OSNOVE ELEKTROTEHNIKE II 1 UNI

Zapiski predavanj

Šolsko leto	2010/2011
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Skeniranje	Damjan Sirnik

UREJANJE DOKUMENTA	
VERZIJA	01.01
DATUM	23.5.2012

OPOMBE

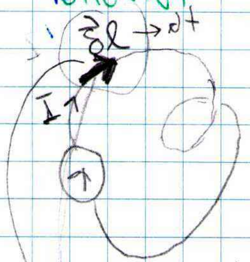
MAGNETIKA

21.2.11

MAGNETOSTATIKA

$\vec{J} = \vec{J}(T)$

TOKOVNI ELEMENT KOT GRADNIK (SPLOŠNE TOČKASTE STRUKTURE)

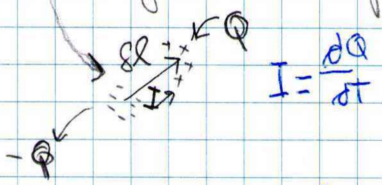


JAKOST TOKOVNEGA ELEMENTA

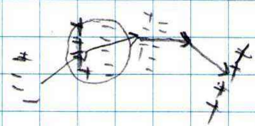
$I\delta l$

Težje za tanko žico moramo pa se od ostale strukture odpeljat

Odsek- gradnik neke splošne strukture



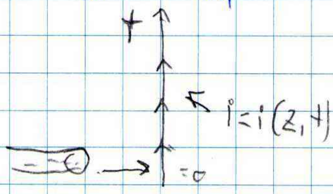
$I = \frac{dQ}{dt}$

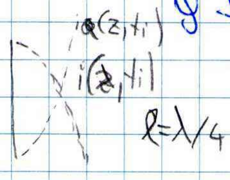


} Ni kopičenja nabojev
⇒ iz kontinuitetne enačbe

Če stalno enakomerno kopičenja ni.

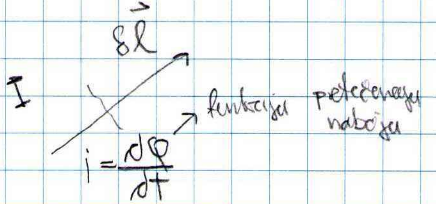
$\frac{\partial}{\partial t}$: antena





$\oint \vec{J}\cdot d\vec{a} = \frac{dQ_{not}}{dt}$

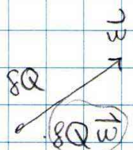
Če je $\vec{J} = \vec{J}(T) \to \oint_A \vec{J}\cdot d\vec{a} = 0$

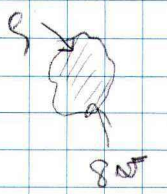


$\frac{q\delta Q}{\delta t} = I$

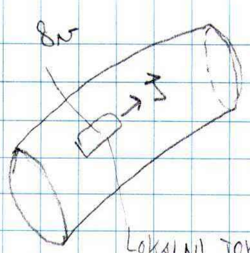
Naboj, ki v določenem času preleti neko razdaljo je

$I\delta l = \frac{\delta Q}{\delta t}\vec{w}\,\delta t = \delta Q\vec{w}$

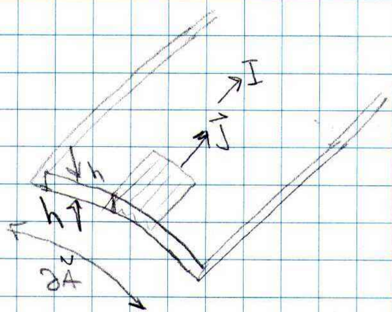




$$\delta Q \vec{w} = \frac{\delta v}{\rho\, \delta v\, \vec{w}} = \vec{J}\, \delta v$$



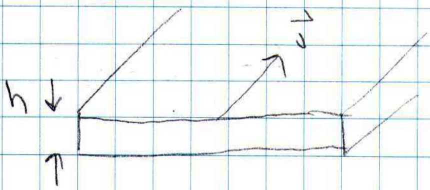
PLOŠČATI VODNIK - FOLIJE (FOLIJSKI VODNIK)



Tok: $|\vec{J}| = \frac{I}{h\check{A}}$

$$\vec{J}\,\delta v = \vec{J} h\, \delta a = \boxed{\vec{K}\, \delta a}$$

$\vec{K}$ → Tokovna obloga



PP:

$$K = Jh = \frac{I}{h\check{a}} = \frac{I}{\check{a}}$$

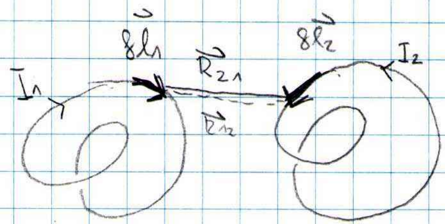
$\check{a} = 1\,nm$, $I = 10\,mA$

$$\boxed{K = \frac{I}{\check{a}} = 10\,A/m}$$

el	δQ	$q\,\delta l$	$\xi\,\delta a$	$\rho\, dv$
mag	$\delta Q \vec{w}$	$I\,\delta\vec{l}$	$\vec{K}\,\delta a$	$\vec{J}\, dv$

AMPEROVA MAGNETNA SILA (1820)

21.2.11



$$\delta F_{m_1}^{(2)} = \frac{\mu_0}{4\pi R_{12}^3} I_1 \delta\vec{l}_1 \times (I_2 \delta\vec{l}_2 \times \vec{R}_{12})$$

$$\delta F_{m_2}^{(1)} = \frac{\mu_0}{4\pi R_{21}^3} I_2 \delta\vec{l}_2 \times (I_1 \delta\vec{l}_1 \times \vec{R}_{21})$$

$I_1 \delta\vec{l}_1$, $I_2 \delta\vec{l}_2$

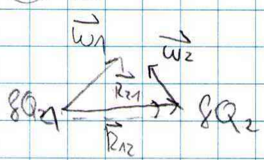
$\vec{R}_{21} = \vec{r}_2 - \vec{r}_1$

$\vec{R}_{12} = -\vec{R}_{21}$

$$\frac{|\vec{R}_{12}|}{R_{12}^3} = \frac{1}{R_{12}^2}$$

$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$

$\boxed{\mu_0 \varepsilon_0 c_0^2 = 1}$



$$\delta F_{m_1}^{(2)} = \frac{\mu_0}{4\pi R_{12}^3} \delta Q_1 \vec{w}_1 \times (\delta Q_1 \vec{w}_2 \times \vec{R}_{12})$$

$$\delta F_{m_2}^{(1)} = \frac{\mu_0}{4\pi R_{21}^3} \delta Q_2 \vec{w}_2 \times (\delta Q_2 \vec{w}_1 \vec{R}_{21})$$

$$\delta F_{e_1}^{(2)} = \frac{\delta Q_1 \delta Q_2 \cdot \vec{R}_{12}}{4\pi \varepsilon_0 R_{12}^3}, \quad \delta F_{e_2}^{(1)} = \frac{\delta Q_1 \delta Q_2 \cdot \vec{R}_{21}}{4\pi \varepsilon_0 R_{21}^3}$$

$\boxed{\delta F_{e_2}^{(1)} + \delta F_{e_1}^{(2)} = \vec{0}}$ → VZAJEMNI SILI

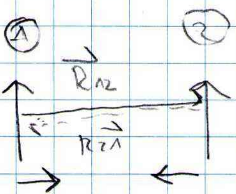
$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$\vec{b} \times (\vec{a} \times -\vec{c}) = -(\vec{b} \cdot \vec{c})\vec{a} + (\vec{b} \cdot \vec{a}) \cdot \vec{c}$$

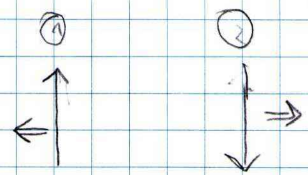
$\boxed{\delta \vec{F}_{m_1}^{(2)} + \delta \vec{F}_{m_2}^{(1)} \neq \vec{0}}$ → MAGNETNI SILI NISTA VZAJEMNI

KONSTELACIJE

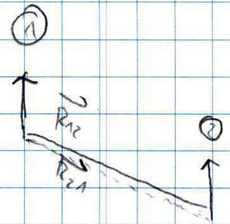
a)



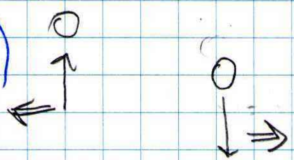
b)

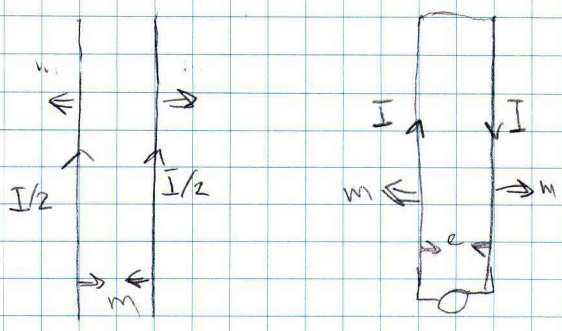


c)

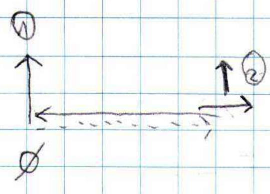


d)



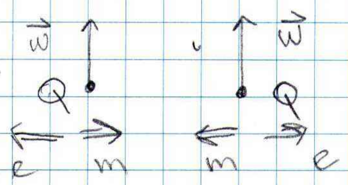


a)



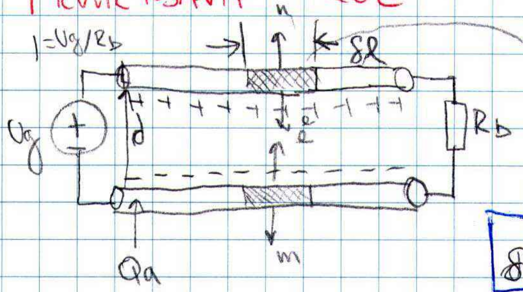
24.2.11

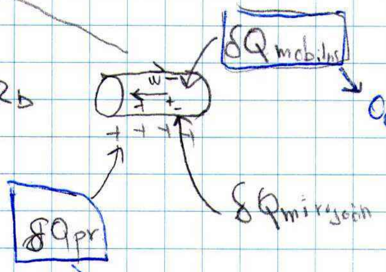
PRIMERJAVA CoL IN AMPEROVE SILE I.



$$\left.\begin{aligned} |\vec{F_m}| &= F_m = \frac{\mu_0 (Q\omega)^2}{4\pi d^2} \\ |\vec{F_e}| &= F_e = \frac{Q^2}{4\pi\varepsilon_0 d^2} \end{aligned}\right\} \quad \boxed{\frac{F_m}{F_e} = \mu_0\varepsilon_0\omega^2 = \left(\frac{\omega}{c_0}\right)^2}$$

PRIMERJAVA COL IN AMPEROVE SILE II.





δQ mobilni → Odgovarja za magnetenje.

$\delta Q_{mir} + \delta Q_{mob} = 0$

δQpr → odgovorna za el. silo

Karakteristična upornost linije

$$\left.\begin{aligned} |\delta\vec{F_m}| &= \delta F_m = \frac{\mu_0 (\delta Q_{mob}\cdot\omega)^2}{4\pi d^2} \\ |\delta\vec{F_e}| &= \delta F_e = \frac{(\delta Q_{pr})^2}{4\pi\varepsilon_0 d^2} \end{aligned}\right\} \quad \frac{\delta F_m}{\delta F_e} = \left(\frac{\omega}{c}\right)^2 \left(\frac{\delta Q_{mob}}{\delta Q_{pr}}\right)^2 = \frac{(I\,\delta l)^2}{C_0^2\,(c\,\delta l\, U_g)^2}$$

$$= \frac{\mu_0\varepsilon_0}{R_b^2}\cdot\frac{1}{\left(\frac{\pi\varepsilon_0}{\ln\frac{d}{a}}\right)^2} = \frac{\frac{\mu_0}{\varepsilon_0}\left(\frac{1}{\pi}\ln\frac{d}{a}\right)^2}{R_b^2} = \frac{\left(\frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\ln\frac{d}{a}\right)^2}{R_b^2}$$

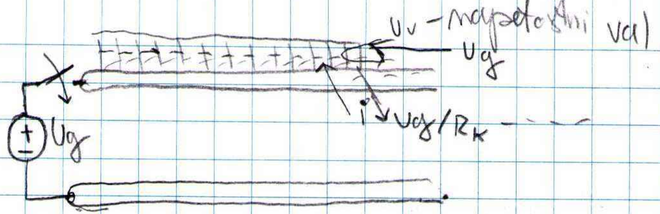
R_k

PP: R_k, $d/a = 20$

$\Rightarrow R_k = \frac{1}{\pi}\cdot 120\pi \ln 20$

$\approx 360\,\Omega$

$$\boxed{\frac{\delta F_m}{\delta F_e} = \left(\frac{R_k}{R_b}\right)^2}$$



Karakteristična upornost kabla tej linije. 50 ohmski kabel - to je njegova karakteristika. Nanj valovi prihajajo tudi tako antena

$$\boxed{\sqrt{\frac{\mu_0}{\varepsilon_0}} = R_v}$$ - valovna upornost.

Za prazen prostor je to $120\pi \approx 377\,\Omega$

Razmerje med el. in mag. poljske jakosti

GOSTOTA MAGNETNEGA PRETOKA BIOT - SAVARTOV ZAKON

$I'\delta\vec{l}'$ $\qquad$ $I\delta\vec{l}$

$T' \xrightarrow{\vec{R}} T$

$$\delta\vec{F}_m = \frac{\mu_0}{4\pi R^3} I\delta\vec{l} \times (I'\delta\vec{l}' \times \vec{R})$$

$$\delta\vec{F}_m = I\delta\vec{l} \times \underbrace{\left(\frac{\mu_0 I'\delta\vec{l}' \times \vec{R}}{4\pi R^3}\right)}_{\delta\vec{B}(T)}$$

$$= I\delta\vec{l} \times \delta\vec{B}(T)$$

$\delta Q'$ $\qquad$ δQ

$T' \xrightarrow{\vec{R}} T$

$$\delta\vec{F}_L = \delta Q(\vec{E} + \vec{v} \times \vec{B})$$
$$= \delta Q\vec{E} + \delta Q\vec{v} \times \vec{B}$$

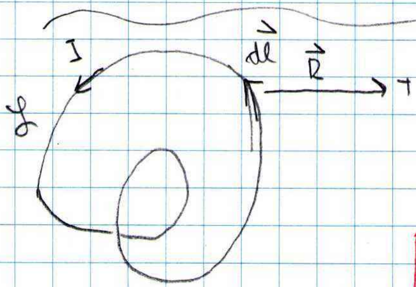
$$\delta\vec{F}_e = \frac{\delta Q\,\delta Q'\vec{R}}{4\pi\varepsilon_0 R^3}$$

$$\delta F_e = \delta Q \underbrace{\left(\frac{\delta Q'\vec{R}}{4\pi\varepsilon_0 R^3}\right)}_{\delta\vec{E}(T)}$$

$= I'\delta\vec{l}'$

$$\boxed{\delta\vec{B}(T) = \frac{\mu_0 I'\delta\vec{l}' \times \vec{R}}{4\pi R^3}}$$

$I'\delta\vec{l}'$ $\quad$ $T' \xrightarrow{\vec{R}} T$



$$d\vec{B}(T) = \frac{\mu_0 I d\vec{l} \times \vec{R}}{4\pi R^3}$$

$$\vec{B}(T) = \int d\vec{B}(T)$$

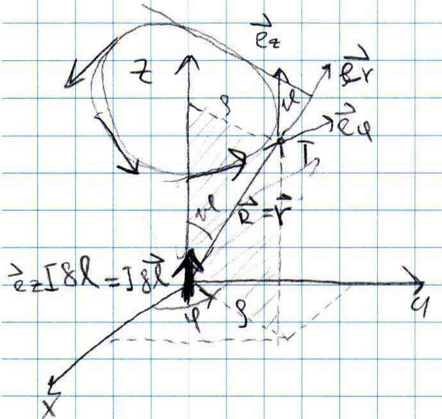
$$\boxed{\vec{B}(T) = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l}' \times \vec{R}}{R^3}}$$

B.S. ZAKON V INTEGRALNI OBLIKI

Velja v vseh točkah povsod poljska jakost

$$\boxed{\delta\vec{E}(T) = \frac{\delta Q'\vec{R}}{4\pi\varepsilon_0 R^3}}$$ $\quad$ $\delta Q'$ $\quad$ $T' \xrightarrow{\vec{R}} T$

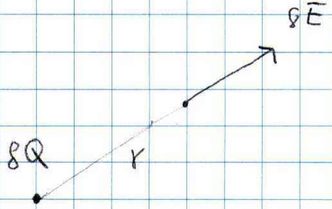
MAGNETNO POLJE TOKOVNEGA ELEMENTA



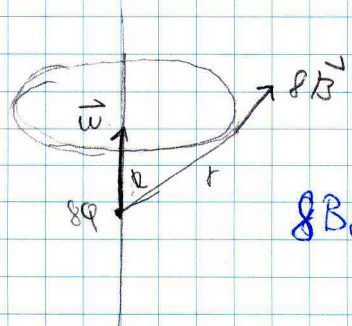
$$\delta\vec{B}(T) = \frac{\mu_0 I \delta\vec{l}\times\vec{R}}{4\pi R^3} = \frac{\mu_0 I \delta l}{4\pi r^2}(\vec{e}_z\times\vec{e}_r)A$$

$$\boxed{= \frac{\mu_0 I \delta l}{4\pi r^2}\sin\vartheta\,\vec{e}_\varphi}$$

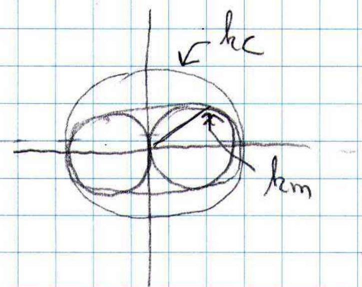
$\delta B_\varphi(T)$



$$E_r = \frac{\delta Q}{4\pi\varepsilon_0 r^2}\cdot\underbrace{1}_{k_e}$$



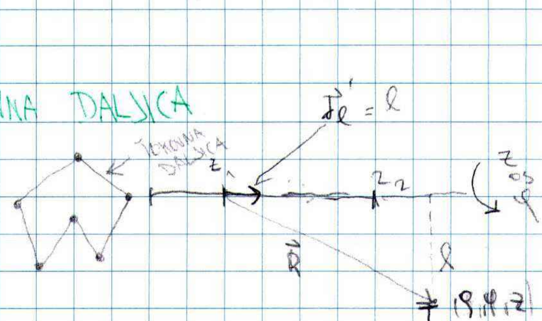
$$\delta B_\varphi = \frac{\mu_0\,\delta Q\,w}{4\pi r^2}\cdot\underbrace{\sin\vartheta}_{k_m}$$



ZGLEDI

1.) DALJICA
2.) PREMICA
3.) OVOJ
4.) VEČ OVOJEV - TULJAVA
5.) TRAK - PLOŠČATI VODNIK

1) TOKOVNA DALJICA



$$dB_\varphi = \frac{\mu_0 I\,dz'\sin\vartheta}{4\pi R^2}$$

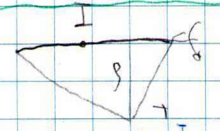
$$z - z' = \rho\tan\vartheta \qquad R = \frac{\rho}{\sin\vartheta}$$

$$dz' = \frac{\rho}{\sin^2\vartheta}d\vartheta$$

$$\frac{\mu_0 I\frac{\rho}{\sin^2\vartheta}d\vartheta\cdot\sin\vartheta}{4\pi\frac{\rho^2}{\sin^2\vartheta}} = \frac{\mu_0 I}{4\pi\rho}\sin\vartheta\,d\vartheta$$

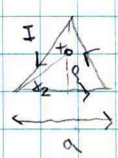
$$B(T) = \int dB(T) = \frac{\mu_0 I}{4\pi\rho}\int_{\vartheta_1}^{\vartheta_2}\sin\vartheta\,d\vartheta$$

$$= \frac{\mu_0 I}{4\pi\rho}\left(\cos\vartheta_1 - \cos\vartheta_2\right)$$



$$B_\varphi(T) = \frac{\mu_0 I}{4\pi\rho}\left(\cos\vartheta_1 - \cos\vartheta_2\right)$$

PP: Polje v težišču zanke v obliki enakostraničnega trikotnika

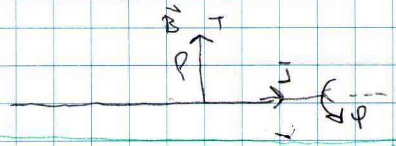


$$B(T) = 3\frac{\mu_0 I}{4\pi\left(\frac{1}{3}\frac{a\sqrt{3}}{2}\right)}\left(2\frac{\sqrt{3}}{2}\right) = \frac{9}{2}\frac{\mu_0 I}{\pi a}$$

2.) POLJE $\vec{B}$ TOKOVNE PREMICE

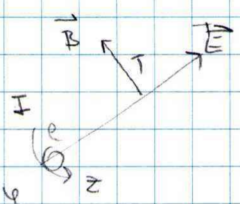
$$\vartheta_1 \to 0,\quad \vartheta_2 \to \pi$$

$$B_\varphi(T) = \frac{\mu_0 I}{2\pi\rho}$$



Premica nastopa kot dejanski kot.

3.) $\vec{E}$ in $\vec{B}$ OB NAELEKTRENI PREMICI

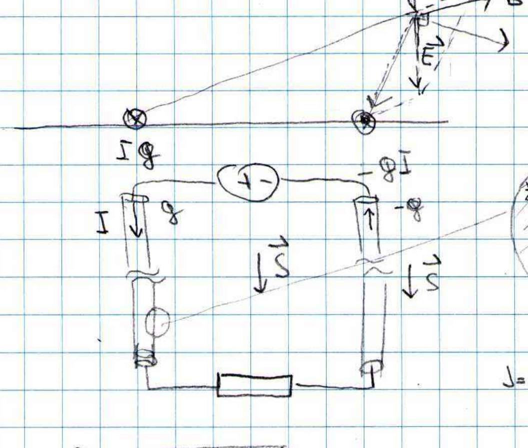


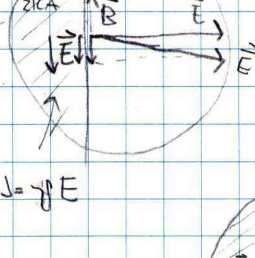
$$B_\varphi = \frac{\mu_0 I}{2\pi\rho}$$

$$E_\rho = \frac{q}{2\pi\varepsilon\rho}$$

POINTINGOV VEKTOR: $\vec{E} \times \frac{\vec{B}}{\mu_0} = \vec{S}$ $[W/m^2]$

4.) POLJE $\vec{B}$ DVOVODA





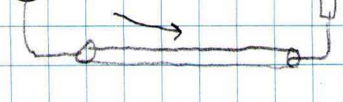
$J = \gamma E$

$$\vec{E} \times \frac{\vec{B}}{\mu_0} = \underbrace{\vec{E}_{\parallel} \times \frac{\vec{B}}{\mu_0}}_{\vec{S}_{\perp}} + \underbrace{\vec{E}_{\perp} \times \frac{\vec{B}}{\mu_0}}_{\vec{S}_{\parallel}}$$

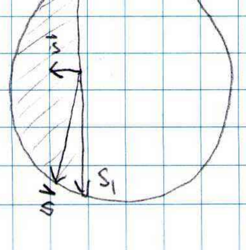
$$|\vec{E}_{\parallel}| \lll |\vec{E}_{\perp}|$$

$$|\vec{S}_{\perp}| \lll |\vec{S}_{\parallel}|$$

E rahlo obrnen v žico. Toliko, kot je prot E dovajano je Joulskih izgub ob žici.



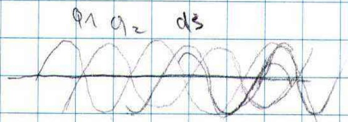
BISTVEN JE PROSTOR OB ŽICI

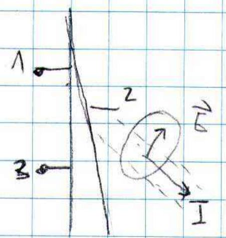


SKOZI PROSTOR OB ŽICI VSTOPA V ŽICO

5) $\vec{E}$ in $\vec{B}$ TROFAZNEGA SISTEMA

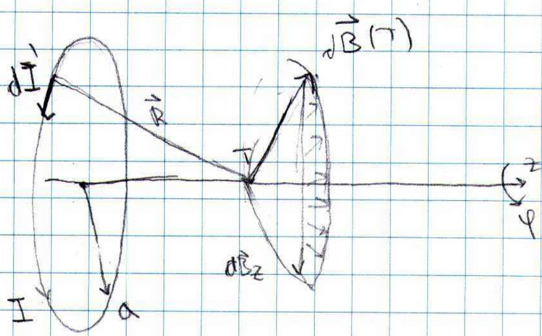
$$q_1 = q_m \cos\omega t \qquad i_1 = I_m \cos(\omega t + \vartheta)$$
$$q_2 = q_m \cos(\omega t - 2\pi/3) \qquad i_2 = I_m \cos(\omega t - 2\pi/3 + \vartheta)$$
$$q_3 = q_m \cos(\omega t + 2\pi/3) \qquad i_3 = I_m \cos(\omega t + 2\pi/3 + \vartheta)$$





6) $\vec{B}$ V OSI KROŽNE TOKOVNE ZANKE (DOKOŽNI ČVOS)

Formula npr. velja tudi za n-kotnik?



$$d\vec{B}(T) = \frac{\mu_0 I\, d\vec{l}' \times \vec{R}}{4\pi R^3}$$

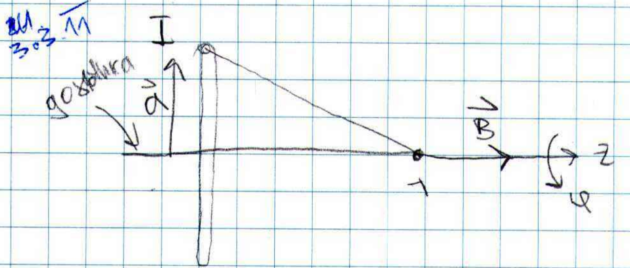
$$d\vec{B}_z(T) = \frac{\mu_0 I\, dl' \times R}{4\pi R^2}\overbrace{\sin\beta}^{a/R} = \frac{\mu_0 I a\, dl'}{4\pi R^3}$$

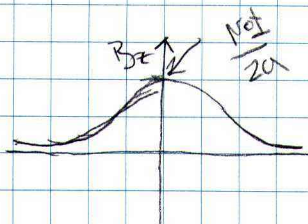
$$B_z(T) = \int dB_z(T) = \frac{\mu_0 I a}{4\pi R^3}\overbrace{\int dl'}^{2\pi a} = \frac{\mu_0 I a^2}{2\pi R^3}$$

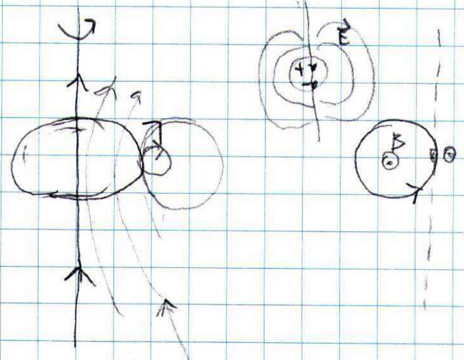
$$B_z(T) = \frac{\mu_0 I a^2}{2R^3} = \frac{\mu_0 I \cdot a^2}{2(a^2+z^2)^{3/2}}$$

PRI $z=0$

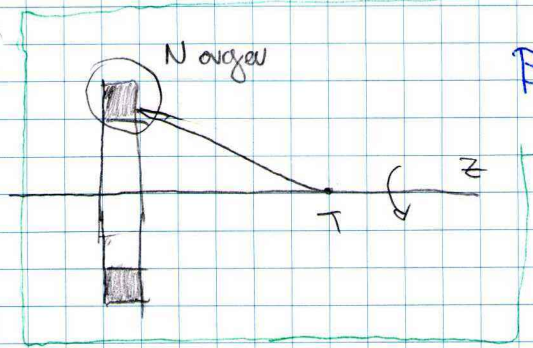
$$\boxed{B(z=0) = \frac{\mu_0 I}{2a}}$$



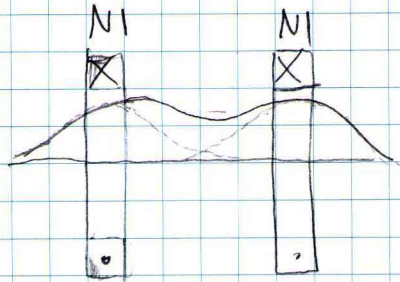




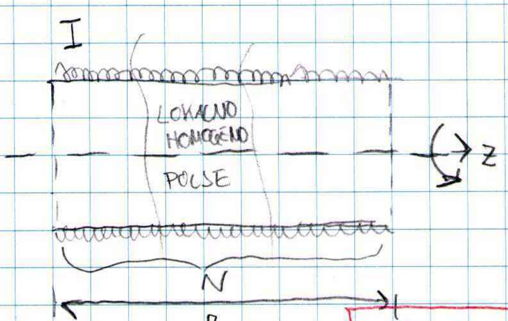
Zaradi tendence zasuka imenujemo zanko tudi magnetni dipol
Spin se navzven obnaša kot tokovna zanka čeprav se nič ne vrti.

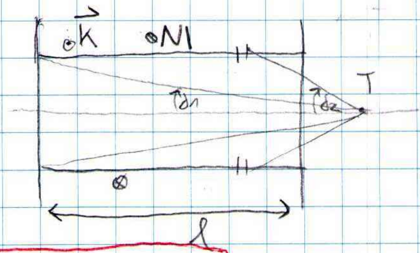


$$B_z(T) \frac{\mu_0 N I}{2} \frac{a}{R^3}$$



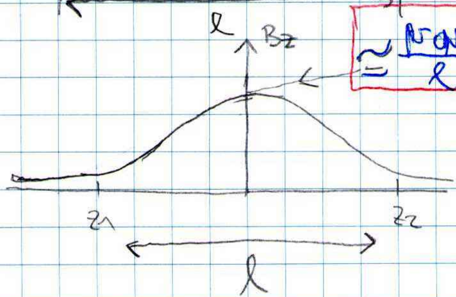
7.) $\vec{B}$ DOLGE TULJAVE



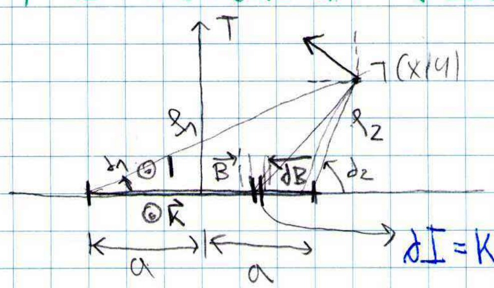


$$B_z(T) = \frac{\mu_0 I N}{2l}(\cos\gamma_1 - \cos\gamma_2)$$

$$\boxed{\simeq \frac{\mu_0 N I}{l} \rightarrow \text{če je } l \gg a} = \mu_0 K$$



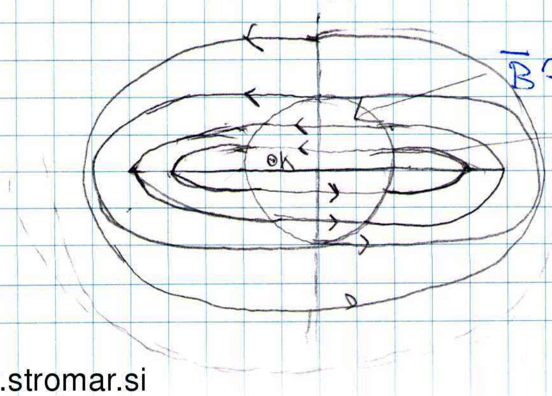
8.) $\vec{B}$ TRAČNEGA VODNIKA



$$K_z = \frac{I}{2a}$$

$$\rightarrow dI = K_z\, dB$$

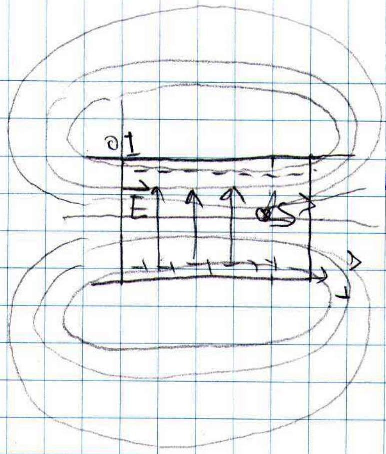
$$\boxed{B_x(T) = \frac{\mu_0 K_z}{2\pi}(\gamma_1 - \gamma_2) \qquad B_y(T) = \frac{\mu_0 K_z}{2\pi} \ln\frac{\rho_1}{\rho_2}}$$



$$\vec{B} \simeq \mp \frac{\mu_0 K_z}{2} \vec{e}_x$$

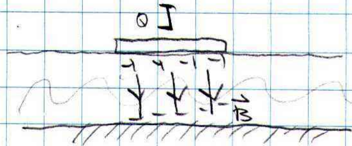
→ Lokalno homogeno polje

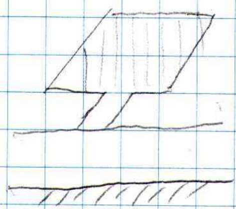
→ ~~Dolži~~ stran postane geslolmica krog



TRAČNI DVOVOD

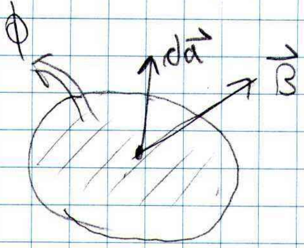
lokalno homogeno polje





TOKOVNA ANTENA

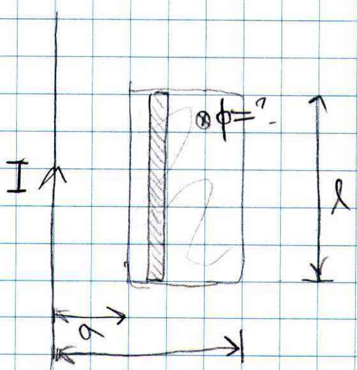
MAGNETNI PRETOK $\phi_m = \phi$



$$\phi = \int_A \vec{B}\, d\vec{a}$$

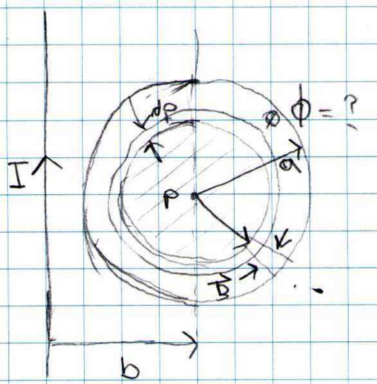
$$\left\{ \begin{array}{l} \int_A \vec{J} \cdot d\vec{a} = i \\ \int_A \vec{E} \cdot d\vec{a} = \phi_E \end{array} \right\}$$

1. ZGLED – PRETOK SKOZI PRAVOKOTNIK OB RAVNEM TOKOVODNIKU



$$\phi = \int \vec{B} \cdot d\vec{a} = \int_a^b \frac{\mu_0 I}{2\pi x}\, l\, dx = \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}$$

2. ZGLED - PRETOK SKOZI KROŽNO OPNO OB RAVNEM TOKOVODNIKU



$$d\phi = \vec{B} \cdot d\vec{a} = \frac{\mu_0 I}{2\pi(b+\rho\cos\vartheta)} \rho d\vartheta d\rho$$

$$\boxed{\phi = 2 \int_{\rho=0}^{a} \int_{\vartheta=-\pi}^{\pi} d\phi = \dots = \mu_0 I \left(b - \sqrt{b^2 - a^2}\right)}$$

(NE)IZVORNOST POLJA $\vec{B}$

$\oint_A J \cdot da = \dots$

$\oint_A \vec{B} \cdot d\vec{a} = ?$

$\oint$ $E, \vec{P}, \vec{D} \dots$

↓

za e rabili preperciavlni naboj[?]

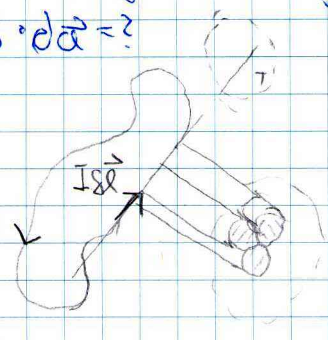
za P ...

Bodi $\delta\vec{B}$ polje tokovnega elementa $I\delta\vec{l}$.

