OSNOVE ROBOTIKE 3

zapiski predavanj AVT

Šolsko leto	2009 / 2010
Izvajalec	Tadej Bajd
Avtor dokumenta	Lea Retelj
Sodelavci	Gregor Treska (skeniranje)

UREJANJE DOKUMENTA

VERZIJA	01	**REVIZIJA**	01
DATUM	20. 6. 2010		
ZADNJI POPRAVLJAL	/		
PREGLEDAL	Gregor Treska		

OPOMBE

Tiskajte na čb in dvignite kontrast (trenutno imamo težave z opremo, boljša verzija je v nastajanju).

POPRAVKI

1. UVOD
2. HOMOGENE TRANSFORMACIJE
matrike 4x4
- lega (pozicija, orientacija)
- premik (translacija, rotacija)
- perspektiva
- premiki glede na referenčni in relativni koord. sis.

(1.) izpitna naloga →

3. DIREKTNI GEOMETRIJSKI MODEL
- (2.) skalarni parametri (Denavit-Hartenberg) (4)
- (3.) vektorski parametri (6)

4. INVERZNI GEOMETRIJSKI MODEL

5. ROTACIJA IN ORIENTACIJA V PROSTORU
- (4.) rotacija okrog poljubne osi – Rodriguez
- Euler & RPY
- (5.) kvaternioni

LITERATURA:

Bajd: OSNOVE ROBOTIKE ← osnova za predavanja
Lenarčič, Bajd: ROBOTSKI MEHANIZMI
Bajd, Mihelj: ROBOTIKA
Bajd, Mihelj ... : ROBOTICS

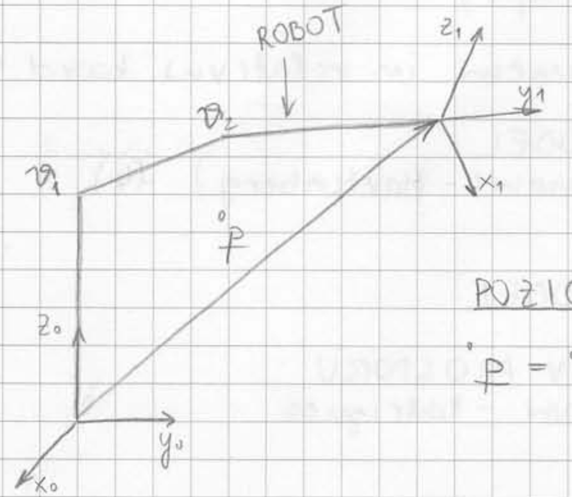
IZPIT: 5 nalog (katere, so označene na levi)
ustni del → pregled pisnega dela
izpeljave niso nujne

POSEBNA PREDAVANJA

1. marec 10.30 HELLA
8. marec Ude: Humanoidna robotika 11.15
15. marec Mihelj: Haptični roboti
22. marec Kontnik: DAX & industrijska robotika
29. marec Kamnik: Mobilna robotika, gradbeni robot
5. april
12. april IJS: Lenarčič, Stanisic
19. april Munih: Rehabilitacijska robotika
26. april
3. maj Žlajpah: Servisna robotika

VAJE
petek 26. 2.
prinočnik
1. ABB-varjenje
2. Robot Studio
3. DH
4. EPSON-vid
5. Stäubli-Phonton
6. Humanoidni robot

HOMOGENE TRANSFORMACIJSKE MATRIKE



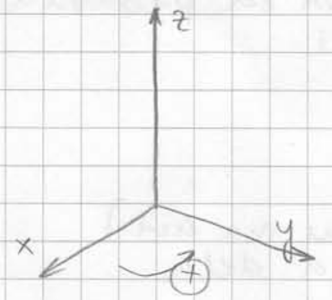
lega $\underline{H}$ 4×4
orientacija $\underline{R}$ 3×3
pozicija $\underline{p}$

POZICIJA

$$^0\underline{p} = {}^0p_x\underline{i} + {}^0p_y\underline{j} + {}^0p_z\underline{k} = \begin{bmatrix}{}^0p_x\\ {}^0p_y\\ {}^0p_z\end{bmatrix} = \begin{bmatrix}{}^0p_x\\ {}^0p_y\\ {}^0p_z\\ 1\end{bmatrix}$$

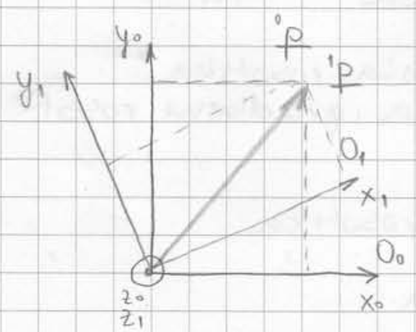
v računalništvu

ORIENTACIJA:



pozitivna smer zasuka je v nasprotni smeri urinega kazalca, ko gledamo pozitivne osi koordinatnega sistema.

Da določimo orientacijo zgornjega koordinatnega sistema narišemo skupaj.



O_0: $^1\underline{p} = {}^1p_x{}^1\underline{i} + {}^1p_y{}^1\underline{j} + {}^1p_z{}^1\underline{k}$

O_1: $^0\underline{p} = {}^0p_x{}^0\underline{i} + {}^0p_y{}^0\underline{j} + {}^0p_z{}^0\underline{k} \quad / {}^0\underline{i}$

$$^0\underline{p}\,{}^0\underline{i} = {}^0p_x = {}^1\underline{p}\,{}^0\underline{i} = {}^1p_x{}^1\underline{i}\,{}^0\underline{i} + {}^1p_y{}^1\underline{j}\,{}^0\underline{i} + {}^1p_z{}^1\underline{k}\,{}^0\underline{i}$$

$$^0\underline{p}\,{}^0\underline{j} = {}^0p_y = {}^1\underline{p}\,{}^0\underline{j} = {}^1p_x{}^1\underline{i}\,{}^0\underline{j} + {}^1p_y{}^1\underline{j}\,{}^0\underline{j} + {}^1p_z{}^1\underline{k}\,{}^0\underline{j}$$

V matrični obliki

$$\begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix} = {}^0\underline{\underline{R_1}} \begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix}$$

$$\begin{array}{c} {}^0\underline{\underline{R_1}} = \underbrace{\begin{array}{ccc} {}^1x & {}^1y & {}^1z \end{array}}_{O_1} \end{array}$$

$$\begin{array}{cc} {}^0\underline{\underline{R_1}} = \underbrace{\begin{bmatrix} {}^1i\,{}^0i & {}^1j\,{}^0i & {}^1k\,{}^0i \\ {}^1i\,{}^0j & {}^1j\,{}^0j & {}^1k\,{}^0j \\ {}^1i\,{}^0k & {}^1j\,{}^0k & {}^1k\,{}^0k \end{bmatrix}}_{\substack{{}^1x \quad {}^1y \quad {}^1z \\ O_1}} & \left.\begin{array}{c} {}^0x \\ {}^0y \\ {}^0z \end{array}\right\} O_0 \end{array}$$

Orientacija O_1 glede na O_0

$${}^0\underline{\underline{R_1}} = \begin{bmatrix} \cos\vartheta_{ii} & & \\ & \ddots & \\ & & \cos\vartheta_{kk} \end{bmatrix}$$

$${}^1p = {}^1p_x\,{}^1i + {}^1p_y\,{}^1j + {}^1p_z\,{}^1k \quad / {}^1i, {}^1j, {}^1k$$

$$\begin{bmatrix} {}^1p_x \\ {}^1p_y \\ {}^1p_z \end{bmatrix} = {}^1\underline{\underline{R_0}} \begin{bmatrix} {}^0p_x \\ {}^0p_y \\ {}^0p_z \end{bmatrix}$$

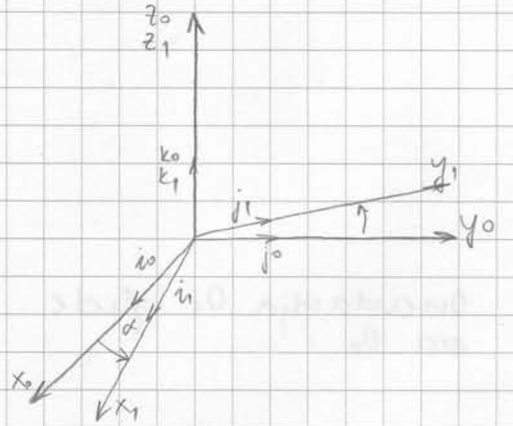
$$\begin{array}{cc} {}^1\underline{\underline{R_0}} = \underbrace{\begin{bmatrix} {}^0i\,{}^1i & {}^0j\,{}^1i & {}^0k\,{}^1i \\ {}^0i\,{}^1j & {}^0j\,{}^1j & {}^0k\,{}^1j \\ {}^0i\,{}^1k & {}^0j\,{}^1k & {}^0k\,{}^1k \end{bmatrix}}_{\substack{x_0 \quad y_0 \quad z_0 \\ O_0}} & \left.\begin{array}{c} x_1 \\ y_1 \\ z_1 \end{array}\right\} O_1 \end{array}$$

${}^1i\,{}^0j = {}^0j\,{}^1i \longrightarrow$ KOMUTATIVNOST

Sledi:

$$\boxed{{}^1\underline{\underline{R_0}} = \left({}^0R_1\right)^{-1} = {}^0\underline{\underline{R_1}}^T}$$

ORTOGONALNE MATRIKE
DETERMINANTA JE +1



$${}^0k\,{}^1k = 1$$

$${}^0i\,{}^1i = \cos\alpha$$

$${}^0j\,{}^1j = \cos\alpha$$

$${}^0i\,{}^1j = \cos(90°+\alpha) = -\sin\alpha$$

$${}^0j\,{}^1i = \cos(90°-\alpha) = \sin\alpha$$

$$\underline{\underline{R}}_{z,\alpha} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{R}}_{x,\alpha} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\underline{\underline{R}}_{y,\alpha} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$$

TRANSLACIJA

$$\underline{\underline{R}}_T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^0i\,{}^1i = {}^0j\,{}^1j = 1$$

$$\underline{\underline{R}}_{x\alpha_1} \cdot \underline{\underline{R}}_{x\alpha_2} = \underline{\underline{R}}_{x(\alpha_1+\alpha_2)}$$

$$\underline{\underline{R}}_{x\alpha_1}^{-1} = \underline{\underline{R}}_{x,-\alpha_1}$$

ZAPOREDNE ROTACIJE

O_0 , O_1 , O_2 izhodišča k.s. v isti točki

P ... točka v koordinatnih sistemih

${}^0\underline{p}$ ${}^1\underline{p}$ ${}^2\underline{p}$

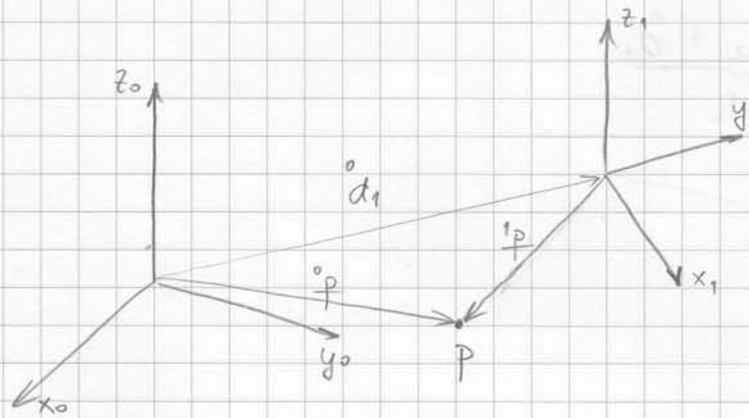
$${}^0\underline{p} = {}^0\underline{\underline{R}}_1 \, {}^1\underline{p}$$

$${}^1\underline{p} = {}^1\underline{\underline{R}}_2 \, {}^2\underline{p}$$

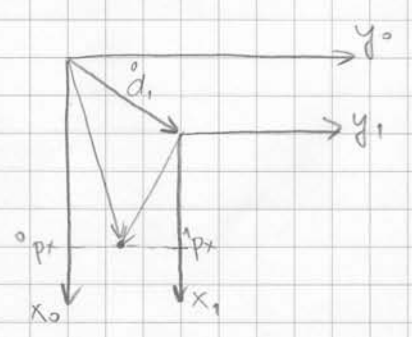
$${}^0p = {}^0\underline{\underline{R}}_1 \cdot {}^1\underline{\underline{R}}_2 \cdot {}^2p = {}^0\underline{\underline{R}}_2 \, {}^2p$$

$$\boxed{{}^0\underline{\underline{R}}_n = {}^0\underline{\underline{R}}_1 \, {}^1\underline{\underline{R}}_2 \cdot \ldots \cdot {}^{n-1}\underline{\underline{R}}_n}$$

LEGA



POGLED od zgoraj pri zasukanem k.s.



