OSNOVE ROBOTIKE
zapiski predavanj

Šolsko leto	2008 / 2009
Izvajalec	Tadej Bajd
Avtor dokumenta	Robert Mesarič

UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	01
DATUM	13. 7. 2009		

ZADNJI POPRAVLJAL
PREGLEDAL

OPOMBE

POPRAVKI

www.stromar.si
zbirka študijske literature na spletu

v dokumentu lahko obstajajo napake

HOMOGENE TRANSFORMACIJE

2.3.2009

→ Cilji:

– lega (pose) $\underline{H}$ 4×4 → pozicija, → orientacija

matrike

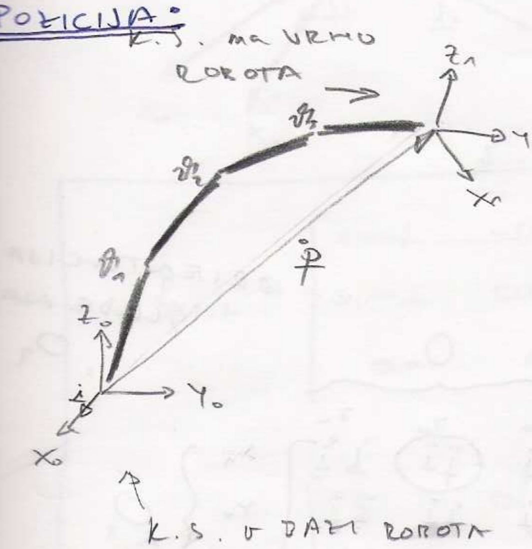
– orientacija $\underline{R}$ 3×3

– premik → translacija, → rotacija

– pozicija $\underline{p}$

vektor

→ POZICIJA:



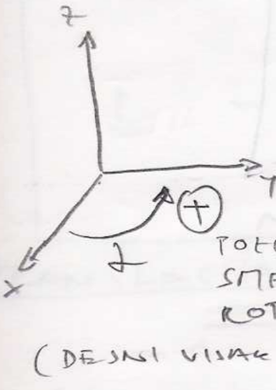
$$^0\underline{p} = {}^0p_x \cdot \underline{i} + {}^0p_y \underline{j} + {}^0p_z \underline{k} =$$

$$= \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix} = \begin{bmatrix} {}^0p_x & {}^0p_y & {}^0p_z \end{bmatrix}^T =$$

$$= \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \\ 1 \end{bmatrix}$$

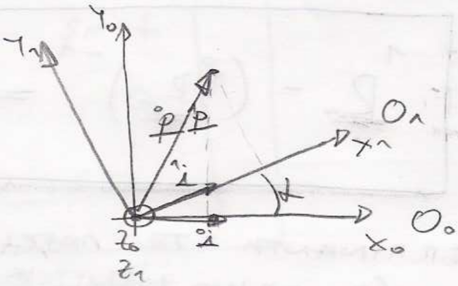
→ ORIENTACIJA:

smer rotacije:



POZITIVNA SMER ROTACIJE

(DESNI VIJAK/ROKA)



$$O_1: {}^1\underline{p} = {}^1p_x {}^1\underline{i} + {}^1p_y {}^1\underline{j} + {}^1p_z {}^1\underline{k}$$

$$O_0: {}^0\underline{p} = {}^0p_x {}^0\underline{i} + {}^0p_y {}^0\underline{j} + {}^0p_z {}^0\underline{k} \quad \Big|\cdot {}^0\underline{i} \quad \Big|\cdot {}^0\underline{j} \quad \Big|\cdot {}^0\underline{k}$$

$$\underline{{}^0\underline{p}\,{}^0\underline{i}} = {}^0p_x = \underline{{}^1\underline{p}\,{}^0\underline{i}} = {}^1p_x {}^1\underline{i}\,{}^0\underline{i} + {}^1p_y {}^1\underline{j}\,{}^0\underline{i} + {}^1p_z {}^1\underline{k}\,{}^0\underline{i}$$

$$\underline{{}^0\underline{p}\,{}^0\underline{j}} = {}^0p_y = \underline{{}^1\underline{p}\,{}^0\underline{j}} = -\|- {}^0\underline{j} + -\|- {}^0\underline{j} + -\|- {}^0\underline{j}$$

$$\underline{{}^0\underline{p}\,{}^0\underline{k}} = {}^0p_z = \underline{{}^1\underline{p}\,{}^0\underline{k}} = -\|- {}^0\underline{k} + -\|- {}^0\underline{k} + -\|- {}^0\underline{k}$$

$$\begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix} = {}^0\underline{R}_1 \begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix}$$

$$
{}^0\underline{\underline{R}}_1 = \begin{array}{c} \overbrace{\begin{array}{ccc} x_1 & y_1 & z_1 \end{array}}^{O_1} \\ \begin{bmatrix} {}^1\underline{i}\cdot{}^0\underline{i} & {}^1\underline{j}\cdot{}^0\underline{i} & {}^1\underline{k}\cdot{}^0\underline{i} \\ {}^1\underline{i}\cdot{}^0\underline{j} & {}^1\underline{j}\cdot{}^0\underline{j} & {}^1\underline{k}\cdot{}^0\underline{j} \\ {}^1\underline{i}\cdot{}^0\underline{k} & {}^1\underline{j}\cdot{}^0\underline{k} & {}^1\underline{k}\cdot{}^0\underline{k} \end{bmatrix} \end{array} \begin{array}{c} x_0 \\ y_0 \\ z_0 \end{array} \Bigg\} O_0
$$

ORIENTACIJA SLEDI O_1 NA O_0

$$
{}^0\underline{\underline{R}}_1 = \begin{bmatrix} \cos\vartheta_{ii} & & \\ & \ddots & \\ & & \cos\vartheta_{kk} \end{bmatrix}
$$

→ DRUGA SITUACIJA:

$$
O_1: {}^1\underline{p} = {}^1p_x\,{}^1\underline{i} + {}^1p_y\,{}^1\underline{j} + {}^1p_z\,{}^1\underline{k} \qquad |\cdot{}^1\underline{i} \quad |\cdot{}^1\underline{j} \quad |\cdot{}^1\underline{k}
$$

$$
{}^1\underline{p}\,{}^1\underline{i} = {}^1p_x = {}^0\underline{p}\,{}^1\underline{i}
$$

$$
{}^1\underline{p}\,{}^1\underline{j} = {}^1p_y = {}^0\underline{p}\,{}^1\underline{j}
$$

$$
{}^1\underline{p}\,{}^1\underline{k} = {}^1p_z = {}^0\underline{p}\,{}^1\underline{k}
$$

ORIENTACIJA O_0 SLEDI NA O_1

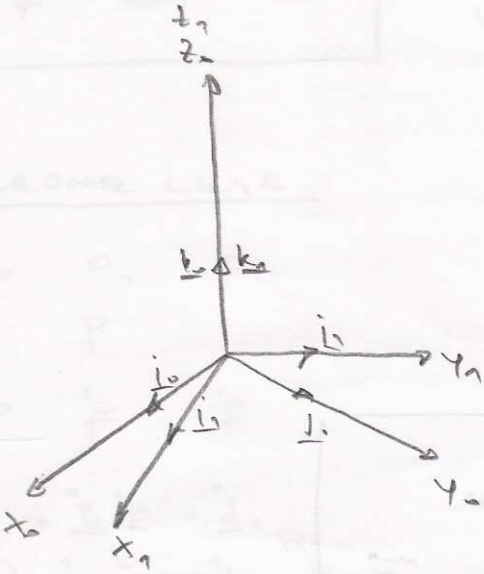
$$
\begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix} = {}^1\underline{\underline{R}}_0 \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix}
\qquad
{}^1\underline{\underline{R}}_0 = \begin{array}{c} \overbrace{\begin{array}{ccc} x_0 & y_0 & z_0 \end{array}}^{O_0} \\ \begin{bmatrix} {}^0\underline{i}\cdot{}^1\underline{i} & {}^0\underline{j}\cdot{}^1\underline{i} & {}^0\underline{k}\cdot{}^1\underline{i} \\ {}^0\underline{i}\cdot{}^1\underline{j} & {}^0\underline{j}\cdot{}^1\underline{j} & {}^0\underline{k}\cdot{}^1\underline{j} \\ {}^0\underline{i}\cdot{}^1\underline{k} & {}^0\underline{j}\cdot{}^1\underline{k} & {}^0\underline{k}\cdot{}^1\underline{k} \end{bmatrix} \end{array} \begin{array}{c} x_1 \\ y_1 \\ z_1 \end{array} \Bigg\} O_1
$$

$${}^1\underline{i}\cdot{}^0\underline{j} = {}^0\underline{j}\cdot{}^1\underline{i}$$ → član nad diagonalo

$$
\boxed{{}^1\underline{\underline{R}}_0 = \left({}^0\underline{\underline{R}}_1\right)^{-1} = \left({}^0\underline{\underline{R}}_1\right)^T}
$$

DETERMINANTA TE MATRIKE JE +1
(ZA DESNO SUČNI K.S)

↳ ROTACIJA OKROG OSI Z:



$\hat{k}\hat{k} = 1$

$\hat{i}\hat{i} = \cos\alpha$

$\hat{j}\hat{j} = \cos\alpha$

$\hat{i}\hat{j} = \quad = -\sin\alpha$

$\hat{j}\hat{i} = \quad = \sin\alpha$

ostali so $\emptyset$

$$\underline{R}_{z,\alpha} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{R}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\underline{R}_{y,\alpha} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

enica je v diagonali in v tistem stolpcu in vrstici okoli katere vrtimo

↳ TRANSLACIJA VZDOLŽ OSI Z:

$\hat{i}\hat{i} = 1$

$\hat{j}\hat{j} = 1$

$\hat{k}\hat{k} = 1$

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\underline{R}_{x,\alpha_1} \cdot \underline{R}_{x,\alpha_2} = \underline{R}_{x(\alpha_1+\alpha_2)}}$$

$$\boxed{\underline{R}^{-1}_{x,\alpha} = \underline{R}_{x,-\alpha}}$$

↳ ZAPOREDNE ROTACIJE:

Izhodišče vseh k.s. v isti točki.

$O_0 \quad O_1 \quad O_2$

P – ena točka

${}^0\underline{p} \quad {}^1\underline{p} \quad {}^2\underline{p}$

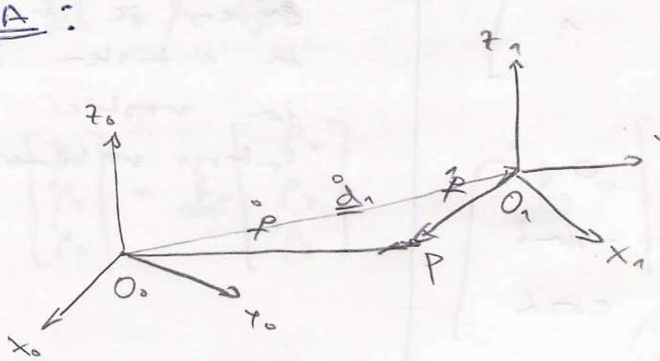
$${}^0\underline{p} = {}^0\underline{\underline{R}}_1 \, {}^1\underline{p}$$

$${}^1\underline{p} = {}^1\underline{\underline{R}}_2 \, {}^2\underline{p}$$

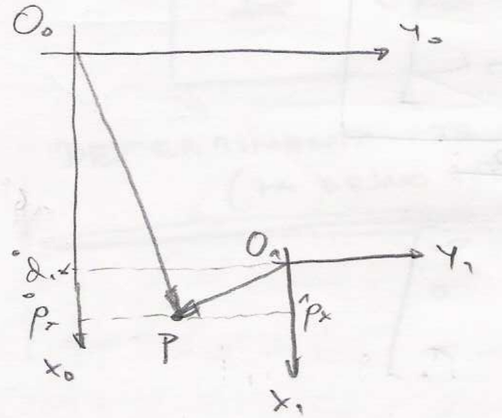
$$\boxed{\begin{aligned} {}^0\underline{\underline{R}}_2 &= {}^0\underline{\underline{R}}_1 \, {}^1\underline{\underline{R}}_2 \\ {}^0\underline{\underline{R}}_n &= {}^0\underline{\underline{R}}_1 \, {}^1\underline{\underline{R}}_2 \cdots {}^{n-1}\underline{\underline{R}}_n \end{aligned}}$$

$${}^0\underline{p} = {}^0\underline{\underline{R}}_1 \, {}^1\underline{\underline{R}}_2 \, {}^2\underline{p}$$

↳ LEGA:



→ PRVA APROKSIMACIJA:



$${}^0p_x = {}^0d_{1x} + {}^1p_x$$

$${}^0\underline{p} = {}^0\underline{d}_1 + {}^1\underline{p}$$

TO VELJA TUDI VEKTORSKO (ZA VSE KOMPONENTE)

$$\boxed{{}^{0}\underline{p} = {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{p} + {}^{0}\underline{d}_1}$$

ENAČBA LEGE

→ ZAPOREDNE LEGE:

$O_0 \quad O_1 \quad O_2$

P

${}^{0}\underline{p} \quad {}^{1}\underline{p} \quad {}^{2}\underline{p}$

$${}^{0}\underline{p} = {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{p} + {}^{0}\underline{d}_1$$

$${}^{1}\underline{p} = {}^{1}\underline{\underline{R}}_2 {}^{2}\underline{p} + {}^{1}\underline{d}_2$$

$${}^{0}\underline{p} = {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{\underline{R}}_2 {}^{2}\underline{p} + \underbrace{{}^{0}\underline{\underline{R}}_1 {}^{1}\underline{d}_2 + {}^{0}\underline{d}_1}$$

$${}^{0}\underline{p} = {}^{0}\underline{\underline{R}}_2 {}^{2}\underline{p} + {}^{0}\underline{d}_2$$

$$\boxed{\begin{aligned} {}^{0}\underline{\underline{R}}_2 &= {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{\underline{R}}_2 \\ {}^{0}\underline{d}_2 &= {}^{0}\underline{d}_1 + {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{d}_2 \end{aligned}}$$

ti dve enačbi hočemo nadomestiti s homogeno transformacijo

$${}^{0}\underline{p} = \underline{\underline{R}}\, {}^{1}\underline{p} + \underline{d}$$

→ HOMOGENE MATRIKE:

$$\begin{bmatrix} {}^{0}\underline{p} \\ 1 \end{bmatrix} = \underline{\underline{H}} \begin{bmatrix} {}^{1}\underline{p} \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^{0}\underline{p} \\ 1 \end{bmatrix} = \begin{bmatrix} \underline{\underline{R}} & \underline{d} \\ \underline{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{1}\underline{p} \\ 1 \end{bmatrix}$$

ZAPOREDNE LEGE:

$$\boxed{\begin{bmatrix} {}^{0}\underline{\underline{R}}_1 & {}^{0}\underline{d}_1 \\ \underline{0} & 1 \end{bmatrix} \begin{bmatrix} {}^{1}\underline{\underline{R}}_2 & {}^{1}\underline{d}_2 \\ \underline{0} & 1 \end{bmatrix} = \begin{bmatrix} {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{\underline{R}}_2 & {}^{0}\underline{\underline{R}}_1 {}^{1}\underline{d}_2 + {}^{0}\underline{d}_1 \\ \underline{0} & 1 \end{bmatrix}}$$

dokaz, da taka matrika deluje

$${}^{0}\underline{\underline{H}}_1 {}^{1}\underline{\underline{H}}_2 = {}^{0}\underline{\underline{H}}_2$$

$$\boxed{{}^{0}\underline{\underline{H}}_n = {}^{0}\underline{\underline{H}}_1 {}^{1}\underline{\underline{H}}_2 \dots {}^{n-1}\underline{\underline{H}}_n}$$

inverzna matrika:

$$^0p = R\,^1p + d \quad \Big| \cdot R^T$$

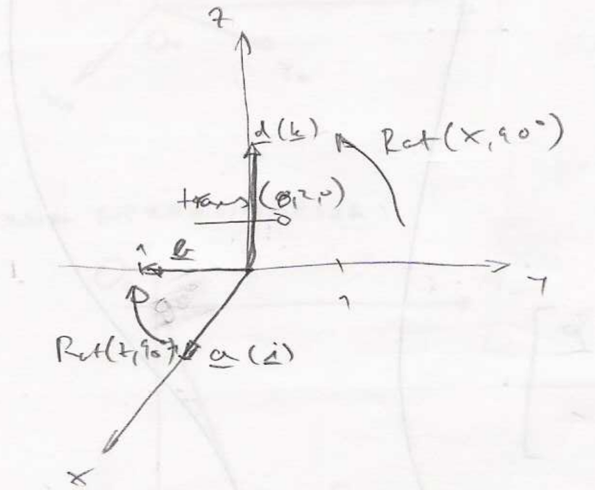
premultiplikacija (množenje z leve)

$$R^T\,^0p = R^T R\,^1p + R^T d$$

$$^1p = R^T\,^0p - R^T d$$

$$\boxed{H^{-1} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}}$$

Premakni enotski vektor i za 90° okoli osi z v smeri urinega kazalca. Nato za +2 translacijsko v smeri osi y in nazadnje za 90° zavrti okrog osi x v nasprotni smeri urinega kazalca.



$b = Rot(z,-90)\,a$

$c = Trans(0,2,0)\,b$

$d = Rot(x,90)\,c$

$d = Rot(x,90)\,Trans(0,2,0)\,Rot(z,-90)\,a =$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} Rot(z,-90)\,a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = k$$

rotacijski vektor → vedno ena enka ali – v vsakem stolpcu in vsaki vrstic

POMEN NAŠE MATRIKE:

$$\begin{bmatrix} o & o & o & x \\ o & o & o & x \\ o & o & o & x \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

LEGA — o ORIENTACIJA, x POZICIJA

PREMIK — o ROTACIJA, x TRANSLACIJA

LEGA:

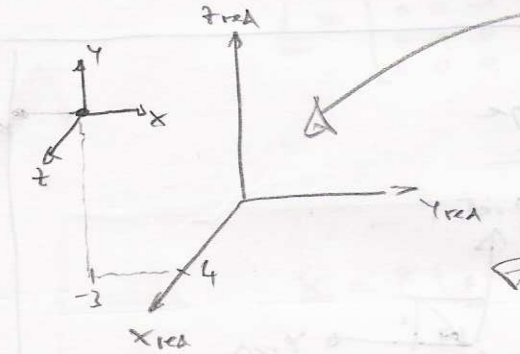
Premik glede na relativni k.s.