$\oint_A \delta\vec{B} \cdot d\vec{a} = ?$

Gostotnice so soosne krožnice

Cevke so soosni toroidi (obroči, rešilni pasovi)



$\oint_A \delta\vec{B} \cdot d\vec{a} = 0$

$\delta\phi \qquad \vec{B} = \sum_{k=1}^{n} \delta\vec{B}_k$

$$\oint \vec{B} \cdot d\vec{a} = \sum_{k=1}^{n} \overbrace{\oint \delta\vec{B}_k \cdot d\vec{a}}^{} = 0$$

$\boxed{\oint E \cdot da = \frac{q_{not}}{\varepsilon_0}}$

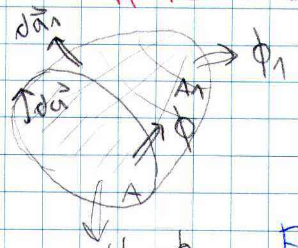
I. Maxwellova enačba

$$\boxed{\oint_A \vec{B} \cdot d\vec{a} = 0}$$

II. MAXWELLOVA ENAČBA

(NASPROTJE ENAČBA, POTEM EKSPERIMENT)

POVRATEK K FLUXU – (NA KRAJ ZLOŽINA)



$\phi' = -\phi$

$$\phi = \oint_A \vec{B}\cdot d\vec{a}$$

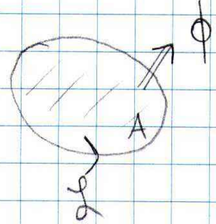
$$\phi_1 = \oint_{A1} \vec{B}\cdot d\vec{a}$$

$A \cup A1 \rightarrow$ Sklenjena ploskev

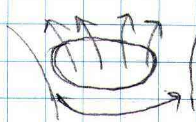
$$\oint \vec{B}\cdot d\vec{a} = \underbrace{\int_A \vec{B}\cdot d\vec{a}'}_{\phi' = -\phi} + \underbrace{\int_{A1} \vec{B_1}\cdot d\vec{a}_1}_{\phi_1} = 0$$

Fluks skozi ploskev ni odvisen od ploskve, ampak traku – roba ploskve (robu ploskve) → Sklenjene pentlje

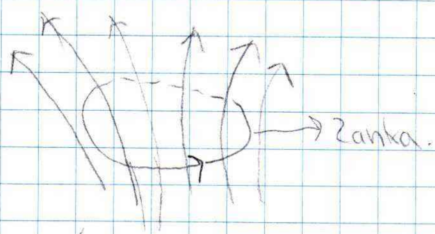
$$-\phi + \phi_1 = 0 \Rightarrow \boxed{\phi_1 = \phi}$$



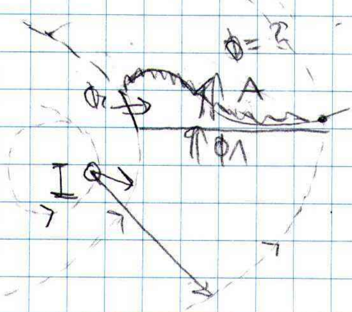
$$\phi_{\text{skozi } A'} = \phi_{\text{skozi } \mathcal{L}''}$$



$= -\frac{dQ}{dt}$

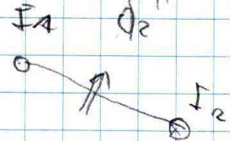


1.) ZGLED – PRETOK SKOZI ZVIJUGAN TRAK OB RAVNEM TOKOVODNIKU



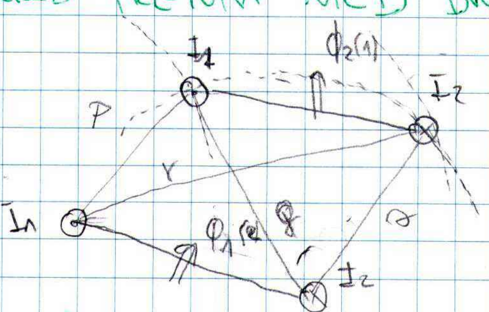
$$\phi + (-\phi_2) + (-\phi_1) = 0$$

$$\phi = \phi_1 = \frac{\mu_0 I l}{2\pi} \ln\frac{b}{a}$$



7.3.11

2.) ZGLED – PRETOKA MED DVOVODOMA



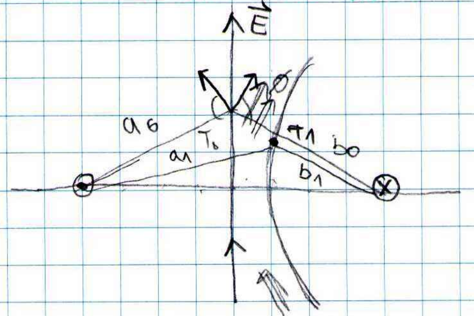
$$\phi_2^{(1)} = \frac{\mu_0 I_1 l}{2\pi}\left(\ln\frac{r}{p} - \ln\frac{s}{q}\right) = \frac{\mu_0 I_1 l}{2\pi} \ln\frac{rq}{ps}$$

$$\phi_1^{(2)} = \frac{\mu_0 I_2 l}{2\pi}\left(\ln\frac{q}{p} + \ln\frac{r}{s}\right) = \frac{\mu_0 I_2 l}{2\pi} \ln\frac{rq}{ps}$$

$$\boxed{L_{21} = \frac{\phi_2^{(1)}}{I_1} = \frac{\phi_1^{(2)}}{I_2} = L_{12}}$$

Medsebojni induktivnosti sta si paroma enaki

3. IZGLED – GOSTOTNICE DVOVODA



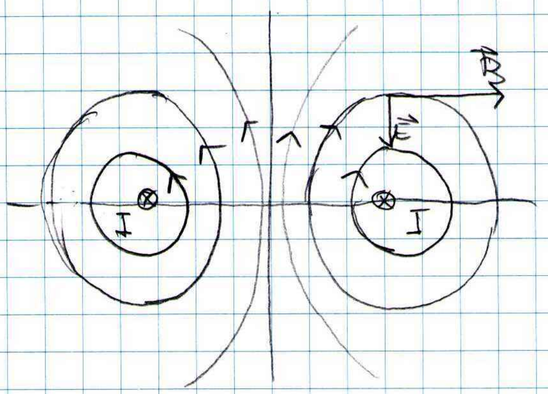
$$\phi = \frac{\mu_0 I l}{2\pi}\left(\ln\frac{a_1}{a_0} + \ln\frac{b_0}{b_1}\right) = \frac{\mu_0 I l}{2\pi}\ln\frac{a_1 b_0}{a_0 b_1}$$

$$\boxed{\phi = \frac{\mu_0 I l}{2\pi}\ln\frac{a_1}{b_1}}$$

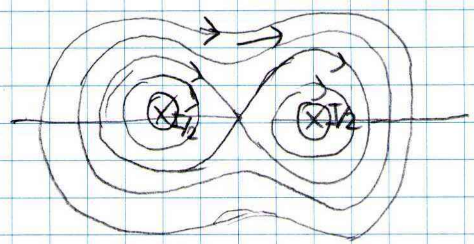
$\frac{a_1}{b_1} = konst$

Pretok nespremenjen, kjer koli na premici če je $a_1 = a_2$ konst

ϕ



4. GOSTOTNICE OB DVOJČKU



$$d\vec{B}(T) = \frac{\mu_0 I\, d\vec{l} \times \vec{B}}{4\pi R^3} \qquad B(T) = \int d\vec{B}(T)$$

$$\phi = \int_A \vec{B}\cdot d\vec{a} \leftarrow \oint_A \vec{B}\cdot d\vec{a} = 0!$$ (mag. polje neizvorno)

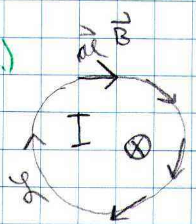
$$\phi_A = \int \vec{B}\cdot d\vec{a} = \phi_L$$

7.3.11

VRTINČNOST MAGNETNEGA POLJA

$\oint_g \vec{E}\cdot d\vec{l} = 0$ $\quad \oint_g \vec{B}\cdot d\vec{l} = ?$ vpr. vrtinčnosti je vpr. vrednosti tega integrala

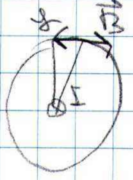
1.)



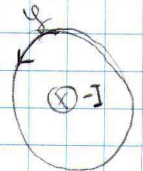
$$\oint_g \vec{B}^{\parallel}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi a} a\, d\varphi = \mu_0 I$$

zavrtinčenost sorazmerna toku, ki [illegible] ga ovija

2.)

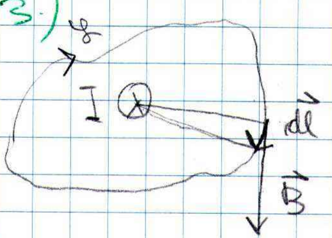


$$\oint \vec{B}\, d\vec{l} = -\mu_0 I = \mu_0(-I) =$$



Rezultat enak toku, ki ga objemamo v pozitivnem smislu

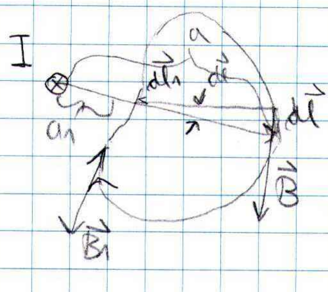
3.)



$$\oint_g \vec{B}\, d\vec{l} = \int \frac{\mu_0 I}{2\pi a} a\, d\vartheta = \mu_0 I$$

$$\vec{B}\cdot d\vec{l} = B\, dl_B = B a \overbrace{d\vartheta}^{\text{dolžina loka pod kotom } \vartheta}$$

4.)



$$\vec{B}\cdot d\vec{l} = -\vec{B}_1\cdot d\vec{l}_1 \qquad \oint_g \vec{B}\cdot d\vec{l} = 0$$

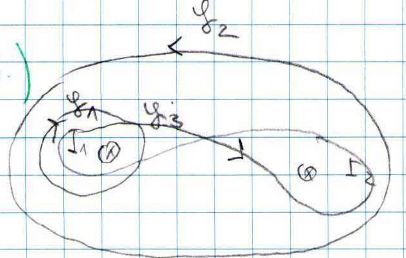
$$\oint_g \vec{A}\cdot d\vec{l} \neq 0 \qquad \oint \vec{A}\cdot d\vec{l} = \langle|\vec{A}|\rangle_t\, l \neq 0$$

dolžina zanke

povpr. vekt. v tang. smeri

$$\oint_g \vec{E}\cdot d\vec{l} = 0 \Rightarrow \langle\vec{E}\rangle_t = 0$$

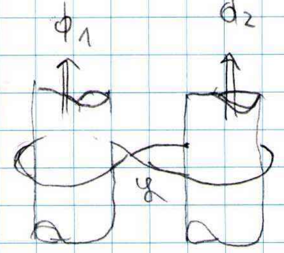
5.)



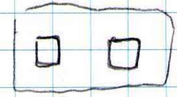
$$\oint_{g_1} \vec{B}\, d\vec{l} = \mu_0 I_1$$

$$\oint_{g_2} \vec{B}\cdot d\vec{l} = \mu_0 (I_2 - I_1)$$

$$\oint_{g_3} \vec{B}\cdot d\vec{l} = \mu_0 (I_1 + I_2)$$

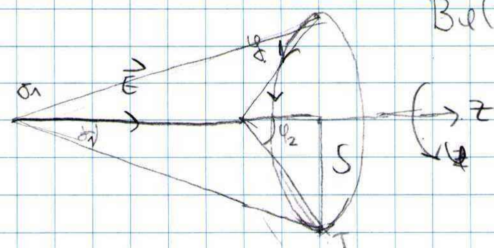


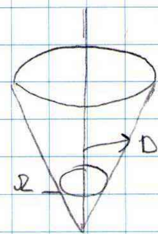
$\phi_g = \phi_1 - \phi_2$



$\oint_g \vec{B}\cdot d\vec{l}$ - tokovne daljice, ko je g krivulja okoli te daljice.

$$B_\varphi(T) = \frac{\mu_0 I}{4\pi S}(\cos\delta_1 - \cos\delta_2)$$





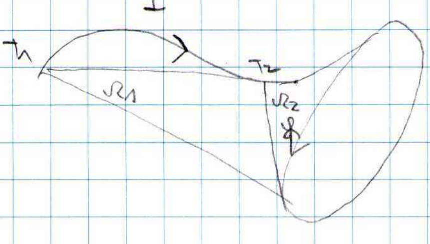
$\Omega = 2\pi(1-\cos\vartheta)$

$\cos\vartheta = 1 - \frac{\Omega}{2\pi}$

$$\oint_g \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{4\pi S}(\cos\delta_1 - \cos\delta_2)\cdot 2\pi S$$

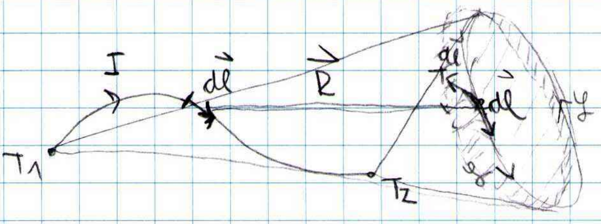
$$= \frac{\mu_0 I}{2}(\cos\delta_1 - \cos\delta_2) = \frac{\mu_0 I}{2}\left(1 - \frac{\Omega_1}{2\pi} - 1 + \frac{\Omega_2}{2\pi}\right) = \frac{\mu_0 I}{4\pi}(\Omega_2 - \Omega_1)$$

I



$$\oint_g \vec{B}\, d\vec{l} = \frac{\mu_0 I}{4\pi}(\Omega_2 - \Omega_1)$$ UPORABEN ZA POLJUBNO KRIVULJE

$$\vec{B}(T) = \int_{T_1}^{T_2} \frac{\mu_0 I\, d\vec{l}' \times \vec{R}}{4\pi R^3}$$



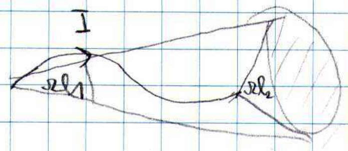
$$\oint_g \vec{B}\cdot d\vec{l} = \oint_g \left(\int_{T_1}^{T_2} \frac{\mu_0 I\, d\vec{l}' \times \vec{R}}{4\pi R^3} \right) d\vec{l}$$

$$= \frac{\mu_0 I}{4\pi} \int_{T_1}^{T_2} \oint \frac{(d\vec{l}' \times \vec{R})\cdot d\vec{l}}{R^3}$$ MEŠANI PSEVDOSKALARNI PRODUKT

$$= \frac{\mu_0 I}{4\pi} \int_{T_1}^{T_2} \oint_g \frac{(d\vec{l}' \times d\vec{l})\cdot \frac{\vec{R}}{R}}{R^2} = \frac{\mu_0 I}{4\pi} \int_{T_1}^{T_2} \oint \underbrace{\frac{(d\vec{l} \times d\vec{l}')\cdot \frac{\vec{R}}{R}}{R^2}}_{\text{Prostorski kot ploskvice } d\Omega \text{ ploskvica}}$$

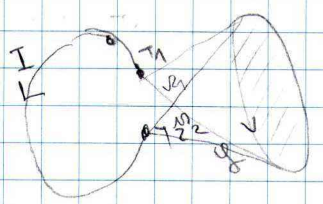
$d\Omega$ prirastku

$\Omega_2 - \Omega_1$



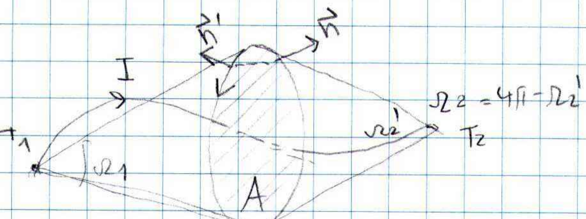
$$\oint_g \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{4\pi}(\Omega_2 - \Omega_1)$$

1.)



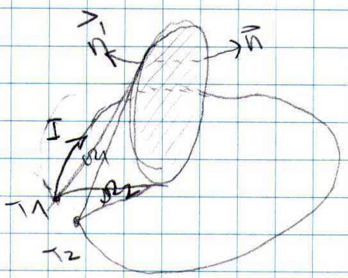
$$\oint_g \vec{B}\cdot d\vec{l} = \lim_{T_1\to T_2}\frac{\mu_0 I}{4\pi}(\Omega_2-\Omega_1) = \emptyset$$

2.)

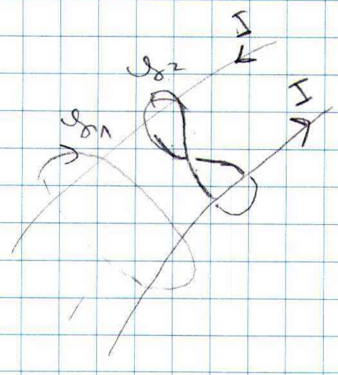


$$\oint \vec{B}\, d\vec{l} = \frac{\mu_0 I}{4\pi}(\Omega_2-\Omega_1) = \frac{\mu_0 I}{4\pi}(4\pi - \Omega_2' - \Omega_1)$$
$$= \mu_0 I - \frac{\mu_0 I}{4\pi}(\Omega_1 + \Omega_2')$$

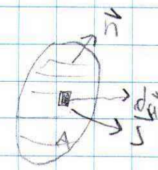
3.)



$$\oint_g \vec{B}\cdot d\vec{l} = \mu_0 I - \lim_{T_2\to T_1}\frac{\mu_0 I}{4\pi}(\Omega_1+\Omega_2') = \mu_0 I$$



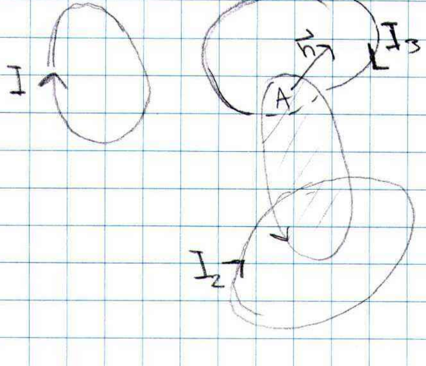
$$\oint_{g_1} \vec{B}\cdot d\vec{l} = 0 \quad ; \quad \oint_{g_2} \vec{B}\cdot d\vec{l} = -2\mu_0 I$$



$\oint \vec{J}\cdot d\vec{a}$

$\sum I$ skozi A v pozit. smislu.

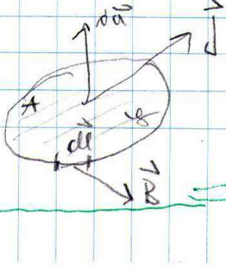
4.)



$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\oint_g \vec{B}\cdot d\vec{l} = \underbrace{\oint_g \vec{B}_1 d\vec{l}}_{\emptyset} + \underbrace{\oint \vec{B}_2 d\vec{l}}_{\mu_0 I_2} + \underbrace{\oint \vec{B}_3 d\vec{l}}_{\mu_0(-I_3)} = \mu_0(I_2 - I_3)$$

$$\boxed{\oint_g \vec{B}\, d\vec{l} = \mu_0 \int_A \vec{J}\cdot d\vec{a}}$$

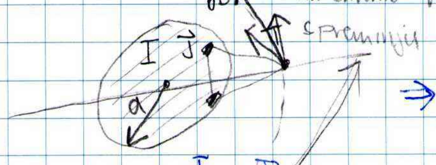


Amperov zakon vrtinčnosti mag. polja

ZGLEDI:

1.) $\vec{B}$ V/OB RAVNEM TOKOVODNIKU KROŽNEGA PRESEKA

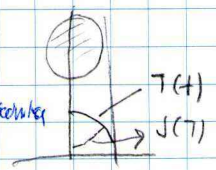
$d\vec{B}$ ok enakomerno razporejen po prerezu, v praksi to ni res ker se s segrevanjem prevodnost spreminja

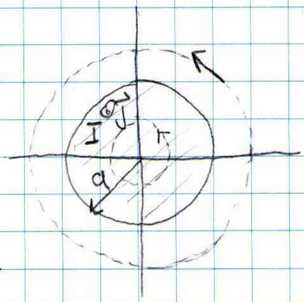


$|\vec{J}| = \frac{I}{\pi a^2}$!

$\Rightarrow \vec{B} = (0, B_\varphi, 0)$

Magn. polje ima samo φ komponenta - okoli osi vodnika

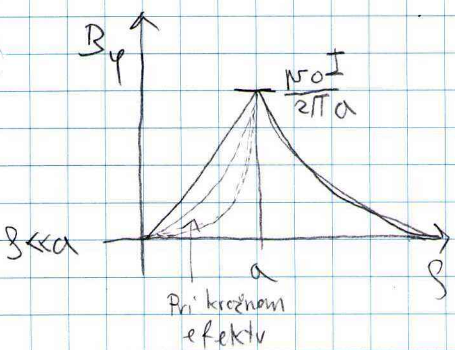


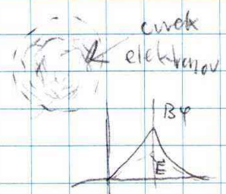


$\rho \gtreqless a$ okoli osi z

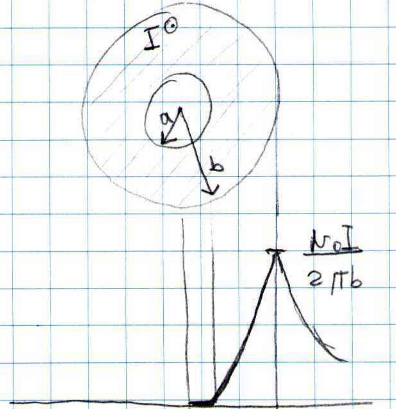
$$\oint_{\rho} \vec{B}\, d\vec{l} = B_\varphi 2\pi \rho = \mu_0 \begin{cases} \frac{I}{\pi a^2}\pi \rho^2, & \rho \le a \\ I, & \rho > a \end{cases}$$

$$B_\varphi = \begin{cases} \frac{\mu_0 I}{2\pi a^2}\rho, & \rho \le a \\ \frac{\mu_0 I}{2\pi \rho}, & \rho > a \end{cases}$$

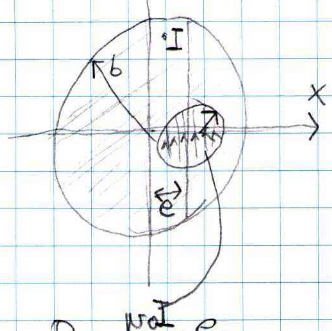




2.) CEVAST TOKOVODNIK

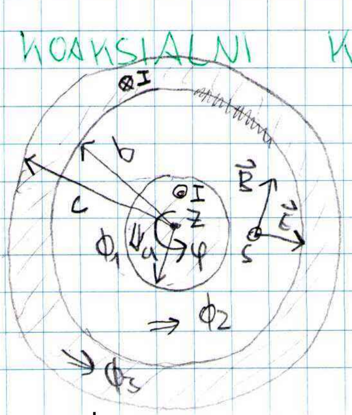


3.) MAKARON



$B_\varphi = \frac{\mu_0 I}{2} e$

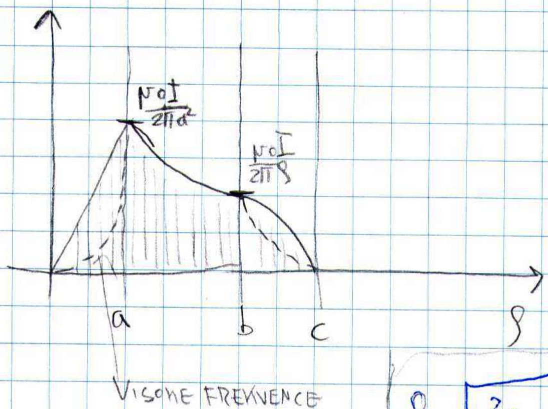
4.) KOAKSIALNI KABEL

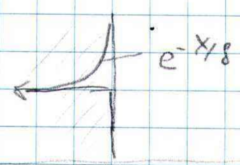


$\vec{B} = (0, B_\varphi, 0)$

$$\oint_{\rho:2\pi\rho} \vec{B} \cdot d\vec{l} = 2\pi\rho B_\varphi = \begin{cases} \mu_0 \frac{I}{\pi a^2}\pi\rho^2, & \rho \le a \\ \mu_0 I, & a < \rho \le b \\ \mu_0 \left(I - \frac{I}{\pi(c^2-b^2)}\pi(\rho^2-b^2)\right), & b < \rho \le c \\ \mu_0 \cdot \emptyset & \rho > c \end{cases}$$

$$B_\varphi = \begin{cases} \frac{\mu_0 I}{2\pi a^2}\rho, & \rho \le a \\ \frac{\mu_0 I}{2\pi\rho}, & a < \rho \le b \\ \frac{\mu_0 I}{2\pi(c^2-b^2)}\left(\frac{c^2}{\rho}-\rho\right), & b < \rho \le c \\ 0, & \rho > c \end{cases}$$





$$\phi_1 = \int_A \vec{B} \cdot d\vec{a} = \int_0^a \frac{\mu_0 I}{2\pi a^2}\rho\, l\, d\rho = \frac{\mu_0 I l}{4\pi}$$

$$\phi_2 = \ldots = \frac{\mu_0 I l}{2\pi}\ln\frac{b}{a}$$

$$\phi_3 = \ldots = \int_b^c \frac{\mu_0 I}{2\pi(c^2-b^2)}\left(\frac{c^2}{\rho}-\rho\right) l\, d\rho = \frac{\mu_0 I l}{2\pi(c^2-b^2)}\left[c^2\ln\frac{c}{b} - \frac{c^2-b^2}{2}\right]$$

$\delta = \sqrt{\frac{2}{\omega\mu\gamma}}$ Globina prodiranja (penetracije)

Za Cu : 50Hz, $\delta \simeq 1$cm

50kHz, $\delta \simeq 1$mm

5MHz $\delta = 10\mu m$

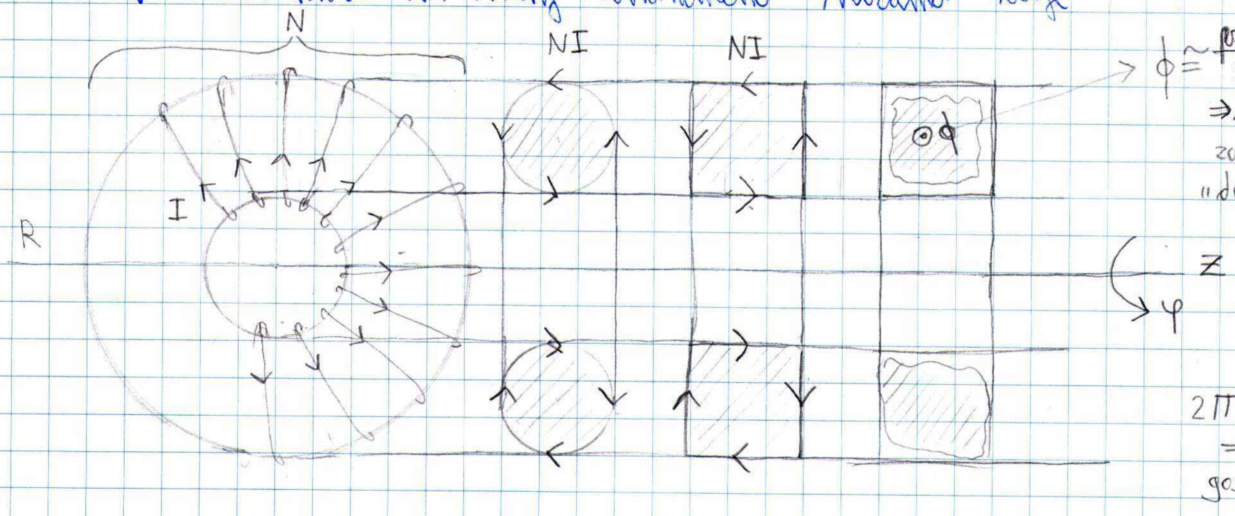
5GHz, $\delta = 10\mu m$

50GHz $\delta = 1$nm

(V KNJIGI GLEJ - TOROIDNO NAVITJE)

5.) MAGNETNO POLJE TOROIDNEGA NAVITJA

Vzamemo toroid in navij enakomerno viramo ovoje



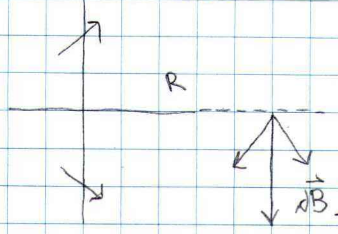
$\phi \simeq \frac{\mu_0 NI}{2\pi\rho} S$

⇒ Aproksimacija za toroido „divjega" preseka

$2\pi\rho_0 = l_s$

⇒ Srednja dolžina gostotnic polja $\vec{B}$

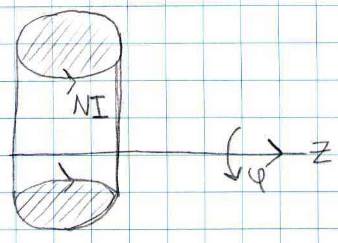
ZRCALNA TOKOVNA ELEMENTA ⇒ njuna vsota je nič



$d\vec{B} \perp R$ → magnetno polje B je vedno pravokotno na to premico ⇒ pravokotno na ravnino φ

$\boxed{\vec{B} = (0, \vec{B}_\varphi, 0)}$ mag. polje ima φ smer

Tok ni samo vzdolž toroida, ampak tudi okrog ⇒ ta tok okrog oz. zunaj je tudi en sam ovoj (čisto malo magnetnega polja je tudi zunaj)



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \cdot \text{objeti tok}$$

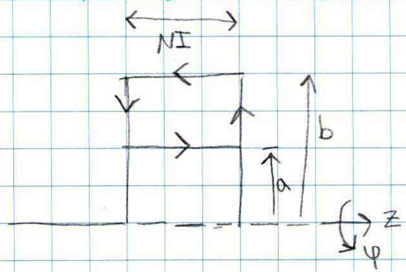
$\mathcal{L}$ krožnica ploskve okoli osi z

$$\oint \vec{B}'' d\vec{l} = \mu_0 \begin{cases} 0, & \mathcal{L}_z \\ NI, & \mathcal{L}_N \end{cases}$$

$$\boxed{B_\varphi = \begin{cases} \frac{\mu_0 NI}{2\pi\rho}, & \text{znotraj} \\ 0, & \text{zunaj} \end{cases}}$$

ZGLEDI NA TEMO TOROIDA

1.) FLUKS V TOROIDNEM NAVITJU PRAVOKOTNEGA PREREZA

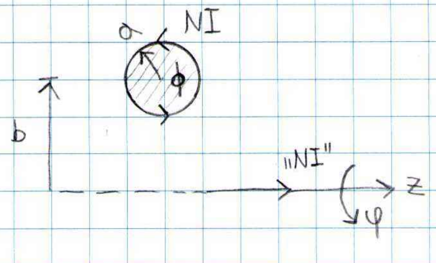


$$d\phi_m = \frac{\mu_0 NI}{2\pi\rho} C\, d\rho$$

$$\boxed{\phi =} \int d\phi = \frac{\mu_0 NIC}{2\pi} \int_a^b \frac{d\rho}{\rho} \boxed{= \frac{\mu_0 NIC}{2\pi} \ln \frac{b}{a}}$$

INDUKTIVNOST: $\psi = N\phi \Rightarrow L = \frac{\mu_0 N^2 C}{2\pi} \ln \frac{b}{a}$ Tako je def. induktivnost ⇒ magnetni sklep

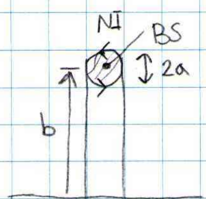
2) FLUKS V TOROIDNEM NAVITJU KROŽNEGA PREREZA



$$\phi_m = \mu_0 (NI)\left(b - \sqrt{b^2 - a^2}\right)$$

$$L = \mu_0 N^2 I \left(b - \sqrt{b^2 - a^2}\right) \rightarrow \text{induktivnost}$$

3.) TOROID MAJHNEGA PREREZA



$a \ll b \rightarrow$ ŠLANK TOROID (npr. 5x manjši, ni nujno pretirano manjši)

$\sqrt{1-x^2} \cong 1 - \frac{x}{2}$

$|x| \ll 1$

$\left(1 - \frac{x}{2}\right)^2 = 1 - x + \frac{x^2}{4} \cong 1 - \frac{x}{2}$

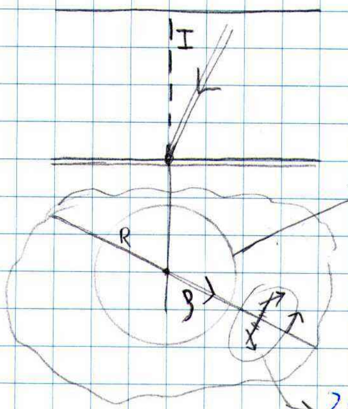
$$\phi = \mu_0 NIb\left(1 - \sqrt{1 - \left(\frac{a}{b}\right)^2}\right) \cong \mu_0 NIb\left(1 - \left(1 - \frac{a^2}{2b^2}\right)\right) = \frac{\mu_0 NI a^2}{2b} \frac{\pi}{\pi}$$

$$\Rightarrow \underbrace{\frac{\mu_0 NI}{2\pi b}}_{\simeq B_S} \cdot \underbrace{\pi a}_{\simeq S}$$

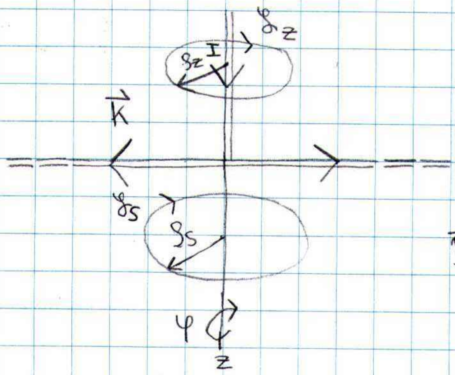
1. KOLOKVIJ DO INDUKCIJE (INDUKCIJE NE BO)

6.) MAGNETNO POLJE PLOSKOVNO RADIALNEGA TOKA

Mišljeno je da je plošča, v eni točki tok vstopa potem pa se razbeži po ploskvi



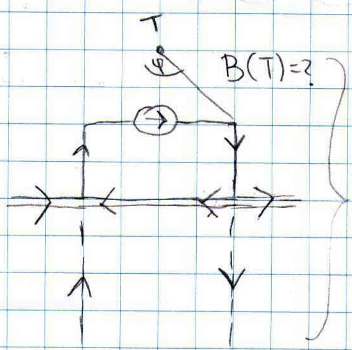
$\vec{K} = \vec{e}_s \frac{I}{2\pi\rho}$ $B_K = ?$



$\vec{B} = \vec{B}_I + \vec{B}_K$

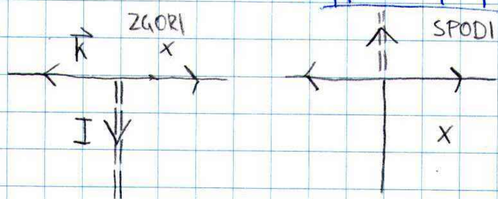
→ Zrcalno prerežemo na polovico

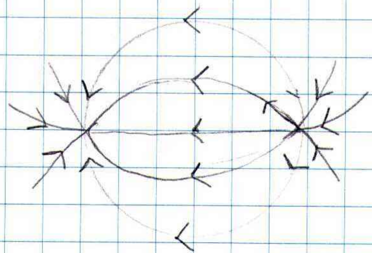
Dva zrcalna tokovna elementa ⇒ njuno magnetno polje je pravokotno na ravnino = B_φ



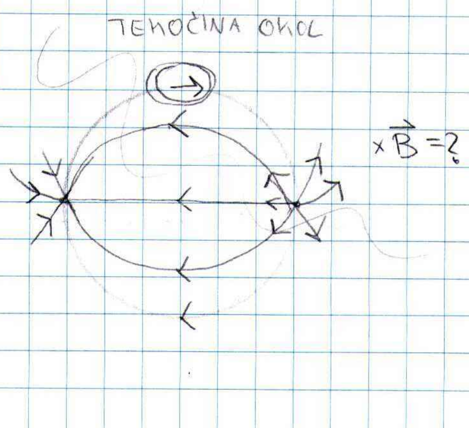
ENKRAT BLO ZA IZPIT/KOLOKVIJ

$$\oint_{\rho_z} \vec{B}\cdot d\vec{l} = 2\pi\rho_z B_\varphi = \boxed{\mu_0 I \Rightarrow B_\varphi = \frac{\mu_0 I}{2\pi\rho_z}}$$





7.) MAGNETNO POLJE SFERIČNO RADIALNEGA TOKA

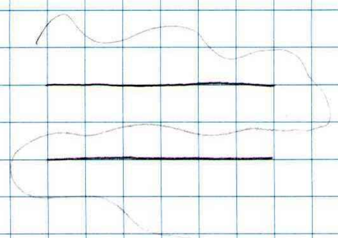


T_B $\cdot T$

$R \to \vec{B}(T) \perp R$, $R' \to \vec{B}(T) \perp R'$ } Povsod je pravokoten samo ničelni vektor

⇒ ALSO: $\boxed{B(T) = 0}$

⇒ Uporabimo Biot-Savartov zakon: $\vec{B}(T) = \frac{\mu_0 I}{4\pi}\int_1^2 \frac{d\vec{l}\times\vec{R}}{R^3}$

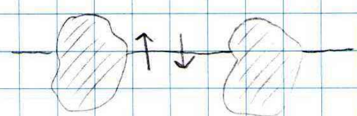


$$\vec{E} = \sum \text{polja točkastih nabojev}$$

$$\vec{j} = \gamma \vec{E} = \sum \text{sferičnih tokov}$$

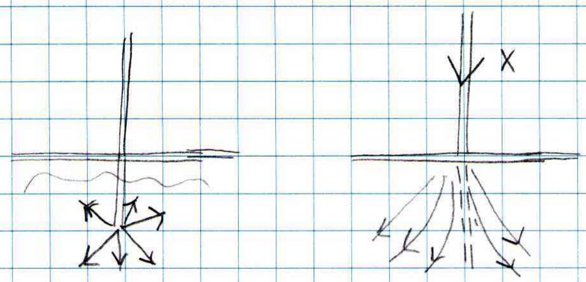
$$\boxed{W_j = W_e}$$

KONDENZATOR V ZRAKU

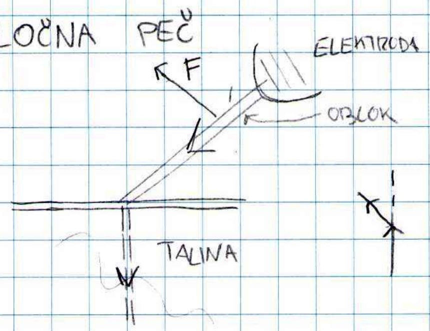


Energija se seli zgolj ob prisotnosti magnetnega polja

V KNJIGI



OBLOČNA PEČ



ODRINJENA VPRAŠANJA

1.) GIBANJE DELCA V $\vec{E}$ IN $\vec{B}$ POLJU (ČET)

