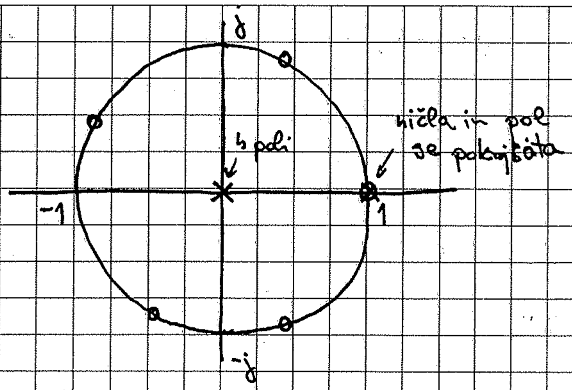
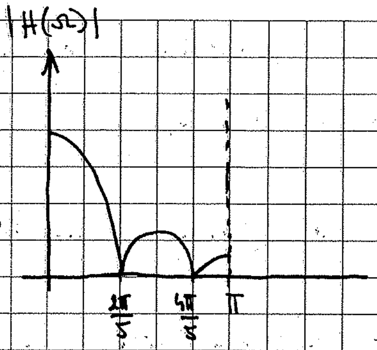
$20 \cdot \log 3 = x$

$10^{x/20} = 3$

③ 7 4 dB

74.



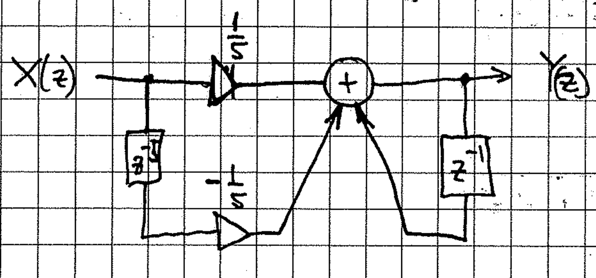


$$\frac{Y(z)}{X(z)} = \frac{1}{5}\,\frac{1-z^{-5}}{1-z^{-1}}$$

$$Y(z)\left(1-z^{-1}\right) = \frac{1}{5}X(z)\left(1-z^{-5}\right)$$

$$Y(z) - Y(z)z^{-1} = \frac{1}{5}X(z) - \frac{1}{5}X(z)z^{-5}$$

$$Y(z) = \frac{1}{5}X(z) - \frac{1}{5}X(z)z^{-5} + Y(z)z^{-1}$$



18.4.2013

Narišite generator kosinusnega in sinusnega signala z normirano krožno frekvenco Ω_1

$x[n] = \cos(\Omega_1 n)$ .... KOSINUS

$h[n] = a^n \cdot \cos(\Omega_1 \cdot n) \cdot u[n]$ .... KOSINUS

$$H(z) = \sum_{n=0}^{\infty} a^n \cdot \cos(\Omega_1 n)\, z^{-n} = \qquad \cos\varphi = \frac{1}{2}\left(e^{j\varphi} + e^{-j\varphi}\right)$$

$$= \frac{1}{2}\left[\sum_{n=0}^{\infty} a^n \cdot e^{j\Omega_1 n} z^{-n} + \sum_{n=0}^{\infty} a^n e^{-j\Omega_1 n} z^{-n}\right] =$$

$$H(z) = \frac{1}{2} \cdot \left[\frac{1 - \cancel{(a e^{j\Omega_1} z^{-1})^{\infty}}}{1 - a e^{j\Omega_1} z^{-1}} + \frac{1 - \cancel{(a e^{-j\Omega_1} z^{-1})^{\infty}}}{1 - a e^{-j\Omega_1} z^{-1}}\right] =$$

$$= \frac{1}{2} \frac{1 - a e^{-j\Omega_1} z^{-1} + 1 - a e^{+j\Omega_1} z^{-1}}{(1 - a e^{j\Omega_1} z^{-1})(1 - a e^{-j\Omega_1} z^{-1})} =$$

$$= \frac{1}{2} \frac{2 - a z^{-1}(e^{-j\Omega_1} + e^{j\Omega_1})}{1 - a e^{j\Omega_1} z^{-1} - a e^{-j\Omega_1} z^{-1} + a^2 z^{-2}}$$

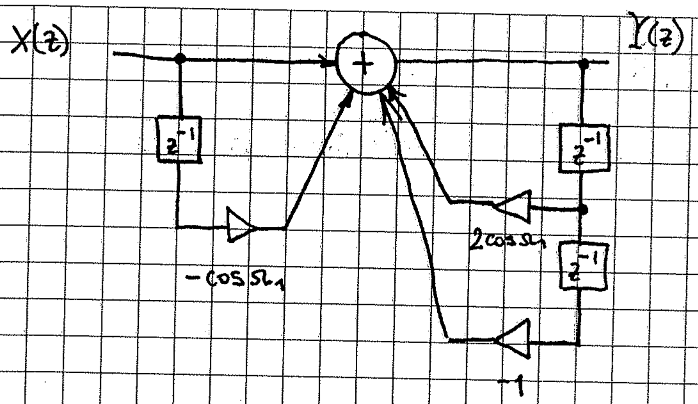
$$H(z) = \frac{1}{\cancel{2}} \frac{\cancel{2} - \cancel{2}\cos\Omega_1 \cdot a z^{-1}}{1 - 2\cos\Omega_1 \cdot a z^{-1} + a^2 z^{-2}} \quad \frac{1 \cdot z^2}{1 \cdot z^2} = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{z^2 - z \cdot \cos\Omega_1 \cdot a}{z^2 - 2\cos\Omega_1 \cdot z \cdot a + a^2}$$

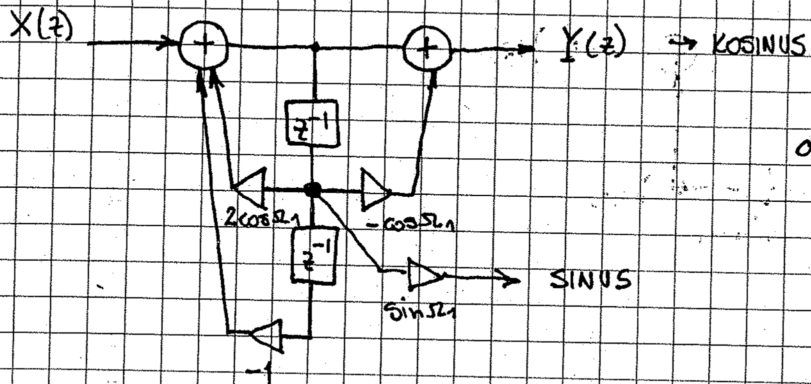
$$Y(z)\left(1 - 2\cos\Omega_1\, a z^{-1} + a^2 z^{-2}\right) = X(z)\left(1 - \cos\Omega_1\, a z^{-1}\right)$$

$$Y(z) = X(z) - a\cos\Omega_1\, X(z) z^{-1} + 2a\cos\Omega_1\, Y(z) z^{-1} - a^2\, Y(z)\, z^{-2}$$

če je $a = 1$: $Y(z) = X(z) - \cos\Omega_1\, X(z) z^{-1} + 2\cos\Omega_1\, Y(z) z^{-1} - Y(z) z^{-2}$



direktna str. I.



direktna str. II.

$h[n] = a^n \cdot \sin(\Omega_1 n)\ u[n]$ .... sinus $\qquad \sin\varphi = \frac{1}{2j}\left(e^{j\varphi} - e^{-j\varphi}\right)$

$$H(z) = \frac{1}{2j}\left(\sum_{n=0}^{\infty} a^n e^{j\Omega_1 n} z^{-n} - \sum_{n=0}^{\infty} a^n e^{-j\Omega_1 n} z^{-n}\right) =$$

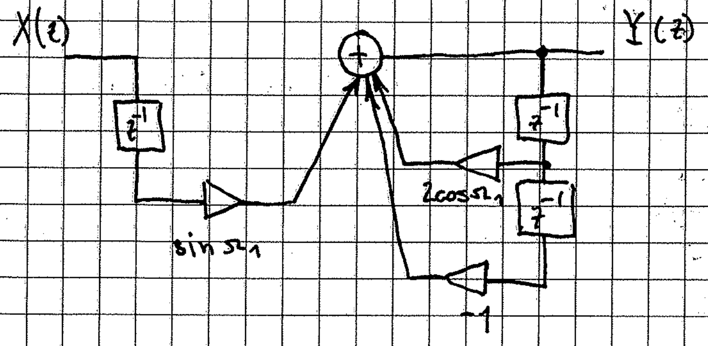
$$= \frac{1}{2j}\left(\frac{1}{1-ae^{j\Omega_1}z^{-1}} - \frac{1}{1-ae^{-j\Omega_1}z^{-1}}\right) = \frac{1}{2j}\,\frac{1-ae^{-j\Omega_1}z^{-1}-1+ae^{j\Omega_1}z^{-1}}{1-az^{-1}(2\cos\Omega_1)+a^2z^{-2}} =$$

$$= \frac{1}{\cancel{2j}}\,\frac{az^{-1}(\cancel{2j}\sin\Omega_1)}{1-2a\cos\Omega_1 z^{-1}+a^2z^{-2}} = \frac{az^{-1}\sin\Omega_1}{1-2a\cos\Omega_1 z^{-1}+a^2z^{-2}}$$

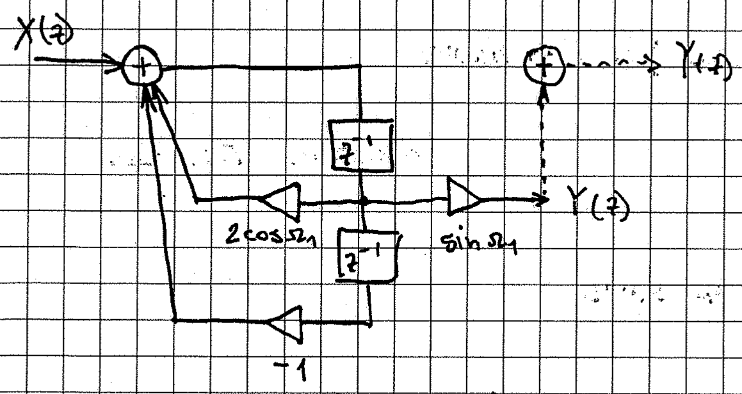
$$Y(z) = a\sin\Omega_1 X(z) z^{-1} + 2a\cos\Omega_1 Y(z) z^{-1} - a^2 Y(z) z^{-2}$$

za $a=1$:

$$Y(z) = \sin\Omega_1 X(z) z^{-1} + 2\cos\Omega_1 Y(z) z^{-1} - Y(z) z^{-2}$$



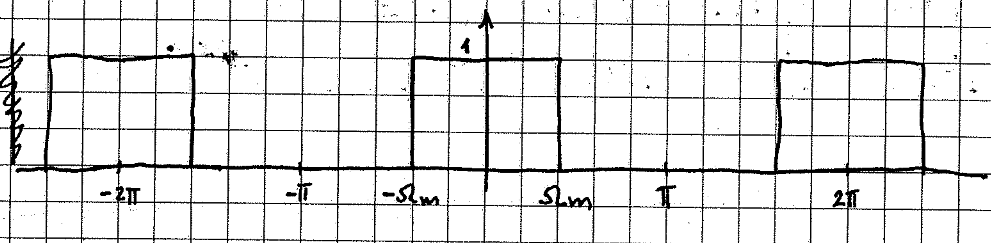
direktna str. I.



25.4.2013

18. Naloga

Izračunajte koeficiente nizkega FIR sita 8. reda $h_n[n]$ z mejno normirano krožno frekvenco $\Omega_m = \frac{\pi}{4}$.



H(Ω) imamo ITDFT

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\Omega)\, e^{+j\Omega n}\, d\Omega \quad \text{..... inv. Fourier transf.}$$

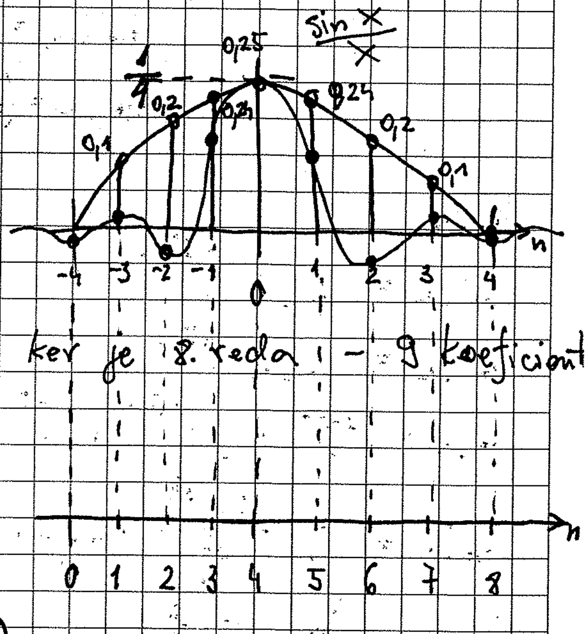
$$h[n] = \frac{1}{2\pi} \int_{-\Omega_m}^{\Omega_m} e^{+j\Omega n}\, d\Omega = \frac{1}{2\pi} \left. \frac{e^{+j\Omega n}}{jn} \right|_{-\Omega_m}^{\Omega_m} =$$

$$= \frac{1}{j2\pi n} \left(e^{j\Omega_m \cdot n} - e^{-j\Omega_m \cdot n} \right) =$$

$$h[n] = \frac{\cancel{2j} \sin(\Omega_m \cdot n)}{\cancel{j2}\pi n}$$

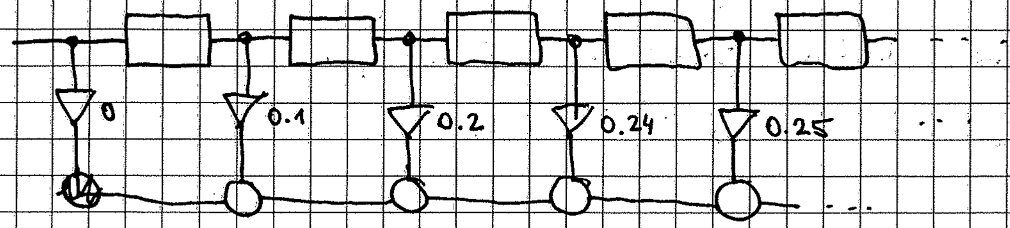
$$h[n] = \frac{\Omega_m}{\pi} \frac{\sin(\Omega_m n)}{\Omega_m n}$$

$$h[n] = \frac{\cancel{\pi}}{4\cancel{\pi}} \frac{\sin\left(\frac{n\pi}{4}\right)}{\frac{n\pi}{4}}$$



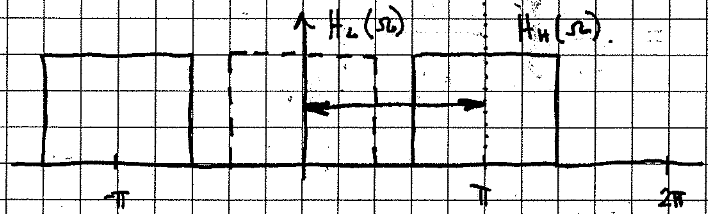
$$h[n] = \{ 0,\ 0.1,\ 0.2,\ 0.25,\ 0.24,\ 0.2,\ 0.1,\ 0 \}$$

shema:



19. NALOGA

Izračunajte koeficiente visokega FIR sita 8. reda $h_H[n]$ z mejno normirano krožno frekvenco $\Omega_m = \frac{4\pi}{5}$. Za izhodišče uporabite koeficiente nizkega sita $h_L[n] = \frac{\Omega_m}{\pi} \frac{\sin(\Omega_m \cdot n)}{\Omega_m \cdot n}$



$$H_H'(\Omega) = H_L(\Omega - \pi)$$
$$H_H''(\Omega) = H_L(\Omega + \pi)$$
$$H_H(\Omega) = \left(H_L(\Omega - \pi) + H_L(\Omega + \pi)\right) / 2$$

delimo z 2, ker dobimo dvojni spekter zaradi dveh premikov

$h_L[n]$

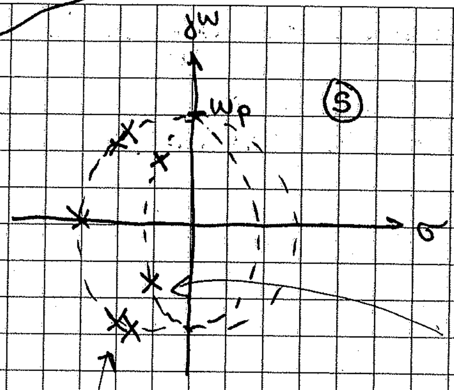
$$h_H'[n] = h_L[n]\, e^{-j\pi n}$$
$$h_H''[n] = h_L[n]\, e^{+j\pi n}$$
$$h_H[n] = \frac{h_L[n]}{2} \cdot \left(e^{-j\pi n} + e^{+j\pi n}\right) = \frac{h_L[n]}{\cancel{2}} \cdot \cancel{2}\cos(\pi n)$$

$$h_H[n] = \frac{1}{4} \frac{\sin\frac{n\pi}{4}}{\frac{n\pi}{4}} \cdot \cos \pi n$$ ... za podatke iz prejšne naloge

$$h_H[n] = \frac{\cancel{\pi}}{5 \cdot \cancel{\pi}} \frac{\sin\frac{n\pi}{5}}{\frac{n\pi}{5}} \cos \pi n$$ ... za podatke iz te naloge

IIR filtri

9. 5. 2013



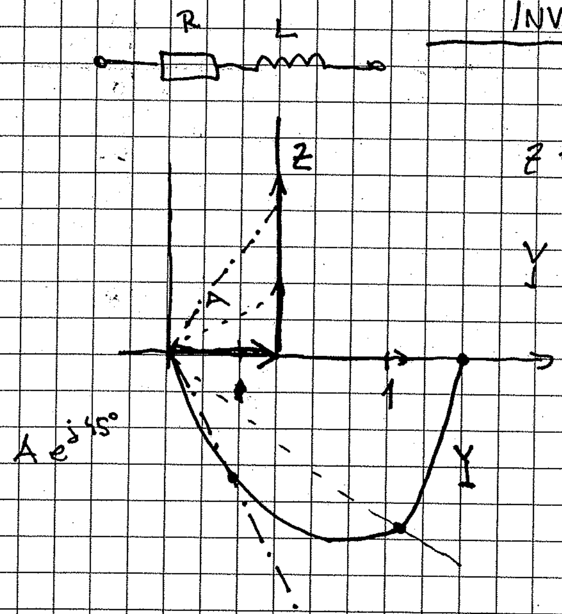
ωp... propusna frekv

Čebišev filter 2. reda (elipsa)

Butterworthov filter 2. reda (kružnica)

3. reda

INVERTIRANJE



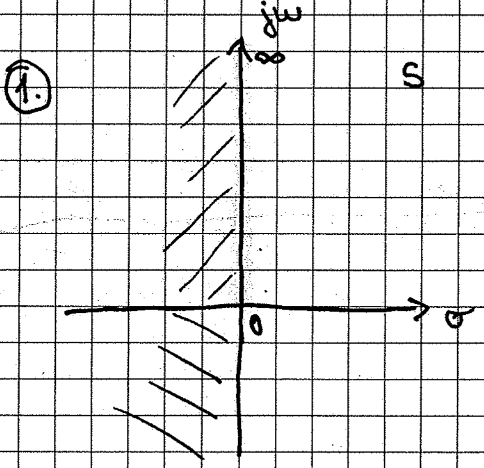
$Z = R + jX_L$

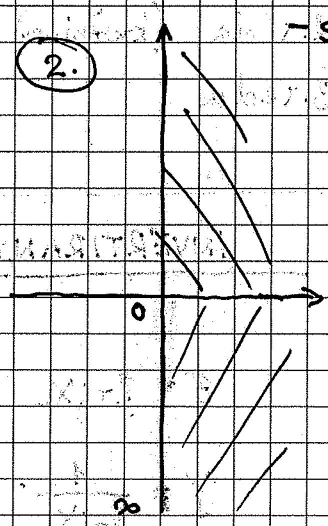
$$Y = \frac{1}{R + jX_L}$$

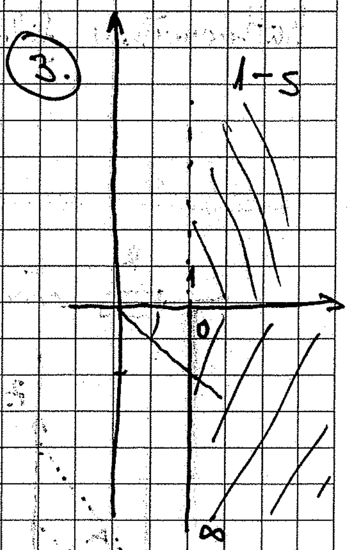
Z invertiranjem pokažite kako bi linearna transformacija

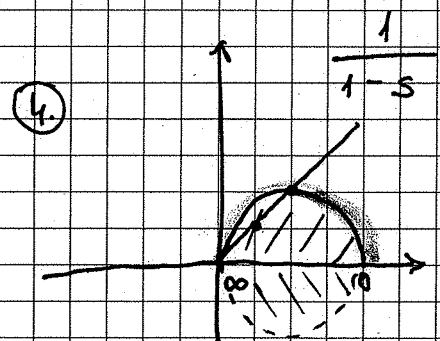
$$z = \frac{1+s}{1-s} \quad \text{preslika } s \text{ ravnino v } z \text{ ravnino.}$$

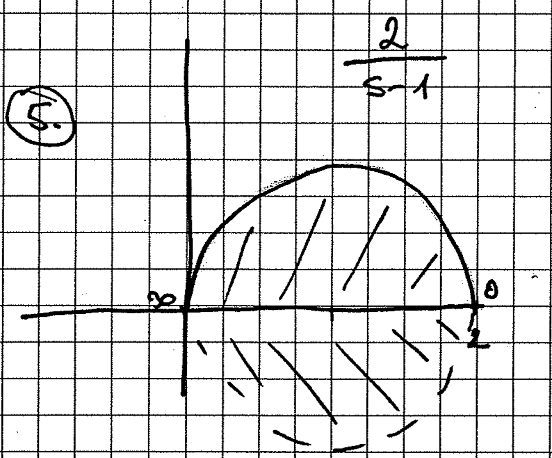
$$z = \frac{1+s}{1-s} = \frac{-(1-s)+2}{1-s} = \frac{2}{1-s} - 1$$

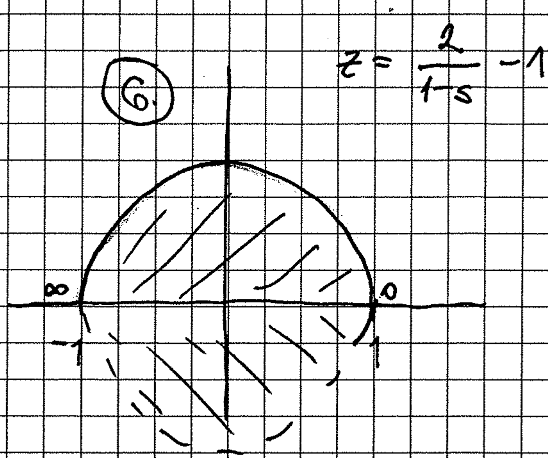












iz formule $z(s)$ izrazimo $s(z)$

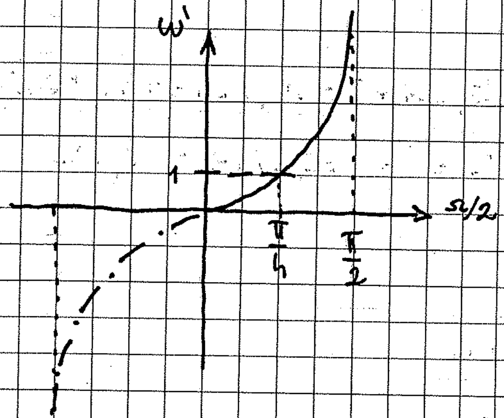
$$s = 1 - \frac{2}{z+1} = \frac{z+1-2}{z+1} = \frac{z-1}{z+1}$$

