Kompleksna analiza

1. Pois■i u(x,y)=realni del in v(x,y)=imaginarni del funkcije w=f(z),
   ■e je z=x+iy, w=u+iv in f(z) = $z^2 e^z$!

   Rez:

   u = $e^x x^2 \mathrm{Cos}[y] - e^x y^2 \mathrm{Cos}[y] - 2 e^x x y \mathrm{Sin}[y]$

   v = $2 e^x x y \mathrm{Cos}[y] + e^x x^2 \mathrm{Sin}[y] - e^x y^2 \mathrm{Sin}[y]$

```
? Re
? Im
? ComplexExpand
```

Re[*z*] gives the real part of the complex number *z*. ≫

Im[*z*] gives the imaginary part of the complex number *z*. ≫

ComplexExpand[*expr*] expands *expr* assuming that all variables are real.
ComplexExpand[*expr*, {x_1, x_2, …}] expands *expr* assuming that variables matching any of the x_i are complex. ≫

```
z = x + y I
ComplexExpand[Re[z]]
ComplexExpand[Im[z]]
```

$x + i y$

x

y

```
z = x + y I
f = z^2 E^z
u = ComplexExpand[Re[f]]
v = ComplexExpand[Im[f]]
D[u, x] == D[v, y]
D[u, y] == -D[v, x]
```

$x + i y$

$e^{x+i y} (x + i y)^2$

$e^x x^2 \mathrm{Cos}[y] - e^x y^2 \mathrm{Cos}[y] - 2 e^x x y \mathrm{Sin}[y]$

$2 e^x x y \mathrm{Cos}[y] + e^x x^2 \mathrm{Sin}[y] - e^x y^2 \mathrm{Sin}[y]$

True

True

2. Pois■i vsaj eno resitev ena■be Sin[z]=3.
   Resitev zapisi v obliki x+iy, kjer sta x in y realna.

Rez:
$\frac{\pi}{2} - i\,\mathrm{Log}\left[3+2\sqrt{2}\right]$

```
? Solve
Clear[z]
ComplexExpand[Solve[Sin[z] == 3, z]]
```

Solve[*expr*, *vars*] attempts to solve the system *expr* of equations or inequalities for the variables *vars*.
Solve[*expr*, *vars*, *dom*] solves over the domain *dom*. Common choices of *dom* are Reals, Integers, and Complexes. ≫

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. ≫

$\left\{\left\{z \to \frac{\pi}{2} - i\,\mathrm{Log}\left[3+2\sqrt{2}\right]\right\}\right\}$

3. Dana je funkcija u(x,y)= $x - 3x^2 + 4x^3 + 3y^2 - 12xy^2$. Dolo■i funkcijo v(x,y) tako da je funkcija f(z) = u(x,y) + i v(x,y) analiti■na in f(0)=0, kjer je z=x+iy. Koliko je v(1,1) ?

Rez: 3

```
Clear[u, v, z, w]
u = x - 3 x^2 + 4 x^3 + 3 y^2 - 12 x y^2
D[u, x, x] + D[u, y, y]
v1 = Integrate[D[u, x], y]
v2 = Integrate[-D[u, y], x]
v = y - 6 x y + 12 x^2 y - 4 y^3 + c /. {c → 0}
Solve[v == 0, c] /. {x → 0, y → 0}
```

$x - 3x^2 + 4x^3 + 3y^2 - 12xy^2$

0

$y - 6xy + 12x^2y - 4y^3$

$-6xy + 12x^2y$

$y - 6xy + 12x^2y - 4y^3$

{}

```
v /. {x → 1, y → 1}
```

3

4. Integral kompleksne funkcije je definiran kot krivuljni integral 2. vrste.
■e je podintegralska funkcija ***analiti■na***, je integral neodvisen od krivulje.
Primerjaj integrale $\int_C F(z)\, dz$ za vse kombinacije krivulje C in funkcije F(z):

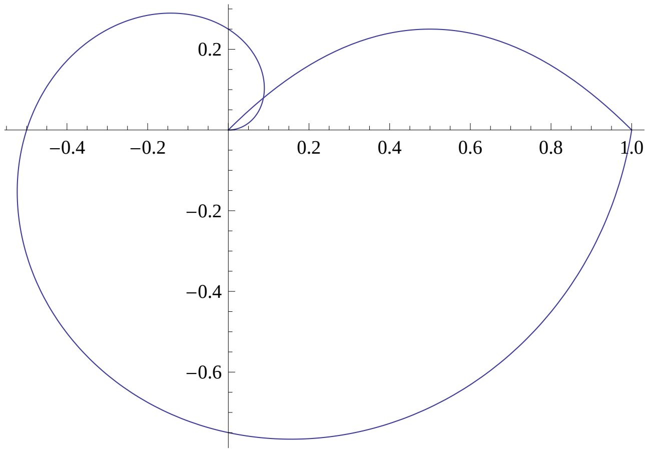
C_1 : spirala $r = \frac{\varphi}{2\pi}$