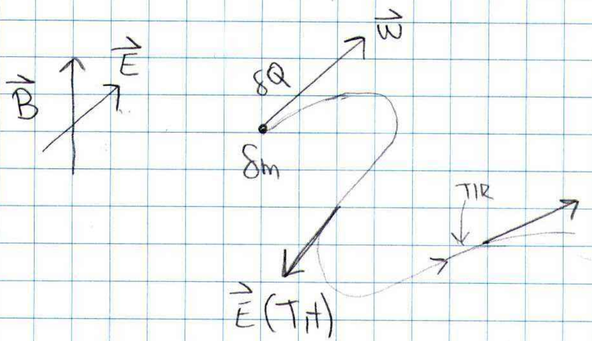
2.) SILI MED ZANKAMA (NE SAMO MED TOKOVNIMA ELEMENTOMA)
→ VPLIV ENE NA DRUGO
→ ZANKI SE ENAKO ČUTITA → SILI STA VZAJEMNI

3.) DELO ZA PREMIK ZANKE

4.) ZANKA V MAGNETNEM POLJU KI DOPUŠČA ROTACIJO → NAVOR NA ZANKO

5.) FINALE → MAGNETNI DIPOL

1. GIBANJE NABITEGA DELCA V $\vec{E}$ IN $\vec{B}$ POLJU



$t_k, T_k, \vec{w}_k, \vec{w}(t_k)$

$\delta m = \frac{\vec{w}_{k+1} - \vec{w}_k}{\delta t} \approx \delta Q(\vec{E}(T_k, t_k) + \vec{w}_B \times \vec{B}(T_k, t_k))$

$\delta t = t_{k+1} - t_k$

$\Rightarrow \vec{w}_{k+1} \rightarrow T_{k+1}$

$\Rightarrow w_{k+2} \rightarrow \dots$

NE SODI V 1. LETNIK

$$\delta \vec{F}_L = \delta Q(\vec{E} + \vec{w} \times \vec{B})$$

$$\delta m \cdot \dot{\vec{w}} = \delta \vec{F}_L = \delta Q(\vec{E} + \vec{w} \times \vec{B})$$

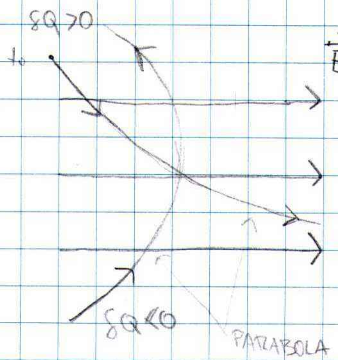
$$\vec{r}(t) = \vec{r}(t_0) + \int_{t_0}^{t} \vec{w}(t)\, \delta t$$

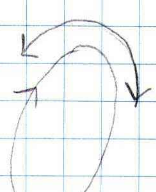
1. A. DELEC V HOMOGENEM POLJU $\vec{E}$

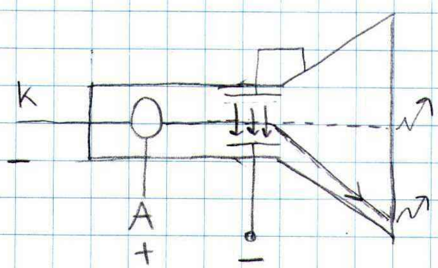
$$\delta m \dot{\vec{w}} = \delta Q \vec{E} \Rightarrow \dot{\vec{w}} = \vec{a} = \frac{\delta Q}{\delta m} \vec{E}$$

$$\vec{w}(t) = \vec{w}(t_0) + \frac{\delta Q}{\delta m} \vec{E}(t - t_0)$$

$$\vec{r}(t) = \vec{w}(t_0)(t - t_0) + \frac{1}{2}\frac{\delta Q}{\delta m} \vec{E}(t - t_0)^2 + \vec{r}(t_0)$$

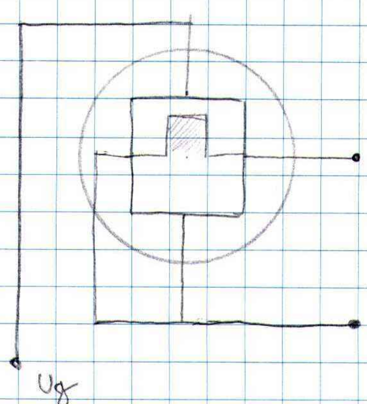






$U_x = U_g \cos \omega t$

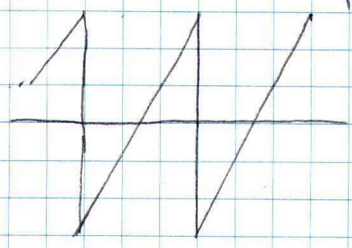
$U_y = U_g \cos(2\omega t + \alpha)$

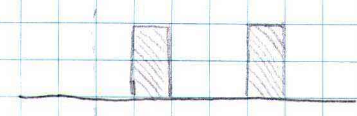


$U_x = U_g \cos \omega t$

$U_y = U_g \cos(\omega t + \alpha)$

„ŽAGA"





OBLAK ELEKTRONOV

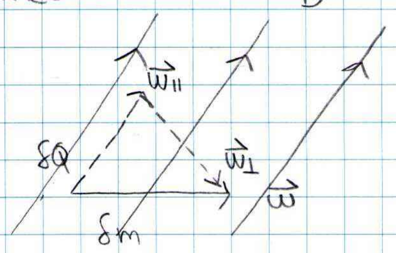
1.B. DELEC V HOMOGENEM POLJU $\vec{B}$

$$\delta m \dot{\vec{w}} = \delta Q (\vec{w} \times \vec{B}) \quad / \cdot \vec{w}$$

$$\Rightarrow \delta m \dot{\vec{w}} \vec{w} = \delta Q (\vec{w} \times \vec{B}) \cdot \vec{w} = \delta Q (\vec{w} \times \vec{w}) \cdot \vec{B} = 0$$

$$\Rightarrow |\vec{w}| = konst$$

RAZCEP:



$$\vec{w} = \vec{w}_\perp + \vec{w}_\parallel$$

$$\delta m (\dot{\vec{w}}_\perp + \dot{\vec{w}}_\parallel) = \delta Q \vec{w}_\perp \vec{B} + \delta Q \overbrace{\vec{w}_\parallel \times \vec{B}}^{\vec{0}}$$

$\boxed{\delta m \dot{\vec{w}}_\parallel = \vec{0}}$ ⇒ Pospešek vzdolž magnetnega polja je nič

Delec se giblje enakomerno. Komponenta hitrosti magnetnega polja se ne spreminja

$w_\parallel(t) = w_\parallel(t_0)$

Enakomerno kroženje ⇐ $\boxed{\delta m \dot{\vec{w}}_\perp = \delta Q \vec{w}_\perp \times \vec{B}}$

Gibanje pri katerem se hitrost ne spreminja.

Sila je vedno pravokotna na hitrost

⇒ Delec se giblje po spirali

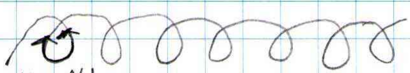
$\cdot / \vec{w}_\perp$: $\Rightarrow |\vec{w}_\perp| = konst$ – Obodna hitrost

$$\Rightarrow \delta m \vec{w}_\perp \cdot \dot{\vec{w}}_\perp \, dt = \frac{\delta m}{2} d(w_\perp^2)$$

RADIALNI POSPEŠEK:

$$\delta m a_r = |\delta Q| w_\perp B$$

$$\delta m \frac{w_\perp^2}{R}$$



1.) ⇒ Ugotoviti je treba smer kroženja

2.) ⇒ Ugotoviti je treba kakšen je radij

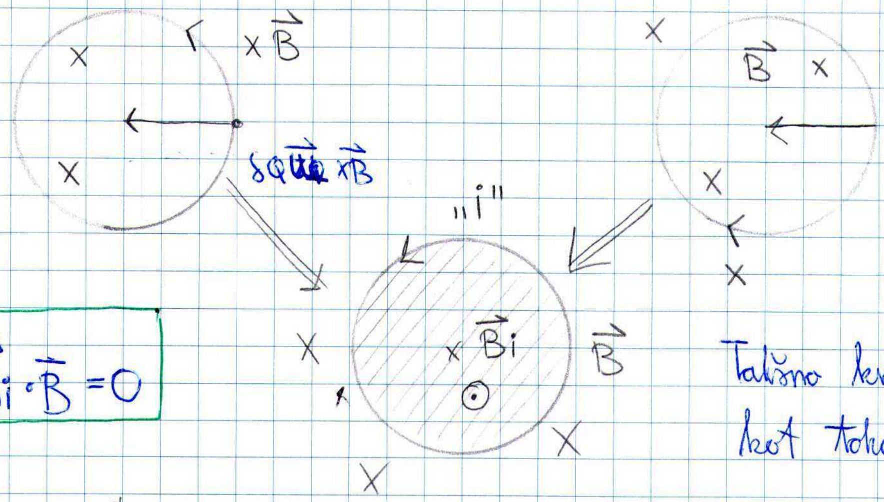
1.) KAKŠEN JE RADIJ

$\boxed{R = \frac{\delta m w_\perp}{|\delta Q| \cdot B}}$ $\Rightarrow \omega = 2\pi f = \frac{w_\parallel}{R}$

⇒ Te radij so zelo majhni

2. KAKŠNA JE SMER KROŽENJA

POZITIVEN DELEC $\delta Q > 0$ NEGATIVEN DELEC $\delta Q < 0$



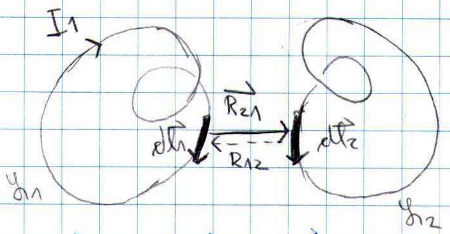
$\delta Q \vec{w}_\perp \times \vec{B}$

$$\boxed{\vec{B_i} \cdot \vec{B} = 0}$$

Takšno kroženje lahko predstavimo kot tokovno zanko

V lakno magnetno polje rahlo manjše $\simeq 10^{-5}$

2. SILA MED TOKOKROGOMA



$$\vec{F}_{m_1}^{(2)} = \oint_{\mathcal{L}_2}\oint_{\mathcal{L}_1} \frac{\mu_0 I_1 I_2}{4\pi R_{12}^3} \vec{dl_1} \times (\vec{dl_2} \times \vec{R_{12}})$$

$$\vec{F}_{m_2}^{(1)} = \oint_{\mathcal{L}_1}\oint_{\mathcal{L}_2} \frac{\mu_0 I_1 I_2}{4\pi R_{21}^3} \vec{dl_2} \times (\vec{dl_1} \times \vec{R_{21}})$$

$$\vec{R_{12}} = \vec{r_1} - \vec{r_2} = -\vec{R_{21}}$$

$$\vec{F}_{m_1}^{(2)} + \vec{F}_{m_2}^{(1)} = \frac{\mu_0 I_1 I_2}{4\pi} \cdot \oint_{\mathcal{L}_1}\oint_{\mathcal{L}_2} \left[\vec{dl_1} \times \left(\vec{dl_2} \times \frac{\vec{R_{12}}}{R_{12}^3}\right) + \vec{dl_2} \times \left(\vec{dl_1} \times \frac{\vec{R_{21}}}{R_{21}^3}\right)\right]$$

Uporabimo: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \cdot \vec{b} - (\vec{a} \cdot \vec{b}) \cdot \vec{c}$

$$[\ldots] = \left(\vec{dl_1} \cdot \frac{\vec{R_{12}}}{R_{12}^3}\right) \cdot \vec{dl_2} - \cancel{(\vec{dl_1} \cdot \vec{dl_2}) \frac{\vec{R_{12}}}{R_{12}^3}} + \left(\vec{dl_2} \cdot \frac{\vec{R_{21}}}{R_{21}^3}\right) \vec{dl_1} - \cancel{(\vec{dl_2} \cdot \vec{dl_1}) \cdot \frac{\vec{R_{21}}}{R_{21}^3}}$$

→ EL. POLJSKA JAKOST

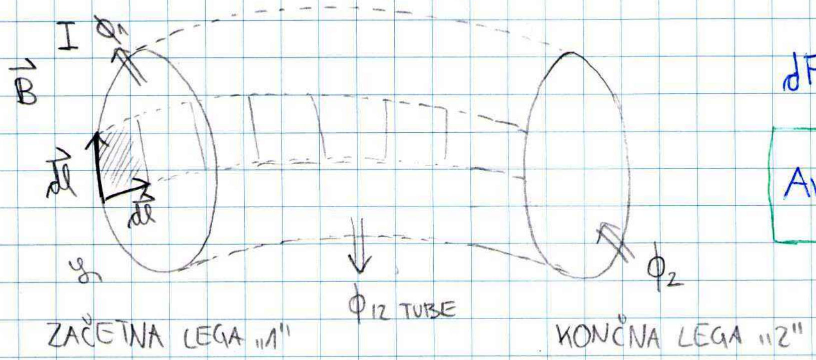
$$\oint_{\mathcal{L}_1}\oint_{\mathcal{L}_2} \left(\vec{dl_2} \cdot \frac{\vec{R_{12}}}{R_{12}^3}\right) \vec{dl_2} = \oint_{\mathcal{L}_2} \left(\oint_{\mathcal{L}_1} \frac{\vec{R_{12}}}{R_{12}^3} \cdot \vec{dl_1}\right) \vec{dl_2} = \oint_{\mathcal{L}_2} \left(\frac{Q\, 4\pi\varepsilon_0}{4\pi\varepsilon_0\, Q} \oint \frac{\vec{R_{12}}}{R_{12}^3} \cdot \vec{dl_1}\right) \vec{dl_2}$$

$$= \oint_{\mathcal{L}_2} \left(\frac{4\pi\varepsilon_0}{Q} \underbrace{\oint_{\mathcal{L}_1} \vec{E} \cdot \vec{dl_1}}_{0}\right) \cdot \vec{dl_2} = 0$$

⇒ Magnetni sili sta vzajemni (recipročnost)

$L_{21} = L_{12}$

3. DELO ZA PREMIK TOKOVNE ZANKE V TUJEM MAGNETNEM POLJU



$$dF_m = I\, d\vec{l} \times \vec{B}$$

$$\boxed{A_m = \oint_{g_1} \int_1^2 (I\, d\vec{l} \times \vec{B}) \cdot d\vec{l}'}$$

Delo magnetne sile za premik iz začetne v končno lego

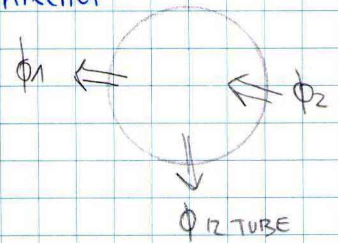
$$= I \oint \int_1^2 \underbrace{(d\vec{l}' \times d\vec{l}) \cdot \vec{B}}_{d\phi_{m\,12\,tube}}$$

$\phi_{12\,tube}$

$$\frac{A}{I} = \phi_2 - \phi_1$$

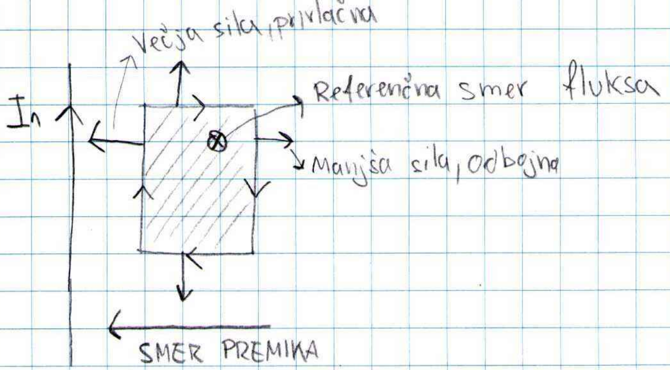
$$\boxed{\Rightarrow A_m = I(\phi_2 - \phi_1)}$$

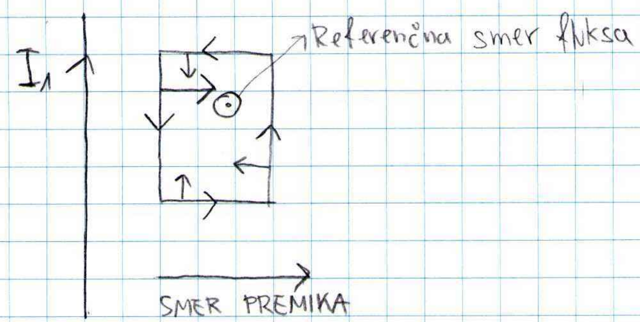
KIRCHOF



ϕ_2 in ϕ_1 sta označena po desnem pravilu glede na tok v zanki

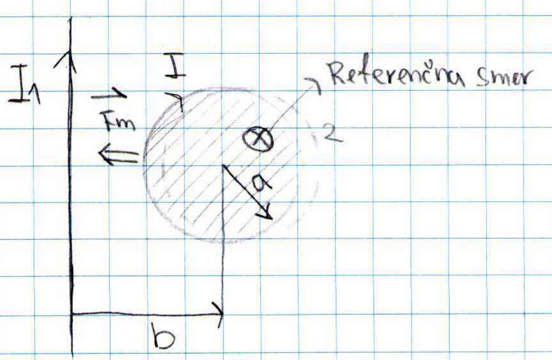
1. ZGLED:





Zanka se premika tako, da se ji fluks v referenčni smeri poveča

2. ZGLED



$$dA_m = I(\phi(b+db) - \phi(b)) = I = \frac{d\phi}{db} \cdot db$$

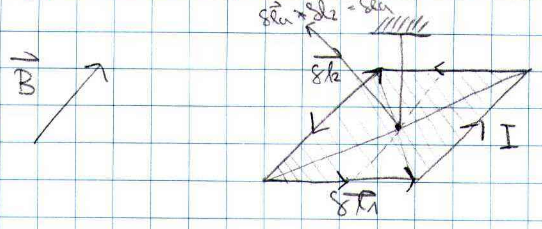
SILA

$$I \frac{d}{db}\left(\mu_0 I_1 \left(b - \sqrt{b^2 - a^2}\right)\right) db < 0$$

$$F_{mb} = \mu_0 I I_1 \left(1 - \frac{b}{\sqrt{b^2 - a^2}}\right) < 0$$

21.3.11

4. NAVOR NA „MAJHNO" TOKOVNO ZANKO (V HOMOGENEM POLJU)



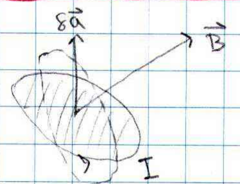
$$\delta\vec{M}_m = -\frac{\delta\vec{l}_2}{2}\times(I\delta\vec{l}_1\times\vec{B}) + \frac{\delta\vec{l}_1}{2}\times(I\delta\vec{l}_2\times\vec{B}) + \frac{\delta\vec{l}_2}{2}\times(-I\delta\vec{l}_1\times\vec{B}) - \frac{\delta\vec{l}_1}{2}\times(-I\delta\vec{l}_2\times\vec{B})$$

$$= \left[\delta\vec{l}_1\times(\delta\vec{l}_2\times\vec{B}) - \delta\vec{l}_2\times(\delta\vec{l}_1\times\vec{B})\right] I$$

$$\Rightarrow \left[(\delta\vec{l}_1\cdot\vec{B})\,\delta\vec{l}_2 - \cancel{(\delta\vec{l}_1\cdot\delta\vec{l}_2)\,\vec{B}} - (\delta\vec{l}_2\cdot\vec{B})\,\delta\vec{l}_1 + \cancel{(\delta\vec{l}_2\cdot\delta\vec{l}_1)\,\vec{B}}\right] I$$

$$= \vec{B}\times(\delta\vec{l}_2\times\delta\vec{l}_1)\, I = I(\delta\vec{l}_1\times\delta\vec{l}_2)\times\vec{B}$$

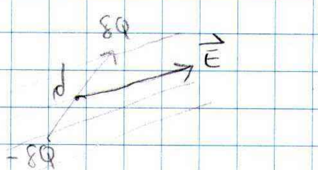
$$\boxed{\delta M_m = I\,\delta\vec{a}\times\vec{B}}$$

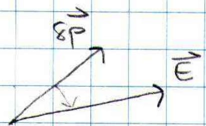


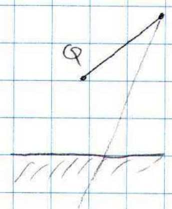
Neglede na to kakšno zanko vzamemo je rezultat enak

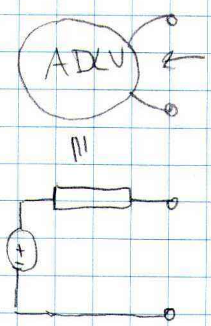
Def. Zanka se postavi tako, da vzame največ fluksa

5. MAGNETNI DIPOL



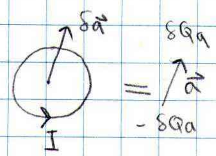




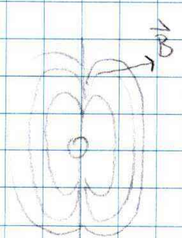


$\delta\vec{p} = \delta Q\,\vec{d}$

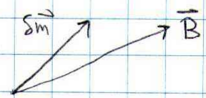
$\delta\vec{M}_e = \delta Q\vec{d}\times\vec{E} = \delta\vec{p}\times\vec{E}$



$\underbrace{I\delta\vec{a}}_{\delta m} = \delta Q_m \vec{d}$



$$\boxed{\delta\vec{m} = I\,\delta\vec{a}}$$

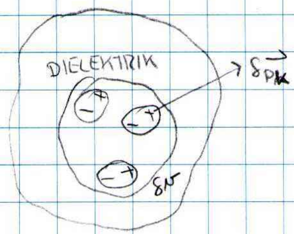


$\delta\vec{m}$ -

Spin lahko modeliramo kot tokovno zanko. Model kot da je

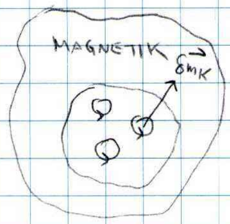
El. in mag. dipol sta modela za pojasnjevanje

VEKTOR MAGNETIZACIJE



$$\vec{P} = \lim_{\text{"}\delta v \to 0\text{"}} \frac{\sum_k \delta \vec{p}_k}{\delta v} \quad [C/m^2]$$

$$\oint \vec{P} \cdot d\vec{a} = -Q_{vez.pol.}$$

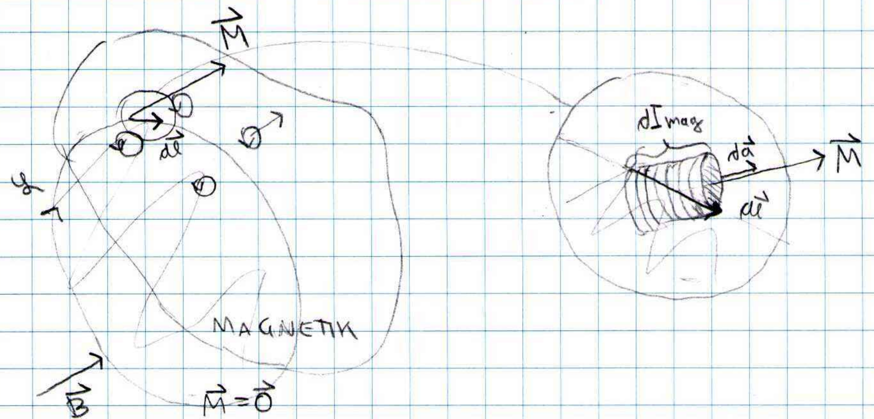


$$\vec{M} = \lim_{\text{"}\delta v \to 0\text{"}} \frac{\sum_k \delta \vec{m}_k}{\delta v} \quad [A/m]$$

$$\oint_g \vec{M} \cdot d\vec{l} \rightarrow \text{VRTINČNOST}$$

WEISSOVE DOMENE

VRTINČNOST VEKTORJA MAGNETIZACIJE



$$\vec{M} \cdot d\vec{l} = \frac{dI_{mag} \cdot d\vec{a}}{dV} \cdot d\vec{l} = dI_{mag}$$

$$\uparrow d\vec{a} \cdot d\vec{l}$$

$$\boxed{\oint_g \vec{M} \cdot d\vec{l} = \oint_g dI_{mag} = I_{mag}}$$

ŽARKOZAVER:

$$\nabla \times \vec{M} = \vec{J}_{mag}$$

↳ Amperovi toki

$$\boxed{\frac{\partial \vec{P}}{\partial t} = J_{pol}}$$

VEKTOR MAGNETNE POLJSKE JAKOSTI

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{not}}{\varepsilon_0} \quad / \varepsilon_0$$

$$\oint_A \vec{P} \cdot d\vec{a} = -Q_{not.pol.}$$

$$\oint_A (\underbrace{\varepsilon_0 \vec{E} + \vec{P}}_{D}) \cdot d\vec{a} = \underbrace{Q_{not} - Q_{not.pol}}_{Q_{not.prosti}}$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\oint \vec{D} \cdot d\vec{a} = Q_{not.prosti}$$

$$Q_{prosti} = Q - Q_{pol}$$

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$$

$$I = I_{pr} + I_{mag}$$

$$\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \int_A \vec{J}_{pr} \cdot d\vec{a}$$

$$\oint_{\mathcal{L}} \vec{B} \cdot d\vec{a} = \mu_0 \int_A \vec{J} \cdot d\vec{a} = \mu_0 I_{skozi\ A\ na\ \mathcal{L}}$$

$$\oint_{\mathcal{L}} \vec{M} \cdot d\vec{l} = I_{mag}$$

TOKI MAGNETIZACIJE = AMPEROVI TOKI

$$\oint_{\mathcal{L}} \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) \cdot d\vec{l} = I - I_{mag} = I_{prosti}$$

MAGNETNA POLARIZACIJA

$$H = \frac{\vec{B}}{\mu_0} - \vec{M} \Rightarrow \boxed{B = \mu_0(\vec{H} + \vec{M})} = \mu_0 \vec{H} + \vec{I}$$

$$\boxed{\oint_{\mathcal{L}} \vec{H} \cdot d\vec{l} = \int_{\mathcal{L}} \vec{J}_{prosti} \cdot d\vec{a} = I_{prosti\ skozi\ A\ na\ \text{"}\mathcal{L}\text{"}}}$$

24.3.11

ELEKTRIČNO POLJE

$$! \ \underbrace{P \propto E}\ !$$

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

$$\vec{D} = \varepsilon_0 \underbrace{(1 + \chi_e)}_{\varepsilon_r} \vec{E}$$

$$\varepsilon$$

$$\vec{D} = \varepsilon E = \varepsilon_0 \varepsilon_r \vec{E}$$

MAGNETNO POLJE

Če je snov linearna (razmere niso nelin. zvezane):

$$!! \ \underbrace{\vec{M} \propto \vec{B}, \vec{H}}\ !!$$

$$\vec{M} = \chi_m \vec{H}$$

$$\vec{B} = \mu_0 \underbrace{(1 + \chi_m)}_{\mu_r} \vec{H}$$

$$\vec{B} = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

MAGNETIKI

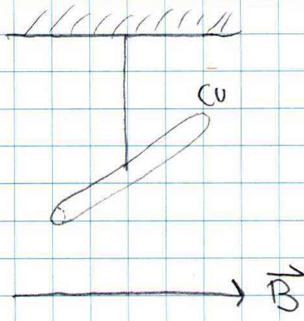
1.) Diamagnetiki (baker, srebro, zlato, voda) 2.) Paramagnetiki (aluminij, zrak) 3.) Feromagnetiki (železo 26, kobalt 27, nikelj 28)

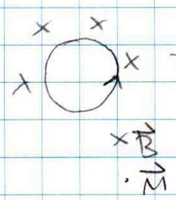
1. DIAMAGNETIKI

Snovi katerih atomi, kot skupek delcev ne izkazuje rezultirajočega dipolskega momenta ⇒ nimajo rezultirajočega dipolskega momenta (paramagnetiki ga imajo)

Poleg dipolskega odziva tudi magnetni odziv

DIA – prečno, PARA – vzdolžno – postavijo se vzdolž, tudi FERO se postavi vzdolž
FERO – železa





Vsak delec se še dodatno giblje zaradi tega magnetnega polja
⇒ zanka je v labilni legi

$\vec{M} \cdot \vec{B} < 0$

$X_m \sim -10^{5}$

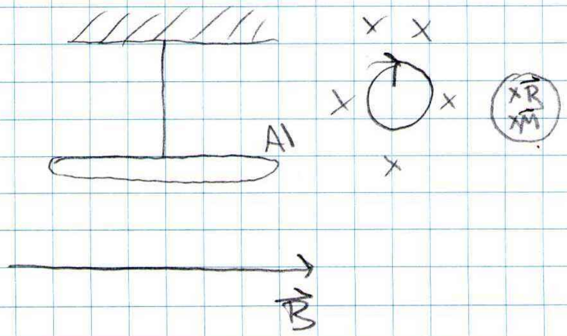
$\mu_r = 1 + X_m \approx 0{,}999xxx$

Vektor nasproten zunanjemu polju. Skalarni produkt negativen.
X_m – mag. susceptibilnost majhna in negativna

2. PARAMAGNETIKI

Snovi, katerih atomi izkazujejo dipolni moment.

Zanke se postavijo v smer mag. polja, zavzamejo stabilno lego



$\vec{M} \cdot \vec{B} > 0$

$X_m \simeq +10^{-5}$

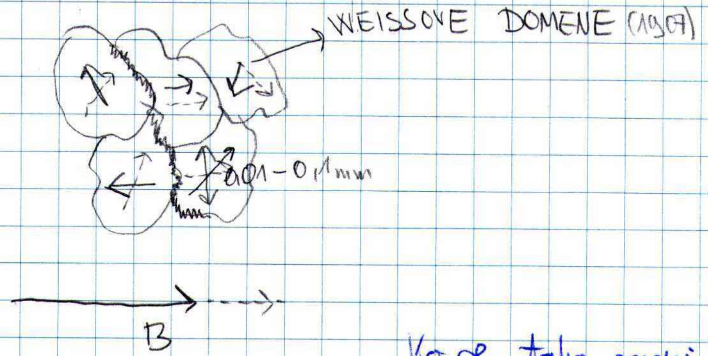
$\mu_r = 1 + X_m = 1{,}0000xx \ldots$

Dia in paramagnetiki so magnetno presvetljeni. Mag. polje se spremeni, če je žica jeklena ipd.

3. FEROMAGNETIKI

Polenevrani paramagnetiki ⇒ Feromagnetiki

$$Fe = \overset{K}{2} + \overset{L}{8} + \overset{M}{14} + \overset{N}{2}$$

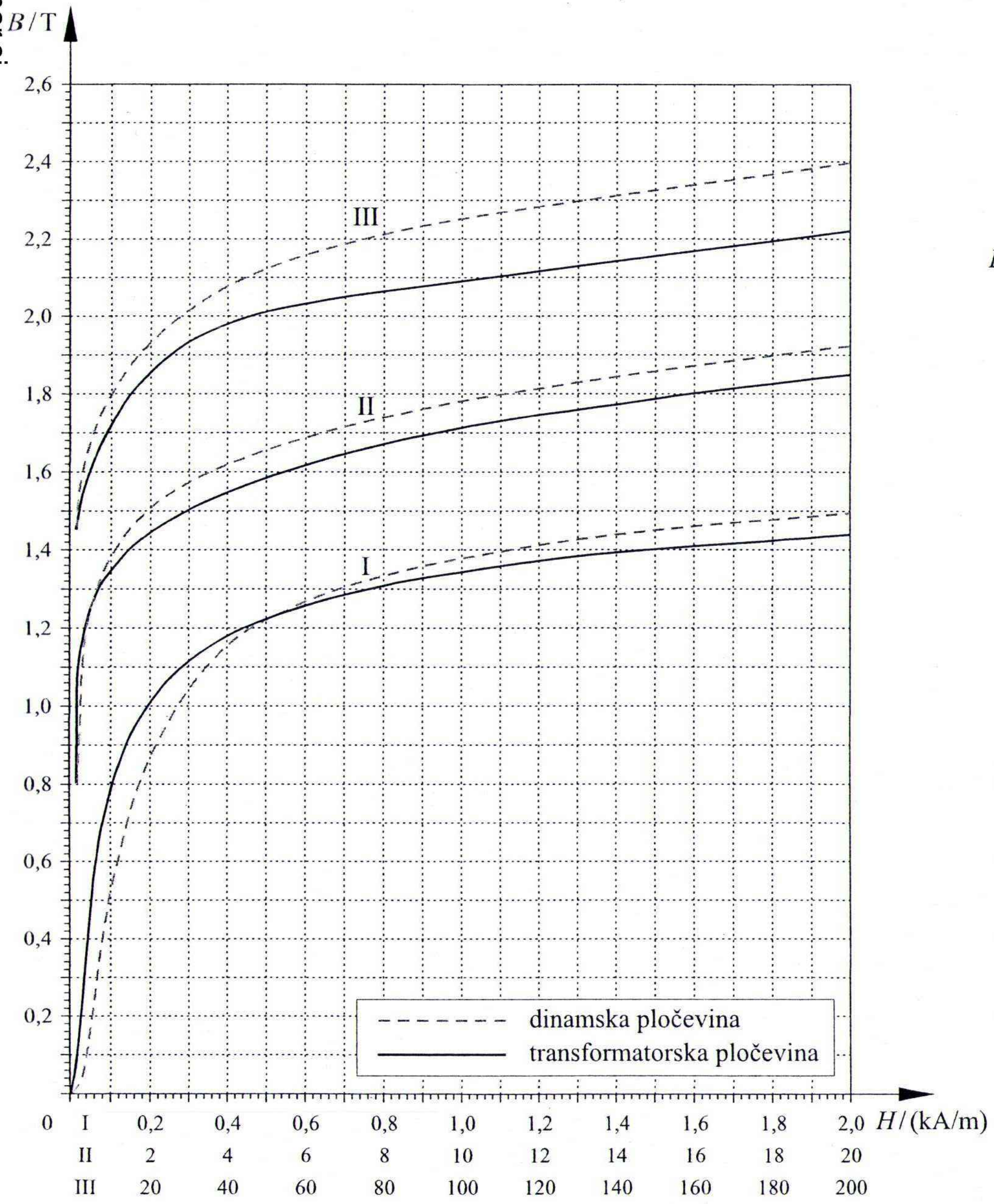


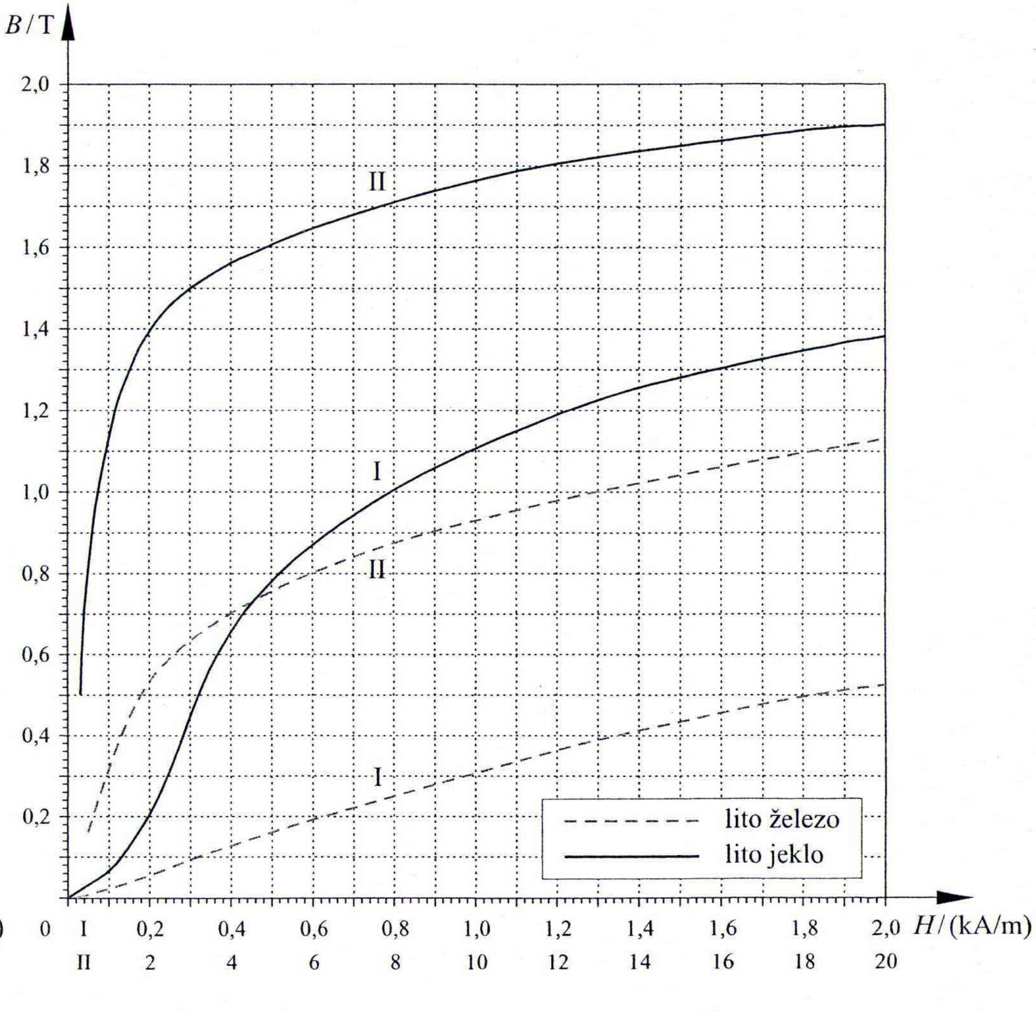
Gruče atomov se razporejajo v Weissovih domenah. Gruče reda velikosti 0,01 % 0,1 mm

Atomi imajo na teh področjih isto orientacijo magnetizacije

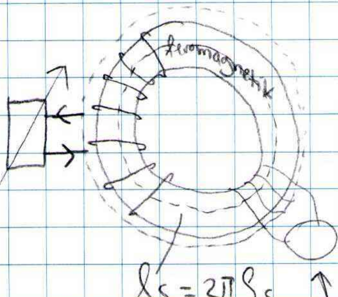
Ko se taka snov izpostavi mag. polju, potem se dogaja sobivanje domen v smer magn. polja

Začetne krivulje magnetenja za mehke magnetne materiale





MERJENJE MAGNETILNIH KRIVULJ



$$\oint_{l_s} \vec{H}\cdot d\vec{l} = \underbrace{2\pi\rho_s}_{l_s} H_s = N\boxed{I}$$ → Merljivo

$$H_s = H = \frac{NI}{l_s}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$B_\varphi = \mu_0(H_\varphi + M_\varphi) \quad ---- \quad B_s = \mu_0 H_s + \mu_0 M_s$$

$$\Rightarrow B = \mu_0 H + \mu_0 M$$

1.) ZAČETNA (DEVIŠKA) MAGNETILNA KRIVULJA

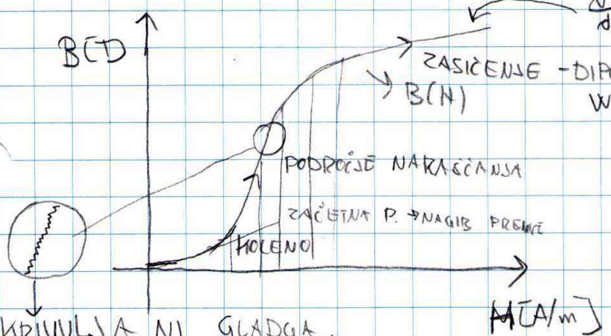
BH krivulja (glej lit.) ⇒ Dobljena pri predhodnem razmagnetenju

(Prepust navitij a Kirijevo temperaturo → Segrejemo železo nad 770°C in ohladimo, dobimo razmagneteno železo).