$^0p_x = {^0d}_{1x} + {^1p}_x$
$^0p_y =$
$^0p_z =$

$^0\underline{p} = {^0d}_1 + {^1p}$

$$\boxed{^0p = {^0\underline{\underline{R}}}_1 \, {^1p} + {^0d}_1}$$

ZAPOREDNE LEGE

$O_0 \quad O_1 \quad O_2$

P - točka

$^0p \quad {^1p} \quad {^2p}$ ← vektorji iz posameznih k.s. do točke

$^0p = {^0R}_1 \, {^1p} + {^0d}_1$
$^1p = {^1R}_2 \, {^2p} + {^1d}_2$

$$^0p = \underbrace{{^0R}_1 \cdot {^1R}_2}_{^0R_2} {^2p} + \underbrace{{^0R}_1 \, {^1d}_2 + {^0d}_1}_{^0d_2}$$

$$\boxed{^0p = {^0\underline{\underline{R}}}_2 \, {^2p} + {^0d}_2}$$

$^0p = R \, {^1p} + d$

$$\begin{bmatrix} ^0p \\ 1 \end{bmatrix} = \underline{\underline{H}} \begin{bmatrix} ^1p \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} ^0p \\ 1 \end{bmatrix} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} ^1p \\ 1 \end{bmatrix}$$

Zaporedna lega

$$\begin{bmatrix} {}^0R_1 & {}^0d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^1R_2 & {}^1d_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} {}^0R_1 {}^1R_2 & {}^0R_1 {}^1d_2 + {}^0d_1 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{\begin{gathered} {}^0H_1 \cdot {}^1H_2 = {}^0H_2 \\ {}^0H_n = {}^0H_1 {}^1H_2 \cdots {}^{n-1}H_n \end{gathered}}$$

Lega & premik

$$\begin{bmatrix} o & o & o & \times \\ o & o & o & \times \\ o & o & o & \times \\ \square & \square & \square & 1 \end{bmatrix}$$

o < orientacija, rotacija

× < translacija, pozicija

□ – perspektiva

$$\begin{bmatrix} {}^0p \\ 1 \end{bmatrix} = H \begin{bmatrix} {}^1p \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^0p \\ 1 \end{bmatrix} = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^1p \\ 1 \end{bmatrix}$$

$${}^0p = R\,{}^1p + d \quad / R^T$$

$$R^T\,{}^0p = R^T R\,{}^1p + R^T d$$

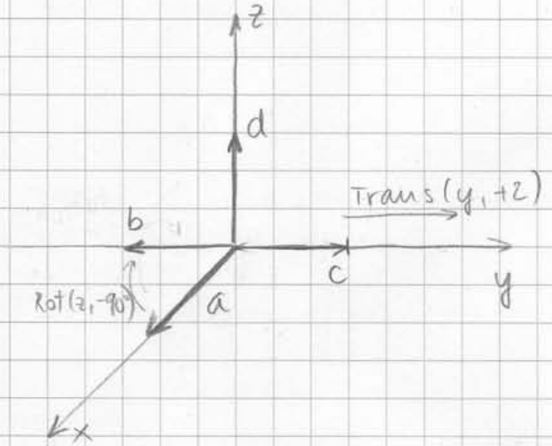
$${}^1p = R^T\,{}^0p - R^T d$$

množenje z levi – PREMULTIPLICIRANJE

desni – POSTMULTIPLICIRANJE

$$\begin{bmatrix} {}^1p \\ 1 \end{bmatrix} = \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^0p \\ 1 \end{bmatrix}$$

PRIMER: Premakni enotski vektor a za 90° okrog z v smeri urnega kazalca. Potem novi vektor za +2 premakni translacijsko v smeri y in nazadnje za 90° zavrti okrog osi x v nasprotni smeri urnega kazalca.



$b = Rot(z,-90°)\,a$

$c = Trans(y,+2)\,b$

$d = Rot(x,90°)\,c$

$$d = Rot(x,90°)\,Trans(y,+2)\,Rot(z,-90°)\,a$$

$$d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} a \\ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \end{array}$$

$$d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Za rotacijsko matriko (na izpitu) velja, da mora biti v vsakem stolpcu natanko ena enka in v vsaki vrstici natanko ena enka.

$$d = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$ ← vektor k, ki kaže v smeri z

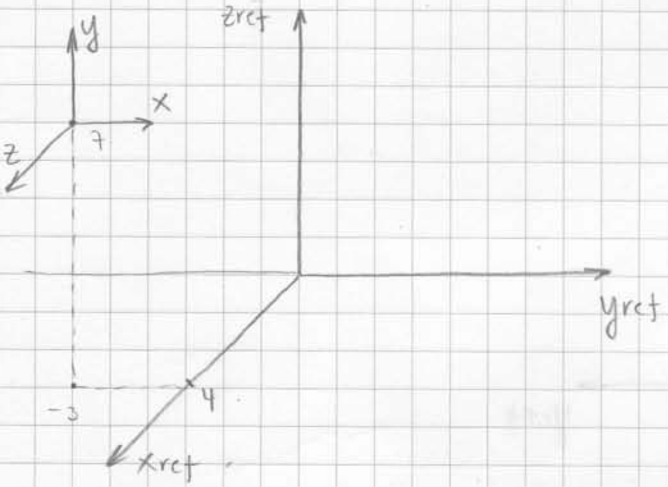
Lega

R ... orientacija
d ... pozicija

$$H = Trans(4,-3,7)\ Rot(y,90^\circ)\ Rot(z,90^\circ)$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} Rot(z,90^\circ) =$$

$$H = \begin{bmatrix} 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{l} \\ x_{ref} \\ y_{ref} \\ z_{ref} \\ \\ \end{array}$$

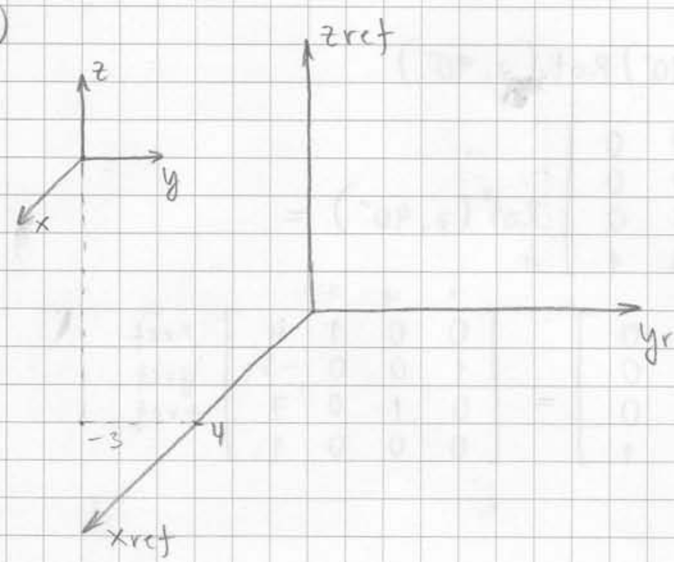


Lega je premik referenčnega k.s.

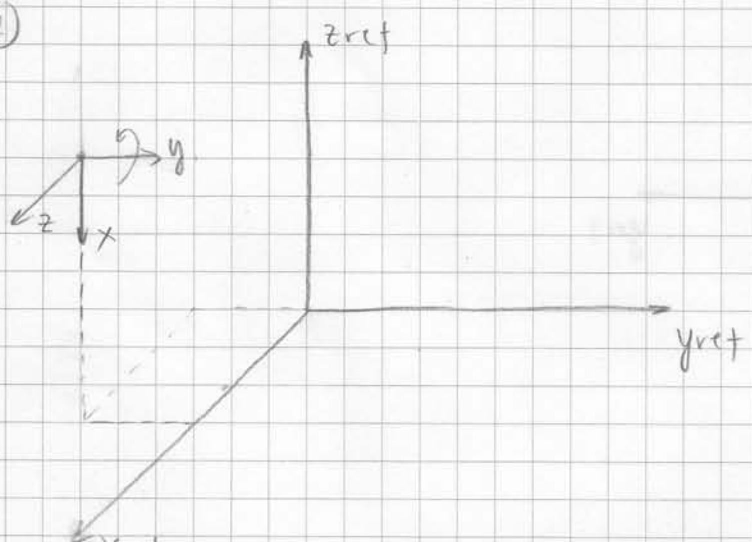
PREMIK REF. K.S → RELATIVNI

Trans(4, -3, 7) Rot(y, 90°) Rot(z, 90°) →

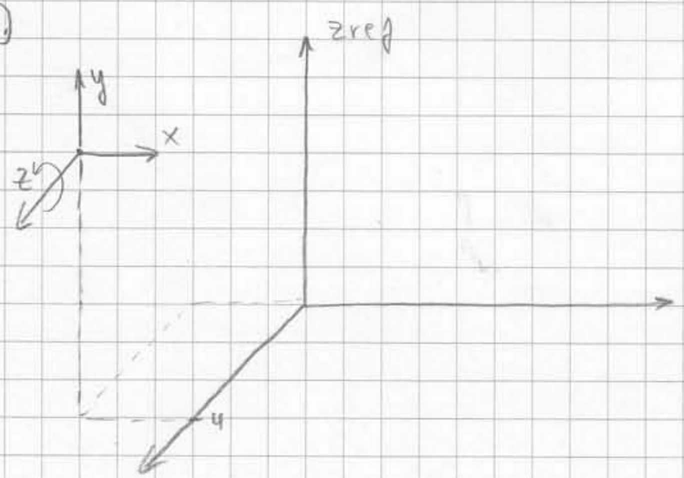
1.



2.



3.

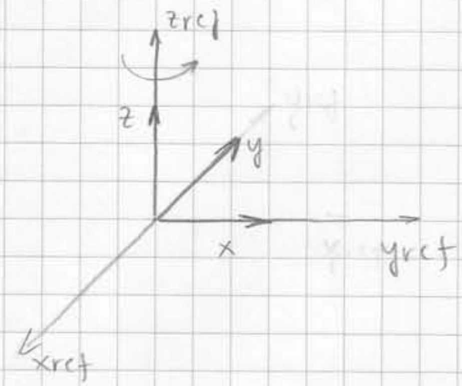


PREMIK REF. K.S → REFERENČNI

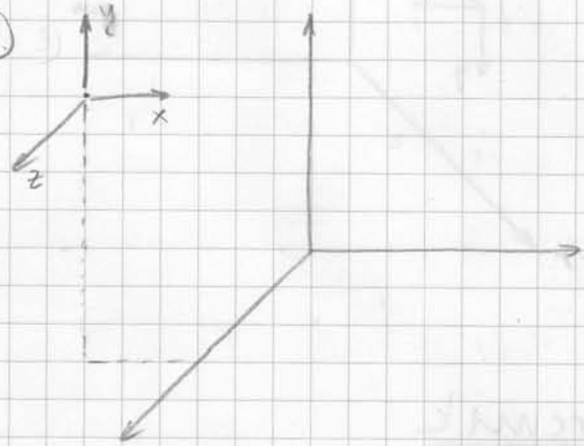
Trans(4,-3,7) Rot(y,90°) Rot(z,90°)

←

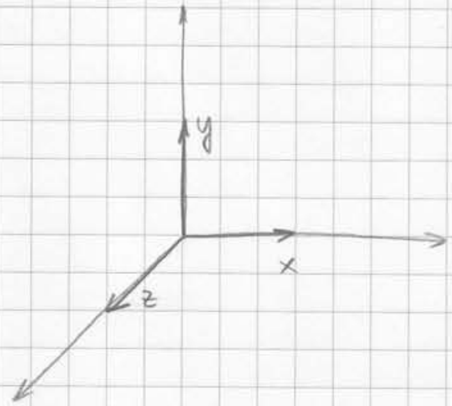
1.