$\underline{\underline{H}} = Trans(4,-3,7)Rot(Y,90)Rot(Z,90) =$

Premik glede na referenčni k.s.

$$= \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot Rot(Z,90) =$$

$$= \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} X_{REF} \\ Y_{REF} \\ Z_{REF} \\ \\ \end{array}$$



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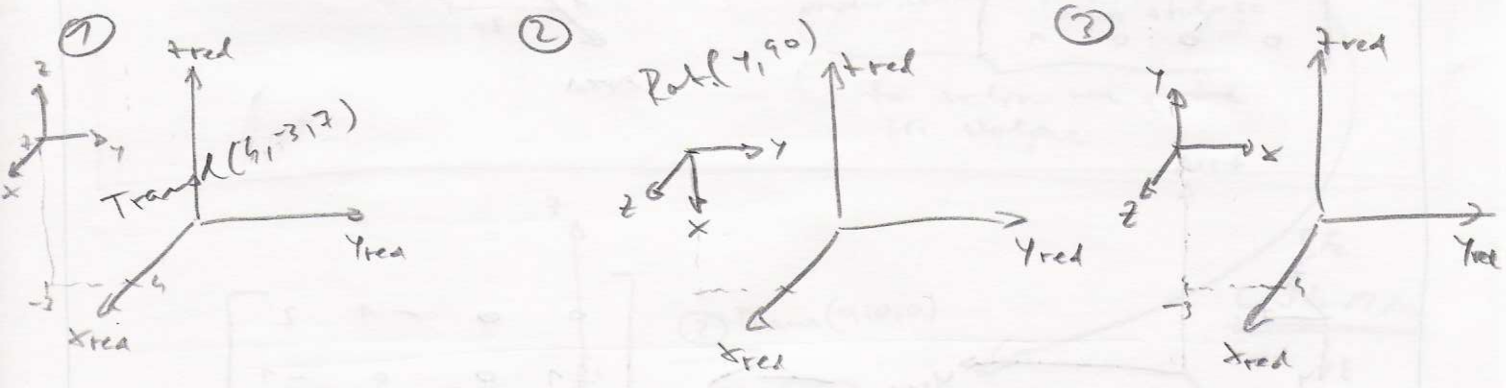
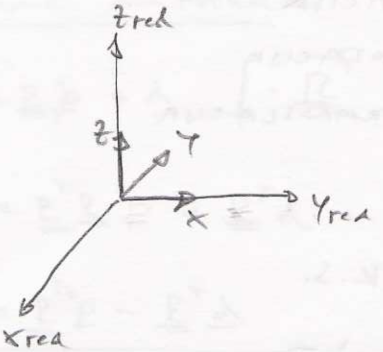


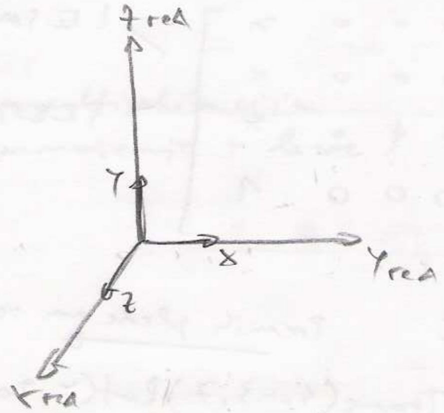
TO JE SAMO ZA ILUSTRACIJO

Premik glede na REL. k.s.: Vedno obračam prejšnji k.s.

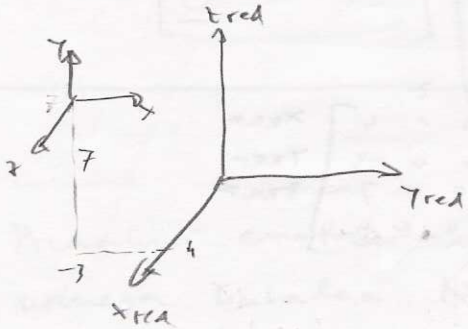
① Rot(z, 90)



② Rot(yred, 90)

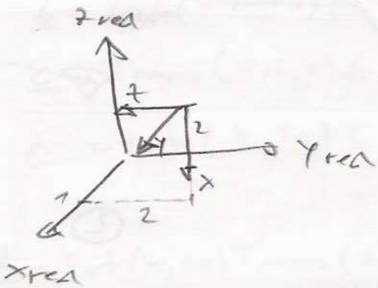


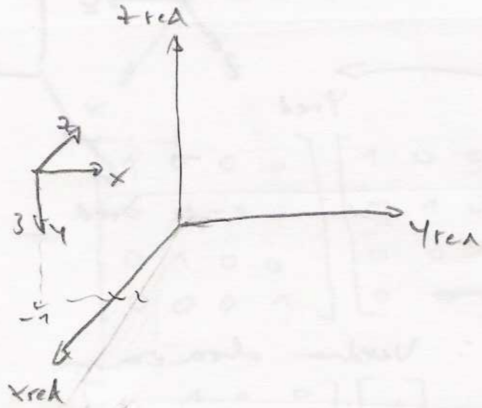
③ Trans(4, -3, 7)



"take home" message

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$





$$\Longrightarrow \begin{bmatrix} 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9.3.2009

PREMIK:

$$\begin{bmatrix} \underline{P} \\ \text{premik} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

① premultiplikacija: množenje z leve

$$\begin{bmatrix} \underline{P} \end{bmatrix} \begin{bmatrix} \underline{L} \end{bmatrix}$$

② postmultiplikacija: množenje z desne

$$\begin{bmatrix} \underline{L} \end{bmatrix} \begin{bmatrix} \underline{P} \end{bmatrix}$$

→ PREMULTIPLIKACIJA - premik glede na REFERENČNI k.s.

$$\underline{L} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cccc|c} x' & y' & z' & & \\ \end{array}$$

$$\underline{L} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} x \\ y \\ z \\ \\ \end{array}$$

$$\underline{P} = \overset{②}{\text{Trans}(0,20,0)} \overset{①}{\text{Rot}(z,90)} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

! OBRATNI VRSTNI RED

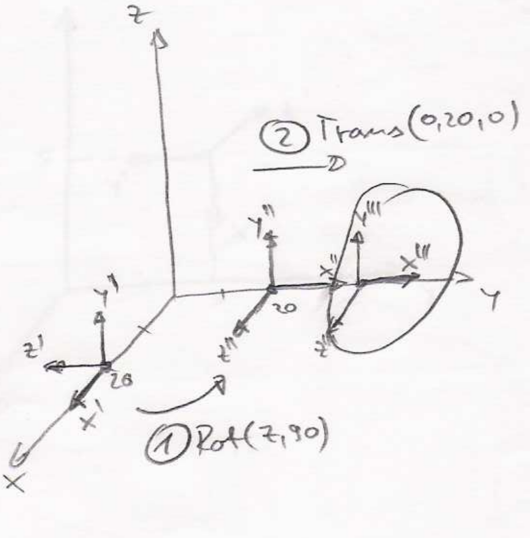
$$\underline{X} = \underline{P}\,\underline{L} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

enka na prvem mestu

prepišem prvi stolpec

to velja za prve tri stolpce

SE UJEMA



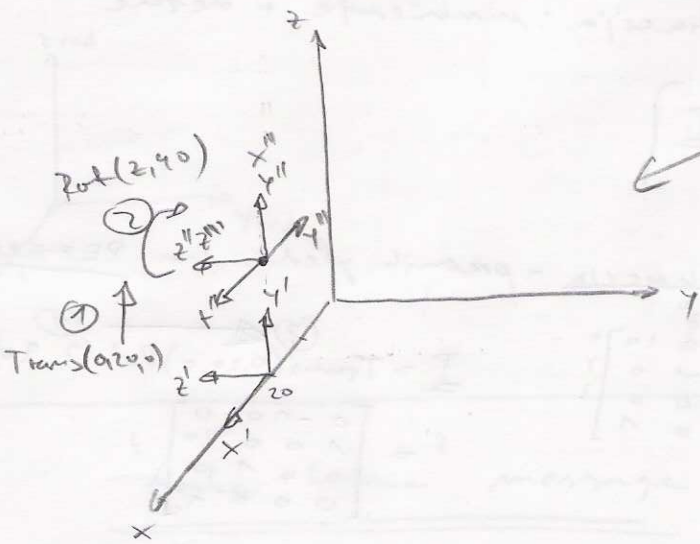
→ **POSTMULTIPLIKACIJA** glede na **RELATIVNI K.S**
(tisti ta zadnji)

$$\underline{\underline{Y}} = \underline{\underline{L}} \cdot \underline{\underline{P}} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{P}} = \underbrace{Trans(0,20,0)}_{1}\ \underbrace{Rot(z,90)}_{2} \longrightarrow$$

! PRAVILNI VRSTNI RED

TO SE UJEMA

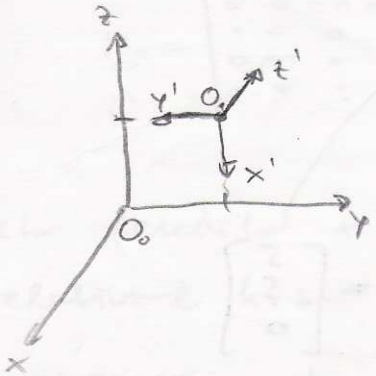


1. NALOGA ZA IZPIT:

a) zač. L, P ? kL
zL, kL ? P
kL, P ? zL

rel. ali rel. k.s

6 TIPOV NALOG



a) Zasuči O_1 za $\frac{\pi}{2}$ v smeri urinega kazalca okrog osi z

b) Opravi translacijo za 3 v negativni Y smeri.

c) Opravi rotacijo za $\frac{\pi}{2}$ v obratni smeri urinega kazalca okrog osi x.

Vsi premiki naj bojo v referenčnem k.s. Zapiši matrike.

1. primer:

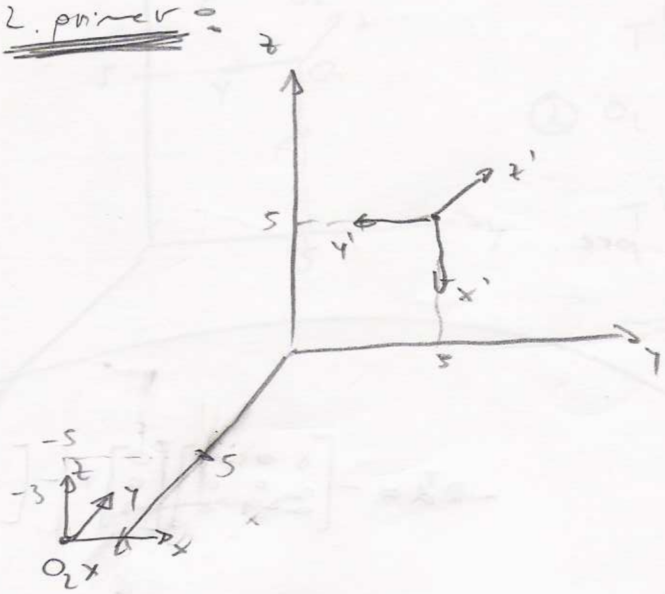
$$\underline{H}_1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{T} = Rot(x, +\tfrac{\pi}{2})\, Trans(0,-3,0)\, Rot(z, -\tfrac{\pi}{2}) =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} Rot(z, -\tfrac{\pi}{2}) =$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{H}_2 = \underline{T} \cdot \underline{H}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. primer:



↳ Reševanje naloge:

→ H_1 in H_2 odčitamo

→ ker je ref.k.o., pomeni da je premultipl.

$\underline{H}_2 = \underline{T} \cdot \underline{H}_1 \;\big|\; \underline{H}_1^{-1}$ post.

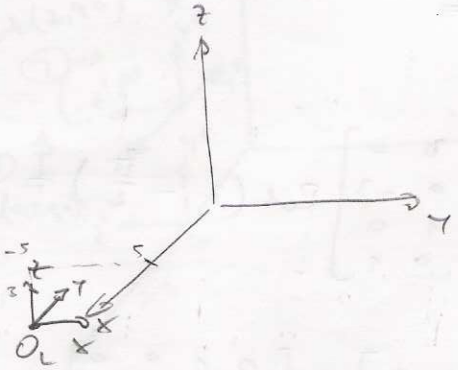
$\underline{T} = \underline{H}_2 \underline{H}_1^{-1}$

$$\underline{\underline{H}}_1^{-1} = \begin{bmatrix} 0 & 0 & -1 & 5 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-R^T d = -\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix} = -\begin{bmatrix} -5 \\ -5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 & 5 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

3. primer:



$$T = Rot(x, \tfrac{\pi}{2})\, Trans(0, -3, 0)\, Rot(z, -\tfrac{\pi}{2})$$

→ Reševanje:

$\underline{\underline{H}}_2$ odčitam s slike

$$\underline{\underline{T}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

računam kot v prvem primeru

$$\underline{\underline{H}}_2 = \underline{\underline{T}}\,\underline{\underline{H}}_1 \;/\; \underline{\underline{T}}^{-1} \text{ pre.}$$

$$\underline{\underline{H}}_1 = \underline{\underline{T}}^{-1}\underline{\underline{H}}_2$$

$$\underline{\underline{T}}^{-1} = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-R^T d = -\begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix} = -\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{H_1} = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & \\ -1 & 0 & 0 & \\ 0 & 0 & 0 & \end{bmatrix} \}$$

3 Isto besedilo + Premiki se odvijajo glede na relativni k.s.

1. primer: H_1 odčitaš iz slike/besedila

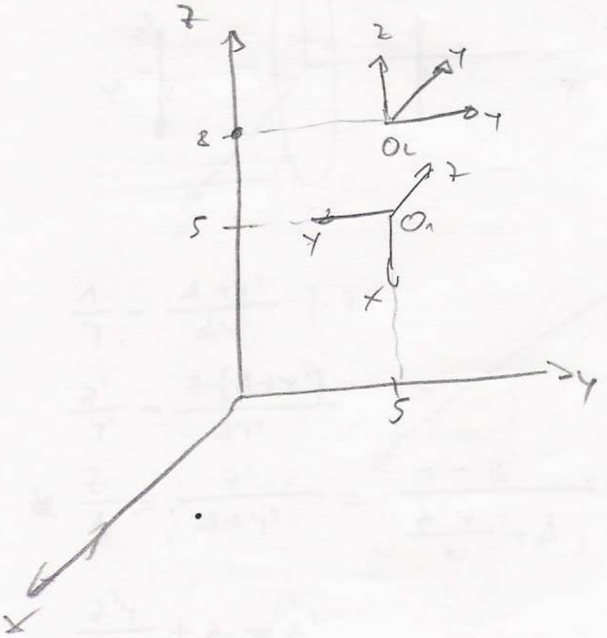
$$\underline{T}' = Rot(z, \tfrac{\pi}{2})\, Trans(0,-3,0)\, Rot(x, \tfrac{\pi}{2}) =$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} Rot(x, \tfrac{\pi}{2}) =$$

$$= \begin{bmatrix} 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{H_2} = \underline{H_1}\, T' = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. primer:



(1) $\underline{H_1}$ in $\underline{H_2}$ iz slike

$\underline{H_2} = \underline{H_1} \cdot \underline{T}' \;/\; \underline{H_1}^{-1}$ pre

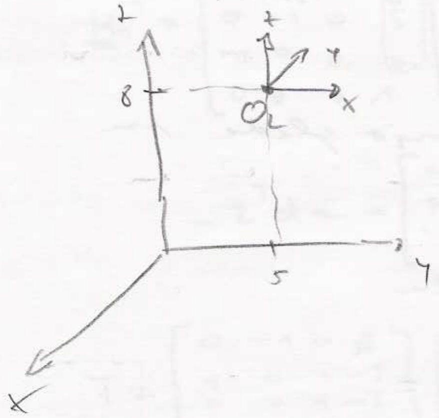
$T' = \underline{H_1}^{-1}\, \underline{H_2}$

(2) O_2 glede na O_1

$$T' = \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ne rabiš računat (odčitaš iz slike, tako da odmisliš ref. k.s. in gledaš O_2 glede na O_1)

3. primer :



$$T' = Rot(z, -\tfrac{\pi}{2})\, Trans(0,-3,0)\, Rot(x, \tfrac{\pi}{2})$$

$$= \begin{bmatrix} 0 & 0 & -1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{H}_2 = \underline{H}_1 \underline{T}' \quad \Big| \cdot (\underline{T}')^{-1} \text{ post}$$

$$\underline{H}_1 = \underline{H}_2 (\underline{T}')^{-1}$$

$$T'^{-1} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$-R^T \cdot d = -\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}$$

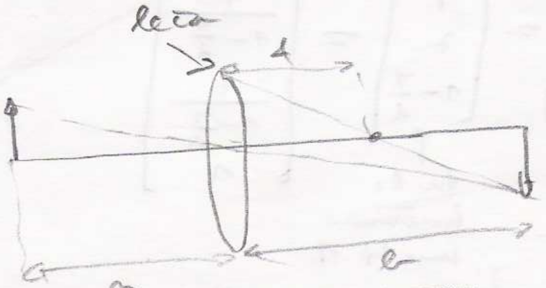
$$\underline{H}_1 = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 5 \\ -1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

PREDAVANJE – HAPTIČNI ROBOTI

- prenašanje informacije – ROBOT – ČLOVEK – v obliki sile, vibracij, gibanja
- človek in robot sta v direktnem kontaktu
- navidezna resničnost

