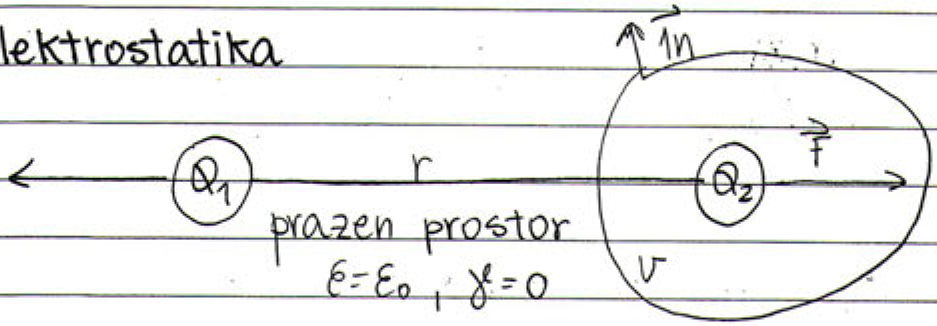
Elektrostatika



enak predznak
→ sila odbojna

sila: $\boxed{\vec{F} = \vec{1}_{odbojna} \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r^2}}$ [N]

sila normirana na enoto elektrine:

$\frac{\vec{F}}{Q_1} = \vec{1}_{odbojna} \frac{1}{4\pi\varepsilon_0} Q_2 = \vec{E}_2$ ← električno polje $\left[\frac{N}{C}\right] = \left[\frac{V}{m}\right]$

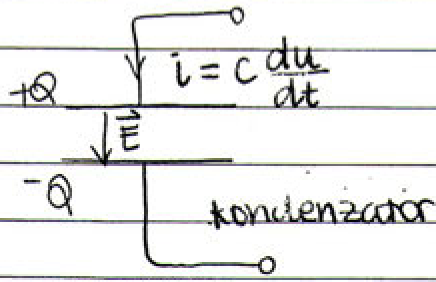
vektor el. poljske jakosti

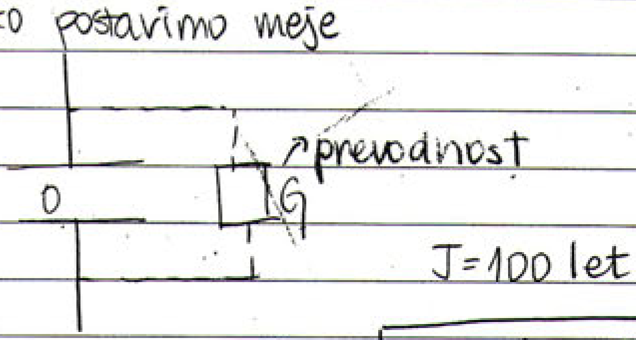
vektor električnega pretoka: $\boxed{\vec{D}_2 = \varepsilon_0 \vec{E}_2}$ $\left[\frac{As}{Vm}\right]\left[\frac{V}{m}\right] = \left[\frac{As}{m^2}\right] = \left[\frac{C}{m^2}\right]$

Gaussov zakon $\boxed{Q_2 = \oint_A \vec{D} \cdot \vec{1}_n \cdot dA = \int_v \rho \, dv}$ [As] = [C]

napetost: $\boxed{U = \pm \int \vec{E} \cdot d\vec{s}}$

odvisno kako postavimo meje





$\boxed{Q = C \cdot U}$ el. energija kondenzatorja $\boxed{W_e = \frac{1}{2} C U^2}$

SNOV:

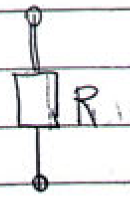
dielektričnost: $\varepsilon \neq \varepsilon_0$ $\varepsilon = \varepsilon_r \varepsilon_0$ $\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{As}{Vm}$ $\left[\frac{As}{Vm}\right] = \left[\frac{F}{m}\right]$

električna prevodnost: $\gamma \neq 0$ [S]

gostota prevodniškega toka: $\boxed{\vec{J} = \gamma \cdot \vec{E}}$ [A/m²]

prevodniški električni tok: $\boxed{I = \int_A \vec{J} \cdot \vec{1}_n \, d\vec{A}}$

$I = G \cdot U = \frac{U}{R}$



↳ elektrika se pretvarja v toploto

moč ki se troši na uporu:

$$\boxed{P = I^2 R = \frac{U^2}{R} = U^2 G}$$

$[W] = [VA]$

Magnetostatika: $\vec{H}$ (mag. poljska jakost) → računski pripomoček

Magnetna poljska jakost

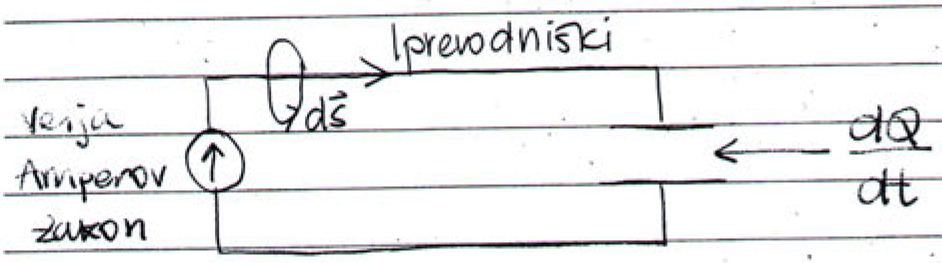
$$\boxed{\oint_s \vec{H} \cdot d\vec{s} = I_{celotni} + I_{\underset{(konduktivni)}{prevodniški}} + I_{konvektivni} + \frac{dQ}{dt}}$$

el. veličina +

$\left[\frac{Vs}{m^2} / \frac{Vs}{Am} \right] = [A/m]$

poljski tok

sprememba elektrine na enoto časa



$\frac{dQ}{dt}$ količina elektrine se povečuje

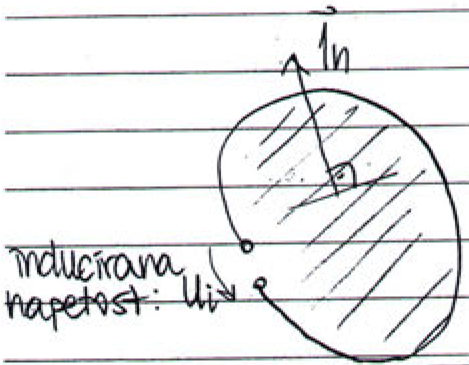
Gostota mag. pretoka $\boxed{\vec{B} = \mu_0 \vec{H}}$ $\boxed{\vec{B} = \mu \vec{H}}$ $[T] = \left[\frac{Vs}{m^2}\right]$

prazen prostor

snov.

$\mu = \mu_r \mu_0$

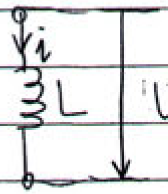
$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$ $\left[\frac{Vs}{Am}\right] = [H/m]$



Faraday-ev zakon

$$\boxed{u_i = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_A \vec{B} \cdot \vec{1}_n \cdot dA}$$

mag. pretok: $\boxed{\Phi = \int_A \vec{B} \cdot \vec{1}_n \cdot dA}$ $[Wb] = Vs = Tm^2$

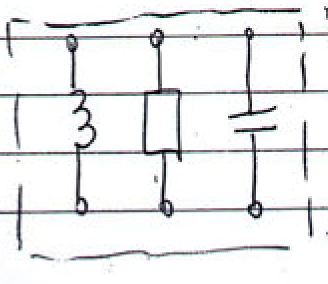


induktivnost:

$$\boxed{u = L \frac{di}{dt}}$$

$$\boxed{W_m = \frac{1}{2} I^2 L}$$

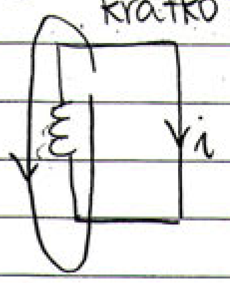
Tuljava in kondenzator lahko hranita energijo. Kondenzator hrani energijo, če ga odklopimo, tuljava pa če je kratko sklenjena

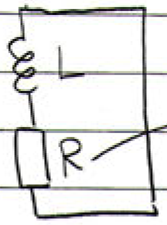


0-D ⇒ nič dimenzij

Pri kond. lahko to traja "∞ časa" (100 let), pri tuljavi pa manj časa (1ms, 1 teden - če prevodna)

tuljava hrani
energijo, če je
kratko sklenjena





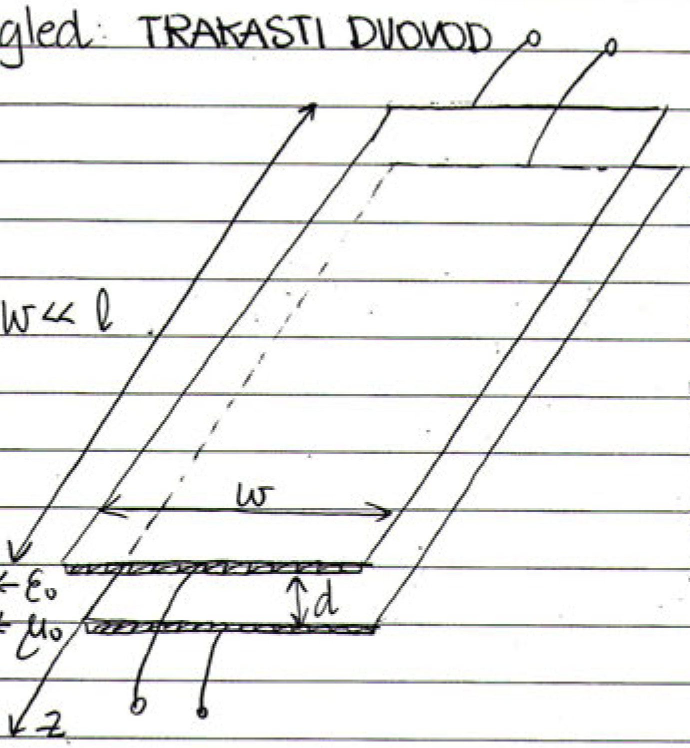
$J_p = 1$teden $J = 1$ms

na tem upori se trosi moč
zgublja se energija

Zgled: TRAKASTI DVOVOD

$\boxed{1\text{-}D = l}$

$d < w \ll l$



kapacitivnost na enoto dolžine

$$\boxed{C/l = \varepsilon \frac{w}{d}}$$

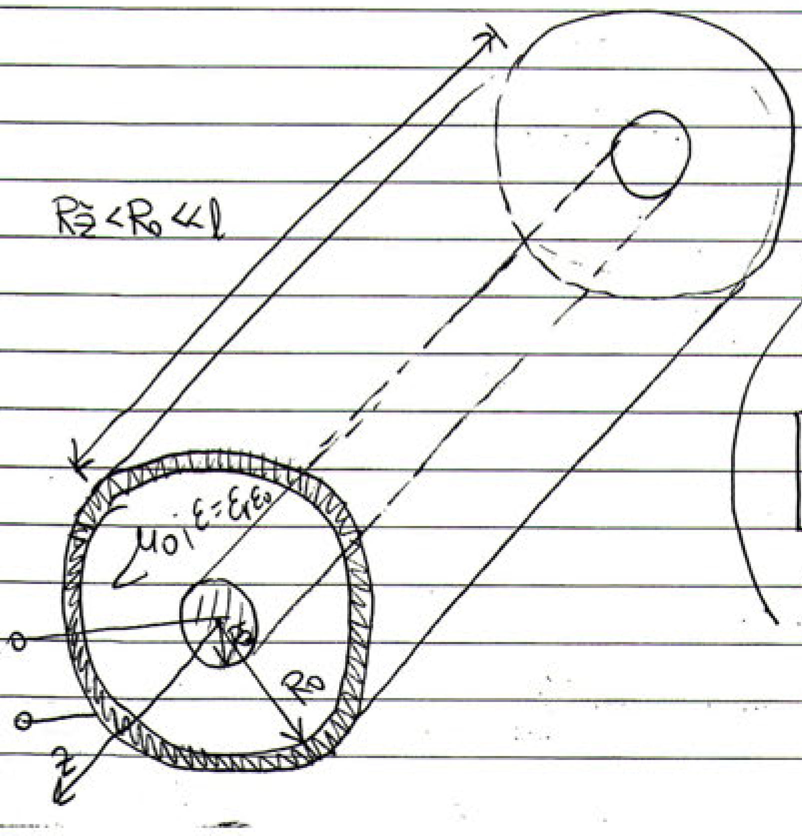
$$\boxed{L/l = \mu \frac{d}{w}}$$

porazdeljene veličine

induktivnost na enoto dolžine

KOAKSIALNI KABEL Rž - radij žice ; Ro - radij oklopa

$R_ž < R_o \ll l$



$$\boxed{C/l = 2\pi\varepsilon / \ln\frac{R_o}{R_ž}}$$

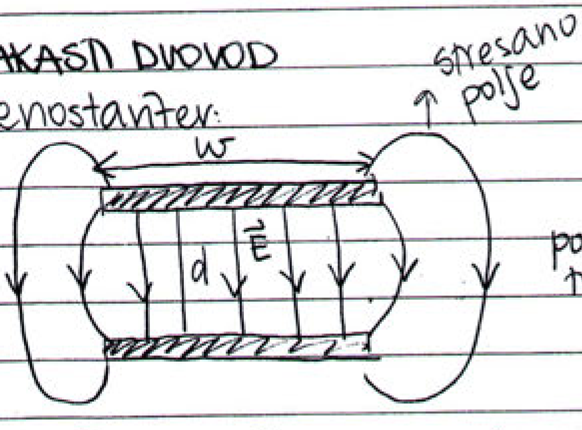
$$\boxed{L/l = \frac{\mu}{2\pi} \ln\frac{R_o}{R_ž}}$$

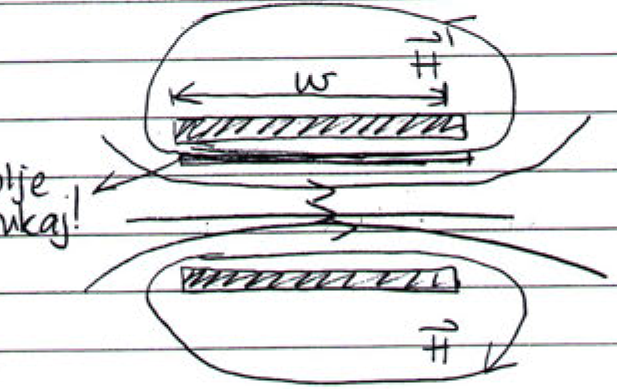
porazdeljene veličine

d<<w => mag. polje skoncentrirano med ploščama

TRAKASTI DVOVOD

poenostavitev:





d<<w => stresanje zanemarimo

$$\boxed{C = \varepsilon \frac{\overset{\text{površina}}{A}}{d} = \varepsilon \frac{wl}{d} = \varepsilon \frac{w}{d}\cdot l}$$

$$\boxed{\oint \vec H d\vec s = I = |\vec H|\cdot w}$$

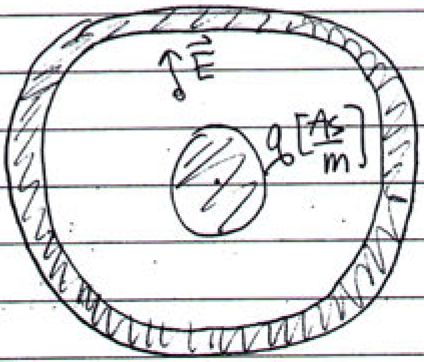
$\vec B = \mu \vec H$

$$|\vec B| = \mu \cdot \frac{I}{w} \Leftarrow$$

$$\boxed{\Phi = |\vec B|\cdot A = |B|\cdot d\cdot l = \mu\frac{I}{w}d\cdot l}$$

$$\boxed{L = \frac{\Phi}{I} = \mu\frac{d}{w}\cdot l}$$

KOAKSIALNI KABEL



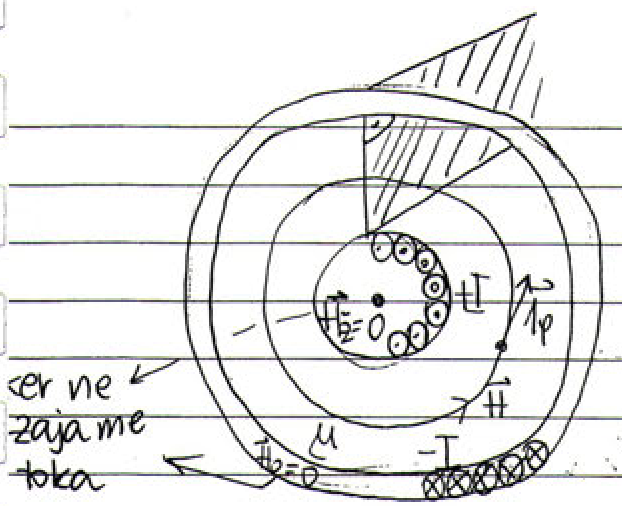
celotna elektrina

$Q = q\cdot l$

$$\boxed{\vec E = \vec 1_n \frac{q}{2\pi\varepsilon}\frac{1}{r}}$$

$$\boxed{U = \int \vec E\cdot d\vec s = \frac{q}{2\pi\varepsilon}\ln\frac{R_o}{R_ž}}$$

$$\underline{\underline{C = \frac{Q}{U}}} = \frac{q\cdot l}{q\cdot\frac{1}{2\pi\varepsilon}\ln\frac{R_o}{R_ž}} = \underline{\underline{2\pi\varepsilon l/\ln\frac{R_o}{R_ž}}}$$



$$\boxed{\vec{H} = \vec{1}_\varphi \frac{I}{2\pi r}}$$

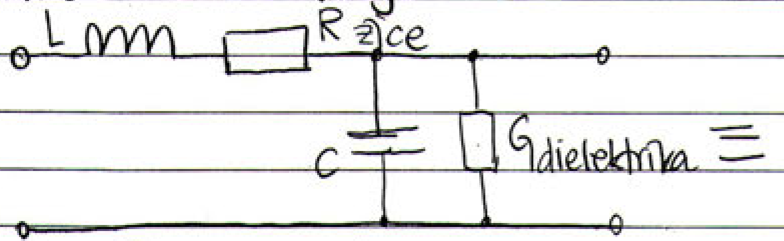
$$\boxed{\vec{B} = \vec{1}_\varphi \mu \frac{I}{2\pi r}}$$

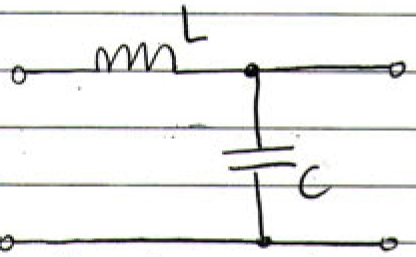
$$\Phi = \int \vec{B}\cdot\vec{1}_n \cdot dA = \int_{R_{\check{z}}}^{R_0}\int_0^l \vec{B}\cdot\vec{1}_\varphi \, dr\, dz = \int_{R_{\check{z}}}^{R_0}\int_0^l \mu \frac{I}{2\pi r} \, dr\, dz$$

$$\Rightarrow \boxed{\Phi = \frac{\mu I}{2\pi} \ln\frac{R_0}{R_{\check{z}}}\cdot l}$$

$$L = \frac{\Phi}{I} \Rightarrow \boxed{L = \frac{\mu}{2\pi} \ln\frac{R_0}{R_{\check{z}}}\cdot l}$$

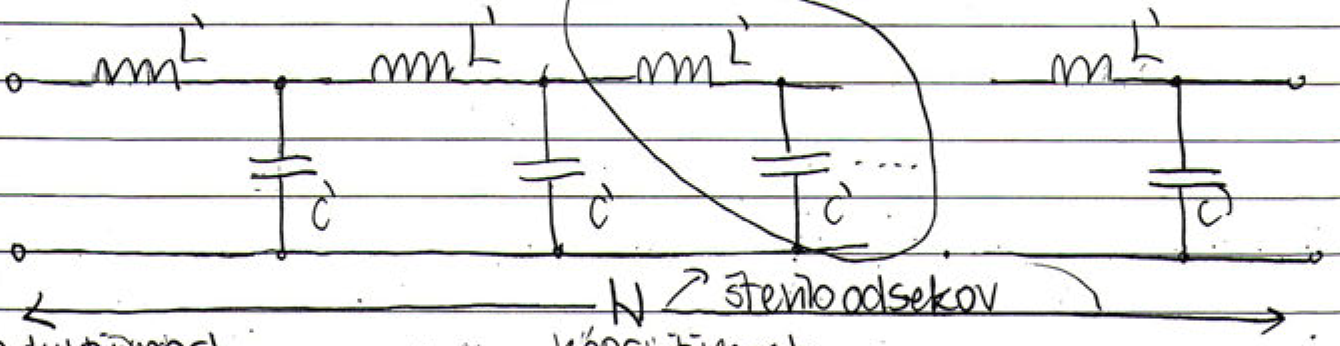
Nadomestno vezje





resnično vezje

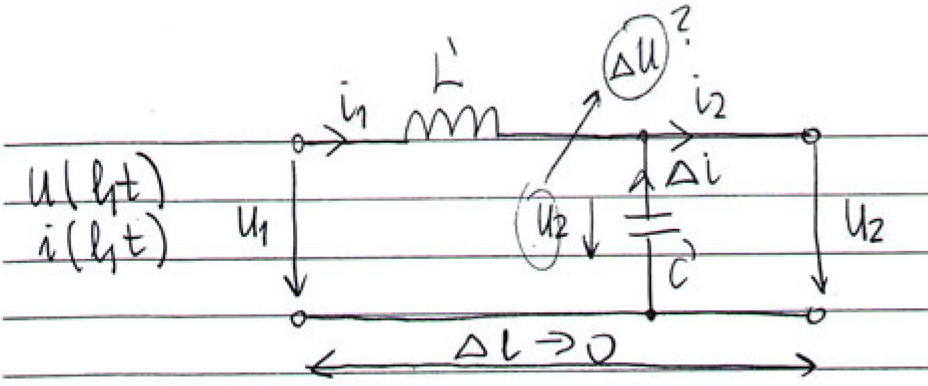
brezizgubno vezje



nduktivnost enega kratkega odseka: $L' = \frac{L}{N}$

kapacitivnost enega kratkega odseka $C' = \frac{C}{N}$ ⟶

Resnica: $N \to \infty$



$$\Delta u = u_2 - u_1 = -L' \frac{di_1}{dt} \qquad L' = L/l \, \Delta l$$

$$\Delta i = i_2 - i_1 = -C' \frac{du_2}{dt} \qquad C' = C/l \cdot \Delta l$$

$$\Delta u = -L' \frac{di_1}{dt} = -L/l \, \Delta l \cdot \frac{di_1}{dt}$$

$$\Delta i = -C' \frac{du_2}{dt} = -C/l \cdot \Delta l \cdot \frac{du_2}{dt}$$

$\Delta l \to 0 \Rightarrow i_1 \approx i$, ko $\Delta l \to 0$

$u_2 \approx u$, ko $\Delta l \to 0$

$$\frac{\Delta u}{\Delta l} = \frac{du}{dl} = -L/l \frac{di_1}{dt} = \frac{du}{dz}$$

$$\frac{\Delta i}{\Delta l} = \frac{di}{dl} = -C/l \frac{du_2}{dt} = \frac{di}{dz}$$

$$\Rightarrow u(z,t): \frac{\partial u(z,t)}{\partial z} = -L/l \frac{\partial i(z,t)}{\partial t}$$

$$i(z,t): \frac{\partial i(z,t)}{\partial z} = -C/l \frac{\partial u(z,t)}{\partial t}$$

u(z,t): $\boxed{\frac{\partial u}{\partial z} = -L/l \frac{\partial i}{dt}}$ Brezizgubna telegrafska enačba v časovnem prostoru

i(z,t): $\boxed{\frac{\partial i}{\partial z} = -C/l \frac{\partial u}{\partial t}}$

za u		za i
$\frac{\partial}{\partial z}$	oz	$\frac{\partial}{\partial t}$
$\frac{\partial}{\partial t}$		$\frac{\partial}{\partial z}$
①		②

Resitev:

$$\Rightarrow \quad \frac{\partial^2 u}{\partial z^2} = -L/l \, \frac{\partial^2 i}{\partial z \partial t} \leftarrow$$

$$\frac{\partial^2 i}{\partial z \partial t} = -C/l \cdot \frac{\partial^2 u}{\partial t^2} \quad \star$$

vstavimo

iz ①: $\boxed{\frac{\partial^2 u}{\partial z^2} = (L/l)(C/l)\frac{\partial^2 u}{\partial t^2}}$ Enodimenzijska valovna enačba

iz ②: $\boxed{\frac{\partial^2 i}{\partial z^2} = (L/l)(C/l)\frac{\partial^2 i}{\partial t^2}}$ (linearna enačba)

$u(z,t) = ?, \; i(z,t) = ?$

Ugibam: $u(z,t) = u(t \pm \frac{z}{v}) \rightarrow$ zato, ker $t[s]$ in $z[m]$

$i(z,t) = i(t \pm \frac{z}{v})$ želimo, da je funkcija ene same spremenljivke

$$\frac{\partial u}{\partial t} = u' \; ; \; \frac{\partial^2 u}{\partial t^2} = u''$$

$$\frac{\partial u}{\partial z} = \pm \frac{1}{v} u' \; ; \; \frac{\partial^2 u}{\partial z^2} = \pm \frac{1}{v^2} u''$$

vstavimo v $\star$

$$\hookrightarrow \frac{1}{v^2} u'' = (L/l)(C/l) u''$$

$$\frac{1}{v^2} = (L/l)(C/l) \quad \Rightarrow \quad \boxed{v = \sqrt{\frac{1}{(L/l)(C/l)}}}$$

konstanta v ni odvisna od oblike dvovoda

$\Uparrow$

Resitev enodimenzijske valovne enačbe

Zgled: TRAKASTI DVOVOD:

$$v = \sqrt{\frac{1}{\mu \frac{d}{w} \varepsilon \frac{w}{d}}} = \frac{1}{\sqrt{\mu \varepsilon}}$$

KOAKSIALNI KABEL:

$$v = \sqrt{\frac{1}{\frac{\mu}{2\pi} \ln \frac{R_0}{R_2} \frac{2\pi\varepsilon}{\ln R_0/R_2}}} = \frac{1}{\sqrt{\mu \varepsilon}}$$

$\Rightarrow$ hitrost svetlobe

$$\boxed{\frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c_0}$$

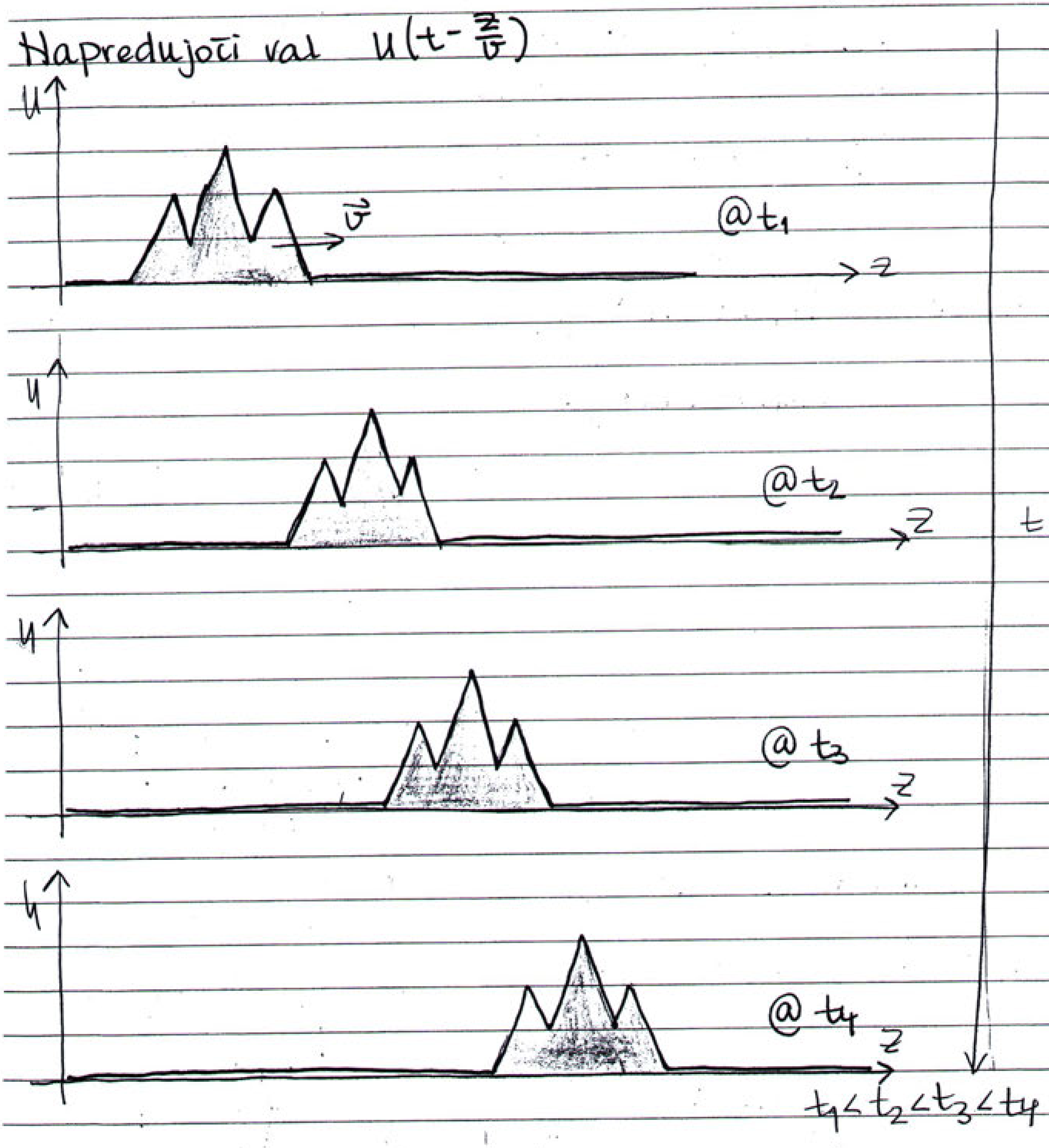
stvari ne mirujejo, ampak se gibljejo s to hitrostjo (se spreminja v odvisnosti od snovi)

Možni rešitvi:

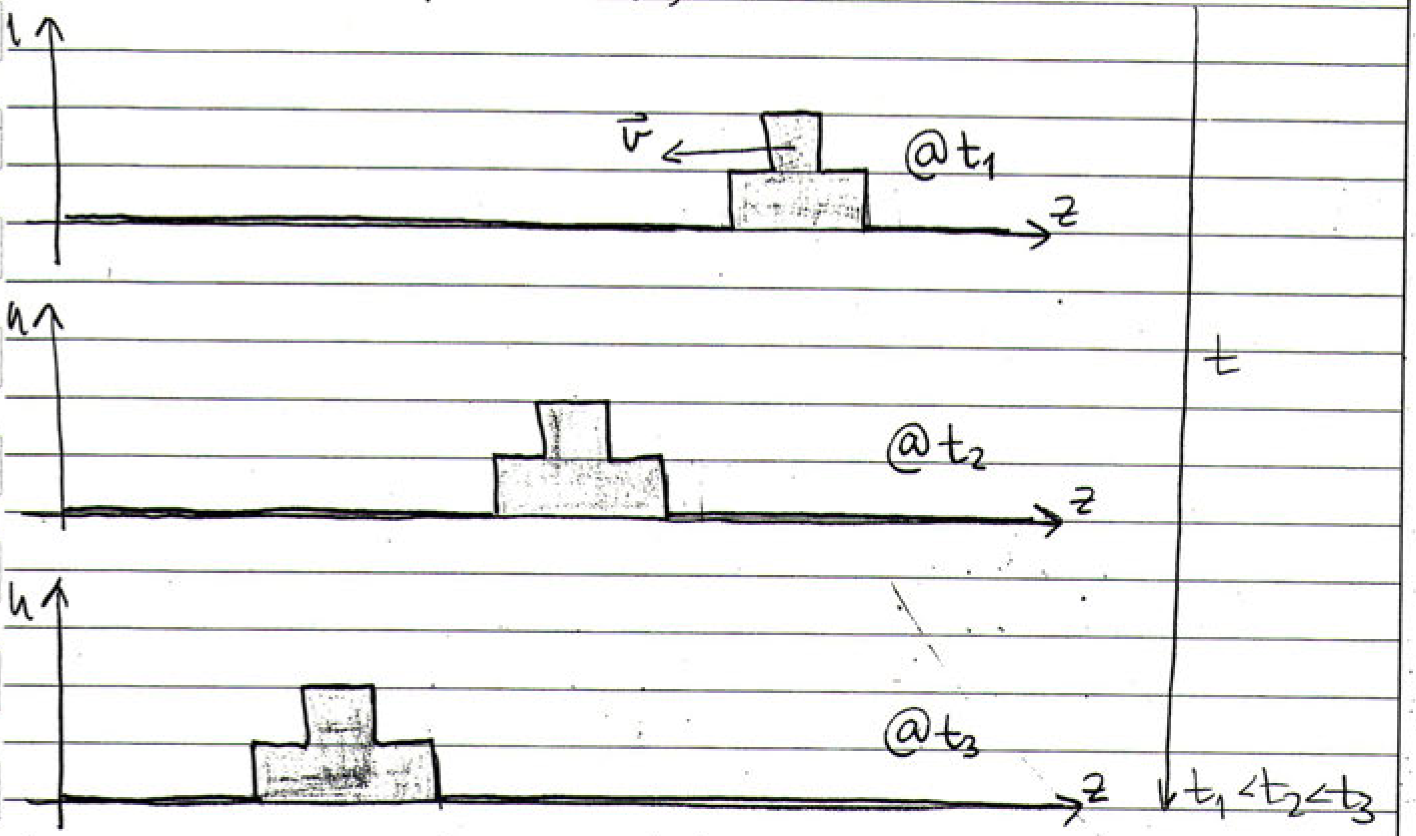
$u(z,t) = u(t - \frac{z}{v})$ → NAPREDUJOČI VAL

$u(z,t) = u(t + \frac{z}{v})$ → ODBITI VAL

Napredujoči val $u(t - \frac{z}{v})$



Odbiti val $u(z,t)=(t+\frac{z}{v})$



Kako vemo, ali je val $^{\text{napredujoči}}$ ali odbiti?

$$\frac{\partial i}{\partial z}=-C/l\,\frac{\partial u}{\partial t}=\pm\frac{1}{v}\frac{\partial i}{\partial t}$$

$$i(z,t)=i(t\pm\frac{z}{v})$$

$$\frac{\partial i}{\partial z}=\pm\frac{1}{v}i'=\pm\frac{1}{v}\frac{\partial i}{\partial t}$$

//

Napredujoči val in odbiti val lahko hkrati obstajata na nekem dvovodu

$$-C/l\cdot u'=\pm\frac{1}{v}i' \,/\int..dt$$

$$-C/l\cdot u=\pm\frac{1}{v}i+\text{konst}$$

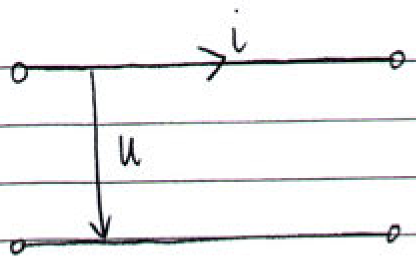
zaradi tege

enosmerna komponenta

samo izmenične:

$$\underline{i}=\mp v\cdot C/l\cdot u=\mp\sqrt{\frac{1}{C/l\cdot L/l}}\cdot C/l\cdot u=\underline{\underline{\mp\sqrt{\frac{C}{L}}u}}$$

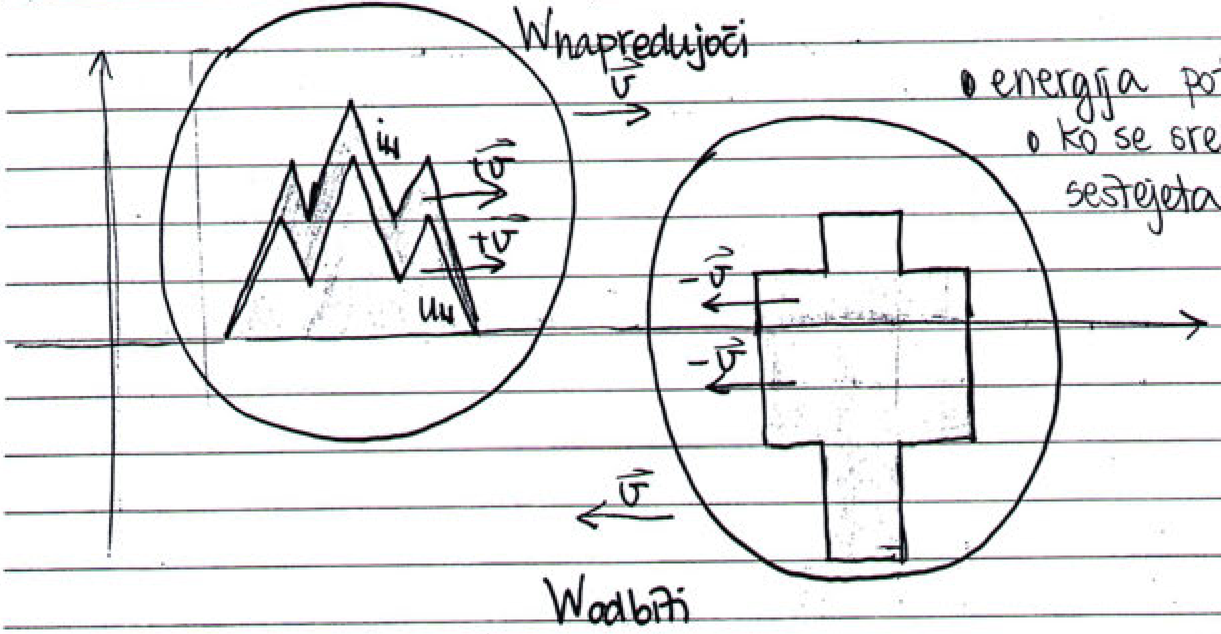
$$v=\sqrt{\frac{1}{L/l\cdot C/l}}$$



Napredujoči val: $\boxed{i = +\,{}^{C}\!/\!_{L} \cdot u}$ $\to$ -L

Odbiti val: $\boxed{i = -\,{}^{C}\!/\!_{L} \cdot u}$ $\to$ L

razlikujeta se po predznaku toka



- energija potuje
- ko se srečata se seštejeta

$u_N = \pm\sqrt{L/C} \cdot i_N = Z_K\, i_N$ $\quad u_0 = \pm\sqrt{L/C} \cdot i_0 = Z_K \cdot i_0$

Karakteristična impedanca: $\boxed{Z_K = \sqrt{L/C}}$ $[\Omega]$!

Zgled: TRAKASTI DVOVOD

$$Z_K = \sqrt{\frac{\mu \frac{d}{w}}{\varepsilon \frac{w}{d}}} = \frac{d}{w}\sqrt{\frac{\mu}{\varepsilon}} \Rightarrow Z_K = 377\Omega \sqrt{\frac{\mu_r}{\varepsilon_r}}\ \frac{d}{w}$$

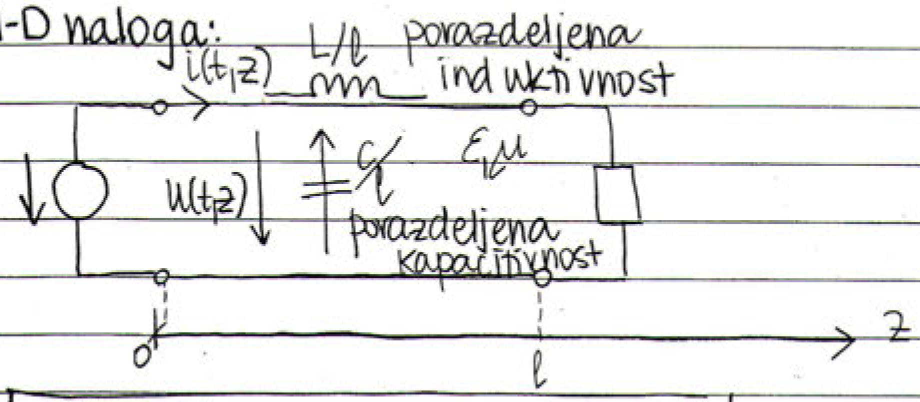
KOAKSIALNI KABEL

$$Z_K = \sqrt{\frac{\frac{\mu}{2\pi} \ln \frac{R_0}{R_Z}}{2\pi\varepsilon / \ln \frac{R_0}{R_Z}}} = \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{\ln R_0/R_Z}{2\pi} \qquad Z_K = 60\Omega \sqrt{\frac{\mu_r}{\varepsilon_r}} \ln \frac{R_0}{R_Z}$$

$$\boxed{\sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi\,\Omega \approx 377\,\Omega}$$

karakteristična impedanca se z obliko dvovoda spreminja

1-D naloga:



$$\boxed{\begin{aligned} \frac{\partial^2 u(t,z)}{\partial z^2} &= C/l \cdot L/l \, \frac{\partial^2 u(t,z)}{\partial t^2} \\ \frac{\partial^2 i(t,z)}{\partial z^2} &= C/l \cdot L/l \, \frac{\partial^2 i(t,z)}{\partial t^2} \end{aligned}}$$

Telegrafska enačba (brezizgubna)

-rešitve enačbe: $u(t,z) = \underbrace{u\left(t \pm \frac{z}{v}\right)}_{\text{poljubna funkcija ene spremenjivke}}$

$$v = \frac{1}{\sqrt{C/l \cdot L/l}} = \frac{1}{\sqrt{\varepsilon\mu}}$$

prazen prostor: $v = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c_0$

dve linearno neodvisni rešitvi:

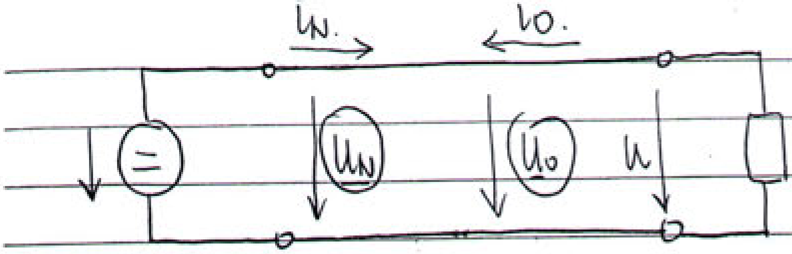
$$u(t,z) = \underbrace{u\left(t - \frac{z}{v}\right)}_{\text{napredujoči val}} + \underbrace{u\left(t + \frac{z}{v}\right)}_{\text{odbiti val}}$$

karakteristična impedanca: $\frac{u}{i} = \pm Z_K$

$$Z_K = \sqrt{\frac{L/l}{C/l}} = \sqrt{\frac{L}{C}}$$

$$\frac{u_{napr.}}{i_{napr.}} = +Z_K \Rightarrow i_N = \frac{u_N}{Z_K} \; *$$

$$\frac{u_{odbiti}}{i_{odbiti}} = -Z_K \Rightarrow i_O = \frac{-u_O}{Z_K} \; *$$



Ohmov zakon: $R_b = \frac{u}{i} \Rightarrow i = \frac{u}{R_b}$

u_N, u_O … podana ↗ na bremenu

$u = u_N + u_O$

$i = i_N + i_O$

iz * $i = \frac{u_N}{Z_K} - \frac{u_O}{Z_K}$

$$i = \underbrace{\frac{u_N + u_O}{R_b}}_{\text{ohmov zakon}} = \frac{u_N - u_O}{Z_K}$$

$$u_O\left(\frac{1}{R_b} + \frac{1}{Z_K}\right) = u_N\left(\frac{1}{Z_K} - \frac{1}{R_b}\right) / Z_K \cdot R_b$$

$$u_O(Z_K + R_b) = u_N(R_b - Z_K)$$

razmerje: $\frac{u_O}{u_N} = \frac{R_b - Z_K}{R_b + Z_K}$] opisuje za kakšno breme gre. razmerje odbitega in napredujočega vala

odbojnost: $\boxed{\Gamma = \frac{u_O}{u_N} = \frac{R_b - Z_K}{R_b + Z_K}}$ ↗ PAZI!

↳ vedno definirana na napetosti u ali el. poljsko jakostjo $\vec{E}$!

↳ če definirana na i ali $\vec{H}$ obraten predznak!

-Zgled

- PRILAGODJENO BREME: $R_b = Z_K \Rightarrow \underline{\Gamma = 0}$
- KRATEK STIK: $R_b = 0 \Rightarrow \underline{\Gamma = -1}$
- ODPRTE SPONKE: $R_b = \infty \Rightarrow \underline{\Gamma = 1}$
- $Z_K = 50\,\Omega, R_b = 100\,\Omega \Rightarrow \Gamma = \frac{100\Omega - 50\Omega}{100\Omega + 50\Omega} = \frac{50\Omega}{150\Omega} = \underline{\underline{0{,}333}}$
- $Z_K = 50\,\Omega, R_b = 10\,\Omega \Rightarrow \Gamma = \frac{10\Omega - 50\Omega}{10\Omega + 50\Omega} = -\frac{40\Omega}{60\Omega} = \underline{\underline{-0{,}667}}$

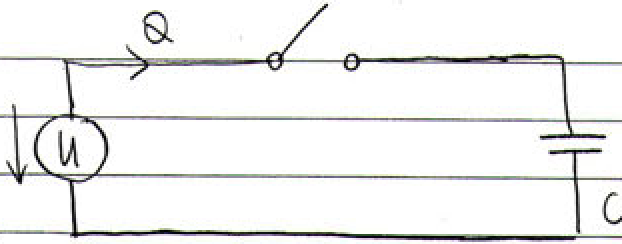
lastnost odbojnosti:

pasivno breme: $\boxed{|\Gamma| \leq 1}$ ↗ odbiti val šibkejši od napredujočega vala

predznak: (-) breme manjše od Z_K

(+) breme večje od Z_K

Ko vklopimo stikalo se prazen kondenzator napolni na napetost U

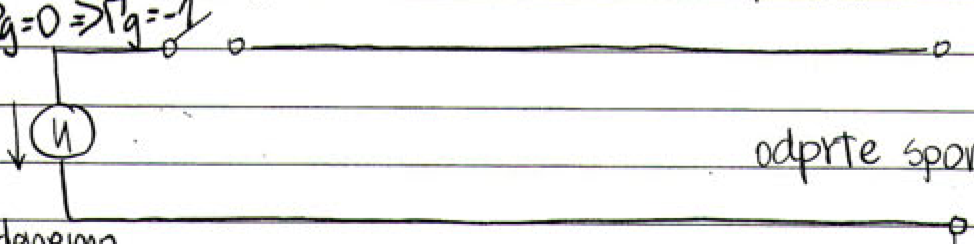


$W_g = Q \cdot U = CU^2$ $W_C = \frac{1}{2} \cdot CU^2$

O-D: Kam je šla energija

ZVONENJE SIGNALA

$R_g = 0 \Rightarrow \Gamma_g = -1$

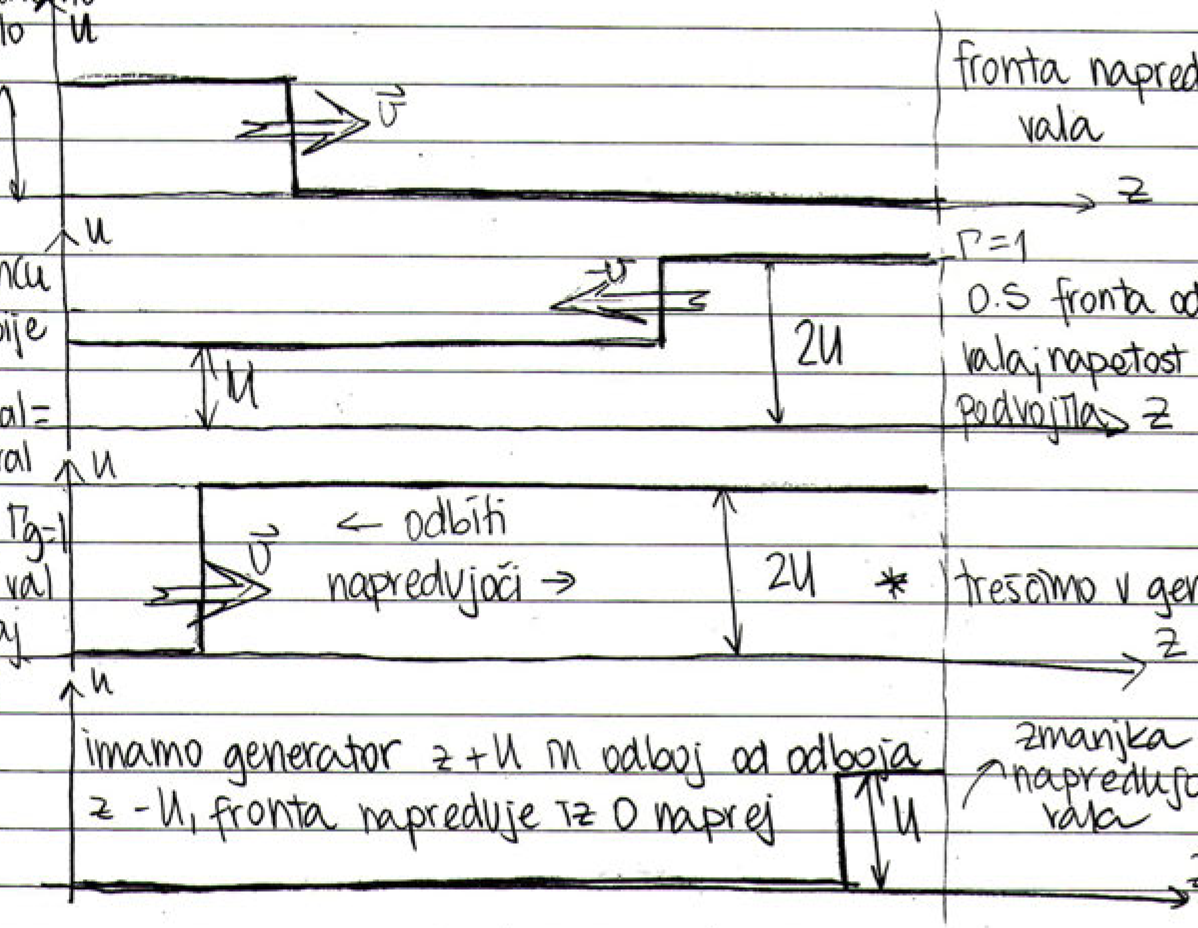


odprte sponke $\Gamma_b = 1$

o sklenemo stikalo

a koncu e odbije os) dbiti val = napr. val

$\Gamma_g = 1$ dbiti val e tukaj dbije in se nniči



fronta napredujočega vala

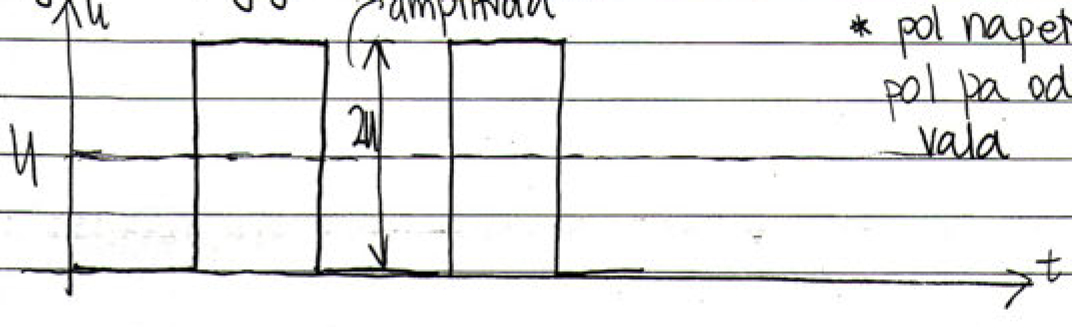
O.S fronta odbitega vala; napetost se podvojila

trešcimo v generator

imamo generator z +U in odboj od odboja z -U, fronta napreduje iz 0 naprej

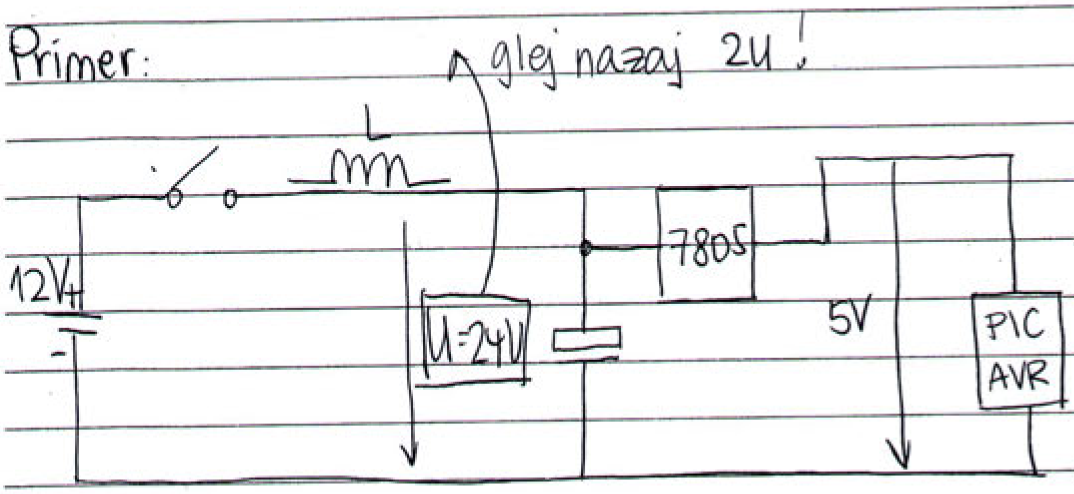
zmanjka napredujočega vala

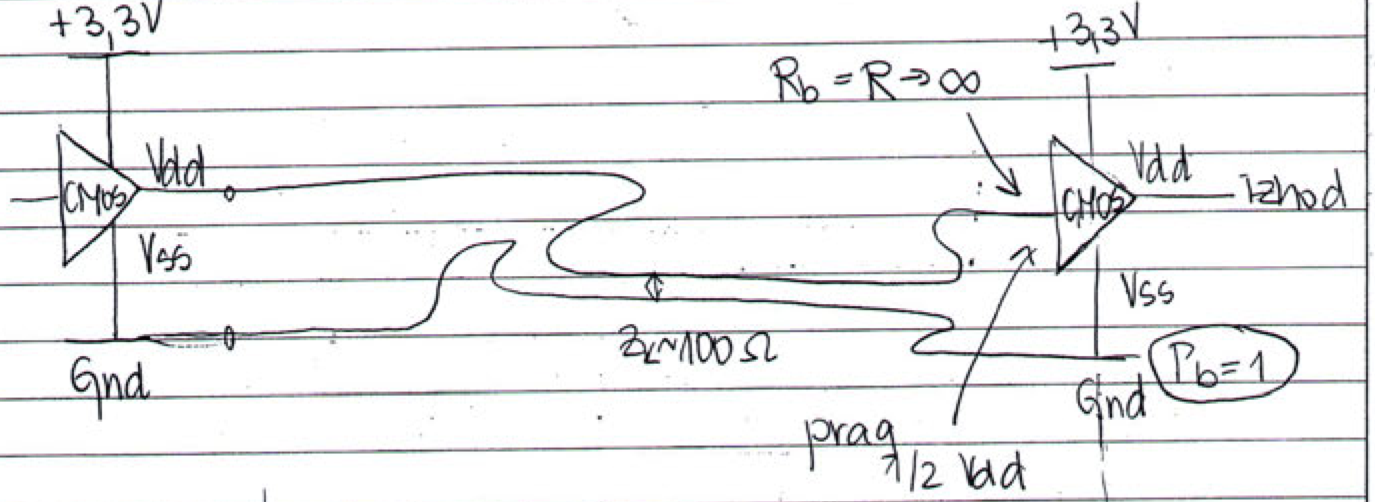
Kaj se dogaja na sponkah?