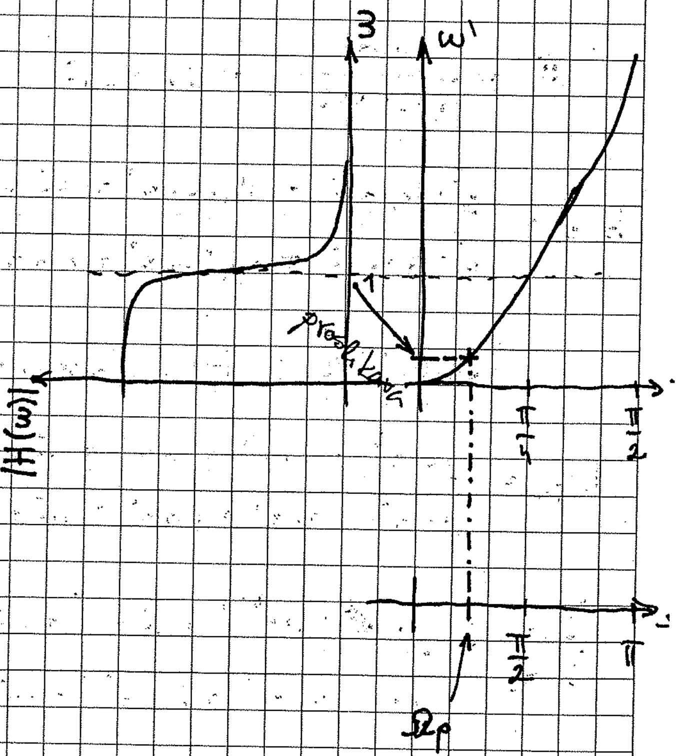
$$z = e^{j\Omega}$$

$$s = j\omega'$$

$$\cancel{j}\omega' = \frac{e^{j\Omega}-1}{e^{j\Omega}+1} = \frac{\cancel{e^{j\frac{\Omega}{2}}}\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right)}{\cancel{e^{j\frac{\Omega}{2}}}\left(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}}\right)} = \frac{\cancel{2j}\,\sin\frac{\Omega}{2}}{\cancel{2}\,\cos\frac{\Omega}{2}}$$

$$\omega' = \tan\frac{\Omega}{2}$$





$$\omega' = \frac{\omega}{\omega_N} \qquad \omega_p' = \frac{\omega_p}{\omega_N} = \tan\frac{\Omega_p}{2}$$

$$\frac{1}{\omega_N} = \tan\frac{\Omega_p}{2} \qquad \omega_N = \frac{1}{\tan\frac{\Omega_p}{2}}$$

p_{s1}, p_{s2} $\quad S = \frac{s}{\omega_N}$ ← mali s

$z = \frac{1+s}{1-s} = \frac{2}{1-s} + 1$ (lažji izračun na kalkulator)

b_0 ... velikost filtra

$$p_{z1} = \frac{1+\frac{p_{s1}}{\omega_N}}{1-\frac{p_{s1}}{\omega_N}} \qquad p_{z2} = \frac{1+\frac{p_{s2}}{\omega_N}}{1-\frac{p_{s2}}{\omega_N}}$$

$$H(z) = b_0 \frac{(z+1)(z+1)}{(z-p_{z1})(z-p_{z2})}$$

$$b_0 = \frac{1}{|H(e^{j0})|}$$

16.5.2013

24. naloga

LTI sistem ima podano sistemsko funkcijo:

$$H(z) = \frac{z^5 - 2.5247147\, z^4 + 1{,}573690\, z^3 + 1{,}5736901\, z^2 - 2{,}5247147\, z + 1}{z^5 - 4{,}4135352\, z^4 + 7{,}890889\, z^3 - 7{,}1347574\, z^2 + 3{,}259770\, z - 0{,}6016686}$$

Koeficiente zapišite z osmimi biti.

1 7

$\boxed{\pm \quad\quad\quad\quad}$

127

množimo ulomek z tako številko, da bo najvišji koeficient enak 127

$$☺ = \frac{127}{7{,}890889} = 16{,}0945$$

množimo $H(z) \cdot$ ☺

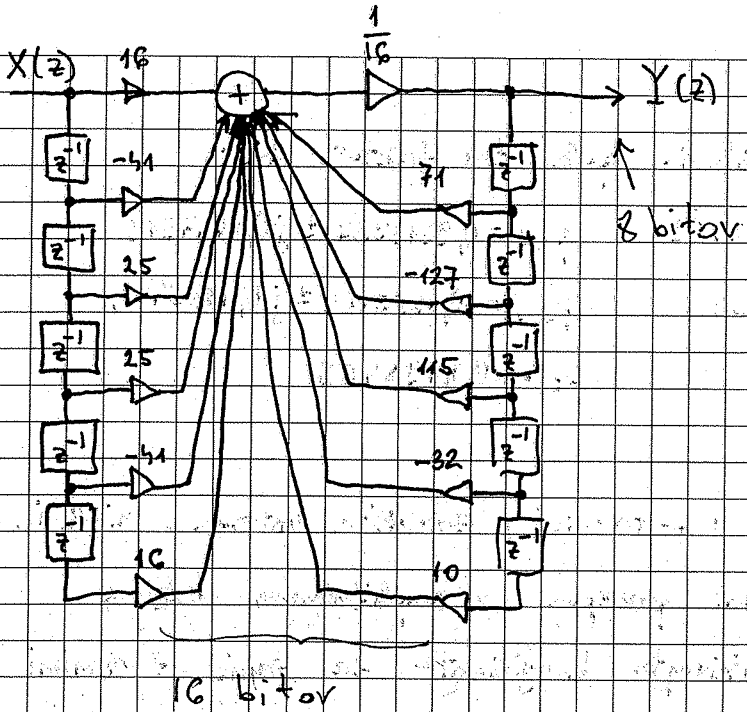
$$H(z) = \frac{z^5 \cdot 16{,}0945 - 40{,}634\, z^4 + 25{,}327\, z^3 + 25{,}327\, z^2 - 40{,}634\, z + 16{,}0945}{z^5 \cdot 16{,}0945 - 71{,}033\, z^4 + 127\, z^3 - 114{,}83\, z^2 + 32{,}464\, z - 9{,}683}$$

sedaj vse cifre zaokrožimo

$$H(z) = \frac{16 z^5 - 41 z^4 + 25 z^3 + 25 z^2 - 41 z + 16}{16 z^5 - 71 z^4 + 127 z^3 - 115 z^2 + 32 z - 10}$$

$$\frac{Y(z)}{X(z)} = \frac{16 - 41 z^{-1} + 25 z^{-2} + 25 z^{-3} - 41 z^{-4} + 16 z^{-5}}{16 - 71 z^{-1} + 127 z^{-2} - 115 z^{-3} + 32 z^{-4} - 10 z^{-5}}$$

$$16\, Y(z) = 16\, X(z) - 41\, X(z) z^{-1} + 25\, X(z) z^{-2} + 25\, X(z) z^{-3} - 41\, X(z) z^{-4} + 16\, X(z) z^{-5} +$$
$$+ 71\, Y(z) z^{-1} - 127\, Y(z) z^{-2} + 115\, Y(z) z^{-3} - 32\, Y(z) z^{-4} + 10\, Y(z) z^{-5}$$


X(z)
16
$\frac{1}{16}$
Y(z)
8 bitov
-41
25
25
-41
16
71
-127
115
-32
10
16 bitov

24. naloga

LTI sistem ima podane ničle in pole ter faktor b_0:

$n_{1,2} = 0{,}9244760 \pm j\,0{,}3812403$ $p_{1,2} = 0{,}9313337 \pm j\,0{,}2647313$

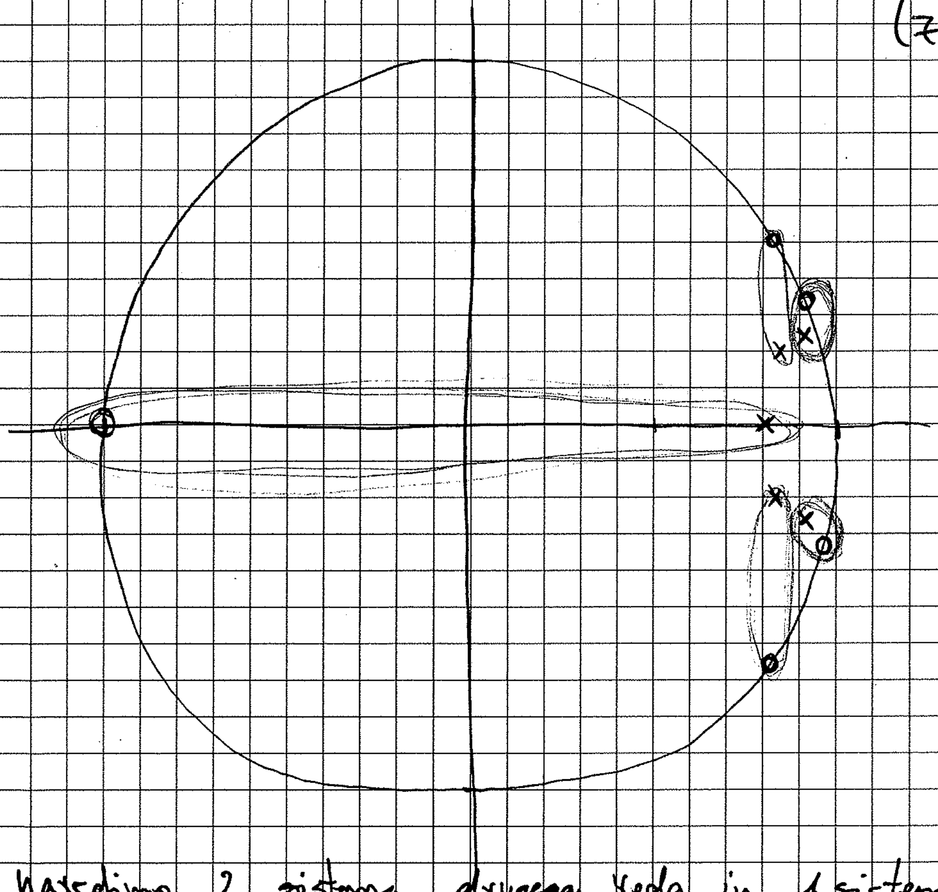
$n_{3,4} = 0{,}8378814 \pm j\,0{,}5458524$ $p_{3,4} = 0{,}8655104 \pm j\,0{,}1836444$

$n_5 = -1$ $p_5 = 0{,}8198469$

$b_0 = 0{,}0071278$

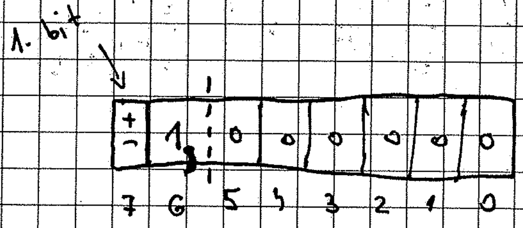
Skicirajte ničle in pole v z-ravnini, združite ustrezne ničle in pole, da dobimo najmanjši možni prerazpon v amplitudnem odzivu posameznih celic. Kvantizirajte koeficiente za zapis z osmimi biti.

$$(z-a-jb)(z-a+jb) = z^2 - 2az + (a^2+b^2)$$



Nastanimo 2 sistema drugega reda in 1 sistem prvega reda

$$H(z) = b_0 \frac{(z-n_1)(z-n_2)}{(z-p_1)(z-p_2)} = \frac{z^2 - \overbrace{(n_1+n_2)}^{\text{vedno} < 2} z + \overbrace{n_1 n_2}^{\leq 1}}{z^2 - \underbrace{(p_1+p_2)}_{\text{vedno} < 2} z + \underbrace{p_1 p_2}_{\leq 1}}$$



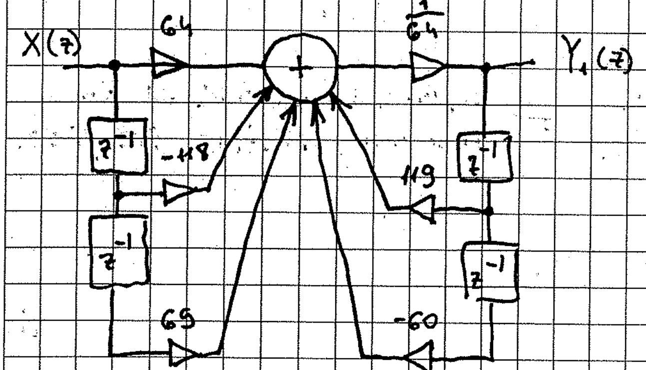
če imamo dvojno ničlo, jo realiziramo kot z sistema prvega reda

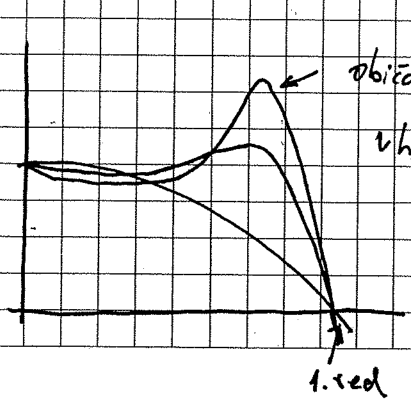
$$H_1(z) = \frac{z^2 - 1{,}848952 \cdot z + 1}{z^2 - 1{,}8626674 \cdot z + 0{,}97465}$$

vse množimo z 64 in z z^{-2}

$$= \frac{64 - 118{,}3\, z^{-1} + 64\, z^{-2}}{64 - 119{,}2\, z^{-1} + 59{,}9\, z^{-2}}$$

$$64\, Y_1(z) = 64\, X_1(z) - 118 X_1(z)\, z^{-1} + 64 X_1(z) z^{-2} + 119\, Y_1(z)\, z^{-1} - 60\, Y_1(z)\, z^{-2}$$





običajno se da b_0 pred vhod v tistega ki ima največjo špico.

1. red

Akta®

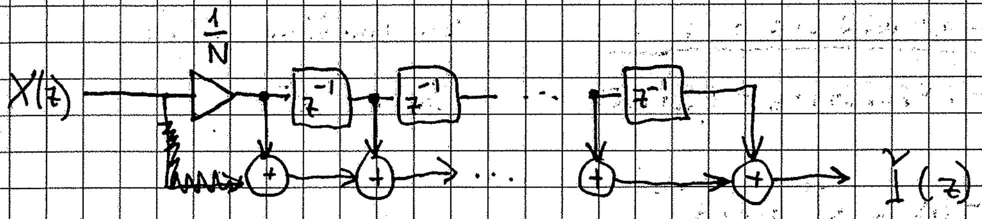
23.5.2013

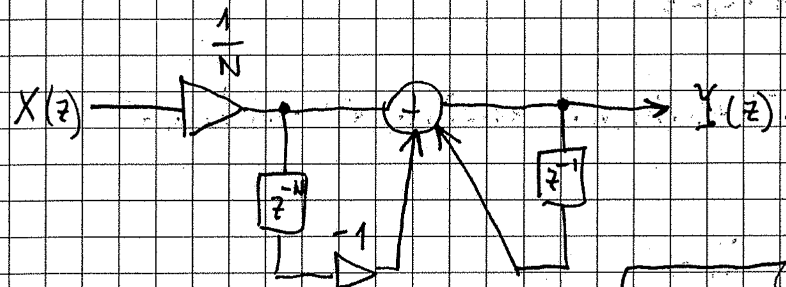
POVPREČEVALNIK - da odstranimo enosmerno komponento

N

$$H(z) = \frac{1}{N}\sum_{n=0}^{N-1} z^{-n} = \frac{1}{N}\,\frac{1-z^{-N}}{1-z^{-1}} = \frac{1}{N}\,\frac{z^{N}-1}{z^{N}-z^{N-1}} = \frac{1}{N}\,\frac{z^{N}-1}{z^{N-1}(z-1)}$$

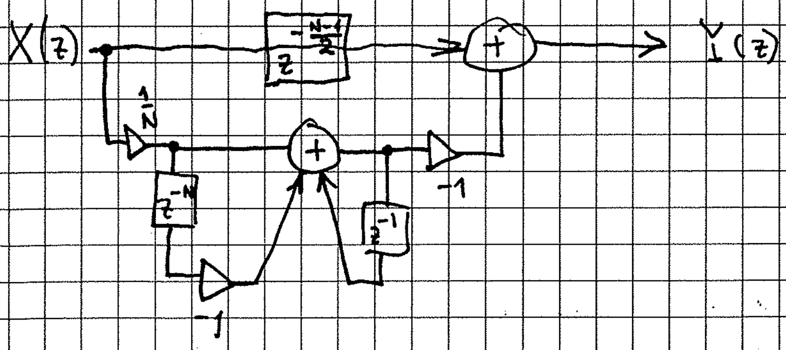
$$Y(z) = \frac{1}{N}\left(X(z) - X(z)z^{-N}\right) + Y(z)z^{-1}$$

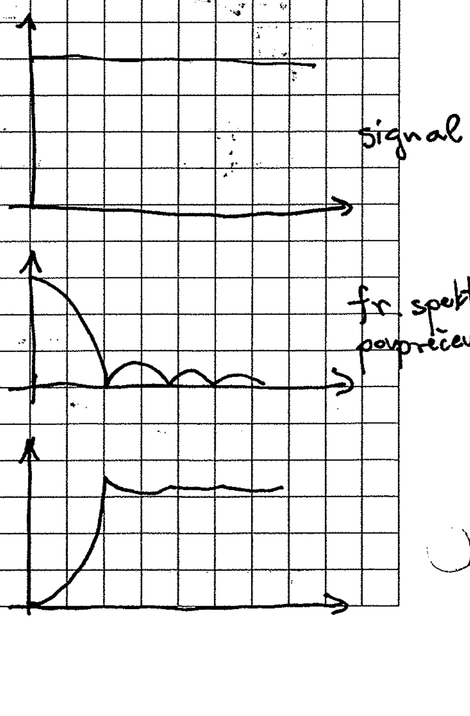


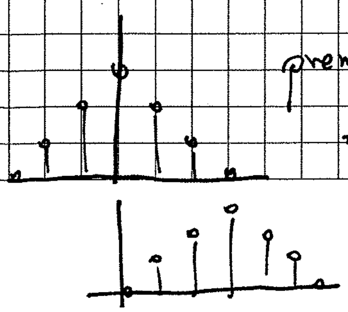


... povprečevalnik

$\boxed{\frac{N-1}{2}}$ = SIGNAL JE ZAMAKNJEN ZA POLOVICO DOLŽINE FIR filtra

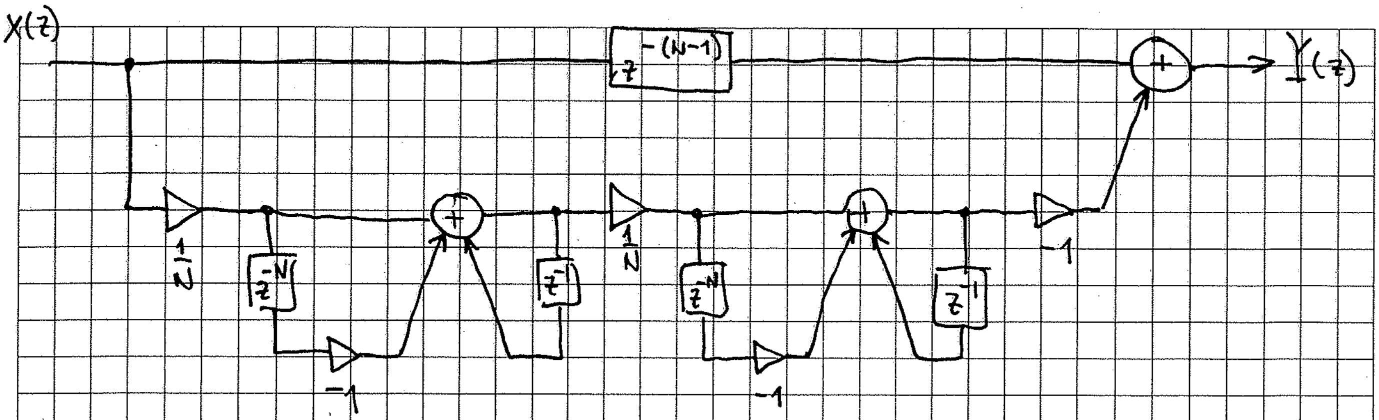






premaknemo za $\frac{N-1}{2}$

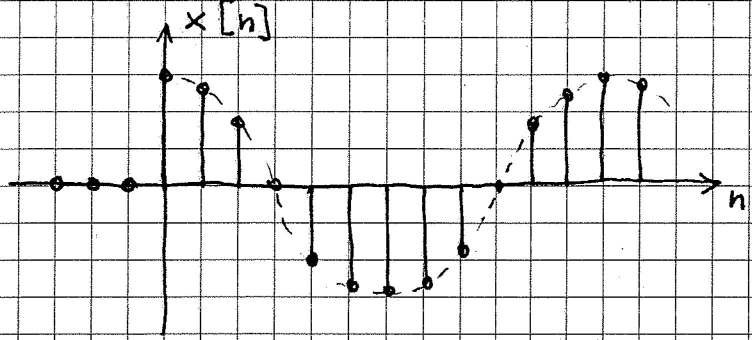
zato zamaknemo tudi člen v filtru


X(z)
z^-(N-1)
Y(z)
1/N
z^-N
z^-1
-1
1/N
z^-N
z^-1
-1
-1

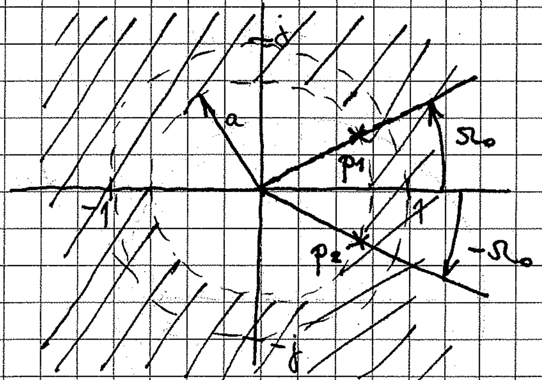
DSS 2.60

4. Dušeni kosinusni signal

$$x[n] = \left(a^n \cos(\Omega_0 n)\right) \cdot u[n]$$



$$\longleftrightarrow \quad \frac{z(z - a\cos\Omega_0)}{z^2 - 2a\cos\Omega_0 \cdot z + a^2}$$



območje konvergence

5. Dušeni sinusni signal

$$x[n] = \left(a^n \sin\Omega_0 n\right) u[n] \quad \longleftrightarrow \quad \frac{z \cdot a \sin\Omega_0}{z^2 - 2a\cos\Omega_0 z + a^2} \qquad |z| > |a|$$