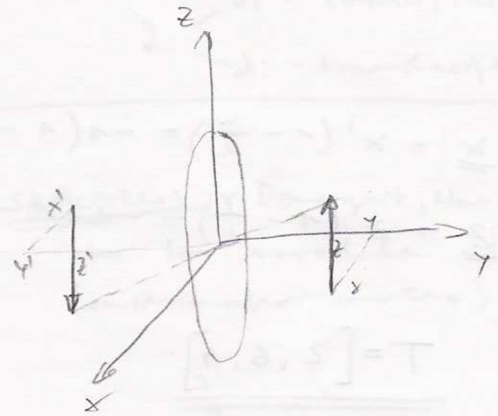
PERSPEKTIVNE TRANSFORMACIJE

16.3.2009



$$\boxed{\frac{1}{a} + \frac{1}{b} = \frac{1}{\lambda}}$$

LEČA v [YZ]:



$$\boxed{\begin{aligned} \frac{1}{Y} - \frac{1}{Y'} &= \frac{1}{\lambda} \\ \frac{Z}{Y} &= \frac{Z'}{Y'} \end{aligned}}$$

$$Y' = \frac{Z'}{Z} Y$$

$$\frac{1}{Y} = \frac{\lambda + Y'}{\lambda Y'} \quad / \cdot Z$$

$$\frac{Z'}{Y'} = \frac{Z(\lambda + Y')}{\lambda Y'}$$

$$* \quad \frac{Z}{\lambda} = \frac{Z'}{\lambda + Y'} = \frac{Z'}{\frac{Z'Y}{Z} + \lambda}$$

$$\frac{Z'Y}{\lambda} + Z = Z'$$

$$Z'\left(1 - \frac{Y}{\lambda}\right) = Z$$

$$\boxed{Z' = \frac{Z}{\left(1 - \frac{Y}{\lambda}\right)}} \qquad \boxed{X' = \frac{X}{1 - \frac{Y}{\lambda}}}$$

$$* \quad \frac{Z}{Z'} = \frac{\lambda}{\lambda + Y'}$$

$$\frac{Y}{Y'} = \frac{\lambda}{\lambda + Y'}$$

$$YY' + Y\lambda = \lambda Y'$$

$$Y'(\lambda - Y) = Y\lambda$$

$$\boxed{Y' = \frac{Y}{1 - \frac{Y}{\lambda}}}$$

PERSPEKTIVNA MATRIKA:

$$\underline{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{d} & 0 & 1 \end{bmatrix} \qquad \underline{P}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{d} & 0 & 1 \end{bmatrix}$$

pojavi se samo eden od teh koeficientov
(ker lečo postavimo v eno od ravnin)

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{d} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1-\frac{y}{d} \end{bmatrix} = \begin{bmatrix} \frac{x}{1-\frac{y}{d}} \\ \frac{y}{1-\frac{y}{d}} \\ \frac{z}{1-\frac{y}{d}} \\ 1 \end{bmatrix}$$

tale hočem imeti 1

PRIMER: Še ena možnost za 1. nalogo na izpitu

$\boxed{\text{1. NALOGA NA IZPITU}}$

$d = 2$
leča $[x, z]$
$T' = [-1, -3, -2]$

$T = ?$

1. NAČIN:

$$y' = \frac{y}{1-\frac{y}{d}}$$

$$y'\left(1-\frac{y}{d}\right) = y$$

$$y' = y\left(1+\frac{y'}{d}\right)$$

$$\underline{\underline{y}} = \frac{y'}{1+\frac{y'}{d}} = -\frac{3}{1-\frac{3}{2}} = \underline{\underline{6}}$$

$$\underline{\underline{x}} = x'\left(1-\frac{y}{d}\right) = -1\left(1-\frac{6}{2}\right) = \underline{\underline{2}}$$

$$\underline{\underline{z}} = z'\left(1-\frac{y}{d}\right) = -2\left(1-\frac{6}{2}\right) = \underline{\underline{4}}$$

$$\underline{\underline{T = [2, 6, 4]}}$$

2. NAČIN:

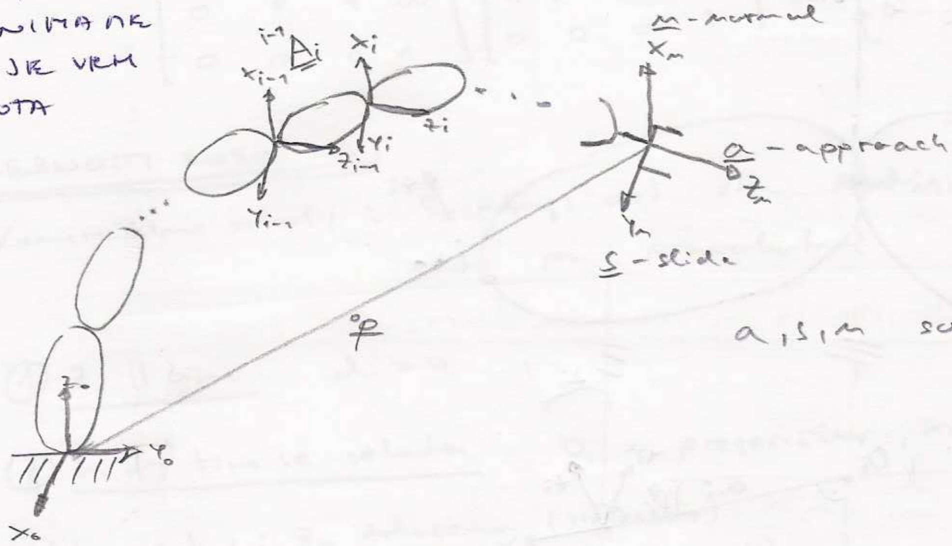
$$\underline{T} = \underline{P}^{-1}\underline{T}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ -2 \\ -\frac{1}{2} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 2 \\ 6 \\ 4 \\ 1 \end{bmatrix}}}$$

DIREKTNI {GEOMETRIJSKI} MODEL ROBOTA

POZNAM PARAMETRE (KOTI, TRANSL) - IŠČEM KJE JE VRH ROBOTA

→ SKALARNI PARAMETRI

→ DENAVIT - HARTENBERG

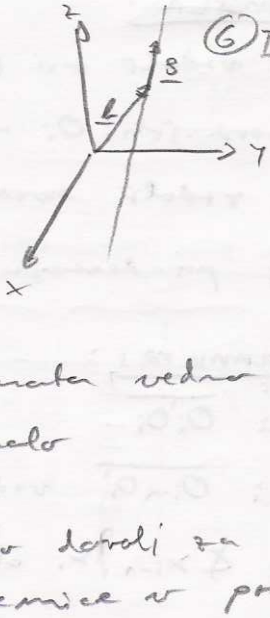


a, s, n so enotski vektorji

$$ {}^0\underline{\underline{T}}_n = \begin{bmatrix} {}^0\underline{n} & {}^0\underline{s} & {}^0\underline{a} & {}^0\underline{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

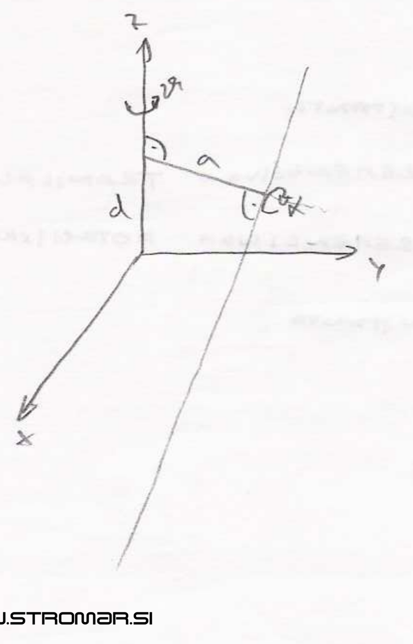
$$\boxed{ {}^0\underline{\underline{T}}_n = {}^0\underline{A}_1(q_1)\,{}^1\underline{A}_2(q_2)\ldots{}^{n-1}\underline{A}_n(q_n) \qquad q_i \begin{cases} \vartheta_i - \text{rotacijski sklep} \\ d_i - \text{translacijski sklep} \end{cases} }$$

Do tega pridemo na dva načina:
- SKALARNI
- VEKTORSKI



⑥ PARAMETROV

SKALARNI: Denavit, Hartenberg sta ta naredila s 4-parametri (minimalno možno)



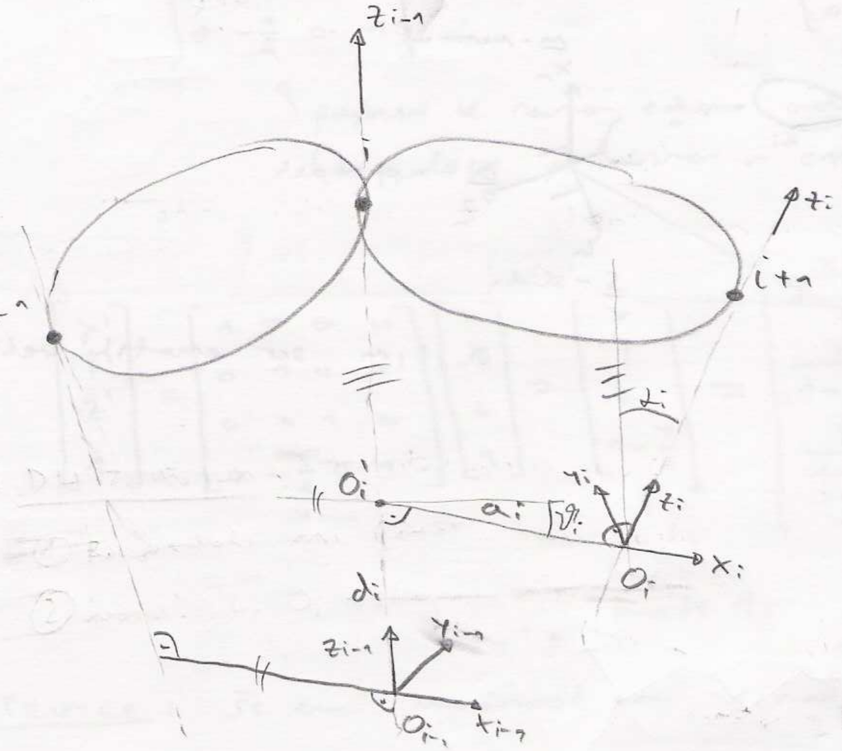
↳ Dve premici imata vedno skupno normalo

↳ Parametri: d, a, ϑ, α so dovolj za določitev premice v prostoru

↳ DENAVIT-HARTENBERGOV PRISTOP:

$\sin\vartheta_i = S\vartheta_i$
$\sin\vartheta_1 = S1$

LENI ROBOTKI



→ DH PRAVILA:

① z_i vzdolž osi $(i+1)$

② koord. izh. O_i – skupna normala z_{i-1}, z_i

③ x_i vzdolž normale ① → $(i+1)$

④ y_i po desnosučnem pravilu

→ DH PARAMETRI:

① a_i: $\overline{O_i' O_i}$ KONSTANTA

② d_i: $\overline{O_{i-1} O_i'}$ vzdolž osi z_{i-1} SPREMENLJIVKA TRANSLACIJSKEGA SKLEPA

③ ϑ_i $\measuredangle x_{i-1}, x_i$ okrog osi z_{i-1} SPREMENLJIVKA ROTACIJSKEGA SKLEPA

④ α_i $\measuredangle z_{i-1}, z_i$ okrog osi x_i KONSTANTA

$$ {}^{i-1}\underline{A}_i(q_i) = Trans(0,0,d_i)\cdot Rot(z_{i-1},\vartheta_i)\cdot Trans(a_i,0,0)\cdot Rot(x_i,\alpha_i) = $$

$$ = \begin{bmatrix} c\vartheta_i & -s\vartheta_i & 0 & 0 \\ s\vartheta_i & c\vartheta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d_i \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\vartheta_i & -s\vartheta_i c\alpha_i & s\vartheta_i s\alpha_i & a_i c\vartheta_i \\ s\vartheta_i & c\vartheta_i c\alpha_i & -c\vartheta_i c\alpha_i & a_i s\vartheta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

↳ POSEBNOSTI ROBOTIKE:

→ Komercialni roboti: Sosednji osi sta ponavadi vzporedni ali pa pravokotni

① $z_i \parallel z_{i-1}$ $\quad d_i = 0$

② z_i in z_{i-1} se sekata $\quad O_i$ v presečišču, $x_i \perp [z_{i-1}, z_i]$

ležišče se sekata in tvorita ravnino

③ bazni k.s.: z_0 določena (STANDARD)

O_0 prvi sklep (naš dogovor)

$x_0 \parallel x_1$ (naš dogovor)

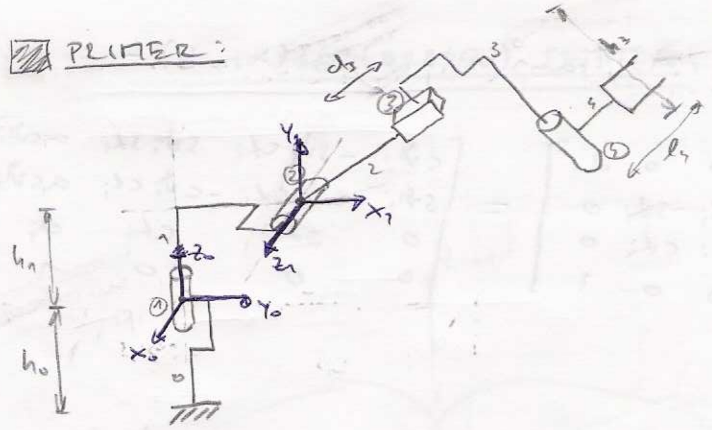
④ k.s. vrha robota: $z_n \parallel z_{n-1}$

⑤ translacijski sklep:

$-z_{i-1}$ vzdolž ~~translacije~~

$-O_{i-1}$ postavimo na začetek translacije

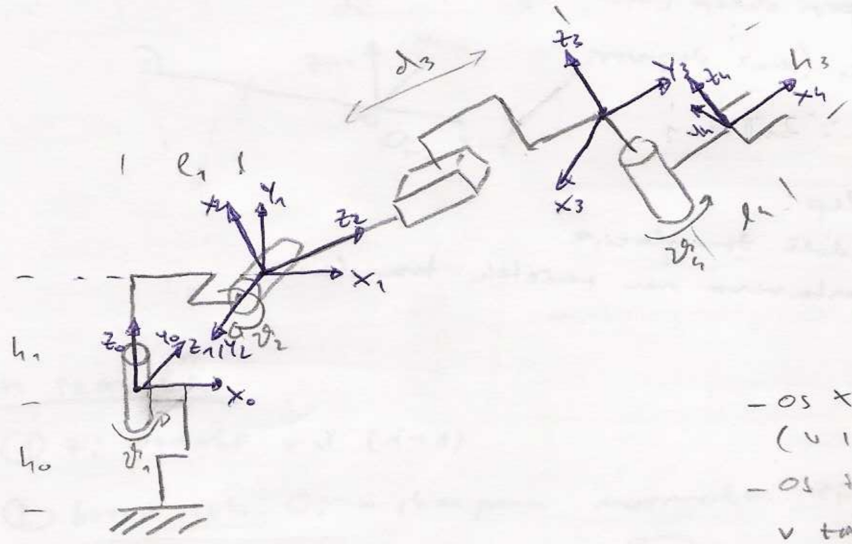
PRIMER:



IZPIT:
→ NARIŠEM ROBOT
MI MORAMO RIŠAT
K.S.

PREDAVANJE – REHABILITACIJSKA ROBOTIKA

23. 3. 2009



– OS x_0 POSTAVIM GLEDE NA x_1 (V ISTI SMERI)

– OS z_2 – OS TRANSL. SKLEPA POSTAVIM V ZAČETEK TRANSLACIJE

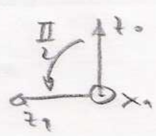
TABELA D-H PARAMETROV:

	x_i		z_{i-1}	
i	a_i	α_i	d_i	ϑ_i
1	l_1	$\frac{\pi}{2}$	h_1	ϑ_1
2	0	$\frac{\pi}{2}$	0	ϑ_2
3	0	$\frac{\pi}{2}$	d_3	$\frac{\pi}{2}$
4	l_4	0	h_3	ϑ_4

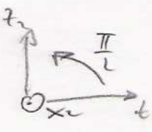
DOF (ŠT. PROSTOSTNIH STOPENJ)

– VSAKA VRSTICA IMA SAMO ENO SPREMENLJIVKO

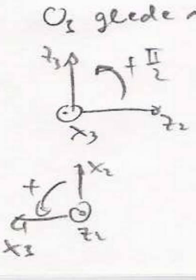
O_1 glede na O_0



O_2 glede na O_1



O_3 glede na O_2



$${}^0\underline{\underline{A}}_1 = Rot(z_1, \vartheta_1)\,Trans(0,0,d_1)\,Rot(x_1,\alpha_1)\,Trans(a_1,0,0)$$

$${}^0\underline{\underline{A}}_1 = \begin{bmatrix} c1 & 0 & s1 & l_1 c1 \\ s1 & 0 & -c1 & l_1 s1 \\ 0 & 1 & 0 & h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c\vartheta_i & -s\vartheta_i c\alpha_i & s\vartheta_i s\alpha_i & a_i c\vartheta_i \\ s\vartheta_i & c\vartheta_i c\alpha_i & -c\vartheta_i s\alpha_i & a_i s\vartheta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

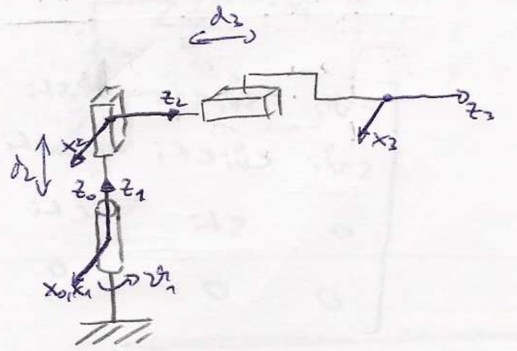
$${}^1\underline{\underline{A}}_2 = \begin{bmatrix} c2 & 0 & s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2\underline{\underline{A}}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^3\underline{\underline{A}}_4 = \begin{bmatrix} c4 & -s4 & 0 & l_4 c4 \\ s4 & c4 & 0 & l_4 s4 \\ 0 & 0 & 1 & -h_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PRIMER: MOŽNA IZPITNA NALOGA:

$\boxed{\text{VALJNI ROBOT}}$ – ker je s svinčnikom je podano



	vzdolž x_i		vzdolž z_{i-1}	
i	a_i	α_i	d_i	ϑ_i
1	0	0	0	ϑ_1
2	0	$-\frac{\pi}{2}$	d_2	0
3	0	0	d_3	0

ker sem naredil x-e vzporedne

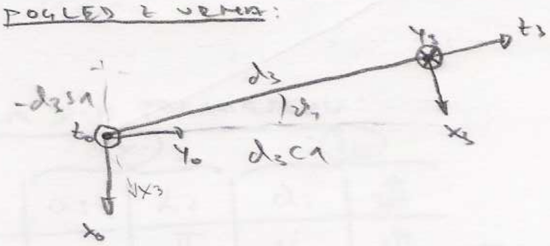
$$\underline{\underline{{}^0\underline{A}_3}} = {}^0\underline{A}_1 \cdot {}^1\underline{A}_2 \cdot {}^2\underline{A}_3 = \begin{bmatrix} C1 & -S1 & 0 & 0 \\ S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} {}^2\underline{A}_3 =$$

$$= \begin{bmatrix} C1 & 0 & -S1 & 0 \\ S1 & 0 & C1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underline{\underline{\begin{bmatrix} C1 & 0 & -S1 & -d_3S1 \\ S1 & 0 & C1 & d_3C1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}}$$

GEOMETRIJSKI MODEL ROBOTA JE LEGA VRHA ROBOTA GLEDE NA BAZNI K.S.

