

Krivulje v ravnini

Za risanje *ravninskih* krivulj so naslednji ukazi:

Krivuljo podano v *eksplicitni* obliki narisemo z ukazom **Plot**.

Krivuljo podano v *implicitni* obliki narisemo z ukazom **ContourPlot**.

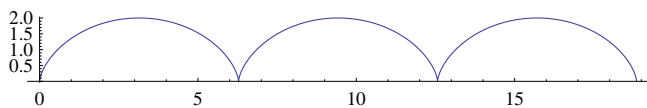
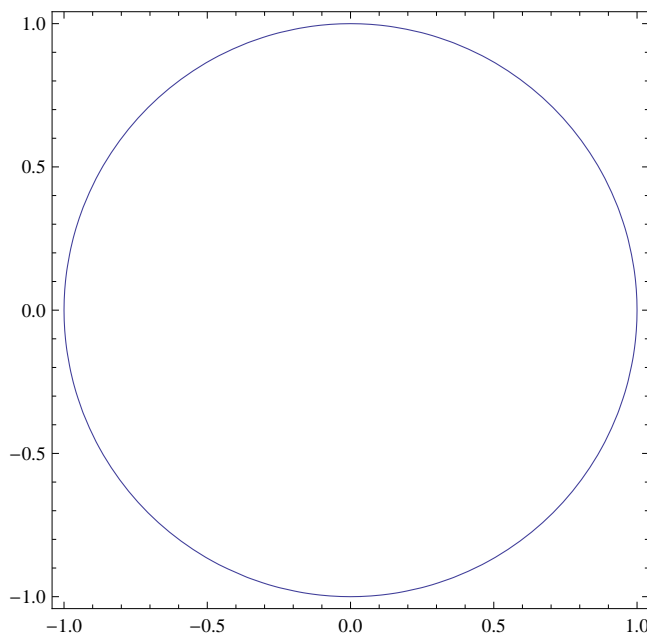
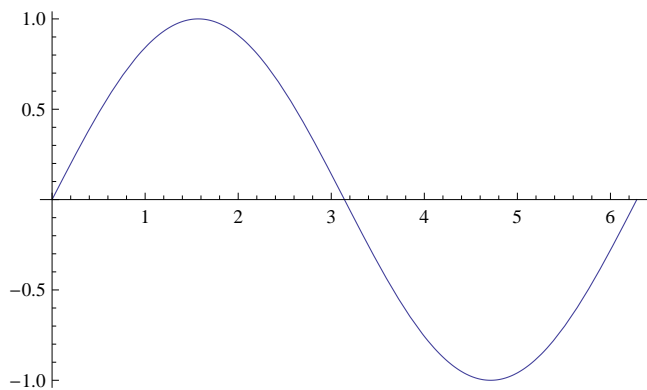
Krivuljo podano v *parametrični* obliki narisemo z ukazom **ParametricPlot**.

(* Primeri *)

```
Plot[Sin[x], {x, 0, 2 Pi}]
```

```
ContourPlot[x^2 + y^2 == 1, {x, -1, 1}, {y, -1, 1}]
```

```
ParametricPlot[{t - Sin[t], 1 - Cos[t]}, {t, 0, 6 Pi}]
```



```
? Plot
? ContourPlot
? ParametricPlot
```

`Plot[f, {x, xmin, xmax}` generates a plot of f as a function of x from x_{min} to x_{max} .
`Plot[{f1, f2, ...}, {x, xmin, xmax}` plots several functions f_i . >>

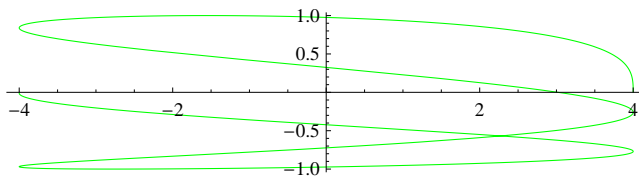
`ContourPlot[f, {x, xmin, xmax}, {y, ymin, ymax}` generates a contour plot of f as a function of x and y .
`ContourPlot[f == g, {x, xmin, xmax}, {y, ymin, ymax}` plots contour lines for which $f = g$.
`ContourPlot[{f1 == g1, f2 == g2, ...}, {x, xmin, xmax}, {y, ymin, ymax}` plots several contour lines. >>