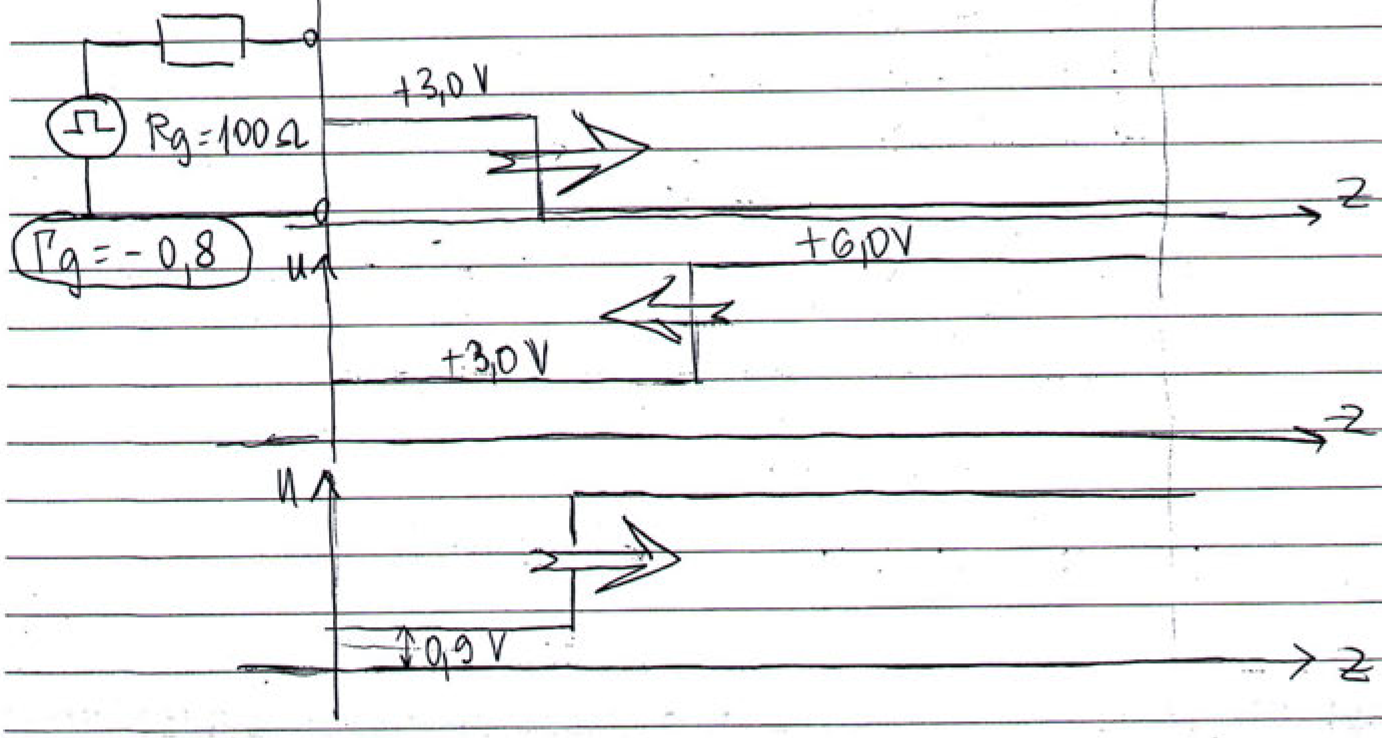


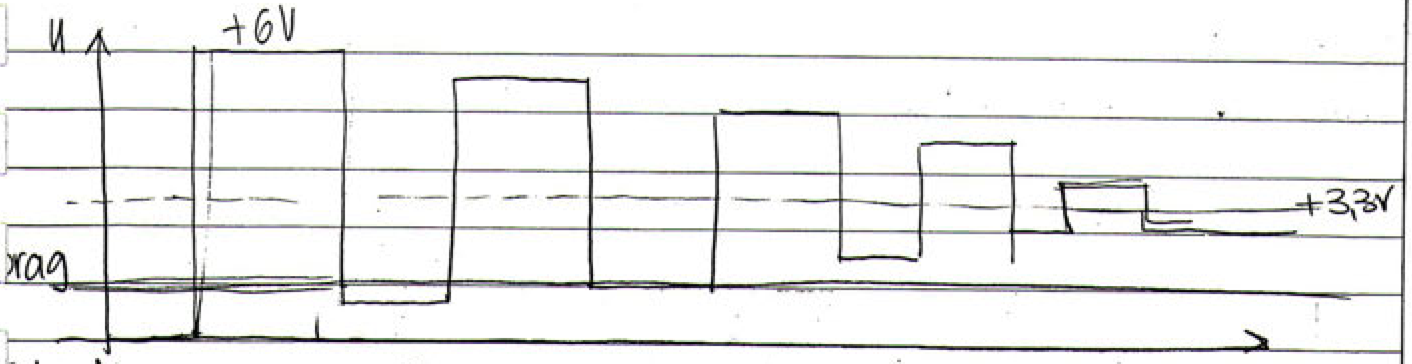
* pol napetosti od odbitega, pol pa od napredujočega vala

Primer:

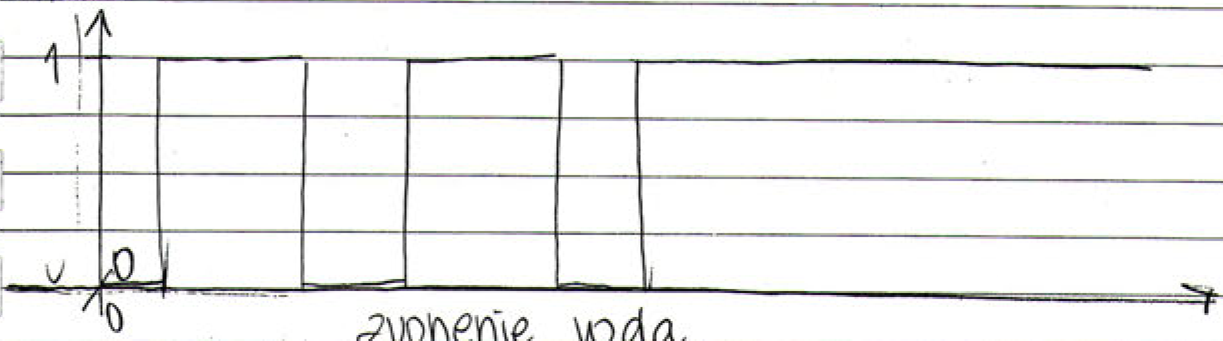

glej nazaj 2u!
L
12V +
-
U=24V
7805
5V
PIC
AVR


+3,3V
CMOS
Vdd
Vss
Gnd
$Z_c \sim 100\,\Omega$
$R_b = R \to \infty$
+3,3V
CMOS
Vdd
Izhod
Vss
Gnd
$\Gamma_b = 1$
prag 1/2 Vdd


$R_g = 100\,\Omega$
$\Gamma_g = -0,8$
+3,0 V
z
u
+6,0 V
+3,0 V
z
u
0,9 V
z



izhod:

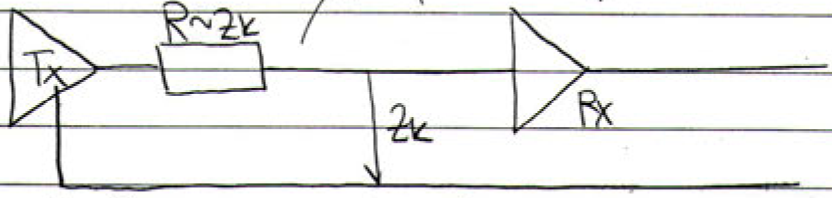


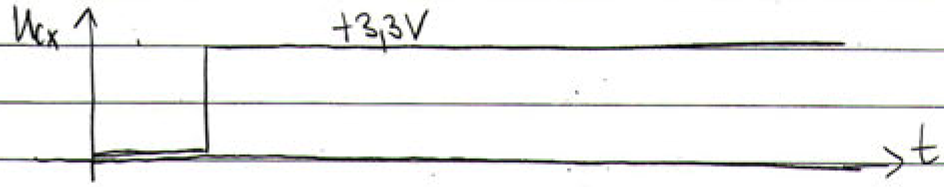
zvonenje voda

Kako to rešimo? Da preprečimo zvonenje voda

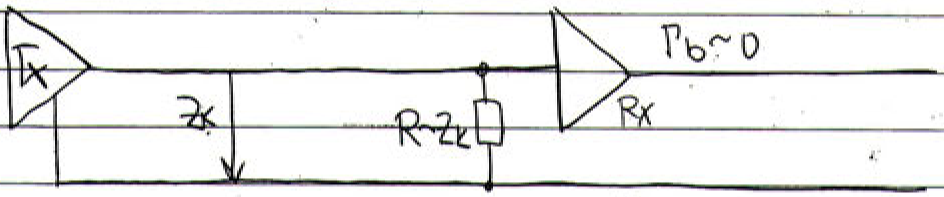
za poredni upori ~ da vodila ne zvonijo

1)

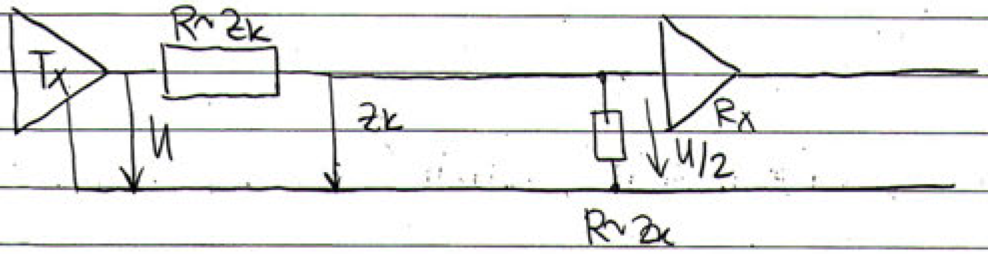




2)

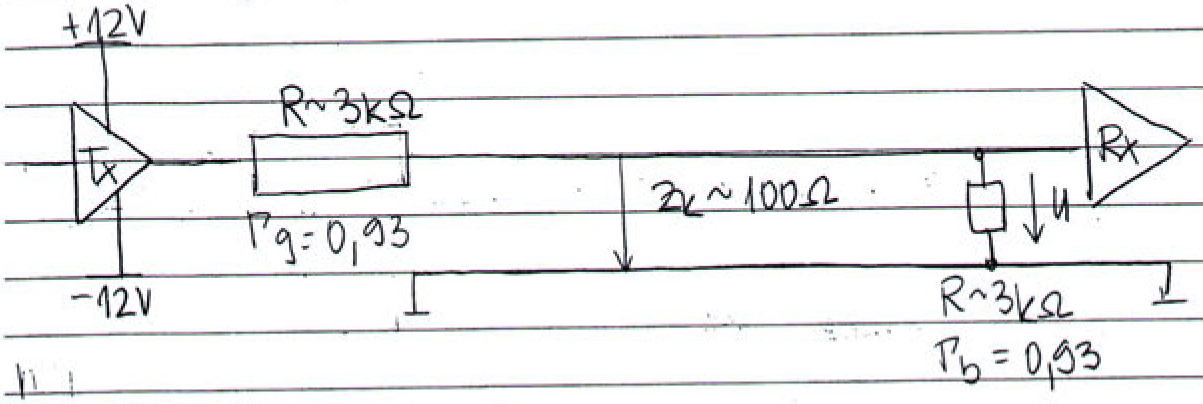


3)

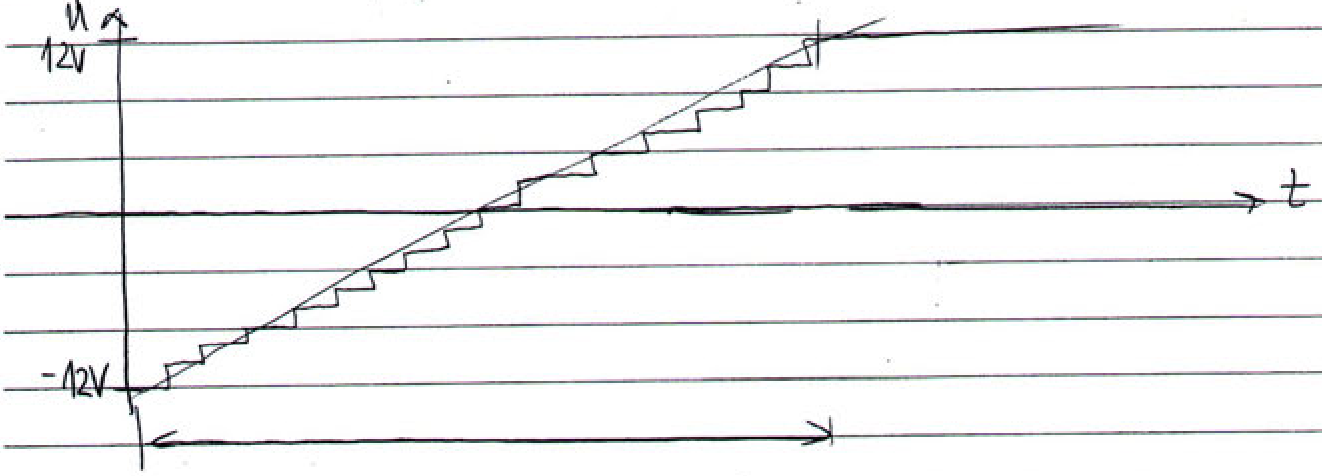


COM port

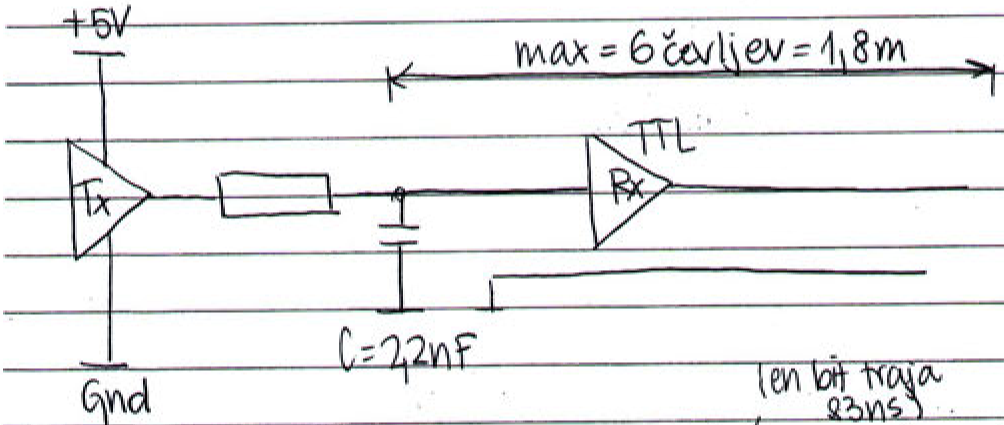
RS-232 ($c<115$ kbit/s)



napetost na sprejemniku:

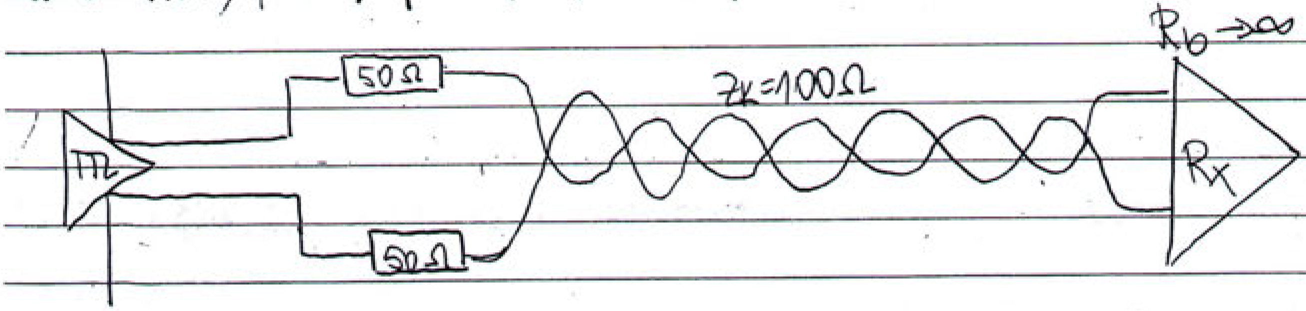


LPT (tiskalnik)

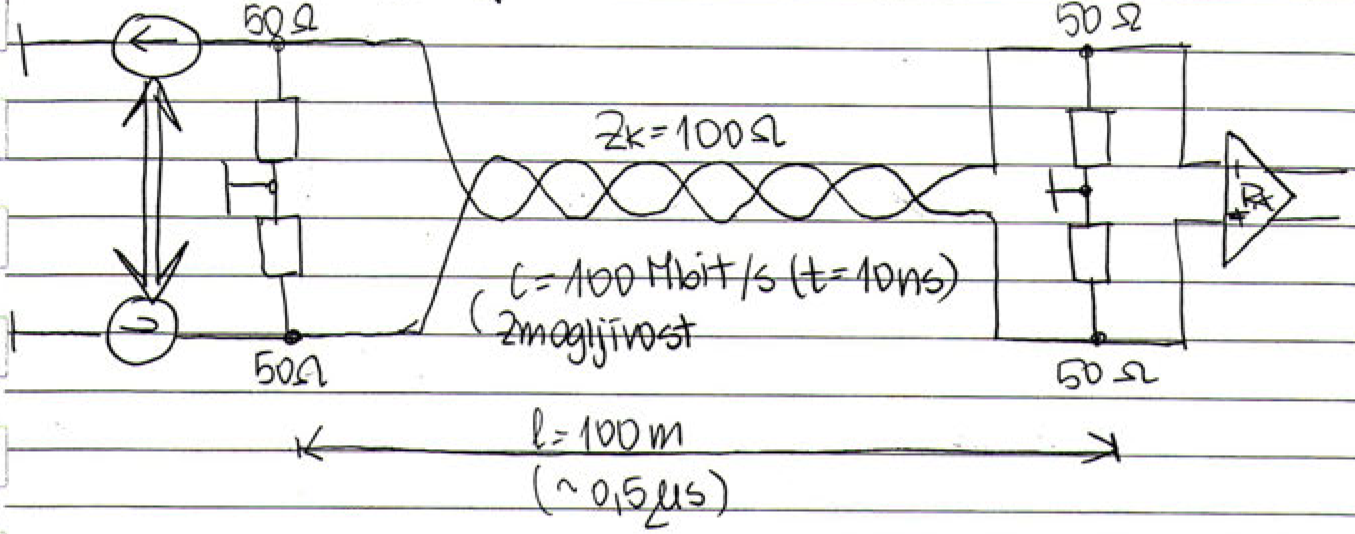


(en bit traja 83ns) zakasnitve

USB (low/full) speed ⇒ 1,5 Mbit/s / 12 Mbit/s ⇒ $l=2m$ (10ns)

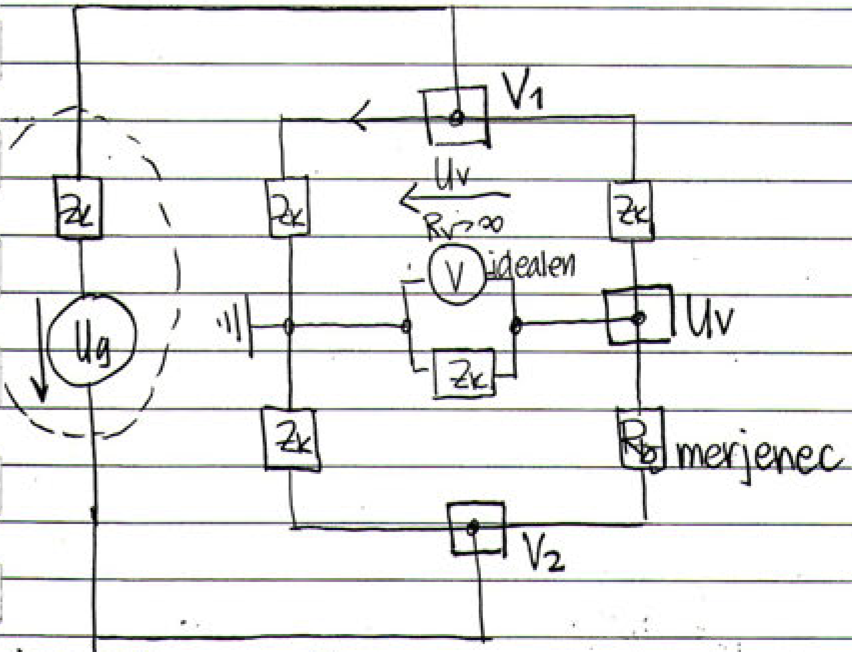


Ethernet / UTP (hi-speed USB 120 Mbit/s)



Meritev odbojnosti Γ?

Mostiček:



Vozliščne enačbe

V_1: $0 = V_1 \frac{1}{Z_k} + (V_1 - U_v)\frac{1}{Z_k} + (V_1 - V_2 - U_g)\frac{1}{Z_k}$ (1)

U_v: $0 = (U_v - V_1)\frac{1}{Z_k} + U_v \frac{1}{Z_k} + (U_v - V_2)\frac{1}{R_b}$ (2)

V_2: $0 = V_2 \frac{1}{Z_k} + (V_2 - V_1 + U_g)\frac{1}{Z_k} + (V_2 - U_v)\frac{1}{R_b}$ (3)

V_1, V_2 izločimo

$$z(1) \Rightarrow -V_1\frac{3}{Z_k} = -\frac{U_v}{Z_k} - \frac{U_2+U_g}{Z_k}$$

$$3V_1 = U_v + U_2 + U_g$$ (vstavimo v (2) in (3))

$$iz\ (2) \Rightarrow 0 = \left(3U_v - (U_v + V_2 + U_g)\right)\frac{1}{Z_k} + \frac{3U_v}{Z_k} + \frac{3U_v}{R_b} - \frac{3V_2}{R_b} =$$

$$= U_v\left(\frac{5}{Z_k} + \frac{3}{R_b}\right) - \frac{V_2}{Z_k} - \frac{3V_2}{R_b} - \frac{U_g}{Z_k} \Big/ \cdot 5$$

$$iz\ (3) \Rightarrow 0 = \frac{3V_2}{Z_k} + \frac{3V_2}{Z_k} - \frac{U_v + V_2 + U_g}{Z_k} + \frac{3U_g}{Z_k} + \frac{3V_2}{R_b} - \frac{3U_v}{R_b} =$$

$$= -\frac{U_v}{Z_k} - \frac{3U_v}{R_b} + \frac{5V_2}{Z_k} + \frac{3V_2}{R_b} + \frac{2U_g}{Z_k}$$

seštejemo:

$$0 = \frac{4U_v}{Z_k} + \frac{4V_2}{Z_k} + \frac{U_g}{Z_k} \Big/ \cdot 3\frac{Z_k}{R_b} \Rightarrow 0 = \frac{12U_v}{R_b} + \frac{12V_2}{R_b} + \frac{3U_g}{R_b}$$

odštejemo:

$$0 = \frac{24U_v}{Z_k} + \frac{12U_v}{R_b} - \frac{12V_2}{R_b} - \frac{3U_g}{Z_k}$$

$+$

$$0 = \frac{24U_v}{Z_k} + \frac{24U_v}{R_b} - \frac{3U_g}{Z_k} + \frac{3U_g}{R_b} \Big/ : 3$$

$$0 = 8U_v\left(\frac{1}{Z_k} + \frac{1}{R_b}\right) + U_g\left(\frac{1}{R_b} - \frac{1}{Z_k}\right) \Big/ \cdot R_b Z_k$$

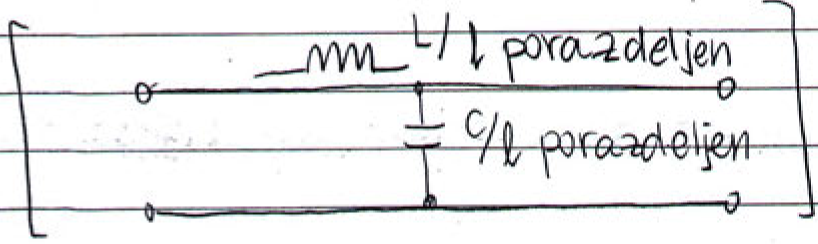
$$0 = 8U_v(R_b + Z_k) + U_g(Z_k - R_b)$$

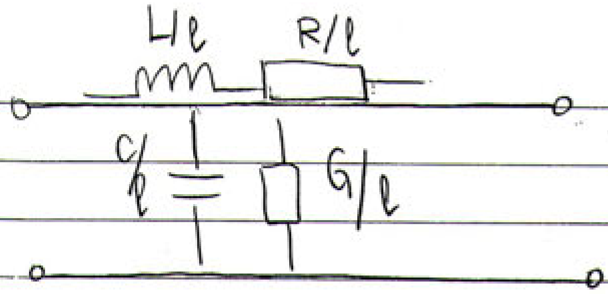
$$U_v = \frac{U_g}{8}\underbrace{\frac{R_b - Z_k}{R_b + Z_k}}_{\Gamma} \Rightarrow U_v = \frac{U_g}{8}\Gamma$$ sorazmernost z odbojnostjo

↳ za merjenje potrebujemo le realne elemente

$$\boxed{\Gamma = \frac{8U_v}{U_g}}$$

brezizgubni vod



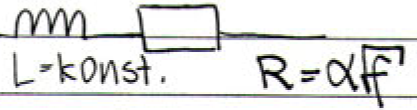


Upoštevati moramo še upornost in prevodnost

⇓

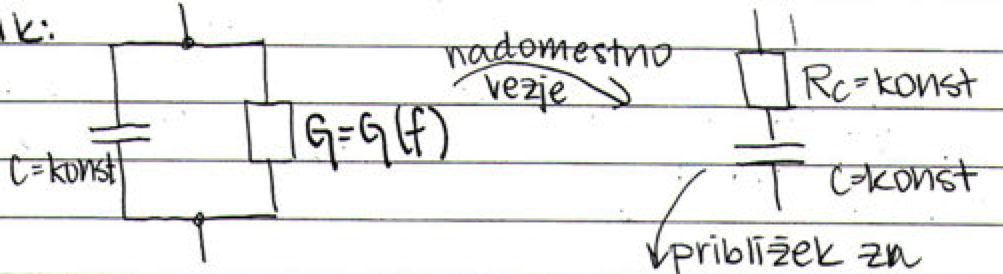
izgubni vod

žica:



upor odvisen od frekvence

dielektrik:



približek za nizke frekvence

Reševanje Telegrafske enačbe v frekvenčnem prostoru

$u(t) = \mathrm{Re}[\hat{U} e^{j\omega t}]$

1) brezizgubna Telegrafska enačba

$$\frac{\partial^2 u}{\partial z^2} = C/l \cdot L/l \, \frac{\partial^2 u}{\partial t^2} = \mu\varepsilon \, \frac{\partial^2 u}{\partial t^2}$$

poskus:

časovni prostor: $u(z,t) = u(t \pm \frac{z}{v})$ nastavek, ki ga ugibamo iz časovnega prostora

$u(z,t) = \mathrm{Re}[\hat{U}_0 e^{j\omega(t \pm \frac{z}{v})}]$ nov nastavek $u = U_0 e^{j\omega(t \pm \frac{z}{v})}$

$$\frac{\partial^2 u}{\partial z^2} = \underset{\uparrow j^2}{-}\frac{\omega^2}{v^2} U_0 e^{j\omega(t \pm \frac{z}{v})} = -\frac{\omega^2}{v^2} u$$

$$\frac{\partial^2 u}{\partial t^2} = -\omega^2 u$$

vstavimo v diferencialno enačbo:

$$-\frac{\cancel{\omega^2}}{v^2} u = -\cancel{\omega^2} \mu\varepsilon u$$

impedanca (ne upornost) bremena

$$\Rightarrow \quad \boxed{v^* = \frac{1}{\sqrt{\mu\varepsilon}}} \quad \boxed{Z_k = \sqrt{\frac{L/l}{C/l}}} \quad \boxed{\Gamma = \frac{Z_b - Z_k}{Z_b + Z_k}}$$

↳ v splošnem primeru je Γ kompleksen

Velja $\boxed{|\Gamma| \leq 1}$ za pasivno breme

Vidimo, da veljajo enake enačbe

kazalec

$$U = U_0 e^{j\omega(t \pm \frac{z}{v})} = U_0 e^{j(\omega t + \varphi)}$$

$$\boxed{\varphi = \omega \cdot \frac{z}{v} = \frac{\omega}{v} \cdot z}$$

[fazna konstanta β] pri izmeničnih veličinah
(valovno število k)

koliko radianov naredi faza na 1 m razdalje $\left[\frac{rad}{m}\right]$

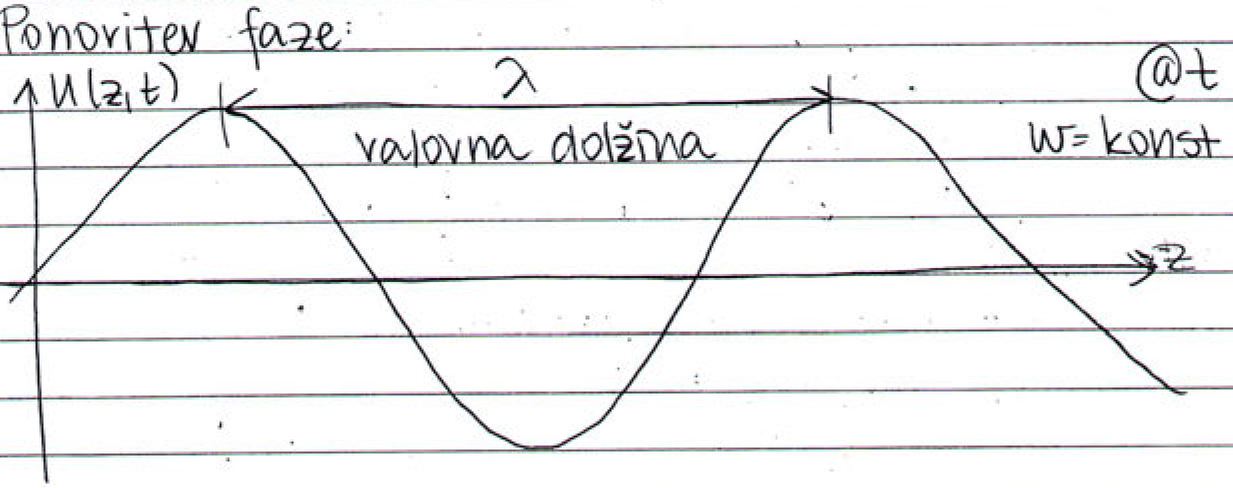
$$\frac{\omega}{v} = \beta = k = \omega\sqrt{\mu\varepsilon}$$

prazen prostor μ_0, ε_0

$$\boxed{k_0 = \frac{\omega}{c_0} = \omega\sqrt{\mu_0\varepsilon_0}}$$

$$\Rightarrow \boxed{\begin{array}{l} U = U_0 e^{j(\omega t + \beta z)} \\ U = U_0 e^{j(\omega t + kz)} \end{array}}$$

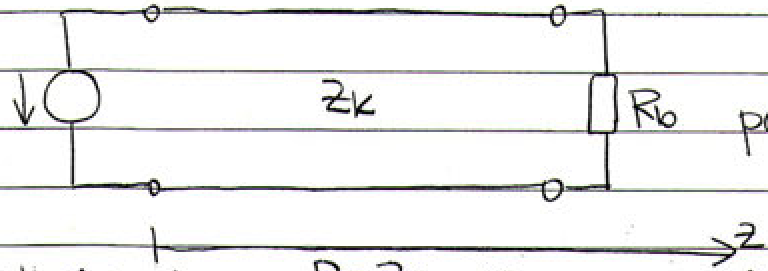
Ponovitev faze:



$$\boxed{\lambda \cdot \beta = \lambda \cdot k = 2\pi}$$

$$\lambda = \frac{2\pi}{k} \quad \text{oz.} \quad k = \frac{2\pi}{\lambda}$$

ponovitev:



$-1 \leq \Gamma \leq 1$

pasivno breme $0 \leq R \leq +\infty$

odbojnost: $\Gamma = \frac{R - Z_k}{R + Z_k} \Leftrightarrow R = Z_k \frac{1+\Gamma}{1-\Gamma}$

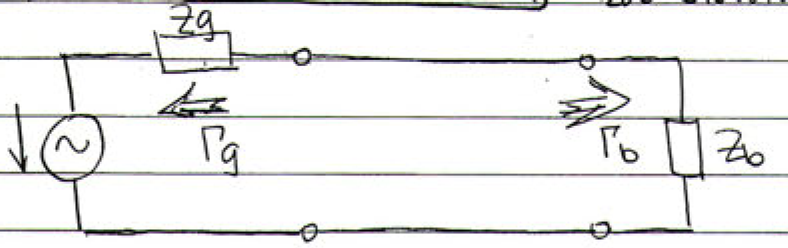
Kaj se dogaja z napredujočim in odbitim valom? (faza nazaduje)

$$\boxed{\begin{aligned} u_N &= Re[U_N e^{j(\omega t - \beta z)}] \\ u_o &= Re[U_o e^{j(\omega t + \beta z)}] \\ \beta &= k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v} \end{aligned}}$$

rešitev: en sam v

$\Rightarrow$ stacionarno stanje

odbojnost najlažje računamo za stacionarno stanje



$z = \frac{1}{y}$ $y = \frac{1}{z}$

$\Gamma = \frac{\frac{1}{y} - \frac{1}{y_k}}{\frac{1}{y} + \frac{1}{y_k}} = \frac{y_k - y}{y_k + y}$

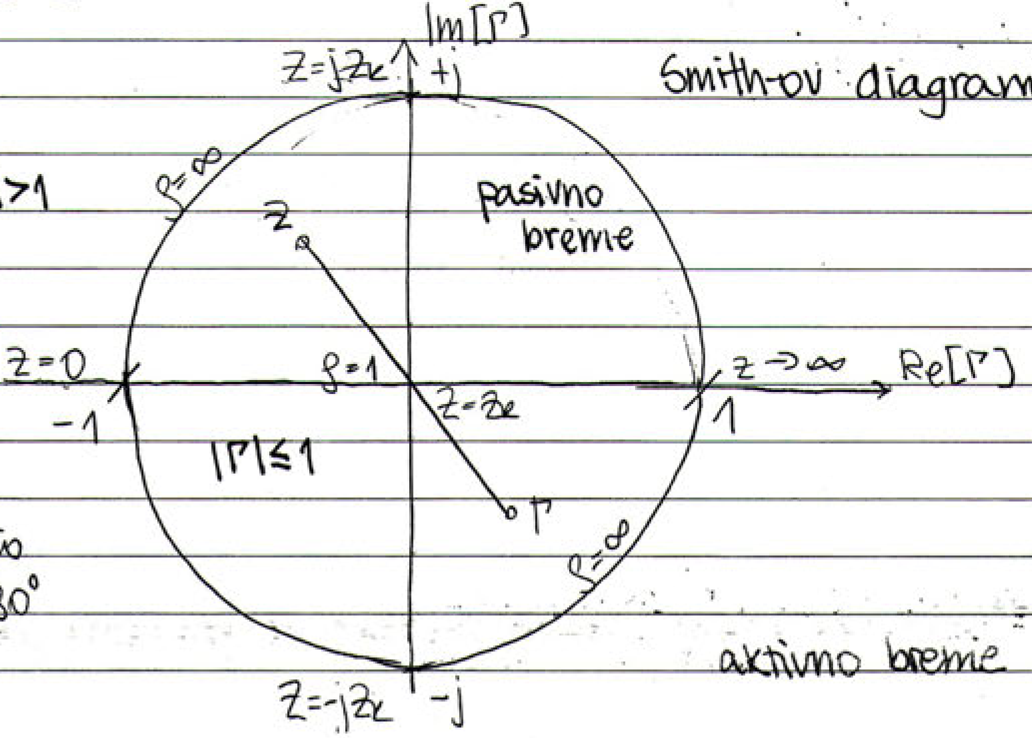
za izmenične veličine: $\boxed{\Gamma = \frac{z - Z_k}{z + Z_k}}$ i $\boxed{\Gamma = \frac{-(y - y_k)}{y + y_k}}$

pasivno breme: $\boxed{Re[z] \geq 0 \Rightarrow |\Gamma| \leq 1}$

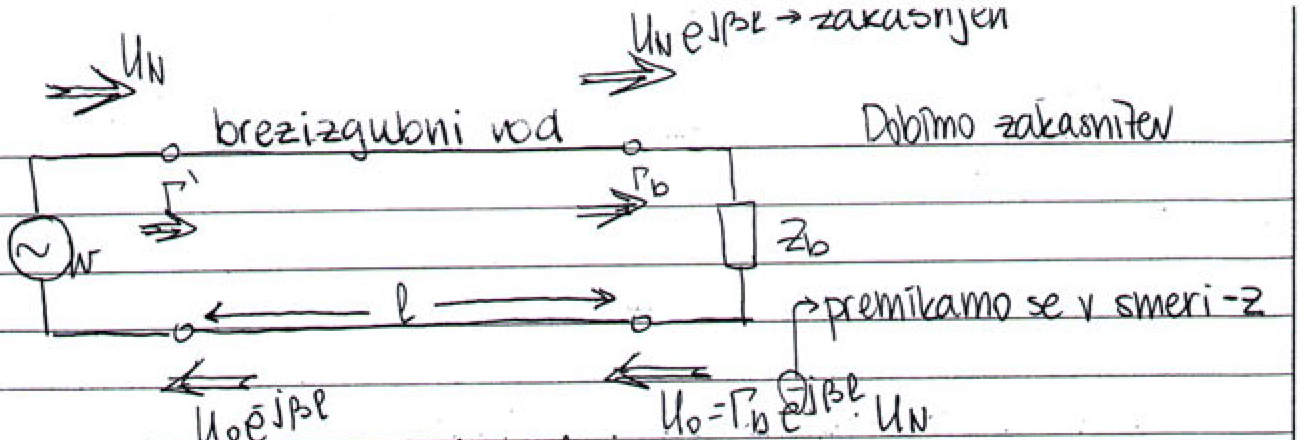
aktivno breme: $\boxed{Re[z] < 0 \Rightarrow |\Gamma| > 1}$ – OJAČAN ODBOJ!

$Z_k = 50\,\Omega$

Smith-ov diagram



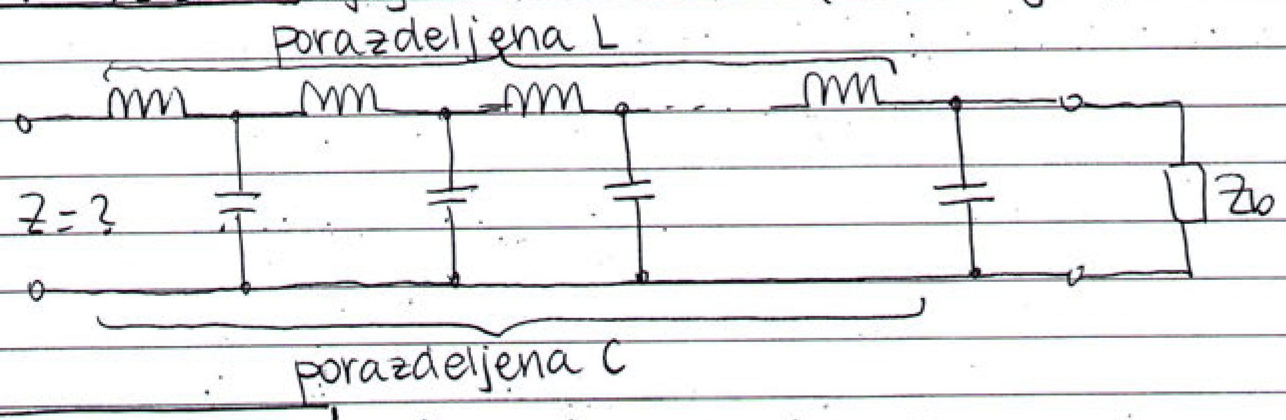
za admitanco
zarneš za 180°



$$\Gamma' = \frac{U_0 e^{-j\beta l}}{U_N} = \frac{(\Gamma_b \cancel{U_N} \cdot e^{-j\beta l}) e^{-j\beta l}}{\cancel{U_N}} = \Gamma_b e^{-j2\beta l}$$

$\boxed{\Gamma' = \Gamma_b e^{-j2\beta l}}$ Γ po vodu ostaja po velikosti enak, njegova faza se vrti (smithov diagram)

BREZIZGUBNI VOD



$$\Gamma_b = \frac{Z_b - Z_k}{Z_b + Z_k}$$

odbojnost se zasuka

$$\Gamma' = \Gamma_b e^{-j2\beta l}$$

faza se zasuka za 2x svetlobno hitrostjo in spet pretvori v impedanco

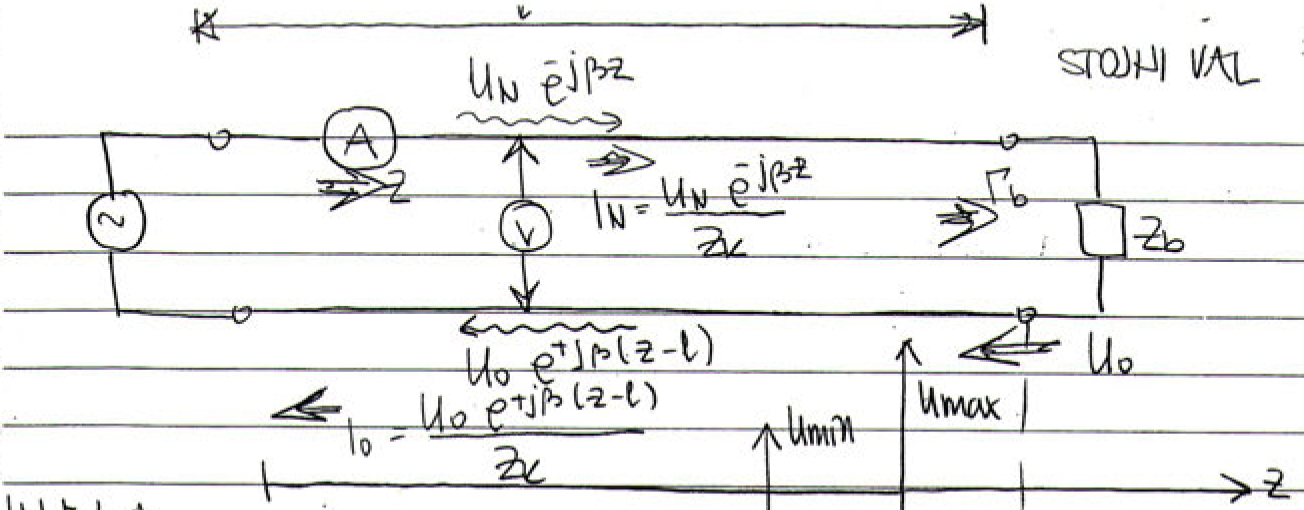
$$Z' = Z_k \frac{1+\Gamma'}{1-\Gamma'} = Z_k \frac{1+\Gamma_b e^{-j2\beta l}}{1-\Gamma_b e^{-j2\beta l}} = Z_k \frac{1 + \frac{Z_b - Z_k}{Z_b + Z_k} e^{-2j\beta l} \Big/ e^{j\beta l}(Z_b+Z_k)}{1 - \frac{Z_b - Z_k}{Z_b + Z_k} e^{-2j\beta l} \Big/ e^{j\beta l}(Z_b+Z_k)}$$

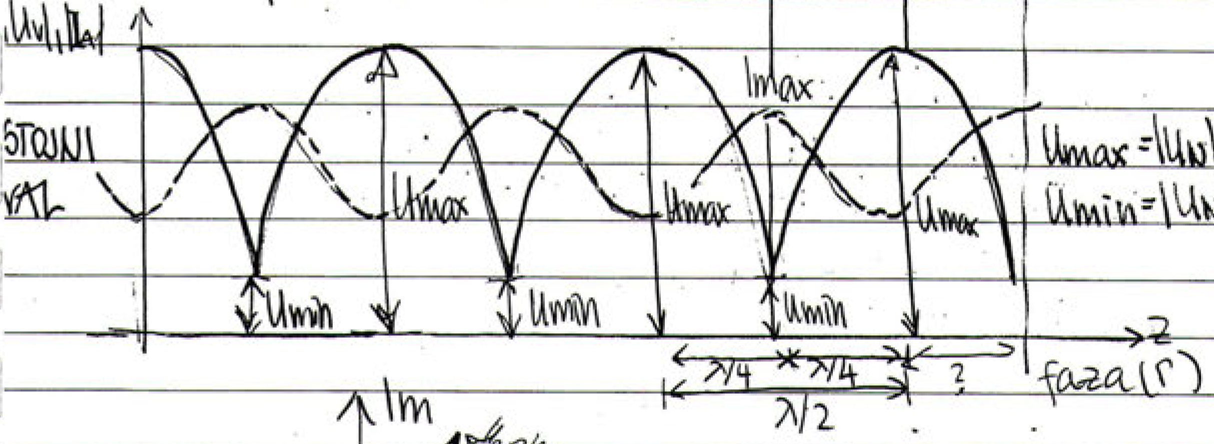
$$Z' = Z_k \frac{Z_b e^{j\beta l} + Z_k e^{j\beta l} + Z_b e^{-j\beta l} - Z_k e^{-j\beta l}}{Z_b e^{j\beta l} + Z_k e^{j\beta l} - Z_b e^{-j\beta l} + Z_k e^{-j\beta l}} =$$

$$= Z_k \frac{2Z_b \cos(\beta l) + 2jZ_k \sin(\beta l)}{2jZ_b \sin(\beta l) + 2Z_k \cos(\beta l)}$$

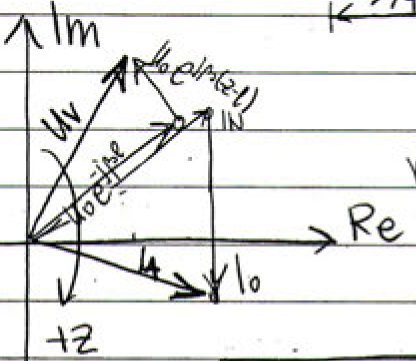
$$Z' = Z_k \frac{Z_b \cos(\beta l) + jZ_k \sin(\beta l)}{Z_k \cos(\beta l) + jZ_b \sin(\beta l)} = \underline{\underline{Z_k \frac{Z_b + jZ_k \tan(\beta l)}{Z_k + jZ_b \tan(\beta l)}}}$$

STOJNI VAL





$U_{max} = |U_N|(1+|\Gamma|)$
$U_{min} = |U_N|(1-|\Gamma|)$



minimumi in maximumi so vedno na istem mestu tega voda

razmerje stojnega vala: valovitost $\boxed{\rho = \frac{U_{max}}{U_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{I_{max}}{I_{min}}}$

$1 \leq \rho \leq \infty$

stojni val toka ima ~~enako razmerje kot~~ stojni val napetosti

$(1-|\Gamma|)\rho = 1+|\Gamma|$

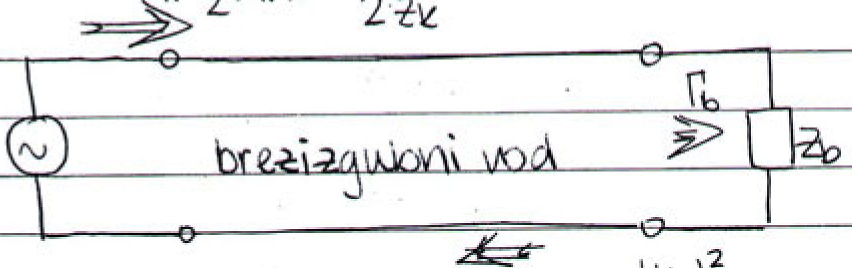
$(\rho-1) = |\Gamma|(\rho+1) \Rightarrow |\Gamma| = \frac{\rho-1}{\rho+1}$ samo velikost $|\Gamma|$ brez faze

aktivno breme:

$|\Gamma|>1 \Rightarrow \boxed{\rho = \frac{|\Gamma|+1}{|\Gamma|-1}} \Rightarrow |\Gamma| = \frac{\rho+1}{\rho-1}$

Moč:

$$P_N = \frac{1}{2} U_N I_N^* = \frac{|U_N|^2}{2 \cdot Z_k}$$



$$\boxed{P_b = P_N - P_o} = \underline{\underline{= P_N(1-|\Gamma|^2)}}$$

moč na bremenu

$$P_o = \frac{1}{2} U_o I_o^* = \frac{|U_o|^2}{2 \cdot Z_k} = |\Gamma_b|^2 \cdot P_N$$

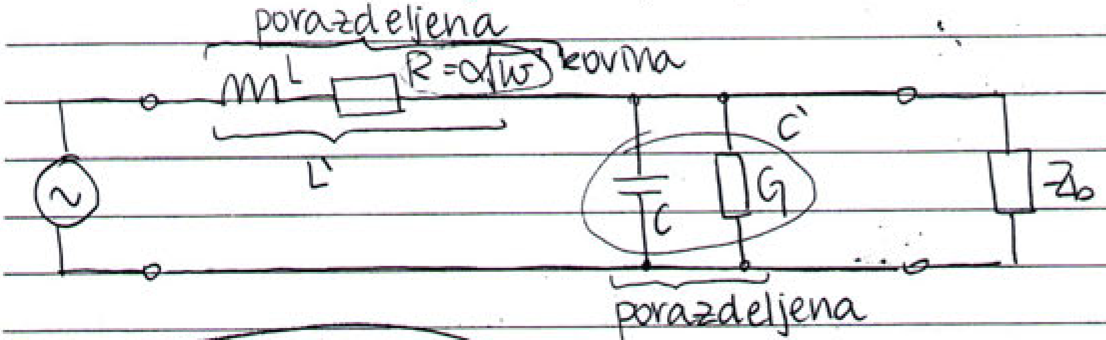
$$\Gamma_{dB} = 10 \log \frac{P_o}{P_N} = 10 \log |\Gamma|^2$$