ZGLED PRAVILA O ODVAJANJU (stran 25-2)

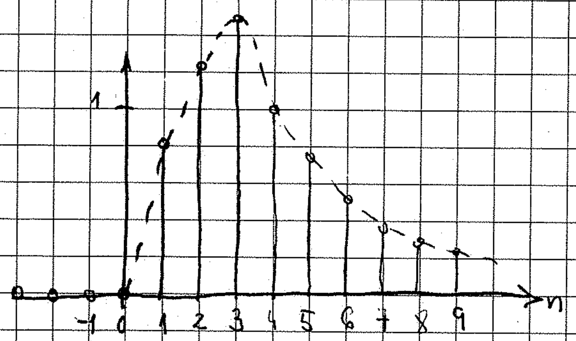
$$X(z) \longleftrightarrow x[n]$$

$$-z\,\frac{dX(z)}{dz} \longleftrightarrow n\,x[n]$$

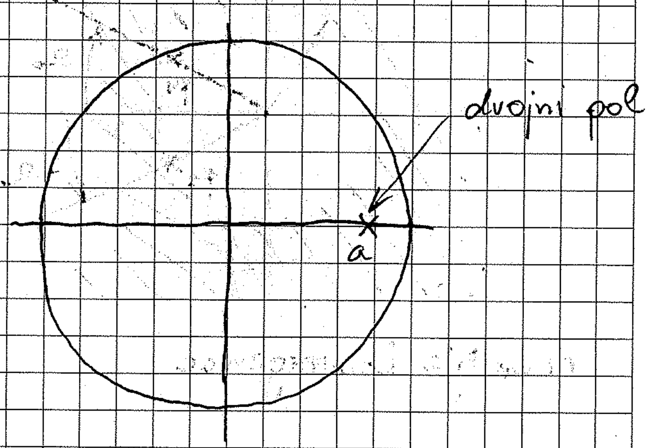
$$x[n] = a^n \cdot u[n] \qquad |a| < 1$$

$$X(z) = \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

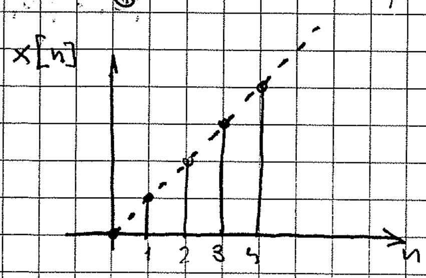
$$-z\,\frac{dX(z)}{dz} = -z\,\frac{\cancel{z}-a-\cancel{z}}{(z-a)^2} = \frac{az}{(z-a)^2} \longleftrightarrow (n \cdot a^n) \cdot u[n]$$

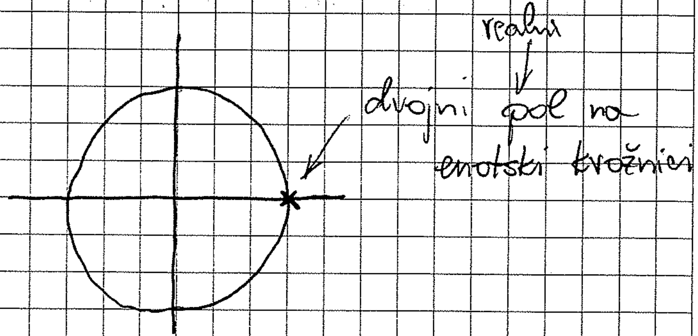


graf za $a = 0.8$

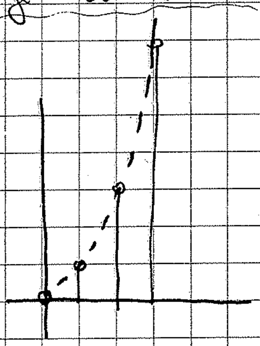


Če je $a = 1$, potem:





če je $a > 1$

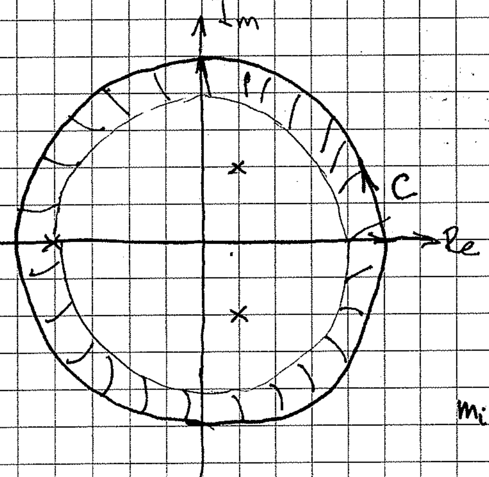


dvojni pol izven območja konvergence

5.5.5. INVERZNA Z - TRANSFORMACIJA

$$x[n] = \frac{1}{2\pi j} \oint_C X(z)\, z^{n-1}\, dz$$

C ... sklenjena krivulja v ROC in obkroža izhodišče v pozitivnem smislu



$$x[n] = \sum_{i=1}^{M} \operatorname*{Res}_{z=p_i} \left[X(z) \cdot z^{n-1}\right]$$

$$\operatorname*{Res}_{z=p_i} \left[X(z)\, z^{n-1}\right] = \lim_{z \to p_i} \left[\frac{1}{(m_i - 1)!} \frac{d^{(m_i-1)}}{dz^{(m_i-1)}} \left((z-p_i)^{m_i} \cdot X(z) \dots\right)\right]$$

m_i je red pola p_i

za $m_i = 1$: $\operatorname*{Res}_{z=p_i} \left[X(z)\, z^{n-1}\right] = \lim_{z \to p_i} \left[(z-p_i)\, X(z)\, z^{n-1}\right]$

$m_i = 2$: $\operatorname*{Res}_{z=p_i} \left[X(z)\, z^{n-1}\right] = \lim_{z \to p_i} \left[\frac{d}{dz}\left((z-p_i)^2 \cdot X(z)\, z^{n-1}\right)\right]$

Zgled:

$$X(z) = \frac{z}{z-a} \qquad \text{ROC} \quad |z| > |a|$$

$$x[n] = \operatorname*{Res}_{z=a}\left[\frac{z}{z-a}\, z^{n-1}\right] = \lim_{z\to a}\left(\cancel{(z-a)}\,\frac{z}{\cancel{(z-a)}}\, z^{n-1}\right)$$

konvergenca zahteva, da velja $n \geq 0$

$$x[n] = a^n \quad \text{za} \quad n \geq 0$$

$$\emptyset \quad \text{za} \quad n < 0$$

Zgled: uporaba tabele z-transformacij

$$X(z) = \frac{3\cdot z}{(z-0{,}6)(z+0{,}5)} \qquad \text{ROC} \quad |z| > 0{,}6 \quad \text{... desnostranski signal}$$

$$X(z) = \frac{A}{z-0{,}6} + \frac{B}{z+0{,}5} = \frac{Az + 0{,}5A + Bz - 0{,}6B}{(z-0{,}6)(z+0{,}5)}$$

$$A + B = 3$$

$$0{,}5A - 0{,}6B = 0$$

$$A = 3 - B = \frac{18}{11}$$

$$2{,}2B = 3$$

$$B = \frac{3}{2{,}2} = \frac{15}{11}$$

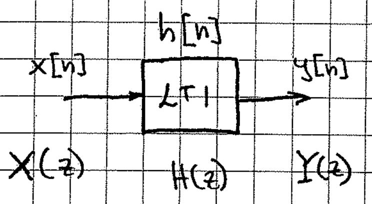
$$X(z) = \frac{18}{11}\frac{1}{z-0{,}6} + \frac{15}{11}\frac{1}{z+0{,}5}$$

$$\updownarrow \text{ZT}$$

$$x[n] = \frac{18}{11}\, 0{,}6^{n-1} \cdot u[n-1] + \frac{15}{11} \cdot 0{,}5^{n-1} \cdot u[n-1]$$

5.6. SISTEMSKA FUNKCIJA $H(z)$

5.6.1. DEFINICIJA SISTEMSKE FUNKCIJE



$$y[n] = x[n] * h[n]$$

$$\updownarrow \text{ZT}$$

$$\boxed{Y(z) = X(z) \cdot H(z)}$$

1. $H(z)$ je z-transform impulznega odziva "mrtvega" sistema

   zakasnilni elementi so 0 za $n \leq 0$

   $$H(z) = \mathcal{Z}\{h[n]\}$$

2. $$\boxed{H(z) = \frac{Y(z)}{X(z)}}$$

5.6.2. DOLOČANJE $H(z)$

1. Iz poznanega impulznega odziva

   $$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

   za kavzalne odzive $\quad H(z) = \sum_{n=0}^{\infty} h[n] \cdot z^{-n}$

2. Iz bločne sheme v z-prostoru

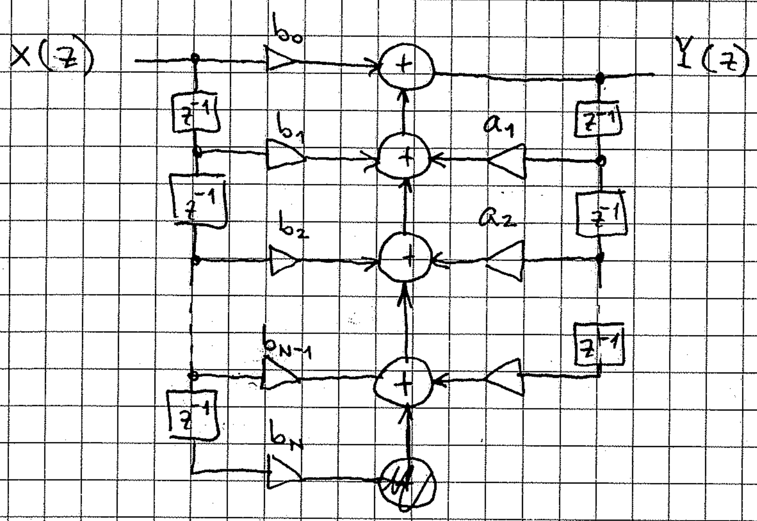
   - seštevalniki in množilniki ostanejo nespremenjeni
   - zakasnilni elementi se spremenijo v množilnike

   $$y[n-m] \longleftrightarrow z^{-m}\, Y(z)$$

18.4.2013

$y[n] \rightarrow \boxed{T} \rightarrow y[n-1] \overset{ZT}{\longleftrightarrow} Y(z) \rightarrow \boxed{z^{-1}} \rightarrow Y(z) \cdot z^{-1}$

Direktna struktura I v z-prostoru



3. S transformacijo diferenčne enačbe

$$y[n] = \sum_{i=0}^{N} b_i \, x[n-i] + \sum_{j=1}^{M} a_j \, y[n-j]$$

$\updownarrow$ z-transf.

$$Y(z) = \sum_{i=0}^{N} b_i \, X(z) \, z^{-i} + \sum_{j=1}^{M} a_j \, Y(z) \, z^{-j}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\sum_{i=0}^{N} b_i \, z^{-i}}{\sum_{j=0}^{M} a_j \, z^{-j}}$$

5.6.3. POLI IN NIČLE SISTEMSKE FUNKCIJE

$$H(z)\cdot \underbrace{\frac{z^N}{z^M}\, z^{M-N}}_{1} = \frac{\sum_{i=0}^{N} b_i\, z^{N-i}}{z^M - \sum_{j=1}^{M} a_j\, z^{M-j}} \cdot z^{M-N}$$

$$\boxed{H(z) = b_0\, \frac{z^N + \frac{b_1}{b_0} z^{N-1} + \frac{b_2}{b_0} z^{N-2} + \dots + \frac{b_N}{b_0}}{z^M - a_1 z^{M-1} - a_2 z^{M-2} - \dots - a_M} \cdot z^{M-N}}$$

Ničle polinoma v števcu $(N(z))$ so ničle sistemske funkcije

z_i ; $i = 1, 2, \dots, N$

$H(z_i) = 0$

Poli so ničle polinoma v imenovalcu $(D(z))$

p_j ; $j = 1, 2, \dots, M$

$|H(p_j)| \longrightarrow \infty$

$$H(z) = b_0\, \frac{(z-z_1)(z-z_2)\cdots(z-z_N)}{(z-p_1)(z-p_2)\cdots(z-p_M)}\, z^{M-N}$$

ničle in poli določajo $H(z)$ do multiplikativne konstante natančno

5.6.4. VPLIV LEGE POLOV IN NIČEL $H(z)$ NA LASTNOSTI SISTEMA

1. STABILNOST

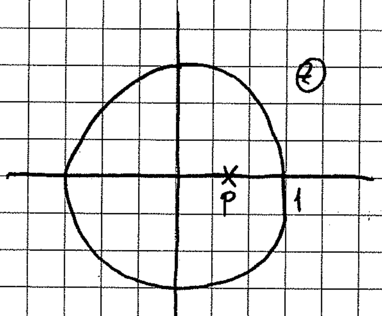
Sistem je stabilen, če so vsi poli v notranjosti enotske krožnice

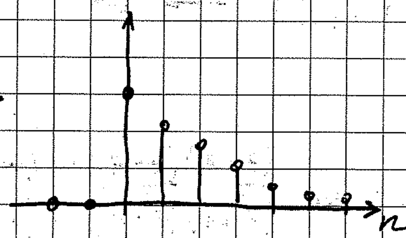
$$|p_j| < 1 \quad \text{za} \quad j = 1, \dots, M$$

Je potreben in zadosten pogoj.

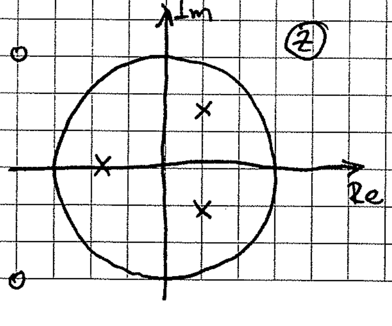
$$H(z) = \sum_{j=1}^{M} \frac{A_j \cdot z}{z - p_j} \longleftrightarrow h[n] = \sum_{j=1}^{M} A_j \cdot p_j^{\,n} \cdot u[n]$$

razbijemo na vsoto delnih ulomkov





Ničle ne uplivajo na stabilnost – ležijo lahko kjerkoli

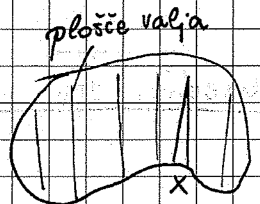


(2.) POLI IN NIČLE DO KONSTANTE b_0 NATANČNO DOLOČAJO FREKVENČNI ODZIV $H(\Omega)$

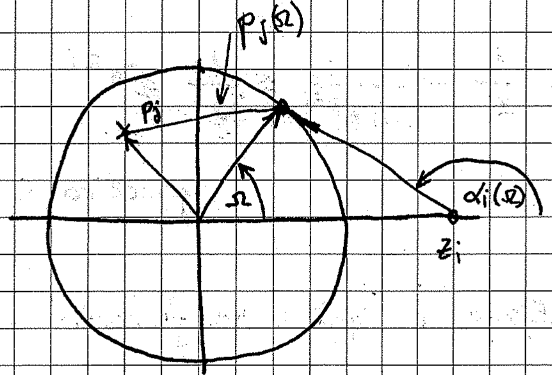
$$H(z) = \frac{\prod_{i=1}^{N}(z-z_i)}{\prod_{j=1}^{M}(z-p_j)} \cdot z^{M-N}$$

$\prod$ .... produkt

$$H(\Omega) = H(z)\Big|_{z=e^{j\Omega}} = b_0 \frac{\prod_{i=1}^{N}(e^{j\Omega}-z_i)}{\prod_{j=1}^{M}(e^{j\Omega}-p_j)} \cdot \underbrace{e^{j\Omega(M-N)}}_{\parallel \atop 1}$$



poli v bližini enotske krožnice povečujejo odziv



$$e^{j\Omega} - z_i = z_i(\Omega) = |z_i(\Omega)|\, e^{j\alpha_i(\Omega)}$$

$$e^{j\Omega} - p_j = p_j(\Omega) = |p_j(\Omega)|\, e^{j\beta_j(\Omega)}$$

$$H(\Omega) = b_0 \frac{\prod_{i=1}^{N} z_i(\Omega)}{\prod_{j=1}^{M} p_j(\Omega)}\, e^{j\Omega(M-N)}$$

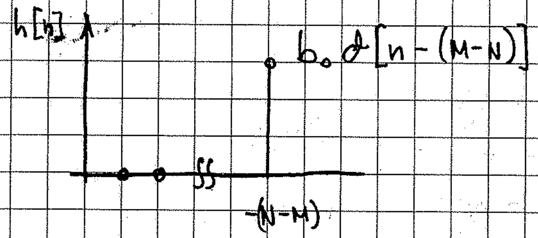
$$H(\Omega) = |H(\Omega)|\, e^{j\phi_H(\Omega)}$$

$$|H(\Omega)| = |b_0| \frac{|z_1(\Omega)|\,|z_2(\Omega)| \cdots |z_N(\Omega)|}{|p_1(\Omega)|\,|p_2(\Omega)| \cdots |p_M(\Omega)|}$$

$$\phi_H(\Omega) = \sum_{i=1}^{N} \alpha_i(\Omega) - \sum_{j=1}^{M} \beta_j(\Omega) + \Omega(M-N) \pm k\pi \qquad k = \begin{cases} 0 \; ; & b_0 > 0 \\ 1 \; ; & b_0 < 0 \end{cases}$$

Poli in ničle v izhodišču ne vpliva na amplitudo, ampak le na fazo.

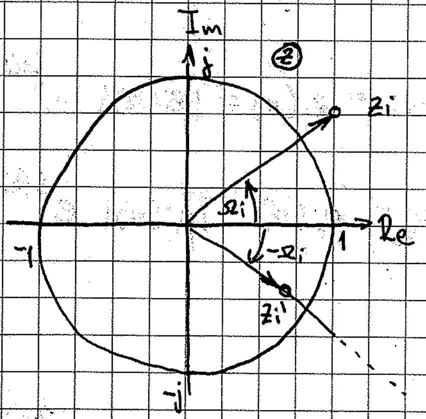
$M-N$ odloča o začetku $h[n]$



$$H(z) = \frac{b_0 z^N}{z^M} z^{M-N} = \left(b_0 z^{N-M} + (b_1 + b_0 a_1) z^{N-M-1}\right) z^{M-N}$$

Nemoremo narediti več ničel kot polov (kavzalnost)

3. PRESLIKAVA ("ZRCALJENJE") NIČEL



$$z_i' = \frac{1}{z_i} = \frac{1}{|z_i| e^{j\Omega_i}} = \frac{1}{|z_i|} e^{-j\Omega_i}$$

preslikava

zrcaljenje ne vpliva na frekvenčni potek

$$H(z) = H_1(z) \cdot H_z(z) \quad ; \quad H_z(z) = (z - z_i)(z - z_i^*)$$

$$H'(z) = H_1(z) \cdot H_z'(z) \quad ; \quad H_z'(z) = \left(z - \frac{1}{z_i}\right)\left(z - \frac{1}{z_i^*}\right)$$

$$H_z'(z) = \frac{z}{z_i}\left(z_i - \frac{1}{z}\right) \cdot \frac{z}{z_i^*}\left(z_i^* - \frac{1}{z}\right) = \frac{z^2}{|z_i|^2}\left(z^{-1} - z_i\right)\left(z^{-1} - z_i^*\right)$$

$$H_z'(\Omega) = H_z'(z)\Big|_{z=e^{j\Omega}} = \frac{e^{j2\Omega}}{|z_i|^2}\left(e^{-j\Omega} - z_i\right)\left(e^{-j\Omega} - z_i^*\right) = \frac{e^{j2\Omega}}{|z_i|^2} H_z(-\Omega)$$

$$\boxed{|H_z'(\Omega)| = \frac{1}{|z_i|^2}|H(-\Omega)| = \frac{1}{|z_i|^2}|H(\Omega)|}$$

soda funkcija

amplituda drugačna

potek je isti

$$\boxed{\phi_z'(\Omega) = 2\Omega - \phi_z(\Omega)}$$

fazni potek je drugačen

Za realno ničlo dobimo: $|H_z'(\Omega)| = \frac{1}{|z_i|}$

$$\phi_z'(\Omega) = \pi + \Omega - \alpha_i(\Omega)$$

Sistemi z minimalno fazo imajo vse ničle znotraj enotskega kroga.