3.

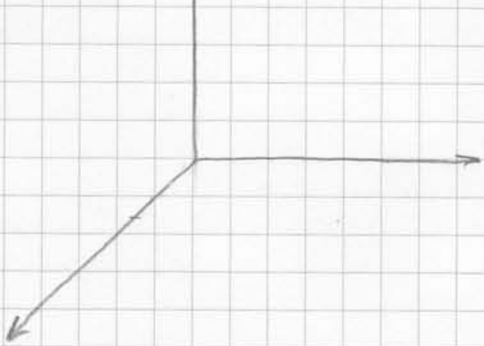


2.

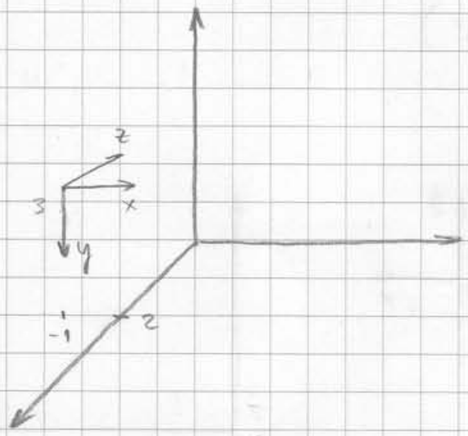


Risanje k.s. iz matrike

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$



zapis matrike iz slike — IZPITNA NALOGA



$$\begin{bmatrix} 0 & 0 & -1 & 2 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Premik

R ... rotacija
d ... translacija

$$\begin{bmatrix} \text{premik} \\ P \end{bmatrix} \begin{bmatrix} \text{pozicija} \end{bmatrix}$$

telo

$$\begin{bmatrix} P \end{bmatrix} \begin{bmatrix} L \\ \text{lega} \end{bmatrix}$$ premultiplikacija

$$\begin{bmatrix} L \end{bmatrix} \begin{bmatrix} P \end{bmatrix}$$ postmultiplikacija

1. PREMULTIPLIKACIJA – premik glede na ref. k.s.

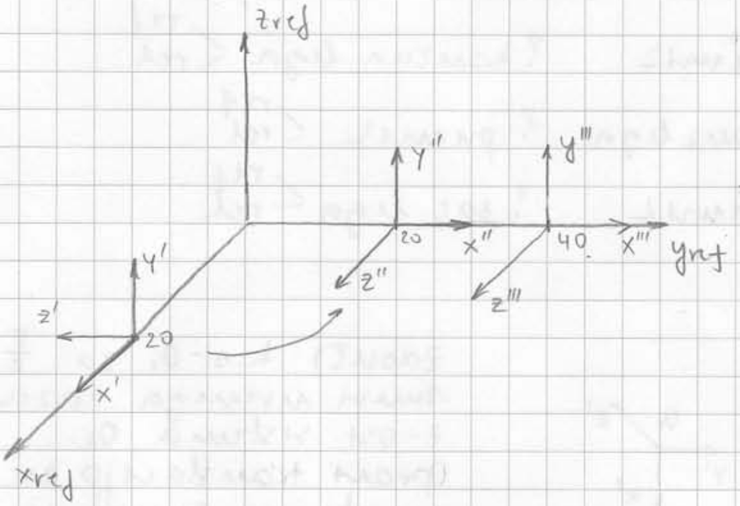
$$L = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P = Trans(0,20,0)\, Rot(z, 90°)$

$\longleftarrow$

$$P = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{X = P \cdot L} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 40 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

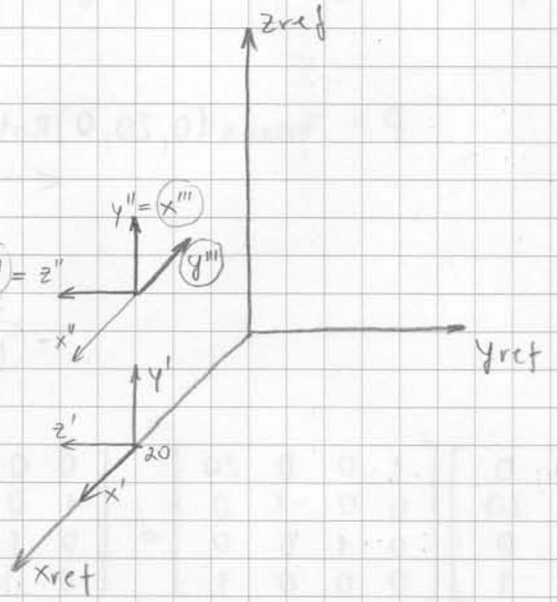


2. POSTMULTIPLIKACIJA – glede na rel. k.s.

$$\boxed{Y = L \cdot P} = \begin{bmatrix} 1 & 0 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 20 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P = Trans(0,20,0)\, Rot(z, 90°)$

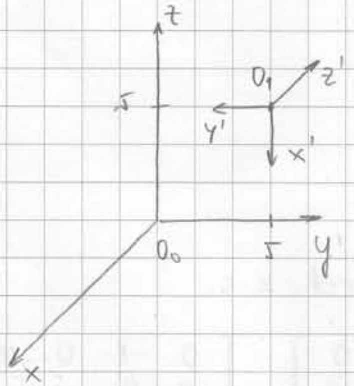
$\longrightarrow$



IZPITNA NALOGA:

1. zač. lega, premik ? končna lega < ref / rel
2. zač. lega, kon. lega ? premik < ref / rel
3. kon. lega, premik ? zač. lega < ref / rel

1.



zasukaj k.s. O_1 za $\frac{\pi}{2}$ v smeri urinega kazalca okrog z-osi sistema O_0.
Opravi translacijo za 3 v negativni smeri y sistema O_0.
Opravi še rotacijo za $\frac{\pi}{2}$ v obratni smeri urinega kazalca okrog osi x ref. k.s.

$$H_1 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) REF. K.S. (obratni vrstni red glede na besedilo)

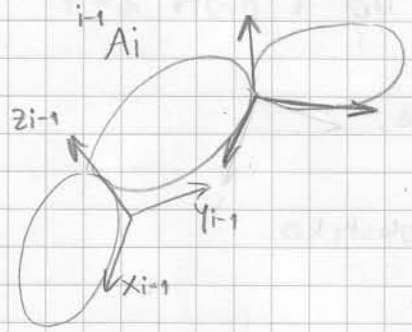
$$T = Rot(x, \tfrac{\pi}{2})\ Trans(0,-3,0)\ Rot(z, -\tfrac{\pi}{2})$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} Rot(z, -\tfrac{\pi}{2})$$

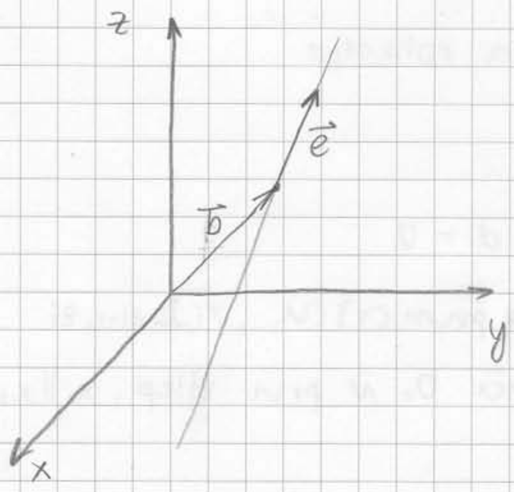
$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2 = T\ H_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 5 \\ -1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 5 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

GEOMETRIJSKI MODEL ROBOTA

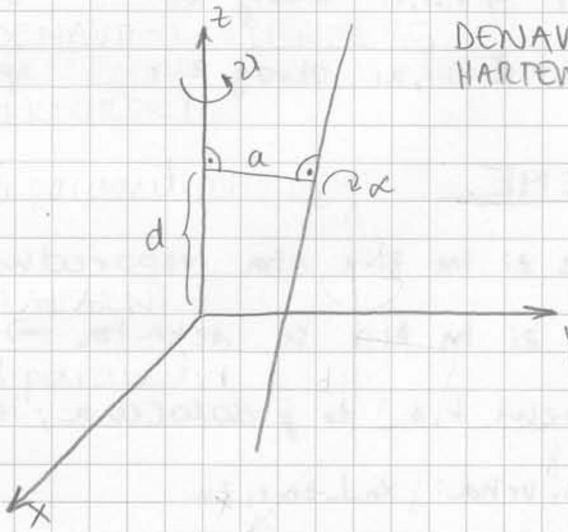


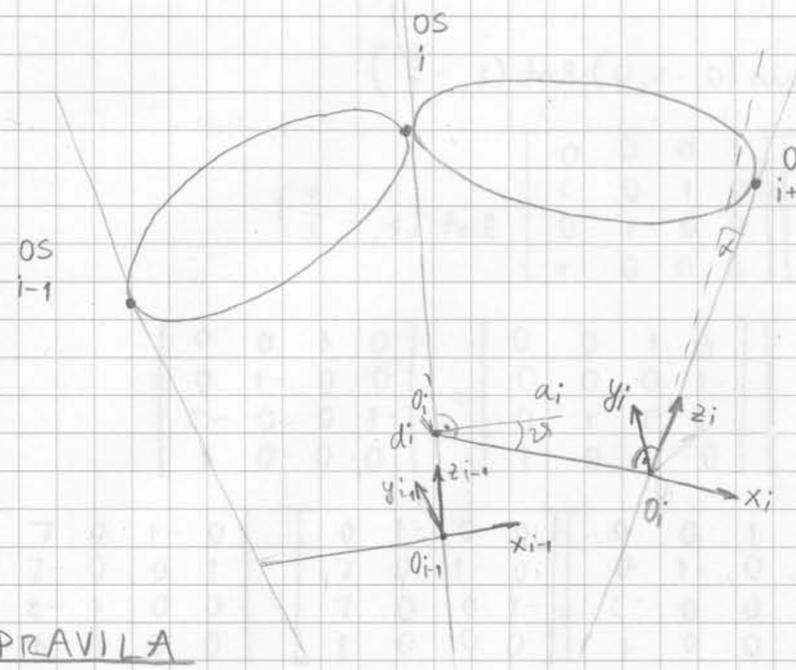
VEKTORSKI (6 parametrov)



SKALARNI (4 parametri)

DENAVIT - HARTENBERG





D-H PRAVILA

1. Izberemo os z_i vzdolž osi $i+1$
2. izhodišče k.s. O_i postavimo na presečišče skupne normale na osi z_i in z_{i+1}
3. x_i os postavimo tako, da gleda od i proti $i+1$ vzdolž skupne normale
4. y_i se prilagodi desnosučnemu k.s.