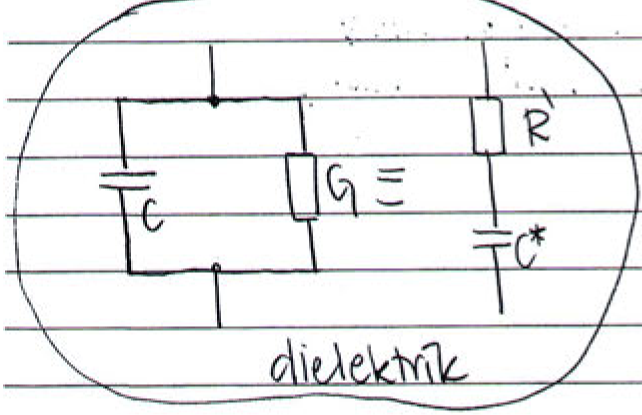
slabljenje odboja: $\boxed{\Gamma_{dB} = 20 \log |\Gamma|}$

Izgube: $\boxed{a_{dB} = 10 \log_{10} \frac{P_b}{P_N} = 10 \log (1 - |\Gamma|^2)}$

razmerje stojnega vala: $\rho_{dB} = 20 \log \frac{U_{max}}{U_{min}} = 20 \log 3$

↳ običajno se NE uporablja v dB !





$$L' = L + \frac{R}{j\omega}$$

$$C' = C + \frac{G}{j\omega}$$

* fazna konstanta je kompleksna, notri je sedaj poleg faze tudi amplituda

$$Z_k = \sqrt{\frac{L'/l}{C'/l}} = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{L + R/j\omega}{C + G/j\omega}} = \sqrt{\frac{j\omega L + R}{j\omega C + G}}$$

majhna

kompleksen, majhen imaginaren del 1% $\text{Im}[Z_k] \ll \text{Re}[Z_k]$

↳ običajno ne opazim

$$K = \frac{L'}{l} \cdot \frac{C'}{l} \cdot \omega = \omega \left(L + \frac{R}{j\omega}\right)/l \cdot \left(C + \frac{G}{j\omega}\right)/l = \beta - j\alpha \leftarrow \text{slabljenje}$$

↑ kompleksno valovno število

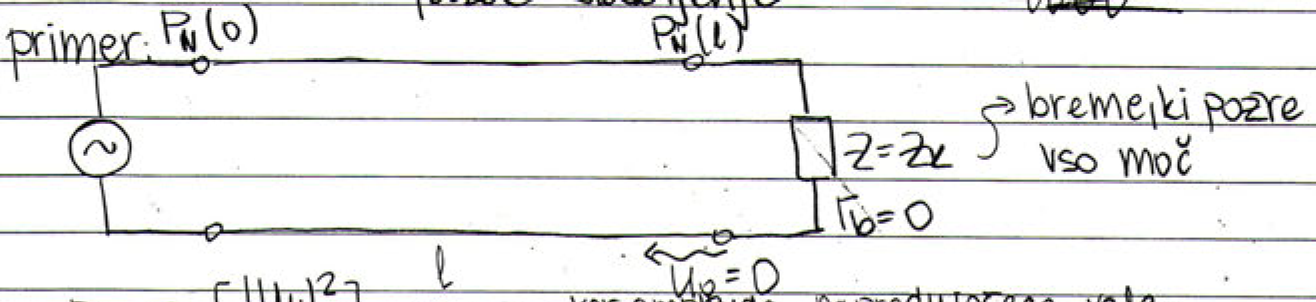
↑ fazna konstanta *

$$u = \mathrm{Re}[U \cdot e^{j(\omega t \pm kz)}] = \mathrm{Re}[U \cdot e^{j(\omega t \pm (\beta - \overset{\overset{\beta - j\alpha\,?}{\parallel}}{\alpha}) z)}]$$

$$u_N = \mathrm{Re}[U_N e^{j\omega t} e^{-j\beta z} e^{-\alpha z}]$$
$$u_0 = \mathrm{Re}[U_0 \cdot e^{j\omega t} \underbrace{e^{\mp j\beta z}}_{\text{faza}} \underbrace{e^{\mp \alpha z}}_{\text{slabljenje}}]$$

amplituda pada z razdaljo ce imamo izgube imamo tudi slabljenje vala

primer:

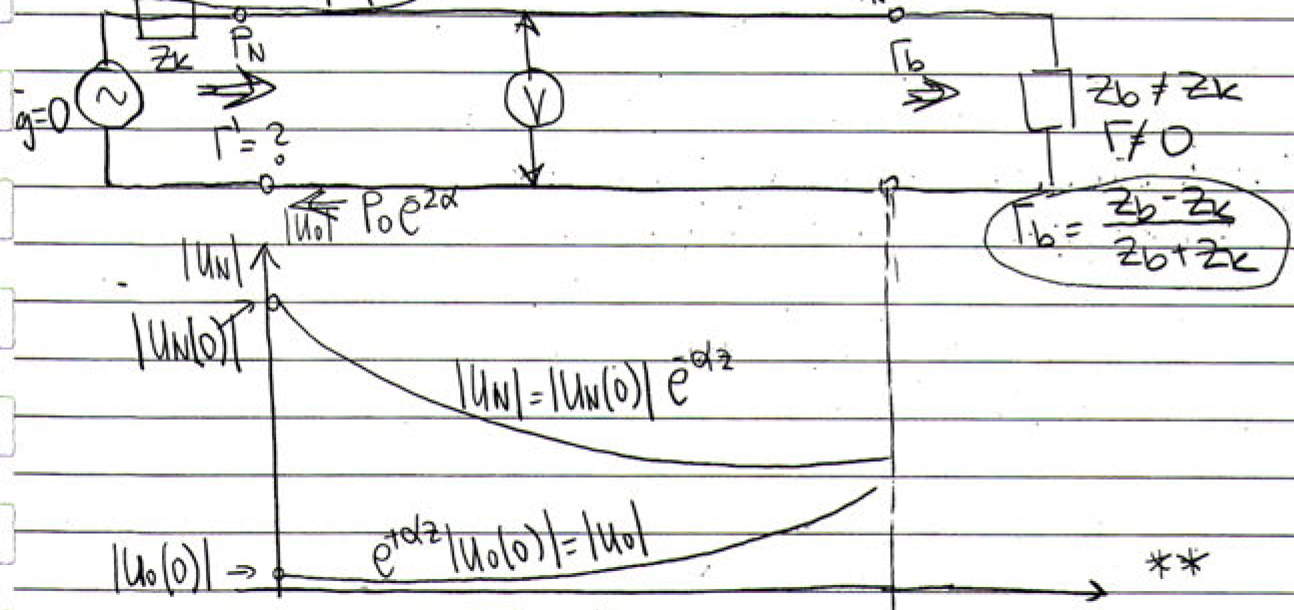


$$P_N = \mathrm{Re}\left[\frac{|U_N|^2}{2 \cdot Z_K}\right] = P_N(\overset{l}{z})$$ → ker amplituda napredujočega vala pada z razdaljo ⇒ $P_N(0) \neq P_N(z)$

slabljenje: $$\frac{P_N(0)}{P_N(l)} = a = \frac{(e^{-\alpha \cdot 0})^2}{(e^{-\alpha l})^2} = e^{2\alpha l}$$

$$a_{dB} = 10 \log_{10} \frac{P_N(0)}{P_N(l)} = 10 \log_{10} e^{2\alpha l} = 10 \frac{\ln e^{2\alpha l}}{\ln 10} = \underline{\underline{\frac{10}{\ln 10} \cdot 2\alpha l \ [dB]}}$$

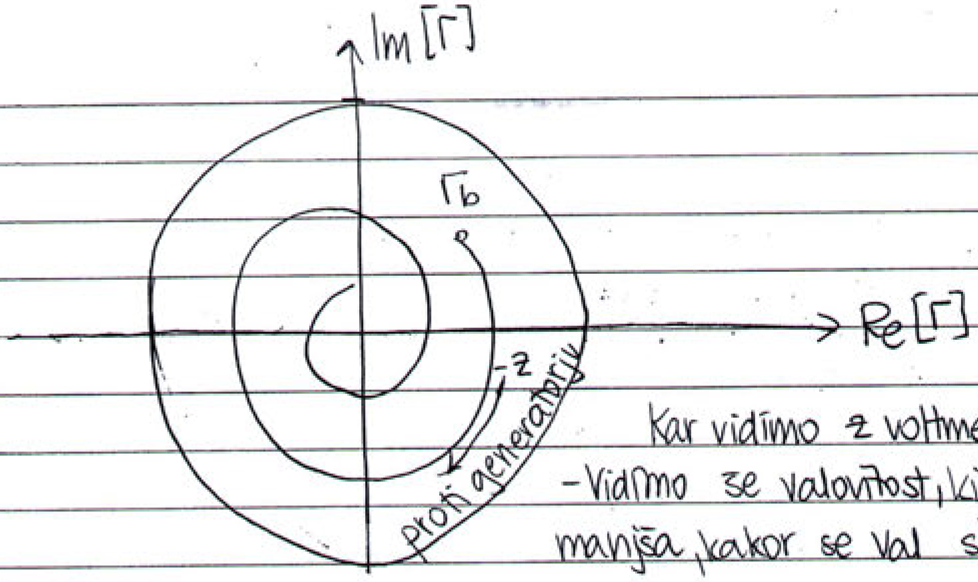
primer: $Z' = Z_K \frac{1+\Gamma'}{1-\Gamma'}$



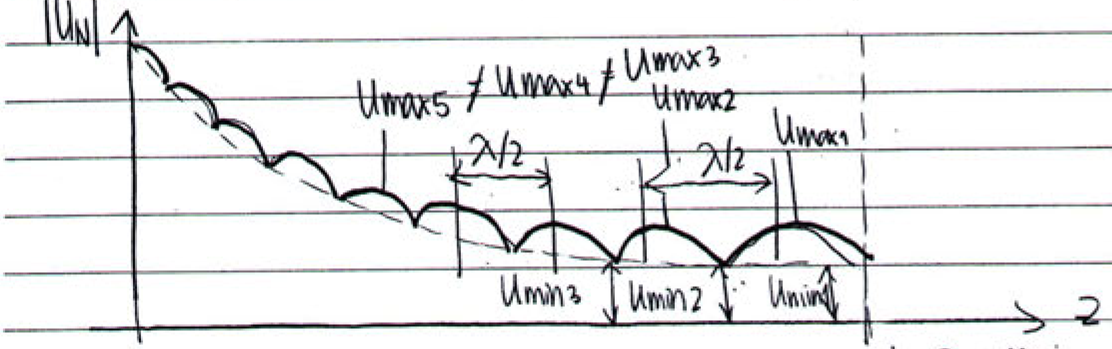
$\Gamma_b = \frac{Z_b - Z_K}{Z_b + Z_K}$

$$\boxed{\Gamma' = \Gamma_b e^{-j2\beta l} e^{-2\alpha l}}$$ vrti se po fazi in spreminja se mu velikost ⇒ glej graf

$|\Gamma'| \leq |\Gamma_b|$ Γ na naši poti ni več konstantno velik. (se manjša)



Kar vidimo z voltmetrom
- Vidimo se valovitost, ki postaja manjša, kakor se val slabi



Maximumi in minimumi so različno veliki, zato ne moremo definirati razmerja

$$\boxed{\text{izgubni vod: } \rho \text{ NI definiran !}}$$

na bremenu: $P_b = (1-|\Gamma_b|^2) P_N \bar{e}^{2\alpha l}$

- Zgled: ↗ žična palica

<u>koaksialni kabel</u>: $f \sim 100 MHz$; $\alpha = \frac{1}{100}\beta$; $R \approx \frac{1}{100}\omega L$

⇒ če: $l < \lambda$ .. obravnavamo kot brezizguben

če: $l > 30\lambda$ .. obravnavamo izgube

- Zgled: svetlobno vlakno: $f \sim 194 THz$; $\lambda_0 = 1{,}55 \mu m$

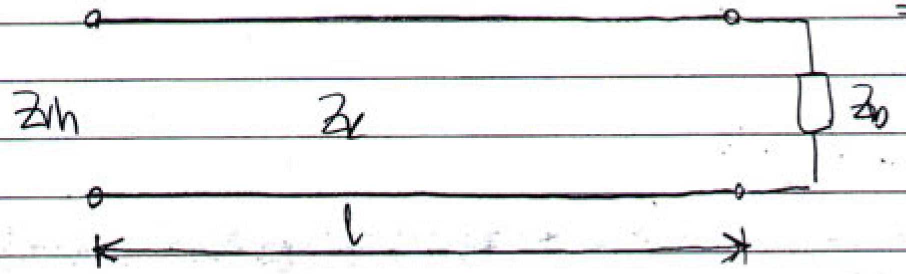
$l < 10^8 \lambda$ .. brezizgubno

$l > 3 \cdot 10^8 \lambda$ .. izgubni vod

Reševanje po korakih:

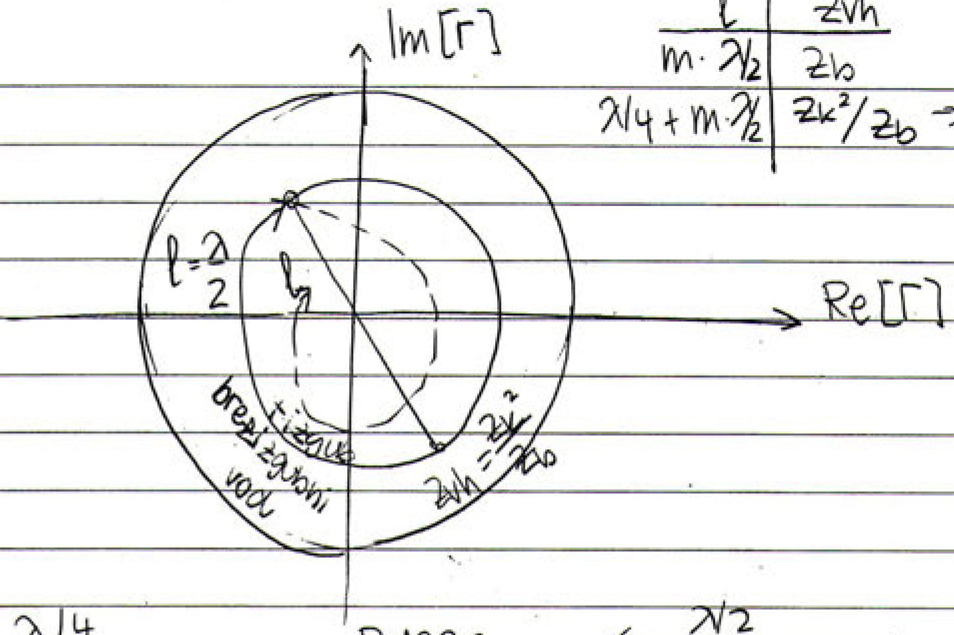
$\Gamma_b = \frac{Z_b - Z_c}{Z_b + Z_c} \Rightarrow Z' = Z_c \frac{1+\Gamma'}{1-\Gamma'}$

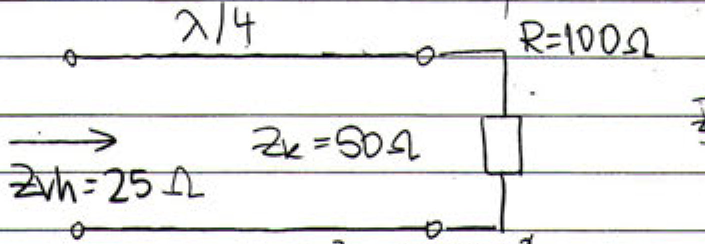
$\Rightarrow \Gamma' = \Gamma_b \bar{e}^{j\beta l} \bar{e}^{2\alpha l}$

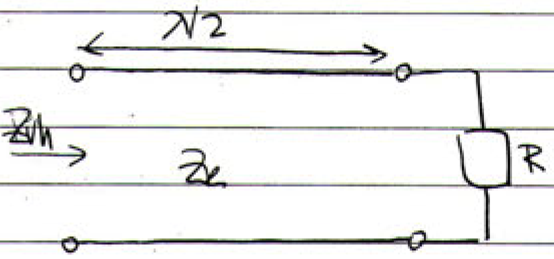


$m = 1, 2, 3 \ldots$

l	Z_{vh}
$m \cdot \lambda/2$	Z_b
$\lambda/4 + m \cdot \lambda/2$	Z_k^2/Z_b → inverter (inverzija bremena)

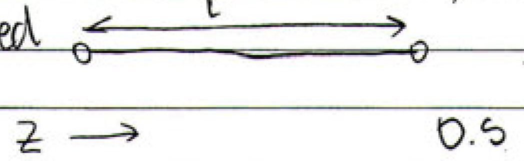


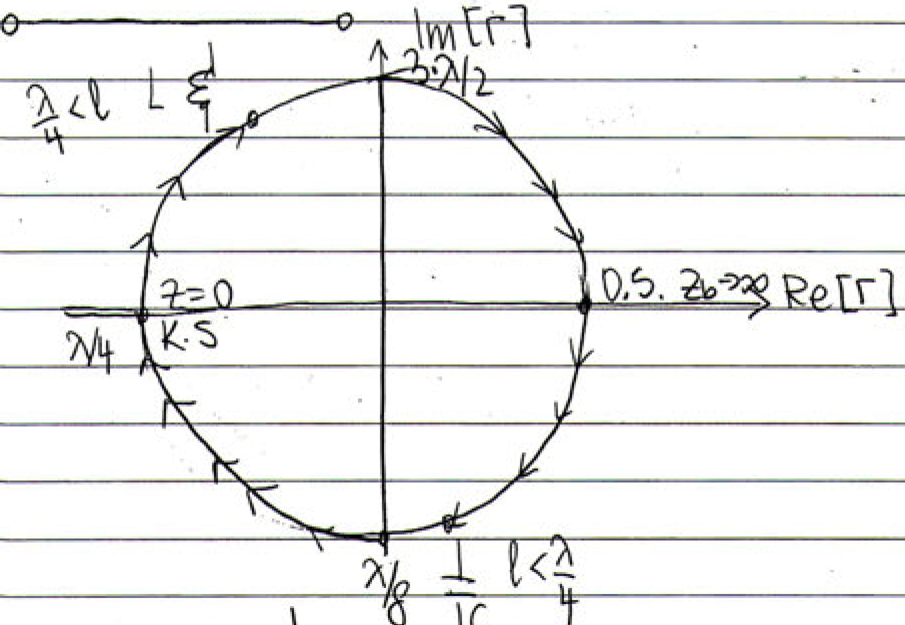




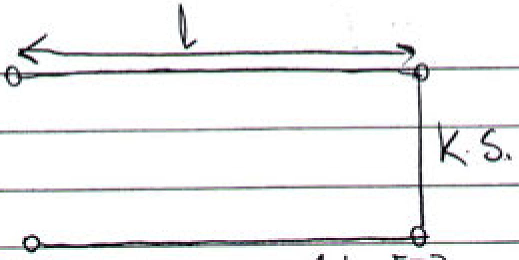
$$Z_{vh} = \frac{Z_k^2}{R} = \frac{2500\,\Omega^2}{100\,\cancel{\Omega}} = 25\,\Omega$$

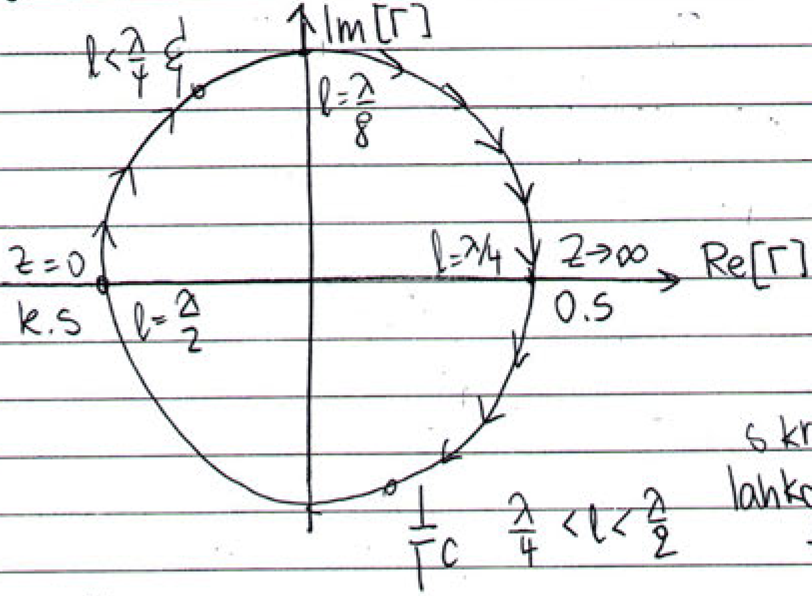
zgled





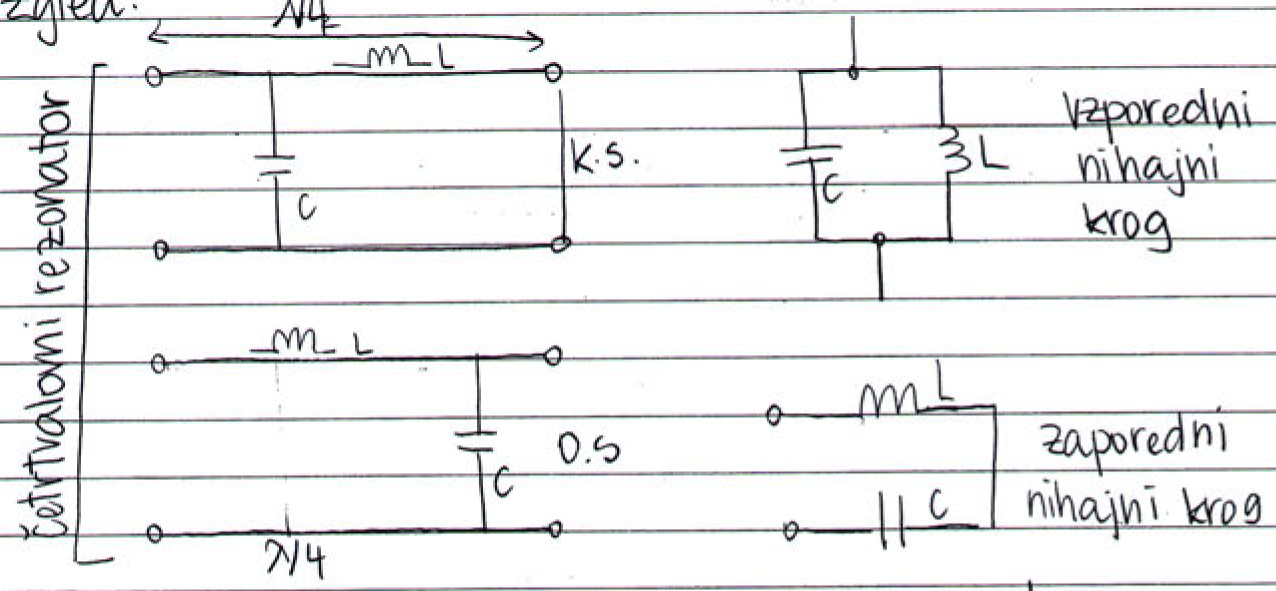
↳ z odprtimi sponkami lahko realiziramo kondenzator

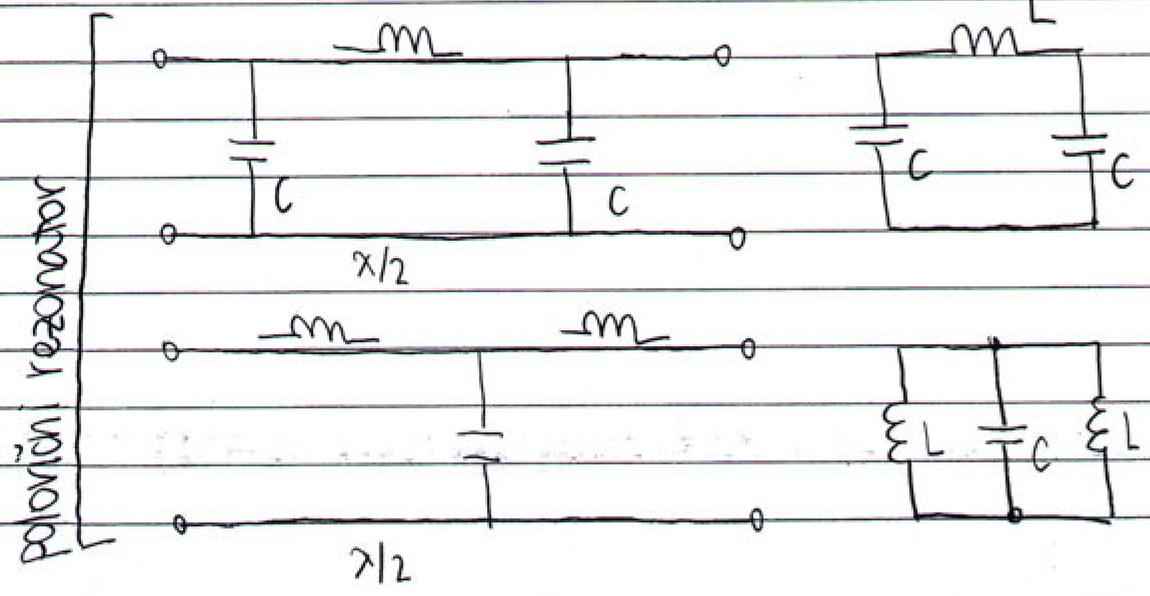




s kratkim stikom lahko realiziramo tuljavo

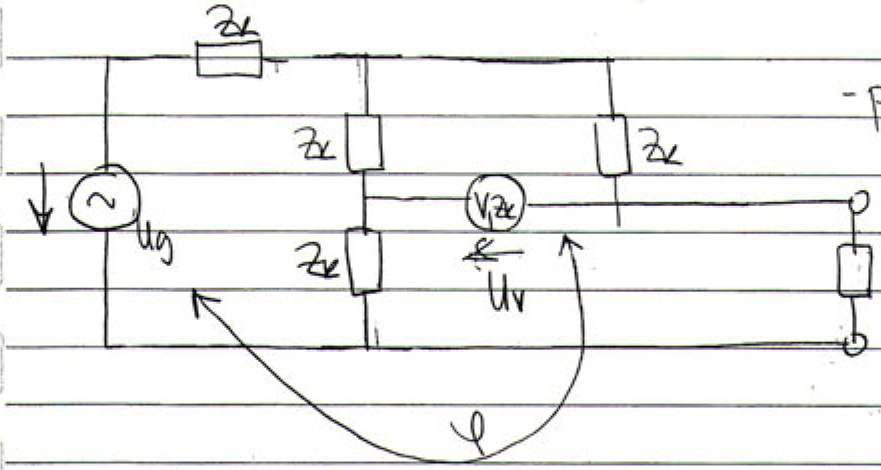
- Zgled:





mostiček

MERJENJE V FREKVENČNEM PROSTORU



- paziti moramo kaj nam meri voltmeter
  - ↳ ali meri le velikost Γ (|Γ|), ali tudi fazo
    - ↳ vektorski V-meter

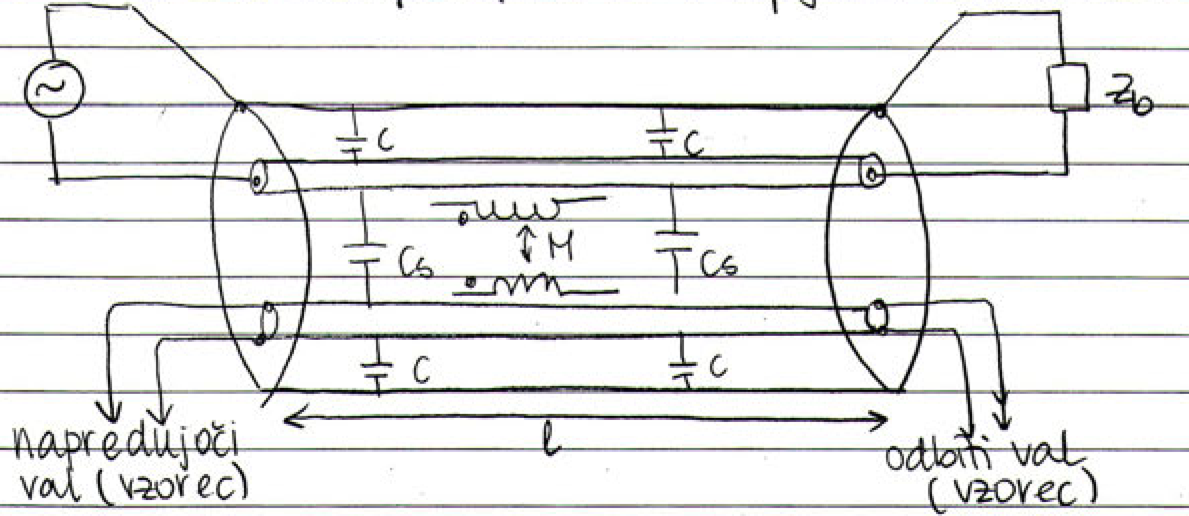
$$U_v = \frac{U_g}{8}\,\frac{Z_b - Z_k}{Z_b + Z_k} = \frac{U_g}{8}\,\Gamma$$

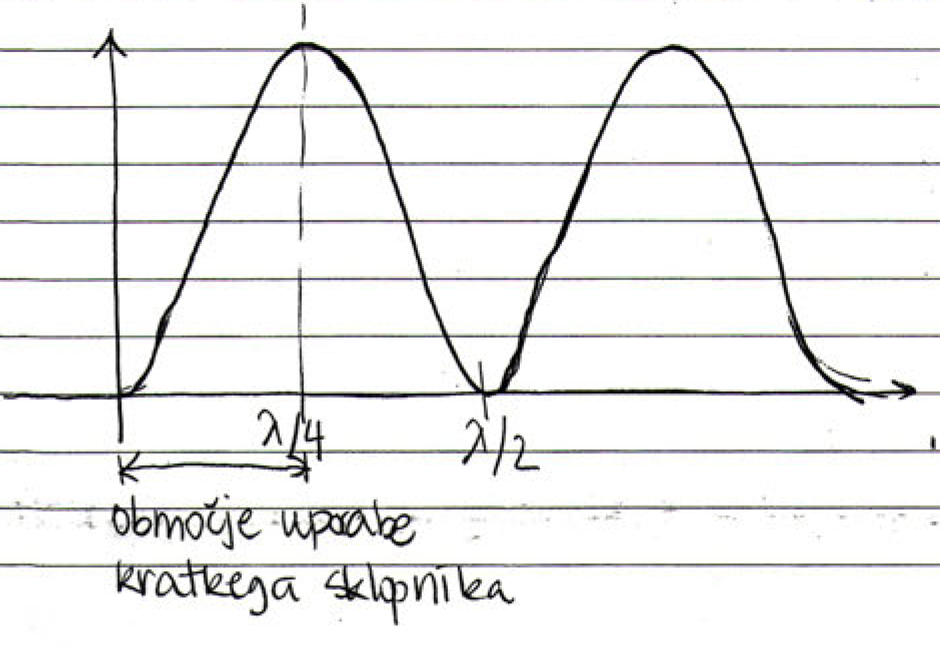
voltmeter: $|U_v| = \frac{|U_g|}{8}|\Gamma| \Rightarrow |\Gamma|$

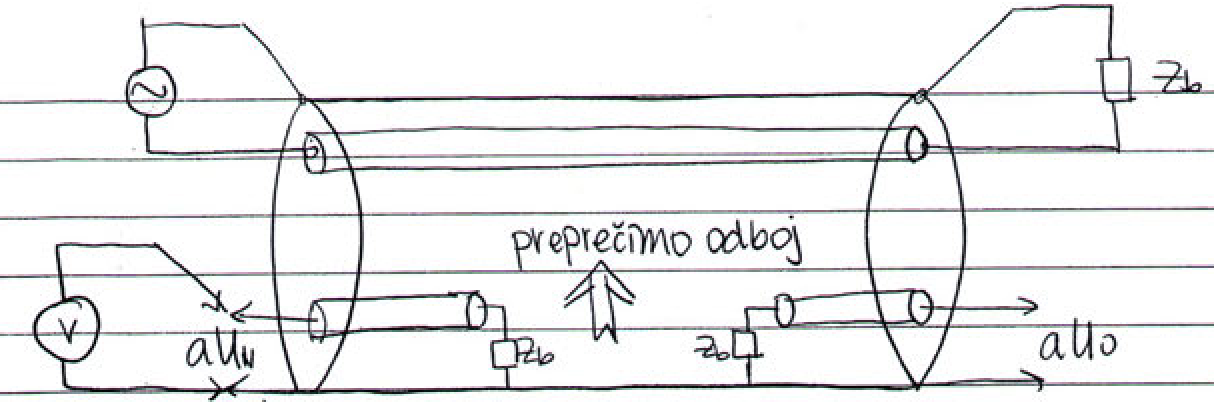
enako ko enosmerno. Pazimo le na fazo odbojnosti

voltmeter + faza φ: $U_v = \frac{U_g}{8}\Gamma \Rightarrow \Gamma$: vektorski voltmeter

Proti-smerni sklopnik (moč se sklaplja v obratno smer)

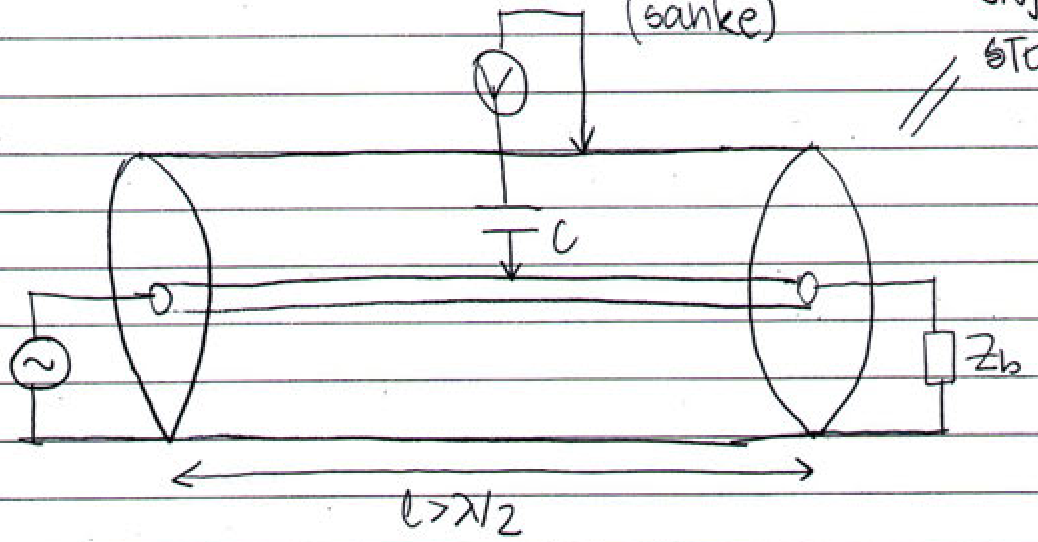






skalarni : |Γ|
vektorski Γ

za merjenje
stojnega vala
STOJNI VAL



vsaj 1x max in vsaj 1x min
za računanje $\rho = \frac{U_{max}}{U_{min}}$

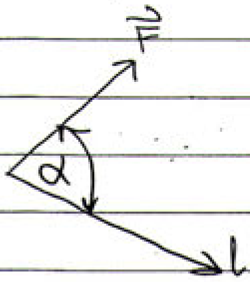
1D.. skalarne veličine
3D.. skalararne veličine + vektorske veličine

masa m.. skalar
hitrost $\vec{v}$.. vektor

Računanje z vektorji

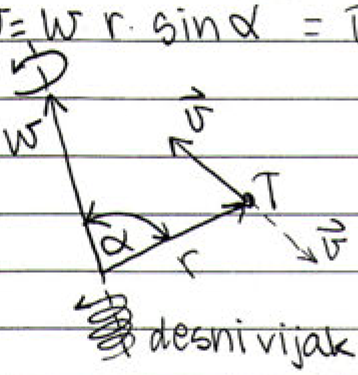
$W = F \cdot l \cdot \cos\alpha = \vec{F} \bullet \vec{l}$

↑ skalarni produkt



$v = w \, r \cdot \sin\alpha = \vec{w} \times \vec{r}$

↑ vektorski produkt



predznak? PRAVILO DESNEGA VIJAKA

$\vec{v} \perp \vec{w}$

$\vec{v} \perp \vec{r}$

Koordinatni sistem

- 3D (g_1, g_2, g_3)
- pravokotni

$\vec{F} = (F_1, F_2, F_3);\ \vec{l} = (l_1, l_2, l_3)$ skalarni

$W = \vec{F} \circ \vec{l} = F_1 l_1 + F_2 l_2 + F_3 l_3$

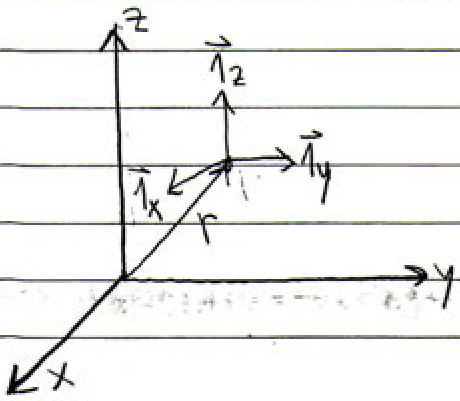
$\vec{w} = (w_1, w_2, w_3);\ \vec{r} = (r_1, r_2, r_3)$

$$\vec{v} = \vec{w} \times \vec{r} = \begin{vmatrix} 1_{g_1} & 1_{g_2} & 1_{g_3} \\ w_1 & w_2 & w_3 \\ r_1 & r_2 & r_3 \end{vmatrix}$$

predznak

- desnoročni (pove predznak)

Kartezični koordinatni sistem (x,y,z)



$-\infty < x\,[m] < +\infty$

$-\infty < y\,[m] < +\infty$

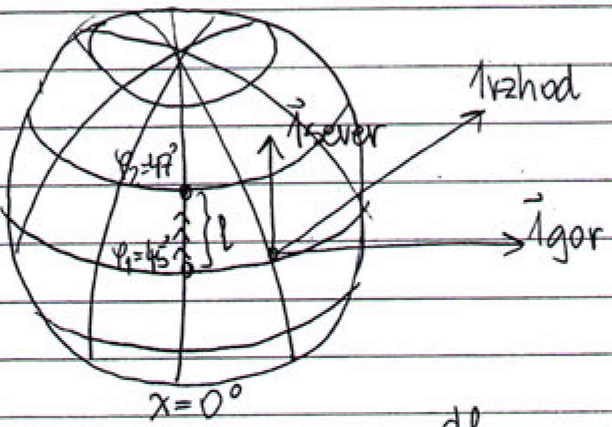
$-\infty < z\,[m] < +\infty$

$\vec{r}(x, y, z) = \vec{1}_x \cdot x + \vec{1}_y \cdot y + \vec{1}_z \cdot z$

desnoročnost: $\vec{1}_x \times \vec{1}_y = \vec{1}_z$

$\boxed{\vec{1}_{g_1} \times \vec{1}_{g_2} = \vec{1}_{g_3}}$

(λ, φ, h) → nadmorska višina



$-180° < \lambda[°] < +180°$

$-90° < \varphi[°] < +90°$

$-6378 < h[km] < +\infty$

$\vec{1}_{vzhod} \times \vec{1}_{sever} = \vec{1}_{gor}$

$\lambda \quad \varphi \quad h$

obseg Zemlje

če $h = 0$ in $\lambda =$ konst: $\frac{dl}{d\varphi} = 111\,km/° = \frac{40000\,km}{360°} = \frac{1}{h_\varphi} h_\varphi$

$\boxed{dl = h_\varphi \cdot d\varphi}$ $\quad [km]\, dl \neq d\varphi\, [°]$

[m] [°]

Lamè-jev koeficient

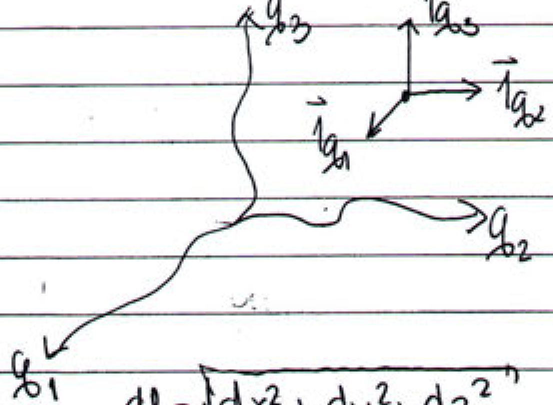
V = 10 litrov goriva

poraba avta = ? Koliko goriva na 100km? $V/l = ?$

$$\frac{dV}{dl} = \frac{dV}{d\varphi} \cdot \frac{d\varphi}{dl} = \frac{dV}{d\varphi} \cdot \frac{1}{h_\varphi} = \frac{\cancel{1°}}{111km} \cdot \frac{10l}{\cancel{1°}} = \frac{10l}{111km\,\frac{9}{10}} \cdot \frac{9}{10} = \frac{9l}{100km}$$

da bo 100km

poljuben koordinatni sistem (q_1, q_2, q_3)



desnoročnost: $\vec{1}_{q_1} \times \vec{1}_{q_2} = \vec{1}_{q_3}$

Lamè-jevi koeficienti:

$q_1 \to h_1$

$q_2 \to h_2$

$q_3 \to h_3$

$dl = \sqrt{dx^2 + dy^2 + dz^2}$

$dx = \frac{\partial x}{\partial q_1} dq_1 \qquad dy = \frac{\partial y}{\partial q_1} dq_1 \qquad dz = \frac{\partial z}{\partial q_1} dq_1$

pot v smeri x

premik v smeri q_1

$$dl = h_1 \cdot dq_1 = dq_1 \sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial y}{\partial q_1}\right)^2 + \left(\frac{\partial z}{\partial q_3}\right)^2}$$

element dolžine

Kako izračunamo Lamè-jeve koeficiente?

$$h_1 = \sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial y}{\partial q_1}\right)^2 + \left(\frac{\partial z}{\partial q_1}\right)^2}$$

$$h_2 = \sqrt{\left(\frac{\partial x}{\partial q_2}\right)^2 + \left(\frac{\partial y}{\partial q_2}\right)^2 + \left(\frac{\partial z}{\partial q_2}\right)^2}$$

$$h_3 = \sqrt{\left(\frac{\partial x}{\partial q_3}\right)^2 + \left(\frac{\partial y}{\partial q_3}\right)^2 + \left(\frac{\partial z}{\partial q_3}\right)^2}$$

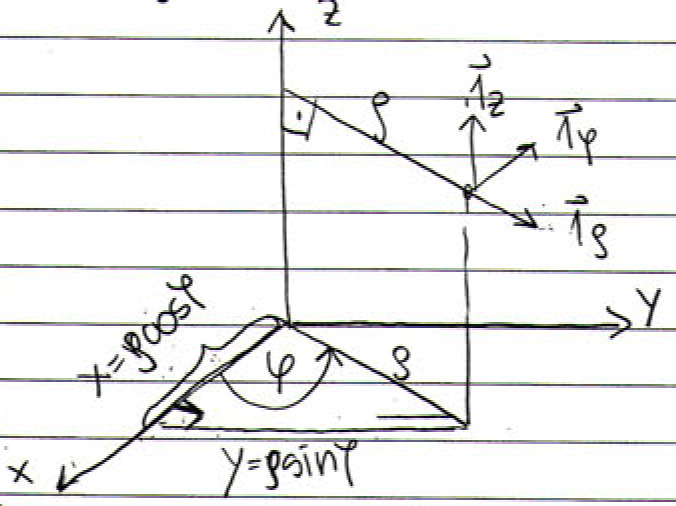
povezava s kartezičnim koordinatnim sistemom:

$$\left.\begin{matrix} x(q_1, q_2, q_3) \\ y(q_1, q_2, q_3) \\ z(q_1, q_2, q_3) \end{matrix}\right\rbrace \text{ kakšne funkcije so?}$$

element ploskvice: $dA = \overbrace{dx\,dy}^{\text{kartezični koor. sist}} = \overbrace{h_1 h_2\, dq_1\, dq_2}^{\text{poljubni koor. sist.}}$

element volumna: $dV = dx\,dy\,dz = h_1 h_2 h_3\, dq_1\, dq_2\, dq_3$

Valjni koordinatni sistem (ρ, φ, z)



$$\vec{1}_\rho \times \vec{1}_\varphi = \vec{1}_z$$

$0 \leq \rho[m] < +\infty$

$0 \leq \varphi[rad] < 2\pi$

$-\infty < z[m] < +\infty$

Povezave med koordinatami:

$\rho = \sqrt{x^2+y^2}$

$\varphi = \arctan(y/x)$ (KVADRANT?)

$z = z$

obratna smer

$x = \rho\cos\varphi$

$y = \rho\sin\varphi$

$z = z$

Lamè-jevi koeficienti:

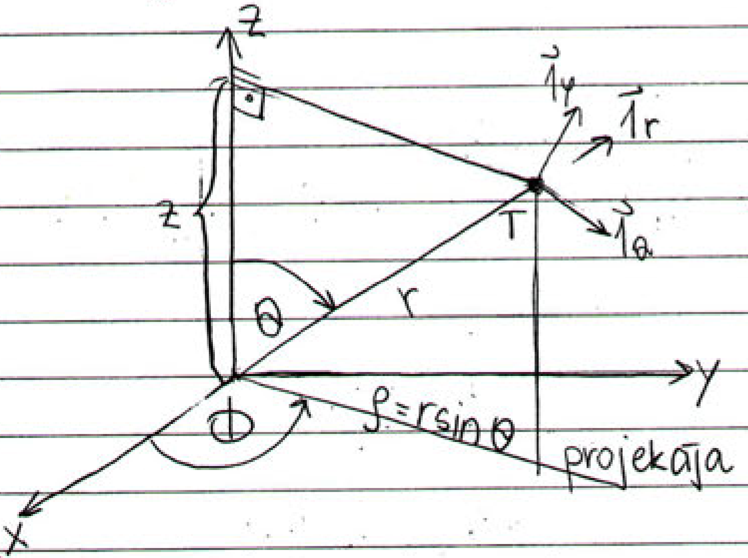
$$\underline{\underline{h_\rho}} = \sqrt{\left(\frac{\partial x}{\partial \rho}\right)^2 + \left(\frac{\partial y}{\partial \rho}\right)^2 + \left(\frac{\partial z}{\partial \rho}\right)^2} = \sqrt{\cos^2\varphi + \sin^2\varphi + 0} = \underline{\underline{1}} \text{ [neimenovano]}$$

$$\underline{\underline{h_\varphi}} = \sqrt{\left(\frac{\partial x}{\partial \varphi}\right)^2+\left(\frac{\partial y}{\partial \varphi}\right)^2+\left(\frac{\partial z}{\partial \varphi}\right)^2} = \sqrt{\rho^2\sin^2\varphi+\rho^2\cos^2\varphi+0} = \underline{\underline{\rho}}\ [\mathrm{m/rad}]$$

$$\underline{\underline{h_z}} = \sqrt{\left(\frac{\partial x}{\partial z}\right)^2+\left(\frac{\partial y}{\partial z}\right)^2+\left(\frac{\partial z}{\partial z}\right)^2} = \sqrt{0+0+1} = \underline{\underline{1}}\ [\text{neimenovano}]$$

smer: $\vec{1}_\rho$: $dl = d\rho$ [m]
smer $\vec{1}_\varphi$: $dl = \rho\, d\varphi$ [m]
smer $\vec{1}_z$: $dl = dz$ [m]

Krogelni koordinatni sistem (r, θ, ϕ)



$\vec{1}_r \times \vec{1}_\theta = \vec{1}_\varphi$

$0 \leq r\,[\mathrm{m}] < +\infty$
severni tečaj $0 \leq \theta\,[\mathrm{rad}] \leq \pi$ južni tečaj
$0 \leq \phi\,[\mathrm{rad}] \leq 2\pi$

Povezave med koordinatami:
$r = \sqrt{x^2+y^2+z^2}$
$\theta = \arccos(z/r)$ ← SINGULARNOST?
$\phi = \arctan(y/x)$ ↙ (KVADRANT?)