2. NAČIN:

POGLED Z VRHA:



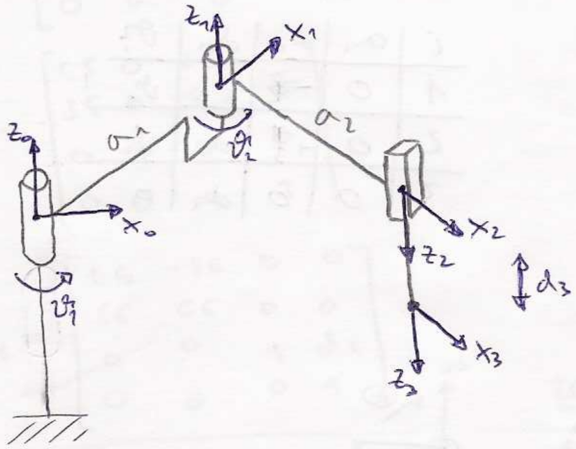
$$\begin{array}{cccccc} & x_3 & y_3 & z_3 & & \\ \left[\begin{array}{c} \\ \\ \\ \\ \end{array}\right. & \cos\vartheta_1 & \cos 90 & \cos(90+\vartheta_1) & -d_3S1 & \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right] \end{array}$$

$$\begin{array}{c} \\ \begin{bmatrix} \cos\vartheta_1 & \cos 90 & \cos(90+\vartheta_1) & -d_3S1 \\ \cos(90-\vartheta_1) & \cos 90 & \cos\vartheta_1 & d_3C1 \\ \cos 90 & \cos 180 & \cos 90 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} x_0 \\ y_0 \\ z_0 \\ \\ \end{array}$$

Tega valda lahko tudi rešujemo, ker je zelo preprost (nobenega drugega).

PRIMER: IZPIT

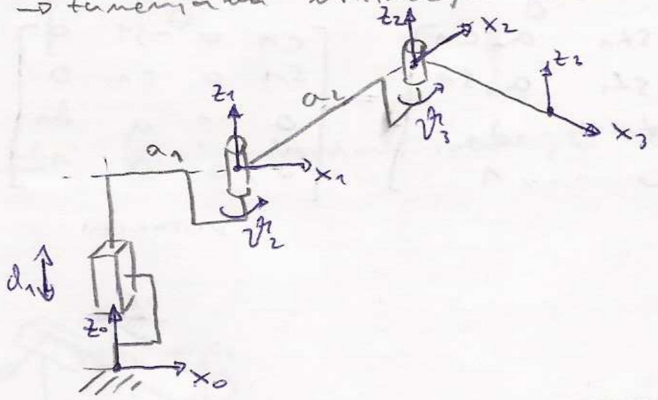
$\boxed{\text{SCARA ROBOT}}$



i	a_i	α_i	d_i	ϑ_i
1	a_1	0	0	ϑ_1
2	a_2	π	0	ϑ_2
3	0	0	d_3	0

MOŽNI PRIMERI:

→ zamenjava vrstnega reda zklepov



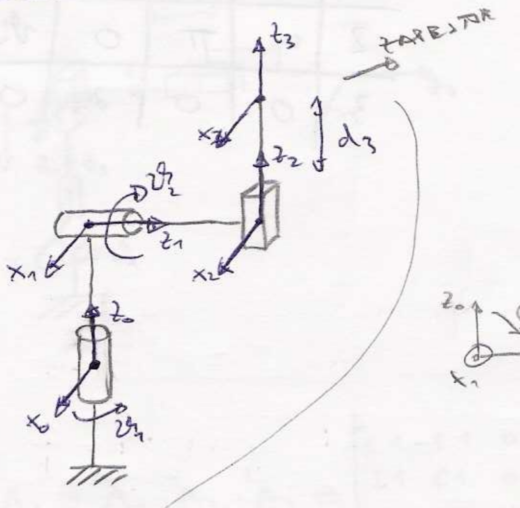
	x_i		z_{i-1}	
i	a_i	α_i	d_i	ϑ_i
1	a_1	0	d_1	0
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

$\boxed{\text{PREDAVANJE - INDUSTRIJSKA ROBOTIKA}}$

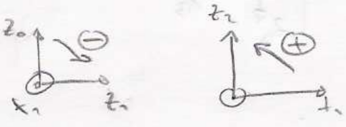
30.3.2009

PRIMER: IZPIT

STANFORDSKI ROBOT



i	a_i	α_i (x_i)	d_i (z_{i-1})	ϑ_i
1	0	$-\frac{\pi}{2}$	d_1	ϑ_1
2	0	$+\frac{\pi}{2}$	d_2	ϑ_2
3	0	0	d_3	0



$$ {}^0\underline{A}_1 = \begin{bmatrix} C1 & -S1C\alpha_1 & S1S\alpha_1 & a_1C1 \\ S1 & C1C\alpha_1 & -C1S\alpha_1 & a_1S1 \\ 0 & S\alpha_1 & C\alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C1 & 0 & -S1 & 0 \\ S1 & 0 & C1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

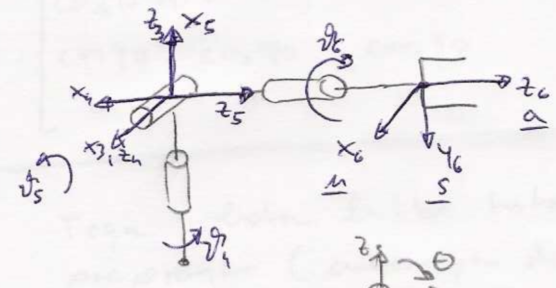
$$ {}^1\underline{A}_2 = \begin{bmatrix} C2 & 0 & S2 & 0 \\ S2 & 0 & -C2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2\underline{A}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

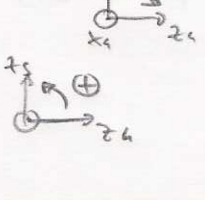
ZAREŽITE

SFERIČNO ZAPESTJE:

Ga pritrdim na vrh Standardskega robota



i	a_i	α_i	d_i	ϑ_i
4	0	$-\frac{\pi}{2}$	0	ϑ_4
5	0	$\frac{\pi}{2}$	0	ϑ_5
6	0	0	d_6	ϑ_6



$$
{}^{3}\underline{\underline{A}}_4 = \begin{bmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

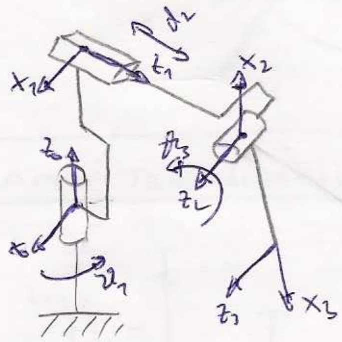
$$
{}^{4}\underline{\underline{A}}_5 = \begin{bmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{5}\underline{\underline{A}}_6 = \begin{bmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Euler $(\varphi, \vartheta, \psi)$

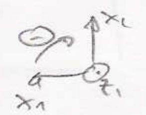
$$
{}^{3}\underline{\underline{A}}_6 = \begin{bmatrix} c4c5c6 - s4s6 & -c4c5s6 - s4c6 & c4s5 & c4s5d_6 \\ s4c5c6 + c4s6 & -s4c5s6 + c4c6 & s4s5 & s4s5d_6 \\ -s5c6 & s5s6 & c5 & c5d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

PRIMER: (Nestandardni robot, ki lahko pride na izpit)



i	a_i	α_i	d_i	ϑ_i
1	0	$-\frac{\pi}{2}$	d_1	ϑ_1
2	0	$-\frac{\pi}{2}$	d_2	$-\frac{\pi}{2}$
3	a_3	0	0	ϑ_3

!!



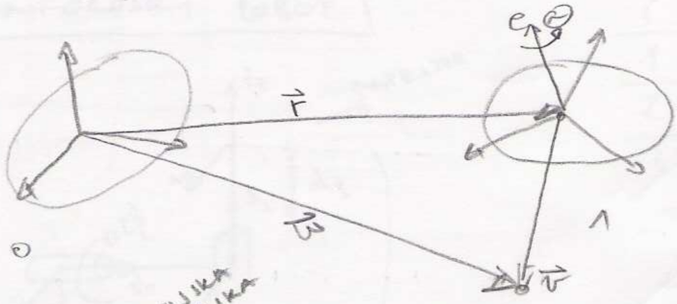
Je treba paziti, ker tu ni vedno $\emptyset$.

PREDAVANJE – MOBILNI ROBOTI

$\boxed{6.4.2009}$

Prof. Lenarčič

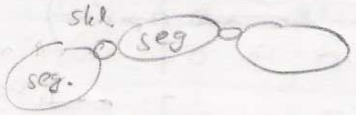
VEKTORSKI PARAMETRI KINEMATIČNEGA PARA



ROTACIJSKA MATRIKA

$$\vec{w} = \vec{r} + A\vec{v}$$

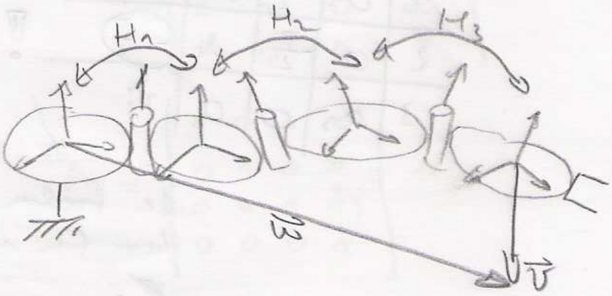
$$\left[\frac{\vec{w}}{1}\right] = \underbrace{\left[\begin{array}{c|c} A & \vec{r} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]}_{H} \cdot \left[\frac{\vec{v}}{1}\right] = \left[\frac{\vec{r} + A\cdot\vec{v}}{1}\right]$$



CILINDRIČNI SKLEP

— TRANSLACIJA

— ROTACIJA

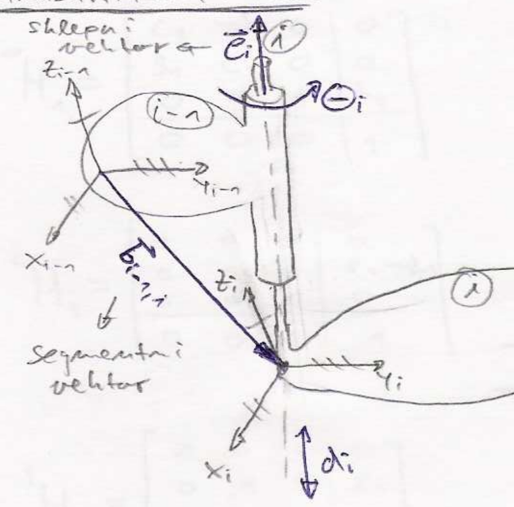


$$\left[\frac{\vec{w}}{1}\right] = H_1 H_2 H_3 \left[\frac{\vec{v}}{1}\right]$$

KAKO POSTAVITI K.S.?

- na začetku telesa - ob sklepu
- vse paralelno

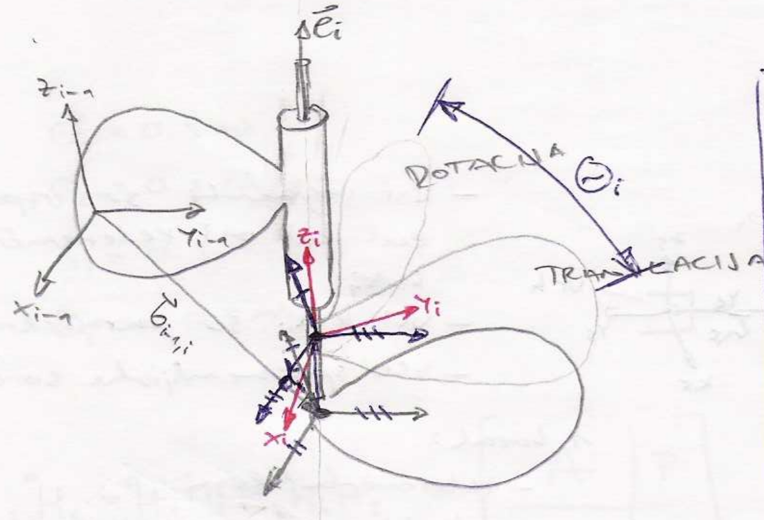
KINEMATIČNI PAR:



$$\left.\begin{array}{l} d_i = 0 \\ \Theta_i = 0 \end{array}\right\} \text{začetna (referenčna) lega}$$

(vsi k.s. so paralelni)

- dva vektorska parametra:
  - $\vec{e}_i$ — sklepni vektor
  - $\vec{b}_{i-1,i}$ — segmentni vektor
- dve spremenljivki:
  - Θ_i
  - d_i



$${}^{i-1}H_i = \left[\begin{array}{c|c} {}^{i-1}A_i & \vec{b}_{i-1,i} + d_i\vec{e}_i \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

SPLOŠNA ENAČBA, KI POVEZUJE TI DVE TELESI (VSEBUJE TRANSLACIJO IN ROTACIJO)

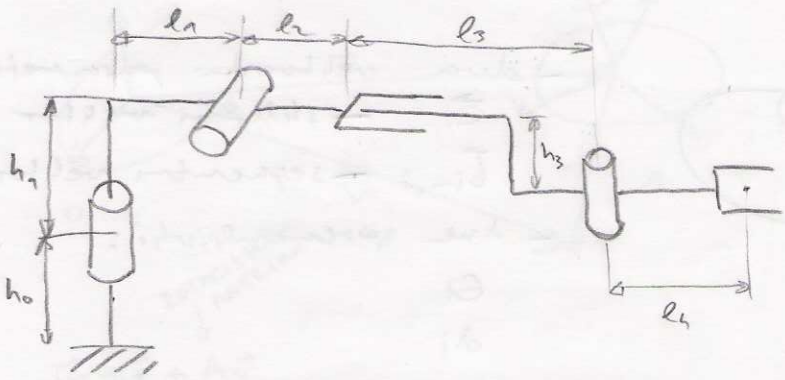
SAMO TRANSLACIJA: $\Theta_i = 0$

$${}^{i-1}H_i = \left[\begin{array}{c|c} I & \vec{b}_{i-1,i} + d_i\vec{e}_i \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

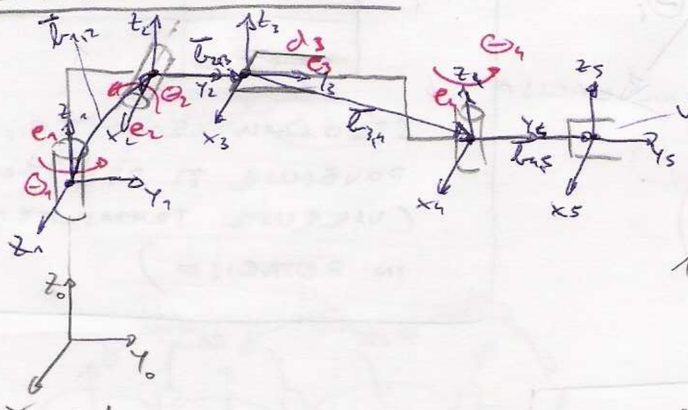
SAMO ROTACIJA: $d_i = 0$

$${}^{i-1}H_i = \left[\begin{array}{c|c} {}^{i-1}A_i & \vec{b}_{i-1,i} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

PRIMER:



REFERENČNA LEGA:



red

- vsi segmenti so vzporedni eni od osi referenčnega k.s.;
- vsi k.s. so paralelni;
- vse spremenljivke so enake 0

1. korak:
 - referenčna lega
 - izhodišča v središča sklepov

2. korak:
 - določimo vektorske parametre sklepov vektorje $\bar{e}_i$

3. korak:
 - določimo segmentne vektorje

SKLEPNI VEKTORJI:

	1	2	3	4
e_i	0	1	0	0
	0	0	1	0
	1	0	0	1

SEGMENTNI VEKTORJI:

	1	2	3	4	5
$\bar{b}_{i-1,i}$	0	0	0	0	0
	0	l_1	l_2	l_3	l_4
	h_0	h_1	0	$-h_3$	0

4. korak:
 - napišemo matrike

SPREMENLJIVKE:

	1	2	3	4
d_i	0	0	d_3	0
θ_i	θ_1	θ_2	0	θ_3

V začetni legi vse spr. enake 0!

$$
{}^{0}H_1 = \left[\begin{array}{ccc|c} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & h_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad {}^{1}H_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & C_2 & -S_2 & l_1 \\ 0 & S_2 & C_2 & h_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

$$
{}^{2}H_3 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2+d_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad {}^{3}H_4 = \left[\begin{array}{ccc|c} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & l_3 \\ 0 & 0 & 1 & -h_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

$$
{}^{4}H_5 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_4 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]
$$

$\Theta_1 = 0.5$ rd

$\Theta_2 = 0.1$ rd

$d_3 = 0.5$ m

$\Theta_3 = -0.5$ rd

$$
{}^{0}H_5 = {}^{0}H_1\,{}^{1}H_2\,{}^{2}H_3\,{}^{3}H_4\,{}^{4}H_5 = \left[\begin{array}{c|c} A & \vec{r} \\ \hline 0\;0\;0 & 1 \end{array}\right]
$$

$\boxed{\text{PREDAVANJE – HUMANOIDNI ROBOTI IN UČENJE KOMPLEKSNIH GIBANJ}}$

20.4.2009

DIREKTNI GEOM. MODEL VEKTORSKI

1. REF. LEGA:
   - ref. k.s. $[x_0, y_0, z_0]$
   - vsak segment vzporeden eni izmed osi x_0, y_0 ali z_0
   - transl. sklepi: $d_i = 0$

