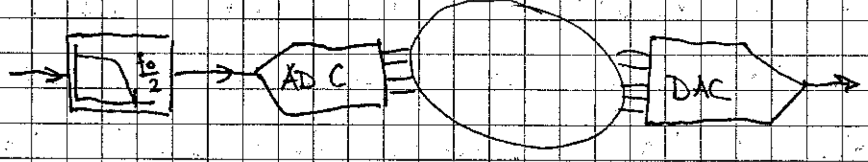
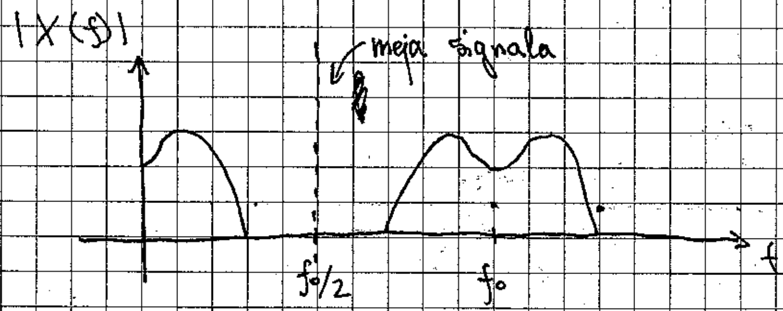
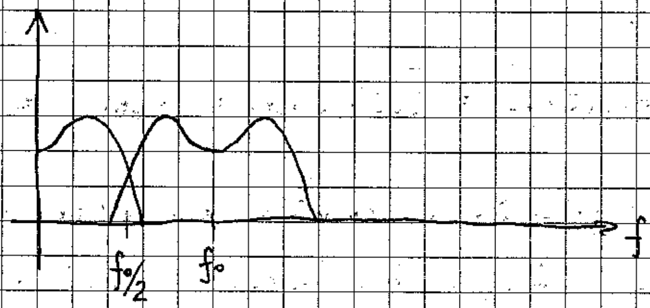
28.2.2013

DISKRETNI SIGNALI IN SISTEMI - VAJE



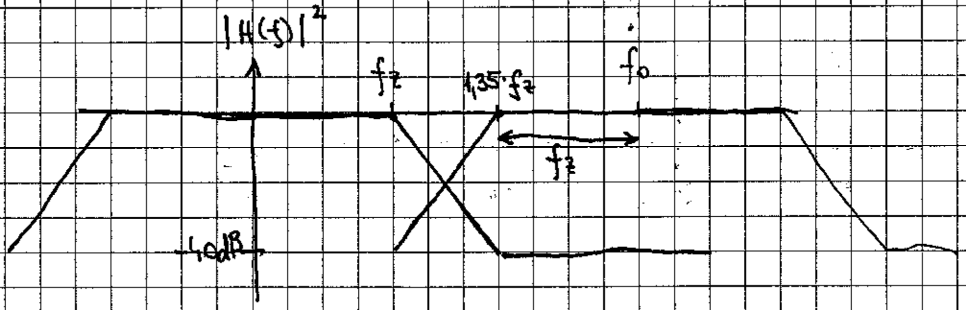




I. KOLOKVIJ : petek 19.4.2013 ob 15:00 v PR02

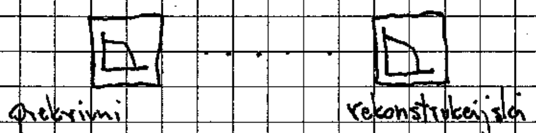
FILTRI

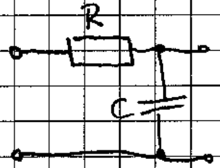
1. Določite min vzorčno frekvenco f_o za rekonstrukcijo govornega signala v frekvenčnem območju od 0 ÷ 3400Hz. Maximalna velikost tujega spektra sme biti −40 dB. Za prekrivni in rekonstruirani ~~signal~~ filter uporabite nizkoprepustni filter z normiranim potekom na sliki.



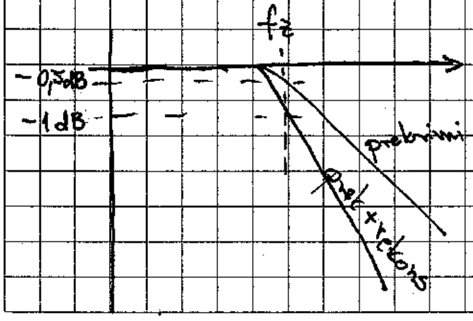
$$f_o = 1{,}35 \cdot f_z + f_z = 2{,}35 \cdot f_z = 2{,}35 \cdot 3{,}4\,kHz = 8\,kHz = 8\,kS/s$$

2. Vzorčiti želimo signal v frekvenčnem območju od 100Hz do 8kHz. Določite frekvenco pola nizkega sita 1. reda (RC člen), da bo dušenje pri zgornji frek. meji 1dB. Kolikšna mora biti frekvenca vzorčenja, da bo tuji spekter vsaj za 40dB manjši od spektera v osnovnem pasu, če bo pri rekonstrukciji uporabljen enak filter?





$$H(\omega) = \frac{1}{1+j\omega RC} = \frac{1}{1+j\frac{\omega}{\omega_p}} = \frac{1}{1+j\frac{f}{f_p}}$$



$$|H(f_z)|^2 = \frac{1}{1+\left(\frac{f_z}{f_p}\right)^2} = -0{,}5\,dB$$

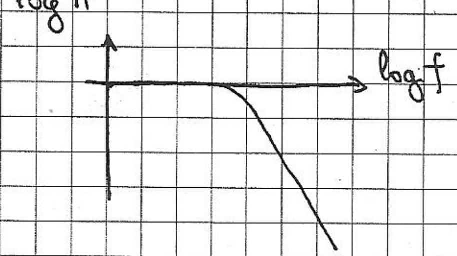
$$|H(f_z)|^2_{dB} = 10 \cdot \log |H(f)|^2$$

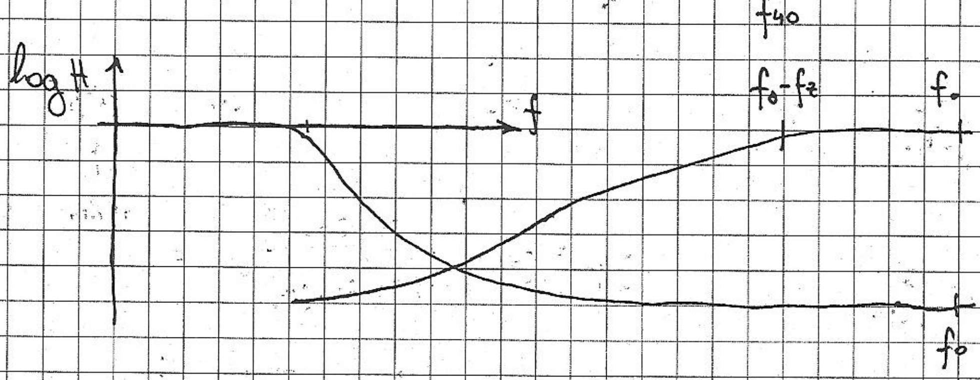
$$10^{\frac{|H(f)|^2_{dB}}{10}} = 10^{-\frac{0,5}{10}} = 10^{-0,05}$$

$$|H(f_z)|^2 + \left(\frac{f_z}{f_p}\right)^2 |H(f_z)|^2 = 1$$

$$\left(\frac{f_z}{f_p}\right)^2 = \frac{1-|H(f_z)|^2}{|H(f_z)|^2}$$

$$f_p = f_z\sqrt{\frac{|H(f_z)|^2}{1-|H(f_z)|^2}} = 8kHz\sqrt{\frac{10^{-0,05}}{1-10^{-0,05}}} = \underline{23kHz}$$





$$f = f_p\sqrt{\frac{1-|H_{40}|^2}{|H_{40}|}} = 23kHz\sqrt{\frac{1-10^{-4}}{10^{-4}}}$$