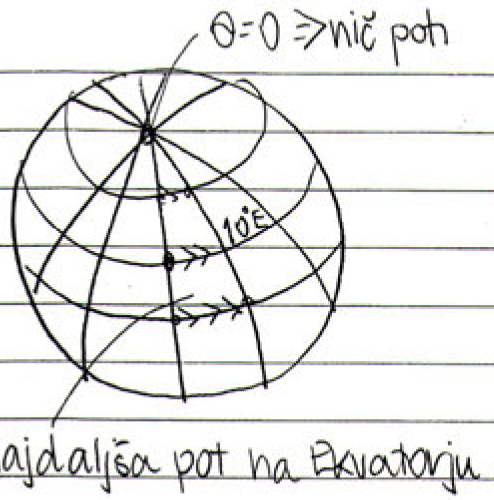
obratno
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

Lamé-jevi koeficienti:

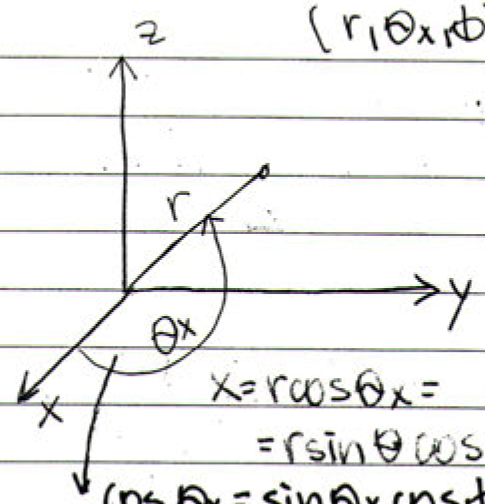
$$\underline{\underline{h_r}} = \sqrt{\left(\frac{\partial x}{\partial r}\right)^2+\left(\frac{\partial y}{\partial r}\right)^2+\left(\frac{\partial z}{\partial r}\right)^2} = \sqrt{\sin^2\theta\cos^2\phi+\sin^2\theta\sin^2\phi+\cos^2\phi} = \underline{\underline{1}}\ [\text{neim}]$$

$$\underline{\underline{h_\theta}} = \sqrt{r^2\cos^2\theta\cos^2\phi+r^2\cos^2\theta\sin^2\phi+r^2\sin^2\theta} = \underline{\underline{r}}\ [\mathrm{m/rad}]$$

$$\underline{\underline{h_\phi}} = \sqrt{r^2\sin^2\theta\sin^2\phi+r^2\sin^2\theta\cos^2\phi+0} = \underline{\underline{r\sin\theta}}\ [\mathrm{m/rad}]$$



prevrnjen krogelni
koordinatni sistem
(r, Θ_x, Φ)

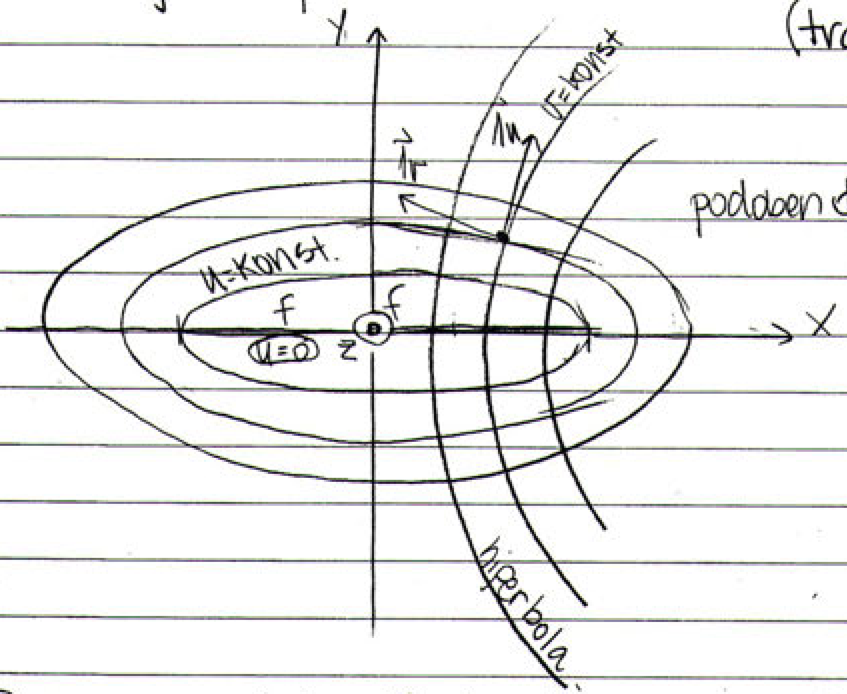


$x = r\cos\Theta_x =$
$= r\sin\Theta\cos\Phi$

$\cos\Theta_x = \sin\Theta_x\cos\Phi$

$\sin\Theta_x = \sqrt{1 - \sin^2\Theta\cos^2\Phi}$

Valjni eliptični koordinatni sistem (u, v, z)
(trakovi)



$0 < u\,[\text{neim}] < +\infty$
podoben Φ $0 \leq v\,[\text{rad}] \leq 2\pi$
$-\infty \leq z\,[\text{m}] < +\infty$

Povezave med koordinatami:
$x = f\cosh u\cos v$
$y = f\sinh u \cdot \sin v$
$z = z$

primerjava z valjnim k.s.
$x = \rho\cos\varphi$
$y = \rho\sin\varphi$
$z = z$

Lamé-jevi koeficienti:

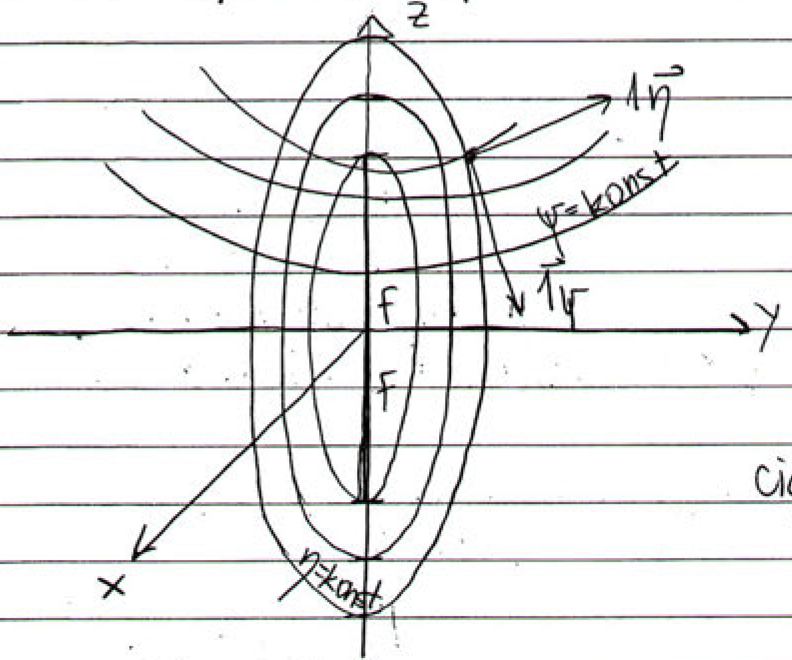
$\boxed{\cosh^2 u = \sinh^2 u + 1}$

$\underline{h_u} = \sqrt{f^2\sinh^2 u\cos^2 v + f^2\cosh^2 u\sin^2 v + 0} \underline{=} f\underline{\underline{\sqrt{\sinh^2 u + \sin^2 v}}}$ [m]

$\underline{h_v} = \sqrt{f^2\cosh^2 u\sin^2 v + f^2\sinh^2 u\cos^2 v + 0} \underline{=} f\underline{\underline{\sqrt{\sinh^2 u + \sin^2 v}}}$ [m/rad]

$\underline{h_z} = \sqrt{0 + 0 + 1} = 1$ [neim]

Palica ⇒ cigara (η, ψ, Φ) podolgovat elipsoid
(prolate elipsoid)



cigara ⟷ krogelni
$\eta \leftrightsquigarrow r$
$\psi \leftrightsquigarrow \theta$
$\Phi = \Phi$

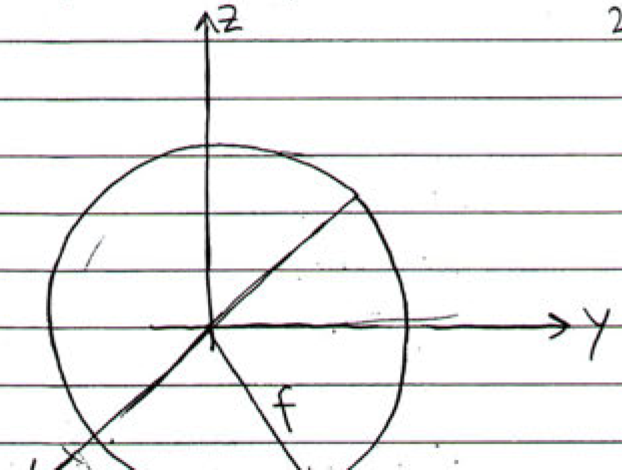
Povezave med koordinatami:

$x = f \sinh\eta \cdot \sin\psi \cos\Phi$

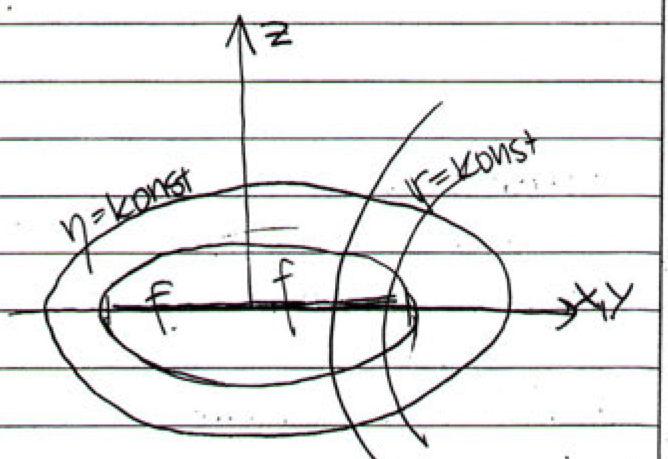
$y = f \cdot \sinh\eta \cdot \sin\psi \cdot \sin\Phi$

$z = f \cdot \cosh\eta \cdot \cos\psi$

Disk -> polpet (η, ψ, Φ)
(oblate elipsoid)



2D:



Povezave med koordinatami:

$x = f \cdot \cosh\eta \, \sin\psi \cdot \cos\phi$

$y = f \cdot \cosh\eta \cdot \sin\psi \cdot \sin\phi$

$z = f \cdot \sinh\eta \cdot \cos\psi$

Skalarna funkcija: $\overset{\leftarrow \text{potencial}}{V}(x,y,z) = V(\vec{r})$

$\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}$ ... 3 različni odvodi

Vektorska funkcija: $\vec{E}(x,y,z) = \vec{E}(\vec{r}) =$

$= \vec{1}_x E_x(x,y,z) + \vec{1}_y E_y(x,y,z) + \vec{1}_z E_z(x,y,z)$

$\frac{\partial E_x}{\partial x}, \frac{\partial E_x}{\partial y}, \frac{\partial E_x}{\partial z}, \dots, \frac{\partial E_z}{\partial z}$ 9 različnih odvodov

OE1: $\vec{E} = -\vec{1}_s \frac{\partial V}{\underset{\substack{\uparrow \\ \max}}{\partial s}}$] smer kamor se potencial največ spreminja

smerni odvod
↓
gradient

$\underset{[V/m]}{\vec{E}} = -\underset{[m^{-1}]}{\text{grad}}\, \underset{[V]}{V(\vec{r})} = -\left[\vec{1}_x \frac{\partial V}{\partial x} + \vec{1}_y \frac{\partial V}{\partial y} + \vec{1}_z \frac{\partial V}{\partial z}\right] = -\vec{\nabla} V(\vec{r})$

nabla: → gradient!

$\underset{[m^{-1}]}{\vec{\nabla}} = \vec{1}_x \frac{\partial}{\partial x} + \vec{1}_y \frac{\partial}{\partial y} + \vec{1}_z \frac{\partial}{\partial z}$

samo kartezični koordinatni sistem, kjer $h_x = h_y = h_z = 1$

poljubni koordinatni sistem (q_1, q_2, q_3)

OSTALI: UPOŠTEVAJ ŠE LAMÉ-JEVE KOEFICIENTE → glej naprej

$-\vec{E} = -\text{grad}\, V = \vec{\nabla} V = \vec{1}_{q_1} \frac{\partial V}{\partial l_1} + \vec{1}_{q_2} \frac{\partial V}{\partial l_2} + \vec{1}_{q_3} \frac{\partial V}{\partial l_3}$

$$\text{grad}\, V = \vec{1}_{q_1} \left(\overset{1/dl_1}{\frac{1}{h_1}} \frac{\partial V}{\partial q_1}\right) + \vec{1}_{q_2} \frac{1}{h_2} \frac{\partial V}{\partial q_2} + \vec{1}_{q_3} \frac{1}{h_3} \frac{\partial V}{\partial q_3}$$

• valjni koordinatni sistem $(\rho, \varphi, z) \Rightarrow h_\rho = 1,\ h_\varphi = \rho,\ h_z = 1$

$$\text{grad}\, V(\rho, \varphi, z) = \vec{1}_\rho \frac{\partial V}{\partial \rho} + \vec{1}_\varphi \frac{1}{\rho} \frac{\partial V}{\partial \varphi} + \vec{1}_z \frac{\partial V}{\partial z} \; [V/m]$$

• sferni koordinatni sistem $(r, \theta, \phi) \Rightarrow h_r = 1,\ h_\theta = r,\ h_\phi = r\sin\theta$

$$\text{grad}\, V(r, \theta, \phi) = \vec{1}_r \frac{\partial V}{\partial r} + \vec{1}_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} + \vec{1}_\phi \frac{1}{r\sin\theta} \frac{\partial V}{\partial \phi}$$

ponovitev:

koordinatni sistem: - zahteve:

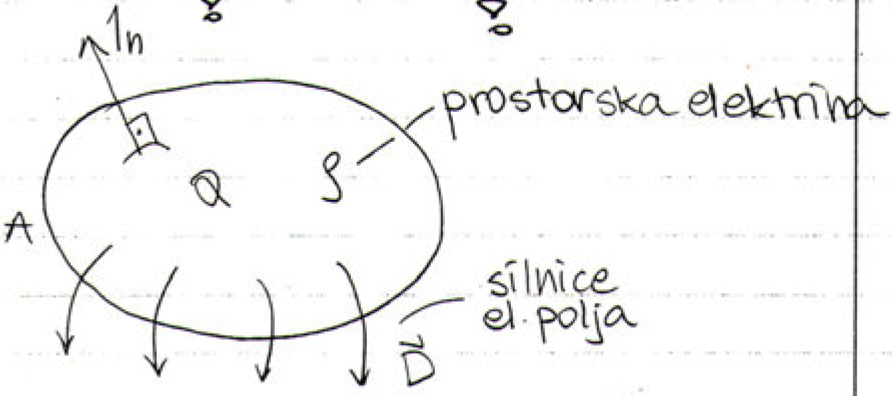
- 3D
- pravokotni $\vec{A}\bullet\vec{B}$ (ni mešanih členov)
- desnoročni $\vec{A}\times\vec{B}$ (predznak)

krivočrtni? $h_1, h_2, h_3 \neq 1$ h_1, h_2, h_3 .. lamejevi koeficienti

$$\underset{[V/m]}{\vec{E}} = -\text{grad } V = -\vec{\nabla} \underset{[V]}{V}$$

$$\text{grad } V = \vec{1}_{q_1}\frac{1}{\underset{\nabla}{\underline{\underline{h_1}}}}\frac{\partial V}{\partial q_1} + \vec{1}_{q_2}\frac{1}{\underset{\nabla}{\underline{\underline{h_2}}}}\frac{\partial V}{\partial q_2} + \vec{1}_{q_3}\frac{1}{\underset{\nabla}{\underline{\underline{h_3}}}}\frac{\partial V}{\partial q_3}$$

Gaussov zakon:



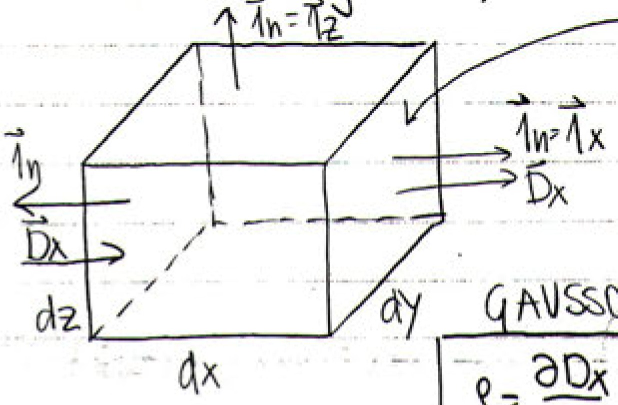
$$\boxed{\oint_A \vec{D}\cdot\vec{1}_n\, dA = Q = \int_V \rho\, dv = \int_V \text{div}\,\vec{D}\cdot dv}$$

$$\boxed{\rho = \lim_{\Delta v \to 0}\frac{\oint_A \vec{D}\cdot\vec{1}_n\, dA}{\Delta v} = \text{div}\,\vec{D}}$$

prostorska elektrina

silnice el. polja če to stlačimo v zelo majhno, nimamo več dimenzij

izvornost (divergenca):



$dv = dx\,dy\,dz$

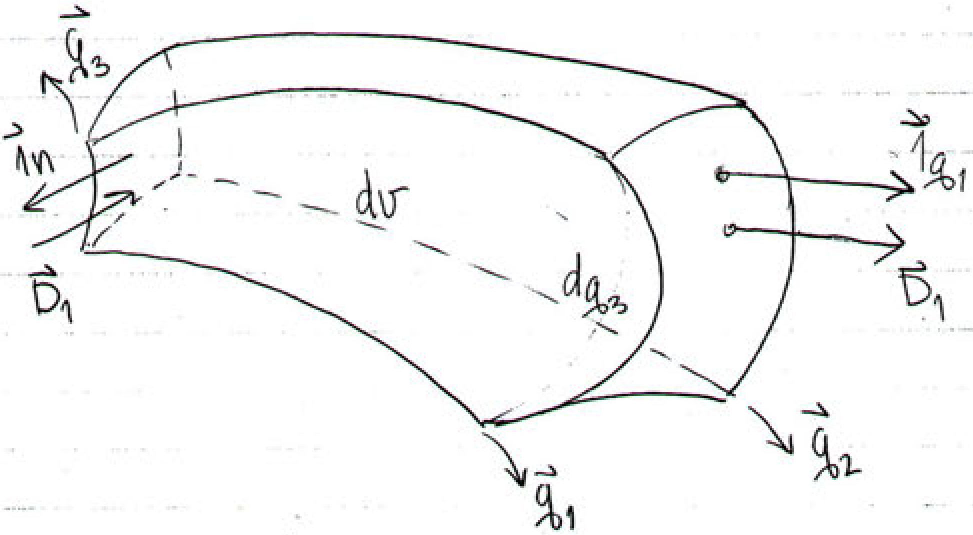
$dA = dy\,dz$

GAUSSOV IZREK

$$\boxed{\rho = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \vec{\nabla}\bullet\vec{D}}$$

razlika dveh nasprotnih ploskvic

kocka v krivočrtnem



$dv = h_1 h_2 h_3 \, dq_1 \, dq_2 \, dq_3$
$dA = h_2 h_3 \, dq_2 \, dq_3$

$h_1(\vec{r}), h_2(\vec{r}), h_3(\vec{r})$ .. lahko se odvajajo

$$\rho = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial(h_2 h_3 D_1)}{\partial q_1} + \frac{\partial(h_1 h_3 D_2)}{\partial q_2} + \frac{\partial(h_1 h_2 D_3)}{\partial q_3}\right]$$

↳ IZVORNOST V KRIVOČRTNEM SISTEMU

valjni koordinatni sistem (ρ, φ, z):

$h_\rho = 1, h_\varphi = \rho, h_z = 1$

$$\vec{D} = \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho \overset{D_\rho}{d\rho}) + \frac{\partial}{\partial \varphi} D_\varphi + \frac{\partial}{\partial z}(\rho D_z)\right]$$

$$\operatorname{div}\vec{D} = \frac{\partial D_\rho}{\rho} + \frac{D_\rho}{\rho} + \frac{1}{\rho}\frac{\partial D_\varphi}{\partial \varphi} + \frac{\partial D_z}{\partial z}$$

$$\boxed{\operatorname{div}\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\varphi}{\partial \varphi} + \frac{\partial D_z}{\partial z}}$$

krogelni koordinatni sistem: (r, θ, ϕ):

$h_r = 1, h_\theta = r, h_\phi = r\sin\theta$

$$\mathrm{div}\,\vec{D} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial r}(r^2 \sin\theta D_r) + \frac{\partial}{\partial \theta}(r\sin\theta D_\theta) + \frac{\partial}{\partial \phi}(r D_\phi)\right]$$

$$\boxed{\mathrm{div}\,\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}}$$

⇒ Gaussov izrek

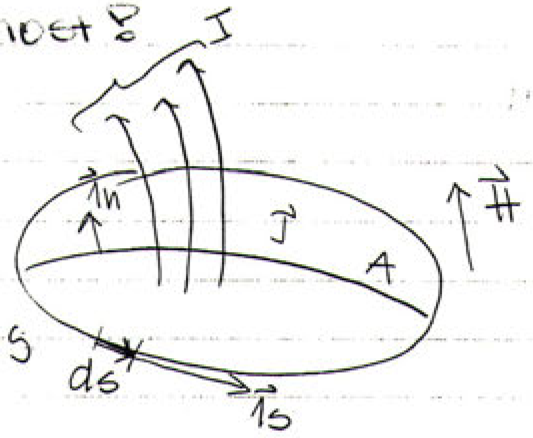
$$\boxed{\int_v \mathrm{div}\,\vec{D}\cdot dv = \oint_A \vec{D}\cdot\vec{1}_n\cdot dA}$$!

Gaussov zakon v diferencialni obliki: !

$$\boxed{\int_v \rho\, dv \Rightarrow \rho = \mathrm{div}\,\vec{D}}$$

slabost: singularnost !

Amperov zakon:



[A/m²]

$$\boxed{\oint_S \vec{H}\cdot d\vec{s} = I = \int_A \vec{J}\cdot\vec{1}_n\cdot dA}$$

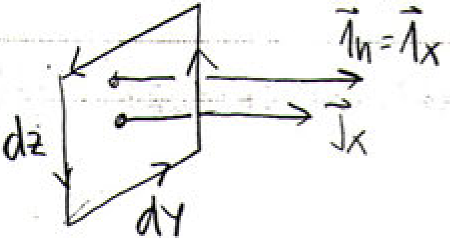
[A/m] [A] celotni tok

$\underline{\vec{1}_n = \vec{1}_x}$, $\underline{\Delta A \to 0}$ spet bi imeli radi dimenzij ⇒ dif. majhna oblika

$$\oint \vec{H}\cdot d\vec{s} = J_x \Delta A$$

vrtinčenje; rotor (curl)

$$J_x = \lim_{\Delta A \to 0} \frac{\oint \vec{H}\cdot d\vec{s}}{\Delta A} = \boxed{\vec{1}_x \cdot \mathrm{rot}\,\vec{H}}$$



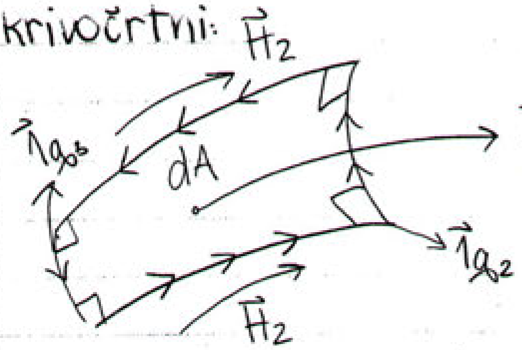
$$\vec{J}_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} -$$

$$\vec{J} = \mathrm{rot}\,\vec{H} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \vec{\nabla} \times \vec{H}$$

Stokesov izrek:

$$\boxed{\oint_s \vec{H} \cdot d\vec{s} = \int_A \mathrm{rot}\,\vec{H} \cdot \vec{1}_n \, dA}$$

krivočrtni:



$dA = h_2 h_3 \, dq_2 \, dq_3$

$dl_2 = h_2 \, dq_2$

$dl_3 = h_3 \, dq_3$

$$J_1 = \frac{1}{h_2 h_3} \left(\frac{\partial}{\partial q_2} (h_3 H_3) - \frac{\partial}{\partial q_3} (h_2 H_2) \right)$$

$$\mathrm{rot}\,\vec{H} = \vec{J} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \cdot \vec{1}_{q_1} & h_2 \cdot \vec{1}_{q_2} & h_3 \cdot \vec{1}_{q_3} \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1 H_1 & h_2 H_2 & h_3 H_3 \end{vmatrix}$$

valjni koordinatni sistem (ρ, φ, z):

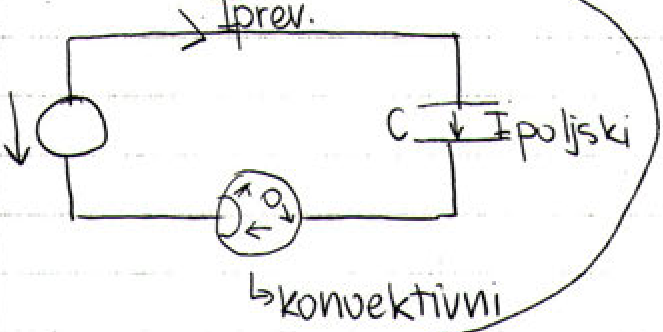
$$\mathrm{rot}\,\vec{H} = \frac{1}{\rho} \begin{vmatrix} \vec{1}_\rho & \rho \cdot \vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\varphi & H_z \end{vmatrix}$$

krogelni koordinatni sistem (r, Θ, Φ):

$$\mathrm{rot}\,\vec{H} = \frac{1}{r \cdot \sin\Theta} \begin{vmatrix} \vec{1}_r & r \cdot \vec{1}_\Theta & r\sin\Theta \cdot \vec{1}_\Phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \Theta} & \frac{\partial}{\partial \Phi} \\ H_r & r H_\Theta & r\sin\Theta \, H_\Phi \end{vmatrix}$$

$$\boxed{\mathrm{rot}\,\vec{H} = \underset{\uparrow}{\vec{J}} = \vec{J}_{prevodniški} + \vec{J}_{konvektivni} + \underbrace{\frac{\partial \vec{D}}{\partial t}}_{poljski}}$$

celotni tok



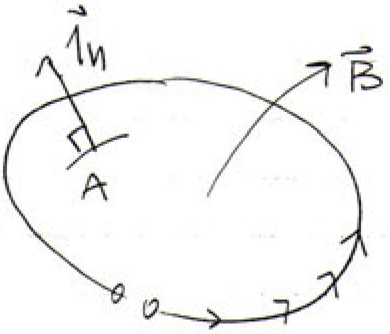
$$\vec{J}_{prevodniški} = \underset{\uparrow}{\gamma} \cdot \vec{E}$$

prevodno število

Amperov zakon v diferencialni obliki:

$$\boxed{\mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Faradayev zakon:



$$u_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_A \vec{B}\cdot\vec{1}_n\,dA = \oint_s \vec{E}\cdot d\vec{s}$$

$$\oint_s \vec{E}\cdot d\vec{s} = \int_A \mathrm{rot}\,\vec{E}\cdot\vec{1}_n\,dA$$

$$u_i = \oint_s \vec{E}\cdot d\vec{s}$$

Uporaben zgled: $\frac{d(\vec{1}_n A)}{dt} = 0$?

$$\hookrightarrow \frac{d}{dt}\int_A \vec{B}\cdot\vec{1}_n\,dA = \int_A \left(\frac{d\vec{B}}{dt}\right)\cdot\vec{1}_n\,dA$$

kadar ploskvica ni funkcija časa

Faradayev zakon v diferencialni obliki:

$$\boxed{\mathrm{rot}\,\vec{E} = -\frac{d\vec{B}}{dt}}$$

MAXWELLOVE ENAČBE V DIFERENCIALNI OBLIKI:

$$\boxed{\begin{array}{ll} ①\ \mathrm{rot}\,\vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t} & \text{Amper} \\ ②\ \mathrm{rot}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \text{Faraday} \\ ③\ \mathrm{div}\,\vec{D} = \rho & \text{Gauss} \end{array}}$$

$\Delta X \to 0$

$\Delta A \to 0$

$\Delta V \to 0$

!

težava: vse funkcije morajo biti zvezne in odvedljive !

Elektromagnetna naloga:

podatki ⇒ izvori: $\vec{J}$, ρ

$\vec{E} = ?$, $\vec{H} = ?$

$\varepsilon \mu$ nista funkciji koordinat

↑

preprosta snov: linearna, izotropna ∀ homogena

$\vec{B} = \mu \vec{H}$, $\vec{D} = \varepsilon \vec{E}$

Harmonske veličine: !

$\frac{\partial}{\partial t} = j\omega$

① $\mathrm{rot}\,\vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t}$ ⇒ $\mathrm{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$ / rot ↗ (za H)

② $\mathrm{rot}\,\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ ⇒ $\mathrm{rot}\,\vec{E} = -j\omega\mu\vec{H}$ / · rot ↗ (za E)

③ $\varepsilon \cdot \mathrm{div}\,\vec{E} = \rho$ ⇒ $\varepsilon\,\mathrm{div}\,\vec{E} = \rho$

Velja:

1) $\mathrm{div}(\mathrm{grad}\,V) = \vec{\nabla}\cdot(\vec{\nabla}V) = \Delta V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ → skalarni Laplace

2) $\mathrm{rot}(\mathrm{grad}\,V) = \vec{\nabla}\times(\vec{\nabla}V) = 0$ gradientno polje nima vrtincev !

3) $\mathrm{grad}(\mathrm{div}\,\vec{F}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{F})$

4) $\mathrm{div}(\mathrm{rot}\,\vec{F}) = \vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = 0$ polje vrtincev nima izvorov !

5) $\mathrm{rot}(\mathrm{rot}\,\vec{F}) = \vec{\nabla}\times(\vec{\nabla}\times\vec{F})$

6) → $\mathrm{grad}(\mathrm{div}\,\vec{F}) - \mathrm{rot}(\mathrm{rot}\,\vec{F}) = \Delta\vec{F}$ = vektorski Laplace

samo v kartezičnih koordinatah:

$$\Delta\vec{F} = \vec{1}_x \Delta F_x + \vec{1}_y \Delta F_y + \vec{1}_z \Delta \vec{F}_z$$

z②

$$\mathrm{rot}(\mathrm{rot}\,\vec{E}) = -j\omega\mu\,\mathrm{rot}\,\vec{H} \overset{①}{=} -j\omega\mu\,(\vec{J} + j\omega\varepsilon\vec{E})$$

$$\mathrm{rot}(\mathrm{rot}\,\vec{E}) = -j\omega\mu\vec{J} + \omega^2\mu\varepsilon\vec{E} \overset{6)}{=} \mathrm{grad}\,(\mathrm{div}\,E) - \Delta\vec{E} \overset{③}{=}$$

$$= \mathrm{grad}\,(\overset{?\to\rho/\varepsilon}{\varepsilon\rho}) - \Delta\vec{E}$$

$$\boxed{\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = \mathrm{grad}\,(\rho\varepsilon) + j\omega\mu\vec{J}}\ \boxed{?}$$

ni uporabna ker ρ in $\vec{J}$ morata biti vezni !

prostor brez izvorov: $\rho = 0$, $\vec{J} = 0$

$\boxed{\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0}$: valovna enačba (nehomogena)

$$\mathrm{rot}(\mathrm{rot}\,\vec{H}) = \mathrm{rot}\,\vec{J} + j\omega\varepsilon\,\mathrm{rot}\,\vec{E} = \mathrm{rot}\,\vec{J} + j\omega\varepsilon(-j\omega\mu\vec{H})$$

$\Downarrow$

$$\underbrace{\mathrm{grad}(\mathrm{div}\,\vec{H})}_{0} - \Delta\vec{H} = \mathrm{rot}\,\vec{J} + \omega^2\mu\varepsilon\vec{H}$$

od ④ $\mathrm{div}\,(\mathrm{rot}\,\vec{E}) = 0$

$\mathrm{div}\,\vec{B} = 0$

$$\boxed{\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = -\mathrm{rot}\,\vec{J}}\ \boxed{?}$$

neuporabna!

prostor brez tokov: $\vec{J} = 0$

$\boxed{\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = 0}$ nehomogena valovna enačba

OET1:

f-ja treh koordinat

Elektrostatika: $\vec{E}(\vec{r}) = -\mathrm{grad}\,V(\vec{r}) \Rightarrow \mathrm{rot}\,\vec{E} = 0 \Rightarrow \omega = 0$ X

OET 2:

Magnetostatika: $\vec{H}(\vec{r}) = -\mathrm{grad}\,V_m(\vec{r}) \Rightarrow \mathrm{rot}\,\vec{H} = 0 \Rightarrow$ $\omega = 0$ Elektrostatika X

*

$\Downarrow$ $\vec{J} = 0$ ni izvora

↗ različno od *

Vektorski potencial

$B = \text{rot}\,\vec{A} = \text{rot}\,\vec{V}_m'$ → $\text{div}\,\vec{A}$ = poljubna?

→ Coulombova izbira, $\text{div}\,\vec{A} = 0$

→ Lorentzova izbra: $j\omega\varepsilon\mu V + \text{div}\,\vec{A} = 0$

$\Rightarrow \text{div}\,\vec{A} = -j\omega\varepsilon\mu V$

↳ $\boxed{\vec{H} = \frac{1}{\mu}\,\text{rot}\,\vec{A}}$

$$\text{rot}\,\vec{E} = -j\omega\mu\vec{H} = -j\omega\,\text{rot}\,\vec{A}$$

$$\text{rot}\,(\vec{E} + j\omega\vec{A}) = 0$$

↗ poljuben grad => ker rot grad = 0 ?

$\vec{E} + j\omega\vec{A} = -\text{grad}\,V$ ← skalarni potencial

skalarni potencial

$\boxed{\vec{E} = -j\omega A - \text{grad}\,V}$ $/\cdot\,\text{div}$ ↳ to je dogovor, saj $\text{rot}(\text{grad}\,V) = 0$

$\vec{J}, \rho \xRightarrow{1\,\text{korak}} A, V \xRightarrow{2.\,\text{korak}} \vec{E}, \vec{H}$: računski postopek

$$\text{rot}\,\vec{H} = \text{rot}\left(\frac{1}{\mu}\,\text{rot}\,\vec{A}\right) = \vec{J} + j\omega\varepsilon(-j\omega\vec{A} - \text{grad}\,V) \qquad (\vec{A}, V)$$

$$\frac{1}{\mu}\,\text{rot}\,(\text{rot}\,\vec{A}) = \frac{1}{\mu}(\text{grad}(\text{div}\,\vec{A}) - \Delta\vec{A}) = \vec{J} + \omega^2\varepsilon\vec{A} - j\omega\varepsilon(\text{grad}\,V) \;/\cdot\mu$$

$$\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J} + \text{grad}(\text{div}\,\vec{A}) + j\omega\mu\varepsilon\,\text{grad}\,V =$$

$$= -\mu\vec{J} + \text{grad}\,(\underbrace{j\omega\mu\varepsilon V + \text{div}\,A}_{=0;\ \text{Lorentzova izbra}})$$

valovna enačba z nehomogenim delom

Valovna enačba: za vektorski potencial

$\boxed{\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}}$! vektorski potencial za Lorentzovo izbiro ga poganjajo samo tokovi ! (ne elektrine)

izvori: div
vrtinci: rot !

ρ.. prostorska elektrina

③ M.E.

$\mathrm{div}\vec{E} = -j\omega\,\mathrm{div}\,\vec{A} - \mathrm{div}(\mathrm{grad}V)$

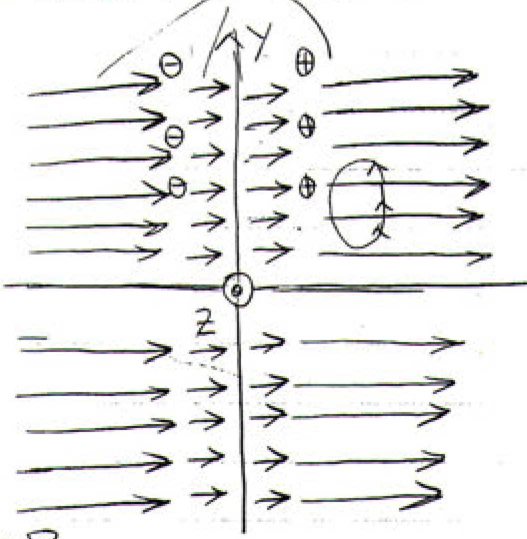
$\frac{\rho}{\varepsilon} = -j\omega(-j\omega\mu\varepsilon V) - \Delta V = -\omega^2\mu\varepsilon V - \Delta V$

valovna enačba: za skalarni potencial

$\boxed{\Delta V + \omega^2\mu\varepsilon V = -\rho/\varepsilon}$!

elektrostatske probleme rešjam z ugasnjeno frekvenco (ω=0)
⇒ DOBIMO ŽE ZNANE ENAČBE

- enačbi sta povezani preko ρ in $\vec{J}$;
  čim večji tok zmanjkuje elektrine



⇒ ne bo vrtincev

- zgled: $\vec{F} = \vec{1}_x \cdot \alpha x^2$

izvori = ?, vrtinci = ?

$\mathrm{rot}\vec{F} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \alpha x^2 & 0 & 0 \end{vmatrix} = \underline{\underline{0}}$

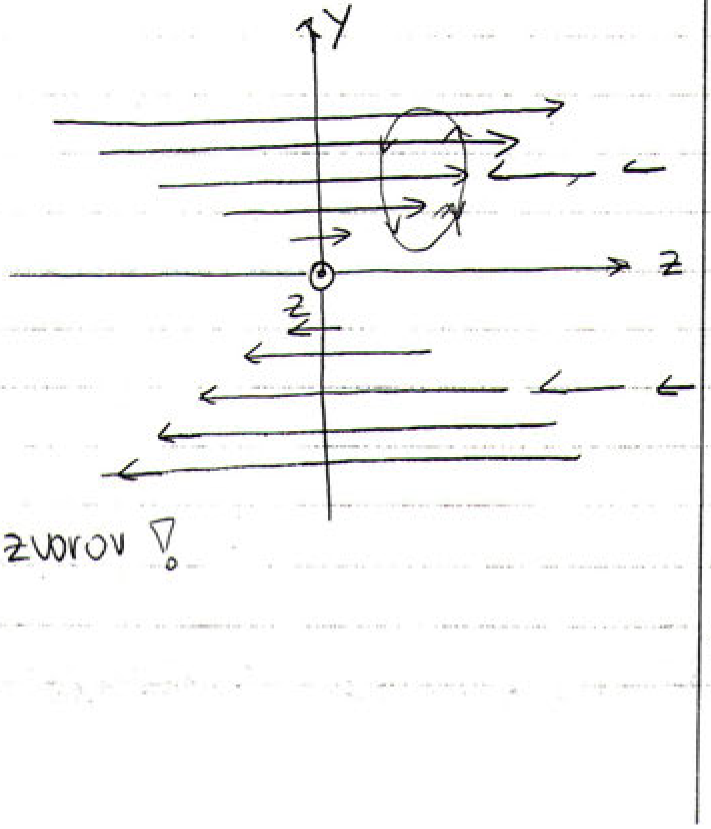
⇒ ni vrtincev!

$\mathrm{div}\vec{F} = \frac{\partial F_x}{\partial x} = \underline{\underline{2\alpha x}}$ so izvori!

- zgled: $\vec{F} = \vec{1}_x \alpha y$

$\mathrm{rot}\vec{F} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \alpha y & 0 & 0 \end{vmatrix} =$

$= 0 - 0 - \vec{1}_z\alpha = \underline{\underline{-\vec{1}_z\alpha}}$

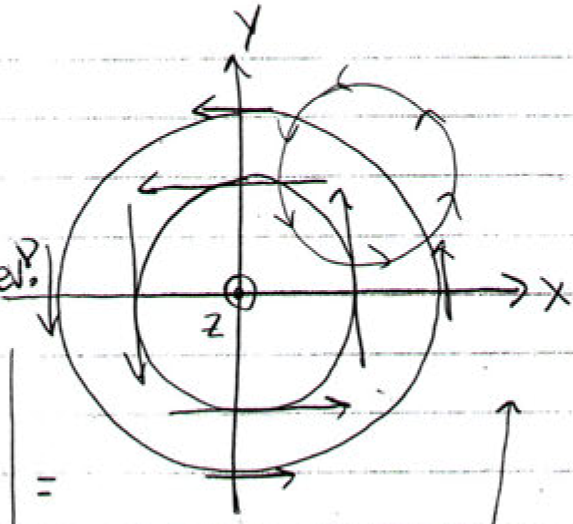


$\mathrm{div}\,\vec{F} = \frac{\partial}{\partial x}(\alpha y) = \underline{\underline{0}}$ ni izvorov !

-Zgled: (ϱ, φ, z): $\vec{F} = \vec{1}_\varphi \cdot \frac{\alpha}{\varrho}$

→ SINGULARNOST?

pri $\varrho = 0 \Rightarrow$ zato ni vrtincev?



$$\mathrm{rot}\vec{F} = \frac{1}{\varrho} \begin{vmatrix} \vec{1}_\varrho & \varrho\vec{1}_\varphi & \vec{1}_z \\ \frac{\partial}{\partial\varrho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ 0 & \varrho\frac{\alpha}{\varrho} & 0 \end{vmatrix} =$$

$$= \frac{1}{\varrho}\vec{1}_z \frac{\partial}{\partial\varrho}\left(\cancel{\varrho}\frac{\alpha}{\cancel{\varrho}}\right) = \underline{\underline{0}}$$ ni vrtincev?

$$\mathrm{div}\vec{F} = \frac{1}{\varrho}\left[\frac{\partial}{\partial\varphi}\left(1 \cdot 1 \cdot \frac{\alpha}{\varrho}\right)\right] = \underline{\underline{0}}$$ ni izvorov?

↗ veljajo na zelo majhnem prostoru

Maxwell (diferencialno)

① $\text{rot}\,\vec{H} = \vec{J} + j\omega\vec{D}$ .. Amperov

② $\text{rot}\,\vec{E} = -j\omega\vec{B}$; $\vec{B} = \mu\vec{H}$ .. Faradayev

③ $\text{div}\,\vec{D} = \rho$; $\vec{D} = \varepsilon\vec{E}$ .. Gaussov

---

brez izvorov: $\underline{\rho = 0}$; $\underline{\vec{J} = 0}$

$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0$

$\Delta\vec{H} + \omega^2\mu\varepsilon\vec{H} = 0$

valovni enačbi

Imamo izvore: $\rho \neq 0$; $\vec{J} \neq 0$

$\vec{B} = \text{rot}\,\vec{A}$ $(\vec{H} = \frac{1}{\mu}\text{rot}\,\vec{A})$ A.. magnetni potencial

$\vec{E} = -j\omega\vec{A} - \text{grad}\,V$ V.. skalarni potencial

$\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A} = -\mu\vec{J}$

$\Delta V + \omega^2\mu\varepsilon V = -\rho/\varepsilon$

Zakaj so te veličine potenciali?

Energija?

DET 1: Elektrostatika

↗ koliko energije prispeva elektrina

$$W_e = \frac{1}{2}\int_v \vec{E}\cdot\vec{D}\,dv = \frac{1}{2}\int_v \rho V\,dv$$

⇒ ne pove kje se ta energija nahaja

$\omega = 0$; $\frac{d}{dt} = 0$

$\Delta V = -\rho/\varepsilon$; $\vec{E} = -\text{grad}\,V$ $\overset{\varepsilon\cdot\vec{E}}{\parallel}$

$\vec{E}\cdot\vec{D} = \underbrace{(-\text{grad}\,V)\cdot(-\varepsilon\,\text{grad}\,V)}_{-\vec{D}} = \varepsilon\,\text{grad}\,V\cdot\text{grad}\,V$

odvajanje

$\text{div}\,(\varepsilon V\,\text{grad}\,V) = \vec{\nabla}\cdot(\varepsilon V\vec{\nabla}V) = \varepsilon\vec{\nabla}V\cdot\vec{\nabla}V + \varepsilon V\Delta V$

$\text{div}\,(V(-\vec{D})) = \vec{E}\cdot\vec{D} + \varepsilon V(-\rho/\varepsilon) = \vec{E}\cdot\vec{D} - \rho V \;/ \int_v dv$

$$-\int_v \text{div}\,(V\cdot\vec{D})\,dv = \int_v \vec{E}\cdot\vec{D}\,dv - \int_v \rho V\,dv = \oint V\cdot\vec{D}\cdot\vec{1}_n\,dA = 0$$

Gauss

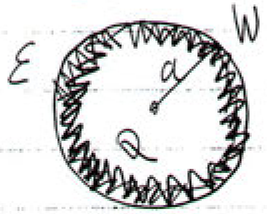
A→∞ celoten prostor

=

izberem $V(\infty) = 0$

A.. vektorski potencial

-Zgled:



$$V = \frac{Q}{4\pi\varepsilon r}$$

$$\vec{E} = \vec{1}_r \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2}$$

$$W = \frac{1}{2}\int_v \varepsilon \vec{E}\cdot\vec{E}\,dv = \frac{1}{2}\int_a^{\overset{r}{\infty}} \int_0^{\overset{\theta}{\pi}} \int_0^{2\pi \to \phi} \varepsilon \left(\frac{Q}{4\pi\varepsilon}\frac{1}{r^2}\right)^2 \overbrace{r^2 \sin\theta \, d\theta \, d\phi}^{dv} =$$

$$(-\cos\theta)_0^{\pi} \qquad -\frac{1}{r} \to -\frac{1}{\infty}$$

$$= \frac{1}{\cancel{2}}\, 2\pi \cdot \overset{\pi}{2}\, \frac{Q^2}{16\pi^2 \varepsilon}\left(\overset{\uparrow}{0} + \frac{1}{a}\right) = \frac{Q^2}{8\pi\varepsilon a}$$

$$W = \frac{1}{2}\int_v \rho \cdot V dv = \frac{1}{2}\sum_i Q_i \cdot V_i = \frac{1}{2} Q \cdot \frac{Q}{4\pi\varepsilon a} = \frac{Q^2}{8\pi\varepsilon a}$$

OET2: Magnetostatika

$$W_m = \frac{1}{2}\int_v \vec{H} \cdot B\, dv$$

$$w = 0,\ \frac{d}{dt} = 0 \Rightarrow \text{rot}\,\vec{H} = J + \cancel{j\omega\vec{D}}$$

$$\vec{H}\cdot\vec{B} = \vec{H}\cdot \text{rot}\,\vec{A}$$

$$\text{div}\,(\vec{A}\times\vec{H}) = \vec{\nabla}\cdot(\vec{A}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{H})$$

$$\text{div}\,(\vec{A}\times\vec{H}) = \vec{H}(\text{rot}\,\vec{A}) - \vec{A}(\text{rot}\,\vec{H}) = \vec{H}\cdot\vec{B} - \vec{A}\cdot\vec{J} \;/\; \int_v dv$$

$$\int_v \text{div}(\vec{A}\times\vec{H})dv = \oint_{\underset{A\to\infty}{A}} (\vec{A}\times\vec{H})\cdot\vec{1}_n dA = \underbrace{\int_v \vec{H}\cdot\vec{B}dv - \int_v \vec{J}\cdot\vec{A}\,dv}_{=}$$

izberem $A(\infty) = 0$

$$\boxed{W_m = \frac{1}{2}\int_v \vec{H}\cdot\vec{B}\,dv = \frac{1}{2}\int_v \vec{J}\cdot\vec{A}\,dv} \rightarrow$$ normirana energija za vsak tok

Elektrodinamika:

$$w \neq 0,\ \frac{d}{dt} \neq 0 \qquad \overset{\text{konst}}{\overset{\uparrow}{v}} = \frac{dv}{dt} = 0,\ \frac{d\varepsilon}{dt} = 0,\ \frac{d\mu}{dt} = 0$$

$$W = W_e + W_m = \frac{1}{2}\int_v (\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})dv$$

$$\frac{dW}{dt} = \frac{1}{2}\int_v \left[\frac{d}{dt}(\vec{E}\cdot\vec{D} + \vec{H}\cdot\vec{B})\right] dv$$

$\vec{A}$.. potencial: vsakemu toku pripiše energijo (ker ima energija smer $\Rightarrow \vec{A}$ vektor)

$$\frac{d}{dt}(\vec{E}\cdot\vec{D}) = \frac{\partial\vec{E}}{\partial t}\vec{D} + \vec{E}\frac{\partial\vec{D}}{\partial t} = 2\vec{E}\frac{\partial\vec{D}}{\partial t}$$

$$\frac{d}{dt}(\vec{H}\cdot\vec{B}) = \frac{\partial\vec{H}}{\partial t}\vec{B} + \vec{H}\frac{\partial\vec{B}}{\partial t} = 2\vec{H}\frac{\partial B}{\partial t}$$

Maxwell. enačbe

$$\Rightarrow \frac{dW}{dt} = \int_v \left(\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\frac{\partial\vec{B}}{\partial t}\right)dv = \leftarrow \begin{matrix} ① \ \mathrm{rot}\,\vec{H} = \vec{J} + \partial\vec{B}/\partial t \\ ② \ \mathrm{rot}\,\vec{E} = -\frac{\partial B}{\partial t} \Rightarrow \frac{\partial B}{\partial t} = -\mathrm{rot}\,\vec{H} \end{matrix}$$

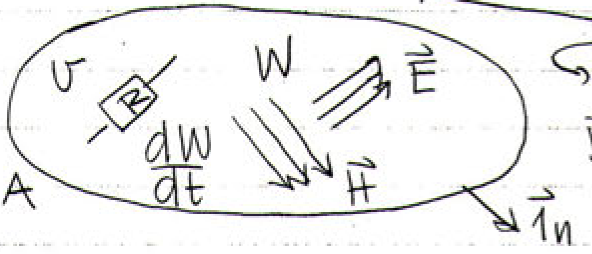
$$= \int_v (\vec{E}\circ\mathrm{rot}\,\vec{H} - \vec{E}\cdot\vec{J} - \vec{H}\cdot\mathrm{rot}\,\vec{E})dv \leftarrow \vec{E}\,\mathrm{rot}\,\vec{H} - \vec{H}\cdot\mathrm{rot}\,\vec{E}$$

$$\vec{E}\cdot\mathrm{rot}\,\vec{H} - \vec{H}\,\mathrm{rot}\,\vec{E} = \vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{H}\cdot(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\cdot(\vec{H}\times\vec{E})$$

$$\Rightarrow \frac{dW}{dt} = \int_v [-\mathrm{div}(\vec{E}\times\vec{H}) - \vec{E}\cdot\vec{J}]\,dv = \leftarrow \text{Gauss ①}$$

$$\boxed{\frac{dW}{dt} = -\oint_A (\vec{E}\times\vec{H})\cdot\vec{1}_n\,dA - \underbrace{\int_v \vec{E}\cdot\vec{J}\,dv}}$$ Poynting-ov izrek

energija, ki potuje P, ki se troši na uporu



Sprememba energije po času je enaka moči, ki se troši na uporu in energiji, ki potuje.

$$\frac{dW}{dt} = -\underbrace{\oint_A (\vec{E}\times\vec{H})\cdot\vec{1}_n\,dA} - \int_v \vec{E}\cdot\vec{J}\,dv$$

potujoča energija
smer: ven iz v

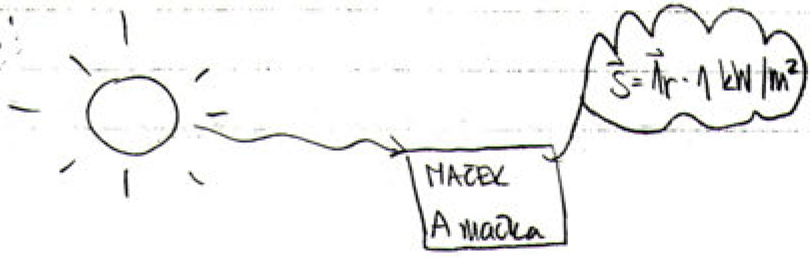
Poyntingov vektor (gostota moči!) $\boxed{\vec{S} = \vec{E}\times\vec{H}}$

↳ pove kam potuje energija

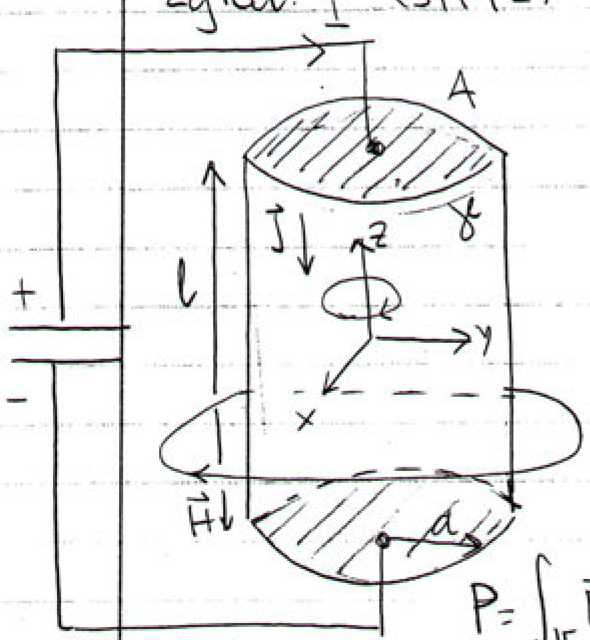
$$\left[\frac{V}{m}\right]\cdot\left[\frac{A}{m}\right] = \left[\frac{W}{m^2}\right] \qquad V\cdot A = W$$

$$\vec{E} \qquad \vec{H}$$

Zgled:



-zgled: (ρ,φ,z)



$$\vec{J} = -\vec{1}_z \frac{I}{A} = \gamma \vec{E}$$

$$\vec{E} = -1_z \frac{I}{\gamma A}$$

$$U = |\vec{E}| \cdot l = \frac{Il}{\gamma A}$$

$$P = U \cdot I = \frac{I^2 l}{\gamma A} = I^2 \left(\frac{l}{\gamma A}\right) = I^2 R$$

$$P = \int_v \vec{E} \cdot \vec{J}\, dv = \int_l \int_A \frac{I^2}{\gamma A} dA\, dl = \frac{I^2 l}{\gamma A}$$

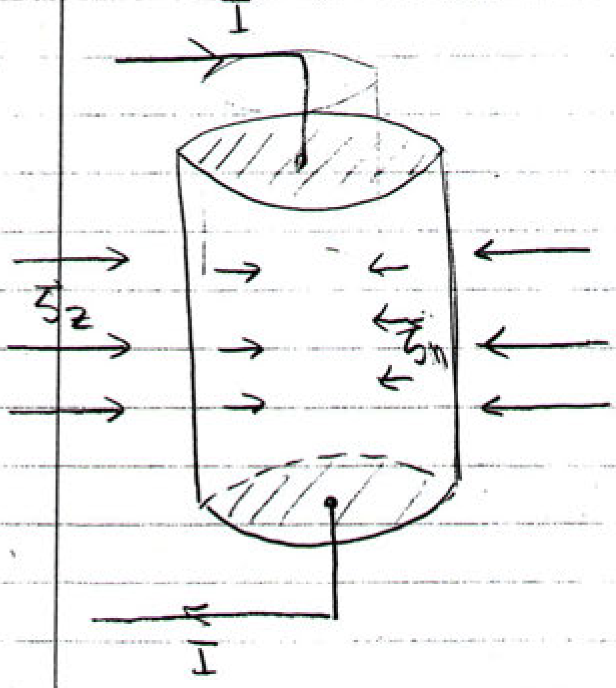
otranji ← $\vec{H}_n = \vec{1}_\varphi \frac{I}{2\pi a^2} \rho$

zunajni ← $\vec{H}_z = -1_\varphi \frac{I}{2\pi \rho}$

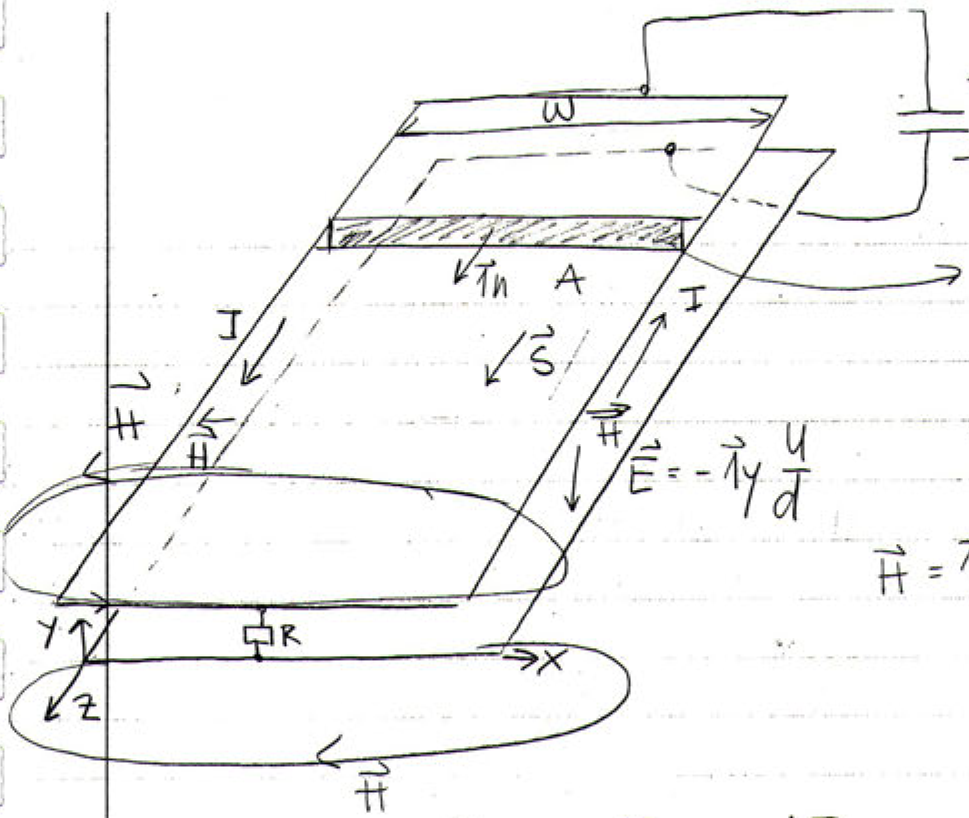
$$\vec{S}_n = -\vec{1}_\rho \frac{I^2 \rho}{2\pi \gamma A a^2}$$

$$\vec{S}_z = -\vec{1}_\rho \frac{I^2}{2\pi \gamma A \rho}$$

moč vstopa bocno v upor (ne po žicah)



moč ne teče po žicah, ampak po prostoru med zicami

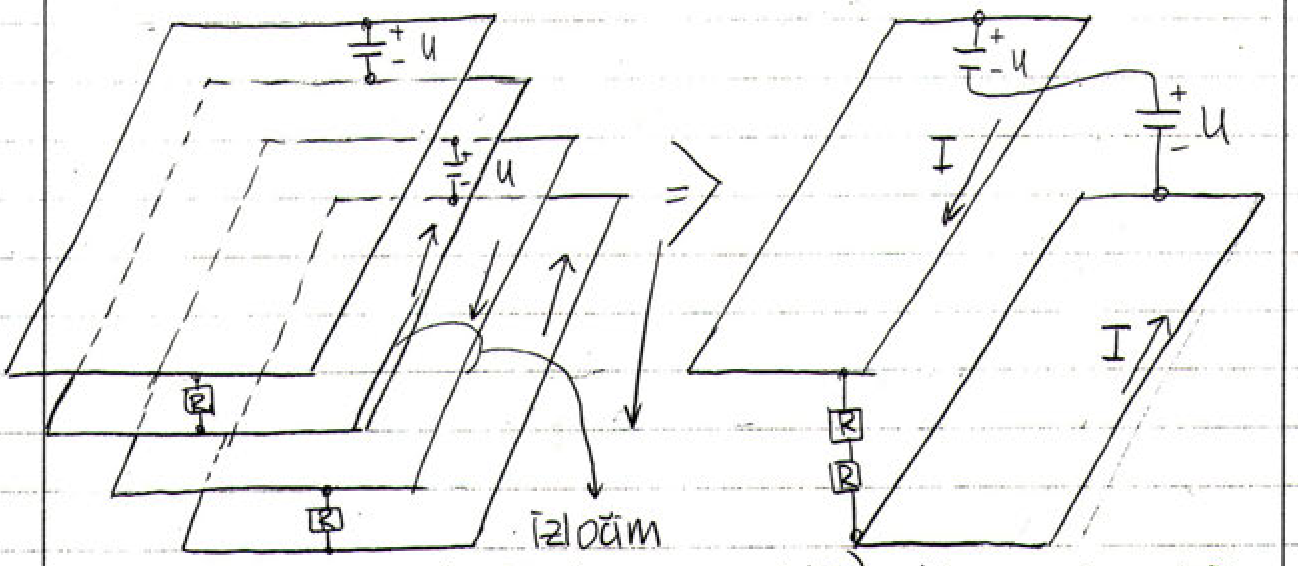


$\vec{K}=\vec{1}_z\frac{I}{w}$

$\vec{H}=\vec{1}_x\frac{I}{w}$ (med ploščami.

