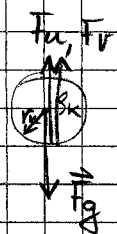
7. $\rho_k = 1600 \frac{\text{kg}}{\text{m}^3}$ $F_u = 6\pi r \eta v$

$\rho_m = 1400 \text{ kg/m}^3$

linearni zakon upora

$r_k = 12 \text{ mm}$

$\eta_m = 200 \text{ Pa/s} = 200 \text{ Ns/m}^2$



ρ_m, η

a) $v_k = ? \leftarrow \text{ravnovesje}$

$F_g - F_u - F_v = 0$

$m g - 6\pi r \eta v - \rho_m \frac{4}{3} \pi r^3 \cdot g$

$\rho_k \frac{4}{3} \pi r^3 g - 6\pi r \eta v - \rho_m \frac{4}{3} \pi r^3 \cdot g$

$\frac{4}{3} \rho_k r^3 \cdot g = 6\eta v + \rho_m \frac{4}{3} r^3 \cdot g$

$$v = \frac{\frac{4}{3} r^3 g (\rho_k - \rho_m)}{6\eta} = \frac{2}{9} \frac{r^2 g (\rho_k - \rho_m)}{\eta}$$

$$= \frac{(12 \cdot 10^{-3} \text{ m})^2 \cdot 9,81 \text{ m/s}^2 (2000 \text{ kg/m}^3)}{200 \text{ Ns/m}^2} = \underline{\underline{314 \frac{\mu\text{m}}{\text{s}}}}$$

b) v kolikšnem času po začetku pospeševanja doseže hitrost $v = 32 \frac{\mu\text{m}}{\text{s}}$

$\frac{N_1}{N_0} = \frac{32 \mu\text{m/s}}{0}$

$t_1 = ?$

$F_g - F_u - F_v = m \cdot a$

$\frac{4}{3} (\rho_k - \rho_m) \pi r^3 g - 6\pi r \eta v = \frac{4}{3} \pi r^3 \rho_k \cdot a$

$$a = \left(\frac{\rho_k - \rho_m}{\rho_k} \right) g - \frac{9 \eta v}{2 r^2 \rho_k} = \underbrace{\left(1 - \frac{\rho_m}{\rho_k} \right) g}_B - \underbrace{\frac{9 \eta}{2 r^2 \rho_k} \cdot v}_C$$

$\frac{dv}{dt} = B - C v$

$\int_{v_0}^{v_1} \frac{dv}{(B - C v)} = \int_{t_0}^{t_1} dt$

$B - C v = u$

$du = -C dv$

$dv = -\frac{du}{C}$

$$t_1 = -\frac{1}{C} \ln \frac{B - C v_1}{B - C v_0} = \frac{2}{9} \frac{r^2 \rho_k}{\eta} \ln \frac{(1 - \frac{\rho_m}{\rho_k}) g + \frac{9 \eta}{2 r^2 \rho_k} v_1}{(1 - \frac{\rho_m}{\rho_k}) g + \frac{9 \eta}{2 r^2 \rho_k} v_0}$$

$$= 512 \cdot 10^{-6} \ln \frac{1,23 + 0,125}{1,23} = \underline{\underline{49,55 \cdot 10^{-6} \text{ s}}}$$