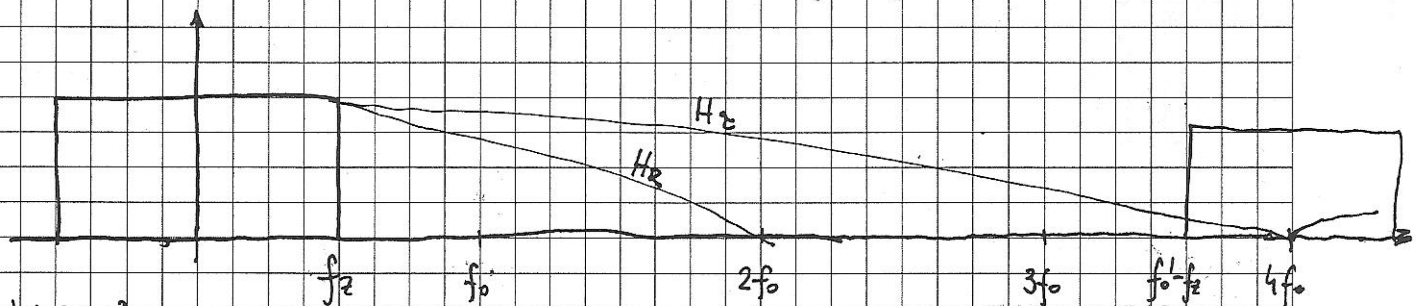
$f_{40} = 2,3MHz$

$$f_0 = f_{40} + f_z \doteq 2,3MHz$$

7.3.2013

REKONSTRUKCIJA

3. Diskretni signal želimo rekonstruirati z uporabo preprostega RC člena – nizko sito 1. reda. Diskretni signal je dobljen s kritičnim vzorčenjem analognega signala z zg. frekvenco $f_z = 10\,kHz$. Za realizacijo uporabimo digitalno interpolacijo s katero povišamo vzorčno frekv. na $f_0' = 4 \cdot f_0$.
Izračunajte frekvenco pola f_p RC sita tako, da slabljenje tega sita pri f_z znašalo 1dB! Koliko znaša dušenje najnižje rekonstruiranega spektra pri najvišji freq. v prvem ponovljenem spektru, če je širina impulzov $x_r(t)$ (izhod D/A C), kar enaka periodi vzorcev v interpoliranem signalu?



$|H(f_z)|^2 = -1dB = 10^{-0,1}$

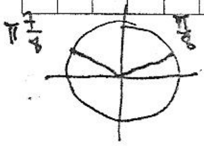
70kHz 80kHz

$$|H(f)|^2 = \frac{1}{1+\left(\frac{f}{f_p}\right)^2} \qquad f_p = f_z\sqrt{\frac{|H(f_z)|^2}{1-|H(f_z)|^2}} =$$

$$\left(\frac{f}{f_p}\right)^2 = \frac{1-|H(f)|^2}{|H(f)|^2} \qquad = 10\,kHz\sqrt{\frac{10^{-0,1}}{1-10^{-0,1}}} = 19,65\,kHz$$

$$H = H_\tau \cdot H_R \qquad H_\tau(f) = \frac{\tau}{T}\frac{\sin(\pi\tau f)}{\pi\tau f} = \frac{\sin(\pi\tau f)}{\pi\tau f}$$

$$|H_\tau(f)| = \frac{\sin(\pi\tau f)}{\pi\tau f} = \frac{\sin\left(\pi\frac{1}{f_0'}\cdot 7f_z\right)}{\pi\frac{1}{f_0'}\cdot 7f_z} = \frac{\sin\left(\pi\frac{7}{8}\frac{f_z}{f_z}\right)}{\pi\frac{7}{8}} =$$



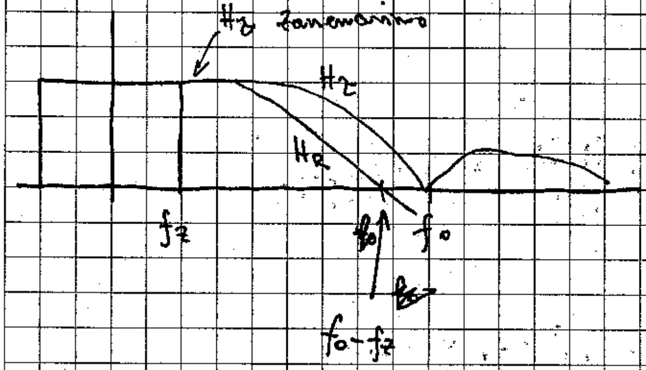
$$= \frac{\pi\frac{1}{8}}{\pi\frac{7}{8}} = \frac{1}{7}$$

$$|H_z(f_0' - f_z)|^2 = 10 \log \left(\frac{1}{7}\right)^2 = -17\ \text{dB}$$

$$|H_R(f_0' - f_z)|^2 = \frac{1}{1+\left(\frac{7 f_z}{f_p}\right)^2} = \frac{1}{1+\left(\frac{70\,\text{kHz}}{20\,\text{kHz}}\right)^2} = 0{,}075 = -11{,}2\ \text{dB}$$

$$|H(f_0' - f_z)|^2 = -17\,\text{dB} - 11{,}2\,\text{dB} = -28{,}2\ \text{dB}$$

4. Izračunajte frekvenco vzorčenja, da bo rekonstrukcijsko zveznega signala zadoščalo RC sito. Dušenje na zgornji mejni frekvenci f_z je 0,2 dB, dušenje prve zrcalne frekvence pa je 40dB. Širina impulzov je enaka periodi vzorčenja.