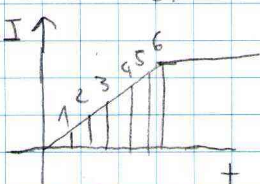
Začne je razmagnetena ⇒ $M=0$

$$\vec{M} = \lim \frac{\sum \delta\vec{m}}{\delta V}$$

↳ Nekaj 100 Weissovih področij



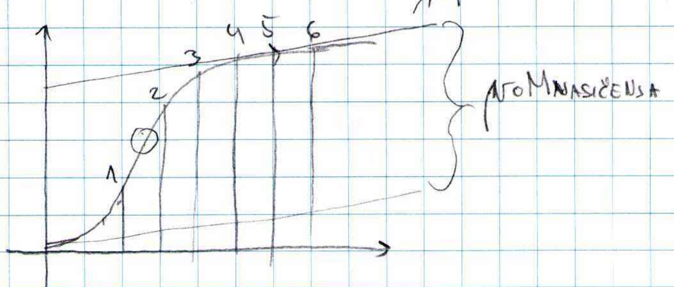
KRIVULJA NI GLADKA, MAJHNI ZASUKI, DA SE SIH Z MIKROFONOM SLIŠAT



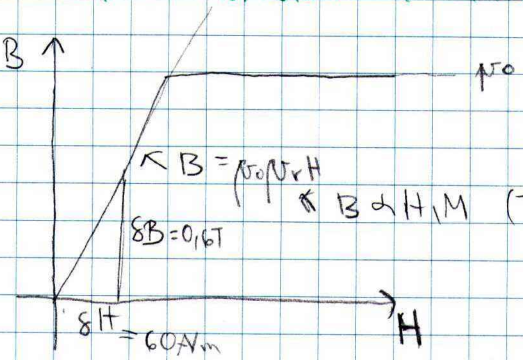
Začetna krivulja dobimo tako, da tok iz 0 pričnemo dvigovati (monotono) (stalno počasi naraščal)

⇒ Monotono se dviguje tudi H in potem B itd.

NB! Počasi dvigujemo tok, da ne izzivamo indukcije, ki bi meritve pokvarila



2. LINEARIZACIJA MAGNETILNE KRIVULJE

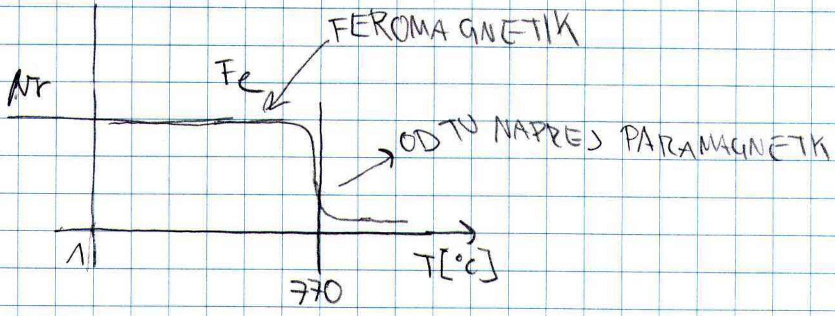


$B \propto H, M$ (Tuki je dejansko magnetenje linearno)

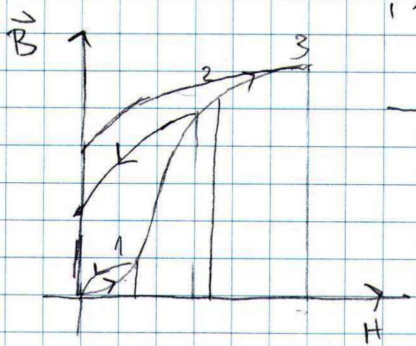
$$\mu_0 \mu_r = \frac{\delta B}{\delta H} \cong 10^{-2} \rightarrow \mu_r = \frac{10^{-2}}{4\pi \cdot 10^{-7}} = \cong 8000$$

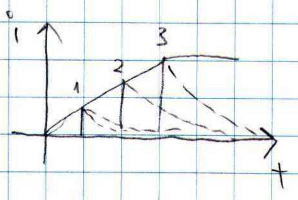
Tuki popišemo 10-tinko, ne atoma pa precesija Weissove ploskve
⇒ So seveda tudi določene izgube

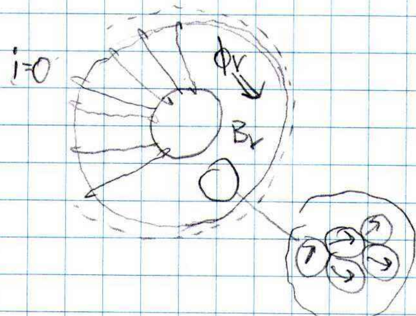
3. COURIJEVA TEMPERATURA



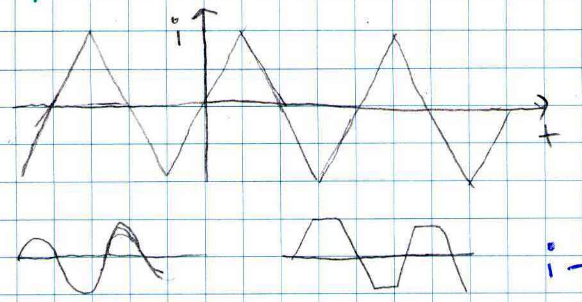
4. REMANENTNA GOSTOTA (PREOSTALA GOSTOTA)



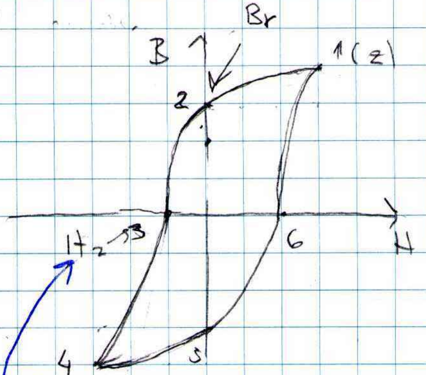




5.) HISTEREZNA ZANKA



i - periodični tok



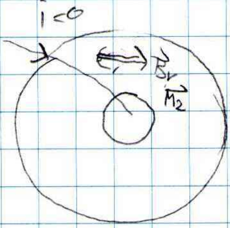
Koercitivna jakost

$\boxed{T2}$:

$B_2 = \mu_0 H_2 + \mu_0 M_2$

$H_2 = 0$

$B_2 = \mu_0 M_2$

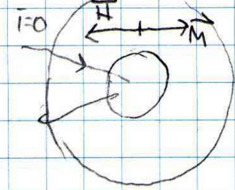


$\boxed{T3}$:

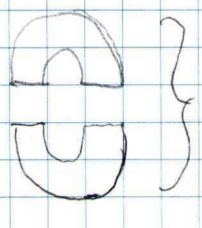
$B_3 = \mu_0 H_2 + \mu_0 H_3$

$B_3 = 0$

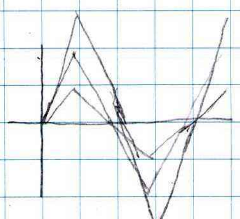
$\boxed{H_3 = -M_3}$

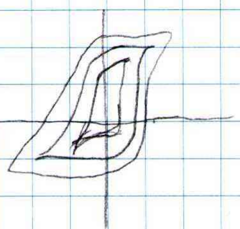


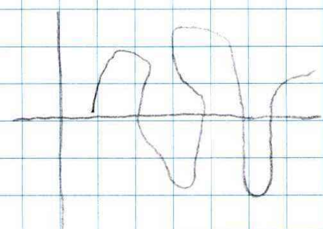
$H_c \sim 10$ A/m – MEHKOMAGNETNI MATERIALI
$H_c \sim 10^5$ A/m – TRDOMAGNETNI MATERIALI → Trajni magneti



Prežagan toroid ⇒ Trajni magnet

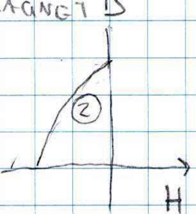


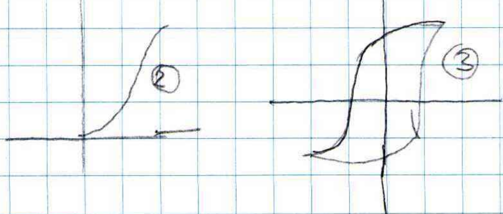




Magnetilna krivulja je nesimetrična

TRAJNI MAGNET



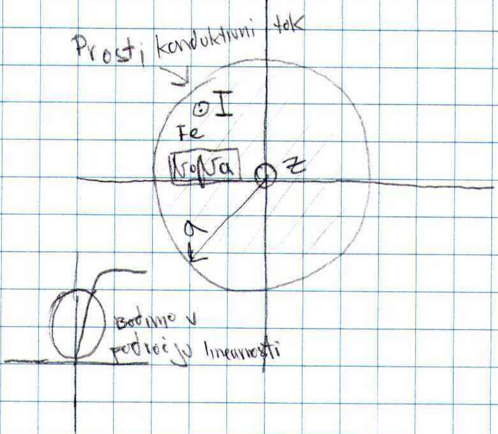


Te tri krivulje so pomembne za mag. materiale

Pomembna je histerezna

kako smo prišli do tega

ZGLED: MAGNETNO POLJE V/OB JEKLENEM VODNIKU KROŽNEGA PRESEKA



$$\oint_{\mathcal{L}} \vec{M}\cdot d\vec{l} = \int_A \vec{J}_{prosti}\cdot d\vec{a}$$

$$\vec{B} = (0, B_\varphi, 0) \rightarrow \vec{M} = (0, M_\varphi, 0) \;,\; \vec{H} = (0, H_\varphi, 0)$$

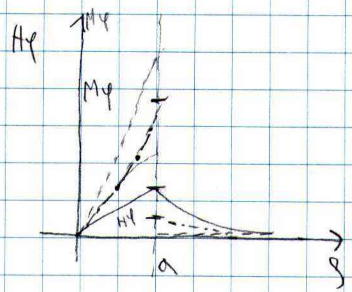
$\mathcal{L}$: Krožnica okoli z-osi polmera $\rho \lessgtr a$

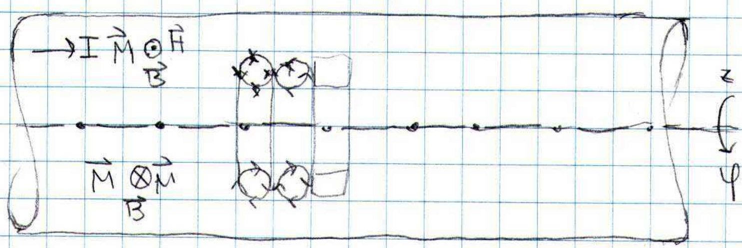
$$\oint_{\mathcal{L}} \vec{M}\cdot d\vec{l} = 2\pi\rho H_\varphi = \begin{cases} \frac{I}{\pi a^2}\pi\rho^2, & \rho \le a \\ I, & \rho > a \end{cases}$$

$$H_\varphi = \begin{cases} \frac{I}{2\pi a^2}\rho, & \rho < a \\ \frac{I}{2\pi\rho}, & \rho > a \end{cases}$$

$$B_\varphi = \mu_0\mu_r H_\varphi = \begin{cases} \frac{\mu_0\mu_r I}{2\pi a^2}\cdot\rho, & \rho < a \\ \frac{\mu_0 I}{2\pi\rho}, & \rho > a \end{cases}$$

$$M_\varphi = \chi_0 H_\varphi = (\mu_r - 1) H_\varphi = \begin{cases} \frac{(\mu_r-1)I}{2\pi a^2}\rho, & \rho < a \\ 0, & \rho > a \end{cases}$$

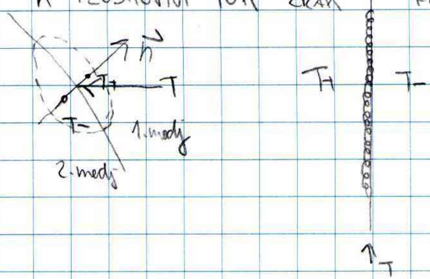




$$B_\varphi = \frac{\mu_0\mu_r I}{2\pi a^2}\rho = \underbrace{\frac{\mu_0 I}{2\pi a^2}\rho}_{\text{Mag. polje zaradi toka v žici}} + \underbrace{\frac{(\mu_r-1) I\mu_0}{2\pi a^2}\rho}_{\text{Mag. polje zaradi amperovih tokov}}$$

MEJNI POGOJI

$\vec{K}$ PLOSKOVNI TOK ZRAK FEROMAGNETIK



$$\oint_A \vec{B}\cdot d\vec{a} = 0 \Rightarrow B_n(T_+) - B_n(T_-) = 0$$

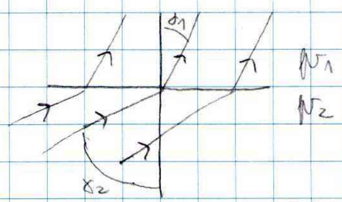
$$\oint_g \vec{H}\cdot d\vec{l} = \int_A \vec{J}_p\cdot d\vec{a} \Rightarrow \vec{n}\times\left[\vec{H}(T_+) - \vec{H}(T_-)\right] = \vec{K}_p(T)$$

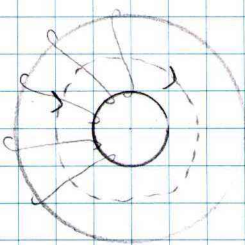
$$H_{\tau 1}(T_+) - H_{\tau 1}(T_-) = K_{\tau 2}(T)$$
$$H_{\tau 2}(T_+) - H_{\tau 2}(T_-) = K_{\tau 1}(T)$$

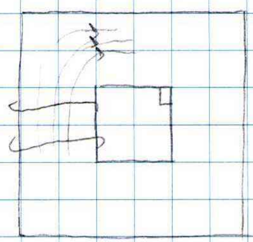
POSEBEN PRIMER: MEJA BREZ OBLOGE

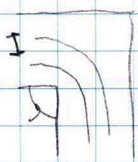
$$\left.\begin{aligned} B_n(T_+) &= B_n(T_-) \\ H_\tau(T_+) &= H_\tau(T_-) \end{aligned}\right\} \quad \boxed{\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\mu_1}{\mu_2}}$$

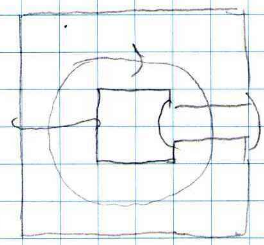
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\gamma_1}{\gamma_2}$$





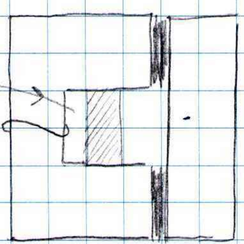






STRESANO POLJE

Namenoma se nepije



RABMO SAMO EL TOK ZA SPROSTITEV

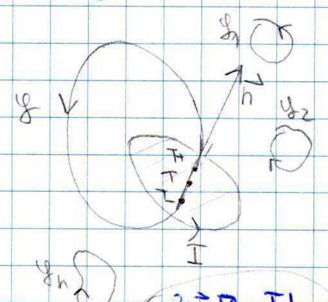
31.3.11

MAGNETNI POTENCIAL IN MAGNETNA NAPETOST

$\oint_g \vec{E}\cdot d\vec{l} \Rightarrow \vec{E} = -\vec{n}\,\frac{\partial V}{\partial n}$, $V(T)=$

$V(T)=\int_{T}^{T_\infty} \vec{E}\cdot d\vec{l}$, $U_{12}=\int_{T_1}^{T_2} \vec{E}\cdot d\vec{l}$

$\oint_g \vec{H}\cdot d\vec{l}$ = objeti tok!



Če se dogovorimo, da so zanke g takšne, da zanke g ne prestopajo, potem velja:

$\oint_g \vec{H}\cdot d\vec{l} = 0$

⇒ Vpeljemo količino, ki jo imenujemo

SKALARNI MAGNETNI POTENCIAL V_m,

da velja:

$\boxed{H = -\vec{n}\,\frac{\partial V_m}{\partial n}}$ $\boxed{V_m(T) = \int_{T}^{T_\infty} \vec{H}\cdot d\vec{l}}$ $\boxed{U_{m12} = \int_{T_1}^{T_2} \vec{H}\cdot d\vec{l} = \Theta_{12}}$

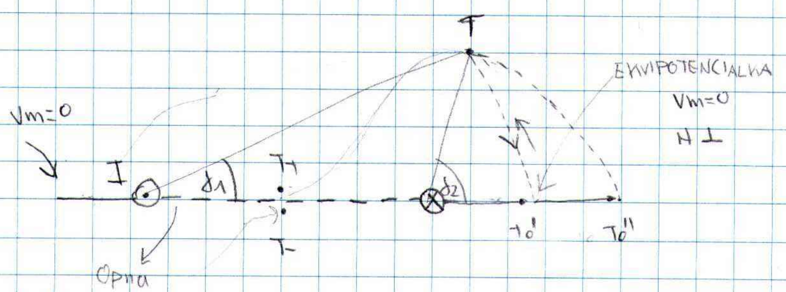
$\oint \vec{H}\cdot d\vec{l} = I$!
$T_+ \times T_-$
$V_m(T_+) - V_m(T_-) = I$

To smo izvedli brez kakršne fizikalne podlage. Samo zato, da zdaj lahko potem mag. veličine si predstavljamo

Funkcija potenciala mu je NEZVEZNA ob legi opni

ZGLEDI:

1.) DVOVOD DVEH TANKIH ŽIC (ravnina prostora brez toka)



$V_m(T_0') = V_m(T_0'')$

$V_m(T) - V_m(T_0') = \int_{T}^{T_0'} \vec{H}\cdot d\vec{l} = \underbrace{\int_{T} \vec{H}_1 d\vec{l}}_{-\frac{I}{2\pi}\alpha_1} + \underbrace{\int_{T}^{T_0'} \vec{H}_2\, d\vec{l}}_{\frac{I}{2\pi}\alpha_2}$

$= \frac{I}{2\pi}\overbrace{(\alpha_2 - \alpha_1)}^{\alpha}$

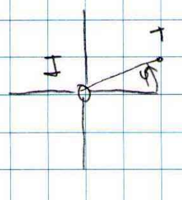
$= \frac{I}{2\pi}\alpha$

$\boxed{V_m(T) = \frac{I}{2\pi}\alpha}$

$V_m(T_+) = \frac{I}{2\pi}\pi = \frac{I}{2}$

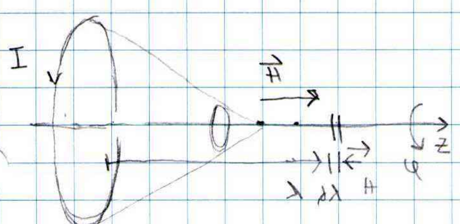
$V_m(T_-) = \frac{I}{2\pi}(-\pi) = -\frac{I}{2}$

POTENCIAL ENE ŽICE



$$\boxed{V_m(T) = \frac{-I}{2\pi}\varphi + C}$$

2.) POTENCIAL V OSI KROŽNEGA OVOJA

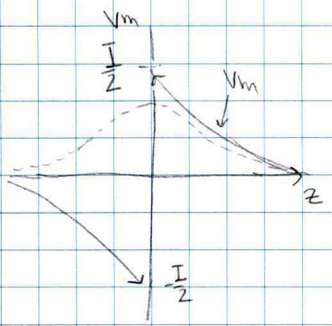


$$V_m(T) = \int_T^{T\infty} \vec{H} \cdot \vec{dl} \,, \quad H_z = \frac{B_z}{\mu_0} = \frac{I}{2} \frac{a^2}{(a^2+z)^{3/2}}$$

$$V_m(T) = \int_z^{\infty} \frac{I}{2} \frac{a^2}{(a^2+z^2)^{3/2}} d\lambda$$

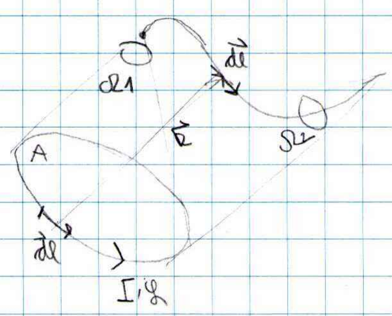
$$= \frac{Ia^2}{2} \left[\frac{\lambda}{a^2\sqrt{a^2+\lambda^2}} \right]_z^{\infty} = \frac{1}{2}\underbrace{\left(1 - \frac{z}{\sqrt{a^2+z^2}}\right)}_{\frac{\Omega}{2\pi}}$$

$\cos\gamma$



$$\boxed{V_m(T) = \frac{I}{4\pi}\Omega}$$

3. POTENCIAL V OKOLICI TOKOVNE ZANKE

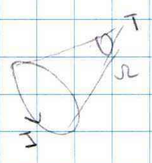


$$V_m(T_1) - V_m(T_2) = \int_{T_1}^{T_2} \vec{H}\vec{dl} = \int_{T_1}^{T_2}\left(\frac{I}{4\pi}\oint \frac{\vec{dl}' \times \vec{R}}{R^3}\right)\cdot \vec{dl} = \ \vdots$$

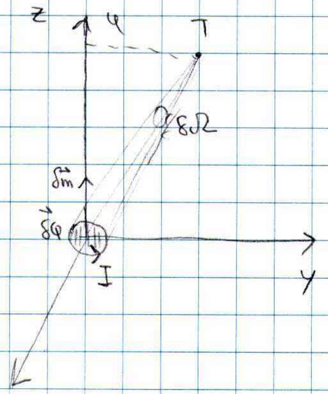
$$= \frac{I}{4\pi}(\Omega_1 - \Omega_2)$$

$T_2 \to T_\infty \,, \ V_m(T_\infty) = 0$

$$\boxed{V_m(T) = \frac{I}{4\pi}\Omega}$$



4.) MAGNETNO POLJE MAGNETNEGA DIPOLA (TOKOVNE ZANKICE)



$$\delta V_m = \frac{I}{4\pi}\delta\Omega$$

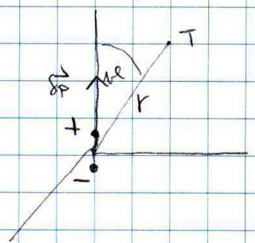
$$= \frac{I}{4\pi}\delta\Omega_{max}\cos\vartheta = \frac{I}{4\pi}\frac{\delta a}{r^2}\cos\vartheta =$$

$$\frac{I\delta a}{4\pi}\frac{\cos\vartheta}{r^2}$$

$$\delta H_r = -\frac{\partial}{\partial r}\delta V_m$$

$$\delta H_\vartheta = -\frac{1}{r}\frac{\partial}{\partial \vartheta} V_m$$

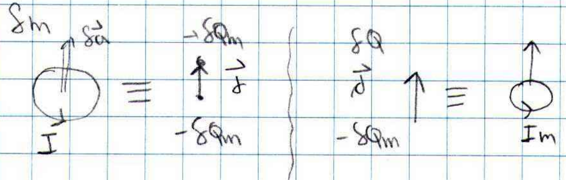
* EL. DIPOL



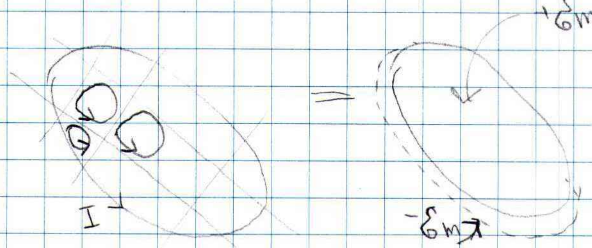
$$\delta V = \frac{\delta p}{4\pi\varepsilon_0}\frac{\cos\vartheta}{r^2}$$

$$\delta E_r = -\frac{\partial}{\partial r}\delta V$$

$$\delta E_\vartheta = -\frac{1}{r}\frac{\partial}{\partial \vartheta}\delta V$$



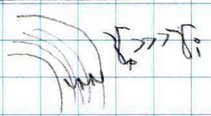
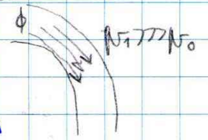
$$I\delta\vec{a} = \delta Q_m\vec{d}$$



Zanko lahko razumemo kot dvojni sloj magnetnih

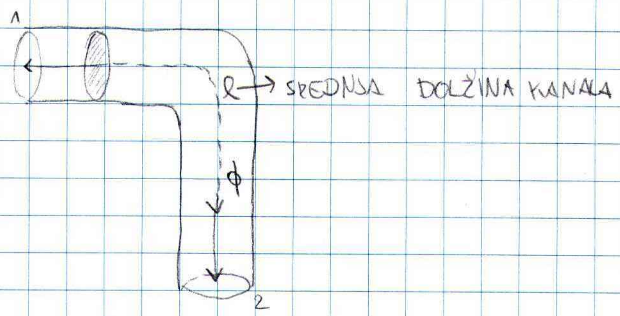
MAGNETNA VEZJA

DUALNOST TOKOVNEGA IN MAGNETNEGA POLJA

Tokovno polje	Magnetno polje
$\vec{J}$, $I = \int_A \vec{J} \cdot d\vec{a}$, $\oint_A \vec{J} \cdot d\vec{a} = 0$	$\vec{B}$, $\phi = \int_A \vec{B} \cdot d\vec{a}$, $\oint_A \vec{B} \cdot d\vec{a} = 0$
$J_n(T+) = J_n(T-)$ $\quad \sum(\pm) I_k = 0$	$B_n(T+) = B_n(T-)$ $\quad \sum(\pm) \phi_k = 0$
$\oint_L \vec{E} \cdot d\vec{l} = 0$, ... -II-, V, U, ...	! $\oint_L \vec{H} \cdot d\vec{l} = 0$! ..., V_m, Θ, ...
$\sum(\pm) U_l = 0$, $E_t(T+) = E_t(T-)$	$\sum(\pm) \Theta_l = 0$, $H_t(T+) = H_t(T-)$
$\vec{J} = \gamma \vec{E}$ 	$\vec{B} = \mu \vec{H}$ MAGNETNI OHMOV ZAKON 
„spec. el. prevodnost"	„specifična magnetna prevodnost"
⇓	⇓
ELEKTRIČNA VEZJA	MAGNETNA VEZJA

ELEMENTI MAGNETNIH VEZIJ

- MAGNETNI UPOR

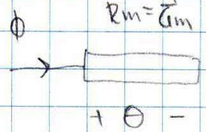


$$\Theta_{12} = \int_1^2 \vec{H} \cdot d\vec{l} = Hl \,!$$

$$\phi = \int_A \vec{B} \cdot d\vec{A} = BS$$

1.) LINEAREN UPOR

$\vec{B} = \mu H$
└ Permeabilnost kanala

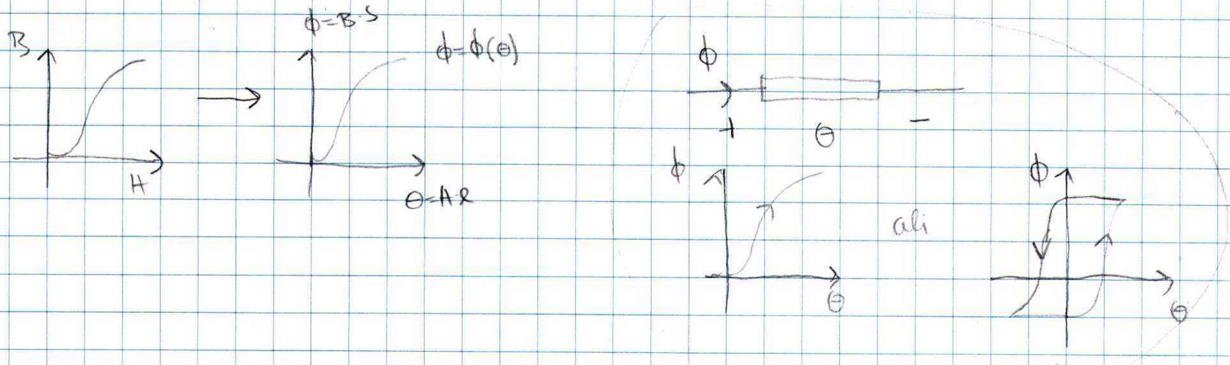


$\Theta = R_m \phi$

$$R_m = \frac{\Theta_{12}}{\phi} = \frac{Hl}{\mu HS} = \frac{l}{\mu S} \qquad \left(R = \frac{l}{\gamma S}\right)$$

2.) NELINEAREN UPOR

Odnos H in B → MAGNETILNA KRIVULJA (deviška in histereza)

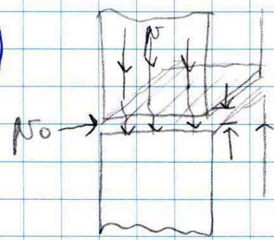


3.) UPORNOST ZRAČNE REŽE

4.4.

$\vec{B} = \mu_0 \vec{H}$ $(\vec{J} = \gamma \vec{E})$

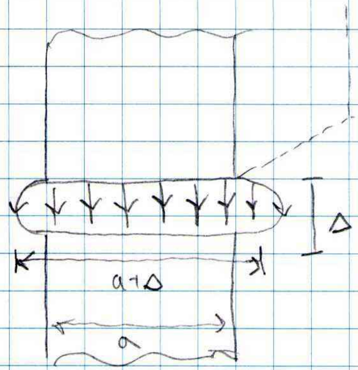
$\vec{B} = \mu_0 \vec{H}$



Špranja je lahko nekaj kotnega ali pa nekaj paraleltnega

$$\boxed{R_m = \frac{\Delta}{\mu_0 S}}$$

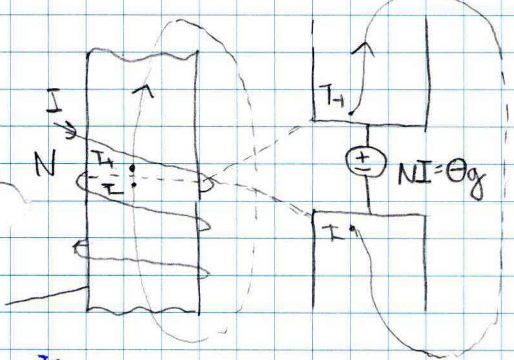
CARTERJEV FAKTOR

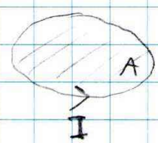


$$R_m = \frac{\Delta}{\mu_0 (a+\Delta)^2} = \frac{\Delta}{\mu_0 a^2} \frac{1}{(1+\frac{\Delta}{a})^2} = C$$

$$\boxed{R_m = \frac{\Delta}{\mu_0 a^2} \cdot C}$$

- VIR MAGNETNE NAPETOSTI



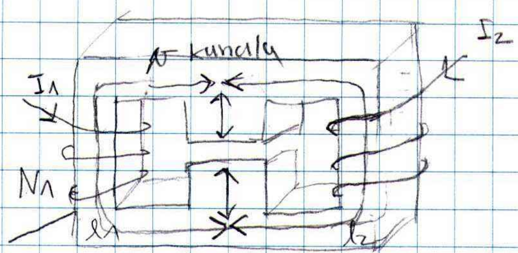


$$\int_{T-}^{T+} \vec{H} \cdot d\vec{l} = NI$$

$$V_m(T+) - V_m(T-) = NI = \Theta$$

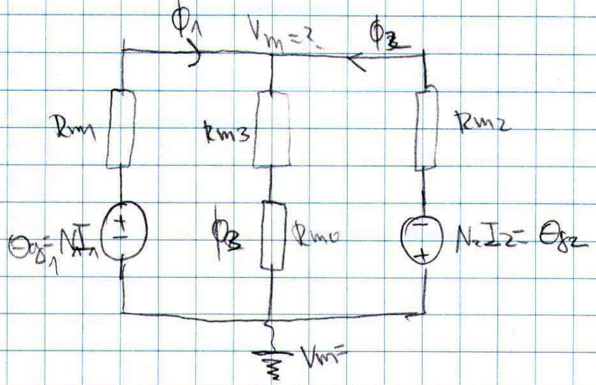
PREPROSTO NELINEARNO VEZJE BO NA KOLOKVIJU

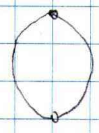
ZGLED: (iz knjige)



$$R_{m3} = \frac{l_3}{\mu S} \quad , k = 1,2,3$$

$$R_{m0} = \frac{\Delta}{\mu_0 S}$$





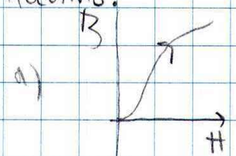
$$\phi_1 + \phi_2 + \phi_3 = 0$$

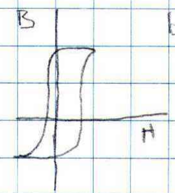
$$\frac{\Theta_{g1} - V_m}{R_{m1}} + \frac{\Theta_{g2} - V_m}{R_{m2}} + \frac{0 - V_m}{R_{m3} + R_{m0}} = 0$$

$$\Rightarrow V_m \rightarrow \phi_k$$

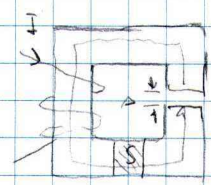
NELINEARNO MAGNETNO VEZJE

ZA KOLOKVIJ:

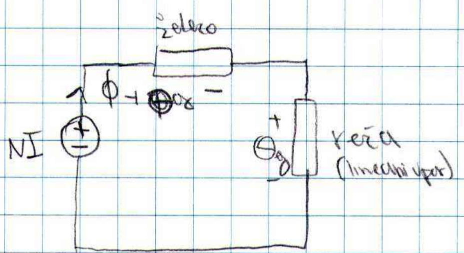




b)

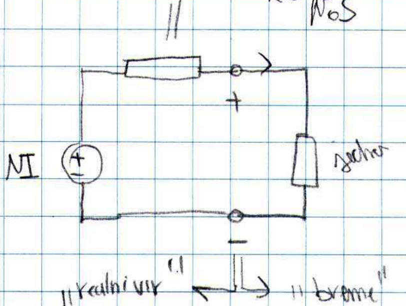


KOLOKVIJ

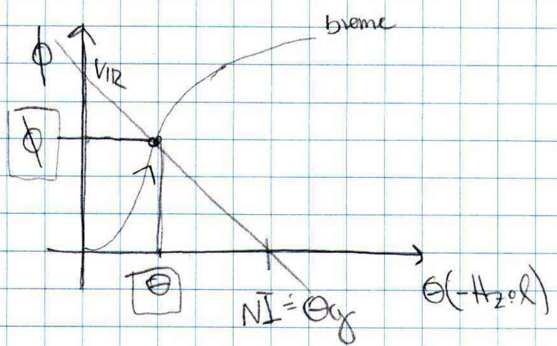


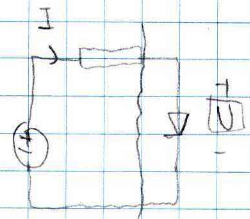
$-NI + R_m \phi + \Theta = 0$	VIR
$\Theta = \Theta(\phi) \rightarrow \phi = \phi(\Theta)$	BREME

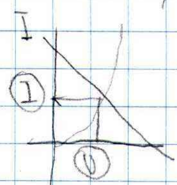
$$R_{m0} = \frac{\Delta}{\mu_0 S}$$



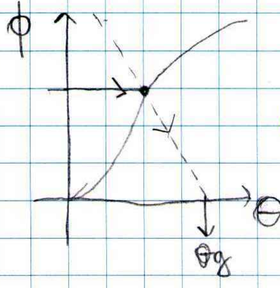
a)