$$\vec{S}=\vec{E}\times\vec{H}=-\vec{1}_y\frac{U}{d}\times\vec{1}_x\frac{I}{w}=\vec{1}_z\frac{UI}{d\cdot w}$$

elotna
el moč $P=\int_A\vec{S}\cdot\vec{1}_n\,dA=\int_0^w\int_0^d\vec{1}_z\frac{UI}{d\cdot w}\cdot\vec{1}_z\,dx\,dy=\underline{\underline{U\cdot I}}$ ✓



izločim

(uničujoči prehodi ±I) → tokova se izničita

enosmerne trenutne veličine: $\underline{P=U\cdot I}$

$\underline{\vec{S}=\vec{E}\times\vec{H}}$

↳ dokaz da kljub temu če plošči izločimo moč vseeno teče ⇒ moč ne teče po žicah, ampak med njima

---

kazalci za harmonske veličine

$P=\frac{1}{2}U\cdot I^*$

$P=U_{eff}\cdot I_{eff}^*$

$\vec{S}=\frac{1}{2}\vec{E}\times\vec{H}$

$\vec{S}=\vec{E}_{eff}\,\vec{H}_{eff}^*$

vršne vrednosti

pomemben fazni kot

$$\boxed{\begin{array}{l}\Delta \vec{A} + \underset{k^2}{\omega^2 \mu\varepsilon} \vec{A} = -\mu\vec{J} \\ \Delta V + \underset{k^2}{\omega^2 \mu\varepsilon} V = -\rho/\varepsilon\end{array}}$$

GREENOV SKALARNI IZREK

$V(\vec{r})$ .. iskana funkcija

$U(\vec{r})$ .. ugibanje

$\mathrm{div}(U \cdot \mathrm{grad} V) = \mathrm{grad}\, U \cdot \mathrm{grad} V + U \Delta V$

$\mathrm{div}(V \cdot \mathrm{grad}\, U) = \mathrm{grad}\, V \cdot \mathrm{grad}\, U + V \Delta U$

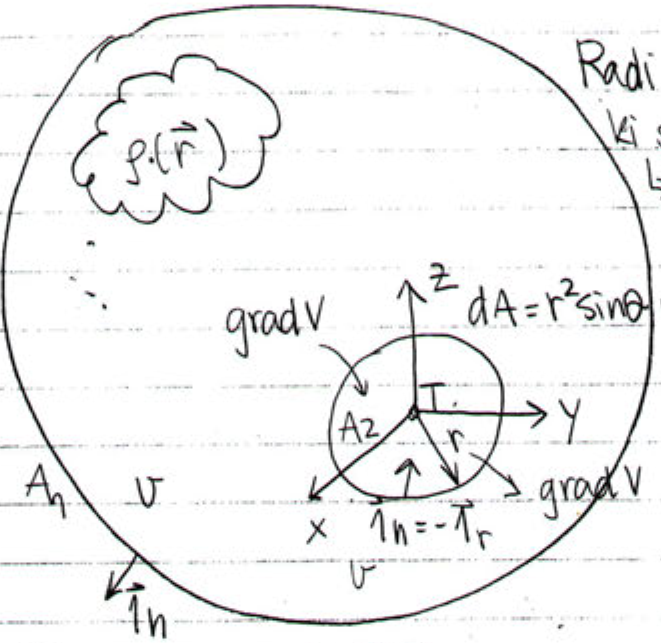
Greenov skalarni izrek:

$\mathrm{div}(U \,\mathrm{grad}\, V - V \,\mathrm{grad}\, U) = U\Delta V \cdot V \Delta U \;/ \int_v dv$

$$\int_v \mathrm{div}(U \,\mathrm{grad} V - V \mathrm{grad}\, U) dv = \oint_A (U \cdot \mathrm{grad}\, V - V \cdot \mathrm{grad}\, U) \cdot \vec{1}_n dA =$$

$$= \oint_A \left(U \frac{\partial V}{\partial n} - V \frac{\partial U}{\partial n}\right) dA = \int_v U \Delta V dv - \int_v V \Delta U dv$$

$$\Delta V = -\rho/\varepsilon - \omega^2 \mu \varepsilon V = -\rho/\varepsilon - k^2 V$$



Radi bi izračunali V v točki T_1 ki se nahaja izhodišču

↳ znajo se pojaviti singularnosti zato jo izločimo s ploskvico A_2

$dA = r^2 \sin\theta \cdot d\Phi \, d\theta$

$A_1 \to \infty$ $A_2 \to 0$

$r \to \infty$ $r \to 0$

$\omega = 0$ → Elektrostatika: $k = 0$

$\Delta V = -\rho/\varepsilon$; $V(\infty) = 0$

ugibam $U = \frac{1}{r}$ → da rešuje $\Delta U = 0$

$$\underline{\underline{\Delta u = \operatorname{div}(\operatorname{grad} u)}}$$

(ϱ, θ, ϕ):

$$\operatorname{grad} u = \vec{1}_r \cdot \frac{-1}{r^2} + \vec{1}_\theta \cdot \frac{1}{r} \cdot 0 + \vec{1}_\phi \frac{1}{\sin\theta} \cdot 0 = \underline{\underline{-\vec{1}_r \cdot \frac{1}{r^2}}} \quad \checkmark$$

$$\Delta u = \operatorname{div}\left(-\vec{1}_r \cdot \frac{1}{r^2}\right) = \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left(r^2 \sin\theta \cdot \frac{-1}{r}\right) = \underline{\underline{0}} \quad //$$

Green:

$$\oint_{A_1+A_2} \left(\overset{r^{-1}}{u} \overset{r^{-2}}{\frac{\partial v}{\partial n}} - \overset{r^{-1}}{v} \overset{r^{-2}}{\frac{\partial u}{\partial n}} \right) \overset{r^2}{dA} = \int_v u \Delta v \, dv - \overbrace{\int_v v \underset{\Delta u = 0}{\underset{\uparrow}{\Delta u}} \, dv}^{0}$$

$A_1 \to \infty$

$$v(\infty) \to 0(r^{-1})\, ; \; \frac{\partial v}{\partial n} \to 0\,(r^{-2})$$

$$\oint_{A_2} \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n} \right) dA = \int_v u \left(-\frac{\varrho}{\varepsilon}\right) dv$$

$$\oint_{\substack{A_2 \\ r \to 0}} \left(\underbrace{\frac{1}{r} \frac{\partial v}{\partial n}}_{\to 0} - v \frac{1}{r^2} \right) r^2 \sin\theta \, d\theta \, d\phi =$$

$$= -\oint_{\substack{A_2 \\ r \to 0}} v \sin\theta \, d\theta \, d\phi = 4\pi v$$

$$-4\pi v = \int_v \frac{1}{r}\left(-\frac{\varrho}{\varepsilon}\right) dv \Rightarrow v(r=0) = \frac{1}{4\pi\varepsilon} \int_v \frac{\varrho}{r} \, dv = \frac{1}{4\pi\varepsilon} \sum_i \frac{Q_i}{r_i}$$

$k = \omega\sqrt{\mu\varepsilon}$ .. valovno število

elektrodinamika: $\omega \neq 0$

$$\Rightarrow u = \frac{1}{r} e^{-jkr}$$

(ϱ, θ, ϕ)

$$\operatorname{grad} u = \vec{1}_r \frac{\partial u}{\partial r} + \vec{1}_\theta \frac{1}{r} \frac{\partial u}{\partial \theta} + \vec{1}_\phi \frac{1}{r \sin\theta} \frac{\partial u}{\partial \phi} =$$

$$= \vec{1}_r \left(-\frac{1}{r^2} e^{-jkr} + \frac{-jk}{r} e^{-jkr} \right) = \underline{\underline{-\vec{1}_r \left(\frac{1}{r^2} + \frac{jk}{r} \right) e^{-jkr}}}$$

$$\underline{\underline{\Delta u}} = \operatorname{div}\left(-\frac{1}{r}\left(\frac{1}{r^2} + \frac{jk}{r} \right) e^{-jkr} \right) = \frac{1}{r \cancel{\sin\theta}} \frac{\partial}{\partial r} \left(r^2 \cancel{\sin\theta} \left(\frac{1}{r^2} + \frac{jk}{r} \right) e^{-jkr} \right) =$$

$$= -\frac{1}{r^2} \frac{\partial}{\partial r} \left((1 + jkr) e^{-jkr} \right) = -\frac{1}{r^2} \left(\cancel{jk} e^{-jkr} + (\cancel{1} + jkr)(-jk) e^{-jkr} \right) =$$

U mora reševati takšno enačbo

$$= -\frac{k^2}{r} e^{-jkr} = \underline{\underline{-k^2 U}} \Rightarrow \Delta U + k^2 U = 0$$

$$\overbrace{\oint_{A_1\to\infty}\left(U\frac{\partial V}{\partial n} - V\frac{\partial U}{\partial n}\right)dA_1}^{0} + \oint_{A_2\to 0}\left(U\frac{\partial V}{\partial n} - V\frac{\partial U}{\partial n}\right)dA_2 =$$

$$= \int_v U\Delta V\,dv - \int_v V\Delta U\,dv = \int_v \left(U\left(-\frac{\rho}{\varepsilon} - k^2 V\right) - \underline{V\left(-k^2 U\right)}\right) dv =$$

$\Delta V = -\rho/\varepsilon - k^2 V$

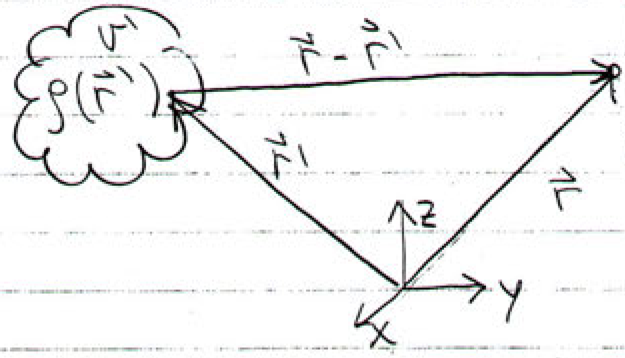
$\Delta U = -k^2 U$

$$= \int_v U\left(-\frac{\rho}{\varepsilon}\right)dv = -\frac{1}{\varepsilon}\int_v \frac{\rho}{r}e^{-jkr}\,dv$$

$$\oint_{\substack{A_2\\ r\to 0}}\Big(\underbrace{\overset{1/r}{U}\frac{\partial V}{\partial n}}_{\to 0} - V\left(-\vec{1}_r\left(\frac{1}{r^2}+\frac{jk}{r}\right)e^{-jkr}\right)\Big)\left(\overset{\text{normala}}{-\vec{1}_r}\right)dA_2 = \underline{4\pi V}$$

$$\Rightarrow \boxed{V(r=0) = -\frac{1}{4\pi\varepsilon}\int_v \frac{\rho(r)}{r}\, e^{-j\overset{k=\omega\sqrt{\mu_0\varepsilon_0}}{k}r}\,dv}$$

enačba za zakasnjeni skalarni potencial



$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon}\int_v \rho(\vec{r})\cdot\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv}$$

↳ enačba za poljubno porazdelitev elektrinida potencial v poljubni točki

vektorska enačba (x,y,z):

⇒ ENAČBA ZA ZAKASNJEN POTENCIAL

$$\Delta\vec{A} = \vec{1}_x\Delta A_x + \vec{1}_y\Delta A_y + \vec{1}_z\Delta A_z$$ (samo za kartezični ks)

$$\Delta\vec{A} + k^2\vec{A} = -\mu\vec{J}$$

$$\Delta A_x + k^2 A_x = -\mu J_x \Rightarrow A_x(\vec{r}) = \frac{\mu}{4\pi}\int_{v'} J_x(\vec{r}')\cdot\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv'$$

$$\Delta A_y + k^2 A_y = -\mu J_y \Rightarrow A_y(\vec{r}) = \frac{\mu}{4\pi}\int_{v'} J_y(\vec{r}')\,\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\,dv'$$

$$\Delta A_z + k^2 A_z = -\mu J_z \Rightarrow A_z(\vec{r}) = \int_{v'} J_z(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dv'$$

$$\Rightarrow \boxed{\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \int_{v'} \vec{J}(\vec{r}') \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} dv'}$$ enacba za zakasnjeni vektorski potencial

↳ dokaz, velja le za kartezicne koordinate

ponovitev:

Maxwellove enačbe v diferencialni obliki

① $\text{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$

② $\text{rot}\,\vec{E} = -j\omega\mu\vec{H}$

③ $\text{div}(\varepsilon\vec{E}) = \rho$

izvor: $\rho, \vec{J}$ => polja $\vec{E}, \vec{H}$

Neposredna pot?

računamo preko potencialov $V, \vec{A}$

$\vec{B} = \text{rot}\,\vec{A} \Rightarrow \vec{H} = \frac{1}{\mu}\text{rot}\,\vec{A}$

$\underline{\vec{E} = -j\omega\vec{A} - \text{grad}\,V}$

Lorentzova izbira = $\text{div}\,\vec{A}$

$$\left.\begin{aligned} \Delta V + \omega^2\mu\varepsilon V &= -\rho/\varepsilon \\ \underline{\Delta\vec{A} + \omega^2\mu\varepsilon\vec{A}} &= -\mu\vec{J} \end{aligned}\right\}$$ analitske rešitve

koordinate izvora

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon}\int_{v'} \rho(\vec{r}\,') \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\,dv'$$ skalarni potencial

koordinate veličine, ki jo računamo

razdalja od izvora do točke, kjer gledamo

$\vec{r}\,'$ ... koordinate izvora $\qquad k^2 = \omega^2\mu\varepsilon$

$\vec{r}$ .. koordinate potenciala $\qquad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

$$\vec{A} = \frac{\mu}{4\pi}\int_{v'} \vec{J}(\vec{r}\,') \frac{e^{-jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|}\,dv'$$ vektorski potencial

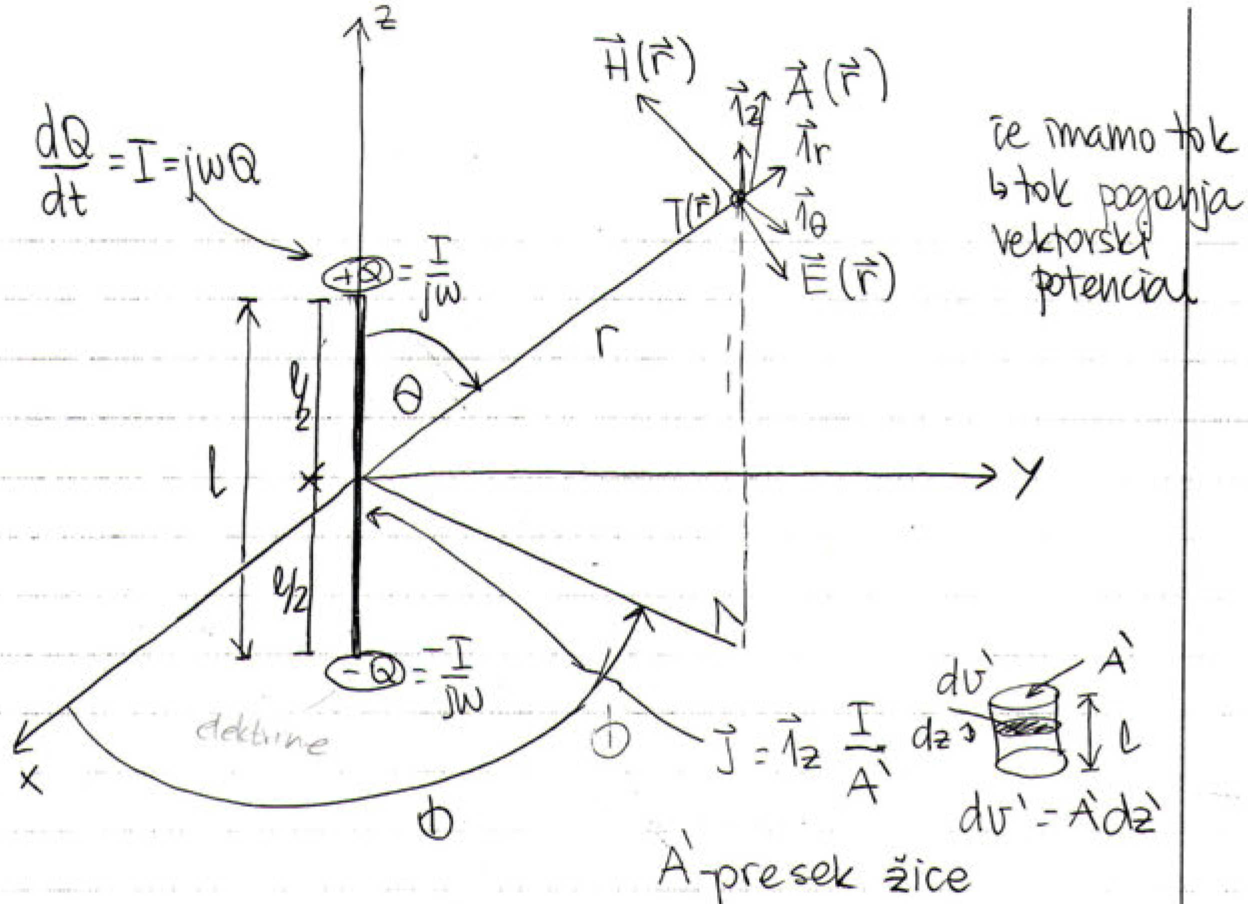
-zgled: (r, θ, ϕ)

$h_r = 1$

$h_\theta = r$

$h_\phi = r\sin\theta$

$$\vec{A}(r) = \frac{\mu}{4\pi}\int_{v'} \vec{1}_z \frac{I}{A} \frac{e^{jk|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} A'\,dz'$$



poenostavitve:

1. $r >> l \Rightarrow \frac{1}{|\vec r - \vec r'|} \approx \frac{1}{r}$

2. $k^2 = \omega^2 \mu \varepsilon$; $k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ ... valovno število

$\frac{1}{k} >> l \Rightarrow e^{-jk|\vec r - \vec r'|} \approx e^{-jkr}$ (majhna fazna napaka)

$\frac{\lambda}{2\pi} >> l$

$$\vec A(\vec r) = \frac{\mu}{2\pi} I \int_{-l/2}^{l/2} \vec 1_z \cdot \frac{e^{-jkr}}{r} dz' = \vec 1_z \frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r}$$

smernik $\vec 1_z$ ni smernik krogelnega koord. sistema

$$\left.\begin{array}{l} \vec 1_z \cdot \vec 1_r = \cos\theta \\ \vec 1_z \cdot \vec 1_\theta = -\sin\theta \\ \vec 1_z \cdot \vec 1_\phi = 0 \end{array}\right] \Rightarrow \vec 1_z = \vec 1_r \cos\theta - \vec 1_\theta \sin\theta$$

$$\vec A(\vec r) = (\vec 1_r \cos\theta - \vec 1_\theta \sin\theta) \frac{\mu I l}{4\pi} \frac{e^{-jkr}}{r}$$

vektorski potencial ~~žice~~ TOKOVE DALJICE v krogelnem k.s.

$$\mathrm{rot}\,\vec A = \frac{1}{h_1 h_2 h_3} \left| \begin{array}{ccc} \vec 1_{q_1} h_1 & \vec 1_{q_2} h_2 & \vec 1_{q_3} h_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{array} \right|$$

$$\vec{H}=\frac{1}{\mu}\,\mathrm{rot}\,\vec{A}=\frac{-1}{\mu}\,\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & r\vec{1}_\theta & r\sin\theta\vec{1}_\phi\\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\\ \frac{\mu Il}{4\pi}\frac{\bar{e}^{jkr}}{r}\cos\theta, & \cancel{r}\frac{\mu Il}{4\pi}\frac{\bar{e}^{jkr}}{\cancel{r}}(-\sin\theta), & r\sin\theta\end{vmatrix}$$

$$=\frac{1}{\cancel{\mu}}\,\frac{1}{r^{\cancel{2}}\cancel{\sin\theta}}\,\frac{\cancel{\mu}Il}{4\pi}\left[\vec{1}_r(0)+r\cdot\vec{1}_\theta\cdot 0+\vec{1}_\phi\,\cancel{r}\cdot\cancel{\sin\theta}\left(\sin\theta\cdot jk\,e^{-jkr}+\sin\theta\,\frac{\bar{e}^{jkr}}{r}\right)\right]$$

sevanje !

$$\boxed{\vec{H}=\vec{1}_\phi\,\frac{Il}{4\pi}\,\bar{e}^{jkr}\left(\left(\frac{jk}{r}\right)+\left(\frac{1}{r^2}\right)\right)\sin\theta}$$

Mag. polje tokove daljice

↳ Biot-Savartov zakon

(OET2: $\omega=0$: Biot-Savart)
↳ $\bar{e}^{jkr}=1$

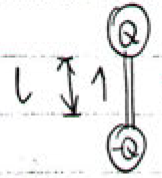
$$\vec{E}=-j\omega\vec{A}-\mathrm{grad}\,V\overset{ME①}{=}\frac{1}{j\omega\varepsilon}\left(\mathrm{rot}\,\vec{H}-\vec{J}\right)$$

↙ $\vec{J}=0$

↑ „V" poganja tockasti dipol Ql.

$$\vec{E}=\frac{1}{j\omega\varepsilon}\,\frac{1}{r^2\sin\theta}\begin{vmatrix}\vec{1}_r & \vec{1}_\theta\cdot r & \vec{1}_\phi\cdot r\cdot\sin\theta\\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi\\ 0 & r\cdot 0 & r\sin\theta\cdot\frac{Il}{4\pi}\bar{e}^{jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\end{vmatrix}=$$

$$=\frac{1}{j\omega\varepsilon}\,\frac{1}{r^2\sin\theta}\,\frac{Il}{4\pi}\Big[\vec{1}_r\bar{e}^{jkr}\left(jk+\frac{1}{r}\right)2\sin\theta\cos\theta+\vec{1}_\theta\cdot r\left(jk\,\bar{e}^{jkr}\left(jk+\frac{1}{r}\right)\right)+\bar{e}^{jkr}\cdot\frac{1}{r^2}\sin^2\theta\Big]$$



$I=j\omega Q$; $Q=\frac{I}{j\omega}$

$$\boxed{\vec{E}=\frac{Ql}{4\pi\varepsilon\cdot j\omega}\,\bar{e}^{jkr}\left[\vec{1}_r\left(\frac{jk}{r^2}+\frac{1}{r^3}\right)2\cos\theta+\vec{1}_\theta\left(-\frac{k^2}{r}+\frac{jk}{r^2}+\frac{1}{r^3}\right)\sin\theta\right]}$$

sevanje na velikih razdaljah prevlada, ne izgine, le počasi upada

$$\boxed{\vec{E} = \frac{Ql}{4\pi\varepsilon} \underset{\uparrow \text{ zakasnitev}}{e^{-jkr}} \left[\vec{1}_r \left(\underset{\circ\circ\circ}{\frac{2jk}{r^2}} + \underset{=}{\frac{2}{r^3}} \right) \cos\Theta + \vec{1}_\Theta \left(\underset{\approx}{-\frac{k^2}{r}} + \underset{\circ\circ\circ}{\frac{jk}{r^2}} + \underset{=}{\frac{1}{r^3}} \right) \sin\Theta \right]} \quad \nabla_{o}$$

točkasti elektrostatični dipol

SEVANJE ∇_{o}

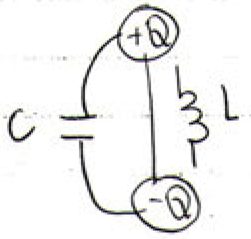
	energetika f = 50 Hz	GSM telefon f = 900 MHz	vidna svetloba f = 600 THz ($\lambda = 0{,}5\ \mu m$)
$k = \frac{\omega}{c} = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c}$ $k = \left\lvert\frac{k}{r}\right\rvert$ @ r = 1m	$1{,}05 \cdot 10^{-6}\ \frac{rad}{m}$	$18{,}9\ \frac{rad}{m}$	$12{,}5 \cdot 10^{6}\ \frac{rad}{m}$
$\left\lvert\frac{k^2}{r}\right\rvert$ @ r = 1m	$1{,}1 \cdot 10^{-12}\ \frac{rad^2}{m^3}$	$360\ \frac{rad^2}{m^3}$	$157 \cdot 10^{12}\ \frac{rad^2}{m^3}$
r = 1m	samo statični členi = 10^6 sevanje	sevanje = = 360 x statika	sevanje = = 10^{14} x statika
r = ? enako veliki členi $k = 1/r$	samo statika 950 km	5,3m ↗ statika v apu ↘ dinamika	80nm => samo sevanje

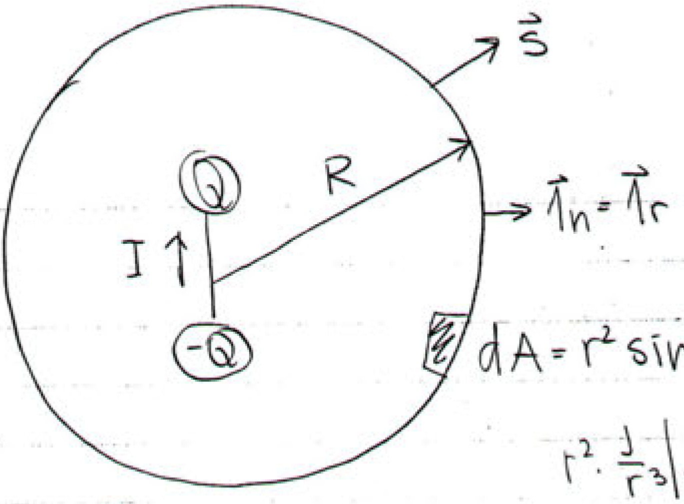
Poyntingov vektor:

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2} \frac{Il}{4\pi j\omega\varepsilon} e^{-jkr} \left[\vec{1}_r \left(\frac{2jk}{r^2} + \frac{2}{r^3} \right) \cos\Theta + \right.$$

$$\left. + \vec{1}_\Theta \left(-\frac{k^2}{r} + \frac{jk}{r^2} + \frac{1}{r^3} \right) \sin\Theta \right] \times \vec{1}_\phi \frac{I^* l}{4\pi} \overset{*}{e^{+jkr}} \left(-\frac{jk}{r} + \frac{1}{r^2} \right) \sin\Theta$$

$$\vec{S} = \frac{|I|^2 l^2}{32\pi^2 \omega\varepsilon} \left[\vec{1}_r \left(\underbrace{\frac{k^3}{r^2}}_{\text{delovna moč}} - \underbrace{\frac{j}{r^5}}_{\text{jalovo}} \right) \sin^2\Theta + \vec{1}_\Theta \underbrace{\left(\frac{2jk^2}{r^3} + \frac{2j}{r^5} \right)}_{\text{jalovo}} \cos\Theta \sin\Theta \right]$$





$dA = r^2 \sin\theta \, d\theta \, d\phi$

Koliko je delovne moči?

$r^2 \cdot \frac{j}{r^3}\Big|_{R\to\infty} \to 0$

$$P = \int_{4\pi, R\to\infty} \vec{S}\cdot\vec{1}_n \cdot dA = \int_0^{\pi}\int_{0, R\to\infty}^{2\pi} \frac{|I|^2 l^2}{32\pi^2 \omega\varepsilon}\left(\frac{k^3}{r^2} - \frac{j}{r^5}\right)\sin^2\theta \, r^2 \sin\theta \, d\theta \, d\phi$$

$$P = \frac{|I|^2 l^2}{32\pi^2 \omega\varepsilon} 2\pi k^3 \int_0^{\pi} \sin^2\theta \, d\theta$$

nova sprem. $u = \cos\theta$
$du = -\sin\theta \, d\theta$
$1-u^2 = \sin^2\theta$

$$= \frac{|I|^2 l^2}{16\pi\omega\varepsilon} k^3 \underbrace{\int_{-1}^{1}(1-u^2)du}_{2-\frac{2}{3}=\frac{4}{3}}$$

$$\boxed{P = \frac{|I|^2 l^2}{12\pi\omega\varepsilon}k^3 = \frac{Z|I|^2 l^2}{12\pi}k^2} = \frac{1}{2}|I|^2 R_s \Longrightarrow R_s = \frac{Z l^2}{6\pi}k^2 = $$

$$= \frac{2\pi Z}{3}\left(\frac{l^2}{\lambda^2}\right)$$

sevalna upornost

$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega = 120\pi\Omega$

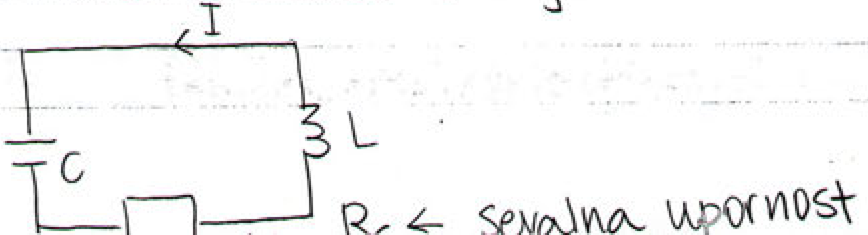
predpostavka: $l \ll \lambda$

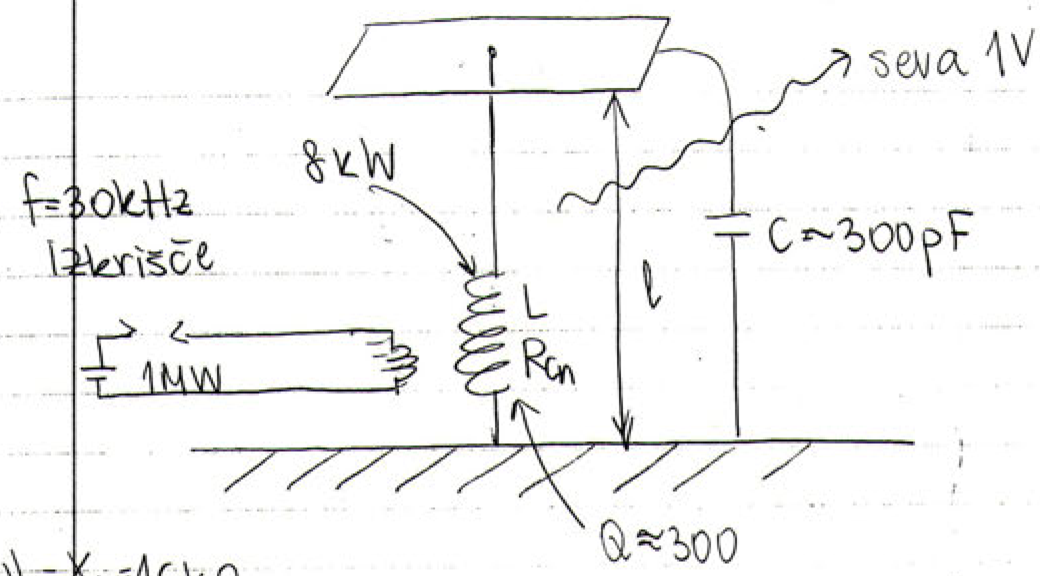
$\omega\varepsilon = \omega\sqrt{\mu\varepsilon}\underbrace{\sqrt{\frac{\varepsilon}{\mu}}}_{1/Z} = \frac{k}{Z}$

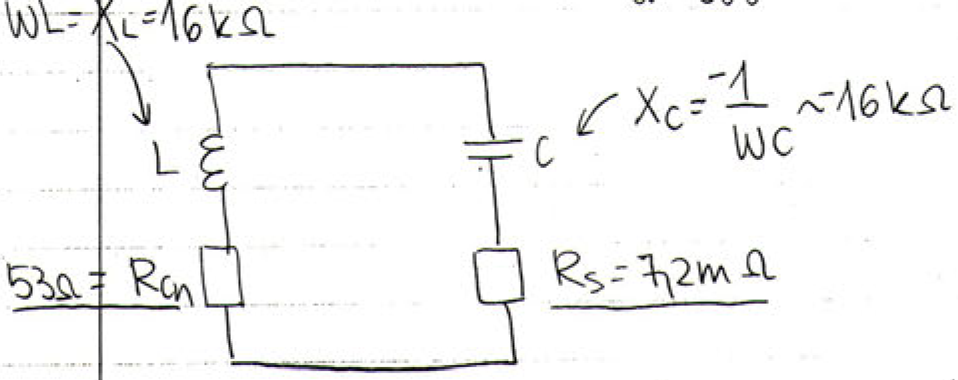
$\sqrt{\frac{\mu}{\varepsilon}}$ .. valovna impedanca

$k = \frac{2\pi}{\lambda}$ $k = \frac{\omega\varepsilon}{Z}$

- pravilno nadomestno vezje:







$$X_C = -\frac{1}{\omega C} = -\frac{1}{3\cdot 10^4 \cdot 2\pi \frac{rad}{s} \cdot 3\cdot 10^{-10} As} = -16\,k\Omega$$

$$Q = \frac{\omega L}{R_{cu}} \Rightarrow R_{cu} = \frac{\omega L}{Q} = \frac{X_L}{Q} = \frac{16\,k\Omega}{300} = 53\,\Omega$$

$$\lambda = \frac{c_0}{f} = \frac{3\cdot 10^8\,m/s}{3\cdot 10^4\,Hz} = \underline{\underline{10\,km}} >> l = 30\,m$$

$$R_s = \frac{2\pi Z_0}{3}\left(\frac{l}{\lambda}\right)^2 = \frac{2\pi\cdot 120\pi\Omega}{3}\left(\frac{30}{10^4}\right)^2 = 800\Omega\,\frac{900}{10^8} = 7{,}2\,m\Omega$$

izkoristek sevanja:

$$\eta = \frac{R_s}{R_{cu}+R_s} = \frac{7{,}2\cdot 10^{-3}\,\Omega}{53\,\Omega} = \underline{\underline{1{,}3\cdot 10^{-4} = 0{,}013\%}}$$

$$\boxed{\eta = \frac{R_s}{R_s+R_{cu}}}$$

ε, μ

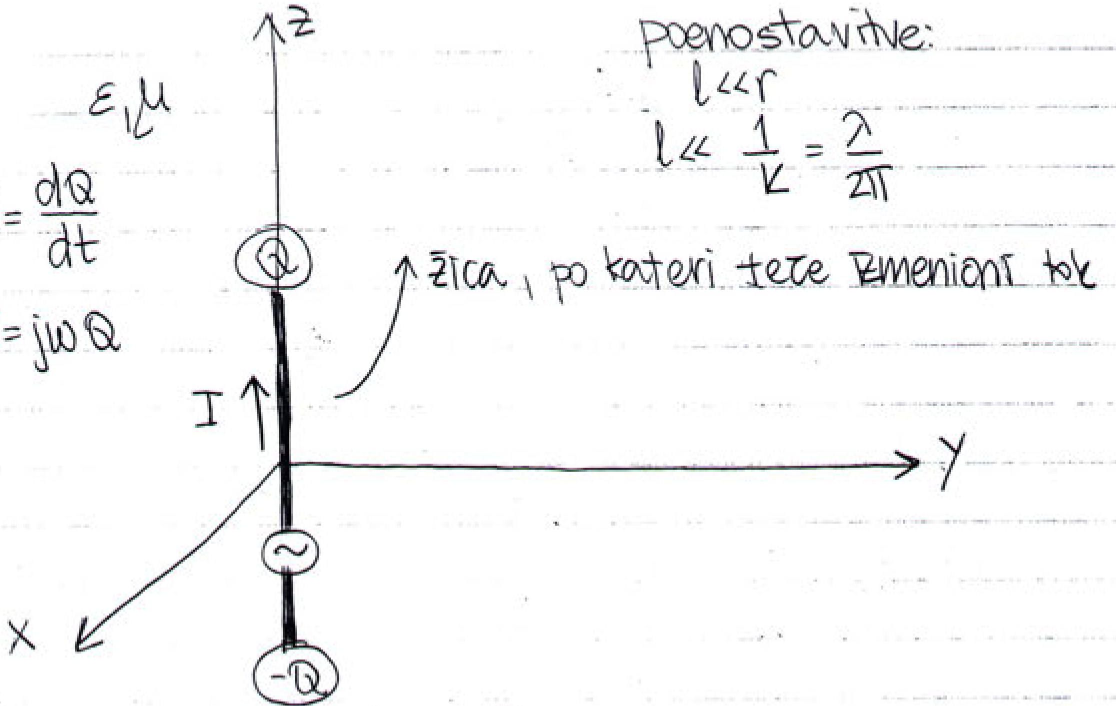
$I = \frac{dQ}{dt}$

$I = j\omega Q$

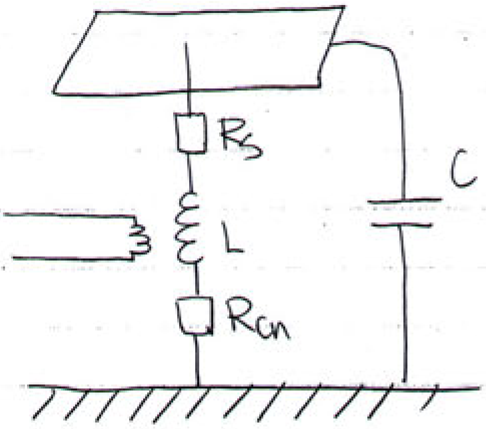
poenostavitve:

$l << r$

$l << \frac{1}{k} = \frac{\lambda}{2\pi}$



Teslov transformator: Primer žice



R_s .. sevalna upornost (ni upoštevano sevanje)

(r, θ, ϕ)

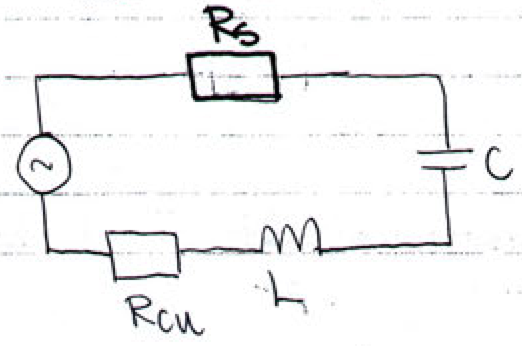
pretok moči, ki potuje v neskončnost od naše naprave

→ sevanje

$$\vec{H} = \vec{1}_\phi \frac{Il}{4\pi} e^{-jkr}\left(\boxed{\frac{jk}{r}} + \frac{1}{r^2}\right)\sin\theta$$

→ sevanje

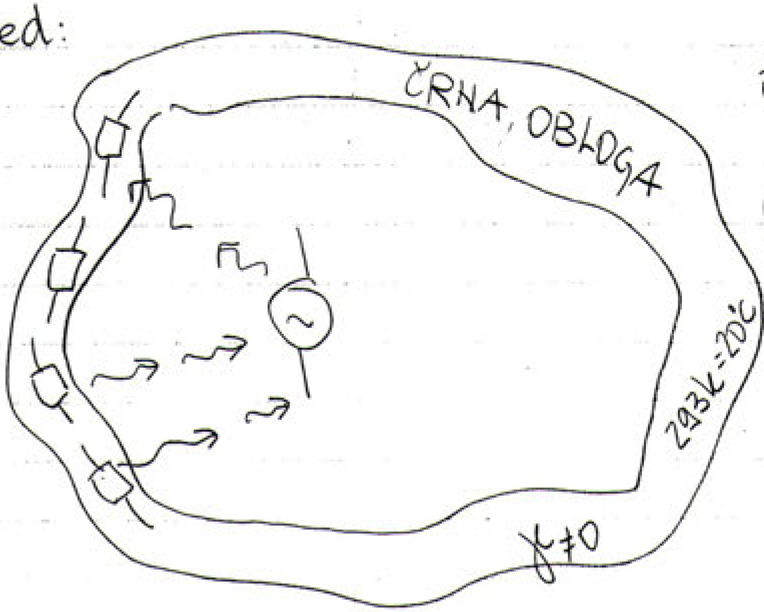
$$\vec{E} = \frac{Ql}{4\pi\varepsilon} e^{-jkr}\left[\vec{1}_r\left(\frac{2jk}{r^2} + \frac{2}{r}\right)\cos\theta + \vec{1}_\theta\left(\boxed{-\frac{k^2}{r}} + \frac{jk}{r^2} + \frac{1}{r^3}\right)\sin\theta\right]$$



$R_s = \frac{2\pi Z}{3}\left(\frac{l}{\lambda}\right)^2$ majhna

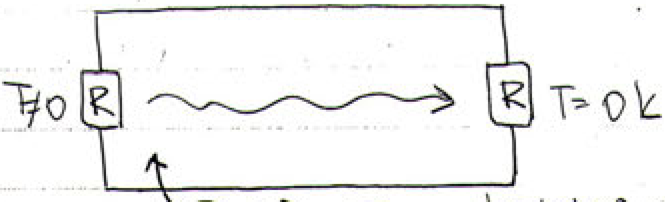
$Z = \sqrt{\frac{\mu}{\varepsilon}}$

- Zgled:



črna obloga
vpije sevanje
(da imamo pretvorbo
el. energije v toploto
↳ upori, ki so preko
sevalnih členov
sklopljeni na napravo)

prazen prostor: ε_0, μ_0 } ⇒ $T = 2{,}7\ K$
$\gamma = 0$



$P = \Delta f\, k_B T$ toplotno gibanje elektronov →
↳ izmenična napetost

$r \ll \frac{1}{k}$ ⇒ statika (OET)
$r \sim \frac{1}{k}$ ⇒ komplicirane enačbe
$r \gg \frac{1}{k}$ ⇒ sevanje

!

---

velikost ni odvisna
↑ od razdalje

velike razdalje
(r, θ, ϕ)

$$\frac{\partial}{\partial r}\left(e^{-jkr}\right) = -jk\, e^{-jkr}$$

odvod faze je velik ! vsi drugi majhni

Vsi ostali odvodi so odvisni od razdalje

$$\frac{\partial}{\partial r} \Longrightarrow -jk$$

$$\frac{1}{r}\frac{\partial}{\partial \theta} \Longrightarrow 0$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \Longrightarrow 0$$

$$r \gg \frac{1}{k}$$

$$\mathrm{rot}\vec{A} = \frac{1}{h_1h_2h_3}\begin{vmatrix} \vec{1}_{g_1}h_1 & \vec{1}_{g_2}h_2 & \vec{1}_{g_3}h_3 \\ \partial/\partial g_1 & \partial/\partial g_2 & \partial/\partial g_3 \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix} \overset{\downarrow}{\approx}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & \vec{1}_\theta \cdot r & r\sin\theta\,\vec{1}_\phi \\ -jk & 0 & 0 \\ \underset{\downarrow\atop 0}{A_r} & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

0 , ker ne bo nikjer razlicen od 0

$$\vec{A} = (\vec{1}_r\cos\theta - \vec{1}_\theta\sin\theta)\,\frac{\mu I l}{4\pi}\cdot\frac{1}{r}$$ priblizek nam da samo sevano polje

$$\boxed{\vec{H} = \frac{1}{\mu}\mathrm{rot}\vec{A} = \vec{1}_\phi\, jk\,\frac{Il}{4\pi}\,\frac{\sin\theta}{r}}$$ magnetno sevano polje ▽ o

↳ samo sevanje

$$\vec{E} = -j\omega\vec{A} - \mathrm{grad}V = -j\omega\vec{A} - \vec{1}_r\frac{\partial V}{\partial r} \approx -j\omega(\vec{A} - \vec{1}_r(\vec{1}_r\cdot\vec{A}))$$ sevanje

$$\text{če } r \gg \frac{1}{k} : \rho = 0 = \mathrm{div}\,\varepsilon\vec{E} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta\,\varepsilon\,\underbrace{E_r}_{=0})\right]$$

$$\boxed{\vec{E} = \vec{1}_\theta\, j\omega\,\frac{\mu I l}{4\pi}\,\frac{\sin\theta}{r}}$$ električno sevano polje ▽ o

pri sevanem el. in mag. polju nimamo radialnih komponent

$$j\omega\mu = j\omega\sqrt{\mu\varepsilon}\,\frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon} = j\,\frac{k^2}{\omega\varepsilon}$$

$I = j\omega Q$ ⟿ pride $-\frac{k^2}{r}$ sevanje

sevanje: $\left.\begin{matrix}\vec{E} \perp \vec{1}_r \\ \vec{H} \perp \vec{1}_r \\ \vec{E} \perp \vec{H}\end{matrix}\right\}$ Prečno valovanje   transferzalno elektromagnetno valovanje TEM ▽ o

Maxwellove enačbe:

② $\text{rot}\,\vec{E} = -j\omega\mu\vec{H}$

$$\Rightarrow \vec{H} = \frac{j}{\omega\mu}\,\text{rot}\,\vec{E} = \frac{j}{\mu\omega}(-jk)\,\vec{1}_r \times \vec{E} = \frac{k}{\mu\omega}\,\vec{1}_r \times \vec{E} =$$

$$= \frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu}\,\vec{1}_r \times \vec{E} = \sqrt{\frac{\varepsilon}{\mu}}\,\vec{1}_r \times \vec{E} = \frac{1}{Z}\,\vec{1}_r \times \vec{E}$$

① $\text{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$

$$\Rightarrow \vec{E} = \frac{1}{j\omega\varepsilon}\,\text{rot}\,\vec{H} \approx \frac{1}{j\omega\varepsilon}(-jk)\,\vec{1}_r \times \vec{H} = -\sqrt{\frac{\mu}{\varepsilon}}\,\vec{1}_r \times \vec{H} =$$

$$= -Z \cdot \vec{1}_r \times \vec{H}$$

SEVANJE!

sofazna (ni faznega zamika)

$$\text{rot}\,\vec{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi \\ -jk & 0 & 0 \\ A_r & r \cdot A_\theta & r\sin\theta A_\phi \end{vmatrix} \approx \vec{1}_r(-jk) \times \vec{A}$$

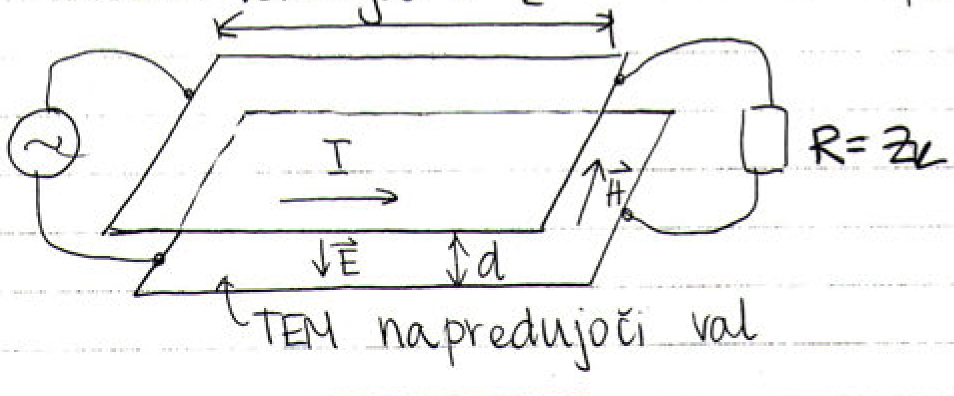
$\vec{1}_r(-jk)$

$$\frac{|\vec{E}|}{|\vec{H}|} = Z = \sqrt{\frac{\mu}{\varepsilon}}$$

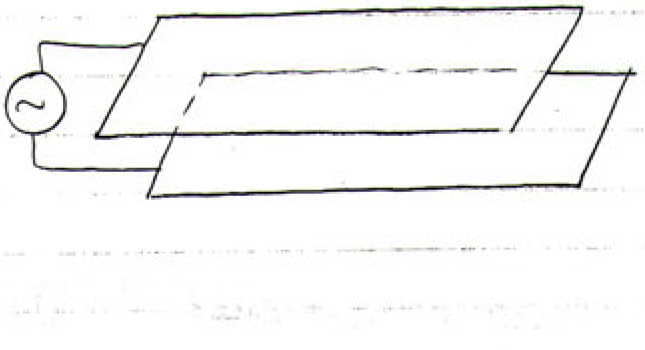
$\vec{E}$ in $\vec{H}$ sta sofazna! Ko ima en max, ima tudi drugi max. Ni faznega zamika

↳ velikost vektorja $l \gtrsim \frac{1}{k}$

$d, w \ll \frac{1}{k}$



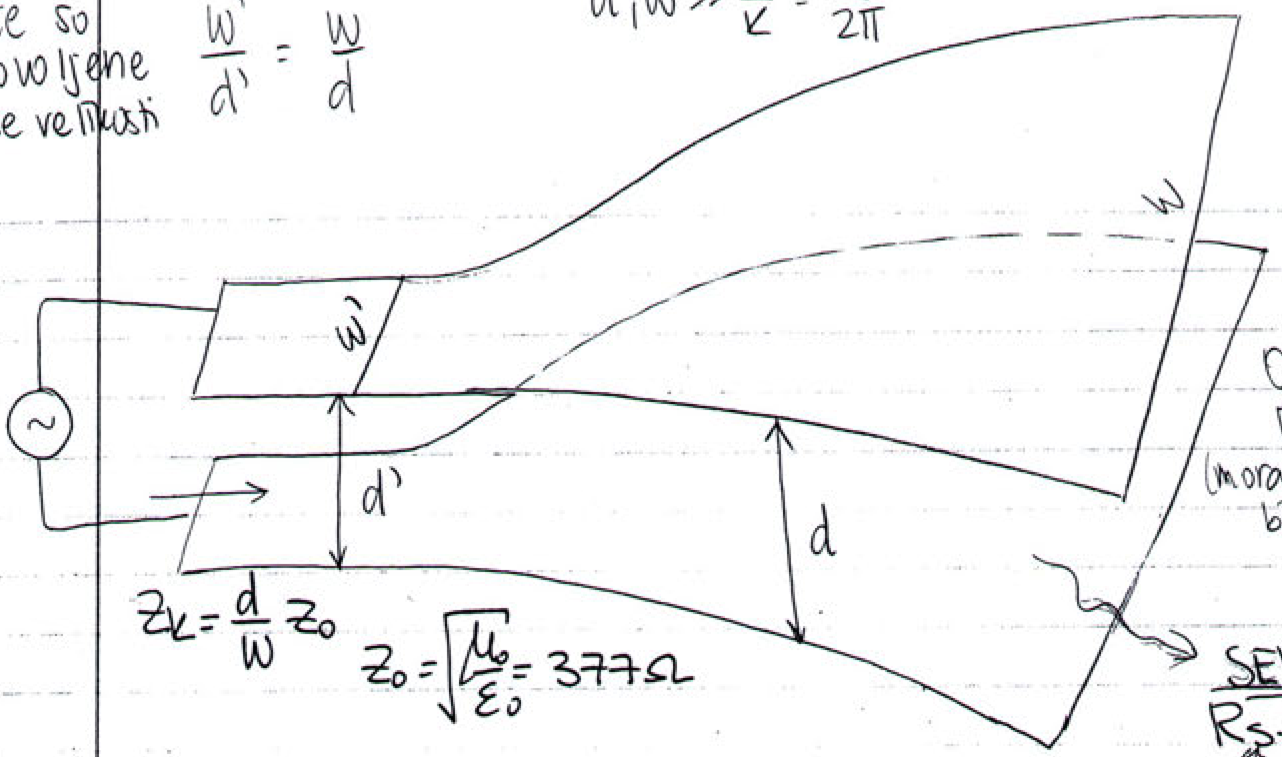
TEM napredujoči val



0.S → $\Gamma = 1$

če so dovoljene vse velikosti $\frac{w'}{d'} = \frac{w}{d}$

$d, w >> \frac{1}{k} = \frac{\lambda}{2\pi}$



$Z_L = \frac{d}{w} Z_0$

$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$

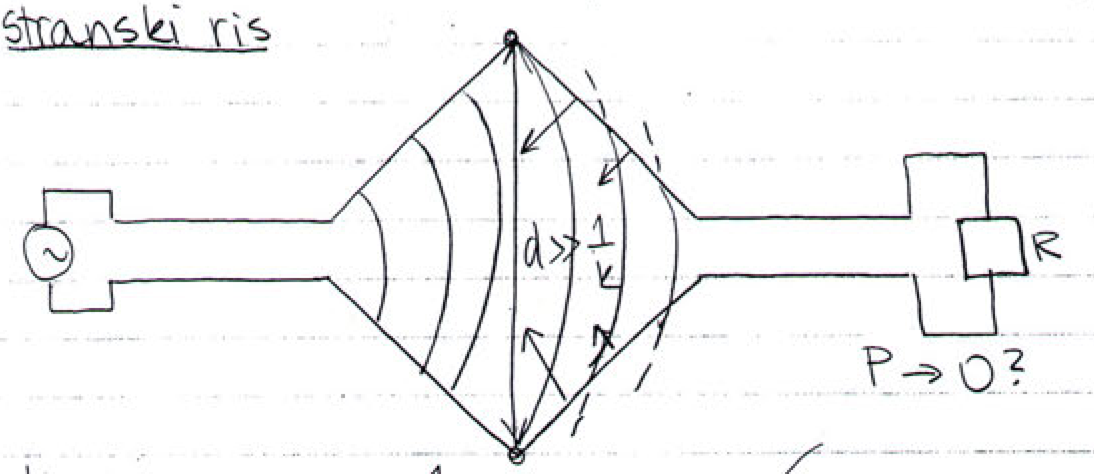
O.S. $\Gamma \to 0$?
(moralo bi biti $\Gamma = 1$)

SEVANJE?