$$|H_R(f_z)|^2 = 10^{-0{,}02}$$

$$|H_R(f)|^2 = \frac{1}{1+\left(\frac{f}{f_p}\right)^2} \rightarrow \left(\frac{f}{f_p}\right) = \sqrt{\frac{1-|H_R(f)|^2}{|H_R(f)|^2}}$$

$$f_p = f_z \sqrt{\frac{|H_R(f_z)|^2}{1-|H_R(f_z)|^2}} = f_z \sqrt{\frac{10^{-0{,}02}}{1-10^{-0{,}02}}} = 4{,}6\, f_z$$

$$|H_z(f)|^2 = \left|\frac{\tau}{T}\,\frac{\sin(\pi \tau f)}{\pi \tau f}\right|^2 = \frac{\sin^2\left(\pi \frac{1}{f_0} \cdot f\right)}{\left(\pi \frac{f}{f_0}\right)^2}$$

$$H^2 = H_R^2 + H_z^2$$

$$-40\,dB = |H(f_0 - f_z)|^2 = \frac{1}{1+\left(\frac{f_0-f_z}{f_p}\right)^2} \cdot \frac{\sin^2\left(\pi \cdot \frac{f_0-f_z}{f_0}\right)}{\underbrace{\left(\pi \cdot \frac{\cancel{f_0} - \cancel{f_z}}{\cancel{f_0}}\right)^2}_{\pi^2}} = 10^{-4}$$

poenostavitve:

$$\pi \frac{f_0 - f_z}{f_0} = \pi \frac{f_0}{f_0} - \pi \frac{f_z}{f_0} = \pi - \pi \frac{f_z}{f_0}$$

$$\sin\left(\pi - \pi \frac{f_z}{f_0}\right) = \sin\left(\pi \frac{f_z}{f_0}\right) \approx \pi \frac{f_z}{f_0}$$

$$1 + \left(\frac{f_0 - f_z}{f_p}\right)^2 \Rightarrow \frac{1}{\left(\frac{f_0}{f_p}\right)^2}$$

$$= \left(\frac{f_p}{f_0}\right)^2 \cdot \left(\frac{f_z}{f_0}\right)^2 \approx 10^{-4}$$

$$\frac{f_p^2 \cdot f_z^2}{10^{-4}} \approx f_0^4 \qquad f_0 = \frac{\sqrt{f_p \cdot f_z}}{10^{-1}} = 10 \cdot \sqrt{f_p \cdot f_z}$$

$$f_0 = 10 \cdot \sqrt{4{,}6 \cdot f_z^2} = 10 \cdot \sqrt{4{,}6} \cdot f_z = 21\, f_z$$

14.3.2013

1. Vzorčite časovni signal $x(t) = \cos(200\pi \cdot t)$ z vzorčno frekv. $f_0 = 1kHz$
Koliko sta normirana frekvenca F in normirana krožna frekvenca Ω signala

$x(t) = \cos(200\pi t)$

$f_0 = 1kHz$

$F = ?$, $\Omega = ?$ $\qquad T_0 = \frac{1}{f_0}$

$$x[n] = x(nT) = \cos(200\pi n T_0)$$
$$= \cos(200\pi \cdot n \cdot 10^{-3})$$
$$= \cos(0{,}2\pi n) = \cos\left(\frac{2\pi}{10} n\right)$$

$$\Omega = \frac{2\pi}{10}$$

$$F = \frac{1}{10}$$

$\cos(\omega t)$ $\qquad \cos(\Omega n)$

$\omega = 2\pi f$ $\qquad \Omega = 2\pi F$

$f = \frac{1}{T}$ $\qquad F = \frac{1}{N}$

$$\cos(\omega n T_0) = \cos\left(\omega n \frac{1}{f_0}\right) = \cos\left(2\pi f \cdot n \cdot \frac{1}{f_0}\right) = \cos\left(2\pi \frac{f}{f_0} n\right) \qquad F = \frac{f}{f_0}$$

$$\Omega = 2\pi \frac{f}{f_0} = \frac{\omega}{f_0}$$

② Uzorčite čas. zvezni signal $x(t) = u(t) - u(t-6ms)$; $f_0 = 1kHz$

Izračunajte njegov spektar.

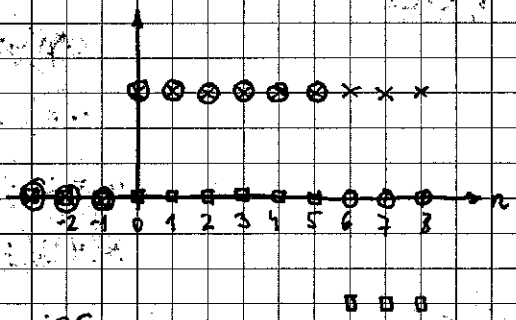
$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad T_0 = \frac{1}{f_0} = 1ms$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

$$\underbrace{x[n]}_{o} = x(nT_0) = u(nT_0) - u(nT_0 - 6ms) =$$

$$= u(n \cdot 1ms) - u(n \cdot 1ms - 6ms) =$$

$$= \underbrace{u[n]}_{\times} - \underbrace{u[n-6]}_{\blacksquare}$$



$$X(\Omega) = \sum_{n=0}^{5\,(N-1)} 1 \cdot e^{-j\Omega n} \overset{\star}{=} \frac{1 - e^{-j\Omega N}}{1 - e^{-j\Omega}} = \frac{1 - e^{-j\Omega 6}}{1 - e^{-j\Omega}} =$$

$$= \frac{e^{-j\Omega 3}\left(e^{+j\Omega 3} - e^{-j\Omega 3}\right)}{e^{-j\Omega/2}\left(e^{+j\Omega/2} - e^{-j\Omega/2}\right)} = \frac{e^{-j\Omega 3}\; 2j \cdot \sin(\Omega 3)}{e^{-j\Omega/2}\; 2 \cdot j \cdot \sin(\Omega/2)}$$

$$X(\Omega) = \frac{\sin(3\Omega)}{\sin(\Omega/2)} \cdot e^{-j\Omega \frac{5}{2}}$$

$$e^{j\varphi} = \cos\varphi + j\sin\varphi$$

$$e^{-j\varphi} = \cos\varphi - j\sin\varphi$$

$$e^{+j\varphi} - e^{-j\varphi} = \cos\varphi + j\sin\varphi - \cos\varphi + j\sin\varphi = 2j\sin\varphi$$

$$e^{j\varphi} + e^{-j\varphi} = 2\cos\varphi$$

$$\cos\varphi = \frac{1}{2}\left(e^{j\varphi} + e^{-j\varphi}\right)$$

vsi vmesni se pokrajšajo

→ ☆ $$\sum_{n=0}^{N-1} a^n = (a^0 + a^1 + a^2 + \dots + a^{N-1}) \frac{1-a}{1-a} = (1 + \cancel{a^1} + \cancel{a^2} + \dots + a^{N-1} - \cancel{a} - \cancel{a^2} - \dots - a^N) \cdot \frac{1}{1-a}$$

$$= \frac{1-a^n}{1-a}$$

dodatno $|a| < 1$, $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1-a^{\infty}}{1-a} = \frac{1}{1-a}$$

3. Imamo LTI sistem in poznamo odziv na enotin impulz

$$h[n] = \begin{cases} 1 & ; \ 0 \le n \le 11 \\ 0 & ; \ \text{sicer} \end{cases}$$