D-H PARAMETRI

za robotiko

1. $a_i = \overline{O_i' O_i}$ vzdolž x_i — const.
2. $d_i = \overline{O_{i-1} O_i}$ vzdolž z_{i-1} — sprem. za translacijski sklep
3. $\alpha_i = \measuredangle z_{i-1}, z_i$ okrog x_i — const.
4. $\vartheta_i = \measuredangle x_{i-1}, x_i$ okrog z_{i-1} — sprem. za rotacijo

IZJEME

1. os z_i in z_{i-1} sta vzporedni $\Rightarrow d_i = 0$
2. os z_i in z_{i-1} se sekata $\Rightarrow O_i$ je v presečišču, $x_i \perp z_{i-1}, z_i$
3. bazni k.s.: z_0 je določena, izhodišče O_0 v prvi sklep, $x_0 \parallel x_1$
4. k.s. vrha:

5. translacijski sklep: O_{i-1} postavimo na začetek translacije

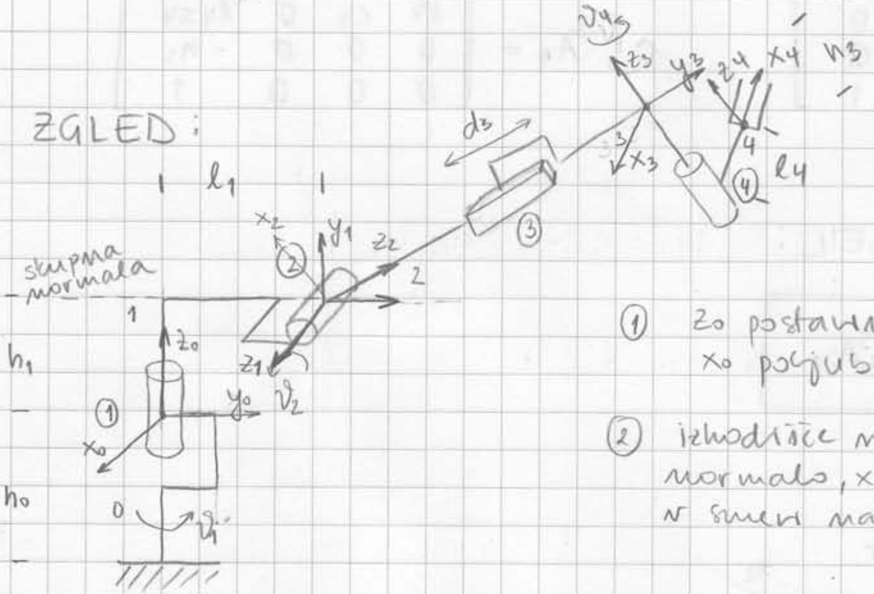
$$^{i-1}A_i(q_i) = Trans(0,0,d_i)\,Rot(z_{i-1},\vartheta_i)\,Trans(a_i,0,0)\,Rot(x_i,\alpha_i)$$

$\sin\vartheta_i = s\vartheta_i = s_i$

$$^{i-1}A_i(q_i) = \begin{bmatrix} c\vartheta_i & -s\vartheta_i & 0 & 0 \\ s\vartheta_i & c\vartheta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{i-1}A_i(q_i) = \begin{bmatrix} c\vartheta_i & -s\vartheta_i c\alpha_i & s\vartheta_i s\alpha_i & a_i c\vartheta_i \\ s\vartheta_i & c\vartheta_i c\alpha_i & -c\vartheta_i s\alpha_i & a_i s\vartheta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

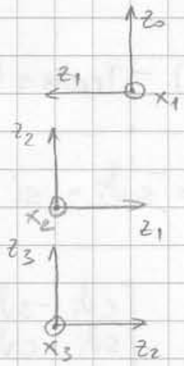
ZGLED:



① z_0 postavimo v prvi sklep
x_0 poljubno

② izhodišče na presečišču skupno normalo, x_1 v smeri normale v smeri naslednjega sklepa

	x_i		z_{i-1}	
i	a_i	α_i	d_i	ϑ_i
1	l_1	$\frac{\pi}{2}$	h_1	ϑ_1
2	0	$\frac{\pi}{2}$	0	ϑ_2
3	0	$\frac{\pi}{2}$	d_3	$\frac{\pi}{2}$
4	l_4	0	$-h_3$	ϑ_4



$$
{}^{0}A_1 = \begin{bmatrix} c1 & 0 & s1 & l_1c1 \\ s1 & 0 & -c1 & l_1s1 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{2}A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

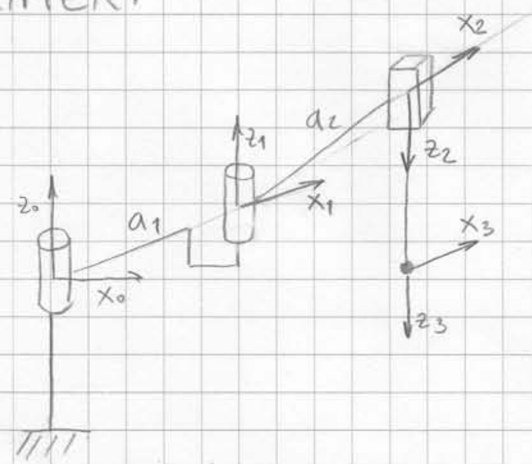
$$
{}^{1}A_2 = \begin{bmatrix} c2 & 0 & s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^{3}A_4 = \begin{bmatrix} c4 & -s4 & 1 & l_4c4 \\ s4 & c4 & 0 & l_4s4 \\ 0 & 0 & 0 & -h_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

GEOMETRIJSKI MODEL:

$$\boxed{T = {}^{0}A_1 \cdot {}^{1}A_2 \cdot {}^{2}A_3 \cdot {}^{3}A_4}$$

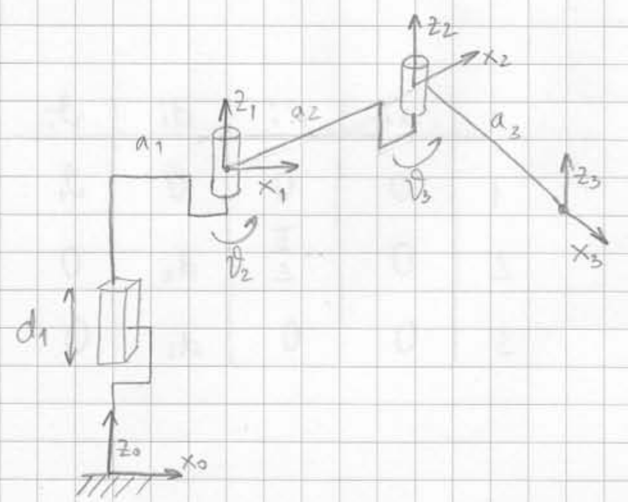
PRIMER:

1.)



i	a_i	α_i	d_i	ϑ_i
1	a_1	0	0	ϑ_1
2	a_2	π	0	ϑ_2
3	0	0	d_3	0

2.) SCARA



i	a_i	α_i	d_i	ϑ_i
1	a_1	0	d_1	0
2	a_2	0	0	ϑ_2
3	a_3	0	0	ϑ_3

$$^0A_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1A_2 = \begin{bmatrix} c2 & -s2 & 0 & a_2 c2 \\ s2 & c2 & 0 & a_2 s2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2A_3 = \begin{bmatrix} c3 & -s3 & 0 & a_3 c_3 \\ s3 & c3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = {}^0A_1 {}^1A_2 {}^2A_3 = \begin{bmatrix} c_2 & -s_2 & 0 & a_1 + a_2 c_2 \\ s_2 & c_2 & 0 & a_2 s_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & a_3 c_3 \\ s_3 & c_3 & 0 & a_3 s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_3 c_3 c_2 - a_3 s_3 c_2 + a_1 + a_2 c_2 \\ s_{23} & c_{23} & 0 & a_3 c_3 s_2 + a_3 s_3 c_2 + a_2 s_2 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

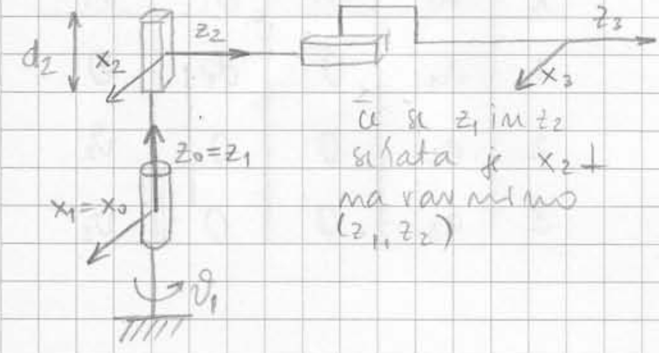
dve rotacijski osi sta vzporedni

$\cos(\vartheta_2 + \vartheta_3) = c_{23}$
$\sin(\vartheta_2 + \vartheta_3) = s_{23}$

$c_{23} = c_2 c_3 - s_2 s_3$
$s_{23} = s_2 c_3 + s_3 c_2$

$$T = \begin{bmatrix} c_{23} & -s_{23} & 0 & a_1 + a_2 c_2 + a_3 c_{23} \\ s_{23} & c_{23} & 0 & a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3.) Valjni robot



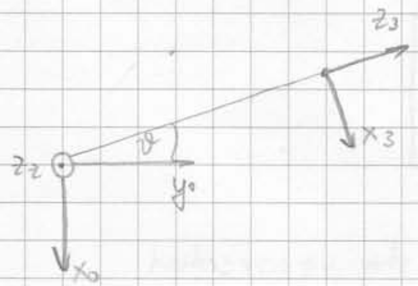
Če se z_1 in z_2 sekata je $x_2 \perp$ na ravnino (z_1, z_2)

i	a_i	α_i	d_i	ϑ_i
1	0	0	0	ϑ_1
2	0	$-\frac{\pi}{2}$	d_2	0
3	0	0	d_3	0

$$
{}^0A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^1A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

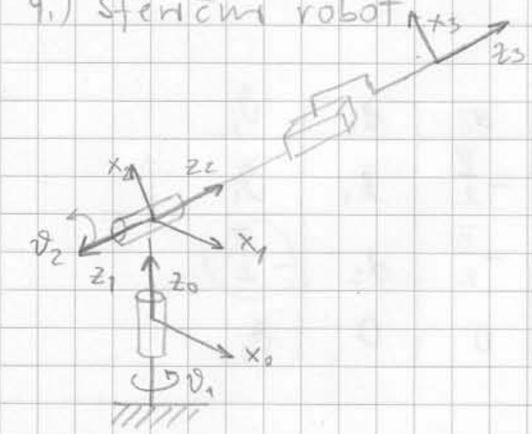
$$
{}^2A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
T = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad {}^2A_3 = \begin{bmatrix} c_1 & 0 & -s_1 & -d_2 s_1 \\ s_1 & 0 & c_1 & d_2 c_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

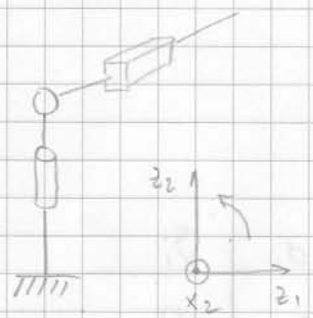


$$
\begin{bmatrix} \cos\vartheta & \cos 90 & \cos(90+\vartheta) & -d_3\sin\vartheta \\ \cos(90-\vartheta) & \cos 90 & \cos\vartheta & d_3\cos\vartheta \\ \cos 90^\circ & \cos 180 & \cos 90 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

4.) Sferični robot



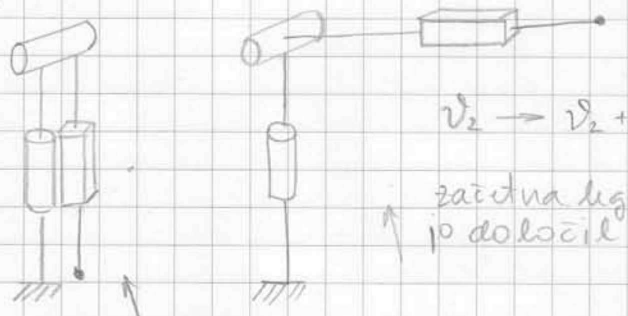
i	a_i	α_i	d_i	ϑ_i
1	0	$\frac{\pi}{2}$	d_1	ϑ_1
2	0	$\frac{\pi}{2}$	0	ϑ_2
3	0	0	d_3	0



$$T = \begin{bmatrix} c_1c_2 & s_1 & c_1s_2 & d_3c_1s_2 \\ s_1c_2 & -c_1 & s_1s_2 & d_3s_1s_2 \\ s_2 & 0 & -c_2 & d_1-d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ZAČETNA LEGA