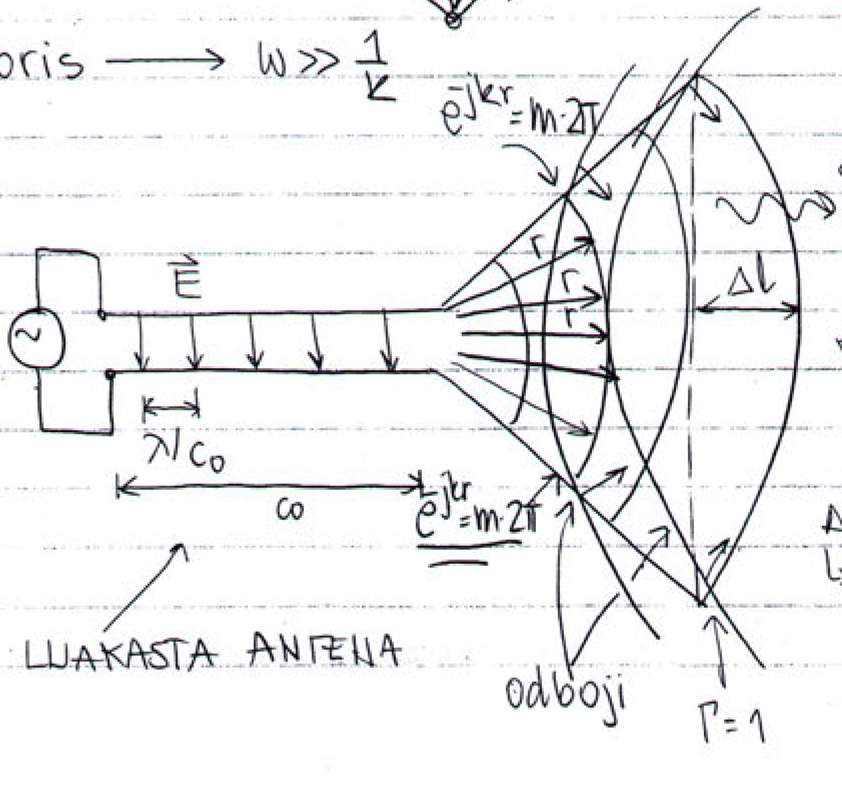
$R_S = Z_L$?

moč se odbije preden pride do bremena! pogoj: $\Delta l > \frac{\lambda}{2}$

stranski ris



tloris ⟶ $w >> \frac{1}{k}$



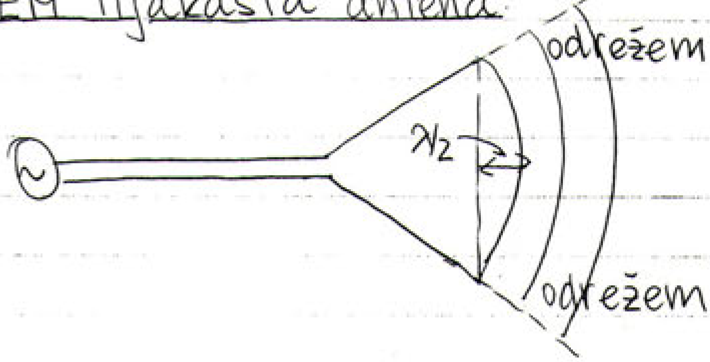
TEM LIJAKASTA ANTENA

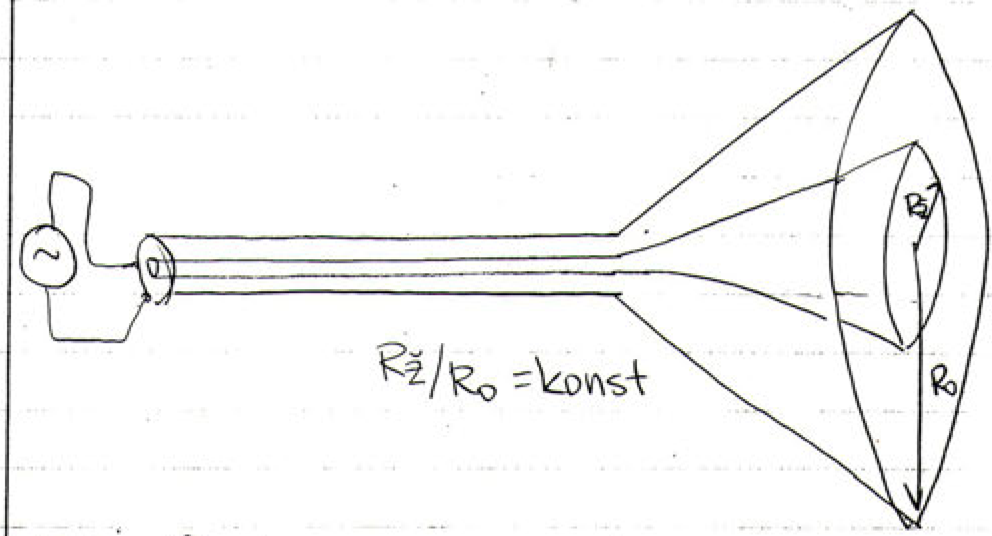
SEVANJE?
se odbije, ampak ne more več noter.

? ODPRTE SPONKE ?
V 3D NE OBSTAJAJO ?

$\Delta l > \frac{\lambda}{2} \Rightarrow R_S = Z_L$
↳ takrat se skoraj nič ne odbije

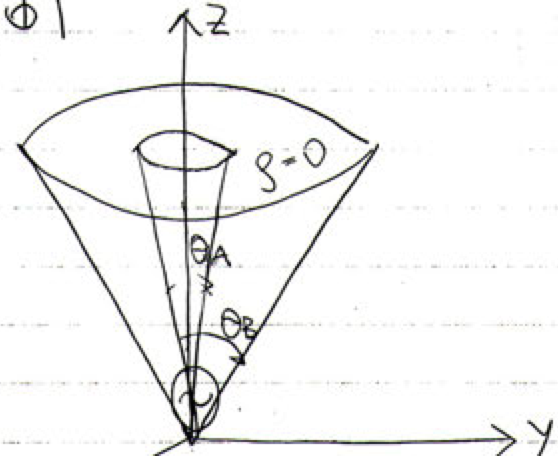
TEM lijakasta antena:





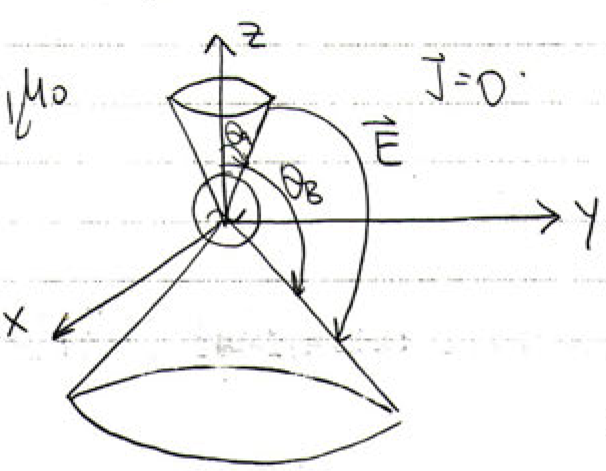
$R_{\check{z}}/R_0 = \text{konst}$

(r, θ, ϕ)



ε_0, μ_0

$\vec{J} = 0$



rešitev Maxwelovih enačb

ugibam: $\vec{E} = \vec{1}_\theta \frac{c}{r\sin\theta} e^{-jkr}$

① $\operatorname{div} \varepsilon\vec{E} = \rho = 0?$ elektrine ni!

$$\operatorname{div} \varepsilon\vec{E} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}\left(r\cancel{\sin\theta}\,\varepsilon \frac{c}{r\cancel{\sin\theta}} e^{-jkr}\right)\cdot\right] = 0 \Rightarrow \rho = 0$$

② $$\vec{H} = \frac{j}{\mu\omega} \operatorname{rot}\vec{E} = \frac{j}{\omega\mu}\frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\vec{1}_\theta & r\sin\theta\cdot\vec{1}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & \frac{c}{\sin\theta}e^{-jkr} & r\sin\theta\cdot 0 \end{vmatrix} =$$

$$= -\vec{1}_\phi \frac{k}{\omega\mu}\frac{c\cdot e^{-jkr}}{r\sin\theta}$$

$$\vec{H} = \vec{1}_\phi \frac{c}{Z_0}\frac{e^{-jkr}}{r\sin\theta}$$

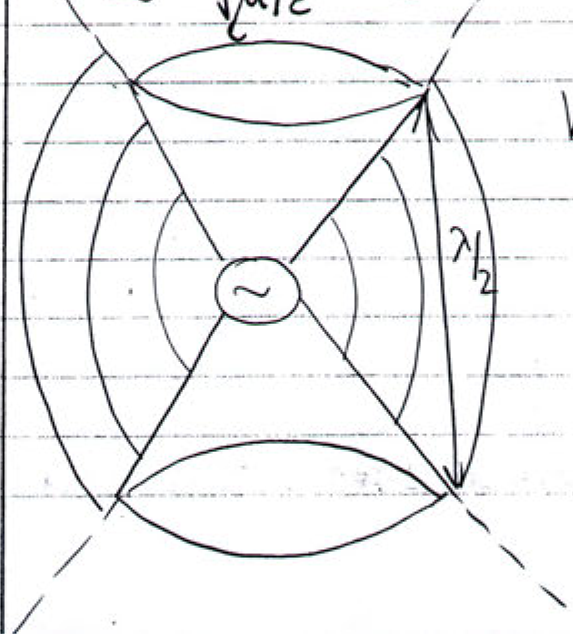
$\vec{E}$ in $\vec{H}$ sta v razmerju valovne impedance ✓

preverimo:

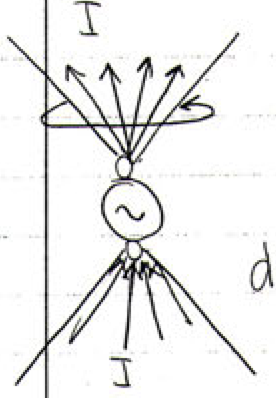
① $$\operatorname{rot}\vec{H} = \vec{J} + j\omega\varepsilon\vec{E} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{1}_r & r\cdot\vec{1}_\theta & r\sin\theta\,\vec{1}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & r\cdot 0 & r\sin\theta\frac{e^{-jkr}}{r\sin\theta}\cdot\frac{c}{Z_0} \end{vmatrix} - j\omega\varepsilon\vec{E}$$

$$= \vec{1}_\theta\cdot jk\frac{c}{Z_0}\frac{e^{-jkr}}{r\sin\theta} - j\omega\varepsilon\vec{1}_\theta c\frac{e^{-jkr}}{r\sin\theta} = \underline{\underline{0}}\ ✓ \Rightarrow \vec{J} = 0$$

$$jk/Z_0 = \frac{j\omega\sqrt{\mu\varepsilon}}{\sqrt{\mu/\varepsilon}} = j\omega\varepsilon$$



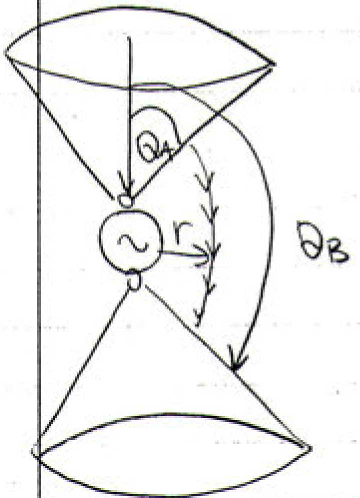
kakšno impedanco vidi generator?



$$I = \oint \vec{H}\cdot d\vec{s} = \int_0^{2\pi} \vec{1}_\phi \frac{C}{Z_0}\frac{e^{-jkr}}{r\sin\theta}\vec{1}_\phi r\sin\theta\, d\phi =$$

$$= \underline{\underline{\frac{2\pi C}{Z_0} e^{-jkr}}} \quad \text{gornji stožec}$$

$d\vec{s} = \vec{1}_\phi r\sin\theta\, d\phi$



$$U = \int_{\theta_A}^{\theta_B} \vec{1}_\theta \frac{C e^{-jkr}}{r\sin\theta}\vec{1}_\theta r\, d\theta =$$

$$= C e^{-jkr}\int_{\theta_A}^{\theta_B}\frac{d\theta}{\sin\theta} = \underline{\underline{C\cdot e^{-jkr}\ln\left(\tan\frac{\theta}{2}\right)\Big|_{\theta_A}^{\theta_B}}}$$

$ds = \vec{1}_\theta \cdot r\, d\theta$

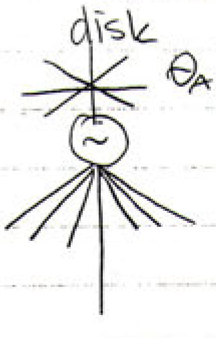
$$\int\frac{d\theta}{\sin\theta} = \ln\left(\tan\frac{\theta}{2}\right) + C'$$

<u>vir</u> <u>pri</u> <u>r=0</u>: $Z_S = \frac{U}{I} = \frac{\cancel{C}\,\cancel{e^{-jkr}}\ln(\tan\theta/2)\Big|_{\theta_A}^{\theta_B}}{\frac{2\pi \cancel{C}}{Z_0}\cancel{e^{-jkr}}}$

sevalna impedanca:

$$Z_S = R_S = \frac{Z_0}{2\pi}\ln\left(\frac{\tan\theta_B/2}{\tan\theta_A/2}\right)$$

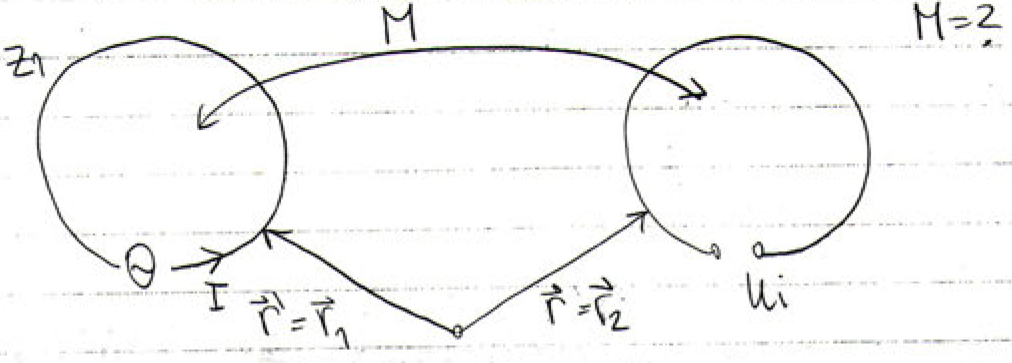
DISCONE



$\theta_A = \frac{\pi}{2}$

$$R_S = 60\Omega \ln\frac{\tan(\theta_B/2)}{\tan(\pi/4)}$$ ↳ $\theta_A = \pi/2$

Primer:



$$\vec{A} = \frac{\mu}{2\pi}\oint_{z_1} I \frac{e^{-jk|\vec r - \vec r'|}}{|\vec r - \vec r'|} d\vec{s}_1 \quad ; \quad \Phi = \int_{A_2 z_2} \vec{B}\cdot\vec{1}_n dA$$

$$\vec{B} = \text{rot}\,\vec{A} \qquad M = \frac{\Phi}{I}$$

$$\Phi = \int_{A_2 z_2} \text{rot}\,\vec{A}\cdot\vec{1}_n\cdot dA \overset{\text{stokes}}{=} \oint_{z_2} \vec{A}\cdot d\vec{s}_2$$

$$M = \frac{\mu}{4\pi}\oint_{z_1}\oint_{z_2} \frac{e^{-jk|\vec r_2 - \vec r_1|}}{|\vec r_2 - \vec r_1|} d\vec{s}_1 \cdot d\vec{s}_2$$

ni vektorskih produktov
↳ ni važno ali je levi ali desni koordinatni sistem

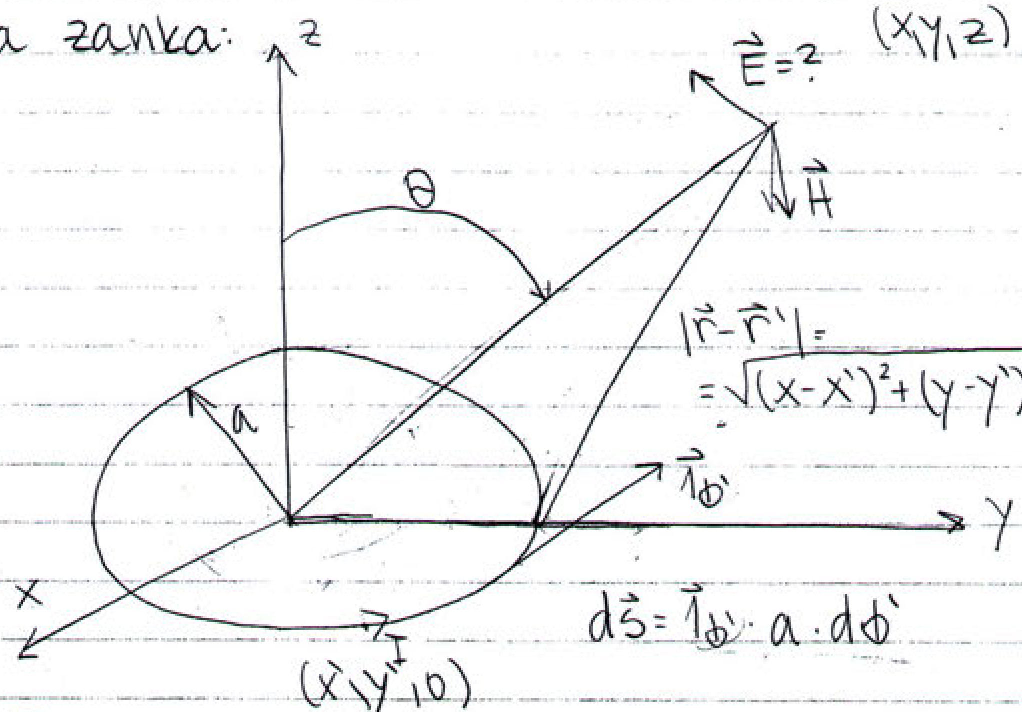
M definiran $w=0$; $jk=0$ v stacionarnem

$$M = \frac{\mu}{4\pi}\oint_{z_1}\oint_{z_2} \frac{d\vec{s}_1\cdot d\vec{s}_2}{|\vec r_2 - \vec r_1|}$$

majhna zanka:

① $a \ll r$

② $a \ll \frac{1}{k}$



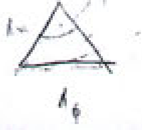
$|\vec r - \vec r'| =$
$= \sqrt{(x-x')^2 + (y-y')^2 + z^2}$

$d\vec{s} = \vec{1}_{\phi'}\cdot a \cdot d\phi'$

$$\vec{A} = \frac{\mu}{4\pi}\int_0^{2\pi} \vec{1}_{\phi'}\cdot I \cdot \frac{e^{-jk|\vec r - \vec r'|}}{|\vec r - \vec r'|}\cdot a \cdot d\phi'$$

$$\vec{1}_{\phi'} = -\vec{1}_x \sin\phi' + \vec{1}_y \cos\phi'$$

$$A = \frac{\mu}{4\pi}\int_v \vec{J}(r) \frac{e^{-jk|\vec r - \vec r'|}}{|\vec r - \vec r'|} dv'$$



$1_\phi = 1_x \sin\phi$

$$|\vec{r}-\vec{r}\,'| = \sqrt{(x-x')^2+(y-y')^2+z^2} =$$
$$= \sqrt{(r\sin\theta\cos\phi - a\cos\phi')^2 + (r\sin\theta\sin\phi - a\sin\phi')^2 + (r\cos\theta)^2} =$$
$$= \sqrt{r^2 + a^2 - 2ra\sin\theta(\cos\phi\cos\phi' - \sin\phi\sin\phi')} =$$
$$= \sqrt{r^2 + \underset{\sim}{a^2} - 2ra\sin\theta\cos(\phi-\phi')} \approx$$

majhen, zanem.

$$\sqrt{1+x} \approx 1+\frac{x}{2} \quad \overset{①}{\approx} \sqrt{r^2 - 2ra\sin\theta\cos(\phi-\phi')} \approx$$
$$\approx r\left(1 - \frac{a}{r}\sin\theta\cos(\phi-\phi')\right)$$
$$\frac{1}{|\vec{r}-\vec{r}\,'|} \overset{②}{\approx} \frac{1}{r}\left(1 + \frac{a}{r}\sin\theta\cos(\phi-\phi')\right)$$

$$1+\frac{1}{x} \approx 1-x \qquad e^{-jk|\vec{r}-\vec{r}\,'|} \overset{①}{\approx} e^{-jkr}\, e^{jka\sin\theta\cos(\phi-\phi')} \overset{②}{\approx}$$
$$\approx e^{-jkr}\left(1 + jka\sin\theta\cos(\phi-\phi')\right)$$

e^{-jx}, $x \ll 1$, $e^{jx} = 1+jx$, $\cos x \approx 1$

$$\vec{A} = \frac{\mu I a}{4\pi}\int_0^{2\pi}(-\vec{1}_x\sin\phi' + \vec{1}_y\cos\phi')\, e^{-jkr}\left(1 + jka\sin\theta\cos(\phi-\phi')\right) \cdot$$
$$\cdot \frac{1}{r}\left(1 + \frac{a}{r}\sin\theta\cos(\phi-\phi')\right) d\phi'$$

$$\vec{A} = \frac{\mu I a}{4\pi}\frac{e^{-jkr}}{r}\int_0^{2\pi}(-\vec{1}_x\sin\phi' + \vec{1}_y\cos\phi')\left(jka + \frac{a}{r}\right)\sin\theta \cdot$$
$$\cdot \underbrace{(\cos\phi\cos\phi' - \sin\phi\sin\phi')}_{\cos(\phi-\phi')}\, d\phi'$$

$$\vec{A} \approx \frac{\mu I a^2}{4\pi}\frac{e^{-jkr}}{r}\left(jk + \frac{1}{a}\right)\sin\theta\,\underbrace{(\vec{1}_y\,\pi\cos\phi - \vec{1}_x\,\pi\sin\phi)}_{\vec{1}_\phi \cdot \pi}$$

A .. površina zanke

$$\boxed{\vec{A} = \vec{1}_\phi\frac{\mu I \pi a^2}{4\pi}e^{-jkr}\left(\frac{jk}{r} + \frac{1}{r^2}\right)\sin\theta}$$ potencial

ZANKA
mag. točkasti dipol

$$\boxed{\vec{E} = -j\omega\vec{A} - \underset{\substack{\Uparrow\\ \shortparallel\\ 0}}{\operatorname{grad} V} = \vec{1}_\phi \cdot \frac{-j\omega\mu}{4\pi}I \cdot A\, e^{-jkr}\left(-\frac{jk}{r} + \frac{1}{r^2}\right)\sin\theta}$$

0; ni elektrin!

Prostor brez izvorov: $\vec{J}=0$, $\rho=0$

$$\left.\begin{array}{l}\Delta\vec{E}+\omega^2\mu\varepsilon\vec{E}=0\\ \Delta\vec{H}+\omega^2\mu\varepsilon\vec{H}=0\end{array}\right\}$$ valovni enačbi (vsebujejo ① in ② Max. enačb)

(x, y, z) $\quad \omega^2\mu\varepsilon=k^2$

$$\Delta\vec{E}=\mathrm{grad}(\mathrm{div}\,\vec{E})-\mathrm{rot}(\mathrm{rot}\,\vec{E})=\vec{1}_x\Delta E_x+\vec{1}_y\Delta E_y+\vec{1}_z\Delta E_z$$

$$\Delta E_x=\frac{\partial^2E_x}{\partial x^2}+\frac{\partial^2E_x}{\partial y^2}+\frac{\partial^2E_x}{\partial z^2}$$

vektorsko enačbo razstavimo na 3 skalarne

$$\Delta\vec{E}+\omega^2\mu\varepsilon\vec{E}=0\Longrightarrow\left\{\begin{array}{l}\Delta E_x+k^2E_x=0\\ \Delta E_y+k^2E_y=0\\ \Delta E_z+k^2E_z=0\end{array}\right]$$ skalarne valovne enač

poskusim z nastavkom:

$$E_x(x, y, z)=X(x)Y(y)Z(z)$$

$$YZ\frac{\partial^2X}{\partial x^2}+XZ\frac{\partial^2Y}{\partial y^2}+XY\frac{\partial^2Z}{\partial z^2}+k^2XYZ=0\Big/\cdot\frac{1}{XYZ}$$

$$\underbrace{\frac{1}{X}\frac{\partial^2X}{\partial x^2}}_{-k_x^2}+\underbrace{\frac{1}{Y}\frac{\partial^2Y}{\partial y^2}}_{-k_y^2}+\underbrace{\frac{1}{Z}\frac{\partial^2Z}{\partial z^2}}_{-k_z^2}+k^2=0$$

$$k_x^2+k_y^2+k_z^2=k^2$$

$$\frac{1}{X}\frac{\partial^2X}{\partial x^2}=-k_x^2\Longrightarrow\frac{\partial^2X}{\partial x^2}+k_x^2X=0$$

$$\frac{1}{Y}\frac{\partial^2Y}{\partial y^2}=-k_y^2$$

$$\frac{1}{Z}\frac{\partial^2Z}{\partial z^2}=-k_z^2$$

Rešitve: $\underline{k_x^2>0}$

$$X=\underbrace{\underbrace{Ae^{+jk_xx}}_{\text{odbiti val}}+\underbrace{Be^{-jk_xx}}_{\text{napredujoči val}}}_{\text{potujoča valova}}$$

DVE MOŽNI REŠITVI

POTUJOCI VAL ↙ ↘ STOJNI VAL

$$X = \underbrace{A' \cos k_x x + B' \sin k_x x}_{\text{stojni val}}$$

$- k_x = j|k_x| \Rightarrow k_x^2 < 0$

$\Rightarrow \frac{\partial^2 X}{\partial x^2} - |k_x|^2 \cdot X = 0$

k_x^2, k_y^2, k_z^2 ni nujno da so pozitivni

↳ če je imaginaren $k_x^2 < 0$ $(k_x = j|k_x|)$

enako velja tudi za k_y in k_z

rešitev: $k_x^2 < 0$

$X = A e^{|k_x|x} + B e^{-|k_x|x}$ eksponentno ushajoče/rastoče polje

$X = A' \cosh|k_x|x + B' \sinh|k_x|x$

③ Maxwellova enačba: $\mathrm{div}(\varepsilon \vec{E}) = 0$

Zgled: $\vec{E} = \vec{1}_x C e^{-jkx}$

$k_x = k;\ k_y = k_z = 0$

$\mathrm{div}(\varepsilon \vec{E}) = \varepsilon \frac{\partial}{\partial x}(C e^{-jkx}) = -jk\varepsilon C e^{-jkx} \neq 0$

Nefizikalni zgled! $\rho \neq 0$

! Zgled: $\vec{E} = \vec{1}_x C e^{-jkz}$ ⇒ Ustreza Maxwellovim enačb! ✓ ← smer $\vec{1}_z$

$k_z = k;\ k_x = k_y = 0;$

$\mathrm{div}(\varepsilon \vec{E}) = \varepsilon \frac{\partial}{\partial x}(C e^{-jkz}) = \underline{\underline{0}}$ ✓

Pripadajoče magnetno polje:

② $\vec{H} = \frac{j}{\omega\mu} \cdot \mathrm{rot}\vec{E} = \frac{j}{\omega\mu} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ C e^{-jkz} & 0 & 0 \end{vmatrix} = \underline{\underline{\vec{1}_y \frac{k}{\omega\mu} C e^{-jkz}}}$

① $\vec{J} = \mathrm{rot}\vec{H} - j\omega\varepsilon\vec{E} = 0$, če ni izvorov!

$\vec{J} = \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & \frac{+kC}{\omega\mu} e^{-jkz} & 0 \end{vmatrix} - j\omega\varepsilon \vec{1}_x C e^{-jkz} = \vec{1}_x \frac{jk^2 C}{\omega\mu} e^{-jkz} - \vec{1}_x j\omega\varepsilon C e^{-jkz} = \underline{\underline{0}}$ ✓

$k^2 = \omega^2 \mu \varepsilon$

$\vec{E} \perp \vec{H};\ \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu}{\varepsilon}} = Z$

↑ sofazna ↑ valovna impedanca

Kam gre moč? Poyntingov vektor:

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_z \frac{kCC^*}{\omega\mu} e^{-jkz}\underbrace{e^{jkz}}_{*} = \underline{\underline{\vec{1}_z \frac{|C|^2}{2Z}}}$$

$\Rightarrow$ <u>TEM (potujoči) val</u>

$$\vec{E} = \vec{1}_E C e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = \vec{1}_E C e^{-j\vec{k}\cdot\vec{r}}$$

↳ napredujoči val

③M.e.: $0 = \operatorname{div}(\varepsilon\vec{E}) = \varepsilon\left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) =$

$* \ \frac{\partial}{\partial x} = -jk_x \ ; \ \frac{\partial}{\partial y} = -jk_y \ ; \ \frac{\partial}{\partial z} = -jk_z$

$$= \varepsilon\big(\vec{1}_E\cdot\vec{1}_x(-jk_x)Ce^{-jk_x x}e^{-jk_y y}e^{-jk_z z} + \vec{1}_E\cdot\vec{1}_y(-jk_y)Ce^{-jk_x x}e^{-jk_y y}e^{-jk_z z} + \vec{1}_E\cdot\vec{1}_z(-jk_z)e^{-jk_x x}e^{-jk_y y}e^{-jk_z z}\cdot C \quad = \underline{\underline{-j\varepsilon\vec{k}\vec{E} = 0}}$$

②M.e.:

$$\vec{H} = \frac{j}{\omega\mu}\operatorname{rot}\vec{E} = \frac{j}{\omega\mu}\begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ -jk_x & -jk_y & -jk_z \\ \vec{1}_x\cdot\vec{E} & \vec{1}_y\vec{E} & \vec{1}_z\vec{E} \end{vmatrix} = \frac{1}{\omega\mu}\vec{k}\times\vec{E} =$$

$\uparrow$

$\operatorname{rot}\vec{E} = -j\vec{k}\times\vec{E}$

$$\underline{\underline{= \frac{k}{\omega\mu}\vec{1}_k\times\vec{E} = \frac{\vec{1}_k\times\vec{E}}{Z}}}$$

Definiramo valovni vektor, $\vec{k} = \vec{1}_k\cdot k$

$$\vec{k} = \vec{1}_x k_x + \vec{1}_y k_y + \vec{1}_z k_z$$

$$|\vec{k}| = k = \omega\sqrt{\mu\varepsilon}$$

$$\vec{1}_x(-jk_x) + \vec{1}_y(-jk_y) + \vec{1}_z(-jk_z) = -j\vec{k}$$

lastnosti za TEM val: $\vec{k} \perp \vec{E}$

$\vec{k} \perp \vec{1}_E$

$\vec{H} \perp \vec{k}, \vec{H} \perp \vec{E}$

$\vec{S} \perp \vec{E}$ & $\vec{S} \perp \vec{H} \Rightarrow$ TEM val

(ni vzdolžnih komponent)

①M.e: $\vec{J} = 0 = \mathrm{rot}\,\vec{H} - j\omega\varepsilon\vec{E} = -j\vec{k} \times \left(\frac{1}{\omega\mu}\vec{k}\times\vec{E}\right) - j\omega\varepsilon\vec{E} = \underline{\underline{0}}$ ✓

Poyntingov vektor:

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^{*} = \frac{1}{2}\vec{1}_E\, C e^{-j\vec{k}\vec{r}} \times \left(\frac{\vec{1}_k \times \vec{1}_E\, C^{*} e^{+j\vec{k}\cdot\vec{r}}}{Z}\right) = \underset{\uparrow}{\vec{1}_k}\frac{CC^{*}}{2Z} = \vec{1}_k\frac{|C|^2}{2Z}$$

smer valovnega vektorja!
kam bo sa moč

-zgled: ε_0, μ_0

$\vec{k} = (\vec{1}_x + \vec{1}_y + \vec{1}_z)\cdot 2\frac{\mathrm{rad}}{\mathrm{m}}$ $\qquad k = |\vec{k}| = \sqrt{4\frac{\mathrm{rad}^2}{\mathrm{m}^2} + 4\frac{\mathrm{rad}^2}{\mathrm{m}^2} + 4\frac{\mathrm{rad}^2}{\mathrm{m}^2}} = 2\sqrt{3}$

$f = ?$

$$k = \frac{\omega}{c_0} = \frac{2\pi f}{c_0} \Rightarrow f = \frac{k c_0}{2\pi} = \frac{3{,}5\frac{\mathrm{rad}}{\mathrm{m}}\cdot 3\cdot 10^8\,\mathrm{m/s}}{2\pi\,\mathrm{rad}} = \underline{\underline{166\,\mathrm{MHz}}}$$

$|\vec{S}| = 1\,\mathrm{W/m^2}$ kam kaže S?

$$\vec{S} = \vec{1}_k \cdot |\vec{S}| = \frac{\vec{1}_x + \vec{1}_y + \vec{1}_z}{\sqrt{3}} = 1\,\mathrm{W/m^2}$$

$$\vec{1}_k = \frac{\vec{1}_x + \vec{1}_y + \vec{1}_z}{\sqrt{3}}$$

$$|\vec{S}| = \frac{|\vec{E}|^2}{2Z_0} \Rightarrow |\vec{E}| = \sqrt{2Z_0|\vec{S}|} = \sqrt{2\cdot 377\,\Omega\cdot 1\,\mathrm{W/m}} = \sqrt{754\,\mathrm{V^2/m^2}} = \underline{\underline{27\,\mathrm{V/m}}}$$

↳ smer $\vec{E} \perp \vec{S}$

$\vec{E}$ leži v ravnini xy $\qquad$ pogoj: $\vec{E}\cdot\vec{k} = 0$ $\quad$ TEM

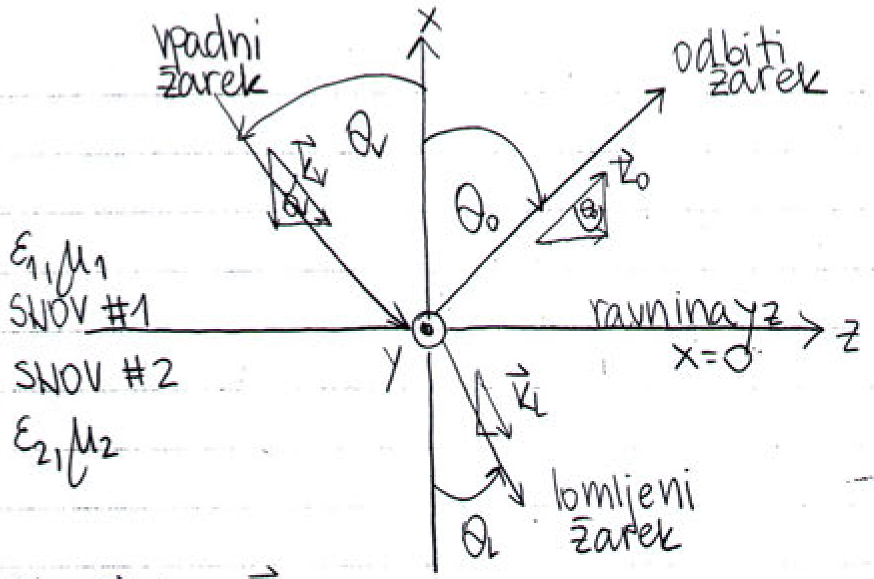
$\vec{E} = \vec{1}_x \cdot E_x + \vec{1}_y \cdot E_y$

$(\vec{1}_x E_x + \vec{1}_y E_y)\cdot(\vec{1}_x + \vec{1}_y + \vec{1}_z)\cdot 2\frac{\mathrm{rad}}{\mathrm{m}} = 0$

$(E_x + E_y + 0)\cdot 2\frac{\mathrm{rad}}{\mathrm{m}} = 0 \Rightarrow E_x = -E_y$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \sqrt{2E_x^2} \Rightarrow E_x = \pm\sqrt{\frac{|\vec{E}|^2}{2}} = \pm\frac{|\vec{E}|}{\sqrt{2}} = \underline{\underline{\pm 19\,\mathrm{V/m}}}$$

poljubna faza!



$\vec{k}_v = \vec{1}_x k_{vx} + \vec{1}_z k_{vz}$

$\vec{k}_o = \vec{1}_x k_{ox} + \vec{1}_z k_{oz}$

$\vec{k}_L = \vec{1}_x k_{Lx} + \vec{1}_z k_{Lz}$

enak pojav na meji

$\Rightarrow$ neodvisen od y, z

faza k_v se enako spreminja kot k_o, kot k_L

$\Rightarrow k_{vz} = k_{oz} = k_{Lz} = \beta$

$\vec{k}_v = \vec{1}_x k_{vx} + \vec{1}_z \beta = \vec{1}_v \omega\sqrt{\mu_1\varepsilon_1}$

$\vec{k}_o = \vec{1}_x k_{ox} + \vec{1}_z \beta = \vec{1}_o \omega\sqrt{\mu_1\varepsilon_1}$

$\vec{k}_L = \vec{1}_x k_{Lx} + \vec{1}_z \beta = \vec{1}_L \omega\sqrt{\mu_2\varepsilon_2}$

$$\left.\begin{array}{l} k_{vx}^2 + \beta^2 = k_1^2 \\ k_{ox}^2 + \beta^2 = k_1^2 \end{array}\right\} k_{ox} = -k_{vx} \Rightarrow \Theta_o = \Theta_v$$

odbojni zakon

$k_{Lx}^2 + \beta^2 = k_2^2$

$$\sin\Theta_v = -\frac{\beta}{k_v} = -\frac{\beta}{\omega\sqrt{\mu_1\varepsilon_1}}$$

lomni zakon (Snell)

$$\sin\Theta_L = -\frac{\beta}{k_L} = -\frac{\beta}{\omega\sqrt{\mu_2\varepsilon_2}}$$

↓

$$\frac{\sin\Theta_L}{\sin\Theta_v} = \frac{\sqrt{\mu_1\varepsilon_1}}{\sqrt{\mu_2\varepsilon_2}} = \frac{\sqrt{\mu_{r1}\mu_0\varepsilon_{r1}\varepsilon_0}}{\sqrt{\mu_{r2}\mu_0\varepsilon_{r2}\varepsilon_0}} = \frac{\sqrt{\mu_{r1}\varepsilon_{r1}}}{\sqrt{\mu_{r2}\varepsilon_{r2}}} = \frac{n_1}{n_2}$$

lomni količnik: $n = \sqrt{\mu_r\varepsilon_r}$ kolikokrat se hitrost svetlobe zmanjša v snovi

$v = \frac{c_0}{n}$

$k = n k_0 = n \omega \sqrt{\mu_0 \varepsilon_0}$

$\lambda = \frac{c_0}{f} = \frac{c_0}{n f_0} = \frac{\lambda_0}{n}$ ← valovna dolžina v praznem prostoru

$\left.\begin{matrix}\mu \\ \varepsilon\end{matrix}\right\} \Rightarrow \left\{\begin{matrix}n \\ z\end{matrix}\right.$

impedan~~ca~~: $z = \sqrt{\frac{\mu}{\varepsilon}}$

impedanca praznega prostora: $z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\sin\Theta_L = \frac{n_1}{n_2} \sin\Theta_v$ — majhen

POPOLNI ODBOJ!

$\boxed{\sin\Theta_L > 1}$???

majhen ↓ ← velik

$k_{Lx}^2 + \beta^2 = k_2^2 \Rightarrow k_{Lx}^2 = k_2^2 - \beta^2 < 0$

$k_{Lx} = \pm j \sqrt{\beta^2 - k_2^2}$

↑ čisto imaginarno

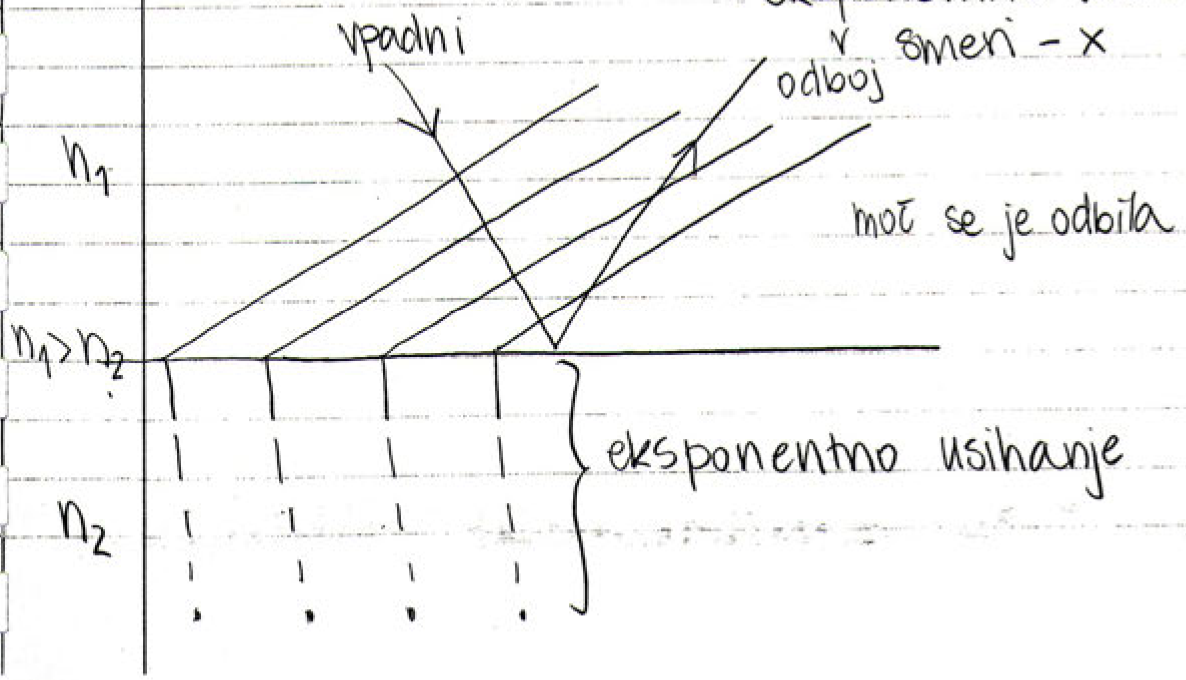
$k_{Lx} = \pm j \, |k_{Lx}|$

$\rightarrow \vec{k}_L = \vec{1}_x \cdot j \, |\vec{k}_{Lx}| + \vec{1}_z \cdot \beta$

$\Rightarrow \vec{E}_L = \vec{1}_{E_L} \cdot C_L \, e^{-jk_x x} \, e^{-jk_z \cdot z} = \vec{1}_{E_L} C_L \, e^{-j(j|\vec{k}_x| x)} \, e^{-j\beta z} =$

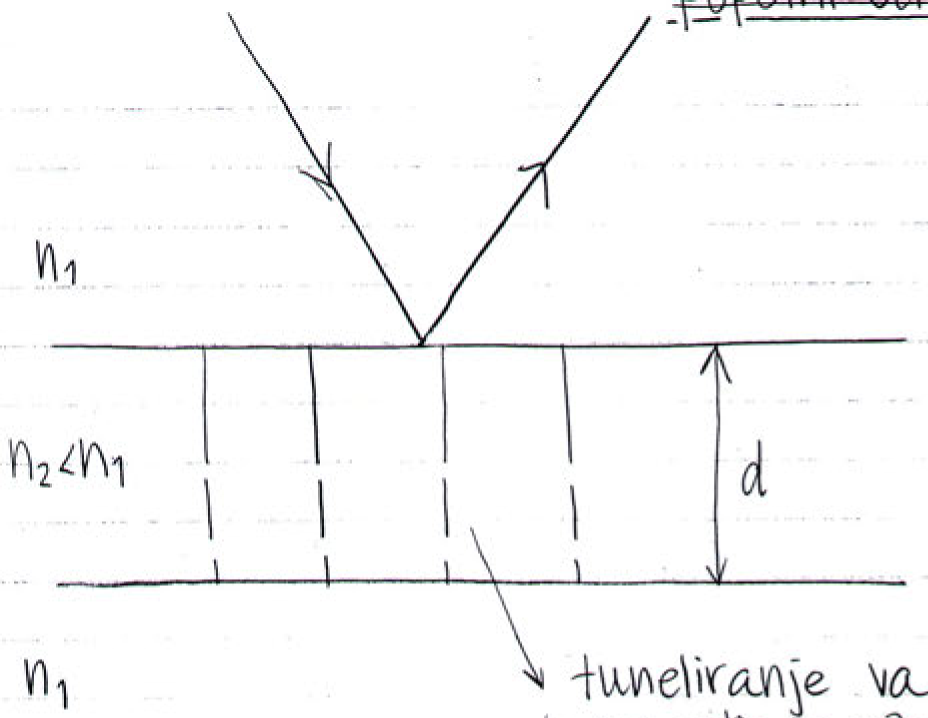
$= \vec{1}_{E_L} C_L \boxed{e^{|\vec{k}_{Lx}|}} \, e^{-j\beta z}$

→ eksponentno usihanje v smeri - x



Tuneliranje

~~popolni odboj~~



tuneliranje valovanja
(nekaj polja, ki usiha, lahko še vseeno pride ven)

$$e^{jkx}+e^{-jkx}=\cos kx-\cancel{j\sin kx}+\cos kx+\cancel{j\sin kx}=\underline{2\cos kx}$$

-Potujoči val

$\vec{E} = \vec{1}_E \cdot C e^{j\vec{k}\vec{r}}$

$\vec{k} = \vec{1}_x k_x + \vec{1}_y k_y + \vec{1}_z k_z$

$k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2 \mu \varepsilon$

$\vec{k} \perp \vec{E}$ $\quad \vec{E} \perp \vec{H}$

$\vec{k} \parallel \vec{S}$ $\quad \vec{k} \perp \vec{H}$ $\quad \vec{\nabla} = -j\vec{k}$ ↱ velja samo za en sam potujoči val

↳ $\vec{E}$ in $\vec{H}$ sofazna $\quad \vec{k}_z = -\vec{1}_z \cdot k$ $\quad \vec{k}_z = \vec{1}_z k$

$\vec{S}$ realen

1D Stojni val

$\vec{E} = \vec{1}_x \cdot C \cos kz = \vec{1}_x C \left(e^{jkz} + e^{-jkz}\right) \frac{1}{2}$ $\qquad Z = \sqrt{\frac{\mu}{\varepsilon}}$

?

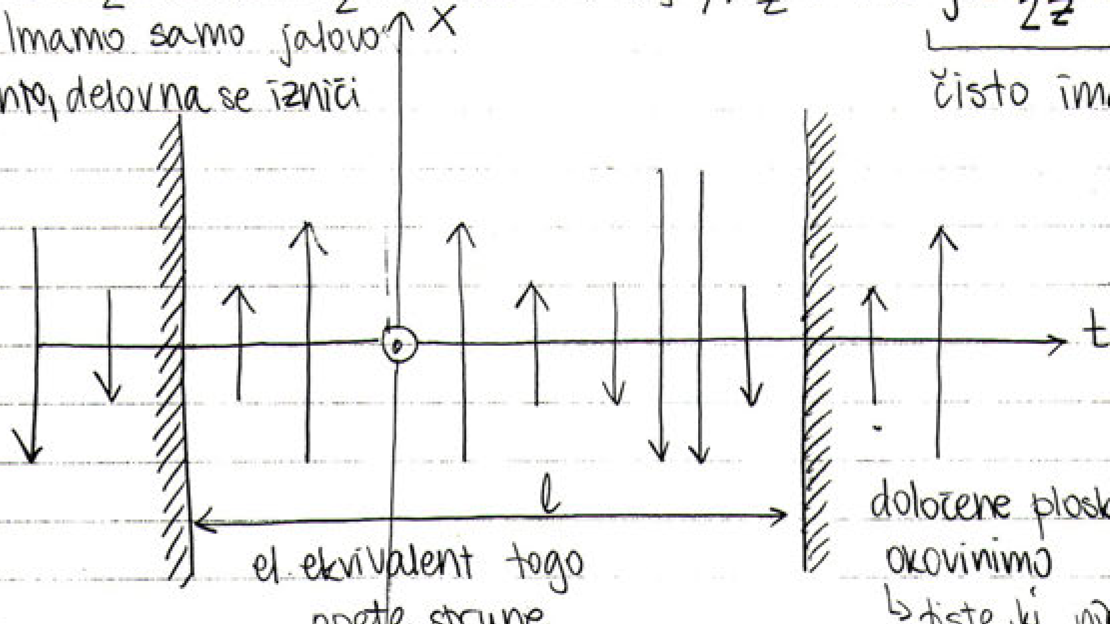
valovni vektor $\quad$ valovni vektor $\qquad$ $\vec{E}, \vec{H}$ v kvadraturi

$$\vec{H} = \frac{j}{\omega\mu} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cos hz & 0 & 0 \end{vmatrix} = \vec{1}_y \frac{jC}{\omega\mu} k(-\sin kz) = -j\vec{1}_y \frac{C}{Z} \sin kz$$

Pri stojnem valu sta si el. in mag. polje v kvadraturi (-j je 90° faznega zasuka). Prej sta bila sofazna.