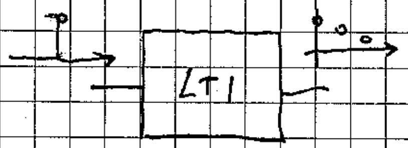
odziv sistema $y[n] = ?$

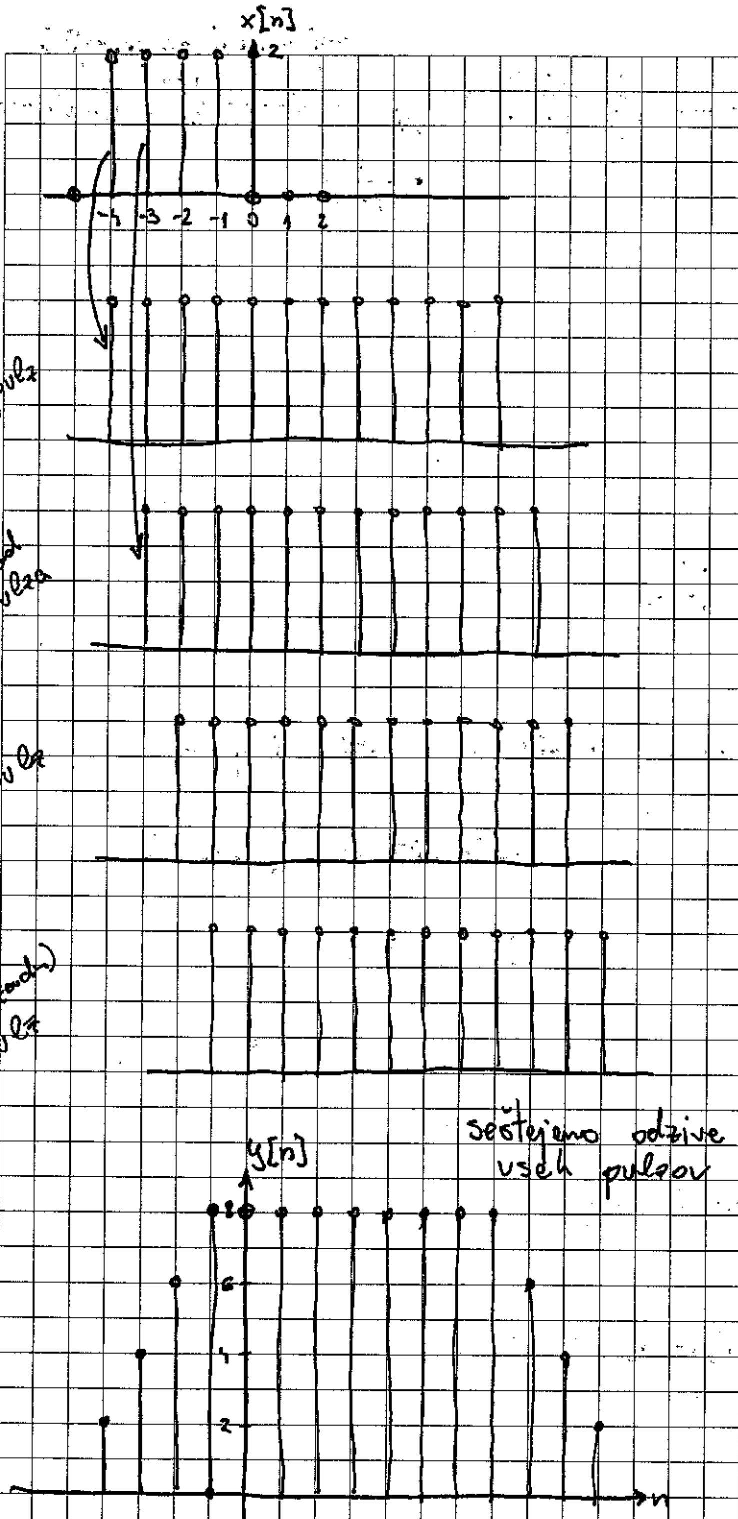
$Y(\Omega) = ?$

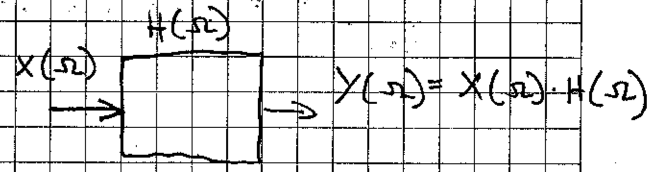
$$x[n] = \begin{cases} 2 & ; \ -4 \le n < 0 \\ 0 & ; \ \text{sicer} \end{cases}$$

$$(x * h)(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \, d\tau$$

$$(x * h)[n] = \sum_{m=-\infty}^{\infty} x[m] \cdot h[n-m]$$




x[n]
2
-4 -3 -2 -1 0 1 2
za 1 pulz
zavod 2 pulza
3 pulz
4 (zadnji) pulz
y[n]
8
6
4
2
n
seštejemo odzive vseh pulzov


H(Ω)
X(Ω)
Y(Ω) = X(Ω)·H(Ω)

21.3.2013

9. naloga

Izračunajte DFT kosinusnega signala, ki je vzorčen z njegovo
a.) štirikratno b.) dvokratno
frekvenco. Za izračun uporabite 8 vzorcev.

$$X_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt \qquad \text{(DFS)}\ DFT = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\Omega_0 n}$$ za periodične

$$\Omega_0 = \frac{2\pi}{N}$$

$$X_\omega = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

FFT

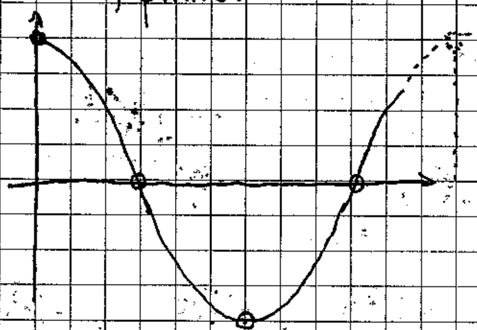
$$TDFT = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} = X(\Omega)$$

za aperiodične signale

$$DFT = \sum_{n=0}^{N-1} x[n]\, e^{-j\Omega_0 n}$$

če nas zanima energija damo $\frac{1}{N}$ spredaj
če je brez $\frac{1}{N}$; potem je močnostni spekter

a.) primer



$$x[n] = \{1, 0, -1, 0\} \qquad N=4$$

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\Omega_0 n} =$$

$$= \frac{1}{4}\sum_{n=0}^{3} x[n]\, e^{-jk\frac{2\pi}{4}\cdot n} =$$

$$= X[k] = \frac{1}{4}\left(1 e^{-jk\frac{2\pi}{4}\cdot 0} + 0\cdot e^{-jk\frac{2\pi}{4}\cdot 1} - 1\cdot e^{-jk\frac{2\pi}{4}\cdot 2} + 0\cdot e^{-jk\frac{2\pi}{4}\cdot 3}\right) =$$

$$= \frac{1}{4}\left(1 - e^{-jk\pi}\right)$$

$$X[3] = \frac{1}{4}\left(1 - e^{-j3\pi}\right) = \frac{1}{2}$$

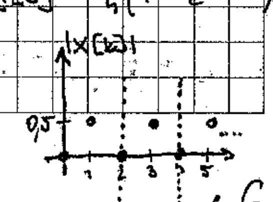
$$X[0] = \frac{1}{4}\left(1 - e^{-j0\pi}\right) = 0$$

$$X[1] = \frac{1}{4}\left(1 - e^{-j1\pi}\right) = \frac{1}{2}$$

$$X[2] = \frac{1}{4}\left(1 - e^{-j2\pi}\right) = 0$$

k	$X[k]$	$\lvert X[k]\rvert$	$\varphi[k]$
0	0	0	0
1	1/2	0,5	0
2	0	0	0
3	1/2	0,5	0
4	0		