Sistemi z maximalno fazo imajo vse ničle zunaj enotskega kroga

Ničle na krožnici so same sebi imune – jih nemoremo preslikat

min. faza → ničle bližje polov → manjša faza

4. FAZNI SUKALNIK (ALL-PASS FILTER (network))

$$|H(\Omega)| = \text{konstanta} = 1 \quad \text{za} \quad -\pi < \Omega < \pi$$

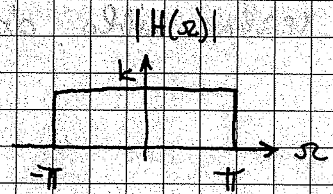
običajno

1. Vsi poli ležijo znotraj enotske krožnice

$$|p_j| < 1 \qquad j = 1, \dots, M$$

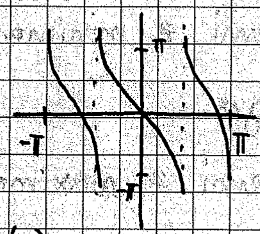
2. Vsak pol je uparjen z ničlo zunaj enotskega kroga

$$z_j = \frac{1}{p_j} \qquad j = 1, \dots, M$$



za $|H(\Omega)| = 1$ moramo izbrati

$$|b_0| = \prod_{j=1}^{M} |p_j|$$



$$H(z) = \prod_{j=1}^{M/2} \frac{\left(z - \frac{1}{p_j}\right)\left(z - \frac{1}{p_j^*}\right)}{(z - p_j)(z - p_j^*)} = \prod_{j=1}^{M/2} H_j(z)$$

$$H_j(\Omega) = \frac{\left(e^{j\Omega} - \frac{1}{p_j}\right)\left(e^{j\Omega} - \frac{1}{p_j^*}\right)}{(e^{j\Omega} - p_j)(e^{j\Omega} - p_j^*)} = \frac{e^{j\Omega}(p_j - e^{-j\Omega})\, e^{j\Omega}(p_j^* - e^{-j\Omega})}{p_j\,(e^{j\Omega} - p_j) \cdot p_j^*\,(e^{j\Omega} - p_j^*)} =$$

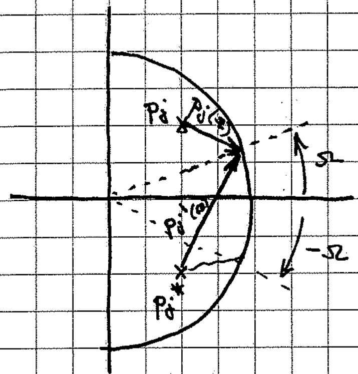
$$= \frac{e^{j2\Omega}(e^{-j\Omega} - p_j^*)(e^{-j\Omega} - p_j^*)}{|p_j|^2 (e^{j\Omega} - p_j)(e^{j\Omega} - p_j^*)}$$

$$e^{j\Omega} - p_j = p_j(\Omega) = |p_j(\Omega)|\, e^{j\beta_j(\Omega)}$$

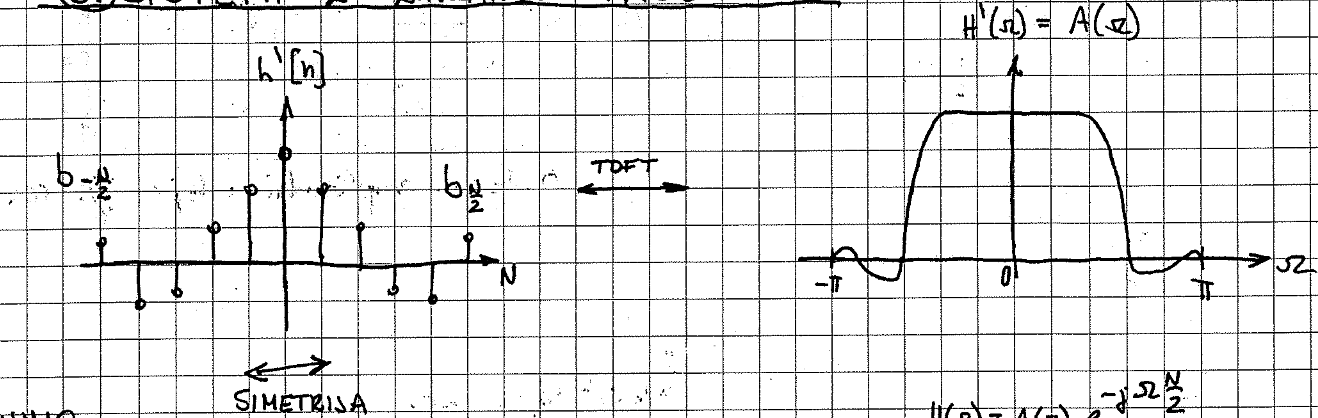
$$e^{-j\Omega} - p_j^* = p_j^*(\Omega) = |p_j(\Omega)|\, e^{-j\beta_j(\Omega)}$$

$$H_j(\Omega) = \frac{e^{j2\Omega}}{|p_j|^2} \; \frac{|p_j|\, e^{-j\beta_j(\Omega)}}{|p_j|\, e^{j\beta_j(\Omega)}} \; \frac{|p_j'(\Omega)|\, e^{-j\beta_j'(\Omega)}}{|p_j'(\Omega)|\, e^{+j\beta_j'(\Omega)}} =$$

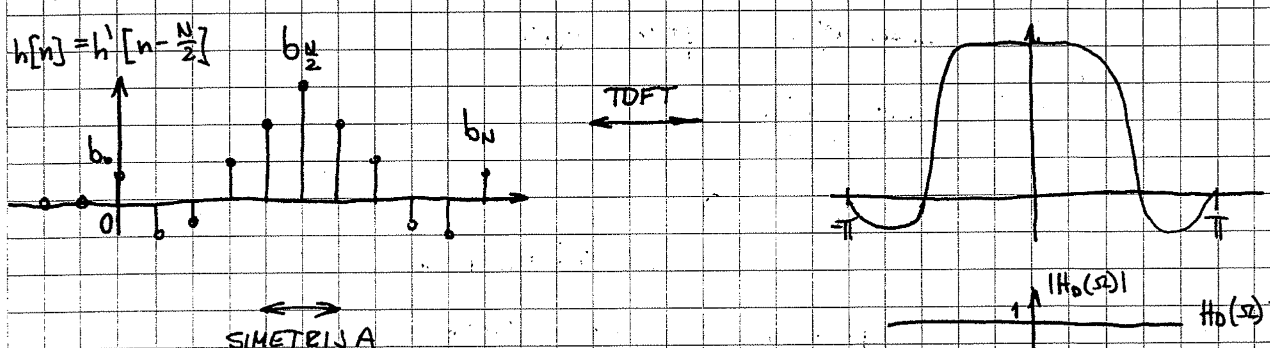
$$= \frac{1}{|p_j|^2}\, e^{j(2\Omega - 2\beta_j(\Omega) - 2\beta_j'(\Omega))}$$

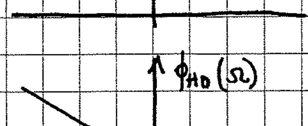


5. SISTEMI Z LINEARNO FAZO



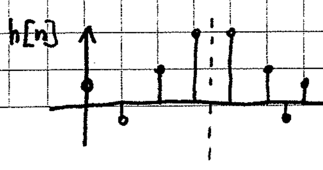
ZAKASNIMO

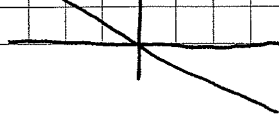




$H_D(\Omega) = e^{-j\Omega\frac{N}{2}}$

sistem lahko tudi nima enakega





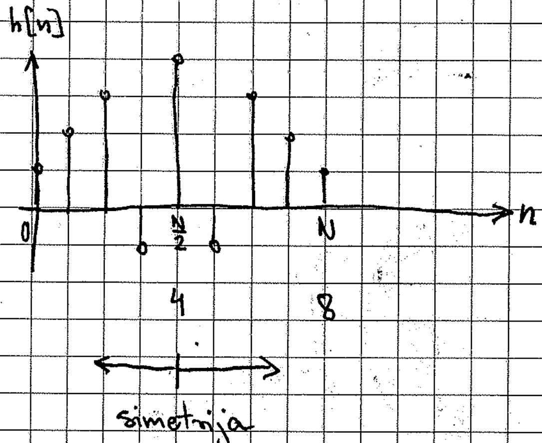
$\phi_{HD}(\Omega) = -\Omega\frac{N}{2}$

22.4.2013

Tip 1 : Simetrični koeficienti $b_i = b_{N-i}$

$N = $ sod - obstaja centralni impulz $b_{\frac{N}{2}}$

$$H(\Omega) = \sum_{n=0}^{N} b_n e^{-j\Omega n}$$



$$H(\Omega) = \sum_{n=0}^{\frac{N}{2}-1} b_n e^{-j\Omega n} + b_{\frac{N}{2}} e^{-j\frac{N}{2}\Omega} + \sum_{n=\frac{N}{2}+1}^{N} b_n e^{-j\Omega n}$$

$$= e^{-j\frac{N}{2}\Omega} \underbrace{\left[b_{\frac{N}{2}} + \sum_{n=0}^{\frac{N}{2}-1} b_n e^{j\left(\frac{N}{2}-n\right)\Omega} + \sum_{n=\frac{N}{2}+1}^{N} b_n e^{+j\left(\frac{N}{2}-n\right)\Omega} \right]}_{A(\Omega)\ \ldots\ \text{amplitudni odziv (je realen)}} =$$

$$= e^{-j\frac{N}{2}\Omega} \cdot A(\Omega)$$

$\left.\begin{array}{ll} \frac{N}{2} - n = k & n = \frac{N}{2} - k \\ k(n=0) = \frac{N}{2} & \ldots \text{sp. meja} \\ k(n=\frac{N}{2}-1) = 1 & \ldots \text{zg. meja} \end{array}\right\}$ substitucija za prvo vsoto

$\left.\begin{array}{ll} \frac{N}{2} - n = -k & n = \frac{N}{2} + k \\ k(n=\frac{N}{2}+1) = 1 & \text{sp. meja} \\ k(n=N) = \frac{N}{2} & \text{zg. meja} \end{array}\right\}$ substitucija za drugo vsoto

$$A(\Omega) = b_{\frac{N}{2}} + \sum_{k=\frac{N}{2}}^{1} b_{\frac{N}{2}-k}\, e^{jk\Omega} + \sum_{k=1}^{\frac{N}{2}} b_{\frac{N}{2}+k}\, e^{-jk\Omega}$$

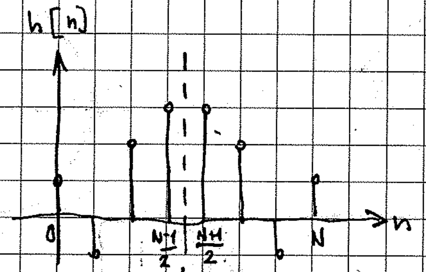
$$= b_{\frac{N}{2}} + \sum_{k=1}^{N/2} b_{\frac{N}{2}-k}\left(e^{jk\Omega} + e^{-jk\Omega}\right) = \boxed{b_{\frac{N}{2}} + 2\sum_{k=1}^{N/2} b_{\frac{N}{2}-k} \cdot \cos k\Omega}$$

odziv amplitudnega dela

Tip 1 je primeren za nizka sita.

Tip 2: Simetrični koeficienti $b_i = b_{N-i}$

N = lih



$$H(\Omega) = \sum_{n=0}^{N} b_n e^{-j\Omega n} = e^{-j\frac{N}{2}\Omega} \cdot \sum_{n=0}^{\frac{N-1}{2}} b_n e^{j\left(\frac{N}{2}-n\right)\Omega} + e^{-j\frac{N}{2}\Omega} \sum_{n=\frac{N+1}{2}}^{N} b_n e^{j\left(\frac{N}{2}-n\right)\Omega}$$

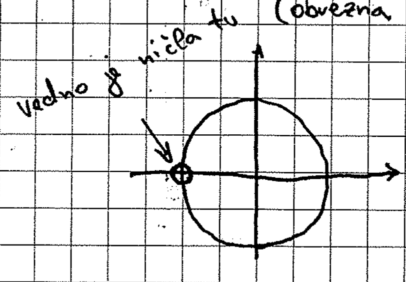
substituciji

$$\frac{N}{2} - n = k + \frac{1}{2} \qquad \frac{N}{2} - n = -k - \frac{1}{2}$$

$$A(\Omega) = \sum_{k=0}^{\frac{N-1}{2}} b_{\frac{N-1}{2}-k}\, e^{j\left(\frac{1}{2}+k\right)\Omega} + \sum_{k=0}^{\frac{N-1}{2}} b_{\frac{N+1}{2}+k}\, e^{-j\left(\frac{1}{2}+k\right)\Omega}$$

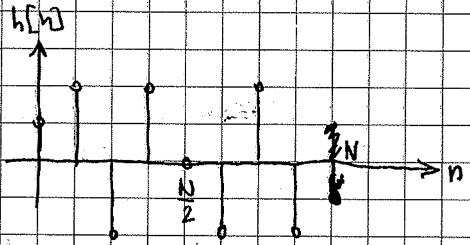
(obvezna ničla)

$$\boxed{A(\Omega) = 2 \cdot \sum_{k=0}^{\frac{N-1}{2}} b_{\frac{N-1}{2}-k} \cdot \cos\left(\left(\frac{1}{2}+k\right)\Omega\right)}$$

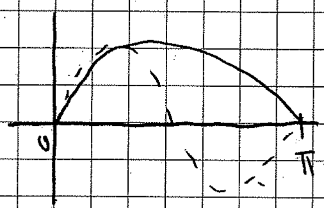


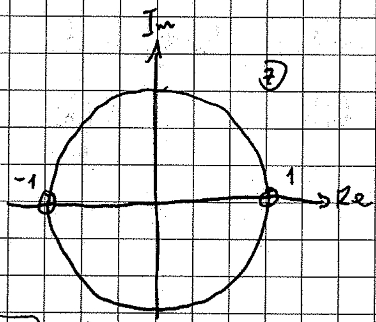
Tip 3: Antisimetrični koeficienti $b_i = -b_{N-i}$

N = sod



$$\boxed{A(\Omega) = 2 \cdot \sum_{k=1}^{\frac{N}{2}} b_{\frac{N}{2}-k} \sin(k\Omega)}$$

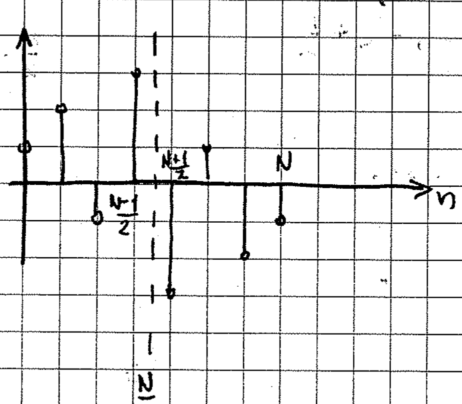




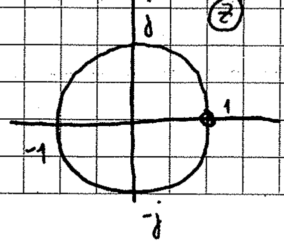
Vezja za pasovna sita!

Tip 4: Antisimetrični koeficienti $b_i = -b_{N-i}$

N = lih



$$A(\Omega) = 2 \cdot \sum_{k=0}^{\frac{N-1}{2}} b_{\frac{N-1}{2}-k} \sin\left(\left(\tfrac{1}{2}+k\right)\Omega\right)$$



$$H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_2 z^{-(N-2)} + b_1 z^{-(N-1)} + b_0 z^{-N}$$

ničle sistemske funkcije:

$$b_0 + b_1 \left(\frac{1}{z}\right) + b_2 \left(\frac{1}{z}\right)^2 + \cdots + b_2 \left(\frac{1}{z}\right)^{N-2} + b_1 \left(\frac{1}{z}\right)^{N-1} + b_0 \left(\frac{1}{z}\right)^N = 0 \quad \Big| \cdot z^N$$

$$b_0 z^N + b_1 z^{N-1} + b_2 z^{N-2} + \cdots + b_2 z^2 + b_1 z + b_0 = 0$$

ničle nastopajo recipročno z_i je ničla $\Rightarrow \frac{1}{z_i}$ je tudi ničla $H(z)$

Vsi poli so v izhodišču (FIR odziv)

Ničle pa nastopajo v recipročnih parih, razen pri ± 1.

Možne lege ničel:

a) par na realni osi : $z = a$ in $z = \frac{1}{a}$

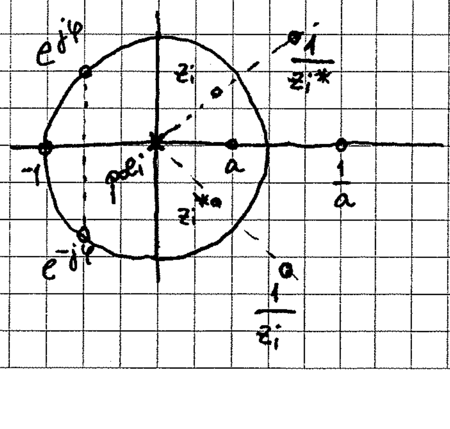
b) kompleksno konjugiran par na enotski krožnici

$z = e^{j\varphi}$ in $z = e^{-j\varphi}$

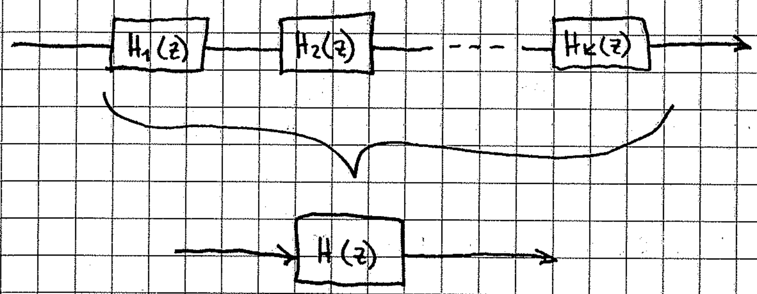
c) kompleksno konjugiran par izven krožnice (štiri kompl. ničle)

$|z| \neq 1$

z_i, z_i^*, $\frac{1}{z_i}$, $\frac{1}{z_i^*}$



6. KASKADNA VEZAVA SISTEMOV



$$H(z) = H_1(z) \cdot H_2(z) \cdot \ldots \cdot H_K(z)$$

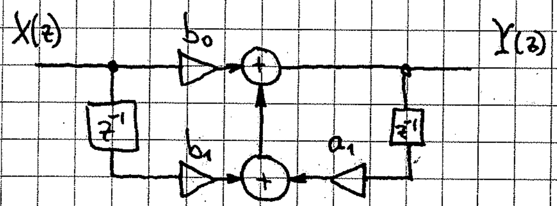
7. Vzorčni sistemi imajo število polov večje, kvečjemu enako število ničel

$$\boxed{M \geq N}$$

6. DISKRETNI FILTRI

6.1. BLOČNA SHEMA IN DIAGRAM PRETOKA SIGNALOV

(signal flow graph)



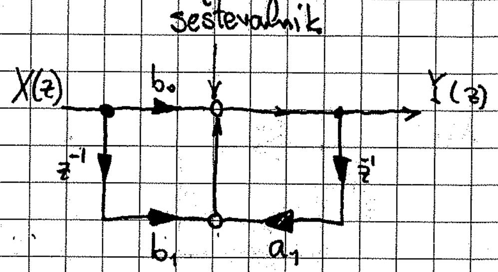
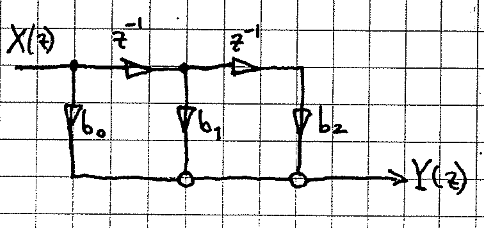


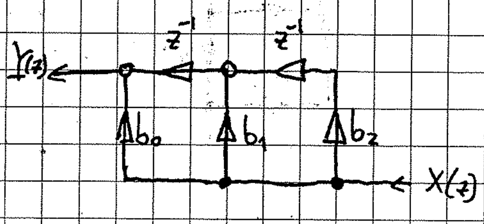
diagram pretoka signala

6.1.1. TRANSPOZICIJA DIAGRAMA PRETOKA

- obrnemo smeri pretoka v vseh vejah
- razvejišče spremenimo v seštevalnike
- seštevalniki postanejo razvejišča
- položaj sponke vhodnega in izhodnega se zamenjata

FIR filter : $Y(z) = b_0 + b_1 z^{-1} + b_2 z^{-2}$



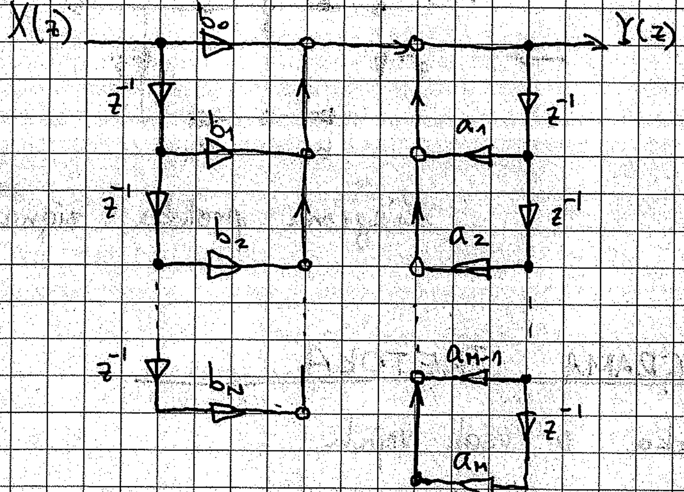


$$Y(z) = b_0 X(z) + z^{-1}\left(b_1 X(z) + z^{-1} \cdot b_2 X(z)\right)$$

6.2. STRUKTURE DISKRETNIH FILTROV

6.2.1. DIREKTNA STRUKTURA I in II.

$$Y(z) = \sum_{i=0}^{N} b_i \cdot z^{-i} X(z) + \sum_{j=1}^{M} a_j \cdot z^{-j} \cdot Y(z)$$

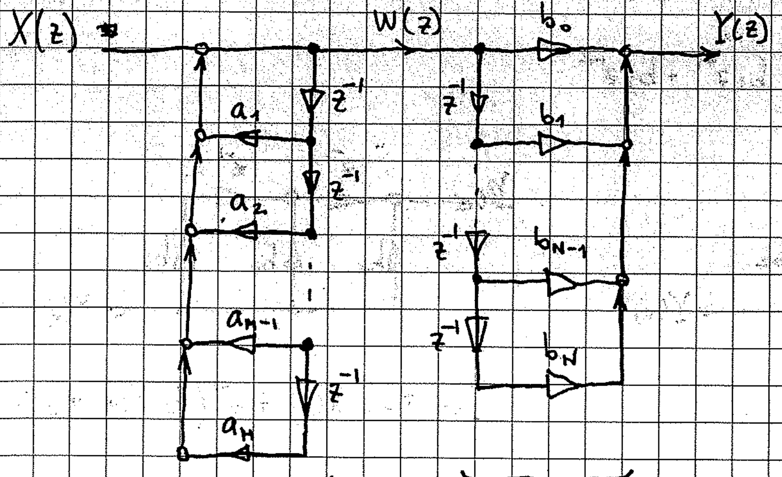


$$H(z) = \frac{N(z)}{D(z)}$$

$$H_1(z) = \sum_{i=0}^{N} b_i z^{-i} = N(z)$$

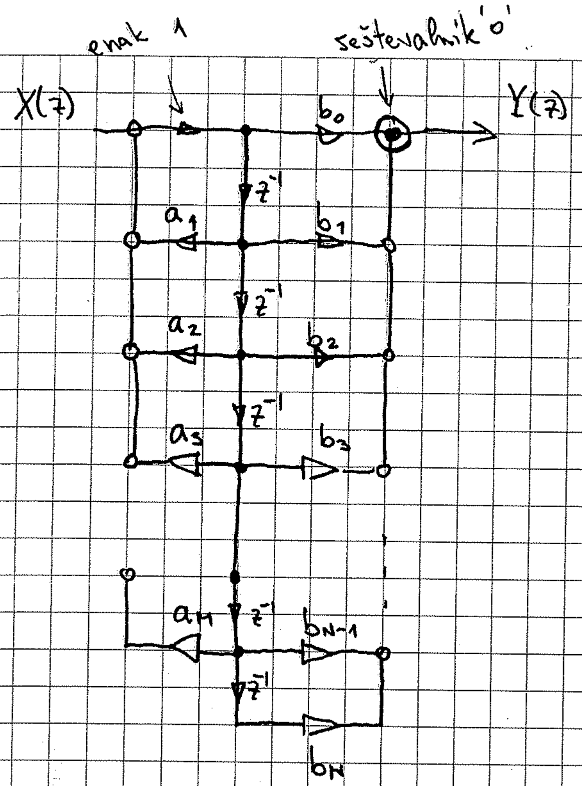
$$H_2(z) = \frac{1}{1 - \sum_{j=1}^{M} a_j z^{-j}}$$

$$H(z) = H_1(z) \cdot H_2(z) = H_2(z) \cdot H_1(z)$$



$$W(z) = H_2(z) \cdot X(z)$$

$$Y(z) = H_1(z) \cdot W(z)$$



direktna struktura II.
(kanonična oblika)
[porabi najmanj elementov za realizacijo]

$$w[n] = x[n] + \sum_{j=1}^{M} a_j\, w[n-j]$$

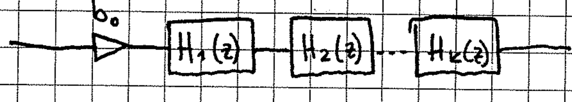
$$y[n] = \sum_{j=0}^{N} b_j\, w[n-j]$$

w... spremenljivka sistema

6.2.2. KASKADNA VEZAVA

uporabljamo jo za IIR filtre

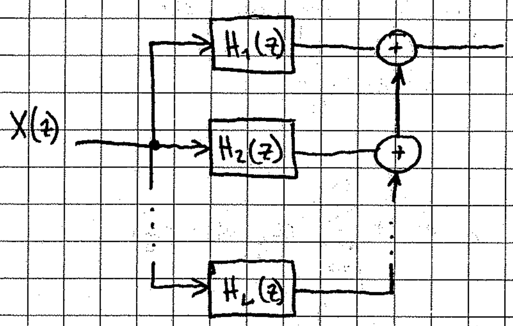
kaskado celic prvega in drugega reda



$$H(z) = b_0 \prod_{i=1}^{k} H_i(z)$$

6.2.3. VZPOREDNA VEZAVA

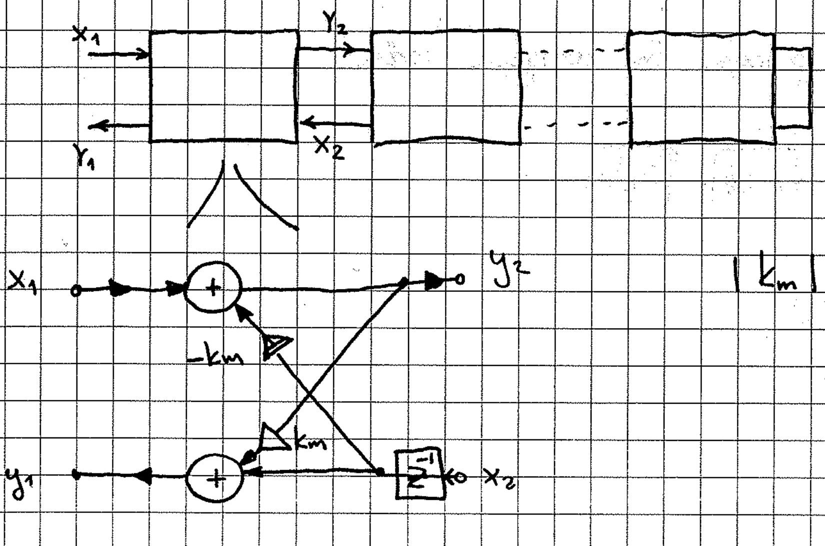
$$H(z) = \sum_{i=1}^{L} \frac{b_{0i} + b_{1i} z^{-1}}{1 - a_{1i} z^{-1} + a_{2i} z^{-2}}$$