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* = \frac{1}{2} \vec{1}_x C \cos kz \times (+j\vec{1}_y) \frac{C^*}{Z} \sin kz = \underbrace{j\vec{1}_z \frac{C \cdot C^*}{2Z} \cos kz \sin kz}_{\text{čisto imaginaren}}$$

Imamo samo jalovo komponento, delovna se izniči



el. ekvivalent togo opete strune

določene ploskve lahko okovinimo
↳ tiste, ki nimajo tangencialne komponente

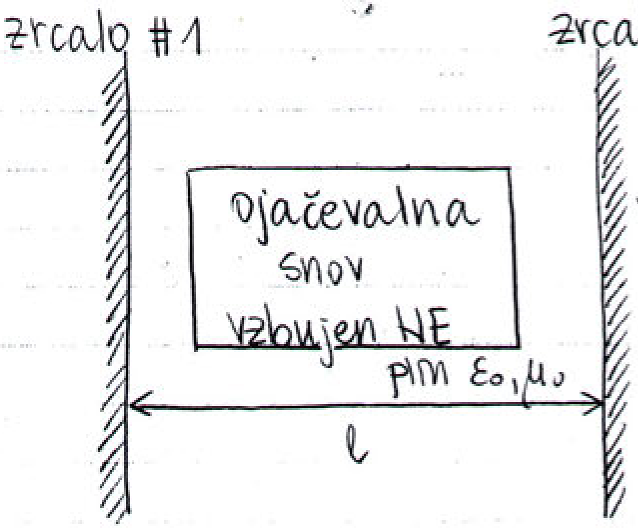
$\vec{E}_t = 0 \Rightarrow$ kovina

$\vec{E}_t = 0 \Rightarrow$ kovina

Fabry-Perot-ov rezonator

Kovinskih plošč ne moremo postavljati na poljubno razdaljo oz. če imam plošče na določeni razdalji, bo to dobro samo za določen k oz. frekvenco

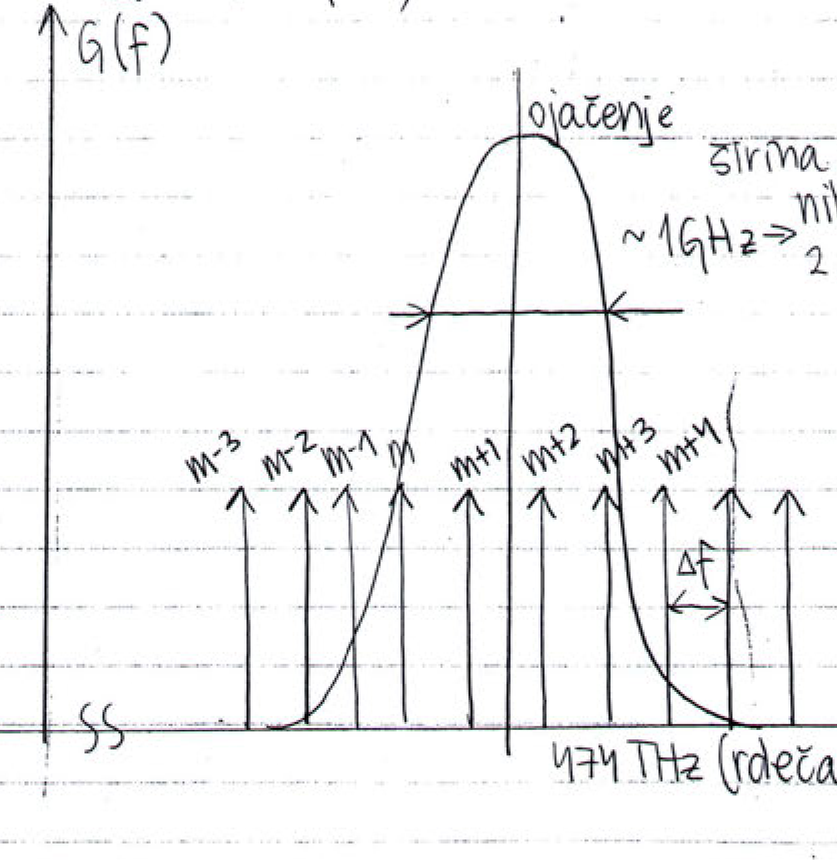
$kl = m\pi$; $m = 1, 2, 3, 4 \ldots$ $\quad m\pi = kl = \frac{\omega}{c_0} l = \frac{2\pi f}{c_0} l \Rightarrow \underline{f = m\frac{c_0}{2l}}$



svetloba niha, želimo da vsaj ena izmed frekvenc svetlobe pade med resonančne frekvence

$l = 33\,\text{cm}$

$$\Delta f = \frac{c_0}{2l} = \frac{3 \cdot 10^8\,\text{m/s}}{2 \cdot 0{,}33\,\text{m}} = 4{,}5 \cdot 10^8\,\text{s}^{-1} = \underline{450\,\text{MHz}}$$



TEM_{00m} ↗ št. roda
$\quad xy\ z$

širina ojačanja ~1GHz ⇒ niha na 2 ali 3 robovih → m-1, m, m+1

Izgubni kabel:

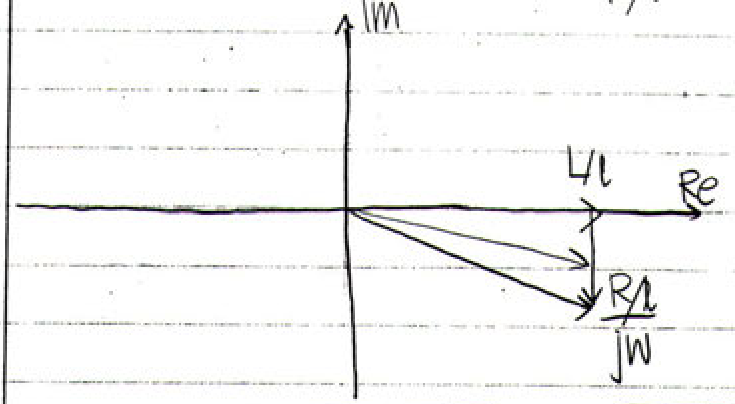
$$Z_k = \sqrt{\frac{L/l + \frac{R/l}{jw}}{C/l}} \approx \sqrt{\frac{L/l}{C/l}}$$

$$K = w\sqrt{\left(L/l + \frac{R/l}{jw}\right)\cdot C/l} = \beta \overset{\text{IZGUBE!}}{\boxed{-j\alpha}} \rightarrow \beta \approx w\sqrt{L/l \cdot C/l}$$

$$\rightarrow \alpha \neq 0$$

$$\alpha \approx \beta \cdot \frac{R/l}{2w\,L/l} = w\sqrt{L/l \cdot C/l}\,\frac{R/l}{2w\,L/l} =$$

$$= \frac{R/l}{2\sqrt{\frac{L/l}{C/l}}} = \underline{\underline{\frac{R/l}{2Z_k}}}$$



$$\frac{R/l}{w} \ll L/l$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

$$x \ll 1$$

$$K = w\sqrt{\left(L/l + \frac{R/l}{jw}\right)(C/l)} = w\sqrt{(L/l)(C/l)\left(1 - j\frac{R/l}{w\,L/l}\right)} \approx$$

$$\approx w\sqrt{(L/l)(C/l)}\left(1 - j\frac{R/l}{2w\,L/l}\right)$$

$$\alpha = \frac{R/l}{2Z_k} \quad i \quad U(l) = U(0)\,e^{-j\beta l}\,e^{-\alpha l}$$

$$a_{dB/m} = 20\log_{10}\left|\frac{U(0)}{U(l)}\right| = 20\log_{10} e^{\alpha l}\ [dB]$$

$$a = \frac{20}{\ln 10} \alpha l = \frac{20}{\ln 10} \frac{R/l}{2 Z_k} l$$

$$a/l = \frac{10}{\ln 10} \frac{R/l}{Z_k} \left[\frac{dB}{m}\right]$$

$$\boxed{Z_k = \frac{Z_0}{2\pi\sqrt{\varepsilon_r}} \ln \frac{R_0}{R_{\check{z}}}}$$

$$R/l = R_{\check{z}}/l + R_0/l = \frac{1}{2\pi R_{\check{z}} \delta \gamma_{cu}} + \frac{1}{2\pi R_0 \delta \gamma_{cu}}$$

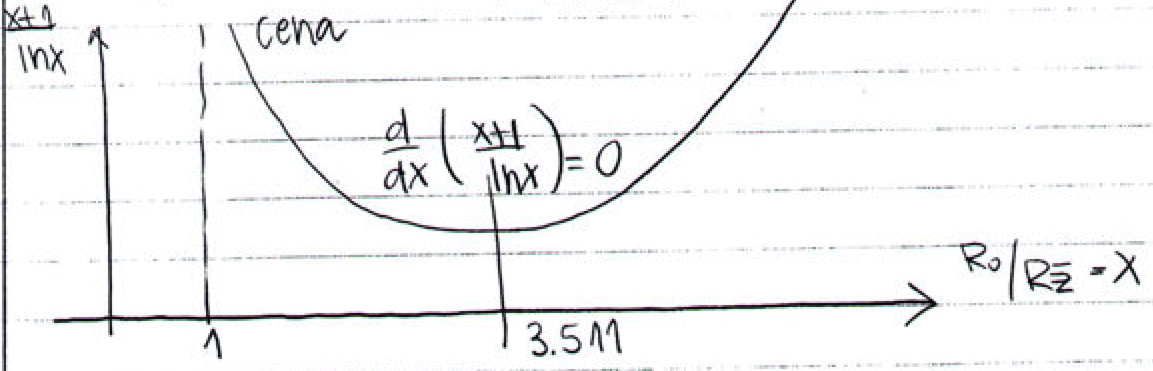
$$R/l = \frac{1}{2\pi} \sqrt{\frac{\omega\mu}{2\gamma_{cu}}} \left(\frac{1}{R_{\check{z}}} + \frac{1}{R_0}\right)$$

$$a/l = \frac{10}{\ln 10} \frac{\frac{1}{2\pi}\sqrt{\frac{\omega\mu}{2\gamma_{cu}}}\left(\frac{1}{R_{\check{z}}}+\frac{1}{R_0}\right)}{\frac{Z_0}{2\pi\sqrt{\varepsilon_r}} \ln \frac{R_0}{R_{\check{z}}}} = \underline{\underline{\frac{10}{\ln 10} \sqrt{\varepsilon_r} \frac{\sqrt{\frac{\omega\mu}{2\gamma}}}{Z_0} \frac{\left(\frac{1}{R_{\check{z}}}+\frac{1}{R_0}\right)}{\ln \frac{R_0}{R_{\check{z}}}} \left[\frac{dB}{m}\right]}}$$

$\varepsilon_r \to 1$; votel dielektrik (penast dielektrik)

cena $\sim R_0^2$

$$a/l = \frac{10}{\ln 10} \frac{\sqrt{\varepsilon_r}}{R_0} \frac{\sqrt{\frac{\omega\mu}{2\gamma}}}{Z_0} \left(\frac{\frac{R_0}{R_{\check{z}}} + 1}{\ln \frac{R_0}{R_{\check{z}}}}\right) \left[\frac{dB}{m}\right]$$



$f = 100\ MHz$ $\delta = 6.8 \cdot 10^{-6}\ m$ $R_{\check{z}} = 0.5 mm$ $\varepsilon_r = 2$ polietilen

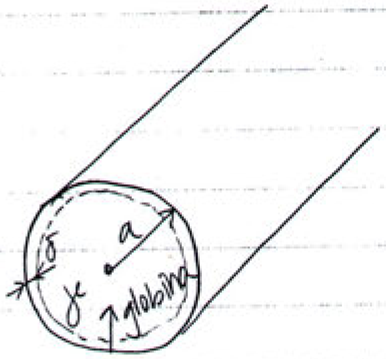
$\gamma_{cu} = 56 \cdot 10^6\ S/m$ $R_0 = 1.75\ mm$ $Z_k = 50\ \Omega$ @ $\frac{R_0}{R_{\check{z}}} = 3.5$

$$R_p = \sqrt{\frac{\omega\mu}{2\gamma}} = \frac{1}{\delta\gamma} = \frac{1}{380} = \underline{\underline{2.6\,m\Omega}}$$

$$R/l = \frac{1}{2\pi} \cdot \overbrace{2{,}6\,m\Omega}^{R_p} \left(\frac{1}{R_z} + \frac{1}{R_o}\right) = \frac{1}{2\pi} \cdot 2.6 \cdot 10^{-3}\Omega \left(\frac{10^3}{0.5m} + \frac{10^3}{1.75m}\right) \approx \underline{\underline{1\Omega/m}}$$

$$a/l = \frac{10}{\ln 10} \cdot \frac{R/1}{Z_l} = \frac{10}{2.305} \frac{1\cancel{\Omega}/m}{50\cancel{\Omega}} \Rightarrow a/l = 0.08\frac{dB}{m} = \underline{\underline{8\,dB/100m}}$$

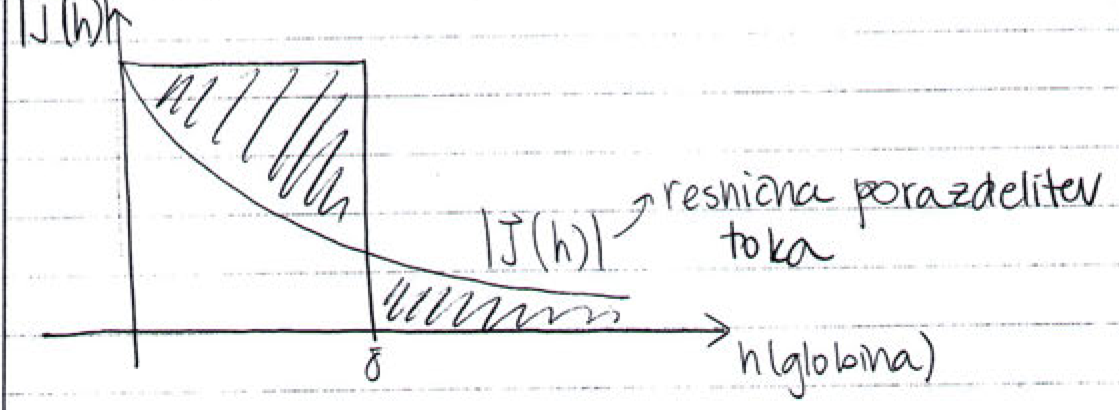
dober prevodnik : $\gamma > \omega\varepsilon$ $\delta = \sqrt{\frac{2}{\omega\mu\gamma}}$



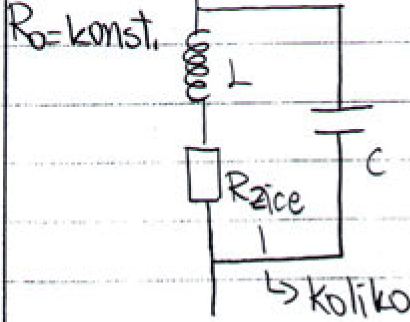
Cu
f= 100 MHz
$\delta = 6.8\,\mu m$

upornost žice z enosmerno $R_= = \frac{l}{\pi a^2} \cdot \frac{1}{\gamma}$

upornost žice z izmenično $R_\sim = \frac{l}{2\pi a \delta} \cdot \frac{1}{\gamma} > R_=$; $R_\sim = R_0\sqrt{\omega}$



RESNIČNA TULJAVA

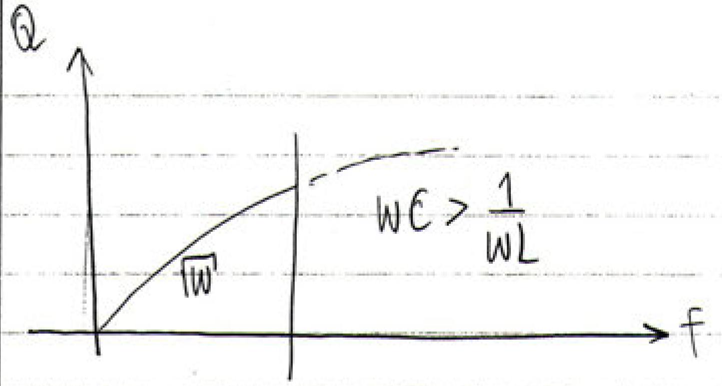


$$Q = \frac{\omega L}{R_{žice}} = \frac{\omega L}{R_0\sqrt{\omega}} = \frac{L}{R_0}\sqrt{\omega}$$

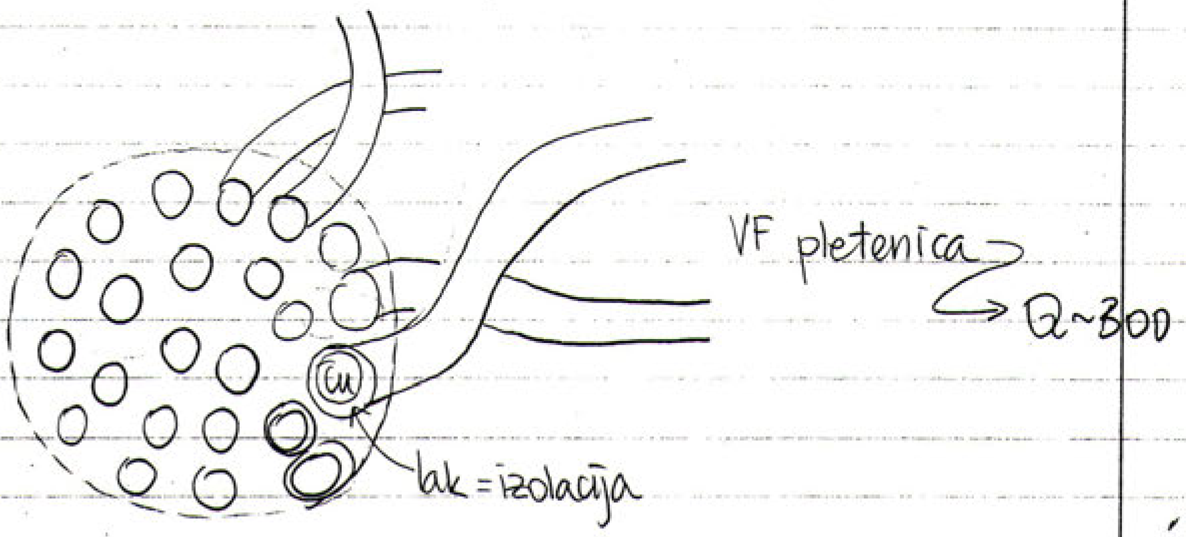
tuljava Cu žica

$\boxed{\mu_{jedra} = \mu : Q \sim 100}$

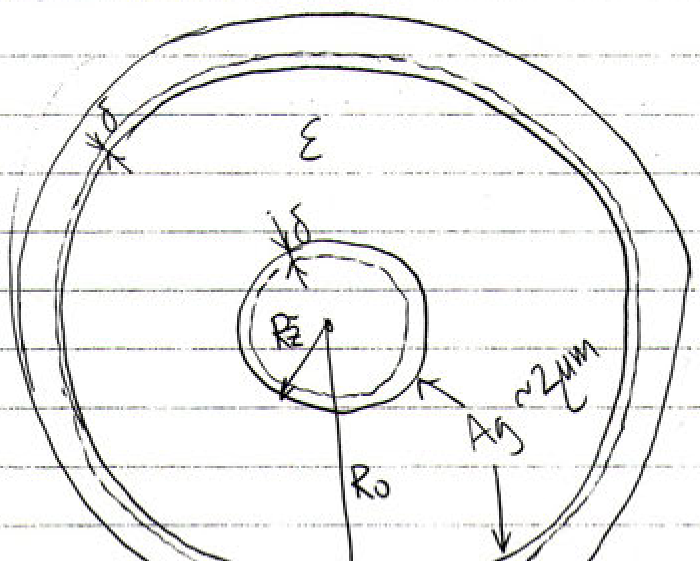
↳ koliko je kvaliteta tuljave (omejuje kvaliteto)



Kako povečamo kvaliteto tuljave?
↳ naredimo žico iz več vodnikov



koaksialen kabel.



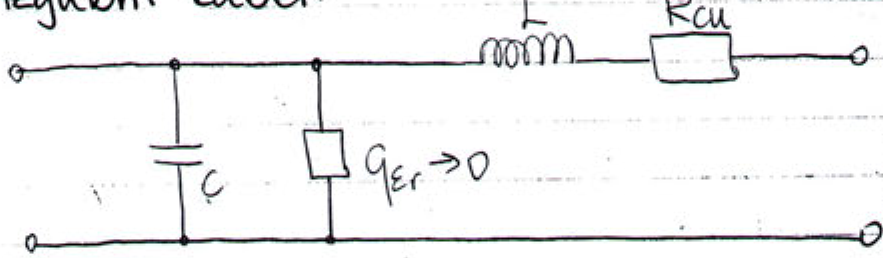
karakteristicna impedanca:

$$Z_k \approx \frac{Z_0}{2\pi\sqrt{\varepsilon_r}} \ln\frac{R_0}{R_z} \quad \left(\begin{matrix}\text{brezizgubni}\\ \text{kabel}\end{matrix} \; ! \right)$$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\,\Omega$$

$$\Rightarrow Z_k \approx \frac{60\,\Omega}{\sqrt{\varepsilon_r}} \ln\frac{R_0}{R_ž}$$

izgubni kabel:



$\omega L \sim 100 R_{cu}$

Z_k kompleksen

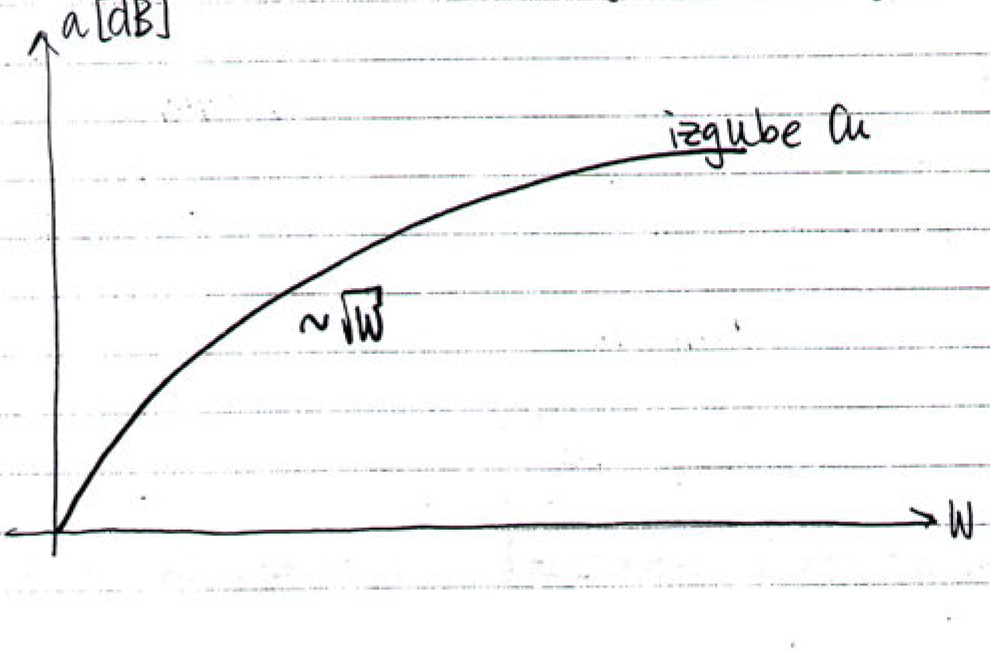
$|Re[Z_k]| \sim 1000 |Im[Z_k]|$

slabljenje kabla:

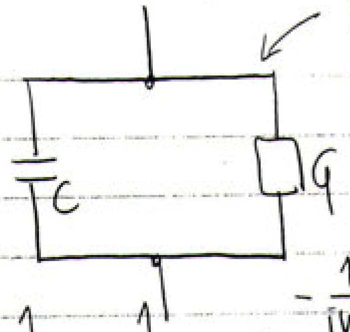
$$a[dB] = \frac{10}{\ln 10}\,\frac{R/l}{Z_k}\cdot l = \overset{\text{konst.}}{C_a}\sqrt{\varepsilon_r}\sqrt{\omega}\,\frac{1}{R_0}\left(\frac{R_0/R_ž+1}{\ln(R_0/R_ž)}\right)$$

minimum
$R_0/R_ž = 3.511$

$$R_{cu} = R_{žile} + R_{oklopa} = \frac{l}{2\pi R_ž \delta}\cdot\frac{1}{\gamma} + \frac{l}{2\pi R_0 \delta}\cdot\frac{1}{\gamma}$$



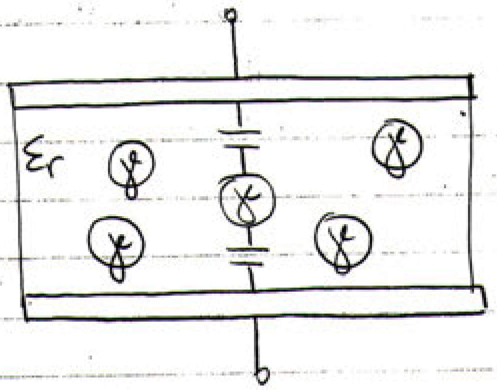
ni uporabno !



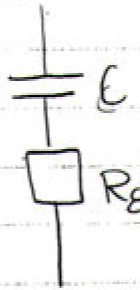
$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{\frac{1}{j\omega C}+R_{\varepsilon r}} = \frac{-\frac{1}{j\omega C}+R_{\varepsilon r}}{\frac{1}{\omega^2C^2}+\underbrace{R_{\varepsilon r}^2}_{\text{zanemarimo}}} \approx j\omega C + R_{\varepsilon r}\,\omega^2 C^2$$

$$\underline{Y} = j\omega C\,(1-jR_{\varepsilon r}\,\omega C)$$

slab dielektrik

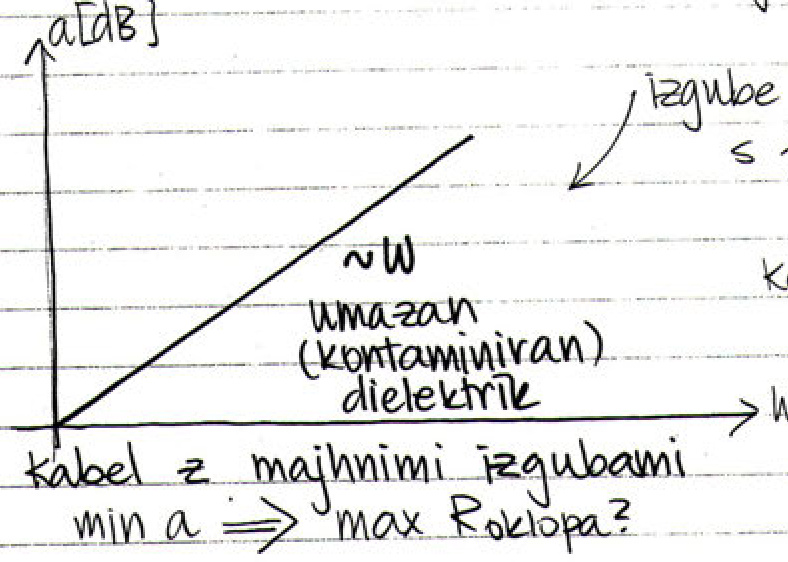


resnicno vezje:



$R_{\varepsilon r} \ll \frac{1}{\omega C}$

$$Z = \frac{1}{j\omega C} + R_{\varepsilon r}$$



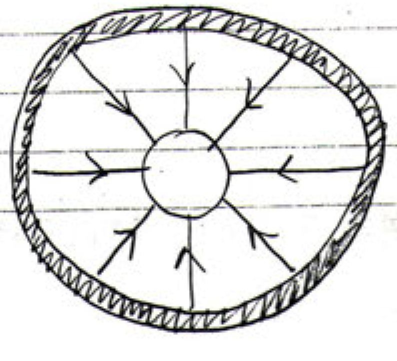
izgube premo sorazmerne s frekvenco

umazan (kontaminiran) dielektrik

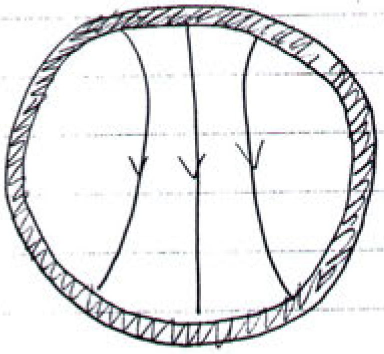
kako ugotovimo ali je kabel dober? - Izmerimo potek !

kabel z majhnimi izgubami

min a ⟹ max R_{oklopa}?



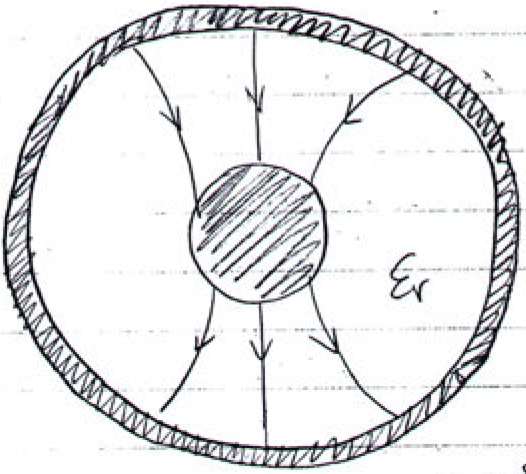
TEM rod: nacin razširjanja



rod TE_{11} (ρ, φ, z)

ustreza TE_{10} v ___?___ snovi

$f_m > 0$



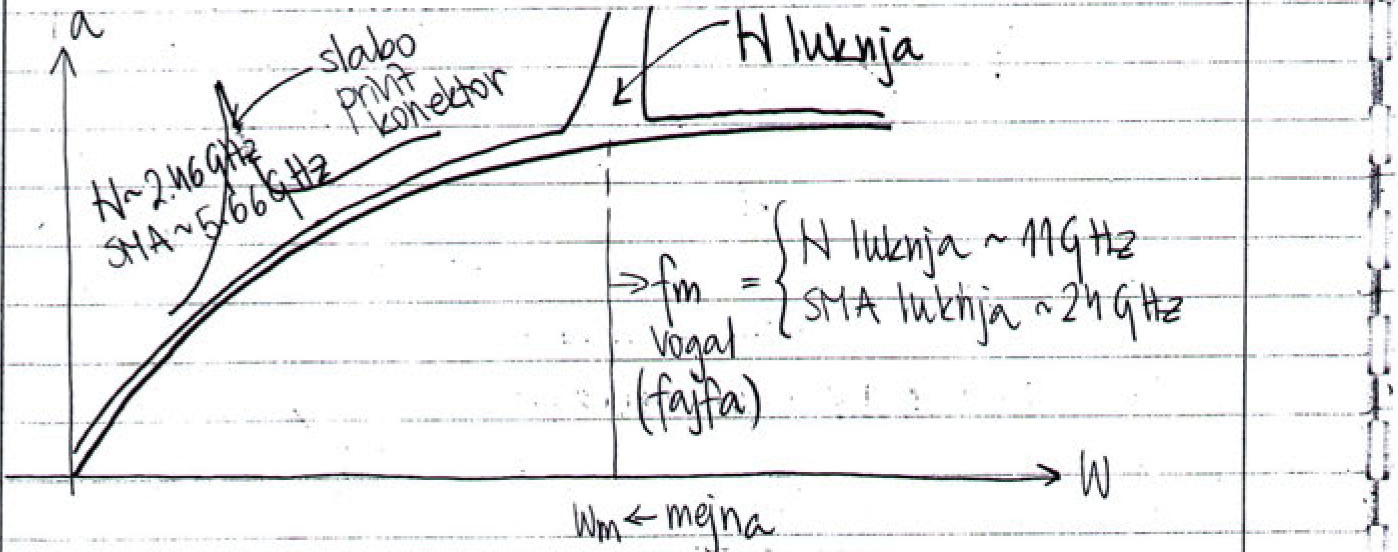
ustreza rodu TE_{11} v koaksu

$R_{žile} = 0.5\,mm$

$R_{oklopa} = 1.75\,mm$

$\varepsilon_r = 2$

$$f_m = \frac{c_0/\sqrt{\varepsilon_r}}{\pi(R_{žile} + R_{oklopa})} = \frac{\frac{3\cdot 10^8\,m/s}{\sqrt{2}}}{\pi(0.5\cdot 10^{-3}m + 1.75\cdot 10^{-3}m)} = \frac{3\cdot 10^8\,Hz \cdot 10^3}{\sqrt{2}\cdot\pi\cdot 2.25} \approx \underline{\underline{30\,GHz}}$$

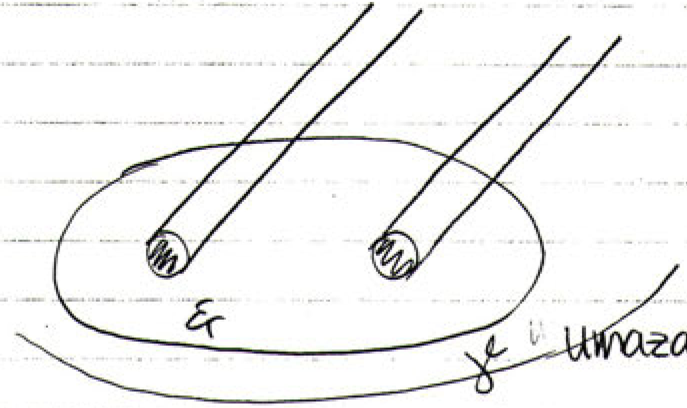


$\rightarrow f_m$ vogal (fajfa) $= \begin{cases} \text{N luknja} \sim 11\,GHz \\ \text{SMA luknja} \sim 24\,GHz \end{cases}$

navadni dvovod: TEM (osnovni rod) + višji rodovi !

palica

Ethernet



umazanija $\xrightarrow{da}$ dielektrične izgube !

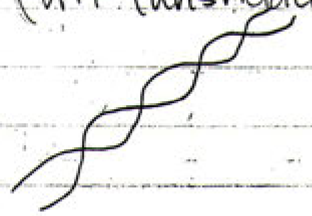
$R_{dvovoda} \sim 2 \cdot R_{kabla}$

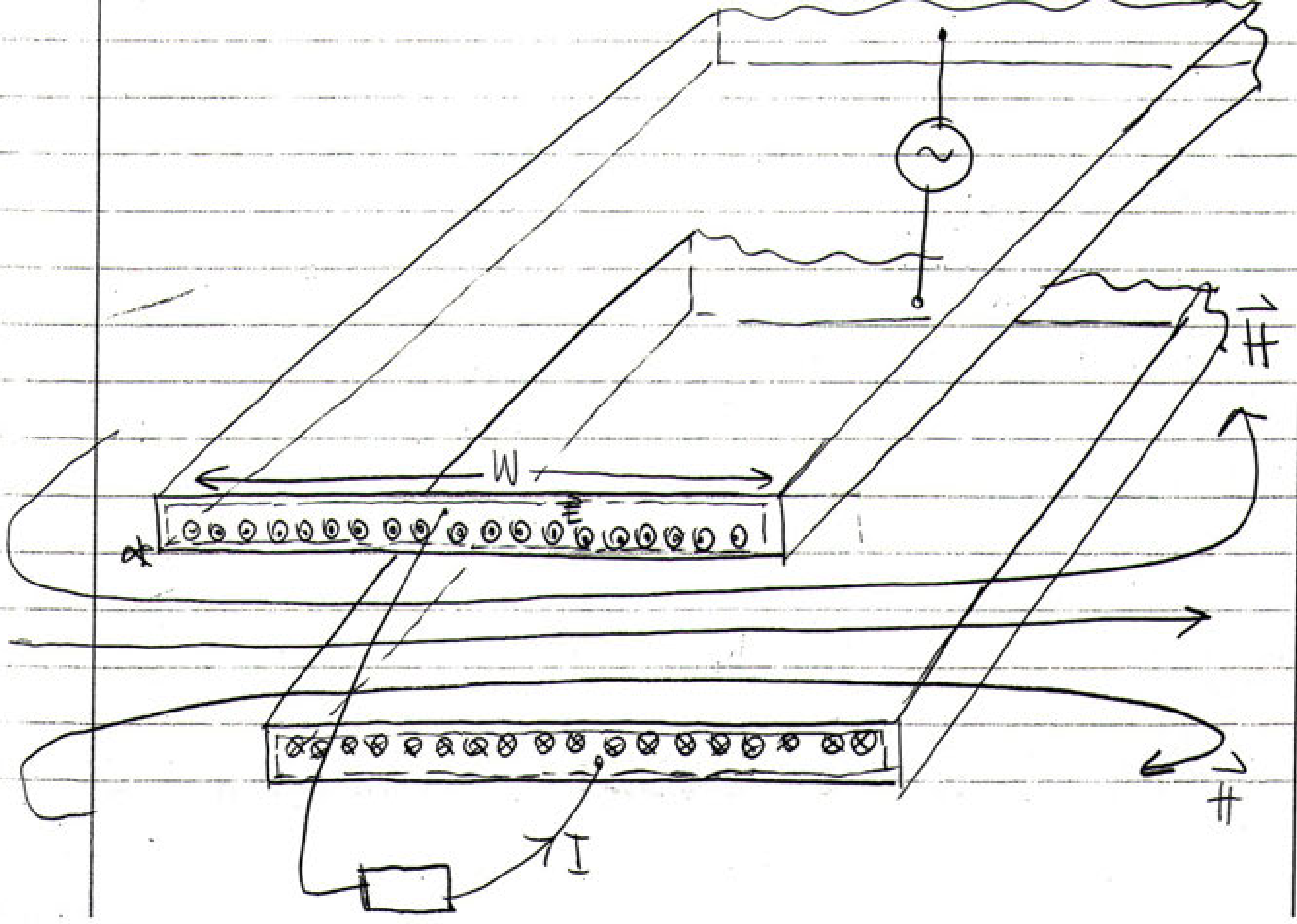
$Z_{k\,dvovoda} \sim 4 Z_{k\,kabla}$

$\alpha_{dvovoda} \sim \frac{1}{2}\alpha_{kabla}$

brez oklopa

(UTP (unshelded twisted pair))





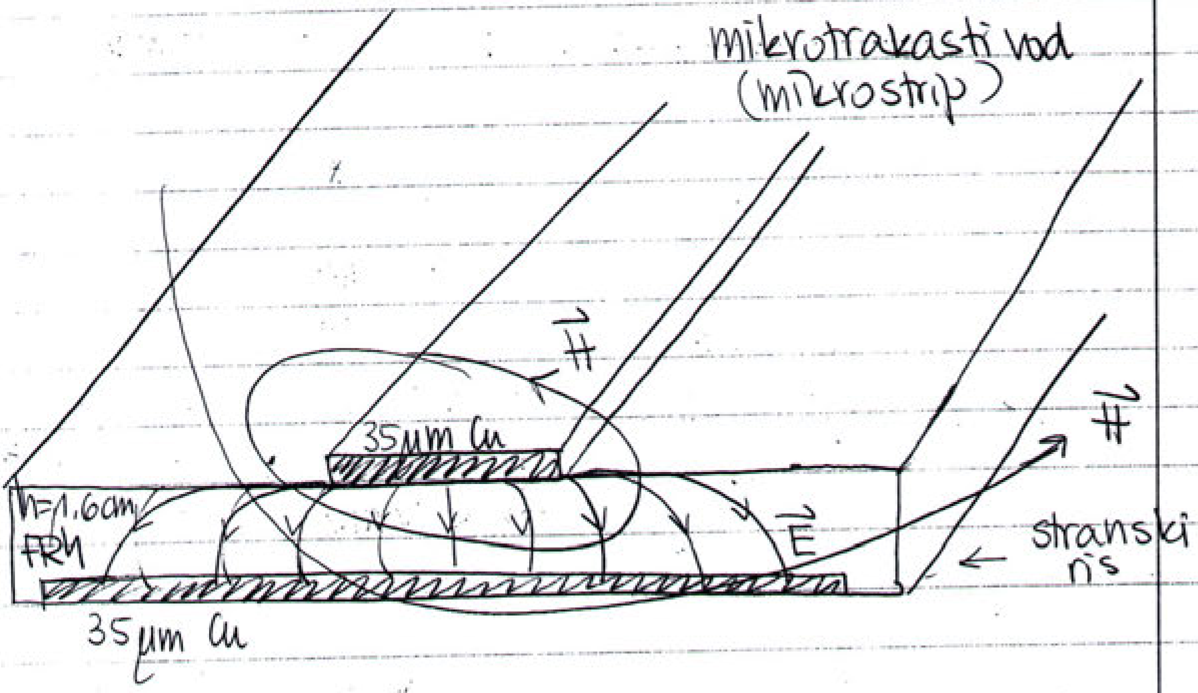
R posameznega vodnika

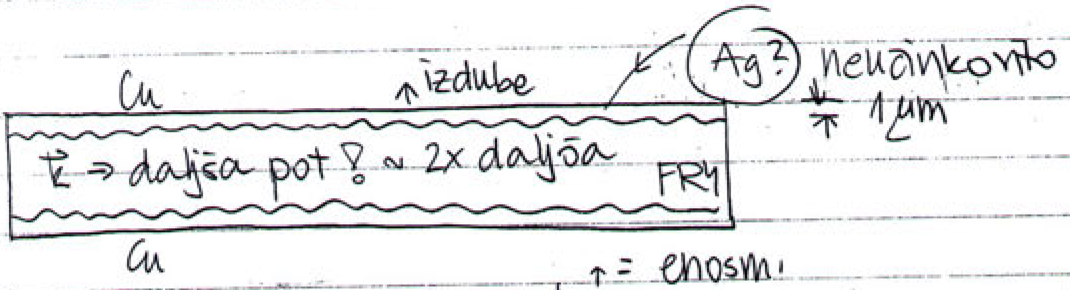
$$* \; R_g = \frac{1}{w\delta}\cdot\frac{1}{\gamma}$$

$$R_s = \frac{1}{w\delta}\cdot\frac{1}{\gamma}$$

$$Z_k = \frac{h}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{h}{w}\frac{Z_0}{\sqrt{\varepsilon_r}}$$

↳ brez stresanja $w >> h$





$\vec{k}$ ⇒ daljša pot ! ~ 2x daljša

navadno tiskano vezje

$$\varepsilon_r: FR4 = \begin{cases} \uparrow = \text{enosm.} \\ < 1\,\text{MHz} \;\; \varepsilon_r \sim 5 \\ 100\,\text{GHz} \;\; \varepsilon_r \sim 4 \end{cases}$$

FR4 $\varepsilon_r \simeq 4.6$

$\underline{Z_k = 50\,\Omega}$ ⇒ MORAMO UPOŠTEVATI STRESANJE!

$w = ?$

$$\underline{\underline{W}} = \frac{h}{Z_k}\cdot\frac{Z_0}{\sqrt{\varepsilon_r}} = \frac{1.6\,\text{cm}}{50\,\Omega}\cdot\frac{120\pi\,\Omega}{\sqrt{4.5}} = 1.6\cdot 3.5\,\text{cm} = \underline{\underline{5.5\,\text{cm}}}\;?$$

NAROBE!

razmerje premajhno $\frac{w}{h} = 3.5$

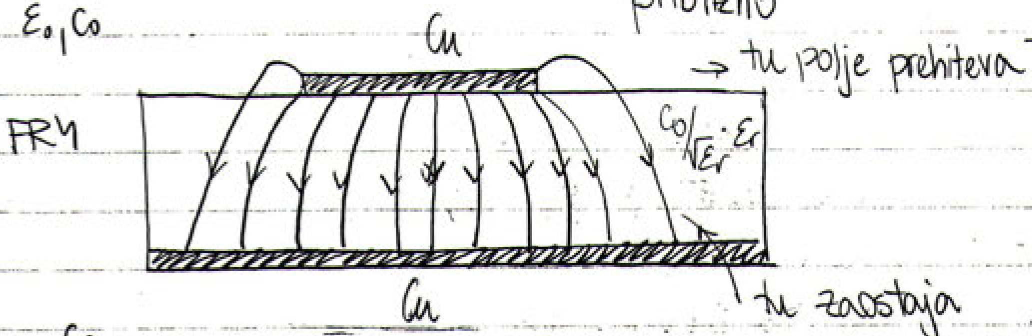
stresanje $\left.\begin{matrix} C/l \uparrow \\ L/l \downarrow \end{matrix}\right\} Z_k = \sqrt{\frac{L/l}{C/l}} \Downarrow$ ZARADI STRESANJA

Wheelerjevi približki $\xrightarrow{Z_k=50\Omega}$

FR4: $\varepsilon_r \sim 4.5 \Longrightarrow \frac{w}{h} \approx 2$

teflonski laminat $\varepsilon_r \sim 2.5 \xrightarrow{Z_k=50\Omega} \frac{W}{h} \approx 3$

osnovni vod
~TEM
približno



} vzdolžni komponen
$\vec{E}, \vec{H}$

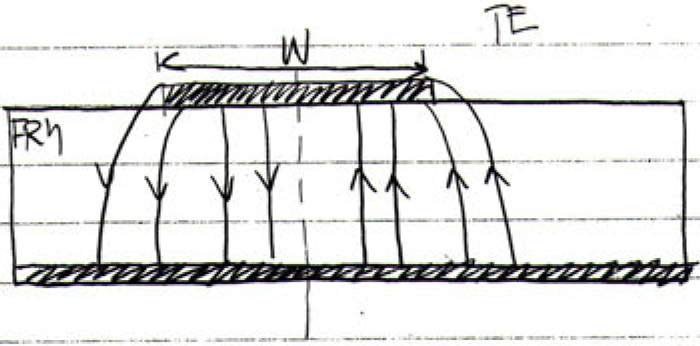
f(jeh)

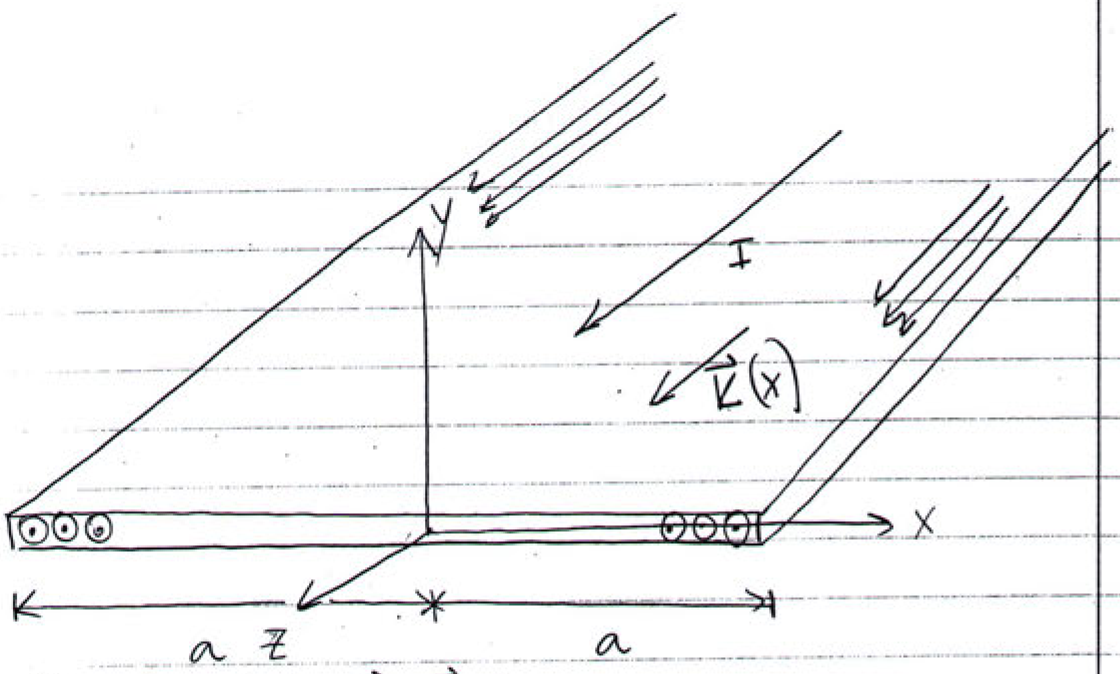
$c_0 \neq \frac{c_0}{\sqrt{\varepsilon_r}}$

ne morem definirati U

tudi Z_k se ne da definirati

naslednji višji red → f-ja W





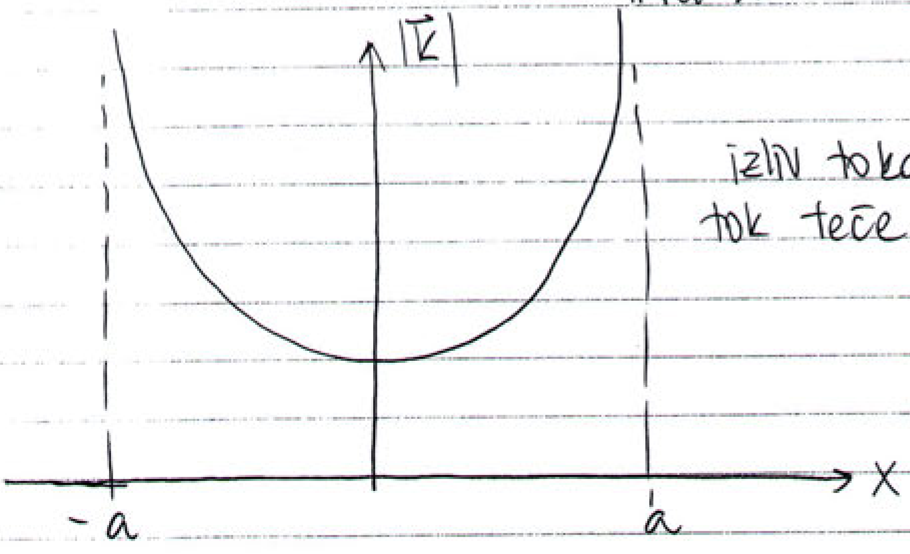
vektorski potencial: $\vec{A} = \vec{1}_z A_z$

na vodniku $\frac{\partial A_z}{\partial x} = 0$

valjni eliptični koordinatni sistem

$$\left.\begin{aligned} x &= a \cdot \cosh u \cdot \cos v \\ y &= a \cdot \sinh u \sin v \end{aligned}\right\} \frac{\partial}{\partial v} = 0$$

$$\Rightarrow \vec{k}(x) = \vec{1}_z \frac{I}{\pi\sqrt{a^2-x^2}}$$



izliv toka na rob

tok teče na robu

Vrsta voda	slabljenje
koaks @ 1GHz	~0.1 dB/m
mikrotrakasti vod	~1dB/m

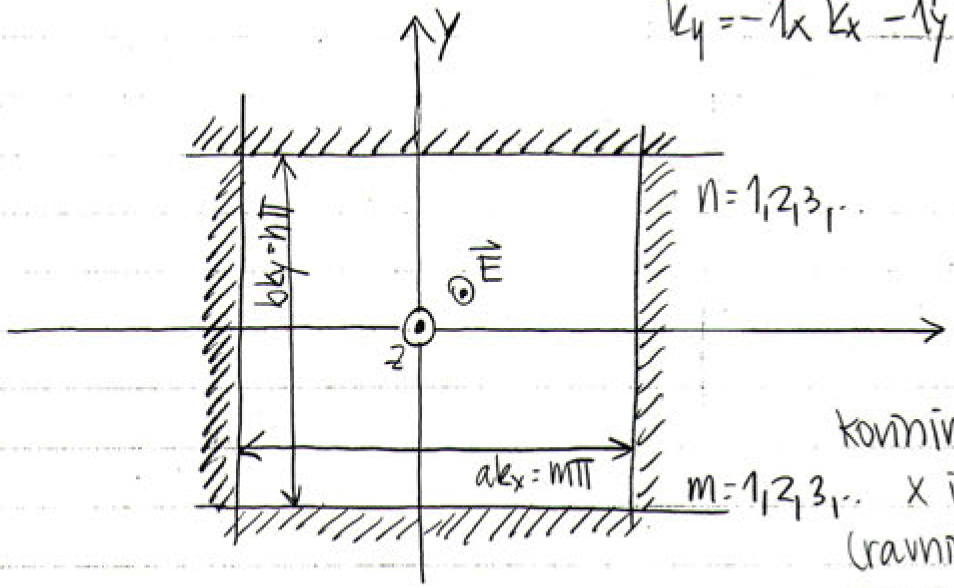
2D stojni val

$$\vec{E} = \vec{1}_z C \cos k_x x \; C \cdot \cos k_y y = \vec{1}_z C \cdot \underbrace{\frac{1}{2}\left(e^{jk_x x} + e^{-jk_x x}\right) \frac{1}{2}\left(e^{jk_y y} + e^{-jk_y y}\right)}$$

$$\vec{k}_1 = \vec{1}_x k_x + \vec{1}_y \cdot k_y$$
$$\vec{k}_2 = -\vec{1}_x k_x + \vec{1}_y k_y$$
$$\vec{k}_3 = \vec{1}_x k_x - \vec{1}_y k_y$$
$$\vec{k}_4 = -\vec{1}_x k_x - \vec{1}_y k_y$$

Iz stirih valov lahko sestavimo 2D stojni val

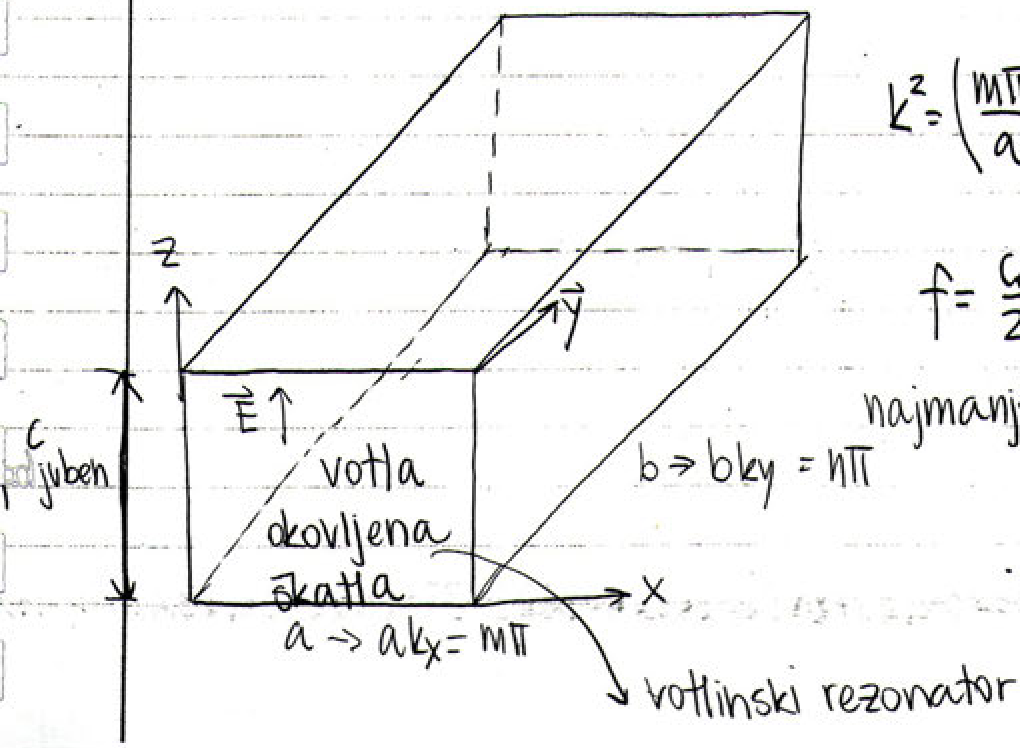
$$k_x^2 + k_y^2 = k^2 = \omega^2 \mu \varepsilon$$



$$k_x = \frac{m\pi}{a}$$

$$k_y = \frac{n\pi}{b}$$

kovinimo lahko po x in y osi (ravnine ⊥ na x in ravnine ⊥ na y)



$$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \frac{\omega^2}{c_0^2} = \left(\frac{2\pi f}{c_0}\right)^2$$

$$f = \frac{c_0}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$?