Alkta



6

10. naloga

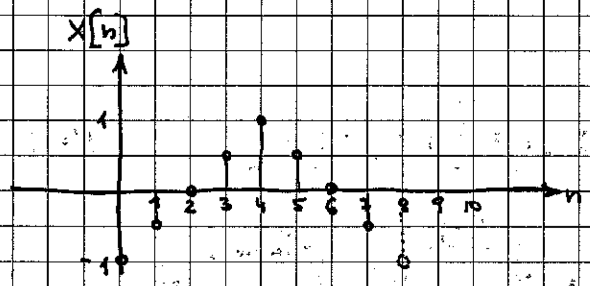
Z uporabo DFT izračunajte amplitudni spekter trikotnega periodičnega signala.

$$x(t) = \begin{cases} 1 + \frac{4}{T_p} t & ; \ 0 \leq t - kT_p < \frac{T_p}{2} \\ 3 - \frac{4}{T_p} t & ; \ \frac{T_p}{2} \leq t - kT_p < T_p \end{cases}$$

Časovno diskr. periodični signal dobite z vzorčenjem ene periode signala z vzorčevalno frekvenco $f_0 = 8/T_p$

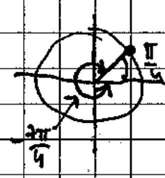
$$x[n] = x(n \cdot T_0) = \begin{cases} -1 + \frac{\cancel{4}\,2}{\cancel{T_p}} \cdot n \cdot \frac{\cancel{T_p}}{\cancel{8}\,2} & ; \ 0 \leq n \cdot \frac{\cancel{T_p}}{8} - k\cancel{T_p} < \frac{\cancel{T_p}}{2} \\ 3 - \frac{\cancel{4}}{\cancel{T_p}} \cdot n \frac{\cancel{T_p}}{\cancel{8}\,2} & ; \ \frac{\cancel{T_p}}{2} \leq n \cdot \frac{\cancel{T_p}}{8} - k\cancel{T_p} < \cancel{T_p} \end{cases}$$

$$x[n] = \begin{cases} -1 + \frac{n}{2} & ; \ 0 \leq n - k8 < 4 \\ 3 - \frac{n}{2} & ; \ 4 \leq n - k8 < 8 \end{cases}$$



$$x[n] = \{\underset{n=0}{-1}, \underset{n=1}{-0.5}, \underset{n=2}{0}, \underset{n=3}{0.5}, \underset{n=4}{1}, \underset{n=5}{0.5}, \underset{n=6}{0}, \underset{n=7}{-0.5}\}$$

$N = 8$



$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{N} \cdot n} = \frac{1}{8} \sum_{n=0}^{7} x[n] e^{-jk\frac{2\pi}{8} n} =$$

$$= \frac{1}{8}\left(-1 \cdot e^{-jk\frac{\pi}{4} 0} - 0.5 e^{-jk\frac{\pi}{4} \cdot 1} + 0.5 e^{-jk\frac{\pi}{4} \cdot 3} + 1 e^{-jk\frac{\pi}{4} 4} + 0.5 e^{-jk\frac{\pi}{4} \cdot 5} - 0.5 \cdot e^{-jk\frac{\pi}{4} \cdot 7}\right)$$

$$= \frac{1}{8}\left(-1 - 0.5 e^{-jk\frac{\pi}{4}} + 0.5 e^{-jk\frac{3\pi}{4}} + 1 e^{-jk\pi} + 0.5 e^{-jk\frac{5\pi}{4}} - 0.5 e^{-jk\frac{7\pi}{4}}\right) \quad -\frac{5\pi}{4} = \frac{3\pi}{4} \quad -\frac{7\pi}{4} = \frac{\pi}{4}$$

$$X[k] = \frac{1}{8}\left(-1 + e^{-jk\pi} + 0.5\left(-e^{-jk\frac{\pi}{4}} - e^{jk\frac{\pi}{4}} + e^{-jk\frac{3\pi}{4}} + e^{jk\frac{3\pi}{4}}\right)\right.$$

k	$X[k]$
0	0
1	$-\frac{1}{8}(2+\sqrt{2})$
2	0
3	$\frac{1}{8}(\sqrt{2}-2)$
4	0
5	$\frac{1}{8}(\sqrt{2}-2)$
6	0
7	$-\frac{1}{8}(2+\sqrt{2})$

konjugirana

konjugirana

$$X[1] = \frac{1}{8}\left(-1-1+0{,}5\left(-\sqrt{2}-\sqrt{2}\right)\right) = \frac{1}{8}\left(-2-\sqrt{2}\right)$$

$$X[2] = \frac{1}{8}\left(0+0{,}5\left(0+0\right)\right) = 0$$

$$X[3] = \frac{1}{8}\left(-1-1+0{,}5\left(\sqrt{2}+\sqrt{2}\right)\right) = \frac{1}{8}\left(-2+\sqrt{2}\right)$$

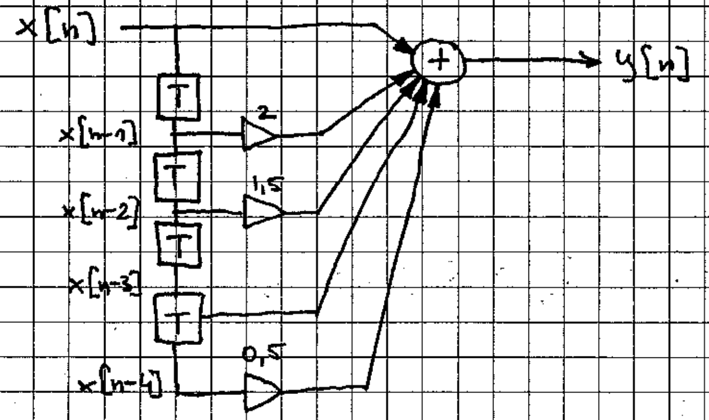
scilab: FFT([-1 -0.5 0 0.5 1 0.5 0 -0.5], -1)

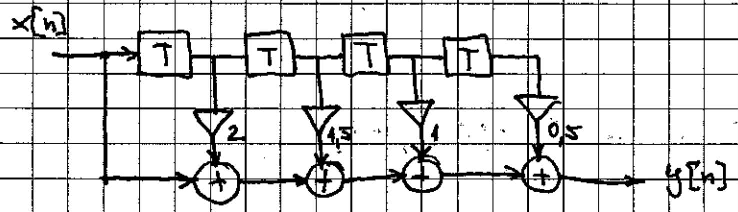
28.3.2013

13. NALOGA

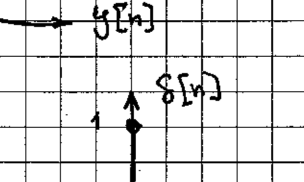
Določite impulzne odzive LTI sistemov, podanih z diferenčno enačbo in narišite njihove sheme. Za vsak sistem določite ali je rekurziven ali ne, ter ali je FIR ali IIR.