2. K.S. SKLEPOV $[x_i, y_i, z_i]$:
   - samo izhodišče (ker so vsi vzporedni baznemu)
     - točki na vrh robota

3. DOLOČANJE VEKTORSKIH PARAMETROV:
   - e_i
   - $b_{i-1,i}$

4. MATRIKE:

ROT:

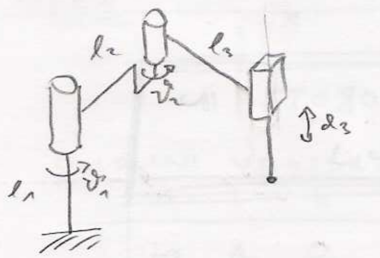
$$ {}^{i-1}\underline{\underline{H}}_i = \begin{bmatrix} R_x, R_y, R_z & \vec{b}_{i-1,i} \\ 0 & 1 \end{bmatrix} $$

TRANS:

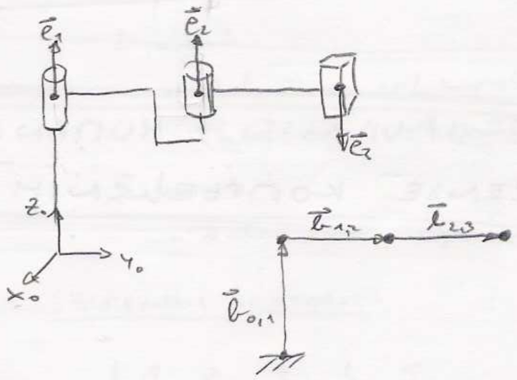
$$ {}^{i-1}\underline{\underline{H}}_i = \begin{bmatrix} \underline{I} & d_i\vec{e}_i + \vec{b}_{i-1,i} \\ 0 & 1 \end{bmatrix} $$

5. $ {}^{0}\underline{\underline{H}}_n = {}^{0}\underline{\underline{H}}_1 \cdots {}^{n-1}\underline{\underline{H}}_n $

PRIMER: SCARA ROBOT



$\equiv$



i	1	2	3
d_i	0	0	d_3
Θ_i	ϑ_1	ϑ_2	0

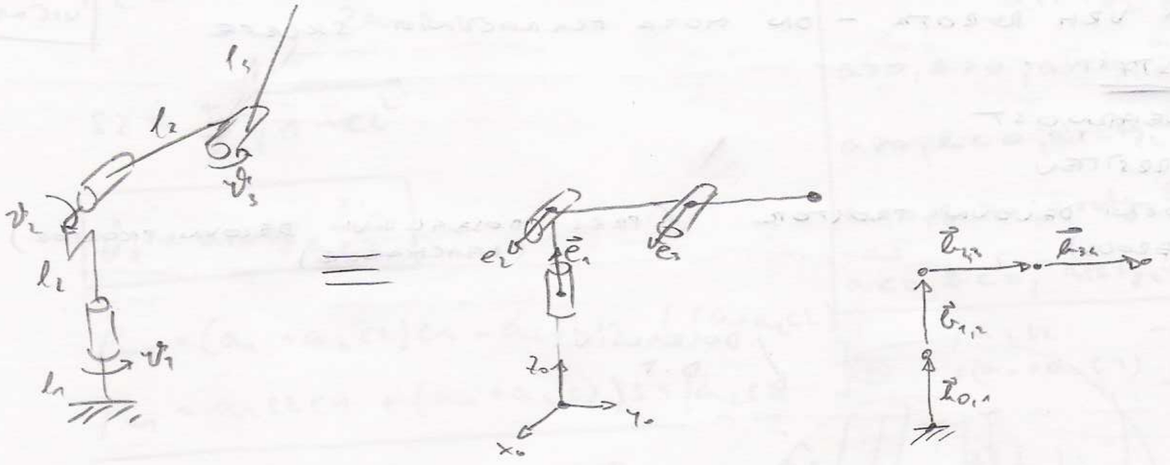
i	1	2	3
$\vec{e}_i$	0	0	0
	0	0	0
	1	1	-1

i	1	2	3
$\vec{b}_{i-1,i}$	0	0	0
	0	l_2	l_3
	l_1	0	0

$$ {}^{0}\underline{\underline{H}}_1 = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{1}\underline{\underline{H}}_2 = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ s2 & c2 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad {}^{2}\underline{\underline{H}}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}\underline{\underline{H}}_3 = {}^{0}\underline{\underline{H}}_1 \cdot {}^{1}\underline{\underline{H}}_2 \cdot {}^{2}\underline{\underline{H}}_3 $$

PRIMER: ANTROPOMORFNI ROBOT



i	1	2	3
ϑ_i	ϑ_1	ϑ_2	ϑ_3
d_i	0	0	0

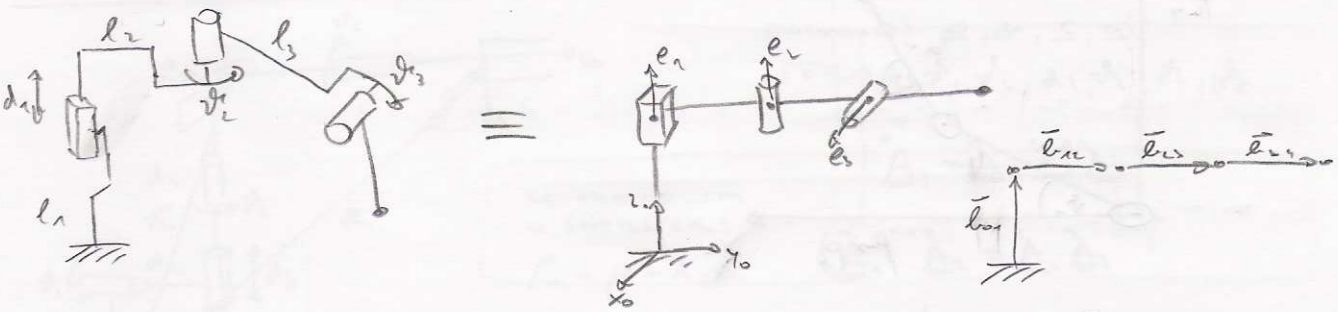
i	1	2	3
$\vec{e}_i$	0	1	1
	0	0	0
	1	0	0

i	1	2	3	4
$\vec{l}_{i-1,i}$	0	0	0	0
	0	0	l_3	l_4
	l_1	l_2	0	0

$$^0\underline{\underline{H}}_1 = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{i} \quad ^1\underline{\underline{H}}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c2 & -s2 & 0 \\ 0 & s2 & c2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2\underline{\underline{H}}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c3 & -s3 & l_3 \\ 0 & s3 & c3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{i} \quad ^3\underline{\underline{H}}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PRIMER: "IZPITNI" ROBOT (ni pogost v realnosti – si ga prof. izmislil)



i	1	2	3
ϑ_i	0	ϑ_2	ϑ_3
d_i	d_1	0	0

$$^0\underline{\underline{H}}_1 = \begin{bmatrix} & I & & 0 \\ & & & 0 \\ & & & l_1+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{i} \quad ^1\underline{\underline{H}}_2 = \begin{bmatrix} c2 & -s2 & 0 & 0 \\ s2 & c2 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	1	2	3
$\bar{e}_i$	0	0	1
	0	0	0
	1	1	0

$$^2\underline{\underline{H}}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c3 & -s3 & l_3 \\ 0 & s3 & c3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{i} \quad ^3\underline{\underline{H}}_4 = \begin{bmatrix} & I & & 0 \\ & & & l_4 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i	1	2	3	4
$\vec{l}_{i-1,i}$	0	0	0	0
	0	l_2	l_3	l_4
	l_1	0	0	0

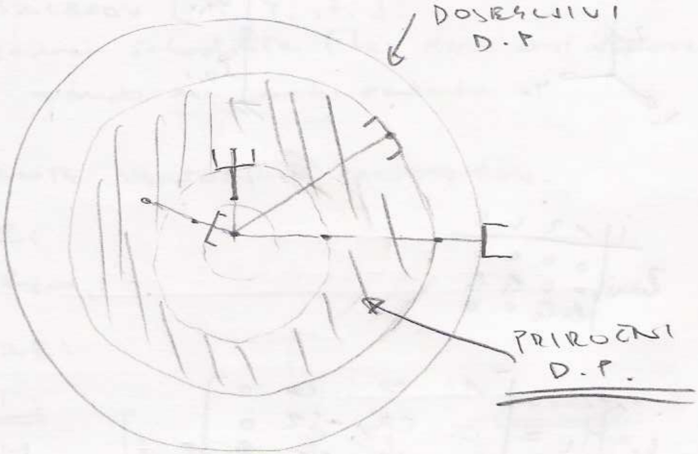
INVERZNI GEOMETRIJSKI MODEL ROBOTA

USTNI IZPIT, ČE HOČEŠ VIŠAT OCENO

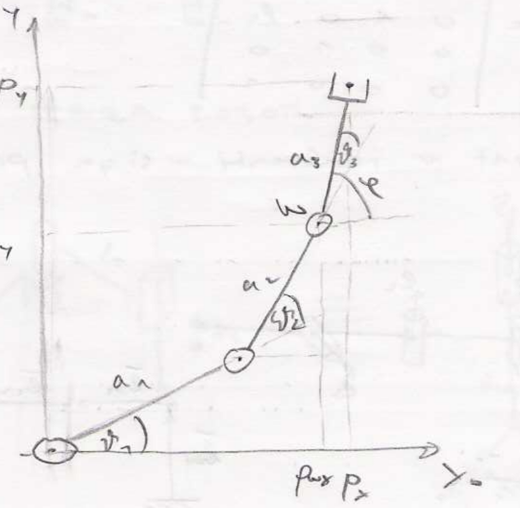
- POZNAM VRH ROBOTA – ON MORA PRERAČUNAT SKLEPE

LASTNOSTI:

- NELINEARNOST
- VEČ REŠITEV
- PRIROČNI DELOVNI PROSTOR (DEXTEROUS) (PREJ DOSEGLJIVI DELOVNI PROSTOR) (REACHABLE)



RAVNINSKI ROBOT:



ZNANO: p_x, p_y, φ

RAČUNAMO: $\vartheta_1, \vartheta_2, \vartheta_3$

$$\varphi = \vartheta_1 + \vartheta_2 + \vartheta_3$$

$$P_x = a_1 C1 + a_2 C12 + a_3 C123$$
$$P_y = a_1 S1 + a_2 S12 + a_3 S123$$

ZAPESTJE: w

$$\left.\begin{aligned} P_{wx} &= P_x - a_3 c\varphi = a_1 c1 + a_2 c12 \quad |\,|^2 \\ P_{wy} &= P_y - a_3 s\varphi = a_1 s1 + a_2 s12 \quad |\,|^2 \end{aligned}\right] \oplus$$

$$P_{wx}^2 + P_{wy}^2 = a_1^2 + a_2^2 + 2a_1 a_2 c2 \leftarrow$$

$C1C12 + S1S12 =$
$= \cos(\vartheta_1 - (\vartheta_1 + \vartheta_2)) = c2$

BRONŠTAJN

$$c2 = \frac{p_{wx}^2 + p_{wy}^2 - a_1^2 - a_2^2}{2a_1a_2}$$

$$s2 = \pm\sqrt{1 - c2^2}$$

$$\boxed{\vartheta_2 = \mathrm{arctg}_2 \frac{s2}{c2}}$$

BRONŠTEJN

$$\mathrm{arctg}_2 \frac{a}{b}$$

$a>0, b>0$; $\mathrm{arctg}_2 \frac{a}{b} = \mathrm{arctg} \frac{a}{b}$

$a>0, b<0$; $\mathrm{arctg}_2 \frac{a}{b} = \pi + \mathrm{arctg} \frac{a}{b}$

$a<0, b>0$; $\mathrm{arctg}_2 \frac{a}{b} = \mathrm{arctg} \frac{a}{b}$

$a<0, b<0$; $\mathrm{arctg}_2 \frac{a}{b} = \mathrm{arctg} \frac{a}{b} - \pi$

$$p_{wx} = (a_1 + a_2c2)c1 - a_2 s2 s1 \quad \Big| (a_1+a_2c2) \quad \Big| a_2s2$$

$$p_{wy} = a_2 s2 c1 + (a_1 + a_2c2)s1 \quad \Big| a_2s2 \quad \Big| (a_1+a_2c2)$$

$\oplus$ $\quad$ $\oplus$

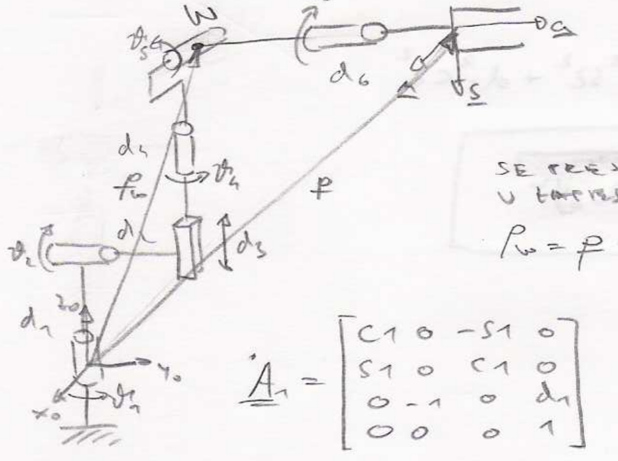
$$p_{wx}(a_1+a_2c2) + p_{wy}a_2s2 = \left[(a_1+a_2c2)^2 + (a_2s2)^2\right]c1$$

$$c1 = \frac{p_{wx}(a_1+a_2c2) + p_{wy}a_2s2}{p_{wx}^2 + p_{wy}^2}$$

$$s1 = \frac{p_{wy}(a_1+a_2c2) - p_{wx}a_2s2}{p_{wx}^2 + p_{wy}^2}$$

$$\boxed{\vartheta_1 = \mathrm{arctg}_2 \frac{s1}{c1}} \qquad \boxed{\vartheta_3 = \varphi - \vartheta_1 - \vartheta_2}$$

↳ STANFORDSKI ROBOT S ZAPESTJEM:



PODANO: p, n, s, a

IŠČEM: $\vartheta_1, \vartheta_2, d_3, \vartheta_4, \vartheta_5, \vartheta_6$

$${}^0\underline{A}_3 = {}^0\underline{A}_1\, {}^1\underline{A}_2\, {}^2\underline{A}_3$$

SE PRESELIM V ZAPESTJE:

$$p_w = p - d_6 a$$

$$({}^0\underline{A}_1)^{-1}\, {}^0\underline{A}_3 = {}^1\underline{A}_2\, {}^2\underline{A}_3$$

$${}^0\underline{A}_1 = \begin{bmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{A}^{-1} = \begin{bmatrix} \underline{R}^T & -\underline{R}^T d \\ \underline{0} & 1 \end{bmatrix}$$

$$\begin{bmatrix} c1 & s1 & 0 & 0 \\ 0 & 0 & -1 & d_1 \\ -s1 & c1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^0\underline{R}_3 & \begin{matrix} p_{wx} \\ p_{wy} \\ p_{wz} \end{matrix} \\ \underline{0} & 1 \end{bmatrix} = \begin{bmatrix} c2 & 0 & s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} p_{wx}c1 + p_{wy}s1 \\ d_1 - p_{wz} \\ -p_{wx}s1 + p_{wy}c1 \end{bmatrix} = \begin{bmatrix} d_3 s2 \\ -d_3 c2 \\ d_2 \end{bmatrix}} \longrightarrow$$

$$t = tg\frac{\vartheta_1}{2}$$

$$c1 = \cos\vartheta_1 = \cos^2\frac{\vartheta_1}{2} - \sin^2\frac{\vartheta_1}{2} = \frac{\cos^2\frac{\vartheta_1}{2} - \sin^2\frac{\vartheta_1}{2}}{\cos^2\frac{\vartheta_1}{2} + \sin^2\frac{\vartheta_1}{2}} = \frac{1-t^2}{1+t^2}$$

MAT. PRIROČNIK

$$s1 = \sin\vartheta_1 = 2\sin\frac{\vartheta_1}{2}\cos\frac{\vartheta_1}{2} = \frac{2\sin\frac{\vartheta_1}{2}\cos\frac{\vartheta_1}{2}}{\cos^2\frac{\vartheta_1}{2} + \sin^2\frac{\vartheta_1}{2}} = \frac{2t}{1+t^2}$$

$$-p_{wx}\frac{2t}{1+t^2} + p_{wy}\frac{1-t^2}{1+t^2} = d_2$$

$$(d_2 + p_{wy})t^2 + 2p_{wx}t + (d_2 - p_{wy}) = 0$$

$$t = \frac{-p_{wx} \pm \sqrt{p_{wx}^2 + p_{wy}^2 - d_2^2}}{d_2 + p_{wy}}$$

$$\boxed{\vartheta_1 = 2\,arctg_2\frac{-p_{wx} \pm \sqrt{p_{wx}^2 + p_{wy}^2 - d_2^2}}{d^2 + p_{wy}}}$$

$$\frac{p_{wx}c1 + p_{wy}s1}{d_1 - p_{wz}} = \frac{d_3 s2}{-d_3 c2}$$

$$\boxed{\vartheta_2 = arctg_2\frac{p_{wx}c1 + p_{wy}s1}{p_{wz} - d_1}}$$

$$(p_{wx}c1 + p_{wy}s1)^2 + (d_1 - p_{wz})^2 = d_3^2 s2^2 + d_3^2 c2^2$$

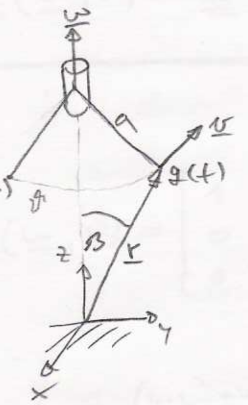
$$\boxed{d_3 = +\sqrt{(p_{wx}c1 + p_{wy}s1)^2 + (d_1 - p_{wz})^2}}$$

ROTACIJA & ORIENTACIJA

4.5.2009

RODRIGUESOVA FORMULA – V MATRIČNI OBLIKI

– Rotacija okoli poljubne osi



$v = \omega \cdot a$

$v = \omega \cdot r \cdot \sin\beta$

$\underline{v} = \underline{\omega} \times \underline{r}$

$$g(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$$

$\boxed{\dot{\underline{g}} = \underline{\omega} \times \underline{g}}$ $\quad |\underline{\omega}|^2 = 1$! NORMIRAN VEKTOR $\underline{\omega}$!