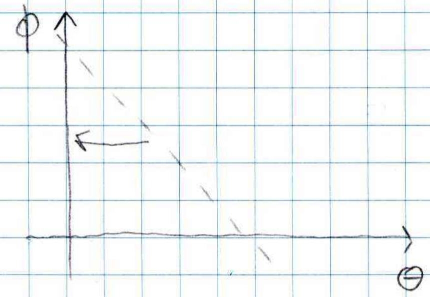


a1) želimo Φ

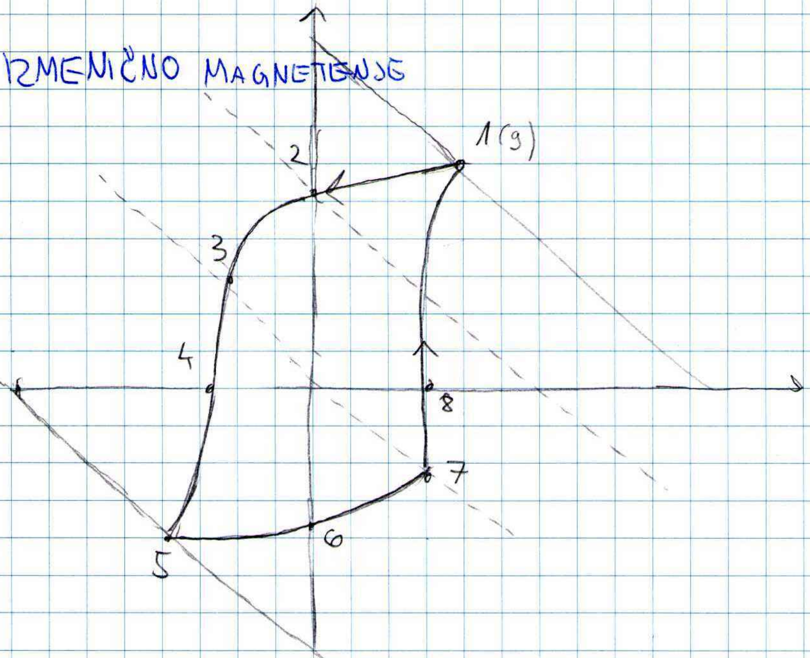


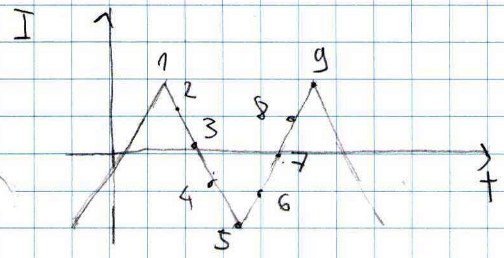
$$\boxed{\frac{\delta\Phi}{\delta\Theta} = \frac{1}{R_{oz}} = G_{oz}}$$

a2) imam Θ_g iščem Φ



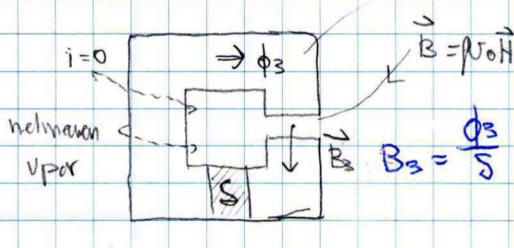
B.) IZMENIČNO MAGNETENJE





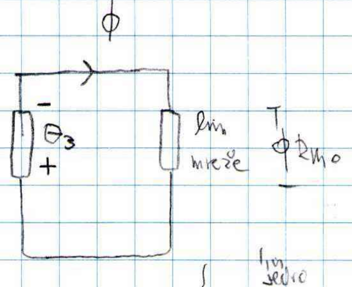
TOČKA 3:

$B = \mu_0(H + M)$



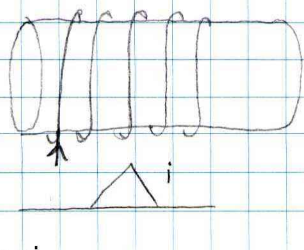
$\vec{B} = \mu_0 \vec{H}$

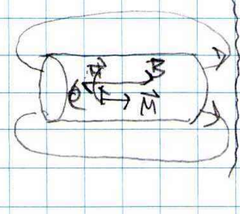
$B_3 = \frac{\Phi_3}{S}$

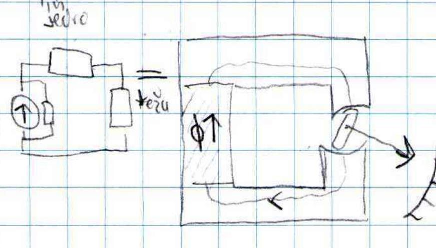


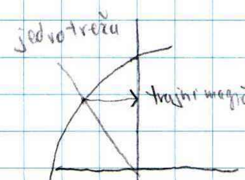
$$\boxed{\Theta_3 + \Phi R_{m_0} = 0}$$

$$\boxed{\Theta_3 = -R_m \cdot \Phi}$$









Vzamemo predmet in ga vržemo – Ali znamo ob tem nekaj povedat tudi v ELM

7.4.M

DINAMIČNO POLJE

3M $\oint_g \vec{E}\,d\vec{l} = 0 \; \left(= -\frac{d\phi_g}{dt}\right)$ Ta nadgradnja je prideLek FARADAYA – 1831

$-\frac{d}{dt}\int_A \vec{B}\cdot d\vec{a}$

Rodil se je

4M $\oint_g \vec{H}\cdot d\vec{l} = \int_A \vec{J}_P\, d\vec{a} \left(+\int_A \frac{\partial \vec{D}}{\partial t}\cdot d\vec{a}\right)$ MAXWELL [1873]

$\frac{d\phi_e}{dt}$

$\varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$

Ubistvu je prideLek FARADAYA ZAPISAL MAXWELL

Svetloba je magnetno polje
Hertz pokazal da se da energijo da prepeljat iz enga na drugo mesto brez fizične povezave

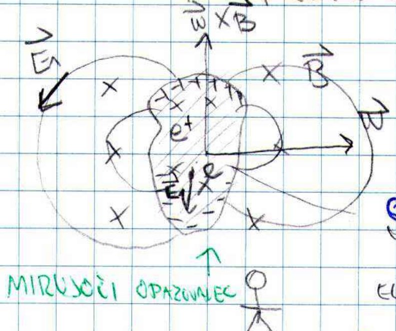
$F_L = Q(\vec{E} + \vec{w}\times\vec{B})$ → Kako polje enega vpliva na drugega - FARADAY To MAXWELLA navdušila da začne razmišljat o polju in matematično zapiše → Preskok → Mat. zapis, potem šele eksperiment

7.4.M

FARADAYEVA – ELEKTROMAGNETNA INDUKCIJA

Vzamemo telo - prevodno telo

Prevodnik se giblje čez magnetno polje
Sila deluje na protone, ampak ti se ne morejo nihamer

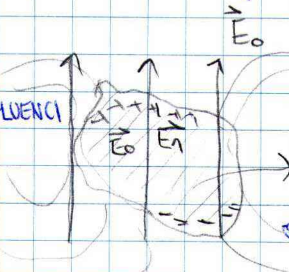


$\vec{F}_m = Q(\vec{w}\times\vec{B})$

$\underbrace{Q\vec{E}_1}_{\text{EL. SILA}} + \underbrace{Q\,\vec{w}\times\vec{B}}_{\text{MAGN. SILA}} = \vec{0}$

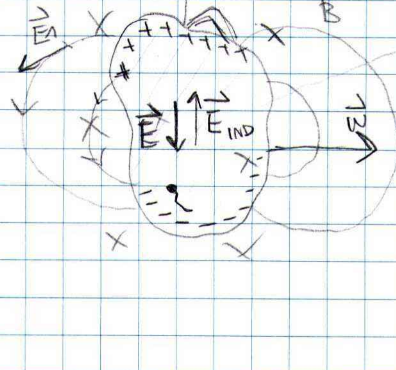
$\Rightarrow \vec{E}_1 = -\vec{w}\times\vec{B}$

Enak pojav kot pri EL. INFLUENCI



$\vec{E} = \vec{E}_0 + \vec{E}_1 = \vec{0}$
$\vec{E}_0 = -\vec{E}_1$
$Q(\vec{E}_0 + \vec{E}_1) = 0$

GIBAJOČI OPAZOVALEC



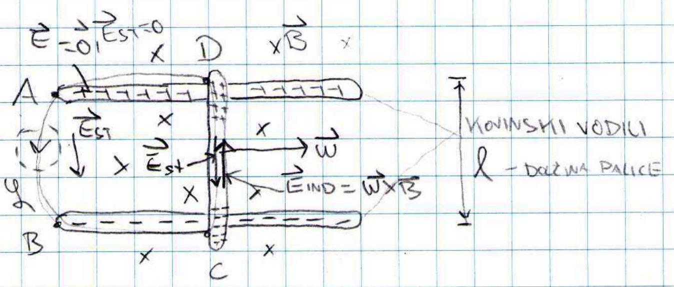
$Q(\underbrace{\vec{E}_1 + \vec{E}_{IND}}_{\vec{E}}) = \vec{0}$

$\vec{E}_1 = -\vec{E}_{IND}$

$\vec{E}_{IND} \equiv \vec{w}\times\vec{B}$ → Če odmislimo oba opazovalca

LINEARNI GENERATOR

7.4.11

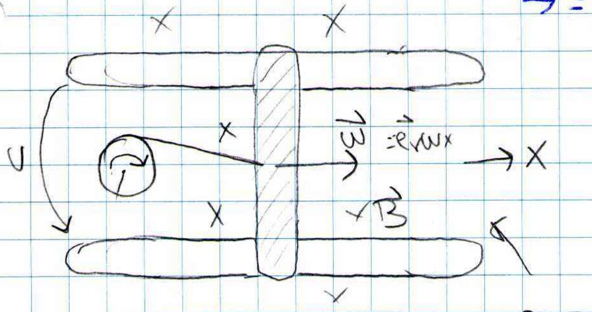


COLOUMBOVA POLJSKA JAKOST

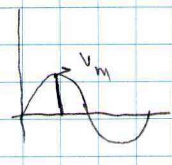
$$\vec{E}=\vec{E}_{ST}+\vec{E}_{IND}$$
$$=\vec{E}_{COL}+\vec{E}_{FAR} \rightarrow \text{FARADAYEVA POLJSKA JAKOST}$$

$$\oint_{\mathcal{L}} \vec{E}_{ST}\cdot d\vec{l} = \underbrace{\int_A^B \vec{E}_{ST}\cdot d\vec{l}}_{U} + \underbrace{\int_B^C \vec{E}_{ST}\cdot d\vec{l}}_{\emptyset} + \underbrace{\int_C^D \vec{E}_{ST}\cdot d\vec{l}}_{=-\int_C^D(\vec{w}\times\vec{B})\cdot d\vec{l}} + \underbrace{\int_D^A \vec{E}_{ST}\cdot d\vec{l}}_{\emptyset} = 0$$

$$\hookrightarrow = -wBl \Rightarrow U-wBl=0 \Rightarrow \boxed{U=wBl}$$

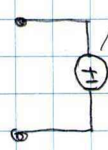


$$\text{ex}\Rightarrow\ w_x = w_{max}\cos(wt+\gamma)$$
$$\Rightarrow U_t = \underbrace{w_{max}Bl}_{U_{MAX}}\cos(wt+\gamma)$$



$U=\vec{w}\times Bl$

$U(t)=w_x(t)Bl$



NAPETOST MED TOČKAMA - SMER

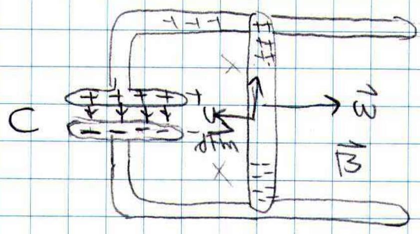
LAHKO PREDPOSTAVIMO KOT HARMONIČNI GENERATOR

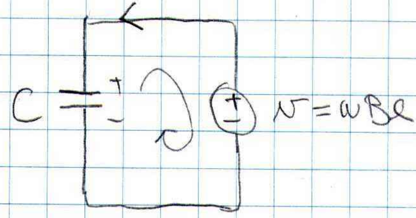
$U(t)=V_m\cos(wt+\vartheta)$

V_m → Amplituda

ϑ → „začetni kot"

OBREMENJENI VIR





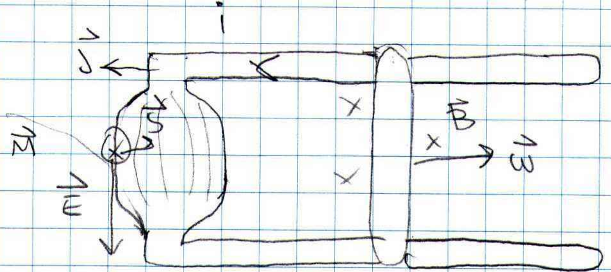
$\oint \vec{E}_{IND} \cdot d\vec{l}$

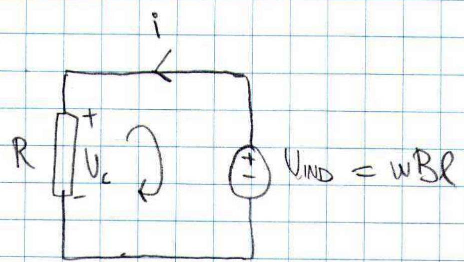
$i = C \frac{du_i}{dt} = C \frac{du_{ind}}{dt}$

$-U_C + U_{IND} = 0 \Rightarrow U_C = U_{IND}$

$p_e = U_C i \; , \; W_e' = p_e$

$i\, d\vec{l} \times \vec{B} = d\vec{F}_m$



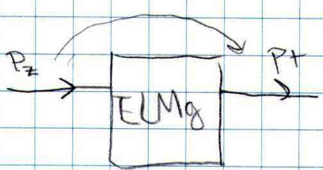


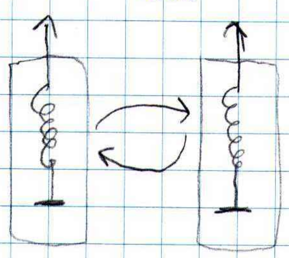
$-\underset{RI}{U_R} + U_{IND} = 0 \Rightarrow \boxed{i = \frac{U_{IND}}{R}}$

$|\vec{F}_m| = i l B$

$\vec{F}_z + \vec{F}_m = \vec{0} \Rightarrow |\vec{F}_z| = |\vec{F}_m| = iBl$

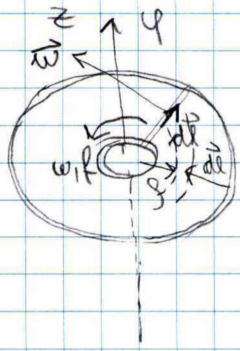
$P_z = |\vec{F}_z||\vec{w}| = iBwl = U_{IND} \cdot I = P_t$





FARADAYEV ENOSMERNI GENERATOR

11.4.11



$$(\vec{w} \times \vec{B}) \cdot \vec{dl}$$

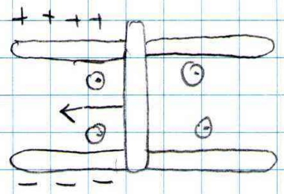
$$\boxed{\vec{w} = (0, w\rho, 0)}$$

$$\vec{dl} = (d\rho, 0, 0)$$

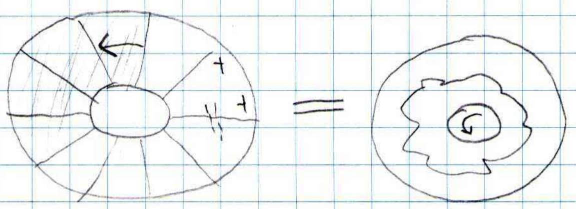
$$U_i = \int_a^b (\vec{w} \times \vec{B}) \cdot \vec{dl} = \int_a^b (\vec{dl} \times \vec{w}) \cdot \vec{B}$$

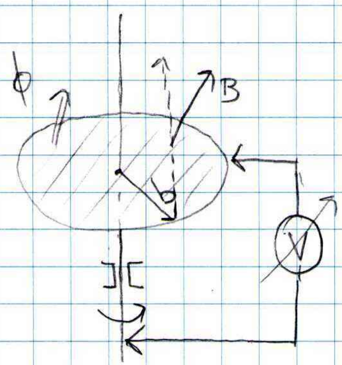
$$= \int_a^b B_z w \rho \, d\rho = \frac{w B_z (b^2 - a^2)}{2}$$

$$\boxed{U_i = \frac{w B_z (b^2 - a^2)}{2}}$$

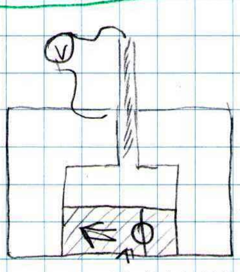


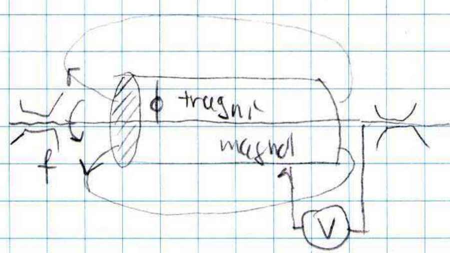
VRTEČI DISK IN KOLOBAR

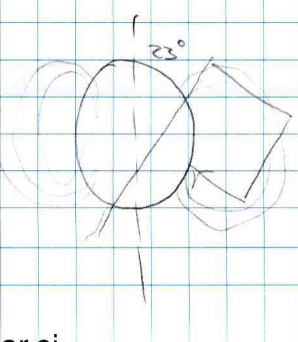




$$\boxed{U_{ind} = \frac{w B_z b^2}{2}} \Rightarrow \frac{w B_z \pi b^2}{2\pi} = \boxed{f \phi}$$

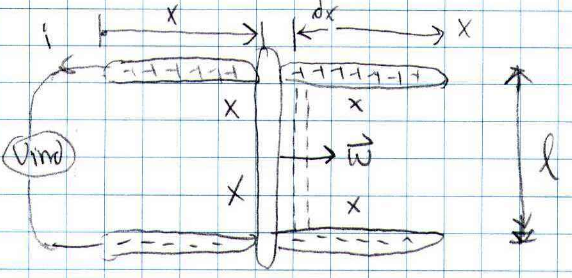






ZEMLJA

LENZOVO PRAVILO



$$U_{ind} = wBl = \frac{dx}{dt}Bl = \frac{(x+dx)Bl - xBl}{dt} = \frac{d\Phi}{dt} = -\frac{d\Phi_1}{dt}$$

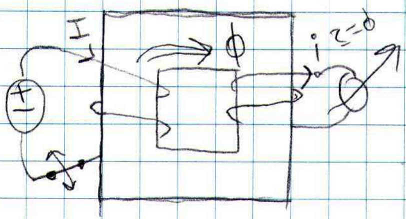
$$\boxed{U_{ind} = -\frac{d\Phi_1}{dt}}$$

V zanki v katerem se fluks inducira v smeri nasprotni vzroku ~~nastanka~~ fluksa v zanki.
⇒ Zaviranje povečevanja fluksa

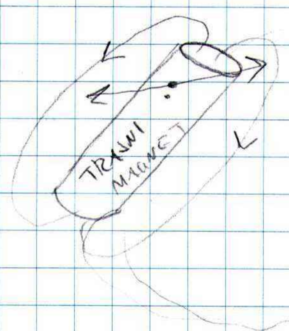
$$v = wBl = U_{ind} = \left(\oint_C (\vec{w} \times \vec{B}) \cdot d\vec{l}\right)$$

FARADAYEV INDUKCIJSKI TOK (1831)

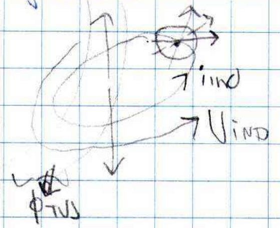
Elektroda, elektrolit – te besede Faradayev izum

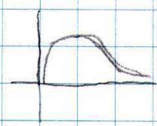


Štartna regulator je bila ob vklopu-izklopu stikala → sprememba fluksa

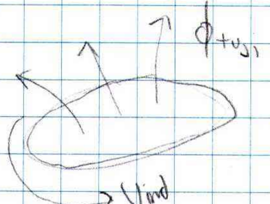


Hitreje kot se fluks premika, večji so odkloni
→ Gibanje zanke



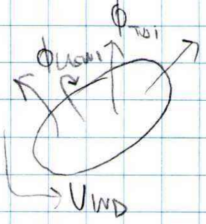


$$\boxed{U_{ind} \propto \frac{d\Phi_{tuj}}{dt}}$$



SAMOINDUKCIJA

$\leftarrow$ Henry (1852)
$\leftarrow$ Faraday (1834)

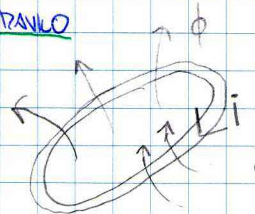


$$\boxed{U_{ind} \propto \frac{d}{dt}\left(\phi_{TUJI} + \phi_{LASTNI}\right)}$$

ϕ

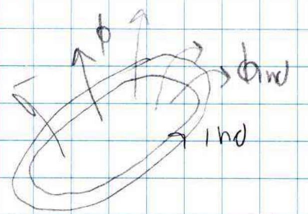
Medsebojna ... Lastna INDUKTIVNOST

LENZOVO PRAVILO



$\frac{d\phi}{dt} > 0$

Rezultirajoči fluks se upira povečanju fluksa skozi zanko

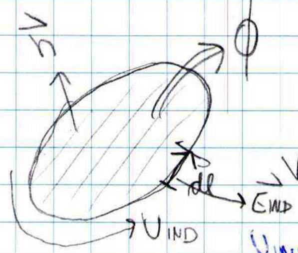


$\frac{d\phi}{dt} < 0$

NEUMANN (1845)

$$|U_{ind}| = \frac{d\phi}{dt} \oplus \text{Lenzovo pravilo}$$

$$U_{ind} = \pm \frac{d\phi}{dt} \leftarrow \text{ali „+" ali „−"}$$



V TEM PRIMERU −

$U_{IND} = \frac{d\phi}{dt}$

! Minus zaradi Lenzovega pravila in zaradi predhodnega dogovora kako smo krog označili

$$U_{ind} = \oint_{\ell} \vec{E}_{ind} \cdot \vec{dl}$$

$$\phi = \int_A \vec{B} \cdot \vec{da}$$

$$U_{IND} = \oint_{\ell} \vec{E}_{ind} \cdot \vec{dl} = -\frac{d}{dt}\int \vec{B}\,\vec{da} = -\frac{d\phi}{dt}$$

$$\boxed{U_{IND} = -\frac{d\phi}{dt}} \qquad \boxed{U_{IND} = \oint_{\ell} \vec{E}_{ind}\,\vec{dl} = -\frac{d}{dt}\int \vec{B}\,\vec{da}}$$

$$\left.\begin{aligned} &\oint_{\ell} \vec{E}_{st} \cdot \vec{dl} = 0 \\ &\vec{E} = \vec{E}_{st} + \vec{E}_{ind} \end{aligned}\right\} \quad \boxed{\oint_{\ell} \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\oint_A \vec{B} \cdot \vec{da}}$$

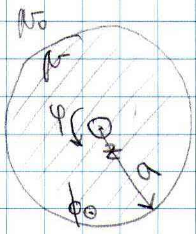
III. MAXWELLOVA ENAČBA (1867) (1831 – Rojstvo) $\rightarrow$ 36 let

$$\oint_L \vec{H}\cdot d\vec{l} = \int_A \vec{J}_{pr}\cdot d\vec{a} + \int_A \frac{\partial D}{\partial t}\cdot d\vec{a}$$ (To tema za kasneje)

$\vec{E}_{IND} = -\frac{\partial \vec{A}}{\partial t}$ → vektorski mag. pot.

ZGLEDI:

1.) $\vec{E}_{IND}$ V/OB FEROMAGNETNEM STEBRU KROŽNEGA PREREZA

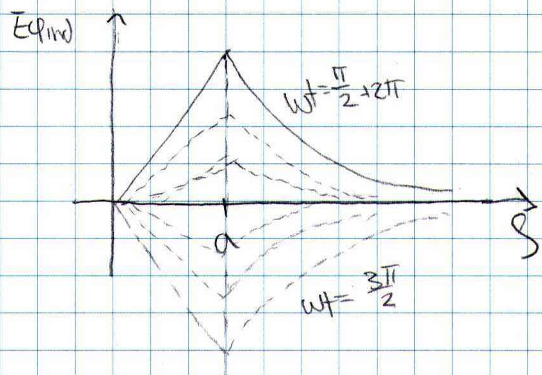


L krožnica polmera ρ okrog osi z

$$\oint_{L(2\pi\rho)} \vec{E}\cdot d\vec{l} = E_\varphi 2\pi\rho = -\frac{d}{dt}\begin{cases}\frac{\phi}{\pi a^2}\rho^2, & \rho\le a\\ \phi, & \rho>a\end{cases}$$

$$E_{\varphi IND} = E_\varphi = \begin{cases}\frac{-\frac{d\phi}{dt}}{2\pi a^2}\rho, & \rho\le a\\ \frac{-\frac{d\phi}{dt}}{2\pi\rho}, & \rho>a\end{cases}$$

$$E_\varphi = \underbrace{E_{\varphi A}}_{\emptyset} + E_{\varphi IND}$$



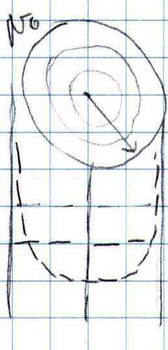
PRIMER: $\phi(t) = \phi_m \cos\omega t$

$$-\frac{d\phi}{dt} = \omega\phi_m \sin\omega t$$

$$\boxed{E_{\varphi ind} = \begin{cases}\frac{\omega\phi_m}{2\pi a^2}\rho\sin\omega t, & \text{not}\\ \frac{\omega\phi_m}{2\pi\rho}\sin\omega t, & \text{zun.}\end{cases}}$$

$\boxed{J = \gamma E_{ind} \rightarrow \text{VRTINČNI TOK}}$ zaradi njih imamo vrtinčne izgube

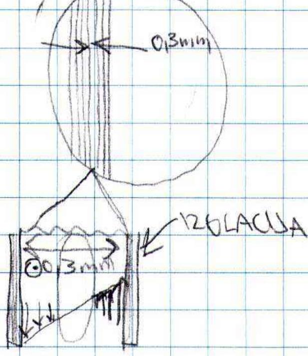
$$p = \frac{J^2}{\gamma}$$



Izliv fluksa na površino

Za preprečitev tega v praksi izvajamo lamelacijo

1. UKREP LAMELACIJA:

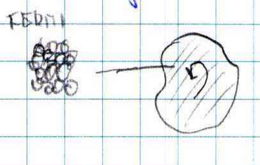


lamele prevlečene oz. polakirane → medsebojno izolirane

Vrtinčne izgube reda 1 W na kg

2. FERITI - 2. NAČIN

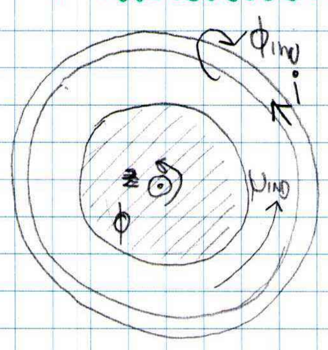
Namenjeni tudi zmanjšanju vrtinčnih tokov.



Relevantni so pri visokih frekvencah

(Imamo tudi histerezne izgube)

2.) STEBER S KROŽNIMI OVOJEM



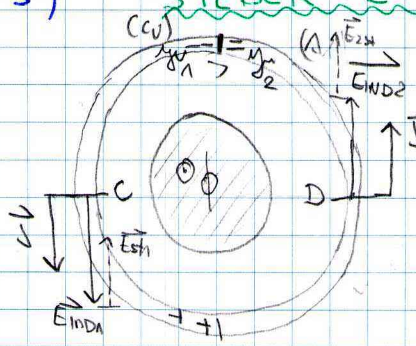
$v_{ind} = -\frac{d\phi_m}{dt} = \omega\phi_m \sin\omega t$

$i \simeq \frac{v_{ind}}{R}$

Če je samoindukcija zanemarljiva

$|\phi_{ind}| << |\phi|$

3.) STEBER Z DVODELNIM KROŽNIM OVOJEM

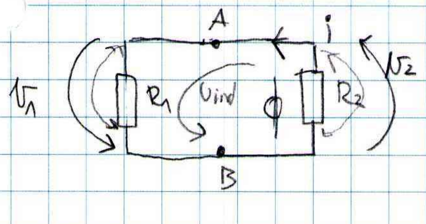


$\vec{E} = \vec{E}_{st} + \vec{E}_{ind}$

$v_{ind} = -\frac{d\phi}{dt} = \omega\phi_m \sin\omega t$

$\Sigma R = R_1 + R_2$

$i \simeq \frac{v_{ind}}{\Sigma R}$



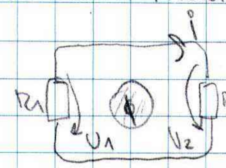
(C) $\vec{J} = \gamma_1(\overbrace{\vec{E}_{ind1} + \vec{E}_{st1}}^{\vec{E}_1})$

(D) $\vec{J} = \gamma_2(\underbrace{\vec{E}_{ind2} + \vec{E}_{st2}}_{E_2})$

$|\vec{E}_2| > |\vec{E}_1|$

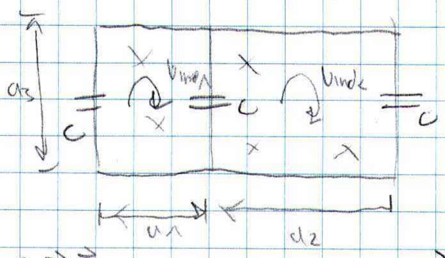
$\boxed{\begin{aligned} v_1 &= iR_1 \\ v_2 &= iR_2 \end{aligned}}$

KOLOKVIJSKA NALOGA



$\Rightarrow$ Treba je bilo izračunat U_1 in U_2

KOLOKVIJSKA



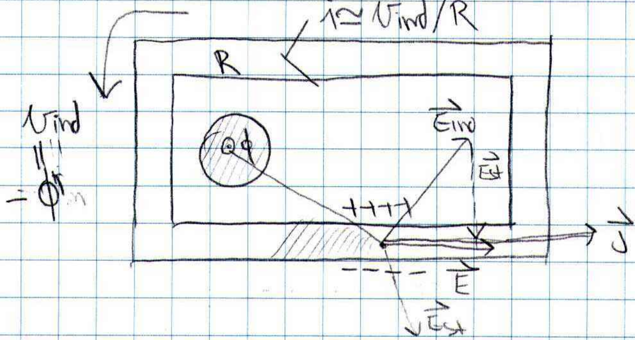
$-U_1+U_2-U_{ind\,1}=0$

$-U_2+U_1-U_{ind\,2}=0$

$\oint_{\partial_1} \vec{E}\,d\vec{l} = -U_1+U_2=U_{ind}$

$\Rightarrow$ Ulje se meje berana o zaporedno/vzporedno vezanih kondenzatorjih

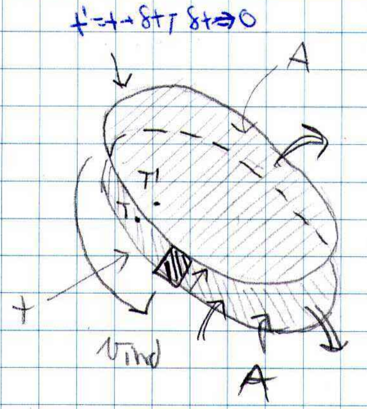
4.) STEBER S PRAVOKOTNIM OKVIRJEM/ZANKO



Na okvirju se zgodi influenca

$J=\gamma\vec{E}=\gamma(\vec{E}_{ind}+\vec{E}_{st})$

5.) TRANSFORMATORSKA IN GIBALNA INDUCIRANA NAPETOST



$$U_{ind}=-\frac{d\Phi}{dt}=-\lim_{\delta t\to 0}\frac{\Phi(t+\delta t)-\Phi(t)}{\delta t}$$

$$=-\lim_{\delta t\to 0}\frac{\delta t\int_{A'}\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}+\int_{A'}\vec{B}(T',t)\cdot d\vec{a}-\int_{A}\vec{B}(T,t)\cdot d\vec{a}}{\delta t}$$

$\Phi(t+\delta t)=\int_{A'}\vec{B}(T',t+\delta t)\cdot d\vec{a}$

$\Phi(t)=\int_{A}\vec{B}(T,t)\cdot d\vec{a}$

$f(x+\delta x)\cong f(x)+\delta x\, f'(x)$

$\vec{B}(T',t+\delta t)=\vec{B}(T',t)+\delta t\frac{\partial\vec{B}}{\partial t}$

Fluks skozi plašč, ki ga ta zanka zariše v kratkem času δt

$\oint(\vec{v}\cdot dt\cdot d\vec{l})\cdot\vec{B}$

$$\Rightarrow -\int_A\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}-\oint_{\partial}(\vec{v}\times d\vec{l})\cdot\vec{B}=-$$

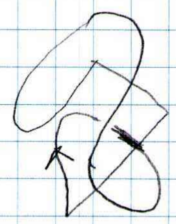
$$\boxed{U_{IND}=-\frac{d\Phi_m}{dt}=\underbrace{-\int_A\frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}}_{(U_{ind})_{transformatorska}}+\underbrace{\oint(\vec{v}\times\vec{B})\cdot d\vec{l}}_{(U_{ind})_{gibalna}}}$$

ZAPIS OBEH PRISPEVKOV K INDUKCIJI

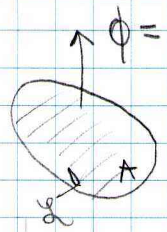
$\rightarrow$ Hitrost tega elementa zanke

6.) MAGNETNI SKLEP – POSPLOŠITEV POJMA MAGNETNI PRETOK

18.4.11



$\phi = ?$

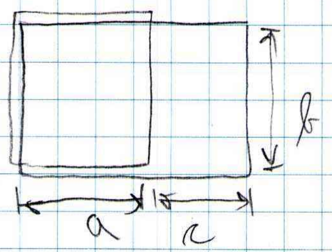


$\phi = \phi_A = \phi_L$

$\boxed{\phi \rightarrow \Psi \ (PSI)}$

$\Psi_L = ?$

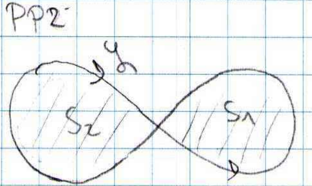
PP1



$\Psi_L = \phi_{(a+c)\times b} + \phi_{a\times b} = 2\phi_{a\times b} + \phi_{c\times b}$

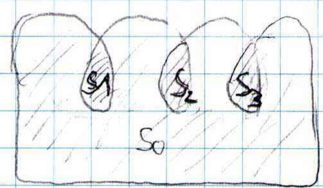
$u_{ind} = -\dfrac{d\Psi}{dt}$

PP2



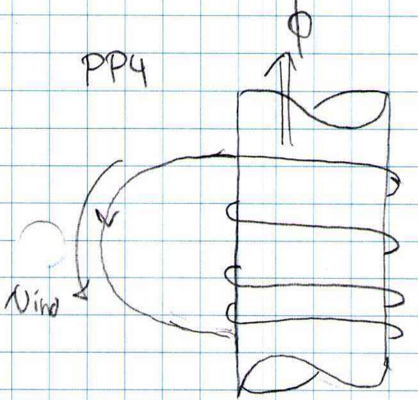
$\Psi_L = \phi_{S1} + \phi_{S2}$

PP3



$\Psi = 2(\phi_{S1} + \phi_{S2} + \phi_{S3}) + \phi_{S0}$

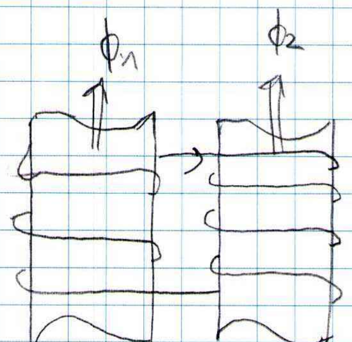
PP4



$\Psi_L = -4\phi$

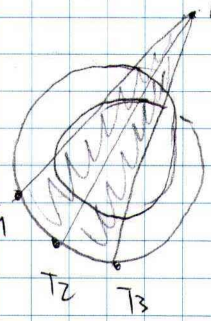
$u_{ind} = -\dfrac{d\Psi_L}{dt} = 4\phi'$

PP5:



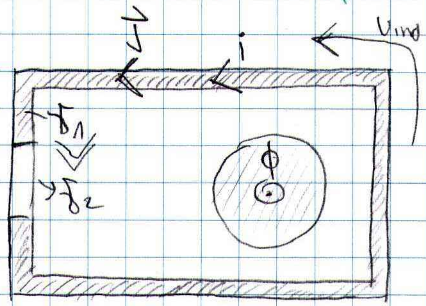
$\Psi_L = -3\phi_1 + 4\phi_2$

$\Psi_{\gamma} = \Psi_A$, $A = ?$



Rešitev ni ena sama, vsih je neskončno

7. INDUKCIJA V ODPRTIH ZANKAH



$\gamma_2 <<< \gamma_1$

$U_{ind} = -\frac{d\phi}{dt}$

$|\vec{J}_{n1}| = |\vec{J}_{n2}|$

$|\gamma_1 \vec{E}_1| = |\gamma_2 \vec{E}_2|$

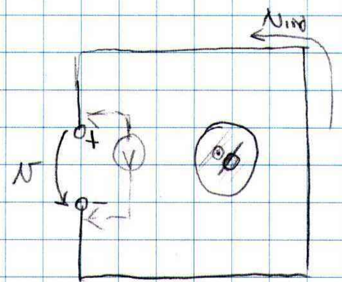
$\Rightarrow |\vec{E}_2| >>> |\vec{E}_1|$

Spomni se mejnega pogoja iz 1. semestra

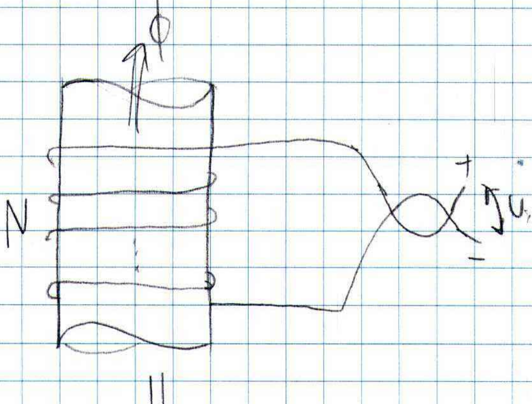
$\left(\frac{\varepsilon_1}{\gamma_1} - \frac{\varepsilon_2}{\gamma_2}\right) J_n = \sigma$

$$u_{ind} = -\frac{d\phi}{dt} = \oint_{\gamma} \vec{E} \cdot d\vec{l} = \int_{T_1 \gamma_1 T_4} \vec{E}_1 \cdot d\vec{l} + \int_{T_4 \gamma_2 T_1} \vec{E}_2 \cdot d\vec{l}$$

$$\approx \int_{T_4 \gamma_2 T_1} \vec{E}_2 \cdot d\vec{l} = -U_2$$



$u \approx u_{ind} = -\frac{d\phi}{dt}$

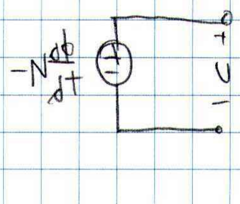


Vprašamo se koliko fluksa objamemo s to pentljo.