6.2.4. DRUGE STRUKTURE

Lestričasti filter - lattice filter

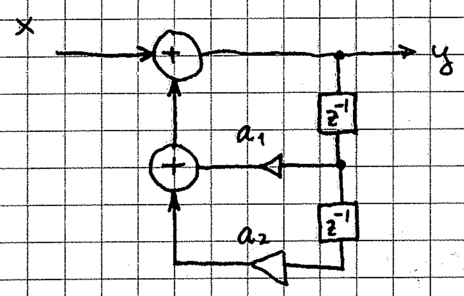
realizacija FIR in IIR filtrov



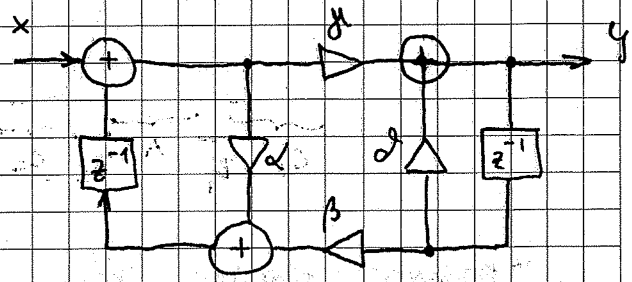
$|k_m| < 1$

6.5.2013

Sklopljena struktura 2. reda (coupled form)



enaka ≡ sistemska funkcija

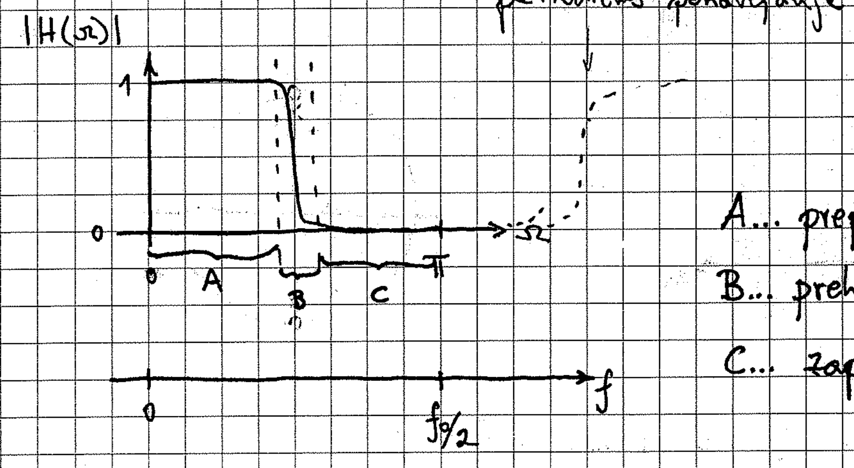


več množilnikov

preglednejša lega polov

6.2.5. ZNAČILNI PREDSTAVNIKI DISKRETNIH FILTROV

- nizko sito

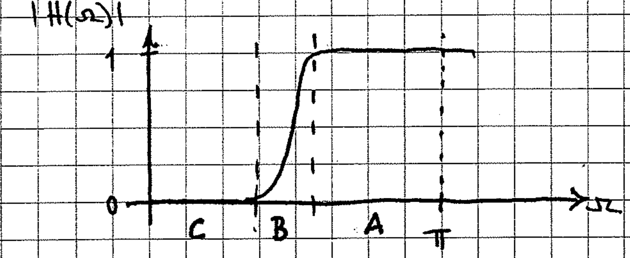


A... prepustni pas

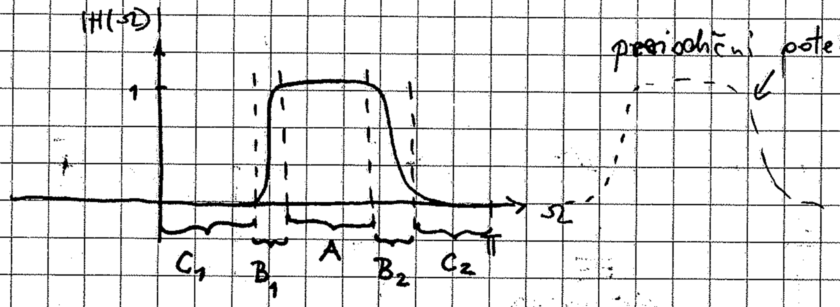
B... prehodni pas

C... zaporni pas

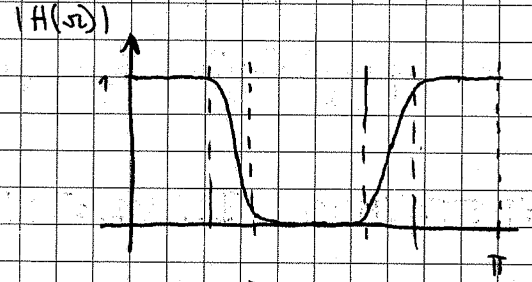
- visoko sito



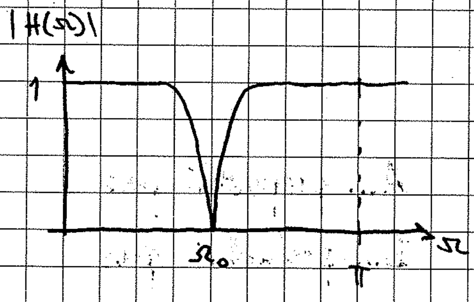
- pasovno prepustno sito (band pass)

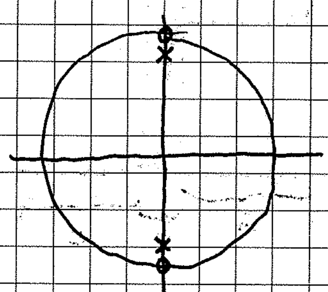


- pasovno zaporno sito (band stop)

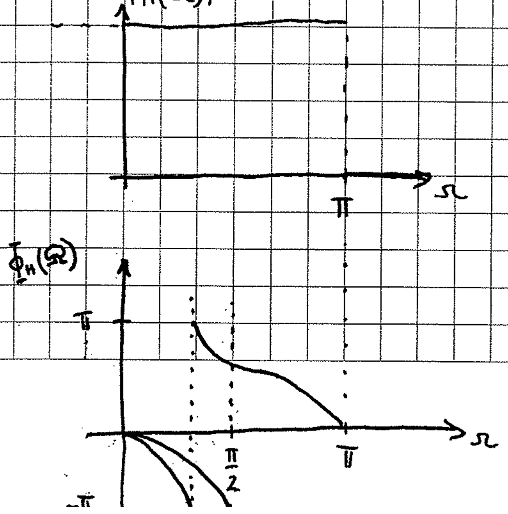


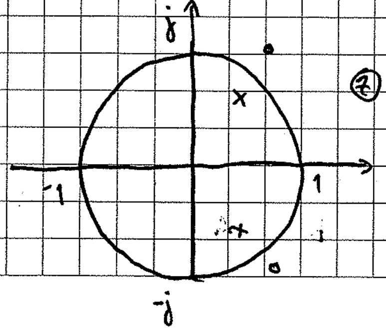
- zarezno sito (notch filter)



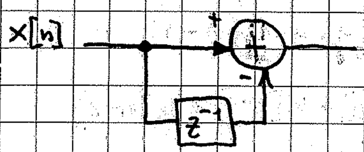


- fazni sukalnik (all pass filter)



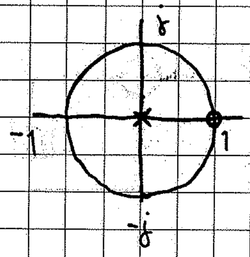


– deferenciator



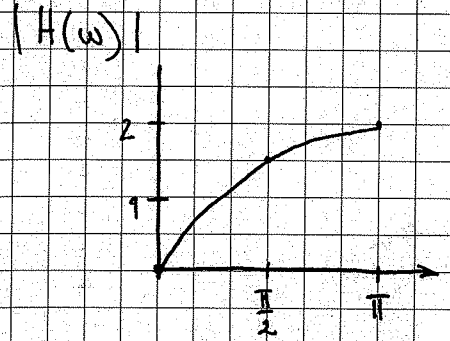
$$y[n] = x[n] - x[n-1]$$

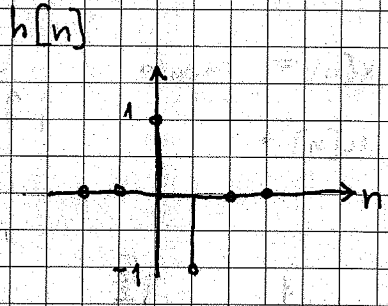
$$H(z) = 1 - z^{-1} = \frac{z-1}{z}$$

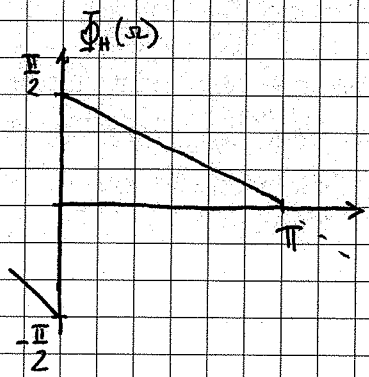


$$H(\Omega) = 2j \sin\frac{\Omega}{2}\, e^{-j\frac{\Omega}{2}}$$

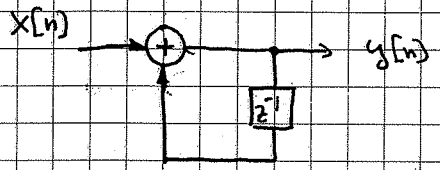
zvezni diferenciator : $y(t) = \dfrac{dx(t)}{dt}$ $\qquad H(\omega) = j\omega$







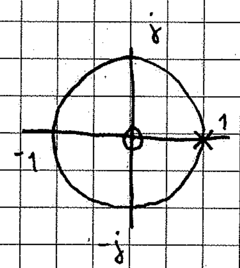
- diskretni sumator (akumulator)



$$y[n] = x[n] + y[n-1]$$
$$= x[n] + x[n-1] + y[n-2]$$

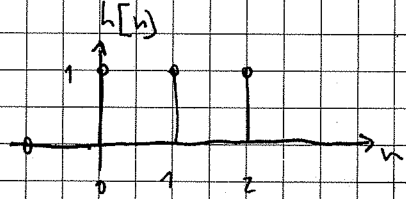
$$y[n] = \sum_{i=-\infty}^{n} x[i]$$

$$H(z) = \frac{z}{z-1}$$

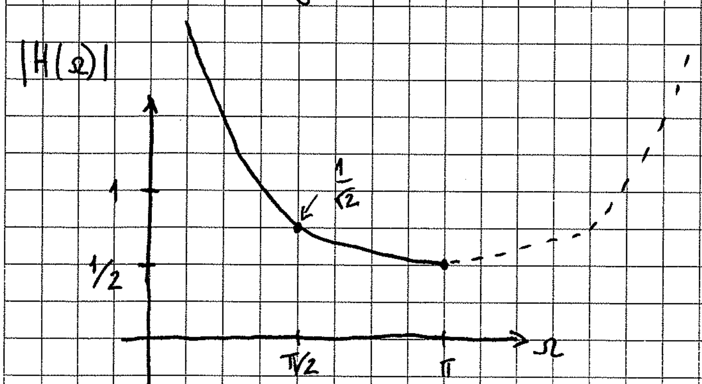


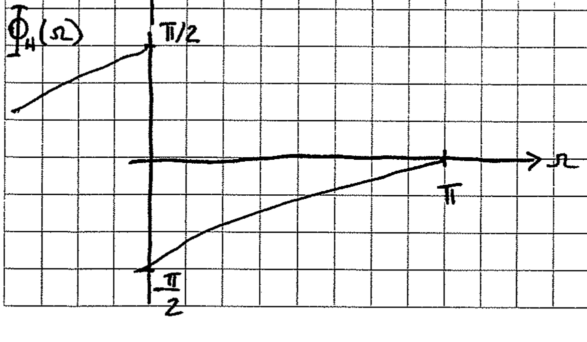
Če damo $x[n]$ = impulz

$h[n] = u[n]$.



$$H(\Omega) = \frac{1}{2j \sin\frac{\Omega}{2}} e^{j\frac{\Omega}{2}}$$





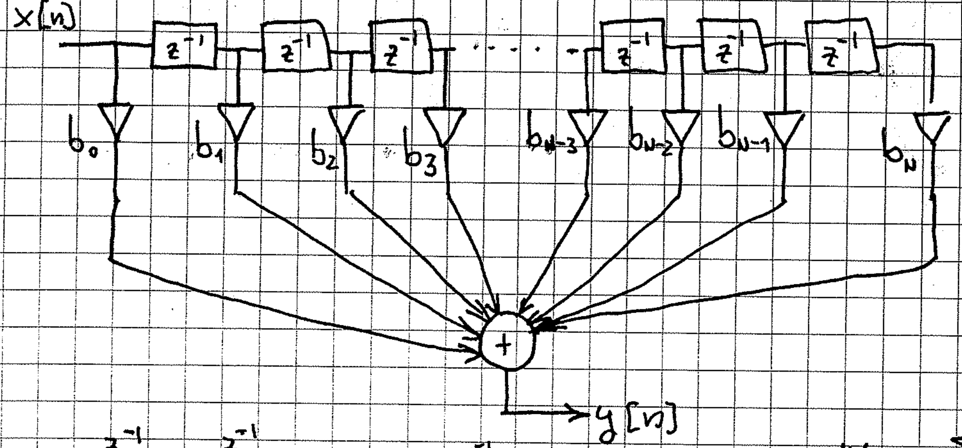
6.3. NAČRTOVANJE FIR FILTROV

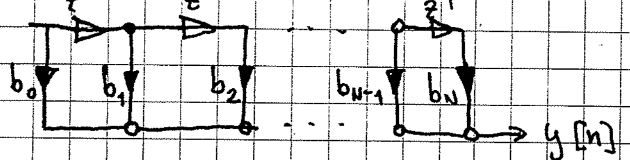
6.3.1. LASTNOSTI

- imajo končen odziv, so vedno stabilni
- dolžina odziva je enaka številu koeficientov, ki je ~~za~~ $N+1$: $b_0, b_1, \dots, b_N$
- poli ležijo v izhodišču z ravnine, in ne vplivajo na potek frekvenčnega odziva, frekvenčni potek je določen z ničlami
- imajo lahko linearno fazo
- primerni za adaptivne izvedbe
  ↳ zadeva se prilagaja na signal

6.3.2. STRUKTURE

a) transferzalni filter (kanonična oblika)





$$H(z) = \sum_{i=0}^{N} b_i z^{-i} = \frac{\sum_{i=0}^{N} b_i z^{N-i}}{z^N}$$

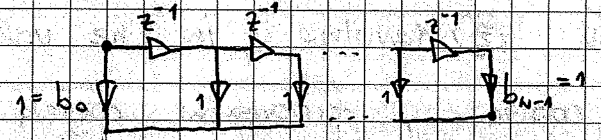
$$h[n] = \sum_{i=0}^{N} b_i \cdot \delta[n-i]$$

b) rekurzivna izvedba

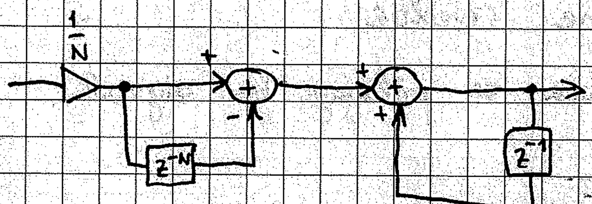
filter za tekoće popriečje (moving average filter)

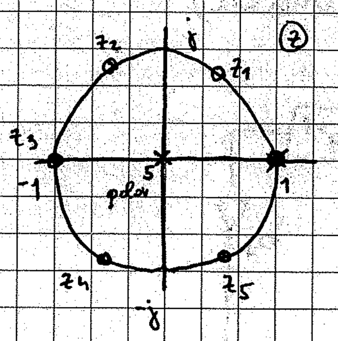
$$H(z) = \frac{1}{N}\sum_{i=0}^{N} z^{-i} = \frac{1}{N}\left(1 + z^{-1} + z^{-2} + \dots + z^{-(N-1)}\right) =$$

$$= \frac{1}{N}\,\frac{1 - z^{-N}}{1 - z^{-1}} = \frac{1}{N}\,\frac{z^{N} - 1}{z^{N-1}(z-1)}$$



$$H(z) = \frac{1}{N}\left(1 - z^{-N}\right)\cdot\frac{1}{1 - z^{-1}}$$

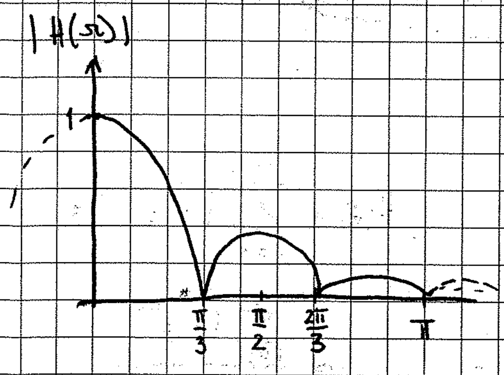




$$z^{N} = 1 = e^{j2k\pi}$$

$$z_k = e^{j\frac{2k\pi}{N}}, \quad k = 1, \dots, N$$

$N = 6$

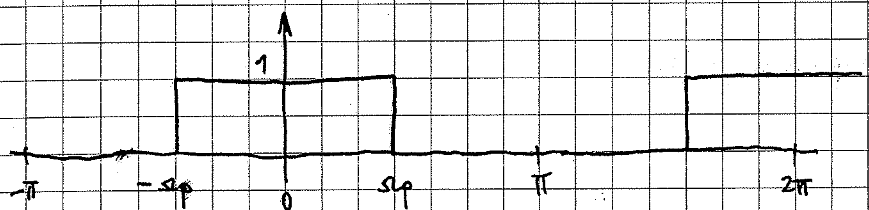


6.3.3 NAČRTOVANJE FIR FILTROV Z OKNENJEM

Idealno nizko sito:

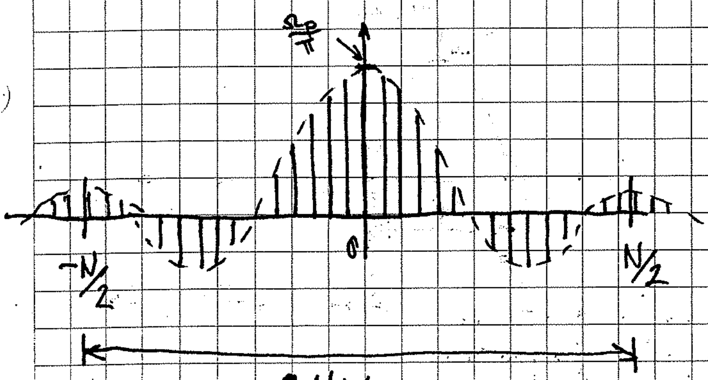
$$H(\Omega) = \begin{cases} 1 & \text{za } |\Omega| < \Omega_p \\ 0 & \text{drugod} \end{cases}$$

za osnovno periodo $-\pi < \Omega < \pi$



$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\Omega)\cdot e^{j\Omega n}\, d\Omega = \frac{1}{2\pi}\int_{-\Omega_p}^{\Omega_p} e^{j\Omega n}\, d\Omega = \frac{\sin(\Omega_p \cdot n)}{\pi \cdot n}$$

$$h[0] = \lim_{n\to 0} \frac{\sin(\Omega_p\cdot n)}{\pi\cdot n} = \frac{\Omega_p}{\pi}$$



$$h[n] = \begin{cases} \dfrac{\sin\left(\Omega_p\left(n-\frac{N}{2}\right)\right)}{\pi\left(n-\frac{N}{2}\right)} & \text{za } 0 \le n \le N \\ 0 & \text{drugod} \end{cases}$$

impulzni odziv ni periodičen; frekvenčni je periodičen

13. 5. 2013

Rezanje predstavimo z množenjem s pravokotnim oknom.