$\boxed{a)}$ $y[n] = x + 2x[n-1] + 1{,}5\,x[n-2] + x[n-3] + 0{,}5\,x[n-4]$





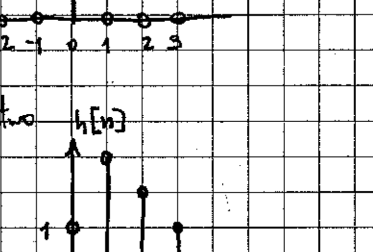
$h[n] = y[n]\Big|_{x[n]=\delta[n]}$



takoj lahko napišemo h[n] direktno

↓

koeficienti diferenčne enačbe



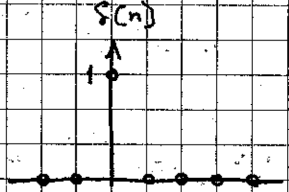
$h[n] = \{1,\ 2,\ 1.5,\ 1,\ 0.5\}$

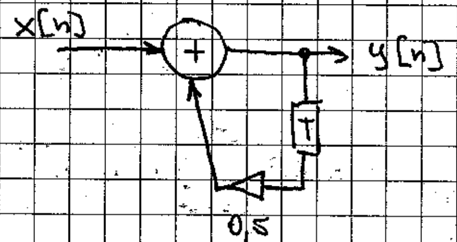
FIR FINITE }
IIR INFINITE } RESPONSE

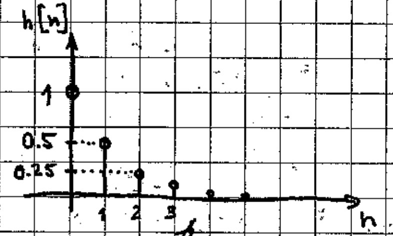
ker je končen je FIR

ni rekurziven

b.) $y[n] = x[n] + 0{,}5\, y[n-1]$







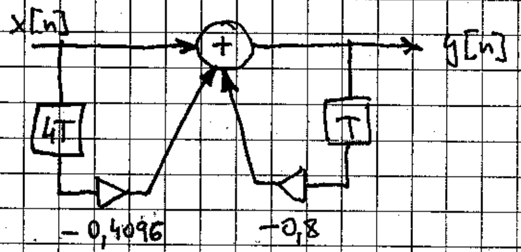
$$h[n] = \begin{cases} 0{,}5^n & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

bliža se ničli, vendar je ne doseže

IIR, ker ima neskončen h[n] odziv

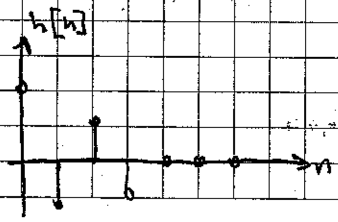
ker ima povratno vezavo je rekurziven

c.) $y[n] = x[n] - 0{,}4096\, x[n-4] - 0{,}8\, y[n-1]$



n	x[n]	y[n]
−4	0	0
−3	0	0
−2	0	0
−1	0	0
0	1	1 − 0,4 · 0 − 0,8 · 0 = 1
1	0	−0,8
2	0	+0,64
3	0	−0,512
4	0	−1 · 0,4096 + 0,8 · 0,512 = 0
5	0	0
6	0	0
		⋮

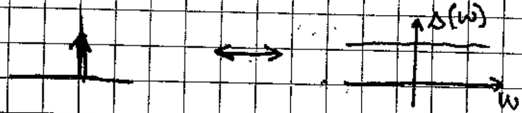
FIR SISTEM, REKURZIVEN



4.4.2013

$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$ $\qquad$ $x[n] \rightarrow \boxed{LTI} \rightarrow y[n]$

$\delta(t)$ $\qquad h(t) \leftrightarrow H(\omega)$ $\qquad \delta[n]$ $\qquad h[n] \leftrightarrow H(\Omega)$



$$H(\Omega) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\Omega n} \quad \dots \text{TDFT}$$

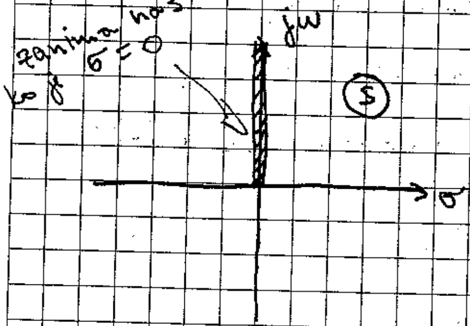
$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

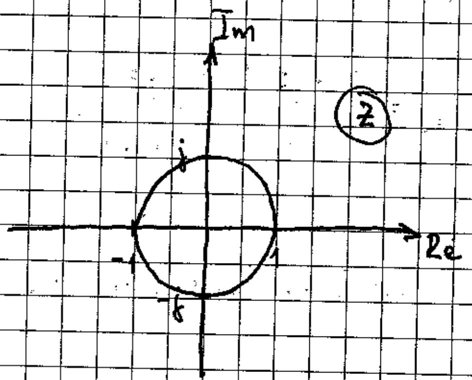
$$= \int_{-\infty}^{\infty} h(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt \qquad s = \sigma + j\omega$$

$$e^{j\Omega} = z \quad \dots z \in \mathbb{C}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt$$





$$H(\omega) = H(s)\Big|_{s=j\omega}$$

primer

$$h[n] = \begin{cases} 1 & ; \ 0 \le n < N \\ 0 & ; \ \text{sicer} \end{cases}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n} = \sum_{n=0}^{N-1} z^{-n} = \frac{1 - z^{-N}}{1 - z^{-1}} \cdot \frac{z^n}{z^n} =$$

$$= \frac{z^N - 1}{z^N - z^{N-1}} = \frac{z^n - 1}{z^{n+1}(z-1)}$$

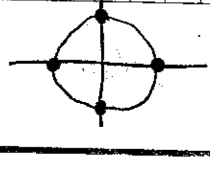
ničle: $z^n - 1 = 0$

$z^n = 1$ ... n rezultatov

$z^n = e^{j2k\pi}$

$(z^N)^{\frac{1}{N}} = (e^{j2k\pi})^{\frac{1}{N}}$

$z = e^{j\frac{2k\pi}{N}}$

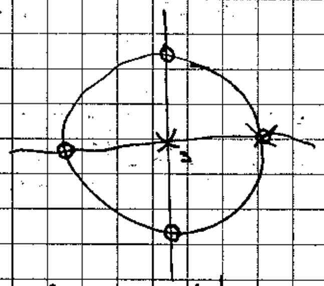


$n_k = e^{j\frac{2k\pi}{N}}$ ... to so ničle   če je $N=4$ $\rightarrow$ $n_0 = 1$ $n_1 = j$