`ParametricPlot[{fx, fy}, {u, umin, umax}` generates
 a parametric plot of a curve with x and y coordinates f_x and f_y as a function of u .
`ParametricPlot[{{fx, fy}, {gx, gy}, ...], {u, umin, umax}` plots several parametric curves.
`ParametricPlot[{fx, fy}, {u, umin, umax}, {v, vmin, vmax}` plots a parametric region.
`ParametricPlot[{{fx, fy}, {gx, gy}, ...], {u, umin, umax}, {v, vmin, vmax}` plots several parametric regions. >>

Slika v Mathematici je objekt, ki ga lahko poimenujemo. Obliko krivulje (barva, debelina ...) opisemo z opcijo **PlotStyle**.

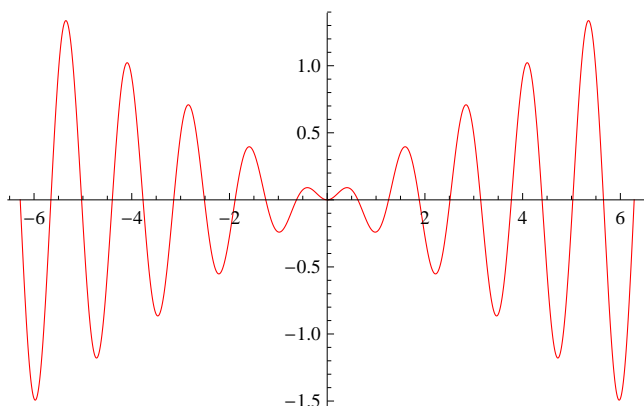
1.a) Narisi krivuljo, dano v parametrični obliki
 $x(t) = 4 \cos(t^{1.5})$, $y(t) = \sin(t)$. Krivulja naj bo zelene barve. Slika naj ima ime **slikal**.

```
Slikal = ParametricPlot[{4 Cos[t^1.5], Sin[t]}, {t, 0, 2 Pi}, PlotStyle -> {Green}]
```



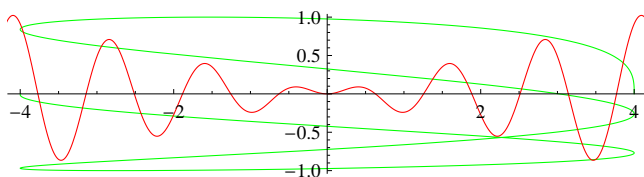
1.b) Narisi krivuljo $y(x) = x/4 \sin(5x)$. Krivulja naj bo rdečee barve. Slika naj ima ime **slika2**.

```
Slika2 = Plot[x / 4 Sin[5 x], {x, -2 Pi, 2 Pi}, PlotStyle -> {Red}]
```



1.c) S pomočjo ukaza **Show** narisi obe krivulji na isti sliki.

```
Show[Slika1, Slika2]
```



1.d) Epicikloida je krivulja, ki jo opiše točka kroznice, ki se kotali po drugi kroznici zunaj nje.

Njena parametrična enačba je

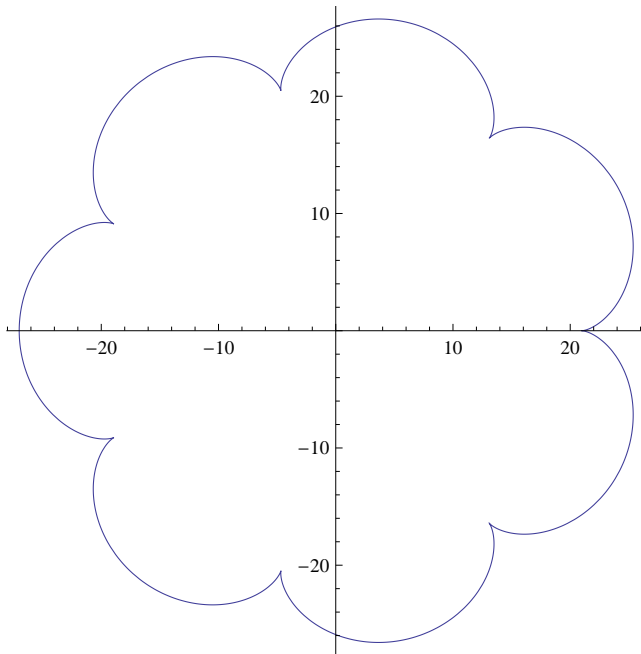
$$x = (R+r)\cos\varphi - r\cos\left(\frac{R+r}{r}\varphi\right), \quad y = (R+r)\sin\varphi - r\sin\left(\frac{R+r}{r}\varphi\right)$$

kjer so R =polmer mirujoče krožnice, r =polmer kotaleče kroznice in φ =polarni kot.