ϑ_2 ∢ $x_1 x_2$ okrog osi z_1



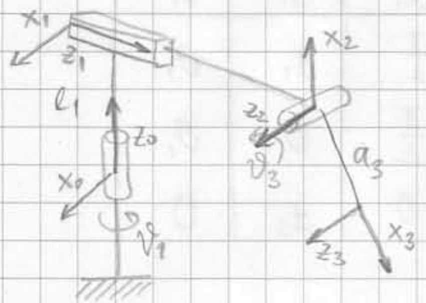
$\vartheta_2 \to \vartheta_2 + \frac{\pi}{2}$

začetna lega, ki bi jo določil proizvajalec

večkrat neizvedljiva začetna lega (omejitve v sklepih)

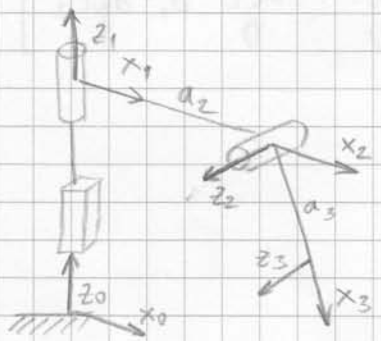
IZPIT
- eden od nalog
- naloga iz skripte (3 roboti)
- robot po smislu

5. 1. "izmišljeni" robot



i	a_i	α_i	d_i	ϑ_i
1	0	$-\frac{\pi}{2}$	l_1	ϑ_1
2	0	$-\frac{\pi}{2}$	d_2	$-\frac{\pi}{2}$
3	a_3	0	0	ϑ_3

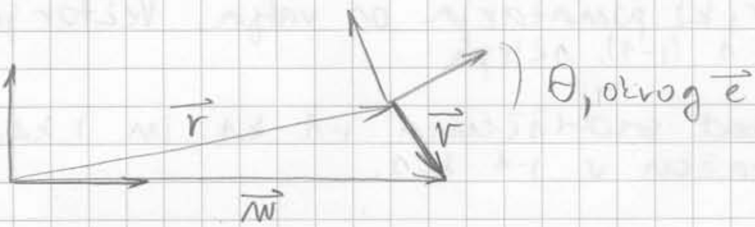
6. 2. "izmišljeni" robot



i	a_i	α_i	d_i	ϑ_i
1	0	0	d_1	0
2	a_2	$\frac{\pi}{2}$	0	ϑ_2
3	a_3	0	0	ϑ_3

! x_1 os ni pripeta na drugi segment

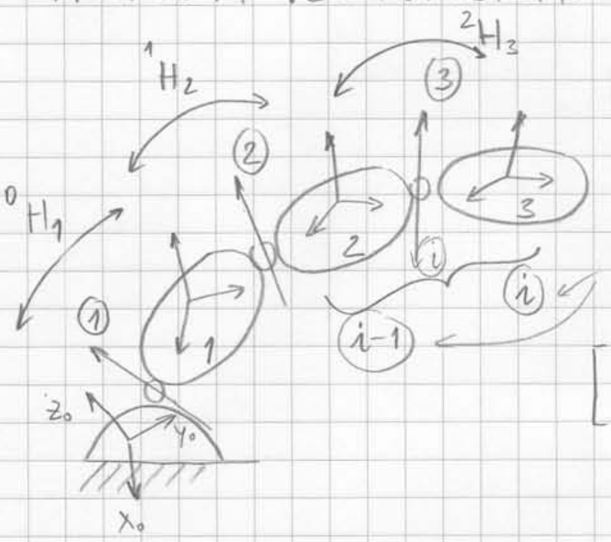
Predavanje prof. Lenarčič



$\vec{w} = \vec{r} + A \cdot \vec{v}$

$$\begin{bmatrix} w \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} & A & & \vec{r} \\ 0 & 0 & 0 & 1 \end{array}\right] = H$$

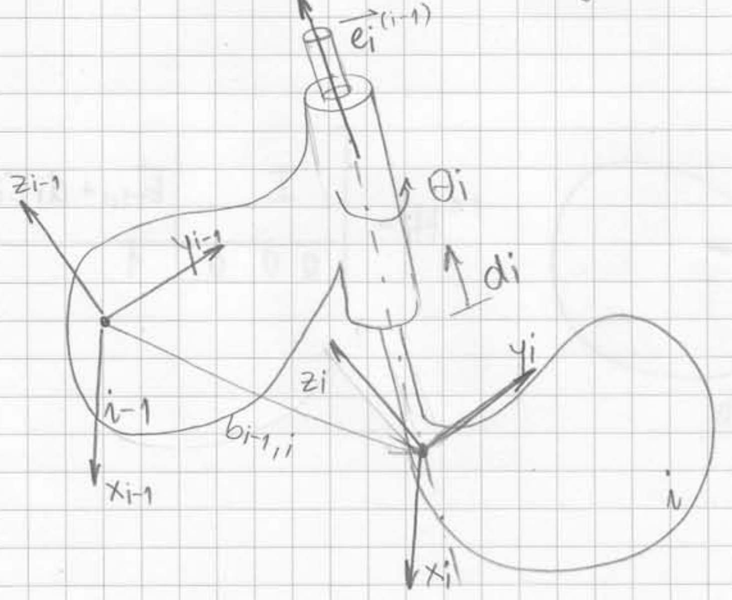
METODA VEKTORSKIH PARAMETROV



kinematični par

$$\begin{bmatrix} w \\ 1 \end{bmatrix} = {}^{0}H_1 \, {}^{1}H_2 \, {}^{2}H_3 \begin{bmatrix} v \\ 1 \end{bmatrix}$$

Referenčna (začetna) lega



k.s. i-1 stoji tako kot stoji kako naj stoji k.s. i-tega sklepa

K.S. i-tega sklepa postavimo v središče sklepa tako, da je vzporeden i-1 sklepu.

VEKTORSKA PARAMETRA

$\vec{e}_i$ – enotski vektor, ki ponazarja os valja. Vektor je izražen v k.s. (i-1). sklepa

$b_{i-1,i}$ – povezava med središčema i-1 k.s. in i k.s. Vektor je izražen v i-1. k.s.

i-to telo se lahko premika gor/dol v smeri $\vec{e}_i$ in okrog vektorja $\vec{e}_i$, i-1 sklep miruje.

SPREMENLJIVKE

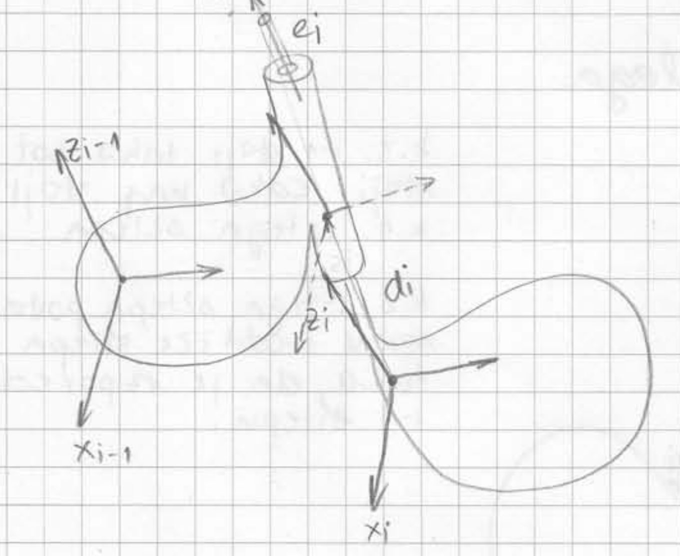
d_i – translacija v smeri $\vec{e}_i$

θ_i – rotacija okrog osi $\vec{e}_i$

začetna lega: $d_i = 0$, $\theta_i = 0$

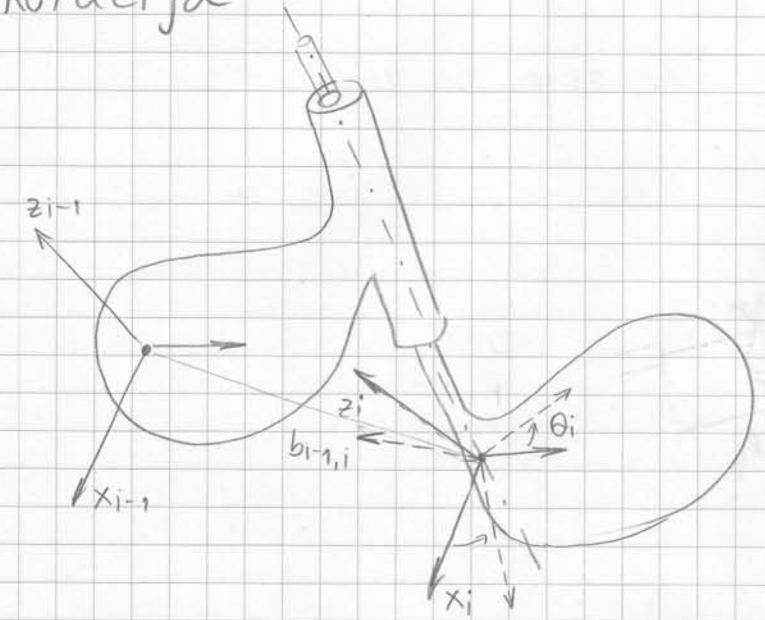
$$ {}^{i-1}H_i = \left[\begin{array}{ccc|c} & I & & b_{i-1,i} \\ \hline 0 & 0 & 1 & 1 \end{array}\right] $$

Translacija



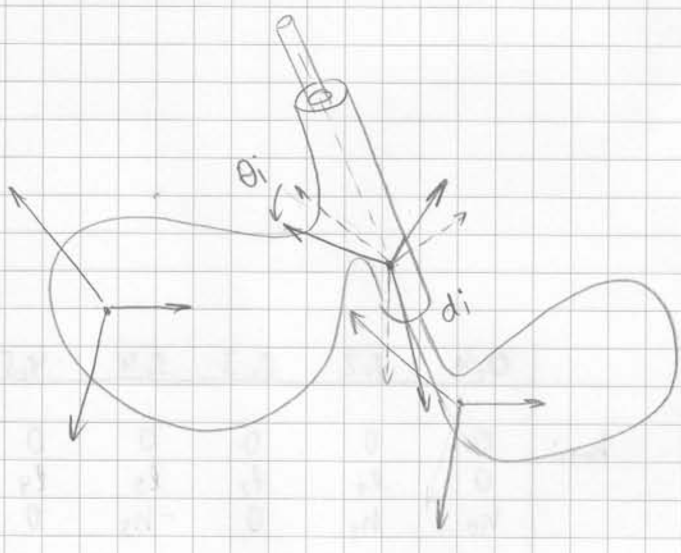
$$ {}^{i-1}H_i = \left[\begin{array}{ccc|c} & I & & \vec{b}_{i-1,i} + d_i\vec{e}_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

Rotacija



$${}^{i-1}H_i = \left[\begin{array}{ccc|c} & {}^{i-1}A_i & & b_{i-1,i} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