$n_2 = -1$ $n_3 = -j$

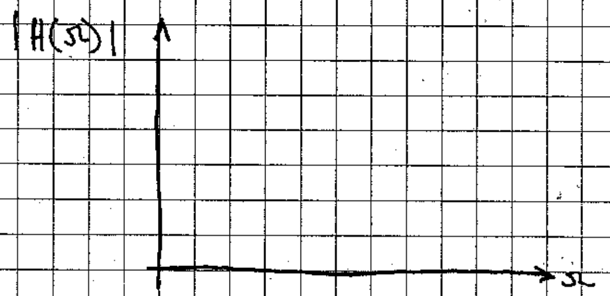
poli: $p_1 = 1$

$p_{...} = 0$



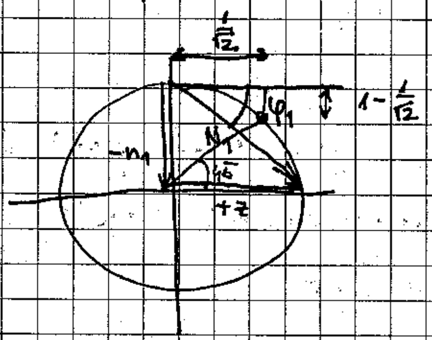
če sta ničla in pol na istem mestu

$$\frac{\cancel{(z-1)}(z-j)(z+1)(z+j)}{z^3\cancel{(z-1)}}$$ se pokrajšata



$$H(z) = \frac{(z-n_1)(z-n_2)(z-n_3)\cdots}{(z-p_1)(z-p_2)(z-p_3)\cdots} =$$

$$= \frac{N_1 e^{j\varphi_1} \; N_2 e^{j\varphi_2} \; N_3 e^{j\varphi_3}}{P_1 e^{j\varphi_{p1}} \; P_2 e^{j\varphi_{p2}} \; P_3 e^{j\varphi_{p3}}}$$



$$H(z) = \frac{N_1 N_2 N_3}{P_1 P_2 P_3} e^{j(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_{p1} - \varphi_{p2} - \varphi_{p3})}$$

$$|H(z)| = \frac{N_1 N_2 N_3}{P_1 P_2 P_3}$$

$$|H(0)| = \frac{\sqrt{2} \cdot 2 \cdot \sqrt{2}}{1 \cdot 1 \cdot 1} = 4$$

$$|H(\tfrac{\pi}{2})| = \frac{0 \cdot \cdots}{1 \cdot 1 \cdot 1} = 0$$

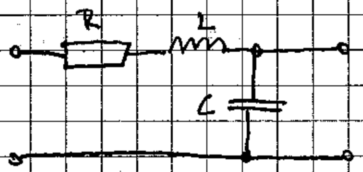
$$|H(\pi)| = \frac{0 \cdot \cdots}{1 \cdot 1 \cdot 1} = 0$$

$$\varphi(0) = -\frac{\pi}{4} + 0 + \frac{\pi}{4} - (3 \cdot 0) = 0$$

$$\varphi(\tfrac{\pi}{4}) = \operatorname{atan}\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} + \operatorname{atan}\frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} + \operatorname{atan}\frac{1+\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} - (3 \cdot \tfrac{\pi}{4}) =$$

Akta

primer za $H(s)$

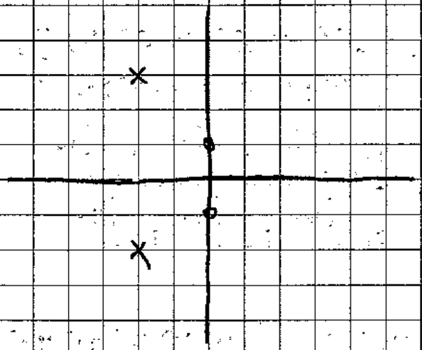


$$H(s) = \frac{X_C}{X_C + X_L + R} = \frac{\frac{1}{sC}}{\frac{1}{sC} + sL + R} = \frac{1}{1 + s^2LC + sRC} =$$

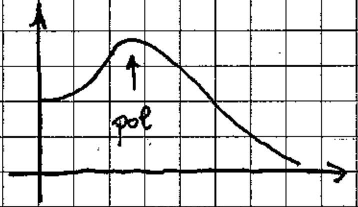
$$= \frac{1}{LC\left(s^2 + s\frac{R}{L} + \frac{1}{LC}\right)}$$

$$p_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

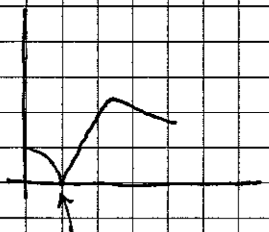
$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4\frac{1}{LC}}}{2}$$



$$|H(s)| = \frac{N_1 \cdot N_2}{p_1 \cdot p_2}$$



Če mamo ničle



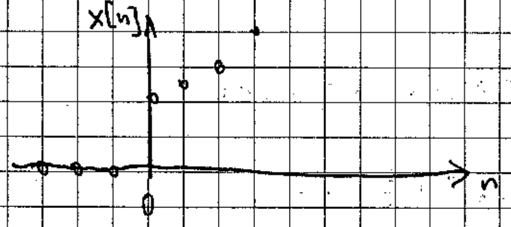
zaradi nič
pade na ničlo

če je polov več, gre odziv proti nič
če je ničel in polov enako, je odziv neka konstantna vrednost

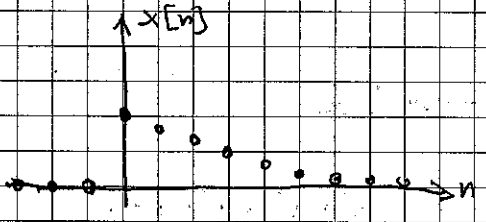
11.4.2013

15. naloga

a) $a > 1$ $\quad x[n] = a^n u[n]$



$a < 1$



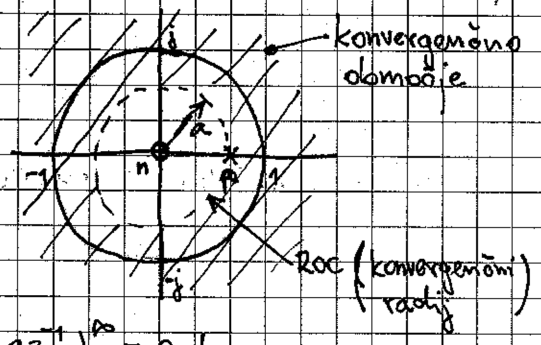
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n} = \sum_{n=0}^{\infty} a^n z^{-n} =$$

$$= \frac{1-(az^{-1})^{\infty}}{1-az^{-1}} = \frac{1}{1-az^{-1}} \Big/ \cdot z$$

$$= \frac{z}{z-a}$$

ničla: $n = 0$

pol: $p = a$



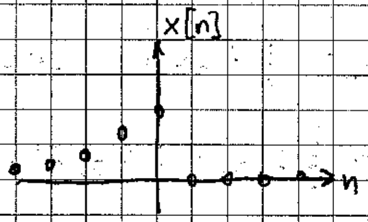
$(az^{-1})^{\infty} = 0 \;\Big|\; |az^{-1}| < 1$