$$w[n] = \begin{cases} 1 & \text{za} \quad |n| \le \frac{N}{2} \\ 0 & \text{drugod} \end{cases}$$

$$h[n] = h_i[n] \cdot w[n]$$ ← okenska funkcija

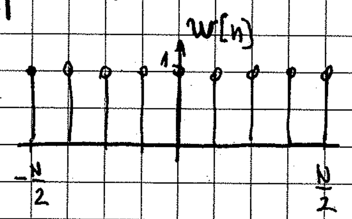
↑ idealni impulzni odziv

↕ TDFT

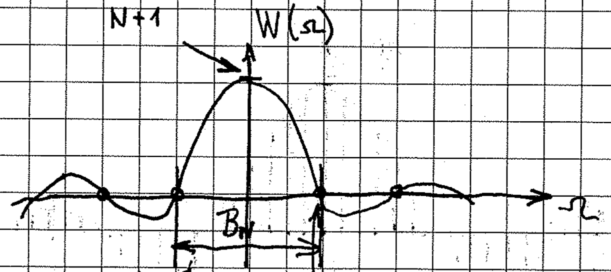
$$H(\Omega) = \frac{1}{2\pi} H(\Omega) * W(\Omega) = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} H_i(\Omega - \lambda) \cdot W(\lambda)\, d\lambda$$

$$W(\Omega) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} e^{-j\Omega n} = \frac{\sin\left(\frac{N+1}{2}\Omega\right)}{\sin\left(\frac{\Omega}{2}\right)}$$

pravokotno okno



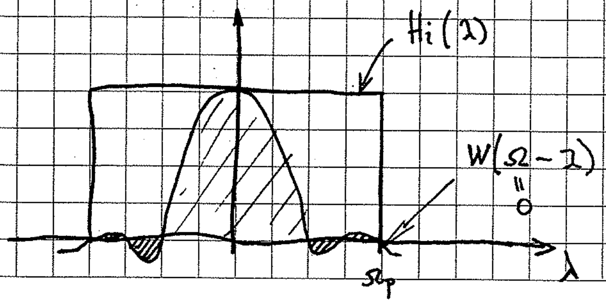
↔

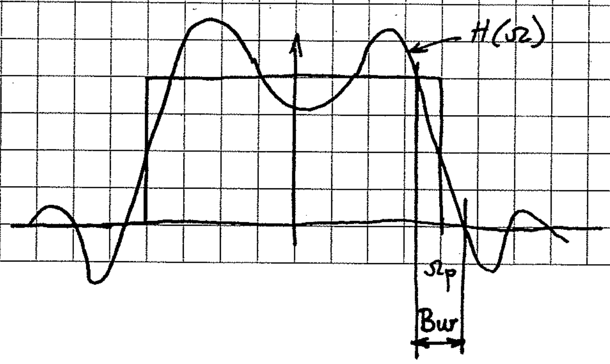


razdalja med tema ničlama je pasovna širina B_N

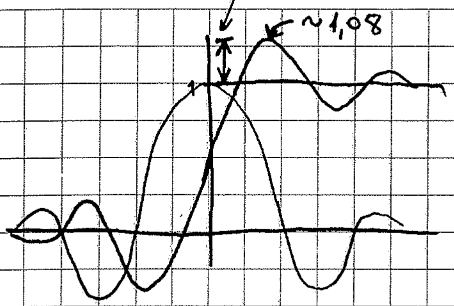
ničla: $\pi = \frac{N+1}{2}\Omega \Rightarrow \Omega = \frac{2\pi}{N+1}$

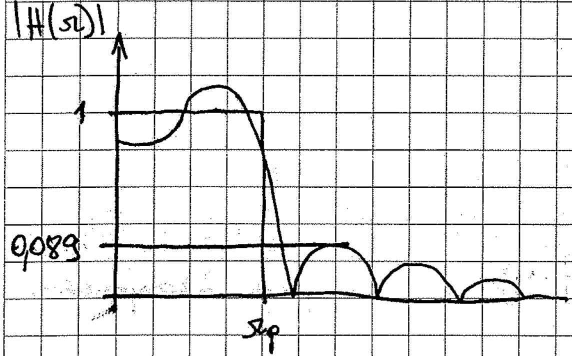
$$B_W = \frac{4\pi}{N+1}$$





preuzpon (Gibbsov fenomen)





dušenje : $20 \log (0{,}089) = -20{,}96$

$\doteq -21\,dB$

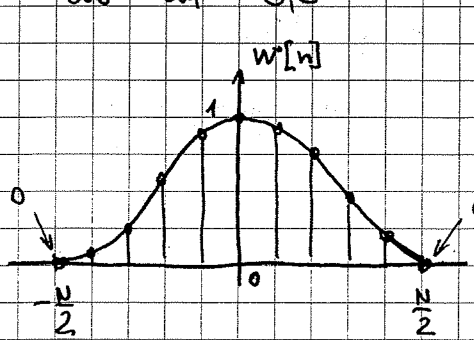
Zaradi tega raje namesto trdih oken uporabimo mehka okna

KOSINUSNA OKNA

$$w[n] = \begin{cases} a_0 + a_1 \cdot \cos\left(\frac{2\pi}{N} \cdot n\right) & \text{za } |n| \leq \frac{N}{2} \\ 0 & \text{za } |n| > \frac{N}{2} \end{cases}$$

von Hannovo okno :

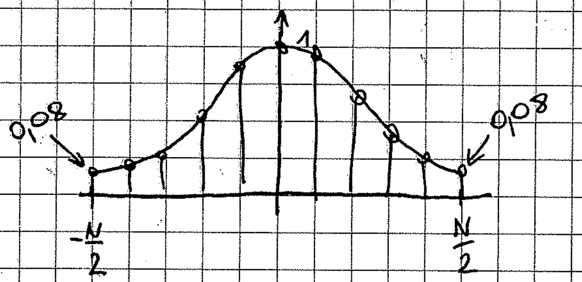
$a_0 = a_1 = 0{,}5$



$B_w = \frac{8\pi}{N}$ dušenje stranskega snopa 43,9 dB

Hammingovo okno:

$a_0 = 0{,}54$ $a_1 = 0{,}46$



$B_w = \frac{8\pi}{N}$

dušenje 54,4 dB

Blackmanovo okno:

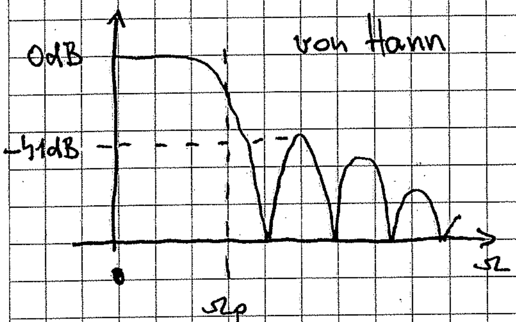
$0{,}42 + 0{,}5 \cdot \cos\left(\frac{2\pi}{N} \cdot n\right) + 0{,}08 \cos\left(\frac{4\pi}{N} \cdot n\right)$ ... ima 3 komponente

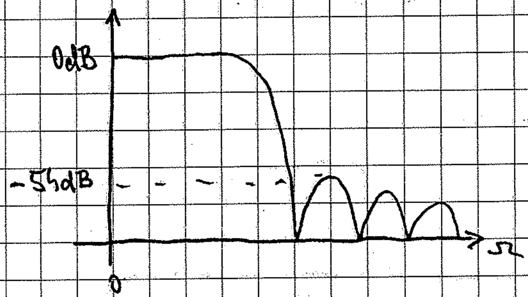
$B_w = \frac{5{,}56\pi}{N}$ dušenje 75,3 dB

Kaiserjevo, Gaussovo, Čebiševo, ...

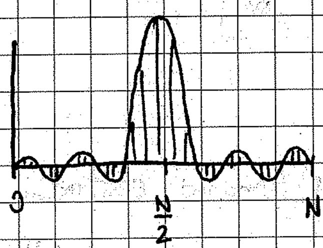
Bistveni pravili pri načrtovanju z okenskimi funkcijami

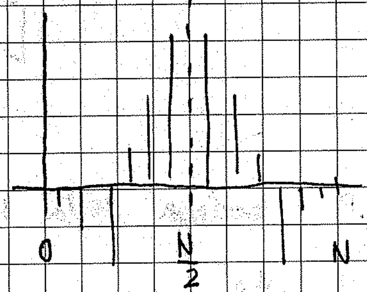
1. Izbira okna vpliva na minimalno slabljenje v zapori, večje slabljenje razširi prehodno področje.
2. Dolžina okna oža centralni snop $W(\Omega)$, z dolžino lahko vplivamo na širino prehodnega področja.





Zaradi simetrije odziva imajo ti filtri LINEARNO FAZO!
(Tip 1)



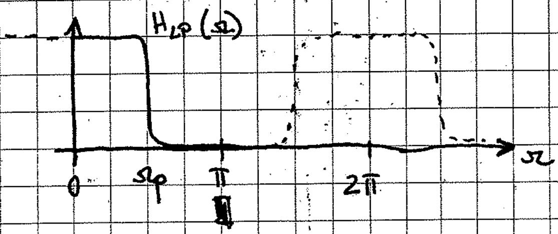


(Tip 2)
tudi ima lin. fazo

6.3.4. FREKVENČNE TRANSFORMACIJE FIR SIT

LP prototip prestavimo po frekvenci za doseganje drugih vrst filtrov

a) HP – visoko sito

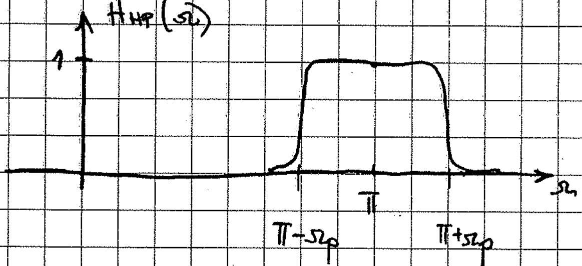


$h_{LP}[n] = b_n$

množenje z $e^{j\pi n}$ premakne spekter za $\pm\pi$ (TDFT)

$e^{j\pi n} = (e^{j\pi})^n = (-1)^n$ ... izmenični signal ±

$$h[n]\, e^{j\pi n} \longleftrightarrow H(\Omega - \pi)$$



$$h_{HP}[n] = (-1)^n \cdot h_{LP}[n] = (-1)^n \cdot b_n$$

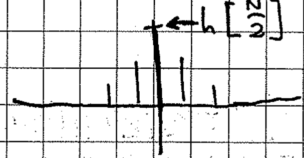
Transformacija deluje za lih in sod tip (Tip1 in Tip2).

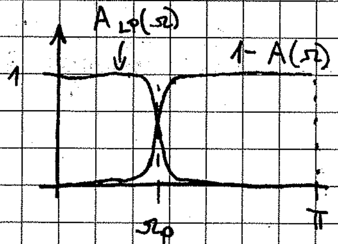
[lih in sodi N]

Za FIR LP sodega reda N (N = sod) (imamo centralni koef.)

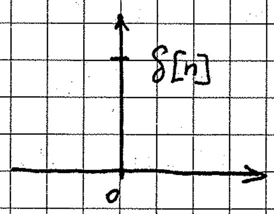
$$H_{LP} = A(\Omega)\, e^{-j\frac{N}{2}\Omega}$$

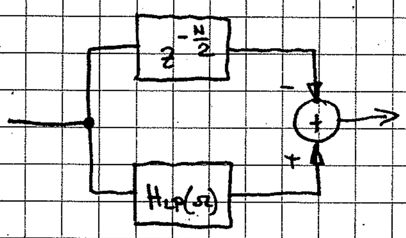
$A(\Omega) \longleftrightarrow$

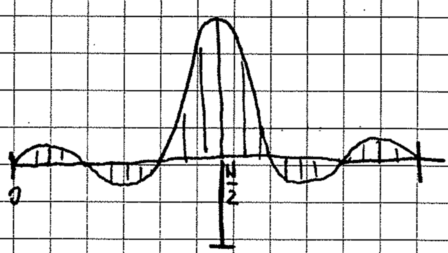




$1 \longleftrightarrow$



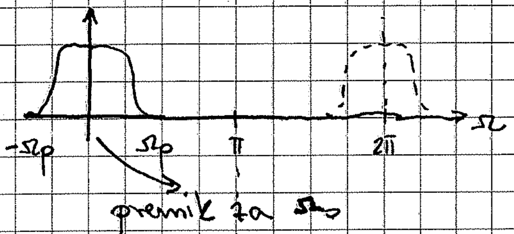


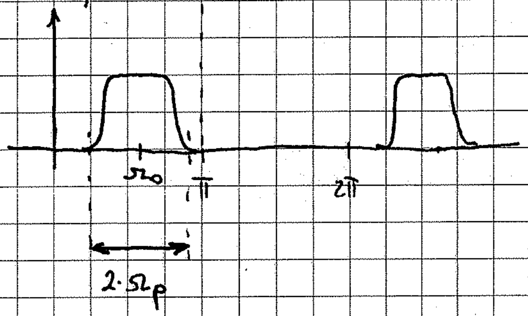


$$H_{HP}(\Omega) \longleftrightarrow h_{HP}[n] = h_{LP}[n] - \delta[\tfrac{N}{2}]$$

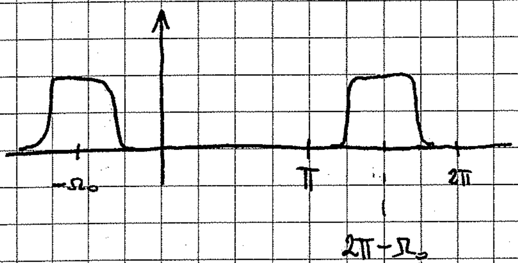
$$h_{LP}[\tfrac{N}{2}] = b_{\frac{N}{2}} - 1$$

b.) BP – pasovno sito





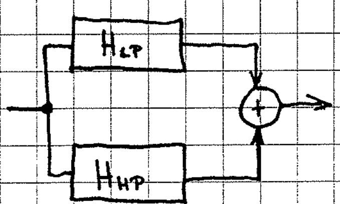
$h[n]\, e^{j\Omega_0 n}$

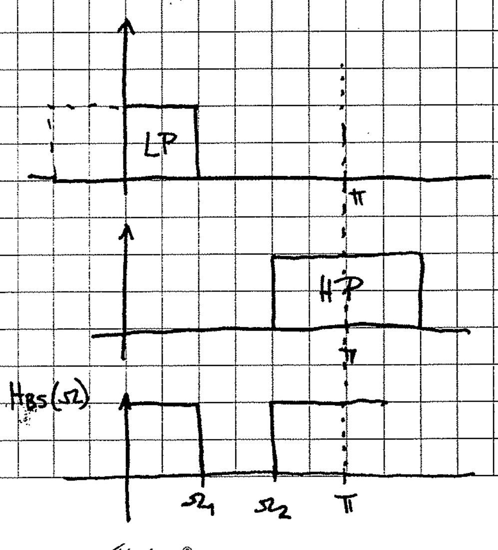


$h[n]\, e^{-j\Omega_0 n}$.

$$h_{BP}[n] = h[n]\left(e^{j\Omega_0 n} + e^{-j\Omega_0 n}\right) = h[n]\cdot 2\cos(\Omega_0 \cdot n)$$

c) BS – pasovno zaporno sito



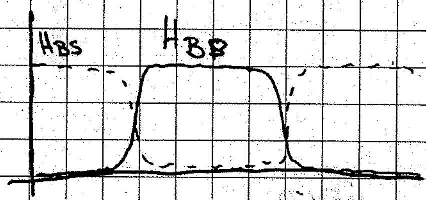


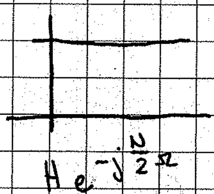
$$H(z) = H_{LP}(z) + H_{HP}(z) = \sum_{i=0}^{N} b_{LP_i} z^{-i} + \sum_{i=0}^{N} b_{HP_i} z^{-i}$$

$$H_{BS}(z) = \sum_{i=0}^{N} (b_{LP_i} + b_{HP_i}) z^{-i}$$

Pogoj je da morata biti filtra istega reda (tip-a)

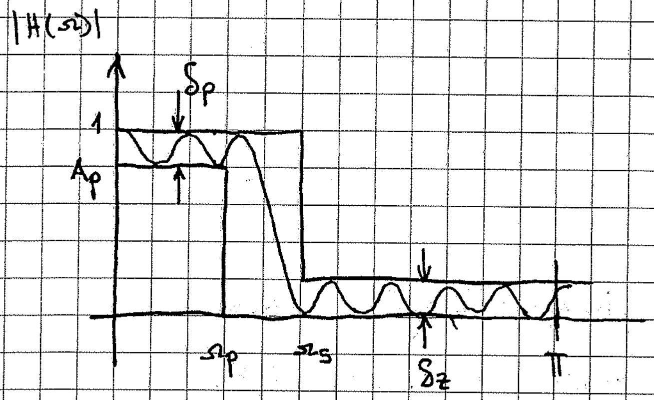
Za sodi red N lahko uporabimo odštevanje





$$H_{BS} = e^{-j\frac{N}{2}\Omega} \cdot A_{BP}(\Omega)$$

6.3.5. NAČRTOVANJE FIR SIT Z ENAKOMERNO VALOVITOSTJO



Parks in McClellan postopek

potrebujemo: N, Ω_p, Ω_s, razmerje $\frac{\delta_p}{\delta_z}$

$$A_p\,[dB]: \quad \delta_p = 1 - 10^{\frac{A_p}{20}}$$

$$A_s\,[dB]: \quad \delta_z = 10^{\frac{A_z}{20}}$$

6.4. NAČRTOVANJE IIR FILTROV

6.4.1. LASTNOSTI

- imajo rekurzivno strukturo
- so potencialno nestabilni
- odziv je neskončen vendar imajo krajše zakasnitve
- nižji red za doseganje zahtevane karakteristike
- večja občutljivost na kvantizacijo koeficientov
- ne moremo doseči linearne faze
- lahko napravimo fazni sukalnik

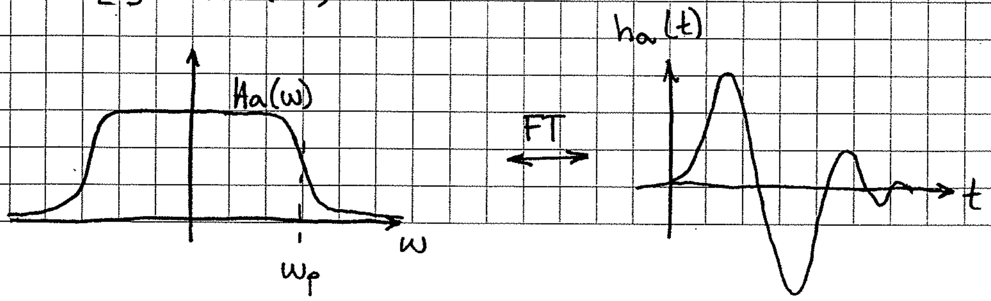
6.4.2. STRUKTURE

- direktna struktura I. in II.
- kaskadna vezava filtrov 2. reda (tudi 1. red)
  2. red - par kompleksnih polov in ničel → kontrola stabilnosti
- lestvičasta (rešetkasta) struktura
- paralelna struktura (uporablja se zelo redko)

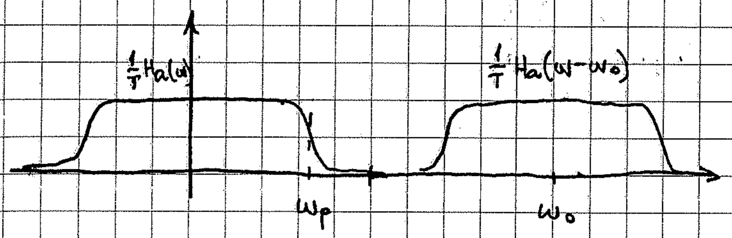
6.4.3. IMPULZNO INVARIANTNA METODA

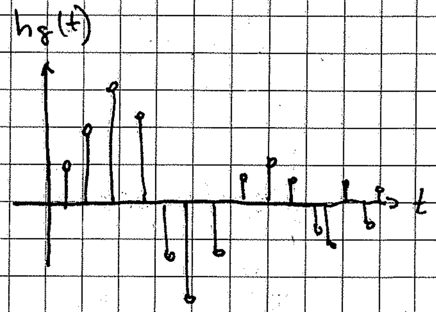
$h_a(t)$ ... impulzni odziv analognega prototipa

$h[n] = h_a(nT)$



20.5.2013





$$H(\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a(\omega - k\cdot\omega_0) = \frac{1}{T}\sum_{k=-\infty}^{\infty} H_a\left(\frac{\Omega - k2\pi}{T}\right)$$

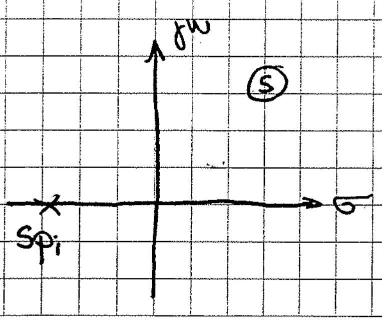
Metoda je uporabna za nizka sita, ki imajo le pole.
To so Butterworthov in Čebišev I.
(ne sme imeti valovitosti v zapori)

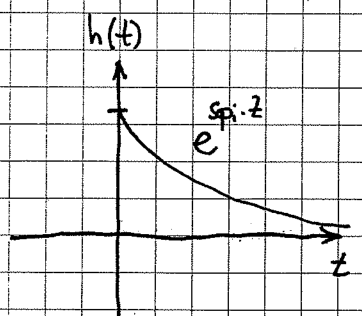
$$H(z) = \sum_{n=0}^{\infty} h_a(nT)\, z^{-n}$$

ponavadi nimamo, ponavadi imamo $H_a(s)$

$$H_a(s) = \sum_{i=1}^{N} \frac{C_i}{s - s_{pi}} = b_0 \prod_{i=1}^{N} \frac{1}{(s - s_{pi})}$$ .... ničel ni, so samo poli

$$C_i = \operatorname*{Res}_{s=s_{pi}} H_a(s) \qquad h_a(t) = u(t)\sum_{i=1}^{N} C_i \cdot e^{s_{pi}\cdot t}$$





$$h[n] = h_a(nT) = u[n] \sum_{i=1}^{N} C_i \, e^{s_{pi} \cdot T \cdot n}$$

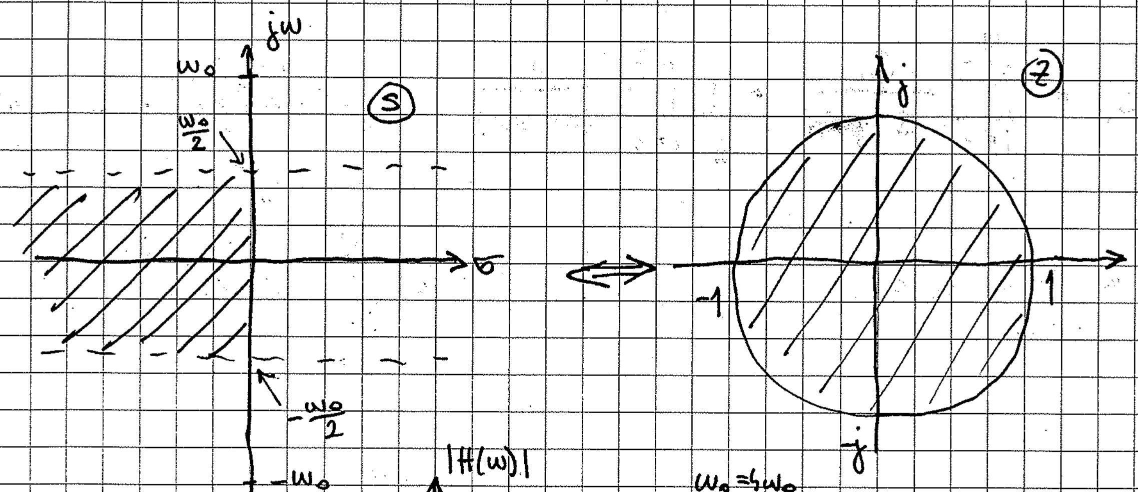
$$\Big\downarrow \; z\text{-transform}$$

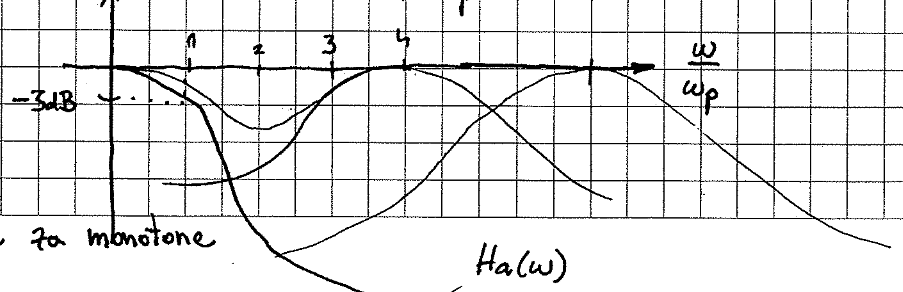
$$H(z) = \sum_{i=1}^{N} C_i \cdot \mathcal{Z}\left\{ u[n] \cdot e^{s_{pi} \cdot T \cdot n} \right\}$$

$$u[n] \cdot e^{s_{pi} \cdot T \cdot n} \xleftrightarrow{z} \frac{1}{1 - e^{s_{pi} T} z^{-1}}$$

$$H(z) = \sum_{i=1}^{N} \frac{C_i}{1 - (e^{s_{pi} T} z^{-1})}$$

$$\boxed{z_{pi} = e^{s_{pi} \cdot T}} \longleftarrow T \ldots \text{ frekvenca vzorčenja}$$





Uporabimo jo lahko le za monotone padajoče.