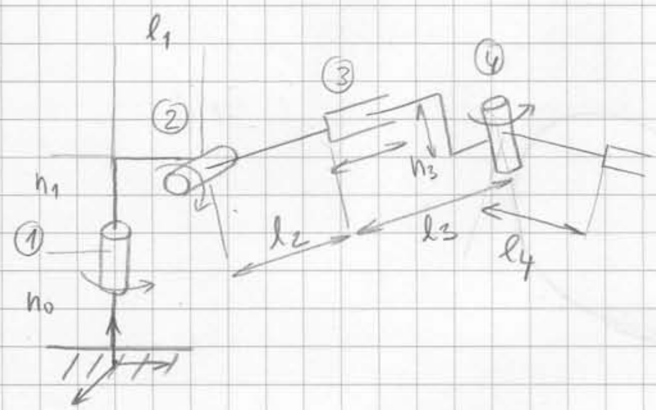
Translacija in rotacija



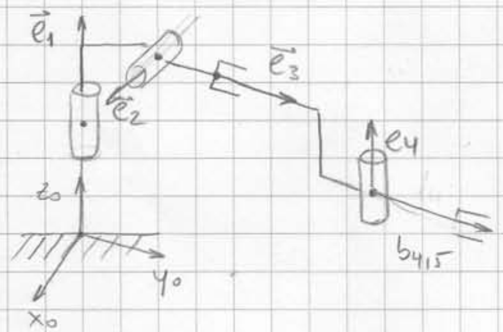
$${}^{i-1}H_i = \left[\begin{array}{ccc|c} & {}^{i-1}A_i & & \vec{b}_{i-1,i} + d_i\vec{e}_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \theta_i \neq \emptyset \quad d_i \neq \emptyset$$

$${}^{i-1}A_i = f(\vec{e}_i, \theta_i)$$

PRIMER



① Referenčna lega



	1	2	3	4
e_i	0	1	0	0
	0	0	1	0
	1	0	0	1

	0,1	1,2	2,3	3,4	4,5
$b_{i-1,i}$	0	0	0	0	0
	0	l_1	l_2	l_3	l_4
	h_0	h_1	0	$-h_3$	0

	1	2	3	4
d_i	0	0	d_3	0
θ_i	θ_1	θ_2	0	θ_4

$$ {}^{0}H_1 = \left[\begin{array}{ccc|c} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & h_0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad {}^{1}H_2 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & c2 & -s2 & l_1 \\ 0 & s2 & c2 & h_1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

$$ {}^{2}H_3 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2+d_3 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad {}^{3}H_4 = \left[\begin{array}{ccc|c} c4 & -s4 & 0 & 0 \\ s4 & c4 & 0 & l_3 \\ 0 & 0 & 1 & -h_3 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

$$ {}^{5}H_4 = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_4 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

19. 4. 2010

1. ref. lega

   $[x_0, y_0, z_0]$

   transl. sklepi v začetni legi

2. središča sklepov
3. $\vec{e_i}$ ← narišemo vektorje v središča sklepov
4. $\vec{b_{i-1,i}}$ ← središča povežemo z vektorji
5. MATRIKA

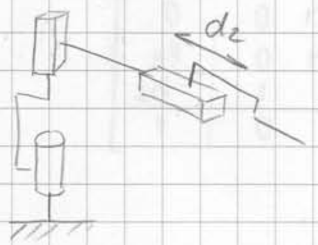
– rot. sklep

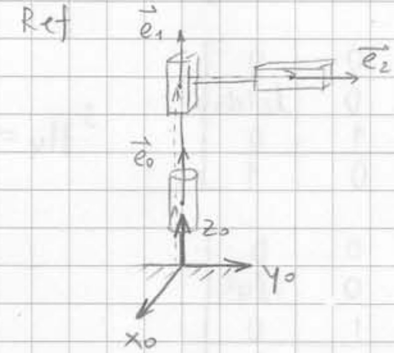
$$ {}^{i-1}H_i = \left[\begin{array}{ccc|c} R_x, & R_y, & R_z & b_{i-1,i} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

– transl. sklep

$$ {}^{i-1}H_i = \left[\begin{array}{ccc|c} & I & & d_i\vec{e_i} + \vec{b_{i-1,i}} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

cilindrični (valjni) robot





i	1	2	3
ϑ_i	ϑ_1	0	0
d_i	0	d_2	d_3

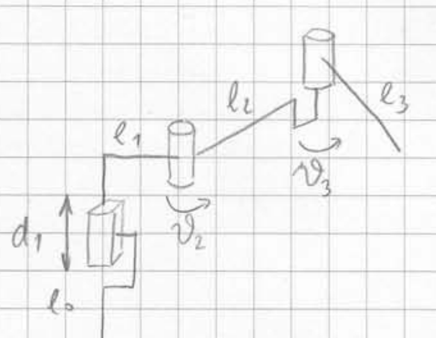
	1	2	3
	0	0	0
$\overline{e_i}$	0	0	1
	1	1	0

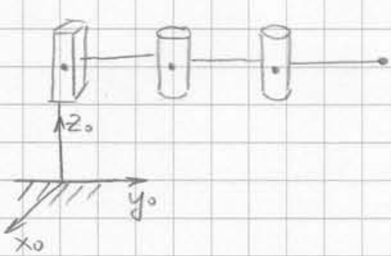
	1	2	3
	0	0	0
$b_{i-1,i}$	0	0	l_3
	l_1	l_2	0

$$^0H_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1H_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_2+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3+d_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

scara robot





	1	2	3
ϑ_i	d_1	0	0
d_i	0	ϑ_2	ϑ_3

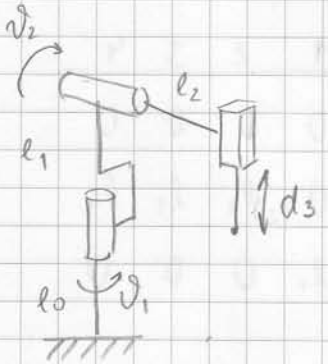
	1	2	3
$\overline{e_i}$	0	0	0
	0	0	0
	1	1	1

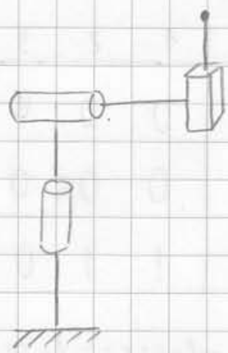
	1	2	3	4
$\overline{b_{i-1,i}}$	0	0	0	0
	0	1	1	1
	1	0	0	0

$$^0H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1H_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2H_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^3H_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

stanfordski robot





	1	2	3
ϑ_i	ϑ_1	ϑ_2	0
d_i	0	0	d_3

	1	2	3
$\overline{e_i}$	0	0	0
	0	1	0
	1	0	1

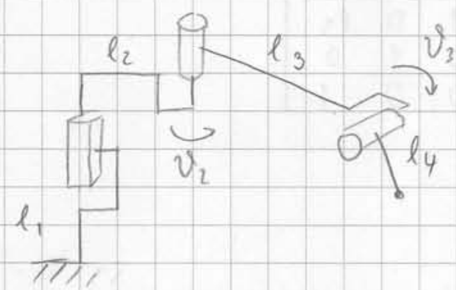
	1	2	3
$\overline{b_{i-1,i}}$	0	0	0
	0	0	l_3
	l_1	l_2	0

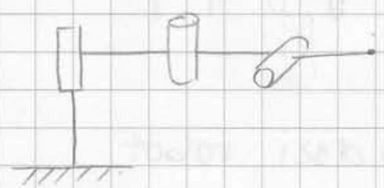
$$^0H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1H_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ 0 & 1 & 0 & 0 \\ -S_2 & 0 & C_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}^{2}H_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{0}H_3 = {}^{0}H_1\, {}^{1}H_2\, {}^{2}H_3 = \begin{bmatrix} c_1c_2 & -s_1 & c_1s_2 & -l_3s_1 + d_3c_1s_2 \\ s_1c_2 & c_1 & s_1s_2 & l_3c_1 + d_3s_1s_2 \\ -s_2 & 0 & c_1 & l_1 + l_2 + d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Robot xy



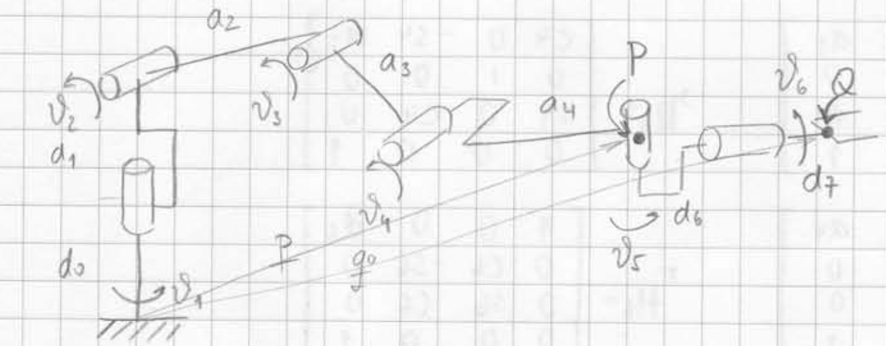


	1	2	3
ϑ_i	0	ϑ_2	ϑ_3
d_i	d_1	0	0

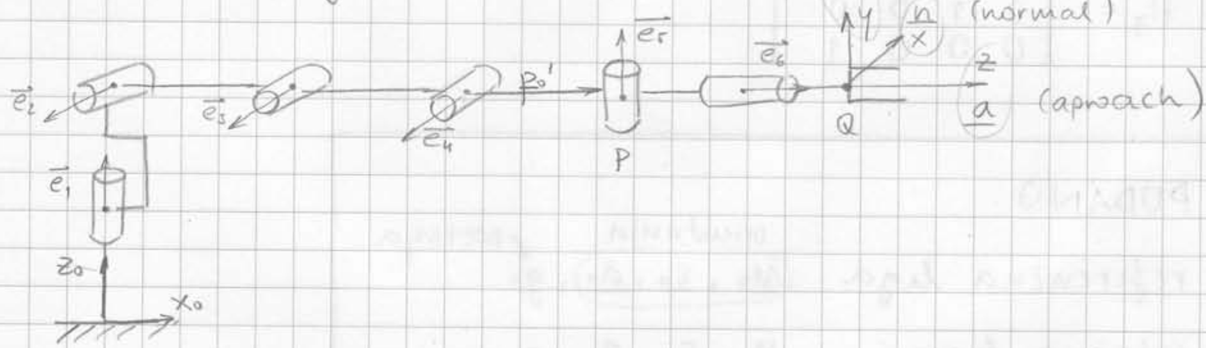
$$ \vec{e}_i = \begin{array}{ccc} 1 & 2 & 3 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{array} \qquad \vec{b}_{i-1,i} = \begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 0 & 0 \\ 0 & l_2 & l_3 & l_4 \\ l_1 & 0 & 0 & 0 \end{array} $$

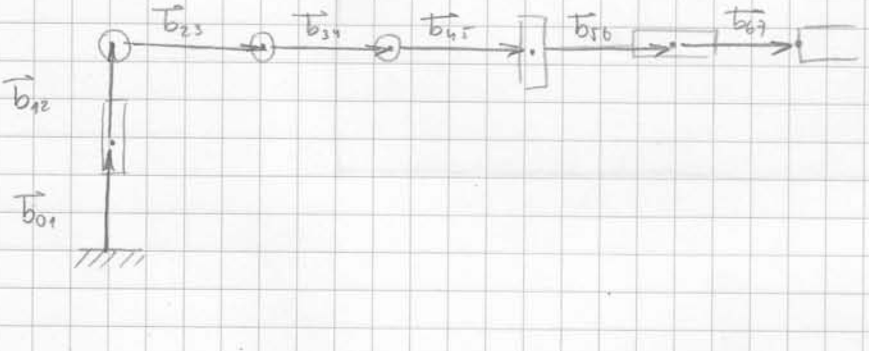
$$ {}^{4}H_0 = \begin{bmatrix} c_2 & -s_2c_3 & s_2c_3 & -l_4s_2c_3 - l_3s_2 \\ s_2 & c_2c_3 & -c_2s_3 & l_4c_2c_3 - l_3c_2 + l_2 \\ 0 & s_3 & c_3 & l_4s_3 + l_1 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