$\frac{a}{|z|} < 1 \qquad a < |z|$

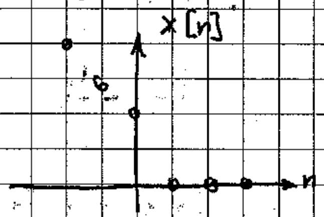
$|z| > a$

b) $x[n] = a^n u[-n]$

$a > 1$



$a < 1$



$$X(z) = \sum_{n=-\infty}^{0} a^n \cdot z^{-n}$$

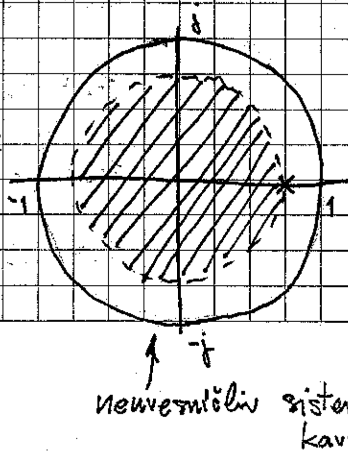
nova spr: $m = -n$

$$= \sum_{m=\infty}^{0} a^{-m} \cdot z^{m} = \sum_{m=0}^{\infty} a^{-m} z^{m}$$

$$= \frac{1-(a^{-1}z)^{\infty}}{1-a^{-1}z} = \frac{1}{1-a^{-1}z} \Big/ \cdot a = \frac{a}{a-z}$$

ničel ni ; pol je: $p = a$

$|a^{-1}z| < 1 \qquad \frac{|z|}{a} < 1 \qquad |z| < a$



Če so vsi poli znotraj enotske krožnice je sistem stabilen.

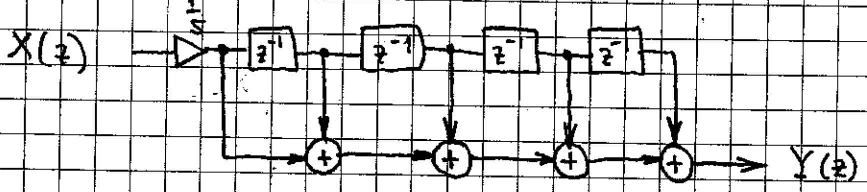
nemogoč sistem... kavzalnost

16. naloga

$$h[n] = \left\{ \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5} \right\}$$

$$y[n] = \frac{1}{5}x[n] + \frac{1}{5}x[n-1] + \frac{1}{5}x[n-2] + \frac{1}{5}x[n-3] + \frac{1}{5}x[n-4]$$

$$Y(z) = \frac{1}{5}X(z) + \frac{1}{5}X(z)z^{-1} + \frac{1}{5}X(z)z^{-2} + \frac{1}{5}X(z)z^{-3} + \frac{1}{5}X(z)z^{-4}$$



$$\frac{Y(z)}{X(z)} = \frac{1}{5} \cdot \frac{1 + z^{-1} + z^{-2} + z^{-3} + z^{-4}}{1} \cdot \frac{z^4}{z^4} = \frac{1}{5} \cdot \frac{z^4 + z^3 + z^2 + z + 1}{z^4}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} = \sum_{n=0}^{4} \frac{1}{5} z^{-n} = \frac{1}{5} \sum_{n=0}^{4} (z^{-1})^n = \frac{1}{5} \cdot \frac{1-(z^{-1})^5}{1-z^{-1}} = \frac{1}{5} \cdot \frac{1-z^{-5}}{1-z^{-1}} \cdot \frac{z^5}{z^5}$$

$$= \frac{z^5 - 1}{z^5 - z^4} = \frac{z^5 - 1}{z^4(z-1)}$$

poljubno celo število

$$z^5 - 1 = 0 \rightarrow z^5 = 1 = e^{j2\pi \cdot k} \quad /^{\frac{1}{5}}$$

$$z = e^{j\frac{2\pi k}{5}}$$

poli:

$p_1 = 1$

$p_{2,3,4,5} = 0$

$$z_0 = 1$$

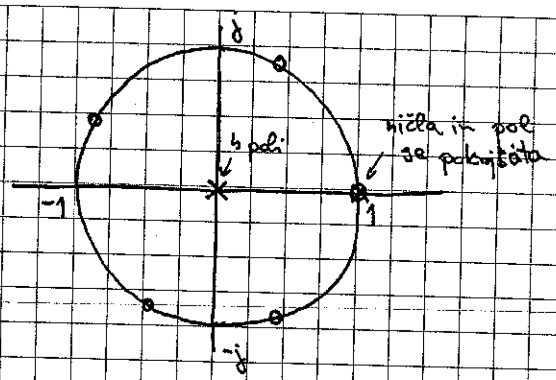
$$z_1 = e^{j\frac{2\pi}{5}}$$

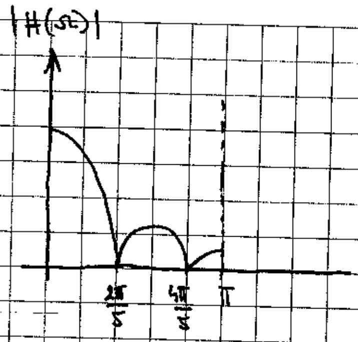
$$z_2 = e^{j\frac{4\pi}{5}}$$

$$z_3 = e^{j\frac{6\pi}{5}} = e^{-j\frac{4\pi}{5}}$$

$$z_4 = e^{j\frac{8\pi}{5}} = e^{-j\frac{2\pi}{5}}$$

ničle ... TUDI V KOLOKVIJU





$$\frac{Y(z)}{X(z)} = \frac{1}{5}\,\frac{1-z^{-5}}{1-z^{-1}}$$

$$Y(z)\left(1-z^{-1}\right) = \frac{1}{5}X(z)\left(1-z^{-5}\right)$$

$$Y(z) - Y(z)\,z^{-1} = \frac{1}{5}X(z) - \frac{1}{5}X(z)\,z^{-5}$$

$$Y(z) = \frac{1}{5}X(z) - \frac{1}{5}X(z)\,z^{-5} + Y(z)\,z^{-1}$$