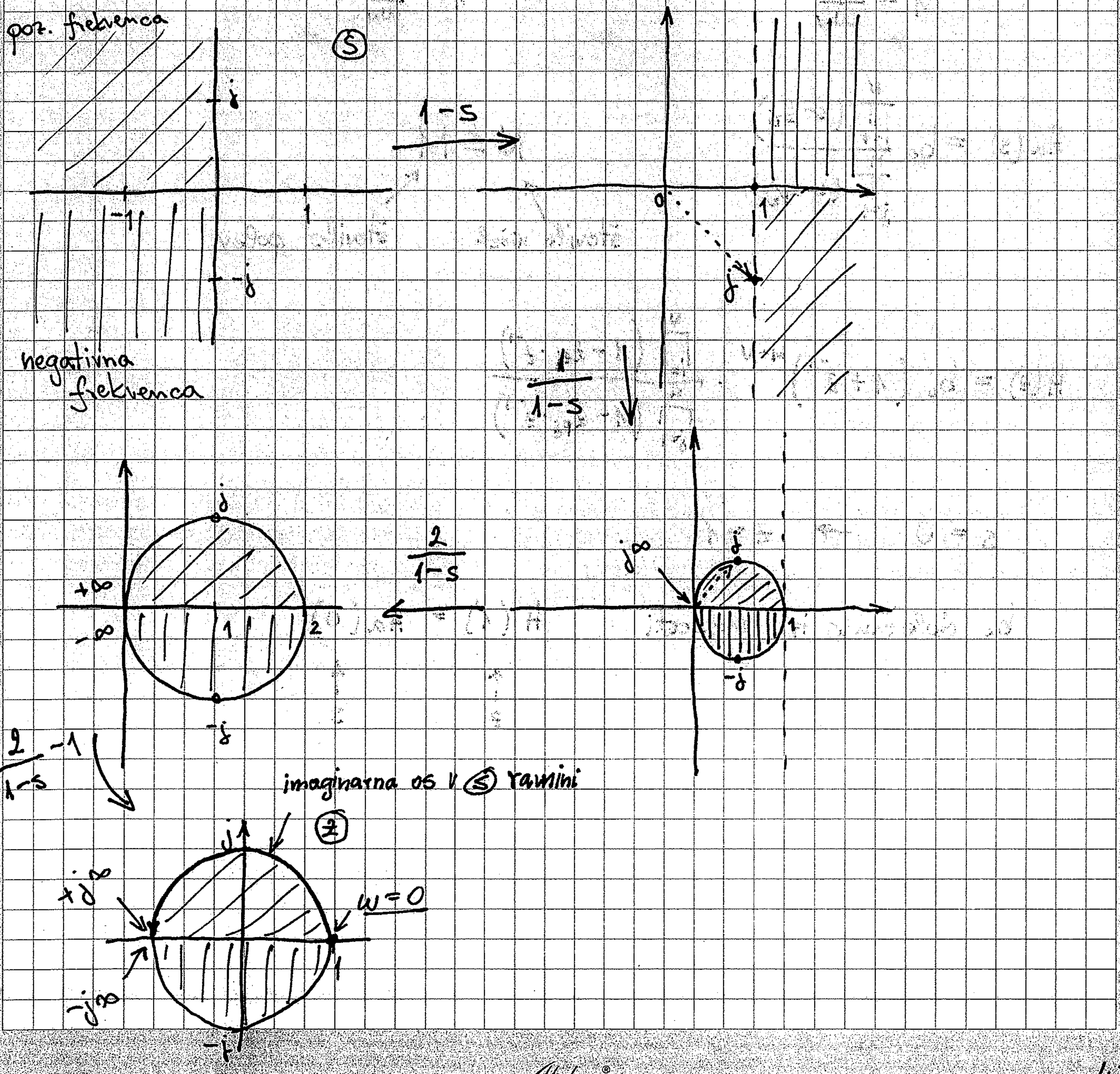
lahko samo pada – ne sme naraščat

6.4.4. BILINEARNA TRANSFORMACIJA

$$z = \frac{1+S}{1-S}$$ .... veliki S , predstavlja normirano frekvenco

S je normiran $\frac{s}{\omega_N}$

$$z = 2\,\frac{1}{1-S} - 1$$



$$S = \frac{z-1}{z+1} = \frac{s}{\omega_N} \qquad s = \omega_N \frac{z-1}{z+1}$$

$$H(z) = H_a\left(\omega_N \frac{z-1}{z+1}\right) \qquad H_a \ldots\ldots H \text{ analogna}$$

raje neposredno določimo pole in ničle

$$z_{pi} = \frac{1 + \frac{s_{pi}}{\omega_N}}{1 - \frac{s_{pi}}{\omega_N}} \qquad z_{zi} = \frac{1 + \frac{s_{zi}}{\omega_N}}{1 - \frac{s_{zi}}{\omega_N}}$$

$$H_a(s) = b_0 \frac{\prod_{i=1}^{N} (s - s_{zi})}{\prod_{j=1}^{M} (s - s_{pj})} \qquad N \leq M$$

N – število ničel, M – število polov

$$H(z) = b_0 \left(1 + z^{-1}\right)^{M-N} \cdot \frac{\prod_{i=1}^{N} \left(1 - z_{zi} \cdot z^{-1}\right)}{\prod_{j=1}^{M} \left(1 - z_{pj} \cdot z^{-1}\right)}$$

$$s = 0 \rightarrow z = 1$$

b_0 določimo iz enakosti $\underset{\uparrow \atop z}{H(1)} = \underset{\uparrow \atop s}{H_a(0)}$

Stiskanje frekvenc

$$\omega \in (0, \infty) \longrightarrow \Omega \in (0, \pi)$$

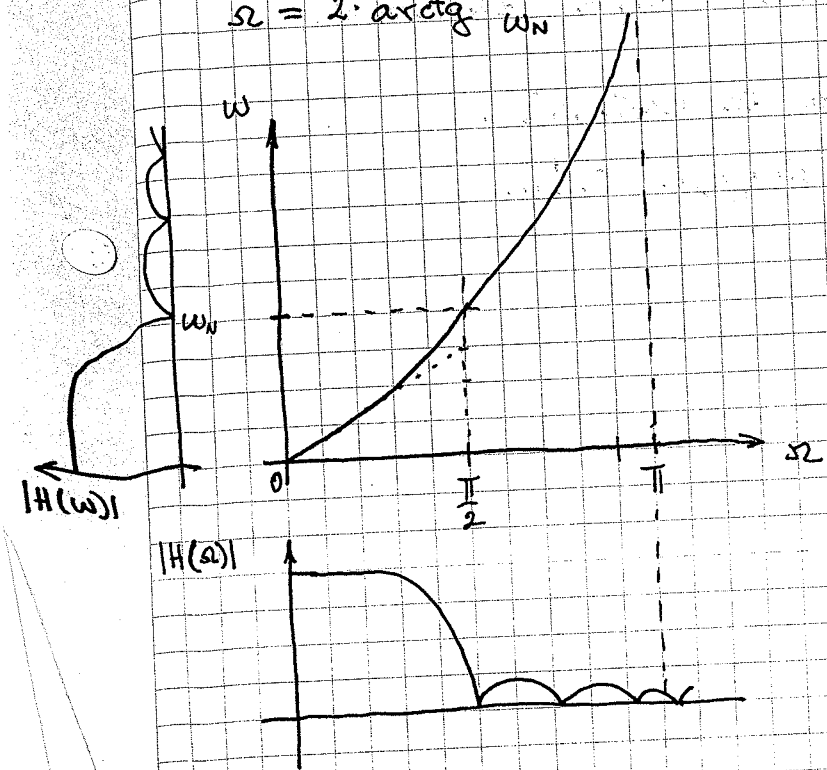
$$s = \omega_N \cdot \frac{z-1}{z+1} \qquad s = j\omega \qquad z = e^{j\Omega}$$

$$j\omega = \omega_N \frac{e^{j\Omega} - 1}{e^{j\Omega}+1} = \omega_N \frac{e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}}{e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}}} \cdot \cancel{\frac{e^{j\frac{\Omega}{2}}}{e^{j\frac{\Omega}{2}}}}$$

$$\cancel{j}\omega = \cancel{j} \cdot \omega_N \frac{\sin\frac{\Omega}{2}}{\cos\frac{\Omega}{2}}$$

$$\boxed{\omega = \omega_N \cdot tg\frac{\Omega}{2}}$$

$$\Omega = 2 \cdot arctg \frac{\omega}{\omega_N}$$



Postopek:

1. Na osnovi Ω_p, Ω_s, A_p, A_s določimo analogne ω_p, ω_s.

$$\omega_p = \omega_N \cdot tg\left(\frac{\Omega_p}{2}\right) \qquad \omega_s = \omega_N \cdot tg\left(\frac{\Omega_s}{2}\right)$$

2. Na osnovi ω_p, ω_s in A_p, A_s določimo vrsto in red analognega filtra.

3. Z bilinearno preslikavo določimo pole in ničle v (z)-ravnini.

$$z_{pi} = \frac{1 + \frac{s_{pi}}{\omega_N}}{1 - \frac{s_{pi}}{\omega_N}}$$

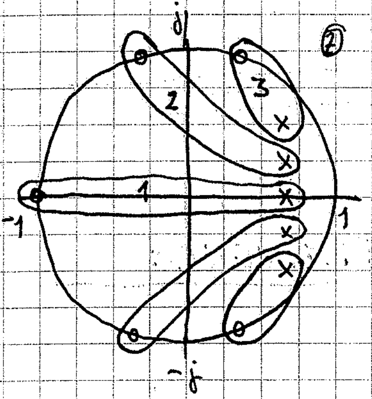
4. Zapišemo $H(z)$.

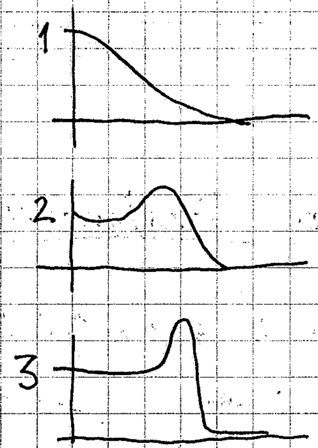
5. Izračunamo b_0, da ustreza danim zahtevam.

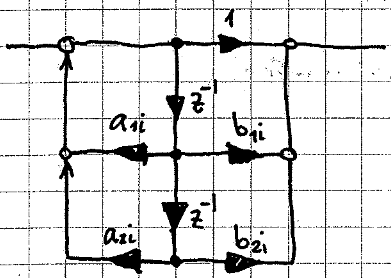
6. Izberemo ustrezno strukturo za realizacijo. Običajno kaskada celic drugega reda.

Primer:

//// ..... uparitev







$$H_i(z) = \frac{1 + b_{1i}\, z^{-1} + b_{2i}\, z^{-2}}{1 - a_{1i}\, z^{-1} - a_{2i} \cdot z^{-2}}$$

z_i, z_i^* in p_i, p_i^* .... konjugiran kompleksni par

$$z_i = |z_i|\, e^{j\Omega_{zi}} \qquad z_i^* = |z_i|\, e^{-j\Omega_{zi}}$$

$$p_i = |p_i|\, e^{j\Omega_{pi}} \qquad p_i^* = |p_i|\, e^{-j\Omega_{pi}}$$

$$(1 - z_i \cdot z^{-1})(1 - z_i^* \cdot z^{-1}) = 1 - 2 \cdot |z_i| \cdot \cos\Omega_{zi} \cdot z^{-1} + |z_i|^2 \cdot z^{-2}$$

$$\boxed{\begin{array}{ll} b_{1i} = -2 \cdot |z_i| \cdot \cos\Omega_{zi} & a_{1i} = +2\,|p_i| \cos\Omega_{pi} \\ b_{2i} = +|z_i|^2 & a_{2i} = -|p_i|^2 \end{array}}$$

po absolutni vrednosti vedno < 1

7. SISTEMI Z RAZLIČNIMI VZORČEVALNIMI FREKVENCAMI

(multirate systems)

7.1. RAZLOGI ZA SPREMEMBO VZORČEVALNE FREKVENCE

- prilagajanje med sistemi z različnimi f_0

  telefonski govor : 8 kHz (300 ÷ 3400 Hz)

  avdio signali : 32 kHz – za radio

  44,1 kHz – CD (zgoščenka)

  48 kHz – DAT (tape-kaseta)

- povečanje učinkovitosti obdelave

  (prilagajanje pasovni širini signala)

- zmanjšanje zahtevnosti sit proti prekrivanju pri rekonstrukciji in zajemanju

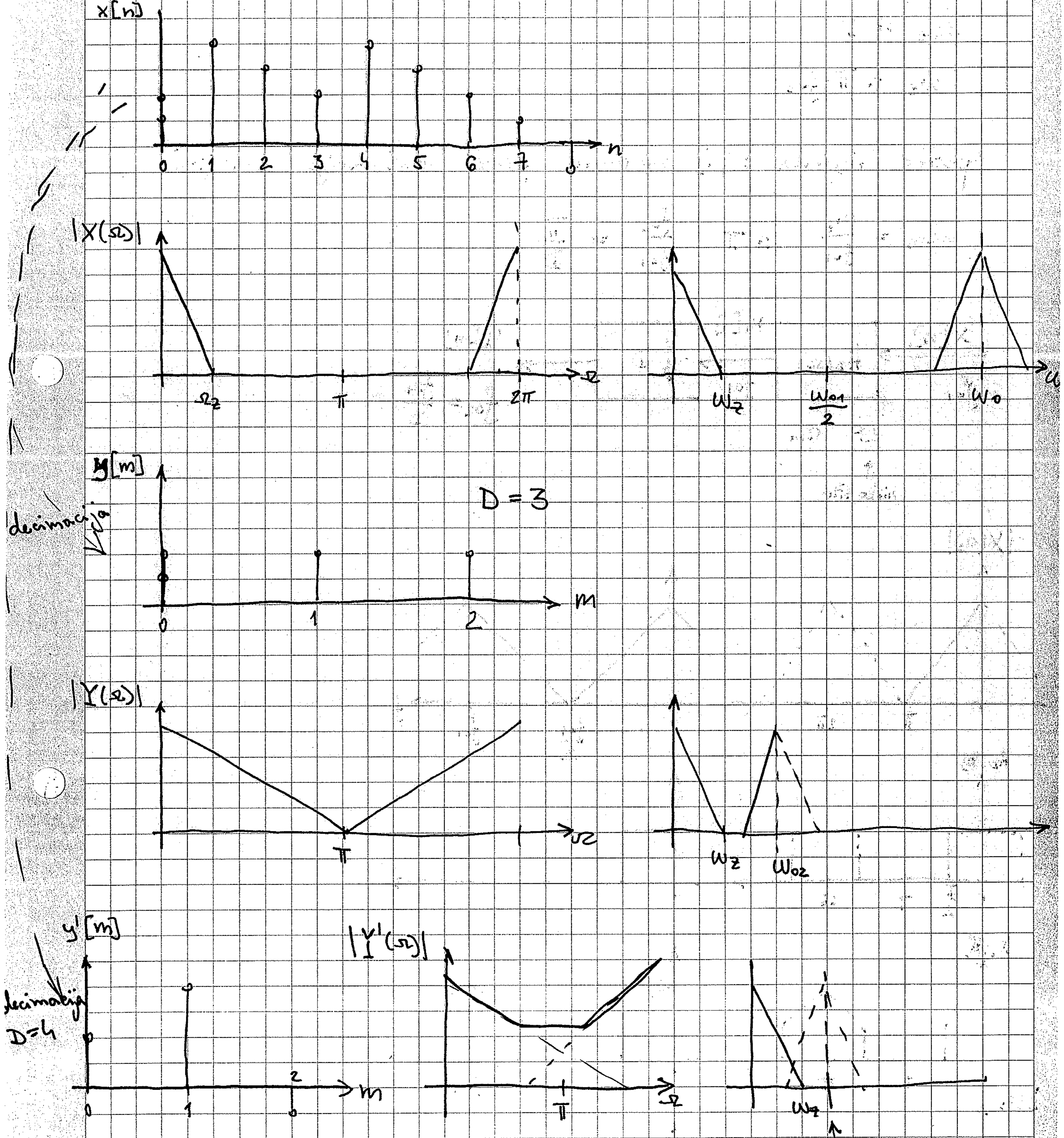
7.2. DECIMACIJA

na D vzorcev spustimo D-1 originalnih

$$x[n] \rightarrow \boxed{\downarrow D} \rightarrow y[m]$$

$$f_{02} = f_{01} \cdot \frac{1}{D} \geq 2 \cdot f_z$$

f_z ... mejna frekvenca analognega signala



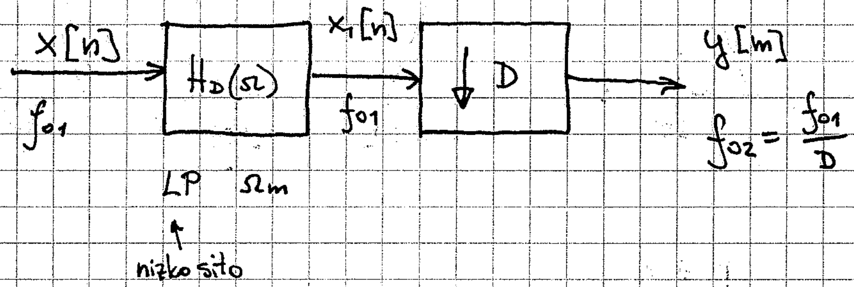
$$\omega_{o2} = \frac{\omega_{o1}}{3} \qquad \omega_{o2}' = \frac{\omega_{o1}}{4}$$

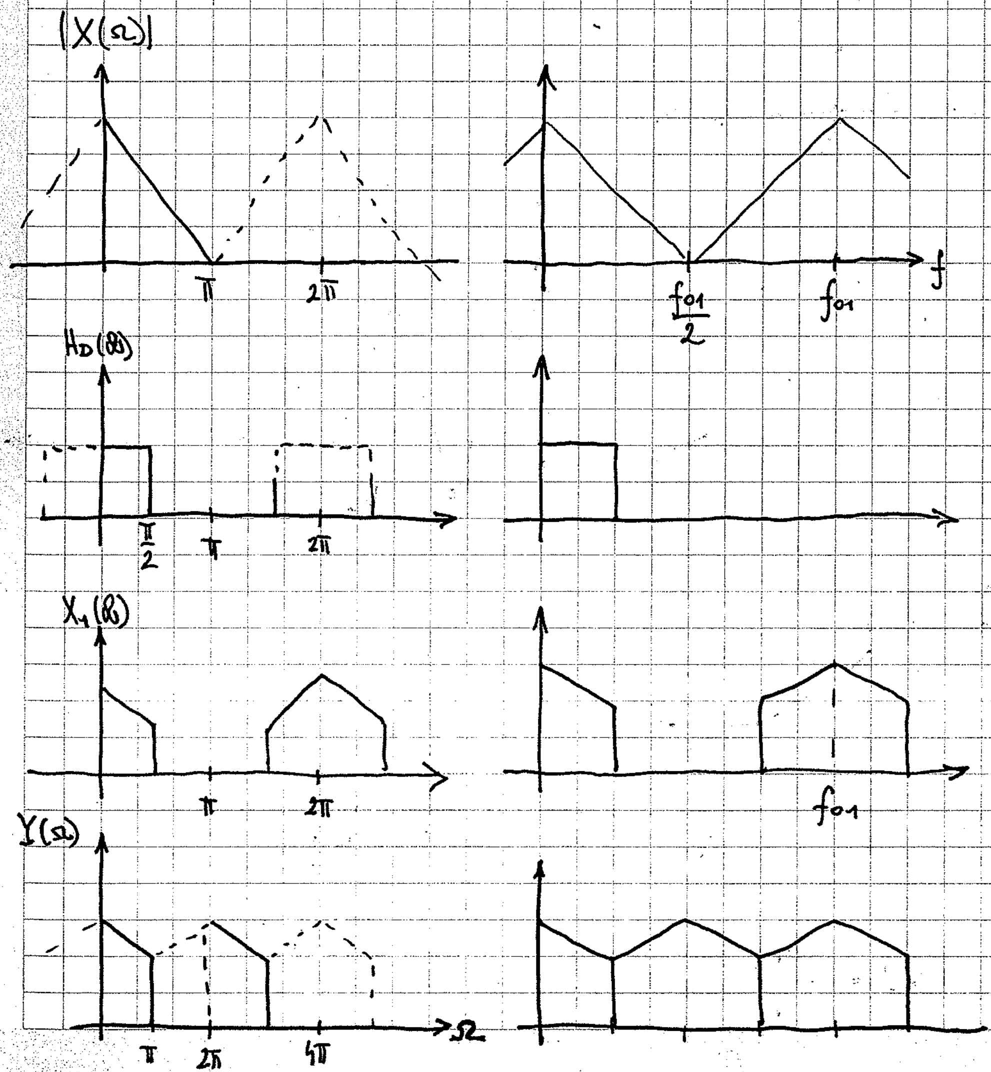
$\Omega_z \leq \pi$

$\omega_z < \pi \cdot f_{02}$

spekter signala $x[n]$ moramo pred decimacijo omejiti

$$\Omega_m = \frac{\omega_z}{f_{01}} \leq \pi \cdot \frac{f_{02}}{f_{01}} = \frac{\pi}{D}$$