Narisi epicikloido s pomočjo ukaza **ParametricPlot**! Razmerje polmerov R/r naj bo celo stevilo.

Merilo v koordinatah x in y se uravnava z opcijo **AspectRatio** (glej "Help").

```
R = 21  
r = 3  
ParametricPlot [  
  {(R+r) Cos[φ] - r * Cos[(R+r) / r * φ], (R+r) Sin[φ] - r * Sin[(R+r) / r * φ]}, {φ, 0, 2 Pi}]  
21  
3
```



Krivulje v prostoru in Ploskve

Za risanje *tri-dimezijskih* objektov uporabljamo ukaza **Plot3D** in **ParametricPlot3D**.

`? Plot3D``? ParametricPlot3D`

`Plot3D[f, {x, xmin, xmax}, {y, ymin, ymax}` generates a three-dimensional plot of f as a function of x and y .

`Plot3D[{f1, f2, ...}, {x, xmin, xmax}, {y, ymin, ymax}` plots several functions. >>

`ParametricPlot3D[{fx, fy, fz}, {u, umin, umax}` produces a

three-dimensional space curve parametrized by a variable u which runs from u_{min} to u_{max} .

`ParametricPlot3D[{fx, fy, fz}, {u, umin, umax}, {v, vmin, vmax}` produces a three-dimensional surface parametrized by u and v .

`ParametricPlot3D[{fx, fy, fz}, {gx, gy, gz} ...]` plots several objects together. >>

3. Narisi krivuljo $(t \cos t, t \sin t, t)$

s pomočjo funkcije `ParametricPlot3D` z zeleno barvo.

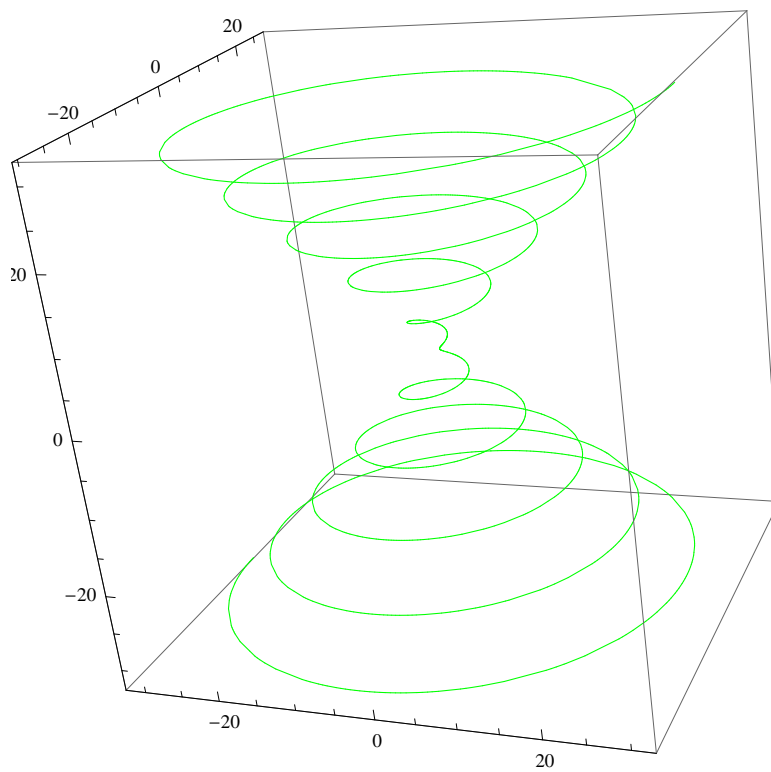
Izračunaj enačbo tangente na to krivuljo v točki $\left(\frac{\pi \sqrt{2}}{8}, \frac{\pi \sqrt{2}}{8}, \frac{\pi}{4}\right)$.

Tangento tudi narisi z modro barvo in
prokazi oba objekta v isti sliki.