$\Psi_{\gamma} = N\phi$

$u \cong -\frac{d\Psi_{\gamma}}{dt} = -N\dot{\phi}$

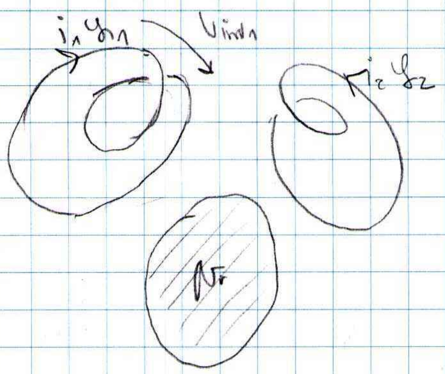
$\Downarrow$



INDUKTIVNOSTI - LASTNE IN MEDSEBOJNE

18.4.11

za linearne sisteme !



$$\vec{B} = \vec{B}^{(1)} + \vec{B}^{(2)}$$

$$\psi_1 = \psi_1^{(1)} + \psi_1^{(2)} = L_{11} i_1 + L_{12} i_2$$

$$\psi_2 = \psi_2^{(1)} + \psi_2^{(2)} = L_{21} i_1 + L_{21} i_2$$

MEDSEBOJNI INDUKTIVNOSTI

$$\frac{\psi_1^{(1)}}{i_1} = L_{11} \qquad \frac{\psi_1^{(2)}}{i_2} = L_{12}$$

$$\frac{\psi_2^{(1)}}{i_1} = L_{21} \qquad \frac{\psi_2^{(2)}}{i_2} = L_{22}$$

LASTNI INDUKTIVNOSTI

v_{ik}

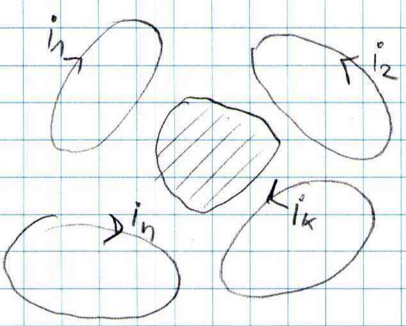
$$v_{ind_1} = -\frac{d\psi_1}{dt} = -L_{11} i_1' - L_{12} i_2'$$

$$v_{ind_2} = -\frac{d\psi_2}{dt} = \underbrace{-L_{21} i_1'}_{\text{TUJA INDUKCIJA}} \underbrace{- L_{22} i_2'}_{\text{SAMOINDUKCIJA}}$$

akumulirano

Induktivnosti merilo za akumulirano magnetna energije

POSPLOŠITEV NA N-ZANK



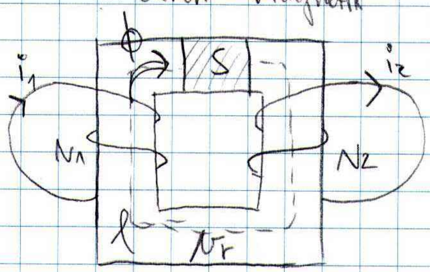
$$\psi_k = \sum_{j=1}^{n} \psi_k^{(j)} = \sum_{j=1}^{n} L_{kj}\, i_j$$

$$\boxed{L_{kj} = \frac{\psi_k^{(j)}}{i_j}}$$

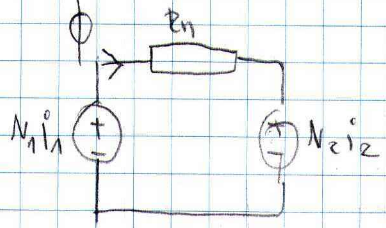
$$v_{ind} = \boxed{v_{ind} = -\frac{d\psi_k}{dt} = -\sum_{j=1}^{n} L_{kj}\, i_j'}$$

ZGLED - TRANSFORMATOR

Linearen magnetik



$$R_m = \frac{l}{\mu_0 \mu_r S}$$



$$\phi = \frac{N_1 i_1 - N_2 i_2}{R_m}$$

$$\Psi_1 = N_1 \phi = \underbrace{\frac{N_1^2}{R_m}}_{L_{11}} i_1 - \underbrace{\frac{N_1 N_2}{R_m}}_{L_{12}} i_2$$

$$\Psi_2 = -N_2 \phi = -\underbrace{\frac{N_1 N_2}{R_m}}_{L_{21}} i_1 + \underbrace{\frac{N_2^2}{R_m}}_{L_{22}} i_2$$

Če bi izbrali drug ref. smer bi se koeficienti nasprotno predznačili

Če so medsebojne induktivnosti negativne ako se lastni in sosedov fluks tepeta

KASNEJE

$L_{12} = L_{21}$ ⇒ To bo kasneje pravilo

$L_{jk} = L_{kj}$ Ponavadi medsebojni induktivnosti vedno enaki

$|L_{12}| = |L_{21}| = \sqrt{L_{11} L_{22}}$ ⇒ Enaka geometrijski sredini obeh lastnih
$(k=1)$

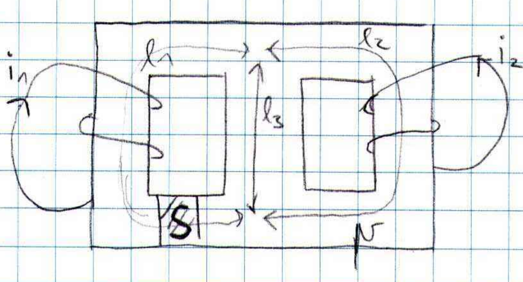
⇒ TO NI SPLOŠNO ⇒ KASNEJE $L_{12} = L_{21} = k\sqrt{L_{11} L_{22}}$

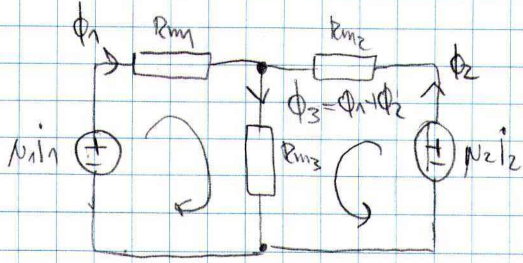
k.

k ↓ Faktor sklopa ↓ med 0 in 1, $0 \le k \le 1$

Idealno 1 ali pa včasih 0 ⇒ Pri telefonskih linijah da ni preslukov

ZGLED - TRISTEBERNO JEDRO Z DVEMA NAVITJEMA





$$R_{mk} = \frac{l_k}{\mu S}$$

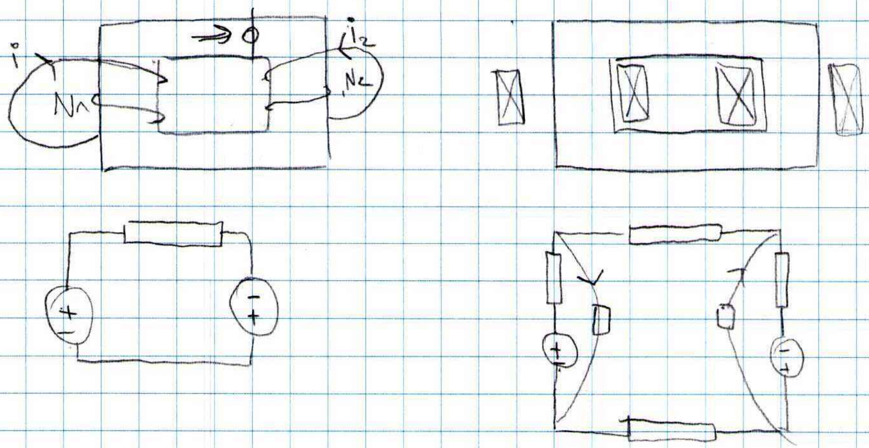
$$R_{m1}\phi_1 + R_{m3}(\phi_1 + \phi_2) = N_1 i_1$$
$$R_{m2}\phi_2 + R_{m3}(\phi_1 + \phi_2) = N_2 i_2$$

$$\Psi_1 = N_1 \phi_1 = L_{11} i_1 + L_{12} i_2$$

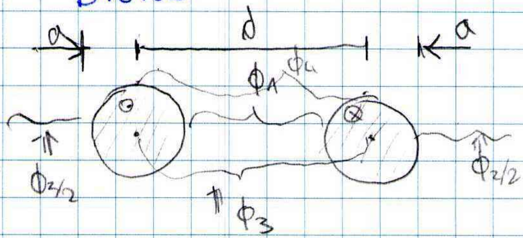
. . . . Napiši rešitev

DILEME IN TEŽAVE DOLOČANJA INDUKTIVNOSTI

22.4.11



DVOVOD



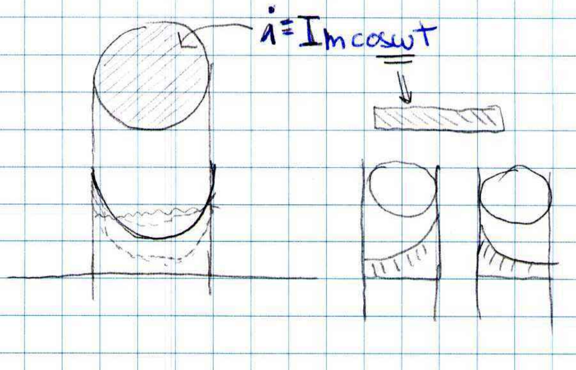
Dilema → mamo nekakšna flukse sam ne vemo katerega vzet. ⇒ V primeru dvovoda ta rešimo s povprečjem

ϕ

$$\phi_x = \dots$$

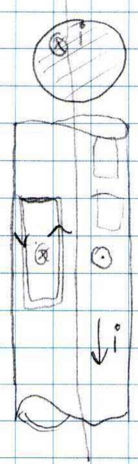
$$\phi_1 = \frac{\mu_0 I l}{\pi} \ln \frac{d-a}{a} \qquad \phi_3 = \frac{\mu_0 I l}{\pi} \left(\frac{1}{2} + \ln \frac{d}{a} \right)$$

$$\phi_2 = \frac{\mu_0 I l}{\pi} \ln \frac{d+a}{a} \qquad \phi_4 = \frac{\mu_0 I l}{\pi} \ln \frac{\sqrt{d^2+a^2}}{a}$$



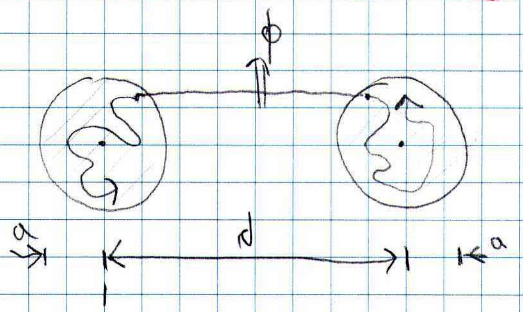
$$\delta = \sqrt{\frac{2}{\omega \mu \gamma}}$$

2A Braker pri 50Hz ta delta → δ cca 1mm



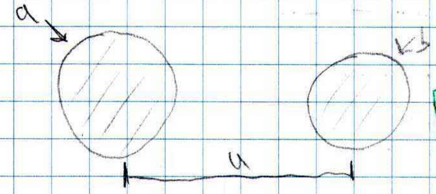
Deluje po Lenzovem smislu

INDUKTIVNOST DVOVODA



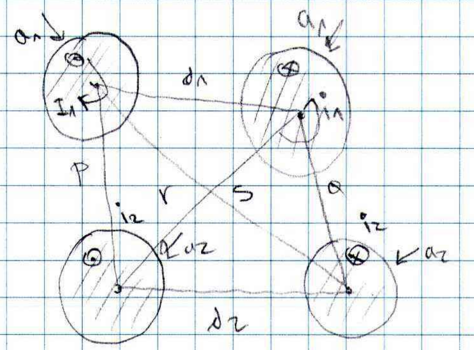
$$\langle\phi\rangle = \frac{\mu_0 I l}{\pi}\left(\frac{1}{4} + \ln\frac{d}{a}\right)$$

$$\boxed{L = \frac{\langle\phi\rangle}{I} = \frac{\mu_0 l}{\pi}\left(\frac{1}{4} + \ln\frac{d}{a}\right)}$$



$$\boxed{L = \frac{\mu_0 l}{\pi}\left(\frac{1}{4} + \ln\frac{d}{\sqrt{ab}}\right)}$$

MEDSEBOJNA INDUKTIVNOST



$$\langle\phi^{(2)}\rangle = \frac{\mu_0 i_2 l}{2\pi}\left(\ln\frac{rs}{pq}\right)$$

$$\boxed{L_{1,2} = \frac{\langle\phi_1^{(2)}\rangle}{i_2} = \frac{\mu_0 l}{2\pi}\left(\ln\frac{rs}{pq}\right)}$$

$$L_{21} = \frac{\langle\phi_2^{(1)}\rangle}{i_1} = L_{12} \ \checkmark\checkmark$$

Pecpor kolišna kot je fl. med zankama

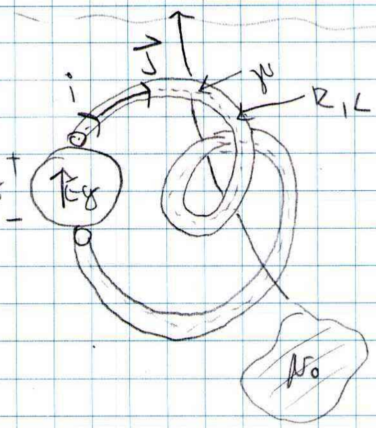
TULJAVA KOT STRNJEN ELEMENT EL. VEZIJ

11.4.11

$i = Cu'$, $W_C = Cu^2/2$, $p_C = ui$

$i = Gu$, $p_r = u \cdot i = Gu^2$

$u = Li'$, $W_L = Li^2/2$, $p_L = ui$



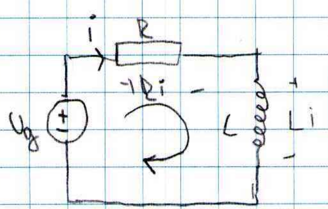
$$\vec J = \gamma(\vec E + \vec E_g) \qquad \vec E = \frac{\vec J}{\gamma} - E_g$$

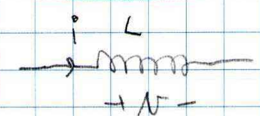
$$\oint_{\mathcal{L}} \vec E \cdot d\vec l = u_{ind} = -\frac{d\psi}{dt}$$

$$\underbrace{\oint_{\mathcal{L}} \frac{\vec J}{\gamma} \cdot d\vec l}_{Ri} - \overbrace{\oint_{\mathcal{L}} \vec E_g \cdot d\vec l}^{u_g}$$

$$Ri - u_g = -\frac{d\psi}{dt} = -\frac{d}{dt}(Li) = -Li'$$

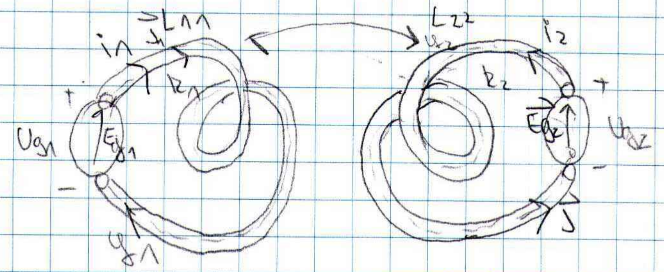
$$\boxed{\underbrace{-u_g + Ri}_{\text{STARO}} + \underbrace{Li'}_{\text{NOVO}} = 0}$$





$u = Li'$

SKLOP DVEH TULJAV



$J_1 = \gamma_1(\vec{E_1} + \vec{E_{g1}})$

$E_1 = \frac{J_1}{\gamma_1} - \vec{E_{g1}}$

$J_2 = \gamma_2(\vec{E_2} + \vec{E_{g2}})$

$E_2 = \frac{J_2}{\gamma_2} - \vec{E_{g2}}$

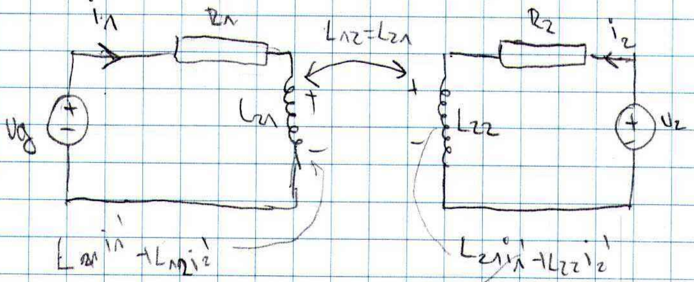
$\oint_{g_1} E_1 \cdot dl = \oint_{g_1} \frac{\vec{J_1}}{\gamma_1} \cdot \vec{dl_1} - \oint_{g_1} \vec{E_{g1}} \cdot \vec{dl}$

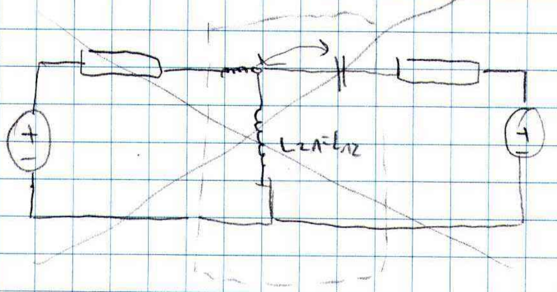
$= R_1 i_1 - U_{g1} = -\frac{d\psi_2}{dt}$

$= -L_{11} i_1' - L_{12} i_2'$

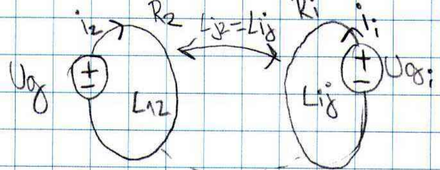
$$\boxed{R_1 i_1 + L_{11} i_1' + L_{12} i_2' - U_{g1} = 0}$$

$$\boxed{R_2 i_2 + L_{21} i_1' + L_{22} i_2' - U_{g2} = 0}$$





SKLOP N-TRANSFORMATORJEV



$\underbrace{R_k i_k + L_{k1} i_1' + \ldots + L_{kj} i_j' + \ldots + L_{k2} i_k' + \ldots}_{\frac{d\psi}{dt}} + L_{k} i_k - U_{gk} = 0$

SKLOPNI FAKTOR

i, j - tuljavi

$$k_{ij}^2 = \frac{\Psi_i(j)\,\Psi_j(i)}{\Psi_i(i)\,\Psi_j(j)} = \frac{L_{ij}L_{ji}}{L_{ii}L_{jj}}$$

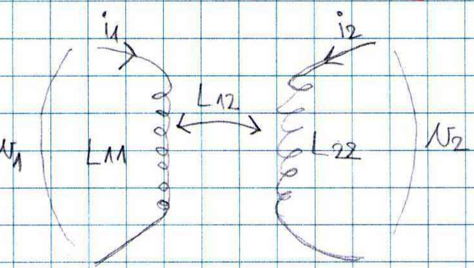
$$\boxed{0 \le k_{ij} = \frac{|L_{ij}|}{\sqrt{L_{ii}L_{jj}}} \le 1}$$

Popolna nepovezanost — Popoln sklep

DRUGO LETO:

$1-k^2=\sigma$ Faktor stresanja

SKLOP DVEH TULJAV IN DOGOVOR O „PIKAH"



$v_1 = L_{11}i_1' + L_{12}i_2'$

$v_2 = L_{21}i_1' + L_{22}i_2'$

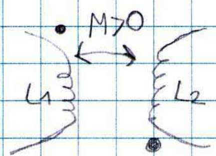
Tak četveropol model za asinhronski motor ipd

(Precej pogosto se pojavlja)

NOV DOGOVOR:

$$\boxed{\begin{matrix} L_{11}=L_1 \\ L_{22}=L_2 \end{matrix}}$$

O PIKAH



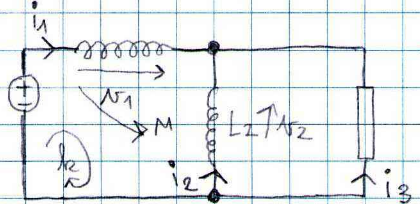
Če sta toka izbrana v pikah kot referenčna, potem se magnetna pretoka podpirata

$k = \frac{M}{\sqrt{L_1L_2}}$

$v_1 = L_{11}i_1' + Mi_2'$

$v_2 = Mi_1' + L_{21}i_2'$

PRIMER:



$v_1 = L_1i_1' + Mi_2'$

$v_2 = Mi_1' + L_2i_2'$

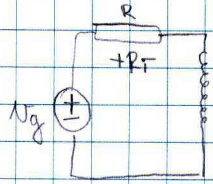
$i_1+i_2+i_3=0$

$-U_g+U_1-U_2=0$

$v_2-Ri_3=0$

MAGNETNA ENERGIJA

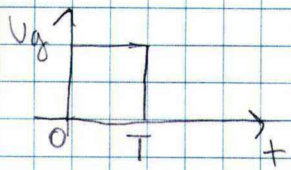
Vloženo delo se manifestira kot neka energija



$-v_g - Ri + v_i' = 0$; $\leftarrow 0 \le t \le T$

$v_g = Ri + L\frac{dI}{dt}$ → SAMOINDUKCIJA

$0 = Ri + Li'$; $t > T$



$t \in [0, T]$

$U_g = Ri + L\frac{dI}{dt} / i$

$U_g i = Ri^2 + Lii' / \int_{t_1}^{t_2} dt$ Močnostna energija tega vezja

$$\underbrace{\int_{t_1}^{t_2} v_g i\, dt}_{A_g(t_1,t_2)} = \underbrace{\int_{t_1}^{t_2} Ri^2\, dt}_{W_t(t_1,t_2)} + \underbrace{\int_{t_1}^{t_2} Lii'\, dt}_{L\int_{t_1}^{t_2} i\, di}$$

$$L\int_{t_1}^{t_2} i\, di = L\frac{i^2}{2}\Big|_{t_1}^{t_2}$$

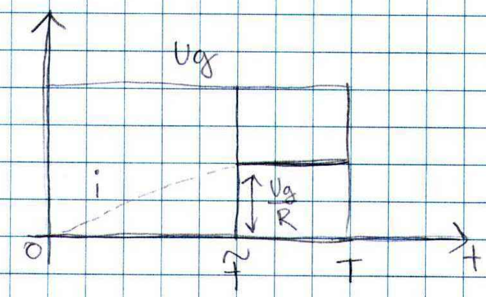
$$\Rightarrow \boxed{A_g(t_1,t_2) = W_t(t_1,t_2) + \frac{L}{2}\left[i^2(t_2) - i^2(t_1)\right]}$$

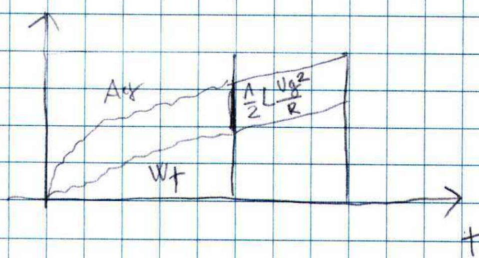
$t_1 = 0,\ t_2 = t^* < \tilde{t} < T$

$A_g(0, t^*) = W_t(0, t^*) + \frac{L}{2} i^2(t^*)$

$\tilde{t} \leftarrow i(\tilde{t}) = \frac{U_g}{R}$

$i(\tilde{t} < t < T) = \frac{U_g}{R}$





za $t'>T$:

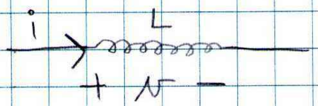
$$A_g(T,t')=0=W_T(T,t')+\frac{L}{2}\left[i^2(t')-I^2\right]$$

$$A_g(T,\infty)=W_T(T,\infty)-\frac{L}{2}I^2=0$$

⇒ Generator ne daje nove energije sistemu. Višek akumulacije se počasi izčrpa

$$\boxed{A_g(t_1,t_2)=W_T(t_1,t_2)+W_m(t_2)-W_m(t_1)}$$

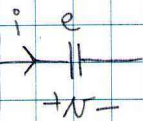
$\boxed{W_m=\frac{1}{2}Li^2}$ $\quad W_m(t)=\frac{1}{2}Li^2(t) \quad W_e(t)=\frac{1}{2}Cu^2(t)$



$v=Li'$

$W_m=\frac{1}{2}Li^2$

$W_m'=\frac{1}{2}L2ii'=iv=p_m$



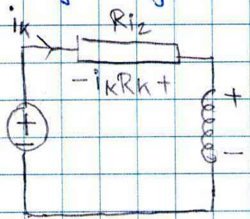
$i=Cu'$

$W_e=\frac{1}{2}Cu^2$

$W_e'=ui=p_e$

GLEJ ZGLED V KNJIGI

Če je tuljav več ⇒ več virov → Glej v knjigi izpeljavo

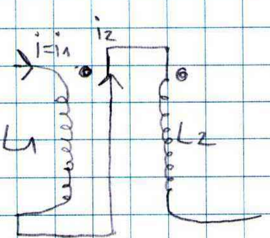


$$A_g(t_1,t_2)=W_T(t_1,t_2)+W_m(t_2)-W_m(t_1)$$

$$\boxed{W_m=\frac{1}{2}\sum_{k=1}^{n}i_k\Psi_k=\frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n}L_{jk}i_ji_k}$$

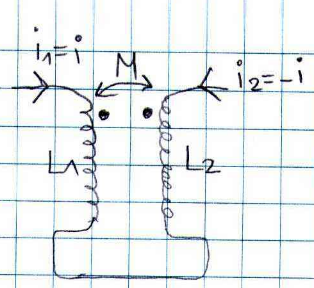
ZGLEDI:

1)



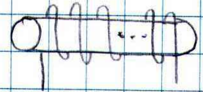
$$W_m=\frac{1}{2}L_1i_1^2+\frac{1}{2}Mi_1i_2+\frac{1}{2}Mi_2i_1+\frac{1}{2}L_2i_2=\frac{1}{2}(L_1+L_2+2M)i^2$$

2.)

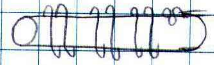


Magn. pretoka se NE podpirata

2a)

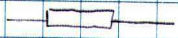


KERAMIKA



BIFILARNO NAVITJE

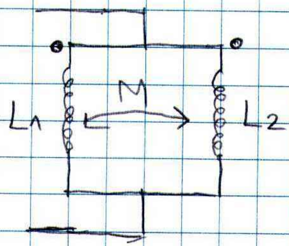
ŽIČNI UPOR



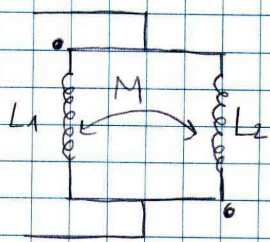
$L \to 0$

Parazitnost

3)



$$\Rightarrow L_{ind} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$



$$L_{ind} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$\boxed{W_m = \frac{1}{2} \sum_{k=1}^{n} \sum_{j=1}^{n} L_{jk} i_k i_j} \Rightarrow \text{Treba gledat kot celota}$$

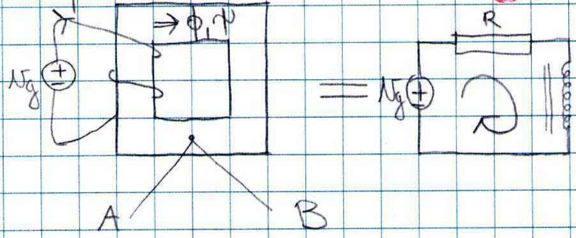
.... $W_m = \frac{1}{2} \mu H^2$

$W_e = \frac{1}{2} Q_1 V_1 + \frac{1}{2} Q_2 V_2 + \ldots \to W_e = \frac{1}{2} E \varepsilon_0$

To je blo za linearne strukture

ENERGIJSKI VLOŽEK ZA MAGNETENJE NELINEARNE STRUKTURE

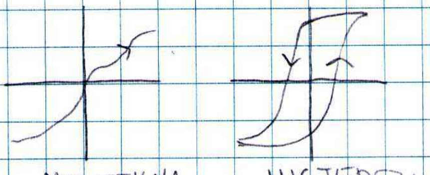
9.5.11



$$-v_g + Ri + \frac{d\psi}{dt} = 0$$

$$v_g = Ri + \frac{d\psi}{dt} \quad / \, i$$

$$v_g i = Ri^2 + i\frac{d\psi}{dt} \quad \int_{t_1}^{t_2} \dots dt$$



MAGNETILNA KRIVULJA

HISTEREZA

$$\boxed{A_g(t_1,t_2) = W_T(t_1,t_2) + \int_{t_1}^{t_2} i\,d\psi}$$

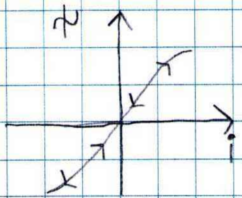
$$\boxed{W_{mag} = \int_{t_1}^{t_2} i\,d\psi}$$

ENERGIJSKI VLOŽEK ZA MAGNETENJE STRUKTURE

NELINEARNOST IN VIŠJI HARMONIKI

$R \to 0 \Rightarrow v_g \simeq \psi'$

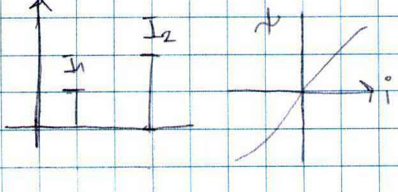
$v_g = V_m \cos\omega t \Rightarrow \psi = \frac{U_m}{\omega}\sin\omega t$



$i = K\psi^3$ $\quad$ $i = K\frac{U_m}{\omega^2}\sin^3\omega t$ $\quad$ $\{\sin^3 x = \frac{3}{4}\sin x - \frac{1}{4}\sin 3x\}$

$i = \frac{3}{4}K\left(\frac{U_m}{\omega}\right)^3\sin\omega t - \frac{1}{4}K\left(\frac{U_m}{\omega}\right)^3\sin 3\omega t = I_{m1}\sin\omega t - I_{m3}\sin 3\omega t$

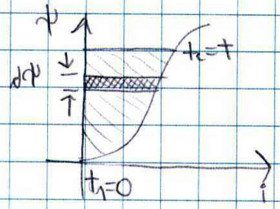
$\psi(-i) = -\psi(i) \Rightarrow$ LIHA F.



$v_g = V_m\cos\omega t$

$i = I_{m1}\sin\omega t + I_{m3}\sin 3\omega t + I_{m5}\sin 5\omega t + \dots$ VRSTA

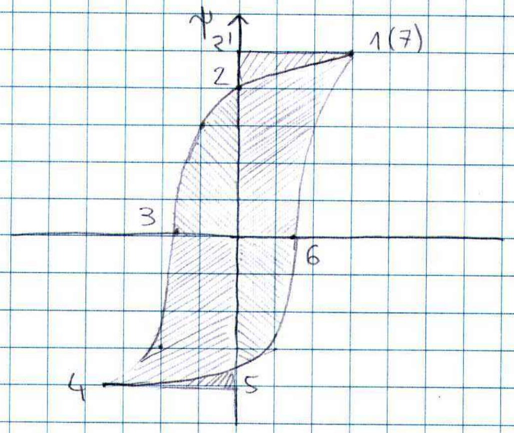
A.) MAGNETENJE PO DEVIŠKI KRIVULJI



$$W_{mag} = \int_{t_1}^{t_2} i\,d\psi$$

$$\boxed{\int_{t_1=0}^{t_2=t} i\,d\psi}$$

B.) MAGNETENJE PO HISTEREZI



$0 > \int i\,d\psi =$ ustreza ploščini lika $\boxed{1,2,2'}$ GENERATORSKI REŽIM

$0 < \int i\,d\psi =$ ustreza ploščini lika $\boxed{2,3,0}$ BREMENSKI REŽIM

$0 < \int i\,d\psi =$ ustreza ploščini lika $\boxed{3,4,5',0}$ BREM.

$0 > \int i\,d\psi =$ ustreza ploščini lika $\boxed{4,5,5'}$ GEN.