$|\omega|^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 = 1$

→ KROGELNI K.S.

$$\underline{g} = \begin{bmatrix} r\sin\beta\cos\vartheta \\ r\sin\beta\sin\vartheta \\ r\cos\beta \end{bmatrix} \qquad \dot{\underline{g}} = \underline{\omega} \cdot \underline{g} \qquad \vartheta = \vartheta(t) \qquad \underline{\omega} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -r\sin\beta\sin\vartheta \\ r\sin\beta\cos\vartheta \\ 0 \end{bmatrix} = \begin{bmatrix} i & j & k \\ 0 & 0 & 1 \\ r\sin\beta\cos\vartheta & r\sin\beta\sin\vartheta & r\cos\beta \end{bmatrix}$$

→ NADOMESTITEV VEKTORSKEGA PRODUKTA

ANTISIMETRIČNA MATRIKA

$$\underline{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \qquad \boxed{\underline{\underline{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}}$$

$\underline{\underline{\omega}}^T = -\underline{\underline{\omega}}$

$\underline{\omega} \times \underline{g} = \underline{\underline{\omega}}\, \underline{g}$

=

$$\begin{bmatrix} i & j & k \\ \omega_1 & \omega_2 & \omega_3 \\ g_1 & g_2 & g_3 \end{bmatrix} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} \omega_2 g_3 - \omega_3 g_2 \\ 0 \\ 0 \end{bmatrix}$$

$\dot{\underline{g}}(t) = \underline{\underline{\omega}}\, \underline{g}(t)$ DE 1. REDA →

$$\boxed{\underline{g}(t) = e^{\underline{\underline{\omega}} t}\, \underline{g}(t)}$$ REŠITEV D.E.

$\vartheta = \omega t \Rightarrow \vartheta = t$

$\omega = 1$

$\Downarrow$

$$\boxed{\underline{g}(\vartheta) = e^{\underline{\underline{\omega}}\vartheta} g(\vartheta)} \qquad \boxed{\underline{\underline{R}}(\underline{\omega}, \vartheta) = e^{\underline{\underline{\omega}}\vartheta}}$$

$$e^{\underline{\underline{\omega}}\vartheta} = \underline{\underline{I}} + \vartheta \underline{\underline{\omega}} + \frac{\vartheta^2}{2!}\underline{\underline{\omega}}^2 + \frac{\vartheta^3}{3!}\underline{\underline{\omega}}^3 + \dots$$

$$\underline{\underline{\omega}}^3 = \underline{\underline{\omega}}\,\underline{\underline{\omega}}\,\underline{\underline{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}\underline{\underline{\omega}} =$$

$$= \begin{bmatrix} -\omega_3^2-\omega_2^2 & \omega_1\omega_2 & \omega_1\omega_3 \\ \omega_1\omega_2 & -\omega_3^2-\omega_1^2 & \omega_2\omega_3 \\ \omega_1\omega_3 & \omega_2\omega_3 & -\omega_2^2-\omega_1^2 \end{bmatrix}\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_1 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} =$$

$$\underline{\underline{\omega}}^3 = \begin{bmatrix} 0 & & \\ -\omega_3^3 - \omega_1^2\omega_3 - \omega_2^2\omega_3 & 0 & \\ \omega_1\omega_3^2 + \omega_2^3 + \omega_1^2\omega_2 & -\omega_1\omega_3^2 - \omega_2^2\omega_1 - \omega_1^3 & 0 \end{bmatrix}$$ ANTISIMETRIČNA

$$\underline{\underline{\omega}}^3 = -\left(\omega_1^2+\omega_2^2+\omega_3^2\right)\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} = -|\underline{\omega}|^2\underline{\underline{\omega}} = -\underline{\underline{\omega}}$$

$\underline{\underline{\omega}}^4 = \underline{\underline{\omega}}^3\underline{\underline{\omega}} = -\underline{\underline{\omega}}^2$

$\underline{\underline{\omega}}^6 = \underline{\underline{\omega}}^3\underline{\underline{\omega}}^3 = \underline{\underline{\omega}}^2$

$\vdots$

$\underline{\underline{\omega}}^3 = -\underline{\underline{\omega}}$

$\underline{\underline{\omega}}^5 = \underline{\underline{\omega}}^3\underline{\underline{\omega}}^2 = -\underline{\underline{\omega}}^3 = \underline{\underline{\omega}}$

$\vdots$

$$e^{\underline{\underline{\omega}}\vartheta} = \underline{\underline{I}} + \left(\vartheta - \frac{\vartheta^3}{3!} + \frac{\vartheta^5}{5!} - \dots\right)\underline{\underline{\omega}} + \left(\frac{\vartheta^2}{2!} - \frac{\vartheta^4}{4!} + \frac{\vartheta^6}{6!} - \dots\right)\underline{\underline{\omega}}^2 =$$

$$= \underline{\underline{I}} + \underline{\underline{\omega}} \sin\vartheta + \underline{\underline{\omega}}^2 (1 - \cos\vartheta)$$

$$\boxed{\underline{\underline{R}}(\underline{\omega}, \vartheta) = \underline{\underline{I}} + \underline{\underline{\omega}}\, s\vartheta + \underline{\underline{\omega}}^2\, v\vartheta}$$ Rodriguesova formula

$$\underline{\underline{R}}(\underline{\omega}, \vartheta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} s\vartheta + \begin{bmatrix} -(\omega_3^2 + \omega_2^2) & \omega_1\omega_2 & \omega_1\omega_3 \\ \omega_1\omega_2 & -(\omega_1^2 + \omega_3^2) & \omega_2\omega_3 \\ \omega_1\omega_3 & \omega_2\omega_3 & -(\omega_1^2 + \omega_2^2) \end{bmatrix} v\vartheta$$

$$\left(1 - (\omega_2^2 + \omega_3^2)\, v\vartheta = 1 - (1 - c\vartheta)(1 - \omega_1^2) = 1 - (1 - c\vartheta - \omega_1^2 + \omega_1^2 c\vartheta) = \right.$$
$$\left. = c\vartheta + \omega_1^2 v\vartheta \right)$$

$$= \begin{bmatrix} c\vartheta + \omega_1^2 v\vartheta & \omega_1\omega_2 v\vartheta - \omega_3 s\vartheta & \omega_1\omega_3 v\vartheta + \omega_2 s\vartheta \\ \omega_1\omega_2 v\vartheta + \omega_3 s\vartheta & \omega_2^2 v\vartheta + c\vartheta & \omega_2\omega_3 v\vartheta - \omega_1 s\vartheta \\ \omega_1\omega_3 v\vartheta - \omega_2 s\vartheta & \omega_2\omega_3 v\vartheta + \omega_1 s\vartheta & \omega_3^2 v\vartheta + c\vartheta \end{bmatrix}$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

→ INVERZNI PROBLEM:

→ SLED MATRIKE = VSOTA DIAGONALNIH ČLENOV

$$\mathrm{trace}(\underline{\underline{R}}) = r_{11} + r_{22} + r_{33} = v\vartheta(\omega_1^2 + \omega_2^2 + \omega_3^2) + 3c\vartheta = 1 - c\vartheta + 3c\vartheta =$$

$$= 1 + 2c\vartheta$$

$$\boxed{\vartheta = \arccos\left(\frac{\mathrm{trace}(\underline{\underline{R}}) - 1}{2}\right)} \qquad \begin{matrix} \vartheta \pm 2\pi n \\ -\vartheta \pm 2\pi n \end{matrix}$$

$$r_{32} - r_{23} = 2\omega_1 s\vartheta$$
$$r_{13} - r_{31} = 2\omega_2 s\vartheta$$
$$r_{21} - r_{12} = 2\omega_3 s\vartheta$$

$$\Rightarrow \boxed{\underline{\omega} = \frac{1}{2s\vartheta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}}$$

PRIMER:

$$\underline{R} = Rot(x,90)\,Rot(y,180)\,\underset{(z,-90)}{Rot(z,270)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} Rot(z,270)$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$\vartheta = \arccos\left(\frac{trace(\underline{R}) - 1}{2}\right) = \arccos\left(-\frac{1}{2}\right) = 120^\circ$$

$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

$$\underline{\omega} = \frac{1}{2 s\vartheta} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Podano imamo $\underline{\omega}$, moramo izračunati $\underline{R}$ z Rodriguesovo formulo.

$$\underline{R} = \underline{\underline{I}} + \underline{\underline{\omega}}\, s\vartheta + \underline{\underline{\omega}}^2\, v\vartheta$$

$$\underline{\underline{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

$$\underline{\underline{\omega}}^2 = \underline{\underline{\omega}}\,\underline{\underline{\omega}} = \frac{1}{3} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} -2 & -1 & -1 \\ -1 & -2 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} s\vartheta + \frac{1}{3} \begin{bmatrix} -2 & -1 & -1 \\ -1 & -2 & 1 \\ -1 & 1 & -2 \end{bmatrix} v\vartheta =$$

$$= \begin{bmatrix} 1 - \frac{2}{3} v\vartheta & -\frac{1}{\sqrt{3}} s\vartheta - \frac{1}{3} v\vartheta & \frac{1}{\sqrt{3}} s\vartheta - \frac{1}{3} v\vartheta \\ \frac{1}{\sqrt{3}} s\vartheta - \frac{1}{3} v\vartheta & 1 - \frac{2}{3} v\vartheta & \frac{1}{\sqrt{3}} s\vartheta + \frac{1}{3} v\vartheta \\ -\frac{1}{\sqrt{3}} s\vartheta - \frac{1}{3} v\vartheta & -\frac{1}{\sqrt{3}} s\vartheta + \frac{1}{3} v\vartheta & 1 - \frac{2}{3} v\vartheta \end{bmatrix} =$$

$\vartheta = 120°$

$s\vartheta = \frac{\sqrt{3}}{2}$

$v\vartheta = \frac{3}{2}$

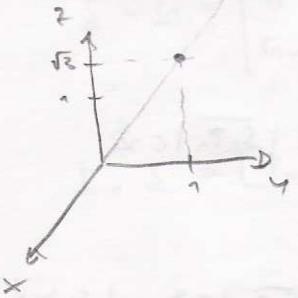
$$= \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}$$

PRIMER: $\boxed{\text{TIPIČNA IZPITNA NALOGA}}$

Z Rodriguesovo f. v matr. obl. napiši matriko ki opisuje rotacijo okrog osi, ki jo opisujeta enačbi:

$x = 0$

$z = \sqrt{3}\, y$



1. NAČIN:

$T = [0 \;\; 1 \;\; \sqrt{3}]$ ni normirana

$$0 + \left(\frac{1}{\omega}\right)^2 + \left(\frac{\sqrt{3}}{\omega}\right)^2 = 1$$

$$1 + 3 = \omega^2$$

$$\omega = 2$$

$$\underline{\omega} = \left[0 \;\; \frac{1}{2} \;\; \frac{\sqrt{3}}{2}\right]$$

2. NAČIN: SMERNI COSINUSI

$$\underline{\omega} = [\cos 90, \cos 60, \cos 30]$$

$$\underline{\underline{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_1 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -\sqrt{3} & 1 \\ \sqrt{3} & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\underline{\underline{\omega}}^2 = \frac{1}{4}\begin{bmatrix} 0 & -\sqrt{3} & 1 \\ \sqrt{3} & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -\sqrt{3} & 1 \\ \sqrt{3} & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 & 0 & 0 \\ 0 & -3 & \sqrt{3} \\ 0 & \sqrt{3} & -1 \end{bmatrix}$$

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & -\sqrt{3} & 1 \\ \sqrt{3} & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} s\vartheta + \frac{1}{4}\begin{bmatrix} -4 & 0 & 0 \\ 0 & -3 & -\sqrt{3} \\ 0 & \sqrt{3} & -1 \end{bmatrix} v\vartheta =$$

$$= \begin{bmatrix} 1 - v\vartheta & -\frac{\sqrt{3}}{2} s\vartheta & \frac{1}{2} s\vartheta \\ \frac{\sqrt{3}}{2} s\vartheta & 1 - \frac{3}{4} v\vartheta & \frac{\sqrt{3}}{4} v\vartheta \\ -\frac{1}{2} s\vartheta & \frac{\sqrt{3}}{4} v\vartheta & 1 - \frac{1}{4} v\vartheta \end{bmatrix}$$

4.5.2009

Orientacija v prostoru

T. Bajd

priredil T. Karčnik

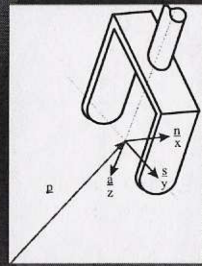
Matrika $\underline{\underline{T6}}$

- Manipulatorji imajo običajno šest prostostnih stopenj.
- Tri prostostne stopnje določajo položaj, tri orientacijo.
- Prijemalo moremo postaviti v poljubno pozicijo in orientacijo.
-

$$\underline{\underline{T6}} = {}^0\underline{\underline{H}}_1 \cdot {}^1\underline{\underline{H}}_2 \cdot {}^2\underline{\underline{H}}_3 \cdot {}^3\underline{\underline{H}}_4 \cdot {}^4\underline{\underline{H}}_5 \cdot {}^5\underline{\underline{H}}_6$$

- $\underline{\underline{T6}}$ tako predstavlja pozicijo in orientacijo vrha manipulatorja.

Koordinatni sistem prijemala



- Izhodišče K.S. je podano z vektorjem $\underline{p}$.
- Enotski vektorji $\underline{n}$, $\underline{s}$ in $\underline{a}$ podajajo orientacijo.
- $\underline{a}$ (approach - približevanje): os z
- $\underline{s}$ (slide - drsenje prstov): os y
- $\underline{n}$ (normal - normala): os x
- $\underline{n} = \underline{s} \times \underline{a}$

40

Struktura $\underline{\underline{T6}}$

$$\underline{\underline{T6}} = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Samo 12 elementov $\underline{\underline{T6}}$ relevantnih, od tega 9 elementov za opis rotacije.
- Poljubno orientacijo prijemala dosežemo s 3 robotskimi sklepi s po 1 DOF.
- Problem: določiti povezavo $9 \to 3$.

Struktura $\underline{\underline{T6}}$, 2.

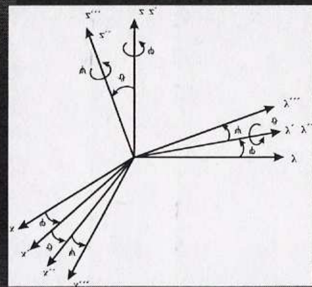
- Ker velja: $\underline{n} = \underline{s} \times \underline{a}$, je $\underline{n}$ redundanten.
- Ker velja:

$$\begin{aligned} \underline{s} \cdot \underline{s} &= 1 \\ \underline{a} \cdot \underline{a} &= 1 \\ \underline{s} \cdot \underline{a} &= 0 \end{aligned}$$

so v $\underline{s}$ in $\underline{a}$ **skupno** trije parametri redundantni.

- Za opis orientacije zadoščajo torej 3 parametri.

Eulerjevi koti



- Opišemo poljubno orientacijo.
- Rotacije okoli **trenutnih** osi.
- Uporaba postmultipliciranja.
- Interpretacija možna tudi v obratni smeri.

$$\underline{\underline{Euler}})\varphi, \vartheta, \psi,(= \underline{\underline{Rot}})z, \varphi(\underline{\underline{Rot}})y', \vartheta(\underline{\underline{Rot}})z'', \psi($$

Eulerjevi koti, izpeljava

$$\mathrm{Rot})z,\varphi(= \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & 0 \\ \sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathrm{Rot})y',\vartheta(= \begin{bmatrix} \cos\vartheta & 0 & -\sin\vartheta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\vartheta & 0 & \cos\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

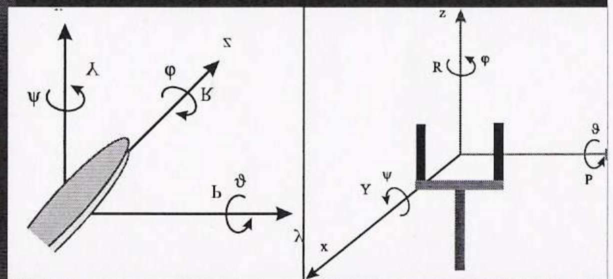
$$\mathrm{Rot})z'',\psi(= \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{Euler}})\varphi, \vartheta, \psi,(= \underline{\underline{Rot}})z, \varphi(\underline{\underline{Rot}})y', \vartheta(\underline{\underline{Rot}})z'', \psi(=$$

$$\begin{bmatrix} \cos\varphi\cdot\cos\vartheta\cdot\cos\psi - \sin\varphi\cdot\sin\psi & -\cos\varphi\cdot\cos\vartheta\cdot\sin\psi - \sin\varphi\cdot\cos\psi & \cos\varphi\cdot\sin\vartheta & 0 \\ \sin\varphi\cdot\cos\vartheta\cdot\cos\psi + \cos\varphi\cdot\sin\psi & -\sin\varphi\cdot\cos\vartheta\cdot\sin\psi + \cos\varphi\cdot\cos\psi & \sin\varphi\cdot\sin\vartheta & 0 \\ -\sin\vartheta\cdot\cos\psi & \sin\vartheta\cdot\sin\psi & \cos\vartheta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

EULERJEVA MATRIKA

RPY notacija



- R (roll) - kotaljenje
- P (pitch) - naklon
- Y (yaw) - odklon

RPY notacija, izpeljava

- Uporaba premultiplikacije.
- Vse transformacije izvajamo glede na referenčni K.S.
- $\underline{\underline{RPY}})\varphi, \vartheta, \psi,(= \underline{\underline{Rot}})z, \varphi(\underline{\underline{Rot}})y, \vartheta(\underline{\underline{Rot}})x, \psi(=$

$$\begin{bmatrix} \cos\varphi\cdot\cos\vartheta & \cos\varphi\cdot\sin\vartheta\cdot\sin\psi - \sin\varphi\cdot\cos\psi & \cos\varphi\cdot\sin\vartheta\cdot\cos\psi + \sin\varphi\cdot\sin\psi & 0 \\ \sin\varphi\cdot\cos\vartheta & \sin\varphi\cdot\sin\vartheta\cdot\sin\psi + \cos\varphi\cdot\cos\psi & \sin\varphi\cdot\sin\vartheta\cdot\cos\psi - \cos\varphi\cdot\sin\psi & 0 \\ -\sin\vartheta & \cos\vartheta\cdot\sin\psi & \cos\vartheta\cdot\cos\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Primer: RPY koti

Enačbe za vrh robota:

$$\underline{\underline{T6}} = \begin{bmatrix} -0.83 & -0.41 & -0.37 & 1.93 \\ -0.55 & 0.61 & 0.56 & 2.62 \\ 0 & 0.67 & -0.74 & 0.24 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} e_{1x} & e_{2x} & e_{3x} & p_x \\ e_{1y} & e_{2y} & e_{3y} & p_y \\ e_{1z} & e_{2z} & e_{3z} & p_x \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left]\begin{matrix} \cos\varphi\cdot\cos\vartheta & \cos\varphi\cdot\sin\vartheta\cdot\sin\psi-\sin\varphi\cdot\cos\psi & \cos\varphi\cdot\sin\vartheta\cdot\cos\psi+\sin\varphi\cdot\sin\psi & p_x \\ \sin\varphi\cdot\cos\vartheta & \sin\varphi\cdot\sin\vartheta\cdot\sin\psi+\cos\varphi\cdot\cos\psi & \sin\varphi\cdot\sin\vartheta\cdot\cos\psi-\cos\varphi\cdot\sin\psi & p_y \\ -\sin\vartheta & \cos\vartheta\cdot\sin\psi & \cos\vartheta\cdot\cos\psi & p_z \\ 0 & 0 & 0 & 1 \end{matrix}\right[$$

Najbolj enostavna rešitev ni najbolj zanesljiva! Uporabimo arctan)(.