Rezultat (enačba tangente): $\left\{ \frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}} \right) t, \frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} \right) t, \frac{\pi}{4} + t \right\}$

```
Clear[er, R, t]
er[t_] = {t Cos[t], t Sin[t], t}
slikaS = ParametricPlot3D[{t Cos[t], t Sin[t], t},
  {t, -10 Pi, 10 Pi}, PlotStyle -> {Green}]
Clear[t]
erpika[t_] = D[er[t], t]
tangenta = er[Pi / 4] + t erpika[Pi / 4]
```

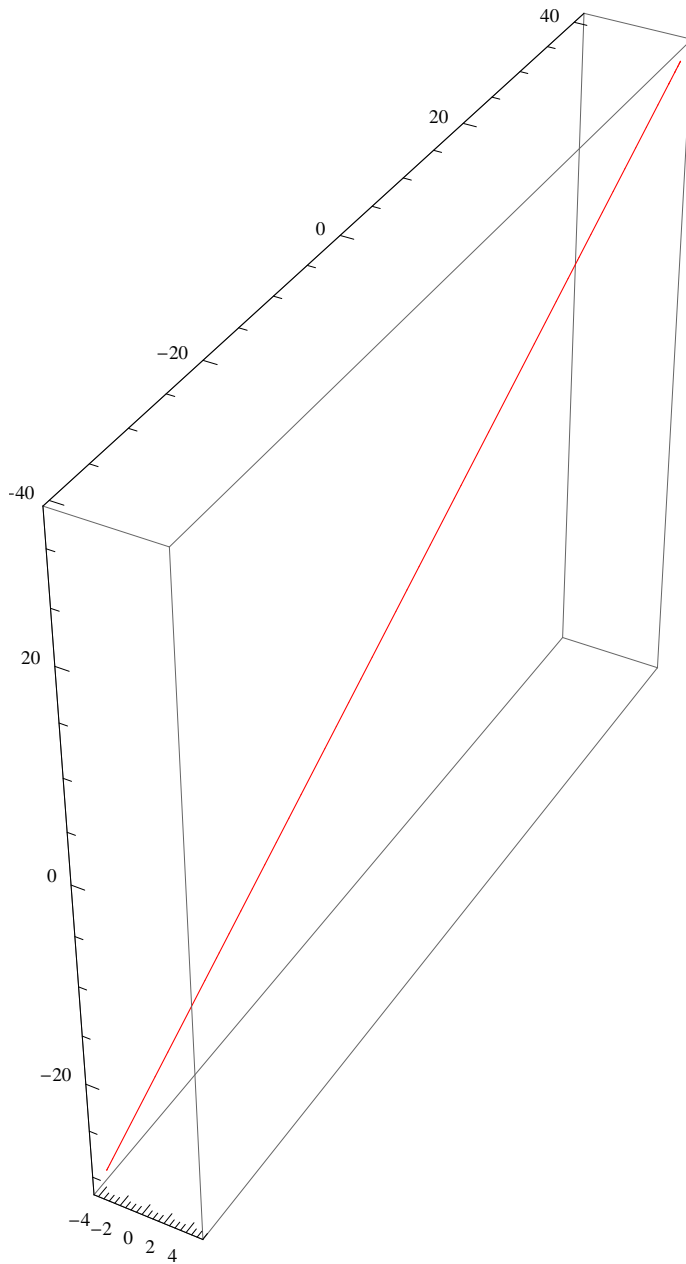
```
{t Cos[t], t Sin[t], t}
```



$$\{\text{Cos}[t] - t \text{Sin}[t], t \text{Cos}[t] + \text{Sin}[t], 1\}$$

$$\left\{ \frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}} \right) t, \frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}} \right) t, \frac{\pi}{4} + t \right\}$$

```
slikaT = ParametricPlot3D[tangenta, {t, -10 Pi, 10 Pi}, PlotStyle -> {Red}]
```



```
Show[slikaS, slikaT]
```

Show::gcomb: Could not combine the graphics objects in Show[slikaS, slikaT]. >>

```
Show[slikaS, slikaT]
```

4. Izračunaj dolžino krivulje iz 3. naloge od točke $t=0$ do točke $7\pi/11$.

Rezultat: 3.59373

? NIntegrate

`NIntegrate[f, {x, xmin, xmax}` gives a numerical approximation to the integral $\int_{x_{\min}}^{x_{\max}} f dx$.