$0 < \int i\,d\psi =$ ustreza ploščini lika $\boxed{5,6,0}$ BREM

$0 < \int i\,d\psi =$ ustreza ploščini lika $\boxed{6,7,2',0}$ BREM

$T = t_2 - t_1 \rightarrow$ Perioda

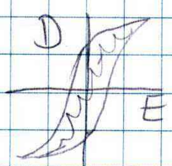
$$W_{mag}(t, t+T) = \oint_{\mathcal{P}} i\,d\psi$$

$\mathcal{P} \rightarrow$ Histerezna pentlja (kaligrafski P)

$$\boxed{P_h = \frac{W_{mag}(t, t+T)}{T}}$$

Tretja vrsta izgub

Dielektrične izgube pri histerezi $\rightarrow$ Na podlagi tega delajo mikrovalovke



u, i, p razmere v toroidni tuljavi

z upoštevanjem histereze feromagnetika

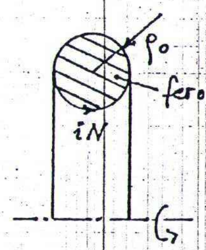
$N = 500,\ \rho_0 = 2cm,\ l = 0{,}5m$

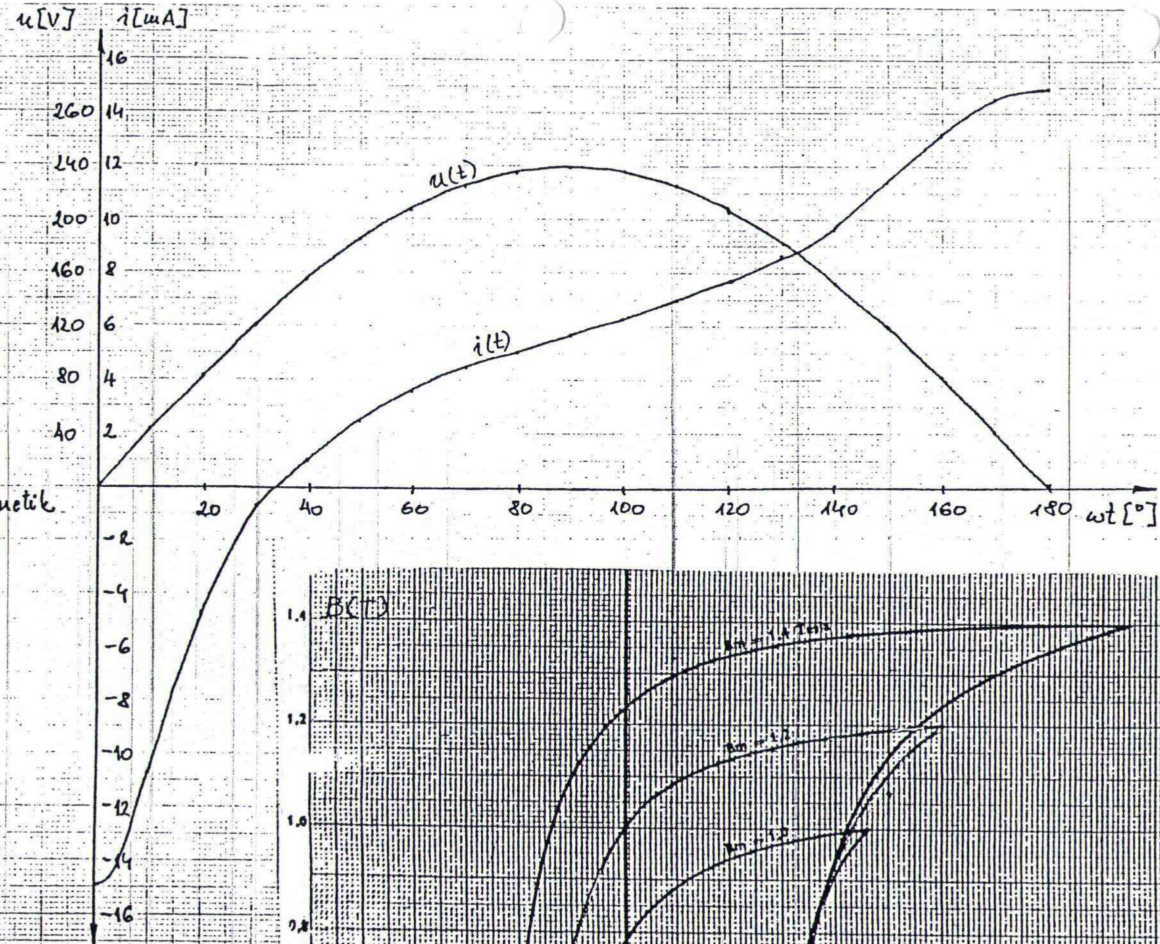
$u(t) = 240 \sin(100\pi t)$; $f = 50Hz$

$$B(t) = -\frac{240}{\omega N \pi \rho_0^2}\cos(100\pi t) = -1{,}2\cos(100\pi t)$$

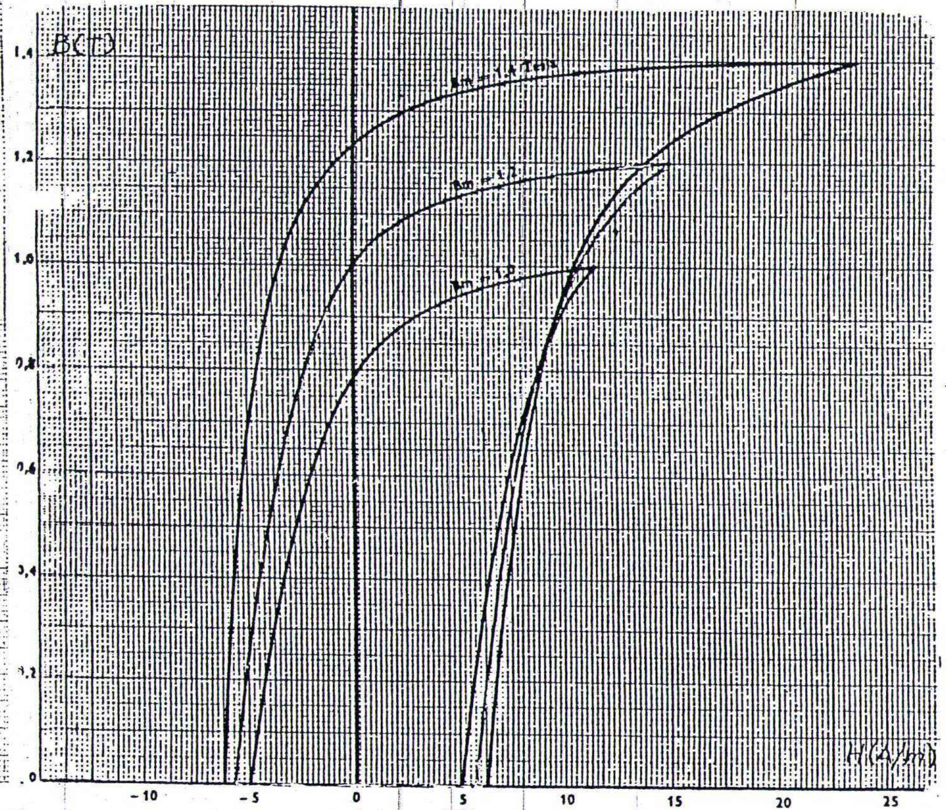
$B(t) \rightarrow H(t)$

$$i(t) = \frac{l}{N} H(t) = 10^{-3} H(t)$$





ωt [°]	B(t) [T]	H(t) [A/m]
0	−1,2	−15,0
10	−1,18	−10,75
20	−1,13	−4,75
30	−1,04	−0,75
40	−0,92	+1,0
50	−0,77	2,5
60	−0,6	3,5
70	−0,41	4,5
80	−0,21	5,0
90	0,0	5,75
100	0,21	6,25
110	0,41	7,0
120	0,6	7,75
130	0,77	8,75
140	0,92	9,75
150	1,04	11,5
160	1,13	13,25
170	1,18	14,5
180	1,2	15,0



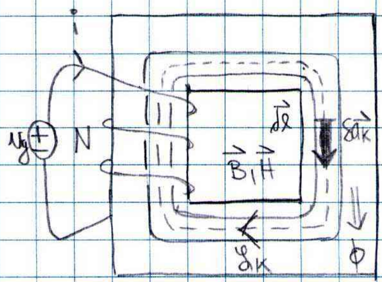
1) $$P = \frac{1}{T}\int_0^T u(t)\cdot i(t)\,dt = \frac{1}{18}\sum_{k=1}^{18} u_k i_k = 0{,}824\ W$$

2) $$P = A_h(B,H)\cdot V\cdot f = A_h(B,H)\cdot \pi\rho_0^2 l\cdot f = 0{,}814\ W$$

$P/m = 0{,}167\ W/kg$ (DO 0,5 W/kg)

GOSTOTA ENERGIJSKEGA VLOŽKA

$$W_{mag}(t_1,t_2) = \int_{t_1}^{t_2} i \, d\Psi$$



$$\Psi = N\phi \, , \, d_t\Psi = N d_t \phi$$

$$\phi = \sum_k \delta\phi_K = \sum_k \vec{B}\cdot\delta\vec{a}_K$$

$$d_t\Psi = N\sum_k d_t\vec{B}\cdot\delta\vec{a}_K$$

$$i = \frac{1}{N}\oint_{g_K} \vec{H}\cdot\vec{dl}$$

$$(\vec{a}\cdot\vec{b})(\vec{c}\cdot\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d})$$ Trije veht. od štirih so kolinearni

$$\Downarrow$$

$$W_{mag}(t_1,t_2) = \int_{t_1}^{t_2}\left(\oint_{g_K}\vec{H}\cdot\vec{dl}\right)\left(\sum_k d_t\vec{B}\cdot d\vec{a}\right) = \int_{t_1}^{t_2}\sum_K \oint_{g_K}(\vec{H}\cdot d_t\vec{B})(\delta\vec{a}_K\cdot\vec{dl})$$

$$= \sum_k \oint_{g_K}\int_{t_1}^{t_2}(\vec{H}\cdot d_t\vec{B})\cdot dV_K$$

$$\boxed{W_{mag}(t_1,t_2) = \int_V\left(\int\vec{H}\cdot d_t\vec{B}\right)dV}$$

$$w_{mag}(t_1,t_2) = \int_{t_1}^{t_2}\vec{H}\cdot d\vec{B} \quad [J/m^3]$$

$$W_{mag}(t_1,t_2) = \int_{t_1}^{t_2} i\,d\Psi \quad [J]$$

GLEJ LIST!

$$u = N\frac{d\phi}{dt} = NS\frac{dB}{dt}$$ $u \rightarrow \sin$, $\frac{dB}{dt} \rightarrow \cos$

Popačenje → Napetost harmonična, tok pa ne

$$P_h = \frac{W_{mag}(t, t+T)}{T} \Rightarrow P_h = \frac{1}{T}\int_t^{t+T} u\,i\,dt$$

Koercitivne jakosti 5A/m → mehkomagnetni materiali

12.5.'11

GOSTOTA MAGNETNE ENERGIJE

$$W_{mag}(t_1,t_2)=\int_{t_1}^{t_2}\vec H\cdot d_t\vec B$$

LINEARNI SISTEM

$\vec B=\mu\vec H$

$$\boxed{W_{mag}(t_1,t_2)=\int_{t_1}^{t_2}\mu\vec H\cdot d\vec H=\frac{\mu}{2}\left(H^2(t_2)-H^2(t_1)\right)}$$

$$\vec H\cdot d\vec H=H_x dH_x+H_y dH_y+H_z dH_z=\frac{1}{2}d\left(H_x^2+H_y^2+H_z^2\right)=\frac{1}{2}dH^2$$

$$\boxed{w_m(t)=\frac{\mu H^2(t)}{2}=\frac{\vec B\cdot\vec H}{2}=\frac{B^2}{2\mu}}$$

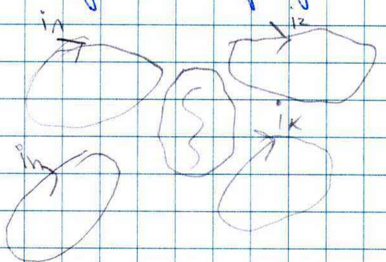
$\left\{w_e(t)=\frac{\varepsilon E^2(t)}{2}\right\}\ldots$

VZAJEMNOST - RECIPROČNOST

$L_{jk}=L_{kj}$

$$W_m=\frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n}L_{jk}i_j i_k=\int_V w_m\,dV=\frac{1}{2}\int_V\vec H\cdot\vec B\,dV$$

Tam kjer ni polja tudi energije ni.



$$\vec H=\vec H^{(1)}+\vec H^{(2)}+\ldots+\vec H^{(n)}$$
$$\vec B=\vec B^{(1)}+\vec B^{(2)}+\ldots+\vec B^{(n)}$$
$$\vec H=\sum_{k=1}^{n}\vec H^{(k)}\qquad\vec B=\sum_{j=1}^{n}\vec B^{(j)}$$

$$\boxed{W_m=\frac{1}{2}\sum_{k=1}^{n}\sum_{j=1}^{n}\int_V\vec H^{(k)}\cdot\vec B^{(j)}\,dV}$$

DOKAZ

$$\underline{L_{jk}}=\frac{1}{i_j i_k}\int\vec H^{(k)}\cdot\vec B^{(j)}dV=\frac{1}{i_j i_k}\int\mu\vec H^{(k)}\vec H^{(j)}dV=\frac{1}{i_j i_k}\int\vec H^{(j)}\cdot\vec B^{(k)}dV=\underline{L_{kj}}$$

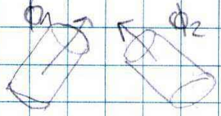
GIBALNI PROCESI

12. 5. 11

$W_m \leftrightarrow \vec{F}_m$

$\{W_e \leftrightarrow \vec{F}_e\}$

1. BREZ VIROV



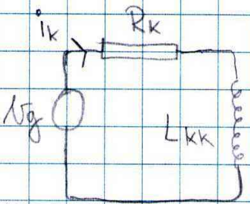
$$\boxed{\vec{F}_m \cdot \delta \vec{l} + \delta W_m = 0} \qquad \vec{F}_m = -\left(\frac{\partial W_m}{\partial x}, \frac{\partial W_m}{\partial y}, \frac{\partial W_m}{\partial z}\right)$$

2. Z VIRI

n = zank s toki i_k in sklepi ψ_k

$t, t+\delta t$ – interval v katerem se ena od zank premakne za nek $\delta \vec{l}$

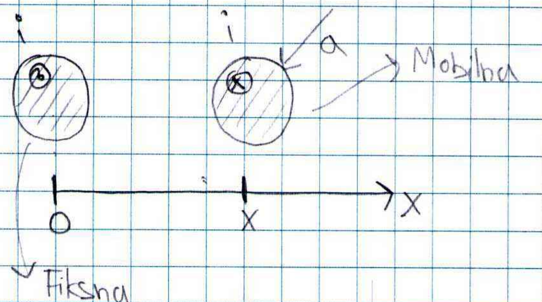
$$\delta A_g = \delta W_T + \overbrace{\underbrace{\frac{1}{2}\sum_{k=1}^{n} i_k \delta\psi_k}_{\delta W_m} + \vec{F}_m \cdot \delta \vec{l}}^{} \qquad W_m = 1/2 \sum_{k=1}^{n} i_k \psi_k$$



$$\delta A_g = \delta W_T + \underbrace{\sum_{k=1}^{n} \int_t^{t+\delta t} i_k \, d\psi_k}_{\delta t \to 0:\ \sum_{k=1}^{n} i_k \delta\psi_k} = \delta W_T + 2\,\delta W_m$$

$$\Rightarrow \boxed{\vec{F}_m \cdot \delta \vec{l} = \delta W_m} \qquad \vec{F}_m = \left(\frac{\partial W_m}{\partial x}, \frac{\partial W_m}{\partial y}, \frac{\partial W_m}{\partial z}\right)$$

ZGLED – Sila med tokovodnikoma simetričnega dvovoda

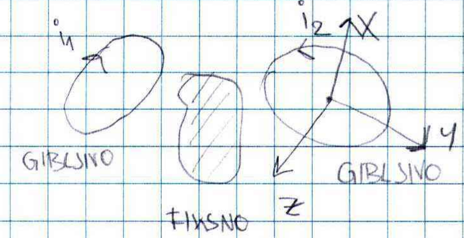


$$W_m = \frac{1}{2} L i^2 = \frac{i^2}{2}\left(\frac{\mu_0 l}{\pi}\left(\frac{1}{4} + \ln\frac{x}{a}\right)\right) = W_m(x)$$

$$F_{mx} = \frac{dW_m}{dx} = \frac{i^2}{2}\frac{\mu_0 l}{\pi x} = \boxed{\frac{\mu_0 i}{2\pi x}\, i l}$$

$$d\vec{F}_m = i\, d\vec{l} \times \vec{B}$$

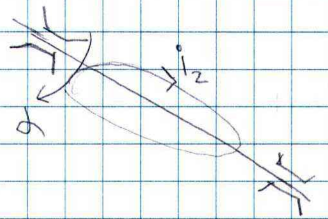
ZGLED – Sila (navor) na sosednji tokovni zanki



$$W_m = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

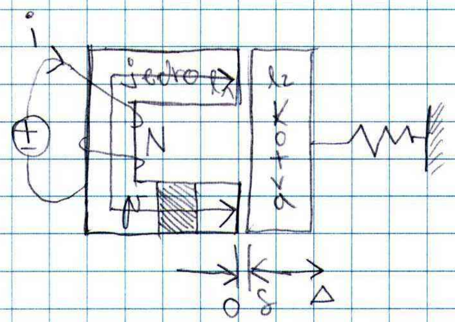
$$F_{mx} = \frac{\partial W_m}{\partial x} = i_1 i_2 \frac{\partial L_{12}}{\partial x}$$

$\left(\frac{y}{z}\right)$ $\left(\frac{y}{z}\right)$



$$\boxed{M_{m\alpha} = i_1 i_2 \frac{\partial L_{12}(\alpha)}{\partial \alpha}}$$

ZGLED – Elektromagnet



$$W_m = \frac{1}{2} L i^2 \qquad l_1 + l_2 = l$$

$$L = \frac{N^2}{R_m}$$

$$R_m = \frac{l}{\mu S} + \frac{2\delta}{\mu_0 S} = R(\delta)$$

$$W_m = \frac{i^2}{2} \frac{N^2}{R_m(\delta)} = W_m(\delta)$$

Dve reži

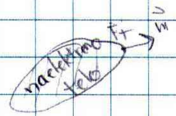
$$\vec{F}_{m\delta} = \frac{dW_m(\delta)}{d\delta} = \frac{(Ni)^2}{2}\left(-\frac{\frac{2}{\mu_0 S}}{R_m^2}\right) = -\frac{\phi^2}{2}\frac{2}{\mu_0 S} = -2\frac{B^2}{2\mu_0}S$$

Pretezna sila

W_{m0}

$$\boxed{\frac{F_m \delta}{2S}} = \frac{B^2}{2\mu_0} = w_{m0}$$

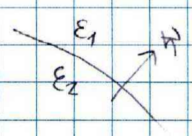
TLAK



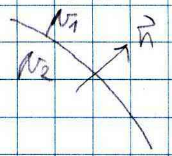
$f_{en} = w_e(T_1)$

$$f_{en} = \frac{\varepsilon_0 E^2}{2}$$ pri 1MV/m je $f_{en} \simeq 4{,}4\ N/m^2$

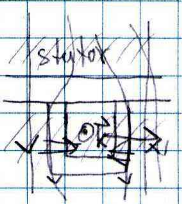
$$f_{mn} = \frac{B^2}{2\mu_0}$$ pri 1T je $f_{mn} \simeq 4 \cdot 10^5\ N/m^2$ (4 atm = 4 bar)



$$f_m = \frac{1}{2}(\varepsilon_1 - \varepsilon_2)\left(E_t^2 + \frac{D_n^2}{\varepsilon_1 \varepsilon_2}\right)$$



$$f_m = \frac{1}{2}(\mu_2 - \mu_1)\left(H_t^2 + \frac{B_n}{\mu_1 \mu_2}\right)$$

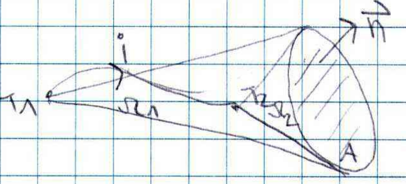


MOTOR B I l

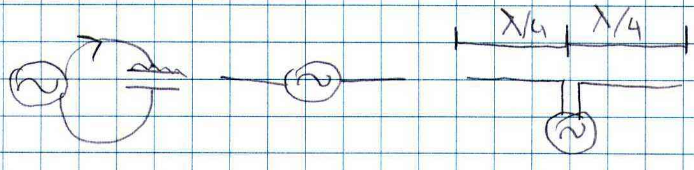
Glavne sile delujejo na stenah utorov. Glavnina sil so - Maxwellove sile

ZADNJA MAXWELLOVA ENAČBA – PREMIKALNI TOK OZ. RAZŠIRJEN AMPEROV ZAKON

$$\oint_g \vec{H}\cdot d\vec{l} = \int_A \vec{J}_{prosti}\cdot d\vec{a} + \dots ?$$



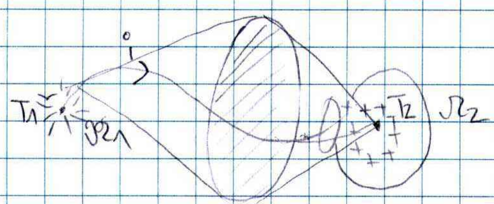
$$\oint_g \vec{B}\cdot d\vec{l} = \frac{\mu_0 i}{4\pi}(\Omega_2-\Omega_1)$$



$$\oint_g \vec{B}\cdot d\vec{l} = \mu_0 \frac{\frac{dQ}{dt}}{4\pi}(\Omega_2-\Omega_1)$$

$$= \mu_0 \frac{d}{dt}\underbrace{\left(\frac{Q}{4\pi}\Omega_2 - \frac{Q}{4\pi\varepsilon_0}\Omega_1\right)\varepsilon_0}_{\int \vec{E}\cdot d\vec{a}}$$

$$\boxed{\oint_g \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{a}}$$



$$\oint_g \vec{B}\cdot d\vec{l} = \frac{\mu_0 i}{4\pi}(4\pi-\Omega_2-\Omega_1) = \mu_0 i = \mu_0 \frac{\frac{dQ}{dt}}{4\pi}(\Omega_2-\Omega_1)$$

$$= \mu_0 i - \mu_0\frac{d}{dt}\underbrace{\left(\frac{Q}{4\pi\varepsilon_0}\Omega_2' + \frac{Q}{4\pi\varepsilon_0}\Omega_1\right)\varepsilon_0}_{-\int_A \vec{E}\cdot d\vec{a}}$$

$$\boxed{\oint_g \vec{B}\cdot d\vec{l} = \mu_0 i + \mu_0\varepsilon_0\frac{d}{dt}\int_A \vec{E}\cdot d\vec{a}}$$

$$\boxed{\oint_g \vec{B}\cdot d\vec{l} = \mu_0\int_A\left(J + \varepsilon_0\frac{\partial E}{\partial t}\right)d\vec{a}}$$ IV. MAXWELLOVA

$$\oint_{\gamma} \vec{B}\cdot d\vec{l} = \mu_0 \Big(\underbrace{\int_A \vec{J}\cdot d\vec{a}}_{\text{Ampere}} + \varepsilon_0 \int_A \frac{\partial E}{\partial t}\cdot d\vec{a} \Big)$$

MAXWELL

$$\int_A \vec{J}\cdot d\vec{a} = \underbrace{\int_A \vec{J}_{prosti}\cdot d\vec{a}}_{i_{prosti}} + \underbrace{\int_A \underbrace{\frac{\partial \vec{P}}{\partial t}}_{\vec{J}_{polarizacijski}}\cdot d\vec{a}}_{i_{polariz}} + \underbrace{\oint_l M\cdot d\vec{l}}_{i_{magnetizacijski}}$$

$$\oint_{\gamma} \frac{\vec{B}}{\mu_0}\cdot d\vec{l} = \int_A \vec{J}_{pr}\cdot d\vec{a} + \int_A \frac{\partial P}{\partial t}\, d\vec{a} + \oint_{\gamma} \vec{M}\cdot d\vec{l} + \varepsilon_0 \int_A \frac{\partial E}{\partial t}\cdot d\vec{a}$$

$$\oint_{\gamma} \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{\vec{H}}\cdot d\vec{l} = \int_A \Big(\vec{J}_{pr} + \frac{\partial}{\partial t}\underbrace{(\varepsilon_0\vec{E} + \vec{P})}_{\vec{D}}\Big)\cdot d\vec{a}$$

$$\boxed{\oint_{\gamma} \vec{H}\cdot d\vec{l} = \int_A \Big(\vec{J}_{pr} + \underbrace{\frac{\partial D}{\partial t}}_{\text{Gostota Maxwellovega premikalnega toka}}\Big)\cdot d\vec{a}} \qquad \oplus \qquad \oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial B}{\partial t}\cdot d\vec{a}$$

Vrtinčnost - Ampere

⇒ Valovna enačba ⇒ Svetloba je EMV

$$\frac{\partial \vec{D}}{\partial t} = \frac{\partial P}{\partial t} + \varepsilon_0 \frac{\partial E}{\partial t}$$

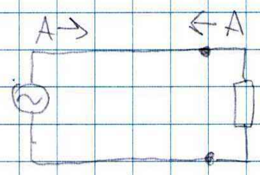
Tolmač se pojem etra - je tako zmotil da je prav

Ta enačba ne pove da tok povzroči magn. polje. Napaka. Namreč samo pove kako sta povezana, ne kdo koga povzroča

BIBLIJA - nasledki

- Rešitve M in ostalih enačb (4: tvornost E, B, vrtinčnost E, B) vse le o povezanosti, ne o povzročanju
kontinuitetna enačba

- Valovanje in difuzija - nesklad!



Karakteristična upornost in ni več dolžina refleksije

$$\boxed{\varepsilon_0\mu_0 c_0^2 = 1}$$

$$\frac{\partial^2 \psi}{\partial x} - \mu\varepsilon \frac{\partial^2 \psi}{\partial t^2} = 0$$

$\mu\varepsilon \to \frac{1}{c^2}$

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} = 0$$

VALOVNA ENAČBA

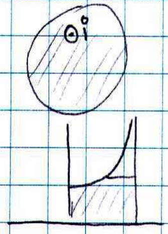
Rešitev: $\psi(t \pm \frac{x}{c})$ vsaka taka f-ja, ki je 2x odvedljiva

VALOVNA FUNKCIJA

$\psi_1(t - \frac{x}{c})$ - vzdolž osi x potuje val

$\psi_2(t + \frac{x}{c})$ - potuje v nasprotno smer $(-x)$

Difuzija → Kvožni efekt



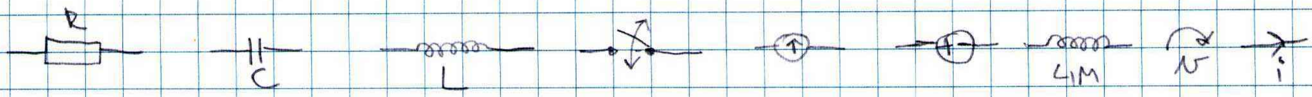
Pomnljev vektor → Parcialni del v žica

Z zdreževanjem dolima

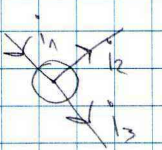
$$\frac{\partial^2 \psi}{\partial x} - k \frac{\partial \psi}{\partial t} = 0 \Rightarrow \text{Difuzijska enačba}$$

To je vse v vesolju, kar je treba, se pride z reševanjem vseh teh enačb

ELEKTRIČNA VEZJA

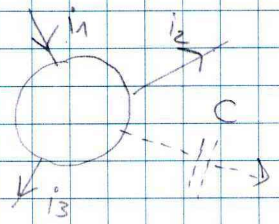


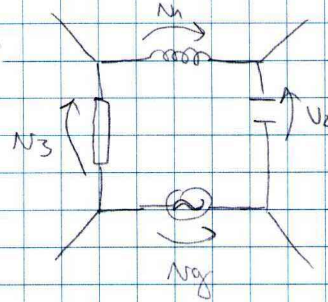
I. in II. Kirchofov zakon



$-i_1+i_2+i_3 = -\frac{dQ_{not}}{dt} = C\frac{dU_T}{dt}$

$-i_1+i_2+i_3=0$





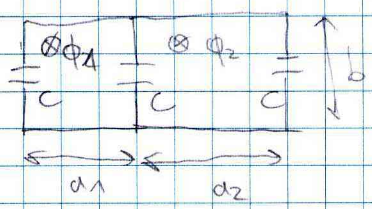
$v_1-v_2-v_g+v_3=-\frac{d\phi}{dt}$

V vezjih strnjenih elementov geometrija ni relevantna, površina zanke = 0, $\phi=0$

$v_1-v_2+v_3-v_g=0$

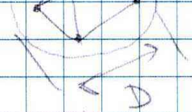
$$\boxed{\sum_{k=1}^{n}(\pm)\, i_k + ?\frac{dQ}{dt}? = 0}$$

$$\boxed{\sum_{k=1}^{n}(\pm)\, v_k + ?\frac{d\phi}{dt}? = 0}$$



Kdaj obravnavamo kot vezje?

Kriterij koncepta vezij:



$D \ll \lambda = \frac{c}{v}$

Višje frekvence za manjše naprave

D $\cancel{\times}$ λ

(A)LV
aktivno linearno vezje
- aktivni elementi
- pasivni el.
- stikala

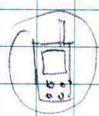
⟸ I. kz
II kz
Vezni tokovi
Vezne napetosti
moči
energije

TELLEGENOV STAVEK

$$\sum_{k=1}^{n} v_k i_k = 0 \qquad v_k i_k = P_{bk} - P_{gk}$$

$$\sum_{k=1}^{n} P_{gk} = \sum_{k=1}^{m} P_{bk} = P \text{ na uporih} + P \text{ elektrenja} + P \text{ za magnetenje}$$

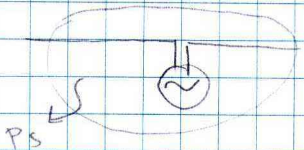
Za D $\cancel{\times}$ λ to ne drži



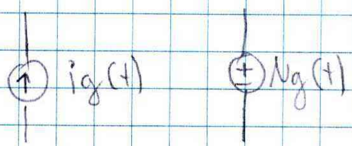
$\rightarrow P_g$

$\rightarrow P_g = P_b + P_s$

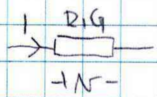
Poyntingov stavek - dodamo še moč sevanja



AKTIVNI ELEMENTI



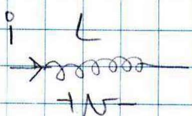
PASIVNI ELEMENTI



$i = Gv$

$p = vi$

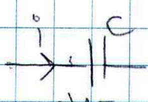
$\int_{t_1}^{t_2} p(t)dt = W(t_2) - W(t_1)$



$v = Li'$

$p_m = vi = \frac{dW_m}{dt}$

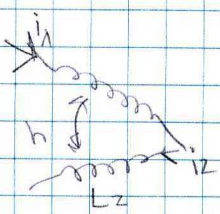
$W_m = \frac{Li^2}{2}$



$i = Cv'$

$p_e = vi = \frac{d}{dt} W_e$

$W_e = \frac{Cv^2}{2}$



$v_1 = L_1 i_1' + M i_2'$

$v_2 = M i_1' + L_2 i_2'$

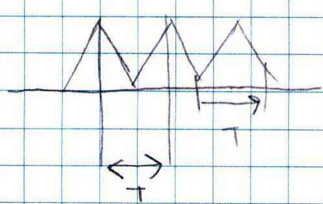
$p_m = v_1 i_1 + v_2 i_2$

$W_m = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$

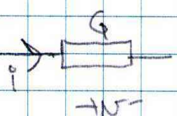
$p_m = W_m'$

SREDNJA IN EFEKTIVNA VREDNOST PERIODIČNEGA TOKA OZ. NAPETOSTI

$v(t+T) = v(t) \quad \forall t$



$$v_{sr} = \frac{1}{T}\int_{t_1}^{t_1+T} v(t)dt = \overline{v}$$



$p = Gv^2$ če periodičen v, je tudi p $p(t'+T) = p(t')$

$$P_{sr} = \frac{1}{T}\int_{t_1}^{t_1+T} Gv^2(t)dt = G\frac{1}{T}\int_{t_1}^{t_1+T} v^2(t)dt = GU_{ef}$$

$$U_{ef}^2 = \frac{1}{T}\int_{t_1}^{t_1+T} v^2(t)dt = \overline{v^2}$$

KONDENZATOR

$W_e = \frac{Cv^2}{2}$ če $v(t+T) = v(t) \Rightarrow \overline{W_e} = C\frac{\overline{v^2}}{2} = \frac{CU_{ef}}{2}$

TULJAVA

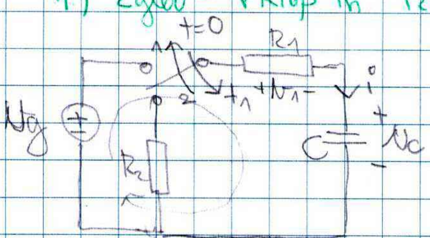
$W_m = \frac{Li^2}{2}$ če $i(t+T) = i(t) \Rightarrow \overline{W_m} = L\frac{\overline{i^2}}{2} = L\frac{I_{ef}^2}{2}$

PREHODNI POJAVI V EL. VEZJIH

19.5.11

Hipni odzivi - vklop/izklop
- Sami mamo tu razlaut zaradi pojavi pri diferencialkah. N

1) Zgled - Vklop in izklop RC vezja



$\underline{v_c(t \le 0) = U_0}$, Predhodno nabit kondenzator

Položaj 1 : $0 \le t \le t_1$

Kirchhoff-zančna enačba

$-U_g + R_1 i + v_c = 0$; $i = Cv_c'$

$\boxed{R_1 C v_c' + v_c = U_g}$ L.D.E (Nehomogena)

Napetost na kond. se zvezno spreminja

$v_c(0) = U_0 = v_c(0+)$

Reševanje DE: $R_1 C v_{ch}' + v_{ch} = 0$; $v_{ch} = Ae^{\lambda t}$

$(\lambda R_1 C + 1)Ae^{\lambda t} = 0$

$\rightarrow \underline{\lambda = -\frac{1}{R_1 C}}$ $\quad \frac{1}{R_1 C} = \tau^{-1}$

$\tau_1 = R_1 C$ → Časovna konstanta

Partikularni del

$v_{cp} = U_g$

$\underline{v_c = Ae^{\lambda t} + U_g}$, Splošna rešitev

$v_c(0) = U_0 = Ae^{0} + U_g$

$\Rightarrow \underline{A = U_0 - U_g}$

$v_c(t \ge 0) = U_g + (U_0 - U_g)e^{-t/\tau_1}$

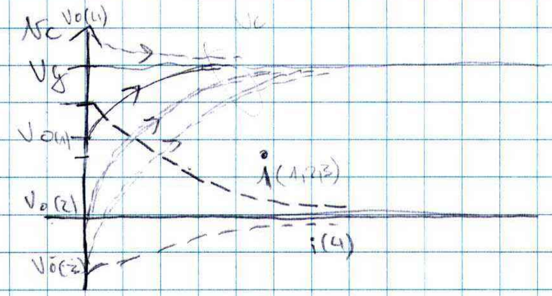
$$\boxed{v_c(t \geq 0) = V_0 + (V_g - V_0)(1 - e^{-t/T_1})}$$

Napetost tik pred vklopom

Neke znotraj tega reda za majhne $x \to 0$. Po definiciji časa ta funkcija narašča

$$i = C v_c' = \frac{V_g - V_0}{RC} C e^{-t/T_1} = \frac{V_g - V_0}{R_1} e^{-t/T}$$

Startna vrednost toka



2. POLOŽAJ $t_1 \leq t < \infty$

$v_c(t_1) = V_1$ (izračunana nova začetna vrednost)

$R_1 i + v_c + R_2 i = 0$

$(R_1 + R_2) i + v_c = 0$, $i = C v_c'$

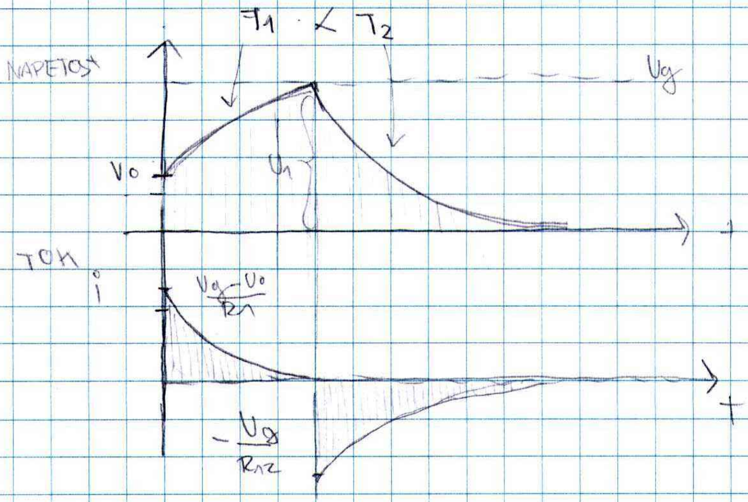
$$\boxed{\underbrace{R_{12} C}_{T_{12}} v_c' + v_c = 0} \quad (4) \quad \boxed{v_c(t \geq t_1) = V_1}$$

$v_c(t \geq t_1) = B e^{-t/RC}$

$v_c(t_1) = V_1 = B e^{-t_1/T_{12}} \Rightarrow B = V_1 e^{t_1/T_{12}}$

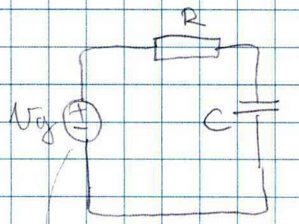
$$v_c(t \geq t_1) = V_1 e^{-(t-t_1)/T_{12}}$$

$$i(t \geq t_1) = -\frac{V_1}{R_{12}} e^{-(t-t_1)/T_{12}}$$

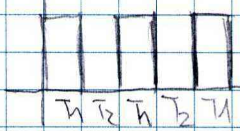


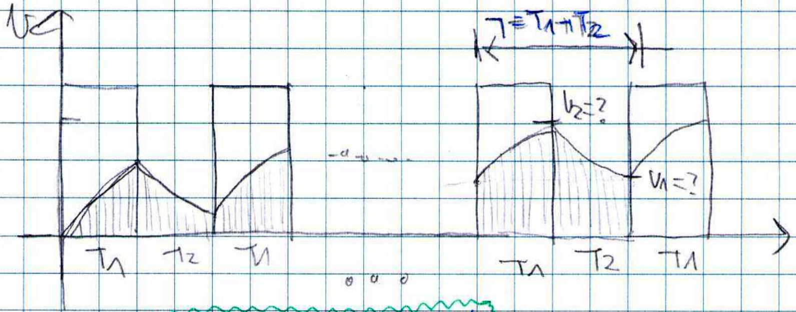
$$W_e = \frac{C v_c^2}{2} \qquad p_e = v_c i$$

2. Zgled - Funkcijski generator



> Funkcijski generator





$$\boxed{v_2 = U_1 + (U_g - U_1)(1 - e^{-T_1/\tau})} \; ; \; \tau = RC$$

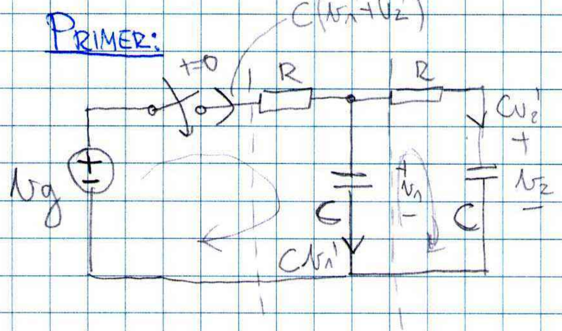
$$\boxed{v_1 = U_2 e^{-T_2/\tau}} \Rightarrow U_1, U_2$$

$T_1 = T_2 = T$

Enkrat bo takšna izpitna naloga

$$\boxed{\begin{aligned} U_2 &= U_1 + (U_g - v_1)(1 - e^{-1}) \\ U_1 &= v_2 e^{-1} \end{aligned}}$$

PRIMER:



Edino kar imamo so Kirchhoffove enačbe

$$-v_1 + RCv_2'' + v_2' = 0$$

$$-v_g + RC(v_1' + v_2') + v_1 = 0 \Rightarrow v_1 = RCv_2' + v_2$$

$$-v_g + RC(v_1' + v_2') + v_1 = 0$$

$$\boxed{(RC)^2 v_2'' + 3(RC) v_2' + v_2 = V_g}$$ Dobili smo DE II. reda

ZAČ. POGOJ $\quad v_2(0-) = v_1(0-) = 0 \Rightarrow \left.\begin{aligned} v_2(0+) &= v_2(0-) \\ v_1(0+) &= v_1(0-) \end{aligned}\right\} \Rightarrow v_2'(0+) = 0$

Do prehodnih pojavov prihaja, ker imamo v teh vezjih akumulativne elemente $\Rightarrow$ Kondenzatorji in tuljave.
V R-vezjih nimamo takih problemov