INVERZNI GEOMETRIJSKI MODEL



Referenčna lega





	1	2	3	4	5	6
ϑ_i	ϑ_1	ϑ_2	ϑ_3	ϑ_4	ϑ_5	ϑ_6
d_i	0	0	0	0	0	0

	1	2	3	4	5	6
	0	0	0	0	0	1
$\vec{e_i}$	0	-1	-1	-1	0	0
	1	0	0	0	1	0

	1	2	3	4	5	6	7
	0	0	a_2	d_3	d_4	d_6	d_7
$b_{i-1,i}$	0	0	0	0	0	0	0
	d_0	d_1	0	0	0	0	0

$$ {}^0H_1 = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^1H_2 = \begin{bmatrix} c2 & 0 & -s2 & 0 \\ 0 & 1 & 0 & 0 \\ s2 & 0 & c2 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^2H_3 = \begin{bmatrix} c3 & 0 & -s3 & a_2 \\ 0 & 1 & 0 & 0 \\ s3 & 0 & c3 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad {}^3H_4 = \begin{bmatrix} c4 & 0 & -s4 & a_3 \\ 0 & 1 & 0 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^4H_5 = \begin{bmatrix} c5 & -s5 & 0 & a_4 \\ s5 & c5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^5H_6 = \begin{bmatrix} 1 & 0 & 0 & d_6 \\ 0 & c6 & -s6 & 0 \\ 0 & s6 & c6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^6H_7 = \begin{bmatrix} 0 & 0 & 1 & d_7 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

PODANO:

orientacija, pozicija

referenčna lega: $\underline{n}_0, \underline{s}_0, \underline{a}_0, \underline{q}_0$

splošna lega: $\underline{n}, \underline{s}, \underline{a}, \underline{q}$

IŠČEM

$\vartheta_1, \vartheta_2, \ldots, \vartheta_6$

$$ \underline{p} = \underline{q} - (d_6 + d_7)\underline{a} $$

$$ \underline{p} = {}^0H_1\,{}^1H_2\,{}^2H_3\,{}^3H_4\,\underline{p}_0' $$

(1) $p = {}^1H_0\, {}^2H_1\, {}^3H_2\, {}^4H_3\; p_0'$

$$({}^0H_1)^{-1} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = {}^2H_1\, {}^3H_2\, {}^4H_3 \begin{bmatrix} a_4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} C1 & S1 & 0 & 0 \\ -S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & -d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} C234 & 0 & -S234 & a_3C23 + a_2C2 \\ 0 & 1 & 0 & 0 \\ S234 & 0 & C234 & a_3S23 + a_2S2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$p_xC1 + p_yS1 = a_4C234 + a_3C23 + a_2C2$$

$$-p_xS1 + p_yC1 = 0$$

$$p_z - d_0 = a_4S234 + a_3S23 + a_2S2 + d_1$$

$\rightarrow \vartheta_1 = \arctan \frac{p_y}{p_x} \leftarrow$ 2 rešitvi

(2) $\underline{a} = {}^0R_1\, {}^1R_2\, {}^2R_3\, {}^3R_4\, {}^4R_5\; \underline{a_0}$

$${}^0R_1^T\; \underline{a} = {}^1R_2\, {}^2R_3\, {}^3R_4\, {}^4R_5\; \underline{a_0}$$

$$\begin{bmatrix} C1 & S1 & 0 \\ -S1 & C1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} C234 & 0 & -S234 \\ 0 & 1 & 0 \\ S234 & 0 & C234 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C5 & -S5 \\ 0 & S5 & C5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$a_xC1 + a_yS1 = C234\,C5$$

$$-a_xS1 + a_yC1 = S5$$

$$a_z = S234 \cdot C5$$

$\rightarrow \vartheta_5 = \arcsin(-a_xS1 + a_yC1)$

$$S234 = \frac{a_z}{C5}$$

$$C234 = \frac{a_xC_1 + a_yS1}{C5}$$

$\arctan_2 \frac{a}{b}$

$a>0$	$b>0$	$\arctan_2 \frac{a}{b} = \arctan \frac{a}{b}$
$a>0$	$b<0$	$\arctan_2 \frac{a}{b} = \pi + \arctan \frac{a}{b}$
$a<0$	$b>0$	$\arctan_2 \frac{a}{b} = \arctan \frac{a}{b}$
$a<0$	$b<0$	$\arctan_2 \frac{a}{b} = -\pi + \arctan \frac{a}{b}$

$$\vartheta_{234} = \vartheta_2 + \vartheta_3 + \vartheta_4 = \arctan_2 \frac{s234}{c234}$$

$\vartheta_1, \vartheta_5, s234, c234 \rightarrow$ podano

$$\left.\begin{aligned} K_1 &= p_x c1 + p_y s1 - a_4\, s234 \\ K_2 &= p_z - d_0 - a_4\, s234 - d_1 \end{aligned}\right\} \Longrightarrow \text{izračunamo } K_1 \text{ in } K_2$$

(3.) $a_2 c_2 + a_3 c_{23} = K_1 \quad /^2$

$a_2 s2 + a_3 s23 = K_2 \quad /^2$ (+)

$$\boxed{c2c23 + s2s23 = \cos((\vartheta_2 + \vartheta_3) - \vartheta_2) = c_3}$$

$$a_1^2 + a_3^2 + 2a_1a_3c_3 = K_1^2 + K_2^2$$

$$\rightarrow \vartheta_3 = \arccos \frac{K_1^2 + K_2^2 - a_2^2 - a_3^2}{2a_1a_3}$$

$$c2 = \frac{K_1(a_2 + a_3c_3) + K_2 a_3 s_3}{K_1^2 + K_2^2}$$

$$s2 = \frac{K_2(a_2 + a_3c_3) - K_1 a_3 s3}{K_1^2 + K_2^2}$$

$\rightarrow \vartheta_2 = \arctan_2 \frac{S2}{C2}$

$\rightarrow \vartheta_4 = \vartheta_{234} - \vartheta_2 - \vartheta_3$

$\underline{s} = {}^1R_0\,{}^2R_1\,{}^2R_3\,{}^3R_4\,{}^4R_5\,{}^5R_6\,\underline{s_0}$

$({}^0R_1\,{}^1R_2\,{}^2R_3\,{}^3R_4)^T\,\underline{s} = {}^4R_5\,{}^5R_6\,\underline{s_0}$ $\quad s_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(4.) $\left.\begin{array}{l} s_x C1C234 + s_y S1C234 + s_z S234 = C5S6 \;/ S5 \\ -s_x S1 + s_y C1 = -C5S6 \;/ -C5 \end{array}\right\} \Rightarrow S6$

$-s_x C1C234 - s_y S1S234 + s_z C234 = C6$

$\rightarrow \vartheta_6 = \arctan_2 \frac{S6}{C6}$

Možnih 8 rešitev pri tem robotu.

KVATERNIONI

enotski vektor vzdolž x osi

$$q = q_0 \cdot 1 + q_1 \underline{i} + q_2 \underline{j} + q_3 \underline{k}$$

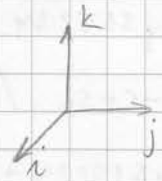
$$p + q = p_0 + q_0 + (p_1 + q_1)\underline{i} + (p_2 + q_2)\underline{j} + (p_3 + q_3)\underline{k}$$

$$w \cdot q = wq_0 + wq_1 i + wq_2 j + wq_3 k$$

$$q^* = q_0 - q_1 i - q_2 j - q_3 k$$

$$i^2 = -1 = j^2 = k^2 = ijk$$

*	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1



$q = q_0 + \underline{q}$ ← včasih pišemo skalarni in vektorski del posebej

(1)

$$pq = (p_0 + p_1 i + p_2 j + p_3 k)(q_0 + q_1 i + q_2 j + q_3 k) =$$

$$pq = p_0 q_0 + q_0(p_1 i + p_2 j + p_3 k) + p_0(q_1 i + q_2 j + q_3 k) +$$
$$+ p_1 q_1 i \cdot i + p_2 q_1 j \cdot i + p_3 q_1 k \cdot i + p_1 q_2 i \cdot j +$$
$$+ p_2 q_2 j \cdot j + p_3 q_2 k \cdot j + p_1 q_3 i \cdot k + p_2 q_3 j \cdot k + p_3 q_3 k \cdot k =$$

skalarni produkt

$$pq = p_0 q_0 + q_0 \underline{p} + p_0 \cdot \underline{q} - p_1 q_1 - p_2 q_2 - p_3 q_3 +$$
$$+ (p_2 q_3 - p_3 q_2) i + (p_3 q_1 - p_1 q_3) j + (p_1 q_2 - p_2 q_1) k$$

vektorski produkt

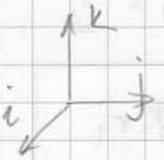
(2)
$$\boxed{p\cdot q = p_0 q_0 - \underline{p}\cdot\underline{q} + p_0\underline{q} + q_0\underline{p} + \underline{p}*\underline{q}} = r$$

(3)
$$r_0 = p_0 q_0 - p_1 q_1 - p_2 q_2 - p_3 q_3$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 q_1 \\ p_0 q_2 \\ p_0 q_3 \end{bmatrix} + \begin{bmatrix} q_0 p_1 \\ q_0 p_2 \\ q_0 p_3 \end{bmatrix} + \begin{bmatrix} p_2 q_3 - p_3 q_2 \\ p_3 q_1 - p_1 q_3 \\ p_1 q_2 - p_2 q_1 \end{bmatrix}$$

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

PRIMER:



(1) $(2+3i-j+5k)(3-4i+2j+k) =$

$= 6+9i-3j+15k-$

$-8-12i\cdot i-4j\cdot i-20ki+$

$+4j+6ij-2j\cdot j+10kj+$

$+2k+3ik-jk+5k\cdot k =$

$= 6+9i-3j+15k-8i-4k+20j+4j+6k+2-10i+$

$+2k+3j-i-5 =$

$= 15-10i-22j+19k$

(2)
$$\left(2+\begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}\right)\left(3+\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}\right) =$$

$$= 6 - \begin{bmatrix} 3 & -1 & 5 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} + \begin{bmatrix} i & j & k \\ 3 & -1 & 5 \\ -4 & 2 & 1 \end{bmatrix} =$$

$$= 6 + 9 + \begin{bmatrix} 1 \\ 1 \\ 17 \end{bmatrix} + \begin{bmatrix} -11 \\ -23 \\ 2 \end{bmatrix} = 15 - 10i - 22j + 19k$$

(3)
$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 & -5 \\ 3 & 2 & -5 & -1 \\ -1 & 5 & 2 & -3 \\ 5 & +1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \\ -22 \\ 19 \end{bmatrix}$$

ROTACIJA

$$\boxed{r_2 = q r_1 q^*}$$

$q = q_0 + \underline{q}$

$r_1 = 0 + \underline{r_1}$ ← navaden vektor

$q^* = q_0 - \underline{q}$

$r_2 = 0 + \underline{r_2}$

$$\boxed{r_2 = R r_1}$$

$$\underline{r_2} = (q_0 + \underline{q})(0 + \underline{r_1})\, q^* =$$

$$= \Big(\underbrace{(-\underline{q}\,\underline{r_1})}_{\text{skalar}} + \underbrace{q_0 \underline{r_1} + \underline{q} \times \underline{r_1}}_{\text{vektor}} \Big)(q_0 - \underline{q}) =$$

$$= \cancel{-\underline{q}\underline{r_1} q_0} + \cancel{q_0 \underline{r_1} \underline{q}} + \overset{0}{(\underline{q} \times \underline{r_1})\underline{q}} + \underline{q}\,\underline{r_1}\,\underline{q} + q_0^2 \underline{r_1} + q_0(\underline{q} \times \underline{r_1}) -$$

$$- q_0(\underline{r_1} \times \underline{q}) - \underline{q} \times \underline{r_1} \times \underline{q}$$

$$\hookrightarrow -(\underline{q} \cdot \underline{q})\underline{r_1} + (\underline{r_1} \underline{q})\underline{q}$$