Primer: RPY koti, ϑ

$$\sin\vartheta = -e_{1z}$$

$$\cos^2\vartheta = \frac{e_{1x}^2+e_{1y}^2+e_{2z}^2+e_{3z}^2}{2}$$

$$\vartheta = \arctan\frac{-e_{1z}}{\sqrt{\frac{1}{2}\left(e_{1x}^2+e_{1y}^2+e_{2z}^2+e_{3z}^2\right)}}$$

- Funkcija arctan)(nad kvocientom je štirikvadrantna in zato lahko določimo ϑ za polni kot.
- Še vedno pa ostanejo težave zaradi numerike; npr. pri majhni vrednosti $\cos\vartheta$. Velike težave pri praktični implementaciji!

Primer: RPY koti, φ

$$\begin{aligned}
e_{2z}e_{3x} &= \cos\vartheta\cdot\sin\psi\cdot)\sin\varphi\cdot\sin\psi+\cos\varphi\cdot\sin\vartheta\cdot\cos\psi(\\
e_{3z}e_{2x} &= \cos\vartheta\cdot\cos\psi\cdot)-\sin\varphi\cdot\cos\psi+\cos\varphi\cdot\sin\vartheta\cdot\sin\psi(\\
e_{2z}e_{3x}-e_{3z}e_{2x} &= \sin\varphi\cdot\cos\vartheta \\
e_{3z}e_{2y} &= \cos\vartheta\cdot\cos\psi\cdot)\cos\varphi\cdot\cos\psi+\sin\varphi\cdot\sin\vartheta\cdot\sin\psi(\\
e_{2z}e_{3y} &= \cos\vartheta\cdot\sin\psi\cdot)-\cos\varphi\cdot\sin\psi+\sin\varphi\cdot\sin\vartheta\cdot\cos\psi(\\
e_{3z}e_{2y}-e_{2z}e_{3y} &= \cos\varphi\cdot\cos\vartheta
\end{aligned}$$

$$\varphi = \arctan\frac{\sin\varphi}{\cos\varphi} = \arctan\frac{e_{2z}e_{3x}-e_{3z}e_{2x}}{e_{3z}e_{2y}-e_{2z}e_{3y}}$$

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Primer: RPY koti, ψ

$$\begin{aligned}
e_{1y}e_{3x} &= \sin\varphi\cdot\cos\psi\cdot)\sin\varphi\cdot\sin\psi+\cos\varphi\cdot\sin\vartheta\cdot\cos\psi(\\
e_{1x}e_{3y} &= \cos\varphi\cdot\cos\vartheta\cdot)-\cos\varphi\cdot\sin\psi+\sin\varphi\cdot\sin\vartheta\cdot\cos\psi(\\
e_{1y}e_{3x}-e_{1x}e_{3y} &= \cos\vartheta\cdot\sin\psi \\
e_{1x}e_{2y} &= \cos\varphi\cdot\cos\vartheta\cdot)\cos\varphi\cdot\cos\psi+\sin\varphi\cdot\sin\vartheta\cdot\sin\psi(\\
e_{1y}e_{2x} &= \sin\varphi\cdot\cos\vartheta\cdot)-\sin\varphi\cdot\cos\psi+\cos\varphi\cdot\sin\vartheta\cdot\sin\psi(\\
e_{1x}e_{2y}-e_{1y}e_{2x} &= \cos\vartheta\cdot\cos\psi
\end{aligned}$$

$$\psi = \arctan\frac{\sin\psi}{\cos\psi} = \arctan\frac{e_{1y}e_{3x}-e_{1x}e_{3y}}{e_{1x}e_{2y}-e_{1y}e_{2x}}$$

Primer: RPY koti, rezultati

$$\underline{\underline{T6}} = \begin{bmatrix} -0.83 & -0.41 & -0.37 & 1.93 \\ -0.55 & 0.61 & 0.56 & 2.62 \\ \mathbf{0} & 0.67 & -0.74 & 0.24 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\vartheta = \arctan\frac{-e_{1z}}{\sqrt{\frac{1}{2}\left(e_{1x}^2+e_{1y}^2+e_{2z}^2+e_{3z}^2\right)}} = 0, \quad \text{ker} \quad e_{1z}=0$$

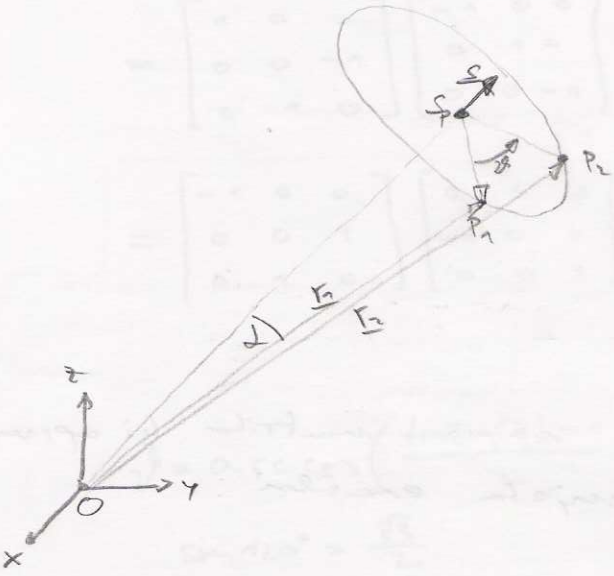
$$\varphi = \arctan\frac{e_{2z}e_{3x}-e_{3z}e_{2x}}{e_{3z}e_{2y}-e_{2z}e_{3y}} = 33.5^o$$

$$\psi = \arctan\frac{e_{1y}e_{3x}-e_{1x}e_{3y}}{e_{1x}e_{2y}-e_{1y}e_{2x}} = -42.2^o$$

RODRIGUESOVA FORMULA – VEKTORSKA OBLIKA

11.5.2009

$\underline{s} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$ enotski vektor!



$$\left.\begin{aligned} \underline{r}_1^T \cdot \underline{s} &= r_1 \cos\alpha \\ \underline{r}_2^T \cdot \underline{s} &= r_2 \cos\alpha \end{aligned}\right\} = |\overline{OS_P}|$$

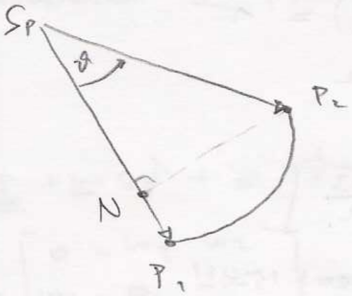
$$\overline{S_P P_1} = \underline{r}_1 - \overline{OS_P}$$

(1) $$\overline{S_P P_1} = \underline{r}_1 - \underbrace{(\underline{r}_1^T \underline{s})}_{\text{velikost}} \underbrace{\underline{s}}_{\text{smer}}$$

(2) $$\overline{S_P P_2} = \underline{r}_2 - (\underline{r}_2^T \underline{s})\underline{s}$$

$$|\overline{S_P P_1}| = r_1 \sin\alpha$$

$$|\overline{S_P P_1}| = |\underline{s} \times \underline{r}_1|$$



$$|\overline{S_P N}| = |\overline{S_P P_2}| c\vartheta = |\overline{S_P P_1}| c\vartheta$$

(3) $$\overline{S_P N} = \overline{S_P P_1} c\vartheta$$

$$|\overline{N P_2}| = |\overline{S_P P_2}| s\vartheta = |\overline{S_P P_1}| s\vartheta = |\underline{s} \times \underline{r}_1| s\vartheta$$

(4) $$\overline{N P_2} = (\underline{s} \times \underline{r}_1) s\vartheta$$

$$\overline{S_P P_2} = \overline{S_P N} + \overline{N P_2}$$

$$\underline{r}_2 - (\underline{r}_2^T \underline{s})\underline{s} = [\underline{r}_1 - (\underline{r}_1^T \underline{s})\underline{s}] c\vartheta + (\underline{s} \times \underline{r}_1) s\vartheta$$

$$\underline{r}_1^T \underline{s} = \underline{r}_2^T \underline{s}$$

$$\boxed{\underline{r}_2 = r_1 c\vartheta + (\underline{s} \times \underline{r}_1) s\vartheta + \underline{s}(\underline{r}_1^T \underline{s})(1 - c\vartheta)}$$

Rodriguesova formula (vektorska)

$$\boxed{\underline{r}_2 = \underline{R}\,\underline{r}_1}$$

PRIMER : IZPITNA NALOGA

$Rot(y, \vartheta)$

a) MATRIČNA OBLIKA:

$$\underline{R} = \underline{I} + \underline{\tilde{\omega}} \sin\vartheta + \underline{\tilde{\omega}}^2 (1 - \cos\vartheta) \qquad \underline{\omega} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\underline{\tilde{\omega}} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\underline{\tilde{\omega}}^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} s\vartheta + \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} (1 - c\vartheta)$$

$$\underline{R} = \begin{bmatrix} c\vartheta & 0 & s\vartheta \\ 0 & 1 & 0 \\ -s\vartheta & 0 & c\vartheta \end{bmatrix}$$

b) VEKTORSKA OBLIKA:

$$\underline{s} \times \underline{r}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ r_{1x} & r_{1y} & r_{1z} \end{vmatrix} = \begin{bmatrix} r_{1z} \\ 0 \\ -r_{1x} \end{bmatrix}$$

$$\underline{r}_1^T \underline{s} = [r_{1x} \; r_{1y} \; r_{1z}] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = r_{1y}$$

$$\begin{bmatrix} r_{2x} \\ r_{2y} \\ r_{2z} \end{bmatrix} = \begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{1z} \end{bmatrix} c\vartheta + \begin{bmatrix} r_{1z} \\ 0 \\ -r_{1x} \end{bmatrix} s\vartheta + \begin{bmatrix} 0 \\ r_{1y} \\ 0 \end{bmatrix} (1 - c\vartheta)$$

$$\underline{R} = \begin{bmatrix} c\vartheta & 0 & s\vartheta \\ 0 & 1 & 0 \\ -s\vartheta & 0 & c\vartheta \end{bmatrix}$$

IZPIT : a) rotacija okoli ene (x, y, z) osi na oba načina

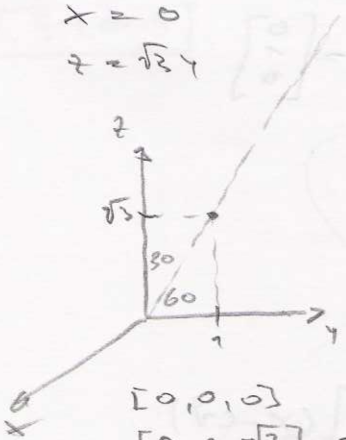
b) rot. okoli poljubne osi na enega od načinov

osi ki ležijo v eni od ravnin z naklonom 30°, 45°, 60° na os

PRIMER: TIPIČNA IZPITNA NALOGA

$x = 0$

$z = \sqrt{3}\, y$



$[0, 0, 0]$

$[0, 1, \sqrt{3}]$ → to se mi vektor s, rabimo enotski vektor

$$\underline{s} = [\cos 0^\circ, \cos 60^\circ, \cos 30^\circ]$$

ali

$$\underline{s} = \frac{1}{2}[0 \;\; 1 \;\; \sqrt{3}]$$

$$\underline{r}_2 = \underline{r}_1 c\vartheta + (\underline{s} \times \underline{r}_1) s\vartheta + \underline{s}(\underline{r}_1^T \underline{s})(1 - c\vartheta)$$

$$\underline{s} \times \underline{r}_1 = \frac{1}{2}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & \sqrt{3} \\ r_{1x} & r_{1y} & r_{1z} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} r_{1z} - \sqrt{3} r_{1y} \\ \sqrt{3} r_{1x} \\ -r_{1x} \end{bmatrix}$$

$$\underline{r}_1^T \underline{s} = \frac{1}{2}[r_{1x} \;\; r_{1y} \;\; r_{1z}]\begin{bmatrix} 0 \\ 1 \\ \sqrt{3} \end{bmatrix} = \frac{1}{2}(r_{1y} + \sqrt{3} r_{1z})$$

$$\begin{bmatrix} r_{2x} \\ r_{2y} \\ r_{2z} \end{bmatrix} = \begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{1z} \end{bmatrix} c\vartheta + \frac{1}{2}\begin{bmatrix} r_{1z} - \sqrt{3} r_{1y} \\ \sqrt{3} r_{1x} \\ -r_{1x} \end{bmatrix} s\vartheta + \frac{1}{4}(r_{1y} + \sqrt{3} r_{1z})\begin{bmatrix} 0 \\ 1 \\ \sqrt{3} \end{bmatrix}(1 - c\vartheta)$$

$$\begin{bmatrix} r_{2x} \\ r_{2y} \\ r_{2z} \end{bmatrix} = \begin{bmatrix} c\vartheta & -\frac{\sqrt{3}}{2} s\vartheta & \frac{1}{2} s\vartheta \\ \frac{\sqrt{3}}{2} s\vartheta & \frac{1}{4} + \frac{3}{4} c\vartheta & \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} c\vartheta \\ -\frac{s\vartheta}{2} & \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} c\vartheta & \frac{3}{4} + \frac{1}{4} c\vartheta \end{bmatrix}\begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{1z} \end{bmatrix}$$

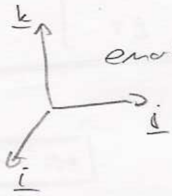
Če bi namesto $1 - c\vartheta$ pisali $v\vartheta$ (vers ϑ), bi dobili enak rezultat, kot pri uporabi matrične R-formule.

OPIS ROTACIJE S KVATERNIONI

RPY, Euler . . . 3
Kvaternioni . . . 4 parametrov   1843 – Hamilton
$\underline{H}$ 9

Kvaternioni so posplošitev kompleksnih števil

$$q = q_0 1 + \underbrace{q_1 \underline{i} + q_2 \underline{j} + q_3 \underline{k}}_{\underline{q}} \qquad (q = q_0 + \underline{q})$$



enotski vektorji $\underline{i}, \underline{j}, \underline{k}$

$i^2 = j^2 = k^2 = -1$ – imaginarno število

$ij = k \qquad ji = -k$
$jk = i$
$ki = j$

desno pravilo

$q^* = q_0 - q_1 \underline{i} - q_2 \underline{j} - q_3 \underline{k}$ konjugirani kvaternion

↳ PRODUKT KVATERNIONOV: SIGURNO NA IZPITU !

$$pq = (p_0 + p_1 \underline{i} + p_2 \underline{j} + p_3 \underline{k})(q_0 + q_1 \underline{i} + q_2 \underline{j} + q_3 \underline{k}) =$$

$$= p_0 q_0 + q_0 (p_1 \underline{i} + p_2 \underline{j} + p_3 \underline{k}) + p_0 (q_1 \underline{i} + q_2 \underline{j} + q_3 \underline{k}) +$$

$$+ p_1 q_1 \underline{i}\underline{i} + p_2 q_1 \overset{-k}{\underline{j}\underline{i}} + p_3 q_1 \overset{j}{\underline{k}\underline{i}} +$$

$$+ p_1 q_2 \overset{k}{\underline{i}\underline{j}} + p_2 q_2 \underline{j}\underline{j} + p_3 q_2 \overset{-i}{\underline{k}\underline{j}} +$$

$$+ p_1 q_3 \overset{-j}{\underline{i}\underline{k}} + p_2 q_3 \overset{i}{\underline{j}\underline{k}} + p_3 q_3 \underline{k}\underline{k} =$$

$$= p_0 q_0 + q_0 \underline{p} + p_0 \underline{q} -$$

$$- p_1 q_1 - p_2 q_2 - p_3 q_3 +$$

$$+ (p_2 q_3 - p_3 q_2) \underline{i} +$$

$$+ (p_3 q_1 - p_1 q_3) \underline{j} +$$

$$+ (p_1 q_2 - p_2 q_1) \underline{k}$$

$$\boxed{pq = \underbrace{p_0 q_0 - \underline{p}\,\underline{q}}_{\text{SKALAR}} + \underbrace{p_0 \underline{q} + q_0 \underline{p} + \underline{p} \times \underline{q}}_{\text{VEKTOR}}}$$