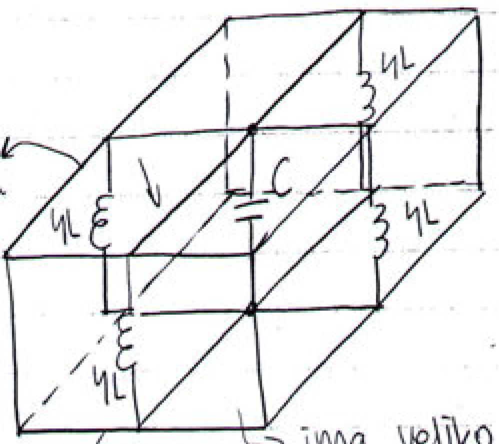
najmanjša f m=1, n=1

$\vec{E} \parallel \vec{1}_z$ } TM_{onm} RESONANCE
$\vec{H} \perp \vec{1}_z$ } $\quad z\,y\,x$ ZA VOTLINSKI REZONATOR

3D Stojni val

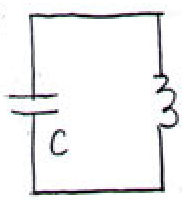
TM011

val se odbija od sten in kroži



$$f = \frac{c_0}{2\,(\sqrt{\varepsilon_r})} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{l}{c}\right)^2}$$

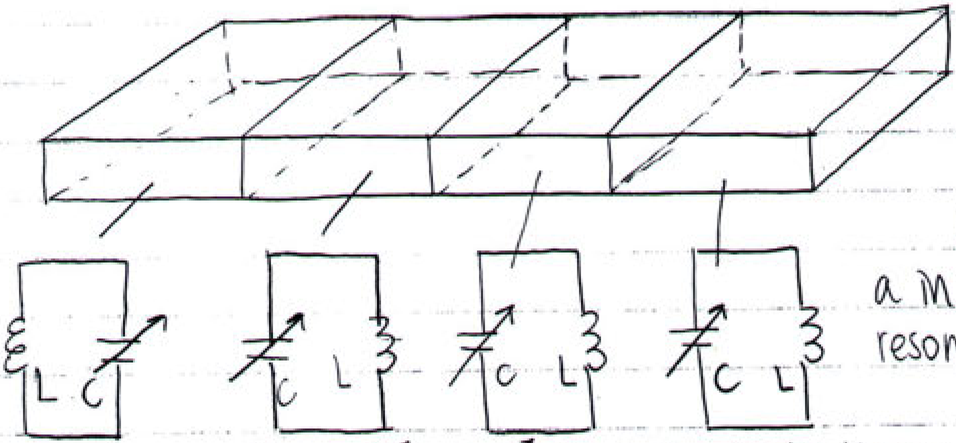
ima veliko resonančnih frekvenc za različne n in m



nihajni krog

3D rezonator → 8 valov ?

4 rezonatorji



TM011
a = 3,5 cm
b = 2,4 cm
c = 1,6 cm

a in b dasta najmanjšo resonančno frekvenco

pasivno sito $\vec{f_0} \rightarrow \vec{C}$ vijaki, ki nastavljajo kapacitivnost
$\vec{\Delta f} \rightarrow \vec{M}$ vijaki za nastavljanje induktivnosti

$Im(\xi)$ $y \rightarrow$ stojni val k_y
$Re(\xi)$ $z \rightarrow$ potujoči val $(\beta = k_z)$

$\vec{k}_1 = -\vec{1}_y k_y + \vec{1}_z \beta$ $\qquad \vec{k}_2 = \vec{1}_y k_y + \vec{1}_z \beta$

$$\vec{E} = \vec{1}_x \cdot C \cos k_y y \, e^{-j\beta z} = \vec{1}_x \cdot C \frac{1}{2}\left(e^{jk_y y} + e^{-jk_y y}\right) e^{-j\beta z}$$

$$\vec{H} = \frac{j}{\omega\mu} \begin{vmatrix} \vec{1}_x & \vec{1}_y & \vec{1}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cos k_y y \, e^{-j\beta z} & 0 & 0 \end{vmatrix} =$$

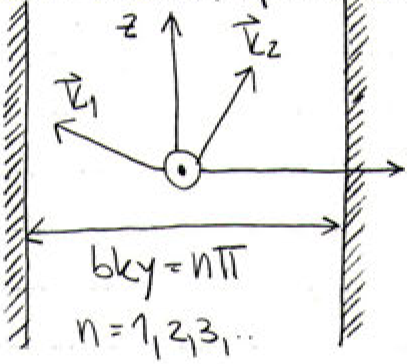
↱ komponenti sta med sabo v kvadraturi

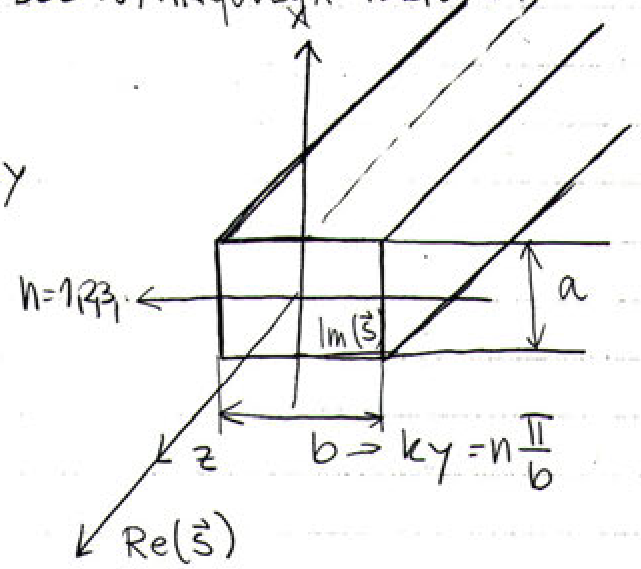
$$= \vec{1}_y \frac{-\beta}{\omega\mu} C \cos k_y y\, e^{-j\beta z} + \vec{1}_z \frac{-jk_y}{\omega\mu} \sin k_y y\, e^{-j\beta z}$$

$$\vec{S} = \frac{1}{2}\vec{E}\times\vec{H}^* = \vec{1}_z \frac{-\beta}{\omega\mu} CC^* k_y y + \vec{1}_y \frac{jk_y}{\omega\mu} CC^* \cos k_y y \sin k_y y$$

POTUJOČI VAL.. REALNI DEL POYTINGOVEGA VEKTORJA

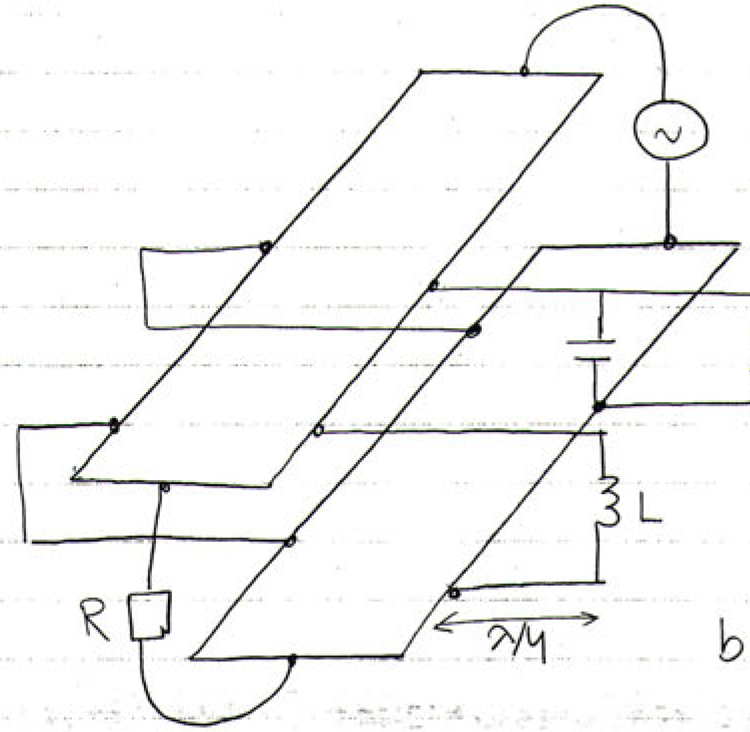
STOJNI VAL.. IMAGINARNI DEL POYTINGOVEGA VEKTORJA





b.. omejen
a.. ni omejen

DRSTIČNIKI.. njihov vpliv
Izračunamo, ko L in C spravimo v resonanco



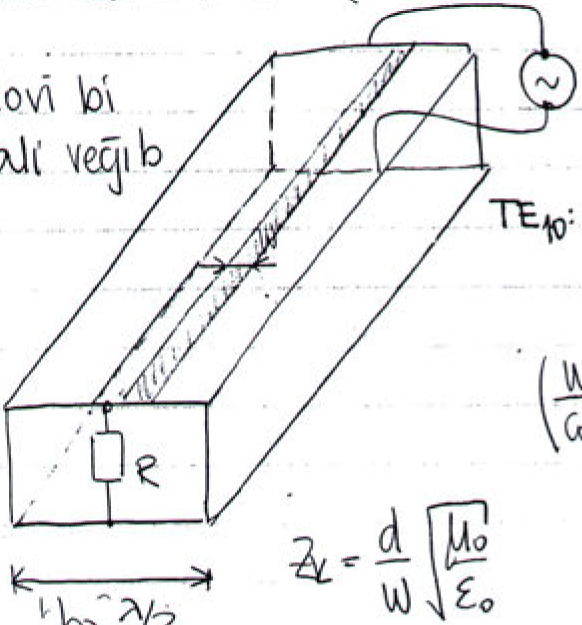
L, C : resonanca

$$b = \frac{\lambda}{4} + w + \frac{\lambda}{4}$$

Če imamo dovolj drstionikov dobimo pravokotno cev

kovinski valovod (metal waveguide):

višji rodovi bi potrebovali večji b



$\beta = \sqrt{k^2 - k_\rho^2} = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

prva rešitev n=1

TE_{10}: $\beta = \sqrt{\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\pi}{b}\right)^2}$

$\left(\frac{\omega}{c_0}\right)^2 - \left(\frac{\pi}{b}\right)^2 \geq 0 : \left(\frac{\omega}{c_0}\right)^2 \geq \left(\frac{\pi}{b}\right)^2$

$Z_k = \frac{d}{w}\sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\frac{2\pi}{\lambda} = \frac{2\pi R}{c_0} \geq \frac{\pi}{b} \Rightarrow b \geq \frac{\lambda}{2}$

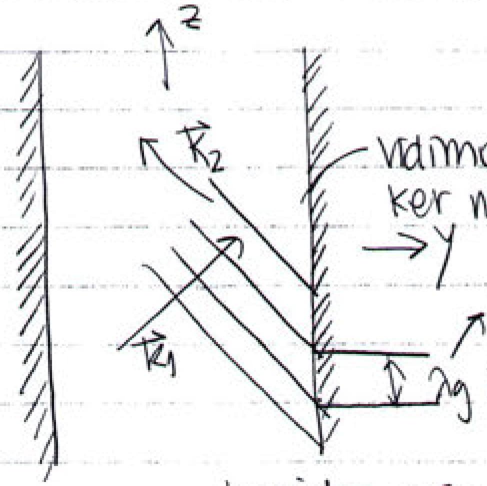
$Z_k \sim 500\,\Omega$

fazna hitrost .. kako hitro se spreminja faza

$v_f = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\left(\frac{\omega}{c_0}\right)^2 - \left(n\frac{\pi}{b}\right)^2}} = c_0\frac{k}{\beta}$

$c_0 = \frac{\omega}{k} \Rightarrow \omega = c_0 k$

$\beta \leq k \Rightarrow \boxed{v_f \geq c_0} \quad \omega \geq \frac{c_0\pi}{b}$



$\lambda_g = \frac{2\pi}{\beta} \geq \lambda = \frac{2\pi}{k}$

vidimo večjo valovno dolžino kot v resnici, ker ne opazujemo v smeri valovanja

$\lambda_g \geq \lambda \Rightarrow$ zato smo dobili, da je $v_g \geq c_0$

navidez večja valovna dolžina

Ko govorimo o hitrosti v kovinskem valovodu moramo biti pozorni ali je to fazna ali skupinska hitrost

Skupinska hitrost

$$v_g = \lim_{\Delta w \to 0} \frac{\Delta w}{\Delta \beta} = \frac{dw}{d\beta} = \underline{\underline{c_0 \frac{\beta}{k} \leq c_0}} \; !$$

↑
group

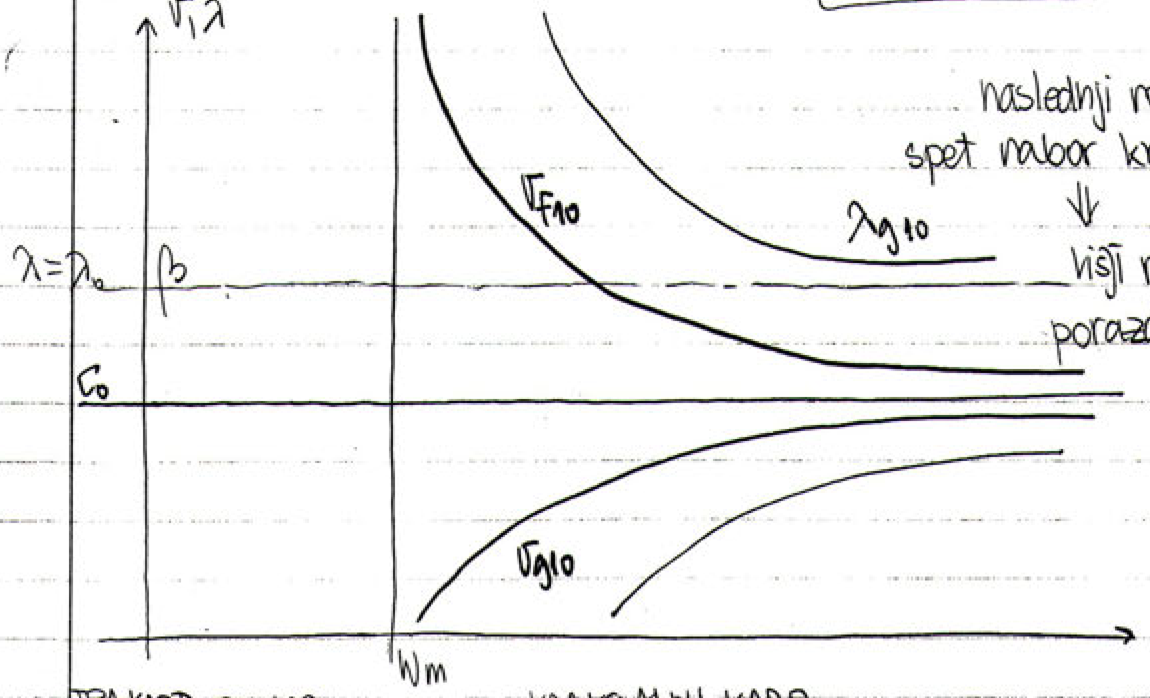
$$\frac{d\beta}{dw} = \frac{d}{dw}\left(\sqrt{\left(\frac{w}{c_0}\right)^2 - \left(n\frac{\pi}{b}\right)^2}\right) = \frac{\cancel{2}\left(\frac{w}{c_0}\right)\frac{1}{c_0}}{\cancel{2}\sqrt{\left(\frac{w}{c_0}\right)^2 - \left(n\frac{\pi}{b}\right)^2}} = \underline{\underline{\frac{w}{c_0^2 \beta}}} =$$

$$\left.\begin{matrix} TE_{nm} \\ TM_{nm} \end{matrix}\right\} \Rightarrow \begin{matrix} \beta_{TE_{nm}} \Rightarrow \overset{\text{mejni}}{w_m} = \overset{\text{kritični}}{w_k} = w_0 \\ \beta_{TM_{nm}} \Rightarrow w_m = w_k = w_0 \end{matrix} \qquad \left(\frac{w_0}{c_0}\right)^2 + \left(n\frac{\pi}{b}\right)^2 = 0$$

$$TE_{10}: \quad \frac{w_m}{c_0} = \frac{\pi}{b} \qquad \boxed{\beta = \frac{1}{c_0}\sqrt{w^2 - w_m^2}} \; !$$

$$\boxed{w_m = c_0 \frac{\pi}{b}}$$

$$\boxed{\beta} = \sqrt{\left(\frac{w}{c_0}\right)^2 - \left(\frac{\pi}{b}\right)^2} = \sqrt{\left(\frac{w}{c_0}\right)^2 - \left(\frac{w_m}{c_0}\right)^2} = \boxed{\frac{\sqrt{w^2 - w_m^2}}{c_0}}$$



naslednji rod ima spet nabor krivulj
⇓
višji rod.. različno porazdelitev polja

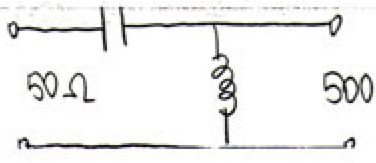
TRAKASTI DVOVOD

$$z_c = \frac{d}{w}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

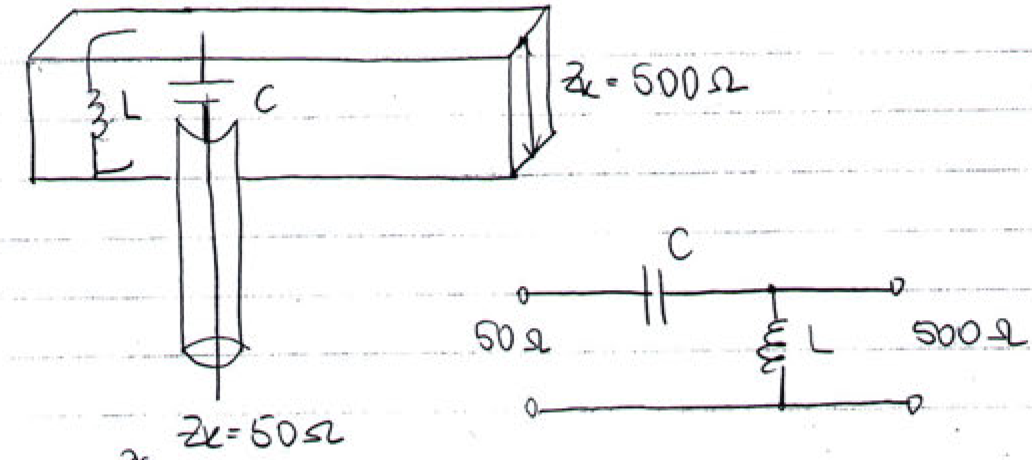
$$z_c \sim 50\Omega$$

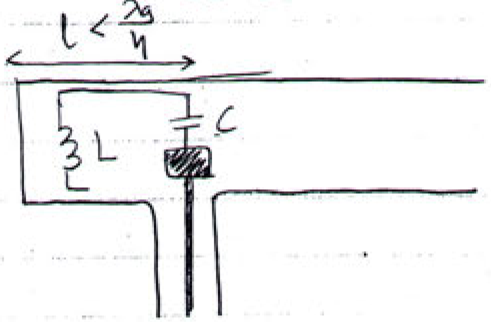
KOAKSIALNI KABEL

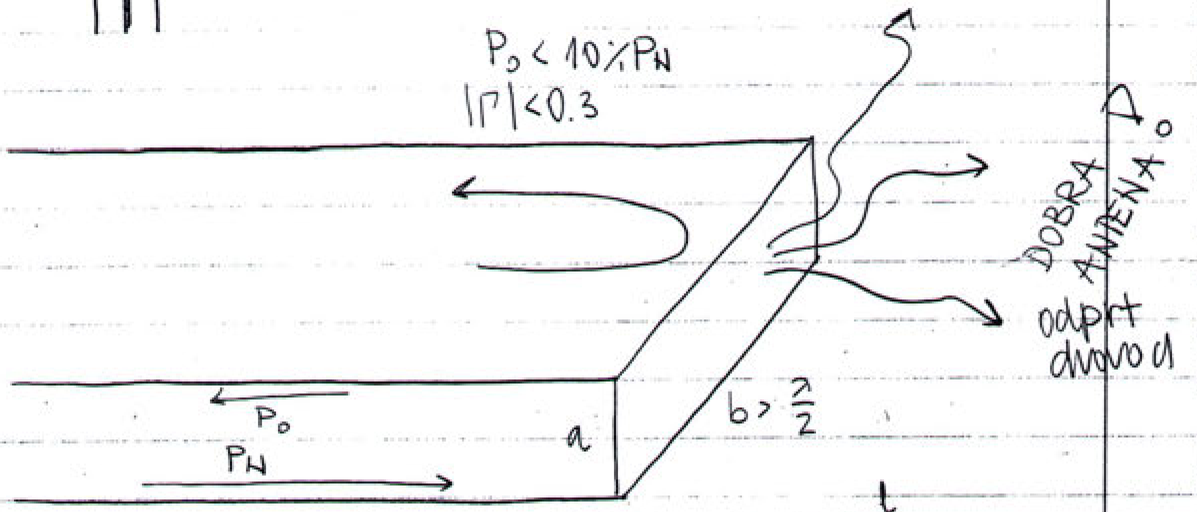
$$l < \lambda_g / n$$



transformacija impedance

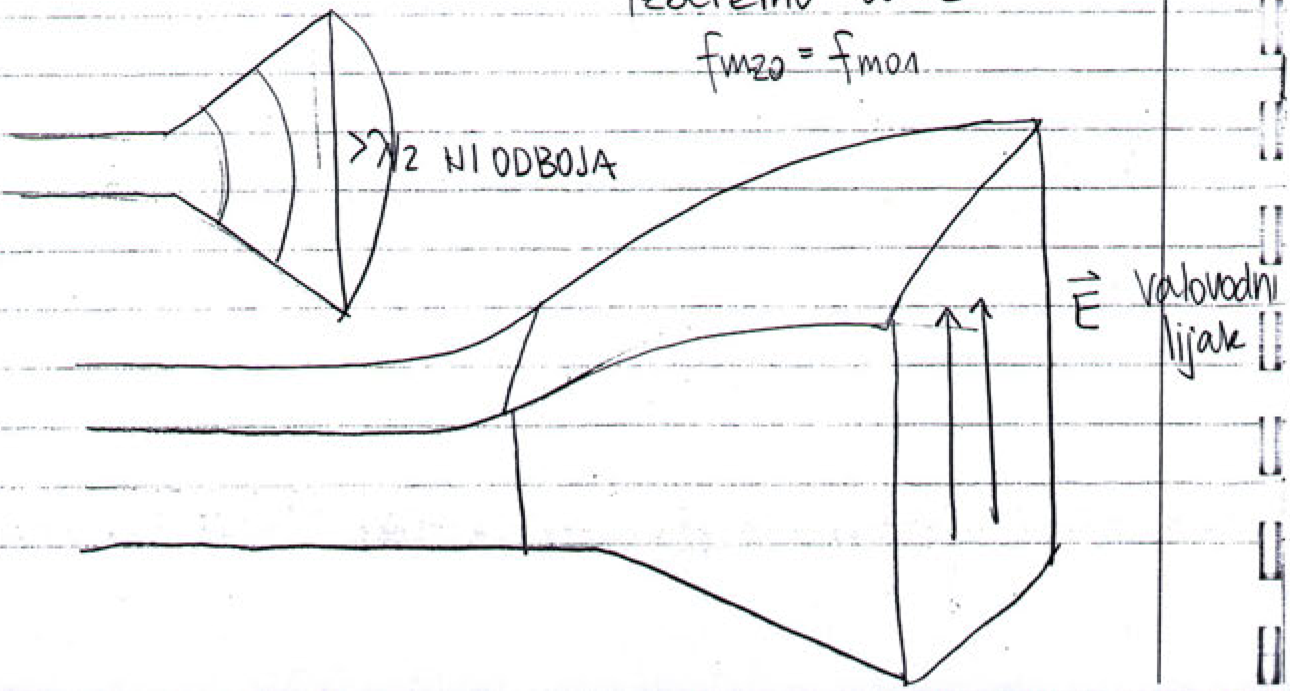






izberemo $a \sim \frac{\lambda}{2}$

$f_{m20} = f_{m01}$



impedanca:

upornost

$$Z = \frac{U}{I} = \frac{E \cdot l}{K \cdot 2\pi a \gamma} = \frac{l}{2\pi a \gamma}(\alpha + j\beta) = R + jX$$

izgube ?

dolžina upora

$$R = \frac{l}{2\pi a}\,\frac{\alpha}{\gamma} = \frac{l}{w} R_p = \frac{l}{2\pi a}\sqrt{\frac{w\mu}{2\gamma}}$$

širina upora

$$\alpha = \sqrt{\frac{w\mu\gamma}{2}}$$

Plastna upornost:

$$R_p = R_{\square} = \sqrt{\frac{w\mu}{2\gamma}}$$

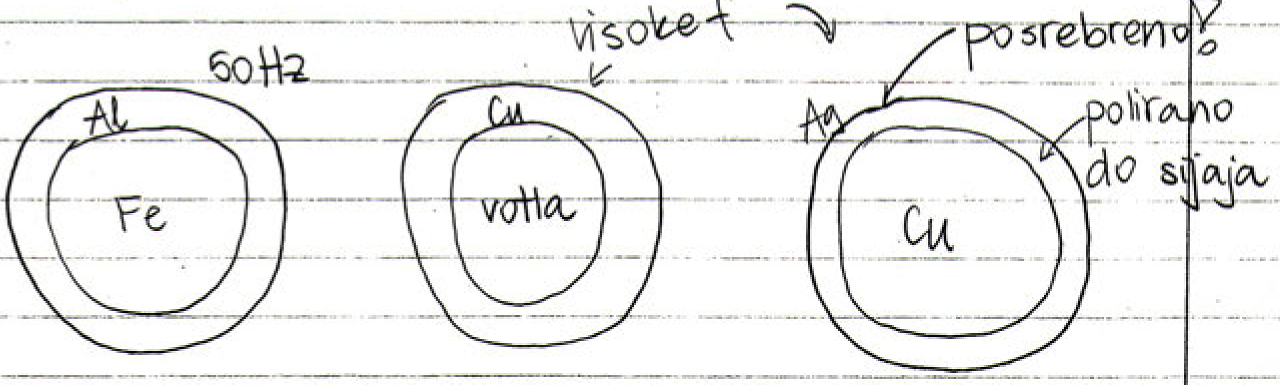
enosmerna: $w = 0$: (tok po celem preseku vodnika)

$$R_{=} = \frac{l}{\underbrace{\pi a^2}_{krog}} \cdot \frac{1}{\gamma} \ll R = \underbrace{\frac{l}{2\pi a}\sqrt{\frac{w\mu}{2\gamma}}}_{} = \frac{l}{\underbrace{2\pi a\delta}_{kolobar}} \cdot \frac{1}{\gamma}$$

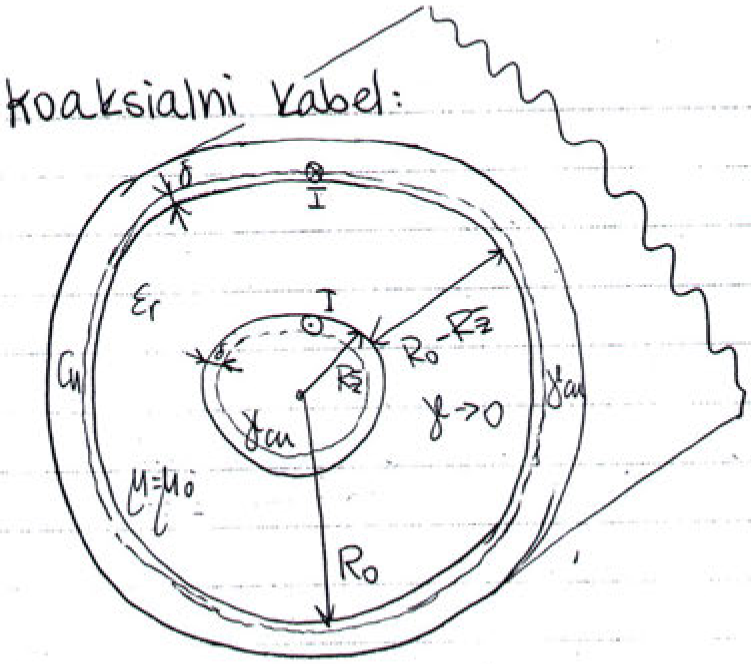
tok samo po koži debeline δ

↳ za visoke frekvence $w \sim 100$ MHz ⇒ več izgub

$$\delta = \sqrt{\frac{2}{w\mu\gamma}}$$



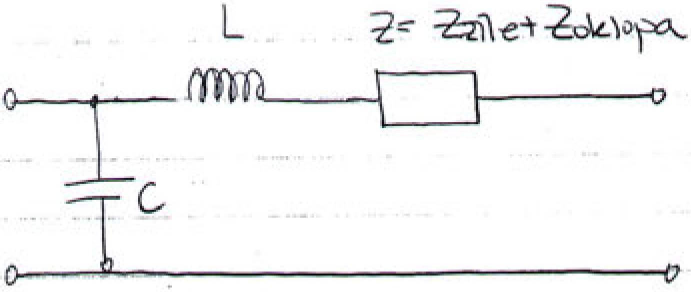
koaksialni kabel:



$R_o - R_ž \gg \delta$

$w L \gg X_{žile}, X_{oklopa}$
zanemarimo

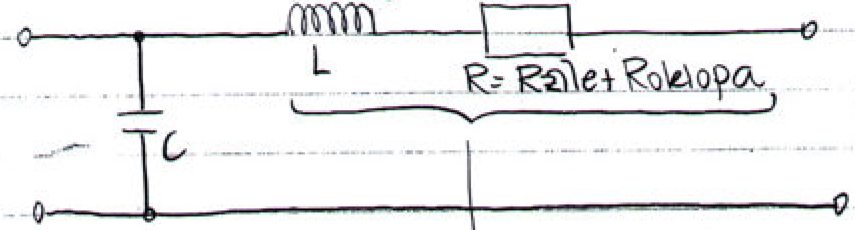
nadomestna vezava:



$C/l = 2\pi\varepsilon / \ln \frac{R_o}{R_ž}$; $L/l = \frac{\mu}{2\pi} \ln \frac{R_o}{R_ž}$

$Z_{žile}/l = \boxed{R_{žile}} + j X_{žile}$; $Z_{oklopa}/l = \boxed{R_{oklopa}} + j X_{oklopa}$

=> nadomestno vezje bo:



$L \gg \frac{R}{jw}$

$\to L' = L + \frac{R}{jw}$

brezizgubni kabel

karakteristicna impedanca: $Z_k = \sqrt{\frac{L/l}{C/l}}$

fazna konstanta: $\beta = w\sqrt{L/l \cdot C/l} = K$

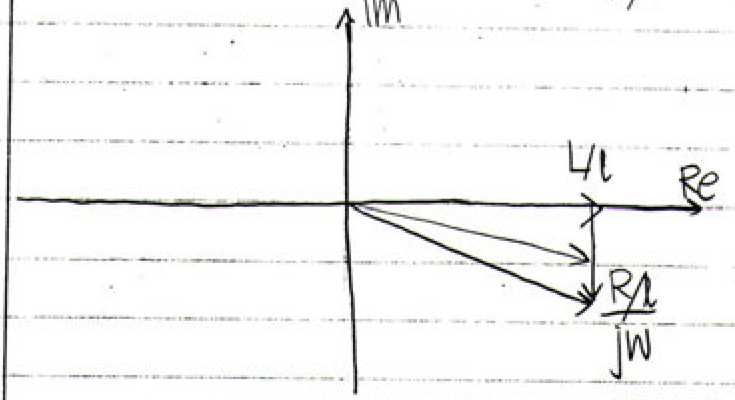
Izgubni kabel:

$$Z_k = \sqrt{\frac{L/l + \frac{R/l}{jw}}{C/l}} \approx \sqrt{\frac{L/l}{C/l}}$$

$$k = w\sqrt{\left(L/l + \frac{R/l}{jw}\right)\cdot C/l} = \beta - \overset{\text{IZGUBE!}}{\boxed{j\alpha}} \quad \begin{aligned} &\rightarrow \beta \approx w\sqrt{L/l \cdot C/l} \\ &\Rightarrow \alpha \neq 0 \end{aligned}$$

$$\alpha \approx \beta \cdot \frac{R/l}{2w\, L/l} = w\sqrt{L/l \cdot C/l}\;\frac{R/l}{2w\, L/l} =$$

$$= \frac{R/l}{2\sqrt{\frac{L/l}{C/l}}} = \underline{\underline{\frac{R/l}{2Z_k}}}$$



$$\frac{R/l}{w} \ll L/l$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

$x \ll 1$

$$k = w\sqrt{\left(L/l + \frac{R/l}{jw}\right)(C/l)} = w\sqrt{(L/l)(C/l)\left(1 - j\frac{R/l}{w\,L/l}\right)} \approx$$

$$\approx w\sqrt{(L/l)(C/l)}\left(1 - j\frac{R/l}{2w\,L/l}\right)$$

$$\alpha = \frac{R/l}{2Z_k} \; ; \quad U(l) = U(0)\, e^{-j\beta l}\, e^{-\alpha l}$$

$$a_{dB/m} = 20\log_{10}\left|\frac{U(0)}{U(l)}\right| = 20\log_{10} e^{\alpha l} \; [dB]$$

$$a = \frac{20}{\ln 10}\alpha l = \frac{20}{\ln 10}\frac{R/l}{2Z_k} l$$

$$a/l = \frac{10}{\ln 10}\frac{R/l}{Z_k}\left[\frac{dB}{m}\right]$$

$$\boxed{Z_k = \frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln\frac{R_0}{R_{\check{z}}}}$$

$$R/l = R_{\check{z}}/l + R_0/l = \frac{1}{2\pi R_{\check{z}}\delta\gamma_{cu}} + \frac{1}{2\pi R_0\delta\gamma_{cu}}$$

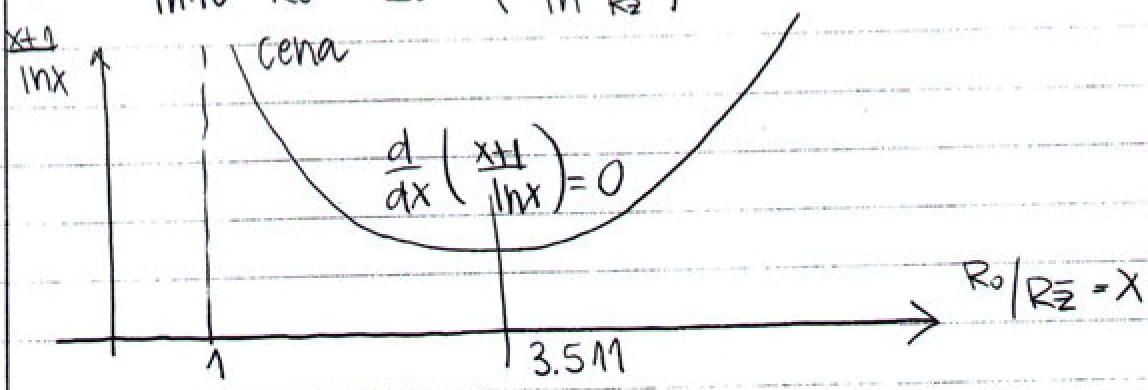
$$R/l = \frac{1}{2\pi}\sqrt{\frac{\omega\mu}{2\gamma_{cu}}}\left(\frac{1}{R_{\check{z}}} + \frac{1}{R_0}\right)$$

$$a/l = \frac{10}{\ln 10}\frac{\frac{1}{2\pi}\sqrt{\frac{\omega\mu}{2\gamma_{cu}}}\left(\frac{1}{R_{\check{z}}}+\frac{1}{R_0}\right)}{\frac{Z_0}{2\pi\sqrt{\varepsilon_r}}\ln\frac{R_0}{R_{\check{z}}}} = \underline{\underline{\frac{10}{\ln 10}\sqrt{\varepsilon_r}\frac{\sqrt{\frac{\omega\mu}{2\gamma}}}{Z_0}\frac{\left(\frac{1}{R_{\check{z}}}+\frac{1}{R_0}\right)}{\ln\frac{R_0}{R_{\check{z}}}}\left[\frac{dB}{m}\right]}}$$

$\varepsilon_r \to 1$; votel dielektrik (penast dielektrik)

cena $\sim R_0^2$

$$a/l = \frac{10}{\ln 10}\frac{\sqrt{\varepsilon_r}}{R_0}\frac{\sqrt{\frac{\omega\mu}{2\gamma}}}{Z_0}\left(\frac{\frac{R_0}{R_{\check{z}}}+1}{\ln\frac{R_0}{R_{\check{z}}}}\right)\left[\frac{dB}{m}\right]$$



$f = 100$ MHz $\delta = 6.8\cdot 10^{-6}$ m $R_{\check{z}} = 0.5$ mm $\varepsilon_r = 2$ polietilen

$\gamma_{cu} = 56\cdot 10^6$ S/m $R_0 = 1.75$ mm $Z_k = 50\,\Omega$ @ $\frac{R_0}{R_{\check{z}}} = 3.5$

rešitev valovne enačbe brez izvorov:

BREZIZGUBNA SNOV

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0$$

$$\rho = 0,\ \vec{J} = 0$$

SNOV Z IZGUBAMI $\vec{J} \neq 0,\ \vec{J} = \gamma\vec{E}$

① M.E. $\text{rot}\,\vec{H} = \vec{J} + j\omega\varepsilon\vec{E}$

$$\text{rot}\,\vec{H} = \gamma\vec{E} + j\omega\varepsilon\vec{E}$$

$$\varepsilon = \varepsilon_0\varepsilon_r$$

$$\text{rot}\,\vec{H} = j\omega\varepsilon_0\underbrace{\left(\frac{\gamma}{j\omega\varepsilon} + \varepsilon_r\right)}_{\varepsilon_r'}\vec{E} = j\omega\varepsilon_0\varepsilon_r'\vec{E} = j\omega\varepsilon'\vec{E}$$

$$\varepsilon_r' = \varepsilon_r - j\frac{\gamma}{\omega\varepsilon_0}$$

enako velika?

$$\varepsilon_r = \frac{\gamma}{\underset{\substack{\uparrow \\ \text{mejni}}}{\omega_m}\varepsilon} \Rightarrow \underline{\underline{\omega_m}} = \frac{\gamma}{\varepsilon_0\varepsilon_r} = \underline{\underline{\frac{\gamma}{\varepsilon}}} \quad ; \varepsilon_0 \sim \frac{1}{4\pi\cdot 9\cdot 10^9}\ \frac{As}{Vm}$$

$$\Delta\vec{E} + \omega^2\mu\varepsilon'\vec{E} = 0 \rightarrow \beta > \alpha$$

$$\Delta\vec{E} + \omega^2\mu\varepsilon\vec{E} - j\omega\mu\gamma\vec{E} = 0$$

pri visokih frekv. dielektrik

prevladа pri nizkih frekvencah prevodnik

PRIMERJAVA: velikostni razred

	snov	ε_r	γ	$f_m = \frac{\omega_m}{2\pi}$
prevodnik	kovina Cu	1	$56\cdot 10^6$ S/m	10^{18} Hz $= 10^6$ THz podrocje rentgenskih žarkov
	morska voda $\mu = \mu_0$	80	5 S/m	$1.2\cdot 10^9$ Hz $= 1.2$ GHz
dielektrik	čisti SiO_2 ↑ čisto kremenovo steklo			1/100 let $\approx 10^{-9}$ Hz

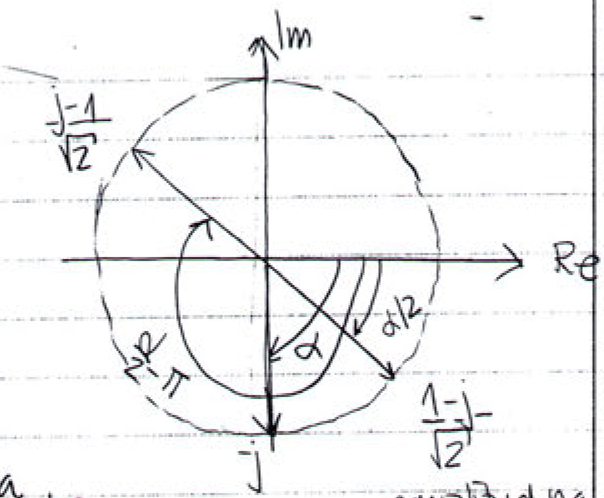
dober prevodnik $\Rightarrow \beta = \alpha$

$\varepsilon_r \ll \frac{\gamma}{\omega \varepsilon_0}$

$\omega \varepsilon \ll \gamma$

približek za valovno enačbo:

$$\Delta \vec{E} - \underbrace{j\omega\mu\gamma}_{k^2} \vec{E} = 0$$



$$\Delta \vec{E} + k^2 \vec{E} = 0$$

fazna konstanta

amplitudna konstanta

$$k^2 = -j\omega\mu\gamma \Rightarrow k = \sqrt{\omega\mu\gamma} \cdot \sqrt{-j} = \pm\left(\sqrt{\frac{\omega\mu\gamma}{2}} - j\sqrt{\frac{\omega\mu\gamma}{2}}\right) = \pm(\beta - j\alpha)$$

$$\vec{E} = \vec{1}_x C e^{-jkz} = \vec{1}_x C e^{\mp j\left(\sqrt{\frac{\omega\mu\gamma}{2}} - j\sqrt{\frac{\omega\mu\gamma}{2}}\right)z}$$

$$\vec{E} = \vec{1}_x C \underbrace{e^{\mp j\sqrt{\frac{\omega\mu\gamma}{2}} \cdot z}}_{\text{faza}} \underbrace{e^{\mp \sqrt{\frac{\omega\mu\gamma}{2}} \cdot z}}_{\text{eksponentno usihanje}}$$

čisto imaginarno

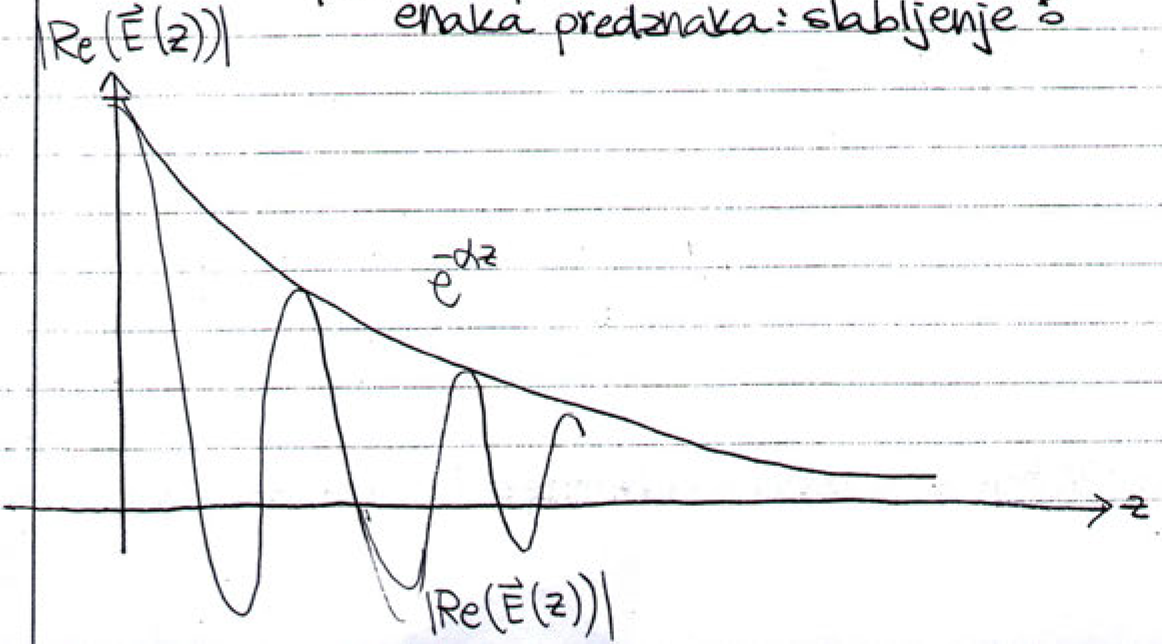
ali eksponentno naraščanje

$$\boxed{\vec{E} = \vec{1}_x C \underbrace{e^{\mp j\beta z}}_{\text{faza}} \underbrace{e^{\mp \alpha z}}_{\text{amplituda}}}$$

(-) napredujoči val v smeri +z

(+) odbiti val v smeri -z

enaka predznaka: slabljenje!



Vdorna gobina δ $(\alpha \neq 0)$

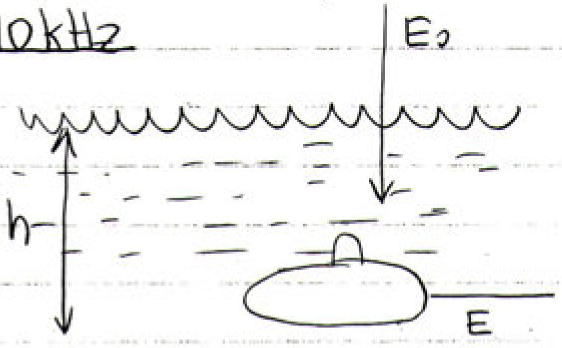
$$|\vec{E}(\delta)| = \frac{|E(0)|}{e} = \frac{1}{e}|E(0)|$$

Zgled	α, β
dober prevodnik $\gamma >> \omega\varepsilon$	$\alpha = \beta = \sqrt{\frac{\omega\mu\gamma}{2}}$
vmesni primer $\gamma \sim \omega\varepsilon$	$\alpha < \beta$
dober dielektrik $\gamma << \omega\varepsilon$	$\beta = \omega\sqrt{\mu\varepsilon}$, $\alpha \to 0$

$$\alpha\delta = 1$$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\underset{\swarrow \gamma}{\cancel{\sigma}}}} \quad (\gamma >> \omega\varepsilon)$$

f=10kHz



$$E = E_0 e^{-\alpha h} = E_0 e^{-h/\delta}$$

$$\delta = \sqrt{\frac{2}{2 \cdot \pi \cdot \underset{\overset{\uparrow}{f}}{10^4}\, s^{-1} \cdot 4\pi \cdot 10^{-7} \frac{\cancel{V}s}{\cancel{A}m} \cdot 5 \frac{\cancel{A}}{\cancel{V}m}}} = \sqrt{5m^2} = \underline{\underline{2.2m}}$$

slabljenje

$$\hookrightarrow a = 20\log_{10}\frac{E_0}{E} = 20\log_{10} e^{+h/\delta} = 20\frac{\ln e^{h/\delta}}{\ln 10} = \frac{20}{\ln 10}\frac{h}{\delta} =$$

$$= \frac{20}{2.305} \cdot \frac{30m}{2.2m} = \underline{\underline{110dB}} \,!$$

Vdorna globina v kovine

Cu: $\mu = \mu_0$ $(\mu_r = 1)$ $\quad \gamma_{cu} = 56 \cdot 10^6$ S/m

Fe: $\mu_r \sim 1000$ $\quad \gamma_{Fe} \sim 1/10 \cdot \gamma_{cu}$

	frekvenca	δ_{cu}	δ_{Fe}	frekvenca	δ_{cu}	δ_{Fe}	
ENERGETIKA	1Hz	6.8cm	6.8mm	1THz	68nm	6.8nm	2
	100 Hz	6.8mm	0.68mm	100 THz	? 6.8nm	0.68nm	°
	10kHz	0.68mm	68μm	svetloba	dimenzije atomov		
ELEKTRONIKA	1MHz radio	68 μm	6.8μm	50Hz	10mm	1mm	
	100MHz	6.8μm	0.68μm				
	10GHz	0.68μm	68nm				

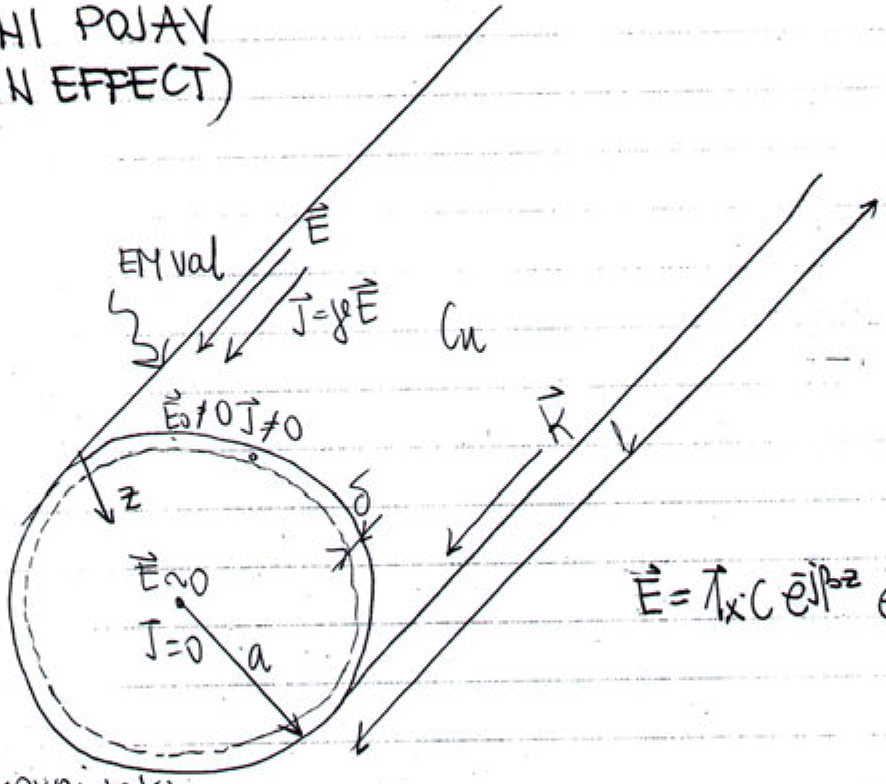
$\underline{f=1\,Hz}$

$$\delta_{cu} = \sqrt{\frac{2}{2\pi \cdot \cancel{s^{-1}} \cdot 4\pi \cdot 10^{-7} \cancel{\frac{Vs}{Am}} \cdot 56 \cdot 10^{6} \cancel{\frac{A}{Vm}}}} = \sqrt{\frac{2m^2}{448}} = \frac{1\,m}{\sqrt{224}}$$

$$\underline{\delta_{cu} = 6.8\,cm \ @\ 1\,Hz}$$

$$\delta_{Fe} = 1/10\ \delta_{cu}$$

KOŽHI POJAV
(SKIN EFFECT)



$$\vec{E} = \vec{1}_x C e^{-j\beta z} e^{-\alpha z}$$

ploskovni tok:

$$\vec{K} = \int_0^\infty \vec{J}\,dz = \int_0^\infty \gamma \vec{E}\,dz = \int_0^\infty \vec{1}_x \gamma C e^{-j\beta z} e^{-\alpha z}\,dz$$

$$\vec{K} = \vec{1}_x \cdot C\gamma \int_0^\infty e^{-(\alpha+j\beta)z}\,dz$$

$$\vec{K} = \underbrace{\vec{1}_x \gamma C}_{\vec{E}(0)} \frac{-1}{\alpha+j\beta} e^{-(\alpha+j\beta)} \Big|_0^\infty = \vec{E}(0) \frac{\gamma}{\alpha+j\beta}$$

ploskovni tok:

$$\boxed{\vec{K} = \frac{\gamma}{\alpha+j\beta}\vec{E}} \qquad \frac{E}{K} = \alpha+j\beta$$