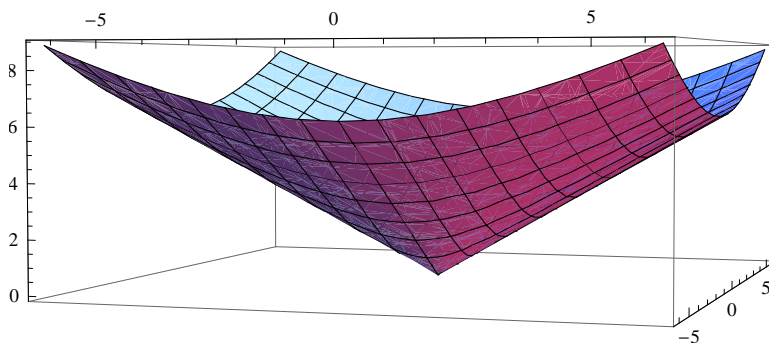
`NIntegrate[f, {x, xmin, xmax}, {y, ymin, ymax}, ...]`

gives a numerical approximation to the multiple integral $\int_{x_{\min}}^{x_{\max}} dx \int_{y_{\min}}^{y_{\max}} dy \dots f$. \gg

```
r[t_] = {t Cos[t], t Sin[t], t}
dolzina = NIntegrate[Sqrt[erpika[t].erpika[t]], {t, 0, 7 Pi / 11}]
{t Cos[t], t Sin[t], t}
3.59373
```

5. a) Narisi stožec $z(x, y) = (x^2 + y^2)^{1/2}$ s pomočjo `Plot3D`.

```
Plot3D[Sqrt[x^2 + y^2], {x, -2 Pi, 2 Pi}, {y, -2 Pi, 2 Pi}]
```

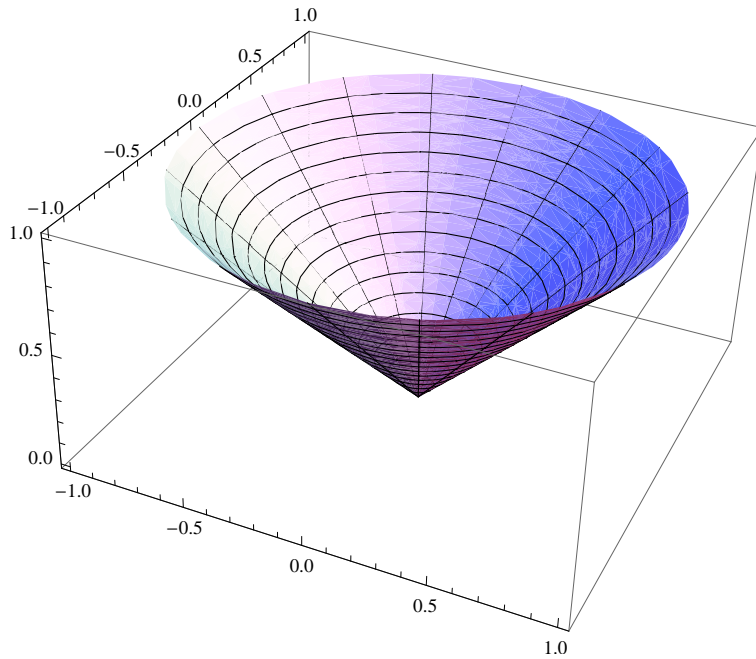


5. b) Lepsa slika stožca nastane z ukazom `ParametricPlot3D`,
 če izberemo za parametra polarne koordinate.

Kako je potrebno spremeniti enačbo, da bo stožec bolj zaprt (odprt)?

```
Clear[x, y, z, r]
x = r Cos[φ]
y = r Sin[φ]
z = r
ParametricPlot3D[{x, y, z}, {φ, 0, 2 Pi}, {r, 0, 1}]

r Cos[φ]
r Sin[φ]
r
```



6. Nariši valj $x^2+y^2=1, -3<z<3$ z ukazom **ParametricPlot3D**.

```
Clear[x, y, z, r]
x = Cos[φ]
y = Sin[φ]
z = r
ParametricPlot3D[{x, y, z}, {φ, 0, 2 Pi}, {r, 0, 1}]
Cos[φ]
Sin[φ]
r
```