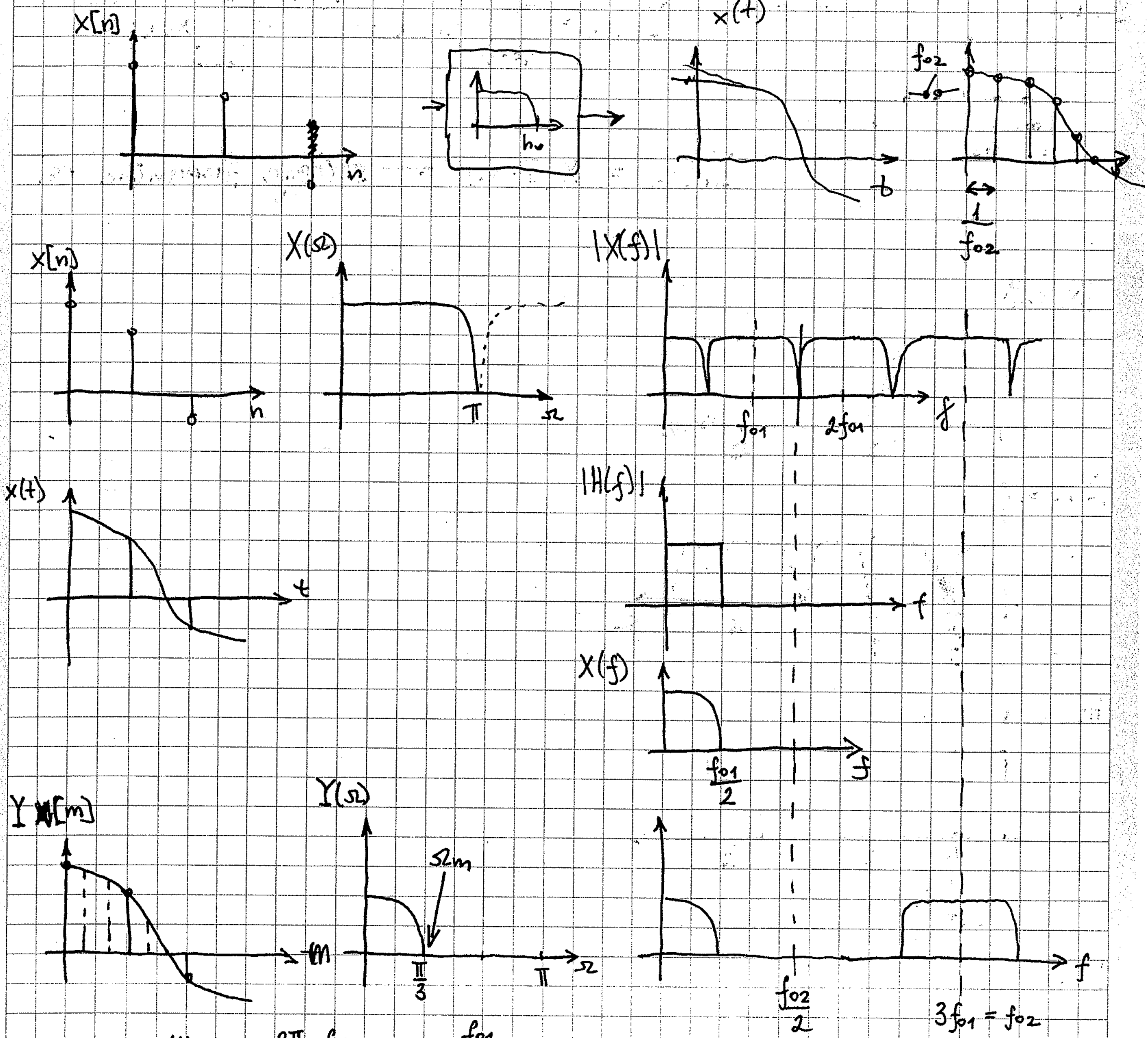


27.5.2013

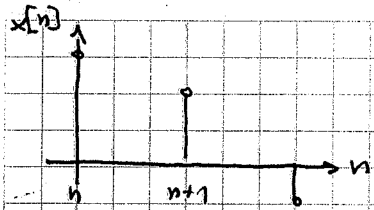
7.3. INTERPOLACIJA

med vzorce damo I-1 novih vzorcev

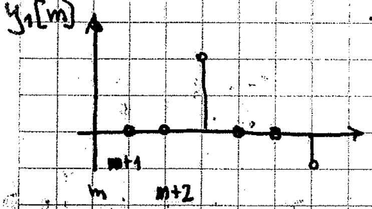
$$I = \frac{f_{02}}{f_{01}}$$



$$\Omega_m = \frac{\omega_z}{f_{02}} = \frac{2\pi \cdot f_{01}}{f_{02} \cdot 2} = \pi \cdot \frac{f_{01}}{f_{02}}$$



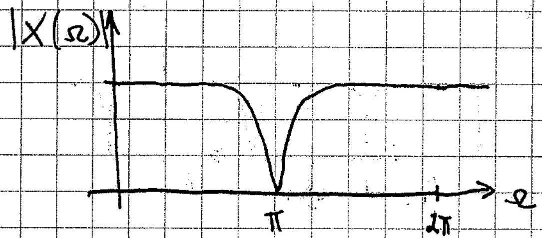
$$y_1[m] = \begin{cases} x[n] & \text{za} \quad m = n \cdot I \\ 0 & \text{za} \quad m \neq n \cdot I \end{cases}$$

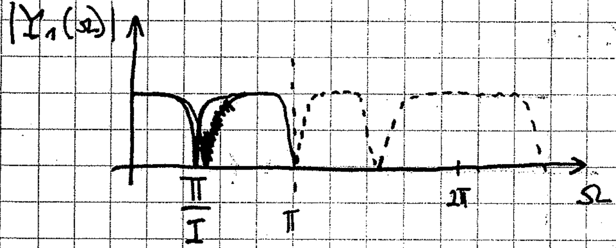


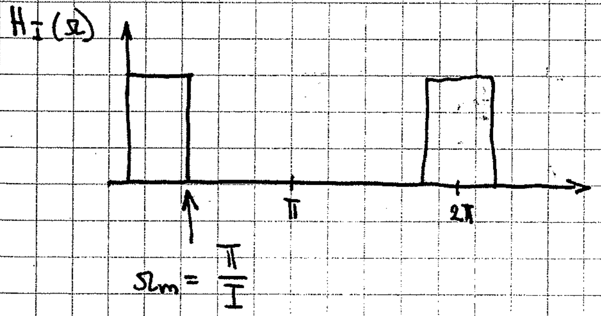
$I = 3$

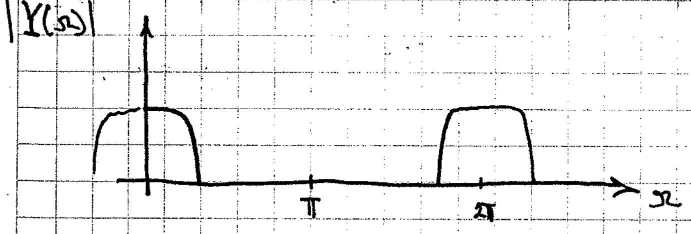
$$Y_1(\Omega) = \sum_{m=-\infty}^{\infty} y_1[m] \cdot e^{-j\Omega m} = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\Omega n \cdot I}$$

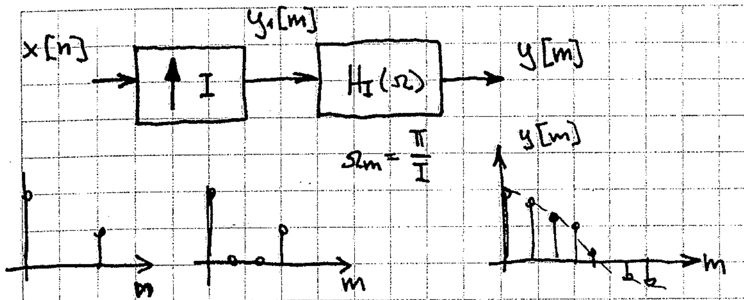
$$Y_1(\Omega) = X(I \cdot \Omega) \leftarrow \text{skrčenje frekvenčne osi}$$









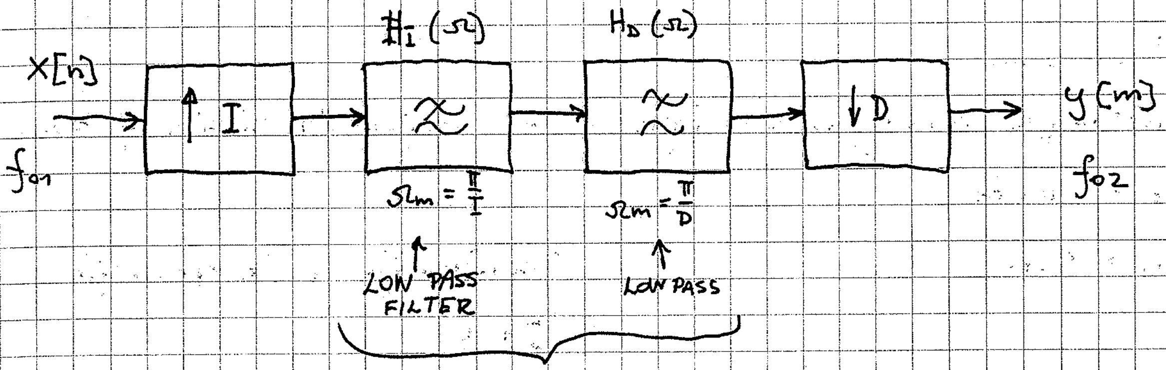


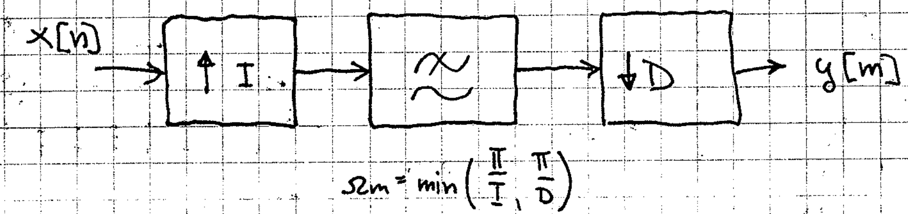
7. 4. SPREMEMBA VZORČNE FREKVENCE ZA RACIONALEN FAKTOR

$$\frac{f_{o2}}{f_{o1}} = k = \frac{I}{D}$$

za $k > 1$ je skupno interpolacija
če $k < 1$ je skupno decimacija

najprej interpoliramo, nato decimiramo.





Zgled:

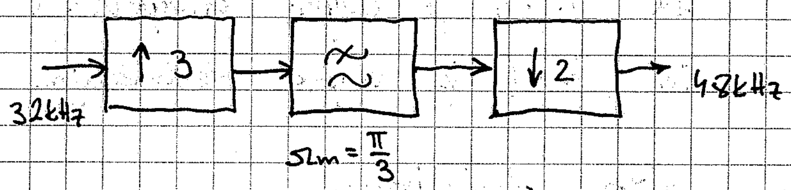
digitalni radio → DAT

$f_{01} = 32\,kHz$ ... radio

$f_{02} = 48\,kHz$ ... DAT

$$f_{02} = f_{01} \cdot \frac{I}{D} \quad \rightarrow \quad \frac{I}{D} = \frac{48\,kHz}{32\,kHz} = \frac{3}{2}$$

$I = 3$, $D = 2$



Zgled:

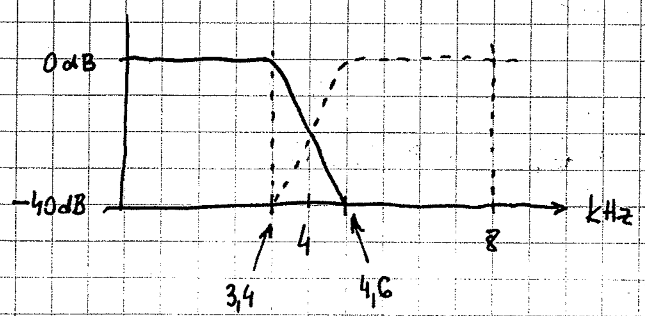
rekonstrukcija analognega signala z enostavnim nizkim filtrom

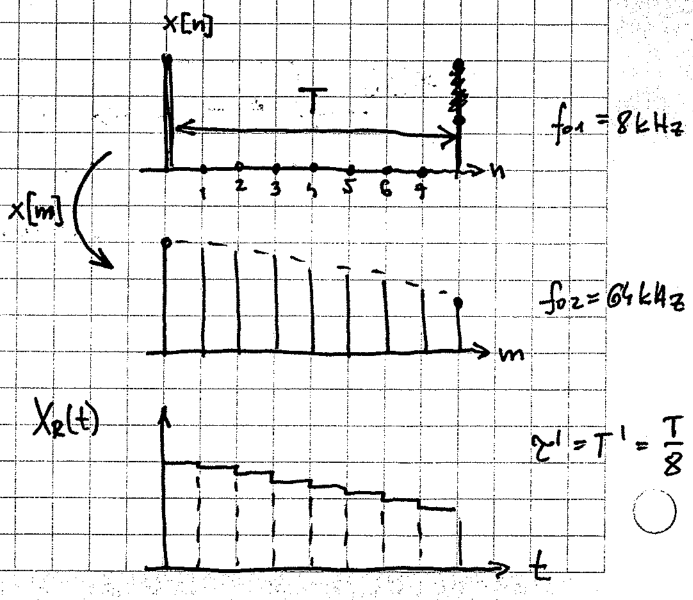
$f_z = 3400\,Hz$

$f_{01} = 8\,kHz$

rekonstrukcija z interpolacijo

$I = 8$





$$|X_R(\omega)| = \frac{\tau'}{T'} \left| \frac{\sin \frac{\omega \tau'}{2}}{\frac{\omega \tau'}{2}} \right| \sum_{k=-\infty}^{\infty} |X(\omega - k \cdot \omega_{0z})|$$

$$\tau' = \frac{1}{f_{0z}} = \frac{1}{2 \cdot I \cdot f_z}$$

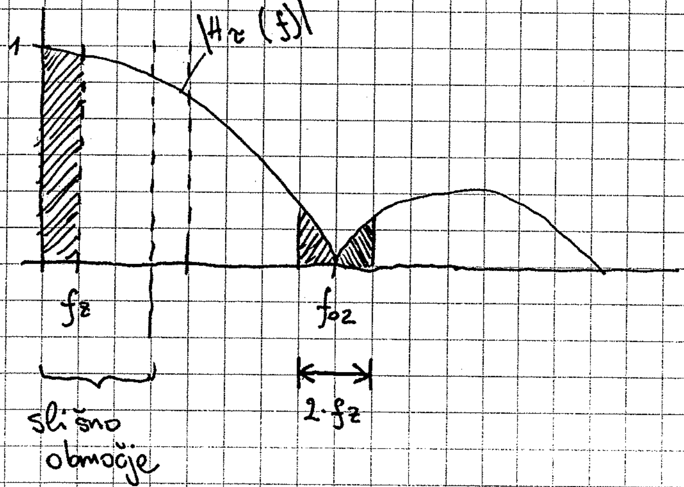
$$|X_R(f)| = \left| \frac{\sin(\pi \cdot \tau' \cdot f)}{\pi \cdot \tau' \cdot f} \right| \sum_{k=-\infty}^{\infty} |X(f - k \cdot f_{0z})|$$

$k = 0$ ... osnovni pas

$$|X_R(f)| = \frac{\sin\left(\pi \frac{f}{f_{0z}}\right)}{\pi \frac{f}{f_{0z}}} \cdot |X(f)|$$

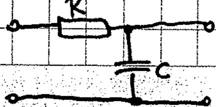
$$f = f_z \qquad |X_R(f_z)| = \frac{\sin \frac{\pi}{16}}{\frac{\pi}{16}} \cdot |X(f_z)| = 0{,}993 \cdot |X(f_z)|$$

$(= -0{,}05\,dB)$

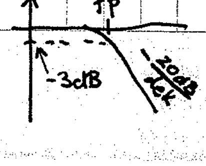


$$|H_z(f)|\Big|_{f_{0z}-f_z} = \frac{\sin\left(\pi \frac{f_{0z}-f_z}{f_{0z}}\right)}{\pi \frac{f_{0z}-f_z}{f_{0z}}} \doteq \frac{\sin\left(\pi - \pi \frac{f_z}{f_{0z}}\right)}{\pi} = \frac{\sin \pi \frac{f_z}{f_{0z}}}{\pi} \doteq \frac{f_z}{f_{0z}} = \frac{1}{16}$$

$(= -24\,dB)$



$$|H_R(f)| = \frac{1}{1 + \left(\frac{f}{f_p}\right)^2}$$



$$f_p = \frac{1}{2\pi RC}$$

$$-40\,dB = |H_{LP}(f)| + |H_Z(f)|$$

$$H_{LP}(f_{oz}) = -16\,dB \longrightarrow \frac{f_{oz}}{f_P} = 6{,}22$$

$$f_P = \frac{f_{oz}}{6{,}22} \doteq 10\,kHz$$

$$|H(f)|\Big|_{f=f_Z} = \frac{1}{1+\left(\frac{4}{10}\right)^2} = 0{,}88 \qquad (=-0{,}5\,dB)$$

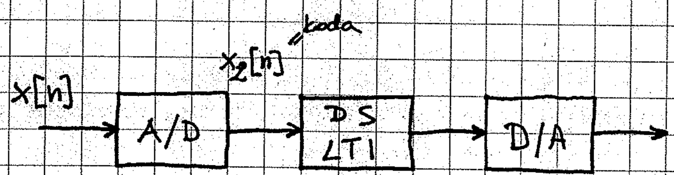
8. VPLIV KVANTIZACIJE SIGNALA IN KOEFICIENTOV NA LASTNOSTI DISKRETNIH SISTEMOV

digitalna izvedba sistemov povzroča dva efekta
- kvantizacijski šum (napaka)
- odstopanje karakteristike realiziranega sistema ($H(\Omega)$)

Vzroki za degradacijo so:
- kvantizacija signala pri A/D pretvorbi
- zaokroževanje delnih rezultatov pri aritmetičnih operacijah

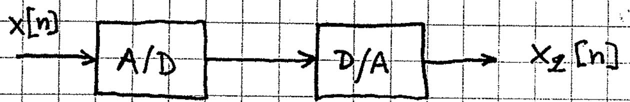
8.1. ANALOGNO DIGITALNA PRETVORBA SIGNALA

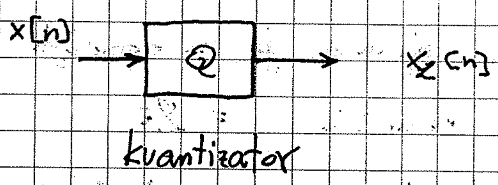
8.1.1. KVANTIZACIJA

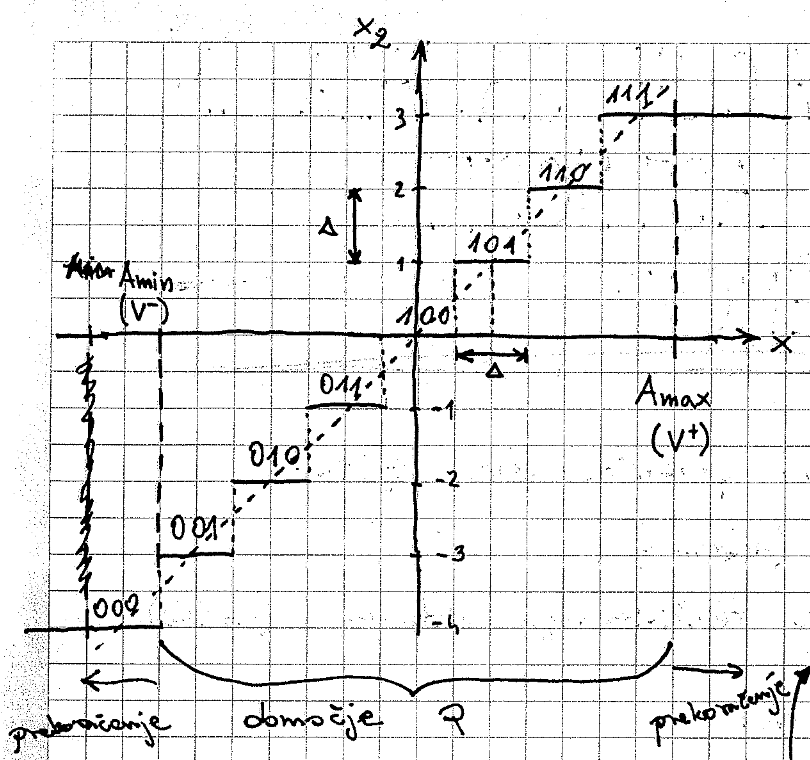


$x_Q[n]$ ... analogna kvantizirana vrednost

sistem zamenjamo z idealno identiteto







	koda z odmikom	dvojiški komplement
3	111	011
2	110	010
1	101	001
0	100	000
-1	011	111
-2	010	110
-3	001	101
-4	000	100

prenos

negacija MSB

L-bitov da 2^L možnih nivojev

$M = 2^L - 1$; ker zavržemo (−4)

za velik L $\quad M \doteq 2^L$

$$x_2[n] = x[n] + e[n]$$

$$e[n] = x_2[n] - x[n]$$

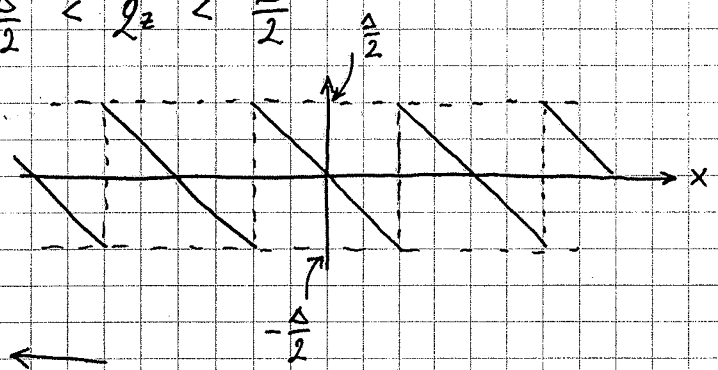
kvantizacijska napaka oz. šum

dve vrsti napake:

- zrnato popačenje (granular noise) – amplituda signala je na območju $A_{min} < x < A_{max}$
  je znotraj območja $= e_z$
- prekoračitveno popačenje (overload noise) – $x < A_{min}$ ali $x > A_{max}$
  e_p

8.1.2. MOČ ZRNATE NAPAKE

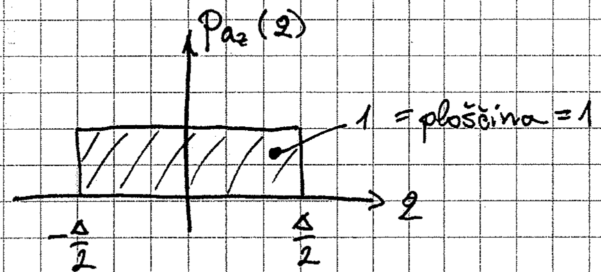
$$-\frac{\Delta}{2} < q_z < \frac{\Delta}{2}$$



prekoračitev

porazdelitev gostote verjetnosti za q_z

$$p_{q_z}(q) = \begin{cases} \frac{1}{\Delta} & \text{za} \quad -\frac{\Delta}{2} < q_z < \frac{\Delta}{2} \\ 0 & \text{drugod} \end{cases}$$



standardna deviacija

$$\sigma_{q_z}^2 = \overline{Q_z^2} = \int_{-\infty}^{\infty} q_z^2 \cdot p_q(q)\, dq = \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q_z^2 \cdot dq = \frac{1}{\Delta} \frac{q^3}{3} \Bigg|_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} =$$

$$= \frac{\Delta^2}{12}$$

$$\boxed{\overline{Q_z^2} = \sigma_z^2 = N_z = \frac{\Delta^2}{12}}$$