C_2 : parabola $y = x - x^2$

$F_1(z) = z^2$

$F_2(z) = \bar{z}^2$

```
C1 = ParametricPlot[{φ Cos[φ], φ Sin[φ]} / (2 π), {φ, 0, 2 π}];
C2 = ParametricPlot[{x, x (1 - x)}, {x, 0, 1}];
Show[C1, C2, AspectRatio → Automatic]
```




```
F1[z_] := z^2;
F2[z_] := ComplexExpand[Conjugate[z^2]];
z1 = φ (Cos[φ] + I Sin[φ]) / (2 π);
z2 = x + I x (1 - x);
```

```
Integrate[F1[z1] D[z1, φ], {φ, 0, 2 π}]
Integrate[F1[z2] D[z2, x], {x, 0, 1}]
(* Integral se da izra■unati kot navaden dolo■eni integral *)
Clear[z]
Integrate[F1[z], z]
(% /. z → 1) - (% /. z → 0)
```

$\frac{1}{3}$

$\frac{1}{3}$

$\frac{z^3}{3}$

$\frac{1}{3}$

```
Integrate[F2[z1] D[z1, φ], {φ, 0, 2 π}]
Integrate[F2[z2] D[z2, x], {x, 0, 1}]
```

$-1 + \frac{2}{\pi^2} + \frac{2 i}{\pi}$

$\frac{4}{15} - \frac{i}{3}$

5. Cauchyjeva integralska formula $\frac{1}{2\pi i} \int_C \frac{f(z)}{z - z_0} dz = f(z_0)$

Izra■unaj integrala $I_1 = \int_C \frac{z^2}{z - (7 + 6 i)}$ in $I_2 = \int_C \frac{z^2}{z - (9 + 6 i)}$

kjer je krivulja C kroznica $|z - 6 i| = 8$.

Integrala izra■unaj na dva na■ina:

1. Po definiciji kot krivuljni integral druge vrste
2. Z uporabo Cauchyjeve integralske formule

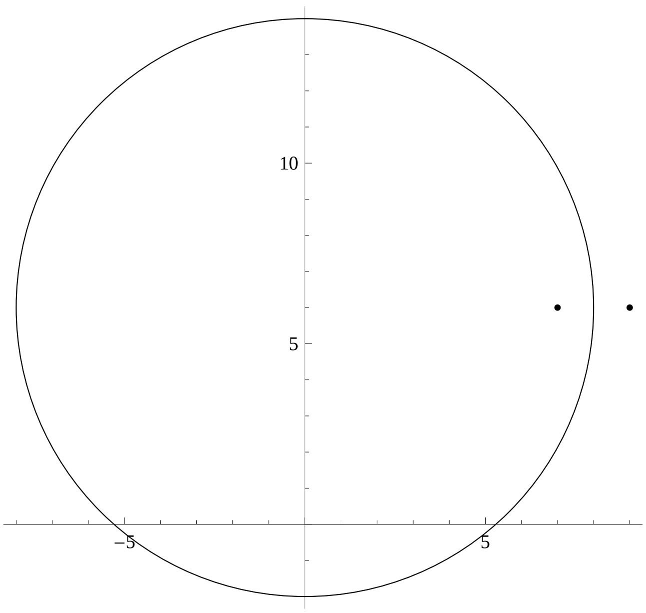
in primerjaj rezultate.

Rez :

$I_1 = (-168 + 26 i) \pi$

$I_2 = 0$

```
Show[
 Graphics[{
  Circle[{0,6},8],
  { PointSize[0.01], Point[{7,6}] },
  { PointSize[0.01], Point[{9,6}] }
  }],
 Axes->True,AspectRatio->Automatic
 ]
```




```
Clear[f]
z=6I+8E^(I*φ);
z1=7+6I;
z2=9+6I;
f[z_]:=z^2
I1 = Integrate[f[z]/(z-z1)*D[z,φ],{φ,0,2π}]
I2 = Integrate[f[z]/(z-z2)*D[z,φ],{φ,0,2π}]
2π*I*f[z1]
```

$(-168+26\,i)\,\pi$

0

$(-168+26\,i)\,\pi$