$2 \underline{q}\,\underline{q}^T \underline{r_1}$

$$\underline{r_2} = q_0^2 \underline{r_1} - (\underline{q}\,\underline{q})\underline{r_1} + 2 q_0 (\underline{q} \times \underline{r_1}) + 2 \underline{q}(\underline{q} \cdot \underline{r_0})$$

$$(\underline{q} \times \underline{r_1}) = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \begin{bmatrix} r_{1x} \\ r_{1y} \\ r_{1z} \end{bmatrix} = \begin{bmatrix} i & j & k \\ q_1 & q_2 & q_2 \\ r_{1x} & r_{1y} & r_{1z} \end{bmatrix}$$

$$\underline{q} \cdot \underline{q}^T = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$$

$$\underline{r_2} = \underline{r_1} \left\{ (q_0^2 - q_1^2 - q_2^2 - q_3^2) I + 2q_0 \begin{bmatrix} 0 & q_3 & -q_2 \\ -q_3 & 0 & q_1 \\ q_2 & -q_1 & 0 \end{bmatrix} + 2 \begin{bmatrix} q_1^2 & q_1q_2 & q_1q_3 \\ q_1q_2 & q_2^2 & q_2q_3 \\ q_1q_3 & q_2q_3 & q_3^2 \end{bmatrix} \right\}$$

$$\underline{\underline{R}} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_2 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}$$

Rodriguetova formula s kvaternionom

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\underline{s} \quad \text{-- enotski}$$

enotski

$$\underline{q}^2 = q_0^2 + q_1^2 + q_2^2 + q_3^2$$

$$q_0 = \cos\frac{\vartheta}{2}$$

$$\underline{q} = \sin\frac{\vartheta}{2}\,\underline{s}$$

$$\underline{r_2} = \cos^2\frac{\vartheta}{2}\,\underline{r_1} - \sin^2\frac{\vartheta}{2}\underbrace{(\underline{s}\cdot\underline{q})}_{1}\underline{r_1} + 2\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2}(\underline{s}\times\underline{r_1}) + 2\sin^2\frac{\vartheta}{2}\cdot\underline{s}(\underline{s}\cdot\underline{r_1})$$

$$2\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2} = \sin\vartheta$$

$$\cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2} = \cos\vartheta$$

$$\cos^2\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2} = 1$$

$$\underline{r_2} = \underline{r_1}\cos\vartheta + (\underline{s}\times\underline{r_1})\sin\vartheta + \underline{s}(\underline{r_1}^T\underline{s})(1-\cos\vartheta)$$

KVATERNIONI

$$q = q_0 + \overbrace{q_1 \cdot \underline{i} + q_2 \underline{j} + q_3 \underline{k}}^{\underline{q}}$$

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\underline{s}$$

↳ enotski $\underline{s}$ in q

$$i^2 = j^2 = k^2 = -1$$

$$ij = k \qquad ji = -k$$

$$pq = p_0 q_0 - \underline{p}\,\underline{q} + p_0 \underline{q} + \underline{p} q_0 + \underline{p} \times \underline{q}$$

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} p_0 & -p_1 & -p_2 & -p_3 \\ p_1 & p_0 & -p_3 & p_2 \\ p_2 & p_3 & p_0 & -p_1 \\ p_3 & -p_2 & p_1 & p_0 \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$\boxed{r_2 = q\, r_1\, q^T} \qquad r_1 = 0 + \underline{r}_1$$

rotacija okrog z-osi

$$q = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$q_0 = \cos\frac{\vartheta}{2} \qquad q_1 = 0 \qquad q_2 = 0 \qquad q_3 = \sin\frac{\vartheta}{2}$$

$$\underline{\underline{R}} = \begin{bmatrix} \cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2} & -2\cos\frac{\vartheta}{2}\sin\frac{\vartheta}{2} & 0 \\ 2\cos\frac{\vartheta}{2}\sin\vartheta/2 & \cos^2\frac{\vartheta}{2} - \sin^2\frac{\vartheta}{2} & 0 \\ 0 & 0 & \cos^2\frac{\vartheta}{2} + \sin^2\vartheta/2 \end{bmatrix}$$

$$\underline{\underline{R}} = \begin{bmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$$\text{trace}\, \underline{\underline{R}} = r_{11} + r_{22} + r_{33} = 3q_0^2 - q_1^2 - q_2^2 - q_3^2$$

samo ortogonalna matrika vrti

samo en...

$$q_1^2 + q_2^2 + q_3^2 = 1 - q_0^2$$

$$\text{trace } \underline{R} = 4q_0^2 - 1$$

$$q_0^2 = \frac{1}{4}(1 + r_{11} + r_{22} + r_{33})$$
$$q_1^2 = \frac{1}{4}(1 + r_{11} - r_{22} - r_{33})$$
$$q_2^2 = \frac{1}{4}(1 - r_{11} + r_{22} - r_{33})$$
$$q_3^2 = \frac{1}{4}(1 - r_{11} - r_{22} + r_{33})$$

$$\underline{s} = \frac{1}{2\sin\vartheta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

$$q_0 = \frac{1}{2}\sqrt{1 + r_{11} + r_{22} + r_{33}}$$
$$q_1 = \frac{1}{2}\,\mathrm{sign}(r_{32} - r_{23})\sqrt{1 + r_{11} - r_{22} - r_{33}}$$
$$q_2 = \frac{1}{2}\,\mathrm{sign}(r_{13} - r_{31})\sqrt{1 - r_{11} + r_{22} - r_{33}}$$
$$q_3 = \frac{1}{2}\,\mathrm{sign}(r_{21} - r_{12})\sqrt{1 - r_{11} - r_{22} + r_{33}}$$

enačbe za ABB

PRIMER: $90°$ okrog z, $90°$ okrog y

obratni vrstni red!

$$R = R_{y90°} \cdot R_{z90°} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$p = \cos 45° + \sin 45° \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$p_0 = \frac{\sqrt{2}}{2} \qquad \underline{p} = \frac{1}{2}\begin{bmatrix} 0 \\ \sqrt{2} \\ 0 \end{bmatrix}$$

$$q_0 = \frac{\sqrt{2}}{2} \qquad \underline{q} = \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix}$$

$$p \cdot g = \frac{1}{2} - \frac{1}{4}\begin{bmatrix}0 & \sqrt{2} & 0\end{bmatrix}\begin{bmatrix}0\\0\\\sqrt{2}\end{bmatrix} + \frac{\sqrt{2}}{4}\begin{bmatrix}0\\0\\\sqrt{2}\end{bmatrix} + \frac{\sqrt{2}}{4}\begin{bmatrix}0\\\sqrt{2}\\0\end{bmatrix} + \frac{1}{4}\begin{bmatrix}i & j & k\\0 & \sqrt{2} & 0\\0 & 0 & \sqrt{2}\end{bmatrix}$$

$$pg = \frac{1}{2} + \frac{1}{2}\begin{bmatrix}0\\0\\1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0\\1\\0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\0\\0\end{bmatrix} = \frac{1}{2} + \frac{1}{2}i + \frac{1}{2}j + \frac{1}{2}k$$

PRIMER: zavrti $\bar{i}$ za $\frac{2\pi}{3}$ okrog osi, ki gre skozi koord. izhod. in točko $(1,1,1)$

$$s_x^2 + s_y^2 + s_z^2 = 1$$

$$s_x = s_y = s_z = s$$

$$3s^2 = 1 \implies s = \frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$g = \frac{1}{2} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{1}{2} + \frac{1}{2}i + \frac{1}{2}j + \frac{1}{2}k$$

$$\underline{r_2} = \frac{1}{2}(1+i+j+k)(i)\frac{1}{2}(1-i-j-k) \quad \leftarrow \text{rotacija}$$

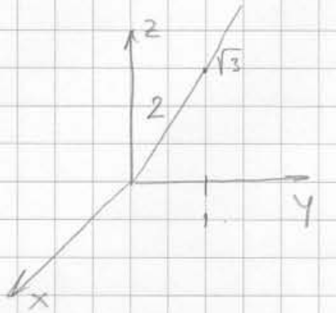
$$r_2 = \frac{1}{4}(i-1-k+j)(1-i-j-k)$$

$$r_2 = \frac{1}{4}(i-1-k+j-i^2-i-ki+ji-ij+j+kj-j^2-jk+k+k^2-jk$$

$$r_2 = \frac{1}{4}(i-1-k+j+1+i+j+k-k+j-i+1+j+k-1-i)$$

$$r_2 = j$$

PRIMER: i za $90°$ okrog $x = 0$, $z = \sqrt{3}y$



$$s = \begin{bmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$p = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$r_2 = p \cdot i \cdot p^*$$

$$p_0 = \frac{\sqrt{2}}{2} \quad p_1 = 0 \quad p_2 = \frac{\sqrt{2}}{4} \quad p_3 = \frac{\sqrt{6}}{4}$$

$$\begin{bmatrix} 0 & \cdot & \cdot \\ \sqrt{2}/2 & \cdot & \cdot \\ \sqrt{2}/4 & \cdot & \cdot \\ -\sqrt{2}/4 & \cdot & \cdot \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2}/2 \\ \sqrt{2}/4 \\ -\sqrt{2}/4 \end{bmatrix}$$

$$r = \begin{bmatrix} 0 & -\sqrt{2}/2 & -\sqrt{6}/4 & \sqrt{2}/4 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/4 & \sqrt{6}/4 \\ \sqrt{6}/4 & \sqrt{2}/4 & 0 & -\sqrt{2}/4 \\ -\sqrt{2}/4 & -\sqrt{6}/4 & \sqrt{2}/4 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ -\sqrt{2}/4 \\ -\sqrt{6}/4 \end{bmatrix} = \begin{bmatrix} \sqrt{12}/16 - \sqrt{12}/16 \\ 2/4 - 2/16 - 6/16 \\ \\ -2/8 \quad -2/8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{3}/2 \\ -1/2 \end{bmatrix}$$

PRIMER:

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.866 \\ 0 & -0{,}866 & 0.5 \end{bmatrix}$$

RPY
$\varphi = 0$
$\vartheta = 0$
$\psi = -60°$

$$q_0 = \frac{\sqrt{3}}{2}$$

$$q_1 = -\frac{1}{2}$$

$$q_2 = 0$$

$$q_3 = 0$$

iz enačb za ABB

R: $q_{z\varphi} = \cos\frac{\varphi}{2} + \sin\frac{\varphi}{2}\,\underline{k}$

P: $q_{y\vartheta} = \cos\frac{\vartheta}{2} + \sin\frac{\vartheta}{2}\,\underline{j}$

Y: $q_{x\psi} = \cos\frac{\psi}{2} + \sin\frac{\psi}{2}\cdot\underline{i}$

$RPY(\varphi, \vartheta, \psi) = q_{z\varphi}\cdot q_{y\vartheta}\cdot q_{x\psi}$

$q_0 = c\frac{\varphi}{2}\,c\frac{\vartheta}{2}\,c\frac{\psi}{2} + s\frac{\varphi}{2}\,s\frac{\vartheta}{2}\,s\frac{\psi}{2}$

$q_1 = c\frac{\varphi}{2}\,c\frac{\vartheta}{2}\,s\frac{\psi}{2} - s\frac{\varphi}{2}\cdot s\frac{\vartheta}{2}\cdot c\frac{\psi}{2}$

$q_2 = c\frac{\varphi}{2}\,s\frac{\vartheta}{2}\,s\frac{\psi}{2} + s\frac{\varphi}{2}\,c\frac{\vartheta}{2}\,s\frac{\psi}{2}$

$q_3 = s\frac{\varphi}{2}\,c\frac{\vartheta}{2}\,s\frac{\psi}{2} - c\frac{\varphi}{2}\,s\frac{\vartheta}{2}\,s\frac{\psi}{2}$

$q_0 = \frac{\sqrt{3}}{2}$

$q_1 = -\frac{1}{2}$

$q_2 = 0$

$q_3 = 0$