$\Rightarrow \boxed{v_2(0+) = 0, \; v_2'(0+) = 0}$ Začetni pogoji

Št. akumulativnih elementov nam določa stopnjo diferencialne

Reševanje DE: $v_{2h} = Ae^{\lambda t}$; $RC = T$

$$\lambda_{1,2} = \frac{-3T \pm \sqrt{9T^2 - 4T^2}}{2T^2} \Rightarrow \lambda_1, \lambda_2$$

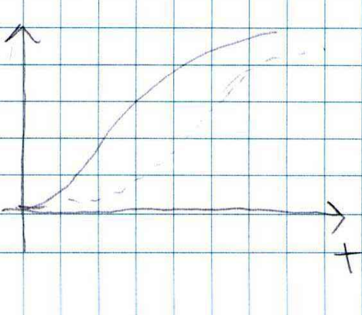
$$\underbrace{(T^2\lambda + 3T\lambda + 1)}_{0} Ae^{\lambda t} = 0$$

$$U_{2h} = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

$$v_{2p} = v_g$$

$$0 = A_1 e^0 + A_2 e^0 + U_g \Rightarrow A_1, A_2$$

$$0 = \lambda_1 A e^0 + \lambda_2 A e^0$$



KAZALEC - KOMPLEKSOR

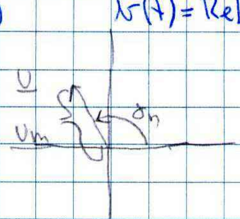
30.05.'11

$v(t) = U_m \cos(\omega t + \vartheta_m)$

$v(t) = Re[\underline{U} e^{j\omega t}]$

$\underline{U} = U_m e^{j\vartheta_m}$

AMPLITUDNI KAZALEC



$$\sum_{k=1}^{n} (\pm) i_k = 0 = \sum_{k=1}^{n} (\pm) Re[\underline{I}_k e^{j\omega t}] =$$

$$= Re\left[\left(\sum_{k=1}^{n} (\pm) \underline{I}_k\right) e^{j\omega t}\right] = 0$$

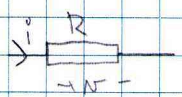
$\sum_{k=1}^{n} (\pm) \underline{I}_k = 0$	$\sum_{v=1}^{\ell} (\pm) \underline{U}_v = 0$

TOKOVNO - NAPETOSTNE RELACIJE

$v(t) = v = U_m \cos(\omega t + \vartheta_u) \Leftrightarrow \underline{U} = U_m e^{j\vartheta_u} \in \mathbb{C}$, $U_m, \vartheta_u \in \mathbb{R}$

$i(t) = i = I_m \cos(\omega t + \vartheta_i) \Leftrightarrow \underline{I} = I_m e^{j\vartheta_i}$

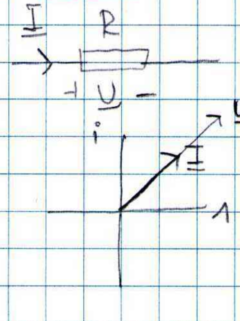
1.) UPOR



$v = Ri$

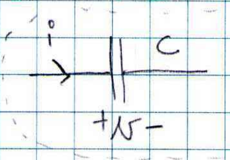
$v = Ri = RI_m e^{j\vartheta_i} = U_m e^{j\vartheta_u}$

$U_m = RI_m$, $\vartheta_u = \vartheta_i$



$\underline{U} = R\underline{I}$

2.) KONDENZATOR



$i = Cv'$

$i = Cv' = C(U_m \cos(\omega t + \vartheta_u))'$

$= -\omega C U_m \sin(\omega t + \vartheta_u)$

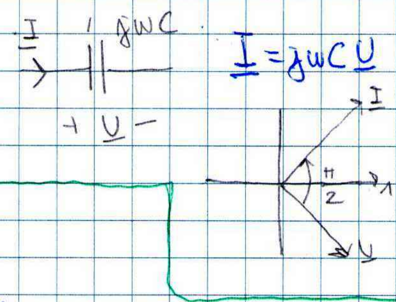
$= \omega C U_m \cos(\omega t + \vartheta_u + \pi/2) =$

$= I_m \cos(\omega t + \vartheta_i)$

$I_m = \omega C U_m$; $\vartheta_i = \vartheta_u + \pi/2$

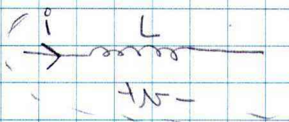
$\underline{I} = I_m e^{j\vartheta_i} = \omega C \underbrace{U_m e^{j\vartheta_u}}_{\underline{U}} \underbrace{e^{j\pi/2}}_{j} = j\omega C \underline{U}$

$v = Re(\underline{U} e^{j\omega t})$

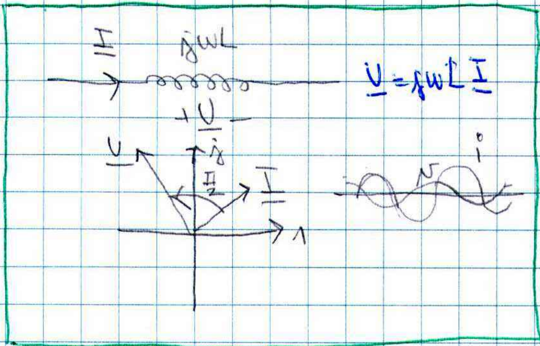


$\underline{I} = j\omega C \underline{U}$

3.) TULJAVA

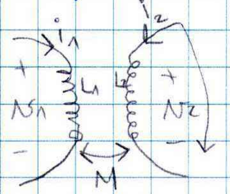


$v = L i'$



$\underline{U} = j\omega L \underline{I}$

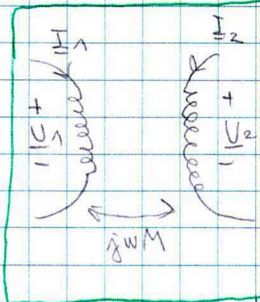
4.) SKLOP TULJAV (DVEH)



$v_1 = L_1 i_1' + M i_2'$

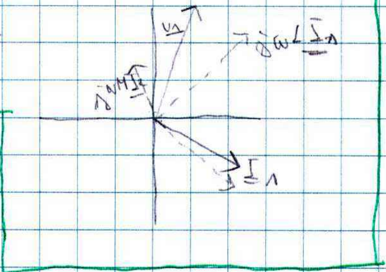
$v_2 = M i_1' + L_2 i_2'$

$\frac{d}{dt}(e^{j\omega t}) = j\omega(e^{j\omega t})$

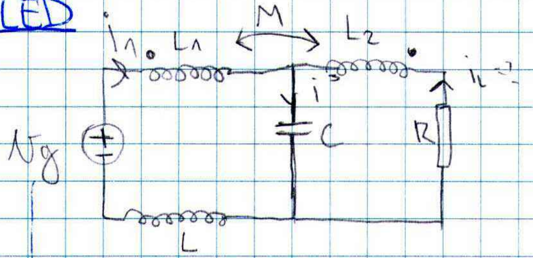


$\underline{U}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$

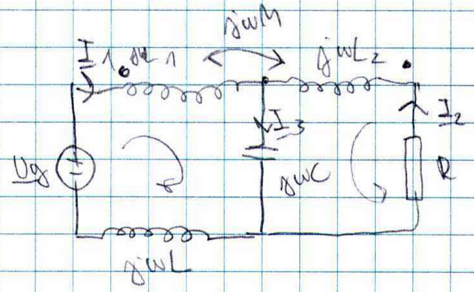
$\underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$



ZGLED



$\equiv$



$v_g = V_{gm}\cos(\omega t + \vartheta_g)$

$i_k = I_{mk}\cos(\omega t + \vartheta_{ik})$

$k = 1, 2, 3 \ldots$

$i_k \Rightarrow \underline{I}_k = I_{mk} e^{j\vartheta_{ik}}$

$\underline{U}_g = U_{gm} e^{j\vartheta_g}$

$-\underline{I}_1 - \underline{I}_2 - \underline{I}_3 = 0$

1: $-\underline{U}_g + j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2 + \frac{\underline{I}_3}{j\omega C} + j\omega L \underline{I}_1 = 0$

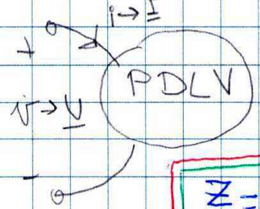
2: $j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2 + \frac{\underline{I}_3}{j\omega C} + R \underline{I}_2 = 0$

$\underline{I}_1 \to i_1$, $\underline{I}_2 \to i_2$, $\underline{I}_3 \to i_3$

IMITANCA (IMPEDANCA IN ADMITANCA)

30.05.'11

PASIVNO DVOPOLNO LIN. VEZJE (Kompl. upornost) (Kompl. prevodnost)



$i = I_m \cos(\omega t + \vartheta_i) \to \underline{I} = I_m e^{j\vartheta_i}$

$u = U_m \cos(\omega t + \vartheta_u) \to \underline{U} = U_m e^{j\vartheta_u}$

$\boxed{\underline{Z} = \underline{U}/\underline{I}}$ IMPEDANCA

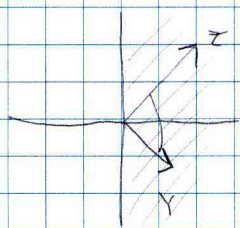
$\boxed{\underline{Y} = \underline{I}/\underline{U} = \underline{Z}^{-1}}$ ADMITANCA

$\boxed{\underline{Z}\,\underline{Y} = 1}$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U_m}{I_m}\frac{e^{j\vartheta_u}}{e^{j\vartheta_i}} = \frac{U_m}{I_m} e^{j(\vartheta_i - \vartheta_u)}$$

$\varphi = \mathrm{abs}(\underline{Z}) = \vartheta_u - \vartheta_i$

$Z = \mathrm{abs}(\underline{Z}) = |\underline{Z}|$

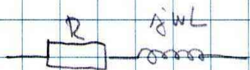


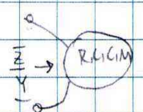
$Re(\underline{Z}) \geq 0$

ELEMENT	Impedanca	Admitanca
Upor	R	$1/R = G$
Kondenzator	$\frac{1}{j\omega C}$	$j\omega C$
Tuljava	$j\omega L$	$\frac{1}{j\omega L}$

ZGLEDI DVOPOLOV

1.)



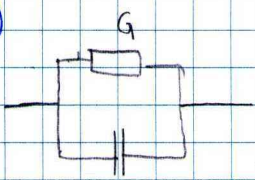


Pri vsaki frekvenci se vezje obnaša z drugačno upornostjo

$\underline{Z} = R + j\omega L$

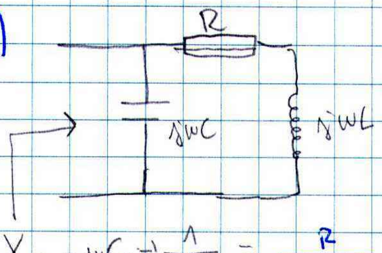
$$\underline{Y} = \frac{1}{R + j\omega L}\cdot\frac{R - j\omega L}{R - j\omega L} = \frac{R - j\omega L}{R^2 + (\omega L)^2} = \frac{R}{R^2 + (\omega L)^2} - \frac{j\omega L}{R^2 + (\omega L)^2}$$

2.)



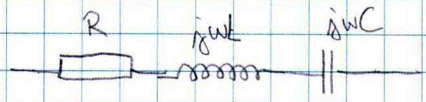
$\underline{Y} = G + j\omega C$ $\quad \underline{Z} = \frac{G - j\omega C}{G^2 + (\omega C)^2}$

3.)

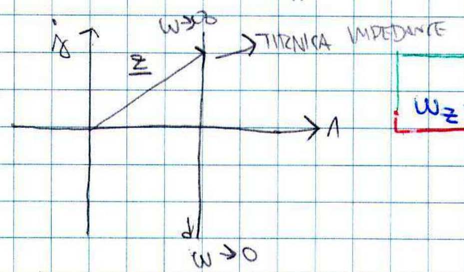


$$\underline{Y} = j\omega C + \frac{1}{R + j\omega L} = \frac{R}{R^2 + (\omega C)^2} + j\omega\left(C - \frac{L}{R^2 + (\omega L)^2}\right)$$

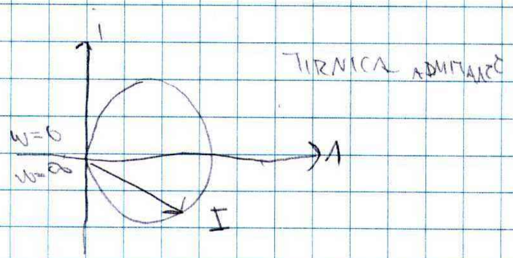
4.)



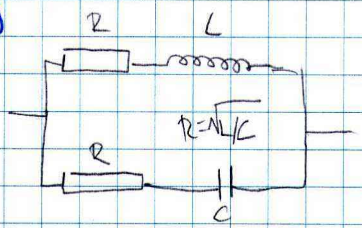
$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$



$$\boxed{\omega_Z = \frac{1}{\sqrt{LC}}}$$



5.)



$$\boxed{\underline{Z} = R}$$

To bo ena od kolokvijskih nalog

$$\underline{Z} = \underbrace{Re(\underline{Z})}_{R} + j\underbrace{Im(\underline{Z})}_{X}$$

REZISTANCA REAKTANCA

$$\underline{Y} = \underbrace{Re(\underline{Y})}_{G} + j\underbrace{Im(\underline{Y})}_{B}$$

KONDUKTANCA SUSCEPTANCA

6.6.XI

Kompenzacijski kondenzatorji - kompenzirajo jalovo moč.

STAVKI V HARMONSKO VZBUJANIH VEZJIH

(v kompleksnem)

1.) SUPERPOZICIJA (KOHERENTNI VIRI)
2.) NADOMESTITEV
3.) TELLEGEN
4.) THEVENIN-NORTON (KOHERENTNI VIRI)
5.) PRILAGODITEV (MAKSIMUM)
6.) RECIPROČNOST

1.) SUPERPOZICIJA

Možna, če so viri koherentni. Če ne pa vezje rešujemo za vsak vir posebej (samo seštejem preslikave)

2.) NADOMESTITEV

Tokovni / napetostni viri

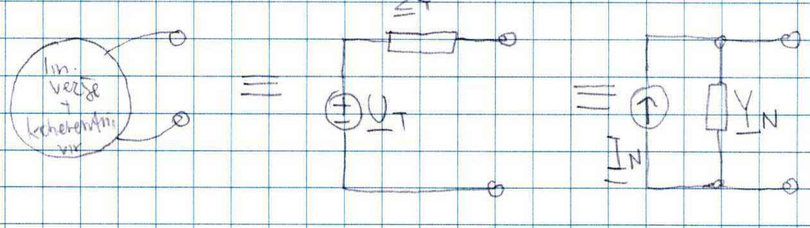
3.) TELLEGEN

$$\sum_{k=1}^{n} i_k u_k = 0 \Rightarrow \sum_{k=1}^{n} \frac{1}{2} \underbrace{\underline{U}_k \underline{I}_k^*}_{\underline{S}_{gk}-\underline{S}_{bk}} = 0$$

$$\sum_{k=1}^{n} \underline{S}_{gk} = \sum_{k=1}^{n} \underline{S}_{bk}$$

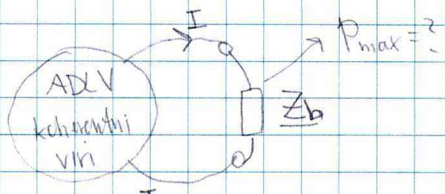
$$\sum_{k} P_{gk} = \sum_{k} P_{bk} \qquad \sum_{k} Q_{gk} = \sum_{k} Q_{bk}$$

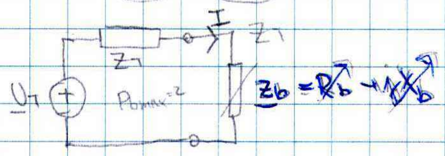
4.) THEVENIN-NORTON



$\underline{U}_T = \underline{U}_0$, $\underline{I}_N = \underline{I}_k$, $\underline{Z}_T \underline{Y}_N = 1$, $\underline{Z}_T = \underline{U}_T / \underline{I}_k = \underline{Z}_{notr.}$ → Pri deaktiviranih virih

5.) PRILAGODITEV





$Z_T = R_T + jX_T$

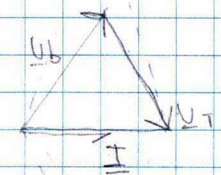
$$P_b = Re(S_b) = Re\left[\frac{1}{2}(R_b + jX_b)|I|^2\right]$$

$$P_b = \frac{1}{2} R_b \frac{|U_T|^2}{(R_T + R_b)^2 + (X_T + X_b)^2} = P_b(R_b, X_b)$$

$$\frac{\partial P_b}{\partial R_b} = 0 \wedge \frac{\partial P_b}{\partial X_b} = 0 \Rightarrow X_b = -X_T \Rightarrow R_b = R_T$$

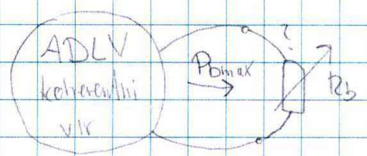
$$Z_T + Z_B = 2R_T$$

$$\boxed{Z_b = Z_T^*}$$



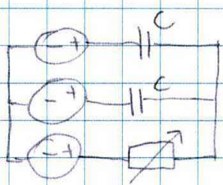
$P_{bmax} = ?$

$$\boxed{P_{bmax} = \frac{1}{8} \frac{|U_T|^2}{R_T}} = \frac{|U_T|^2/2}{R_T} \cdot \frac{1}{4}$$



$$P_b = \frac{1}{2} R_b \frac{|U_T|^2}{(R_T + R_b)^2 + X_T^2} = P_b(R_b)$$

$$\frac{\partial P_b}{\partial R_T} = 0 \Rightarrow R_b = \sqrt{R_T^2 + X_T^2} = |Z_T|$$

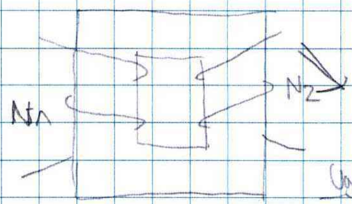


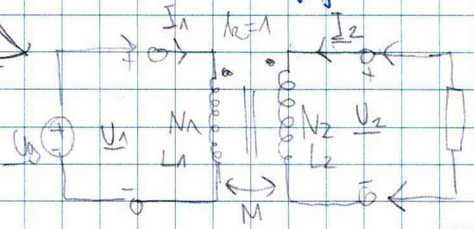
Trifazni sistem

$P_{izg}=0$ $(k=1)$

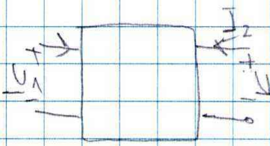
BREZIZGUBNI POPOLNO SKLOPLJENI TRANSFORMATOR → IDEALNI TRANSFORMATOR

Popolno sklopljeni ⇒ Ni stresanja





$$\underline{U}_1 = j\omega L_1 \underline{I}_1 + j\omega M \underline{I}_2$$
$$\underline{U}_2 = j\omega M \underline{I}_1 + j\omega L_2 \underline{I}_2$$



$$\boxed{\underline{I}_1 = \frac{\underline{U}_1}{j\omega M} - \frac{L_2}{M}\underline{I}_2}$$

$$\underline{U}_1 = j\omega L_1\left(\frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M}\underline{I}_2\right) + j\omega M \underline{I}_2$$
$$\boxed{= \frac{L_1}{M}\underline{U}_2 + j\omega\underbrace{\left(M - \frac{L_1 L_2}{M}\right)}_{0}\underline{I}_2}$$

$M^2 = L_1 L_2$

$L_1 = \frac{M^2}{L_2}$

$$\boxed{\underline{U}_1 = \frac{L_1}{M}\underline{U}_2 \qquad \underline{I}_1 = \frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M}\underline{I}_2}$$

$$\begin{bmatrix}\underline{U}_1\\ \underline{U}_2\end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2\end{bmatrix}\begin{bmatrix}\underline{I}_1\\ \underline{I}_2\end{bmatrix}$$ Impedančna matrika

$$\begin{bmatrix}\underline{U}_1\\ \underline{I}_1\end{bmatrix} = \begin{bmatrix} \frac{L_1}{M} & 0 \\ \frac{1}{j\omega M} & \frac{L_2}{M}\end{bmatrix}\begin{bmatrix}\underline{U}_2\\ (-\underline{I}_2)\end{bmatrix}$$ Verižna matrika

1.) $\underline{U}_1 = \frac{L_1}{M}\underline{U}_2 = \frac{N_1}{N_2}\underline{U}_2$, $\frac{\underline{U}_1}{\underline{U}_2} = \frac{N_1}{N_2} = n$ ← Prestava

2.) $Z_b \to \infty \Rightarrow \underline{I}_2 = 0 \Rightarrow \underline{I}_1 = \frac{\underline{U}_2}{j\omega M} - \frac{L_2}{M}0 = \frac{M\underline{U}_1}{j\omega M L_1} = \frac{\underline{U}_1}{j\omega L_1} = \underline{I}_m$ (magnetilni tok)

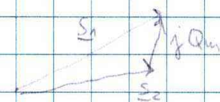
3.) $|\underline{Z}_b|$ - končen: $\underline{I}_1 = \underline{I}_m - \frac{N_2}{N_1}\underline{I}_2 = \underline{I}_m \underbrace{- n^{-1}\underline{I}_2}$ $\qquad \underline{I}_r = \frac{1}{n}\underline{I}_2$

$\underline{I}_r$ → Reakcijski tok = Ravnotežni tok

4.) $\underline{\Theta} = N_1\underline{I}_1 + N_2\underline{I}_2 = N_1\underline{I}_m \underbrace{- N_1\frac{1}{n}\underline{I}_2 + N_2\underline{I}_2}_{0} = N_1\underline{I}_m$

5.) $\underline{Z}_{vh} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{n\underline{U}_2}{\frac{\underline{U}_2}{j\omega M} - \frac{1}{n}\underline{I}_2} \quad \frac{:(-\underline{I}_2)}{:(-\underline{I}_2)} = \frac{n\underline{Z}_b}{\frac{\underline{Z}_b}{j\omega M} + \frac{1}{n}}$

6.) $\left[\underline{S}_1 = \frac{1}{2}\underline{U}_1\underline{I}_1^* = \frac{1}{2}\underline{U}_1\left(\underline{I}_m^* - \frac{1}{n}\underline{I}_2^*\right) = \underbrace{\frac{1}{2}\underline{U}_1\underline{I}_m^*}_{jQ_{1m} \text{ - jalova moč}} + \overbrace{\frac{1}{2}\underline{U}_2(-\underline{I}_2)^*}^{\underline{S}_2 = \underline{S}_b} = jQ_{1m} + \underline{S}_2\right]$



$|\underline{I}_m| \ll |\underline{I}_1|$ reda 100 manjši $\Rightarrow \underline{I}_1 \simeq -\frac{1}{n}\underline{I}_2 \qquad \frac{\underline{U}_1}{\underline{U}_2} = n \qquad \frac{\underline{I}_1}{\underline{I}_2} \simeq -\frac{1}{n}$

Transformator zanimamo za ekonomičnejši prenos energije $\qquad \tan\varphi = \sqrt{1 - \frac{1}{\cos^2\varphi}}$

IDEALNI TRANSFORMATOR

$R_m \to 0$, $\mu_r \to \infty$, $L_1, L_2, M \to \infty$

1.) $\frac{\underline{U}_1}{\underline{U}_2} = n = \frac{N_1}{N_2}$

KAZALČNI DIAGRAM ZA

2.) $\underline{I}_{1m} \to 0$

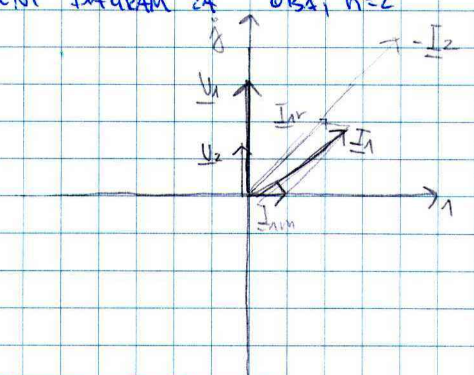
3.) $\underline{I}_1 = -\frac{1}{n}\underline{I}_2 = -\frac{N_2}{N_1}\underline{I}_2$

4.) $\underline{\Theta} \to 0$

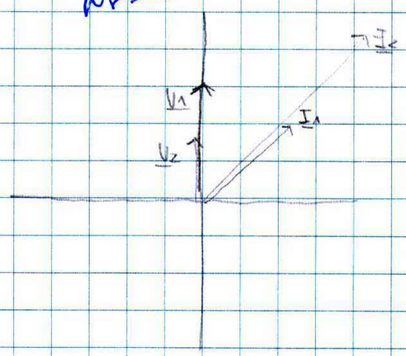
5.) $\underline{Z}_{vh} = n^2 \underline{Z}_b$

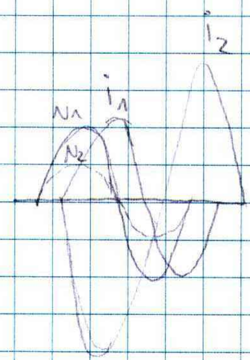
6.) $\underline{S}_1 = \underline{S}_2$

KAZALČNI DIAGRAM ZA OBR, $h=2$



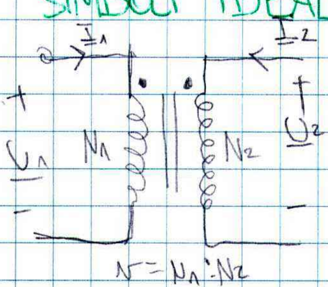
$\mu_r \to \infty$





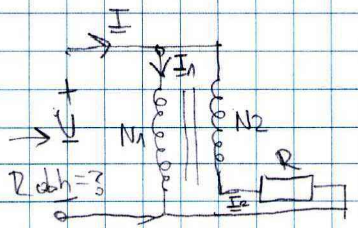
9.6.11

SIMBOLI IDEALNEGA TRANSFORMATORJA



$$\boxed{\frac{\underline{U}_1}{\underline{U}_2} = -\frac{\underline{I}_2}{\underline{I}_1} = n}$$

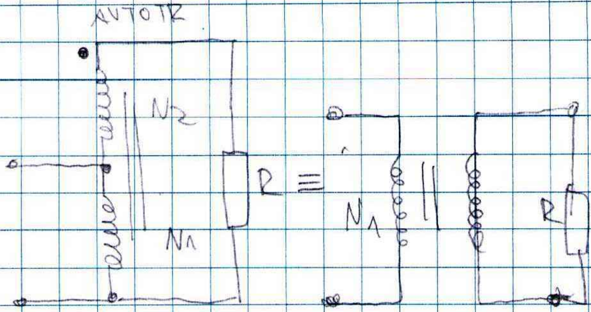
ZGLED – 6.56 P.N.



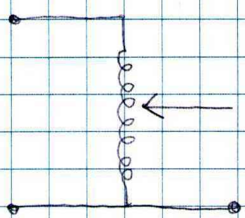
$\underline{I}_1 - \underline{I}_2 = \underline{I}$, $\frac{\underline{I}_2}{\underline{I}_1} = -N$

$\underline{U}_1 = \underline{U}$, $\underline{U}_2 = \frac{\underline{U}_1}{n}$

$R \cdot \underline{I}_2 + \underline{U}_2 + \underline{U} = 0$

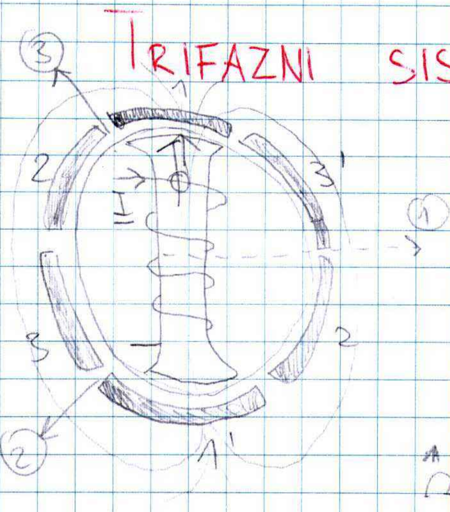


AVTOTRANSFORMATOR



VARIAK
(AVTOTRANSFORMATOR)

Trifazni sistem



$v_1 = U_m \cos(\omega t + \gamma)$

$v_2 = U_m \cos(\omega t + \gamma \mp \frac{2\pi}{3})$

$v_3 = U_m \cos(\omega t + \gamma \pm \frac{4\pi}{3})$

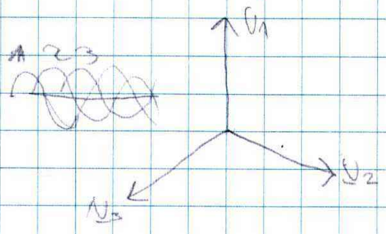
Negativni sistem

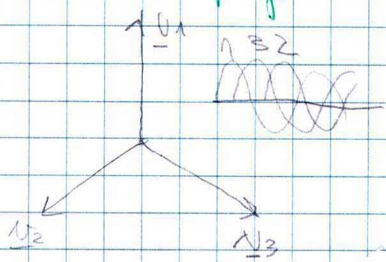
Pozitivni sistem

Pozitivno in negativno fazno zaporedje

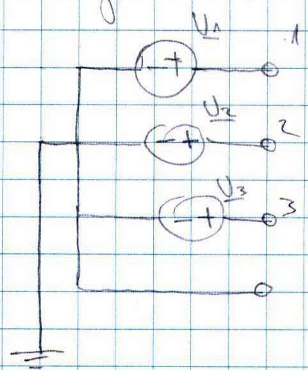
⊕ Fazno zaporedje

⊖ Negativno fazno zaporedje





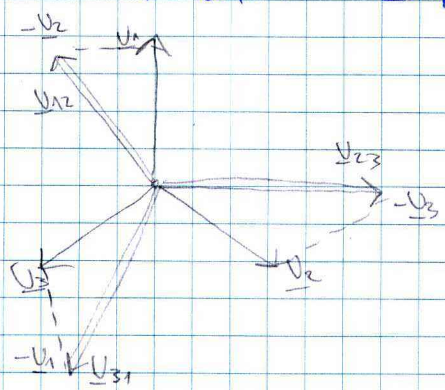
Vezano v skupno točko.
Na generatorski strani točka ozemljena (Spojeno na zvezdo)

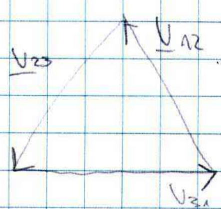


Fazne in medfazne napetosti

$v_{10}, v_{20}, v_{30} \to v_1, v_2, v_3 \to \underline{U}_1, \underline{U}_2, \underline{U}_3$

$v_{12}, v_{23}, v_{31} \to \underline{U}_{12}, \underline{U}_{23}, \underline{U}_{31}$





$\underline{U}_1 + \underline{U}_2 + \underline{U}_3 = 0$

$\underline{U}_{12} + \underline{U}_{23} + \underline{U}_{31} = 0$

Amplituda medfazne napetosti je $\sqrt{3}$ večja od fazne napetosti   Npr 400kV deljeno → 230kV

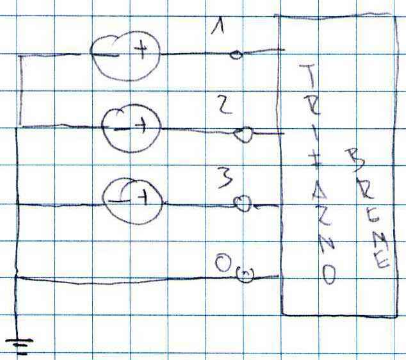
Nivoji: 400kV, 200kV, 110kV, ...

$230\,U_{ef} \cdot \sqrt{3} \simeq 400\,U_{ef}$

Amplitudni kazalec: $\underline{U}$, $v = Re(\underline{U} e^{j\omega t})$, $\underline{U} = U_m e^{j\gamma}$ ← $\underline{S} = \frac{1}{2} \underline{U}\underline{I}^*$

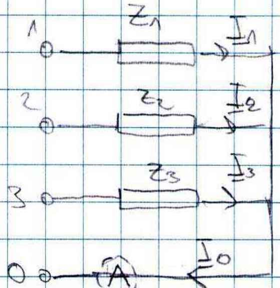
Efektivni kazalec: $\underline{U}$, $v = Re(\sqrt{2}\,\underline{U} e^{j\omega t})$, $\underline{U} = \frac{U_m}{\sqrt{2}} e^{j\gamma} = U_{ef} e^{j\gamma}$ ← $\boxed{\underline{S} = \underline{U}\underline{I}^*}$!

TRIFAZNO BREME IN OSNOVNI NAČINI PRIKLJUČEVANJA



zvezda in trikot oznaka načina

1) ZVEZDNA VEZAVA Z NIČLOVODOM



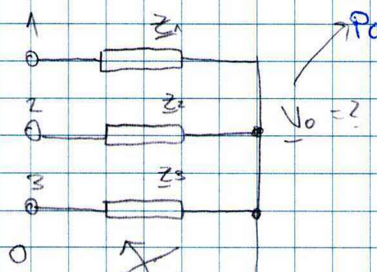
$$\underline{I}_k = \frac{\underline{U}_k}{\underline{Z}_k}$$

$$\underline{I}_0 = \underline{I}_1 + \underline{I}_2 + \underline{I}_3$$

$$\underline{S}_k = \underline{U}_k \underline{I}_k^*$$

$$\underline{S} = \underline{S}_1 + \underline{S}_2 + \underline{S}_3$$

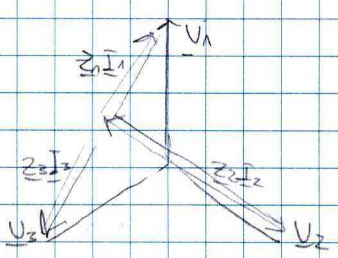
a) zvezdna vezava brez ničlovoda



$$\underline{I}_k = \frac{\underline{U}_k - \underline{V}_0}{\underline{Z}_k} = \underline{Y}_k(\underline{U}_k - \underline{V}_0)$$

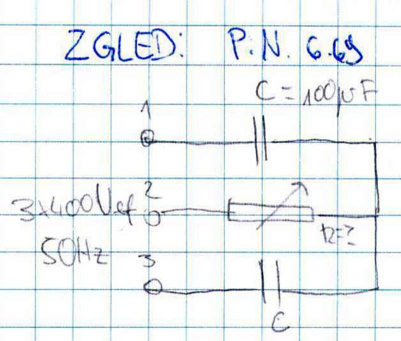
$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

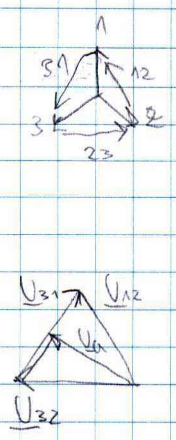
$$\sum_{k=1}^{3} \underline{Y}_k(\underline{U}_k - \underline{V}_0) = 0 \Rightarrow \underline{V}_0 = \frac{\underline{Y}_1\underline{U}_1 + \underline{Y}_2\underline{U}_2 + \underline{Y}_3\underline{U}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}$$

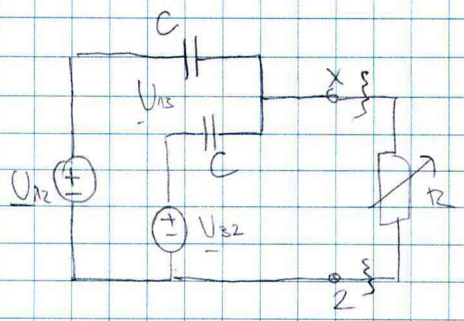


Na to temo naloga na kolokviju..

ZGLED: P.N. 6.63





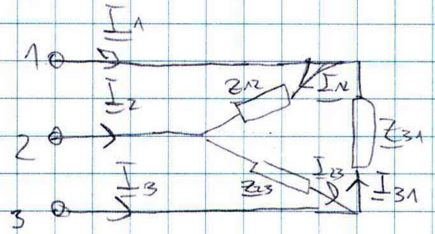


$$\underline{U}_0 = \frac{\underline{U}_{32} - \underline{U}_{12}}{2} + \underline{U}_{12} \quad \frac{\underline{U}_{31}}{2}$$

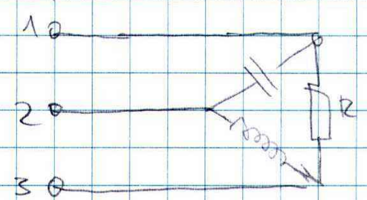
$$\boxed{\begin{gathered} |\underline{U}_0| = 400\sqrt{\tfrac{3}{2}} \\ R = \frac{1}{R\omega C} \end{gathered}}$$

Največja moč ko bo tolikšna napetost

2) TRIKOTNA VEZAVA BREMEN



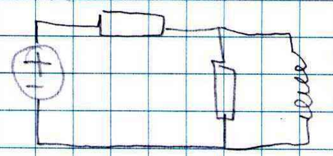
Trikotna vezava pri motorjih.
Tudi za zagon asinhronskih motorjev.
Zagonski tok je veliko večji (10x) kot delovni. (trikot-zvezda-stikalo)
Stari uporabljajo ta stikala

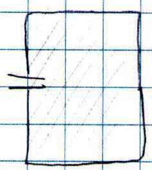


Indukcijske peči
To breme proti omrežju podobno kot trije upori.
Da lahko močna enofazna bremena priključimo na omrežje

Kolokvij II

1. → RLC vezje z impedanco – zadnji naslov
2.) → Vezava zvezda brez povratnega = okoli potenciala
3.) → Vezje brez stikala



4.) →  Ovrednotit [illegible] ob tem vezju

5.) → Induktivnost – Transformator jedro z dvema navitjema – glej zgled v zapiskih