$r = pq$ →

$r = p\,g$

$r_0 = p_0 g_0 - p_1 g_1 - p_2 g_2 - p_3 g_3$

vzdolž $\underline{i}$ $\quad r_1 = p_1 g_0 + p_0 g_1 + p_2 g_3 - p_3 g_2$

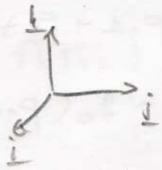
vzdolž $\underline{j}$ $\quad r_2 = p_2 g_0 + p_0 g_2 + p_3 g_1 - p_1 g_3$

vzdolž $\underline{k}$ $\quad r_3 = p_3 g_0 + p_0 g_3 + p_1 g_2 - p_2 g_1$

$$\boxed{\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} \equiv \begin{bmatrix} p_0 & -\underline{p}^T \\ \underline{p} & p_0 \underline{\underline{I}} + \underline{p} \end{bmatrix}}$$

PRIMER: MOŽNA S. NALOGA

$(2 + 3\underline{i} - \underline{j} + 5\underline{k})(3 - 4\underline{i} + 2\underline{j} + \underline{k}) =$



$= 6 + 9\underline{i} - 3\underline{j} + 15\underline{k} -$

$-8\underline{i} - 12\underline{i}^2 + 4\underline{j}\underline{i} - 20\underline{k}\underline{i} +$

$+4\underline{j} + 6\underline{i}\underline{j} - 2\underline{j}^2 + 10\underline{k}\underline{j} +$

$+2\underline{k} + 3\underline{i}\underline{k} - \underline{j}\underline{k} + 5\underline{k}^2 =$ $\quad \boxed{\text{1. NAČIN}}$

$= 6 + 9\underline{i} - 3\underline{j} + 15\underline{k} -$

$-8\underline{i} + 12 - 4\underline{k} - 20\underline{j} +$

$+4\underline{j} + 6\underline{k} + 2 - 10\underline{i} +$

$+2\underline{k} - 3\underline{j} - \underline{i} - 5 = \underline{\underline{15 - 10\underline{i} - 22\underline{j} + 19\underline{k}}}$

$\boxed{\text{2. NAČIN}}$ po enačbi:

$p\,g = p_0 g_0 - \underline{p}\cdot\underline{g} + p_0 \underline{g} + g_0 \underline{p} + \underline{p} \times \underline{g} = (p_0 + \underline{p})(g_0 + \underline{g})$

$$\left(2+\begin{bmatrix}3\\-1\\5\end{bmatrix}\right)\left(3+\begin{bmatrix}-4\\2\\1\end{bmatrix}\right)=$$

$$=6-\begin{bmatrix}3&-1&5\end{bmatrix}\begin{bmatrix}-4\\2\\1\end{bmatrix}+2\begin{bmatrix}-4\\2\\1\end{bmatrix}+3\begin{bmatrix}3\\-1\\5\end{bmatrix}+\begin{bmatrix}i&j&k\\3&-1&5\\-4&2&1\end{bmatrix}=$$

$$=6+9+\begin{bmatrix}1\\1\\17\end{bmatrix}+\begin{bmatrix}-11\\-23\\2\end{bmatrix}=$$

$$=15+\begin{bmatrix}-10\\-22\\19\end{bmatrix}$$

$\boxed{\text{3. NAČIN}}$

$$\begin{bmatrix}r_0\\r_1\\r_2\\r_3\end{bmatrix}=\begin{bmatrix}2&-1&1&-5\\3&2&-5&-1\\-1&5&2&-3\\5&1&3&2\end{bmatrix}\begin{bmatrix}3\\-4\\2\\1\end{bmatrix}=\begin{bmatrix}15\\-10\\-22\\19\end{bmatrix}$$

↳ ROTACIJA:

$$\boxed{v' = q\,v\,q^*} \qquad (\underline{v}' = \underline{R}\,\underline{v})$$

$q = q_0 + \underline{q}$

$q^* = q_0 - \underline{q}$

$v = 0 + \underline{v}$ } v resnici nista kvaterniona, ampak jih moram

$v' = 0 + \underline{v}'$ } kot taka obravnavati

$$p_0 q_0 - \underline{p}\,\underline{q} + p_0\underline{q} + q_0\underline{p} + \underline{p}\times\underline{q}$$

$$v' = (q_0+\underline{q})(0+\underline{v})\,q^* =$$

$$=(\underbrace{-\underline{q}\,\underline{v}}_{\text{skalar}}\underbrace{+q_0\underline{v}+\underline{q}\times\underline{v}}_{\text{vektor}})(q_0-\underline{q})=$$

$$=\cancel{-\underline{q}\,\underline{v}\,q_0}\cancel{+q_0\underline{v}\,\underline{q}}+\underbrace{(\underline{q}\times\underline{v})\underline{q}}_{\emptyset}+(\underline{q}\,\underline{v})\underline{q}+q_0^2\underline{v}+q_0(\underline{q}\times\underline{v})-$$

$$-q_0\underline{v}\times\underline{q}-\underline{q}\times\underline{v}\times\underline{q}$$

$\Downarrow$ $\qquad\qquad\qquad$ $\Downarrow$ BRONŠTAJN

$$+q_0\underline{q}\times\underline{v} \qquad -(\underline{q}\,\underline{q})\underline{v}+(\underline{v}\,\underline{q})\underline{q}$$

$$v' = q_0^2\underline{v}-(\underline{q}\,\underline{q})\underline{v}+2q_0(\underline{q}\times\underline{v})+2\underline{q}(\underline{q}\cdot\underline{v})=$$

$$=(q_0^2-\underline{q}\,\underline{q})\underline{v}+2q_0\begin{bmatrix}0&-q_3&q_2\\q_3&0&-q_1\\-q_2&q_1&0\end{bmatrix}\underline{v}+2\underline{q}\,\underline{q}^T\underline{v}= \longrightarrow$$

$$= \left\{ (q_0^2 - q_1^2 - q_2^2 - q_3^2)\underline{\underline{I}} + 2q_0 \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} + 2 \begin{bmatrix} q_1^2 & q_1 q_2 & q_1 q_2 \\ q_1 q_2 & q_2^2 & q_2 q_3 \\ q_1 q_2 & q_2 q_3 & q_3^2 \end{bmatrix} \right\} \underline{v}$$

$$\underline{\underline{R}} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_3 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_2 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

$$\boxed{q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\underline{s}}$$

ENOTSKI KVATERNION (vedno kadar pomeni zasuk)

ϑ ... kot zasuka

$\underline{s}$ ... enotski vektor okoli katerega vrtimo

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$$

$v' \to r_2$

$v \to r_1$

$$r_2 = q_0^2 r_1 - (\underline{q}\,\underline{q}) r_1 + 2q_0(\underline{q} \times r_1) + 2\underline{q}(\underline{q}\,\underline{v})$$

$$q = q_0 + \underline{q} = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\underline{s}$$

$$r_2 = \cos^2\frac{\vartheta}{2} r_1 - \sin^2\frac{\vartheta}{2} \underbrace{\underline{s}\,\underline{s}}_{1} r_1 + 2\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2}(\underline{s} \times \underline{r}_1) + 2\sin^2\frac{\vartheta}{2}\underline{s}(\underline{s}\,\underline{r}_1)$$

$$\sin\vartheta = 2\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}$$

$$\cos\vartheta = \cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2}$$

$$1 = \cos^2\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2}$$

$$\boxed{\underline{r}_2 = \underline{r}_1\cos\vartheta + (\underline{s} \times \underline{r}_1)\sin\vartheta + \underline{s}(\underline{r}_1^T \underline{s})(1 - \cos\vartheta)}$$

ROTACIJA $\to$ KVATERNIONI

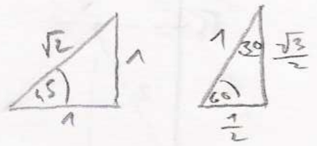
18.5.2009

$$p = p_0 + \underline{p}$$
$$\underline{p} = i p_1 + j p_2 + k p_3$$
$$\boxed{p = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\underline{n}}$$

PRIMER:

Zamenjaj produkt rotacijskih matrik s produktom kvaternionov.

$$Rot(y,90)\,Rot(z,90)$$



$Rot(y,90)$

$$p = \cos 45^\circ + \sin 45^\circ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$p_0 = \frac{\sqrt{2}}{2} \qquad \underline{p} = \frac{\sqrt{2}}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$Rot(z,90)$

$$q_0 = \frac{\sqrt{2}}{2} \qquad \underline{q} = \frac{\sqrt{2}}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$pq = p_0 q_0 - \underline{p}\,\underline{q} + p_0 \underline{q} + q_0 \underline{p} + \underline{p} \times \underline{q}$$
$$= \frac{1}{2} - \frac{1}{2}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$
$$= \frac{1}{2} + \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} =$$
$$= \frac{1}{2} + \frac{1}{2} i + \frac{1}{2} j + \frac{1}{2} k$$

PREIZKUS:

$$\underline{R} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & & \\ 2(q_1 q_2 + q_0 q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

=

$$\underline{R} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

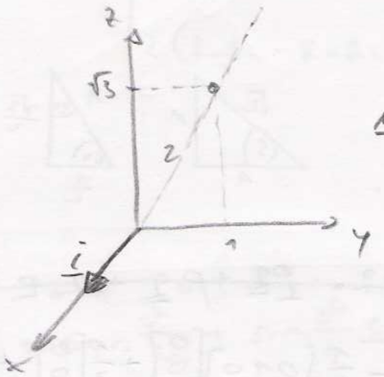
3 PRIMER:

Zasuči enotski vektor i za 90 stopinj okoli dane osi v pozitivni smeri.

$\underline{i}$ $\quad 90^\circ$

$x = 0$

$z = \sqrt{3}\, y$



$$\underline{u} = \left[0, \frac{1}{2}, \frac{\sqrt{3}}{2}\right]$$

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\underline{u} \qquad \frac{\vartheta}{2}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\begin{bmatrix}0\\ \frac{1}{2}\\ \frac{\sqrt{3}}{2}\end{bmatrix} = \frac{\sqrt{2}}{2} + \begin{bmatrix}0\\ \frac{\sqrt{2}}{4}\\ \frac{\sqrt{6}}{4}\end{bmatrix}$$

$$q^2 = \frac{1}{2} + \frac{2}{16} + \frac{6}{16} = 1$$ mora biti enotski kvaternion

$$\boxed{v' = q\,v\,q^*} = \underbrace{\begin{bmatrix}\frac{\sqrt{2}}{2}\\ 0\\ \frac{\sqrt{2}}{4}\\ \frac{\sqrt{6}}{4}\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\\0\end{bmatrix}}_{1.\ \text{množ.}}\cdot\begin{bmatrix}\frac{\sqrt{2}}{2}\\ 0\\ -\frac{\sqrt{2}}{4}\\ -\frac{\sqrt{6}}{4}\end{bmatrix} =$$

(2. množenje)

$$\begin{bmatrix}r_0\\r_1\\r_2\\r_3\end{bmatrix} = \begin{bmatrix}p_0 & -\underline{p}^T\\ \underline{p} & p_0\underline{I} + \underline{\tilde{p}}\end{bmatrix}\begin{bmatrix}q_0\\q_1\\q_2\\q_3\end{bmatrix} = \begin{bmatrix}p_0 & -p_1 & -p_2 & -p_3\\ p_1 & p_0 & -p_3 & p_2\\ p_2 & p_3 & p_0 & -p_1\\ p_3 & -p_2 & p_1 & p_0\end{bmatrix}$$

1. MNOŽENJE:

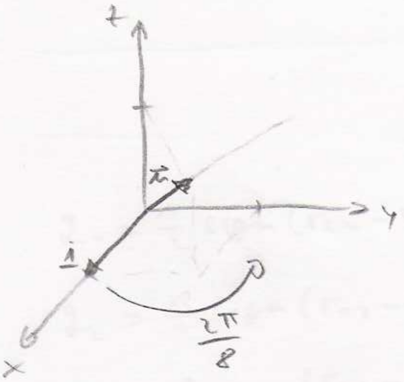
$$\begin{bmatrix}r_0\\r_1\\r_2\\r_3\end{bmatrix} = \begin{bmatrix}-p_1\\p_0\\p_3\\-p_2\end{bmatrix} = \begin{bmatrix}0\\ \frac{\sqrt{2}}{2}\\ \frac{\sqrt{6}}{4}\\ -\frac{\sqrt{2}}{4}\end{bmatrix}$$

2. MNOŽENJE:

$$\begin{bmatrix}r_0\\r_1\\r_2\\r_3\end{bmatrix} = \begin{bmatrix}0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{4}\\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{4} & \frac{\sqrt{6}}{4}\\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{2}}{4} & 0 & -\frac{\sqrt{2}}{2}\\ -\frac{\sqrt{2}}{4} & -\frac{\sqrt{6}}{4} & \frac{\sqrt{2}}{2} & 0\end{bmatrix}\cdot\begin{bmatrix}\frac{\sqrt{2}}{2}\\ 0\\ -\frac{\sqrt{2}}{4}\\ -\frac{\sqrt{6}}{4}\end{bmatrix} = \begin{bmatrix}\frac{\sqrt{12}}{16} - \frac{\sqrt{12}}{16}\\ \frac{2}{4} - \frac{2}{16} - \frac{6}{16}\\ \frac{\sqrt{12}}{8} + \frac{\sqrt{12}}{8}\\ -\frac{2}{8} - \frac{2}{8}\end{bmatrix} = \begin{bmatrix}0\\0\\ \frac{\sqrt{3}}{2}\\ -\frac{1}{2}\end{bmatrix} = v'$$

PRIMER:

Zavrtite enotski vektor i za $\frac{2\pi}{3}$ okoli osi, ki gre skozi koord. izh in točko $T[1,1,1]$.



$$! \ \vec{n} \neq [1,1,1] \ !$$

$$n_x = n_y = n_z = n$$

$$3n^2 = 1$$

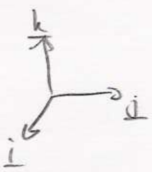
$$n = \frac{1}{\sqrt{3}}$$

$$\underline{n} = \frac{1}{\sqrt{3}}[1,1,1]$$

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\underline{n} =$$

$$= \cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cdot\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$= \frac{1}{2} + \frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{2}\underline{k}$$



$$v' = q v q^* = \frac{1}{4}(1 + \underline{i} + \underline{j} + \underline{k})(\underline{i})(1 - \underline{i} - \underline{j} - \underline{k}) =$$

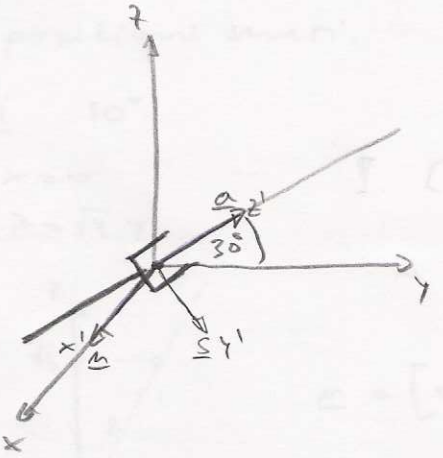
$$= \frac{1}{4}(\underline{i} - 1 - \underline{k} + \underline{j})(1 - \underline{i} - \underline{j} - \underline{k}) =$$

$$= \frac{1}{4}(\underline{i} - 1 - \underline{k} + \underline{j} -$$
$$- i^2 + \underline{i} + \underline{k}\underline{i} - \underline{j}\underline{i} -$$
$$- \underline{i}\underline{j} + \underline{j} + \underline{k}\underline{j} - j^2 -$$
$$- \underline{i}\underline{k} + \underline{k} + k^2 - \underline{j}\underline{k}) =$$

$$= \frac{1}{4}(\cancel{i} - \cancel{1} - \cancel{k} + j +$$
$$+ \cancel{1} + \cancel{i} + j + \cancel{k} -$$
$$- \cancel{k} + j - \cancel{i} + \cancel{1} +$$
$$+ j + \cancel{k} - \cancel{1} - \cancel{i}) =$$

$$= \frac{1}{4}4j = \underline{\underline{j}}$$

→ ORIENTACIJA PRIJEMALA:



Prijemalo v ravnini lista

① ROTACIJSKA MATRIKA:

$\measuredangle n x = 0 \qquad \measuredangle s x = 90° \qquad \measuredangle a x = 90°$

$\measuredangle n y = 90° \qquad \measuredangle s y = 60° \qquad \measuredangle a y = 30°$

$\measuredangle n z = 90° \qquad \measuredangle s z = 150° \qquad \measuredangle a z = 60°$

↓ v resnici 270 (−90) ampak je vseeno, ker bomo računali skalarne produkte

$$\underline{R} = \begin{bmatrix} n_x & s_x & a_x \\ n_y & s_y & a_y \\ n_z & s_z & a_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

② RPY:

$\varphi = 0$

$\vartheta = 0$

$\psi = -60°$

$$RPY(\varphi, \vartheta, \psi) = \underbrace{Rot(z, \varphi)}_{= \underline{I}} \underbrace{Rot(y, \vartheta)}_{= \underline{I}} Rot(x, \psi) =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

③ KVATERNIONI:

$$\text{trace}\,\underline{R} = r_{11} + r_{22} + r_{33} = 3q_0^2 - q_1^2 - q_2^2 + q_3^2$$

$$q_1^2 + q_2^2 + q_3^2 = 1 - q_0^2$$

$$\text{trace}\,\underline{R} = 4q_0^2 - 1$$

$$q_0^2 = \frac{1}{4}(1 + r_{11} + r_{22} + r_{33})$$

$$\underline{w} = \frac{1}{2 \sin\vartheta} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

$$q_1 = \frac{1}{2} \operatorname{sign}(r_{32} - r_{23}) \sqrt{1 + r_{11} - r_{22} - r_{33}}$$

$$q_2 = \frac{1}{2} \operatorname{sign}(r_{13} - r_{31}) \sqrt{1 - r_{11} + r_{22} - r_{33}}$$

$$q_3 = \frac{1}{2} \operatorname{sgn}(r_{21} - r_{12}) \sqrt{1 - r_{11} - r_{22} + r_{33}}$$

<u>NAŠ PRIMER:</u>

$$q_0 = \frac{1}{2} \sqrt{1 + 1 + \frac{1}{2} + \frac{1}{2}} = \frac{\sqrt{3}}{2}$$

$$q_1 = \frac{1}{2} (-1) \sqrt{1 + 1 - \frac{1}{2} - \frac{1}{2}} = -\frac{1}{2}$$

$$q_2 = 0$$

$$q_3 = 0$$

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2} \underline{u} =$$

$$= \cos(-30) + \sin(-30) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{\underline{\vartheta = -60^\circ}}$$